Eigenmode Computation for Biased Ferrite-Loaded Cavity Resonators*

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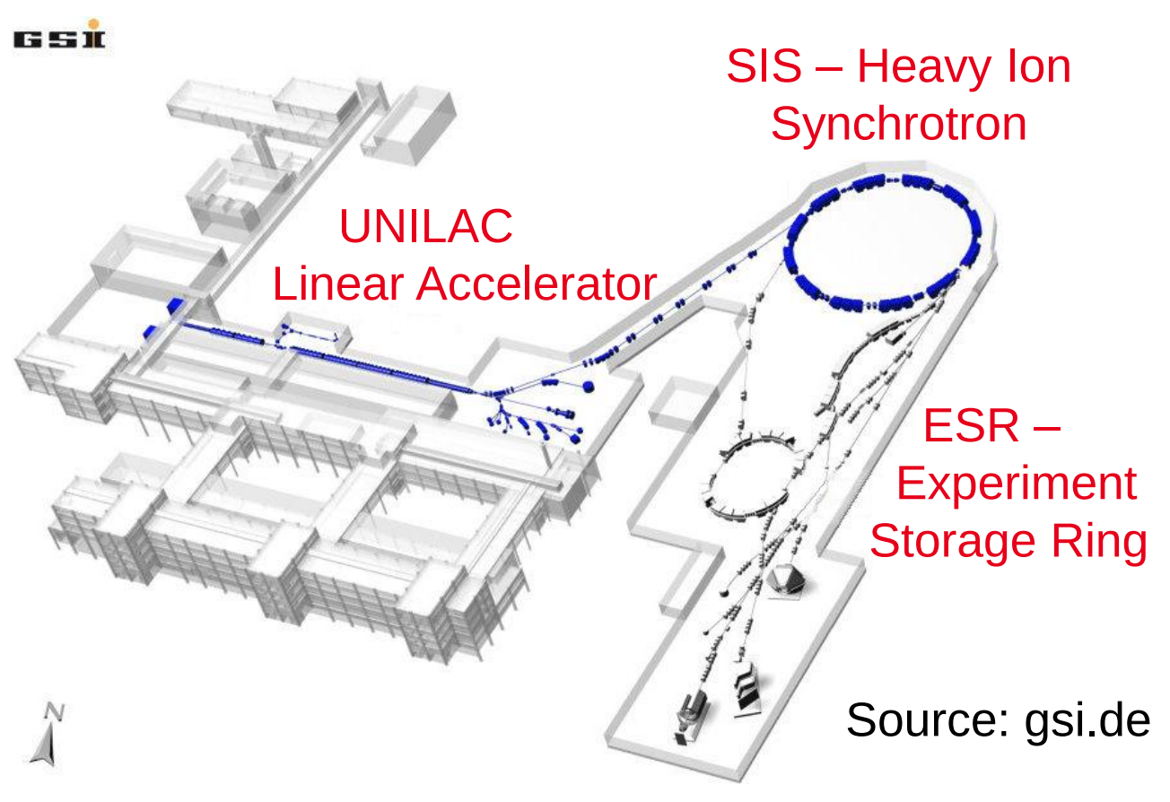
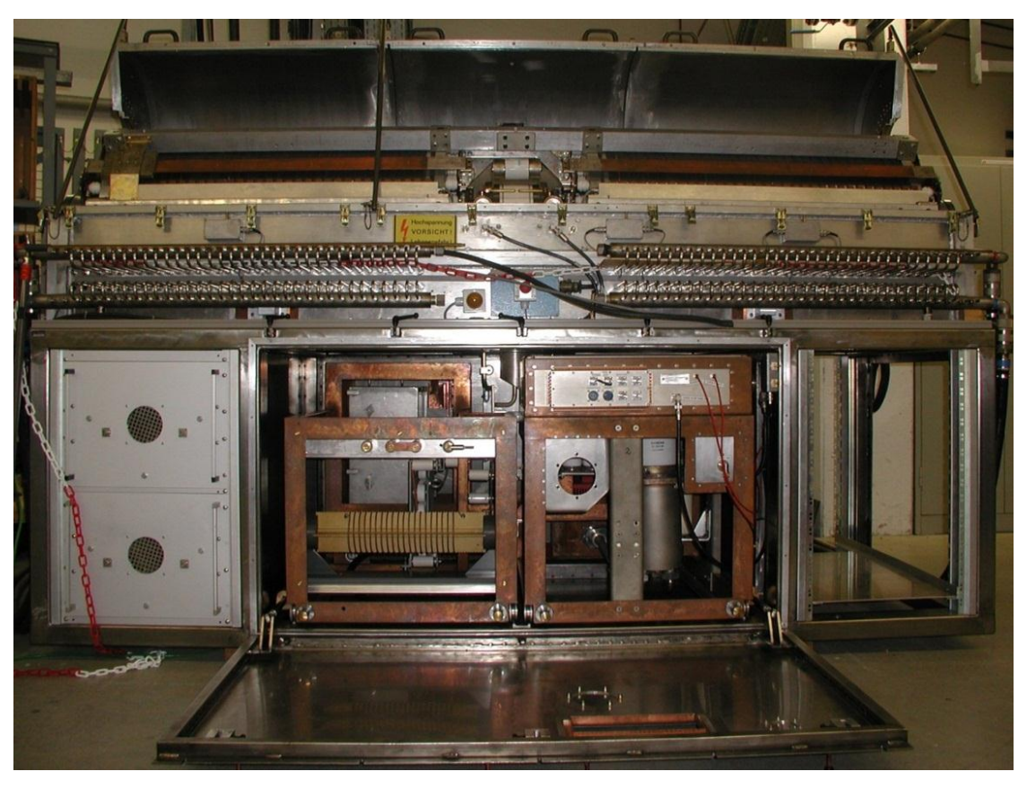
SIS 18 Ferrite Cavity of GSI

