Photo:
Reidar Hahn

Superconducting RF Systems II

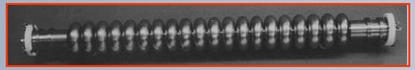
Cavities of different shapes

Erk JENSEN, CERN

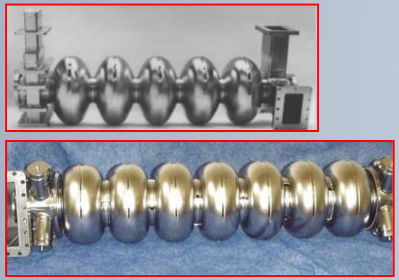
Elliptical cavities – the *de facto* standard for SRF

FERMI 3.9 GHz

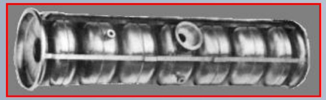
S-DALINAC 3 GHz



CEBAF 1.5 GHz

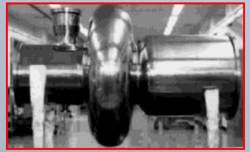


HEPL 1.3 GHz



CESR 0.5 GHz

KEK-B 0.5 GHz



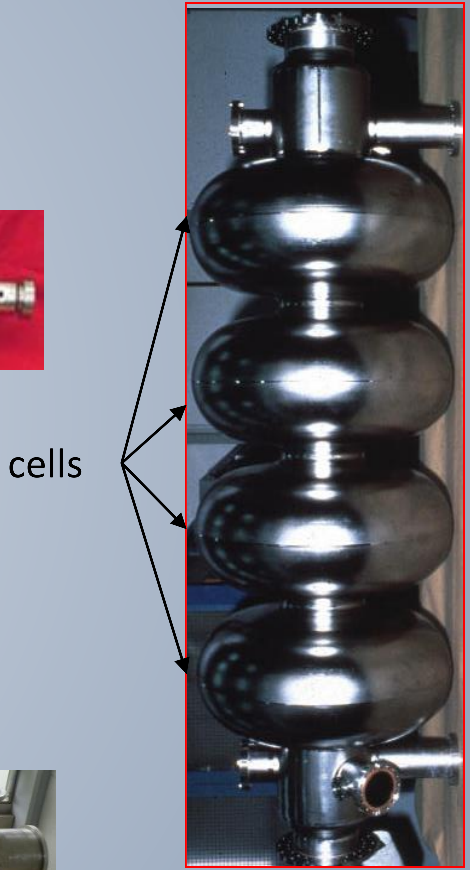
TESLA/ILC 1.3 GHz

SNS $\beta = 0.61, 0.81,$ 0.805 GHz

HERA 0.5 GHz

TRISTAN 0.5 GHz

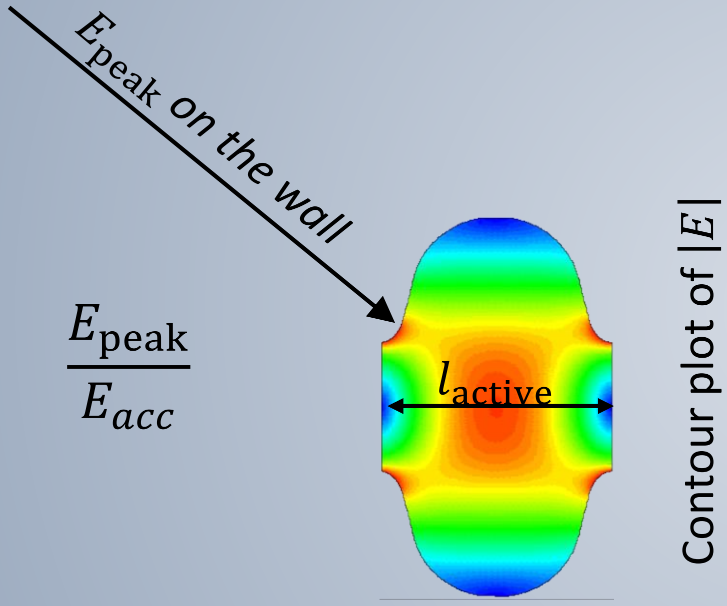
LEP 0.352 GHz



Practical RF parameters 1

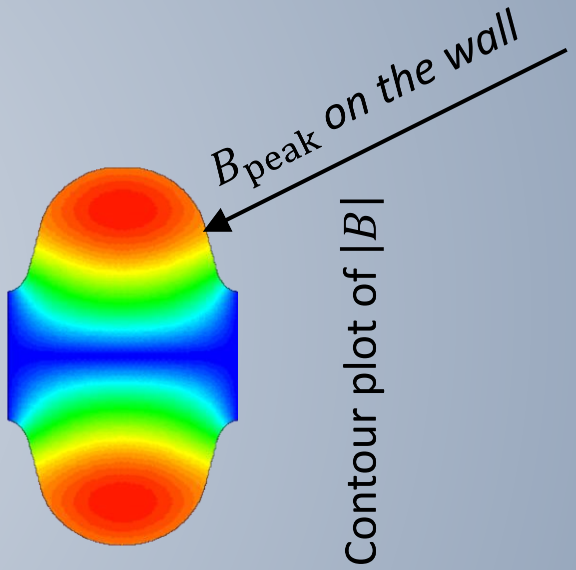
- Average accelerating gradient: $E_{acc} = \frac{\sqrt{\omega W (R/Q)}}{l_{\text{active}}}$

$$\frac{E_{\text{peak}}}{E_{acc}}$$



The ratio shows sensitivity of the shape to the **field emission** of electrons.

$$\frac{B_{\text{peak}}}{E_{acc}}$$



The ratio shows limit in E_{acc} due to the breakdown of superconductivity (**quench**, Nb: ≈ 190 mT).

courtesy: Jacek Sekutovicz/DESY

Practical RF parameters 2

$$G \cdot (R/Q)$$

- Both G and R/Q are purely geometric parameters.
- Like the shunt impedance R, the product $G \cdot (R/Q)$ is a measure of the power loss for given acceleration voltage V_{acc} and surface resistance R_s.

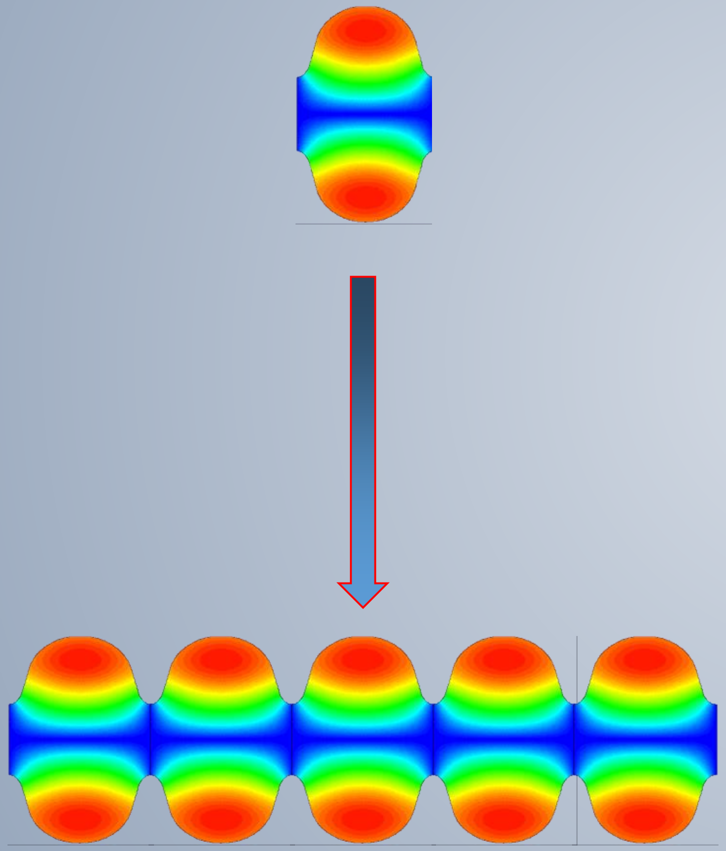
Minimize R_s:
operation at lower T,
better surface cleanliness,
lower residual resistance

$$P_{\text{loss}} = \frac{|V_{acc}|^2 R_s}{2\, \underbrace{G \cdot (R/Q)}}$$

Optimize geometry maximizing $G \cdot (R/Q)$.

courtesy: Jacek Sekutovicz/DESY

Single-cell versus multi-cell cavities



- Advantages of single-cell cavities:
  - It is easier to manage HOM damping
  - There is no field flatness problem.
  - Input coupler transfers less power
  - They are easy for cleaning and preparation

