MODELS TO STUDY COHERENT BEAM-BEAM EFFECTS OF MULTI BUNCH BEAMS

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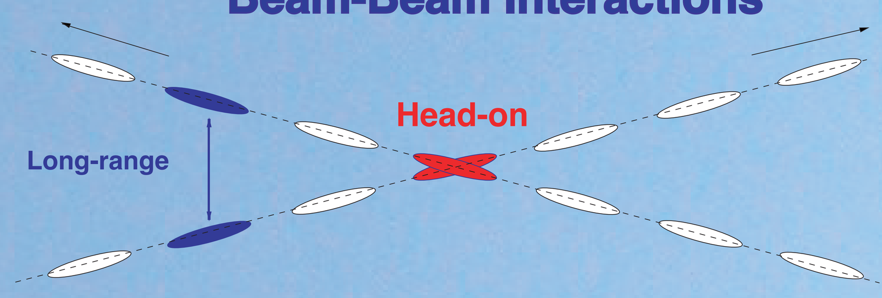
Abstract

During the collision of two charged beams the strong non linear electromagnetic field of one beam perturbs the trajectory of the opposing beam. This effect is called beam-beam interaction. The beam-beam force in first approximation is linearly dependent on the distance to the opposite beam and leads to a coupling between the colliding bunches. Each particle of a bunch is affected by a non-linear single particle effect that leads to the so called incoherent beam-beam effects. However the beam can be affected as a whole and the resulting collective motion leads to the so-called coherent beam-beam effects. For two colliding bunches this can be computed and is well understood. For a large number of bunches colliding in more complicated configurations, as for example the LHC, this will result in a richer spectrum of oscillation frequencies. To compute the possible beam-beam modes for different collision pattern and beam filling schemes, three models were developed and are described focusing on their advantages and/or limitations. The three models give useful and different information about the beam-beam coherent interactions of multi bunch beams. All together they can provide a deeper insight into the underlying physics.

Beam = ultrarelativistic high concentration charged particles

Strong Electromagnetic field:
-defocusing for equal charge beams
-focusing for opposite charge beams

Beam-Beam Interactions

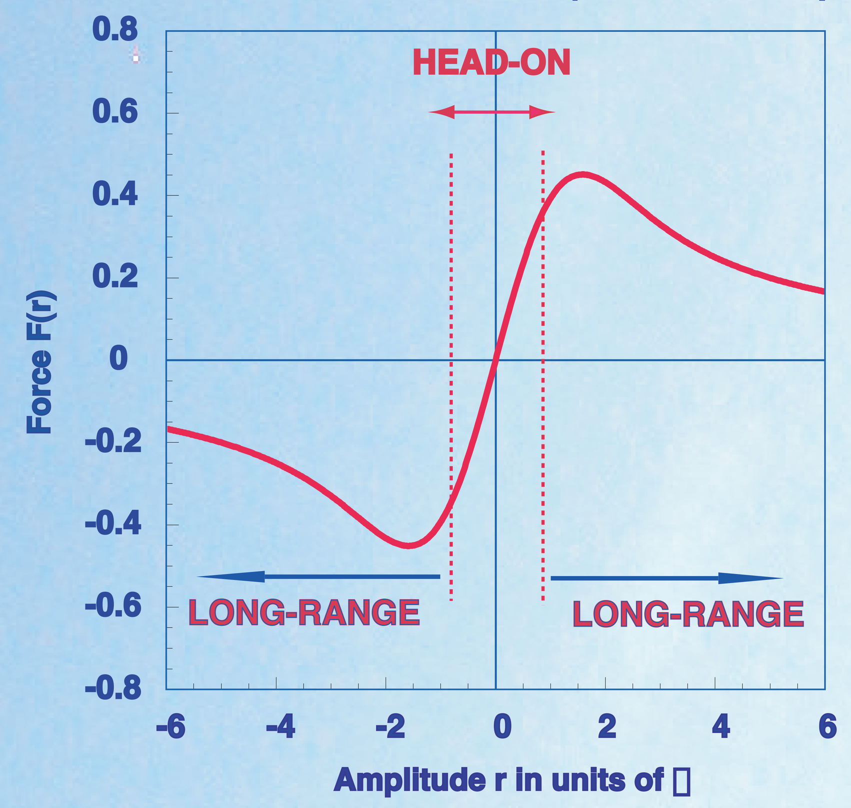


Beam-Beam Effects

INCOHERENT → Perturbation on each single bunch particle

COHERENT → Bunch affected as a whole

BEAM-BEAM FORCE (round beams)

