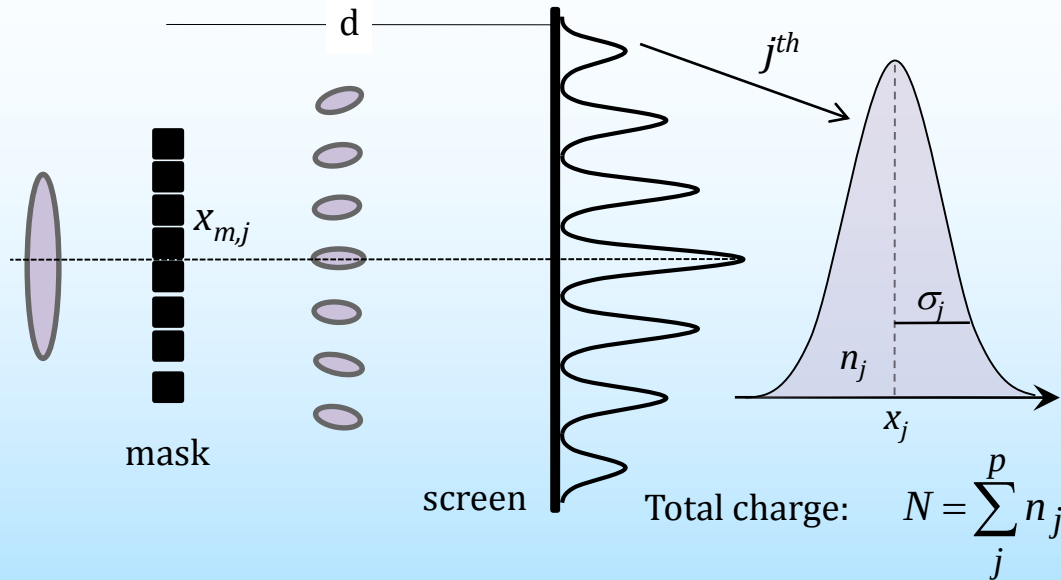


Laboratory Session on Pepperpot Emittance Measurements

**Ref 1: Emittance Formula for Slits and Pepper-pot
Measurement, Min Zhang, FERMILAB-TM-1988**

Your tasks in green frames



In all beamlets:

Mean position:

$$\bar{x} = 1/N \sum_j n_j x_{m,j} = \langle x \rangle$$

Mean divergence:

$$\bar{x}' = 1/N \sum_j n_j \bar{x}'_j = \langle x' \rangle$$

In j^{th} beamlet:

Mean divergence:

$$\bar{x}'_j = 1/n_j \sum x'_{ji}$$

rms divergence:

$$\sigma_{j,x'} = \frac{\sigma_j}{d}$$

$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

$$\langle x^2 \rangle = \frac{1}{N^2} \sum_{j=1}^p n_j (x_{m,j} - \bar{x})^2$$

$$\langle x'^2 \rangle = \frac{1}{N^2} \sum_{j=1}^p n_j \left[\left(\frac{\sigma_j}{d} \right)^2 + (x_{m,j} - \bar{x})^2 \right]$$

$$\langle xx' \rangle^2 = \frac{1}{N^2} \left[\sum_{j=1}^p n_j x_j \bar{x}'_j - N \bar{x} \bar{x}' \right]^2$$

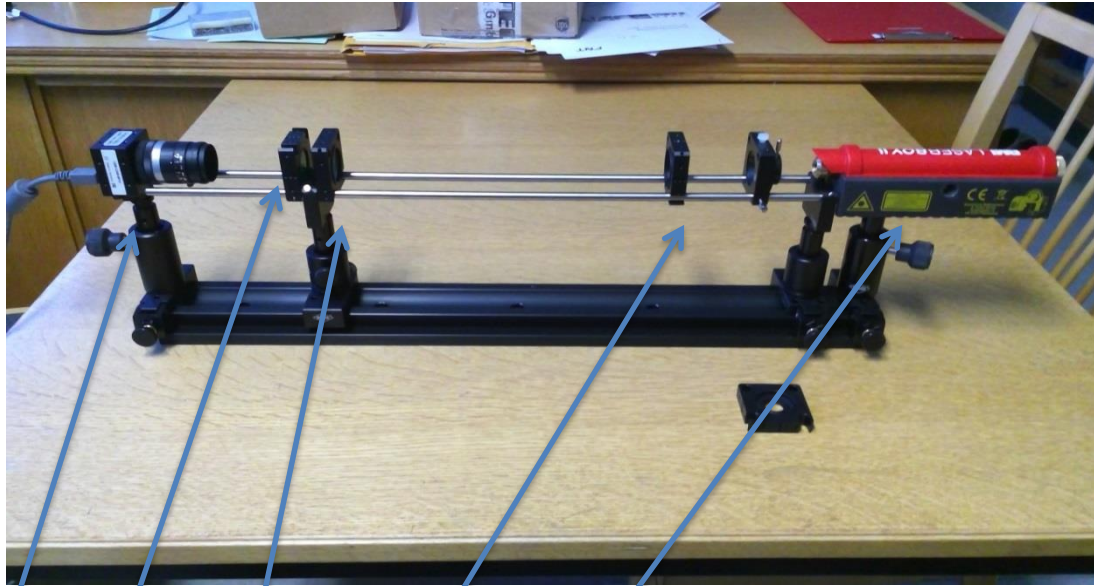
with

$$\sigma_j \equiv \sqrt{\frac{1}{n_j} \sum_{i=1}^{n_j} (X_{ji} - \bar{X}_j)^2},$$

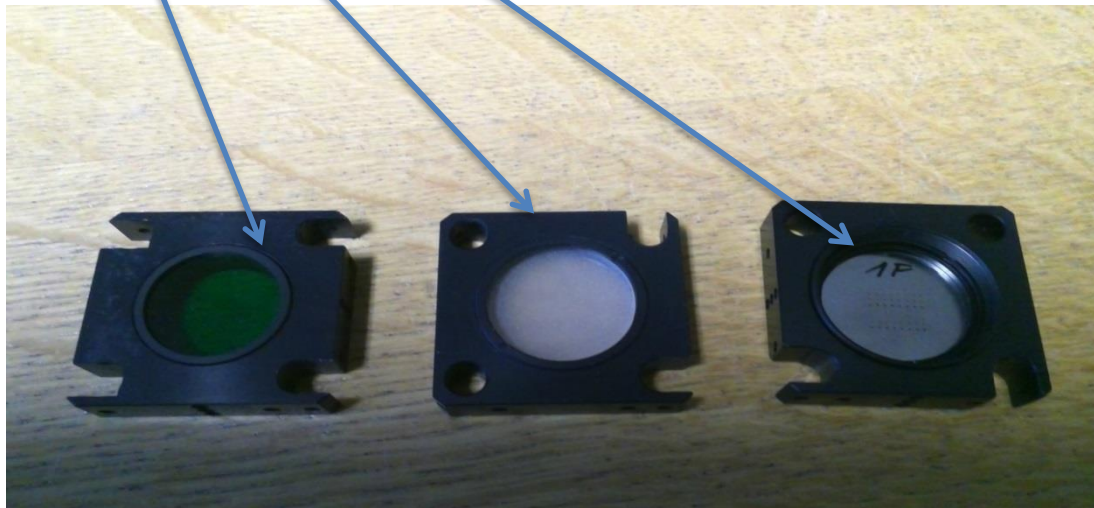
which is the rms spot size of j -th beamlet on screen.

p = number of slits, $x_{m,j}$ = j -th slit position, N = all particles behind the slits
 n_j = number of particles passing through slit (weight of spot intensity)

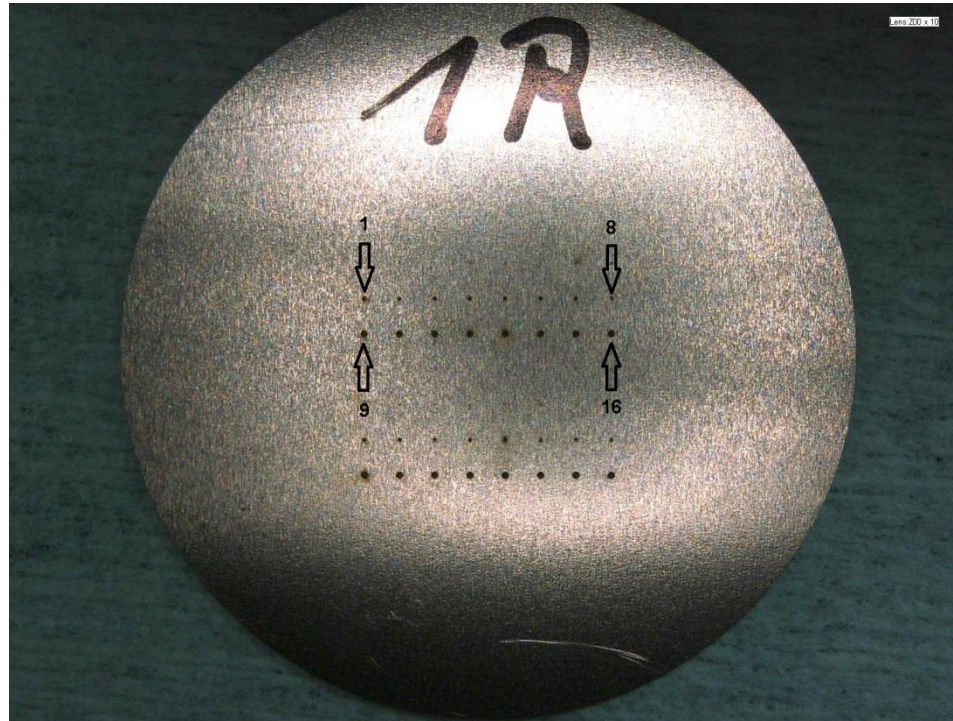
Pepperpot Emittance Measurement



Optical bench with: Camera, Filter, Screen, Pepperpot plate, Laser, Data acquisition (CCD readout), Data analyzation (ImageJ), Evaluation program (Excel)



Pepperpot Emittance Measurement



There are 2 x 4 rows with 8 holes each. The marked rows are more or less in the middle, hole 01-08 have a diameter of $130\ \mu\text{m}$ while holes 09-16 have a diameter of about $240\ \mu\text{m}$. The horizontal distance is $1202\ \mu\text{m}$ in each row. The other rows have smaller diameters, but should not be used in this experiment.

The orientation of the pepperpot in the experiment should be with the 1R side showing to the camera.

