Low Level RF challenges / Timing systems

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INTRODUCTION

- **TIMING/SYNCH:**

In a good-performing accelerator some **events** have to **happen at the same time** (simultaneously for an observer in the laboratory frame) **or** in a **rigidly defined temporal sequence**, within a maximum allowed time error.

The **whole complex** (hardware + software) overseeing this task is the **Timing/Synchronization system** of the facility. It has to keep under control and correct the timing errors of any machine sub-systems.

Specifications, basic concepts, architectures and performances of Timing/Synchronization systems will be discussed all through the next lecture.

- **LOW LEVEL RF challenges:**

Fine, accurate and **"smart" control** of the **fields** in the **RF accelerating cavities** is a crucial task for **successful operation** of any kind of particle accelerator.

In **linear machines**, for instance, RF fields set the bunch energy, the bunch length, the intra-bunch and the bunch to bunch energy spread.

In **circular machines** RF fields set the bunch length, as well as the coherent and incoherent synchrotron frequencies. Moreover, the **interaction** of the **beam** with the **cavity accelerating modes** may lead to various forms of **instabilities**, which may be prevented or cured by **feedback systems** implemented in the **LLRF** and including the RF power sources and the cavities in the loops.

And in the end for **all kind of accelerators** the **phase** and **amplitude** of the **RF fields** probed by the bunches strongly affect their **arrival time** at any chosen target position along the machine.

LOW LEVEL RF is part of the Timing systems

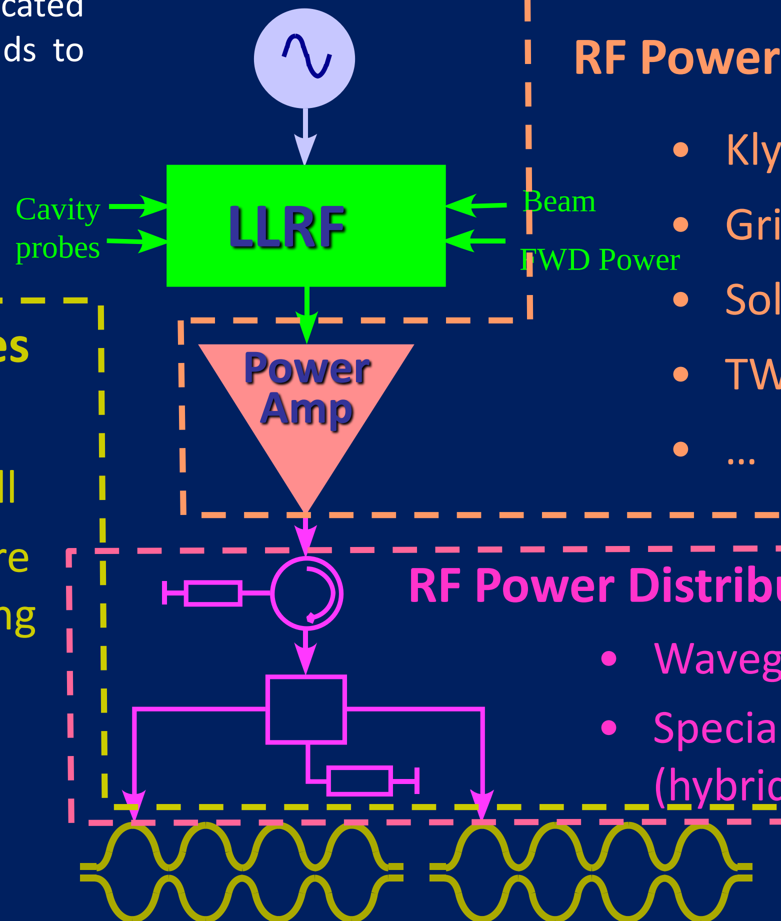
Lecture I Outline

- **INTRODUCTION**
  - ✓ RF system schematics
  - ✓ RLC cavity model
  - ✓ Matching and Tuning of a resonant cavity
- **TASKS OF LINAC LLRF (Examples)**
  - ✓ Beam loading compensation
  - ✓ Pulse compression
- **TASKS OF CIRCULAR ACCELERATORS LLRF (Examples)**
  - ✓ Tuning loops
  - ✓ Consequences of a large cavity detuning caused by heavy reactive beam loading
  - ✓ Extra damping of coherent synchrotron oscillation - Beam phase loop
  - ✓ Active impedance reduction – Direct RF feedback loop
- **STRUCTURE OF A LLRF SYSTEM**
  - ✓ Front-end. Down-conversion. A-to-D conversion
  - ✓ Signal processing
  - ✓ Back-end. Up-conversion
  - ✓ Cascade modulation of the drive signal

CONCLUSIONS AND REFERENCES

RF System Schematics

The RF systems in particle accelerators are the hardware complexes dedicated to the generation of the e.m. fields to accelerate charged particle beams.



RF Power Generation

- Klystrons
- Grid Tubes
- Solid State Amps
- TWTs
- …

Accelerating Structures

- Resonant Cavities
  - Single or multi-cell
  - Room-temperature or Superconducting
- Travelling wave sections
- RF Deflectors (either SW or TW)

RF Power Distribution

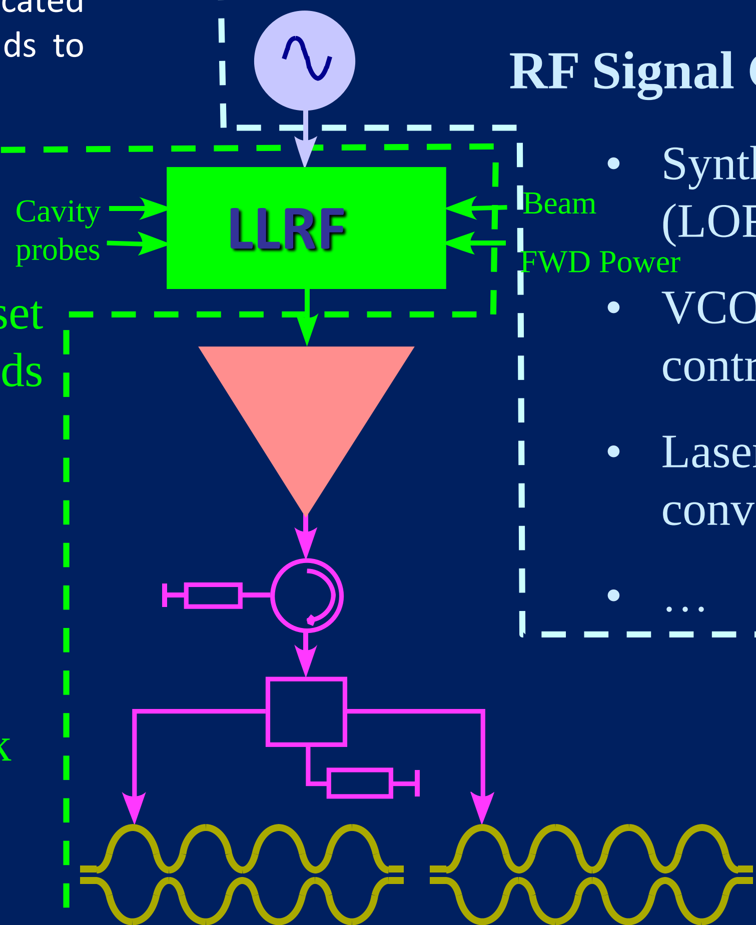
- Waveguide network
- Special Components (hybrids, circulators, …)

RF System Schematics

The RF systems in particle accelerators are the hardware complexes dedicated to the generation of the e.m. fields to accelerate charged particle beams.

Low-level RF control

- Amplitude and phase set of the accelerating fields
- Tuning control of the accelerating structures
- Beam loading compensation
- RF and beam feedback systems
- …



RF Signal Generation

- Synthesized oscillators (LORAN stabilized)
- VCOs (driven by low-level controls)
- Laser-to-voltage reference converters
- …

Low Level RF TASKS

LLRF tasks are very specific for each different accelerator (custom systems). However, two main categories can be identified:

Linear Accelerators (single pass machines):
Beam time structure approximately periodic over macro-cycles (machine rep rate) and micro-cycles (train of bunches)
Accelerating structures either Standing Wave cavities or Travelling Wave iris loaded waveguides

- Amplitude and phase stability (for bunch energy, energy spread, length, emittance, …)
- Beam loading compensation (multi-bunch operation)
- Active cavity tuning (especially delicate in superconducting linacs)
- RF pulse shaping for pulse compression (when required)
- RF based and beam based feedback systems to preserve beam quality and performances over long time scales

Beam quality

Circular Accelerators (multi-pass machines):
Beam time structure exactly periodic with the machine one-turn revolution period.
Accelerating structures are Standing Wave cavities

- Amplitude and phase stability (for bunch length, arrival time, …)
- Beam loading compensation required
- Active cavity tuning (to match reactive beam loading) over broad excursions
- A number of RF based and beam based feedback systems to prevent and cure longitudinal instabilities caused by beam interaction with the accelerating mode impedance (beam phase loop, direct RF FBK, RF FFW, …)
- RF gymnastics in hadron synchrotrons to proper manipulation of the longitudinal phase space

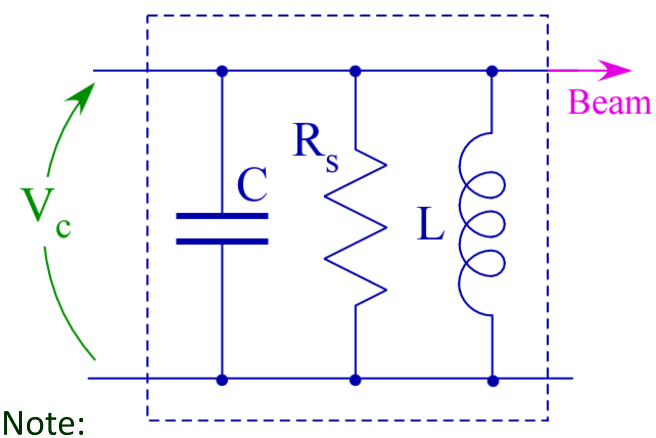
Beam quality and stability

THE CAVITY RLC MODEL

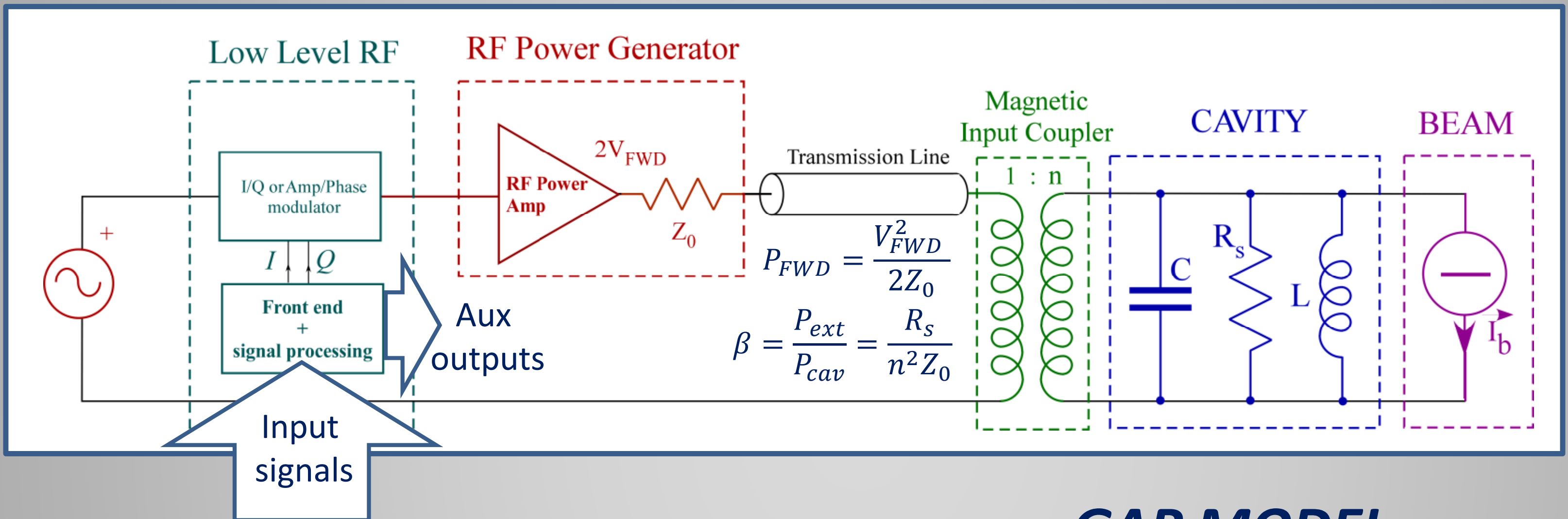
The beam-cavity interaction can be conveniently described through the **resonator RLC model**.

The cavity fundamental mode interacts with the beam current similarly to **a parallel RLC lumped resonator**.

Relations between the RLC model parameters and the mode field integrals $\boldsymbol{\omega_r}$ (mode angular resonant frequency), $\boldsymbol{V_c}$ (maximum voltage gain for a particle across the cavity gap), $\boldsymbol{U}$ (energy stored in the mode), $\boldsymbol{P_d}$ (average power dissipated on the cavity walls) and $\boldsymbol{Q}$ (mode quality factor), are given.

Mode field integrals	Cavity RLC model	RLC parameters
$V_c = \left\| \int_{gap} E_z(z)\, e^{j\omega_r \frac{z}{\beta c}} dz \right\|$ $P_d = \frac{1}{2}\int_S R_{surf} H_t^2 d\sigma$ $U = \frac{1}{4}\int_V (\varepsilon E^2 + \mu H^2) d\tau$ $Q = \frac{\omega_r U}{P_d}; \quad R/Q = \frac{V_c^2}{2\omega_r U}$	 Note: we assume that positive voltages accelerate the beam, that is accordingly modelled as a current generator discharging the cavity	$R_s = \frac{V_c^2}{2P_d}$ $C = \frac{1}{\omega_r R/Q}$ $L = \frac{1}{\omega_r^2 C} = \frac{R/Q}{\omega_r}$

RF SYSTEM MODELS

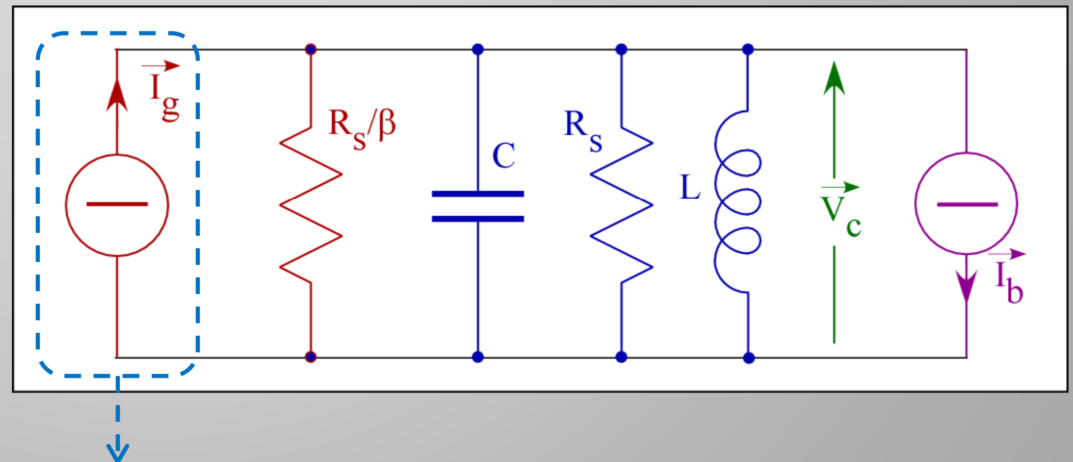


RF
- → Cavity probes
- → FWD power
- → RFL power

- → Beam probes
- → Triggers

Aux
- → Encoders (from tuning systems, RF trombones, …)
- → Temperature sensors
- → …

GAP MODEL



*External excitation modelled with a generator that is **not** independent. Its parameters (amplitude and phase) are properly regulated and continuously adjusted by the LLRF system*

RF CAVITIES: Matching and Tuning

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Optimal **matching** (i.e. choice of the cavity input coupling coefficient) and **tuning** (i.e. fine adjustment of the cavity resonant frequency wrt the RF frequency) can be computed on the base of the **RLC** lumped element equivalent circuit.