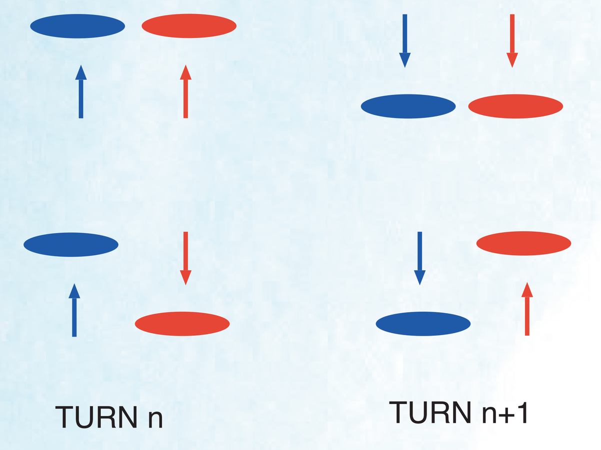


$$F_r(r) = \frac{-Ne^2(1+\beta^2)}{2\pi\epsilon_0 r}\left[1-\exp\left(-\frac{r}{2\sigma^2}\right)\right]$$

Head-on collisions: for small oscillations F(r) is in first approximation **linear**

A Simple case: 1 vs 1

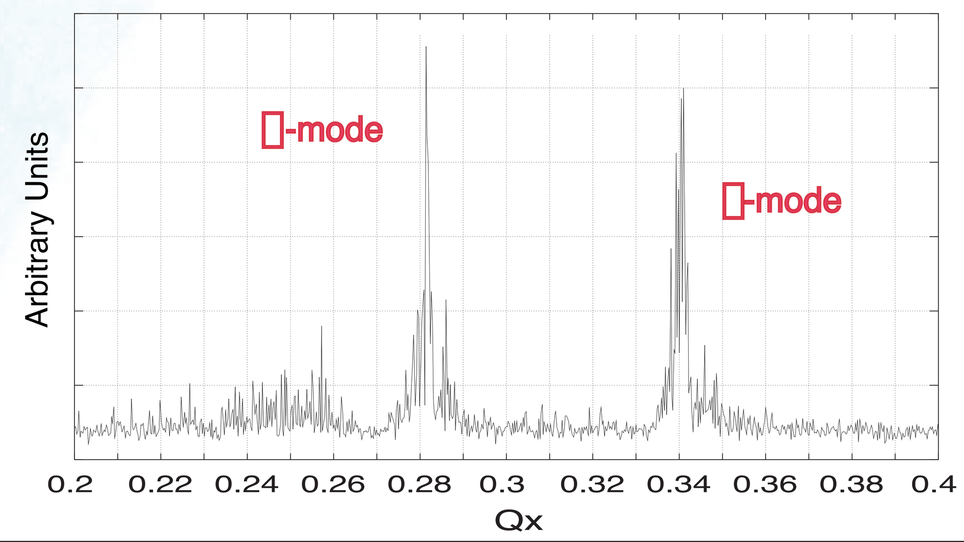
Two bunches colliding head-on are coupled by F(r) and can oscillate at two possible frequencies each frequency corresponds to one oscillating mode:



□-mode: The two bunches oscillate in phase turn after turn

□-mode: The two bunches oscillate out of phase turn after turn

LEP tune



THE ONE TURN MATRIX FORMALISM

bunches are like rigid objects

Particle distribution: Gaussian with fixed RMS (□) defined constant for all bunches of a beam and all times

2 beams

$$X = \begin{pmatrix} x_{1b_1} \\ x'_{1b_1} \\ y_{1b_1} \\ y'_{1b_1} \\ \dots \\ x_{1b_2} \\ x'_{1b_2} \\ y_{1b_2} \\ y'_{1b_2} \\ \dots \end{pmatrix}$$

(beam 1: bch1, bch2,...; beam 2: bch1, bch2,...)