(A real pepperpot has an array of equal holes, this plate is for 1 dim. tests only)

Parameters

● CCD

➤ Phytect USB-CAM 051H

Resolution	2592 x 1944 (5 MPix), 2048 x 1536 (3,1MPix), 1600 x 1200 (2MPix), 1280 x 960 (1,2MPix) 1024 x 768 (0,8MPix), 640 x 480 (VGA)			
Model	USB-CAM-051H	USB-CAM-151H	USB-CAM-052H	USB-CAM-152H
color / monochrom	monochrom		color	
Sensor Format	1/2,5"			
Image Sensor	Aptina MT9P031, CMOS			
Pixel Size	2,2 µm x 2,2 µm			
Color format	Y8		RGB32, RGGB (Raw)	
Lens Holder	C / CS – Mount			
fps	6 fps to 52 fps			
Dynamic Range	8 bit			
Shutter	Rolling			
Light sensitivity	1,4 V/lux-sec			
Interface	USB 2.0 High Speed			
Exposure time	1/10.000 s to 30 s			
Gain	0 dB to 18 dB			
White Balance	-		-6 dB bis +6 dB	
Power supply	4,5 V bis 5,5V DC			
Power Consumption	Circa 250 mA bei 5V			
Feature (optional)	-	ext. Trigger, Digital-Output	-	ext. Trigger, Digital-Output
Temperature range	-5°C bis +45°C			
Dimensions (B x L x H)	36 mm x 36 mm x 25 mm			
Fixing	1/4" and M6x8 on all sides			
Weight	70 g			
Connection	USB Mini-B			
Feature- Connection	-	Hirose HR10A-7R-4P	-	Hirose HR10A-7R-4P

● screen

➤ material: Quarz scatter disk

● calibration target

➤ spacing: 1.202 mm

● Laser: LaserBoyII

BMI Bayerische Laserboy II Wasserwaage 649 015

Allgemeine Informationen

Artikelnummer	ET1117000
EAN	4007368050049
Hersteller	BMI Bayerische
Hersteller-ArtNr	649 015
Hersteller-Typ	649 015
Verpackungseinheit	1 Stück
Artikelklasse	Messlaser



Technische Informationen

Länge der Signalstrecke	30m
Laserklasse	
Sichtbare Signalstrecke	
Rotierende Signalstrecke	

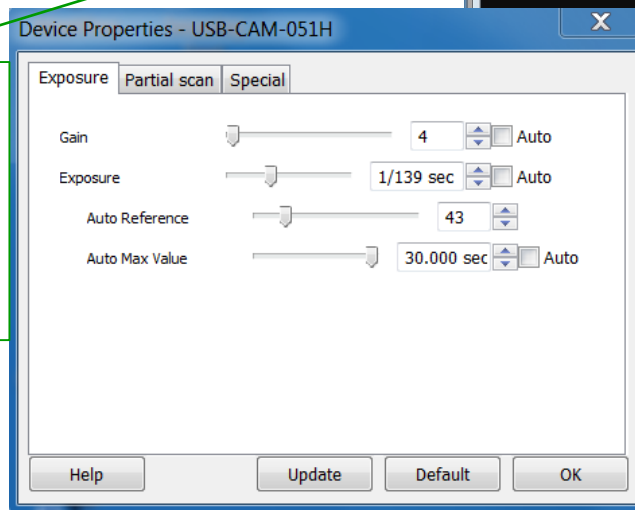
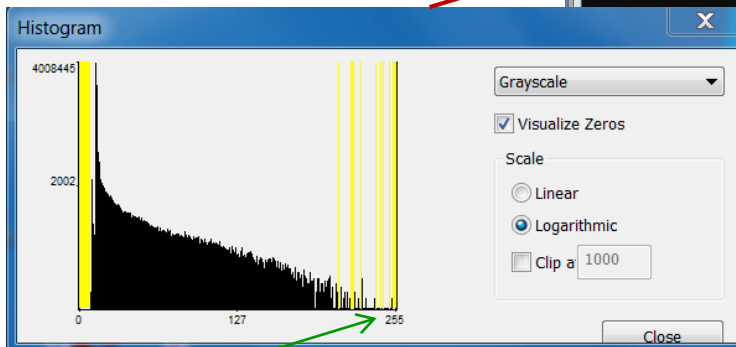
BMI Bayerische Laserboy II Wasserwaage 649 015 Länge der Signalstrecke 30m, Laserklasse 2, Sichtbare Signalstrecke,

CCD Readout: Introduction

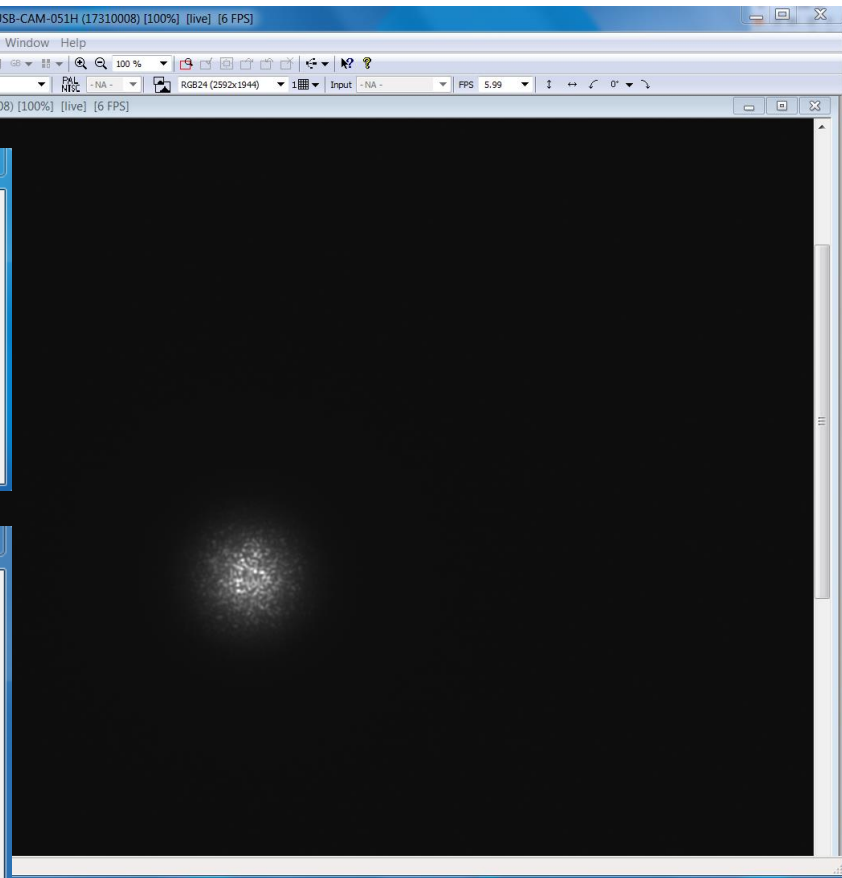
● readout program

PHYTEC Vision Demo 2.2

► histogram of grey values



**Check
always:
Do not
saturate
(255)**



► CCD control parameters

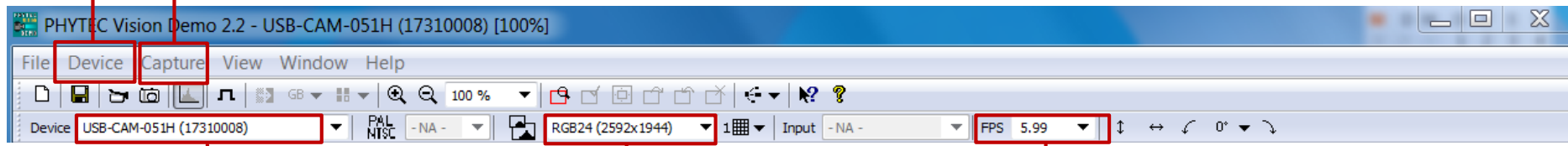
→ Device → Properties

CCD Readout: Introduction

Start/Stop acquisition → Device → Live (Shortcut: Ctrl + L)

CCD control parameters → Device → Properties

Save image → Capture → Save Image (Shortcut: Ctrl + U): save as Jpeg images



CCD type

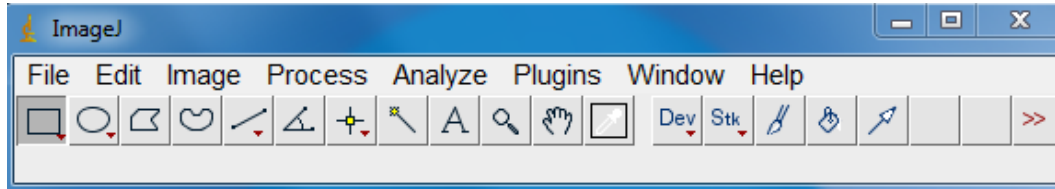
readout format
(RGB, 2592 x 1944 pixel)

readout rate
(5.99 frames per
second)

Maybe less

ImageJ: Introduction

- **press icon** access to start panel

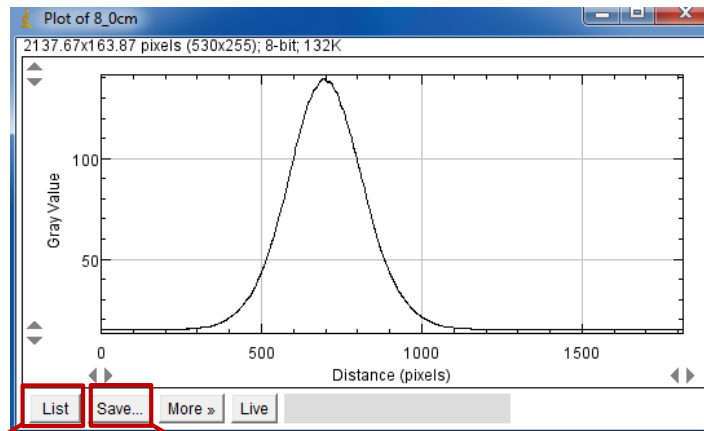


- **load image file** → File → Open (Shortcut: Ctrl + O)

- **select ROI:** in start panel: select left button (below "File"), usually already pre-selected
then with left mouse button: draw rectangular ROI



- **plot horizontal projection** → Analyze → Plot Profile (Shortcut: Ctrl + k)

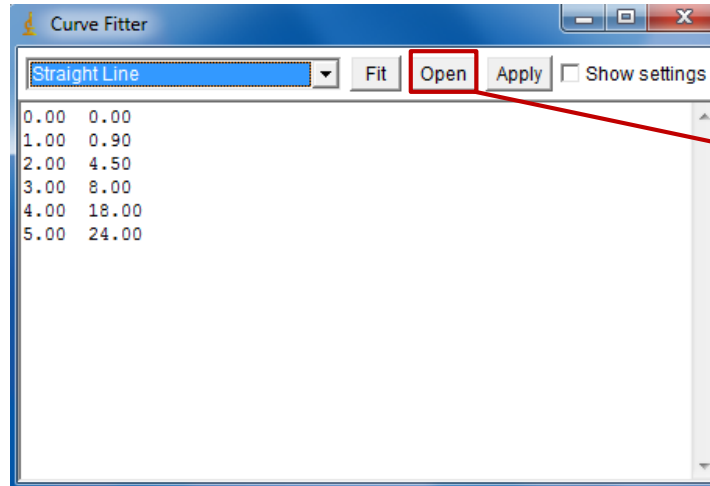


- **save data** -> list data points -> save data as .csv file (required for profile fitting)

ImageJ: Introduction

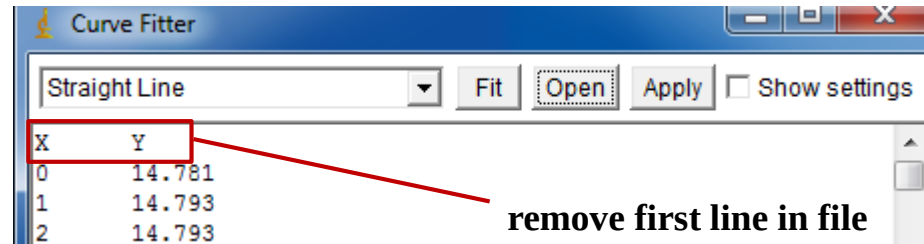
● **profile fitting** → Analyze → Tools → Curve Fitting...