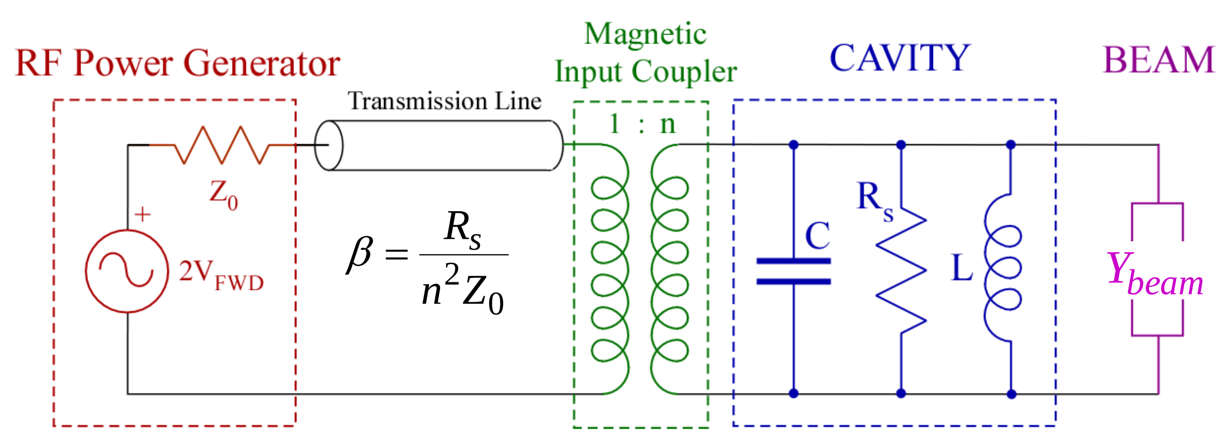
detuning parameter

According to the model the cavity complex impedance can be expressed as: $Z_c = \dfrac{R_s}{1 + jQ\delta}$ *with* $\delta = \dfrac{\omega_{RF}}{\omega_{cav}} - \dfrac{\omega_{cav}}{\omega_{RF}} \approx 2\dfrac{\Delta\omega}{\omega}$

useful tips: $Z_c = |Z_c|e^{j\phi_z}$ *with* $|Z_c| = R_s/\sqrt{1+(Q\delta)^2}$; $\tan\phi_z = -Q\delta \approx -\Delta\omega/\sigma$ ⟶ $\sigma = \omega/(2Q)$ *cavity half-bandwidth*

A beam equivalent complex admittance can be also defined according to: $Y_b = \dfrac{I_b e^{j\varphi_b}}{V_c}$

beam phase relative to the accelerating voltage



$$Y_c = \frac{1}{R_s} + j\frac{\delta}{R/Q}$$

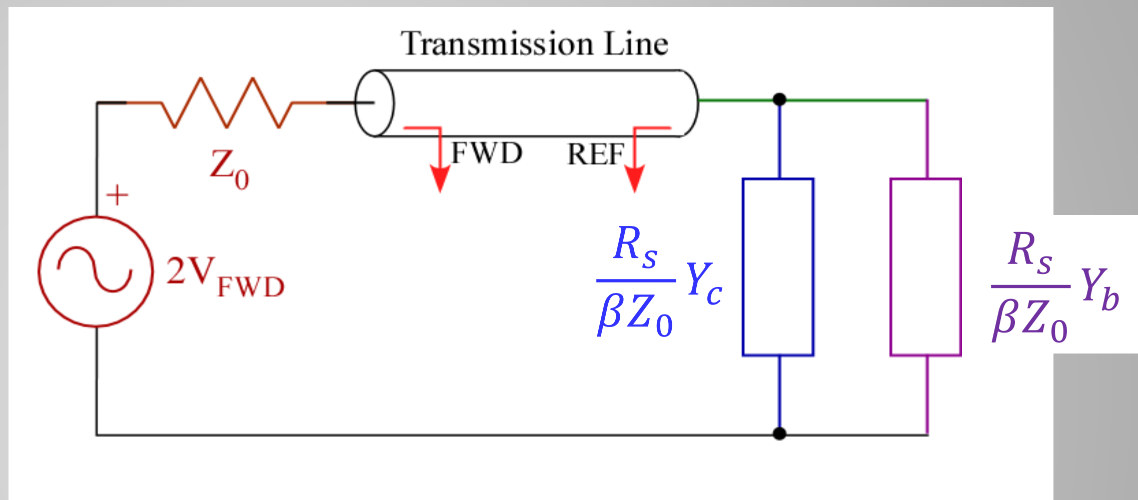
$$Y_b = \frac{I_b e^{j\varphi_b}}{V_c} = \frac{I_b}{V_c}\cos\varphi_b + j\frac{I_b}{V_c}\sin\varphi_b$$

The perfect matching and tuning conditions are met when the beam and cavity impedances are such that the input transmission line is perfectly terminated and no reflected power at the main coupler is present.

RF CAVITIES: Matching and Tuning

Cavity input reflection coefficient

$$\rho = \frac{Z - Z_0}{Z + Z_0} = \frac{1 - YZ_0}{1 + YZ_0}$$



$$Y = \frac{R_s}{\beta Z_0}(Y_c + Y_b)$$

Zero for optimal coupling factor (matching)

Zero for optimal cavity detuning

$$\rho = \frac{\boxed{\beta - 1 - \frac{R_s I_b}{V_c}\cos\varphi_b} - jR_s \boxed{\left(\frac{\delta}{R/Q} + \frac{I_b}{V_c}\sin\varphi_b\right)}}{\beta + 1 + \frac{R_s I_b}{V_c}\cos\varphi_b + jR_s\left(\frac{\delta}{R/Q} + \frac{I_b}{V_c}\sin\varphi_b\right)}$$

on crest acceleration

=0 **=1**

off-crest acceleration < 1

$$\beta = 1 + \frac{R_s I_b}{V_c}\cos\varphi_b$$

$$\delta = -(R/Q)\frac{I_b}{V_c}\sin\varphi_b$$

Transforms summary

Transforms	Fourier - $\mathcal{F}$	Laplace - $\mathcal{L}$
Definition	$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t} dt$	$X(s) = \int_{0}^{+\infty} x(t)\, e^{-st} dt$
Inverse transform	$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)\, e^{j\omega t} d\omega$	$x(t) = \frac{1}{2\pi i} \int_{\gamma - j\cdot\infty}^{\gamma + j\cdot\infty} X(s)\, e^{st} ds$
Transformability conditions	$\int_{-\infty}^{+\infty} \lvert x(t)\rvert^2\, dt \neq \infty$	$x(t) = 0 \text{ if t} < 0;\ x(t)\cdot e^{-\sigma t} \xrightarrow[t\to+\infty]{} 0$
Linearity	$\mathcal{F}\,[a\, x(t) + b\, y(t)] = aX(\omega) + bY(\omega)$	$\mathcal{L}\,[a\, x(t) + b\, y(t)] = aX(s) + bY(s)$
Convolution product	$(x * y)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{+\infty} x(t+\tau)\cdot y(\tau) d\tau$ $\mathcal{F}\,[(x * y)(t)] = X^*(\omega)\cdot Y(\omega)$	$(x * y)(t) \stackrel{\text{def}}{=} \int_{0}^{t} x(t+\tau)\cdot y(\tau) d\tau$ $\mathcal{L}\,[(x * y)(t)] = X^*(s)\cdot Y(s)$
Derivative	$\mathcal{F}\left[\frac{dx}{dt}\right] = j\omega\cdot X(\omega)$	$\mathcal{L}\left[\frac{dx}{dt}\right] = s\cdot X(s)$
Delay	$\mathcal{F}\,[x(t-\tau)] = X(\omega)e^{-j\omega\tau}$	$\mathcal{L}[x(t-\tau)] = X(s)e^{-s\tau}$

TASKS OF LINAC LLRF
(Examples)

Beam Loading

One of the main task of multibunch Linac LLRF systems is the **beam loading compensation**.

While travelling across accelerating structures the **bunches subtract energy** to the accelerating mode and reduce the level of the accelerating fields. If not compensated by playing with the external RF source this effect would result in an **energy spread along the train**.

In this respect various cases can be distinguished:

RF Pulse Length:

- ***Short pulses (≈ 1 μs or less):*** *no feedback, compensation can only be obtained applying feed-forward, i.e. proper RF pulse shaping*
- ***Long pulses (≈ 1 ms) or CW:*** *similar to storage rings, feedback compensation applicable, but transients potentially harmful. This is the case of superconducting Linacs*

RF Accelerating Structures:

- ***Standing Waves (SW):*** *resonant cavities, the RLC model applies. This is the case of superconducting Linacs and proton/ion Linacs*
- ***Travelling Waves (TW):*** *the RLC model does not apply, an alternative waveguide model is more appropriate. This is the case of normal conducting lepton Linacs based on S/C/X band RF technology*

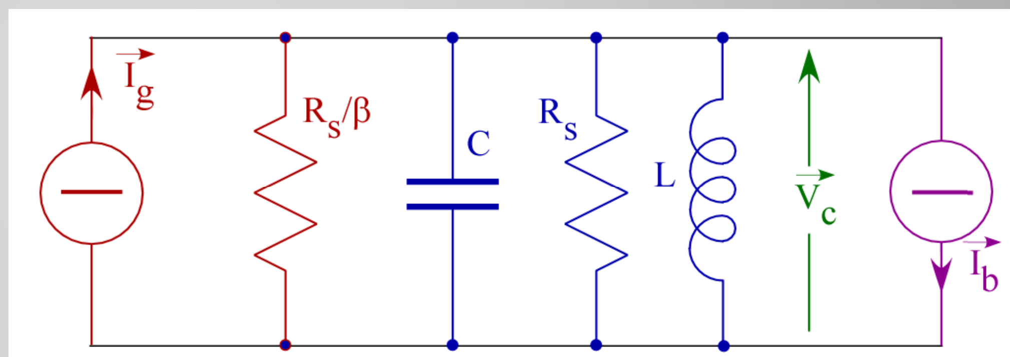
Beam Loading: current pulses in SW accelerating cavities

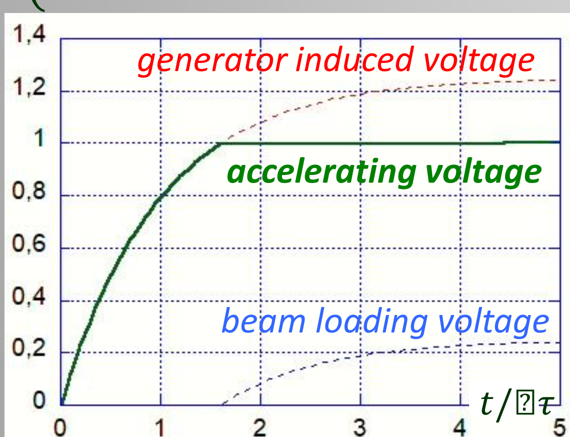
Let's consider a SW cavity accelerating on crest a train of bunches. The RF pulse will start at $t = 0$, while the first bunch will be injected in the cavity with a certain delay ΔT_b.

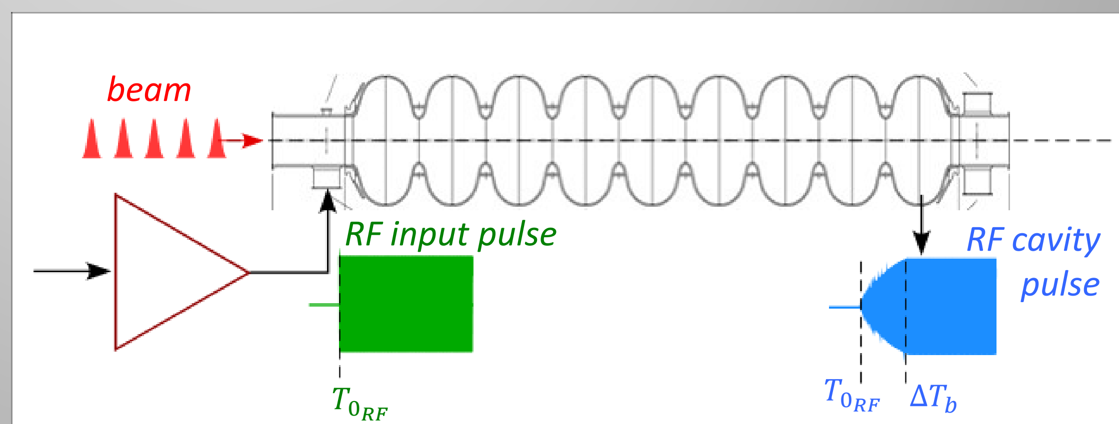
$$V_c = R_L\left[I_g\left(1-e^{-t/\tau}\right)\cdot 1(t) - I_b\left(1-e^{-(t-\Delta T_b)/\tau}\right)\cdot 1(t-\Delta T_b)\right] \quad \text{with} \quad \begin{cases} R_L = R_s/(1+\beta) \\ \tau = 2Q_0/(1+\beta)\omega_r \\ I_b \approx 2q_b/\Delta T_b \end{cases}$$

The accelerating voltage V_c will be **time independent** (no transients across the bunch train) provided that:

$$\begin{cases} I_g = I_b + V_c/R_L \\ e^{\Delta T_b/\tau} = 1 + V_c/(R_L I_b) \end{cases}$$





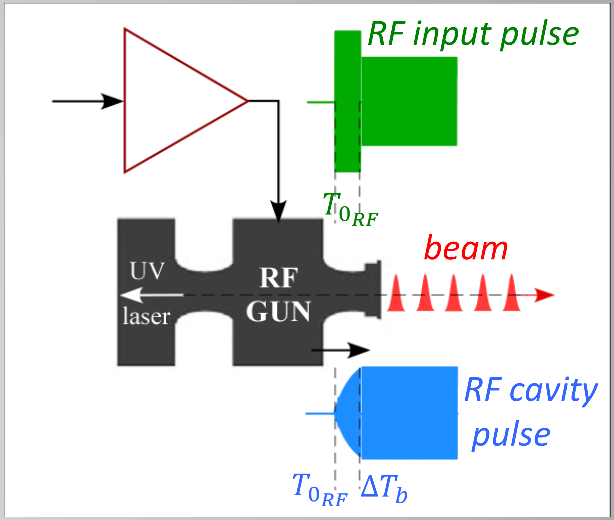


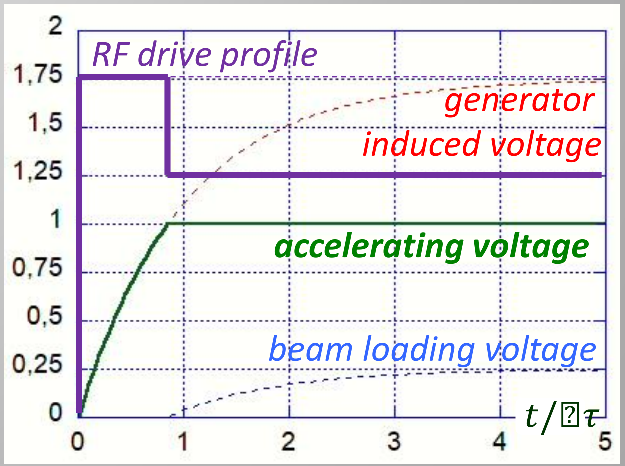
Beam Loading: current pulses in high-gradient SW cavities

The **LLRF system** can compensate the **transient beam loading** effects on a bunch train accelerated on crest by a SW cavity by **properly setting** the **level** and the **pre-delay** of the input RF pulse.

In some special cases, when **high gradients** at the limit of the RF breakdown are required, **reducing the filling time** of the structure is an issue. This can be obtained by:

- **Overloading the cavity**, i.e. setting an **input coupling factor β larger than** what **needed** for matching the beam. The filling time τ is reduced being inversely proportional to the factor $\mathbf{1+\beta}$, at the **price** of generating a certain amount of **RF reflected power** at the cavity input coupler.
- **Overdriving the cavity**, i.e. ramping the fields in the cavity using **more power than** what **needed** to sustain the bunch train acceleration. The target accelerating voltage is reached more rapidly, but as soon as it has been reached the beam has to be injected and **the amplitude of the driving pulse must be reduced** at the equilibrium level to avoid transient beam loading along the train (bunch-to-bunch energy spread). **The proper pulse shaping must be provided by the LLRF system.**

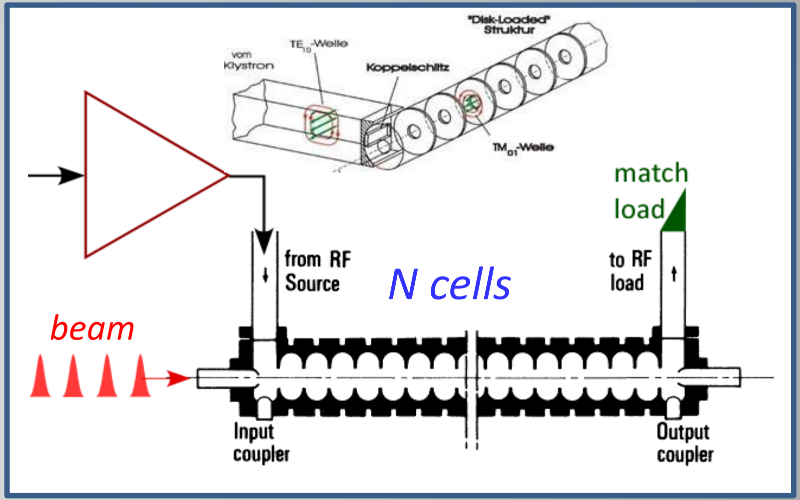


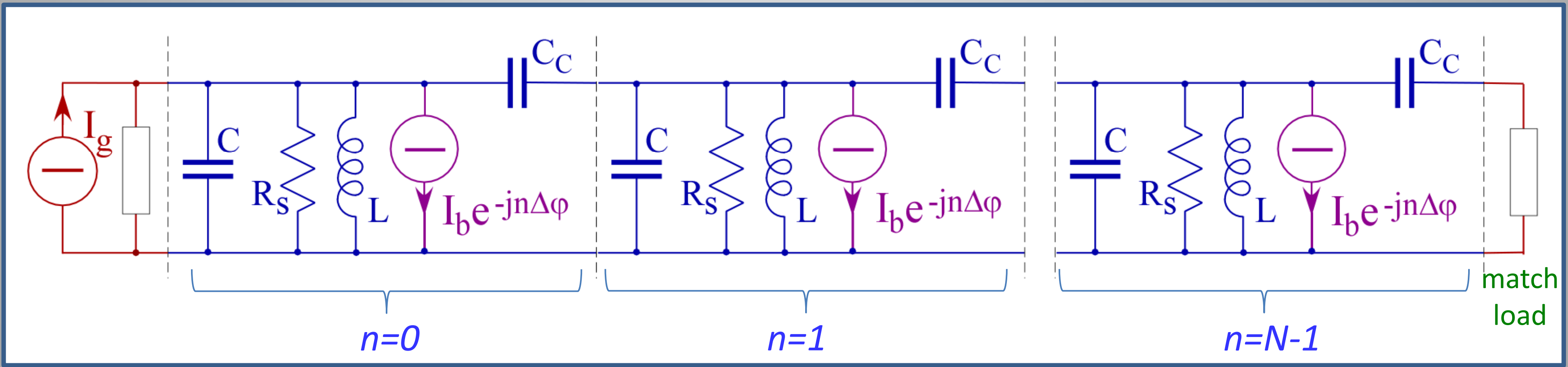


Beam Loading: current pulses in TW accelerating structures

Travelling wave structures are widely used to accelerate **ultra-relativistic particles** (leptons). They are essentially **iris-loaded waveguides** supporting **travelling e.m. fields** with a longitudinal component of the electric field **E**. Phase velocity of the wave matches the particle speed ($v_{ph} = c$), while **group velocity v_g is much lower** (at level few % of c or less) in order to **concentrate more RF energy** in the structure.