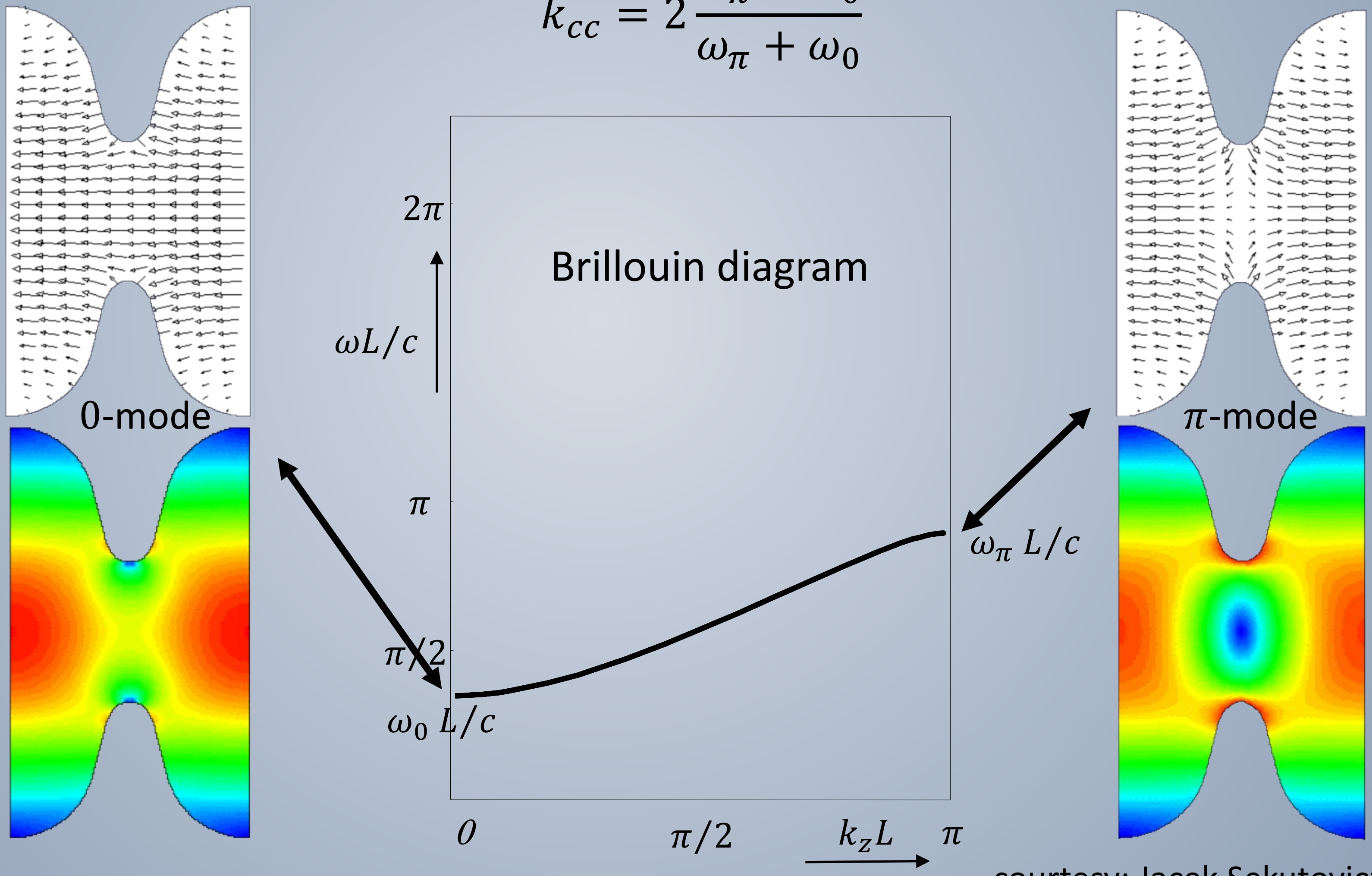
- Advantages of multi-cell cavities:
  - much more acceleration per meter!
  - better use of the power ($R \rightarrow n\ R$)
  - more cost-effective for most applications

courtesy: Jacek Sekutovicz/DESY

Practical RF parameters 3

- **Cell-to-cell coupling** k_{cc} will determine the width of the passbands in multi-cell cavities.

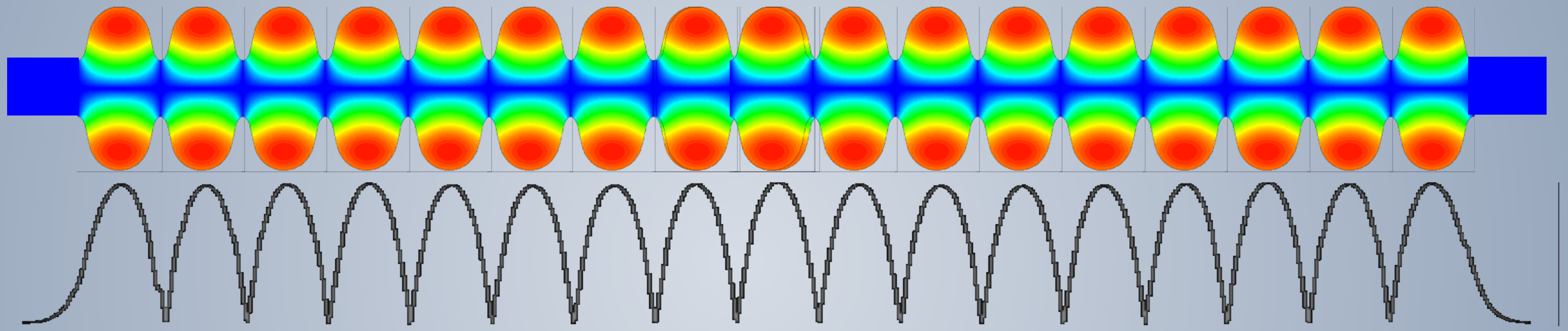
$$k_{cc} = 2\frac{\omega_\pi - \omega_0}{\omega_\pi + \omega_0}$$



courtesy: Jacek Sekutovicz/DESY

Field flatness

- Field amplitude variation from cell to cell in a multi-cell structure
- Should be small for maximum acceleration.



- Field flatness sensitivity factor a_{ff} for a structure made of N cells:

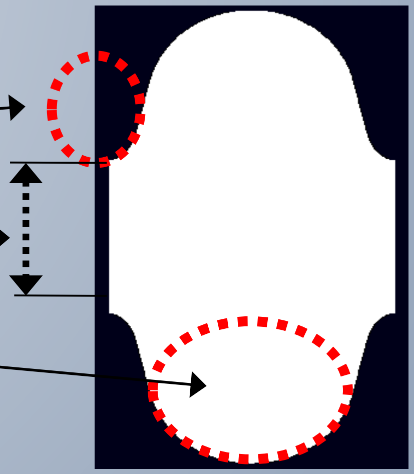
$$\frac{\Delta A_i}{A_i} = a_{ff} \frac{\Delta f_i}{f_i}$$

a_{ff} is related to the cell-to-cell coupling as $a_{ff} = \frac{N^2}{k_{cc}}$ and describes the sensitivity of the field flatness on the errors in individual cells.

courtesy: Jacek Sekutovicz/DESY

Criteria for Cavity Design (1)

- Here: Inner cells of multi-cell structures
- Parameters for optimization:
  - Fundamental mode: $\frac{R}{Q}$, G, $\frac{E_{\text{peak}}}{E_{acc}}$, $\frac{B_{\text{peak}}}{E_{acc}}$, k_{cc}.
  - Higher order modes: $k_\perp$, k_z.
- The elliptical cavity design has distinct advantages:
  - easy to clean (rinse)
  - little susceptible to MP – can be conditioned …
- Geometric parameters for optimization:
  - iris ellipse half axes: a, b:
  - iris aperture radius: r_i,
  - equator ellipse half axes: A, B
- Problem: 7 parameters to optimize, only 5 to play with – some compromise has to be found!



courtesy: Jacek Sekutovicz/DESY

Photo:
Reidar Hahn

Criteria for Cavity Design (2)

Criterion	RF parameter	Improves if	examples
high gradient operation	E_{peak}/E_{acc} B_{peak}/E_{acc} ↓	r_i ↓	TESLA, CEBAF 12 GeV HG
low cryogenic losses	$\frac{R}{Q} \cdot G$ ↑	r_i ↓	CEBAF LL
High I_{beam}	$k_{\perp}, k_z$ ↓	r_i ↑	B-factory RHIC cooling LHeC

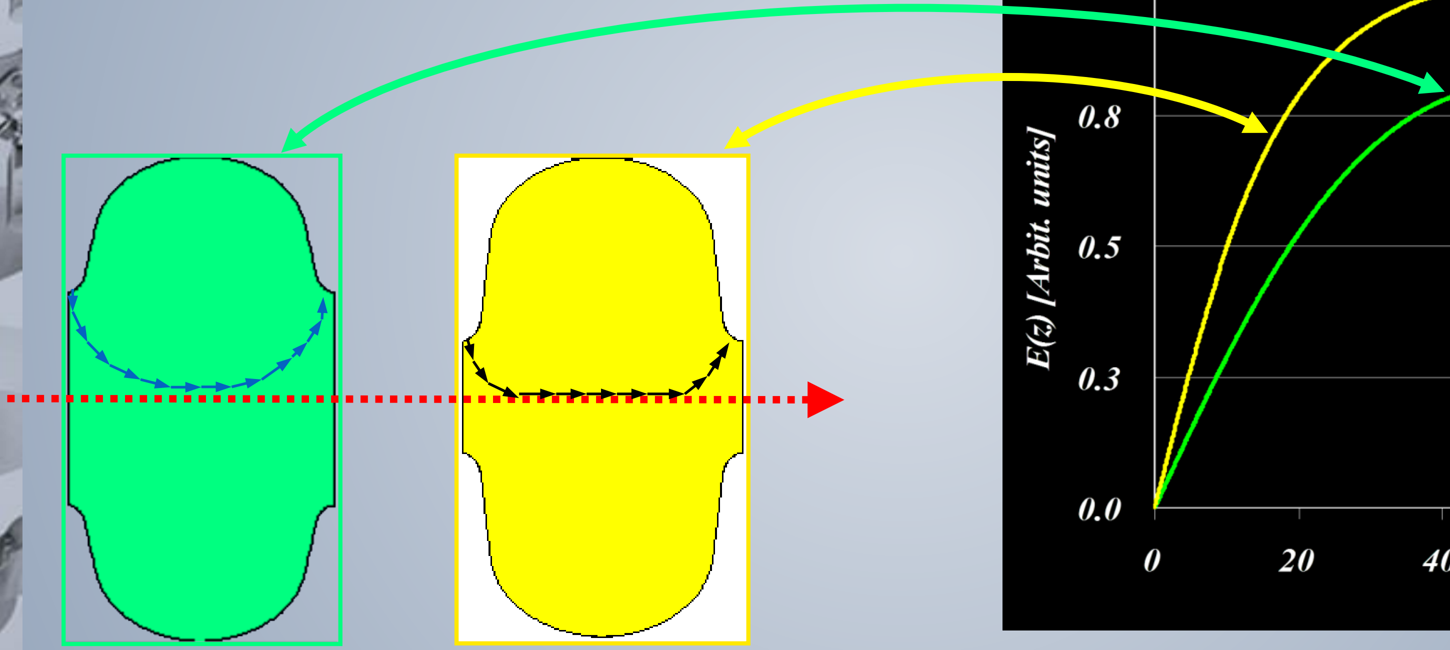
We see here that r_i is a very “powerful” variable to trim the RF-parameters of a cavity.

