

Magnet Vibration and Feedbacks

Andrei Seryi
John Adams Institute

CERN Accelerator School
Beam dynamics and technologies for future colliders

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Two scientific instruments



LIGO, Hanford



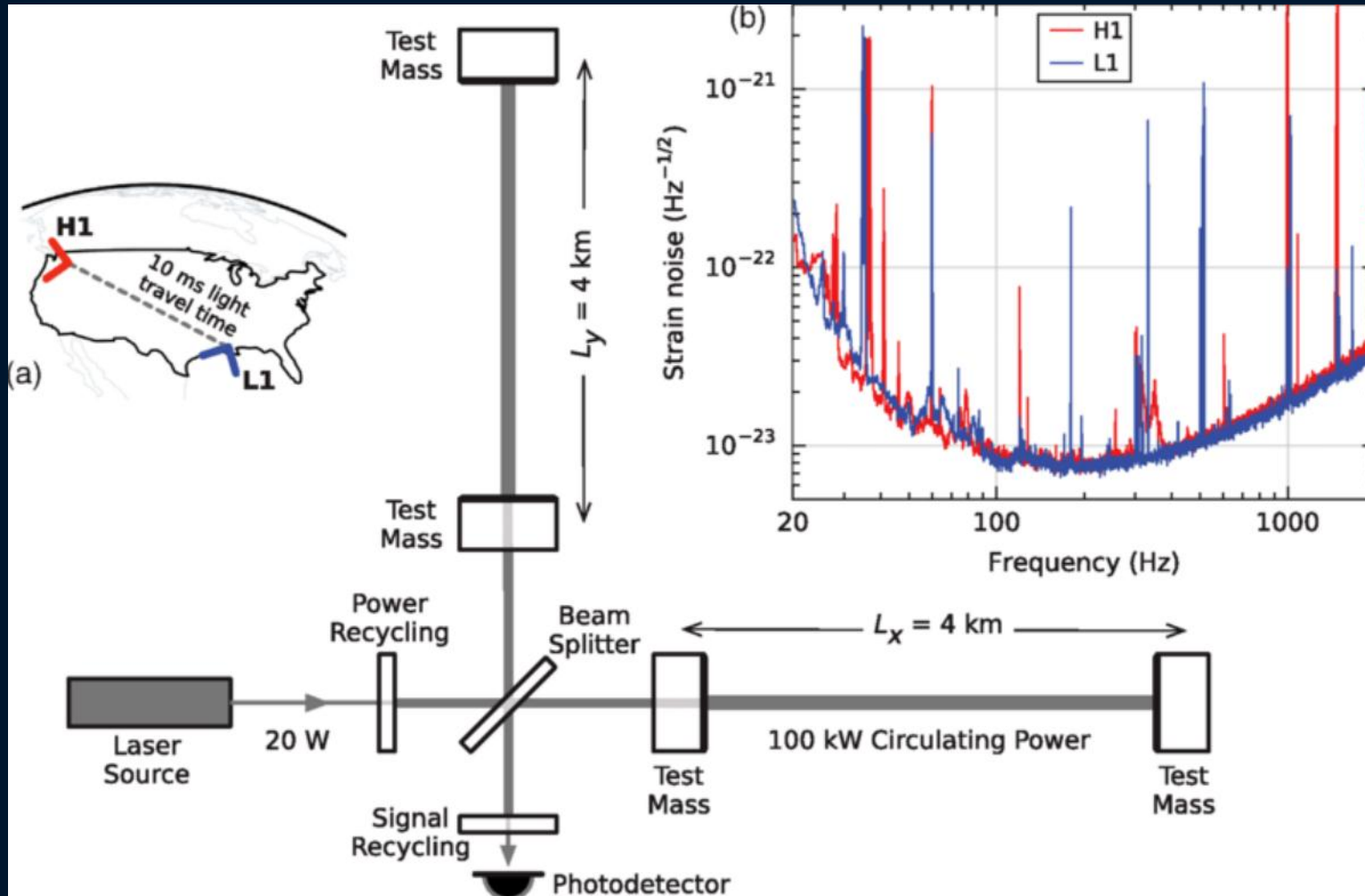
SLC, Stanford

What is in common?

A lot of things

And also sensitivity to seismic noises.

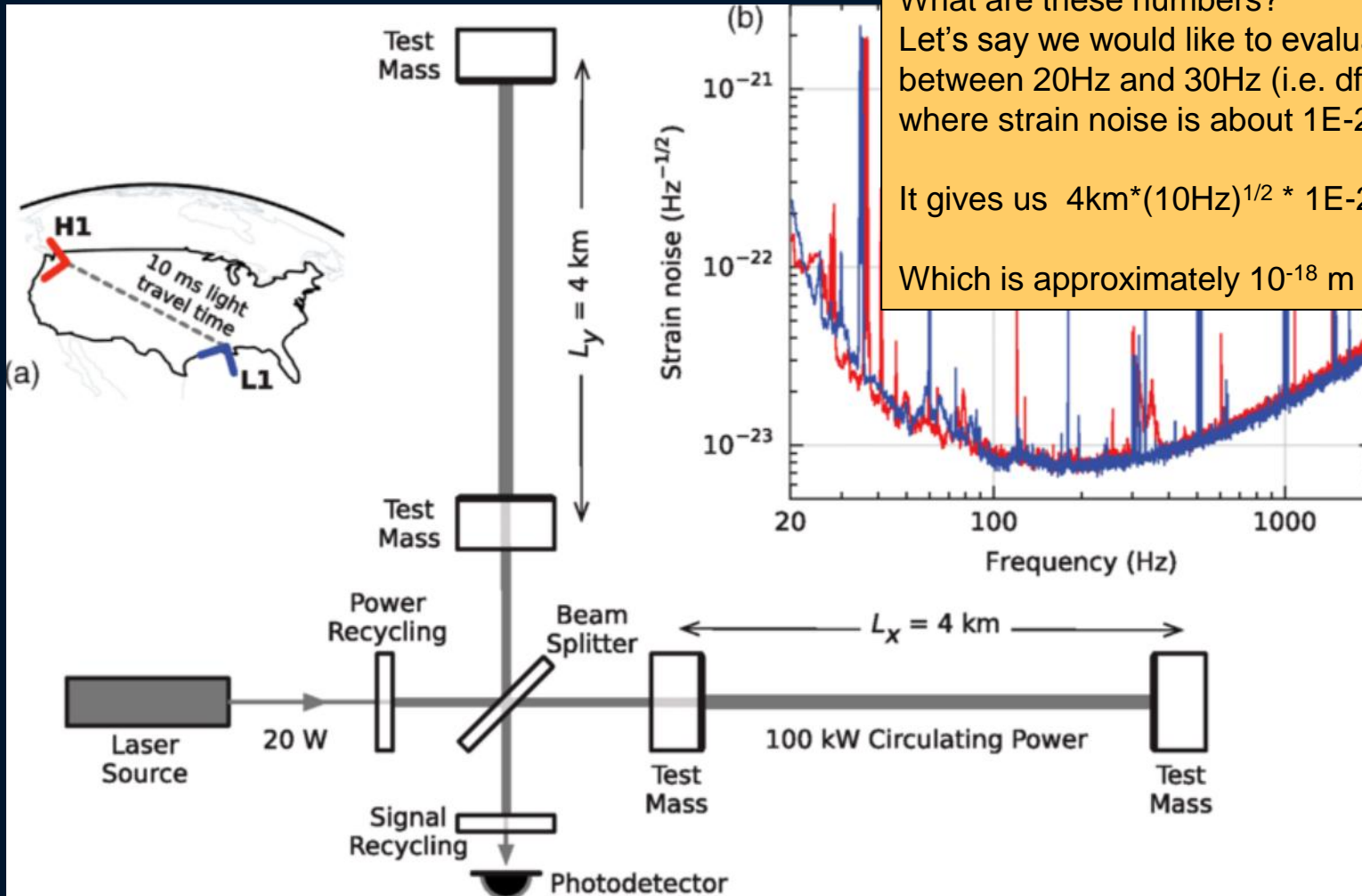
LIGO



LIGO layout and sensitivity curve

Source: PRL 116, 061102 (2016)

LIGO



What are these numbers?
 Let's say we would like to evaluate noise between 20Hz and 30Hz (i.e. $\Delta f = 10\text{Hz}$), where strain noise is about $1\text{E-}22 \text{ Hz}^{-1/2}$

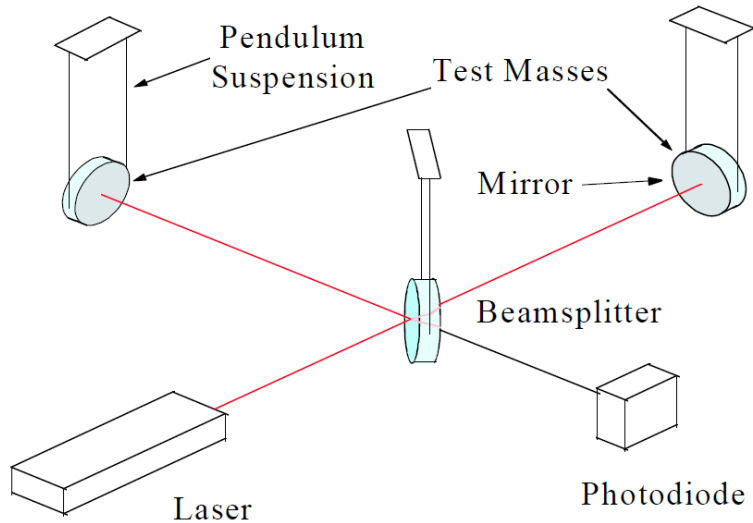
It gives us $4\text{km} * (10\text{Hz})^{1/2} * 1\text{E-}22 \text{ Hz}^{-1/2}$

Which is approximately 10^{-18} m

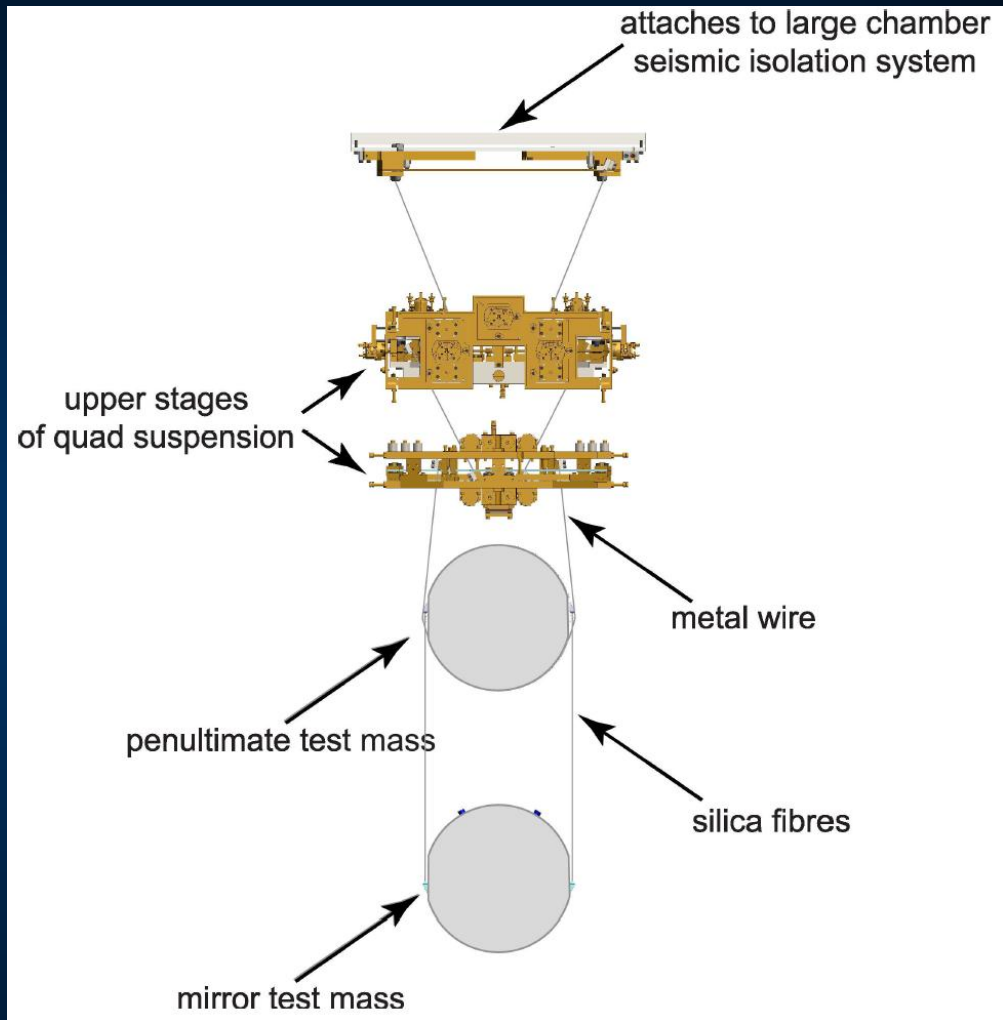
LIGO layout and sensitivity curve

Source: PRL 116, 061102 (2016)

LIGO test mass isolation



Concept



Solution: nested pendulums

Source: [arXiv:1102.3355](https://arxiv.org/abs/1102.3355)

LIGO seismic sensitivity

Gravity gradients, caused by **direct gravitational coupling of mass density fluctuations to the suspended mirrors**, were identified as a potential source of noise in ground-based gravitational-wave detectors in 1972 [312]. The noise associated with gravity gradients was first formulated by Saulson [274] and Spero [290], with later developments by Hughes and Thorne [183] and Cella and Cuoco [93]. These studies suggest that the **dominant source of gravity gradients arise from seismic surface waves**, where density fluctuations of the Earth's surface are produced near the location of the individual interferometer test masses, as shown in Figure 7.

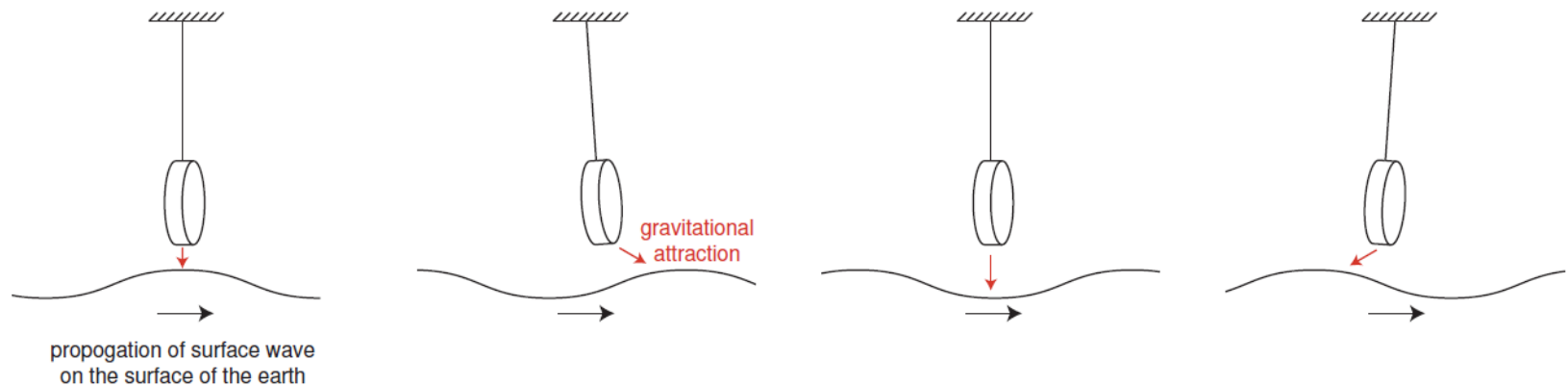


Figure 7: Time-lapsed schematic illustrating the fluctuating gravitational force on a suspended mass by the propagation of a surface wave through the ground.

Source: arXiv:1102.3355

Seismic gravity-gradient noise in interferometric gravitational-wave detectors

Scott A. Hughes

Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125

Kip S. Thorne

*Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125
and Max-Planck-Institut für Gravitationsphysik, Schlatzweg 1, 14473 Potsdam, Germany*

(Received 4 June 1998; published 18 November 1998)



Drawing by Glen Edwards, Utah State University, Logan, UT

When ambient seismic waves pass near and under an interferometric gravitational-wave detector, they induce density perturbations in the Earth, which in turn produce fluctuating gravitational forces on the interferometer's test masses. These forces mimic a stochastic background of gravitational waves and thus constitute a noise source. This seismic gravity-gradient noise has been estimated and discussed previously by Saulson

at noisy times, and (iii) a corresponding estimate of the magnitude of $\beta'(f)$ at quiet and noisy times. We conclude that at quiet times $\beta' \approx 0.35-0.6$ at the LIGO sites, and at noisy times $\beta' \approx 0.15-1.4$. (For comparison, Saulson's simple model gave $\beta = \beta' = 1/\sqrt{3} = 0.58$.) By folding our resulting transfer function into the "standard LIGO seismic spectrum," which approximates $\tilde{W}(f)$ at typical times, we obtain the gravity-gradient noise spectra. At quiet times this noise is below the benchmark noise level of "advanced LIGO interferometers" at all frequencies (though not by much at ~ 10 Hz); at noisy times it may significantly exceed the advanced noise level near 10 Hz. The lower edge of our quiet-time noise constitutes a limit, beyond which

Source for portrait: Caltech web

Human gravity-gradient noise in interferometric gravitational-wave detectors

Kip S. Thorne

*Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125
and Max-Planck-Institut für GravitationsPhysik, Schlatzweg 1, 14473 Potsdam, Germany*

Carolee J. Winstein

*Department of Biokinesiology and Physical Therapy, University of Southern California, Los Angeles, Ca
(Received 5 October 1998; published 24 September 1999)*



Drawing by Glen Edwards, Utah State University, Logan, UT

Among all forms of routine human activity, the one which produces the strongest gravity-gradient noise in interferometric gravitational-wave detectors (e.g. LIGO) is the beginning and end of weight transfer from one foot to the other during walking. The beginning and end of weight transfer entail sharp changes (time scale

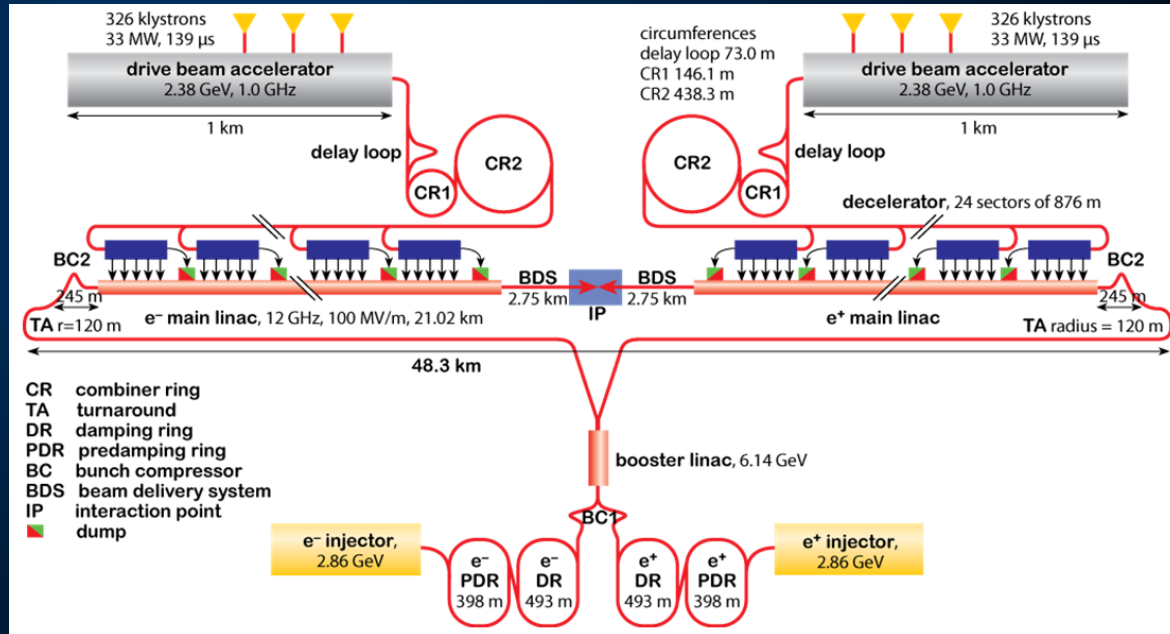
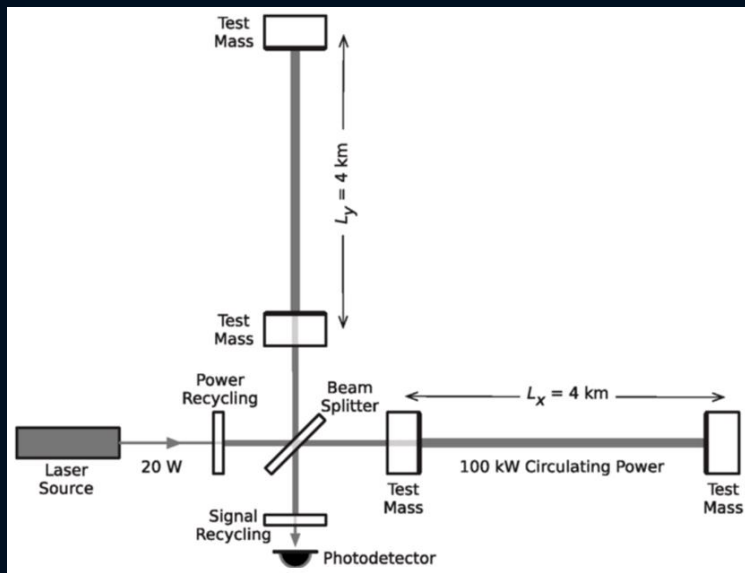
test mass, and we estimate this formula to be accurate to within a factor 3. To ensure that this noise is negligible in advanced LIGO interferometers, people should be prevented from coming nearer to the test masses than $r \approx 10$ m. A $r \approx 10$ m exclusion zone will also reduce to an acceptable level gravity gradient noise from the slamming of a door and the striking of a fist against a wall. The dominant gravity-gradient noise from automobiles and other vehicles is probably that from decelerating to rest. To keep this below the sensitivity of advanced LIGO interferometers will require keeping vehicles at least 30 m from all test masses.

Source for portrait: Caltech web

These two instruments

LIGO: keep two objects placed 4km apart stable* to about $1e-9$ nm

CLIC – Compact Linear Collider: keep $\sim 1e4$ objects distributed over 50km **stable*** to about 10 nm



*) approximately, and in certain frequency range

After this lecture you will have knowledge that would help you to quantify this statement

Let's now discuss stability issues in accelerators more systematically

As stability issues much more severe for linear colliders, we will focus primarily on LCs

The first ever linear collider



SLC e^+e^- Linear Collider

for center of mass energy **50 GeV**

The first ever linear collider



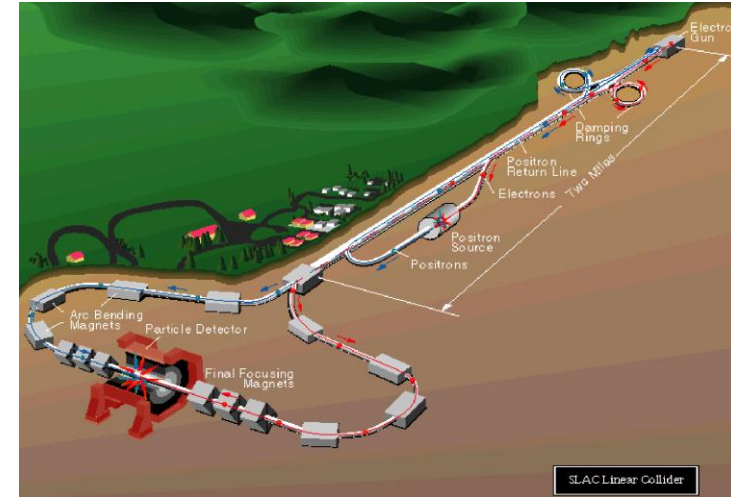
SLC e^+e^- Linear Collider

for center of mass energy **50 GeV**

SLAC Linear Collider

The challenge of Linear Collider – Luminosity

- Energy: initial goal 250GeV CM
 - This is “just” 5 times more than SLC
- But Luminosity: x 10000 !!!
(vs the only so far linear collider SLC)
 - Many improvements needed, to ensure this : generation of smaller beams, their better preservation, ...