For acceleration of heavy ions at the synchrotron SIS18 of the GSI Helmholtzzentrum für Schwerionenforschung in Darmstadt two biased ferrite-loaded cavity resonators are installed.



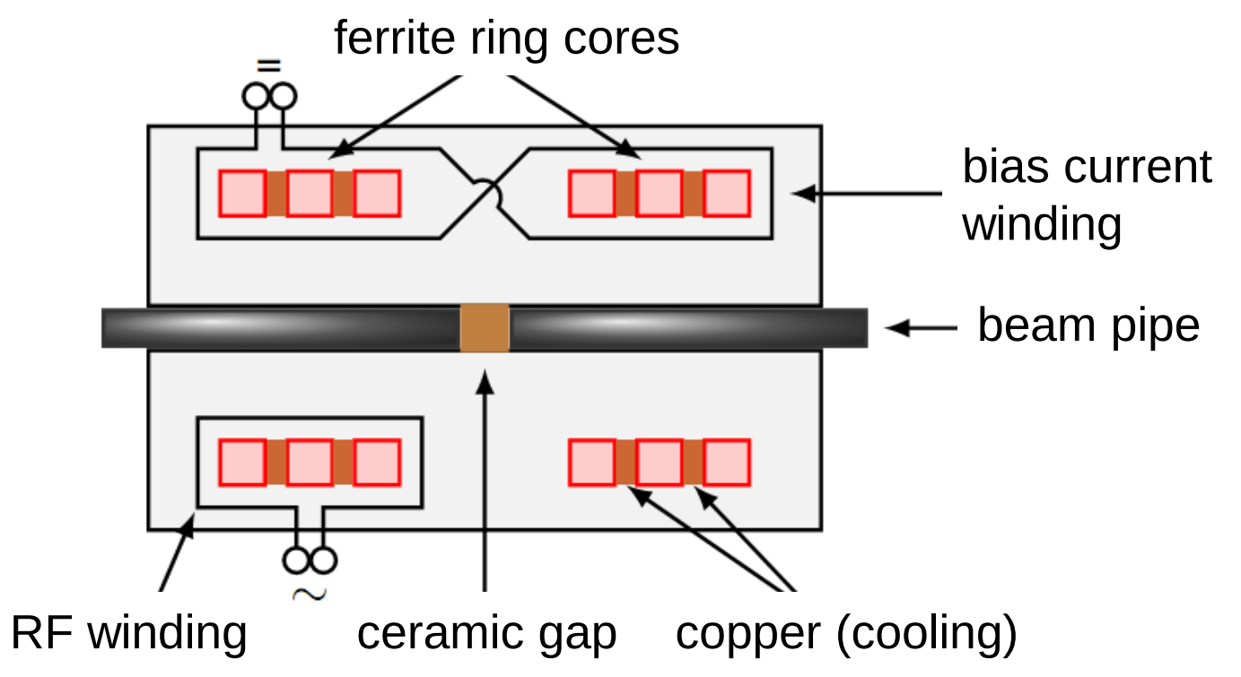
Source: gsi.de



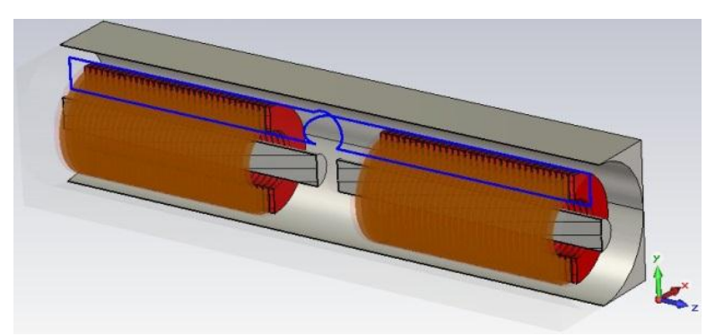
SIS 18 ferrite cavity. Source: GSI, A. Zschau

Cavity Parameters	
Length	3 m
Max. gap voltage	16 kV
Resonance frequency	~ 0.8 – 5.4 MHz
Max. bias current	~ 500 A

During the acceleration phase the resonance frequency has to be adjusted to reflect the increasing speed of the heavy ions.