Of course it has to fit the aperture required for the beam!

courtesy: Jacek Sekutovicz/DESY

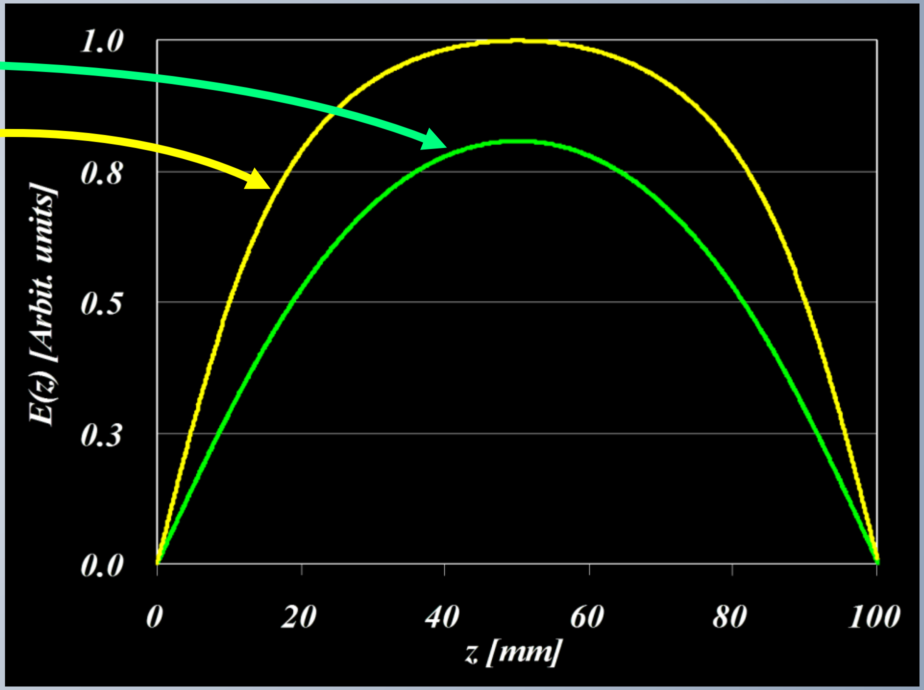
Effect of r_i

- Smaller r_i allows to concentrate E_z where it is needed for acceleration



$r_i = 40$ mm

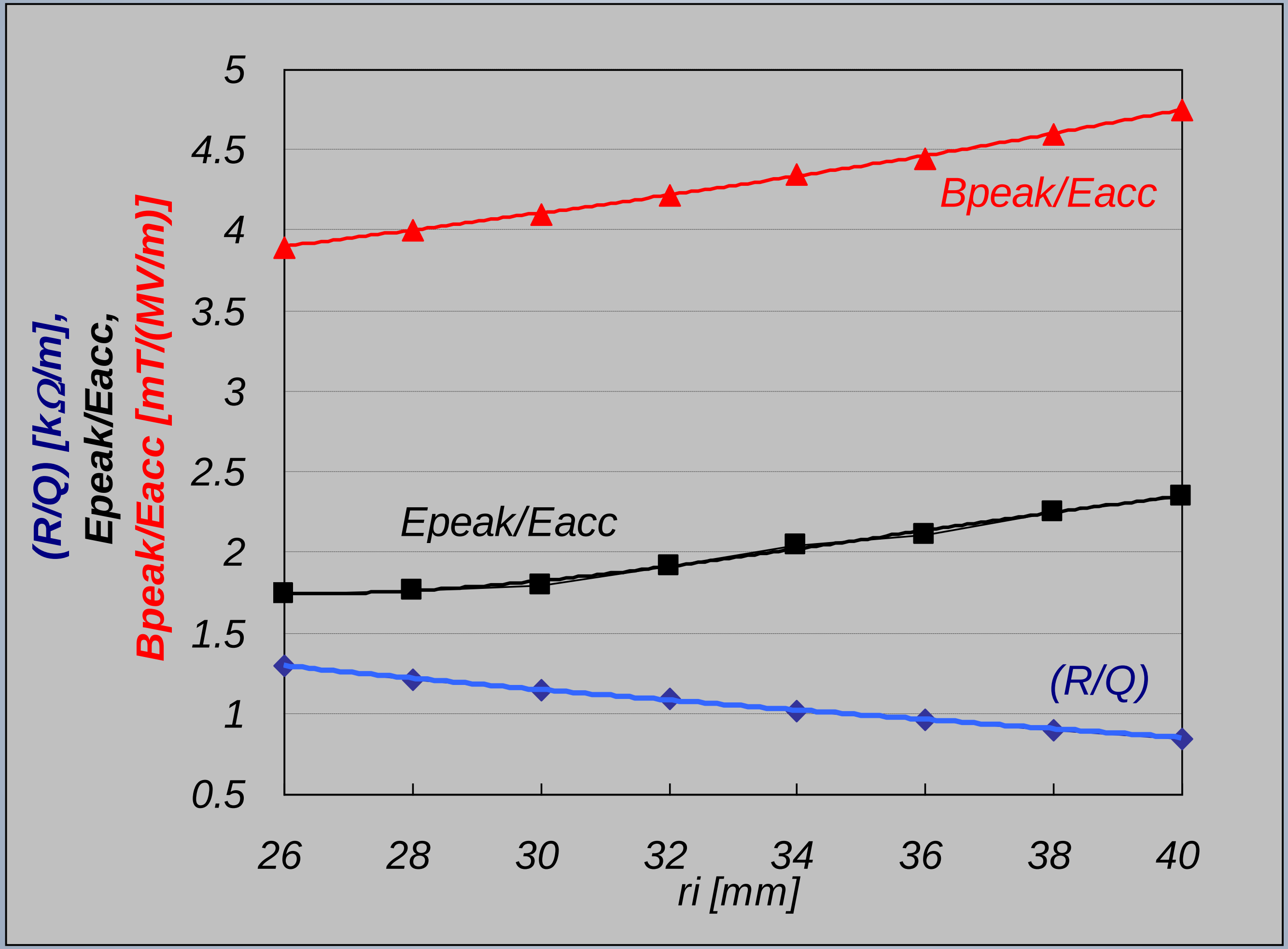
$r_i = 20$ mm



$E_z(z)$ for small and big iris radius

courtesy: Jacek Sekutovicz/DESY

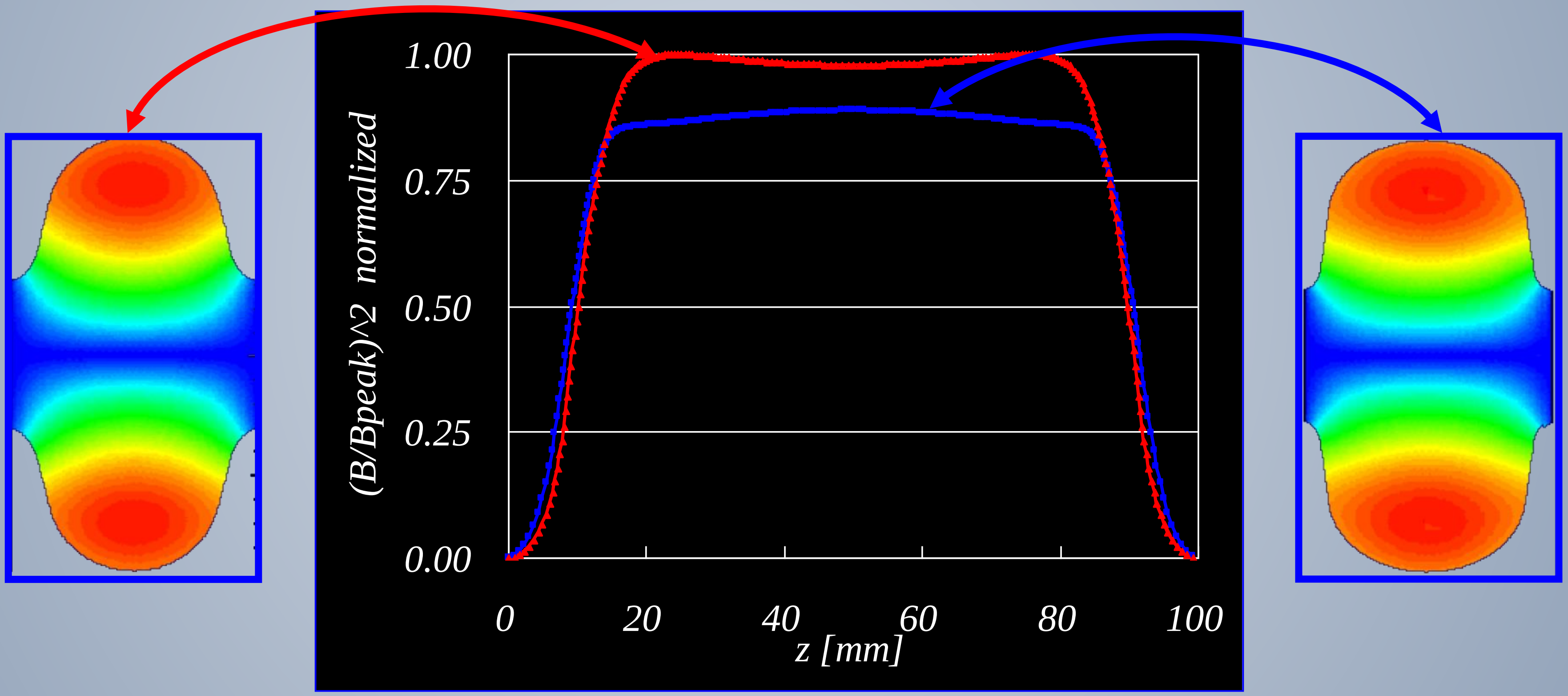
Example: cell optimization at 1.5 GHz