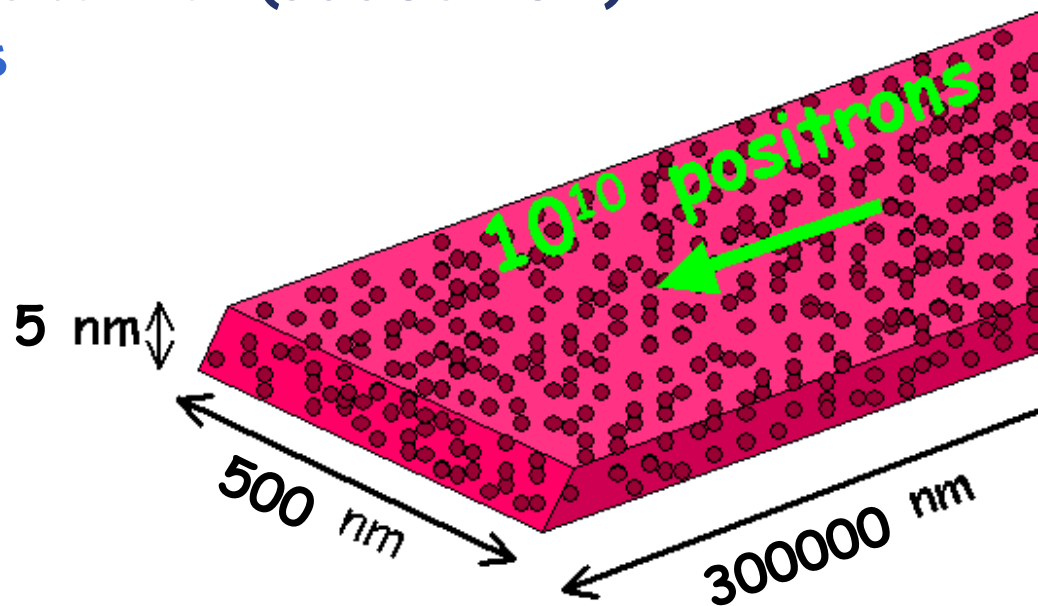
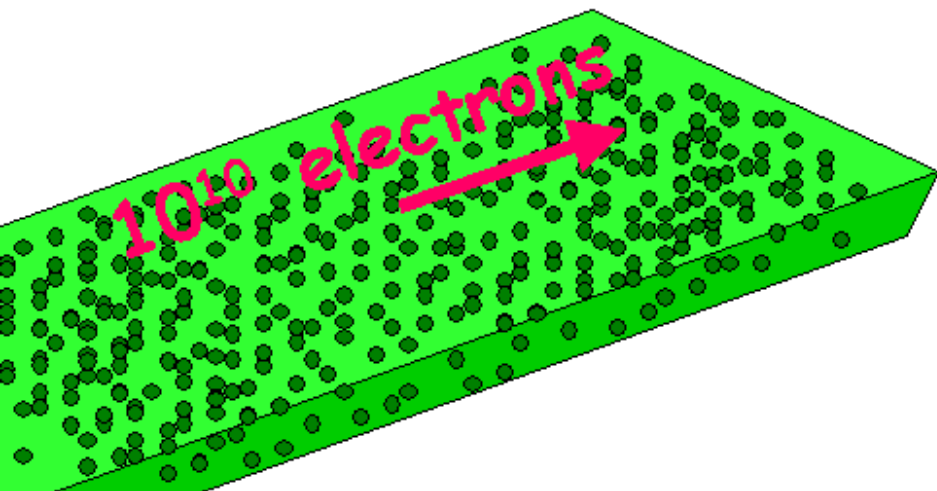


- Technical and natural vibration and natural ground motion continuously **misalign** components of a linear collider => may be a limiting factor

How to get Luminosity

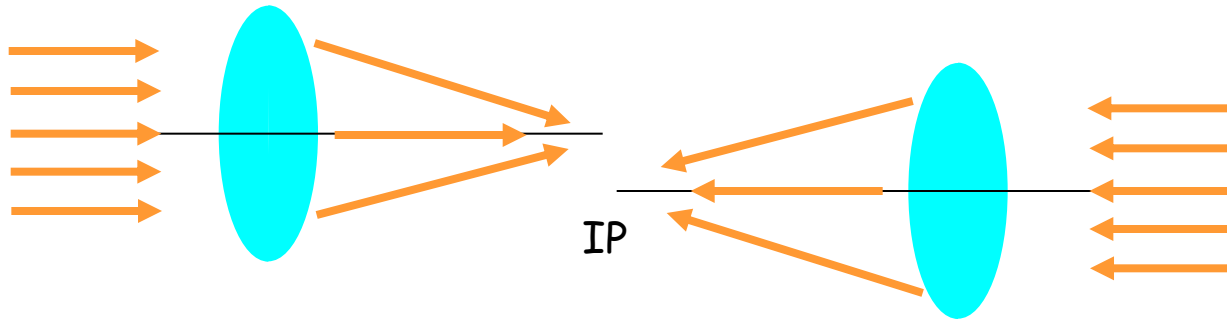
- To increase probability of direct e^+e^- collisions (luminosity) and birth of new particles, beam sizes at IP must be very small
- E.g., ILC beam sizes just before collision (500GeV CM):
 $500 * 5 * 300000$ nanometers
 (x y z)

↑
 Vertical size
 is smallest



$$L = \frac{f_{rep}}{4\pi} \frac{n_b N^2}{\sigma_x \sigma_y} H_D$$

Stability – tolerance to motion of final lenses

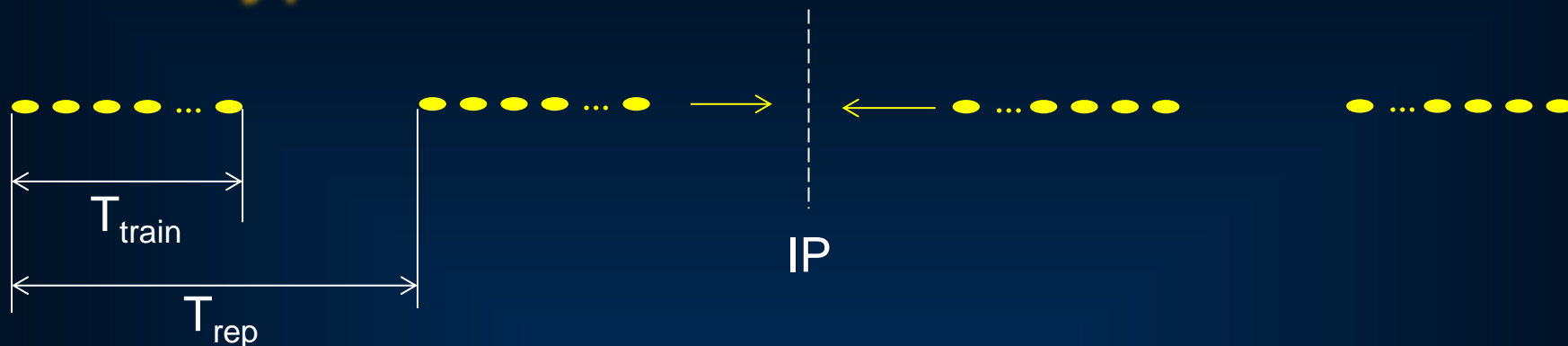


- Displacement of final lenses (final doublet - FD) cause similar displacement of the beams at the Interaction Point (IP)
- Therefore, stability of FD need to be maintained with a fraction of nanometer accuracy
 - Slow (in comparison with repetition rate of collisions) drifts can be corrected
 - Fast motion is more dangerous

Slow and Fast

- **Static misalignments can be corrected**
- **Slowly evolving misalignments can also be corrected**
- **Rapidly evolving misalignments harder to correct**
- **What defines if motion is “slow” or “fast”?**

Typical bunch train formats in LC



Case 1: T_{train} is typically 100 ns, with ~50 bunches per train
 T_{rep} corresponds to ~50 Hz

NC RF

Case 2: T_{train} is typically 1 ms, with ~3000 bunches per train
 T_{rep} corresponds to ~5 Hz

SC RF

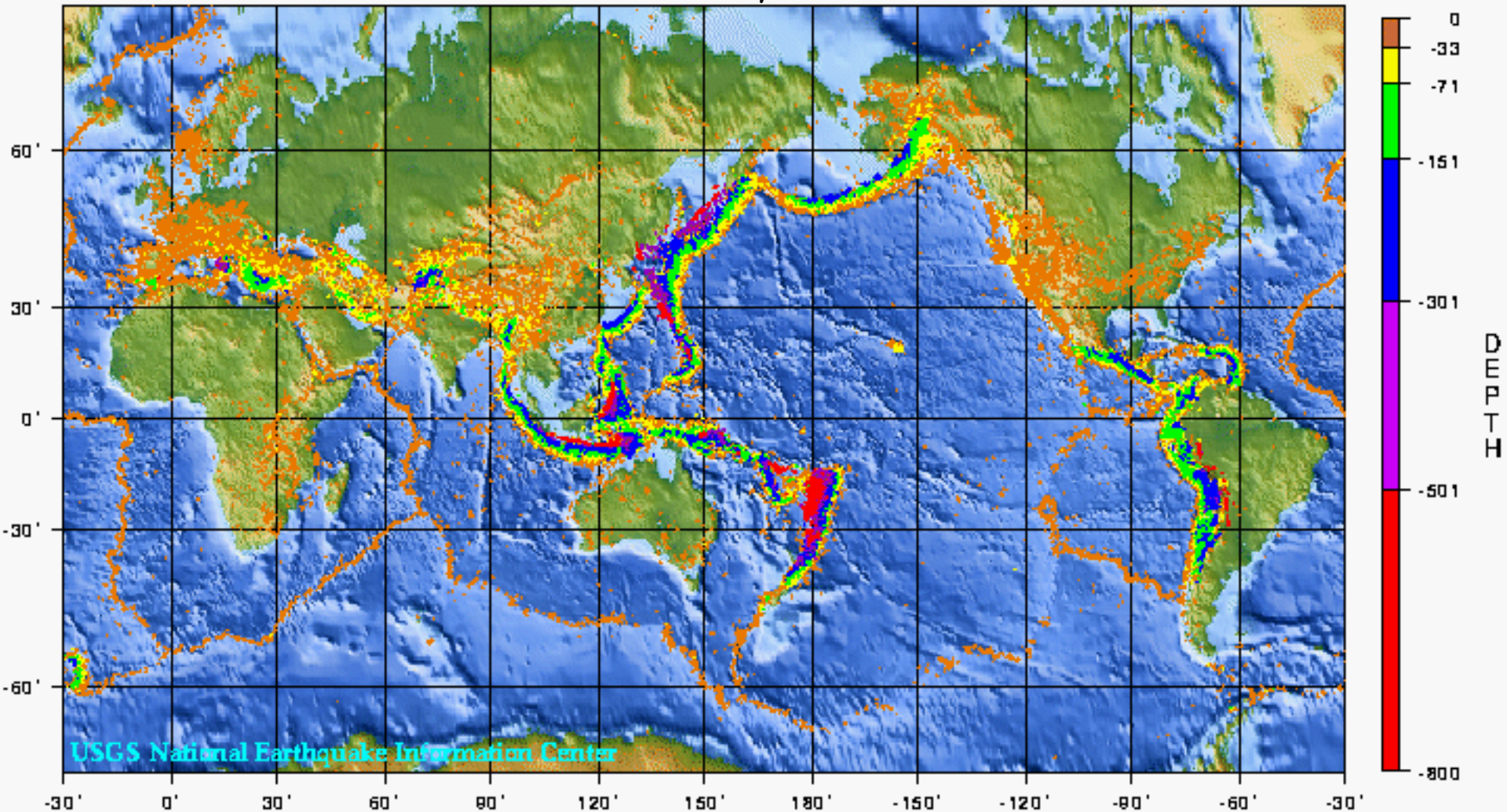
Capability of train-to-train and bunch-to-bunch corrections are quite different in these two cases. Also different which disturbances we consider fast and which slow. In this lecture we mostly focus on Case 1

Slow and Fast

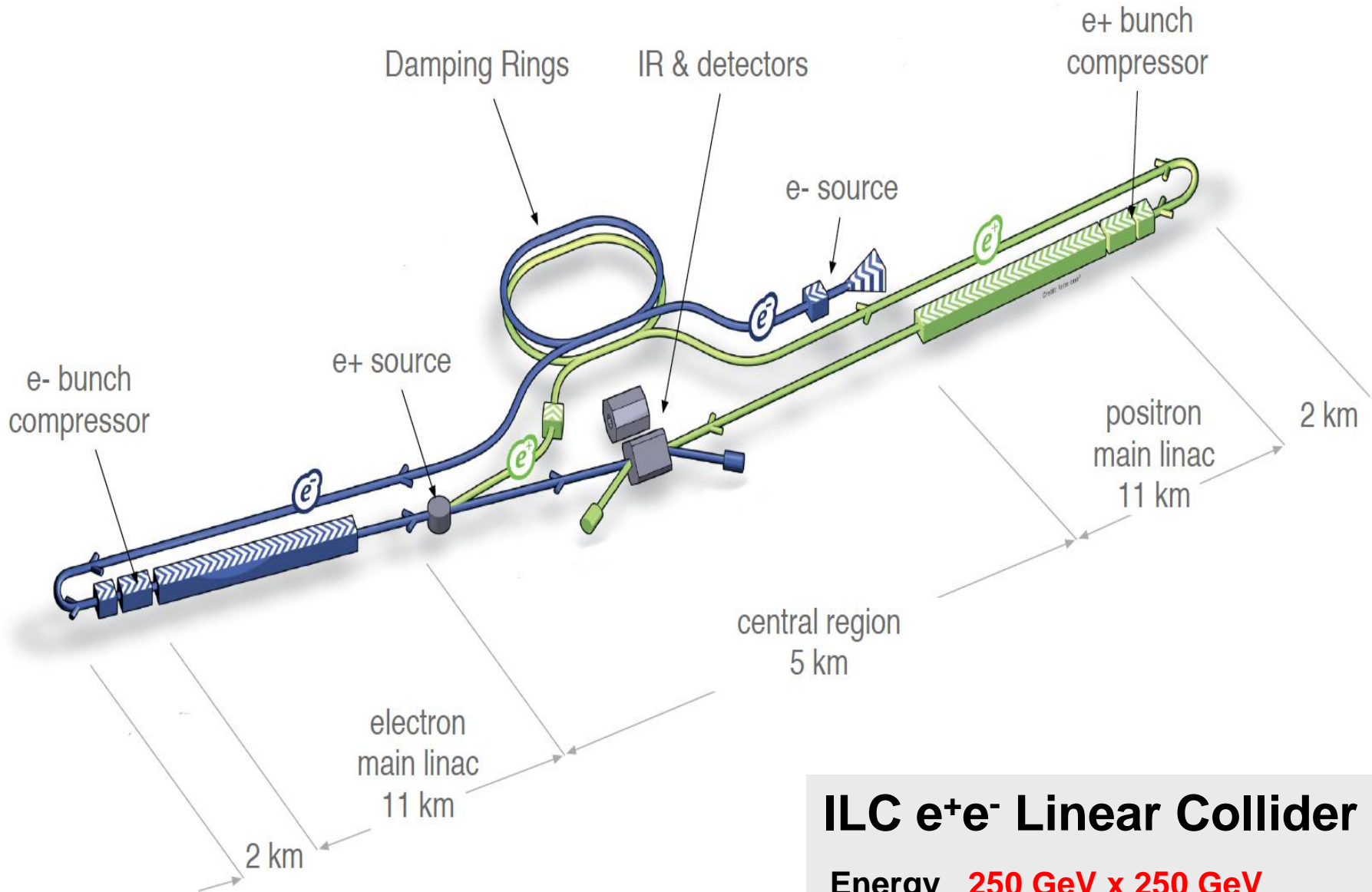
- **Static misalignments can be corrected**
- **Slowly evolving misalignments can also be corrected**
- **Rapidly evolving misalignments harder to correct**
- **What defines if motion is “slow” or “fast”?**
- **Are there other important characteristics in addition to “slow” or “fast”?**

We are concerned not so much about earthquakes...

World Seismicity: 1975-1995



International Linear Collider ILC



ILC e⁺e⁻ Linear Collider

Energy 250 GeV x 250 GeV

ILC - possibly in Japan



- Japanese Mountainous Sites -

Stable granite site



Site-A KITAKAMI



TOHOKU district

私たちは国際リニアコライダー計画を応援しています
We support the International Linear Collider Project.
— 国府工会議所 / 岩手県ILC推進協議会



The final decision will be made by the Government of Japan in the coming years

... and not about slow tidal motion

Variation of LEP ring circumference was noticed, via precise measurement of the beam energy

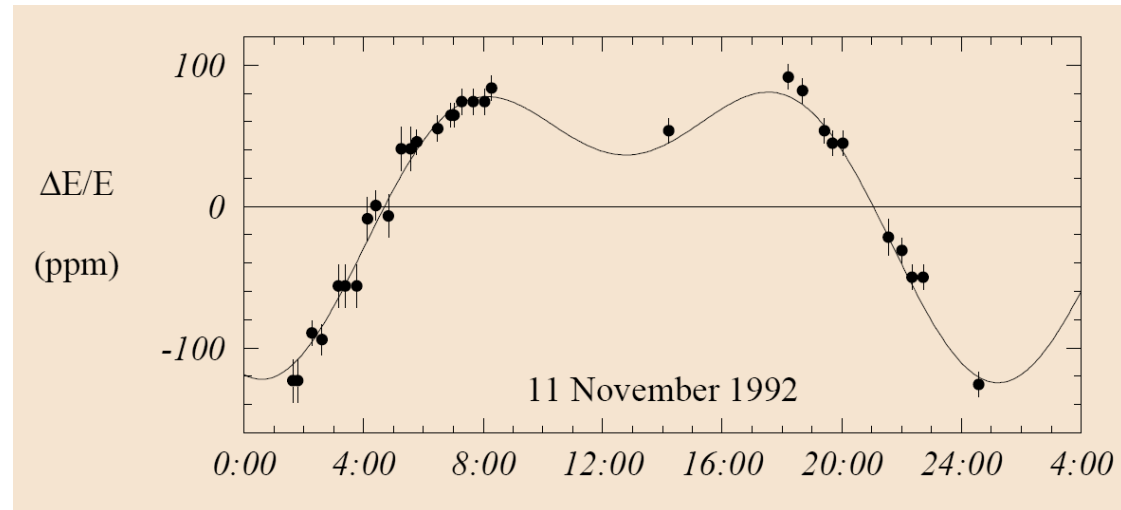
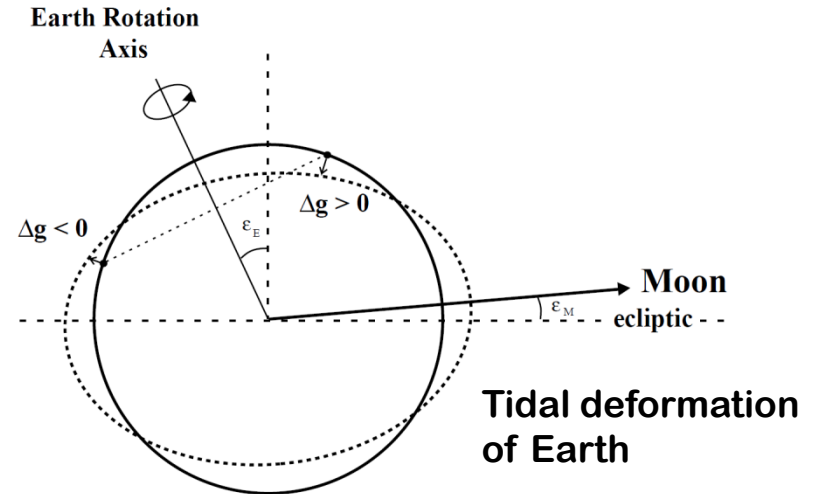
Measured energy variation fit perfectly the predictions based on the tidal model

This is again peculiar example of slow motion, and this time it was noticed by the accelerator.

But this type of effects can be easily corrected for.

We should be more concerned about fast effects, that cannot be corrected.

What is “fast” depend on parameters...



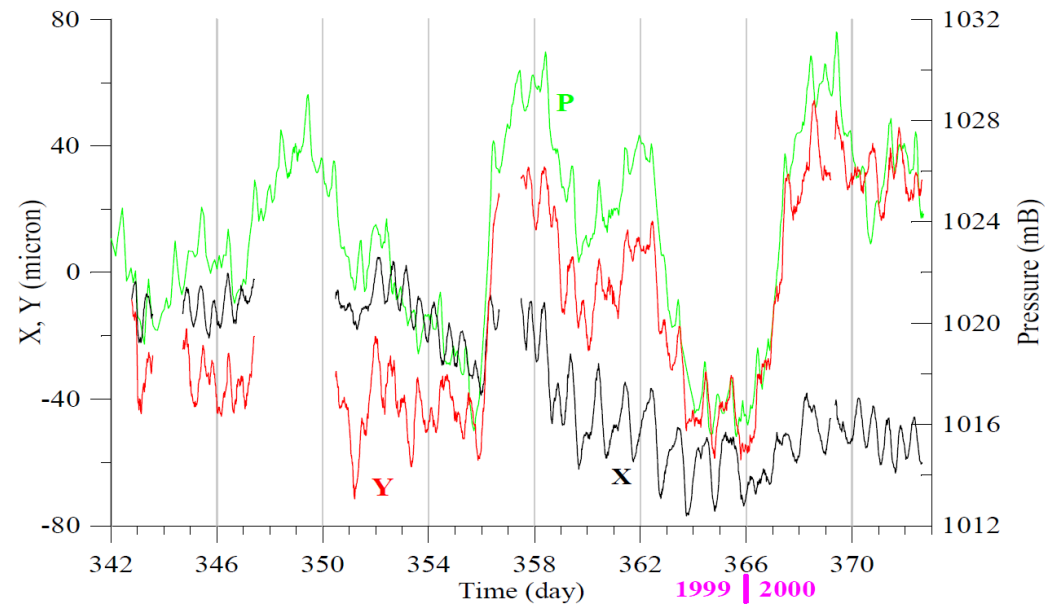
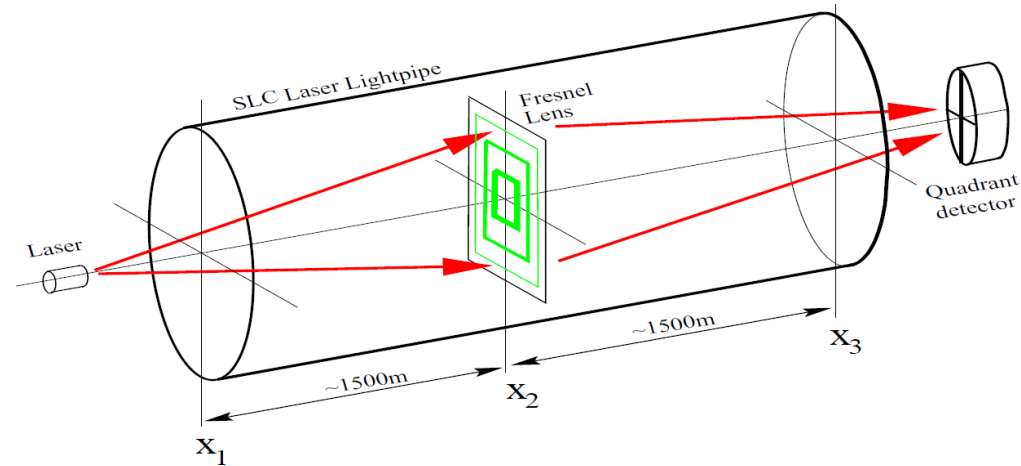
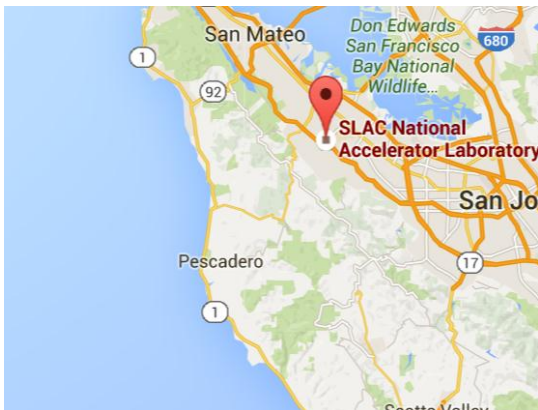
Effects of Terrestrial Tides on the LEP Beam Energy, L. Arnaudon, et al., CERN SL/94-07 (BI)

...and even not about enhanced tidal motion

Deformation of 3km SLAC linac was measured

10 micron tidal component was observed, exceeding by 1000 times what is expected for a uniform elastic Earth

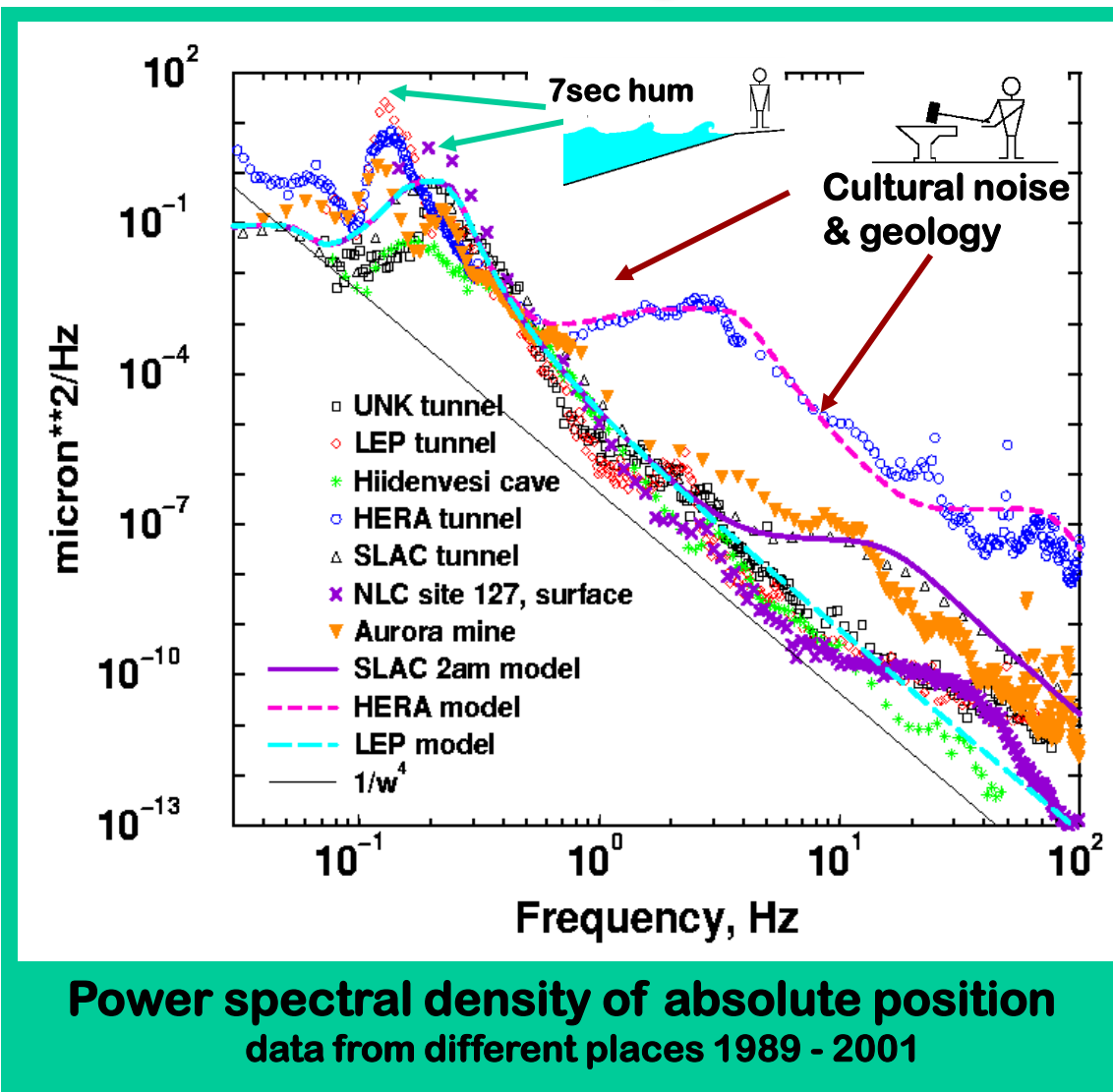
Explained by “Ocean loading” effects, which enhances the tidal deformations locally



This is peculiar, but this motion is slow, long wavelength and usually even not noticed by the accelerator

Primary concern - natural and man-made (cultural) ground motion - one of disturbing factors

- **Fundamental – decrease as $1/\omega^4$**
- **Quiet & noisy sites/conditions**
- **Cultural noise & geology very important**
- **Motion is small at high frequencies...**



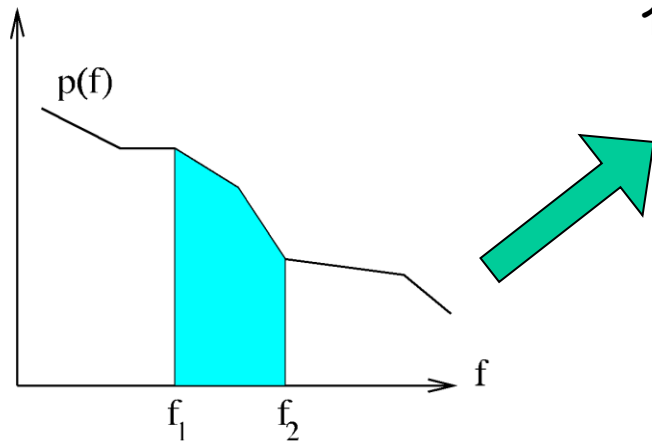
Note to the plot on the previous page: Random signal & Power spectra

- Periodic signals can be characterized by amplitude (e.g. μm) and frequency
- Random signals described by PSD (Power Spectral Density) which usually have units like (m^2/Hz)
- The way to make sense of PSD amplitude is to * by frequency range and take SQRT

Natural ground motion is small at high frequencies

At $F > 1$ Hz the motion can be < 1 nm (i.e. much less than beam size in LC). Is it OK?

