RF Cavity Design

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CERN Accelerator School
Accelerator Physics (Intermediate level)
Darmstadt 2009

Overview

- DC versus RF
  - Basic equations: Lorentz & Maxwell, RF breakdown
- Some theory: from waveguide to pillbox
  - rectangular waveguide, waveguide dispersion, standing waves … waveguide resonators, round waveguides, Pillbox cavity
- Accelerating gap
  - Induction cell, ferrite cavity, drift tube linac, transit time factor
- Characterizing a cavity
  - resonance frequency, shunt impedance,
  - beam loading, loss factor, RF to beam efficiency,
  - transverse effects, Panofsky-Wenzel, higher order modes, PS 80 MHz cavity (magnetic coupling)
- More examples of cavities
  - PEP II, LEP cavities, PS 40 MHz cavity (electric coupling),
- RF Power sources
- Many gaps
  - Why?
  - Example: side coupled linac, LIBO
- Travelling wave structures
  - Brillouin diagram, iris loaded structure, waveguide coupling
- Superconducting Accelerating Structures
- RFQ's

DC VERSUS RF

DC versus RF

DC accelerator

RF accelerator

Lorentz force

A charged particle moving with velocity $\vec{v}$ through an electro-magnetic field experiences a force

$$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad \vec{v} = \frac{\vec{p}}{m\gamma}$$

The energy of the particle is $W = \sqrt{\left(mc^2\right)^2 + \left(pc\right)^2} = \gamma mc^2$

$$W_{kin} = mc^2(\gamma - 1)$$

Change of *W* due to the this force (work done) ; differentiate:

$$W\mathrm{d}W = c^2\vec{p}\cdot\mathrm{d}\vec{p} = qc^2\vec{p}\cdot\left(\vec{E}+\vec{v}\times\vec{B}\right)\mathrm{d}t = qc^2\vec{p}\cdot\vec{E}\,\mathrm{d}t$$

$$\mathrm{d}W = q\vec{v}\cdot\vec{E}\,\mathrm{d}t$$

Note: no work is done by the magnetic field.

Maxwell's equations (in vacuum)

$$\nabla \times \vec{B} - \frac{1}{c^2}\frac{\partial}{\partial t}\vec{E} = \mu_0 \vec{J} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t}\vec{B} = 0 \qquad \nabla \cdot \vec{E} = \mu_0 c^2 \rho$$

<u>why not DC?</u>

1) DC ($\frac{\partial}{\partial t} \equiv 0$): $\nabla \times \vec{E} = 0$ which is solved by $\vec{E} = -\nabla \Phi$

Limit: If you want to gain 1 MeV, you need a potential of 1 MV!

2) Circular machine: DC acceleration impossible since $\oint \vec{E} \cdot \mathrm{d}\vec{s} = 0$

<u>With time-varying fields:</u>

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B} \qquad \oint \vec{E} \cdot \mathrm{d}\vec{s} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot \mathrm{d}\vec{A}$$

Maxwell's equation in vacuum (contd.)

$$\nabla \times \vec{B} - \frac{1}{c^2}\frac{\partial}{\partial t}\vec{E} = 0 \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t}\vec{B} = 0 \qquad \nabla \cdot \vec{E} = 0$$

curl of 3rd and $\frac{\partial}{\partial t}$ of 1st equation:

$$\nabla \times \nabla \times \vec{E} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = 0$$

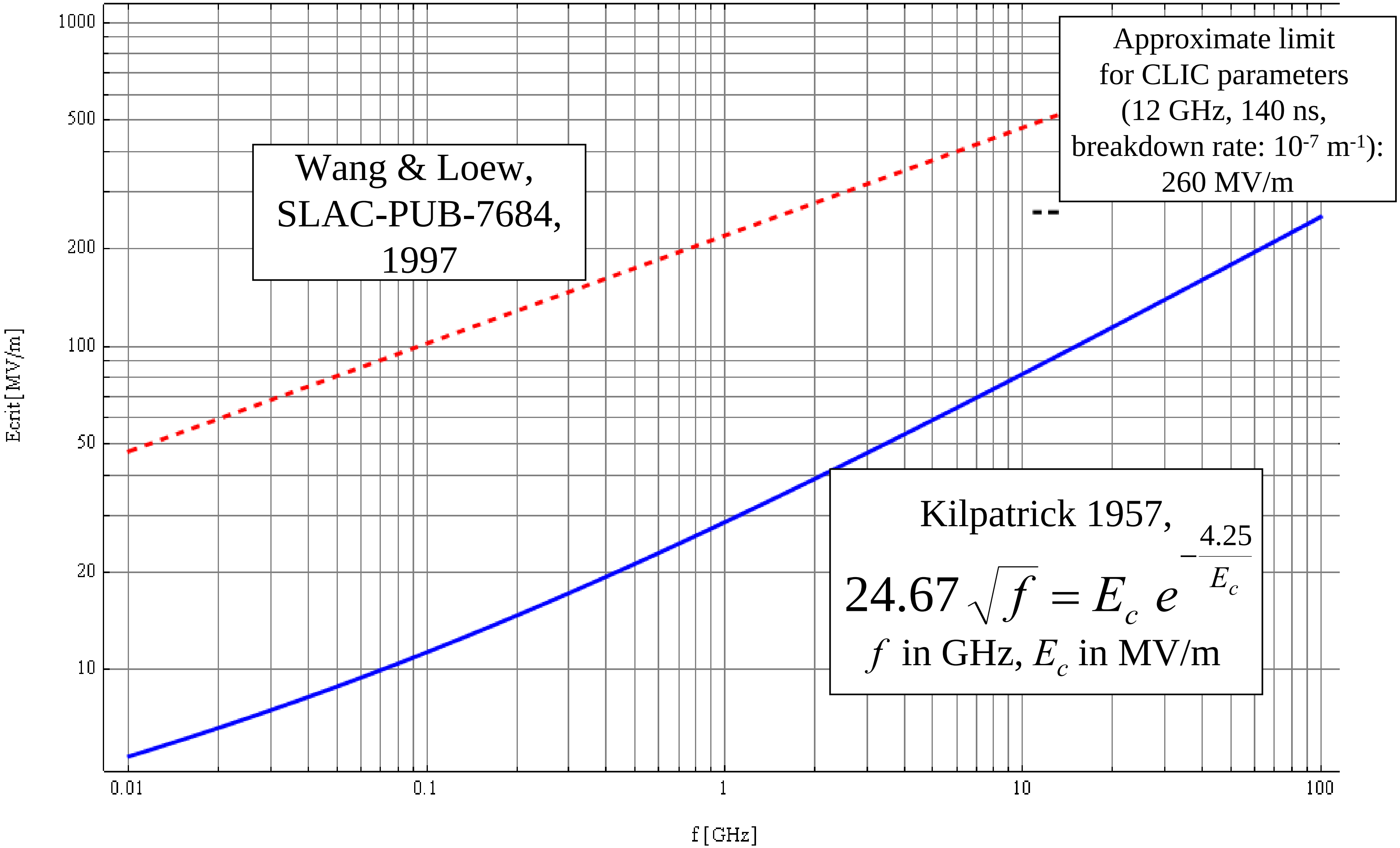
vector identity: $\nabla \times \nabla \times \vec{E} = \nabla\nabla \cdot \vec{E} - \Delta\vec{E}$

with 4th equation : $\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = 0$

i.e. Laplace in 4 dimensions

Another reason for RF: breakdown limit

surface field, in vacuum,Cu surface, room temperature



FROM WAVEGUIDE TO PILLBOX

Homogeneous plane wave

$$\vec{E} \propto \vec{u}_y \cos\left(\omega t - \vec{k} \cdot \vec{r}\right)$$

$$\vec{B} \propto \vec{u}_x \cos\left(\omega t - \vec{k} \cdot \vec{r}\right)$$

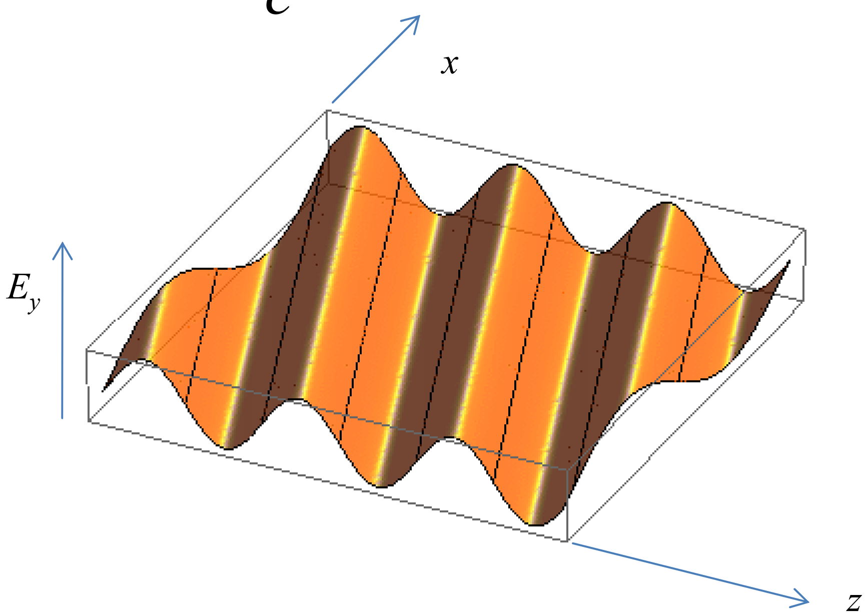
$$\vec{k} \cdot \vec{r} = \frac{\omega}{c}\left(\cos(\varphi) z + \sin(\varphi) x\right)$$

Wave vector $\vec{k}$

the direction of $\vec{k}$ is the direction of propagation,

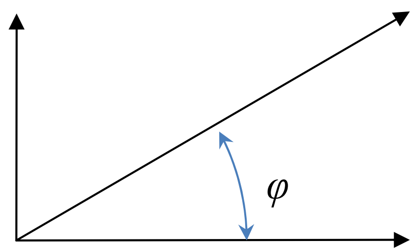
the length of $\vec{k}$ is the phase shift per unit length.

$\vec{k}$ behaves like a vector.



$$k_\perp = \frac{\omega_c}{c}$$

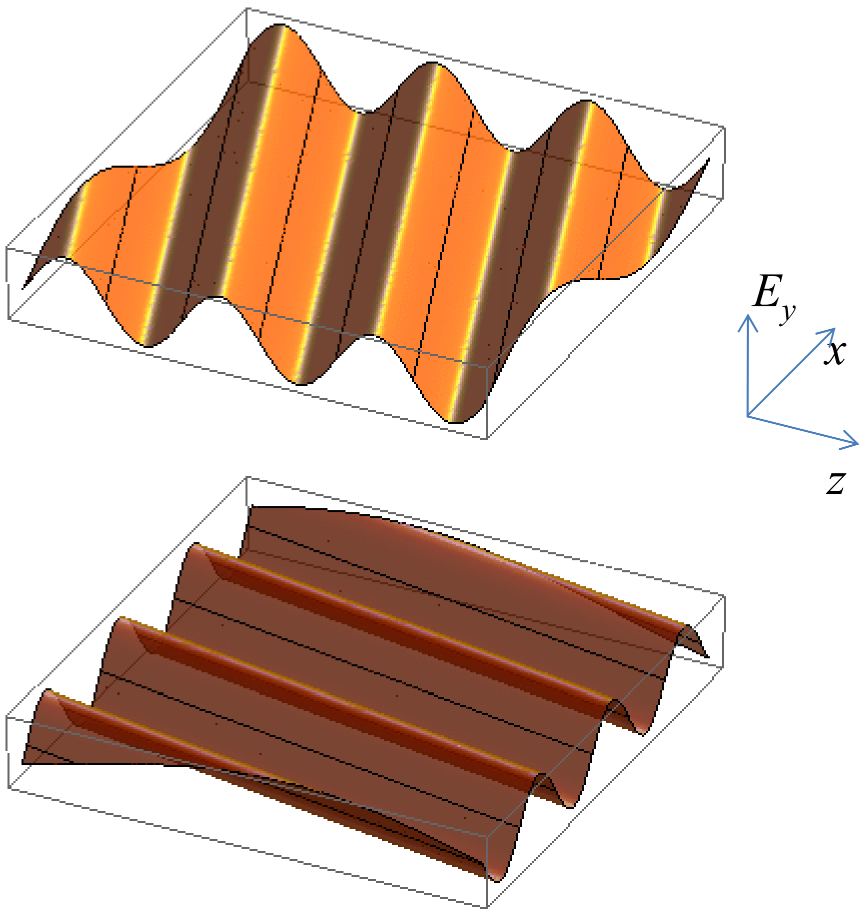
$$k = \frac{\omega}{c}$$

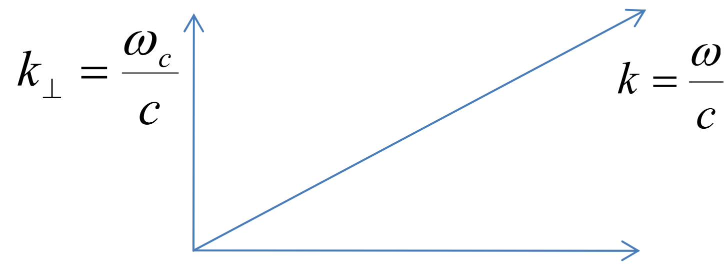


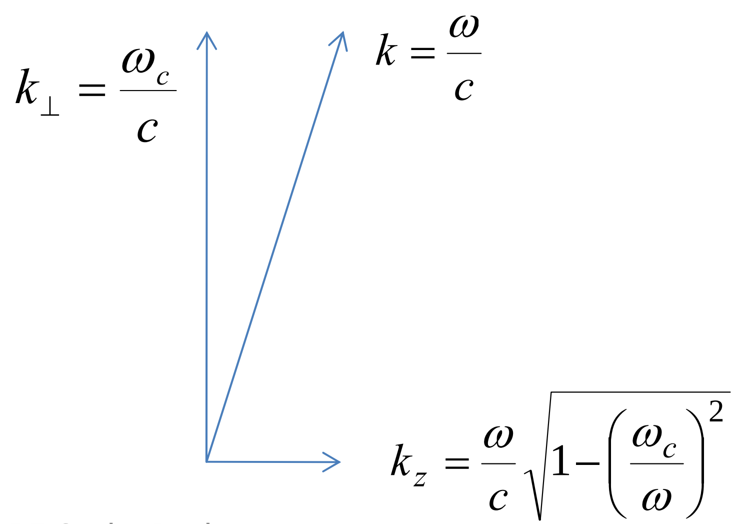
$$k_z = \frac{\omega}{c}\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$

Wave length, phase velocity

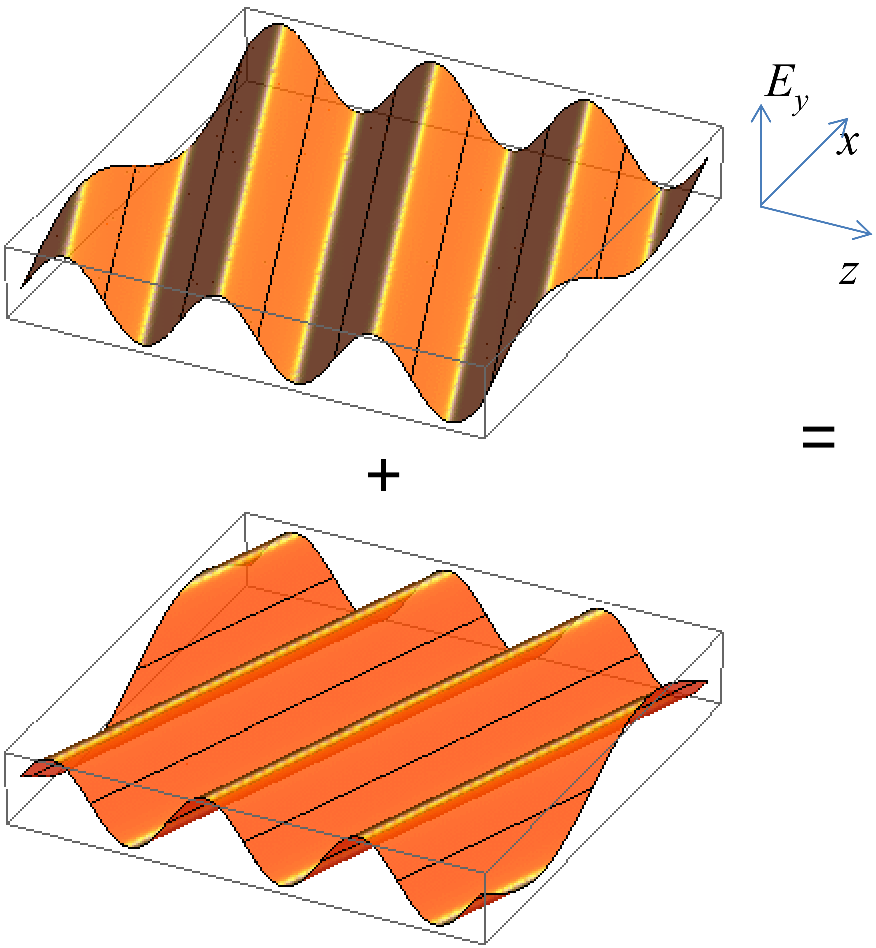
- The components of $\vec{k}$ are related to the wavelength in the direction of that component as $\lambda_z = \dfrac{2\pi}{k_z}$ etc. , to the phase velocity as $v_{\varphi,z} = \dfrac{\omega}{k_z} = f\,\lambda_z$.

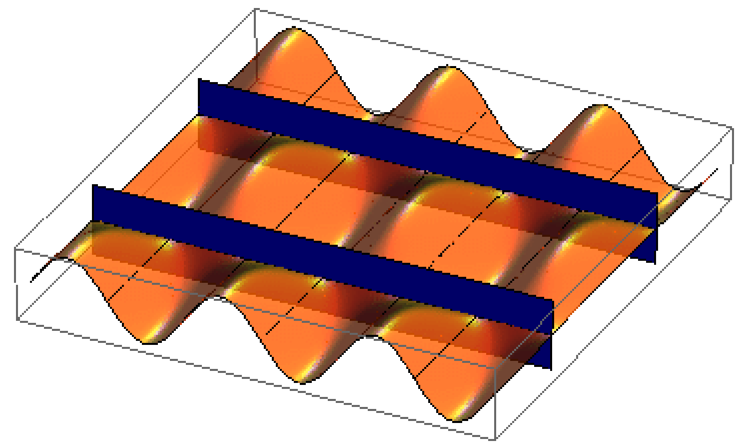






Superposition of 2 homogeneous plane waves





Metallic walls may be inserted where $E_y \equiv 0$

without perturbing the fields.

Note the standing wave in *x*-direction!

This way one gets a hollow rectangular waveguide

Rectangular waveguide

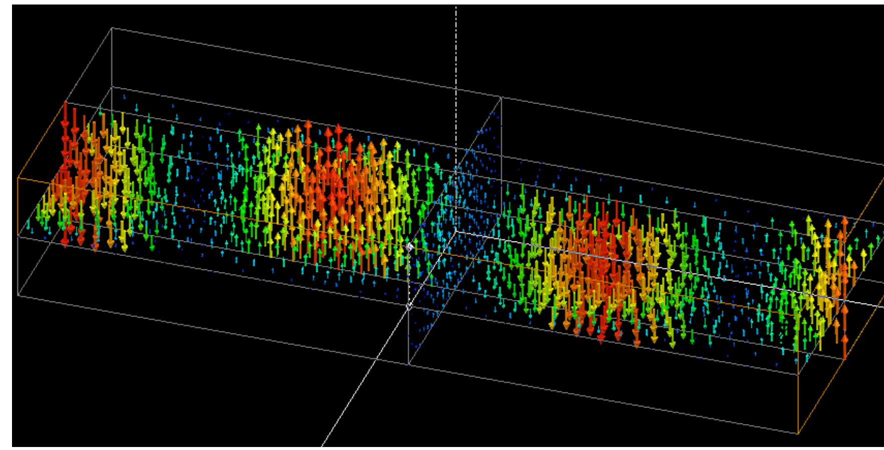
Fundamental (TE_{10} or H_{10}) mode
in a standard rectangular waveguide.