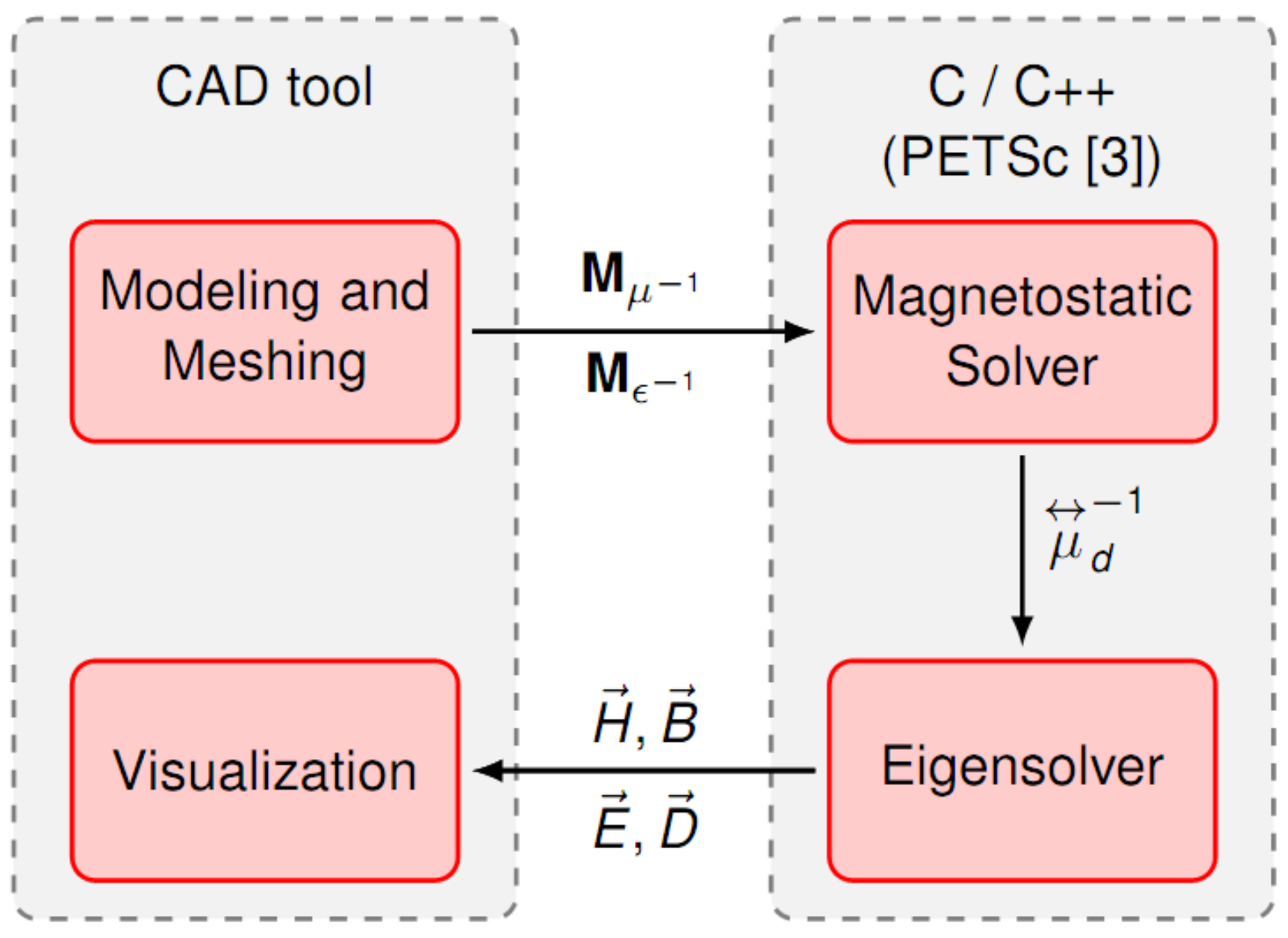
Simplified 2D-model of the SIS 18 ferrite cavity.



Simplified 3D-model of the SIS 18 ferrite cavity.

Computational Model

Implementation
The current implementation is based on the Finite Integration Technique [2] using a hexahedral staircase mesh.



CAD tool

Modeling and Meshing

Visualization

C / C++ (PETSc [3])

Magnetostatic Solver

Eigensolver

$\mathbf{M}_{\mu^{-1}}$

$\mathbf{M}_{\epsilon^{-1}}$

$\overset{\leftrightarrow}{\mu}_d^{-1}$

$\vec{H}, \vec{B}$

$\vec{E}, \vec{D}$

Magnetostatic field problem
Helmholtz decomposition of H-field
$\vec{H} = \vec{H}_i + \vec{H}_h$ with
$\nabla \times \vec{H}_i = \vec{J}$ and $\vec{H}_h = -\nabla\varphi$
↑
(bias) current density

Jacobi-Davidson algorithm
The nonlinear eigenvalue problem is iteratively solved as a sequence of linearized eigenproblems.

General requirements
The solver should support nonlinear and lossy material.
The implementation aims at efficient distributed computing (scalability).

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[2] T. Weiland, "A Discretization Method for the Solution of Maxwell's Equations for Six-Component Fields", Electr. and Comm. AEUE, vol. 31, no. 3, pp. 116-120, 1977.
[3] S. Balay et al., "PETSc Users Manual", ANL-95/11 - Revision 3.2, Argonne National Laboratory, 2011.

