TECHNISCHE UNIVERSITÄT DARMSTADT

Investigation of Numerical Noise in PIC-Codes

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Abstract

In the simulation of space charge dominated beams a combination of efficient FDTD schemes and Particle-in-Cell methods is widely used. A drawback of this procedure is, as reported repeatedly, its sensitivity to numerical noise.

The impact of the bunch shape on the noise level was studied and an 'a priori'-filter for noise suppression is presented.

Causes of Numerical Noise

In simulations of various particle tracking problems with FDTD based programs a strong emittance growth was observed. On the left hand side of Fig. 1 an example for this circumstance is shown. When performing a Discrete Fourier Transformation of the associated fields, an accumulation of field energy in the region of the grid cutoff frequency f_c is observed. The grid cutoff frequency corresponds to the highest resolved mode

$$f_c = \frac{c}{\pi}\sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}$$

Δx, Δy, and Δz being the grid step sizes.

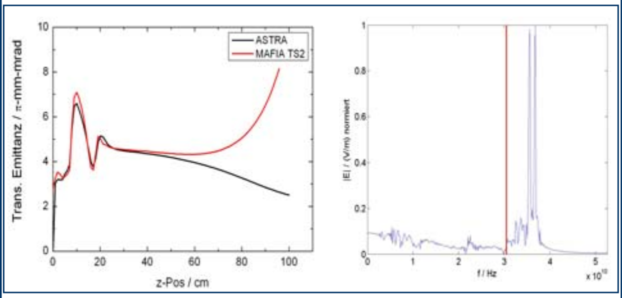

Figure 1: Emittance development in ASTRA and TS2 (left). Accumulation of field energy near the marked grid cutoff frequency (right).

Since investigations showed a strong dependency of the noise level on the bunch shape different Gaussian bunches were tested. The results are plotted in Fig. 2. All bunches have a total length of six standard deviations σ where the length of σ varies from 2/3 Δz to 3 Δz.

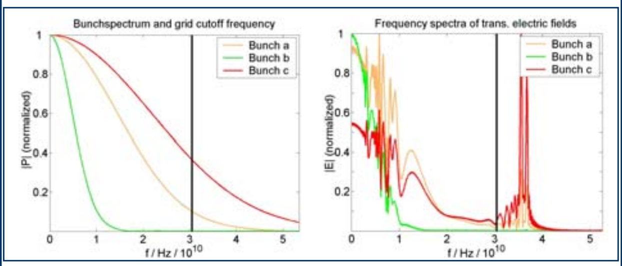

Figure 2: Frequency spectra of three Gaussian bunches of different length (left) and resulting spectra of the long. electrical field (right).

Numerical noise is due to energy fractions above f_c in the exciting grid current.

An a priori-Filter for noise suppression

Numerical noise can be suppressed by modifying the bunch shape in three steps:

- Transformation of bunch signal into frequency domain via FFT
- Erasing frequency fractions above grid cutoff
- Transformation back to time domain via inverse FFT

An example of results for this simple filtering algorithm are shown in Fig. 3.

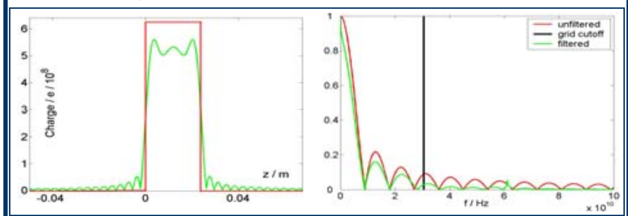

Figure 3: Rectangular and filtered bunch shape (left) and associated frequency spectra of the long. E-Field (right).

The filtering skills can be improved by using windowing functions as the Dolph-Tschebyscheff-Window which have higher attenuations in the blocking zone (Fig. 4). This leads to less ripples in the filtered shape.

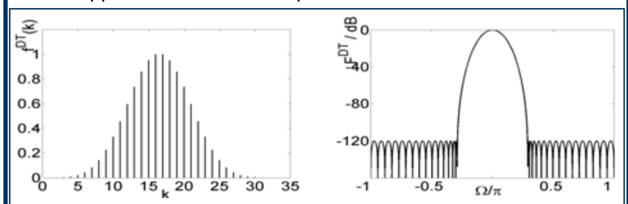

Figure 4: 32-point Dolph-Tschebyscheff-Window (left) and spectral properties (right).

Testing the Filter

By application of the filter almost every bunch can be simulated noise free. Therefore an appropriate mesh must be chosen. In Fig. 5 results for a symmetric bunch of a FWHM of 22 ps with 2 ps rise time are shown.

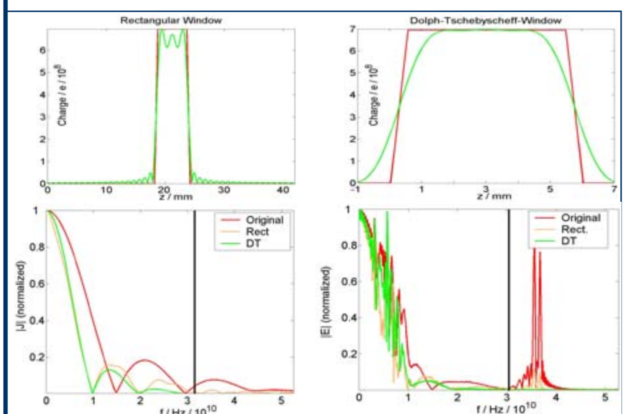

Figure 5: Upper row: Filtered bunches (rectangular and DT-Window) Lower row: Frequency spectra of bunches and long. E-field

Application of the a priori-Filter saves 56 % of the grid points in one dimensions and ensures noise-free simulations.