E.g. forward wave

power flow

-y

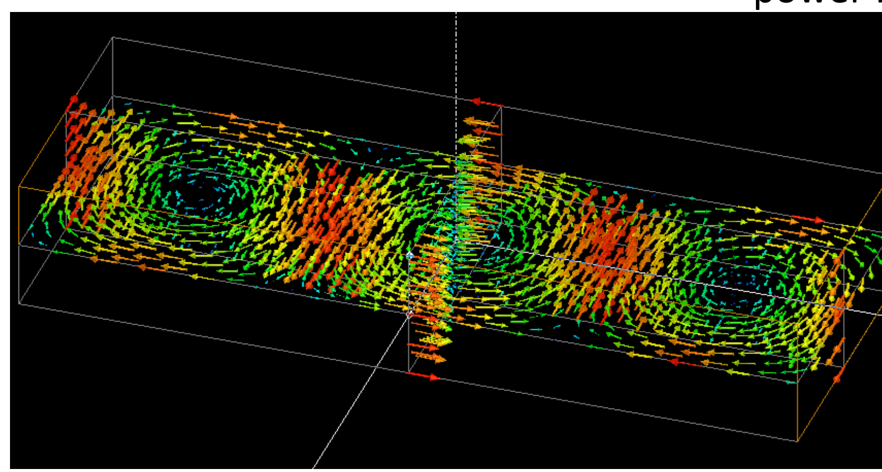
electric field



x

power flow

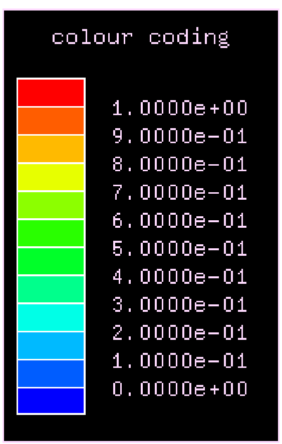
magnetic field



z

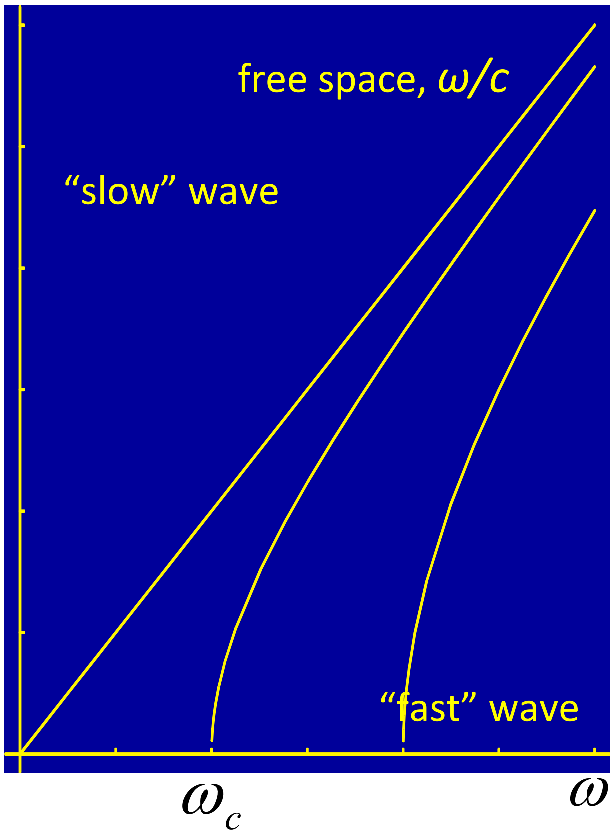
x

power flow: $\frac{1}{2}\mathrm{Re}\left\{ \iint\limits_{\substack{\text{cross}\\\text{section}}} \vec{E}\times\vec{H}^{*}\cdot \mathrm{d}\,\vec{A} \right\}$



Waveguide dispersion

$$k_z = \mathrm{Im}\{\gamma\} = \frac{2\pi}{\lambda_g}$$



e.g.: TE_{10}-wave in rectangular waveguide:

$$\gamma = \mathrm{j}\sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2}$$

$$Z_0 = \frac{\mathrm{j}\omega\mu}{\gamma}$$

$$\lambda_{\mathrm{cutoff}} = 2a$$

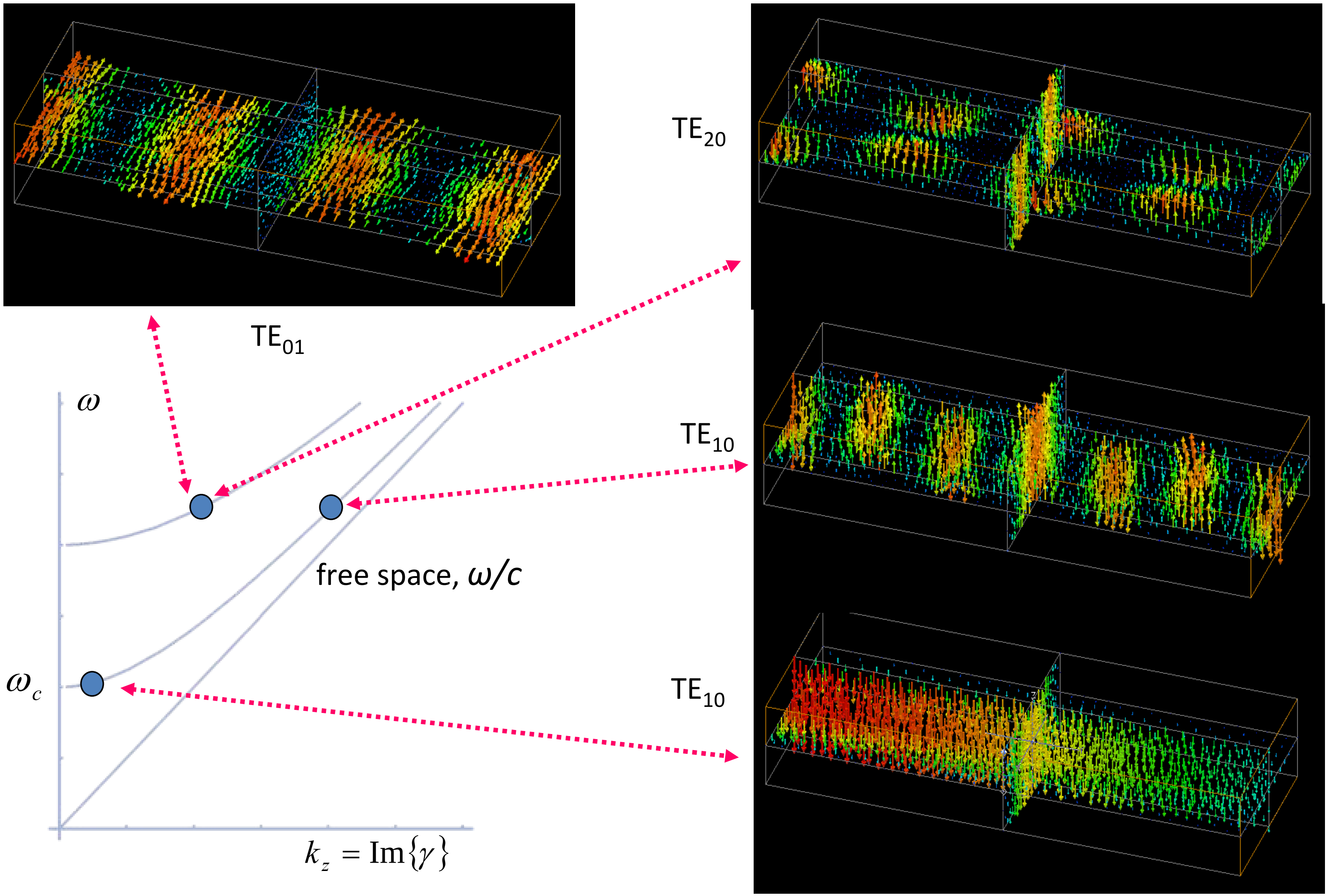
general **cylindrical** waveguide:

$$\gamma = \mathrm{j}\sqrt{\left(\frac{\omega}{c}\right)^2 - k_\perp{}^2}$$

$$Z_0 = \frac{\mathrm{j}\omega\mu}{\gamma} \text{ for TE,} \quad Z_0 = \frac{\gamma}{\mathrm{j}\omega\varepsilon} \text{ for TM}$$

In a hollow waveguide: phase velocity > *c*, group velocity < *c*

Waveguide dispersion (continued: Higher Order Modes)



General waveguide equations:

Transverse wave equation (membrane equation): $\Delta T + \left(\frac{\omega_c}{c}\right)^2 T = 0$

	TE (or H) modes	TM (or E) modes
boundary condition:	$\vec{n} \cdot \nabla T = 0$	$T = 0$
characteristic impedance:	$Z_0 = \frac{\mathrm{j}\omega\mu}{\gamma}$	$Z_0 = \frac{\gamma}{\mathrm{j}\omega\varepsilon}$
ortho-normal eigenvectors:	$\vec{e} = \vec{u}_z \times \nabla T$	$\vec{e} = -\nabla T$
longitudinal field:	$H_z = \left(\frac{\omega_c}{c}\right)^2 \frac{T U(z)}{\mathrm{j}\omega\mu}$	$E_z = \left(\frac{\omega_c}{c}\right)^2 \frac{T I(z)}{\mathrm{j}\omega\varepsilon}$

longitudinal wave equations (transmission line equations):

$$\frac{\mathrm{d}U(z)}{\mathrm{d}z} + \gamma Z_0 I(z) = 0$$

$$\frac{\mathrm{d}I(z)}{\mathrm{d}z} + \frac{\gamma}{Z_0} U(z) = 0$$

propagation constant: $\gamma = \mathrm{j}\frac{\omega}{c}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$

transverse fields:

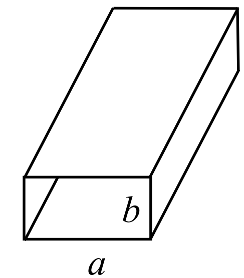
$$\vec{E} = U(z)\vec{e}$$

$$\vec{H} = I(z)\vec{u}_z \times \vec{e}$$

Rectangular waveguide: transverse eigenfunctions

TE (H) modes: $$T_{mn}^{(H)} = \frac{1}{\pi}\sqrt{\frac{ab\varepsilon_m\varepsilon_n}{(mb)^2+(na)^2}}\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

TM (E) modes: $$T_{mn}^{(E)} = \frac{2}{\pi}\sqrt{\frac{ab}{(mb)^2+(na)^2}}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

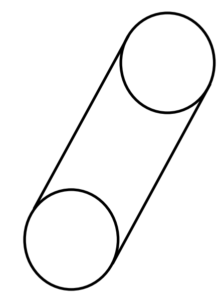


$$\frac{\omega_c}{c} = \sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

Round waveguide: transverse eigenfunctions

TE (H) modes: $$T_{mn}^{(H)} = \sqrt{\frac{\varepsilon_m}{\pi\left(\chi'^2_{mn}-m^2\right)}}\frac{\mathrm{J}_m\left(\chi'_{mn}\frac{\rho}{a}\right)}{\mathrm{J}_m\left(\chi'_{mn}\right)}\begin{Bmatrix}\cos(m\varphi)\\ \sin(m\varphi)\end{Bmatrix}$$

TM (E) modes: $$T_{mn}^{(E)} = \sqrt{\frac{\varepsilon_m}{\pi}}\frac{\mathrm{J}_m\left(\chi_{mn}\frac{\rho}{a}\right)}{\chi_{mn}\mathrm{J}_{m-1}(\chi_{mn})}\begin{Bmatrix}\sin(m\varphi)\\ \cos(m\varphi)\end{Bmatrix}$$

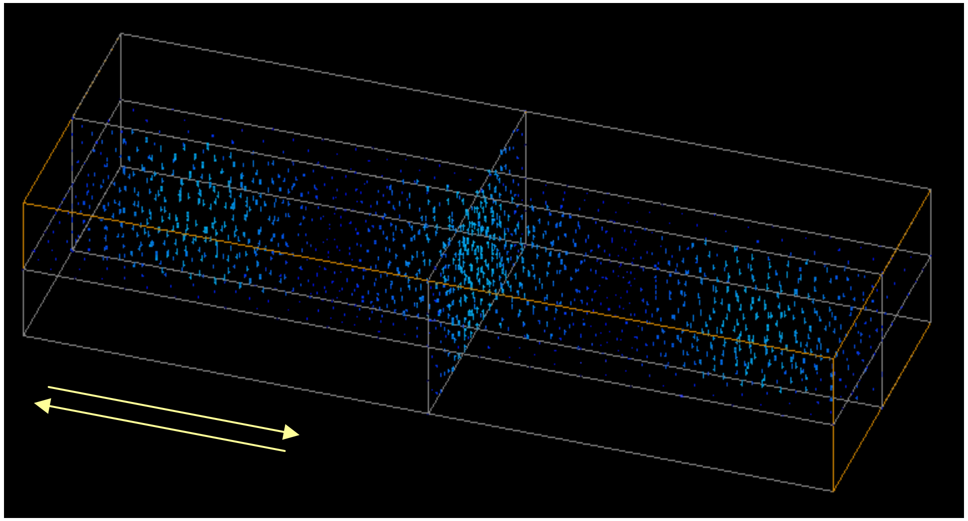


$$\frac{\omega_c}{c} = \frac{\chi_{mn}}{a}$$

where $$\varepsilon_i = \begin{cases}1 & for \quad i=0\\ 2 & for \quad i\neq 0\end{cases}$$

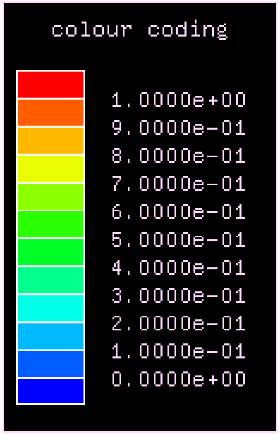
Standing wave – resonator

Same as above, but two counter-running waves of identical amplitude.

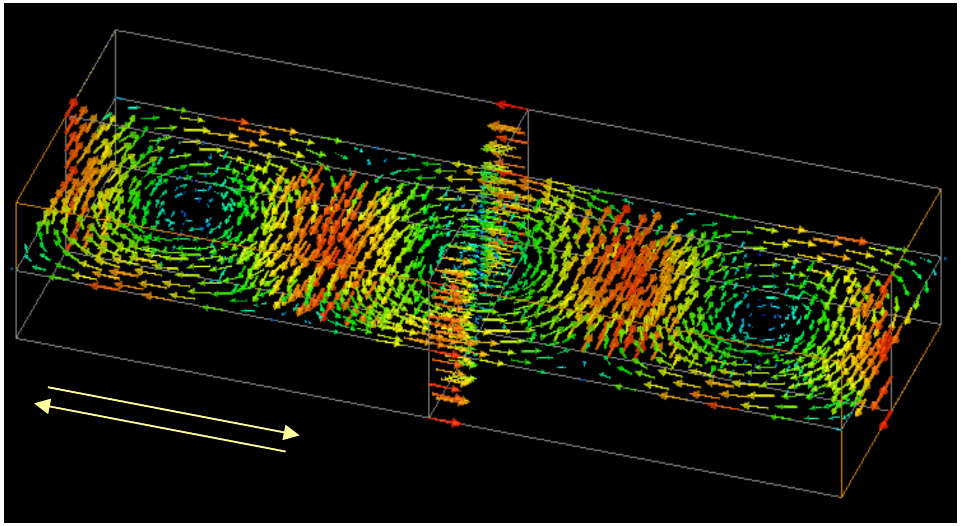


electric field

no net power flow: $$\frac{1}{2}\mathrm{Re}\left\{\iint\limits_{\substack{\text{cross}\\\text{section}}} \vec{E}\times\vec{H}^{*}\cdot\mathrm{d}\,\vec{A}\right\} = 0$$



magnetic field (90° out of phase)

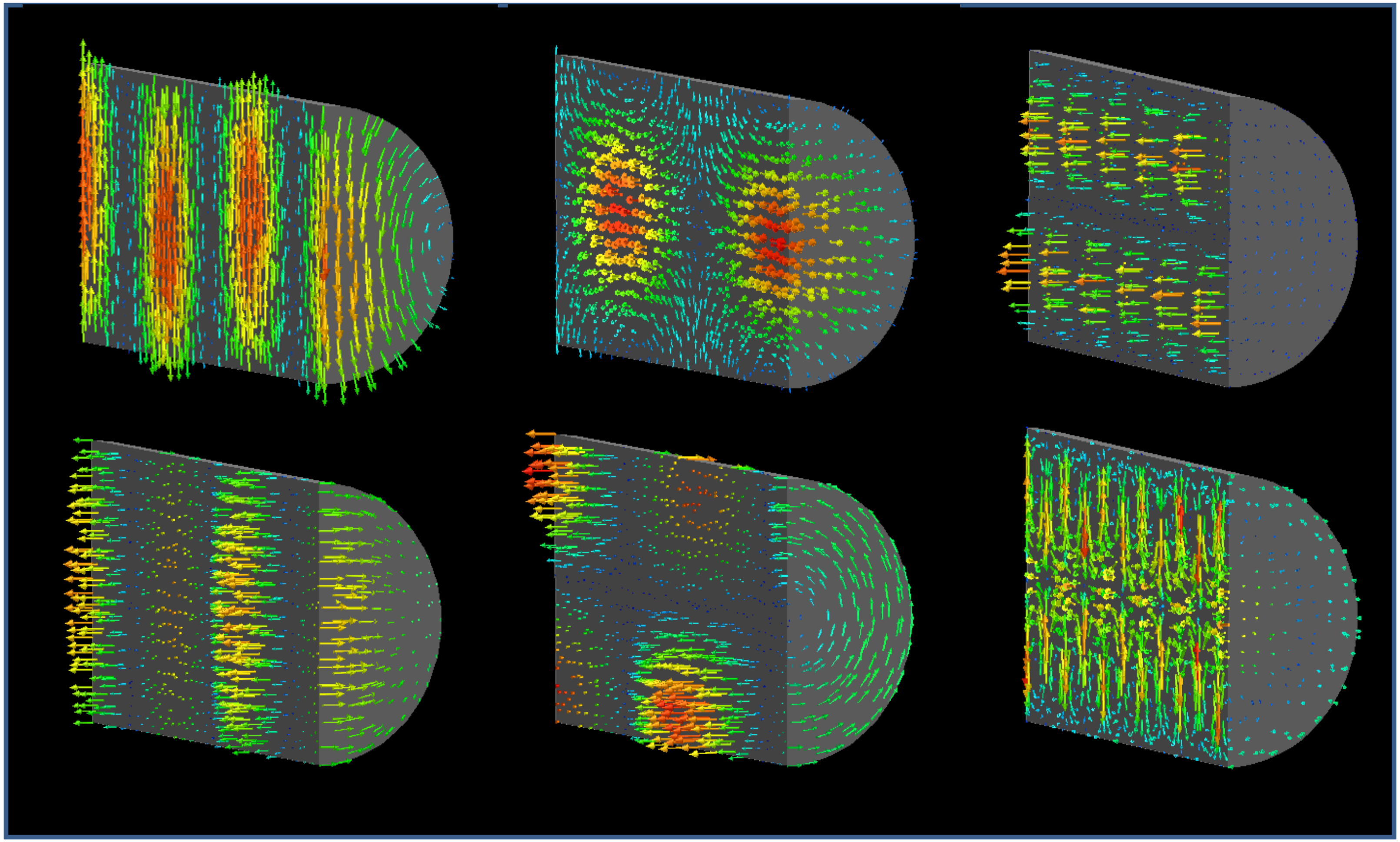


Round waveguide

parameters used in calculation:
f = 1.43, 1.09, 1.13 f_c, a: radius

$\vec{E}$

$\vec{B}$



TE$_{11}$: fundamental mode

$$\frac{f_c}{\text{GHz}} = \frac{87.85}{a/\text{mm}}$$

TM$_{01}$: axial electric field

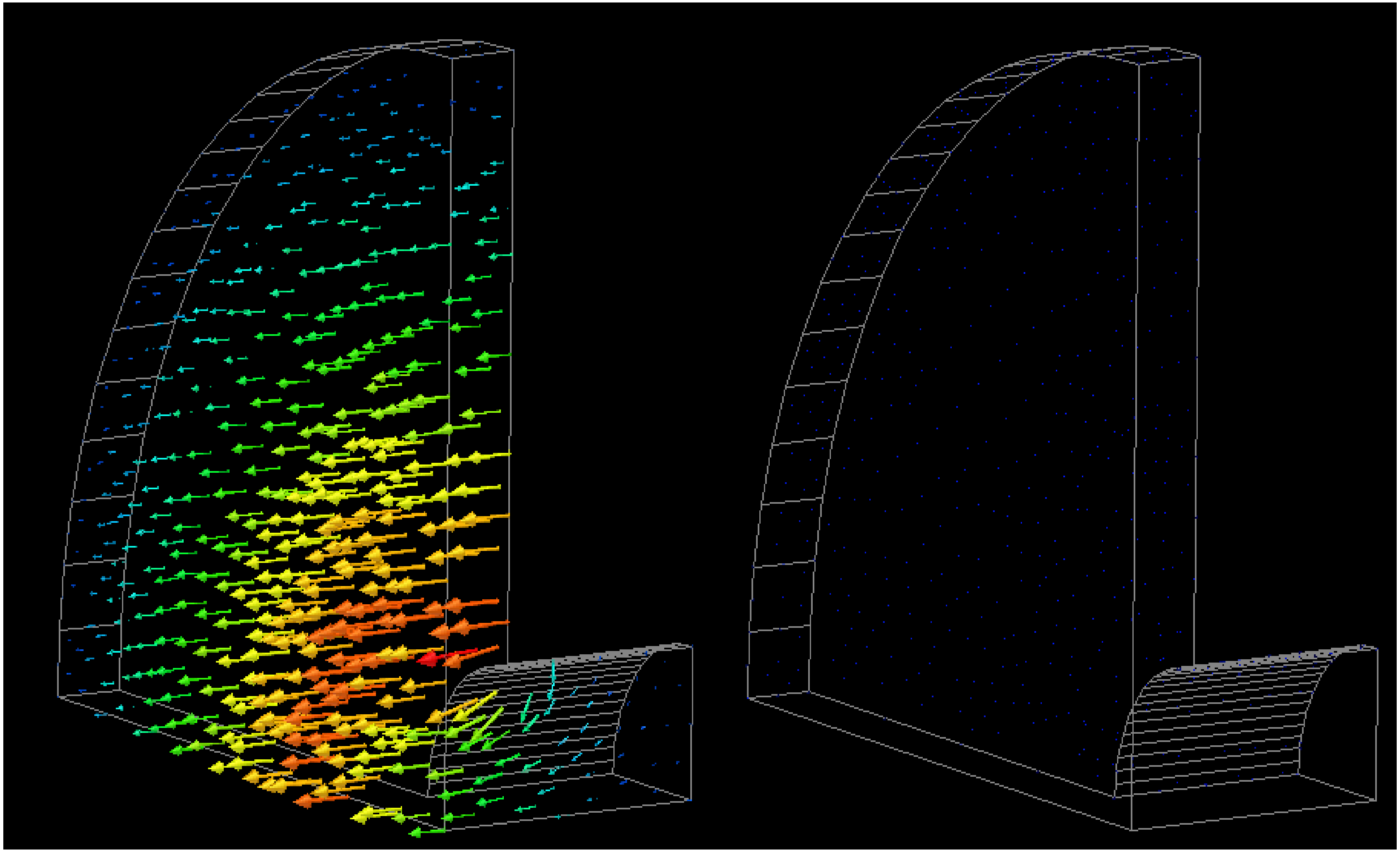
$$\frac{f_c}{\text{GHz}} = \frac{114.74}{a/\text{mm}}$$

TE$_{01}$: lowest losses!

$$\frac{f_c}{\text{GHz}} = \frac{334.74}{a/\text{mm}}$$

Pillbox cavity

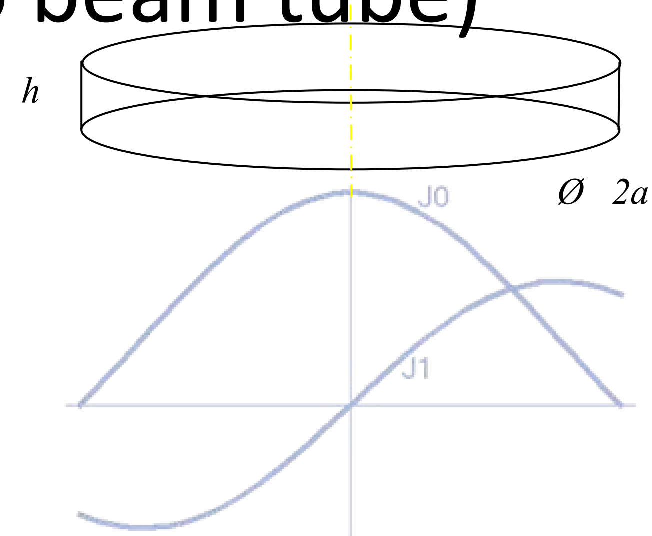
TM$_{010}$-mode (only 1/8 shown)



electric field magnetic field

Pillbox cavity field (w/o beam tube)

$$T(\rho,\varphi)=\sqrt{\frac{1}{\pi}}\frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{\chi_{01}\,J_1\left(\frac{\chi_{01}}{a}\right)} \qquad \chi_{01}=2.40483\ldots$$



The only non-vanishing field components :

$$E_z=\frac{1}{j\omega\varepsilon_0}\frac{\chi_{01}}{a}\sqrt{\frac{1}{\pi}}\frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{a\,J_1\left(\frac{\chi_{01}}{a}\right)}$$

$$B_\varphi=\mu_0\sqrt{\frac{1}{\pi}}\frac{J_1\left(\frac{\chi_{01}\rho}{a}\right)}{a\,J_1\left(\frac{\chi_{01}}{a}\right)}$$

for later:

