Isogeometric Simulation of Lorentz Detuning in Superconducting Accelerator Cavities

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Abstract - Cavities suffer from eigenfrequency shifts due to mechanical deformation caused by the electromagnetic radiation pressure, a phenomenon known as Lorentz detuning. Standard Finite Element Methods fail to achieve a sufficient accuracy due to the poor representation of the geometry and due to the low order basis functions. We propose Isogeometric Analysis for discretising both geometry and fields in a coupled multiphysics simulation approach.

Multi-physics Model for Lorentz Detuning

Step 1

Solve Maxwell's eigenproblem in the undeformed cavity:

$$\begin{cases} \nabla \times \left(\frac{1}{\mu_0}\nabla \times \mathbf{E}_0\right) = \omega_0^2 \epsilon_0 \mathbf{E}_0 & \text{in } \Omega_C \\ \mathbf{E}_0 \times \mathbf{n}_c = 0 & \text{on } \Gamma_{CW} \\ \left(\frac{1}{\mu_0}\nabla \times \mathbf{E}_0\right) \times \mathbf{n}_c = 0 & \text{on } \Gamma_C \end{cases}$$

Design frequency:

$$f_0 = \frac{\omega_0}{2\pi}$$

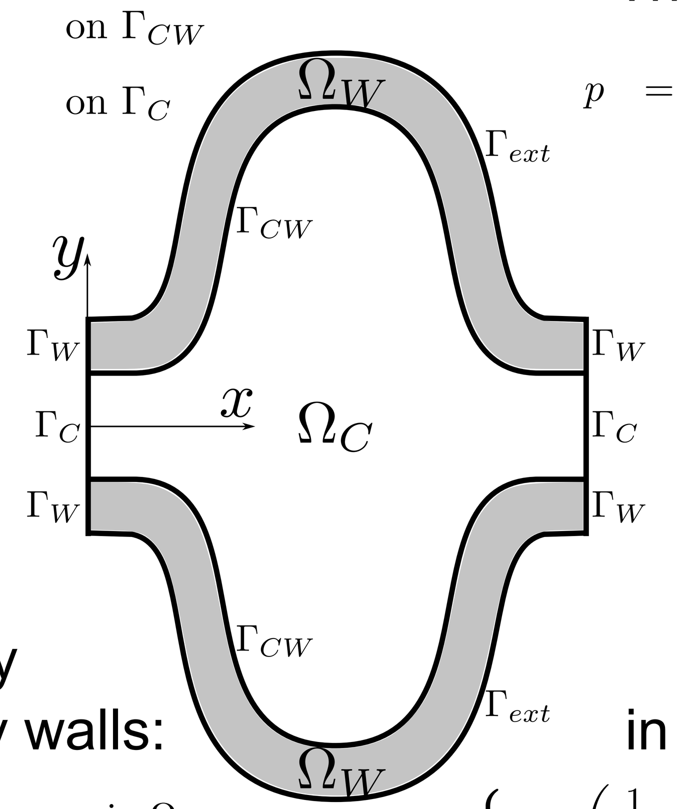
Step 2

Evaluate the magnetic field:

$$\mathbf{H}_0 = \frac{1}{i\omega_0\mu_0}\nabla \times \mathbf{E}_0$$

The radiation pressure:

$$p = -\frac{1}{2}\epsilon_0 (\mathbf{E}_0 \cdot \mathbf{n}_c) \cdot (\mathbf{E}_0^* \cdot \mathbf{n}_c) + \frac{1}{2}\mu_0 (\mathbf{H}_0 \times \mathbf{n}_c) \cdot (\mathbf{H}_0^* \times \mathbf{n}_c)$$



Step 3

Solve linear elasticity problem in the cavity walls:

$$\begin{cases} \nabla \cdot \left(2\eta\nabla^{(S)}\mathbf{u} + \lambda \mathbf{I}\nabla \cdot \mathbf{u}\right) = 0 & \text{in } \Omega_W \\ \mathbf{u} = 0 & \text{on } \Gamma_W \\ \left(2\eta\nabla^{(S)}\mathbf{u} + \lambda \mathbf{I}\nabla \cdot \mathbf{u}\right) \cdot \mathbf{n}_w = p \cdot \mathbf{n}_w & \text{on } \Gamma_{CW} \\ \left(2\eta\nabla^{(S)}\mathbf{u} + \lambda \mathbf{I}\nabla \cdot \mathbf{u}\right) \cdot \mathbf{n}_w = 0 & \text{on } \Gamma_{ext} \end{cases}$$