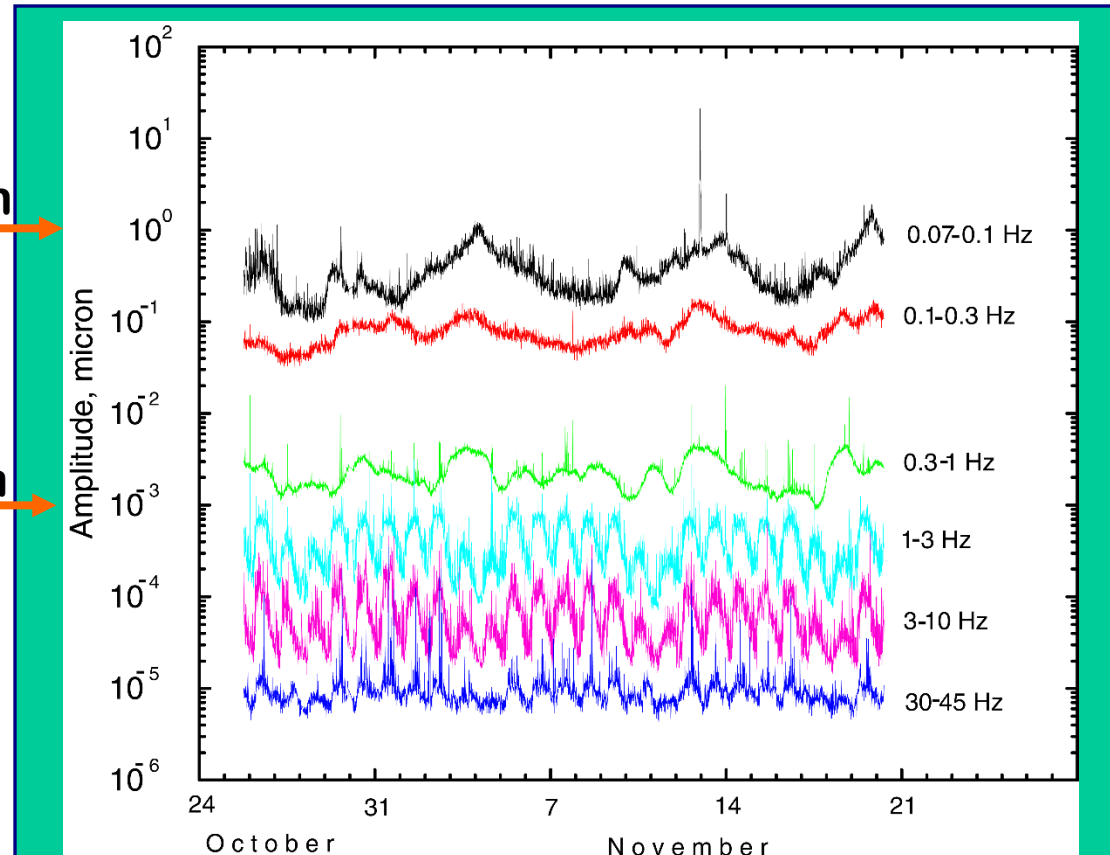
What about low frequency motion? It is much larger...



PSD is in m^2/Hz . Its integral over frequency range give square of rms amplitude of the motion in this frequency band.

1 micron

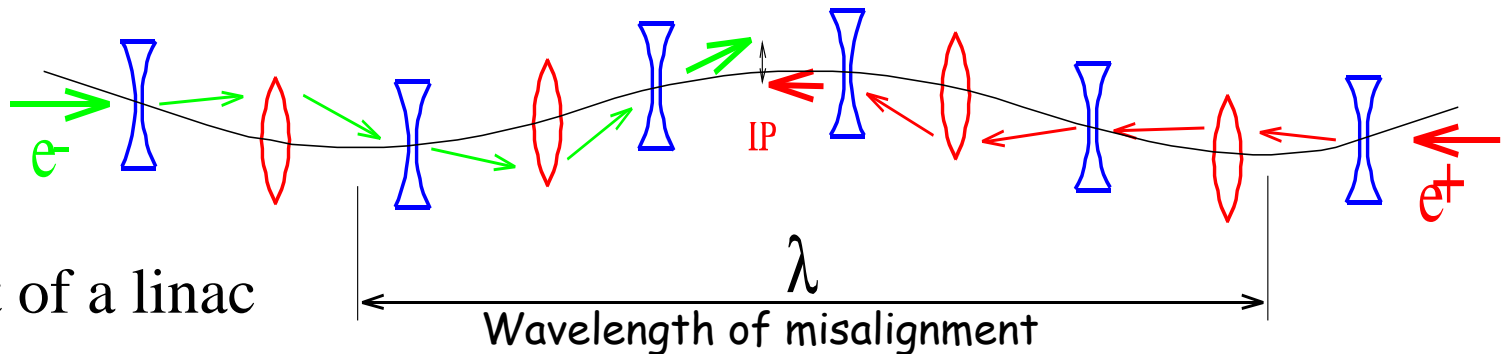
1 nm



Rms displacement in different frequency bands. Hiidenvesy cave, Finland, 1993

Ground motion in time and space

- To find out whether large slow ground motion relevant or not...
- One need to compare
 - Frequency of motion with repetition rate of collider
 - Spatial wavelength of motion with focusing wavelength of collider

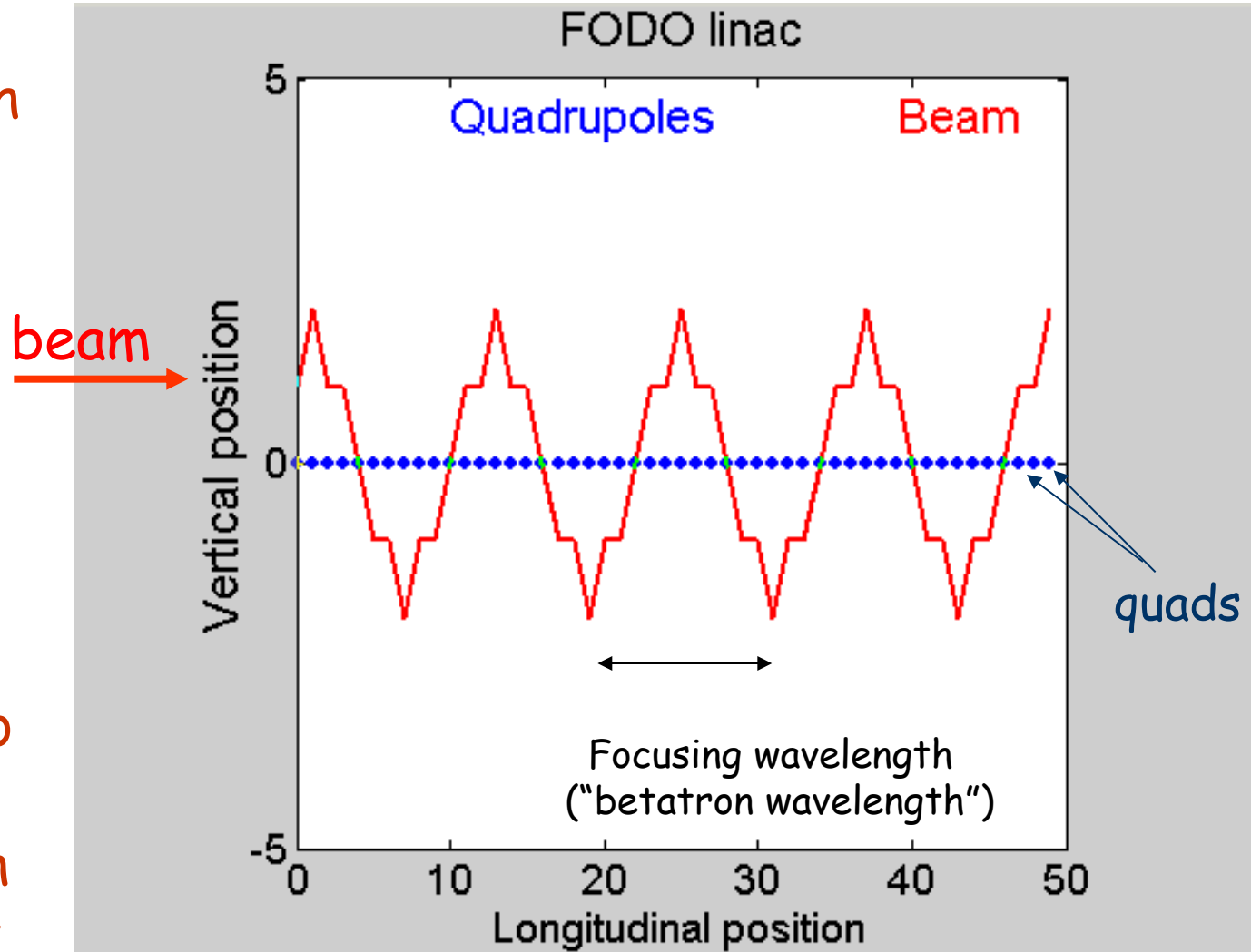


Snapshot of a linac

Focusing wavelength of a FODO linac

FODO linac with beam entering with an offset

Betatron wavelength is to be compared with wavelength of misalignment



Movie of a Misaligned FODO linac

next page

Note the following:

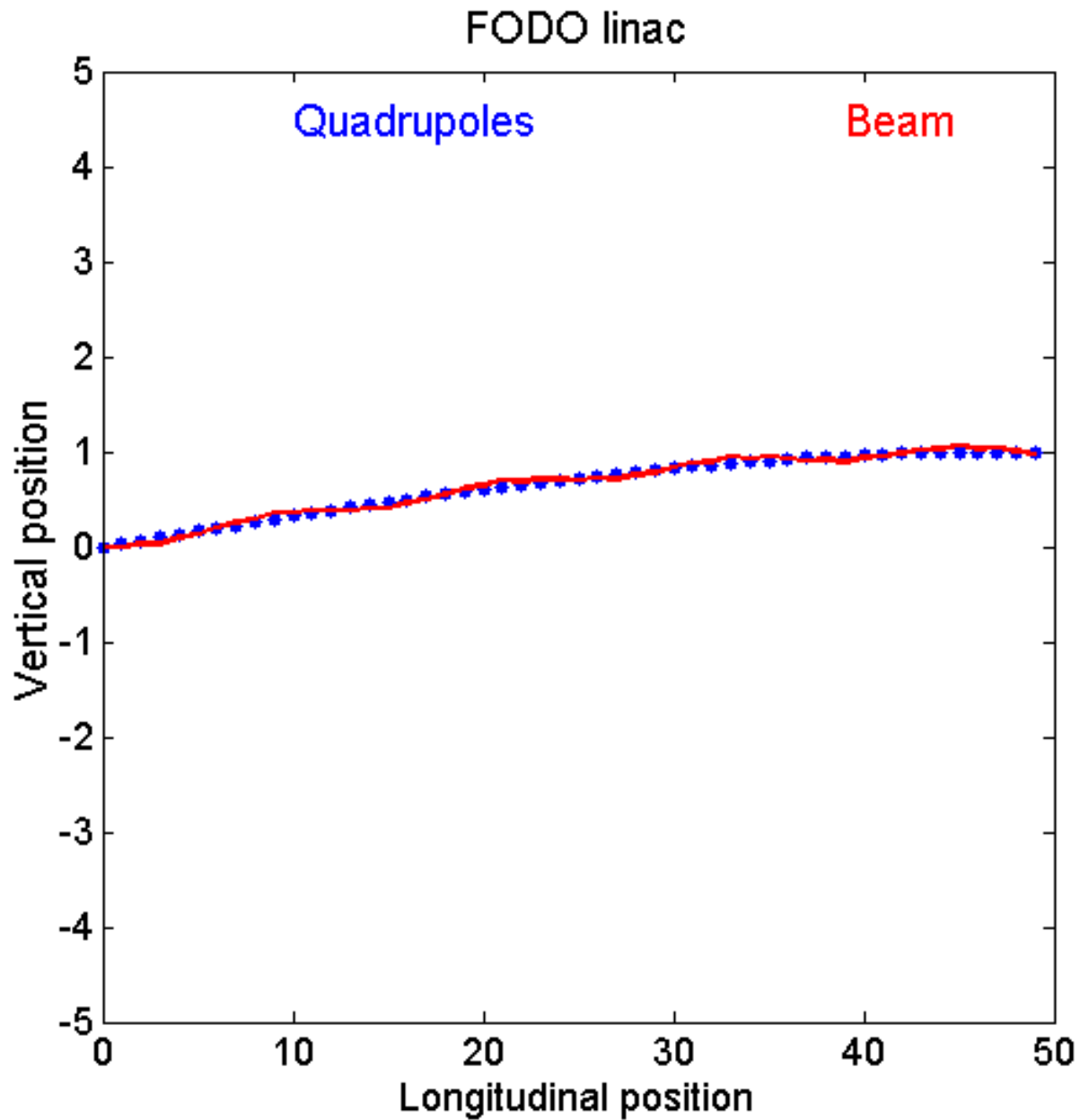
Beam follows the linac if misalignment is more smooth than betatron wavelength

Resonance if wavelength of misalignment \sim focusing wavelength

Spectral response function - how much beam motion due to misalignment with certain wavelength

Below, we will try to understand this behavior step by step...

Movie of a Misaligned FODO linac

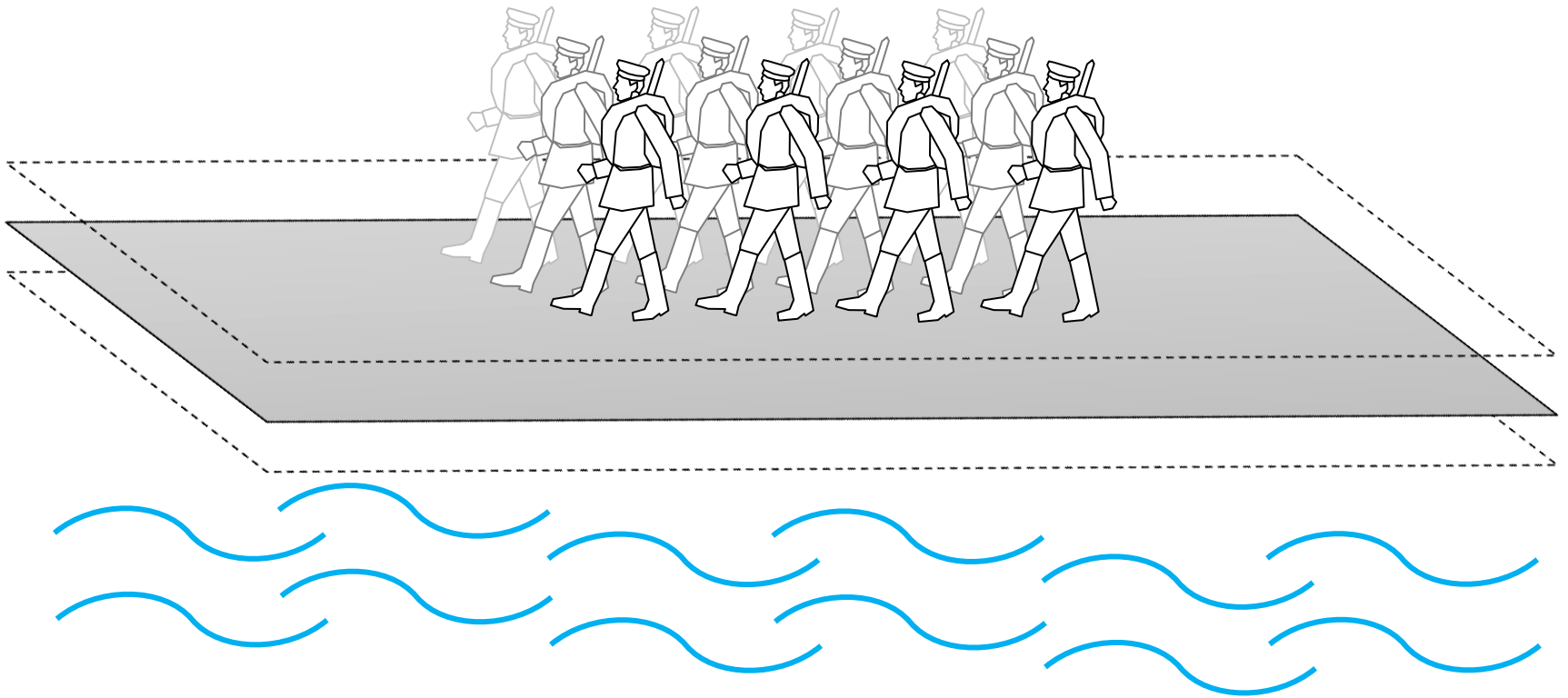


Resonances, bridges & accelerators



Collapse of the “Egyptian bridge” in Saint-Petersburg in 1905, when the squadron of the life guards regiment was passing through the bridge, is usually explained by resonance

Resonances, bridges & accelerators



Resonances, bridges & accelerators



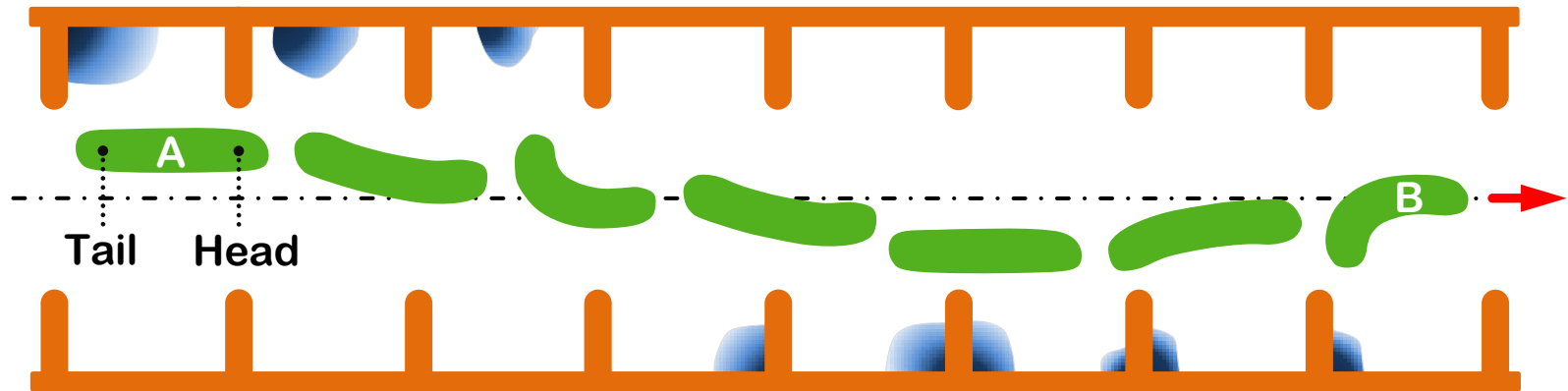
Collapse of the “Egyptian bridge” in Saint-Petersburg in 1905, when the squadron of the life guards regiment was passing through the bridge, is usually explained by resonance

It is not clear if this explanation is correct (most of regiment was on horses) but we know for sure that damaging resonance phenomena do happen, and in accelerators too

Wakefields – fields left by bunch in accelerating structure

Digression

In RF cavity **these fields** can build up resonantly and disrupt the bunch itself in the so called Beam Break Up instability



The problem was that all accelerating structures of 2 mile SLAC linac were exactly the same!

Resonance build up of the effect on the bunch
Beam Break Up instability

Wakefields and how to cure them with a hammer

Digression



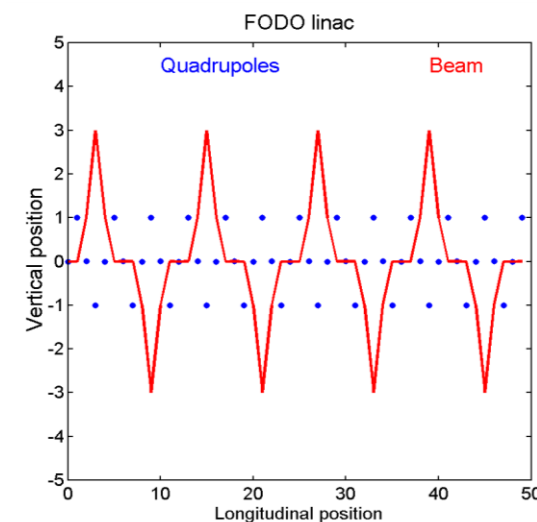
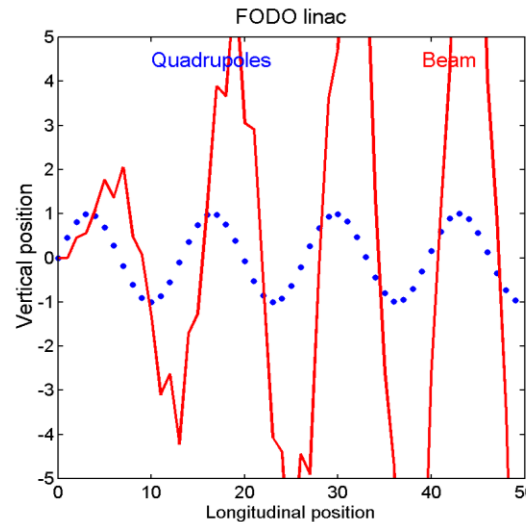
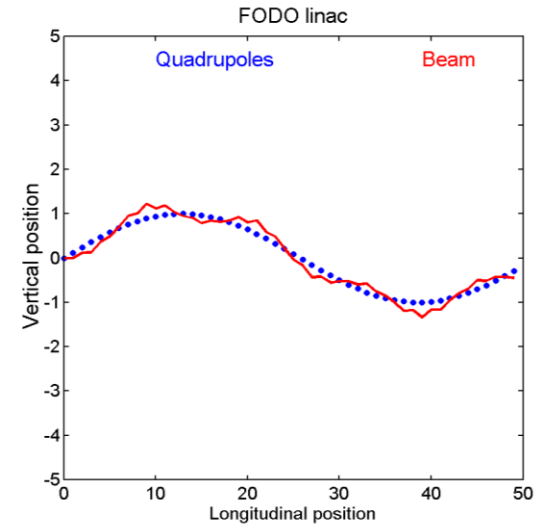
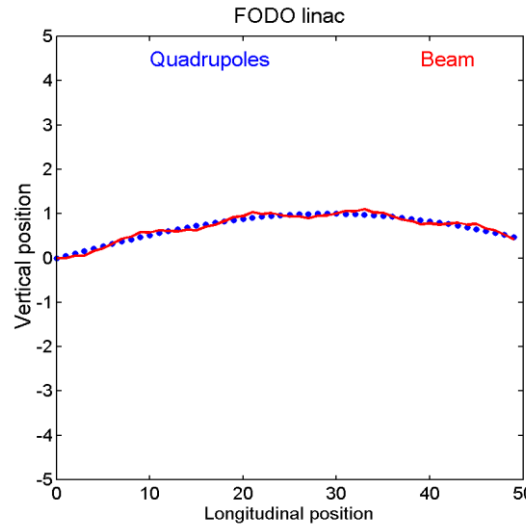
To take care of BBU, it was necessary to “detune” the individual cells of the accelerator to prevent coherent build up of the wakefield forces. The detuning process was accomplished in the SLAC linac by dimpling the radial dimensions of each cell so that the frequencies of the lowest dipole components of the wakefields vary slightly from cell to cell. This “delicate” correction was applied to the as-build accelerator with handheld hammers

To find out whether large slow ground motion relevant or not compare focusing wavelength of the collider with wavelength of misalignment

Beam follows the
linac if misalignment
is more smooth than
focusing wavelength

Resonance appear if
wavelength of
misalignment \sim
focusing wavelength

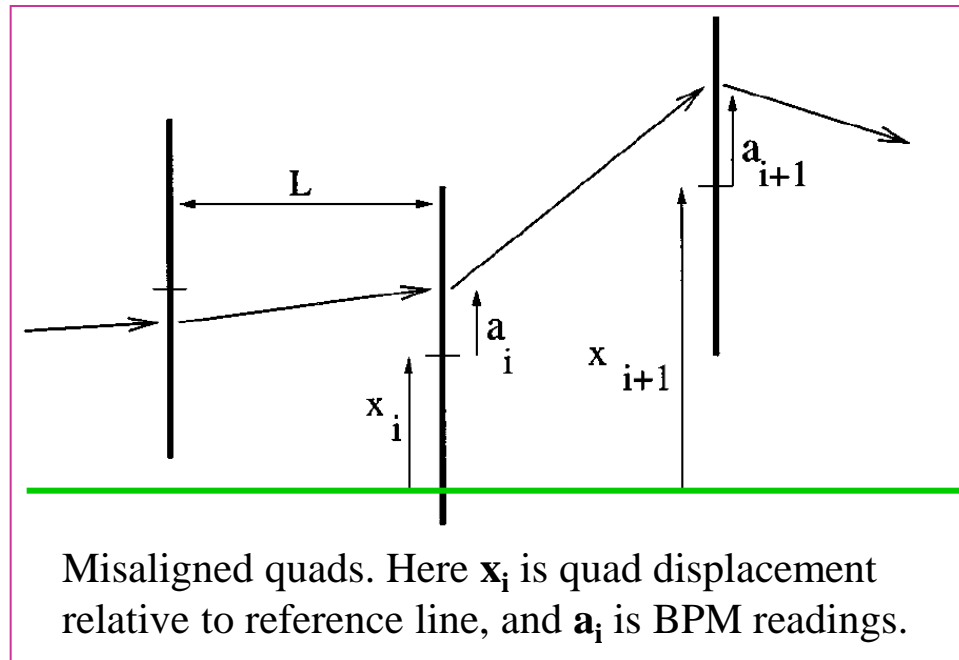
Let's now approach this
case analytically



Example: misaligned FODO linac

How to predict orbit motion or chromatic dilution

Let's consider a beamline consisting of misaligned quadrupoles with position $\mathbf{x}_i(\mathbf{t})=\mathbf{x}(\mathbf{t},\mathbf{s}_i)$ of the i -th element measured with respect to a reference line. Here \mathbf{s}_i is longitudinal position of the quads. If $\mathbf{x}_{\text{abs}}(\mathbf{t},\mathbf{s})$ is a coordinate measured in an inertial frame and the reference line passes through the entrance, then $\mathbf{x}(\mathbf{t},\mathbf{s})=\mathbf{x}_{\text{abs}}(\mathbf{t},\mathbf{s})-\mathbf{x}_{\text{abs}}(\mathbf{t},\mathbf{0})$. We also assume that at $\mathbf{t}=\mathbf{0}$ the quads were aligned $\mathbf{x}(\mathbf{0},\mathbf{s})=\mathbf{0}$.



We are interested to find the beam offset at the exit \mathbf{x}_* or the dispersion $\eta_{\mathbf{x}}$, produced by misaligned quadrupoles. Let's assume that \mathbf{b}_i and \mathbf{d}_i are the first derivatives of the beam offset and beam dispersion at the exit versus displacement of the element i . Then the final offset, measured with respect to the reference line, and dispersion are given by summation over all elements:

$$\mathbf{x}_*(\mathbf{t}) = \mathbf{R}_{11} \mathbf{x}_{\text{inj}}(\mathbf{t}) + \mathbf{R}_{12} \mathbf{x}'_{\text{inj}}(\mathbf{t}) + \sum_{i=1}^{\mathbf{N}} \mathbf{b}_i \mathbf{x}_i(\mathbf{t})$$

$$\eta_{\mathbf{x}}(\mathbf{t}) = \mathbf{T}_{116} \mathbf{x}_{\text{inj}}(\mathbf{t}) + \mathbf{T}_{126} \mathbf{x}'_{\text{inj}}(\mathbf{t}) + \sum_{i=1}^{\mathbf{N}} \mathbf{d}_i \mathbf{x}_i(\mathbf{t})$$

Is it clear why there is no \mathbf{a}_i in this formula?

Where \mathbf{N} is the total number of quads, \mathbf{R} and \mathbf{T} are 1st and 2nd order matrices of the total beamline, and we also took into account nonzero position and angle of the injected beam at the entrance.