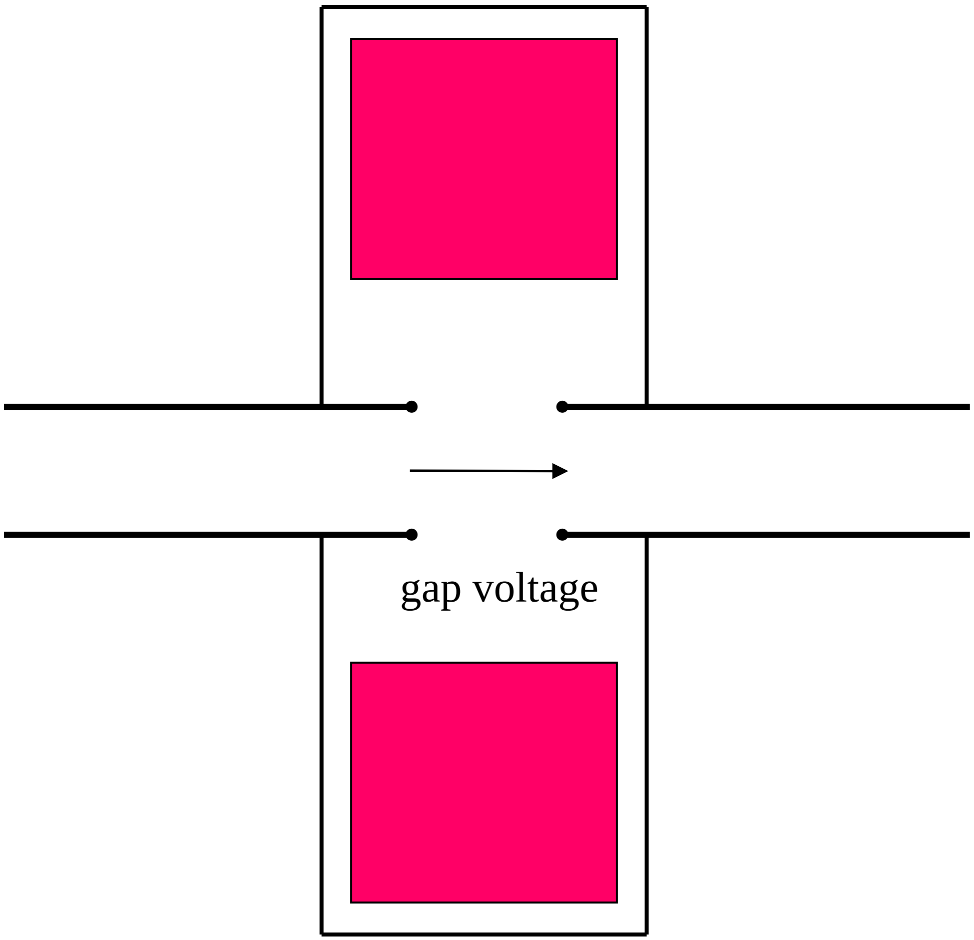
$$\omega_0\big|_{pillbox}=\frac{\chi_{01}\,c}{a} \qquad \eta=\sqrt{\frac{\mu_0}{\varepsilon_0}}=377\,\Omega$$

$$Q\big|_{pillbox}=\frac{\sqrt{2a\,\eta\sigma\chi_{01}}}{2\left(1+\frac{a}{h}\right)}$$

$$\frac{R}{Q}\bigg|_{pillbox}=\frac{4\eta}{\chi_{01}{}^{3}\pi\,J_1^2\left(\chi_{01}\right)}\,\frac{\sin^2\left(\frac{\chi_{01}}{2}\frac{h}{a}\right)}{h/a}$$

ACCELERATING GAP

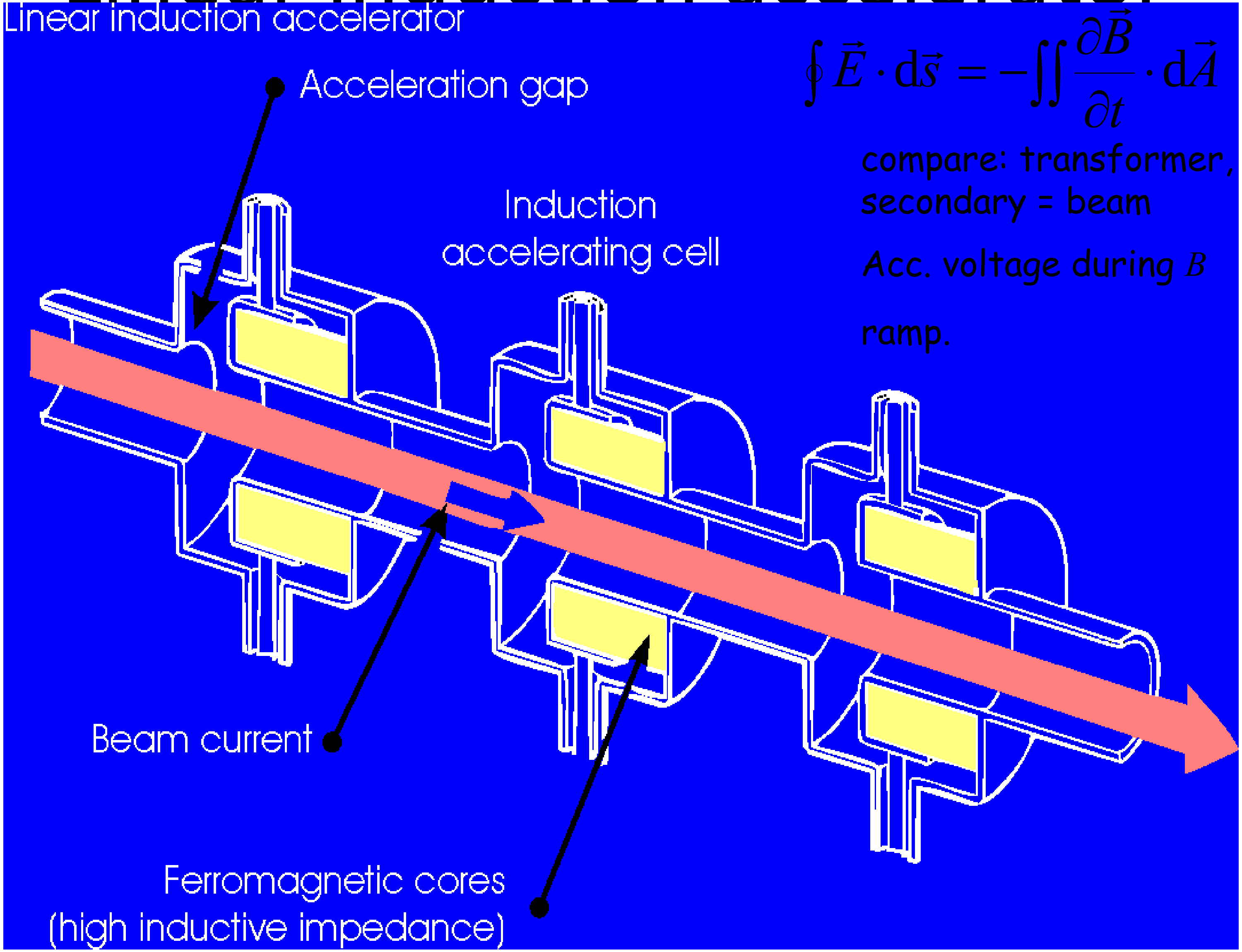
Accelerating gap



- We want a voltage across the gap!
- It cannot be DC, since we want the beam tube on ground potential.
- Use $$\oint \vec{E} \cdot \mathrm{d}\vec{s} = -\iint \frac{\mathrm{d}\vec{B}}{\mathrm{d}t} \cdot \mathrm{d}\vec{A}$$
- The “shield” imposes a
  - upper limit of the voltage pulse duration or – equivalently –
  - a lower limit to the usable frequency.

- The limit can be extended with a material which acts as “open circuit”!
- Materials typically used:
  - ferrites (depending on f-range)
  - magnetic alloys (MA) like Metglas®, Finemet®, Vitrovac®…
- resonantly driven with RF (ferrite loaded cavities) – or with pulses (induction cell)

Linear induction accelerator

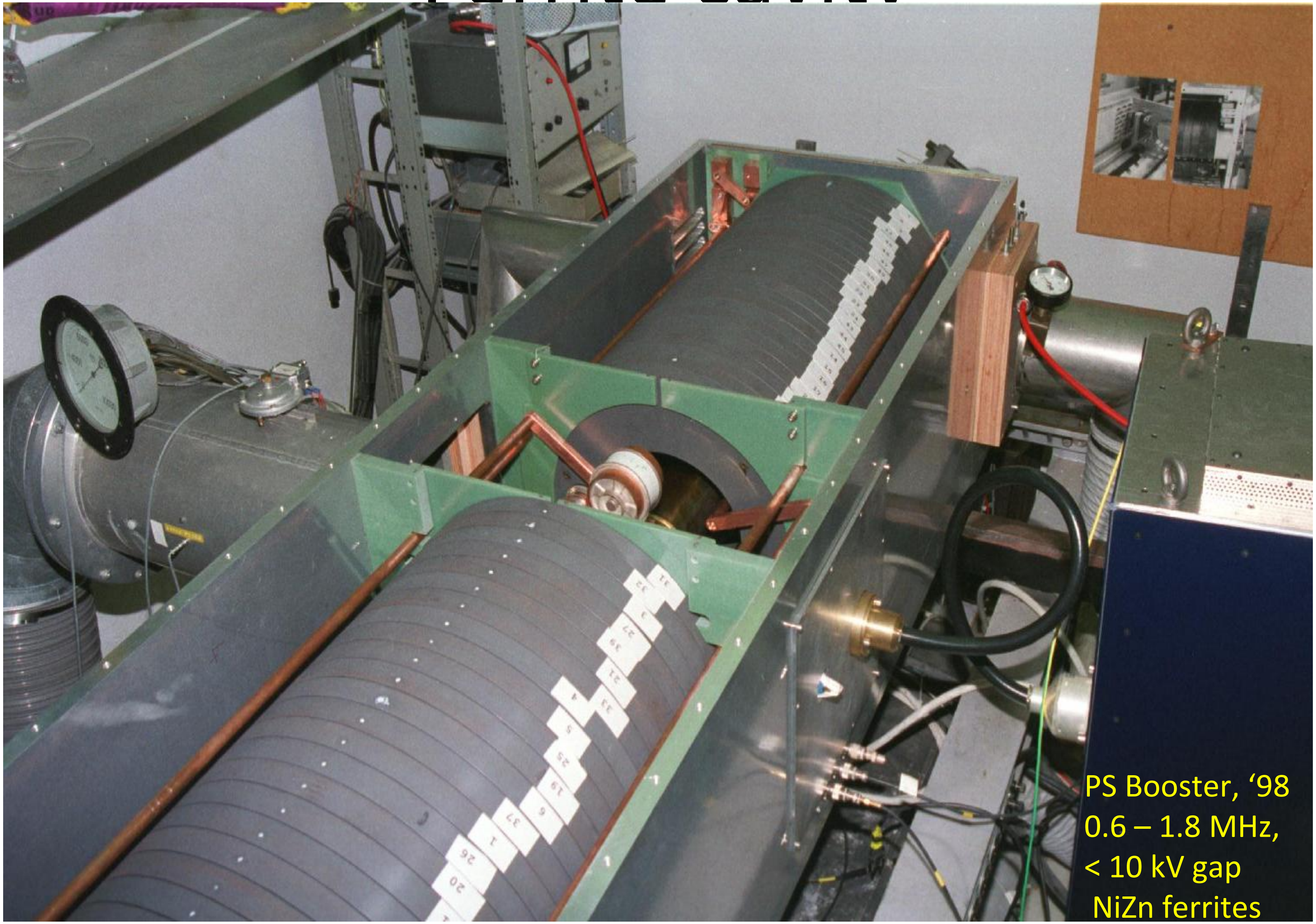


$$\oint \vec{E}\cdot \mathrm{d}\vec{s} = -\iint \frac{\partial \vec{B}}{\partial t}\cdot \mathrm{d}\vec{A}$$

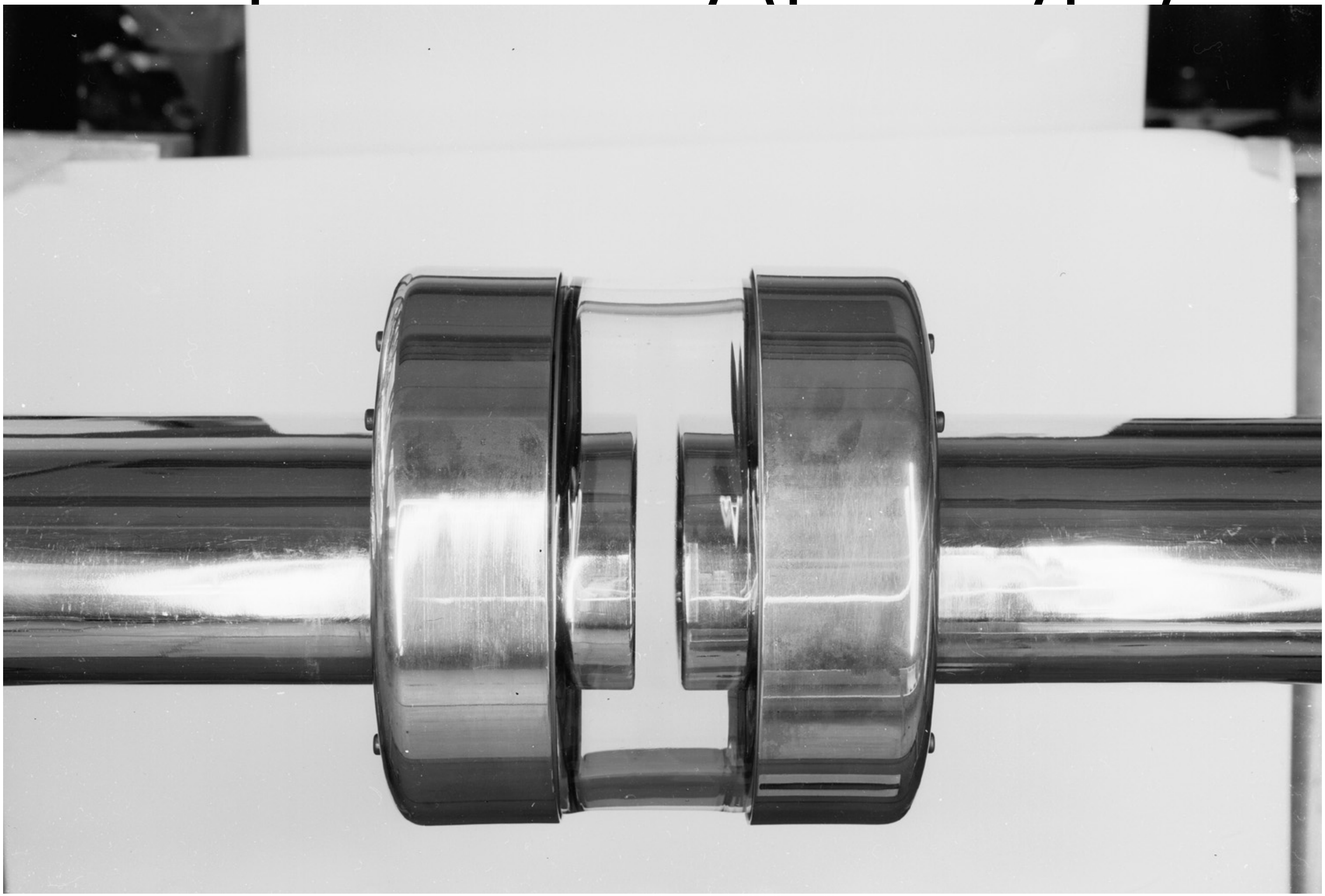
compare: transformer, secondary = beam

Acc. voltage during B ramp.

Ferrite cavity



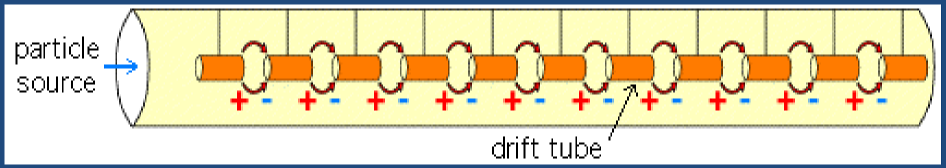
Gap of PS cavity (prototype)



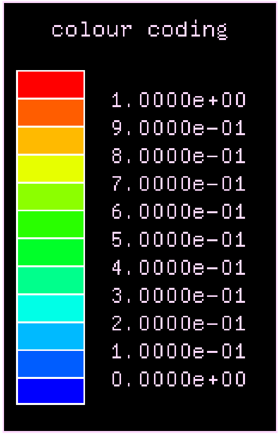
Drift Tube Linac (DTL) – how it works

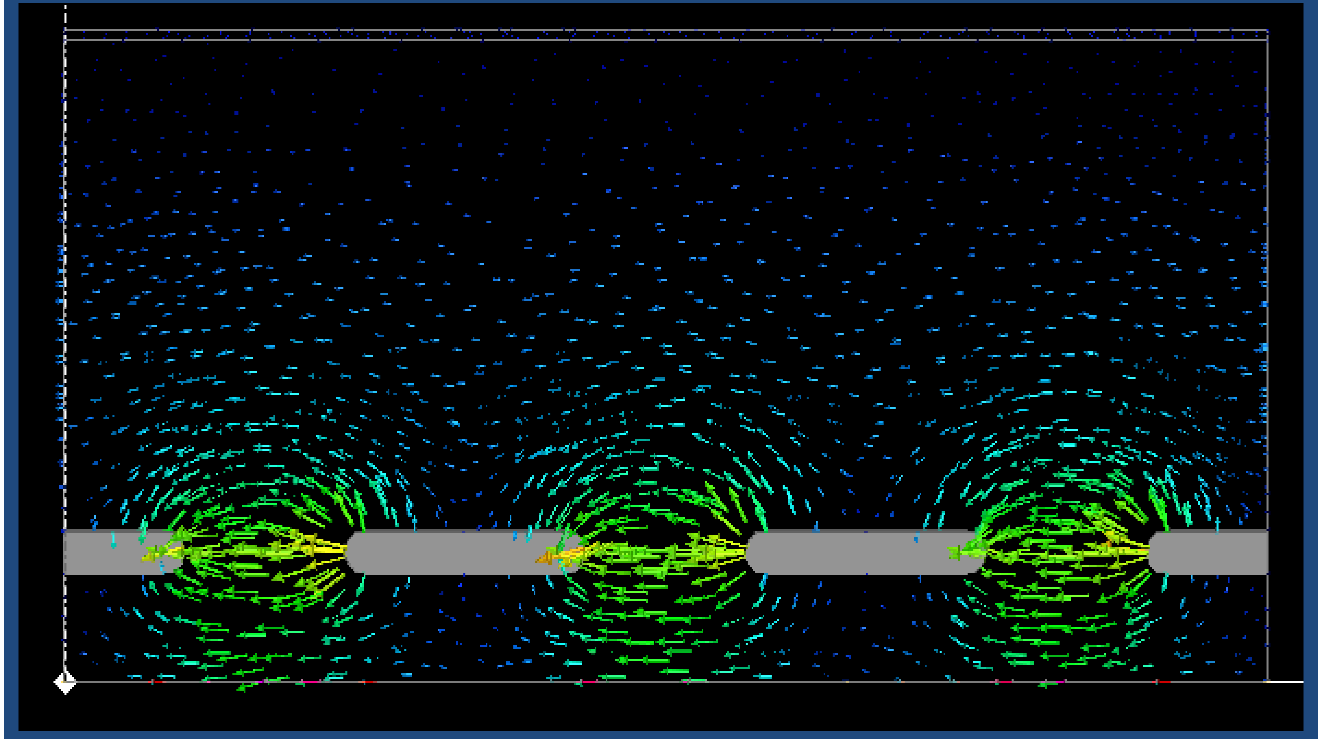
For slow particles !
E.g. protons @ few MeV

The drift tube lengths
can easily be adapted.

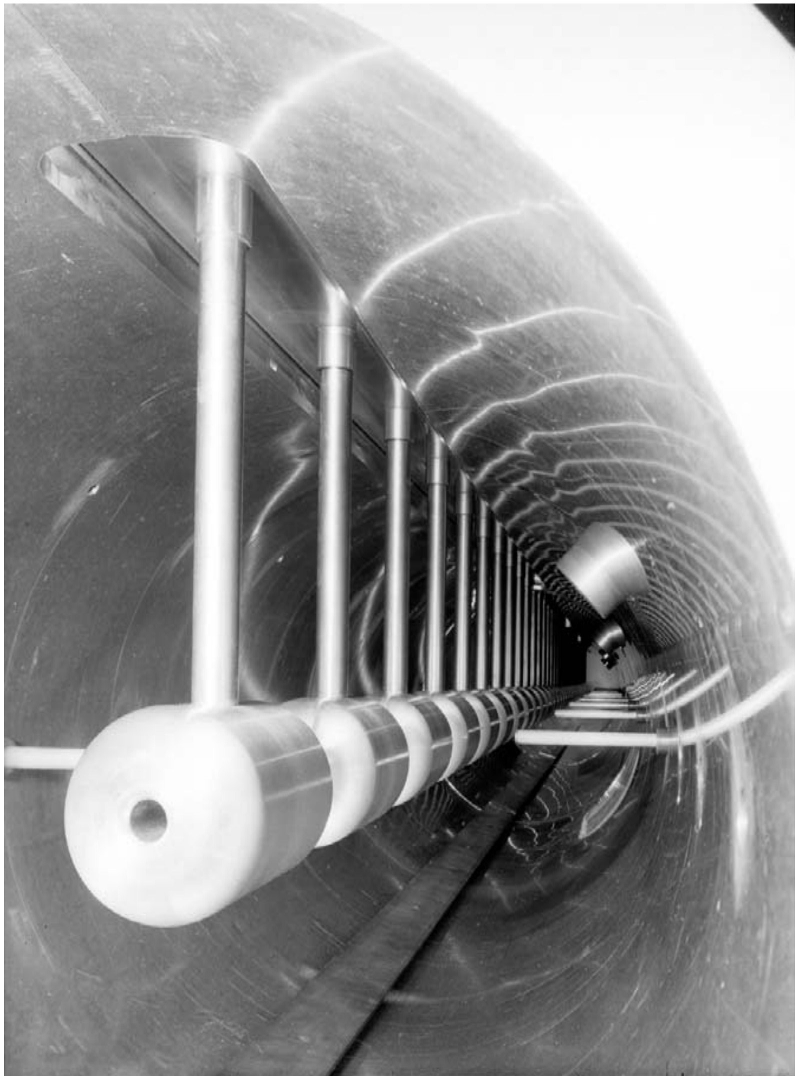


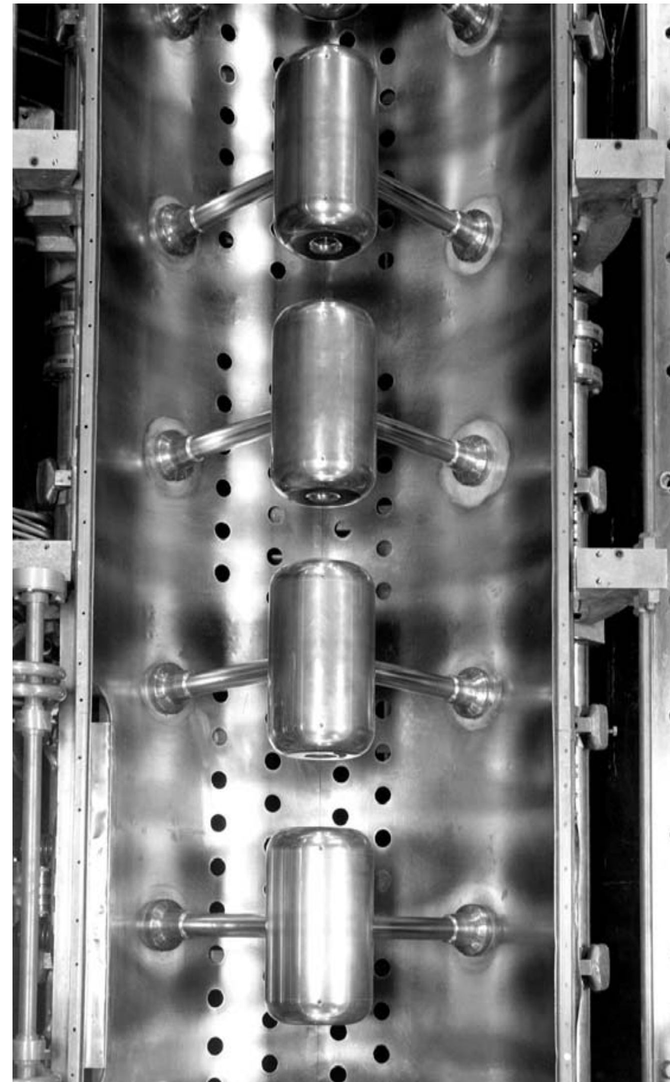
electric field





Drift tube linac – practical implementations





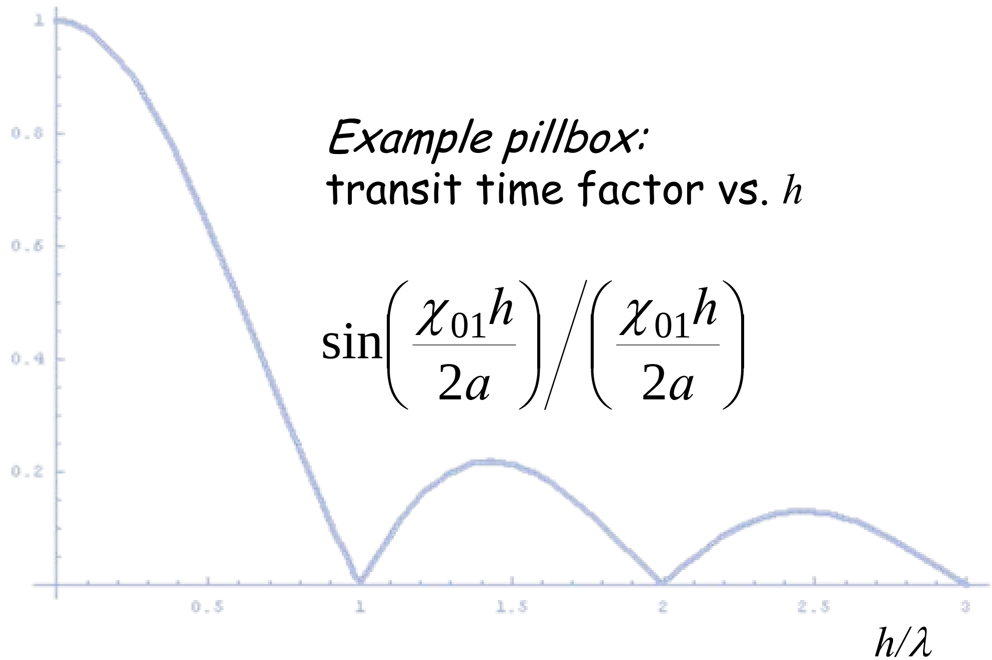
Transit time factor

If the gap is small, the voltage $\int E_z \mathrm{d}z$ is small.