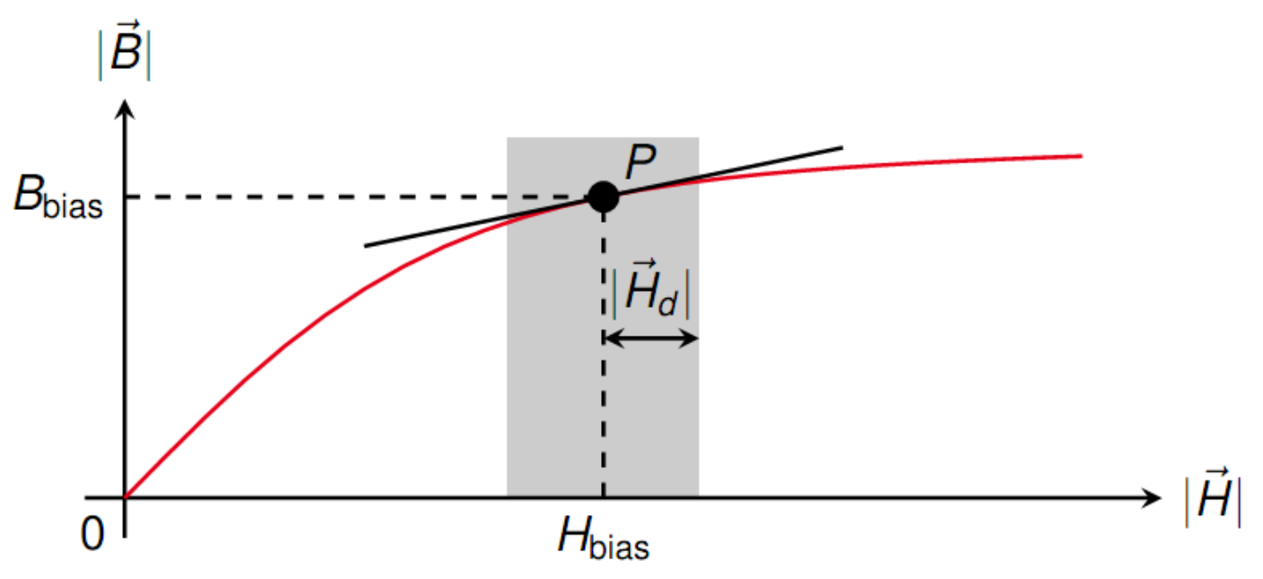
Resonance Frequency Tuning

The magnetic induction inside the accelerating cavity can be decomposed into

$$\vec{B}(t) = \mu_0\mu_{\text{bias}}\vec{H}_{\text{bias}} + \mu_0\overset{\leftrightarrow}{\mu}_d\,\text{Re}\left(\vec{H}_d \cdot e^{-\text{i}\omega t}\right).$$

The eigenvectors are calculated under the assumptions that $|\vec{H}_d| \ll |\vec{H}_{\text{bias}}|$ and that effects of hysteresis are negligible. This allows a linearization of the constitutive equation at the working point.

Modification of bias current
⇒ Modification of differential permeability
⇒ Adjustment of eigenfrequency



Fundamental Relations

Eigenvalue formulation

$$\boxed{\begin{aligned}\epsilon^{-1}\nabla \times \left(\mu_0^{-1}\overset{\leftrightarrow}{\mu}_d^{-1}\nabla \times \vec{E}(\vec{r},t)\right) &= \omega^2\vec{E}(\vec{r},t), \;\; \vec{r} \in \Omega,\\ \vec{n} \times \vec{E}(\vec{r},t) &= 0, \;\; \vec{r} \in \partial\Omega.\end{aligned}}$$

Properties of the differential permeability tensor $\overset{\leftrightarrow}{\mu}_d$

1. Fully occupied (3x3) – tensor, which for a bias magnetic field aligned with the z - axis reduces to the well-known Polder tensor [1]

$$\overset{\leftrightarrow}{\mu}_d = \begin{pmatrix} \mu_1 & \text{i}\,\mu_2 & 0 \\ -\text{i}\,\mu_2 & \mu_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{with} \qquad \mu_{1,2} = \mu_{1,2}\big(\vec{H}_{\text{bias}}, \omega\big).$$