Predicting orbit motion and chromatic dilution ... random case

Let's assume now that the beam is injected along the reference line, then:

$$\mathbf{x}_*(t) = \sum_{i=1}^N \mathbf{b}_i x_i(t) \qquad \eta_x(t) = \sum_{i=1}^N d_i x_i(t)$$

Assume that quads misalignments, averaged over many cases, is zero. Let's find the nonzero variance

$$\langle \mathbf{x}_*^2(t) \rangle = \sum_{i=1}^N \sum_{j=1}^N \mathbf{b}_i \mathbf{b}_j \langle x_i(t) \cdot x_j(t) \rangle \qquad \langle \eta_x^2(t) \rangle = \sum_{i=1}^N \sum_{j=1}^N d_i d_j \langle x_i(t) \cdot x_j(t) \rangle$$

Let's first consider a very simple case.

In case of **random** uncorrelated misalignment we have $\langle x_j(t) \cdot x_j(t) \rangle = \sigma_x^2 \delta_{ij}$ (σ_x is rms misalignment, not the beam size)

So that, for example $\langle \mathbf{x}_*^2(t) \rangle = \sigma_x^2 \sum_{i=1}^N \mathbf{b}_i^2$ And similar for dispersion

Now we would like to know what are these **b** and **d** coefficients.

Predicting x_* and η ...

what are these b_i and d_i coefficients

Let's consider a thin lens approximation. In this case, transfer matrix of i -th quadrupole is $\begin{pmatrix} 1 & 0 \\ -K_i & 1 \end{pmatrix}$
 ($K > 0$ for focusing and $K < 0$ for defocusing)

A quad displaced by x_i produces an angular kick $\theta = K_i x_i$ and the resulting offset at the exit will be $x_* = r_{12}^i K_i x_i$ Where r_{12}^i is the element of transfer matrix from i -th element to the exit

The coefficient b_i is therefore $b_i = r_{12}^i K_i$

The coefficient d_i is the derivative of b_i with respect to energy deviation δ :

$$d_i = \frac{d}{d\delta} (r_{12}^i K_i) = \frac{d}{d\delta} \left(r_{12}^i \frac{K_i(0)}{1+\delta} \right)$$

Which is equal to $d_i = -K_i (r_{12}^i - t_{126}^i)$ Where t_{126}^i is the 2nd order transfer matrix from i -th element to the exit

Transfer matrices for FODO linac

Let's consider a FODO linac... No, let's consider, for better symmetry, a (F/2 O D O F/2) linac. Example is shown in the figure on the right side.

The quadrupole strength is $K_i = K (-1)^{i+1}$ (ignoring that first quad is half the length). The position of the quadrupoles is $S_i = (i - 1)L$ where L is quad spacing.

The betatron phase advance μ per FODO cell is given by $2 \sin\left(\frac{\mu}{2}\right) = |K| L$

The matrix element r_{12}^i from the i-th quad to the exit (N-th quad) is $r_{12}^i = \sqrt{\beta_i \beta_N} \sin(\Psi_i)$

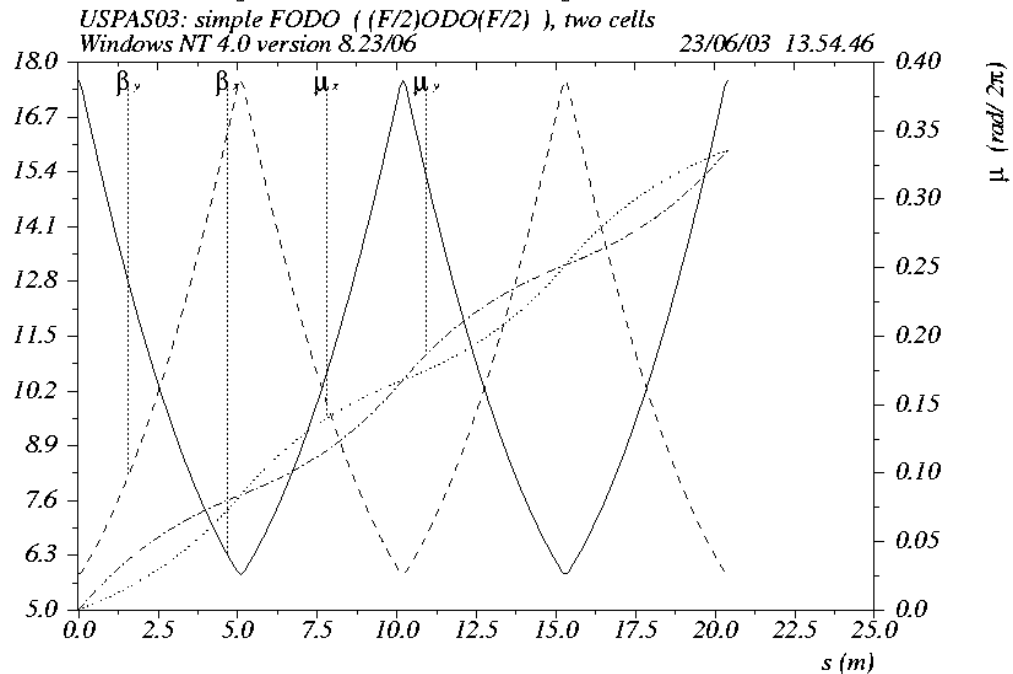
Where Ψ_i is the phase advance from i-th quad and exit. Obviously, $\Psi_i = \frac{\mu}{2} (N - i)$

And here β_i and β_N are beta-functions in the quads. For such regular FODO, the min and max values of beta-functions (achieved in quads) are

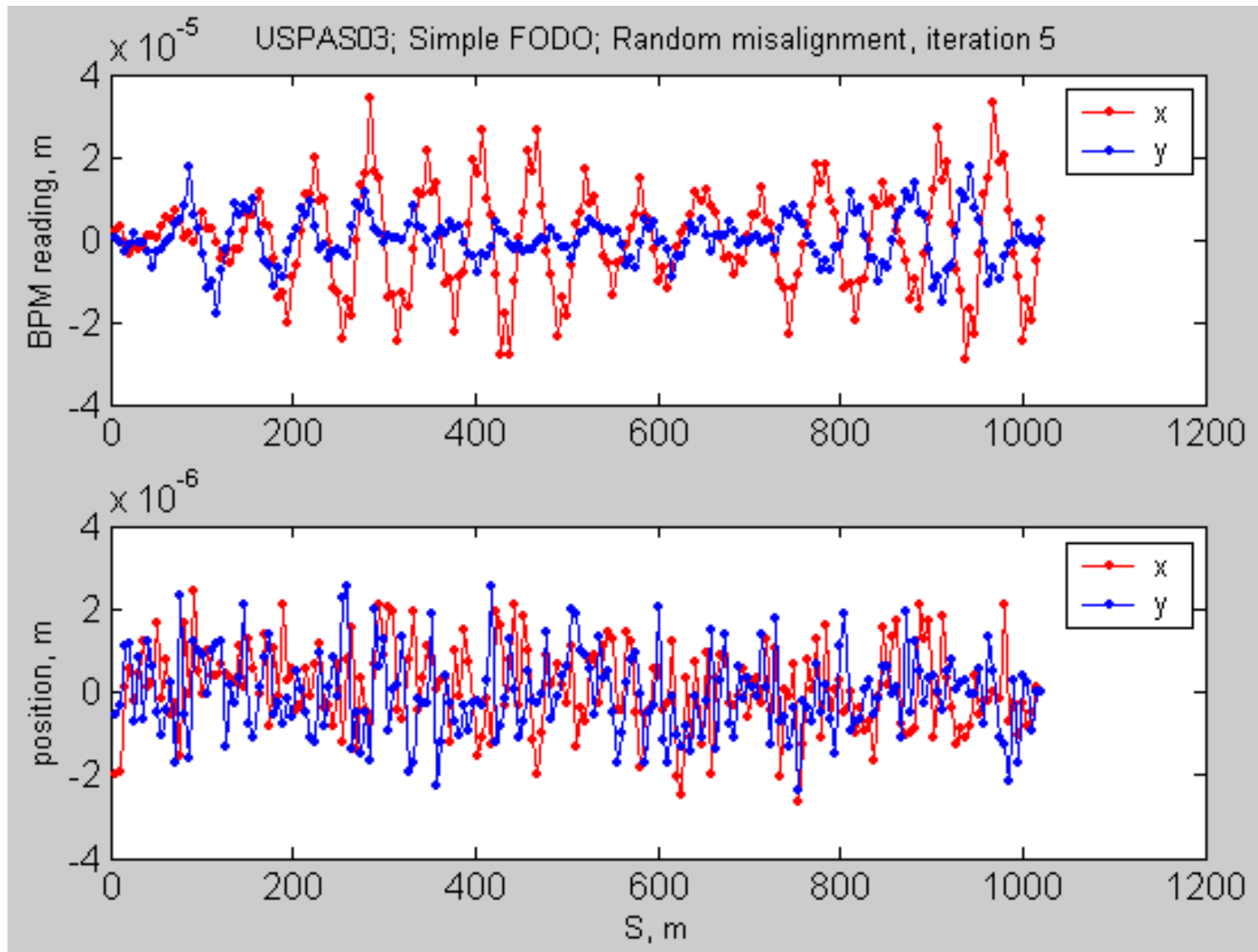
$$\beta_{\max, \min} = \frac{L}{\tan\left(\frac{\mu}{2}\right) \left[1 \pm \mu \sin\left(\frac{\mu}{2}\right) \right]}$$

Since the energy dependence comes mostly from the phase advance (it has large factor of N) and the beta-function variation can be neglected, the second order coefficients are given by

$$t_{126}^i \approx -r_{12}^i (N - i) \tan\left(\frac{\mu}{2}\right) \frac{1}{\tan(\Psi_i)}$$



Example of random misalignments of FODO linac

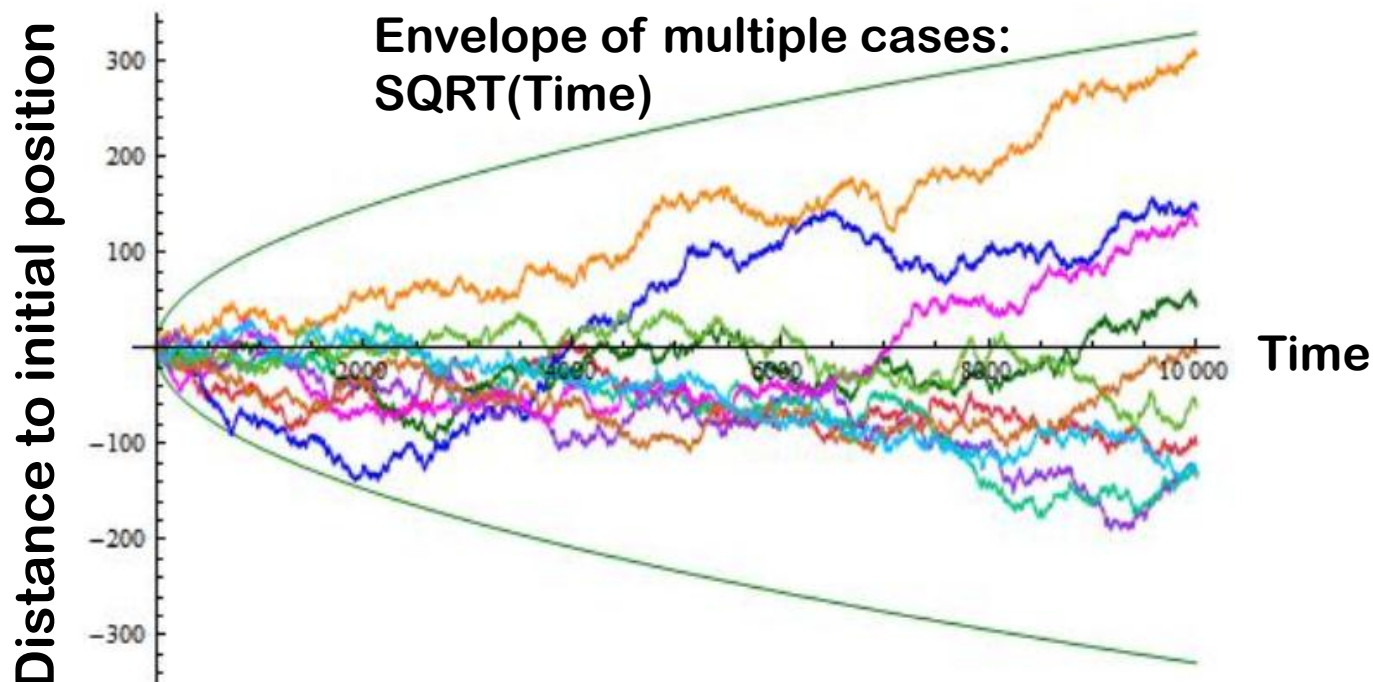


Example of misalignments and orbits

Random misalignment ... is it really possible?

- Now you have everything to calculate \mathbf{b} and \mathbf{d} coefficients and find, for example, the rms of the orbit motion at the exit for the simplest case – completely random uncorrelated misalignments.
- Completely random and uncorrelated means that misalignments of two neighboring points, even infinitesimally close to each other, would be completely independent.
- If we would assume that such random and uncorrelated behavior occur in time also, i.e. for any infinitesimally small Δt the misalignments will be random (no “memory” in the system) then it would be obvious that such situation is physically impossible. Simply because its spectrum correspond to white noise, i.e. goes to infinite frequencies, thus having infinite energy.
- We have to assume that things do not get changed infinitely fast, nor in space, neither in time. i.e., there is some correlation with previous moments of time, or with neighboring points in space.
- Let's consider the random walk (drunk sailor). In this case, together with randomness, there is certain memory in this process: the sailor makes the next step relative to the position he is at the present point.

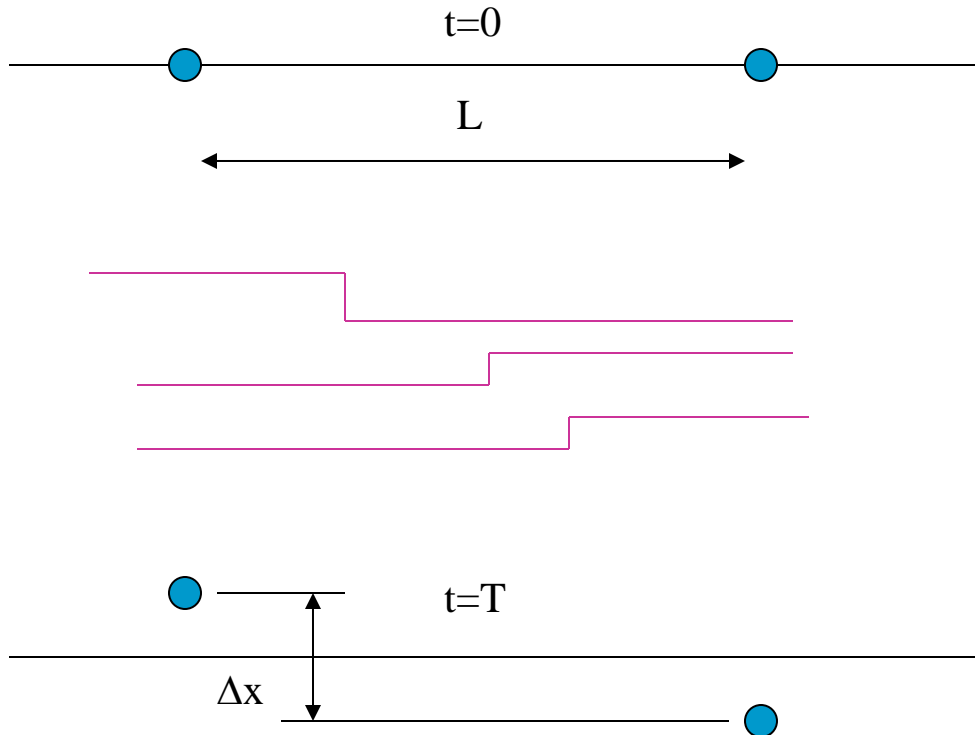
Random walk - diffusive motion



- In this case distance from the initial position ΔX in average is zero
- The rms value, ΔX^2 grows with time linearly
i.e. $\Delta X^2 \sim AT$ (T – elapsed time, A – some constant that depend on the case)
- This is diffusion
- Random walk (drunk sailor) still cannot be applied directly to FODO linac
- Extension of random walk model to multiple points in space and time is described by the “ATL law” [B.Baklakov, V.Parkhomchuk, A.Seryi, V.Shiltsev, et al, 1991]

The ATL motion

According to “ATL law” (rule, model, etc.), misalignment of two points separated by a distance L after time T is given by $\Delta X^2 \sim ATL$ where A is a coefficient which may depend on many parameters, such as site geology, etc., if we are talking about ground motion. (The ATL-kind of motion can occur in other areas of physics as well.)



Such ATL motion would occur, for example, if step-like misalignments occur between points 1 and 2 and the number of such misalignments is proportional to elapsed time and separation between point. You then see that the average misalignment is zero, but the rms is given by the ATL rule.

ATL ground measurements will be discussed later. Let's now discuss orbit motion in the linac for ATL ground motion.

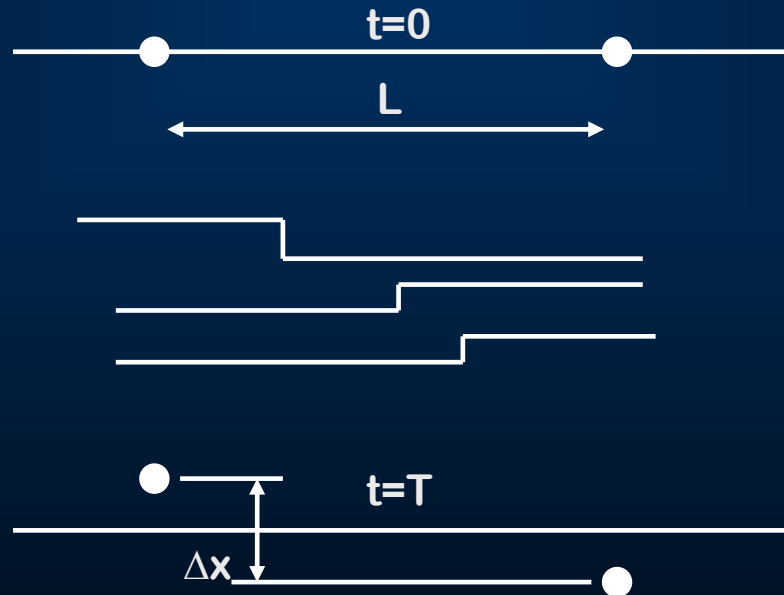
Slow but short λ ground motion

What if we are interested in two separated points?

\Rightarrow ATL motion, or diffusion in space and time

- **Diffusive or ATL motion: $\Delta X^2 \sim ATL$** (T - elapsed time, L - separation between two points)
 - Caused by underground water, dissipation of high frequency motion, temperature, atmosphere, etc.
- **Observed 'A' varies by ~5 orders: 10^{-9} to $10^{-4} \mu\text{m}^2/(\text{m}\cdot\text{s})$**
 - 'A' strongly depends on **geology**
 - **Higher 'A'** in sedimentary geology, lower A in solid rock

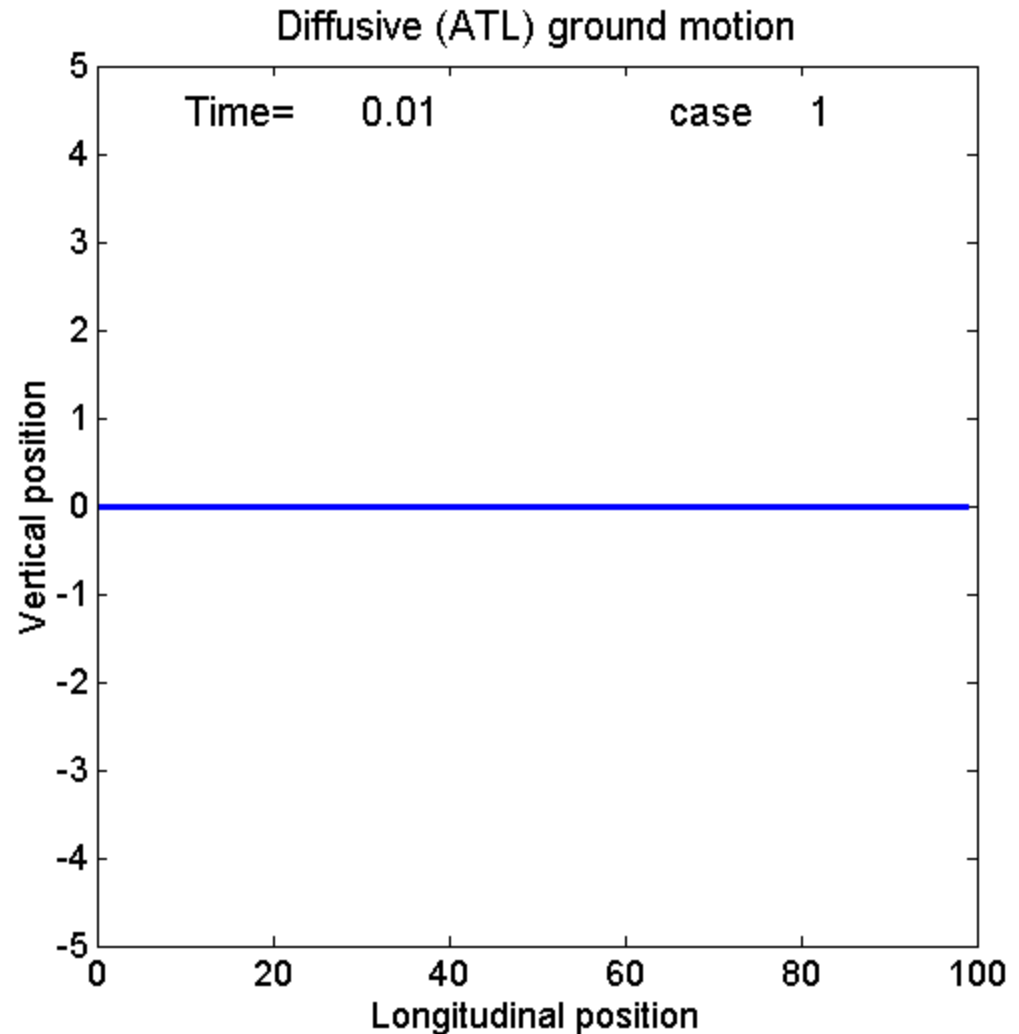
Simple illustration
allowing to
imagine how ATL
motion happens:



Number of random
step-like displacements
between two points is
proportional to L & T

How diffusive ATL motion looks like?

- **Movie of simulated ATL motion**
- **Note that it starts rather fast**
- **$X^2 \sim L$**
- **and it can change direction...**



Predicting orbit motion and chromatic dilution ... ATL case

So, we would like to calculate $\langle x_*^2(t) \rangle = \sum_{i=1}^N \sum_{j=1}^N b_i b_j \langle x_i(t) \cdot x_j(t) \rangle$ for ATL case.

Let's rewrite ATL motion definition. Assume that there is an inertial reference frame, where coordinates of our linac are $\mathbf{x}_{abs}(\mathbf{t}, \mathbf{s})$. Let's assume that at $t=0$ the linac was perfectly aligned, and let's define misalignment with respect to this original positions as $\mathbf{x}(\mathbf{t}, \mathbf{s}) = \mathbf{x}_{abs}(\mathbf{t}, \mathbf{s}) - \mathbf{x}_{abs}(\mathbf{t} = 0, \mathbf{s})$

The ATL rule can then be written as: $\langle (\mathbf{x}(\mathbf{t}, \mathbf{s} + \mathbf{L}) - \mathbf{x}(\mathbf{t}, \mathbf{s}))^2 \rangle = \mathbf{A} \cdot \mathbf{t} \cdot \mathbf{L}$

Take into account that beam goes through the entrance (where $s=0$) without offset and write:

$$x_i = x(\mathbf{t}, s_i) - x(\mathbf{t}, 0) \quad x_j = x(\mathbf{t}, s_j) - x(\mathbf{t}, 0)$$

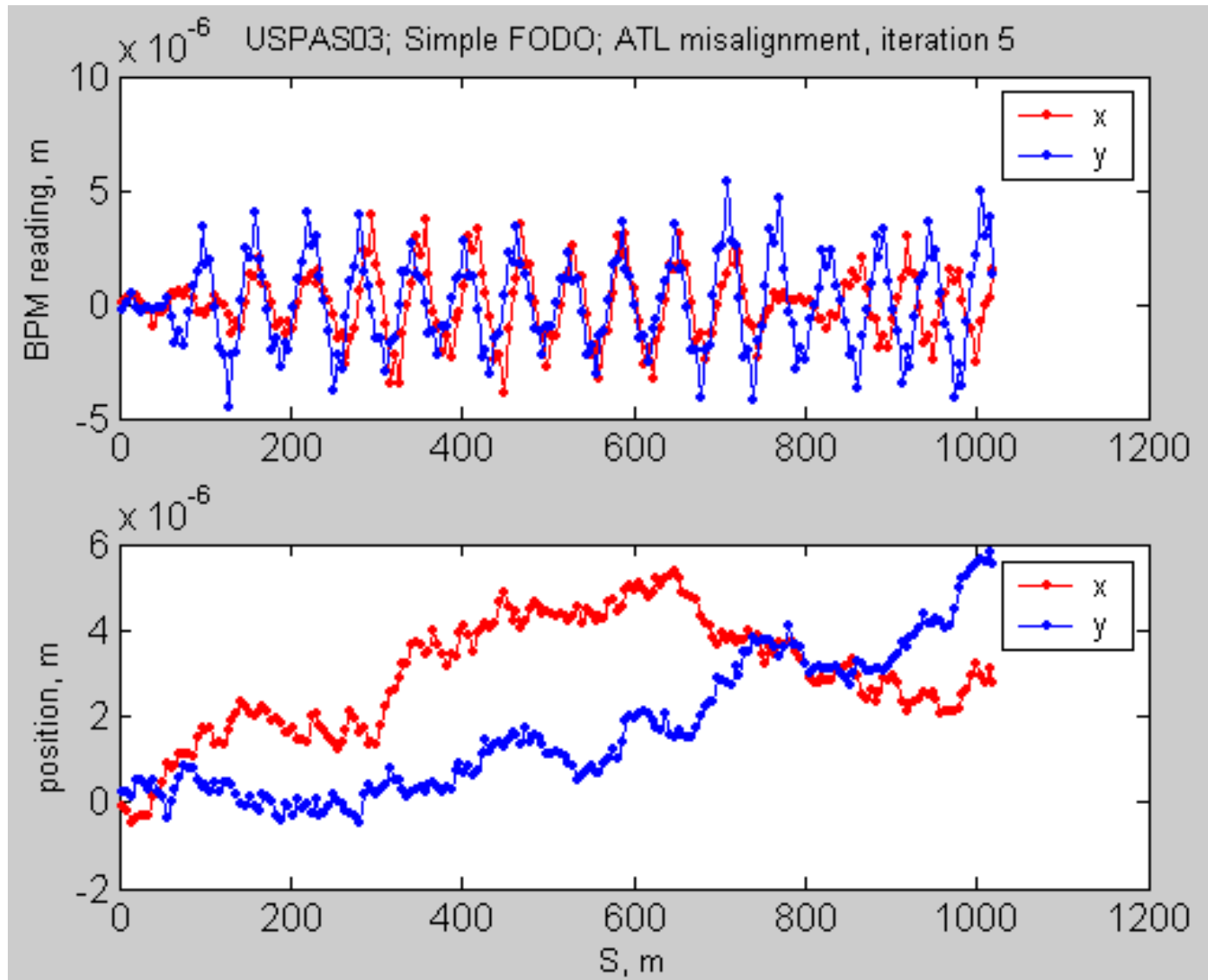
Then rewrite $x_i x_j$ term as $x_i x_j = \frac{1}{2} \left[(x(\mathbf{t}, s_i) - x(\mathbf{t}, 0))^2 + (x(\mathbf{t}, s_j) - x(\mathbf{t}, 0))^2 - (x(\mathbf{t}, s_i) - x(\mathbf{t}, s_j))^2 \right]$

Now use ATL rule and get $\langle x_i x_j \rangle = \frac{1}{2} \cdot \mathbf{A} \cdot \mathbf{t} \cdot (|s_i| + |s_j| - |s_i - s_j|)$

Taking into account $\mathbf{S}_i = (\mathbf{i} - 1)\mathbf{L}$ we have the final result for the rms exit orbit motion in ATL case:

$$\langle x_*^2(t) \rangle = \frac{\mathbf{A} \cdot \mathbf{t} \cdot \mathbf{L}}{2} \sum_{i=1}^N \sum_{j=1}^N b_i b_j ((i-1) + (j-1) - |i-j|)$$

Example of ATL misalignments of FODO linac



Example of misalignments and orbits

Predicting orbit motion for arbitrary misalignments

So, we would like to calculate, for example, $\langle x_*^2(t) \rangle = \sum_{i=1}^N \sum_{j=1}^N d_i d_j \langle x_i(t) \cdot x_j(t) \rangle$ in case of arbitrary properties of misalignments

One can introduce the spatial harmonics $\mathbf{x}(\mathbf{t}, \mathbf{k})$ of wave number $\mathbf{k} = 2\pi/\lambda$, with λ being the spatial period of displacements:

$$x(t, k) = \int_{-\mathcal{L}/2}^{\mathcal{L}/2} x(t, s) e^{-iks} ds$$

The displacement $x(t, s)$ can be written using the back transformation:

$$x(t, s) = \int_{-\infty}^{\infty} x(t, k) (e^{iks} - 1) \frac{dk}{2\pi}$$

which ensures that at the entrance $\mathbf{x}(\mathbf{t}, \mathbf{s}=\mathbf{0})=\mathbf{0}$.