Beam-beam interaction (HO): the bunch receives a kick calculated by using the linear term of the beam-beam kick:

$$B = \begin{pmatrix} 1 & 0 & \dots & 0 & 0 & \dots \\ -k_{bb} & 1 & \dots & k_{bb} & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 & \dots \\ k_{bb} & 0 & \dots & -k_{bb} & 1 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix} \qquad k_{bb} = \frac{2\pi\xi}{\beta_x}$$

$$\xi_{\{x,y\}} = \frac{2r_p N_p}{2\pi\gamma}\frac{\beta_{\{x,y\}}}{\sigma_{\{x,y\}}(\sigma_x+\sigma_y)}$$

In between a **Linear Transfer (LT)** is applied

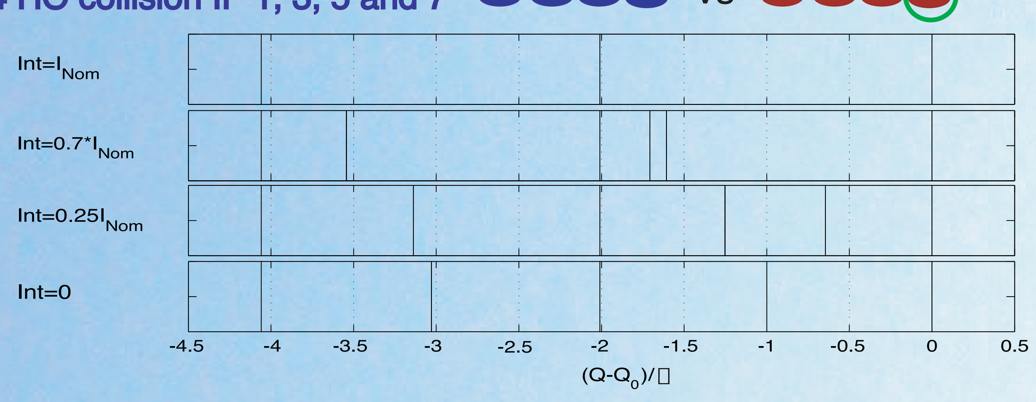
$$T = \begin{pmatrix} \cos(\Delta\mu_x^{b_1}) & \sin(\Delta\mu_x^{b_1}) & \dots & 0 & 0 & \dots \\ -\sin(\Delta\mu_x^{b_1}) & \cos(\Delta\mu_x^{b_1}) & \dots & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \cos(\Delta\mu_x^{b_2}) & \sin(\Delta\mu_x^{b_2}) & \dots \\ 0 & 0 & \dots & -\sin(\Delta\mu_x^{b_2}) & \cos(\Delta\mu_x^{b_2}) & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

$\Delta\mu_{x\,y}$ phase advance in units of 2□

One turn MAP: M= T*B*T*T*B*..... → $X_{1turn} = M^*X_0$

The **eigenvalues** of the one turn map M give the **frequencies** of the system eigenmodes

4 HO collision IP 1, 3, 5 and 7 — 4 3 2 1 vs 1 2 3 4 — Variable Intensity



Advantages:
- fast calculation speed
- Moderate flexibility to changes in the collision and bunch schemes
- get ALL mode frequencies

Disadvantages:
- non-linear terms can be treated only with a linear approximation
- Landau damping cannot be included (rigid bunches)
- higher order modes cannot be evaluated
- does not use a correct field calculation

THE RIGID BUNCH MODEL

bunches are like rigid objects

Particle distribution: Gaussian with constant transverse sizes for all bunches of a beam and all times in both planes x and y

Beam-beam interaction: bunch at (x,y) receives a transverse kick from the opposite bunch (X,Y) calculated by the formula for the coherent beam-beam kick where the bunch transverse sizes are fixed (□x, □y):