A. Mosnier, E. Haebel, SRF Workshop 1991

Equator shape optimization

- B_{peak}/E_{acc} (and G) change when changing the equator shape.

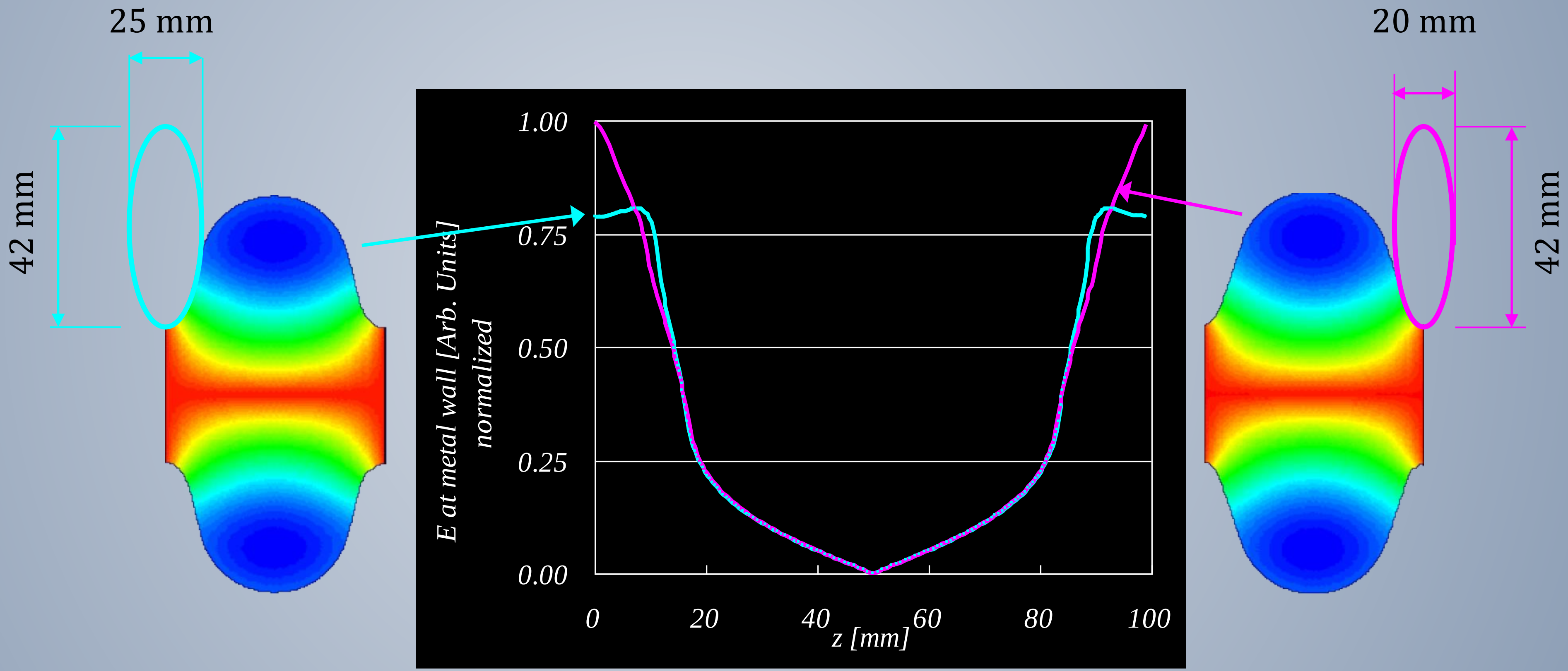


courtesy: Jacek Sekutovicz/DESY

Photo:
Reportedly Reidar Hahn

Iris shape optimization

- E_{peak}/E_{acc} changes with the iris shape

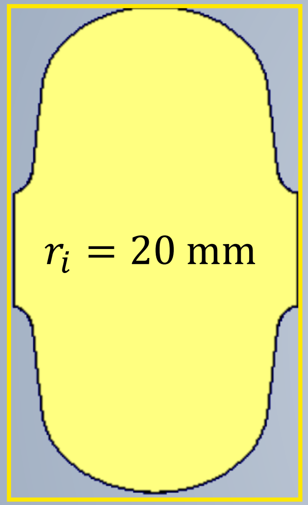


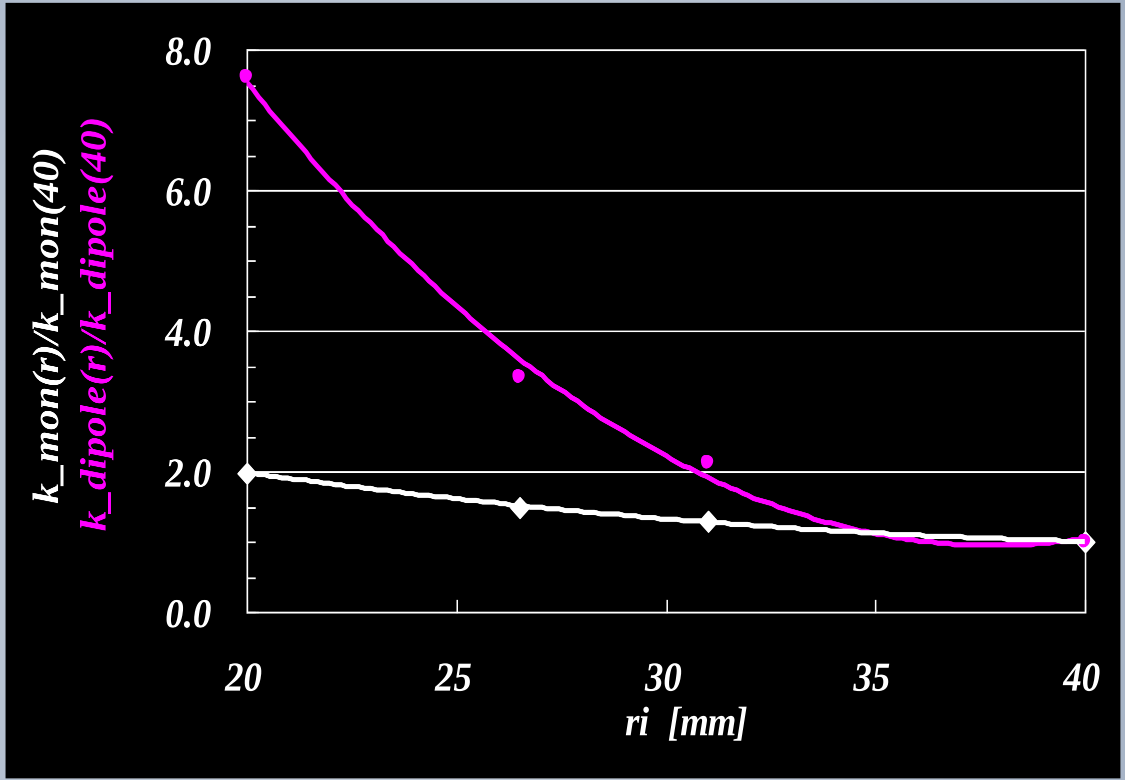
Both cells have the same: f_0, R/Q, and r_i.