Then the variance of dispersion is

$$\langle \eta_x^2(t) \rangle = \sum_i \sum_j d_i d_j \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle x(t, k_1) x^*(t, k_2) \rangle (e^{ik_1 s_i} - 1) (e^{-ik_2 s_j} - 1) \frac{dk_1}{2\pi} \frac{dk_2}{2\pi}$$

We can rewrite it as

$$\langle \eta_x^2(t) \rangle = \sum_i \sum_j d_i d_j \int_{-\infty}^{\infty} P(t, k) (e^{iks_i} - 1) (e^{-iks_j} - 1) \frac{dk}{2\pi}$$

Where we defined the spatial power spectrum of displacements $\mathbf{x}(\mathbf{t}, \mathbf{s})$ as

$$P(t, k) = \lim_{\mathcal{L} \rightarrow \infty} \frac{1}{\mathcal{L}} x(t, k) x^*(t, k) = \lim_{\mathcal{L} \rightarrow \infty} \frac{1}{\mathcal{L}} \left| \int_{-\mathcal{L}/2}^{\mathcal{L}/2} x(t, s) e^{-iks} ds \right|^2$$

Predicting orbit motion for arbitrary misalignments

So, we see that we can write the variance of dispersion (and very similar for the offset) in such a way, that the lattice properties and displacement properties are separated:

$$\langle \eta_x^2(t) \rangle = \int_{-\infty}^{\infty} P(t, k) G(k) \frac{dk}{2\pi}$$

Here $G(k)$ is the so-called spectral response function of the considered transport line (in terms of dispersion):

$$G(k) = g_c^2(k) + g_s^2(k)$$

where

$$g_c(k) = \sum_{i=1}^N d_i [\cos(ks_i) - 1] \quad \text{and} \quad g_s(k) = \sum_{i=1}^N d_i \sin(ks_i)$$

The spectral function for the offset will be the same, but \mathbf{d}_i substituted by \mathbf{b}_i

Refs for this and neighbouring slides: A. Seryi and O. Napoly, Phys. Rev. E 53, 5323 (1996)
A. Seryi and A. Mosnier, Phys. Rev. E 56, 3558 (1997)

2-D spectra of ground motion

Arbitrary ground motion can be fully described, for a linear collider, by a 2-D power spectrum $\mathbf{P}(\omega, \mathbf{k})$

If a 2-D spectrum of ground motion is given, the spatial power spectrum $\mathbf{P}(\mathbf{t}, \mathbf{k})$ can be found as

$$P(t, k) = \int_{-\infty}^{\infty} P(\omega, k) 2[1 - \cos(\omega t)] \frac{d\omega}{2\pi}$$

Example of 2-D spectrum for ATL motion: $P(\omega, k) = \frac{A}{\omega^2 k^2}$ And for $\mathbf{P}(\mathbf{t}, \mathbf{k})$: $P(t, k) = \frac{A \cdot t}{k^2}$

The 2-D spectrum can be used to find variance of misalignment. Again, assume that there is an inertial reference frame, where coordinates of our linac are $\mathbf{x}_{\text{abs}}(\mathbf{t}, \mathbf{s})$. And assume that at $t=0$ the linac was perfectly aligned, and that misalignment with respect to this original positions is $\mathbf{x}(\mathbf{t}, \mathbf{s}) = \mathbf{x}_{\text{abs}}(\mathbf{t}, \mathbf{s}) - \mathbf{x}_{\text{abs}}(t=0, \mathbf{s})$, its variance is given by

$$\langle (\mathbf{x}(\mathbf{t}, \mathbf{s} + \mathbf{L}) - \mathbf{x}(\mathbf{t}, \mathbf{s}))^2 \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(\omega, k) 2 \cdot [1 - \cos(\omega t)] 2 \cdot [1 - \cos(kL)] \frac{d\omega}{2\pi} \frac{dk}{2\pi}$$

You can easily verify, for example, that for ATL spectrum it gives the ATL formula

The (directly measurable !) spectrum of relative motion is given by

$$\rho(\omega, L) = \int_{-\infty}^{\infty} P(\omega, k) 2[1 - \cos(kL)] dk / 2\pi$$

Creating models of 2-D spectrum $P(\omega, k)$

- Use measurements of ATL motion
- Use measurements of fast motion and correlation data
- Solve the contradiction:
 - Spectrum of ATL motion behaves as $1/\omega^2$
 - Spectrum of absolute fast motion behaves as $1/\omega^4$
 - \Rightarrow for some parameters **spectrum of relative ATL motion will be larger than spectrum of absolute motion \Rightarrow this is impossible!**
 - Therefore, ATL can be included into the model only in a corrected way
 - See references for details how it was done

Slow but short λ ground motion – ATL

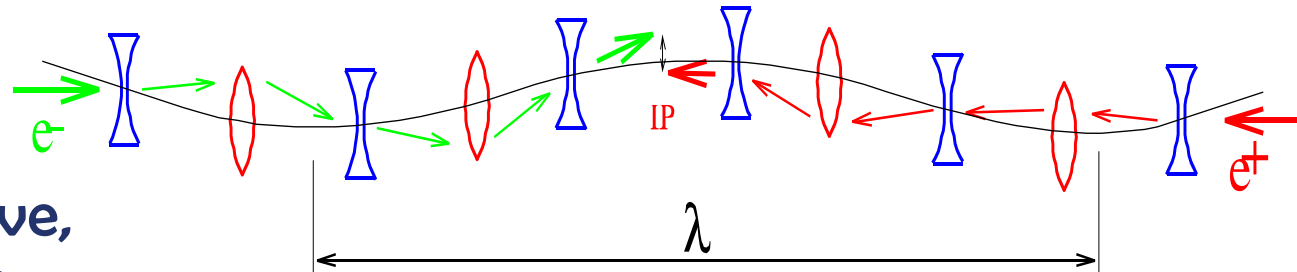
Examples of measured data

- **Diffusive or ATL motion:** $\Delta X^2 \sim A_D T L$ (minutes-month)
(T – elapsed time, L – separation between two points)

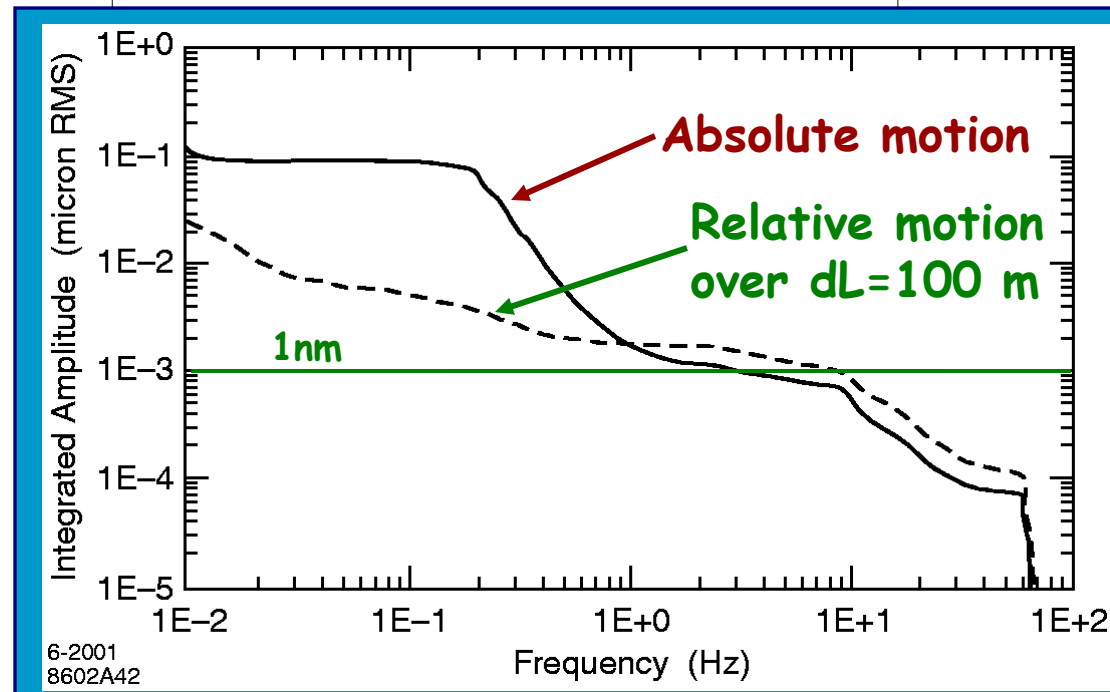
Place	A $\mu\text{m}^2/(\text{m}\cdot\text{s})$
HERA	$\sim 10^{-5}$
FNAL surface	$\sim 1\text{-few}\cdot 10^{-6}$
SLAC*	$\sim 5\cdot 10^{-7}$
Aurora mine*	$\sim 2\cdot 10^{-7}$
Sazare mine	$\sim 10^{-8}$

$\sim 20\mu\text{m}$ displacement
over 20m in one month

Fast motion: examples of absolute motion and relative (correlation data)



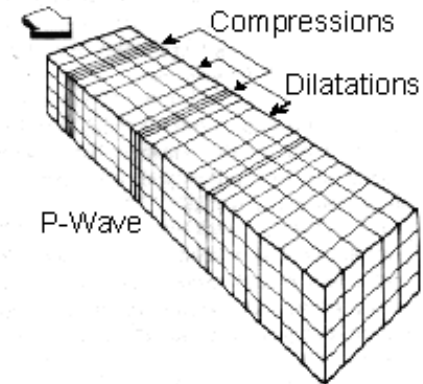
- Care about relative, not absolute motion
- Beneficial to have good correlation (longer wavelength)
- Relative motion can be much smaller than absolute



Integrated (for $F > F_0$) spectra. SLC tunnel @ SLAC

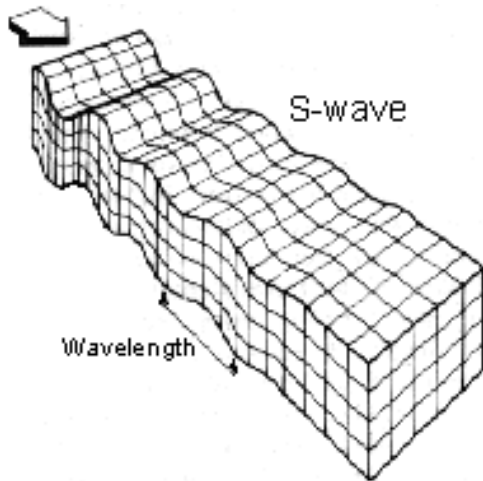
Correlation of ground motion depends on velocity of waves (and distribution of sources in space)

P-wave, (primary wave, dilatational wave, compression wave)
 Longitudinal wave. Can travel through liquid part of earth.



Velocity of propagation

$$v_P = \sqrt{\frac{\lambda + 2G}{\rho}}$$



S-wave, (secondary wave, distortional wave, shear wave)
 Transverse wave. Can not travel through liquid part of earth

Velocity of propagation

$$v_S = \sqrt{\frac{G}{\rho}} \quad \text{typically} \quad v_S \approx \frac{v_P}{2}$$

Here ρ - density, G and λ - Lamé constants:

$$G = \frac{E}{2(1+\nu)} \quad \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

E - Young's modulus, ν - Poisson ratio

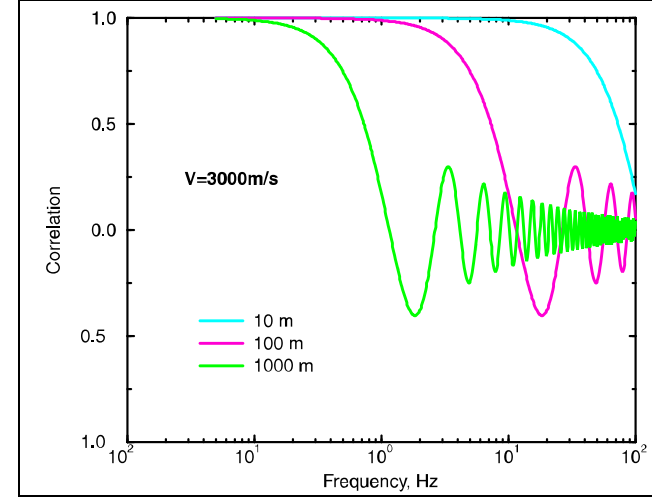
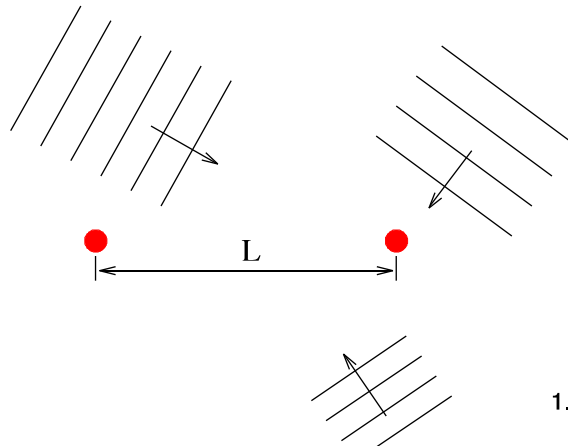
Correlation measurements and interpretation

In a model of plane wave propagating on surface

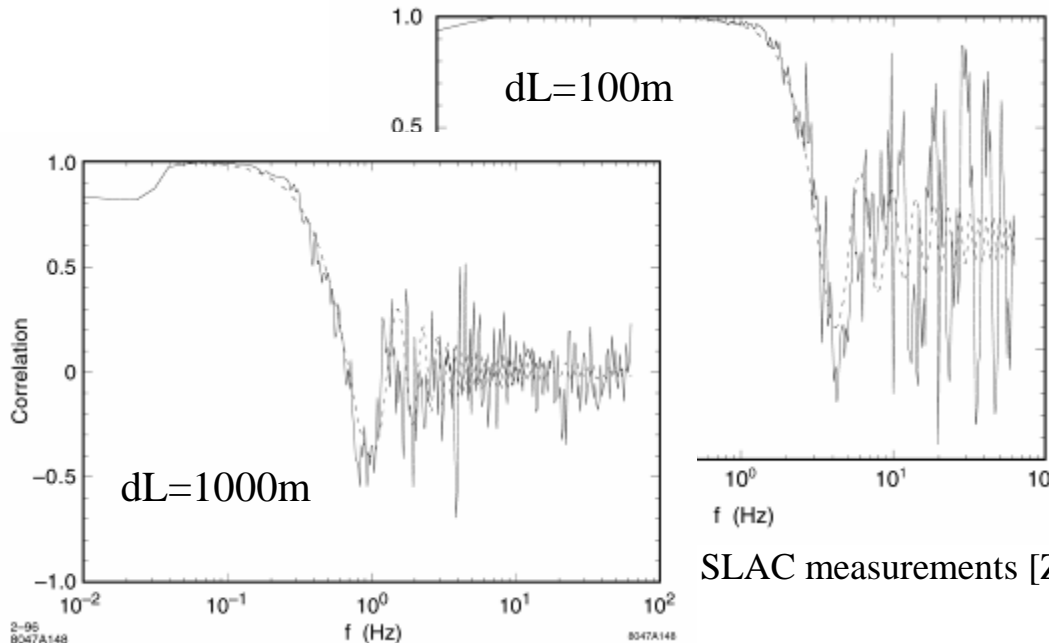
correlation =

$$\langle \cos(\omega \Delta L / v \cos(\theta)) \rangle_{\theta} = J_0(\omega \Delta L / v)$$

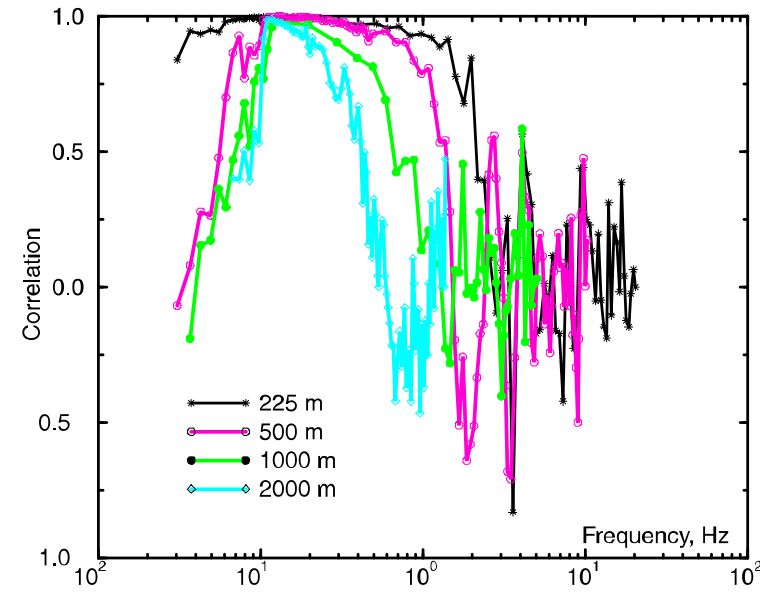
where v - phase velocity



Theoretical curves



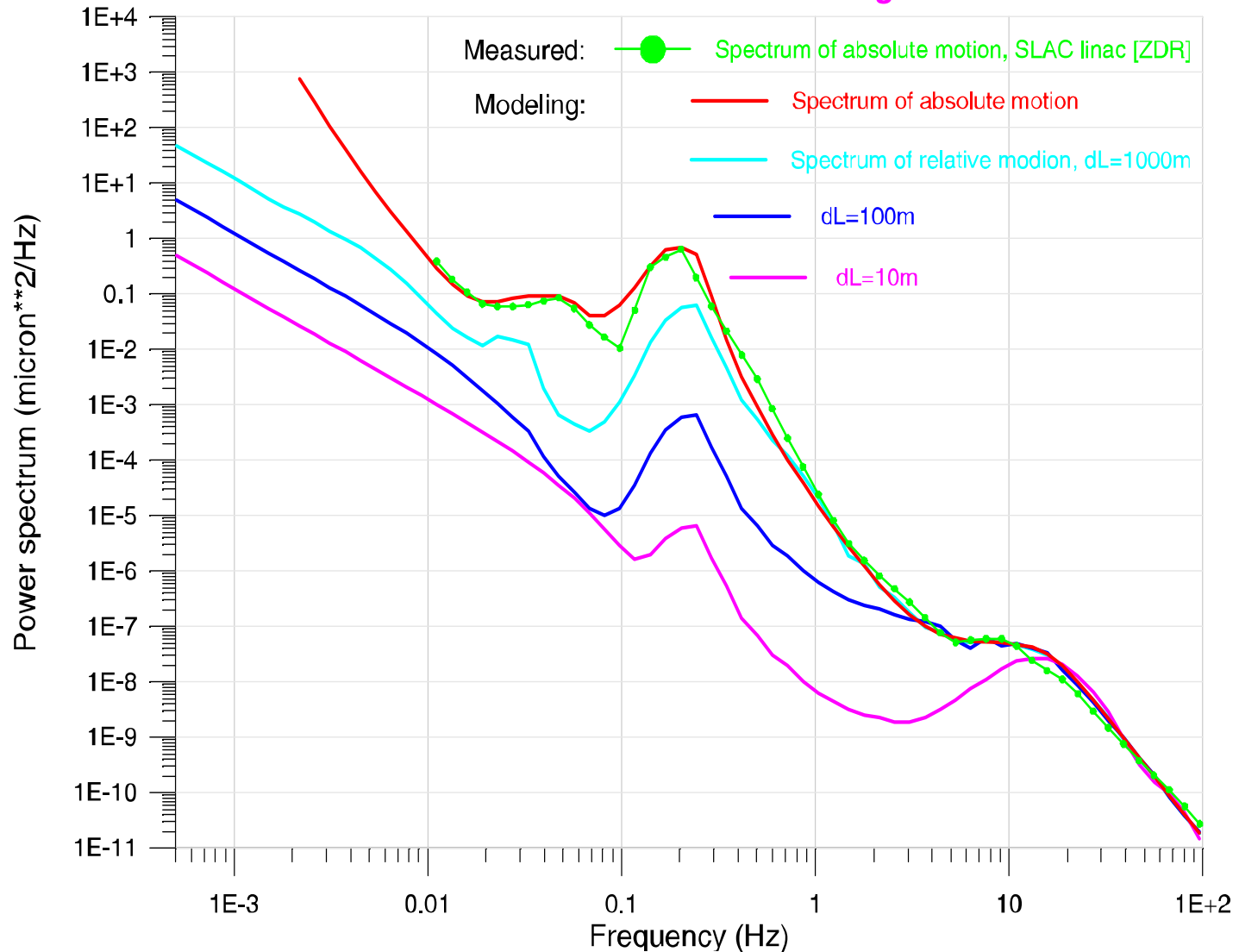
SLAC measurements [ZDR]



LEP measurements

Example of $P(\omega, L)$ spectrum (model)

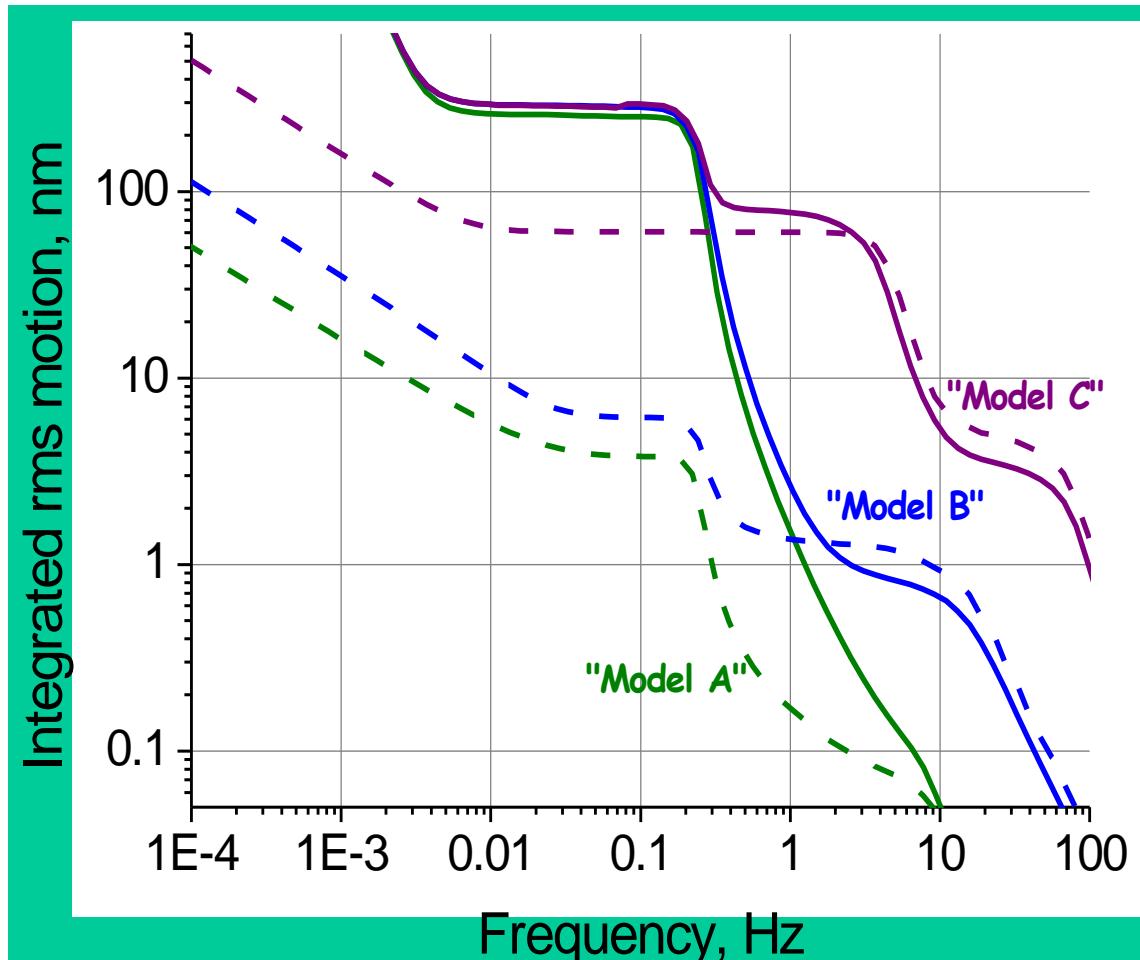
Spectra of absolute and relative motion of two points separated by distance dL for the "2am SLAC site" model of ground motion.