Structure efficiency optimization requires field phase advance per cell ($\Delta\varphi = \omega_{RF}\, L_c/c$) values around $2\pi/3$.





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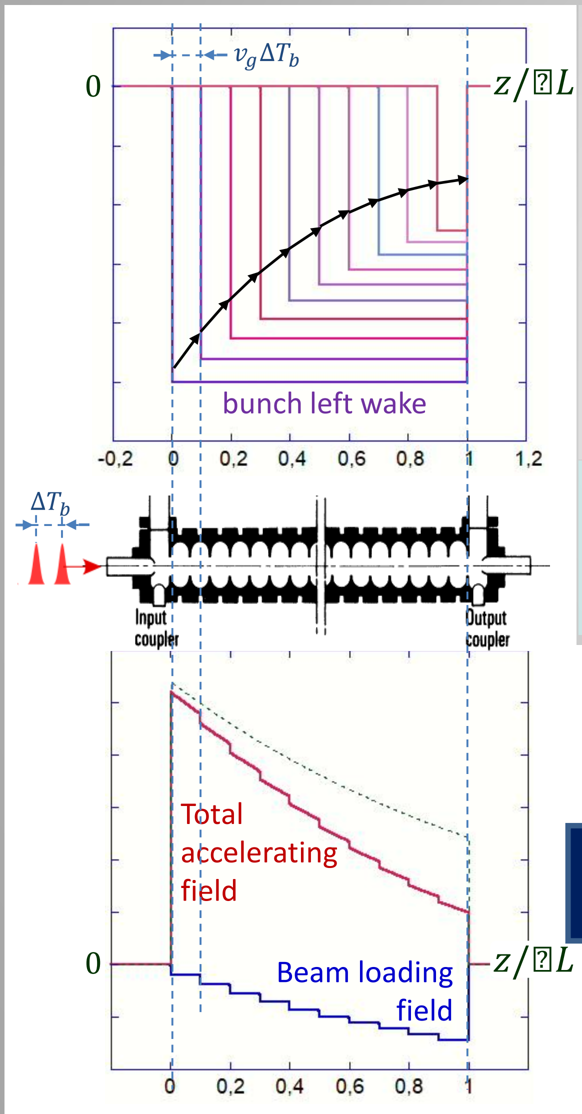
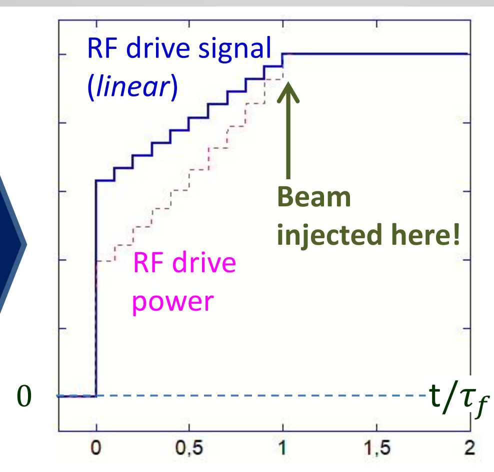
Beam Loading: current pulses in TW accelerating structures



When the 1st bunch enters the structure it leaves a **uniform wake** whose **envelope** starts **moving forward** with the **group velocity** of the structure v_g also experiencing the structure attenuation. Following bunches behave the same.

If the **bunch train** is **longer** than the structure **filling time** the total wake reaches a **stationary configuration**, such that the bunches in the train tail get all the same energy gain, while an energy spread confined to the train head remains as a consequence of the transient beam loading.

Transient beam loading can be **compensated** by **pre-loading** in the structure the same **field profile** as that induced by the beam loading at regime.

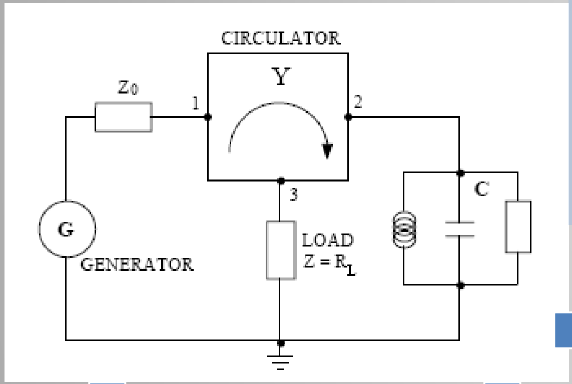


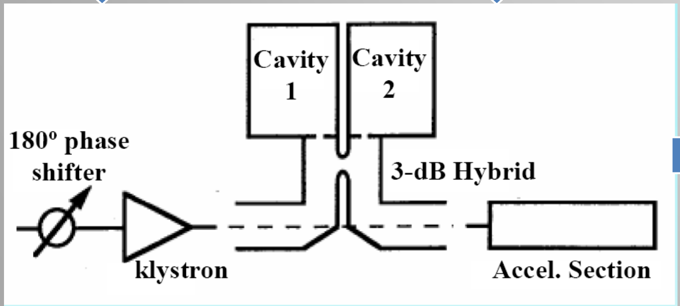
The LLRF system has to be programmed to draw a ramp with a proper rising profile. Power plant non-linearities have to be taken into account, as well as bandwidth limitation coming from the LLRF back-end and the rest of the RF power chain. The whole correction needs to be applied in 1 filling time (< 1μs typical !).

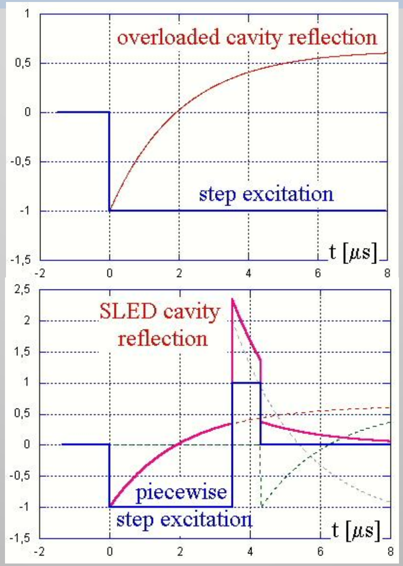
RF pulse compression

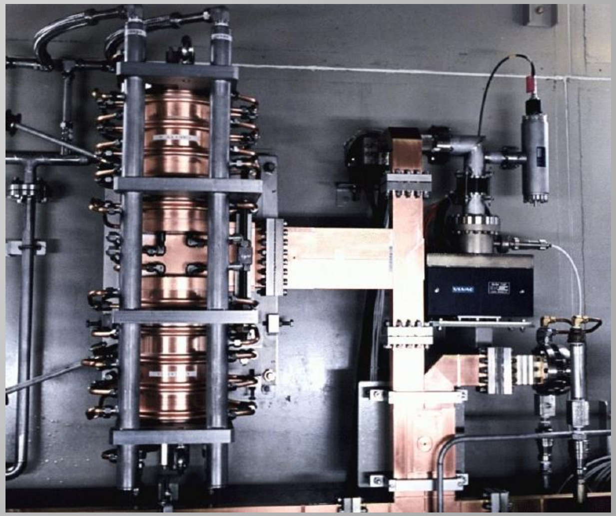
The **Stanford Linac Energy Doubling** (SLED) is a system developed to **compress RF pulses** in order to **increase** the **peak power** (and the available accelerating gradients) for a given total pulse energy. This is obtained by capturing the **pulsed power reflected by a high-Q cavity** properly excited by the RF generator (typically a klystron). In fact the wave reflected by an overcoupled cavity ($\beta \approx 5 \div 10$) peaks at the end of the RF pulse to about twice the incident wave level (with opposite polarity).

By properly tailoring the incident wave with a ≈180° phase jump during the pulse, the reflected power peak is enhanced and the integrated gradient in the accelerating structures downstream the compressor are almost doubled, at the price of a shorter pulse duration.

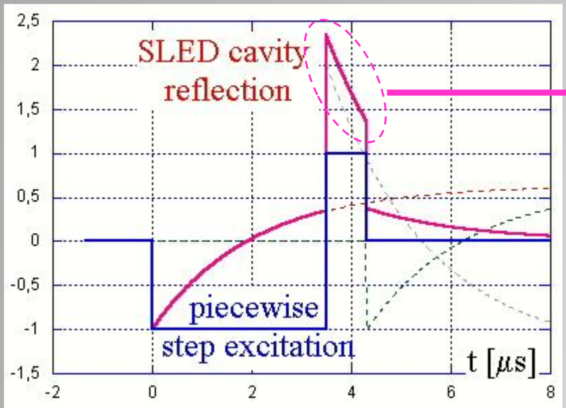






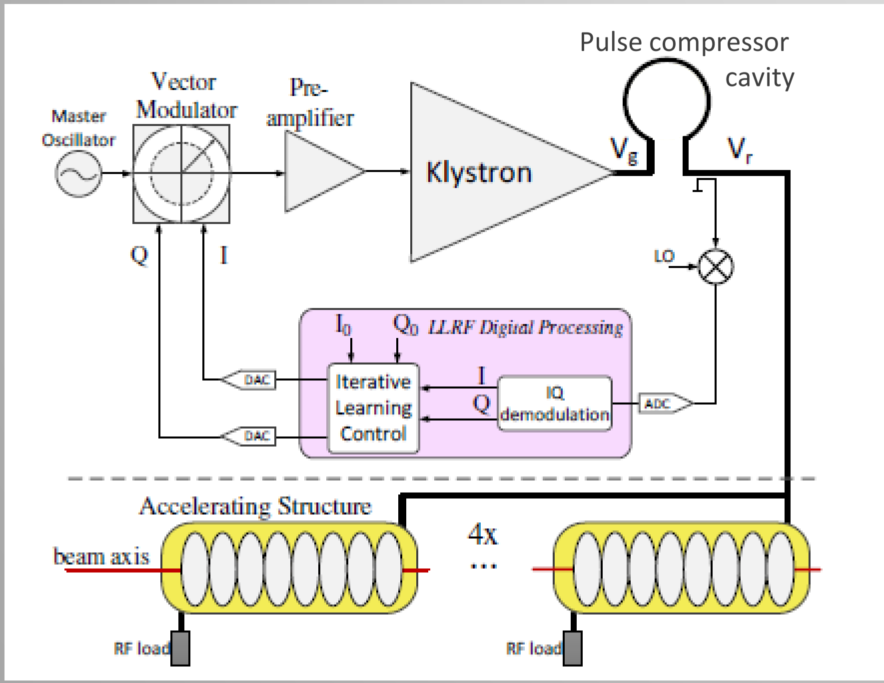


RF pulse compression: flat output pulses



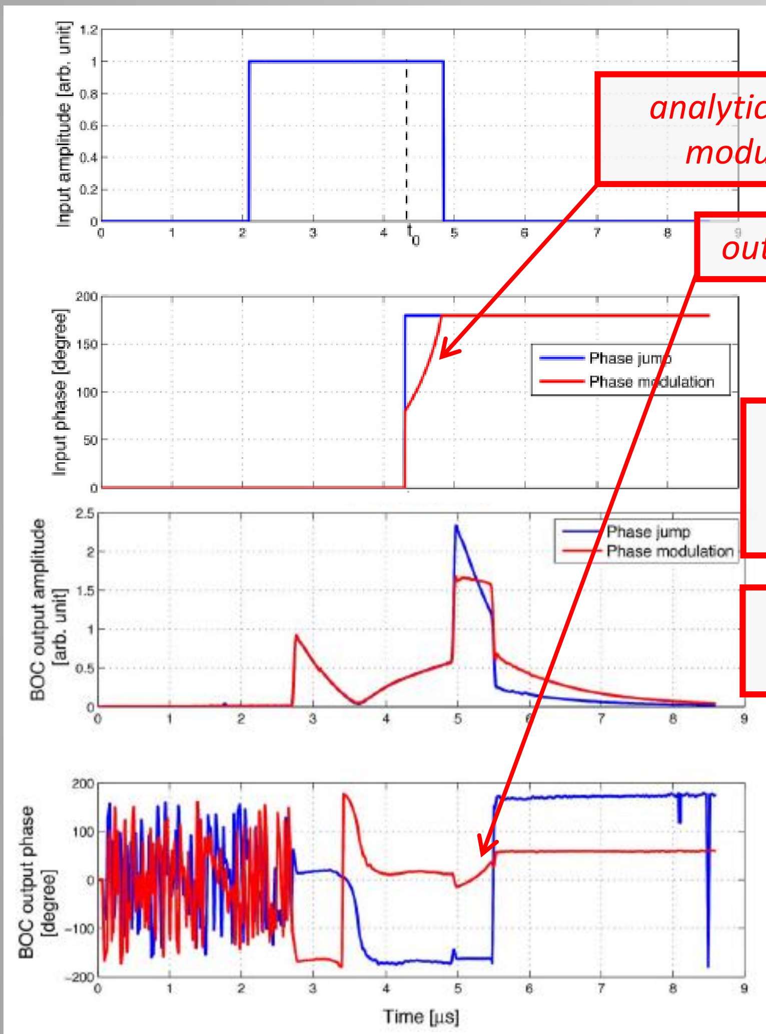
The compressed pulse shape resulting from the basic drive signal (180° phase jump, no amp modulation) is not suitable for multibunch operation. The output amplitude shows a very steep profile, so that the available average gradient in the accelerating structure decreases with time.

A flat compressed pulse can be obtained by applying proper phase/amplitude modulation to the drive signal.



Based on a realistic model of the vector modulator/pre-amplifier/klystron RF chain, the input signal can be computed, generated and applied.
However, better results can be obtained implementing iterative algorithms that look at the actual pulse shape and on this base readjust the RF input characteristics.
This approach (Iterative Learning Control) can be considered as a smart feed-forward control, or a slow feedback system over a certain custom-defined cost function.