2. Non-Hermitian if magnetic losses are taken into account, i.e. $\text{Im}(\mu_{1,2}) \neq 0$.

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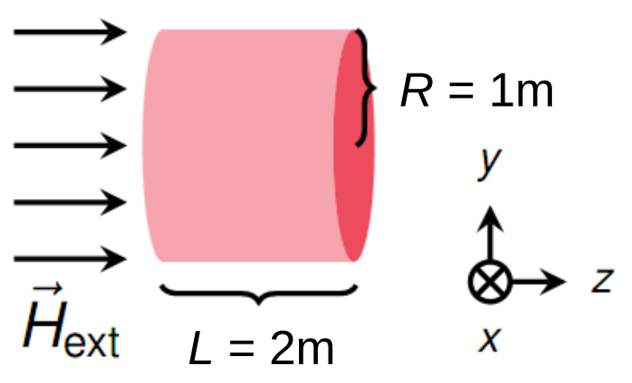
[1] D. Polder, Phil. Mag., 40, p. 99, 1949.

Numerical Examples

Biased cylinder

For verification of the nonlinear eigensolver the following model is considered:

Lossless, ferrite-filled cylindrical cavity resonator, longitudinally biased by a homogeneous magnetic field



Parameters	
$\lvert\vec{H}_{\text{ext}}\rvert$	2750 A / m
μ_r	7
ϵ_r	1

A characteristic equation determining the resonance frequencies can be formulated analytically [4].