If the gap large, the RF field varies notably while the particle passes.

Define the accelerating voltage $V_{gap} = \left| \int E_z e^{\mathrm{j}\frac{\omega}{c}z} \mathrm{d}z \right|$

Transit time factor $\dfrac{\left| \int E_z e^{\mathrm{j}\frac{\omega}{c}z} \mathrm{d}z \right|}{\left| \int E_z \mathrm{d}z \right|}$



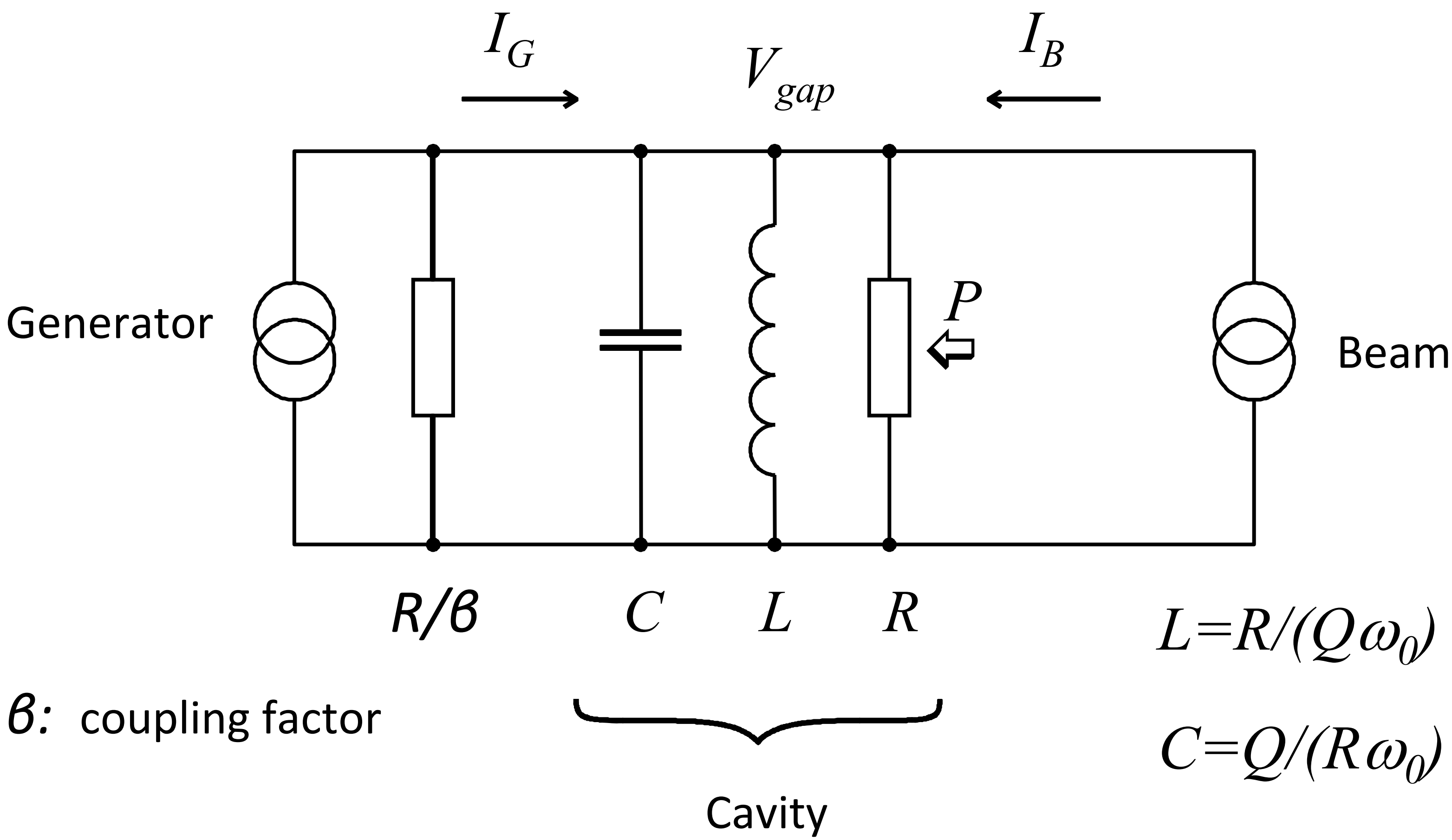
Example pillbox:
transit time factor vs. h

$$\sin\left(\frac{\chi_{01}h}{2a}\right) \Big/ \left(\frac{\chi_{01}h}{2a}\right)$$

CHARACTERIZING A CAVITY

Cavity resonator – equivalent circuit

Simplification: single mode



$L=R/(Q\omega_0)$

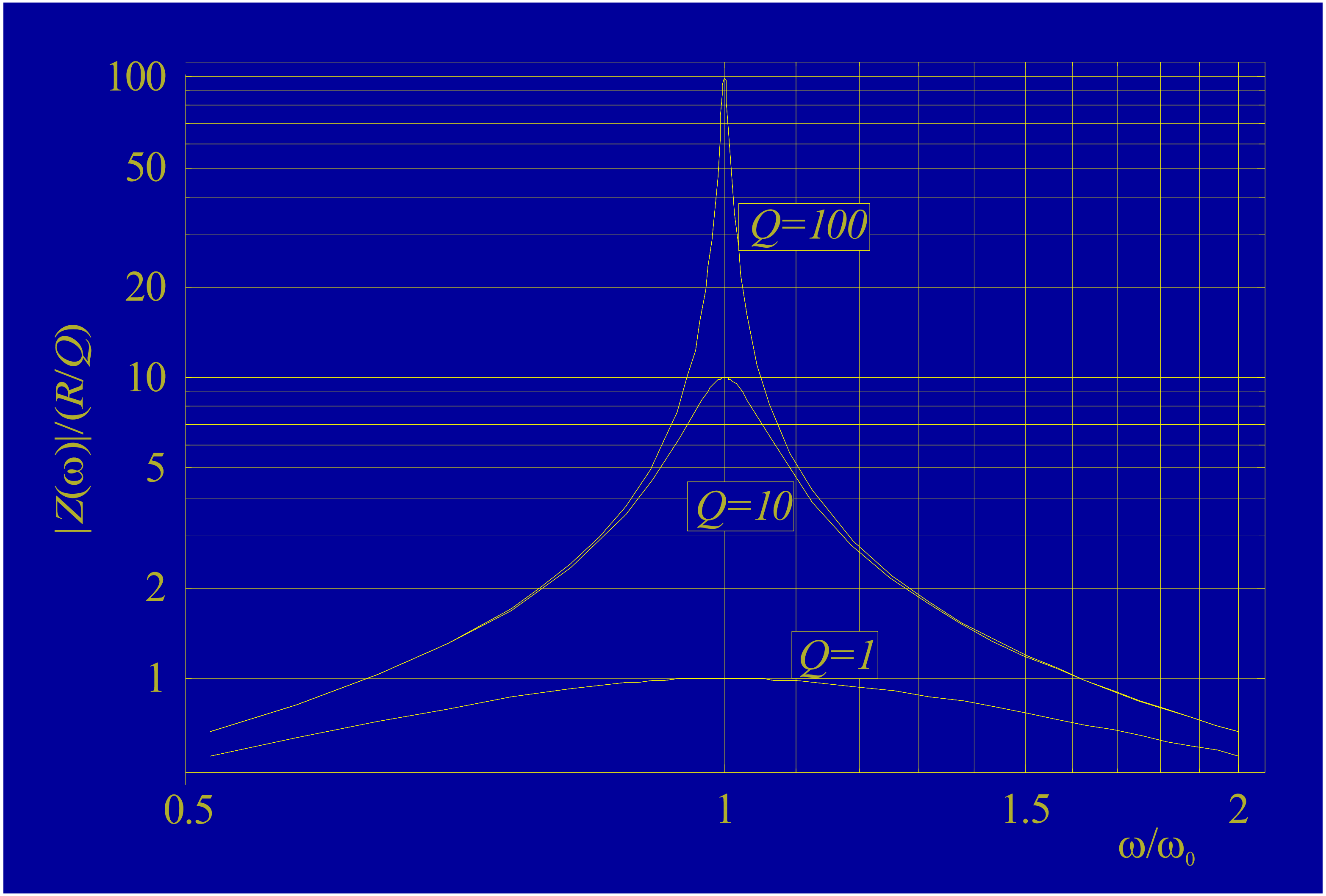
$C=Q/(R\omega_0)$

β: coupling factor

R: Shunt impedance

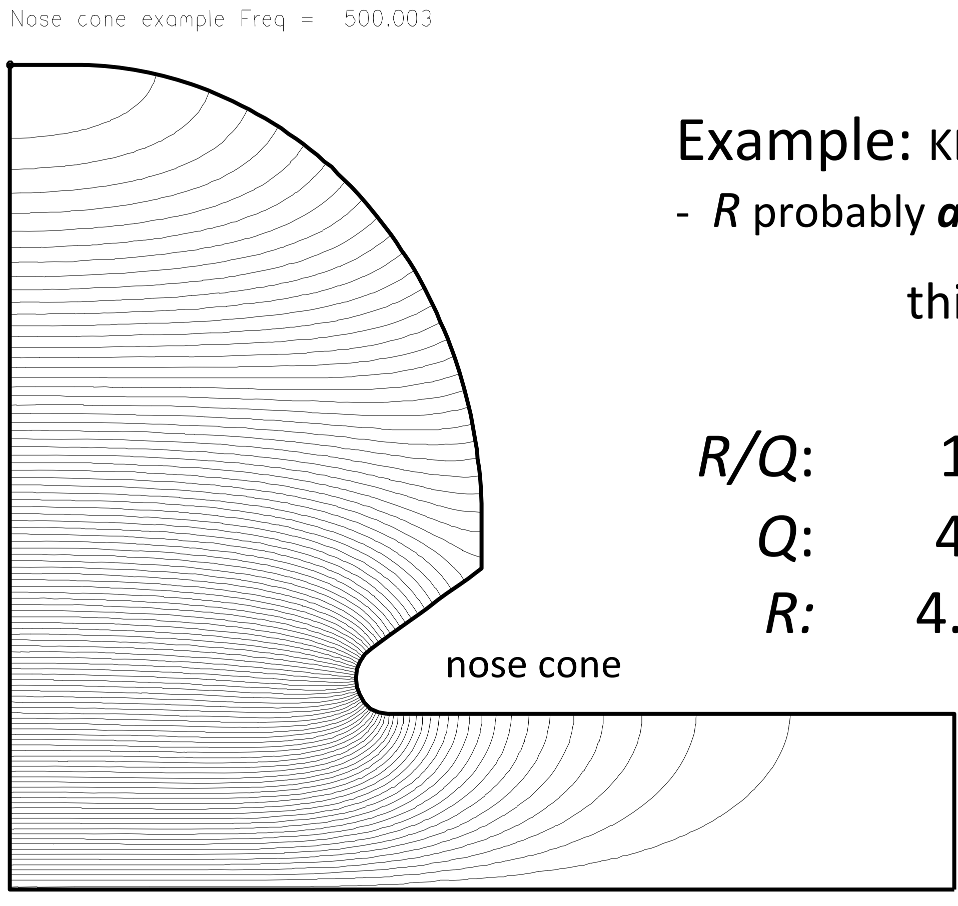
$\sqrt{L/C}$: R-upon-Q

Resonance



Reentrant cavity

Nose cones increase transit time factor, round outer shape minimizes losses.



Example: KEK photon factory 500 MHz

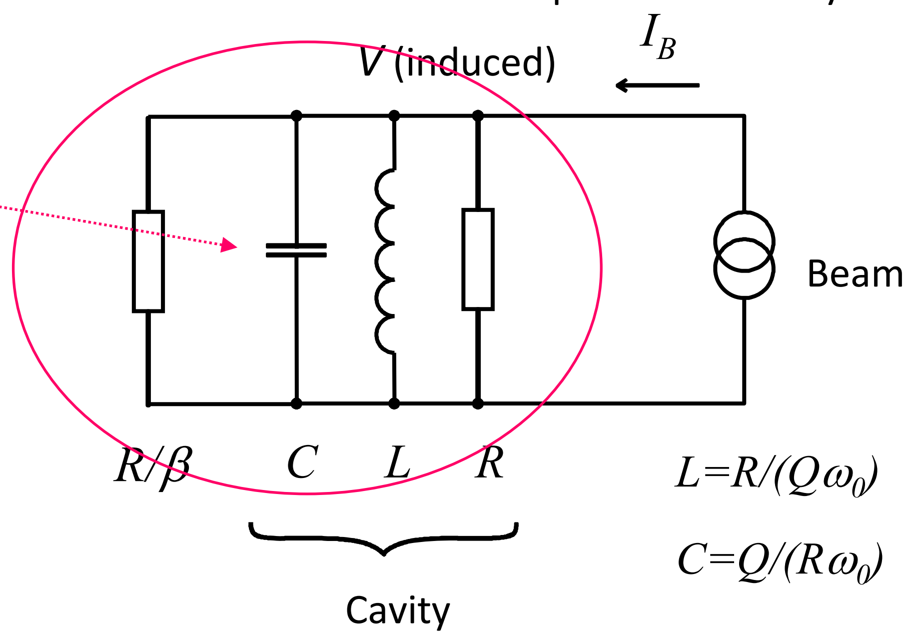
- *R* probably ***as good as it gets*** -

	this cavity	optimized pillbox
R/Q:	111 Ω	107.5 Ω
Q:	44270	41630
R:	4.9 MΩ	4.47 MΩ

Loss factor

$$k_{loss} = \frac{\omega_0}{2}\frac{R}{Q} = \frac{\left|V_{gap}\right|^2}{4W} = \frac{1}{2C}$$

Impedance seen by the beam



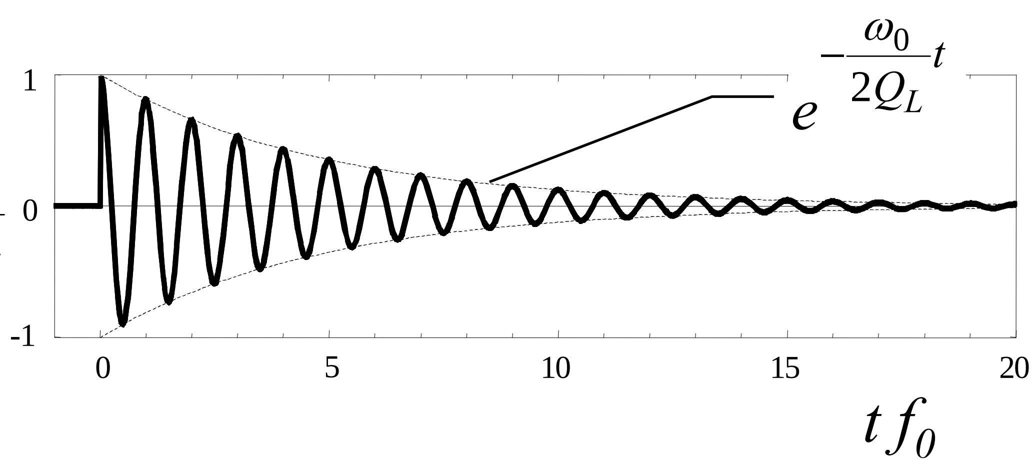
$L=R/(Q\omega_0)$

$C=Q/(R\omega_0)$

Energy deposited by a single charge *q*: $k_{loss}\, q^2$

Voltage induced by a single charge *q:*

$$\frac{V_{gap}}{2\, k_{loss}\, q}$$



Summary: relations V_{gap}, W, P_{loss}

gap voltage

$$V_{gap}$$

$$\frac{R}{Q} = \frac{\left|V_{gap}\right|^2}{2\,\omega_0 W}$$

$$k_{loss} = \frac{\omega_0}{2}\frac{R}{Q} = \frac{\left|V_{gap}\right|^2}{4\,W}$$

$$R_{shunt} = \frac{\left|V_{gap}\right|^2}{2\,P_{loss}}$$

Energy stored inside the cavity

$$W$$

Power lost in the cavity walls

$$P_{loss}$$

$$Q = \frac{\omega_0 W}{P_{loss}}$$

Beam loading – RF to beam efficiency

- The beam current "loads" the generator, in the equivalent circuit this appears as a resistance in parallel to the shunt impedance.
- If the generator is matched to the unloaded cavity, beam loading will cause the accelerating voltage to decrease.
- The power absorbed by the beam is $-\frac{1}{2}\mathrm{Re}\left\{ V_{gap}\ I_B^* \right\}$
  the power loss $P = \frac{\left|V_{gap}\right|^2}{2\,R}$.
- For high efficiency, beam loading shall be high.
- The RF to beam efficiency is $\eta = \frac{1}{1 + \frac{V_{gap}}{R\left|I_B\right|}} = \frac{\left|I_B\right|}{\left|I_G\right|}$.

Characterizing cavities

- Resonance frequency

$$\omega_0 = \frac{1}{\sqrt{L \cdot C}}$$

- Transit time factor

  field varies while particle is traversing the gap

$$\frac{\left| \int E_z \, e^{\mathrm{j}\frac{\omega}{c}z} \mathrm{d}z \right|}{\left| \int E_z \mathrm{d}z \right|}$$

Circuit definition

Linac definition

- Shunt impedance

  gap voltage – power relation

$$\left| V_{gap} \right|^2 = 2\, R_{shunt}\, P_{loss}$$

$$\left| V_{gap} \right|^2 = R_{shunt}\, P_{loss}$$

- Q factor

$$\omega_0 W = Q P_{loss}$$

- R/Q

  independent of losses – only geometry!

$$\frac{R}{Q} = \frac{\left| V_{gap} \right|^2}{2\,\omega_0\, W} = \sqrt{\frac{L}{C}}$$

$$\frac{R}{Q} = \frac{\left| V_{gap} \right|^2}{\omega_0\, W}$$

- loss factor

$$k_{loss} = \frac{\omega_0}{2}\frac{R}{Q} = \frac{\left| V_{gap} \right|^2}{4\, W}$$

$$k_{loss} = \frac{\omega_0}{4}\frac{R}{Q} = \frac{\left| V_{gap} \right|^2}{4\, W}$$

Example Pillbox:

$$\omega_0\Big|_{pillbox} = \frac{\chi_{01}\, c}{a}$$

$$\chi_{01} = 2.4048$$

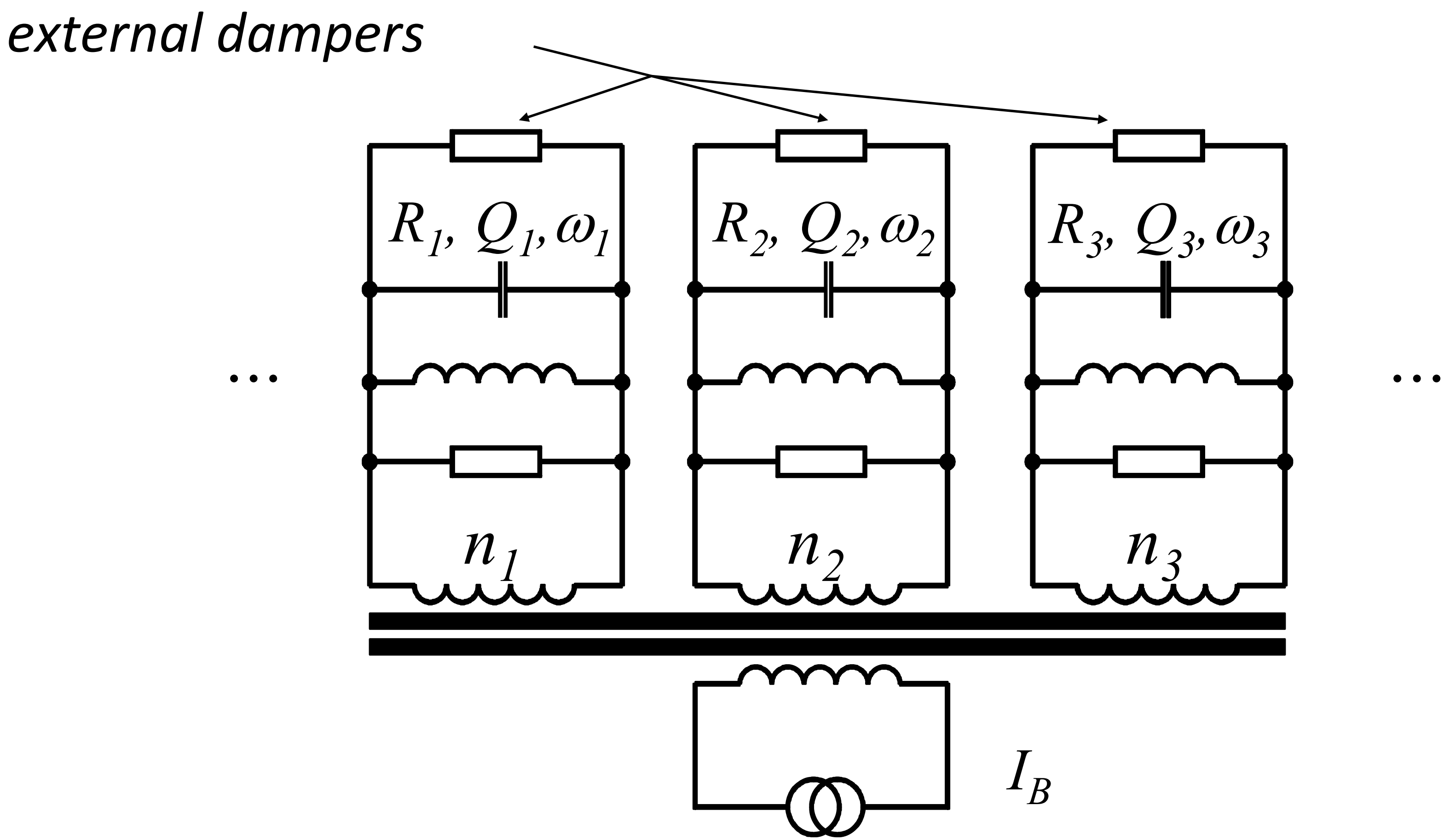
$$Q\Big|_{pillbox} = \frac{\sqrt{2a\eta\sigma\chi_{01}}}{2\left(1+\frac{a}{h}\right)}$$