RF pulse compression: flat output pulses



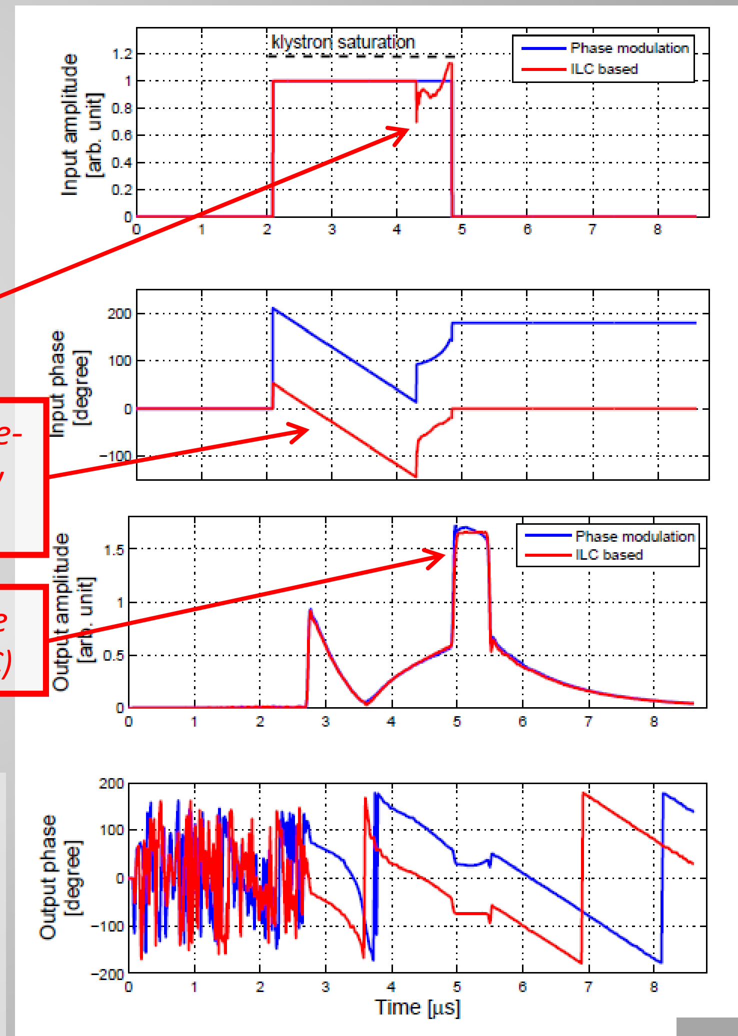
analytical phase modulation

output frequency shift

AM from ILC

input frequency shift pre-correction (SLED cavity detuning required)

output pulse amplitude (≈ 3 times flatter with ILC)



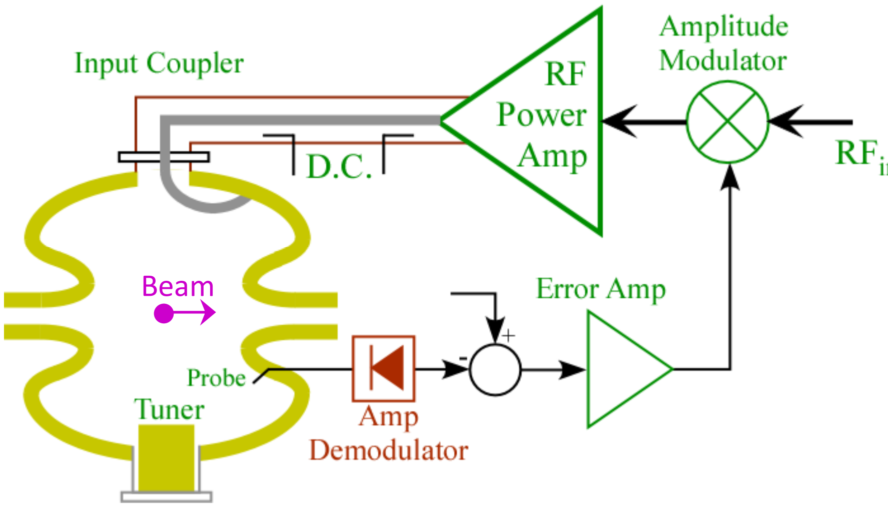
not for free!!!
≈ 20 % lower average gradient
w.r.t unflat compressed pulse

TASKS OF CIRCULAR ACCELERATORS LLRF

(Examples)

AMPLITUDE LOOPS

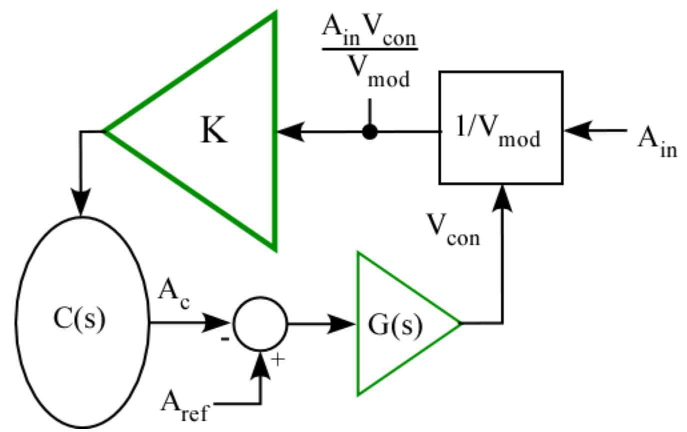
The **automatic regulation** of the generator output level can be obtained by implementing **amplitude loops**. These are feedback systems which detect and correct variations of the level of the cavity voltage.
If the power amplifier is not fully saturated, the regulation can be obtained by controlling the **RF level** of the amplifier **driving signal**.
If the amplifier is saturated, the feedback has to act directly on the high voltage that sets the level of the saturated output power.
Referring to the reported model, the loop transfer function can be written in the form:



$$\frac{A_c}{A_{ref}} = \frac{H(s)}{1+H(s)} \quad with \quad H(s) = \frac{A_{in}}{V_{mod}} K\,C(s)G(s)$$

For little cavity detuning ($Q_L\delta \ll 1$) the cavity response to an amplitude modulated signal is a single pole low-pass:

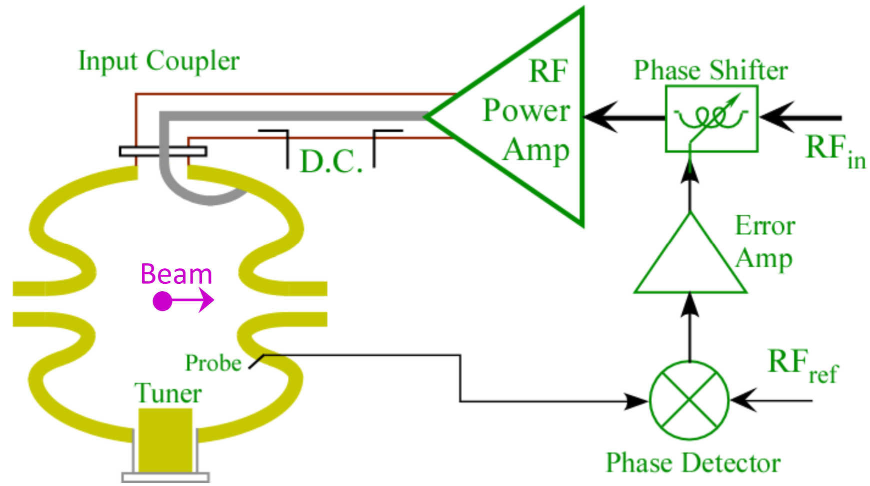
$$C(s) \approx \frac{1}{1+s/\sigma} \quad with \quad \sigma = \frac{\omega_{cav}}{2Q_L}$$



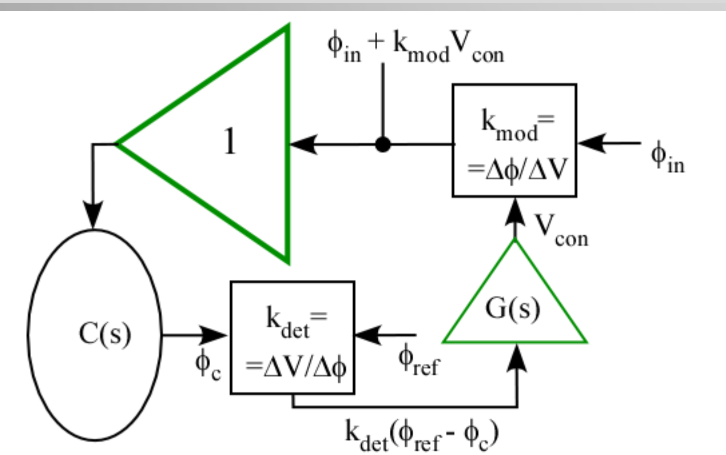
For linac cavities the architecture of amplitude (and phase) loops is almost the same. In short pulse operation the control can only act from pulse to pulse. The loop BW is narrower than pulse rep rate.

RF PHASE LOOPS

The **cavity RF phase** (or the power station RF phase) can be **locked to the reference RF clock** by another **dedicated servo loop**. The need for a phase loop is not strictly related to beam loading effects but more to ensure **synchronization** between different RF cavities or between RF voltage and other sub-systems of the accelerator (such as injection system, beam feedback systems, ...).



The phase is locked to the reference by measuring the relative phase deviation by means of a phase detector and applying a continuous correction through a phase shifter. For loop gain and bandwidth the same considerations expressed in the amplitude loop case hold.



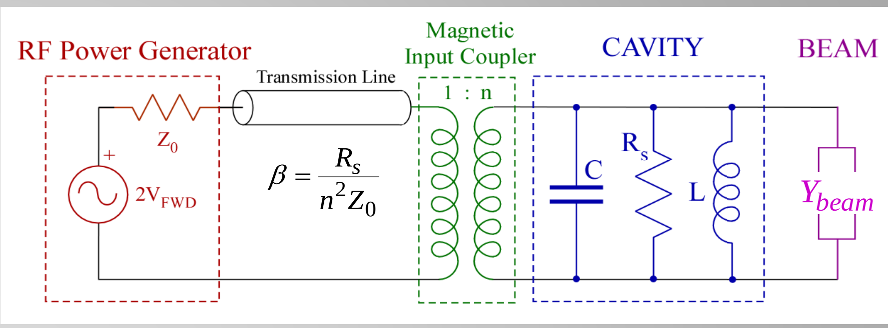
$$\phi_c = \frac{C(s)}{1+H(s)}\phi_{in} + \frac{H(s)}{1+H(s)}\phi_{ref}$$

$$with \qquad H(s) = k_{mod} k_{det} C(s) G(s)$$

$$\phi_c \approx \phi_{ref} \quad if \quad |H(s)| \gg |C(s)|; |H(s)| \gg 1$$

TUNING LOOPS

The **tuning loop regulates** automatically and continuously the **cavity resonant frequency** to **compensate** the **beam susceptance** by checking the RF phase between the cavity voltage and the forward wave on the cavity input line. Detected error signals drive actuators inducing **mechanical deformations of the cavity** by means of dedicated devices (plungers, squeezers, ...).
The loop controls the phase of the transfer function:

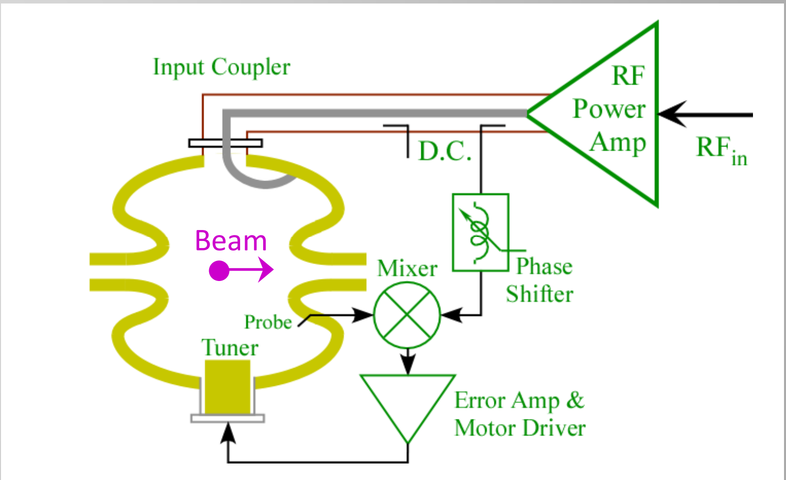


$$T(j\omega) = \frac{V_c}{V_{FWD}} = 2n\frac{[n^2(Y_{cav}+Y_{beam})]^{-1}}{Z_0+[n^2(Y_{cav}+Y_{beam})]^{-1}} =$$

$$= \frac{2n}{1+(R_s/\beta)(Y_{cav}+Y_{beam})} = \frac{2}{1+\beta}\cdot\frac{\sqrt{\beta R_s/Z_0}}{1+jQ_L\delta+(R_L I_b/V_c)e^{j\varphi_s}}$$

with $Q_L = \frac{Q_0}{1+\beta}; \quad R_L = \frac{R_s}{1+\beta}; \quad \varphi_s = \varphi_{beam}$

synchronous phase



$$\angle T(j\omega) = -\tan^{-1}\frac{Q_L\delta+(R_L I_b/V_c)\sin\varphi_s}{1+(R_L I_b/V_c)\cos\varphi_s}$$

The set point of the variable phase shifter in the loop establishes the phase of the transfer function that can be locked to 0 or to any other value φ_0.

TUNING LOOPS

The optimal cavity tuning is obtained by locking the loop to $\varphi_0 = 0$. In this case we have:

$$\angle T(j\omega) = -\tan^{-1}\frac{Q_L\delta + (R_L I_b/V_c)\sin\varphi_s}{1 + (R_L I_b/V_c)\cos\varphi_s} = 0 \;\Rightarrow\; \delta = \frac{\omega_{RF}}{\omega_{cav}} - \frac{\omega_{cav}}{\omega_{RF}} = \boxed{-\frac{I_b\, R/Q}{V_c}\sin\varphi_s}$$

optimal detuning, reactive beam loading perfectly compensated

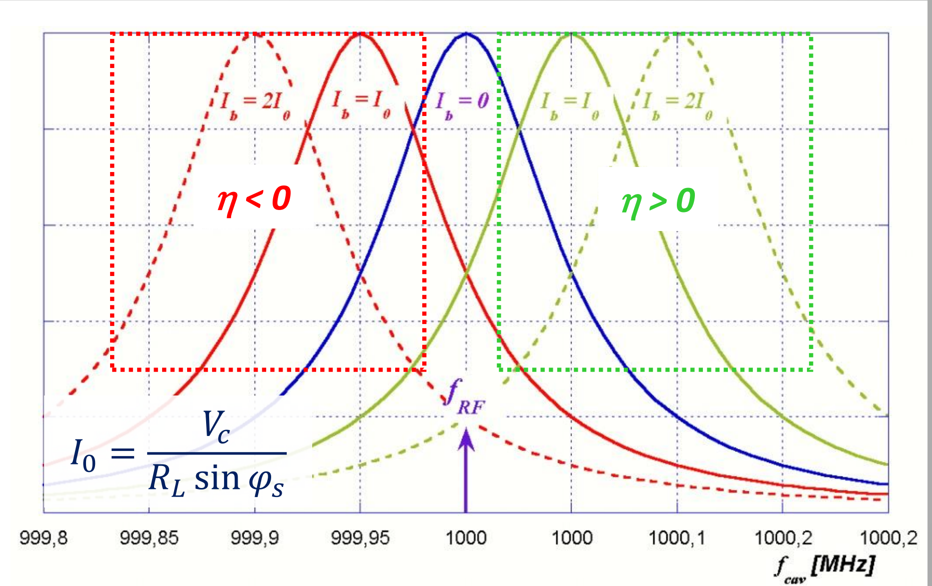
Consequences:

In a synchrotron/storage ring the **sign** of the **synchronous phase $\boldsymbol{\varphi_s}$** is equal to the sign of the **dilation factor $\boldsymbol{\eta}$** defined as:

$$\eta = \frac{\Delta f_{rev}/f_{rev}}{\Delta p/p} = \frac{1}{\gamma^2} - \alpha_c$$

where $\alpha_c = (\Delta L_{ring}/L_{ring})/(\Delta p/p)$ is the ring momentum compaction.

So optimal cavity tuning condition in a $\eta < 0$ ring requires $\boldsymbol{\delta > 0}$. Cavities have to be tuned **below** the frequency of the RF generator. Situation is just the opposite when $\eta > 0$.



The amount of required **detuning** is proportional to the **intensity of the stored current**. It may be very large (much larger than the cavity bandwidth) in high current storage rings. This has very **important consequences** on **beam dynamics** and on the **LLRF system** controls.

Tuning loops are generally very slow since they involve mechanical movements through the action of motors. Typical bandwidth of such systems are of the order of 1 Hz. Tuning systems are also necessary to stabilize the cavity resonance against thermal drifts. In this respect superconducting cavities are peculiar because of their very narrowband and high sensitivity to the microphonic noise in the cryogenic bath.

Coupled Bunch Instabilities

1st Consequence of cavity detuning: instabilities of the longitudinal Coupled Bunch (CB) motion

The elementary synchrotron equation for the longitudinal motion of a particle in a synchrotron or a storage ring is:

impedance dependent in CB

$$\ddot{\varphi}_b + 2\boxed{\alpha}\dot{\varphi}_b + \boxed{\omega_s^2}\varphi_b = 0$$

with

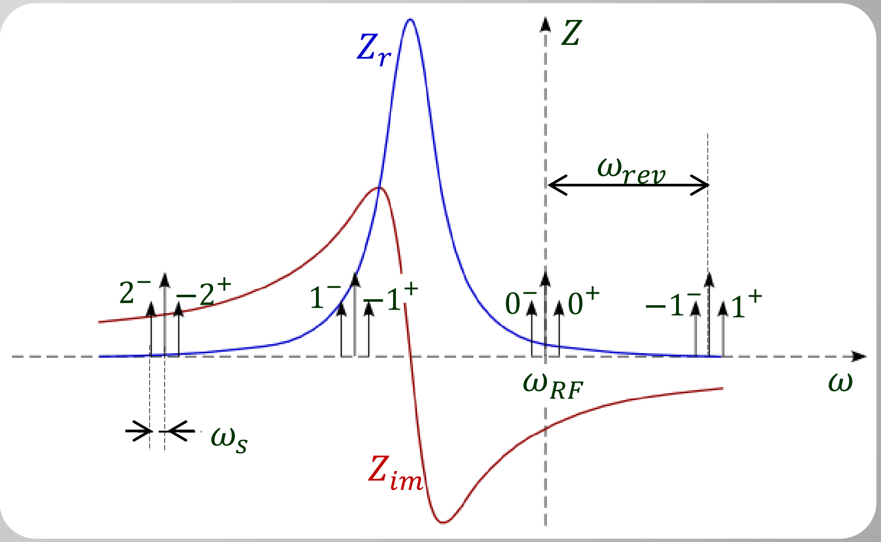
synchrotron frequency

$$\boxed{\omega_s} = \omega_{RF}\sqrt{\frac{\eta V_c \sin\varphi_s}{2\pi\beta^2 h\, E/q}}$$

where α accounts for the eventual presence of frictional terms (such as radiation damping).