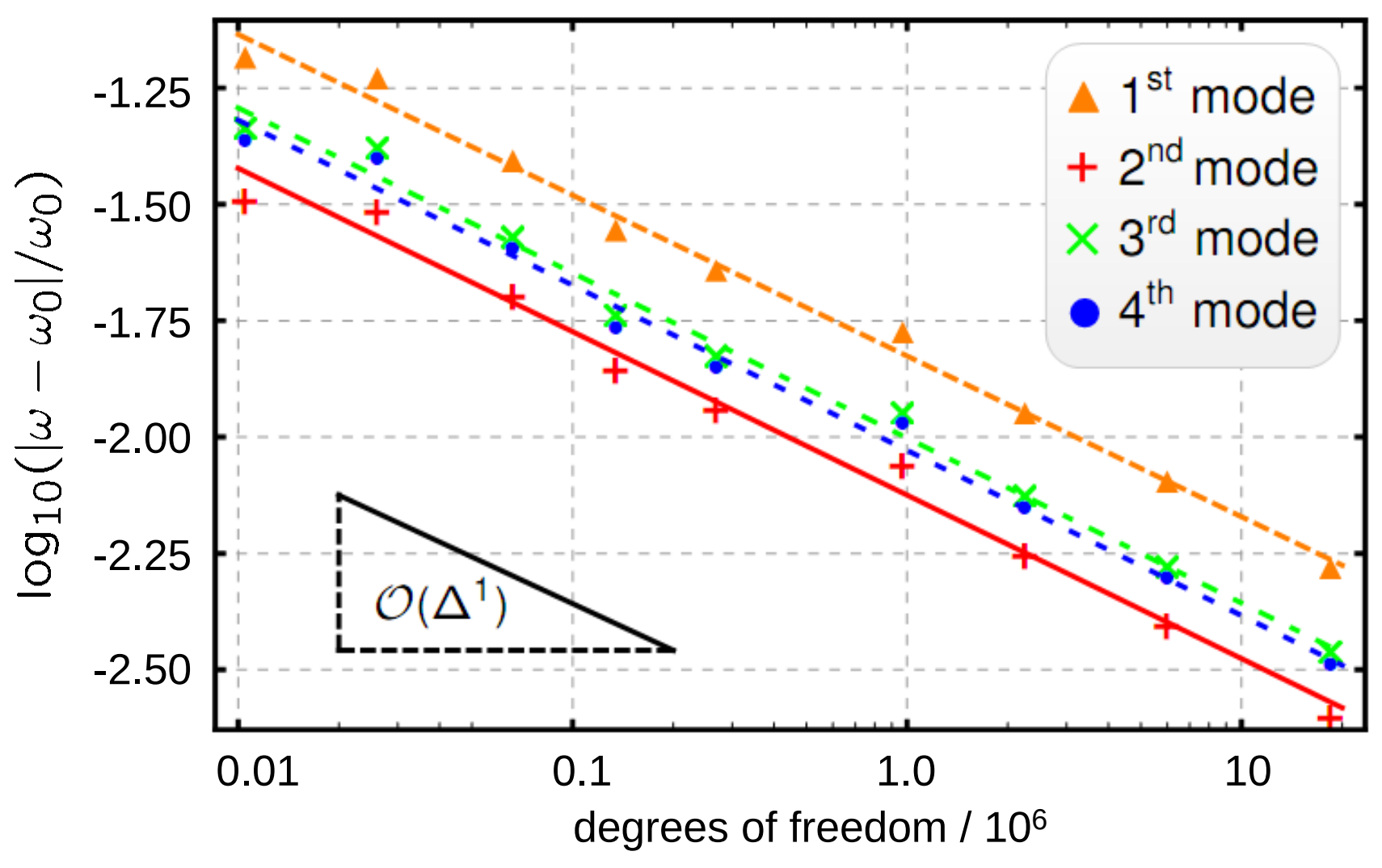


Figure: Relative deviation of the numerically obtained value ω to the analytical result ω_0 as a function of the degrees of freedom.