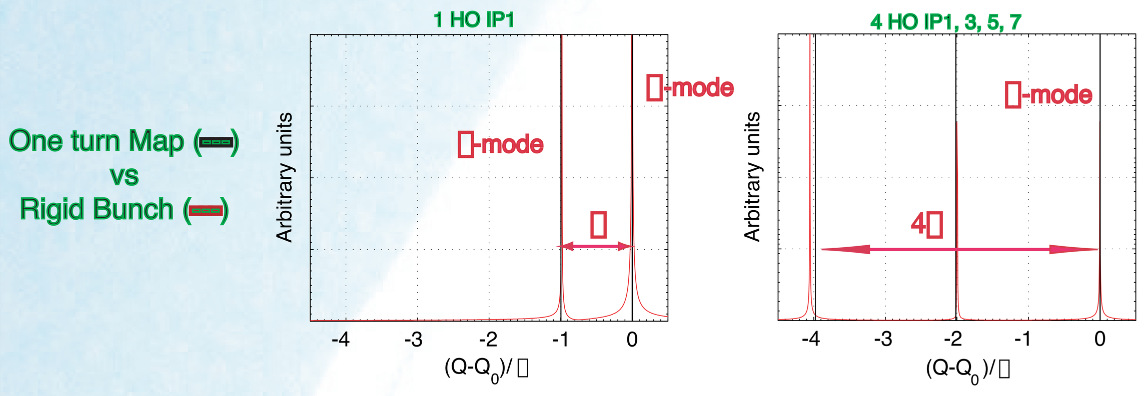
$$\Delta x' = \frac{2r_p N_p}{\gamma}\frac{\beta_x}{\sigma_x^2}F_x(x-X, y-Y, \sigma_x^2, \sigma_y^2)$$

$$F_{\{x,y\}}(x-X, y-Y, \sigma_x^2, \sigma_y^2) = \frac{\{x,y\}}{(x^2+y^2)}\left[1-\exp\left(-\frac{x^2+y^2}{2\sigma_x^2+2\sigma_y^2}\right)\right]$$

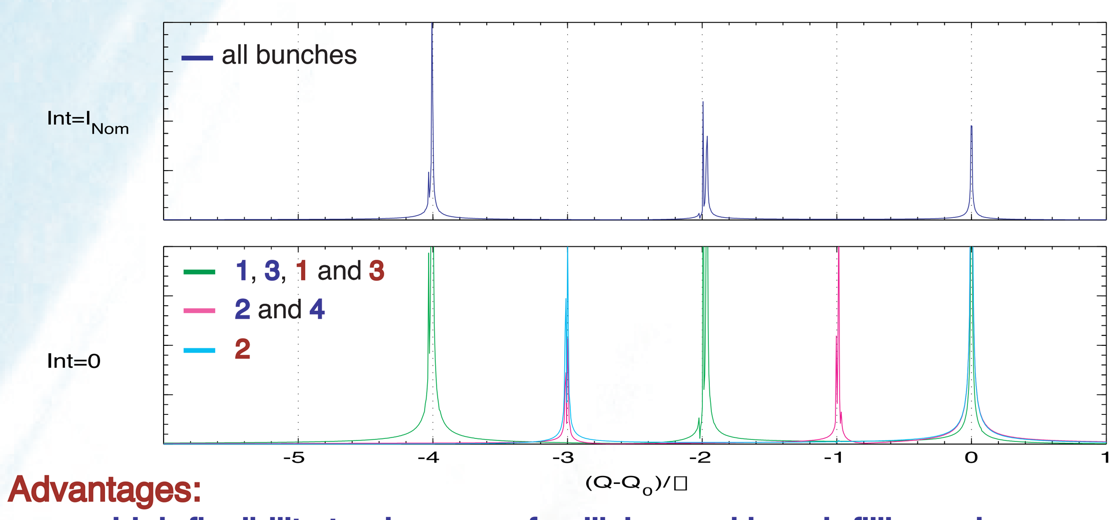
In between the interactions a **linear transfer** is applied

A **Fourier analysis** of the bunch barycentres turn by turn gives the tune spectra of the dipole modes

MULTI HO collisions vs



4 HO collision IP 1, 3, 5 and 7 — 4 3 2 1 vs 1 2 3 4 — Variable Intensity



Advantages:
- very high flexibility to changes of collision and bunch filling scheme
- good calculation speed
- non-linear effects treated with an additional factor

Disadvantages:
- non-linear terms partially treated (field calculation not correct)
- Landau damping not included (rigid bunches)
- higher order modes cannot be evaluated
- does not use a correct field calculation

THE MULTI-PARTICLE MODEL

bunches are dynamic objects

Bunches consist of a **Particle distribution** (at the present Gaussian) of N_{tot} macro particles