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega$$

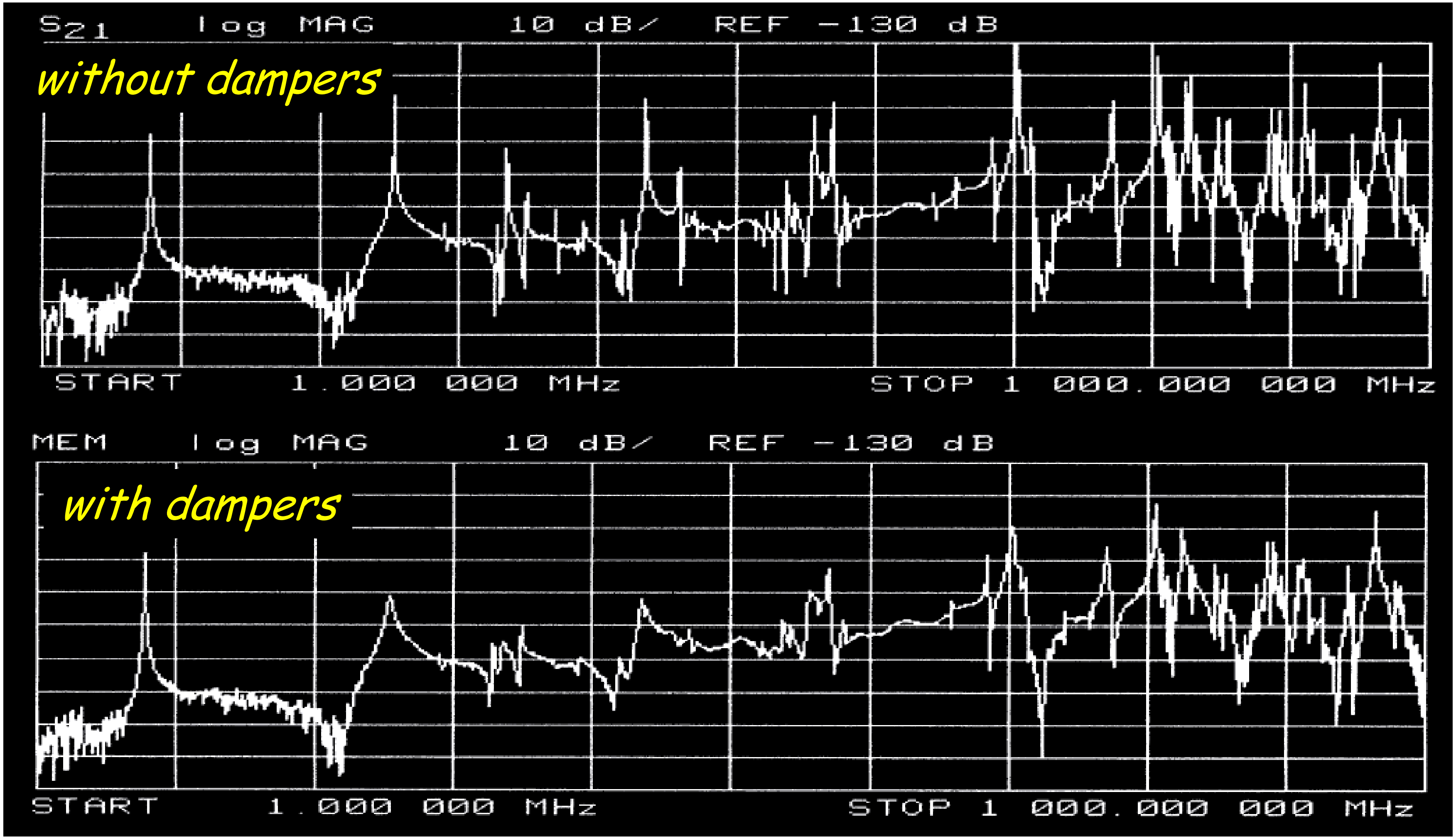
$$\sigma_{\mathrm{Cu}} = 5.8\cdot 10^7\,\mathrm{S/m}$$

$$\frac{R}{Q}\Big|_{pillbox} = \frac{4\eta}{\chi_{01}{}^3\pi\,\mathrm{J}_1^2(\chi_{01})}\,\frac{\sin^2(\frac{\chi_{01}}{2}\frac{h}{a})}{h/a}$$

Higher order modes

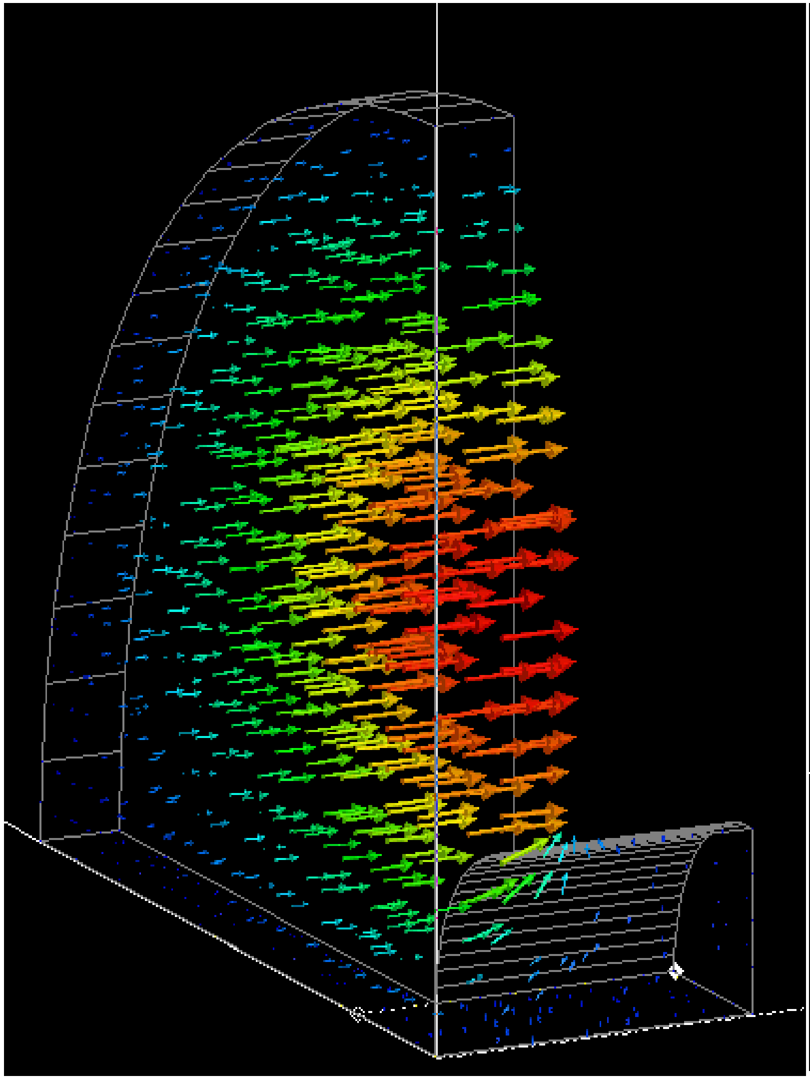


Higher order modes (measured spectrum)

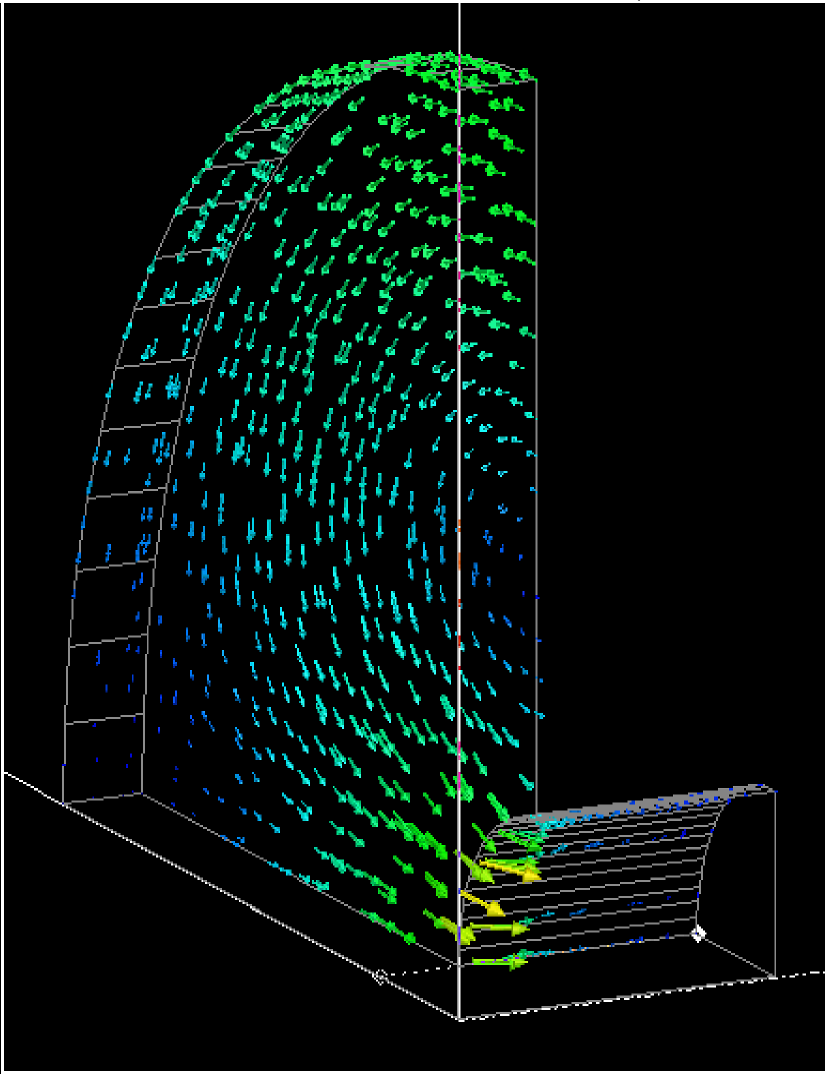


Pillbox: Dipole mode (TM_{110})

(only 1/8 shown)



electric field (@ 0°)



magnetic field (@ 90°)

Panofsky-Wenzel theorem

For particles moving virtually at $v=c$, the integrated transverse force (kick) can be determined from the transverse variation of the integrated longitudinal force!

$$\mathrm{j}\frac{\omega}{c}\vec{F}_{\perp} = \nabla_{\perp} F_{\|}$$

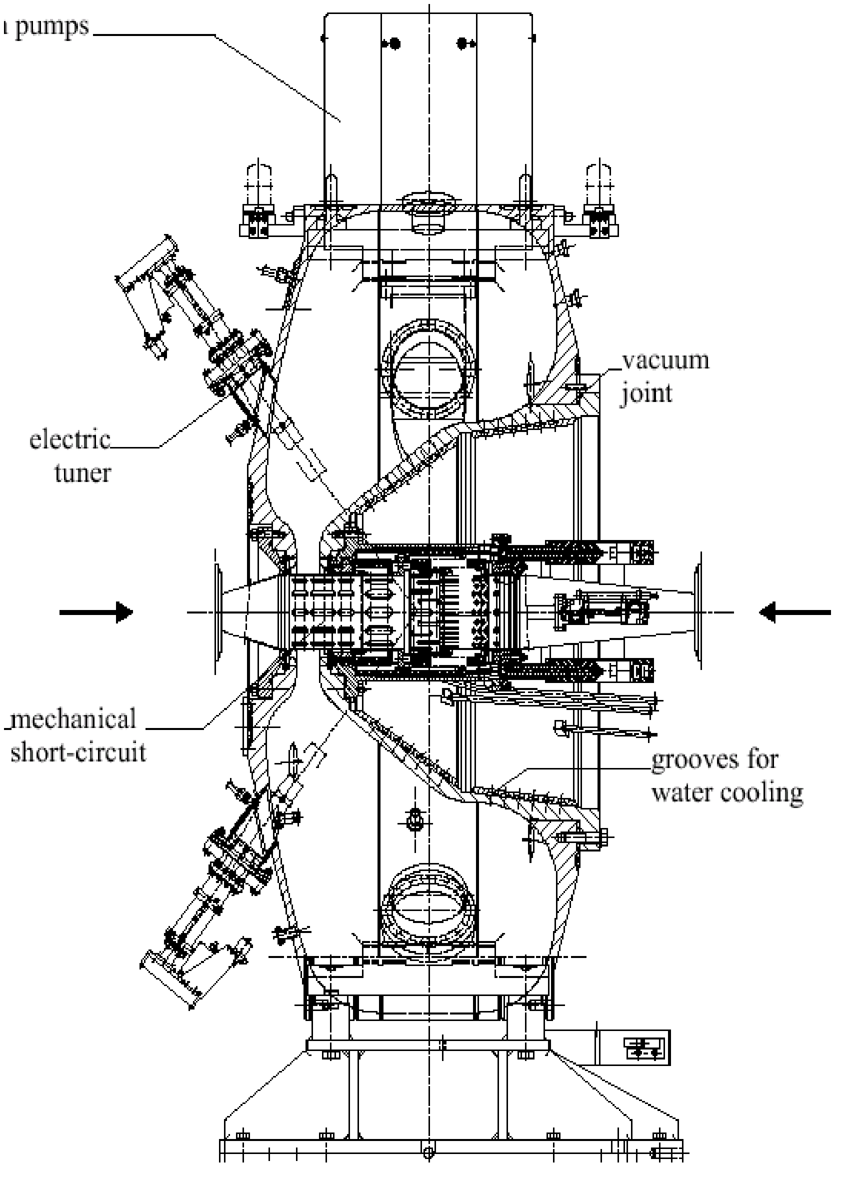
Pure TE modes: No net transverse force !

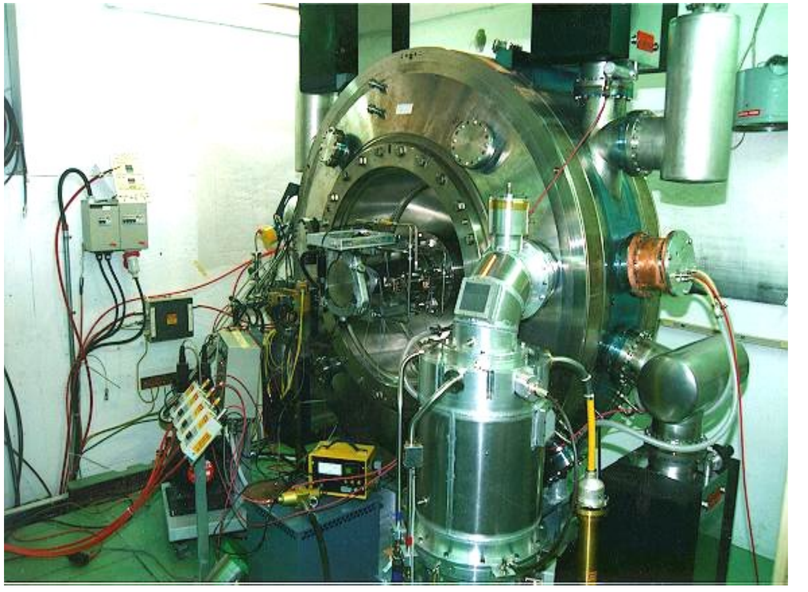
Transverse modes are characterized by

- the transverse impedance in ω-domain
- the transverse loss factor (kick factor) in t-domain !

W.K.H. Panofsky, W.A. Wenzel: "Some Considerations Concerning the Transverse Deflection of Charged Particles in Radio-Frequency Fields", RSI **27**, 1957]

CERN/PS 80 MHz cavity (for LHC)



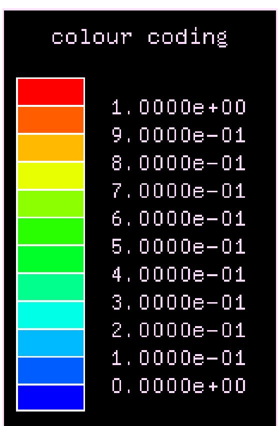


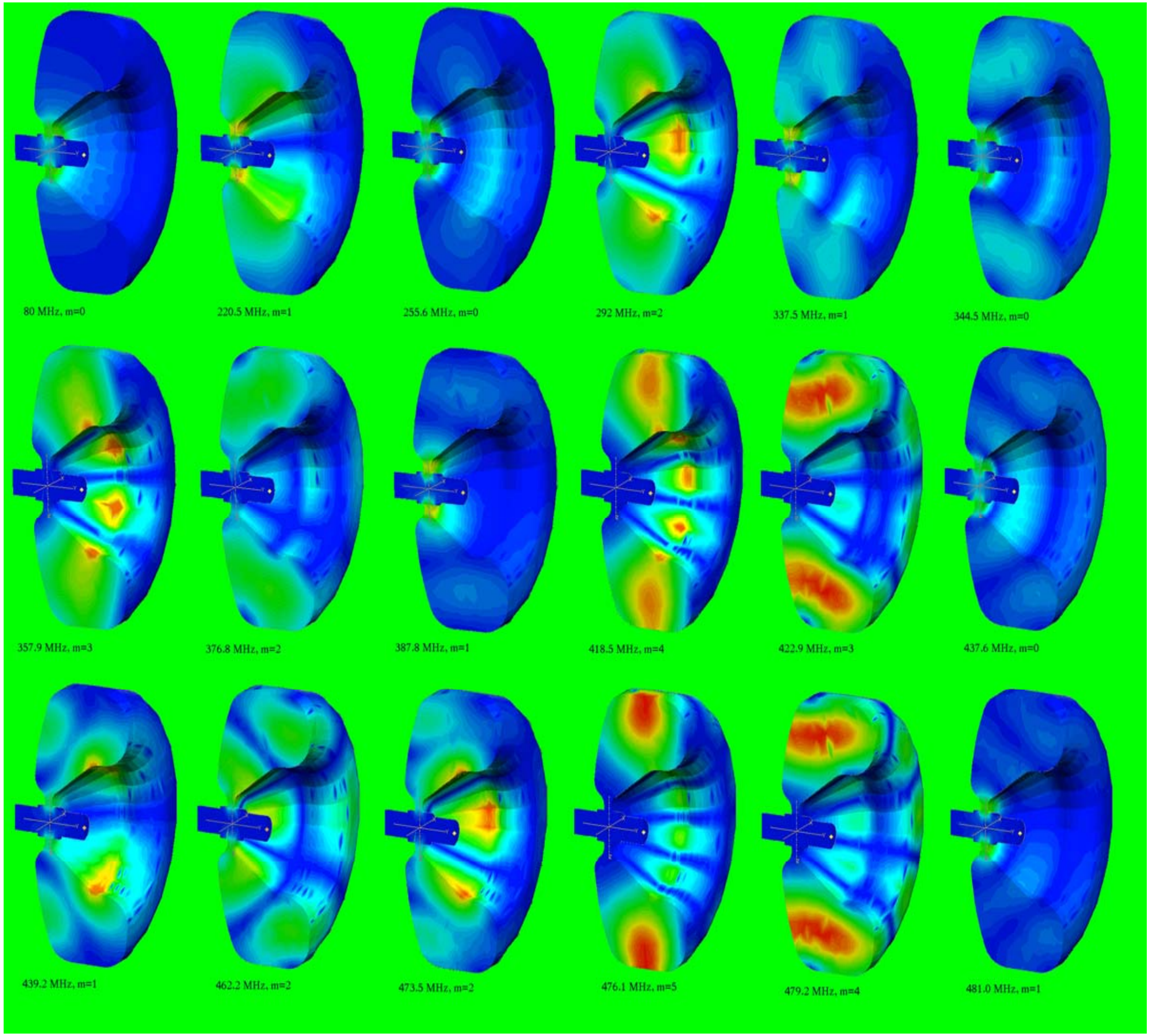
Higher order modes

Example shown: 80 MHz cavity PS for LHC.