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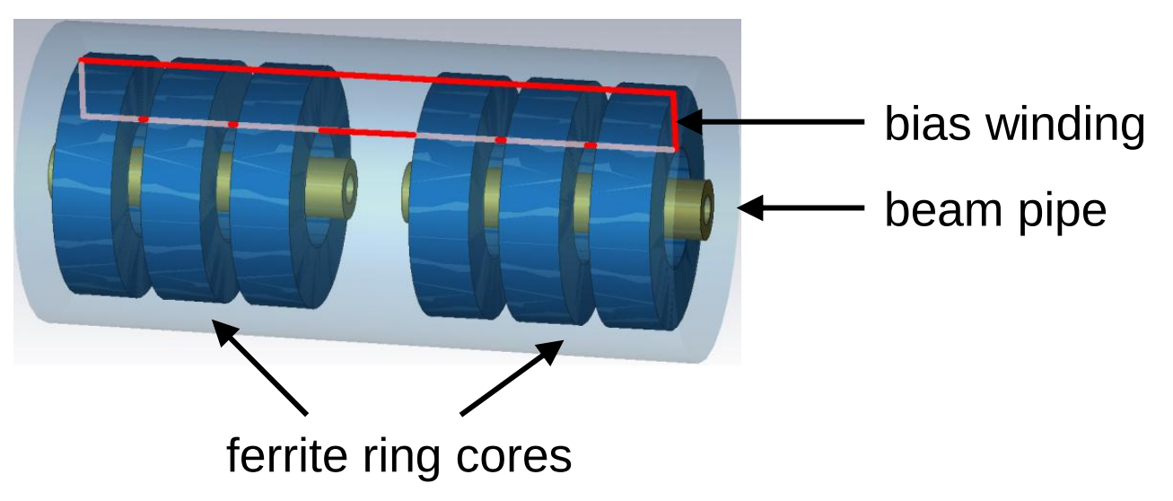
[4] G. C. Chinn, L.W. Epp and G.M.Wilkins, IEEE Transactions on Microwave Theory Techniques, 43, May 1995.