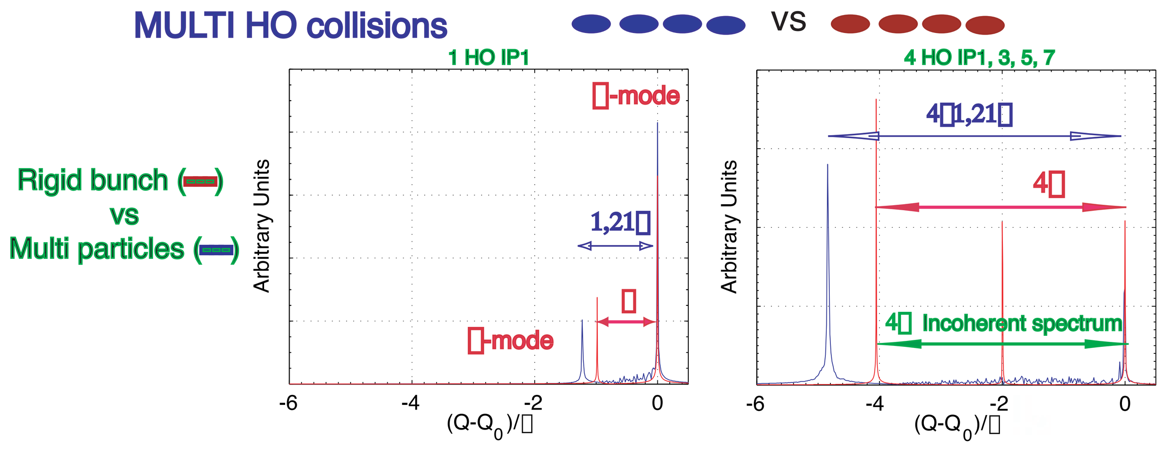
At a **beam-beam interaction** each particle of the involved bunches receives a transverse kick calculated by the formula for the incoherent beam-beam kick where the opposite bunch barycentres (X and Y) and sizes (□x, □y) are now changed and calculated by the particle distribution just before the interaction:

$$\Delta x' = \frac{2r_p N_p}{\gamma}\frac{\beta_x}{\sigma_x^2}F_x(x-X, y-Y, \sigma_x^2, \sigma_y^2)$$

$$F_{\{x,y\}}(x-X, y-Y, \sigma_x^2, \sigma_y^2) = \frac{\{x,y\}}{(x^2+y^2)}\left[1-\exp\left(-\frac{x^2+y^2}{\sigma_x^2+\sigma_y^2}\right)\right]$$

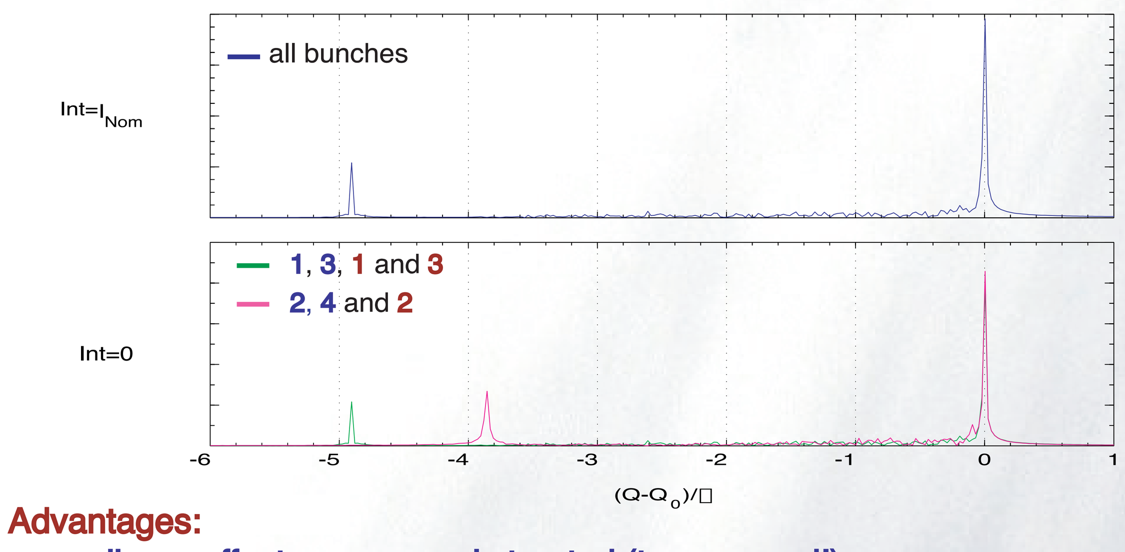
A **Fourier analysis** of the bunch barycentres calculated turn by turn gives the tune spectra of the dipole modes