Color-coded:

$$\left|\vec{E}\right|$$



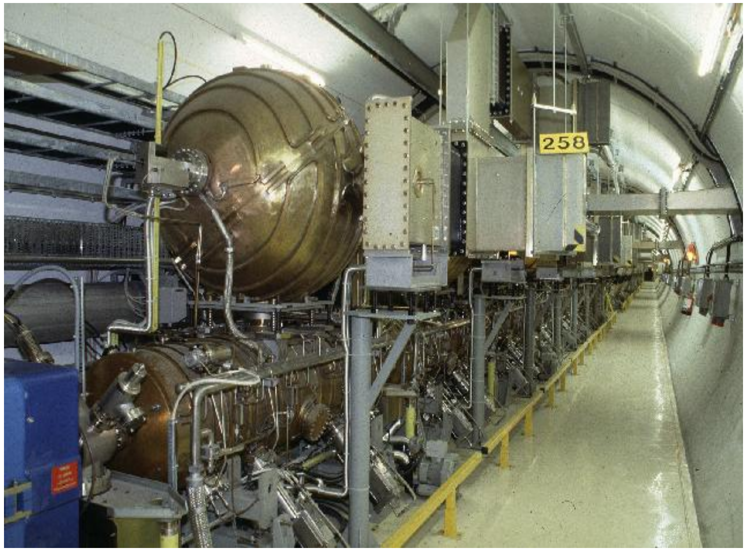


MORE EXAMPLES OF CAVITIES

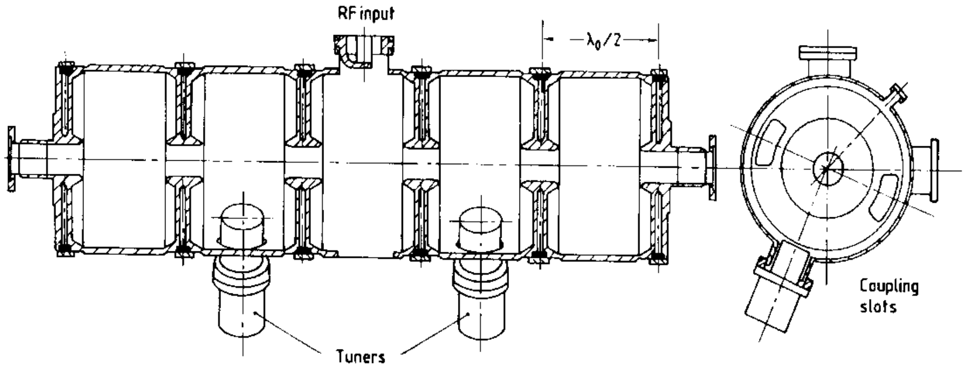
PS 19 MHz cavity (prototype, photo: 1966)

Examples of cavities

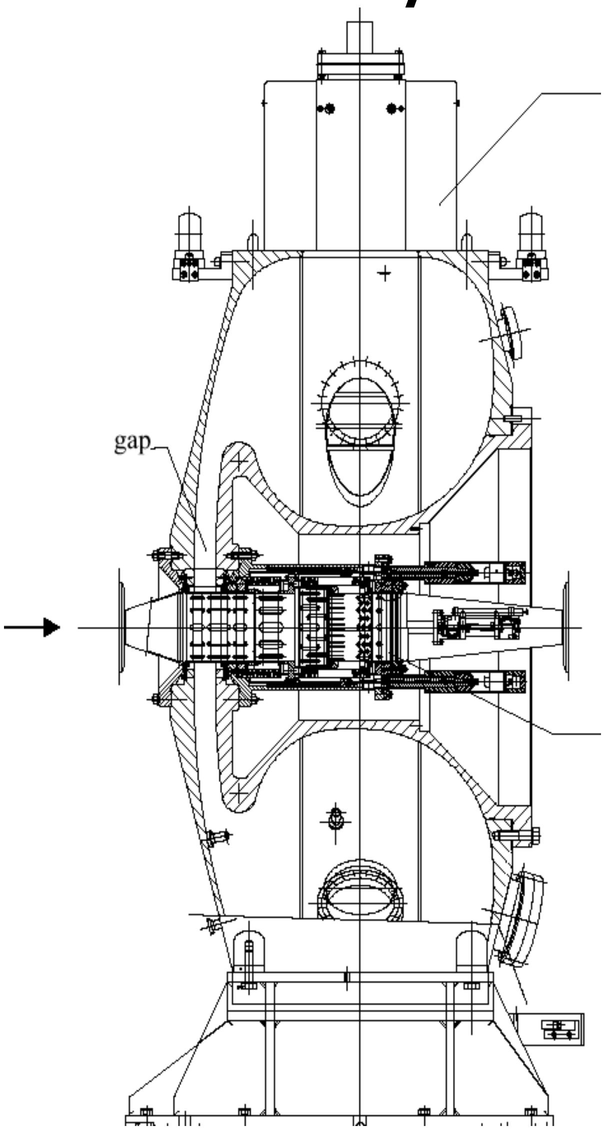
PEP II cavity
476 MHz, single cell,
1 MV gap with 150 kW,
strong HOM damping,

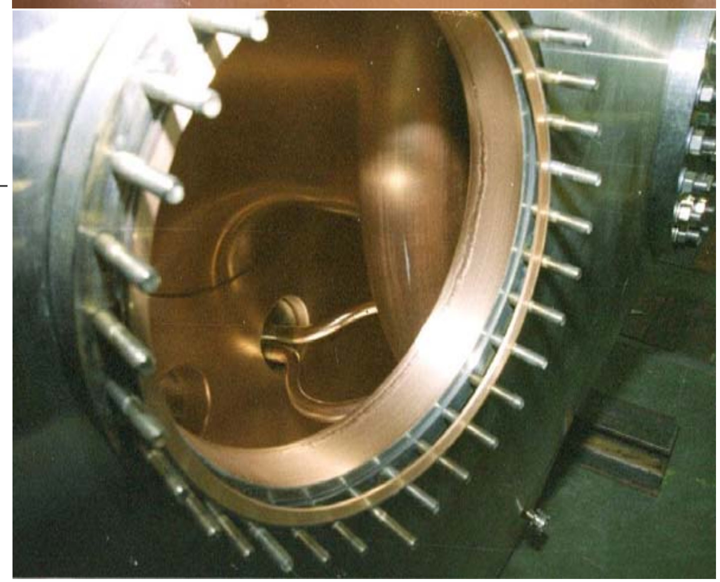


LEP normal-conducting Cu RF cavities,
350 MHz. 5 cell standing wave + spherical
cavity for energy storage, 3 MV



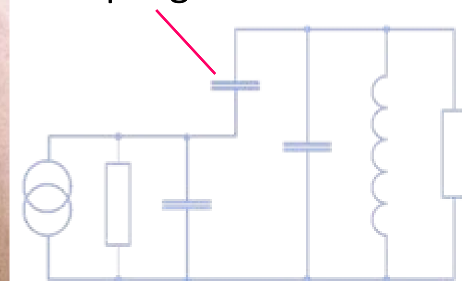
CERN/PS 40 MHz cavity (for LHC)





example for
capacitive coupling

coupling C



cavity

RF POWER SOURCES

RF Power sources

> 200 MHz: Klystrons

Thales TH1801, Multi-Beam Klystron (MBK), 1.3 GHz, 117 kV. Achieved:
48 dB gain, 10 MW peak, 150 kW average, η = 65 %

dB: $$\frac{output\ power}{input\ power} = 10^{4.8}$$

< 1000 MHz: grid tubes

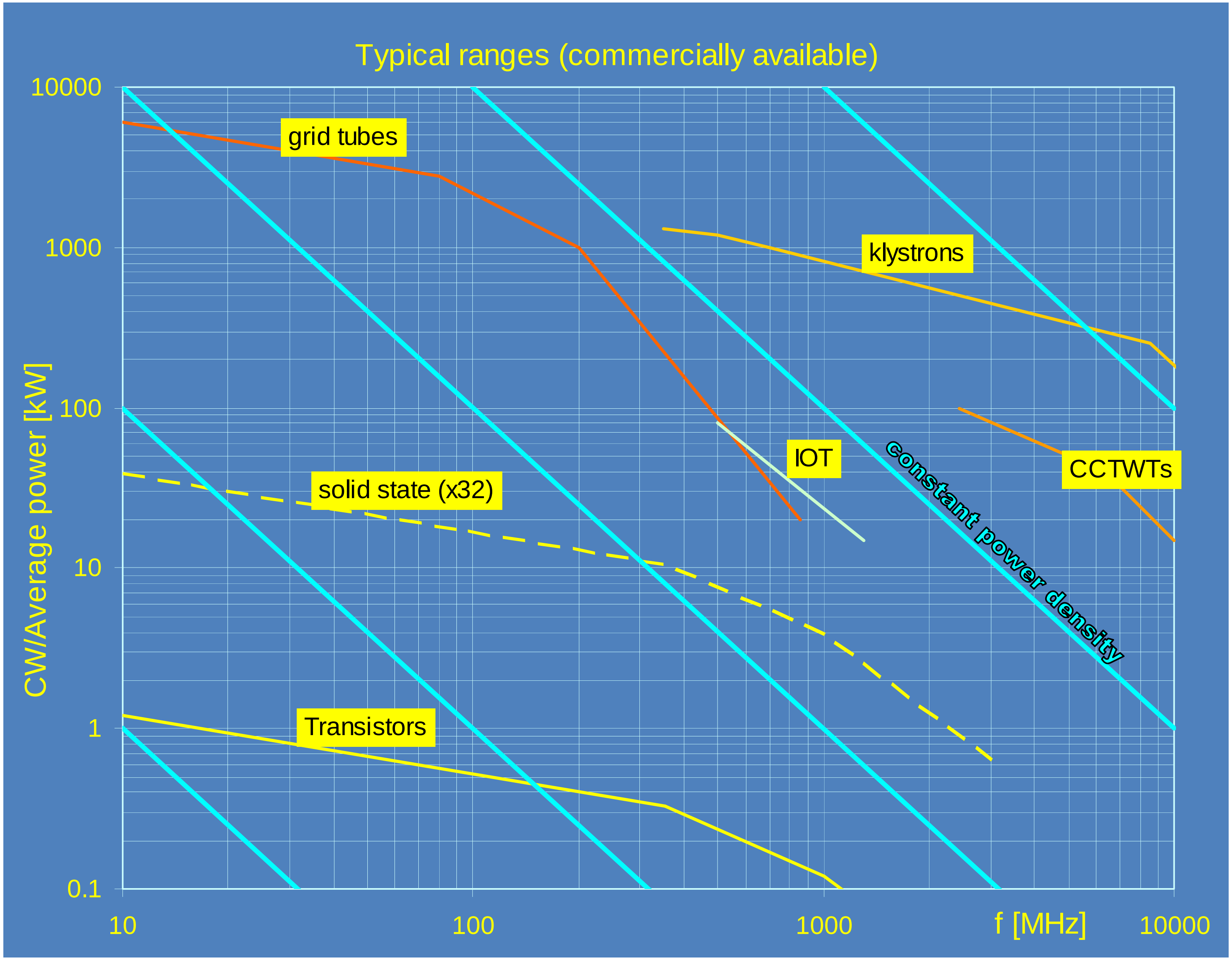
Tetrode

IOT

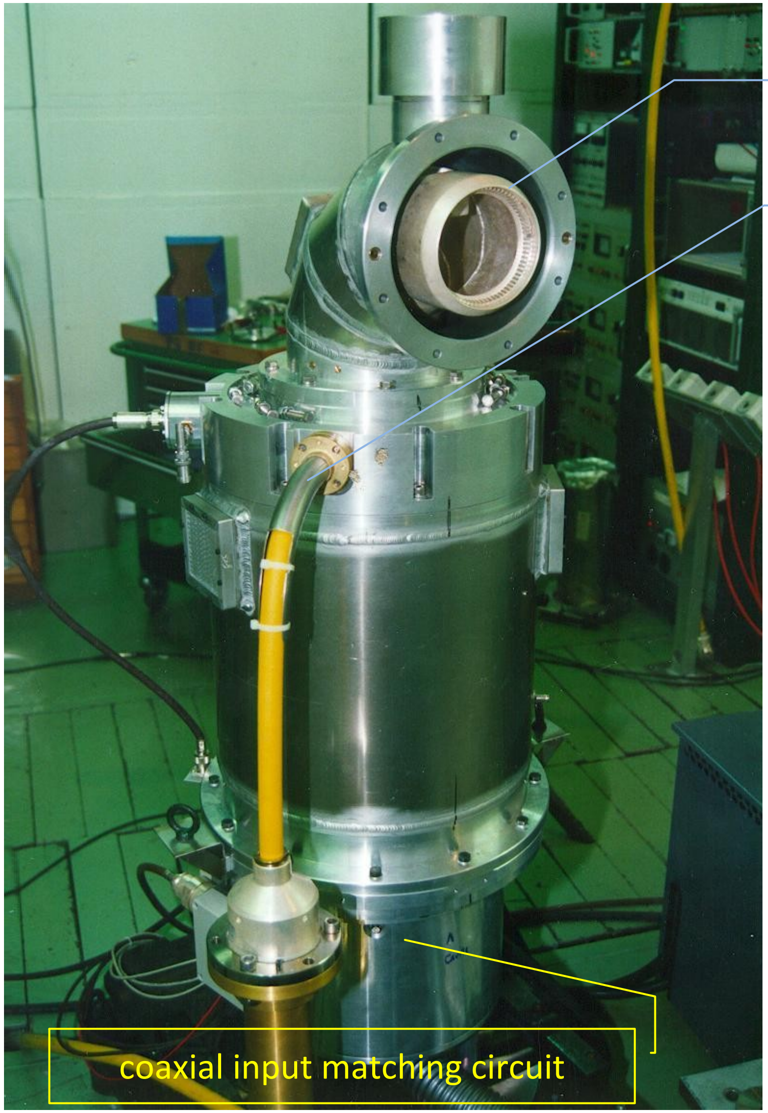
UHF Diacrode

pictures from http://www.thales-electrondevices.com

RF power sources



Example of a tetrode amplifier (80 MHz, CERN/PS)

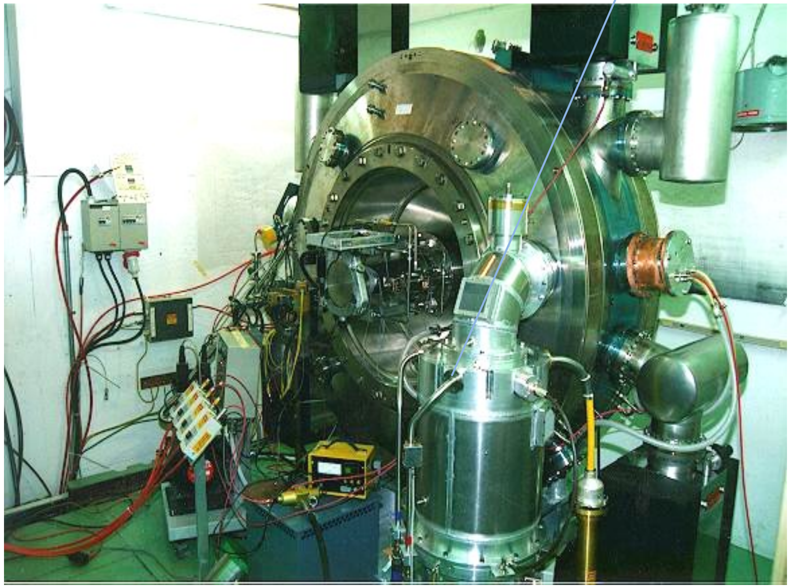


400 kW, with fast RF feedback

18 Ω coaxial output (towards cavity)

22 kV DC anode voltage feed-through with $\lambda/4$ stub

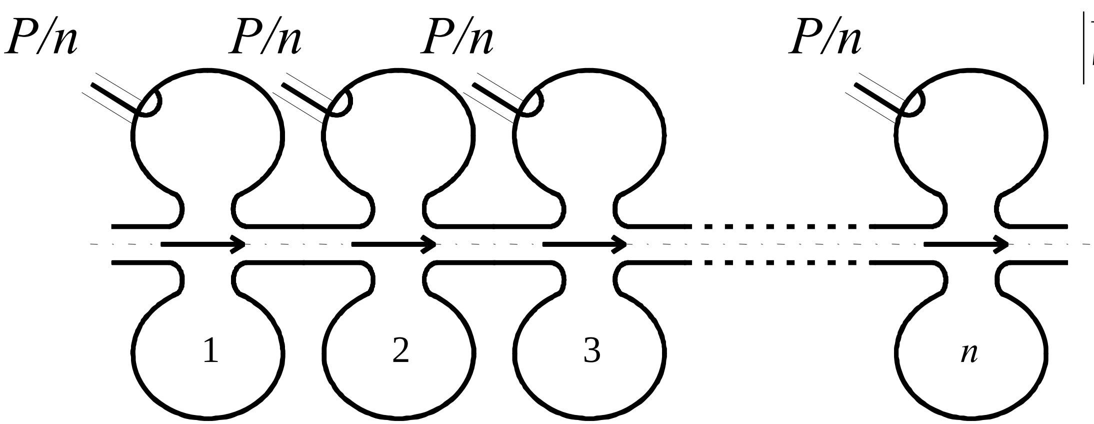
tetrode cooling water feed-throughs



MANY GAPS

What do you gain with many gaps?

- The *R/Q* of a single gap cavity is limited to some 100 **W**.
  Now consider to distribute the available power to *n* identical cavities: each will receive *P/n*, thus produce an accelerating voltage of $\sqrt{2R\,P/n}$.
  The total accelerating voltage thus increased, equivalent to a total equivalent shunt impedance of nR.

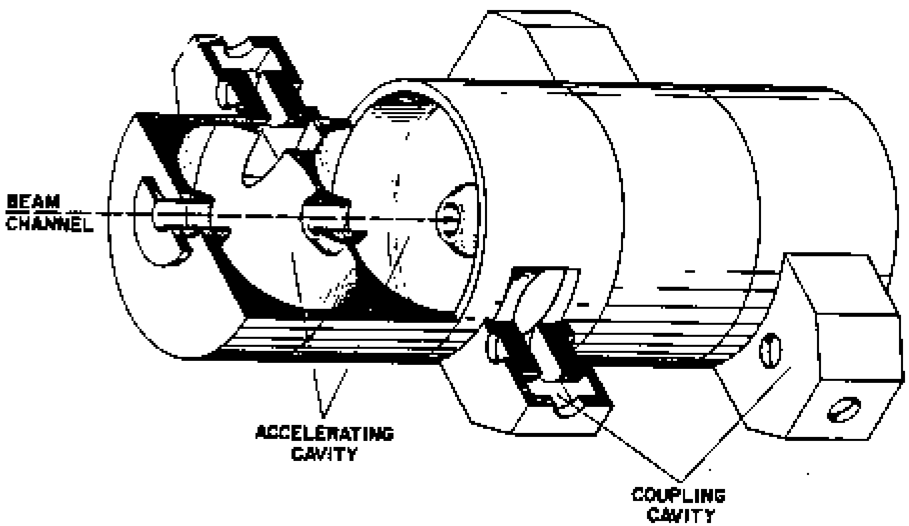


$$\left|V_{acc}\right| = n\sqrt{2R\frac{P}{n}} = \sqrt{2(nR)P}$$

Standing wave multicell cavity

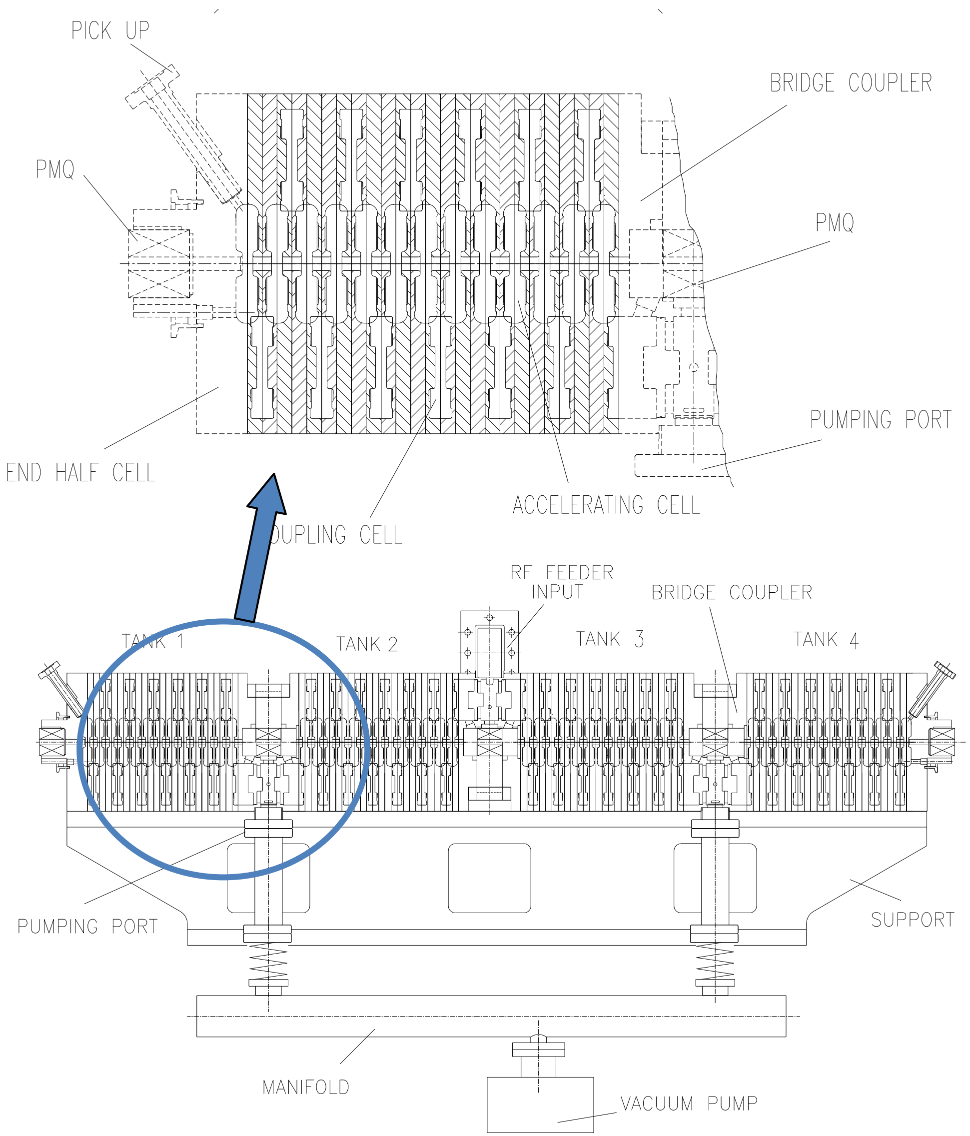
- Instead of distributing the power from the amplifier, one might as well couple the cavities, such that the power automatically distributes, or have a cavity with many gaps (e.g. drift tube linac).

- Coupled cavity accelerating structure (side coupled)



- The phase relation between gaps is important!

Example of Side Coupled Structure



LIBO (= Linac Booster)

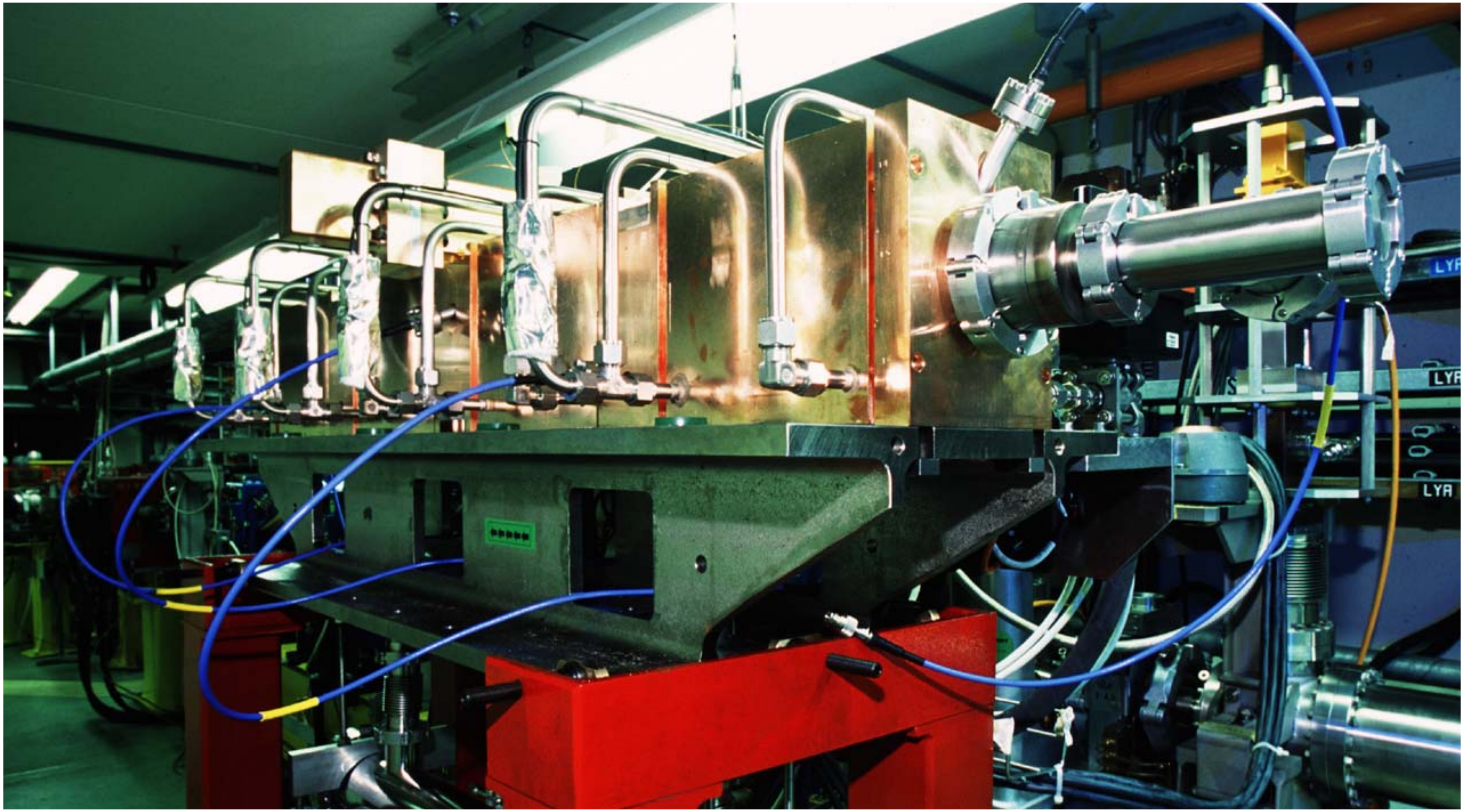
A 3 GHz Side Coupled Structure to accelerate protons out of cyclotrons from 62 MeV to 200 MeV

Medical application: treatment of tumours.