In **N-bunch operation** there are **N** different longitudinal **coupled bunch modes**. Each of them interact with the **ring impedance** (whose main contribution comes from the **accelerating mode** of the **RF cavities**). For each CB mode the **damping coefficient** $\boldsymbol{\alpha}$ and the **coherent synchrotron frequency** $\boldsymbol{\omega_s}$ are **perturbed**, leading to a beam instability whenever α or ω_s^2 become negative.

$$\Delta\alpha \div I_b\left(Z_r^- - Z_r^+\right); \;\; \Delta\left(\omega_s^2\right) \div I_b\left(Z_{im}^- + Z_{im}^+\right)$$



Clearly, a cavity impedance tuned on the excitation frequency ω_{RF} does not perturb the barycenter motion (CB mode #0) and the other CB modes. This is not the case in **heavy beam loading detuning** regime. Reducing the effects of the beam-cavity accelerating mode interaction is **mainly a LLRF system task!**

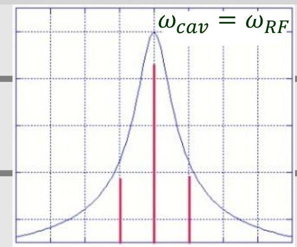
2nd Consequence of cavity detuning: modulation mixing

LLRF servo-loops and feedback loops often need to **apply AM** and **PM modulation** to the **RF drive** signal. The response of a **resonant cavity** to AM and PM excitations depends on its **bandwidth** and **tuning** relative to the carrier:

IN TUNE

$v_i(t) = A_i[1 + a_i(t)]\cos(\omega_{RF}t)$ → $v_o(t) = A_i[1 + a_o(t)]\cos(\omega_{RF}t)$

$v_i(t) = A_i\cos[\omega_{RF}t + \phi_i(t)]$ → $v_o(t) = A_o\cos[\omega_{RF}t + \Delta\phi_o + \phi_o(t)]$



L-transform

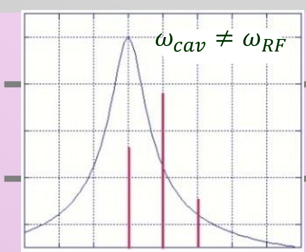
$x(t) \quad \Rightarrow \quad \hat{x}(s)$

$$G(s) = \frac{\hat{a}_o(s)}{\hat{a}_i(s)} = \frac{\hat{\phi}_o(s)}{\hat{\phi}_i(s)} = \frac{1}{1+s/\sigma} \quad \text{with} \quad \sigma = \frac{\omega_{cav}}{2Q_L}$$

OFF-TUNE

$v_i(t) = A_i[1 + a_i(t)]\cos(\omega_{RF}t)$ → $v_o(t) = A_o\left[1 + a_{o,a}(t)\right]\cos\left[\omega_{RF}t + \Delta\phi_o + \phi_{o,a}(t)\right]$

$v_i(t) = A_i\cos[\omega_{RF}t + \phi_i(t)]$ → $v_o(t) = A_o\left[1 + a_{o,p}(t)\right]\cos\left[\omega_{RF}t + \Delta\phi_o + \phi_{o,p}(t)\right]$



$$G_{aa}(s) = \frac{\hat{a}_{o,a}(s)}{\hat{a}_i(s)}; \quad G_{pp}(s) = \frac{\hat{\phi}_{o,p}(s)}{\hat{\phi}_i(s)}; \quad G_{ap}(s) = \frac{\hat{\phi}_{o,a}(s)}{\hat{a}_i(s)}; \quad G_{pa}(s) = \frac{\hat{a}_{o,p}(s)}{\hat{\phi}_i(s)}$$

Modulation Transfer Functions

It may be demonstrated that **direct** and **cross** modulation transfer functions are given by:

$$G_{pp}(s) = G_{aa}(s) = \frac{1}{2}\left[\frac{A(s+j\omega_{RF})}{A(j\omega_{RF})} + \frac{A(s-j\omega_{RF})}{A(-j\omega_{RF})}\right]; \quad G_{ap}(s) = -G_{pa}(s) = \frac{1}{2j}\left[\frac{A(s+j\omega_{RF})}{A(j\omega_{RF})} - \frac{A(s-j\omega_{RF})}{A(-j\omega_{RF})}\right]$$

with $A(s)$ = transfer function in Laplace domain of the filter applied to the modulated signal. If the signal is filtered by a resonant cavity, one has to consider $A(s)=A_{cav}(s)$ given by:

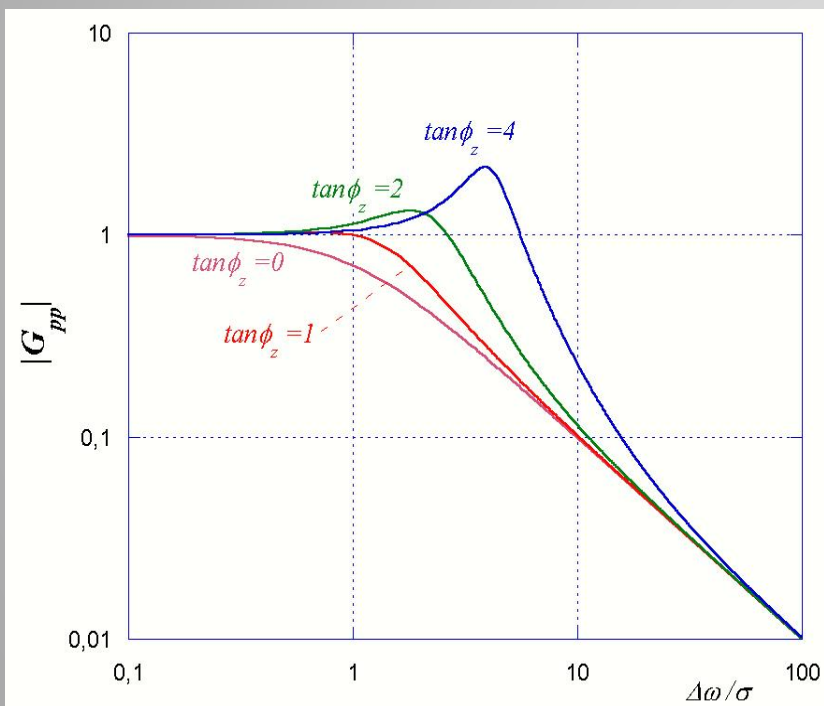
$$A_{cav}(s) = A_0 \frac{2\sigma s}{s^2 + 2\sigma s + \omega_{cav}^2} \quad \text{with} \quad \omega_{cav} \approx \omega_{RF} + \sigma \tan\phi_z$$

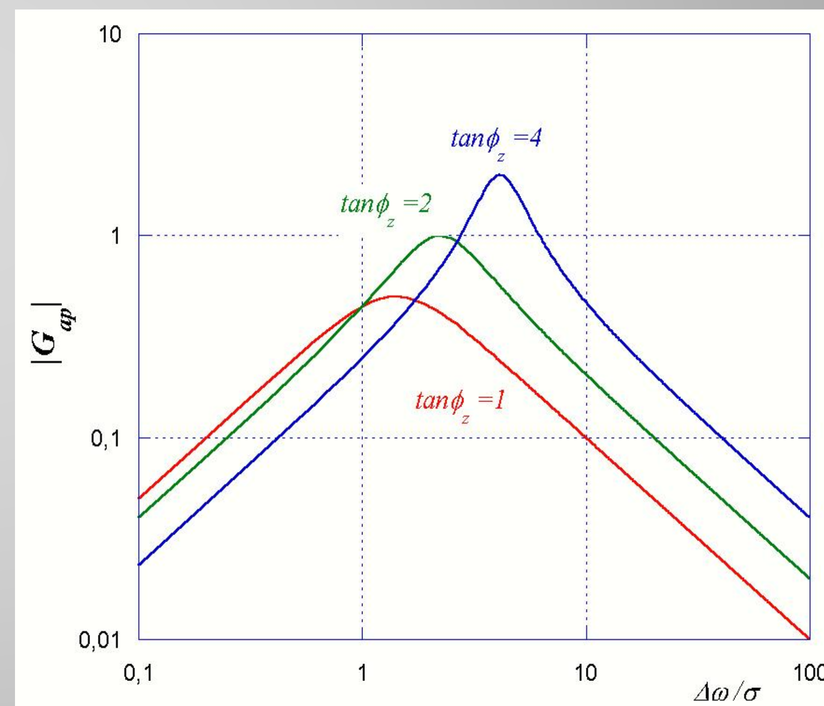
where $\boldsymbol{\phi_z}$ is the ***cavity tuning angle***, i.e. the phase of the cavity transfer function at the carrier frequency $\boldsymbol{\omega_{RF}}$. Finally one gets:

$$G_{pp}(s) = G_{aa}(s) = \frac{\sigma s + \sigma^2\left(1+\tan^2\phi_z\right)}{s^2 + 2\sigma s + \sigma^2\left(1+\tan^2\phi_z\right)}; \quad G_{ap}(s) = -G_{pa}(s) = -\frac{\sigma \tan\phi_z\, s}{s^2 + 2\sigma s + \sigma^2\left(1+\tan^2\phi_z\right)}$$

Modulation Transfer Functions

The general form of the modulation transfer functions features **2 poles** (a complex conjugate pair at large detunes) and **1 zero**, and the **direct transfer function** degenerates to a **single pole LPF** response if the cavity is **perfectly tuned** (cross modulation terms vanish in this case).





Beam Transfer Function

Let's consider the synchrotron motion $\varphi_b(t)$ of a particle in a storage ring where the accelerating voltage is not modulated (i.e. $\varphi_c(t)= \varphi_{clock}$, being $\varphi_b(t)$ measured respect to the same φ_{clock}). The synchrotron equation has the form:

$$\ddot{\varphi}_b + 2\alpha\dot{\varphi}_b + \omega_s^2\varphi_b = 0$$

$$\omega_s = \omega_{RF}\sqrt{\frac{\eta V_c \sin\varphi_s}{2\pi\beta^2 h\, E/q}}$$

where α accounts for frictional terms which may damp or amplify the oscillations.
If the cavity phase $\varphi_c(t)$ is modulated the synchrotron equation becomes (neglecting α):

$$\ddot{\varphi}_b + \omega_s^2\varphi_b = \omega_s^2\varphi_c \xrightarrow{\mathcal{L}} s^2\tilde{\varphi}_b(s) + \omega_s^2\tilde{\varphi}_b(s) = \omega_s^2\tilde{\varphi}_c(s) \Rightarrow B(s) = \frac{\tilde{\varphi}_b(s)}{\tilde{\varphi}_c(s)} = \frac{\omega_s^2}{s^2+\omega_s^2}$$

The "Beam transfer function" $\boldsymbol{B(s)}$ measures the **response** of the **beam** to a **cavity phase modulation** in the Laplace s-domain. The response is **one-to-one at dc** (the beam follows any slow phase motion of the cavities) and **peaks at the synchrotron frequency** (infinite amplitude if no damping is provided).
The function $\boldsymbol{B(s)}$ represents the **forward block** in the **active feedback systems** aimed at generating some damping term $\boldsymbol{\alpha}$ to stabilize the beam.

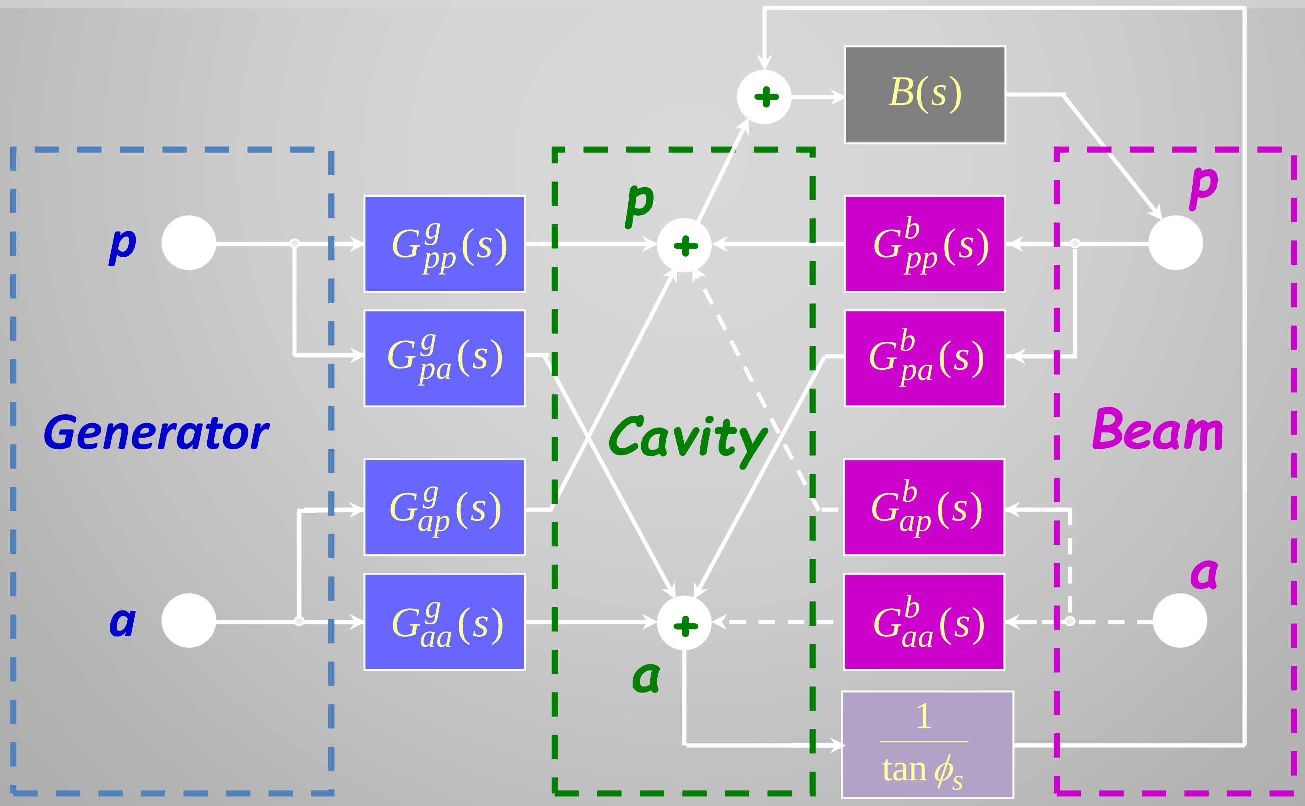
The beam phase also depends on the cavity voltage amplitude, according to:

$$\cos(\varphi_b) = \frac{V_{loss}}{V_c} \Rightarrow \sin(\varphi_b)\,d\varphi_b = \frac{V_{loss}}{V_c^2}\mathrm{d}V_c \Rightarrow d\varphi_b = \frac{1}{\tan\varphi_b}\frac{\mathrm{d}V_c}{V_c}$$

The PEDERSEN model

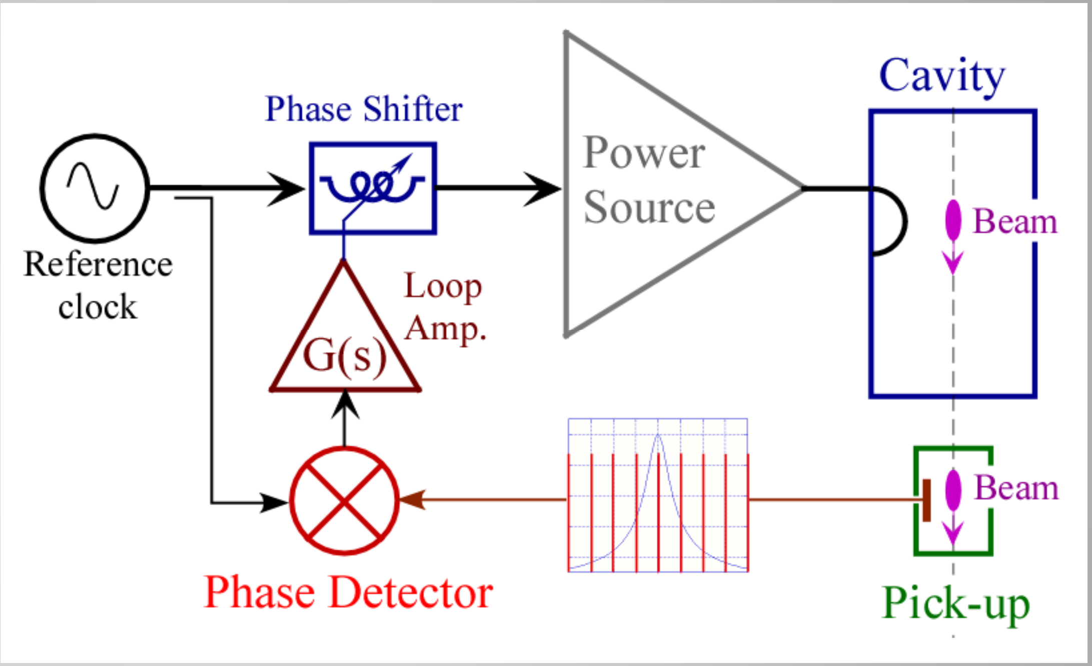
In **circular accelerators** the **beam phase** depends on the **cavity RF phase** through the **beam transfer function**, while the **cavity RF amplitude** and **phase** depend on the **beam phase** through the **beam loading mechanism**. The whole **generator-cavity-beam linear system** can be graphically represented in a **diagram** called **Pedersen Model**.