Ground motion models

- Based on data, build modeling $P(\omega, k)$ spectrum of ground motion which includes:

- Elastic waves
- Slow ATL motion
- Systematic motion
- Cultural noises



Example of integrated spectra of absolute (solid lines) and relative motion for 50m separation obtained from the models

Behavior of spectral functions

Remember that before assuming that beams injected without offset we wrote that

$$\mathbf{x}_*(t) = \mathbf{R}_{11} \mathbf{x}_{inj}(t) + \mathbf{R}_{12} \mathbf{x}'_{inj}(t) + \sum_{i=1}^N b_i \mathbf{x}_i(t) \quad \eta_x(t) = \mathbf{T}_{116} \mathbf{x}_{inj}(t) + \mathbf{T}_{126} \mathbf{x}'_{inj}(t) + \sum_{i=1}^N d_i \mathbf{x}_i(t)$$

It is easy to show that the coefficients b (and d) follow certain rules, which can be found in the next way. By considering a rigid displacement of the whole beam line, it is easy to find the identity

$$\sum_{i=1}^N b_i = 1 - R_{11} \quad \text{and} \quad \sum_{i=1}^N d_i = -T_{116}$$

On the other hand, one can show by tilting the whole beam line by a constant angle that the coefficients satisfy for thin lenses the following identity:

$$\sum_{i=1}^N b_i s_i + R_{12} = s_{exit} \quad \text{and} \quad \sum_{i=1}^N d_i s_i + T_{126} = 0$$

These rules allow to find behavior of the spectral functions at small k :

$$g_c(k \rightarrow 0) \approx O(k^2)$$

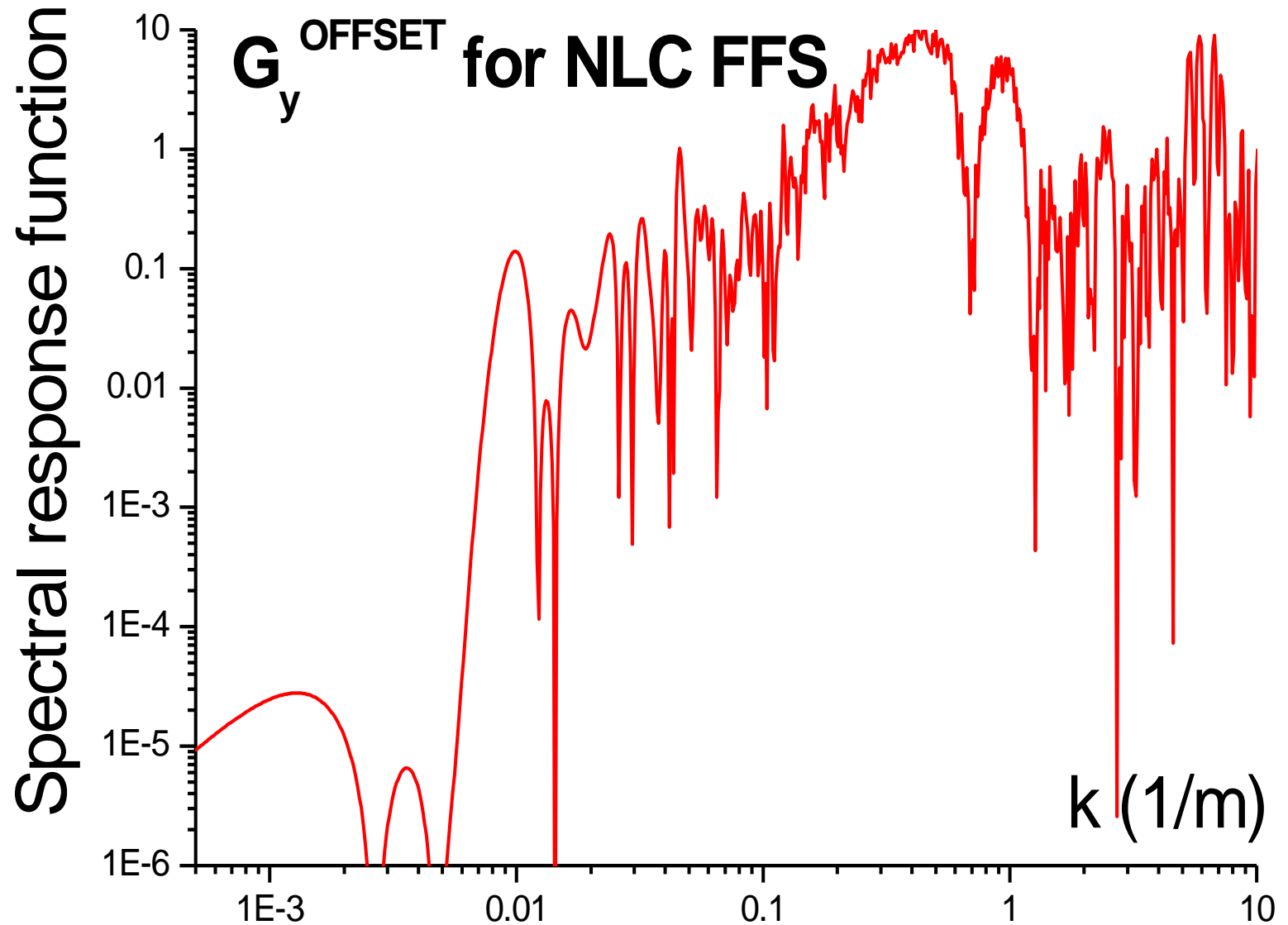
$$g_s(k \rightarrow 0) \approx -k \cdot R_{12} + O(k^3)$$

$$g_s(k) = \sum_{i=1}^N d_i \sin(ks_i)$$

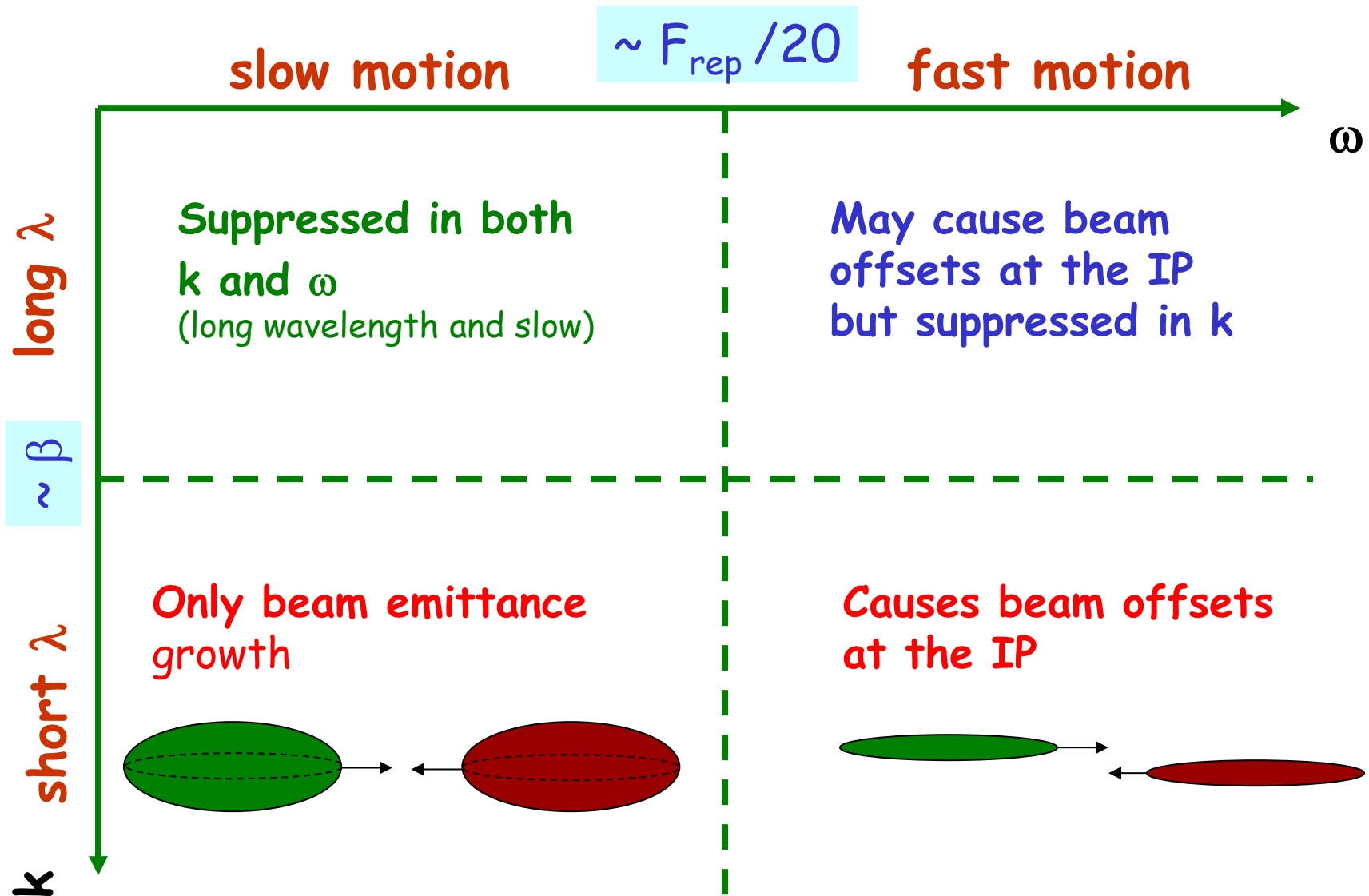
$$g_c(k) = \sum_{i=1}^N d_i [\cos(ks_i) - 1]$$

You see that if R_{12} is zero, effect of long wavelength is suppressed as k^2

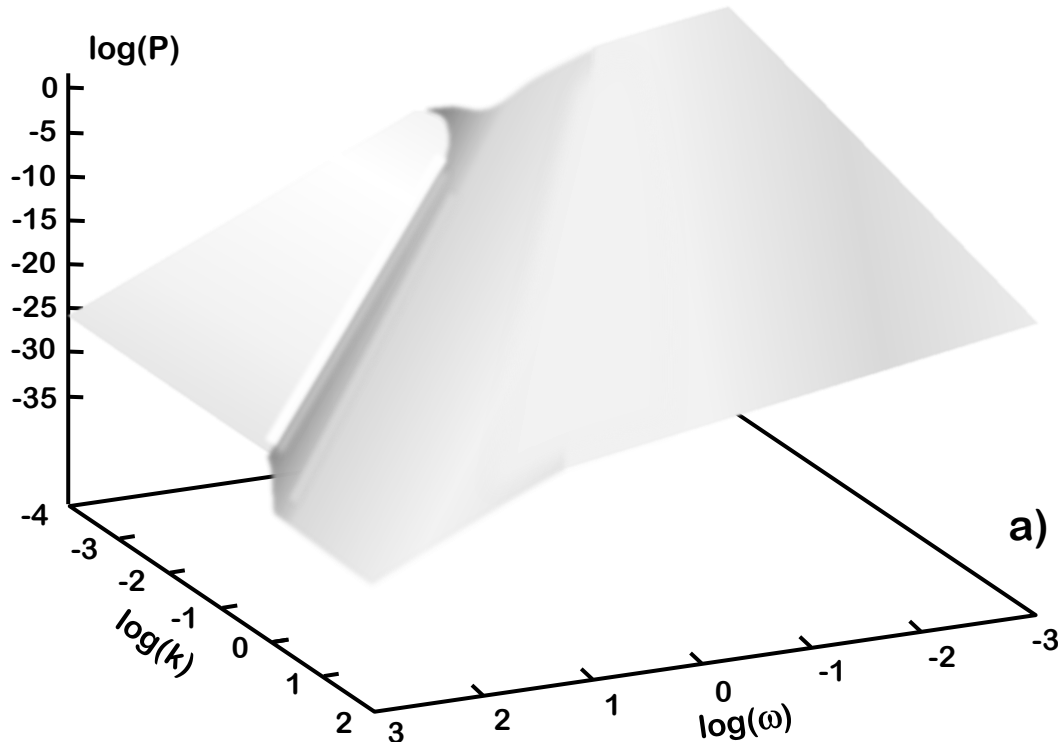
Example of spectral response function



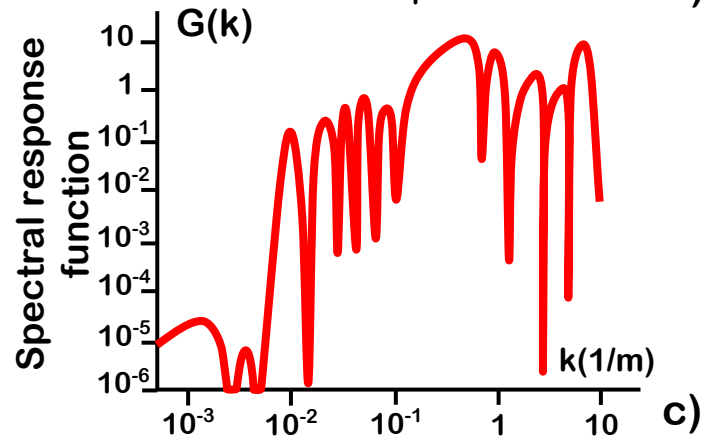
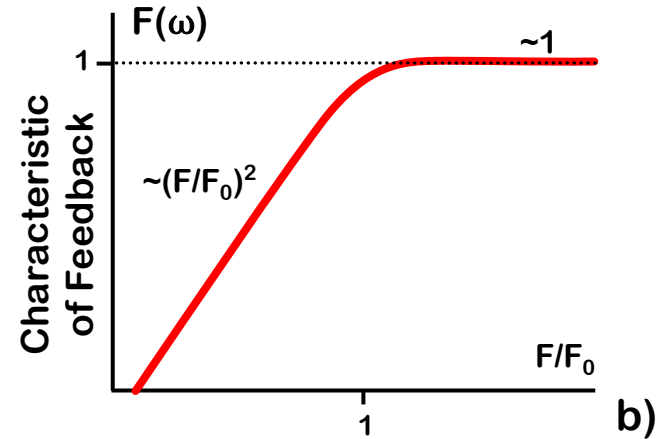
Effects of ground motion in Linear Colliders



Ground motion induced beam offset at IP



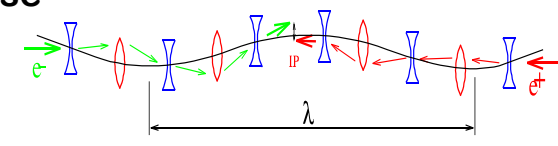
$P(\omega, k)$ - 2D spectrum of ground motion



rms beam offset at IP: $\propto \iint P(\omega, k) \cdot G(k) \cdot F(\omega) \cdot dk \cdot d\omega$

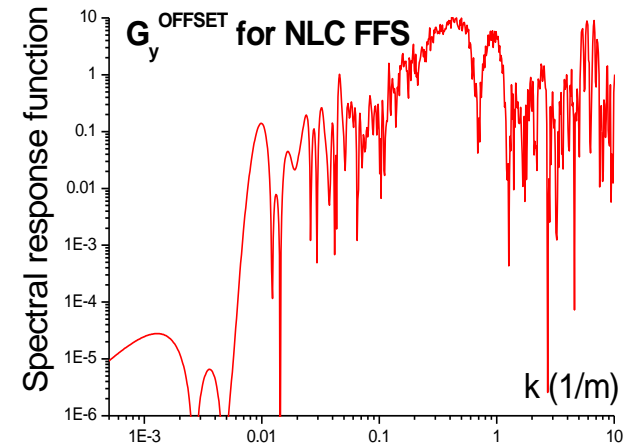
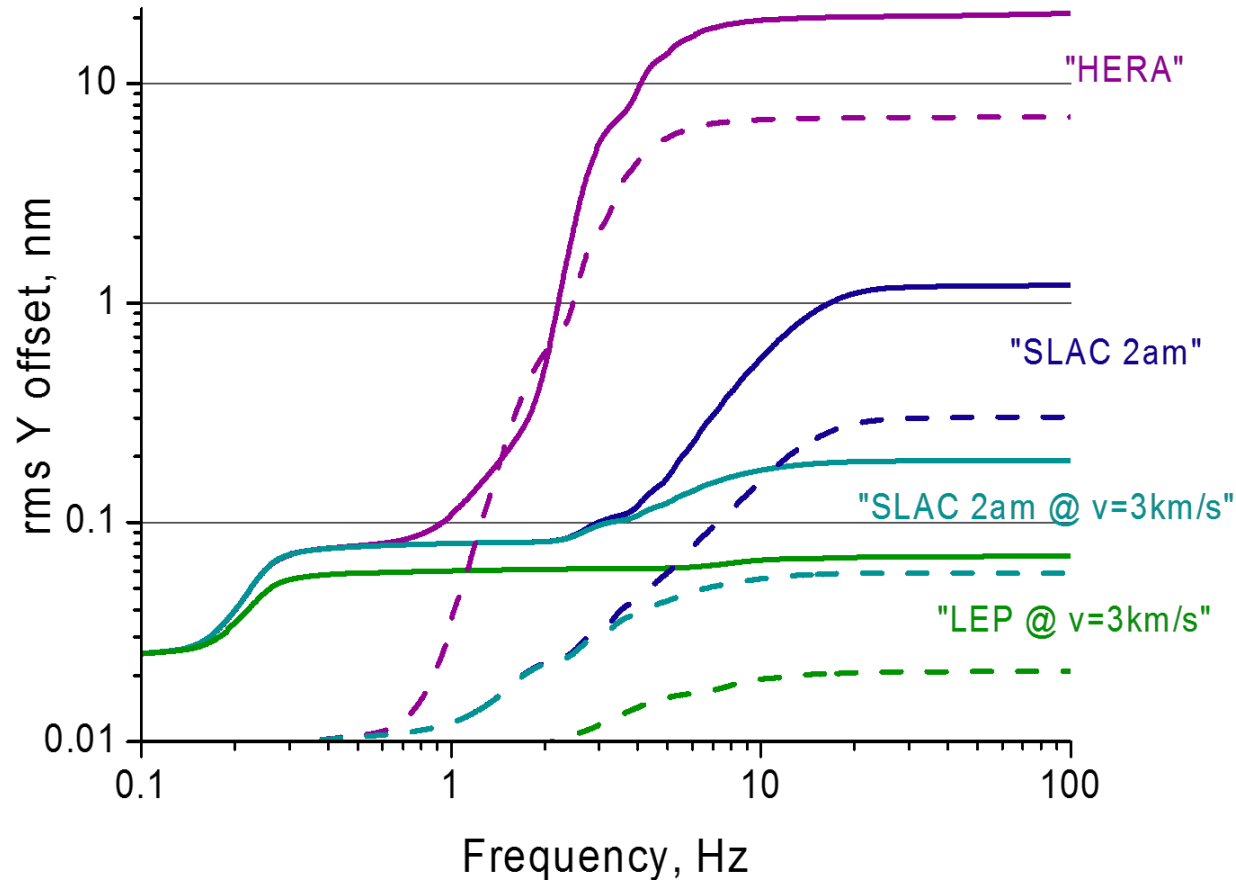
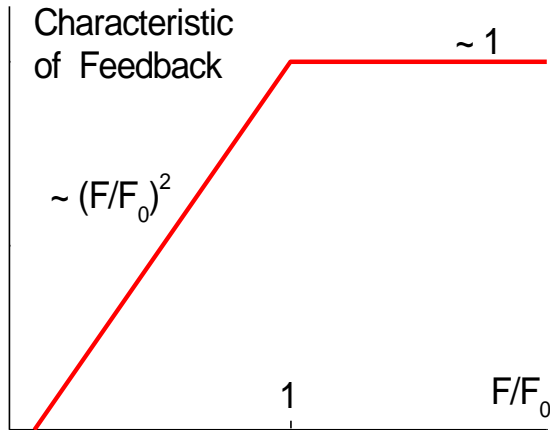
Spectral response function $G(k)$

Performance of inter-bunch feedback $F(\omega)$



- Thank you for your attention!
- There are extra slides but we will likely run out of time
- You are welcome to ask questions about the main part of the lecture or about the additional slides
 - We can also discuss after the class
- In the next lecture we will discuss design and layouts of final focus systems, and also touch again on FF stability

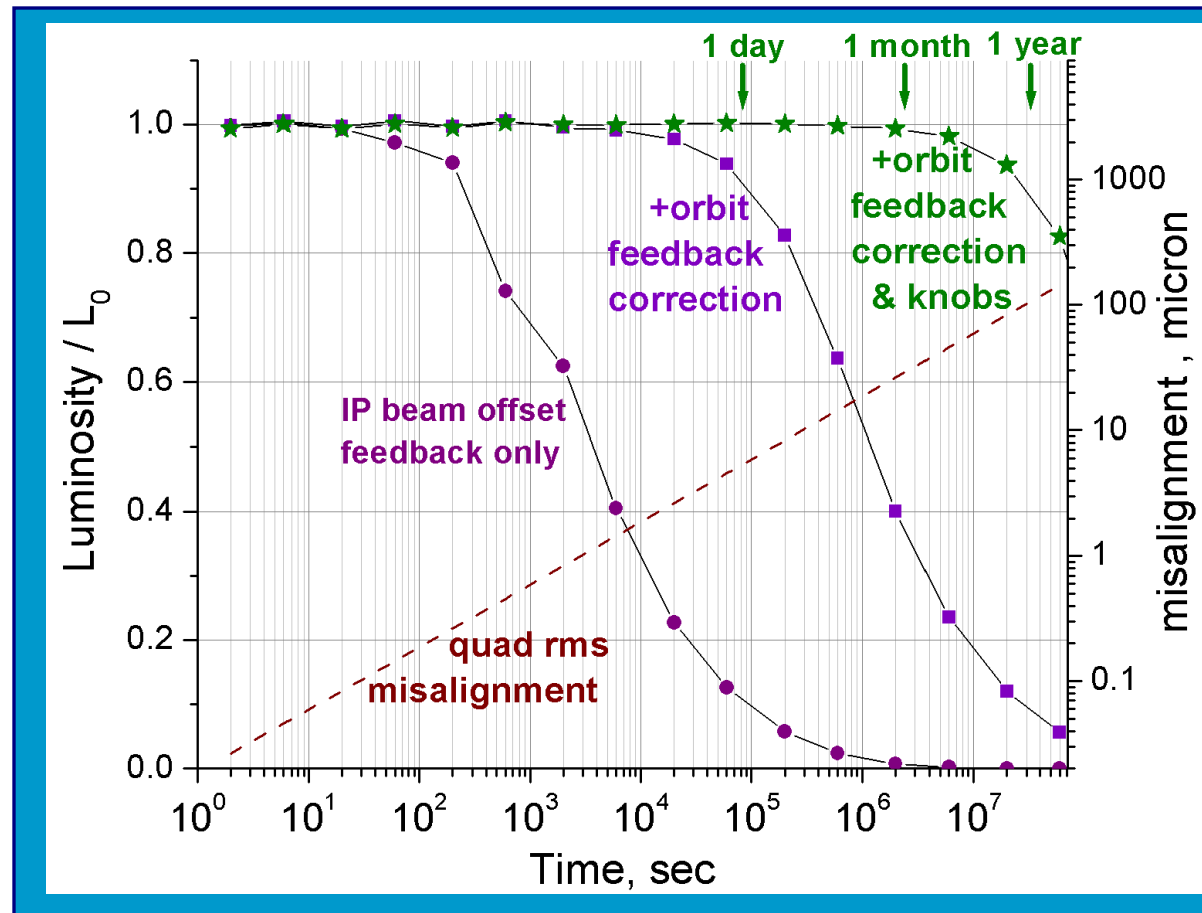
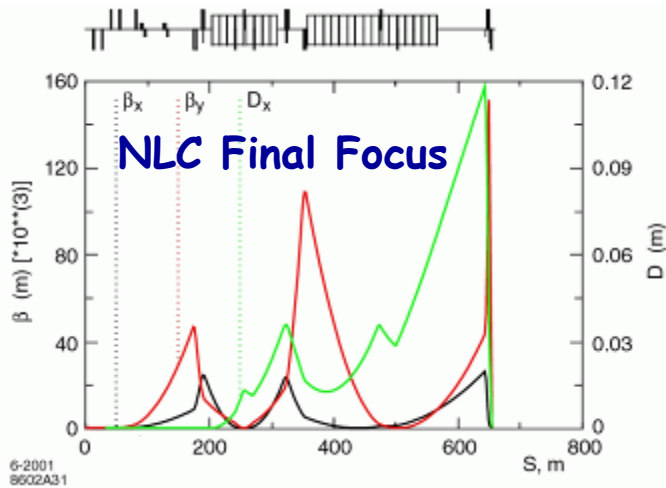
Beam offset at the IP of NLC FF for different GM models



rms beam offset at IP: $\propto \iint P(\omega, k) \cdot G(k) \cdot F(\omega) \cdot dk \cdot d\omega$

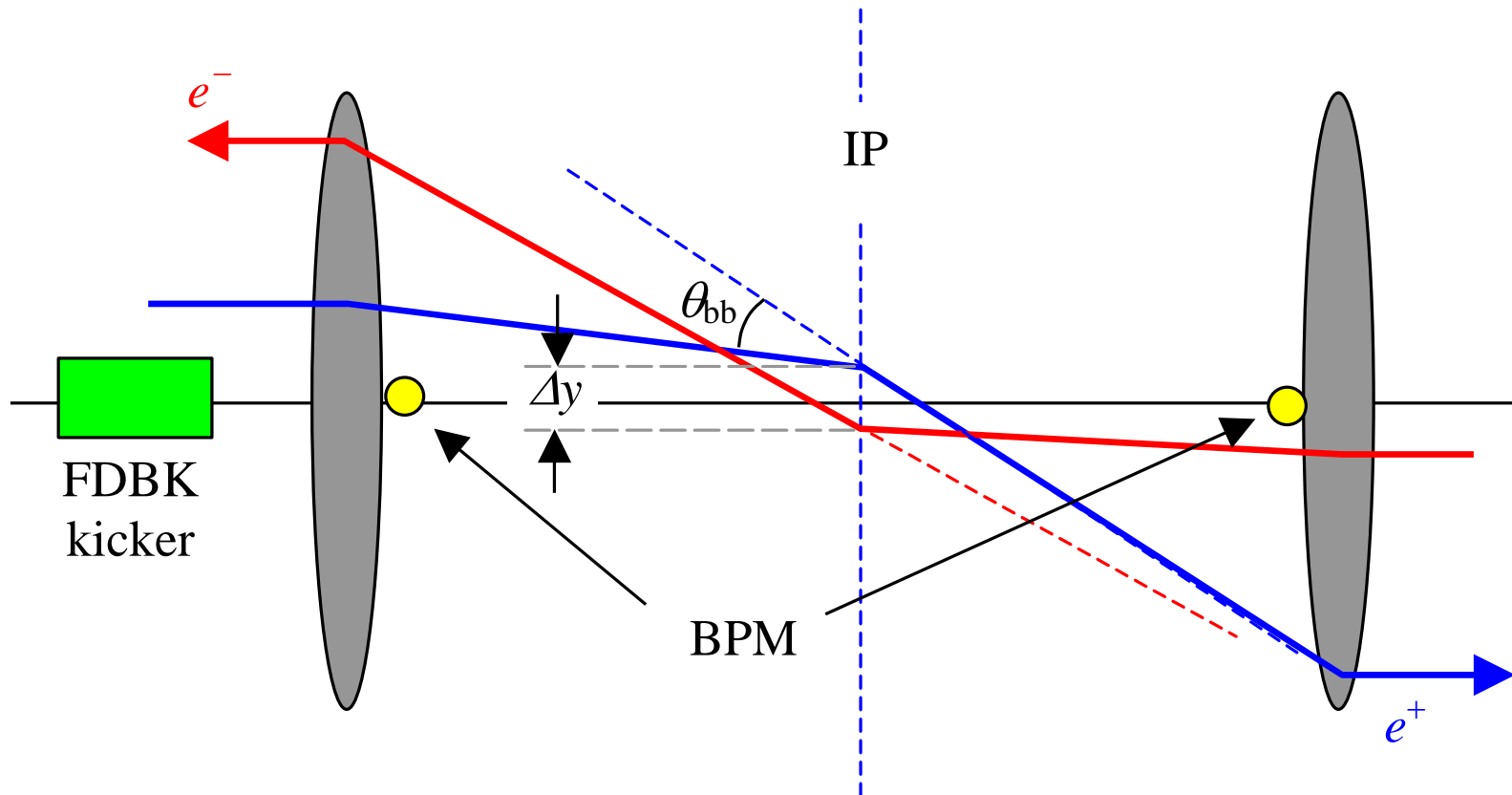
Simulations of feedbacks and Final Focus knobs

IP feedback, orbit feedback and dithering knobs suppress luminosity loss caused by ground motion



- Ground motion with $A=5 \times 10^{-7} \mu\text{m}^2/\text{m/s}$
- Simulated with MONCHOU