Deformed cavity:

$$\Gamma_{CW}^{'} \equiv \{\mathbf{x} + \mathbf{u}(\mathbf{x}), \mathbf{x} \in \Gamma_{CW}\}$$
$$\Omega_W^{'} \equiv \{\mathbf{x} + \mathbf{u}(\mathbf{x}), \mathbf{x} \in \Omega_W\}$$

Step 4

Repeat first step in the deformed cavity:

$$\begin{cases} \nabla \times \left(\frac{1}{\mu_0}\nabla \times \mathbf{E}^{'}\right) = \left(\omega_0^{'}\right)^2 \epsilon_0 \mathbf{E} & \text{in } \Omega_C^{'} \\ \mathbf{E}^{'} \times \mathbf{n}_c^{'} = 0 & \text{on } \Gamma_{CW}^{'} \\ \left(\frac{1}{\mu_0}\nabla \times \mathbf{E}^{'}\right) \times \mathbf{n}_c^{'} = 0 & \text{on } \Gamma_C \end{cases}$$

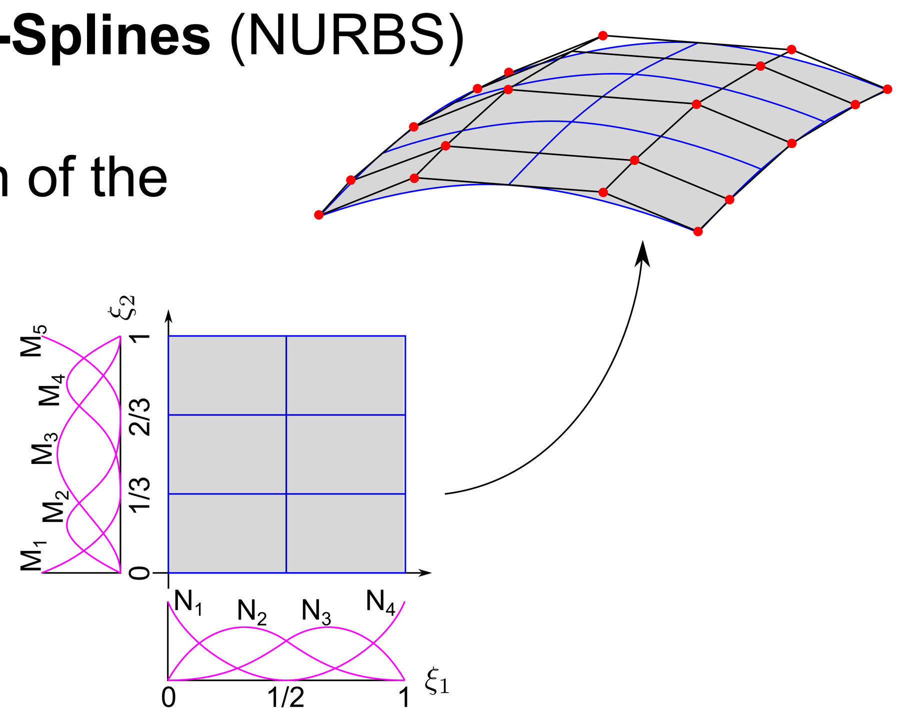
Frequency shift:

$$f_0^{'} = \frac{\omega_0^{'}}{2\pi} \qquad \Delta f_0 = \left|f_0 - f_0^{'}\right|$$

Isogeometric Analysis

Aim: "*bridging the gap between CAD and FEA*"

- Exact representation of CAD geometries using **B-Splines** and **Non-Uniform Rational B-Splines** (NURBS)
- Isoparametric approach
- Elegant and simple description of the deformed geometry
- No need of a re-meshing step
- Global **smoothness**