➤ **load profile data:**



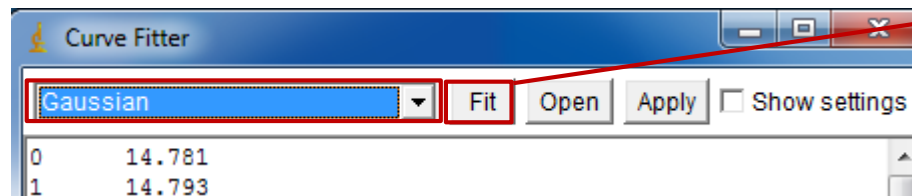
load .csv data file

➤ **delete bad data:**



remove first line in file

➤ **select fit function:**



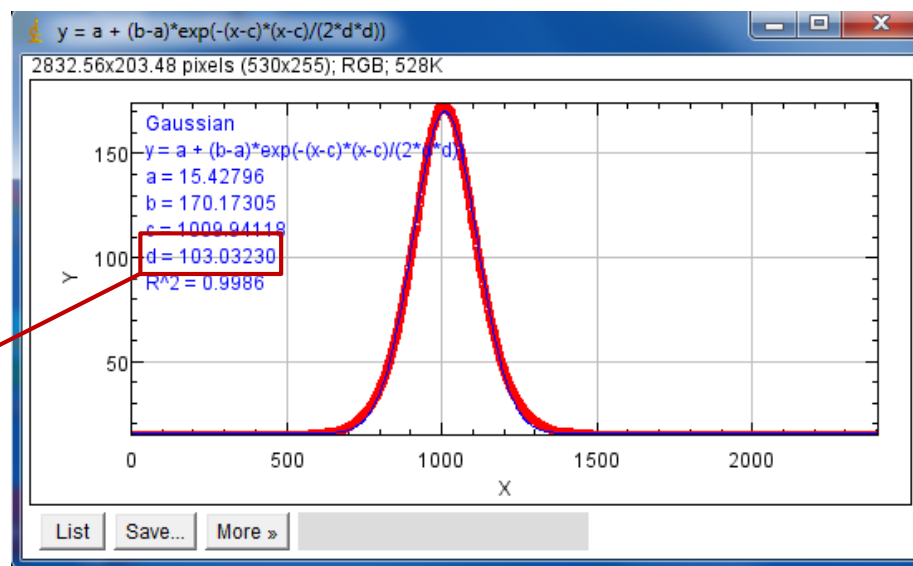
fit profile data

$$y = a + (b - a) \cdot e^{-\frac{(x-c)^2}{2d^2}}$$

ImageJ: Introduction

fit results:

1 σ -width (in pixel)



additional data fitting

- create data file → e.g. Excel or simple ASCII text file with Notepad
- repeat fitting as described before

Pepperpot Measurement

Only the evaluation of the horizontal phase plane is implemented but... on an optical system horizontal and vertical phase space is symmetric.

Steps to be performed:

1. We will use a series of horizontal holes calculating the projection of the image to the horizontal axis. Therefore ...
Make sure the holes are almost aligned and are on a horizontal line
2. Measure the distance between the holes (in pixels) and calculate the scaling factor s [pixels/mm]
To do this: Replace the screen with the pepperpot plate. After this measurement keep the distance screen-camera constant.
Save the image as an 640x480 .bmp file
3. Take the pepperpot image on the screen (make sure the camera focuses on the screen), save it and calculate the emittance.

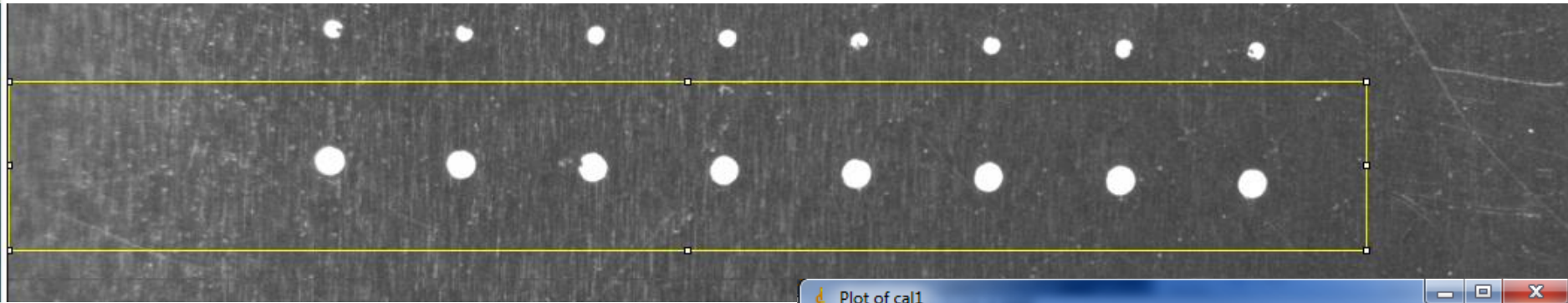
In more details:

Your tasks in green frames

Camera Calibration

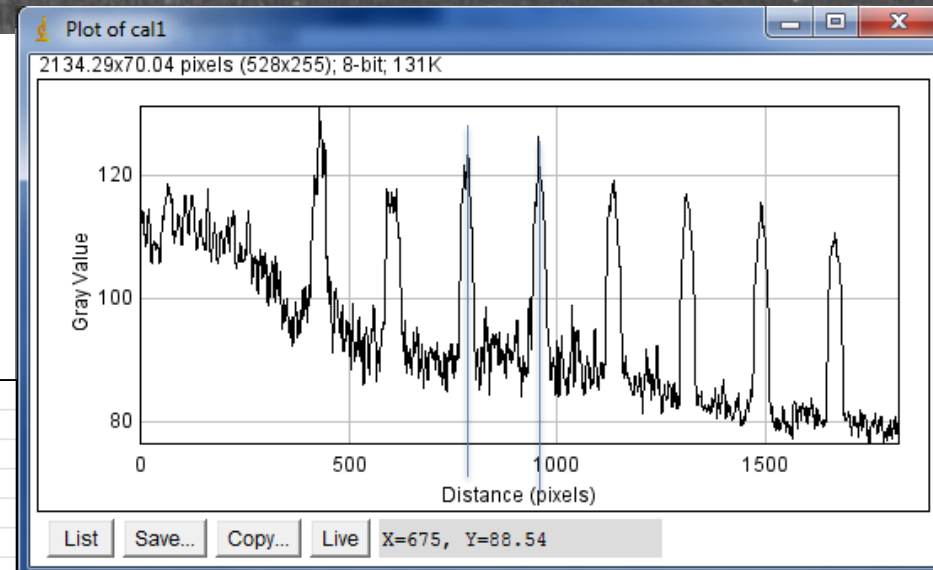
Check: Do
not saturate
(255)

- To 1) and 2): **Camera Calibration!**: Replace the screen with the pepperpot plate. The distance of the holes (1.202 mm) is known and can be used for calibration. Focus the camera and take a picture of all 8 holes and measure the distance in pixels. Hint: Use a white paper to illuminate the holes from the back. Keep the filter in.



Use ImageJ to open the picture, define an ROI (up to the left side =0!!!) and measure the distance between 2 holes. Enter this number into the attached Excel sheet “Pepperpot_calc.xlsx” at c1 and c2. The calibration will be calculated automatically (yellow field).

For Calibration (pixels/mm)			
dist. Of holes:	1.202 mm	dist. Pepperpot-Screen L =	188 mm
c1=	425 pixel		
c2=	673 pixel		
cal:	206.3227953 pixel/mm		



Camera Calibration

Check: Do
not saturate
(255)

- Use the picture also to define the positions of the holes:

Measure the x coordinate in Pixels of the center of each hole. Enter this number into the Excel sheet at xs1 to xs8.

For Calibration (pixels/mm)		
dist. Of holes:	1.202 mm	
c1=	425	pixel
c2=	673	pixel
cal:	206.3227953	pixel/mm

Center Position of hole images of Pepperpot:		
xs1	460	pixel
xs2	702	pixel
xs3	950	pixel
xs4	1192	pixel
xs5	1428	pixel
xs6	1670	pixel
xs7	1906	pixel
xs8	2160	pixel

2134.29x70.04 pixels (528x255); 8-bit; 131K

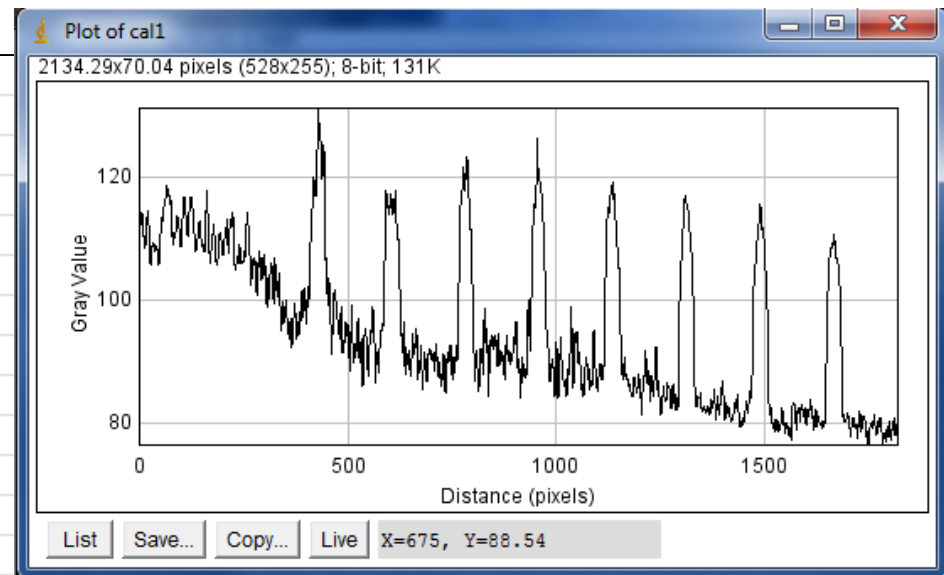
List

Save...

Copy...