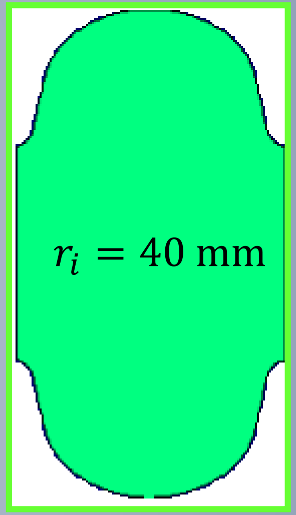
courtesy: Jacek Sekutovicz/DESY

Minimizing HOM excitation

HOMs loss factors ($k_{\mathrm{loss},\perp}$, k_{loss})





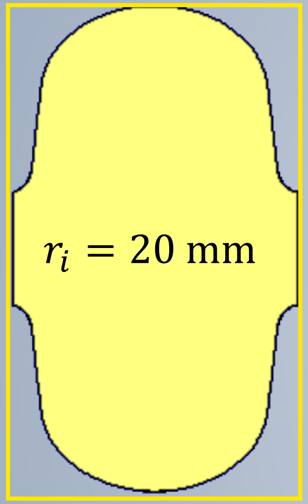


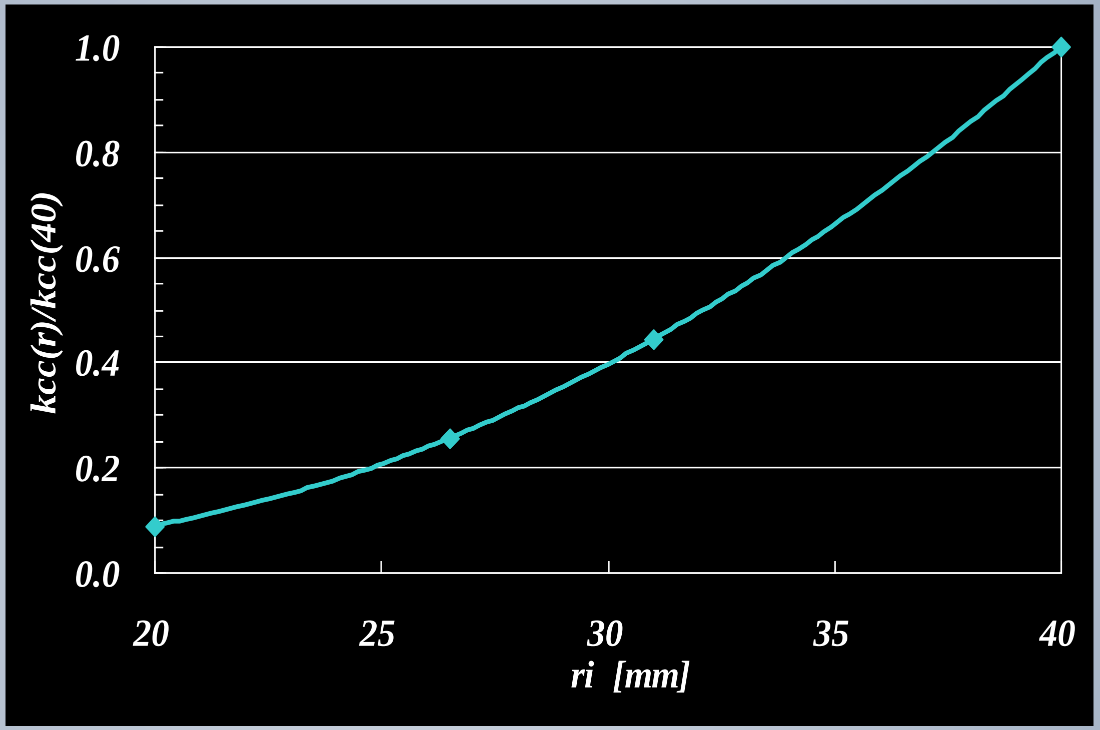
$R/Q = 152\ \Omega$
$B_{\mathrm{peak}}/E_{acc} = 3.5\,\mathrm{mT/(MV/m)}$
$E_{\mathrm{peak}}/E_{acc} = 1.9$

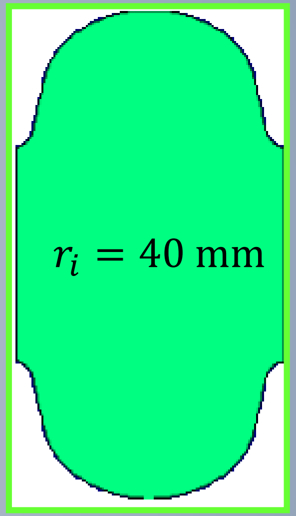
$R/Q = 86\ \Omega$
$B_{\mathrm{peak}}/E_{acc} = 4.6\,\mathrm{mT/(MV/m)}$
$E_{\mathrm{peak}}/E_{acc} = 3.2$

courtesy: Jacek Sekutovicz/DESY

Cell-to-cell coupling k_{cc}







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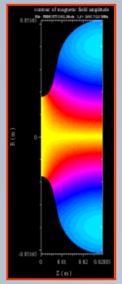
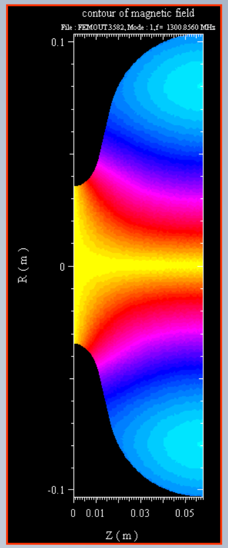
courtesy: Jacek Sekutovicz/DESY

Scaling the frequency



$\times\, \mathbf{2} \;=$



f_π	[MHz]	*2600*
R/Q	[Ω]	*57*
r/Q	[Ω/m]	*2000*
G	[Ω]	*271*

f_π	[MHz]	*1300*
R/Q	[Ω]	*57*
r/Q	[Ω/m]	*1000*
G	[Ω]	*271*

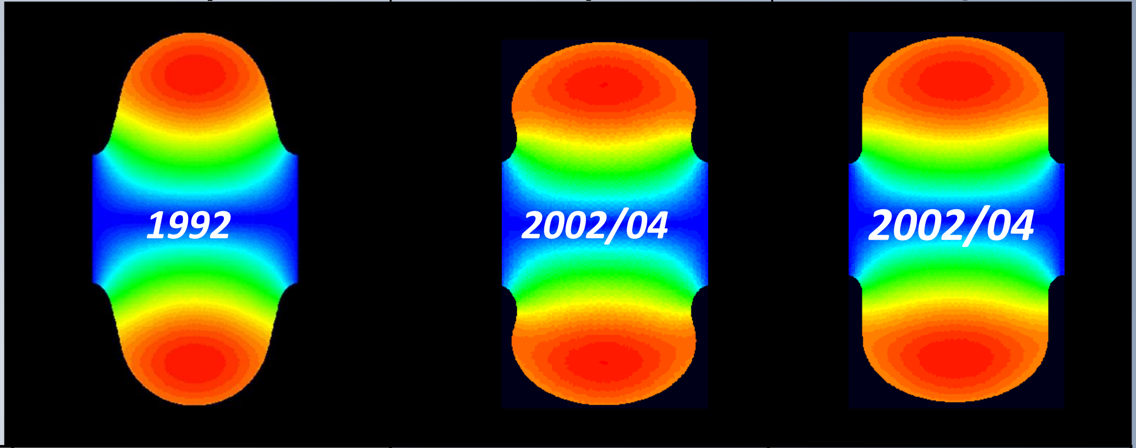
$r/Q = (R/Q)/l \propto f$ (or $(R/Q)/\lambda = \text{const}$)

courtesy: Jacek Sekutovicz/DESY

Historic evolution of inner cell geometry

Example: ILC

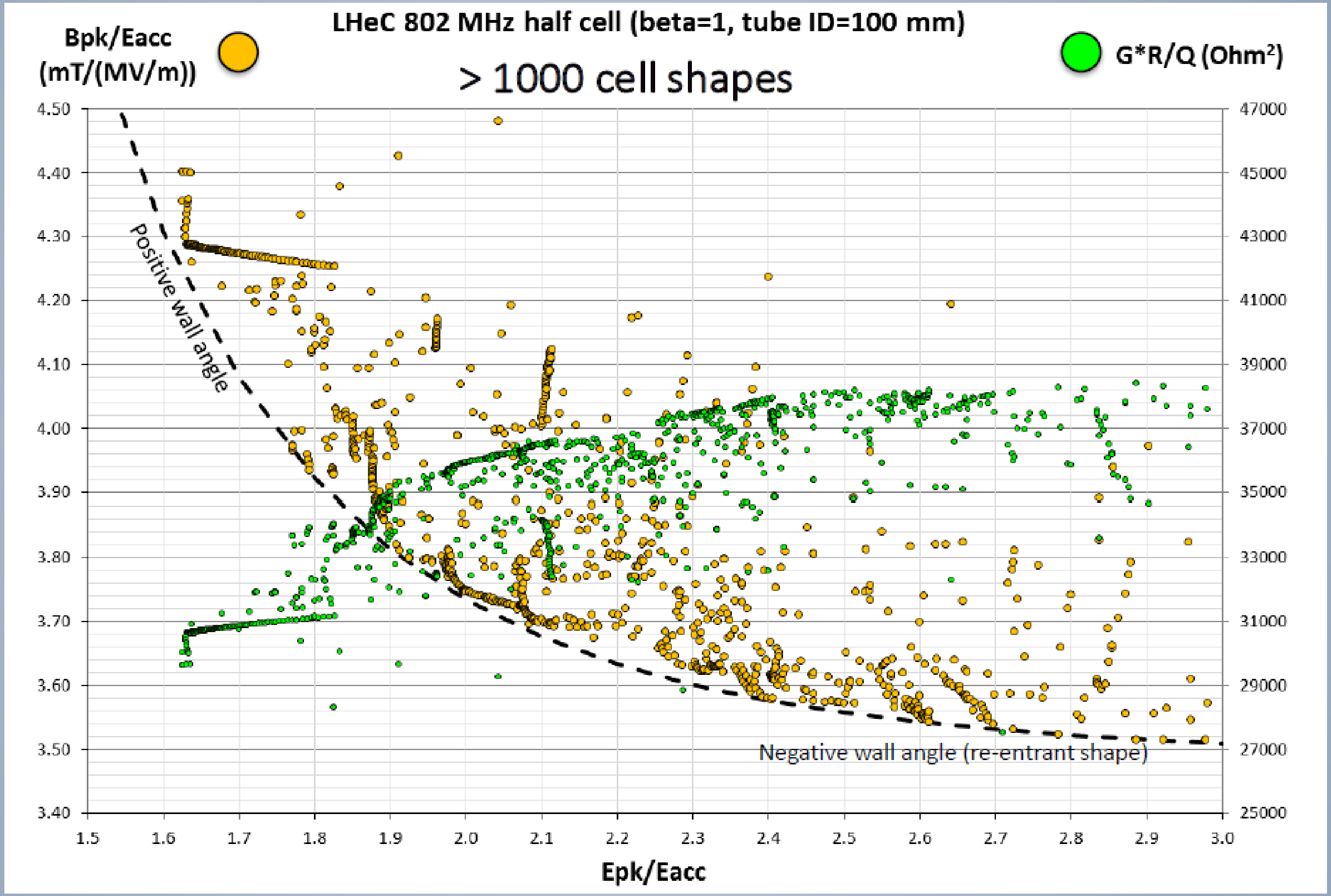
		TESLA, optimized E_{peak}/E_{acc}	Re-entrant, optimized B_{peak}/E_{acc}	Low-loss (LL) optimized B_{peak}/E_{acc}