Prototype of Module 1 built at CERN (2000)

Collaboration CERN/INFN/ Tera Foundation

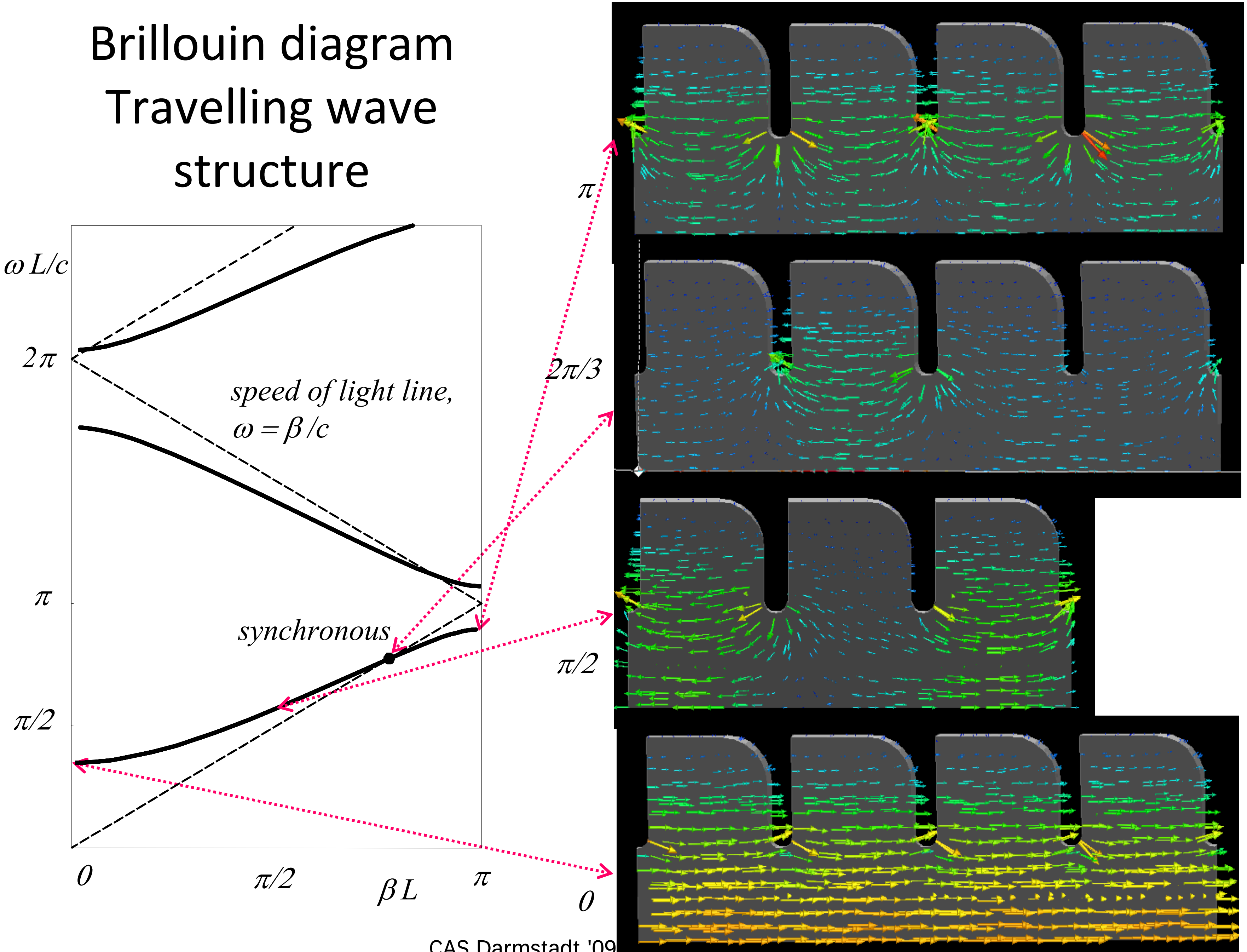
LIBO prototype



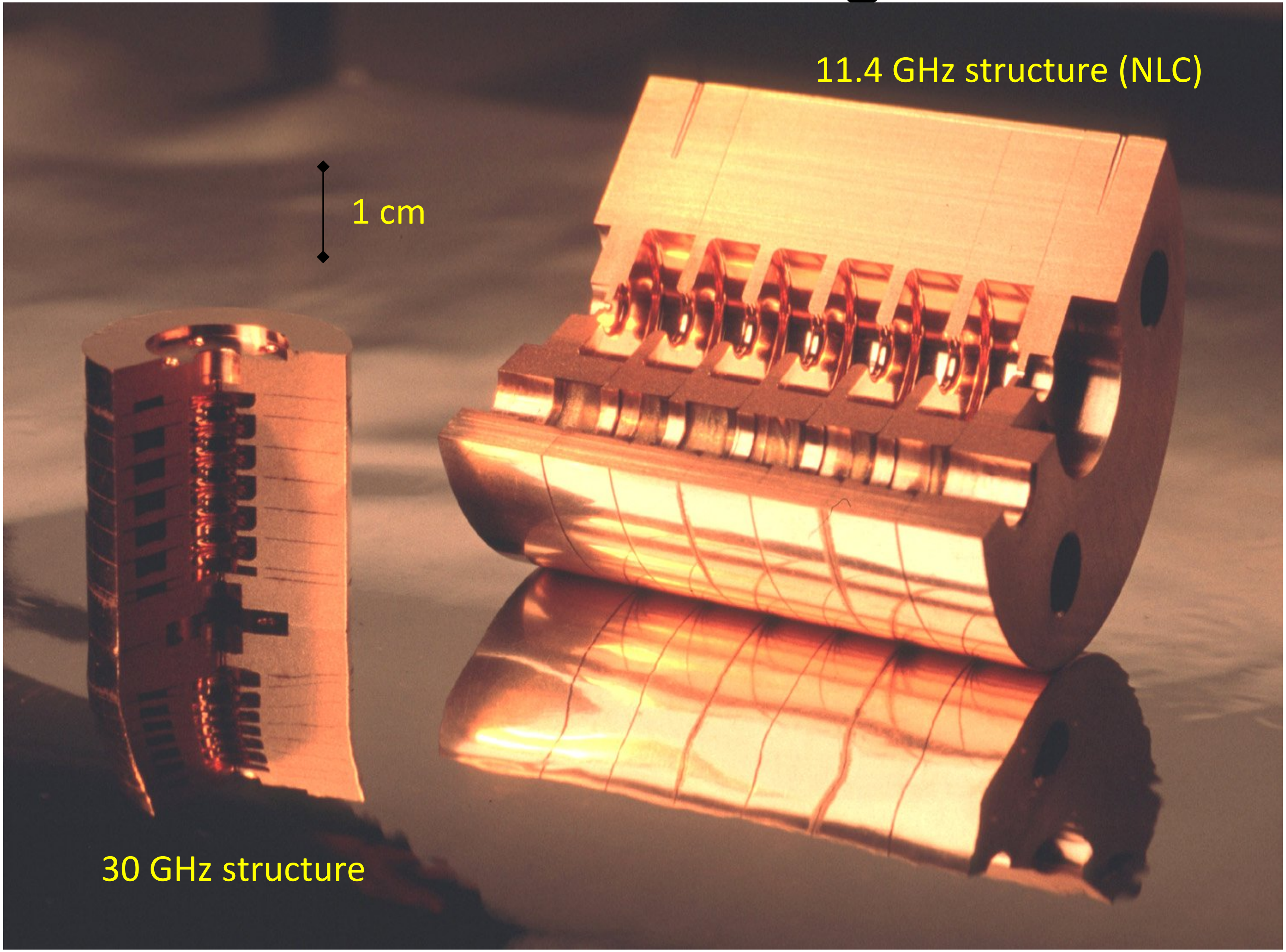
This Picture made it to the title page of CERN Courier vol. 41 No. 1 (Jan./Feb. 2001)

TRAVELLING WAVE STRUCTURES

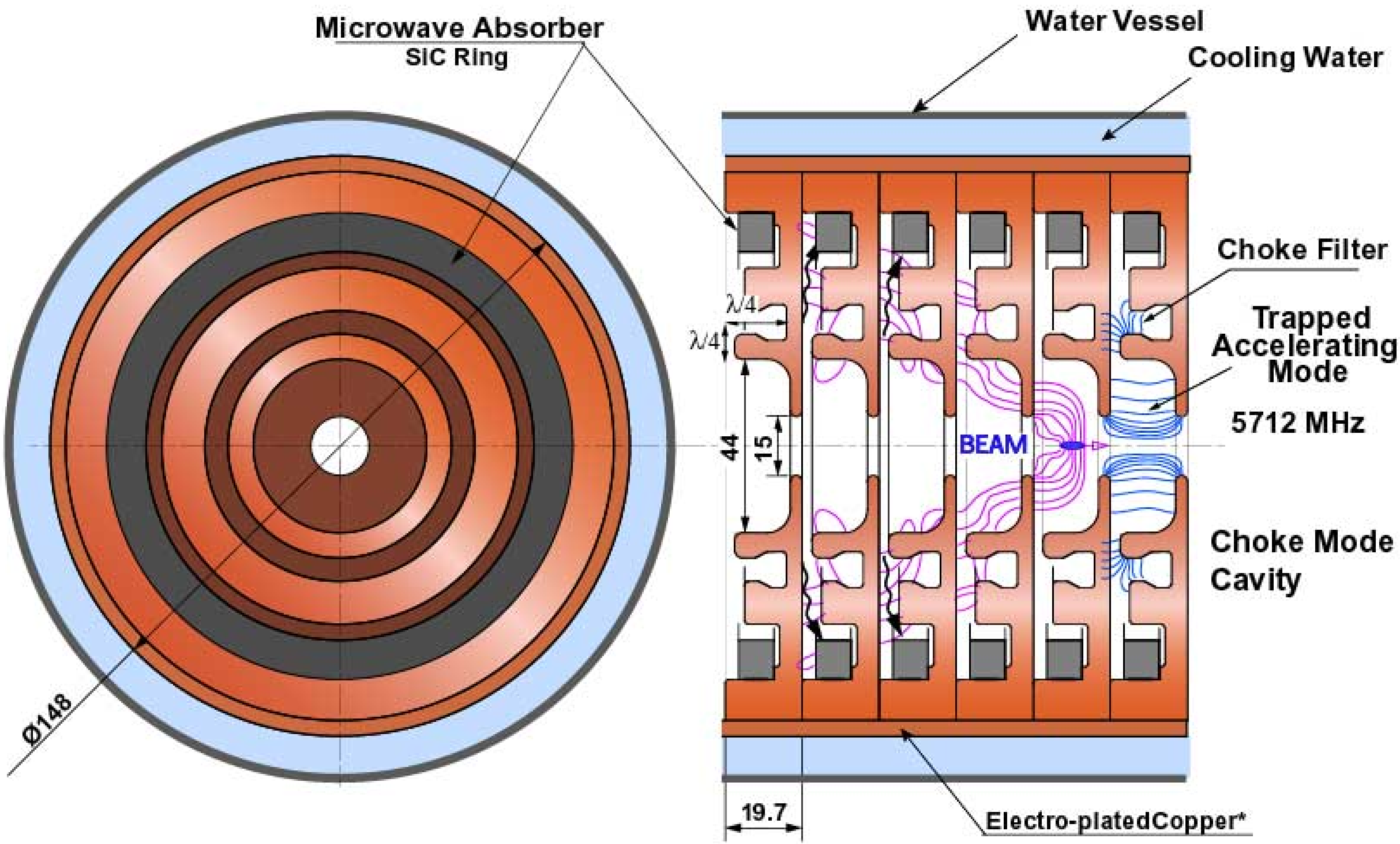
Brillouin diagram Travelling wave structure



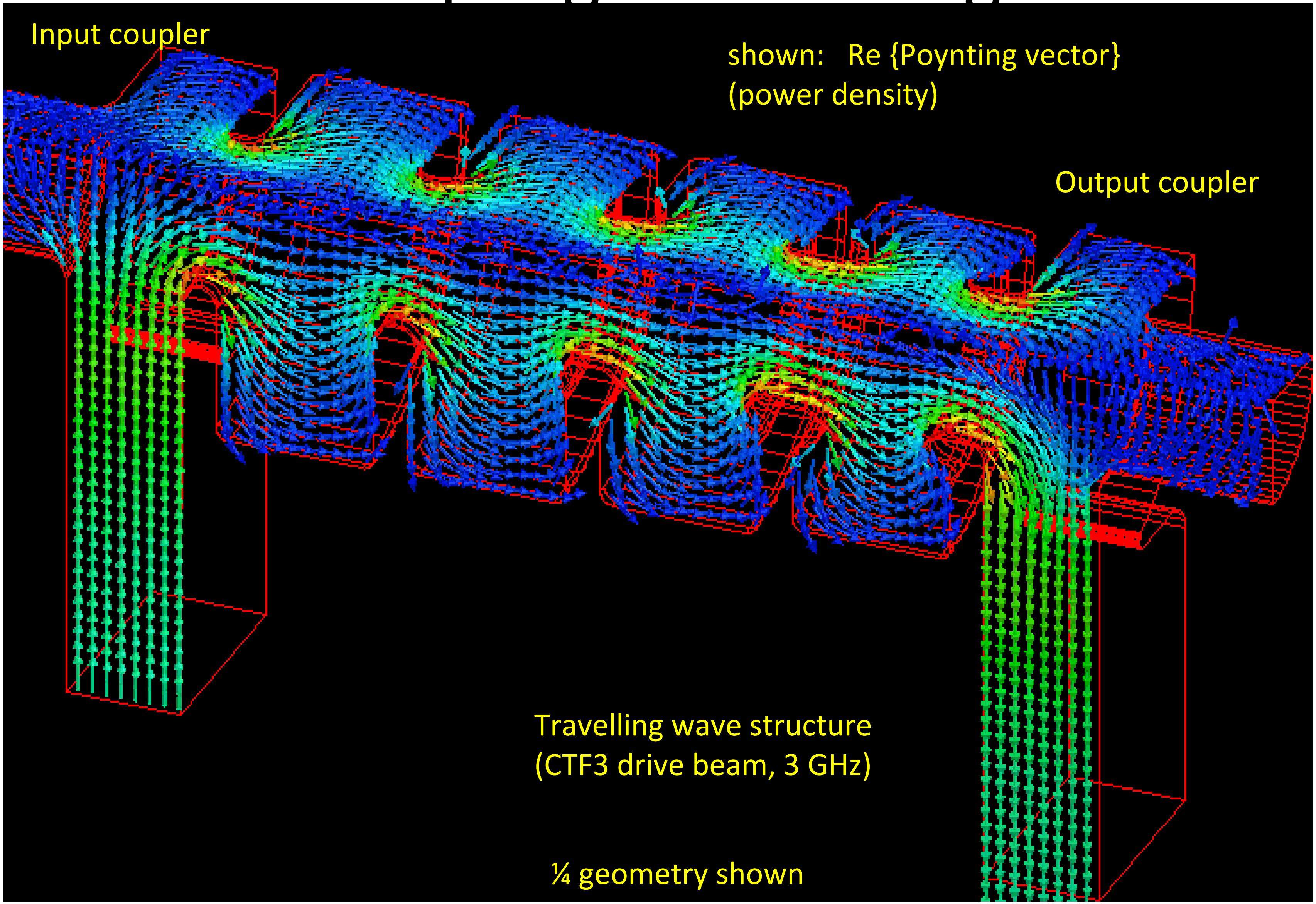
Iris loaded waveguide



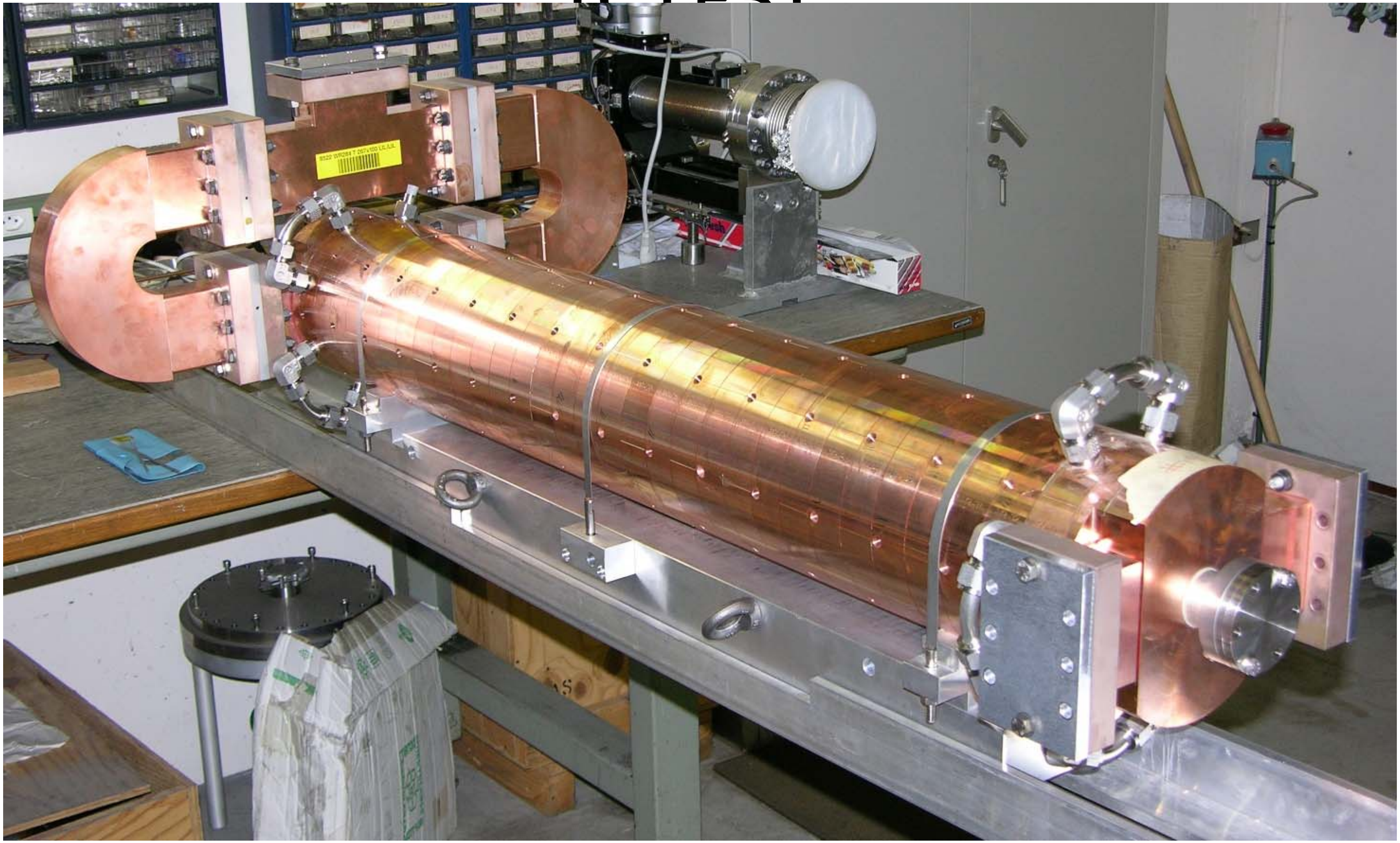
Disc loaded structure with strong HOM damping "choke mode cavity"



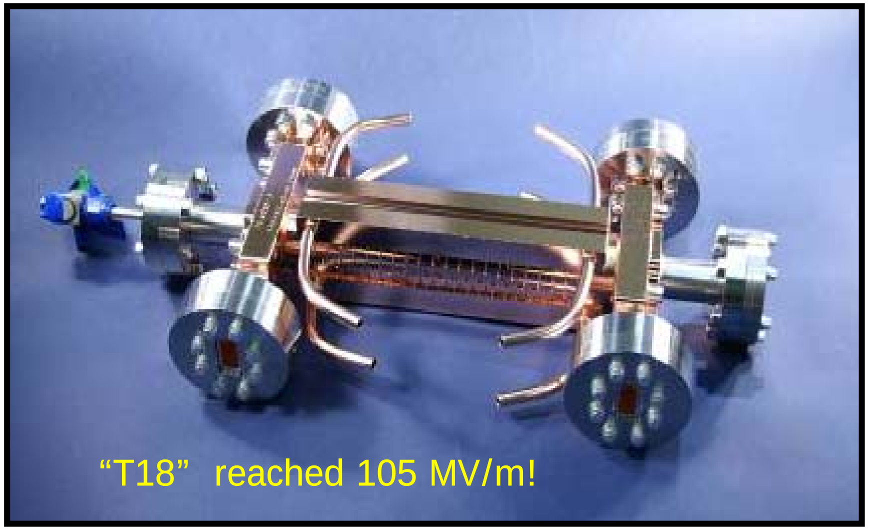
Power coupling with waveguides

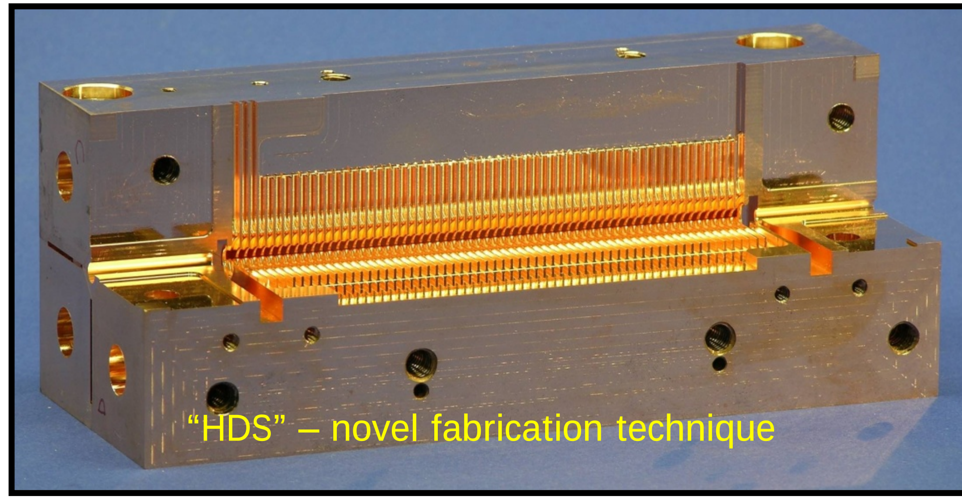


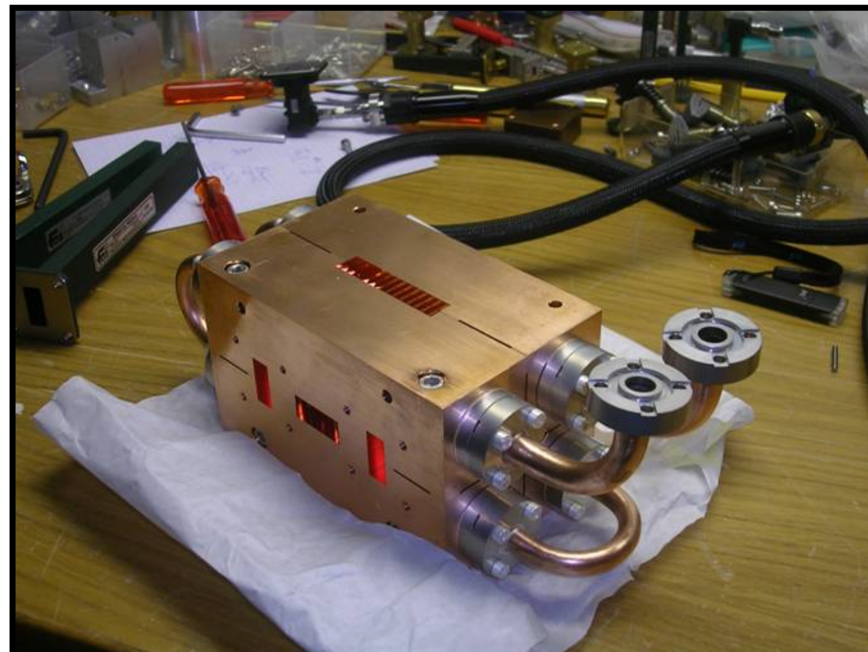
3 GHz Accelerating structure (CTF3)