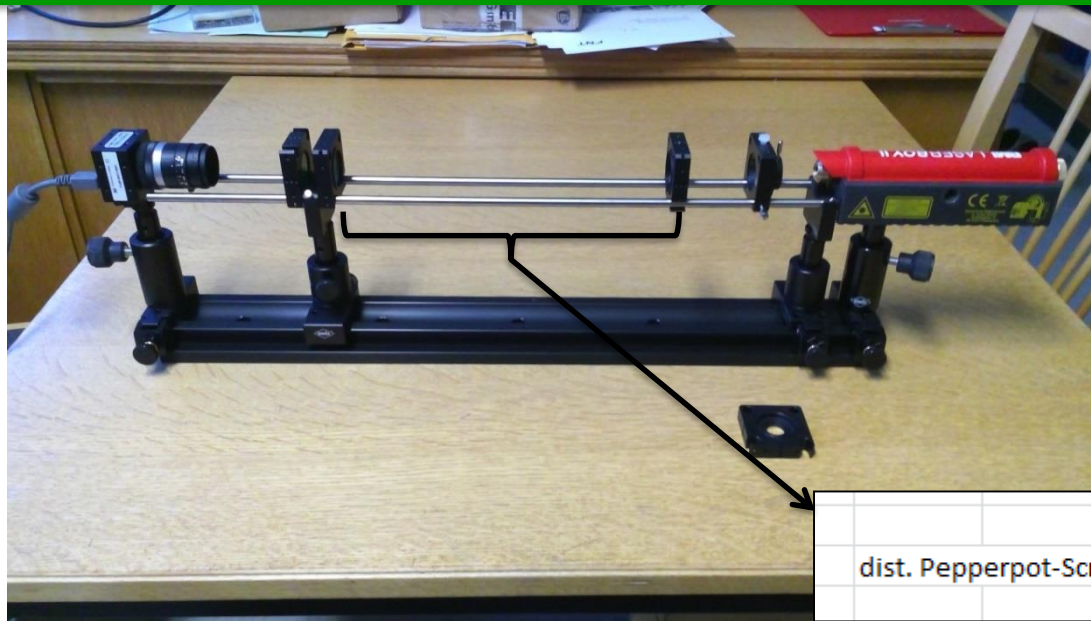
Live

X=675, Y=88.54



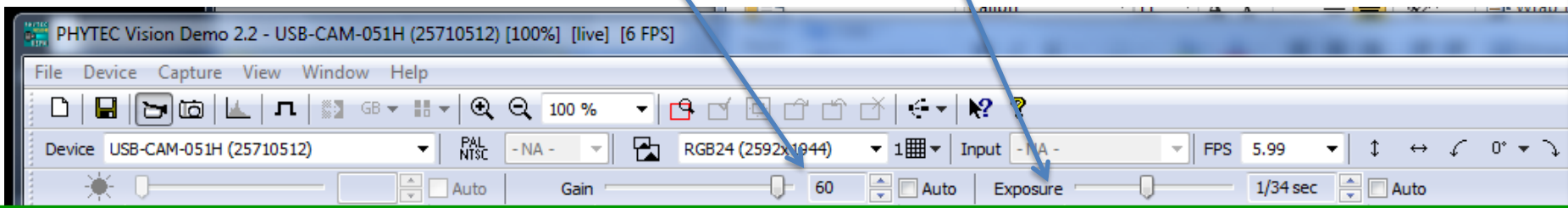
Camera Calibration

Check: Do
not saturate
(255)

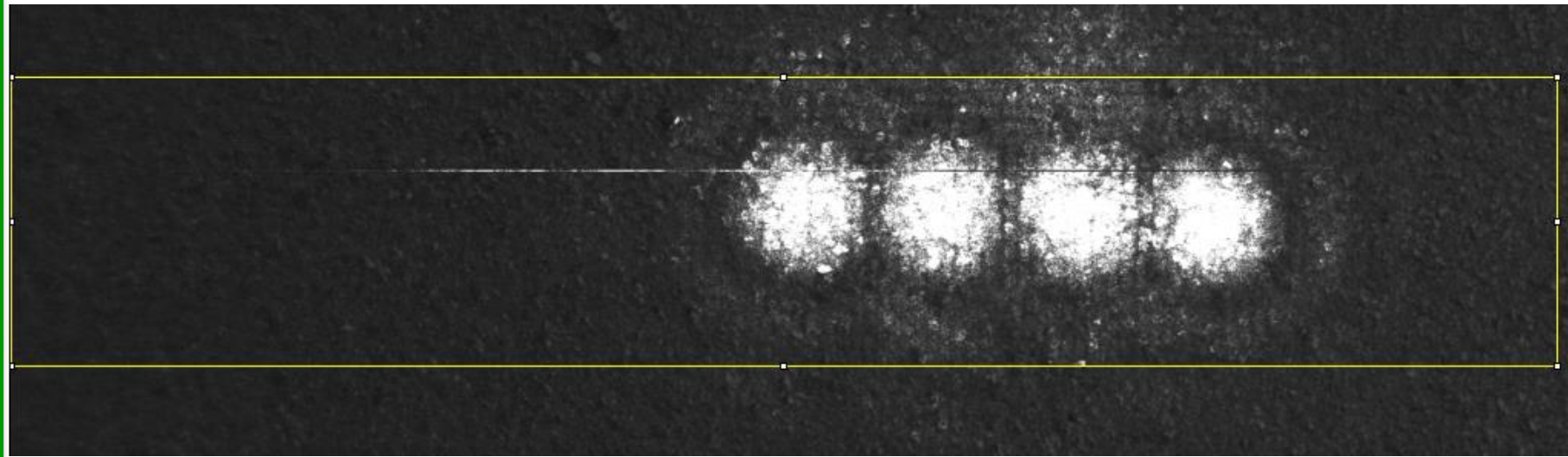


Now put the screen at the **exact location** of the pepperpot (keep camera focus, etc.) and put the pepperpot between the aperture and screen, so that the images of the holes **do not overlap**. Measure the distance L between pepperpot and screen and enter it at L in the Excel sheet. Switch on the laser. Ensure that the one of the rows is now illuminated (4 holes should be visible) by the laser and projected on the screen. Use the rows with larger holes in the middle of the pepperpot.

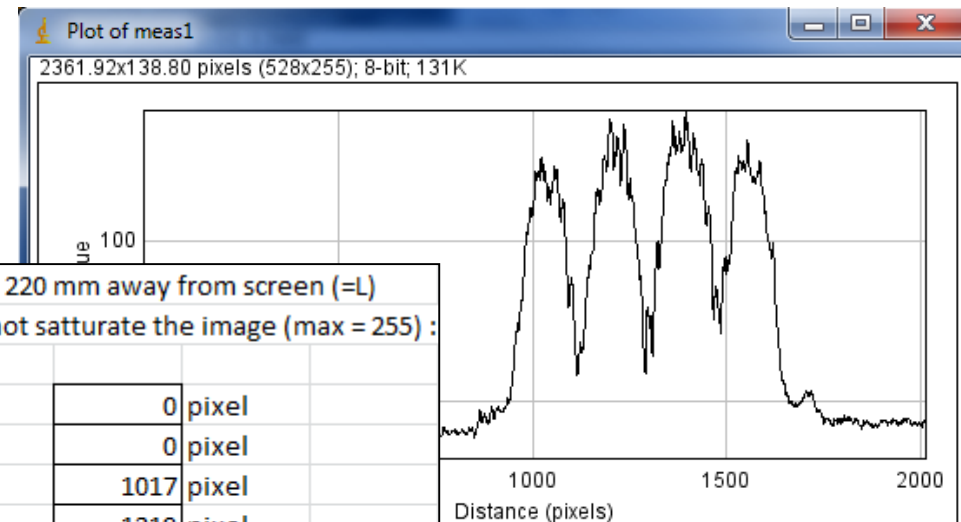
Ensure that no saturation exist, in case adjust gain and/or exposure time of the camera. Save the picture, that is your measurement!



Check: Do not saturate (255)



That is your measurement! Open the picture in ImageJ and define ROI like before (left border is 0). Plot the profile and measure the amplitude and the center position of each visible hole and enter it into Excel as n1-n8 and X1-X8, zeros for holes not visible.



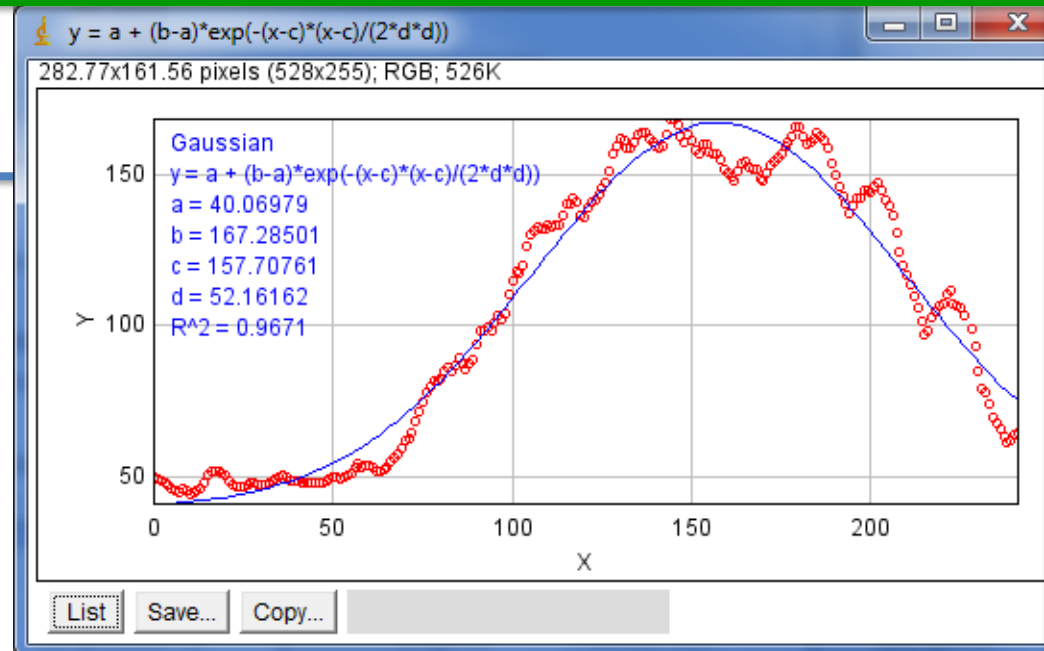
Now put screen on exact position of pepperpot and the pepperpot about 220 mm away from screen (=L)
Measure the intensity and center and RMS of each hole distribution (do not saturate the image (max = 255) :

Amplitude n1	0 bits	Position X1=	0 pixel
Amplitude n2	0 bits	Position X1=	0 pixel
Amplitude n3	172 bits	Position X1=	1017 pixel
Amplitude n4	164 bits	Position X1=	1219 pixel
Amplitude n5	165 bits	Position X1=	1460 pixel
Amplitude n6	46 bits	Position X1=	1719 pixel
Amplitude n7	0 bits	Position X1=	0 pixel
Amplitude n8	0 bits	Position X1=	0 pixel
Sum N =	547 bits		

Now we need the width σ of each hole image. One can do it by hand ($\sigma = \text{FWHM}/2.36$) or can use the fitting algorithm of ImageJ: **See ImageJ**

Introduction.

Enter each RMS value in the Excel table at RMS1-RMS8. Now Excel calculates $\langle x \rangle$, $\langle x' \rangle$, $\langle x^2 \rangle$, $\langle x'^2 \rangle$, $\langle xx' \rangle$ and with $\epsilon^2 = \langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2$ the emittance. It follows the formalism of Ref 1. The result should be around some 10^{-6} rad.