r_i	[mm]	35	30	30
k_{cc}	[%]	1.9	1.56	1.52
E_{peak}/E_{acc}	-	1.98	2.30	2.36
B_{peak}/E_{acc}	[mT/(MV/m)]	4.15	3.57	3.61
R/Q	[Ω]	113.8	135	133.7
G	[Ω]	271	284.3	284
$R/Q \cdot G$	[Ω*Ω]	30840	38380	37970
$k_{\text{loss},\perp}$ ($\sigma_z = 1$ mm)	[V/pC/cm²]	0.23	0.38	0.38
k_{loss} ($\sigma_z = 1$ mm)	[V/pC]	1.46	1.75	1.72

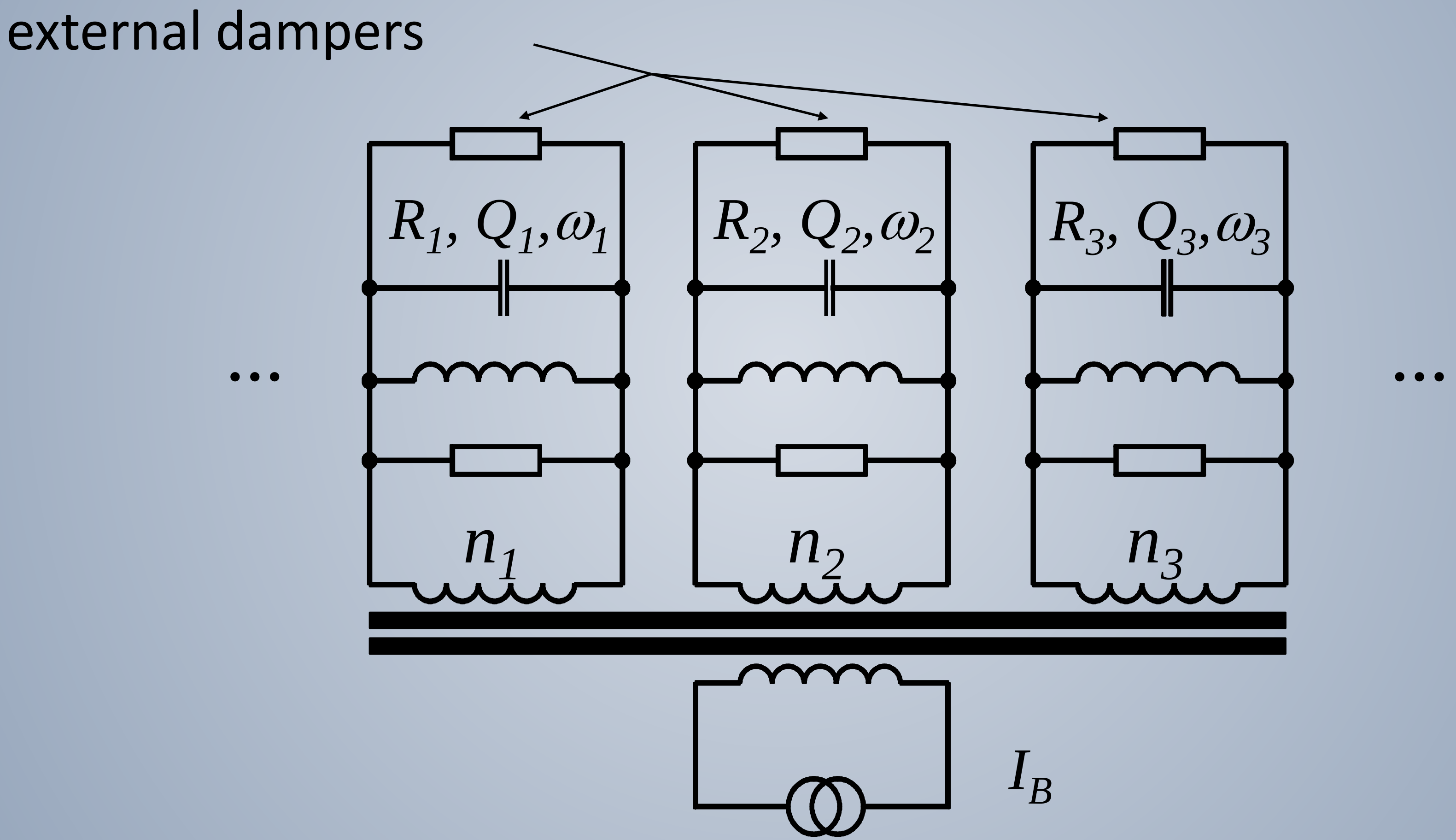
courtesy: Jacek Sekutovicz/DESY

Cavity optimization example



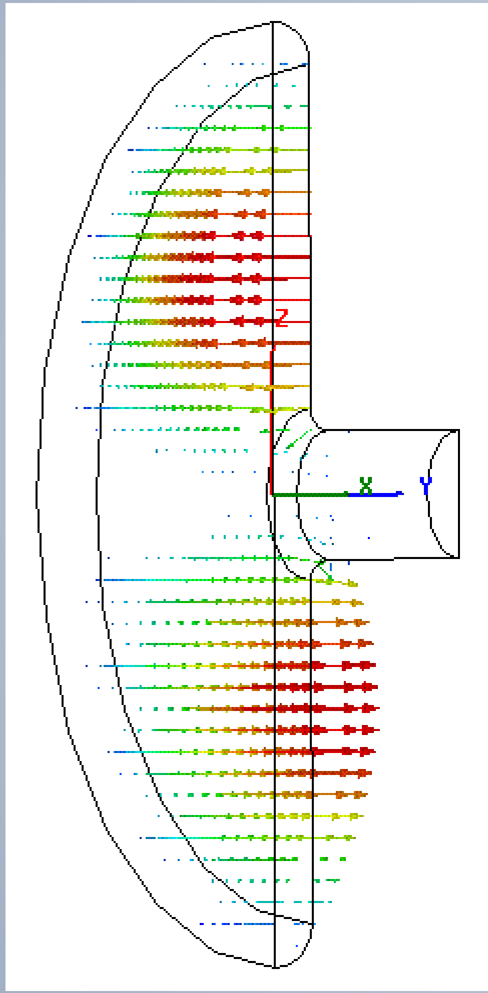
courtesy: Frank Marhauser/JLAB

Higher order modes

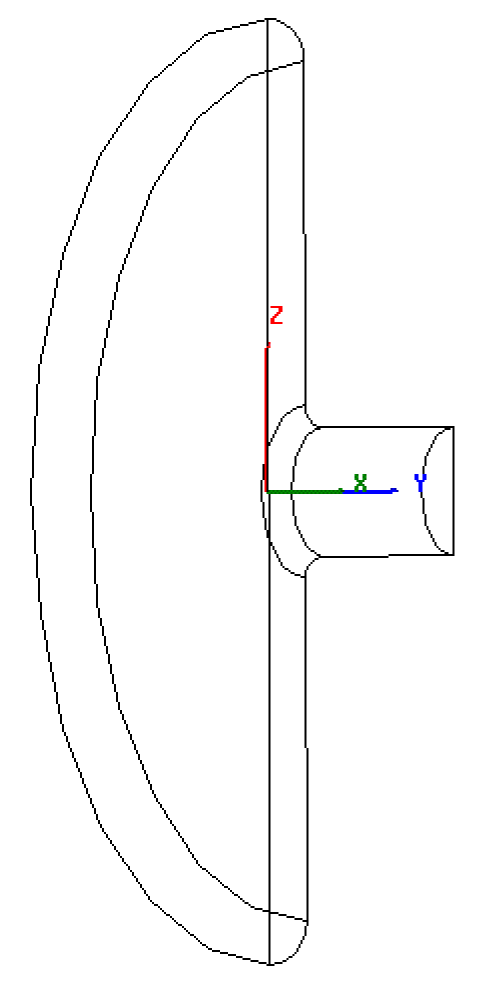


Pillbox: 1st dipole mode

TM_{110}-mode (only 1/4 shown)