The modulation transfer functions vary with the stored current and definitely couple the servo-loops and the beam loops implemented around the system.



Beam Phase Loop

The **beam phase loops** are **feedback systems** aimed at generating a **damping** (=frictional) **term** in the **synchrotron equation** for the **beam barycenter coherent motion** (CB mode #0).
In the basic scheme **the phase of the beam is detected** and, after a manipulation to introduce a **90° phase shift** at the synchrotron frequency, is applied back as a phase modulation on the cavity RF driving signal.



Ideally, if the cavity modulation were exactly proportional to the time derivative of the beam phase we would get:

$$\begin{cases} \varphi_c = k\dot{\varphi}_b \\ \ddot{\varphi}_b + \omega_s^2\varphi_b = \omega_s^2\varphi_c \end{cases} \Rightarrow \quad \ddot{\varphi}_b - \omega_s^2 k\dot{\varphi}_b + \omega_s^2\varphi_b = 0$$

making a frictional term appearing in the synchrotron equation.

Beam PL: Pure Differentiator

A pure differentiator in the Laplace s-domain has a transfer function of the type $G(s) = s/\omega_d$. If beam loading effects can be neglected, the open loop transfer function $H(s)$ and the characteristic equation have the form:

$$H(s) = G(s)\cdot B(s) = \frac{s}{\omega_d}\frac{\omega_s^2}{s^2+\omega_s^2} \quad ; \qquad G(s)\cdot B(s) - 1 = 0 \Rightarrow s^2 - s\frac{\omega_s^2}{\omega_d} + \omega_s^2 = 0$$

The closed loop transfer function will have poles at:

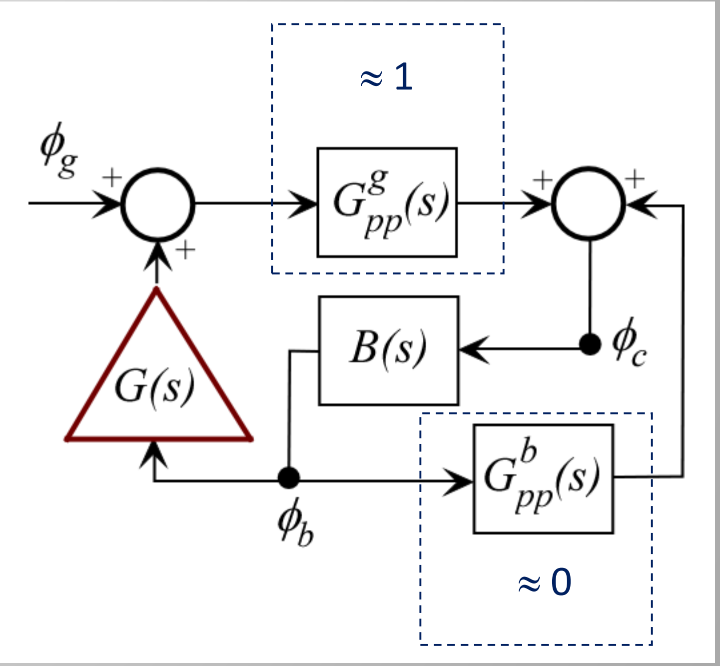
$$s_{1,2} = \frac{\omega_s^2}{2\omega_d} \pm j\omega_s\sqrt{1-\left(\frac{\omega_s}{2\omega_d}\right)^2}$$

Provided that $\omega_d < 0$ (negative feedback), the pole pair has a negative real part. A damping constant α_d is added, given by:

$$\alpha_d = \frac{\omega_s^2}{2|\omega_d|}$$

while critical damping is achievable under the condition:

$$\omega_d = -\frac{\omega_s}{2} \Rightarrow \alpha_d = \omega_s$$

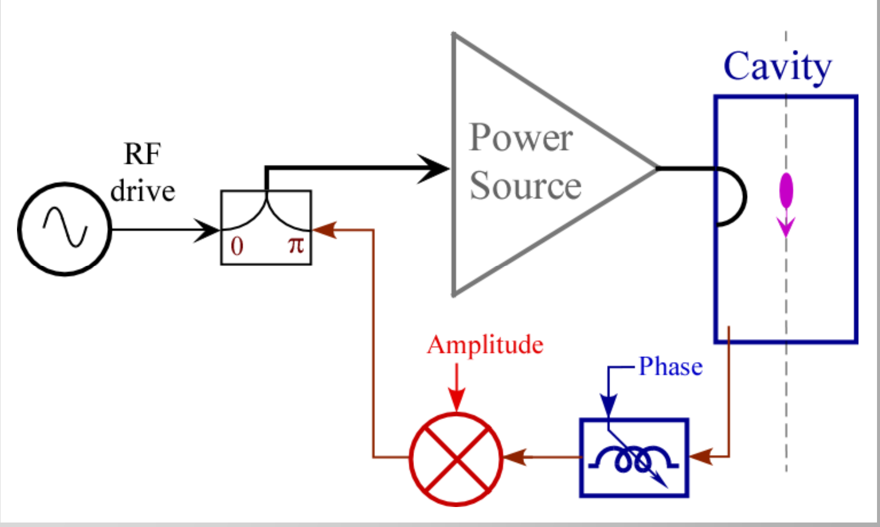


The gain of a pure differentiator circuit **grows linearly with frequency**, which is not a realistic behaviour for any physical system.

Active impedance reduction: the Direct RF Feedback

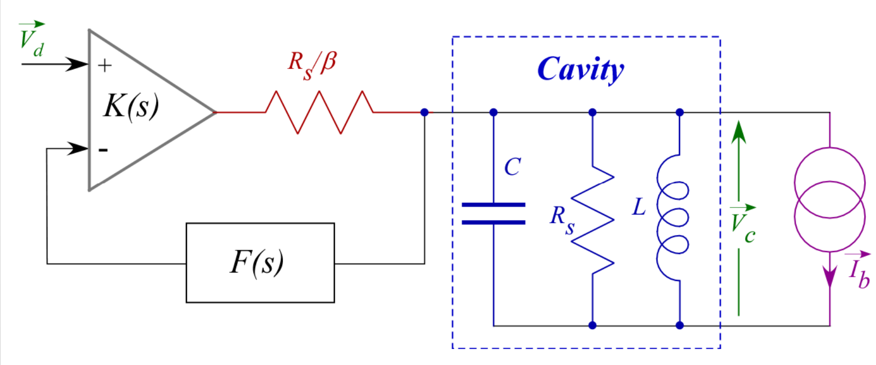
Reducing the accelerating mode impedance is beneficial to avoid driving CB instabilities on modes -1, -2, … and excessive reduction of the mode 0 coherent synchrotron frequency.

The **direct RF feedback reduces the cavity impedance** as **seen by the beam** by subtracting a sample of the cavity voltage to the RF drive. The effective **impedance** is **reduced** by a factor equal to the **open loop gain**. The cavity voltage is related to the beam current and to the RF drive signal by:



$$V_c(s) = -I_b(s)\frac{Z_L(s)}{1+H(s)} + V_d(s)\frac{H(s)/F(s)}{1+H(s)}$$

$$\text{with} \qquad H(s) = K(s)\frac{\beta}{\beta+1}\frac{Z_L(s)}{R_L}F(s)$$



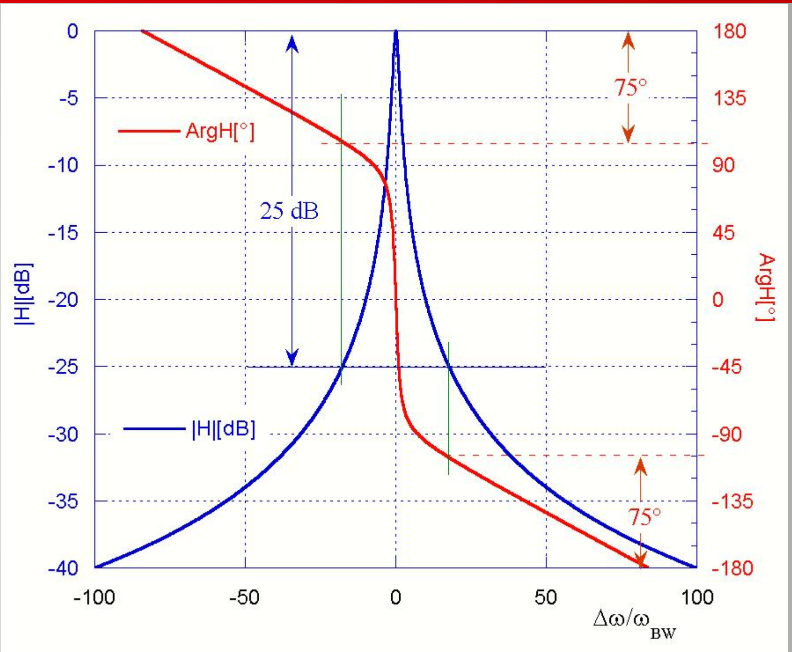
In the limit of large loop gain ($H_0 \gg 1$) the cavity equivalent impedance and the cavity voltage are given by:

$$Z_L'(s) \approx \frac{Z_L(s)}{H_0}; \qquad V_c(s) \approx -I_b\frac{Z_L(s)}{H_0} + \frac{V_d(s)}{F_0} \underset{H_0\to\infty}{\approx} \frac{V_d(s)}{F_0}$$

Active impedance reduction: the Direct RF Feedback

In the limit of **large loop gain** ($H_0 \rightarrow \infty$) the **beam induced voltage is cancelled** and the cavity voltage is entirely due to the RF drive signal (***= zero impedance***). Actually, the **gain** can not be infinite but it **is limited by the total delay $\boldsymbol{\tau_t}$** of the loop path. The physical delay $\boldsymbol{\tau_{ph}}$ (the total length of the connection) and the group delay $\boldsymbol{\tau_g}$ (the derivative of the phase response of the bandwidth limited devices such as the RF power source) contribute both to $\boldsymbol{\tau_t}$ ($100 \div 500\ ns$ typically).

A realistic expression for the open loop gain $H(j\omega)$ is:



$$H(j\omega) = \frac{H_0\, e^{-j\omega\tau_t}}{1 + j\Delta\omega/\omega_{BW}}$$

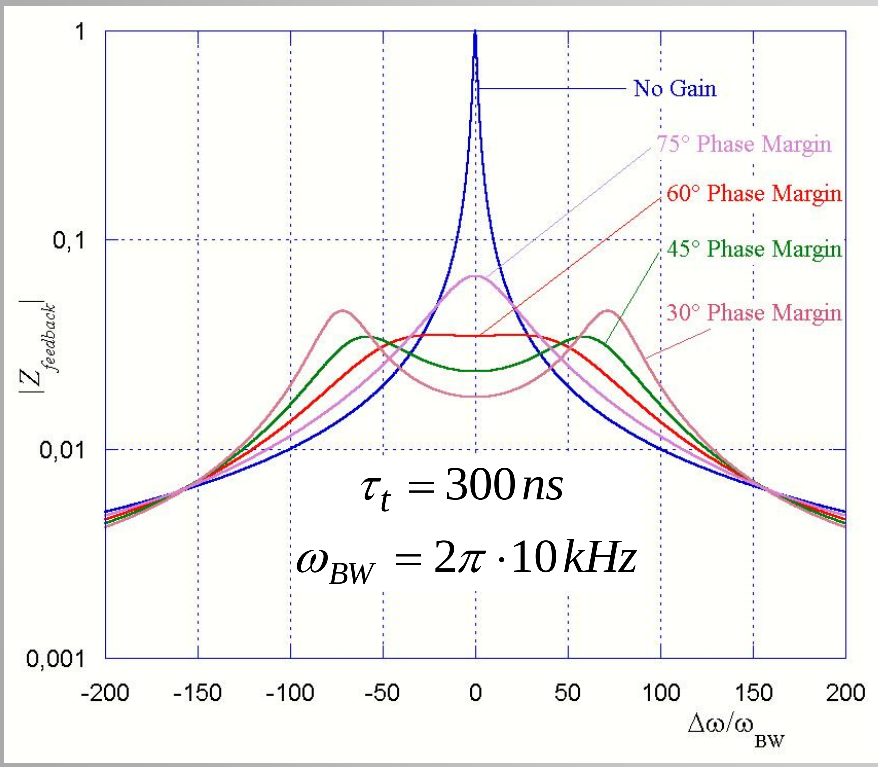
$\omega_{BW} = \omega_r/(2Q_L)$ = *loop half-bandwidth* = *σ*

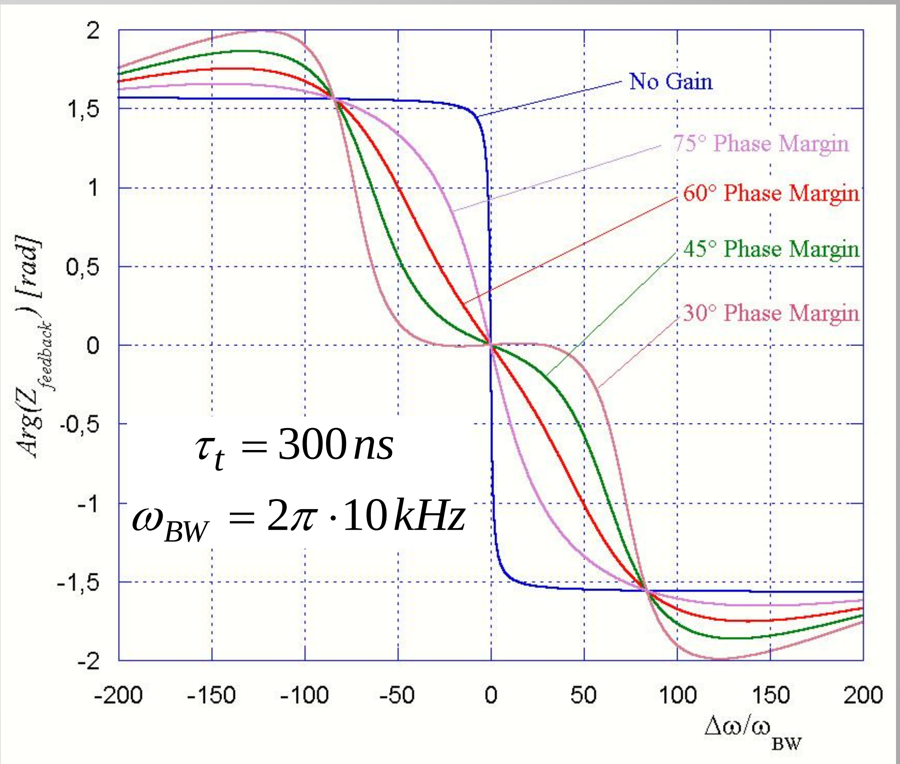
In order to maximize H_0 it is necessary to "trim" the delay with the loop phase shifter to the condition $\boldsymbol{\omega_r\tau_t = 2n\pi}$ ($\boldsymbol{\omega_r}$=cavity resonant frequency), and this condition has to be maintained while the cavity detuning changes to match to different beam current values. Under this condition, being ϕ_M the design loop phase margin, the maximum allowed gain H_0 is given by:

$$H_0 \approx \frac{\pi/2 - \phi_M}{\omega_{BW}\tau_t}$$

Active impedance reduction: the Direct RF Feedback

Depending on the total delay $\boldsymbol{\tau_t}$ and cavity bandwidth $\boldsymbol{\omega_{BW}}$ the equivalent impedance of the cavity accelerating mode is reduced and deformed as shown. Even though it can't be completely cancelled, the reduction is in general sufficient to weaken the beam loading effects to a tolerable level.