Now put screen on exact position of pepperpot and the pepperpot about 220 mm away from screen (=L)
Measure the intensity and center and RMS of each hole distribution (do not saturate the image (max = 255)) :

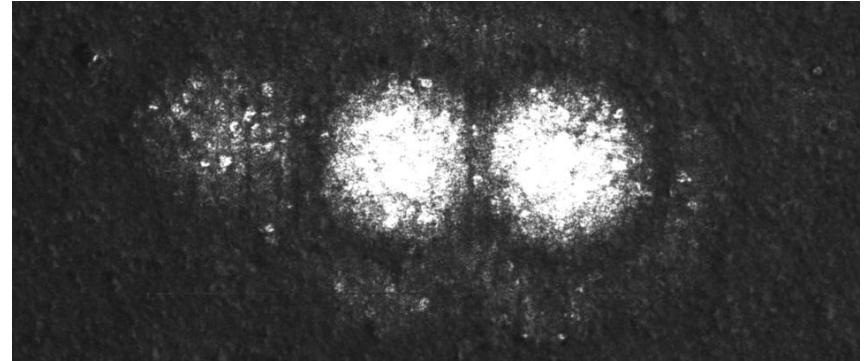
Amplitude n1	0 bits	Position X1=	0 pixel	RMS1	0 pixel
Amplitude n2	0 bits	Position X1=	0 pixel	RMS2	0 pixel
Amplitude n3	172 bits	Position X1=	1017 pixel	RMS3	30.4 pixel
Amplitude n4	164 bits	Position X1=	1219 pixel	RMS4	32.2 pixel
Amplitude n5	165 bits	Position X1=	1460 pixel	RMS5	29.7 pixel
Amplitude n6	46 bits	Position X1=	1719 pixel	RMS6	22.8 pixel
Amplitude n7	0 bits	Position X1=	0 pixel	RMS7	0 pixel
Amplitude n8	0 bits	Position X1=	0 pixel	RMS8	0 pixel
Sum N =	547 bits				

99					
100	$\langle xx' \rangle =$	-0.00029563			
101					
102					
103	Emittance^2 =	9.16246E-07	Emittance =	0.000957 mm rad	
104				9.57E-07 rad	

$$\epsilon_r^2 = \langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2$$

**Check: Do
not saturate
(255)**

Try to reduce the dimension of the
Laser beam by closing the aperture.
What happened to the emittance?



Amplitude n1	0	bits	Position X1=	0	pixel	RMS1	0	pixel
Amplitude n2	0	bits	Position X1=	0	pixel	RMS2	0	pixel
Amplitude n3	0	bits	Position X1=	0	pixel	RMS3	0	pixel
Amplitude n4	164	bits	Position X1=	1219	pixel	RMS4	32.2	pixel
Amplitude n5	165	bits	Position X1=	1460	pixel	RMS5	29.7	pixel
Amplitude n6	0	bits	Position X1=	0	pixel	RMS6	0	pixel
Amplitude n7	0	bits	Position X1=	0	pixel	RMS7	0	pixel
Amplitude n8	0	bits	Position X1=	0	pixel	RMS8	0	pixel
Sum N =	329	bits						

End of experiment

- More about pepperpot in some following slides:

Measurement of beam emittance, at low energy

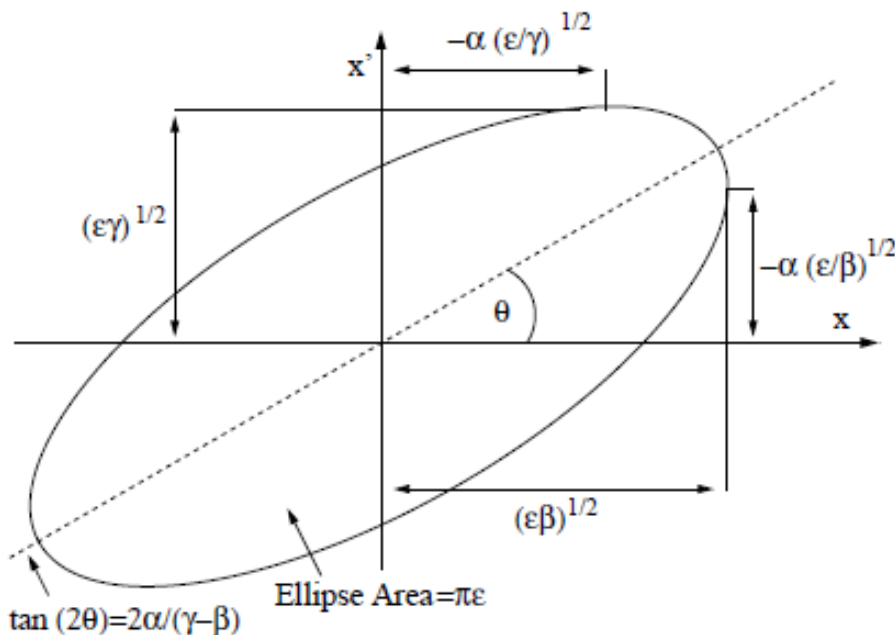
It is very important to understand and control transverse (and longitudinal) beam transport and emittance at the beginning of an accelerator chain since any kind of mismatch may result in emittance growth, halo formation and beam losses.

And, in contrast to electron circular machines, the normalized emittance in hadron machines cannot shrink. Therefore the luminosity is defined by the whole beam transport. **Measurements of the emittance** at the beginning and at every stage are most helpful **to understand problems in the accelerator chain** helping to achieve "good" beams at the experiment

Measurement of beam emittance

$$\varepsilon_{rms}(x, x') = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2}$$

In linear machines and transport lines the optic parameters depend on the incoming beam and the emittance has to be determined by measuring $\langle x \rangle$ and $\langle x' \rangle$. Methods and instruments for this purpose are discussed now.



$$\varepsilon_x = \frac{1}{\beta_x(s)} \left[\sigma_x^2(s) - \left(D_x(s) \frac{\Delta p}{p} \right)^2 \right]$$

In circular accelerators a measurement of the beam profile width σ is often sufficient to determine the beam emittance since $\Delta p/p$ = momentum spread, $\beta(s)$, $D(s)$ and $\Delta p/p$ can be determined sufficiently precise by common diagnostic methods. Minimum invasive instruments are needed not to destroy the circulating beam. These instruments were discussed in "Profile Monitors".

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \varepsilon \cdot \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} \quad \text{with} \quad \varepsilon = \sqrt{\det \sigma} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$$

Federico Roncarolo :

Accuracy of the transverse emittance measurements of the CERN large hadron collider. Thèse EPFL, no 3402 (2005).

Measurement of beam emittance

Due to their nature of measuring the angular and spatial beam distributions at the same time these type of instruments are fully destructive to the beam. They cannot be used in full power high intense beams. However, they are installed in nearly all machines to enable a measurement of the optic parameters at low current to tune the machine and to prove its health.

Most typical devices for emittance measurement use fluorescent screens or Secondary Emission- (SEM-) Harps as detectors.

Many useful information at:

Proceedings of the 6th CARE-HHH-ABI workshop on "Transverse and Longitudinal Emittance Measurement in Hadron-(Pre-)Accelerators". Date: December 10th to 12th, 2008, Castle

Kauzenburg in Bad Kreuznach.

CARE-Conf-08-033-HHH,

http://adweb.desy.de/mdi/CARE/Bad_Kreuznach/Reports/ABI-Workshop_Proceedings_all.pdf

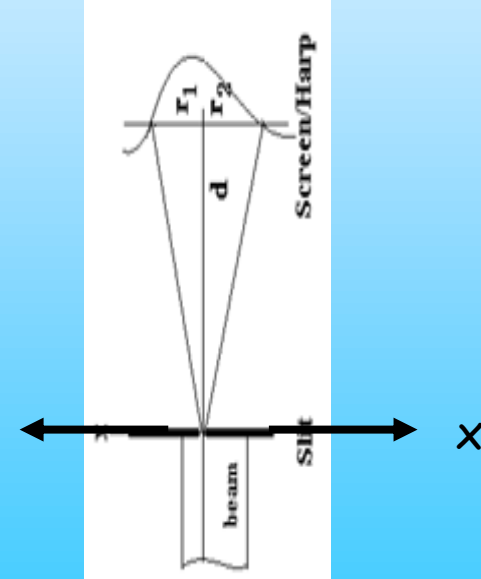
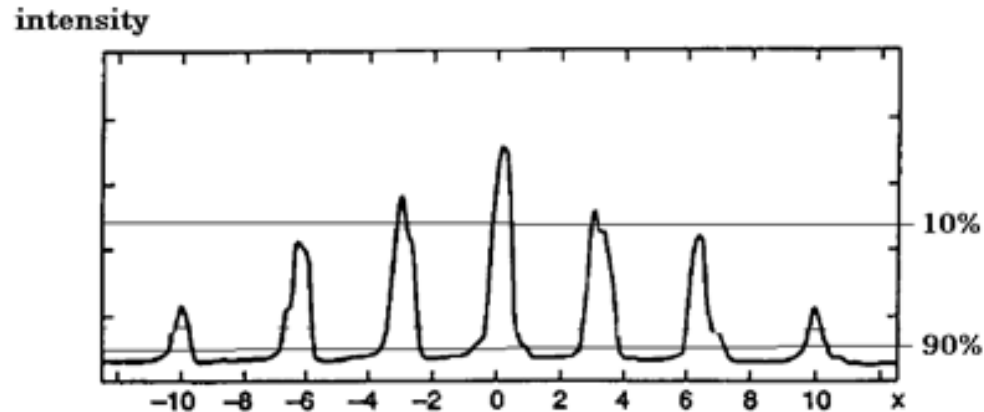
Emittance measurement by Slit + screen/harp

Low energy hadron particles which have a penetration depth of a cm or less can be stopped by a simple metallic plate. A small transversal slit of height h_{slit} selects a beamlet which shines on a screen or harp monitor in a drift distance d .

The width of the measured beamlet is defined by

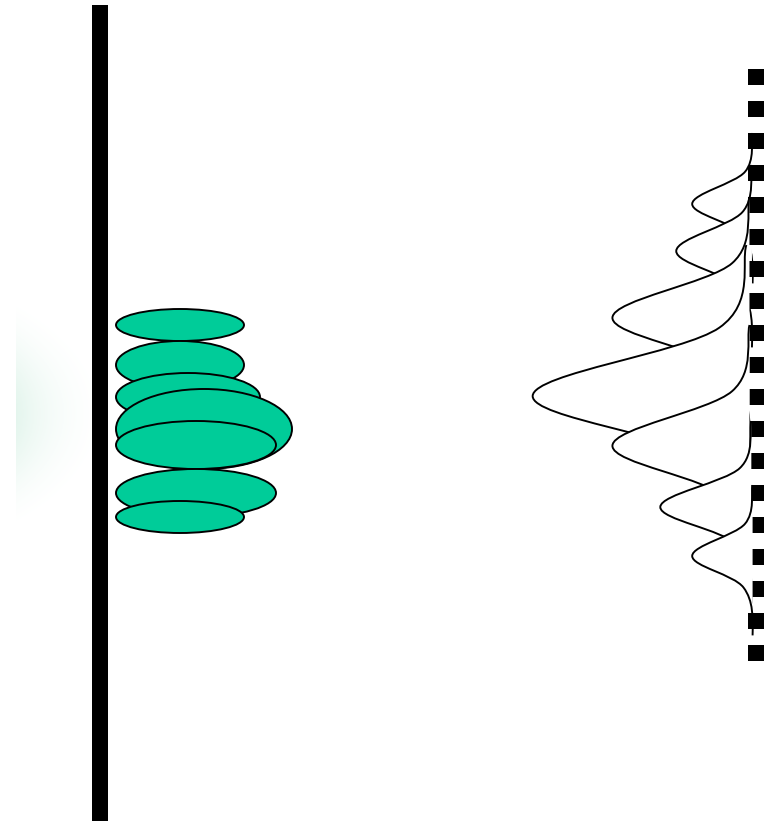
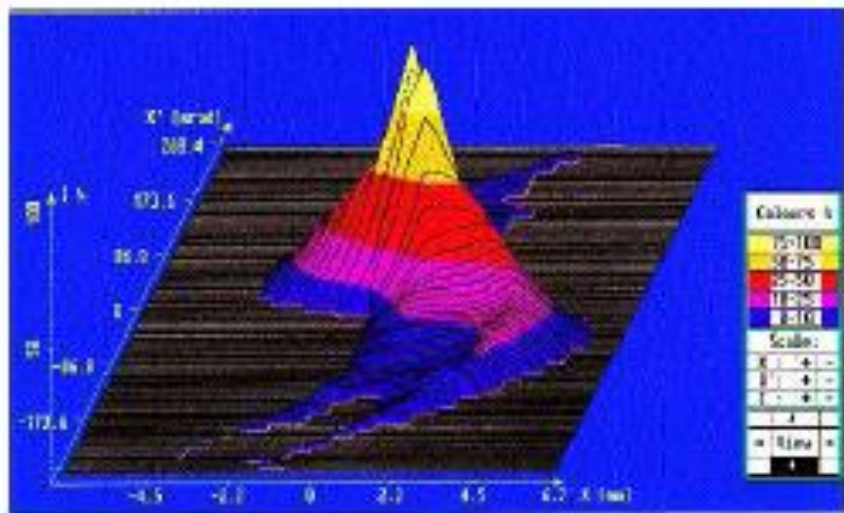
- the divergence of the beam x' at the slit position x_n , on the distance d and
- on the resolution of the system defined by the h_{slit} and
- the resolution of the screen/harp device.