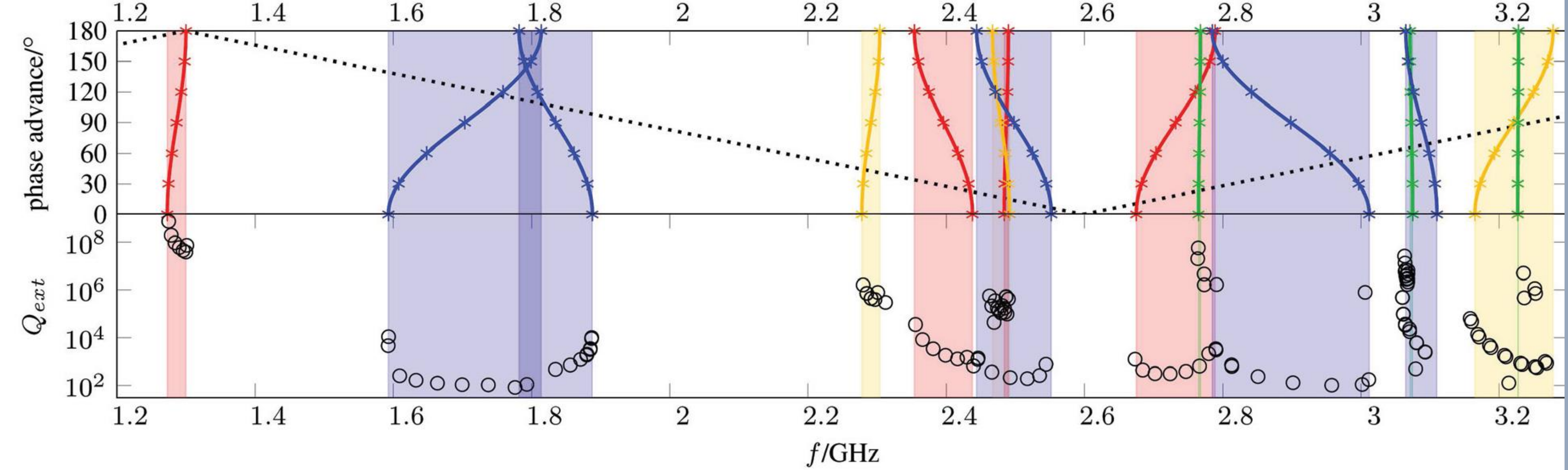


electric field



magnetic field

7-cell 1.3 GHz structure for bERLinPro

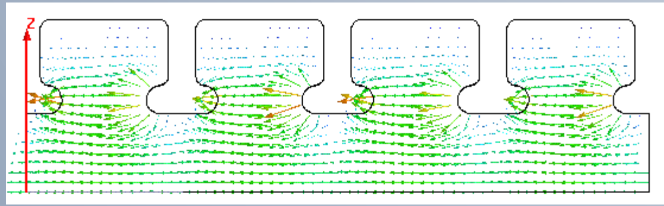


Band diagram (top) and Q-factors (bottom)

Galek et al.: IPAC2013

Reminder:

0-mode



π-mode

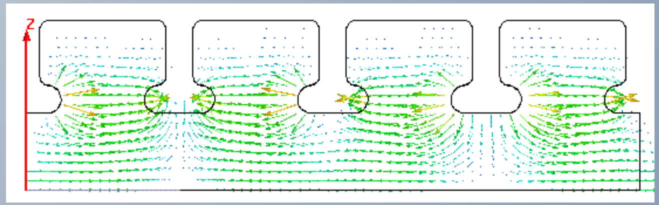
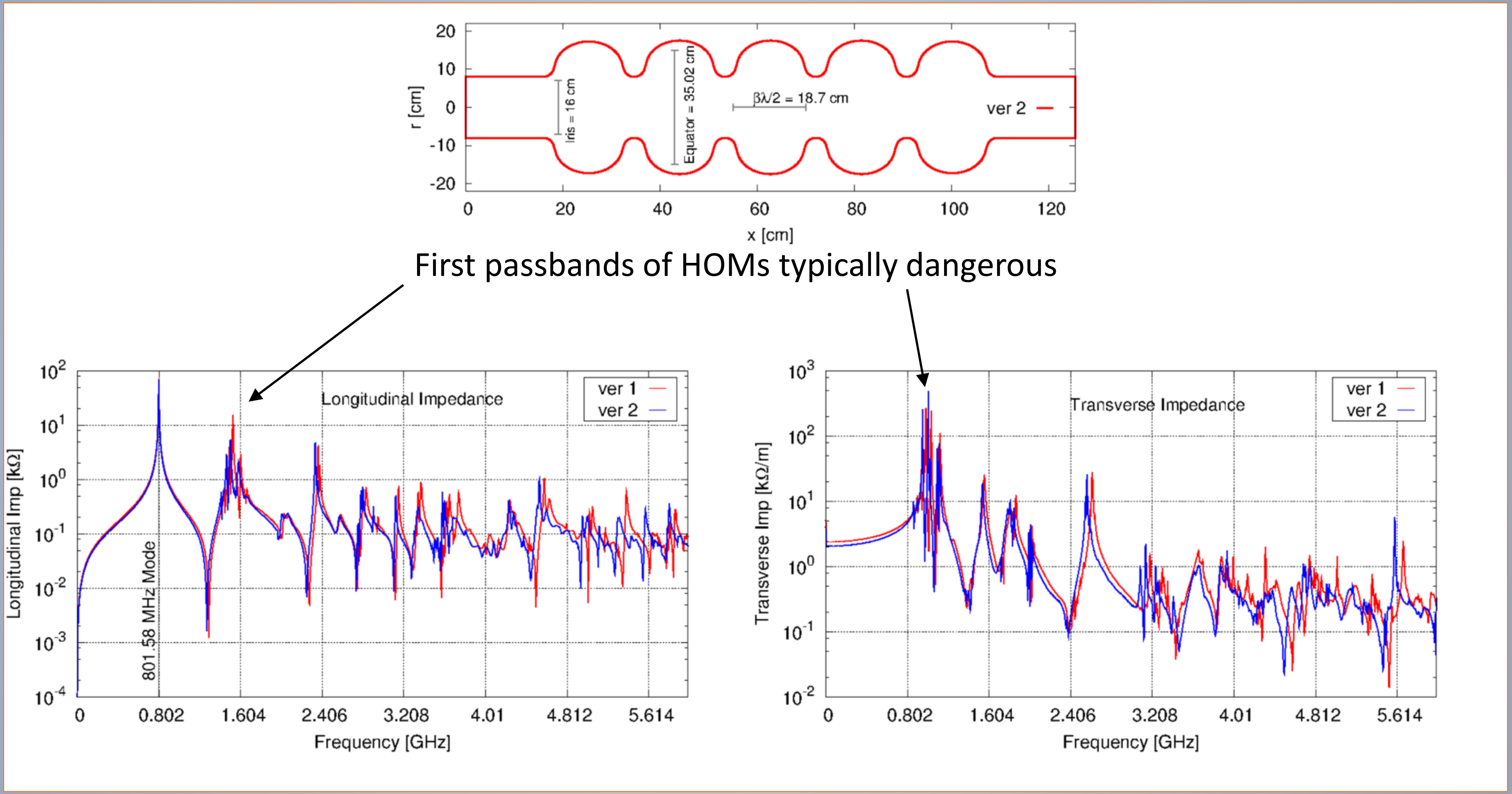


Photo:
Reidar Hahn

HOMs: Example 5-cell cavity

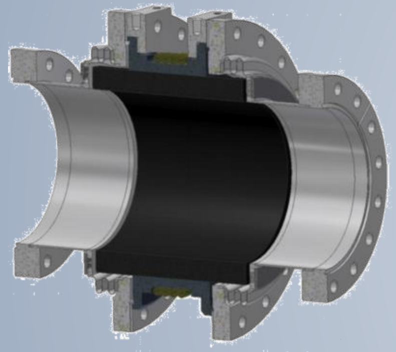


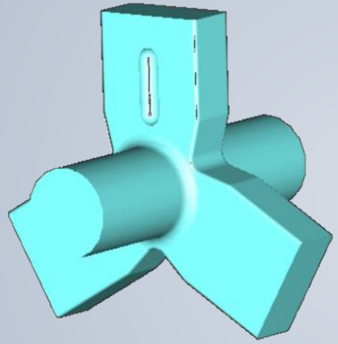
courtesy: Rama Calaga/CERN

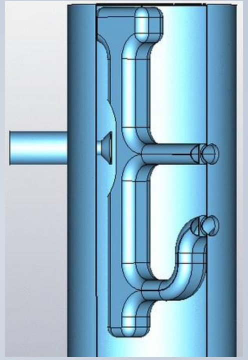

Photo: Reidar Hahn

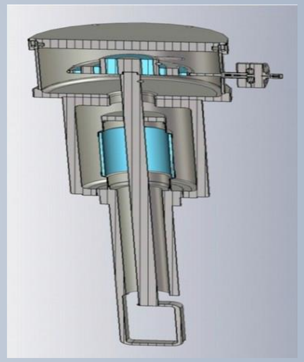
HOM dampers

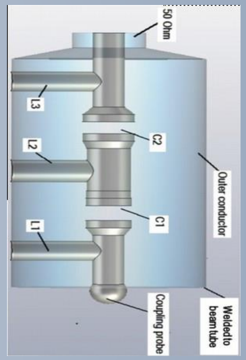
- Ferrite absorbers: broadband damper, room temperature
- Waveguides: better suited for higher frequencies (size!)
- Notch filters: narrow-band; target specific mode