MULTI HO collisions vs



- □Q larger because bunch shape changes during HO
- Landau damping of modes inside the incoherent spectrum

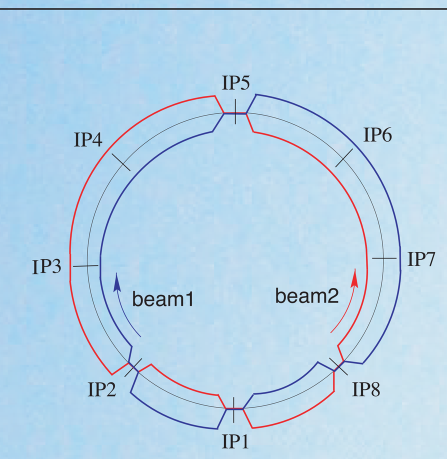
4 HO collision IP 1, 3, 5 and 7 — 4 3 2 1 vs 1 2 3 4 — Variable Intensity



Advantages:
- non-linear effects are properly treated (tune spread!)
- Landau damping can be reproduced
- higher order modes can be reproduced
- correct field calculation (depending on the field solver used)
- high flexibility to different collision patterns and beam filling schemes
- incoherent effects can be studied (emittance growth, beam life time...)

Disadvantages:
- time consuming (concrete results for the LHC only in parallel mode)
- does not give ALL mode frequencies

More complicated....the LHC?!?



LHC geometrical layout

```
# Beam filling scheme #
```

4 Trains 9 bunch each

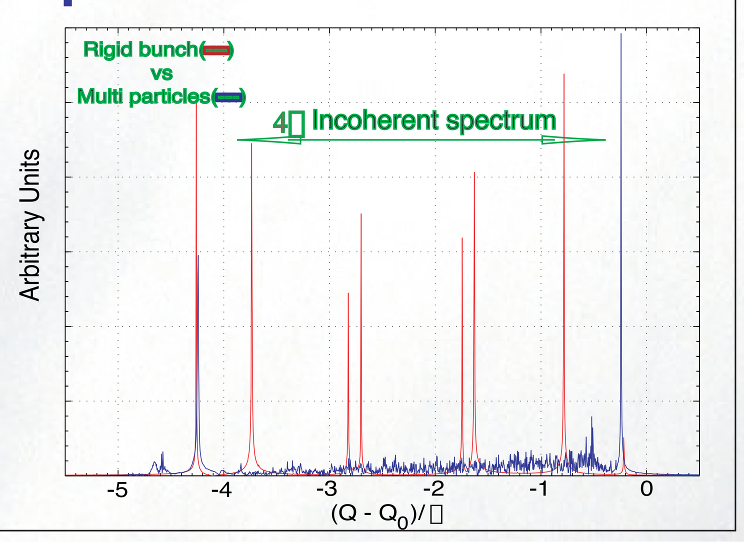
```
9 1 71 0    9 1 71 0
9 1 71 0    9 1 71 0

# Collision Scheme #
```

		$\Delta\mu_x$	$\Delta\mu_y$
1(IP1)	HO		
41	LT	8.046	6.940
81(IP2)	HO		
202	LT	23.015	21.821
321(IP5)	HO		
441	LT	23.533	20.689
561(IP8)	HO		
601	LT	7.716	7.870

*) Higher number of bunches and complex collision scheme give multi-peaks spectrum

*) Landau damping of modes inside the incoherent spectrum is confirmed

Consistent results



CONCLUSIONS

The three methods describe the beam-beam phenomena correctly and consistently within their physical model.
They give useful and different information about the coherent beam-beam interactions of multi bunches.
All together they provide a deeper insight into the underlying physics.

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