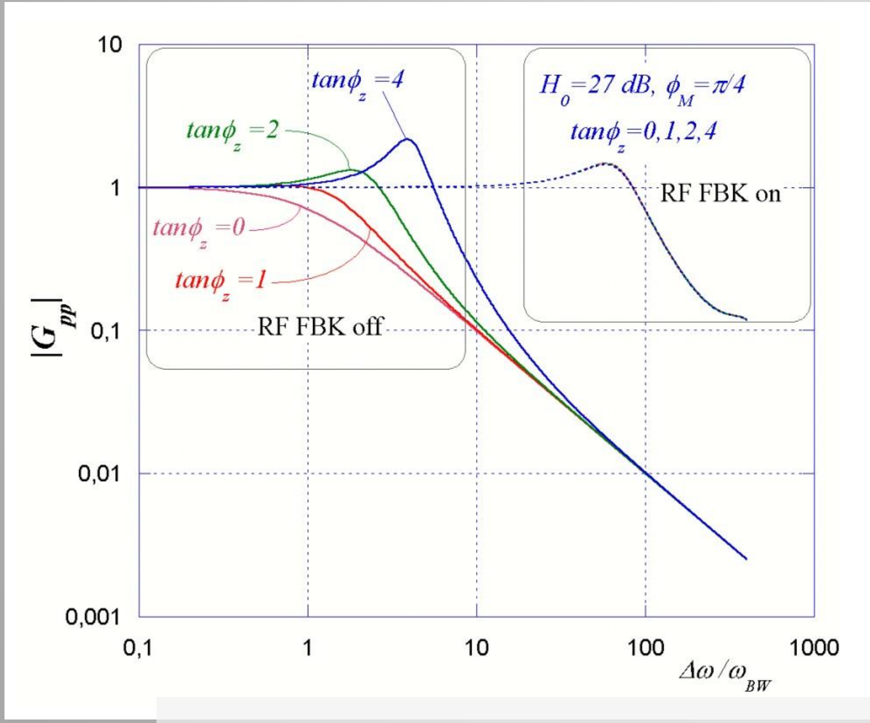
The direct RF feedback does not change the static beam loading aspects. What is changed is the **dynamics** of the **beam loading**, in terms of **reduction** of the **signals** induced by the **beam oscillations** and **modification** of the **modulation transfer functions**.

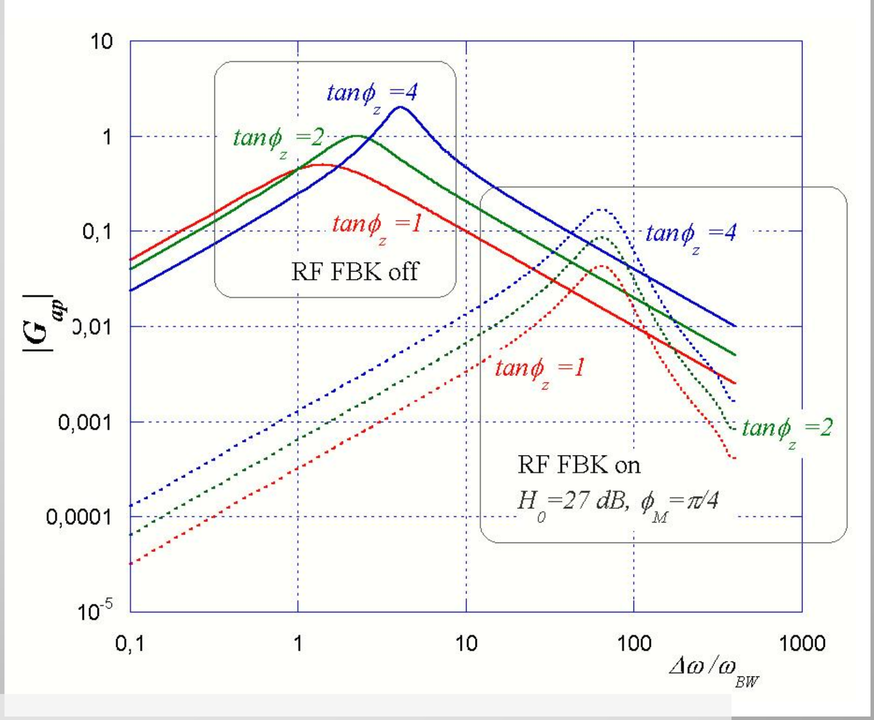
Active impedance reduction: the Direct RF Feedback

The direct and cross modulation transfer functions are in this case given by:

$$G_{pp}(s)=G_{aa}(s)=\frac{1}{2}\left[\frac{Z_L^{'}(s+j\omega)}{Z_L^{'}(j\omega)}+\frac{Z_L^{'}(s-j\omega)}{Z_L^{'}(-j\omega)}\right];\quad G_{ap}(s)=-G_{pa}(s)=\frac{1}{2j}\left[\frac{Z_L^{'}(s+j\omega)}{Z_L^{'}(j\omega)}-\frac{Z_L^{'}(s-j\omega)}{Z_L^{'}(-j\omega)}\right]$$

where the impedance is that **reduced by the feedback**. Typical plots of the module of the direct and cross modulation transfer functions are:

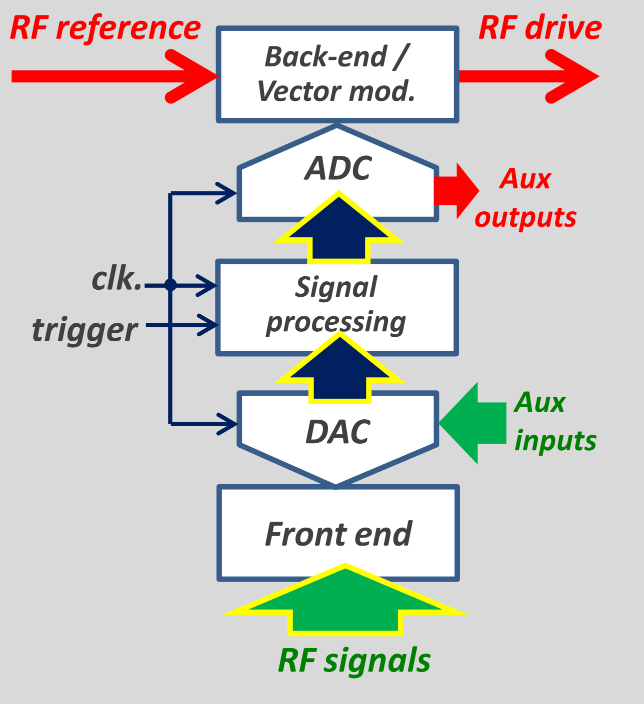




Direct RF feedback is a cure also for AM↔PM mixing!

STRUCTURE OF A LLRF SYSTEM

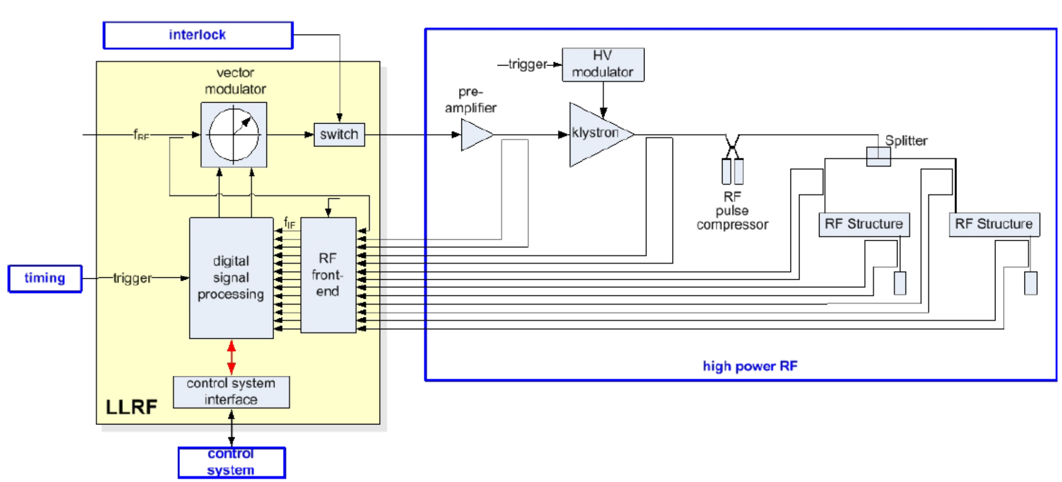
LLRF System Architecture



In general for LINAC applications the architecture of the LLRF systems is pretty standard.
Samples of the most relevant RF signals are routed to a front-end card and down-converted, either directly to baseband or to a suitable IF). Then the signals are processed, in modern system mostly in digital form, in order to control properly the RF drive. In general one LLRF system per power plant is needed.

The main tasks of Linac LLRF systems are:

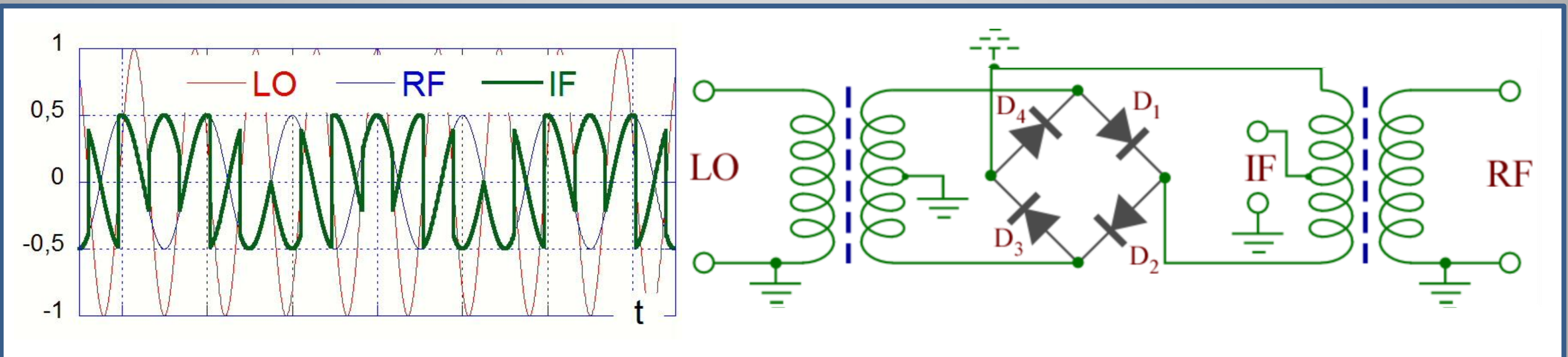
- Set amplitude and phase of the accelerating fields;
- Optimize the RF pulse compression process;
- Compensate beam loading effects;
- Weighted vector sum of RF signals from accelerating structures driven by the same power station;
- Feedforward pulse shaping (iterative algorithms may be required);
- Pulse-to-pulse FBK control of the main RF and beam characteristics;
- Parallel temperature control and tuning of the accelerating structures;
- Integration in the accelerator control and machine protection systems



Down-conversion

Frequency mixing is a **non-linear process** accomplished by dedicated devices. The **Double Balanced Mixer** is the **most diffused RF device** for frequency translation (up/down conversion) and detection of the relative phase between 2 RF signals (LO and RF ports). The LO voltage is differentially applied on a diode bridge switching on/off alternatively the D_1-D_2 and D_3-D_4 pairs, so that the voltage at IF is:

$$V_{IF}(t) = V_{RF}(t) \cdot \mathrm{sgn}\left[V_{LO}(t)\right]$$



$$V_{RF}(t) = V_{RF} \cdot \cos\left(\omega_{RF} t\right); \quad V_{LO}(t) = V_{LO} \cdot \cos\left(\omega_{LO} t\right)$$

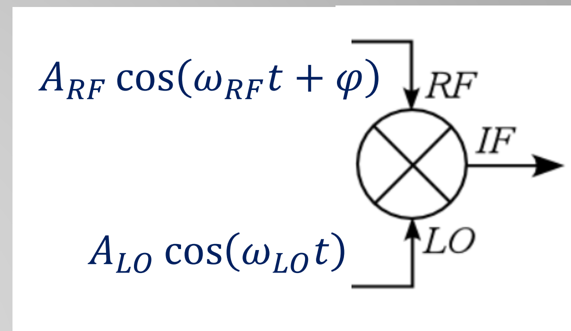
$$V_{RF} << V_{LO}$$

$$V_{IF}(t) = V_{RF}\cos(\omega_{RF}t) \cdot \mathrm{sgn}\left[\cos(\omega_{LO}t)\right] = V_{RF}\cos(\omega_{RF}t) \cdot \sum_{n=odds} \frac{4}{n\pi}\cos(n\omega_{LO}t) =$$

$$= \frac{2}{\pi} V_{RF}\left[\cos((\omega_{LO} - \omega_{RF})t) + \cos((\omega_{LO} + \omega_{RF})t) + intermod \;\; products \;\right]$$

LLRF System Front-end

The lowest frequency appearing at the IF port is $f_{IF} = |f_{RF} - f_{LO}|$, which can be easily extracted by low-pass filtering.



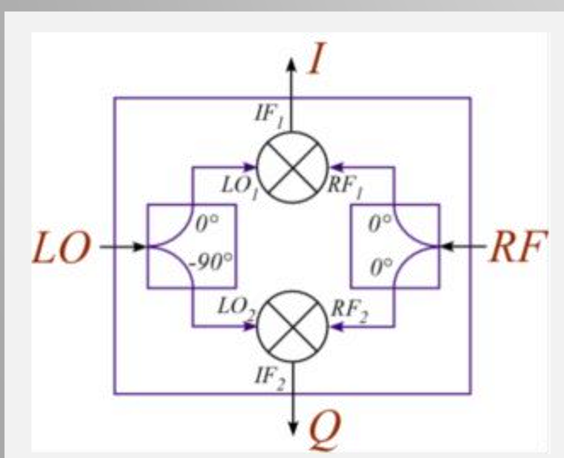
$$A_{RF} \ll A_{LO} \Rightarrow V_{IF}(t) = k_{CL} A_{RF}\cos(\omega_{IF}t+\varphi)$$

If $f_{LO}= f_{RF}$ the IF signal has a DC component given by:

$$\underbrace{V_{IF}\Big|_{DC} = k_{CL} A_{RF}\cos\varphi}_{\text{baseband conversion}}$$

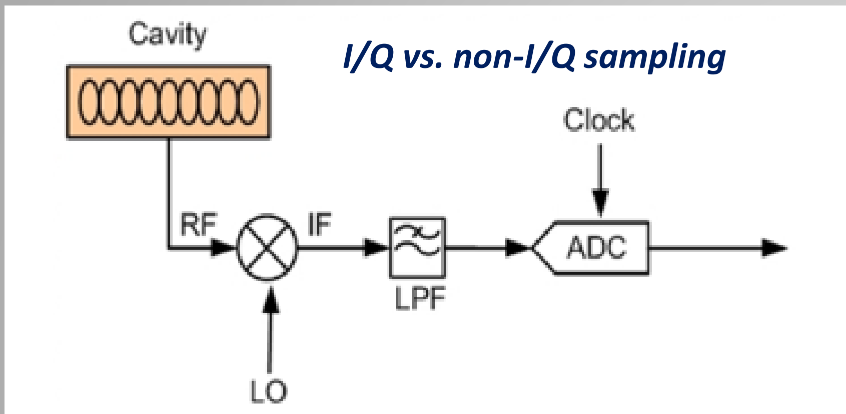
mixer conversion loss

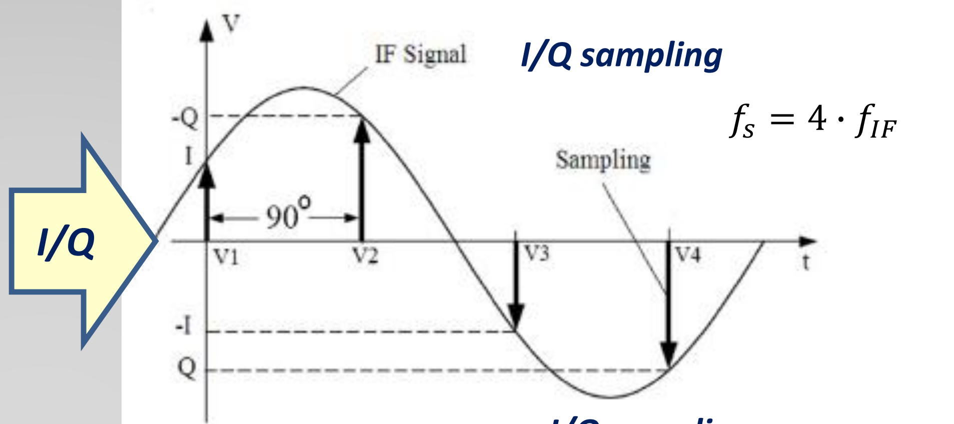
I/Q mixers are used to extract both components of the RF signal (In-phase, out-of-phase) or (amplitude, phase) by using a single device:

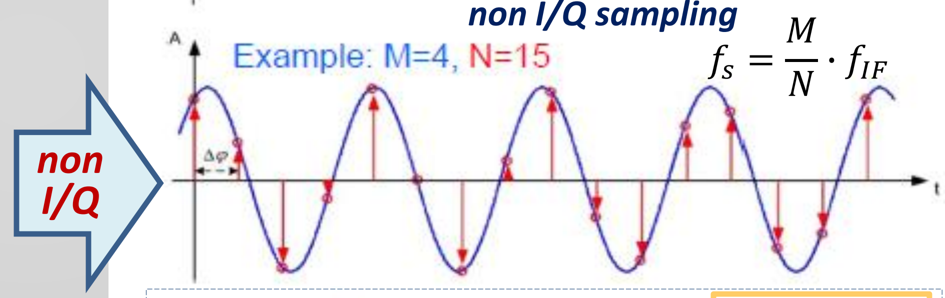


$$\begin{cases} V_I = k_{CL}A_{RF}\cos\varphi \\ V_Q = k_{CL}A_{RF}\sin\varphi \end{cases} \Rightarrow \begin{cases} A_{RF} = \dfrac{\sqrt{V_I^2+V_Q^2}}{k_{CL}} \\ \varphi = \underset{4\ quadrans}{\tan^{-1}}(V_Q, V_I) \end{cases}$$