The height of the signal depends on the bunch current I_0 and on the current distribution in the bunch (the beam profile). The slit is scanned across the beam profile and for each position x_n a beamlet distribution $I(x_n, x')$ is collected.



The Slit and Grid Method

3-dim plot:

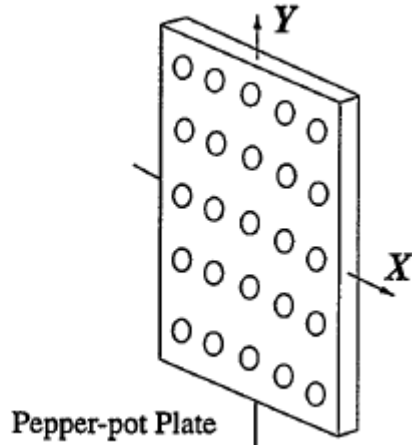
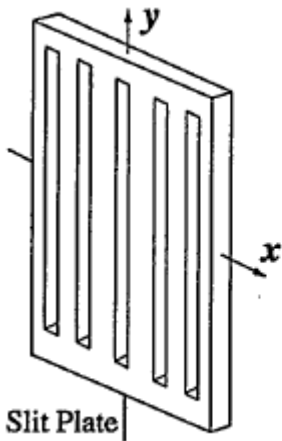


Emittance measurement by Pepperpot

The scanning of the whole beam size in both planes needs quite a lot of time with very stable beam conditions. To overcome the scanning procedure a plate with thin holes in defined distances (pepperpot) produces a lot of beamlets in the x, y plane..

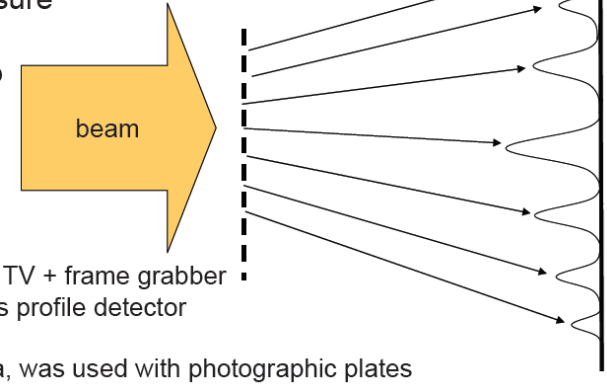
Advantages of pepper-pot method

- Single shot measurement
- Less prone to space charge effect
- Can obtain x and y emittance at one measurement
- Reduced heat load of the plate compare to slowly scanning slits.



● Needs high resolution profile detector

● Must make sure that profiles don't overlap



U. Raich CERN Accelerator School 2005

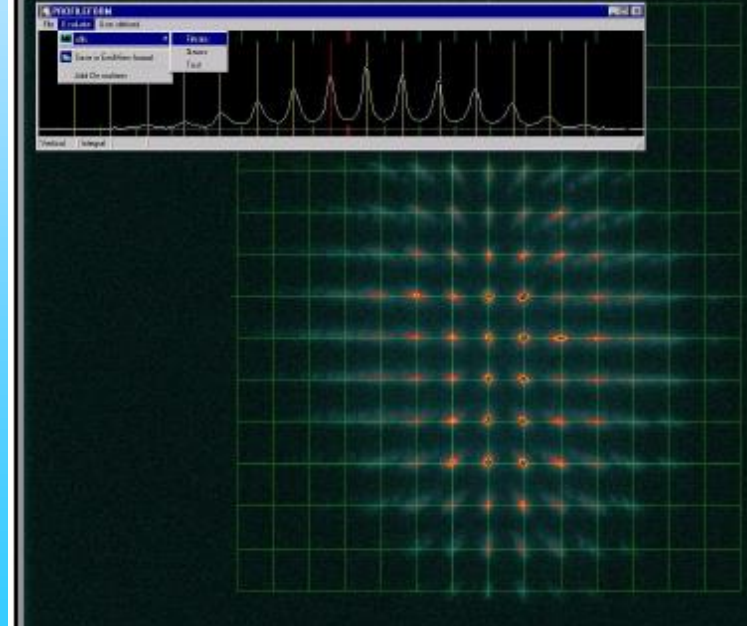
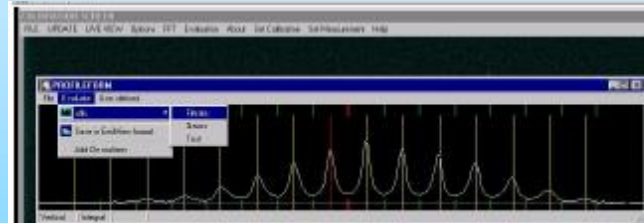


Photo P. Forck

Emittance measurement by Slit + screen/harp

The radii $r_{1,2}/d = x_{1,2}'$ give the angular spread of the beam at the position x_n at **a defined amount of current** included in the emittance ellipse.

The emittance is:

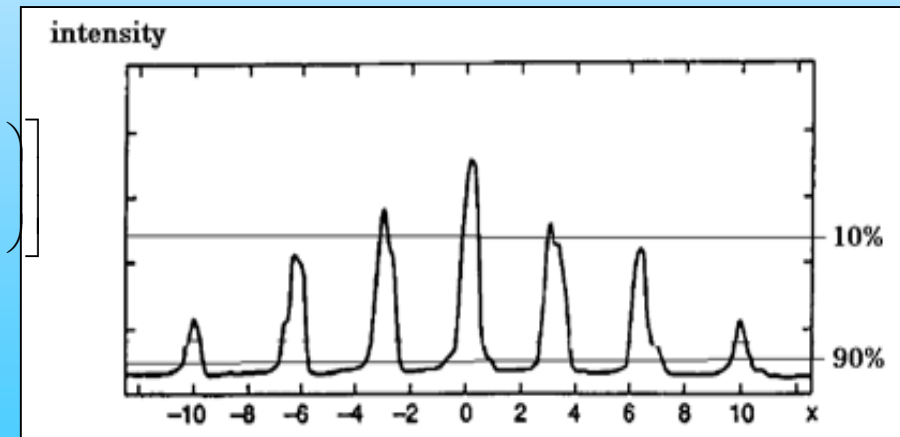
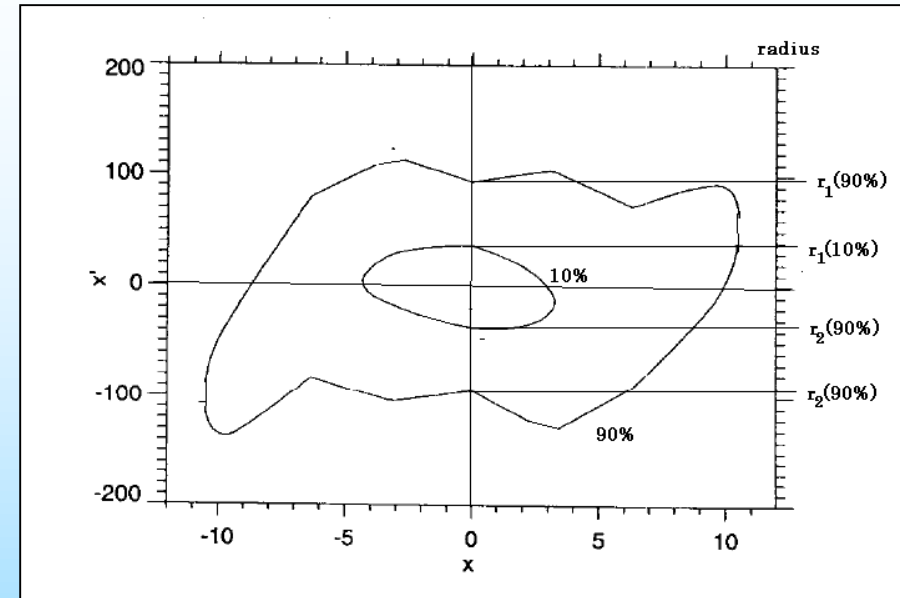
$$\varepsilon_x(x, x') = 1/\pi \int dx dx' = 1/\pi \cdot A_x$$

The rms emittance of the incident beam is:

$$\varepsilon_{rms}(x, x') = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

$$\varepsilon_{rms}^2 \approx \frac{1}{N^2} \cdot \left[\sum_{j=1}^p n_j \left(x_{nj} - \bar{x} \right)^2 \right] \cdot \left[\sum_{j=1}^p \left(n_j \cdot \left(\frac{\sigma_j}{d} \right)^2 + n_j (\bar{x}'_j - \bar{x}')^2 \right) \right] - \frac{1}{N^2} \left[\sum_{j=1}^p n_j x_{nj} \bar{x}'_j - N \bar{x} \bar{x}' \right]^2$$

Calc. details see: M. Zhang, Emittance Formula for Slits and Pepper-pot Measurement, TM-1988, Fermilab, 1996.