Beam-Beam orbit feedback



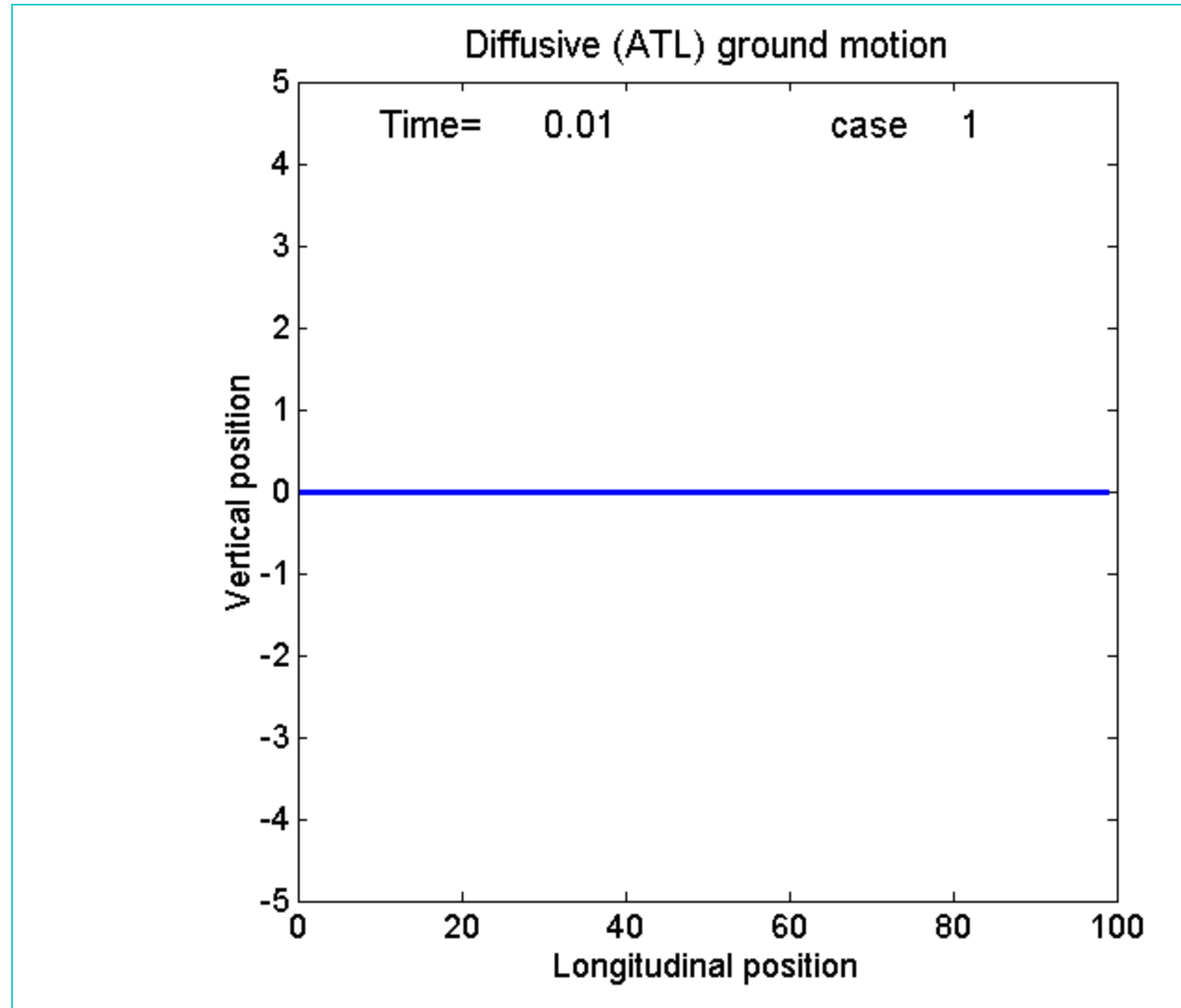
use strong beam-beam kick to keep beams colliding

Slow motion (minutes - years)

- **Diffusive** or **ATL** motion: $\Delta X^2 \sim ATL$
(T – elapsed time, L – separation between two points)
(minutes-month)
- Observed 'A' varies by ~5 orders: 10^{-9} to $10^{-4} \mu\text{m}^2/(\text{m}\cdot\text{s})$
 - **parameter 'A' should strongly depend on geology -- reason for the large range**
 - **Range comfortable for NLC: $A < 10^{-6} \mu\text{m}^2/(\text{m}\cdot\text{s})$**
Very soft boundary! Observed A at sites similar to NLC deep tunnel sites is several times or much smaller.
- **Systematic** motion: ~linear in time (month-years), similar spatial characteristics
- **In some cases can be described as ATTL law :**
 - SLAC 17 years motion suggests $\Delta X^2 = A, T^2 L$ with
 $A, \sim 4 \cdot 10^{-12} \mu\text{m}^2/(\text{m}\cdot\text{s}^2)$ for early SLAC

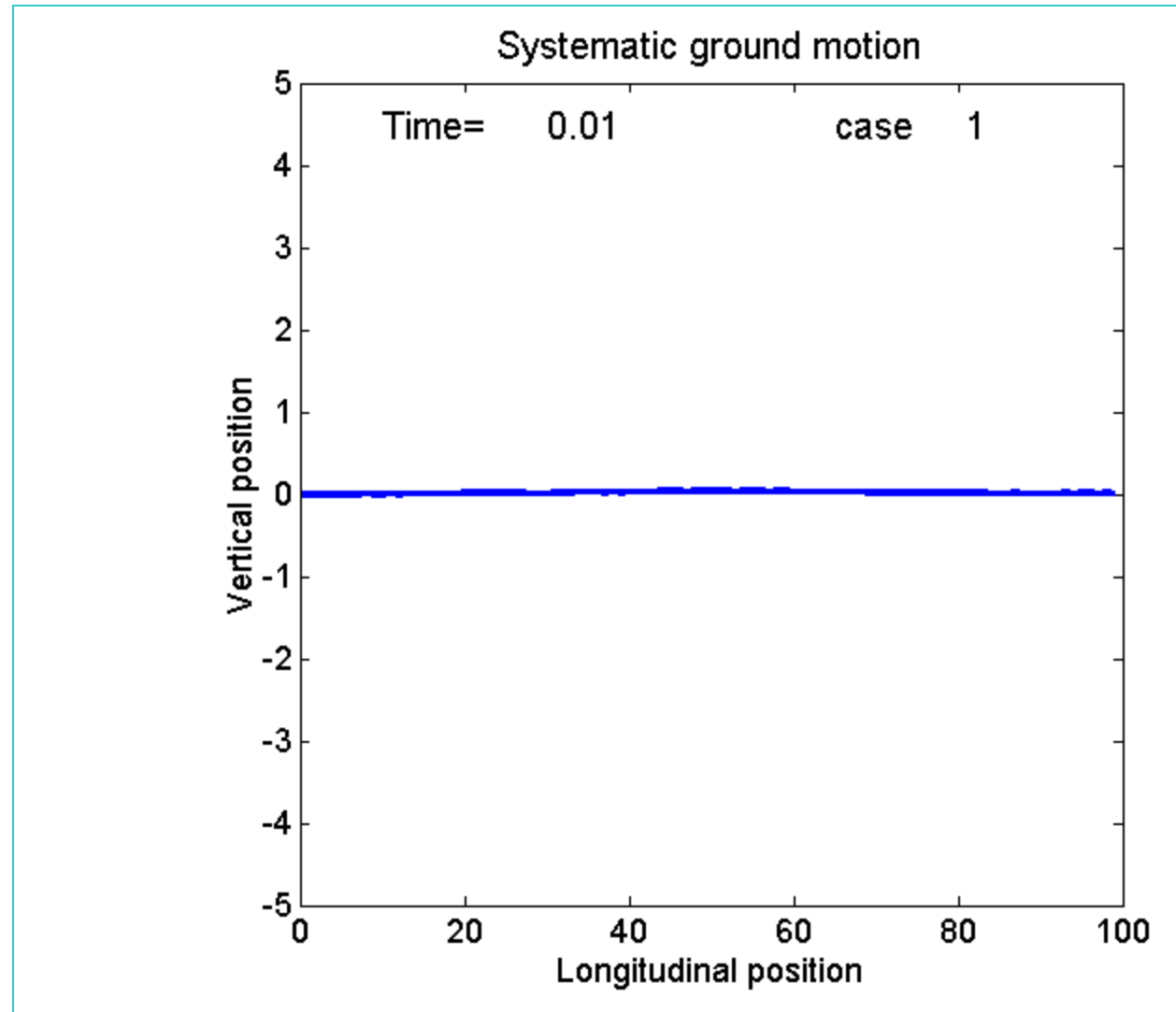
How diffusive ATL motion looks like?

- Movie of simulated ATL motion
- Note that it starts rather fast
- $X^2 \sim L$
- and it can change direction...



How systematic motion looks like?

- Movie of simulated systematic motion
- Note that final shape may be the same as from ATL
- And it may resemble...



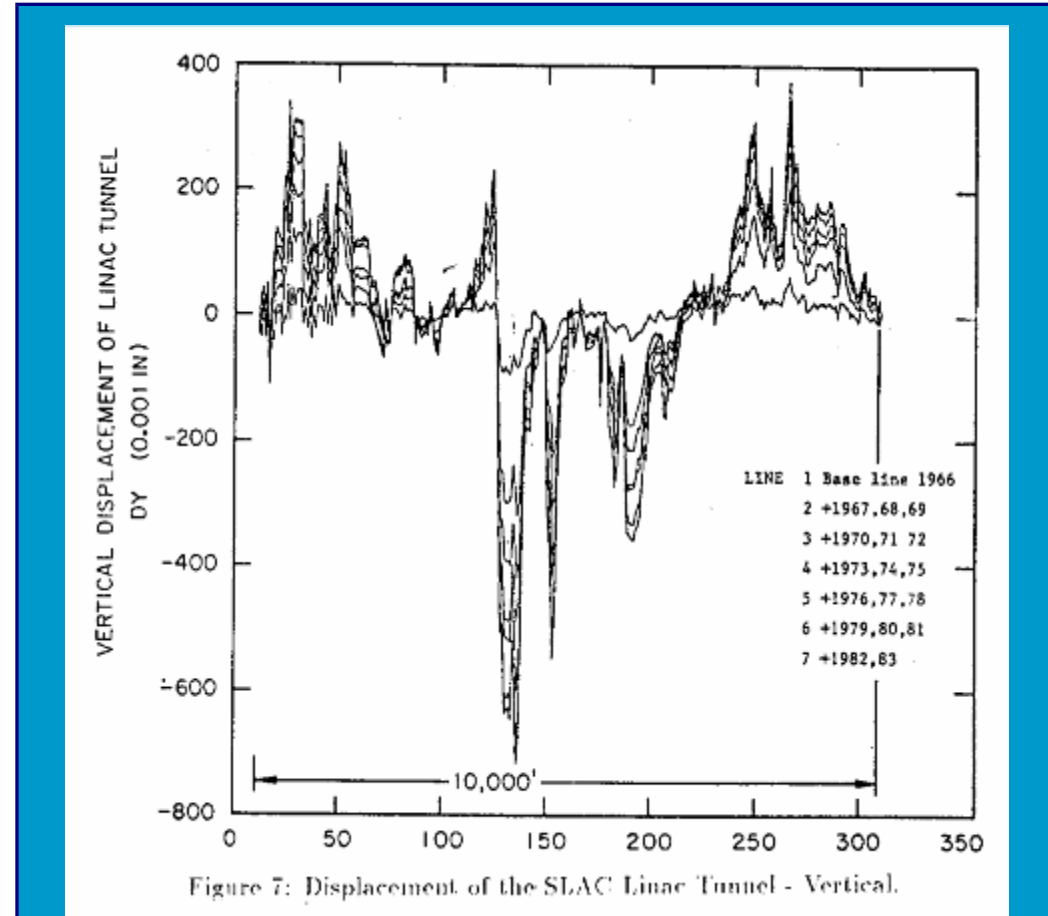
And in billion years...



Systematic motion

SLAC linac tunnel in 1966-1983

- Year-to-year motion is dominated by systematic component
- Settlement...



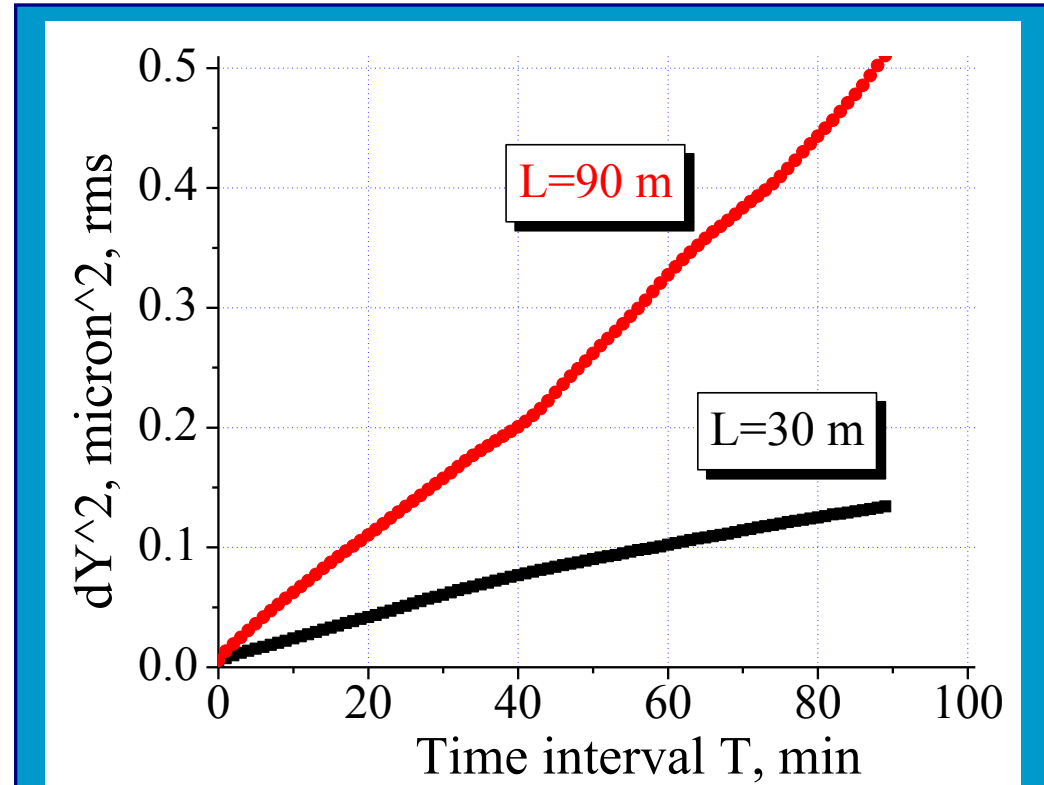
Vertical displacement of SLAC linac for 17 years

Slow motion

example: Aurora mine

- Slow motion in Aurora mine exhibit ATL behavior
- Here $A \sim 5 \cdot 10^{-7} \mu\text{m}^2/\text{m}/\text{s}$

(similar value was observed at SLAC tunnel)



Slow motion in Aurora mine.
Measured by hydrostatic level system.

Slow motion study (BINP-FNAL-SLAC)

Diffusion coefficients A [$10^{-7} \mu\text{m}^2/(\text{m}\cdot\text{s})$]:
 (10-100) for MI8 shallow tunnel in glacial till
 (in absence of dominating cultural motion);
 ~3 or below in deep Aurora mine in dolomite
 and in SLAC shallow tunnel in sandstone

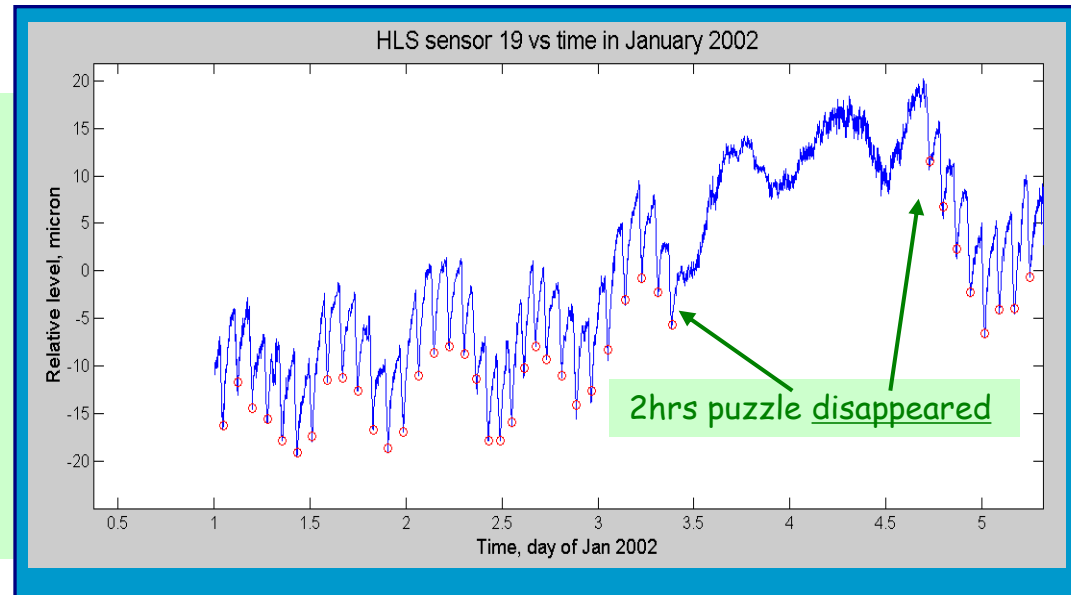
Shallow tunnel in sedimentary/glacial geology - is a risk factor, both because of higher diffusive motion, and because of possibility of cultural slow motion.



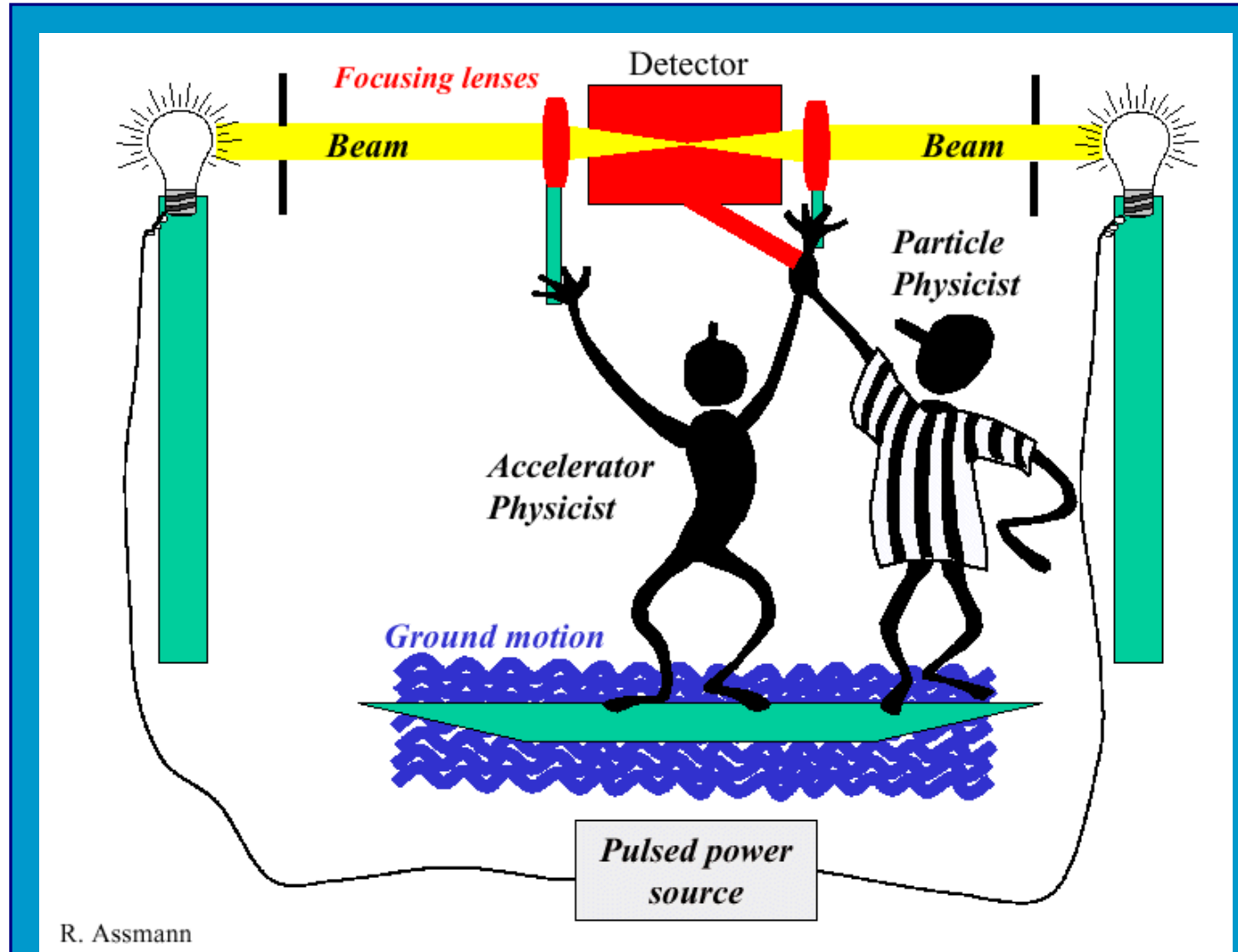
Cultural effects on slow motion:
 "2hour puzzle" - 10 μm motion occurring near one of the ends of the system

Reason: domestic water well which slowly and periodically change ground water pressure and cause ground to move

Large amplitude, rather short period, bad correlation - nasty for a collider

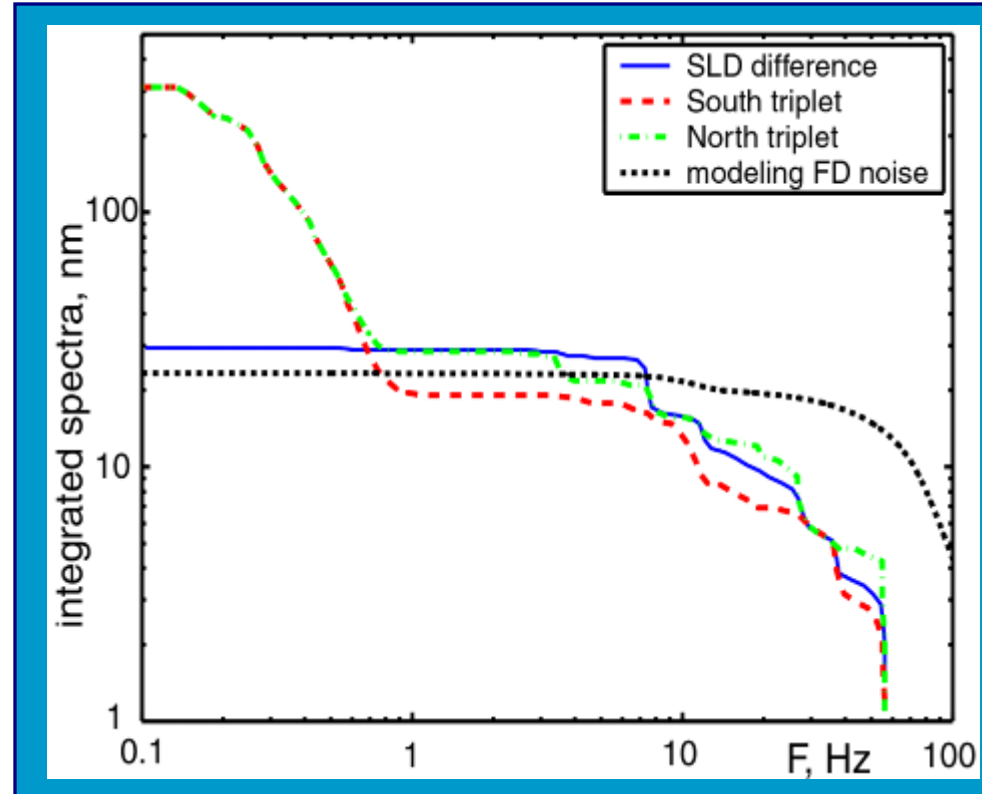
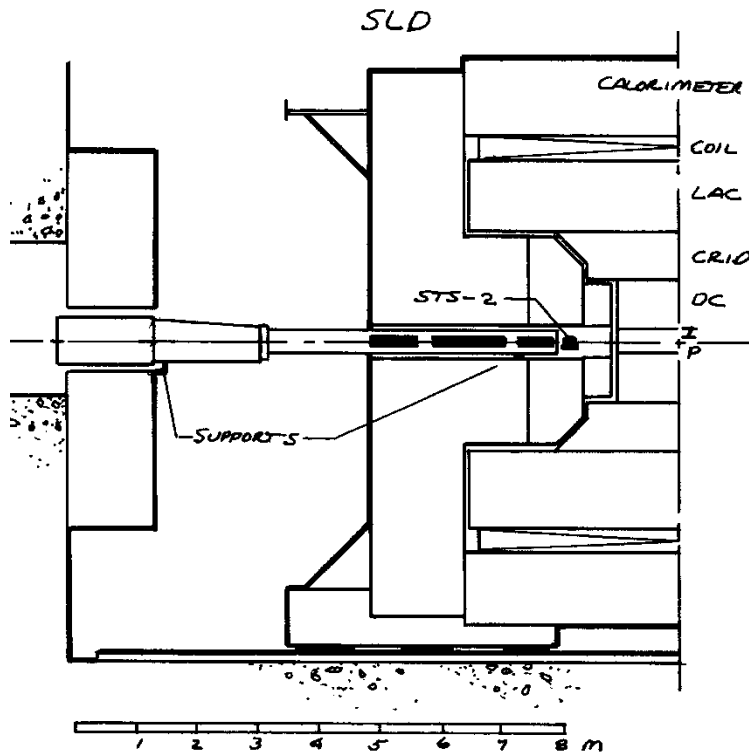


Detector complicates reaching FD stability



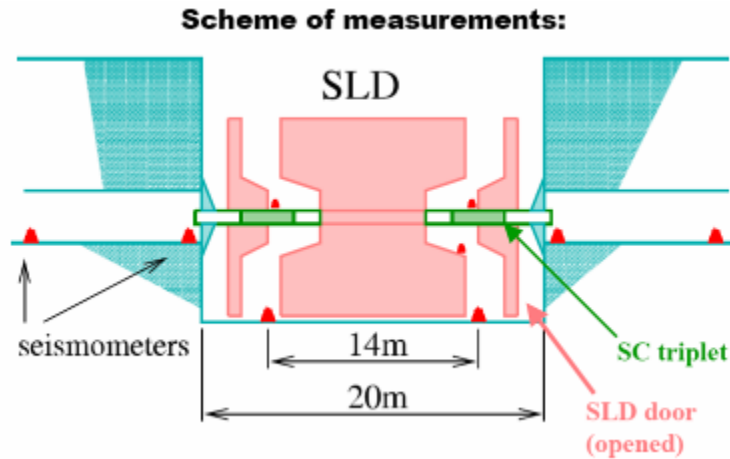
Cartoon from Ralph Assmann (CERN)

Detector is a noisy ground !