Biased cavity with ferrite ring cores



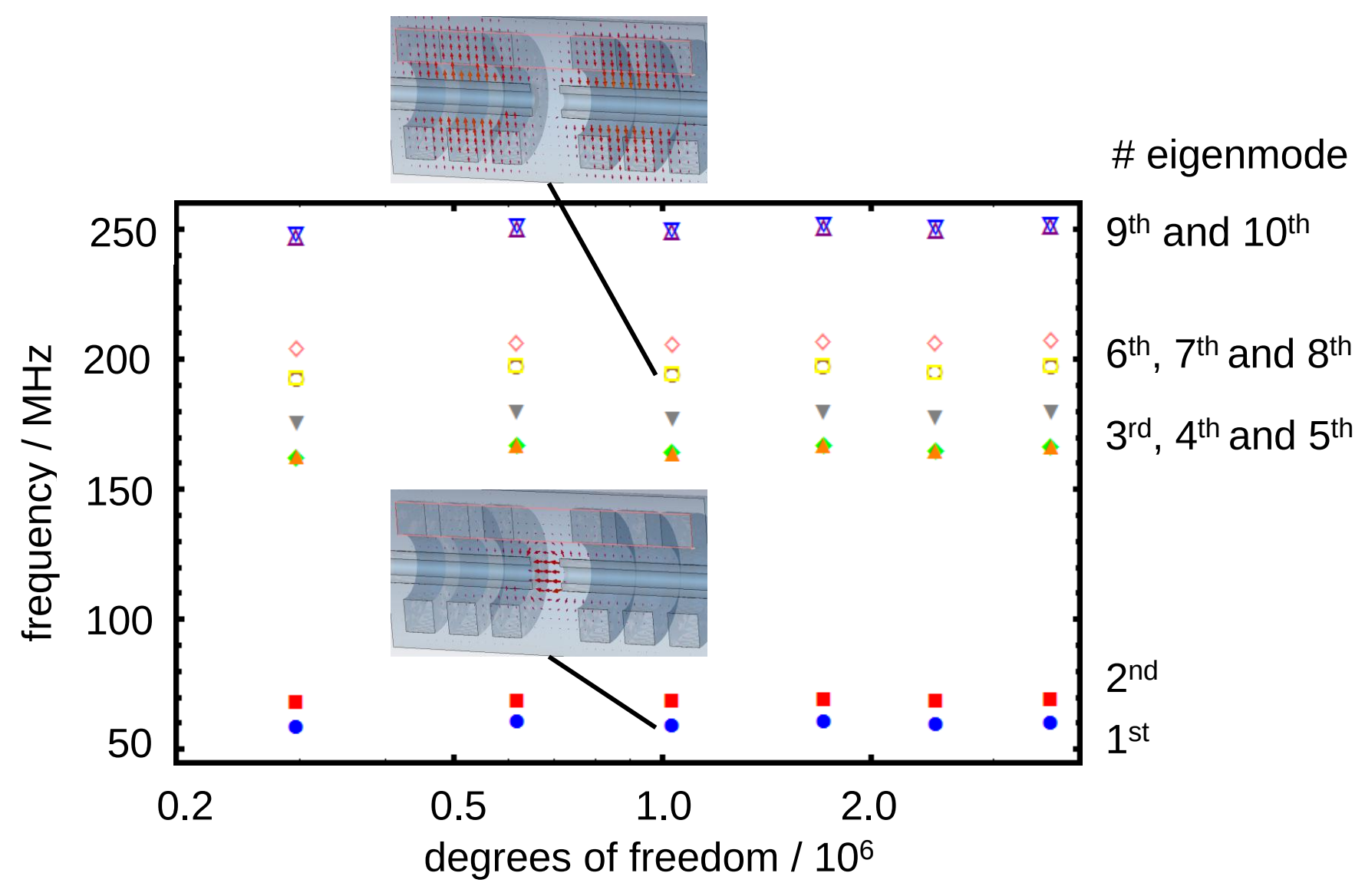


Figure: Results of fully nonlinear computation for lossless resonator

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