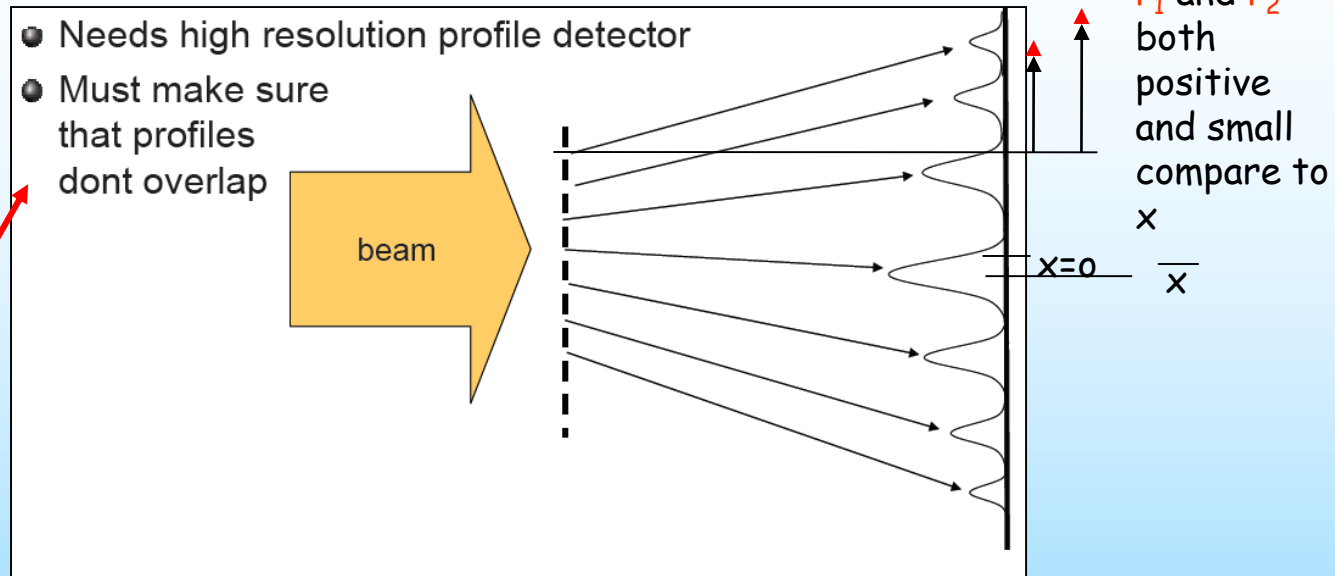


Emittance measurement by Pepperpot/slits

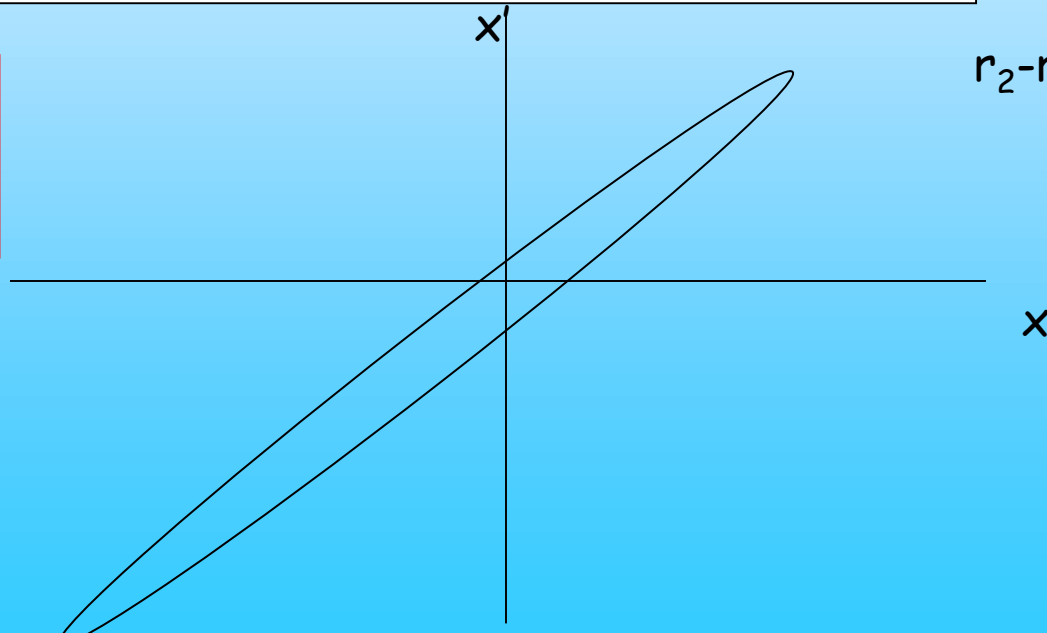
What kind of emittance-picture do you expect from this picture at almost 100%?

Assume $d=1$ unit

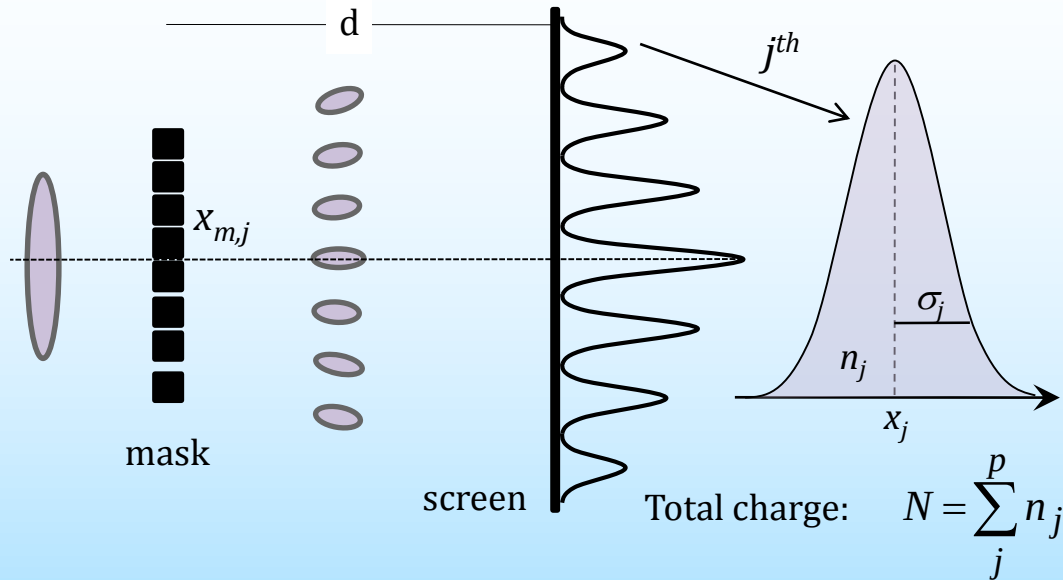
=> very good resolution is required!



$$r_2 - r_1 / d = x_{1,2}'$$



- The intensity of beamlet spots on a phosphor screen is primarily proportional to the number of particles in the beamlets which are hitting the screen
- It is also assumed that the size of slits is all the same and is very small compared with the beam size.
- L is small enough so that there is no overlap between beamlet spots on the screen.



In all beamlets:

Mean position:

$$\bar{x} = 1/N \sum_j n_j x_{m,j} = \langle x \rangle$$

Mean divergence:

$$\bar{x}' = 1/N \sum_j n_j \bar{x}'_j = \langle x' \rangle$$

In j^{th} beamlet:

Mean divergence:

$$\bar{x}'_j = 1/n_j \sum x'_{ji}$$

rms divergence:

$$\sigma_{j,x'} = \frac{\sigma_j}{d}$$

$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

$$\langle x^2 \rangle = \frac{1}{N^2} \sum_{j=1}^p n_j (x_{m,j} - \bar{x})^2$$

$$\langle x'^2 \rangle = \frac{1}{N^2} \sum_{j=1}^p n_j \left[\left(\frac{\sigma_j}{d} \right)^2 + (x_{m,j} - \bar{x})^2 \right]$$

$$\langle xx' \rangle^2 = \frac{1}{N^2} \left[\sum_{j=1}^p n_j x_j \bar{x}'_j - N \bar{x} \bar{x}' \right]^2$$

with

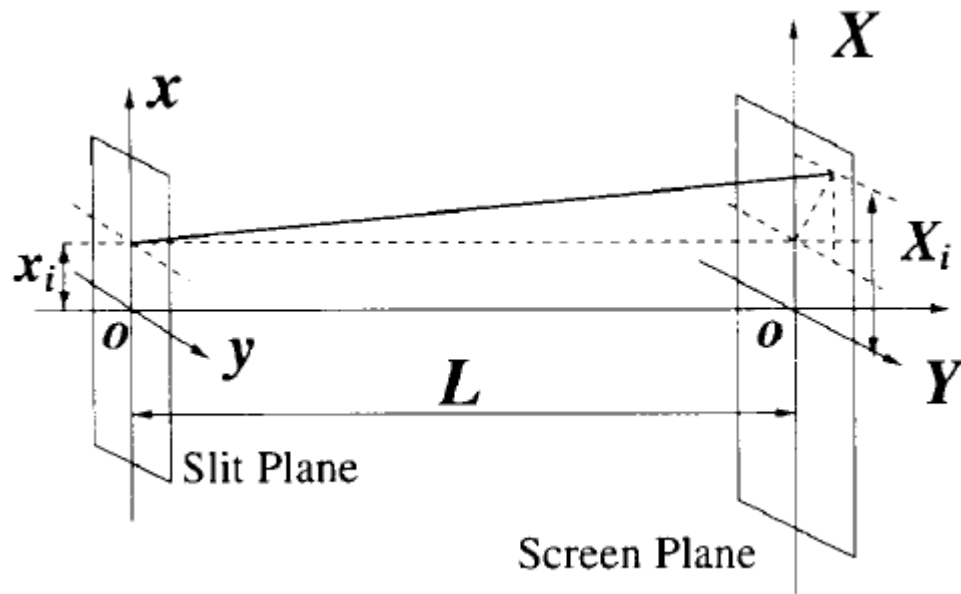
$$\sigma_j \equiv \sqrt{\frac{1}{n_j} \sum_{i=1}^{n_j} (X_{ji} - \bar{X}_j)^2},$$

which is the rms spot size of j -th beamlet on screen.

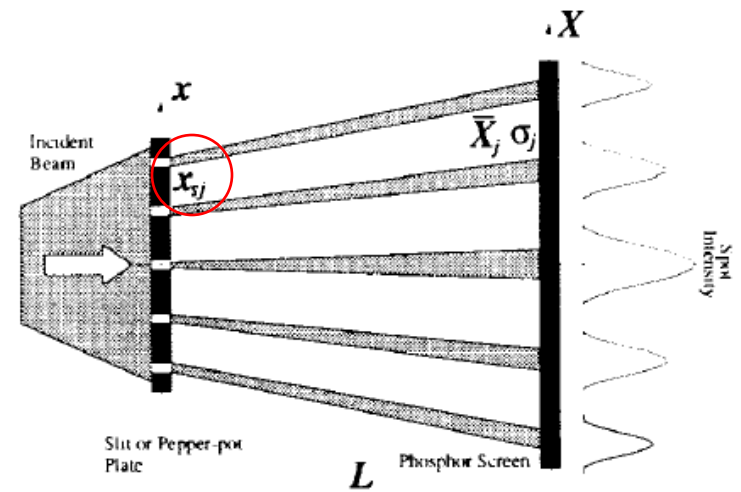
p = number of slits, $x_{m,j}$ = j -th slit position, N = all particles behind the slits
 n_j = number of particles passing through slit (weight of spot intensity)

between beamlet spots on the screen. Our goal is to find an emittance formula (ϵ_x) which employs only geometrical parameters of the slits and the spots on screen, that is, slits position (x_{sj}), mean position of spots on screen ($\bar{X}_j$), and rms size of spots on screen (σ_j).

It has to be made clear that a measured emittance is not beam's real mathematical emittance because of the following two reasons. Firstly, the measured emittance is based on a subset of particles instead of the whole beam. Secondly, there always exists space charge contribution to the final beamlets. So normally a measured emittance is at most an estimation of the real beam emittance.



$$x'_i \equiv \frac{X_i - x_i}{L}.$$



Total rms emittance is defined by

$$\epsilon_x \equiv \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2},$$

with

$$\langle x^2 \rangle \equiv \frac{1}{M} \sum_{i=1}^M (x_i - \bar{x})^2, \quad \bar{x} \equiv \frac{1}{M} \sum_{i=1}^M x_i, \quad \langle x x' \rangle \equiv \frac{1}{M} \sum_{i=1}^M (x_i - \bar{x})(x'_i - \bar{x}').$$

Applying a mask to the beam, say, a slit plate, we have N particles behind the slit. Clearly, $N \leq M$. This is a subset of the initial beam. Then the subset beam's rms emittance is given by Eqs. (3) and (4) with $\langle \bullet \rangle = \frac{1}{N} \sum_{i=1}^N \bullet$. From now on we will only discuss this subset emittance.