Examples (CLIC structures @ 11.4, 12 and 30 GHz)

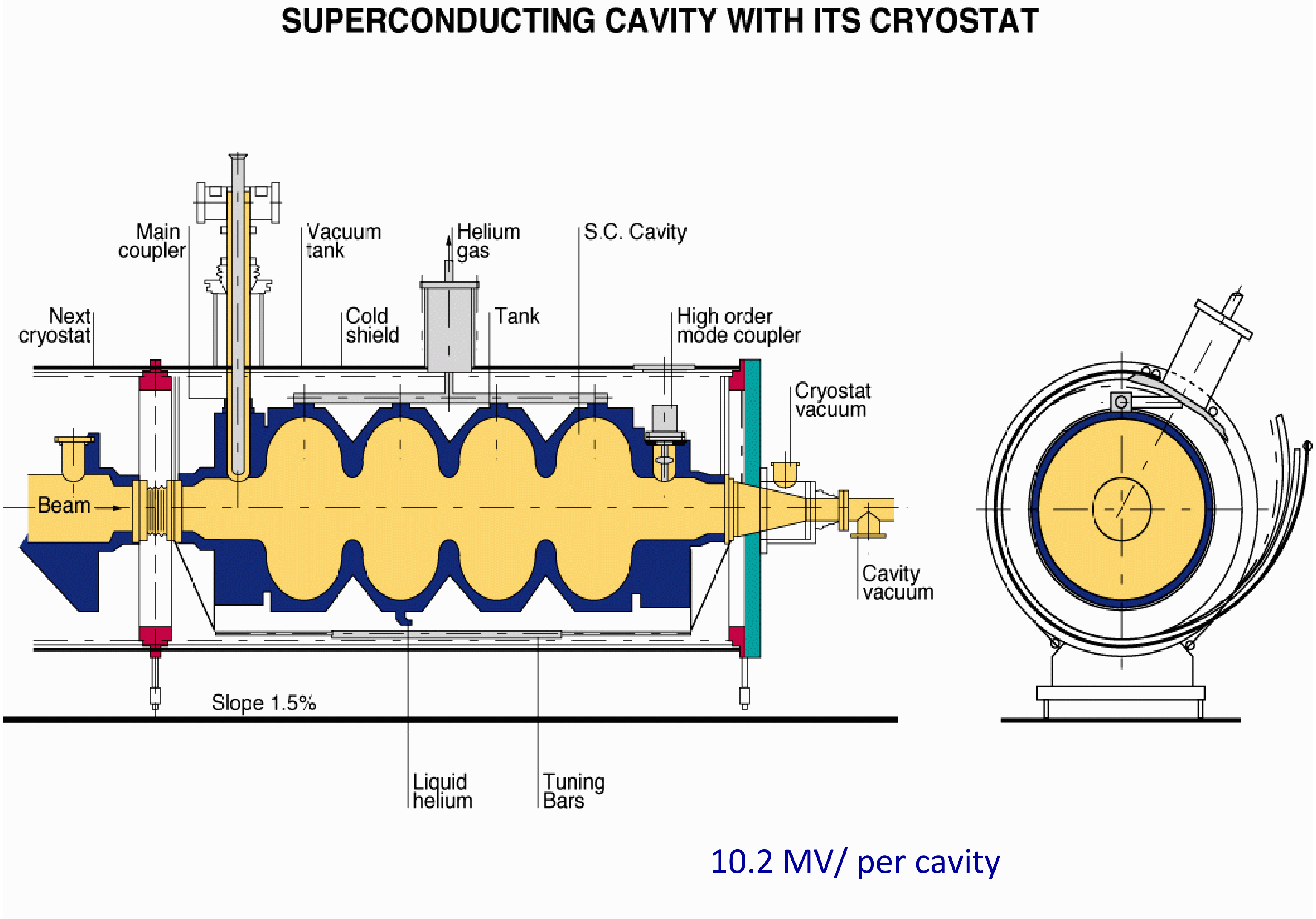






SUPERCONDUCTING ACCELERATING STRUCTURES

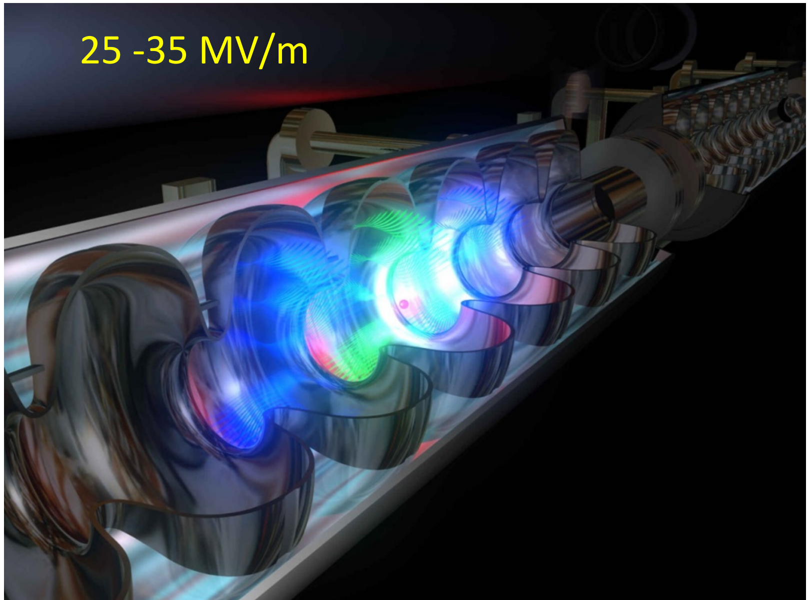
LEP Superconducting cavities

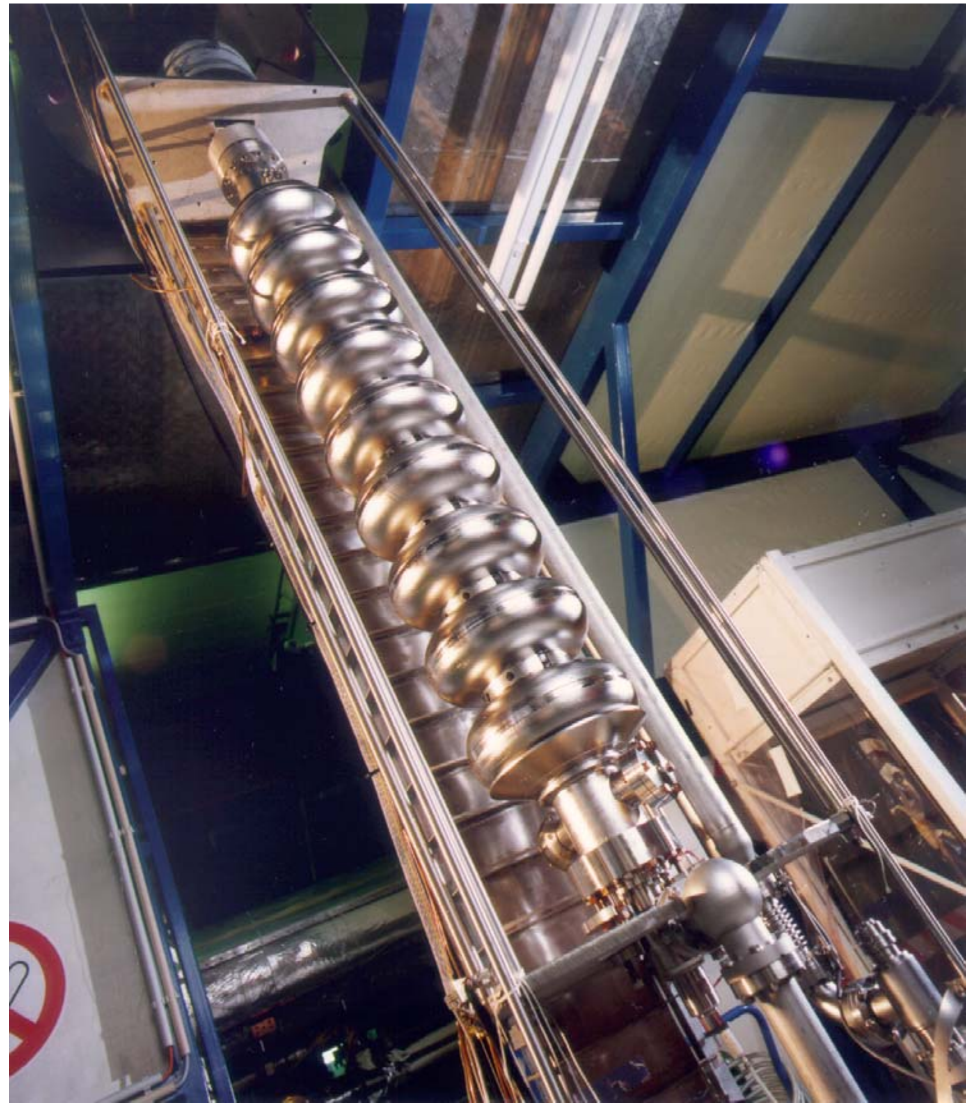


10.2 MV/ per cavity

LHC SC RF, 4 cavity module, 400 MHz

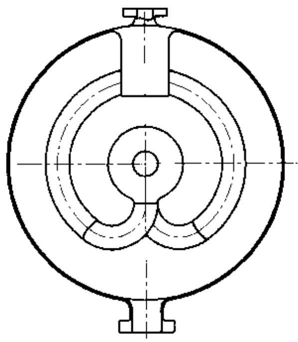
ILC high gradient SC structures at 1.3 GHz



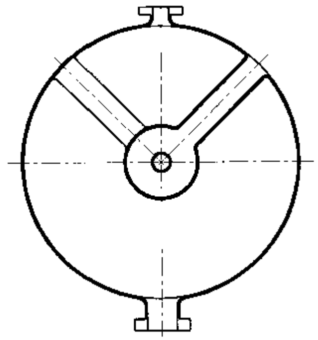


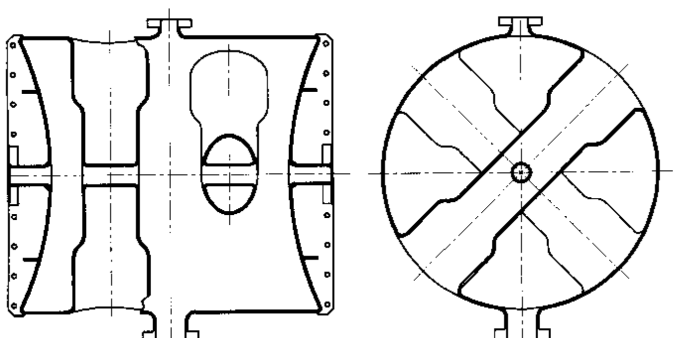
Small β superconducting cavities (example RIA, Argonne)

115 MHz split-ring cavity,



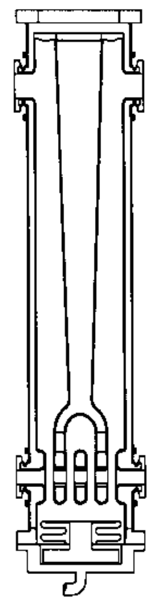
172.5 MHz β = 0.19 “lollipop” cavity



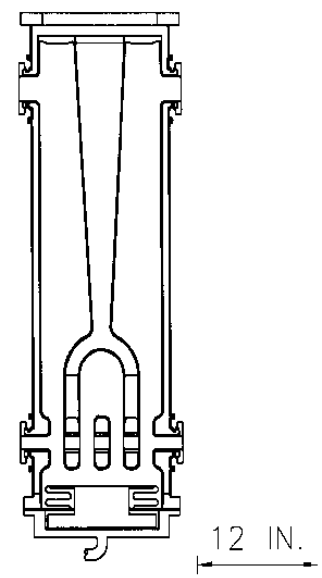


345 MHz β = 0.4 spoke cavity

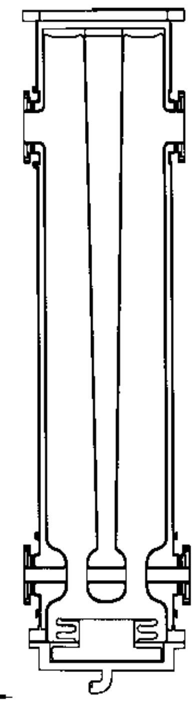
57.5 MHz cavities:



β = 0.021 fork cavity



β = 0.03 fork cavity

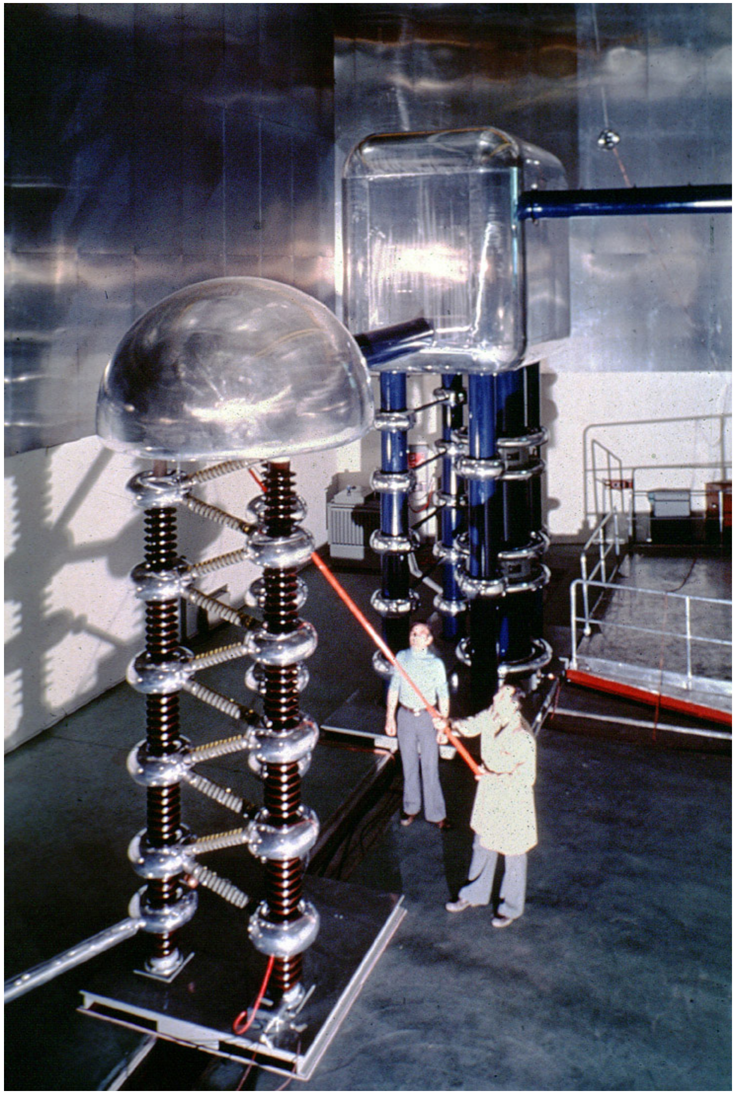


β= 0.06 QWR
(quarter wave resonator)

pictures from Shepard et al.: “Superconducting accelerating structures for a multi-beam driver linac for RIA”, Linac 2000, Monterey

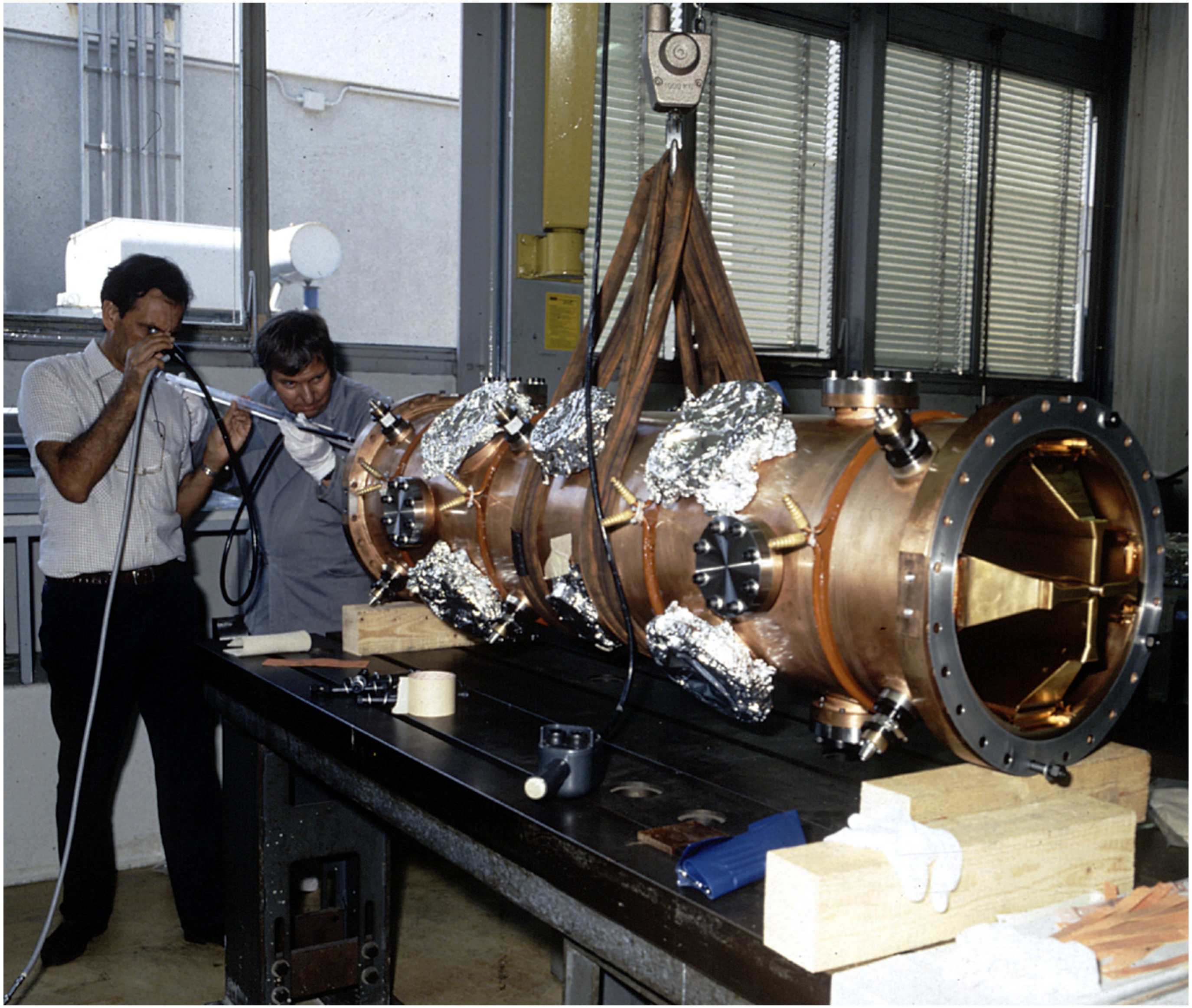
RFQ’S

Old pre-injector 750 kV DC , CERN Linac 2 before 1990



All this was replaced by the RFQ …

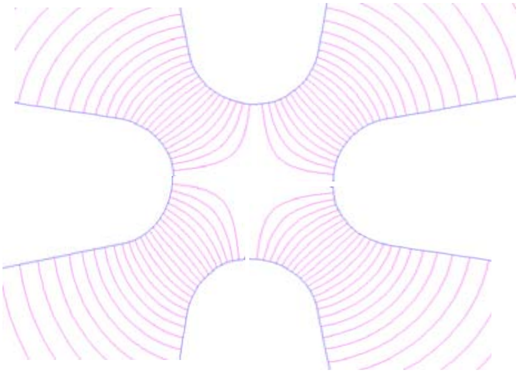
RFQ of CERN Linac 2

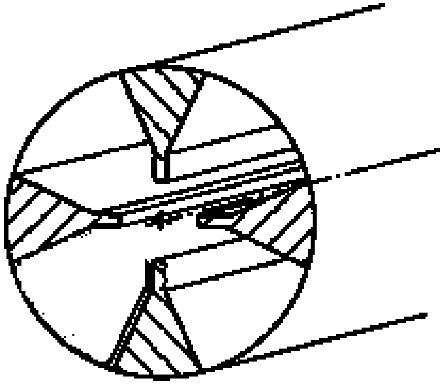


The Radio Frequency Quadrupole (RFQ)

Minimum Energy of a DTL: 500 keV (low duty) - 5 MeV (high duty)
At low energy / high current we need strong focalisation
Magnetic focusing (proportional to β) is inefficient at low energy.
Solution (Kapchinski, 70's, first realised at LANL):

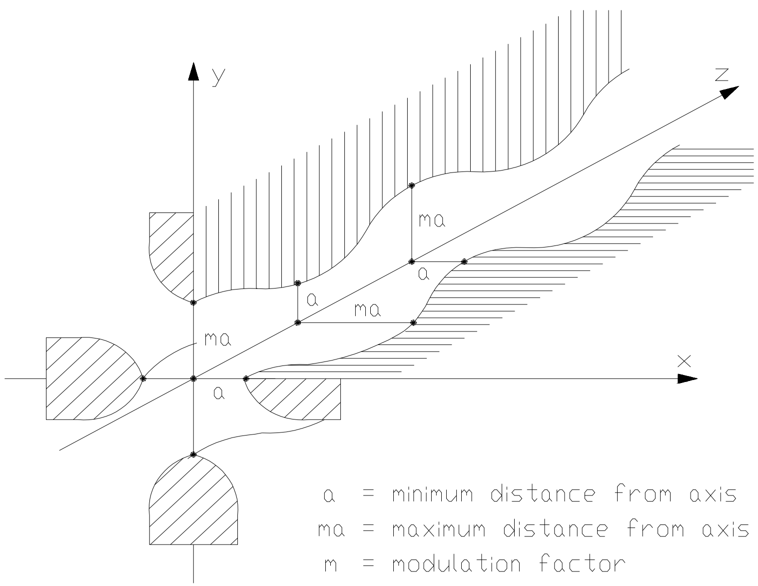
<u>Electric quadrupole focusing + bunching + acceleration</u>

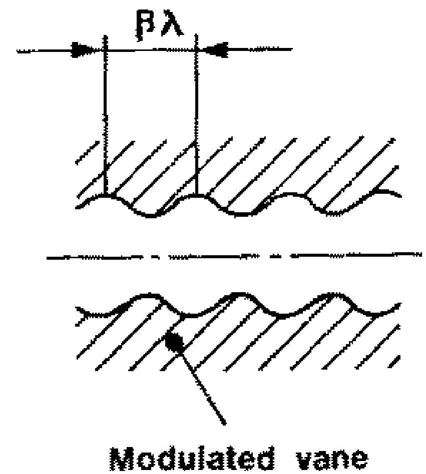




RFQ electrode modulation

The electrode modulation creates a longitudinal field component that creates the "bunches" and accelerates the beam.





A look inside CERN AD's "RFQ-D"