Front-end / digitalization

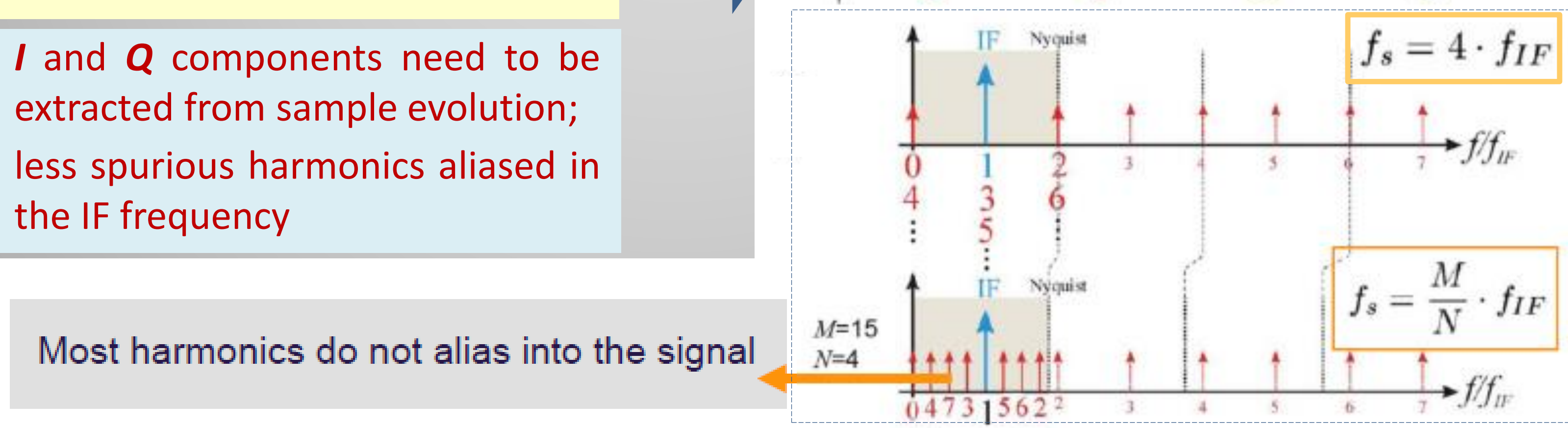






I/Q
- simpler, easy to synchronize
- gives alternating samples of ***I*** and ***Q*** components

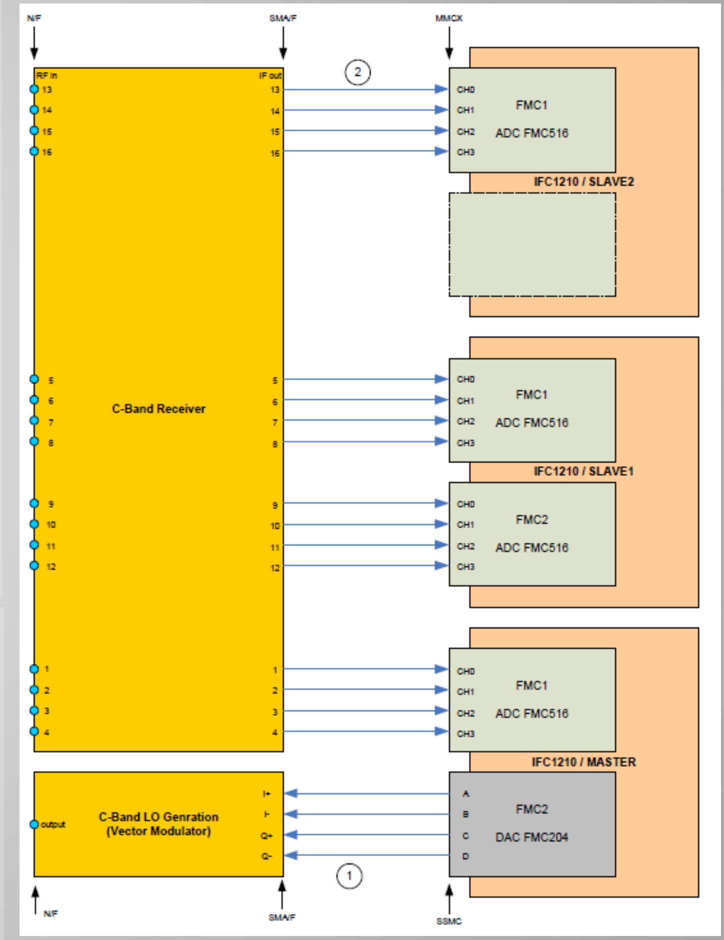
non I/Q
- ***I*** and ***Q*** components need to be extracted from sample evolution;
- less spurious harmonics aliased in the IF frequency



Signal processing

LLRF signal processing capability is another essential feature. The signal manipulations required are specific of any different accelerator and in general include:

- signal weighted sum/subtraction;
- RF amplitude/phase stabilization through feedback topology including PID controllers to optimize closed loop responses;
- signal complex equalization;
- implementation of feed-forward schemes, including recursive optimization algorithms;
- ….

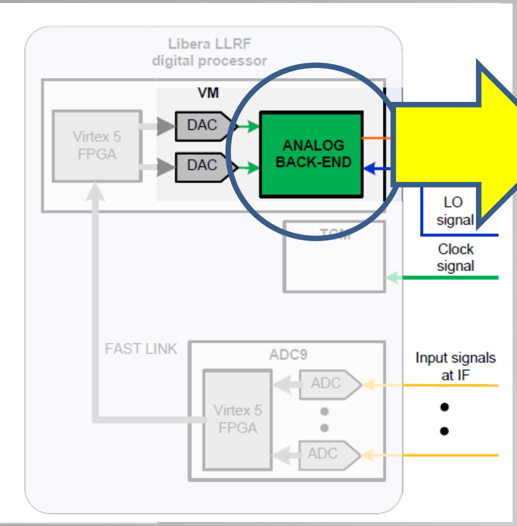


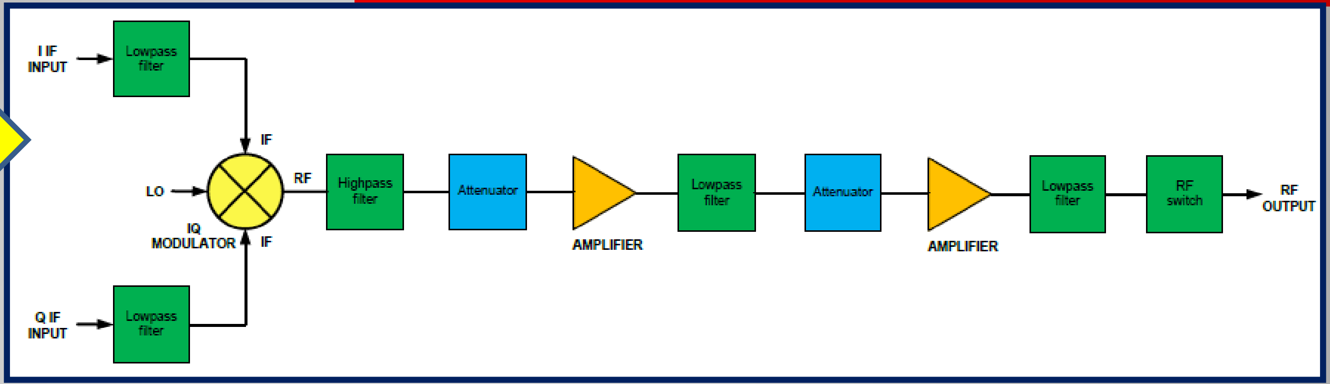
Till 90s most of these functions and many others have been implemented in LLRF by means of fully-analog electronic boards.

In the last ≈25 years the use of digital processing in LLRF has grown more and more. Digital Signal Processors (DSP) have been progressively replaced by Field Programmable Gate Arrays (FPGAs), with improved processing speed and reliability.

FPGAs are “wired logic” processors and allow a very fast and efficient data stream and processing. On the other hand FPGA programming requires dedicated and specialized technical and human resources.

LLRF System Back-end

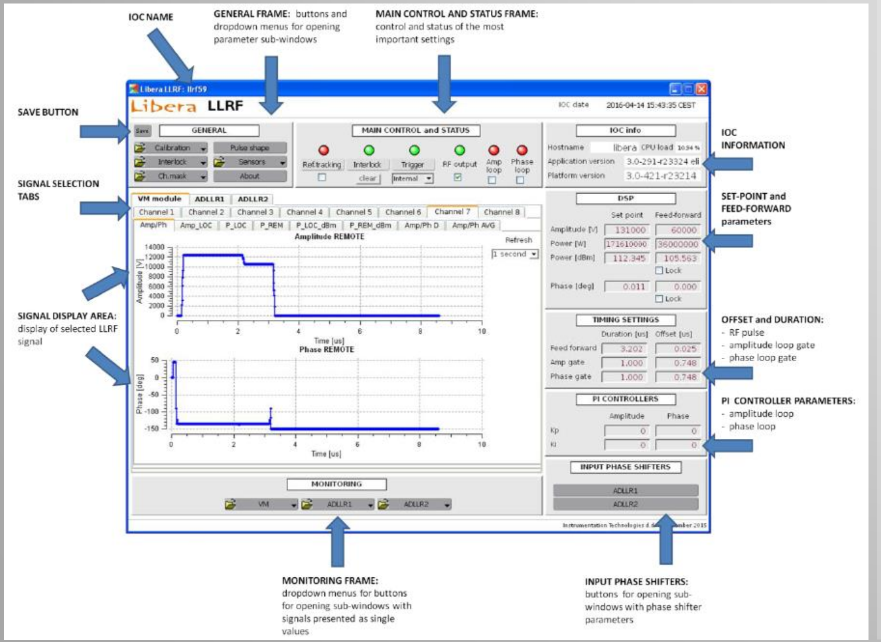




The **LLRF back-end** consists on a **vector modulator** (either I/Q or Amp/phase) generating the RF signal to drive an RF power source feeding a group of accelerating cavity according to the data processing results.

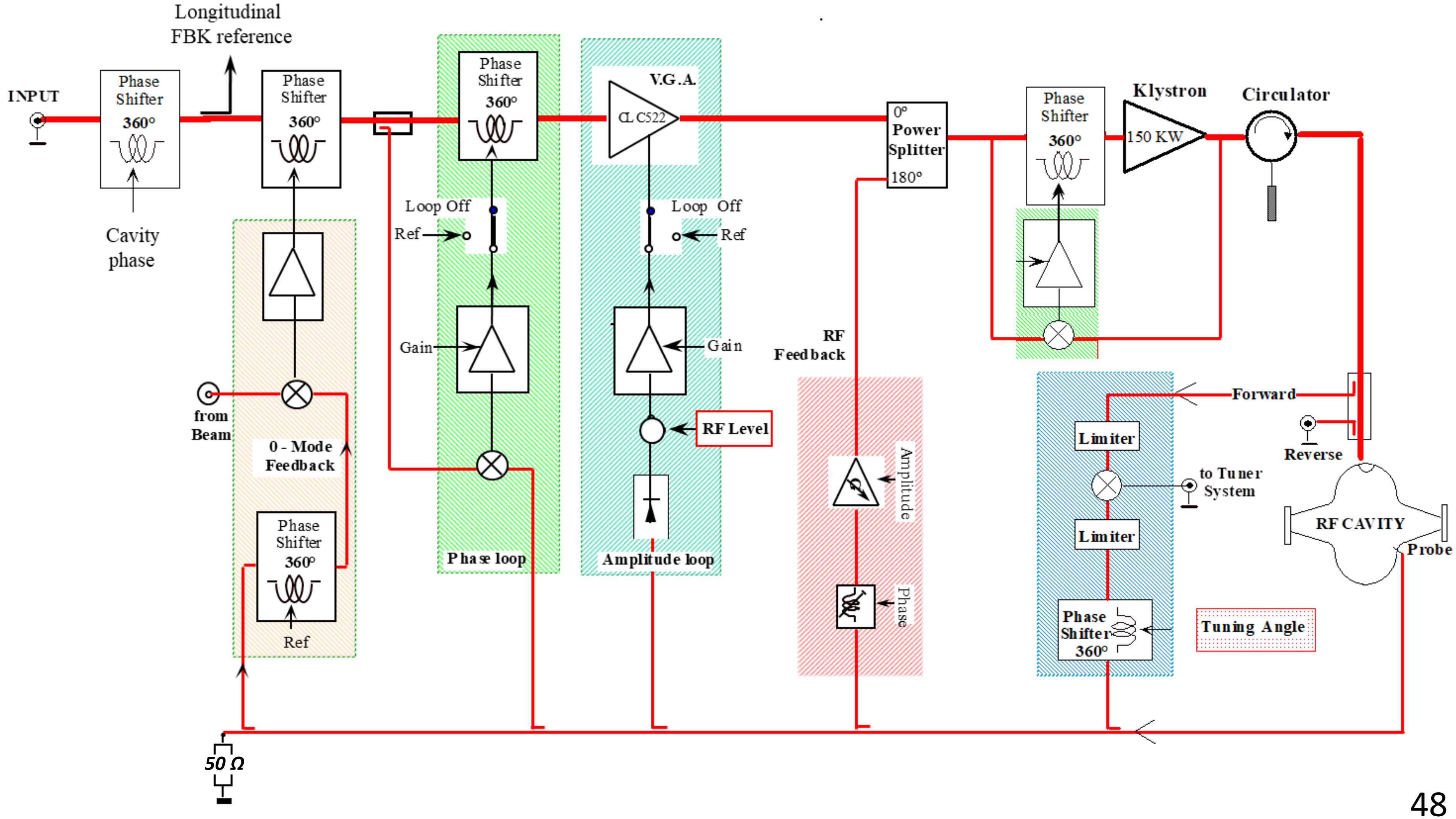
In digital system the back-end modulator is controlled by DACs and the RF line has to include **filters** to reject spurious harmonics produced by **IF aliasing**. **Back-end filtering** may be a dominant component setting the **effective bandwidth** of the whole LLRF system.

The DAC + back-end characteristics set also the capability of generating RF pulses. The DAC sampling frequency and the buffer depth define the temporal resolution of the pulse shaping and the maximum pulse length.



DAΦNE Analog LLRF Control

In circular accelerator the chosen LLRF architecture may be based on **multiple back-end modulators** (either I/Q or Amp/Phase) in a structure of many “concentric” loops. A careful **global stability analysis** is required.



PEP-II LLRF Control Loops

The LLRF control of the SLAC B-factory PEP-II is an example of the historical evolution of the RF controls. **Various technologies** together with **original developments** led to a very complex and effective system.

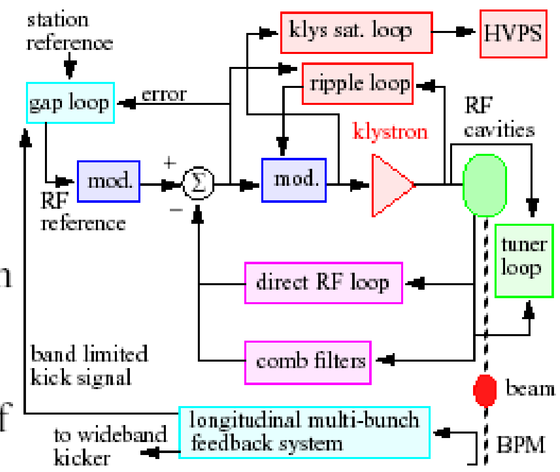
Tuner loops - standard tuning for minimum reflected power

Klystron operating point support

- Ripple loop adjusts a complex modulator to maintain constant gain and phase shift through the klystron/modulator system.
- Klystron saturation loops maintain constant saturation headroom

Direct feedback loop (analog)

- Causes the station to follow the RF reference adding regulation of the cavity voltage
- Extends the beam-loading Robinson stability limit
- Lowers the effective fundamental impedance seen by the beam



Loop technology legend

Magenta - EPICS loop
Green - digital
Red - analog

Comb filter (digital)

- Adds narrow gain peaks at synchrotron sidebands to further reduce the residual impedance

Gap feedback loop (digital)

- Removes revolution harmonics from the feedback error signal to avoid saturating the klystron on gap synchronous phase transients

Longitudinal feedback uses RF as low-frequency "woofer" kicker

CONCLUSIONS

- ✓ RF fields are in the core of the beam physics in particle accelerators. RF precise control is crucial for beam quality and stability. Low level RF systems are the hardware and software complexes devoted to this task.
- ✓ The functionalities to be implemented in the LLRF depend very much on the specific beam and machine characteristics. In this respect LLRFs are essentially custom systems.
- ✓ LLRF design and development requires a combination of expertise in different fields such as beam physics, RF, electronics, control and computational engineering.
- ✓ Many clever ideas and techniques for beam manipulation have been developed already since the pioneeristic era of the proton synchrotrons on the base of analog signal processing.
- ✓ Nowadays digital processing has boosted substantially the data manipulation capability, and LLRF teams need to incorporate more and more know-how and specialists in this field to face the challenges of future high energy and high performances accelerators.

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