Zentrum des Strahls ($X = x?$) x ist hier Slit plane!!!

2.1 $\langle x \rangle$

$$\begin{aligned}
 \langle x \rangle &= \bar{x} = \frac{1}{N} \sum_{i=1}^N x_i \\
 &= \frac{1}{N} \left(\sum_{i=1}^{n_1} x_{1i} + \sum_{i=1}^{n_2} x_{2i} + \cdots + \sum_{i=1}^{n_p} x_{pi} \right) \\
 &\approx \frac{1}{N} (n_1 x_{s1} + n_2 x_{s2} + \cdots + n_p x_{sp}) = \frac{1}{N} \sum_{j=1}^p n_j x_{sj}.
 \end{aligned} \tag{5}$$

x_{sj} ($j = 1, 2, \dots, p$) is j -th slit position. p is the number of slits. n_j is number of particles through the j -th slit, which is proportional to the spot intensity on screen. x_{ji} ($j = 1, 2, \dots, p$ and $i = 1, 2, \dots, n_j$) are positions of particles passing through j -th slit. The whole set of $\{x_{ji}\}$ is exactly equal to $\{x_i | i = 1, 2, \dots, N\}$. This grouping scheme is also made for spot positions $\{X_{ji}\}$ and beam divergences $\{x'_{ji}\}$ in the following discussions. Clearly,

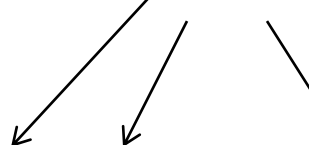
$$n_1 + n_2 + \cdots + n_p = N. \tag{6}$$

2.2 $\langle x' \rangle$

and

$$\bar{X}_j \equiv \frac{1}{n_j} \sum_{i=1}^{n_j} X_{ji}.$$

The latter is the mean position of j-th spot on the screen. The former can be rewritten as

$$\bar{x}'_j = \frac{\bar{X}_j - x_{sj}}{L}$$


X_{sj} ist die x
position des j
Schlitz auf
Maske

using Eq. (9). Then we have

$$\langle x' \rangle = \frac{1}{N} [n_1 \bar{x}'_1 + n_2 \bar{x}'_2 + \cdots + n_p \bar{x}'_p] = \frac{1}{N} \sum_{j=1}^p n_j \bar{x}'_j.$$

2.3 $\langle x^2 \rangle$

$$\begin{aligned} \langle x^2 \rangle &= \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2 \\ &= \frac{1}{N} \left[\sum_{i=1}^{n_1} (x_{1i} - \bar{x})^2 + \sum_{i=1}^{n_2} (x_{2i} - \bar{x})^2 + \cdots + \sum_{i=1}^{n_p} (x_{pi} - \bar{x})^2 \right] \\ &\approx \frac{1}{N} \sum_{j=1}^p n_j (x_{sj} - \bar{x})^2 \end{aligned}$$

2.4 $\langle x'^2 \rangle$

with

$$\sigma_j \equiv \sqrt{\frac{1}{n_j} \sum_{i=1}^{n_j} (X_{ji} - \bar{X}_j)^2},$$

which is the rms spot size of j-th beamlet on screen.

$$\bar{x}'_j = \frac{\bar{X}_j - x_{sj}}{L}$$

$$\langle x' \rangle = \bar{x}' = \frac{1}{N} [n_1 \bar{x}'_1 + n_2 \bar{x}'_2 + \cdots + n_p \bar{x}'_p] = \frac{1}{N} \sum_{j=1}^p n_j \bar{x}'_j.$$

$$\sigma_{x'_j}^2 \stackrel{\text{Eq. (22)}}{=} \left(\frac{\sigma_j}{L} \right)^2.$$

$$\langle x'^2 \rangle = \frac{1}{N} \sum_{j=1}^p \left[n_j \sigma_{x'_j}^2 + n_j (\bar{x}'_j - \bar{x}')^2 \right].$$

2.5 $\langle xx' \rangle$

$$\begin{aligned}
 \langle xx' \rangle &= \frac{1}{N} \sum_{i=1}^N (x_i x'_i - x_i \bar{x}' - \bar{x} x'_i + \bar{x} \bar{x}') \\
 &= \frac{1}{N} \sum_{i=1}^N x_i x'_i - \bar{x} \bar{x}' \\
 &\approx \frac{1}{N} \left(\sum_{i=1}^{n_1} x_{s1} x'_{1i} + \sum_{i=1}^{n_2} x_{s2} x'_{2i} + \cdots + \sum_{i=1}^{n_p} x_{sp} x'_{pi} - N \bar{x} \bar{x}' \right) \\
 &\stackrel{Eq. (13)}{=} \frac{1}{N} (n_1 x_{s1} \bar{x}'_1 + n_2 x_{s2} \bar{x}'_2 + \cdots + n_p x_{sp} \bar{x}'_p - N \bar{x} \bar{x}') \\
 &= \frac{1}{N} \left(\sum_{j=1}^p n_j x_{sj} \bar{x}'_j - N \bar{x} \bar{x}' \right)
 \end{aligned}$$

2.6 ϵ_x^2

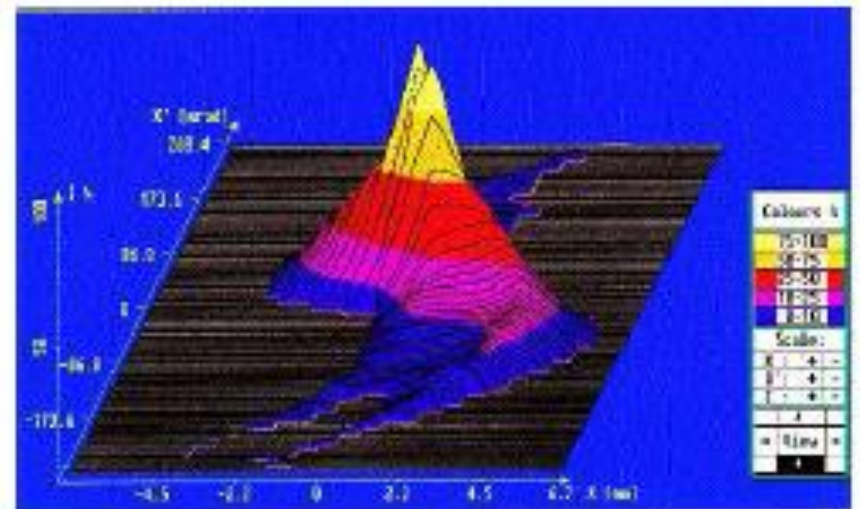
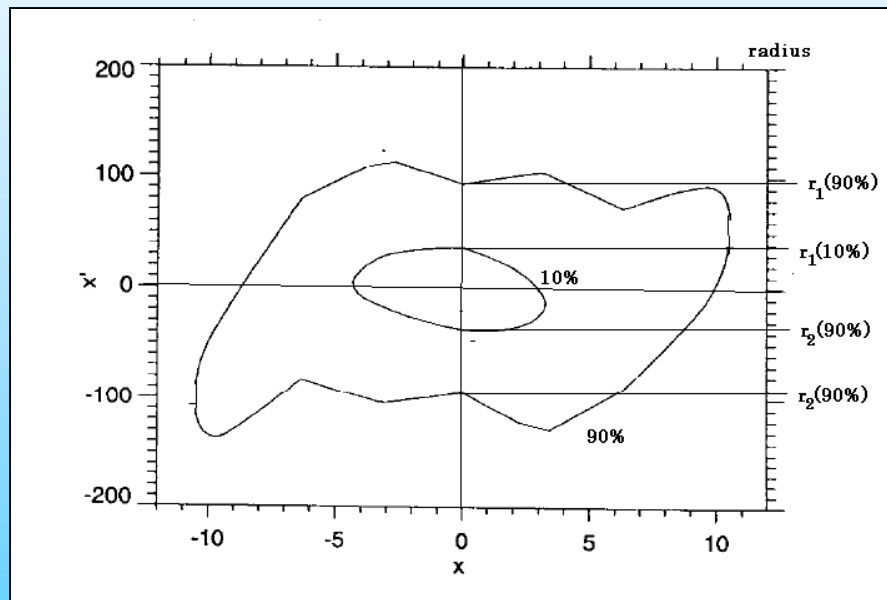
With Eq. (16) for $\langle x^2 \rangle$, Eq. (24) for $\langle x'^2 \rangle$, and Eq. (29) for $\langle xx' \rangle^2$, we obtain

$$\begin{aligned} \epsilon_x^2 &= \langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2 \\ &\approx \frac{1}{N^2} \left\{ \left[\sum_{j=1}^p n_j (x_{sj} - \bar{x})^2 \right] \left[\sum_{j=1}^p [n_j \sigma_{x'_j}^2 + n_j (\bar{x}'_j - \bar{x}')^2] \right] - \left[\sum_{j=1}^p n_j x_{sj} \bar{x}'_j - N \bar{x} \bar{x}' \right]^2 \right\}. \quad (30) \end{aligned}$$

This is the final slit emittance formula. All the terms in the formula are expressed by slit positions and beamlet spots parameters on the screen. Specifically, they are

- x_{sj} – j-th slit's position;
- p – total number of slits;
- n_j – number of particles passing through j-th slit and hitting the screen. Practically it is a weighting of spot intensity;
- $\bar{x}$ – mean position of all beamlets (Eq. (5));
- $\bar{x}'_j$ – mean divergence of j-th beamlet (Eq. (10));
- $\bar{x}'$ – mean divergence of all beamlets (Eq. (11));
- $\sigma_{x'_j}$ – rms divergence of j-th beamlet (Eq. (21)).

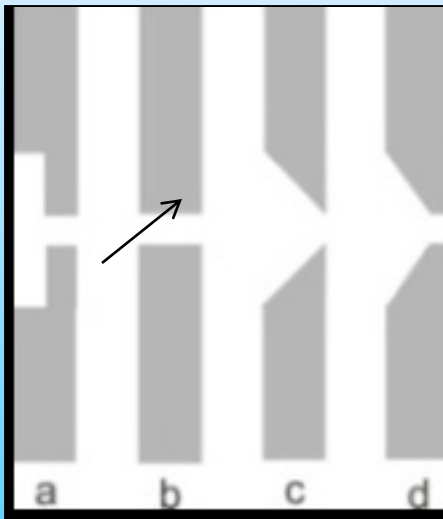
3-dim plot:



Emittance measurement by Slit + screen/harp

The procedures described above neglect the finite resolution of the device.

- A small h_{slit} and small step sizes of the slit positioning improve the resolution.
- When using a grid as detector its resolution can be improved by scanning the grid as well.
- A position of the device in a region with high x' is most helpful for a good resolution.
- In high density beams care has to be taken that space charge in a beamlet might extend the angular spread of its on the way to the detector.



Comparisons (simulations 3 MeV protons) of different geometries (stainless steel) show that

- geometry (d) gives results similar to geometry (a),
- geometry (b) gives the best (smallest) results and
- geometry (c) gives the worse results, with about 30% of the particles that reach the SEM-grid being scattered.

But at (b) the aperture of the slit is smaller than its thickness
=> the measured phase-space distribution is flowed by an angular cut and the emittance is underestimated.