B-Spline basis functions are created from a knot vector

$$\Xi = [\xi_0, ..., \xi_{n+p+1}], \ \xi_i \in [0,1]$$

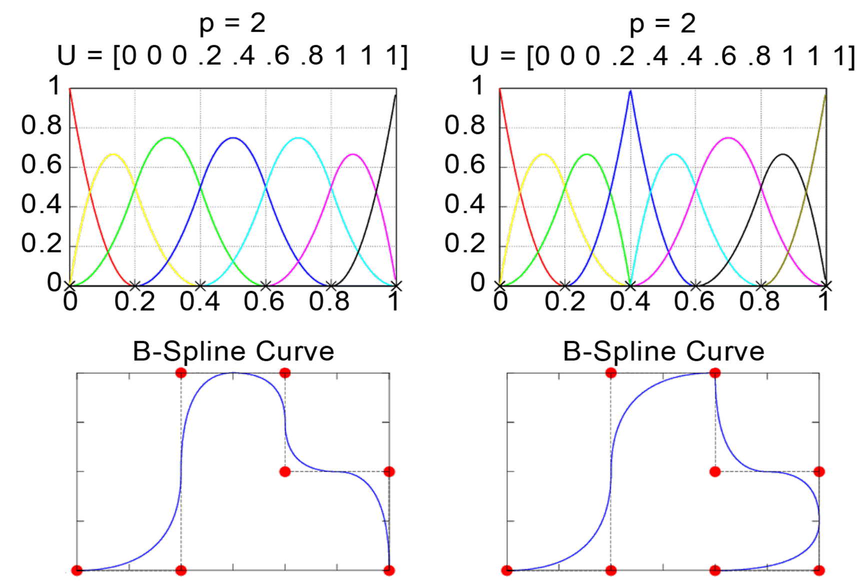
and a degree p.

They are then weighted by control points to create curves, surfaces and volumes:

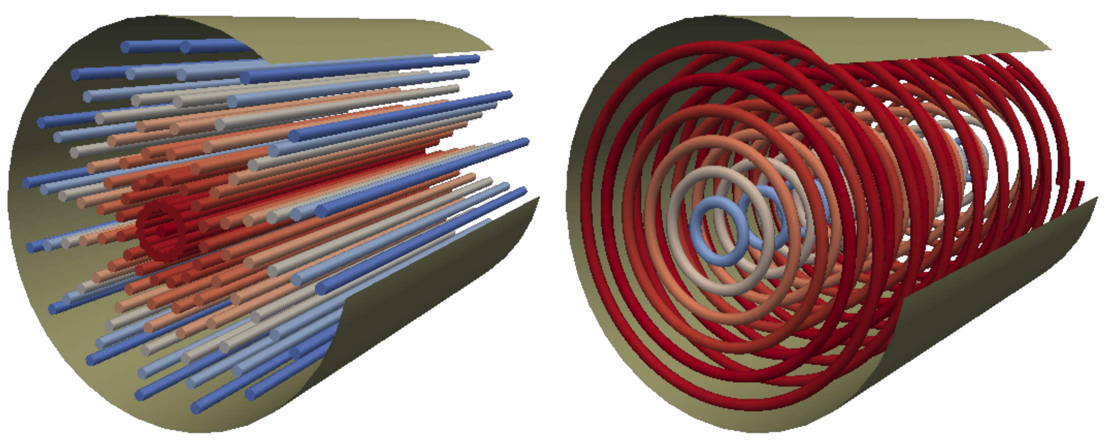
$$\mathbf{C}(\xi) = \sum_{i=0}^{n} N_{i,p}(\xi)\mathbf{P}_i$$

B-Spline properties:

- Point-wise non negative
- Partition of unity
- Convex hull
- Local support
- Up to *p* - 1 regularity
- Repeated knots affect smoothness



