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# New Discretization Scheme for Wake Field Computation in Cylindrically Symmetric Structures

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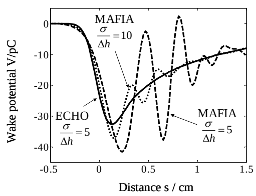
## Abstract

Collective effects due to wake fields are a limiting factor in almost every new front line accelerator. Since the early 80's computer codes such as TBCI and MAFIA have been developed for computing wake fields in realistic accelerator structures. With the advent of linear collider studies and small wavelength FEL projects these codes had to face a severe limitation. For the very short bunches in these new accelerators combined with the need for an analysis of very long sections the discrete dispersion became a serious drawback. This effect of having only discrete field values rather than continous ones can be overcome by special algorithms such as semi-implicit integrators as used e.g. in the wake field code ECHO. In this paper we present a new explicit approach which combines the advantage of explicit algorithms (fast) with the absence of dispersion in beam direction.

## Introduction



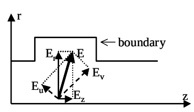

- MAFIA converges to accurate wake potential with increasing mesh resolution
- oscillations due to accumulation of dispersion error

- drawback in wake potential calculations for short bunches in long structures
- accumulation of dispersion error
- time step is limited in FDTD algorithms by the mesh step

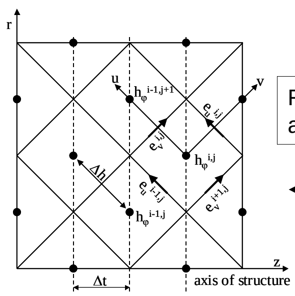
$$\Delta t = \frac{1}{\sqrt{2}} \cdot \Delta h$$

To overcome this problem:

- use the properties of the mesh i.e. no dispersion along diagonals of the mesh
- rotate the mesh by 45° to align these diagonals with the direction of the propagating bunch
  - ⇒ electric field components need to be described in the rotated local basis $(\vec{u}, \vec{v})$.




Results in a rotated mesh with a new indexing scheme




## Algorithm

Boundary value problem for cylindrical symmetric problem

$$\frac{\partial h_\varphi}{\partial t} = -\frac{1}{\mu_\varphi}\left(\frac{\partial e_r}{\partial z} - \frac{\partial e_z}{\partial r}\right)$$

$$\frac{\partial e_r}{\partial t} = \frac{1}{\varepsilon_r}\left(-\frac{\partial h_\varphi}{\partial z}\right)$$

$$\frac{\partial e_z}{\partial t} = \frac{1}{\varepsilon_z}\frac{1}{r}\frac{\partial}{\partial r}\left(r \cdot h_\varphi\right) + j_z$$

- rotate the mesh
- introduce scattered and exciting fields
- discretise analytical equations referring to scheme

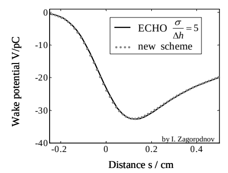
New update scheme

$$h_\varphi^s\Big|_{i,j}^{n+1} = h_\varphi^s\Big|_{i,j}^{n} - \frac{\Delta t}{\Delta h}\frac{1}{\mu_\varphi|_{i,j}}\left(e_u^s\Big|_{i,j}^{n+0.5} - e_u^s\Big|_{i+1,j}^{n-0.5} - \left(e_v^s\Big|_{i,j}^{n+0.5} - e_v^s\Big|_{i-1,j}^{n-0.5}\right)\right) - \left(1 - \frac{1}{\mu_\varphi|_{i,j}}\right)\left(h_\varphi^e\Big|_{i,j}^{n+1} - h_\varphi^e\Big|_{i,j}^{n}\right)$$

$$e_u^s\Big|_{i-1,j}^{n+0.5} = e_u^s\Big|_{i-1,j}^{n-0.5} - \frac{\Delta t}{\Delta h}\frac{1}{\varepsilon_u|_{i-1,j}}\frac{1}{r|_{i-1,j}}\left(-\left(r|_{i,j} \cdot h_\varphi^s\Big|_{i,j}^{n} - r|_{i-1,j} \cdot h_\varphi^s\Big|_{i-1,j}^{n}\right)\right) - \left(1 - \frac{1}{\varepsilon_u|_{i-1,j}}\right)\left(e_u^e\Big|_{i-1,j}^{n+0.5} - e_u^e\Big|_{i-1,j}^{n-0.5}\right)$$

$$e_v^s\Big|_{i,j}^{n+0.5} = e_v^s\Big|_{i,j}^{n-0.5} - \frac{\Delta t}{\Delta h}\frac{1}{\varepsilon_v|_{i,j}}\frac{1}{r|_{i,j}}\left(-\left(r|_{i-1,j+1} \cdot h_\varphi^s\Big|_{i-1,j+1}^{n} - r|_{i,j} \cdot h_\varphi^s\Big|_{i,j}^{n}\right)\right) - \left(1 - \frac{1}{\varepsilon_v|_{i,j}}\right)\left(e_v^e\Big|_{i,j}^{n+0.5} - e_v^e\Big|_{i,j}^{n-0.5}\right)$$

## Results

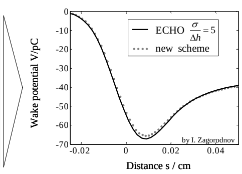



Comparison: ECHO – New Code

- mesh resolution is 5 points per $\sigma$
- bunch width equal to 1mm
- loss factor: $L$=21.034 (ECHO)
  $L$=21.329 (new code)

Comparison: ECHO – New Code

- mesh resolution is 5 points per $\sigma$
- bunch width equal to 0.1mm
- loss factor: $L$=46.972 (ECHO)
  $L$=45.549 (new code)




- MAFIA loss factor is $L$=17.7372 ($\sigma$=1mm)
- conventional FDTD approach should use 160 points per $\sigma$ for equally accurate result
- this finer mesh step and with it the finer time step leads to a factor of approximately 900 in computation time

## Conclusions

- new discretisation scheme combining explicit approach and zero dispersion in beam direction
- no dispersion error accumulation in longitudinal wake potential calculation
- MAFIA demands a very fine mesh
- very accurate results are obtained by rotating the mesh