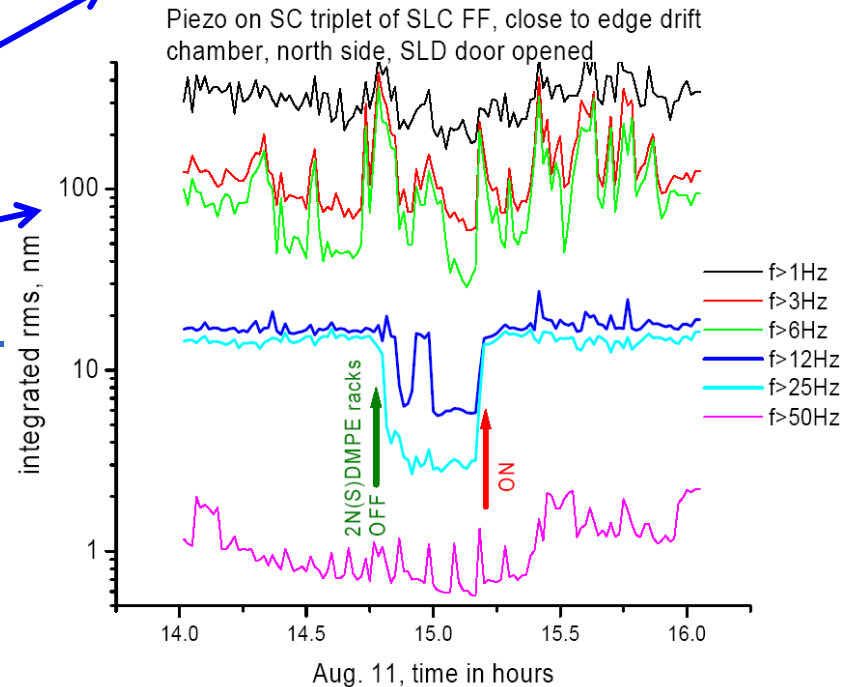
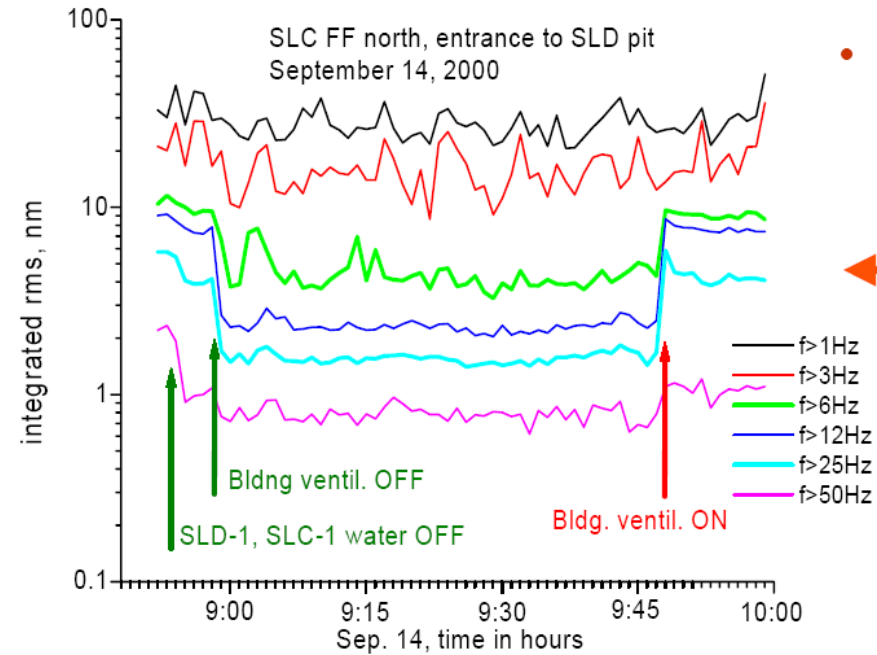


Measured ~30nm relative motion between South and North final triplets of SLC final focus. The NLC detector will be designed to be more quiet. But in modeling we pessimistically assume the amplitude as observed at SLD

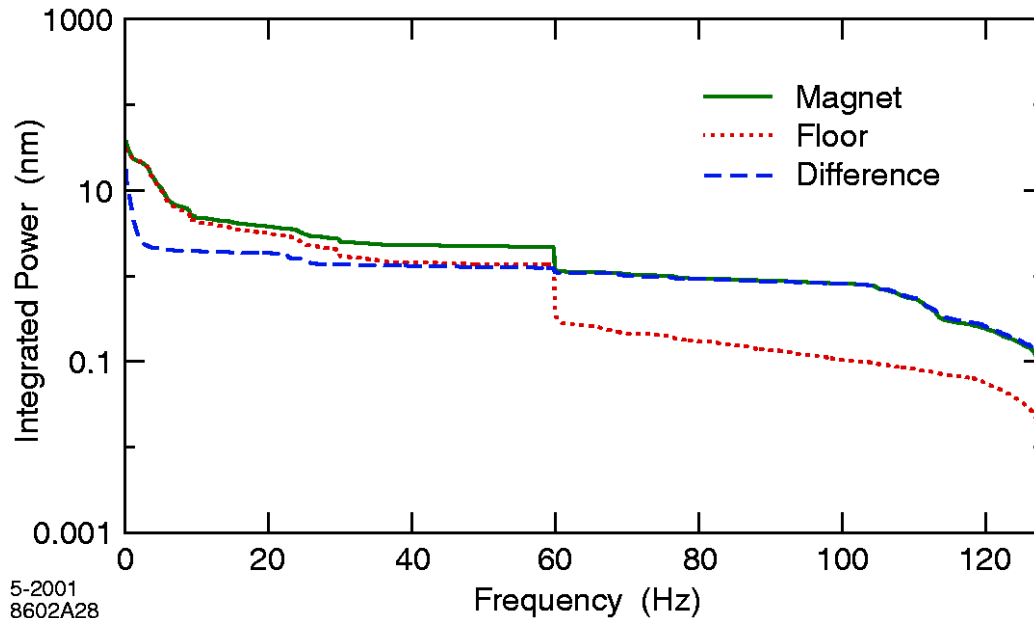
Vibrations at detector



- Floor noise in SLD pit and FF tunnel mostly affected by building ventilation and water compressor station
- Vibration on detector mostly driven by on-SLD door mounted racks, pumps, etc.
- This shows that it may be needed to place noisy detector equipment on separate platform nearby



10 nm goal for BDS component jitter

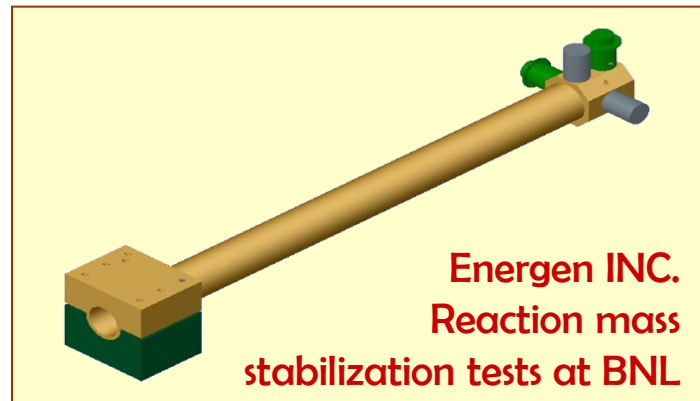
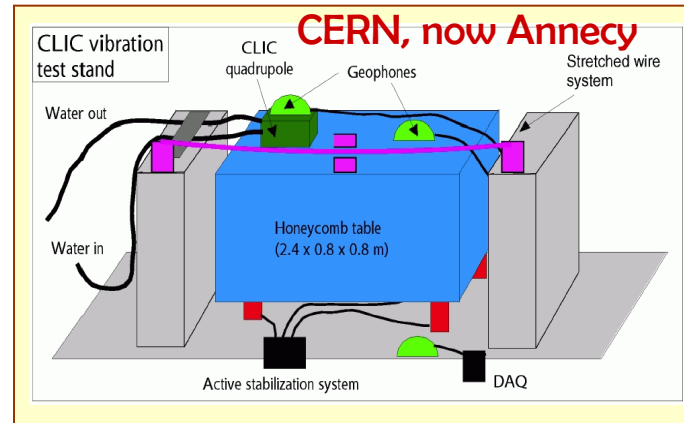
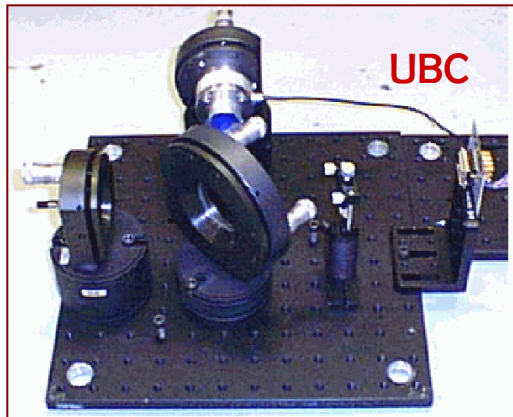
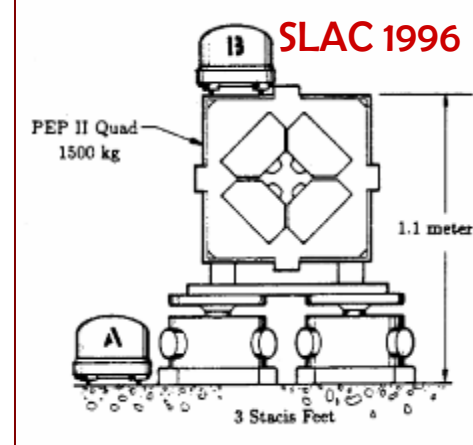
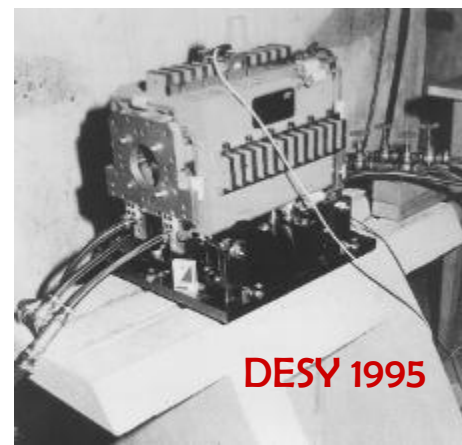


● FFTB quad

- Small (~2nm at 5Hz) difference to ground (on movers, with water flow, etc.)
- Lower frequency is relevant for 5Hz machine (0.2-0.5Hz) but was not studied accurately
- The 10nm goal may be achievable (for BDS area in gm B to B*3)

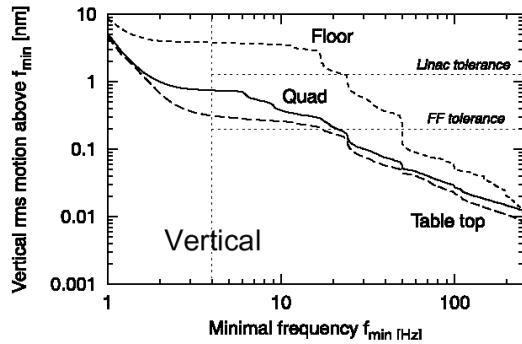
Stabilization studies

- Experience invaluable
- Components of developed hardware may be applicable



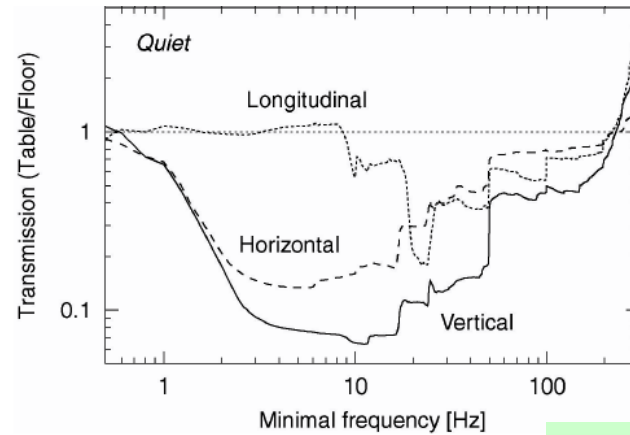
CLIC stability study

Quadrupole vibration:



On magnet top:

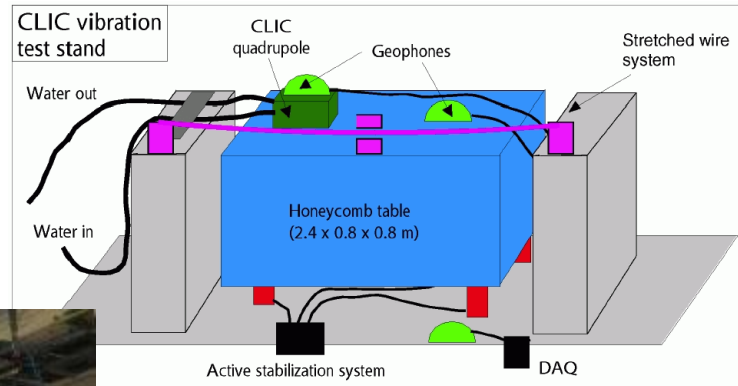
X: (0.4 ± 0.1) nm
 Y: (0.9 ± 0.1) nm
 (0.3 nm on table top)
 Z: (3.2 ± 0.4) nm
 without cooling water.



With nominal flow of cooling water:

Y: (1.3 ± 0.2) nm

Tight vertical linac tolerance demonstrated!



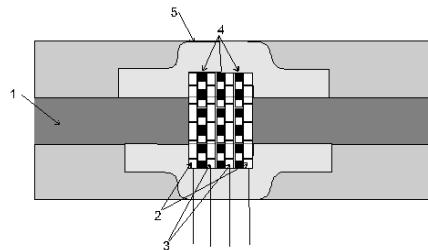
Using commercial STAICIS 2000 (TMC) achieved 1nm stability of a CLIC quadrupole

Nonmagnetic sensors, detector friendly design, would be needed in real system

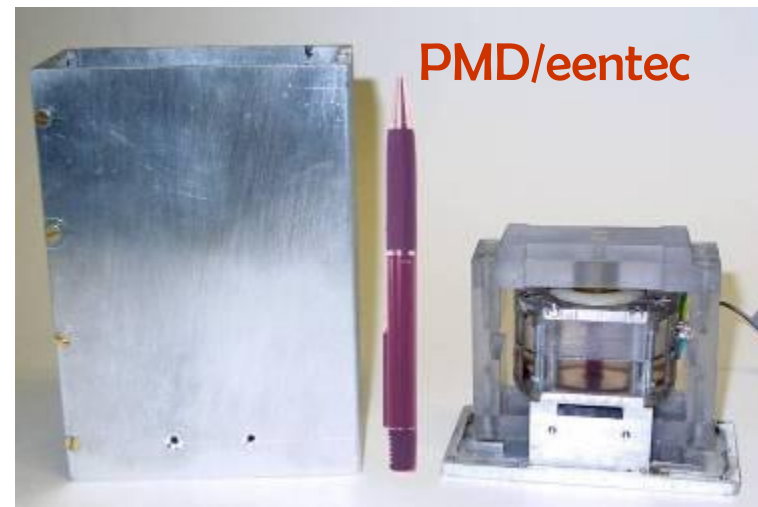


Development of sensors for IR

- Nonmagnetic inertial seismometers
 - SLAC home built – low noise, as good as Mark4 geophone or better
 - Molecular Electronic Transfer sensor – low noise, tested in 1.2T field, but cannot be cooled
- Interferometer methods
- Will need to use these or more advanced sensors to monitor FD motion

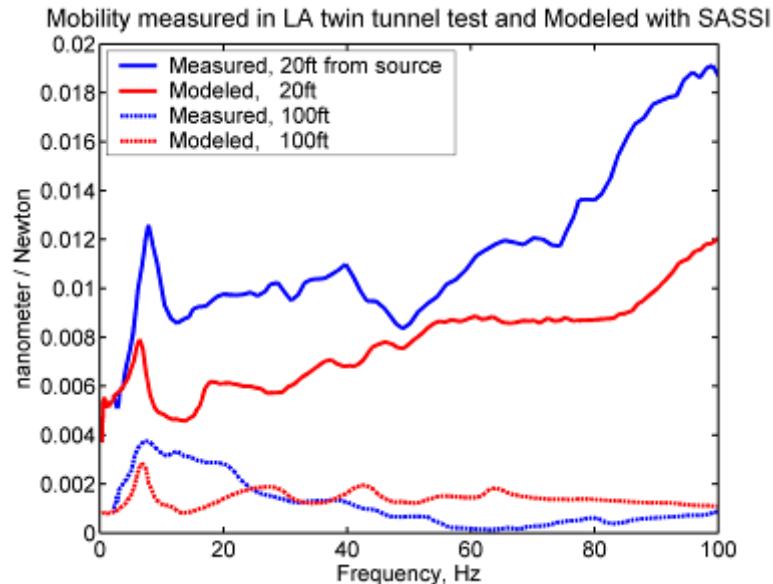
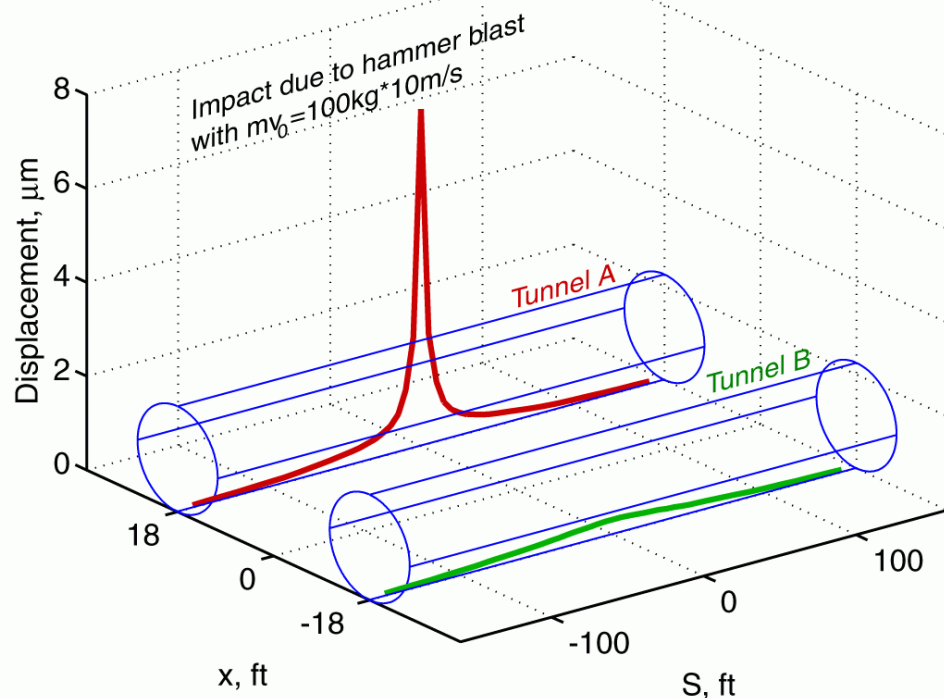
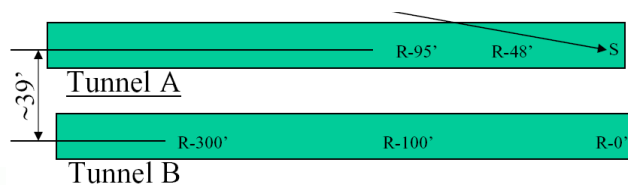


- 1 - Electrolyte channel
- 2 - Platinum mesh anodes
- 3 - Platinum mesh cathodes
- 4 - Microporous spacers
- 5 - Housing



Vibration transmission

- LA twin tunnel: between tunnels and from surface (figs shown)
- Results are valuable for ILC

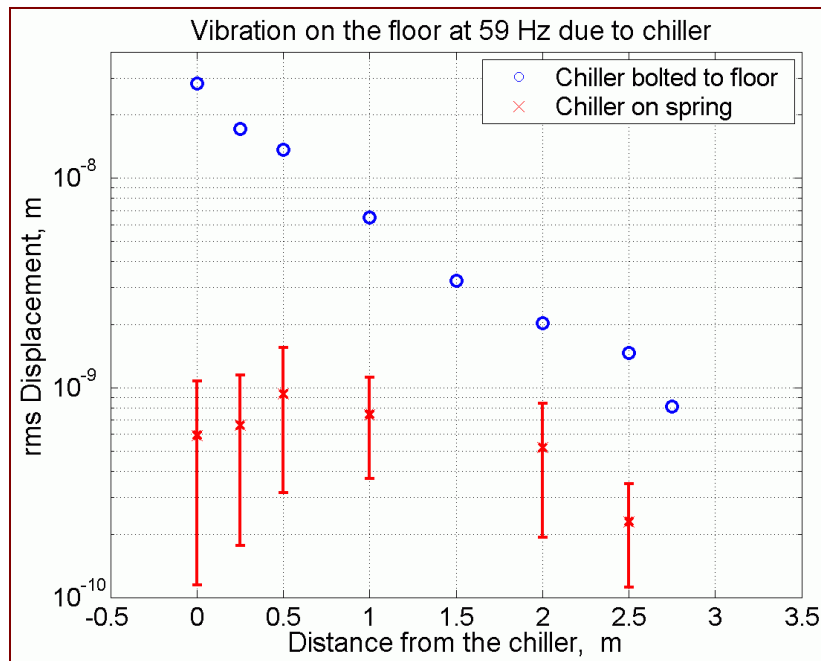


Mobility (response / driving force) measured in LA metro twin tunnel test and modeled with 3D code SASSI.

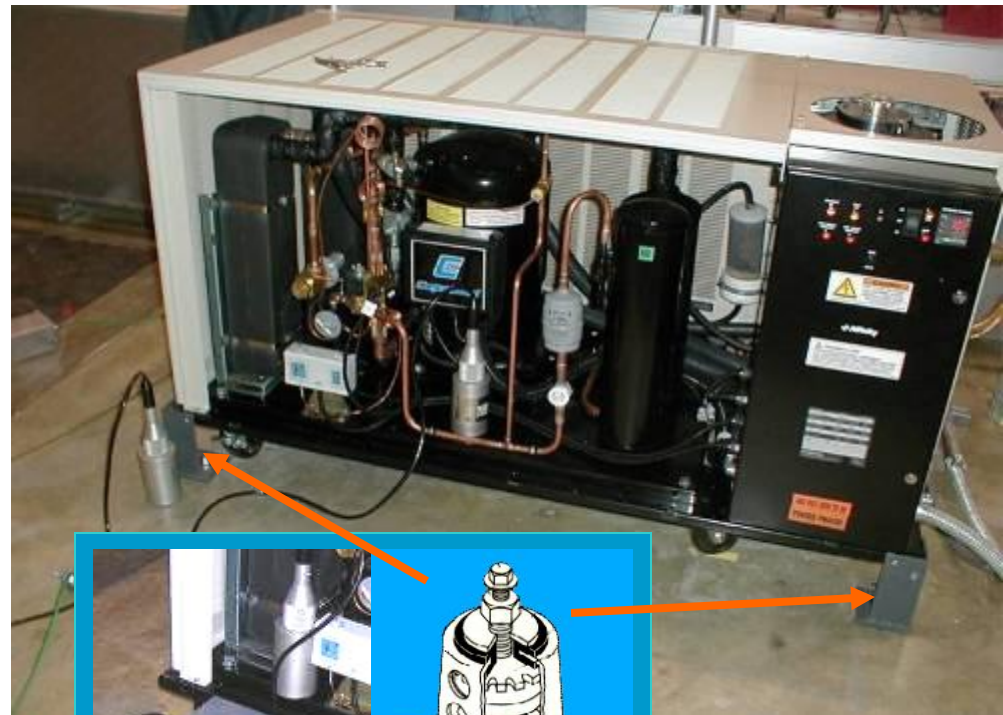


Vibration isolation of vibration sources

- Should be a standard practice for ILC

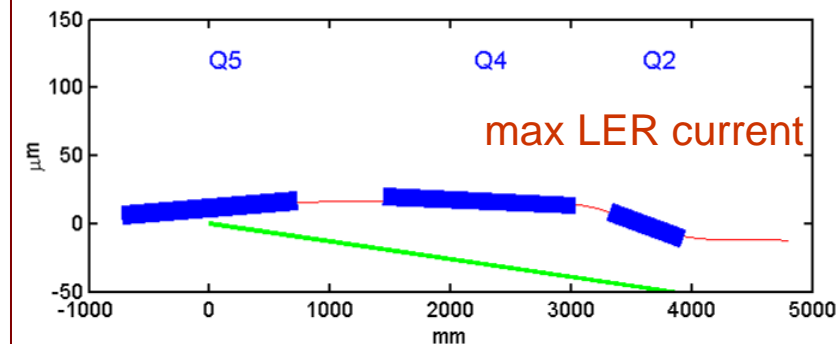
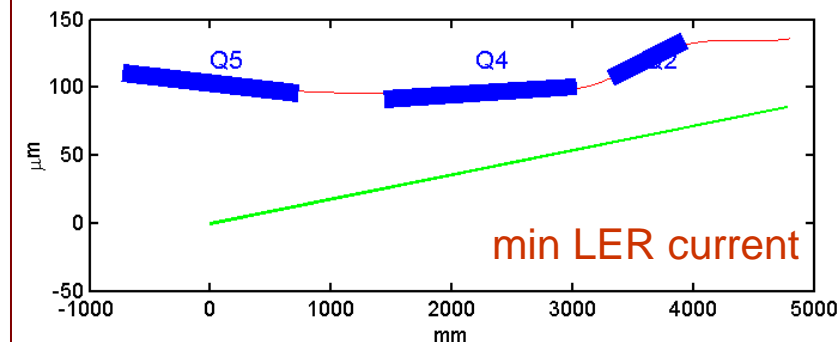
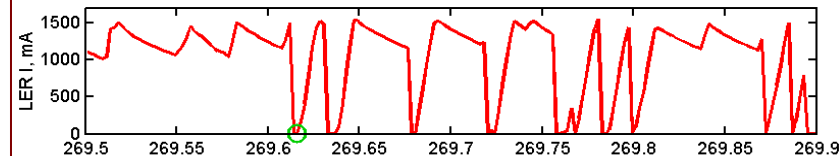
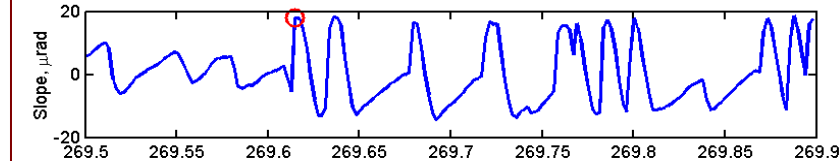
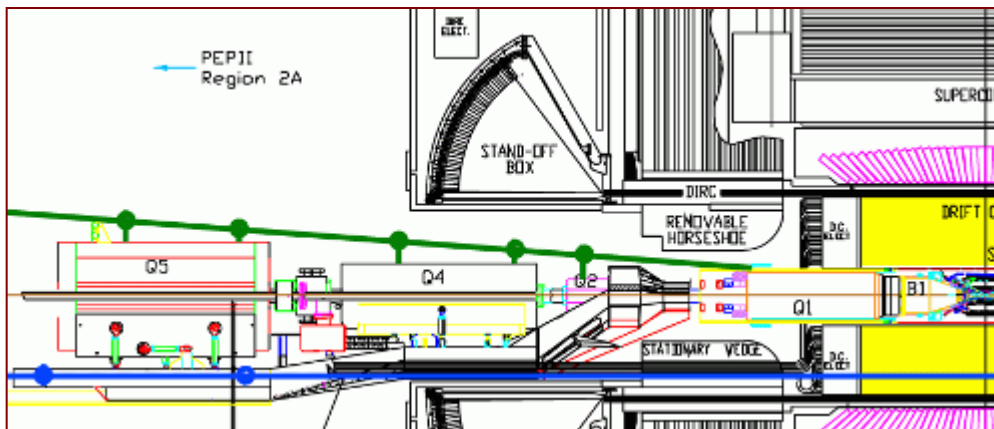


Vibration on the floor vs distance.
For chiller on springs, its vibration effects are indistinguishable on the floor.



IR stability

- Vibration is not the only concern
 - Temperature stability?
 - Wakes heating the IR chamber and deforming it? During 1ms?
 - SR should be well masked in IR, but may it cause deformations in other parts of BDS?
 - Example of PEP-II: IR heated by SR from LER and is moving by 0.1 mm as e⁺ current vary



Current of Low Energy Ring, slope of the girder measured by HLS and wire, and reconstructed position of FD magnets for min and max LER current