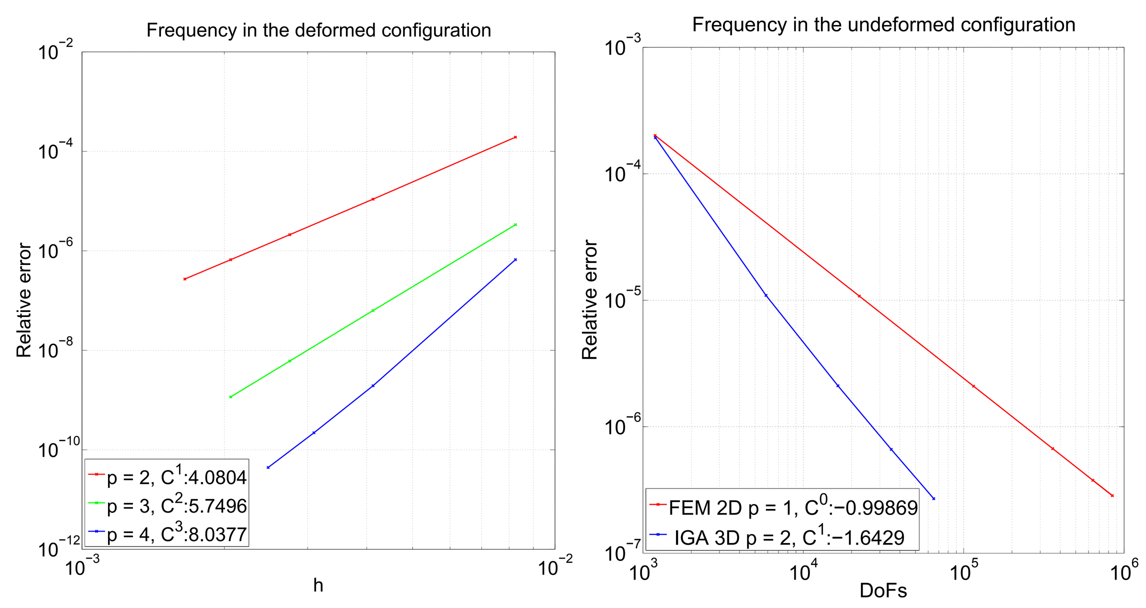
Validation - Pillbox Cavity



Accelerating mode in a cylindrical pillbox cavity.

Steps 1-4 have been applied and the detuning has been computed.

On the left:
Error w.r.t. the exact solution is shown. The multi-physical coupling does not decrease the optimal rate of convergence.



On the right:
IGA guarantees higher accuracy per DoF than Nédélec FEA in 2D.

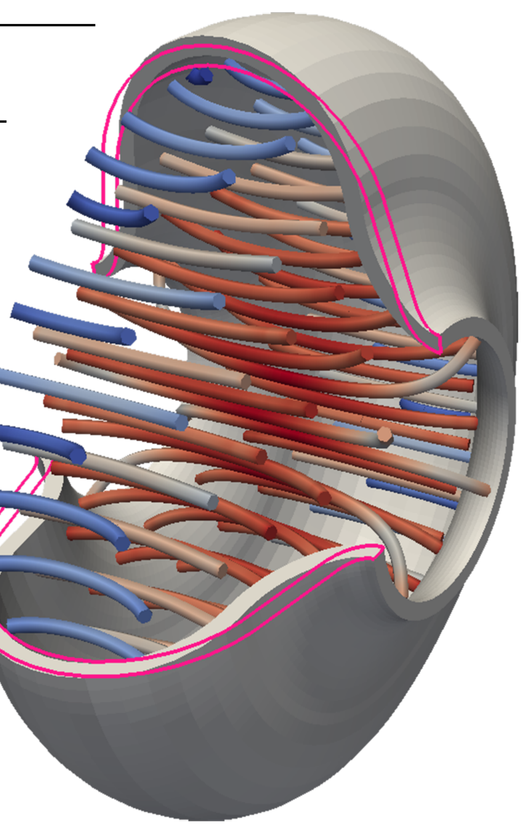
Conclusions

The scheme has been applied to the 1-cell TESLA cavity (on the right). The computed displacement is of the order of 10 nm which is in good accordance to results reported in literature [1].

The corresponding frequency shift is:

$$\Delta f_0 = 216 \text{ Hz}$$

with an accuracy of approximately ± 1 Hz.



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