ferrite absorber


waveguides


notch filter

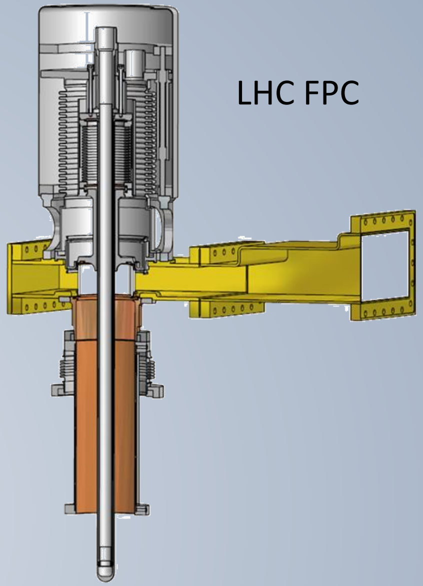

bandpass filter

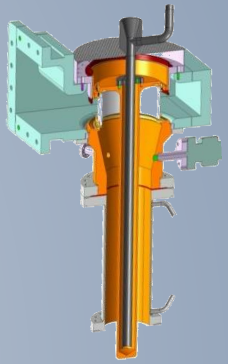

double notch

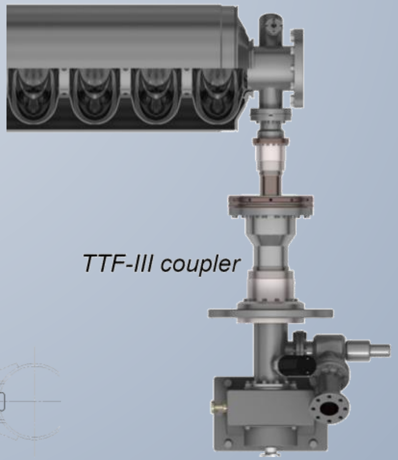
- Multi-cell cavities require broadband dampers!

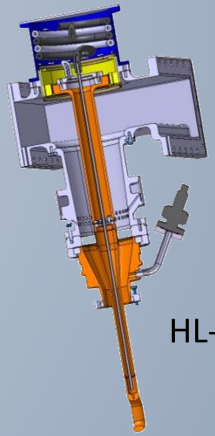
Fundamental Power Coupler – FPC

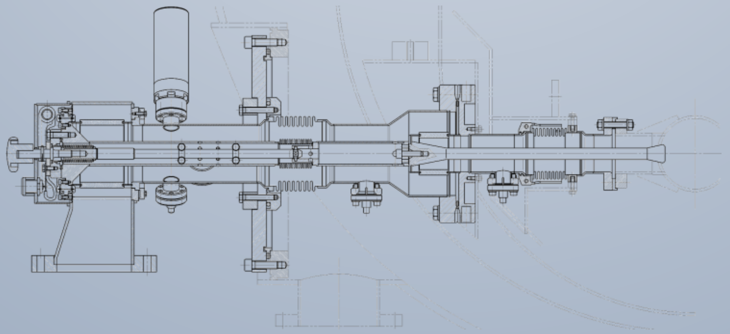
- The **Fundamental Power Coupler** is the connecting part between the RF transmission line and the RF cavity
- It is a specific piece of transmission line that also has to provide the cavity vacuum barrier.
- FPCs are amongst the most critical parts of the RF cavity system in an accelerator!
- A good RF design, a good mechanical design and a high quality fabrication are essential for an efficient and reliable operation.







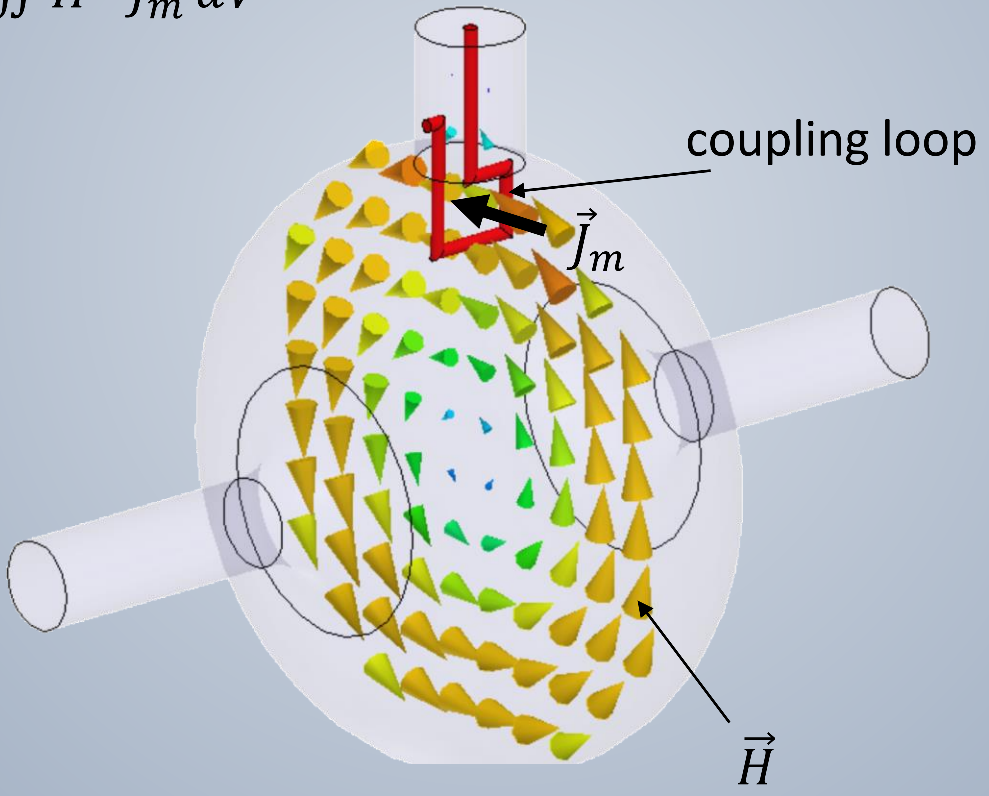




courtesy: Eric Montesinos/CERN

Magnetic (loop) coupling

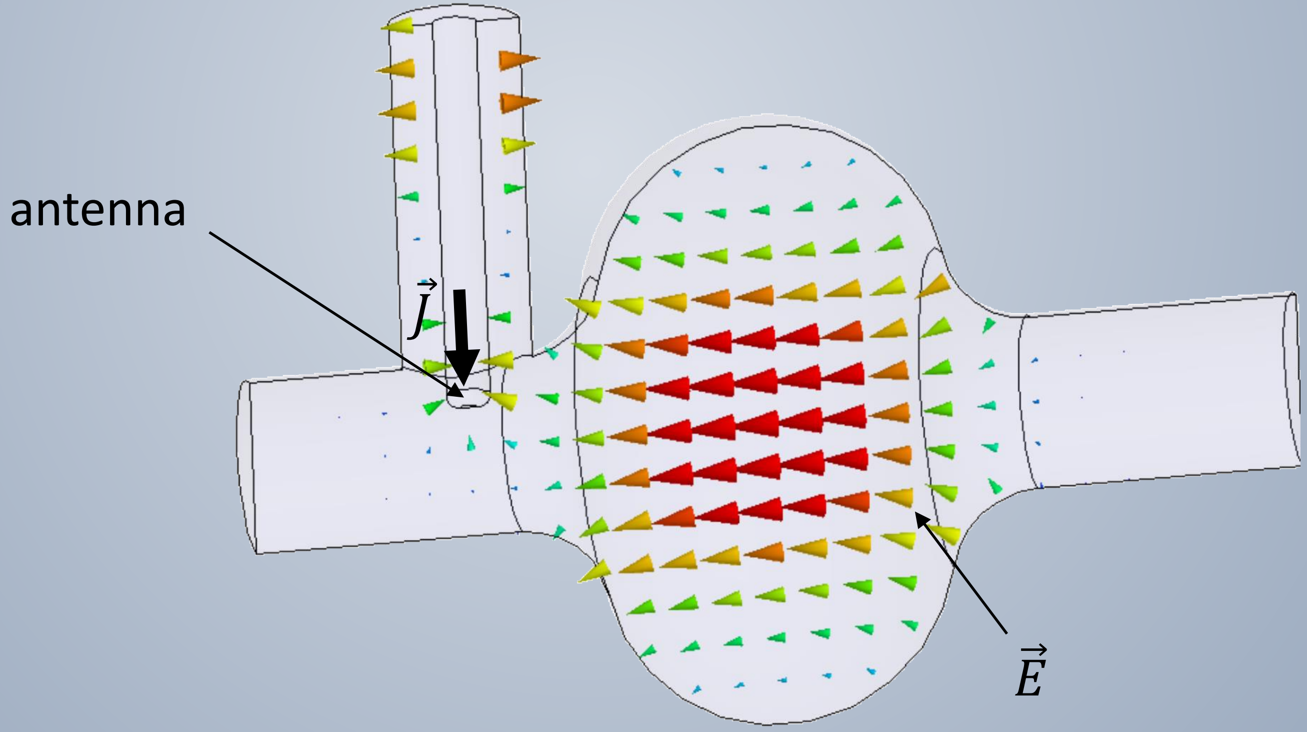
- The magnetic field of the cavity main mode is intercepted by a coupling loop
- The coupling can be adjusted by changing the size or the orientation of the loop.
- Coupling: $\propto \iiint \vec{H} \cdot \vec{J}_m \, dV$



courtesy: David Alesini/INFN

Electric (antenna) coupling

- The inner conductor of the coaxial feeder line ends in an antenna penetrating into the electric field of the cavity.
- The coupling can be adjusted by varying the penetration.
- Coupling $\propto \iiint \vec{E} \cdot \vec{J}\, dV$



courtesy: David Alesini/INFN

Non-elliptical cavities

Useful relations

Protons/H- ($A = 1, Q = 1$)

$$p = \gamma\beta m_0 c = \frac{\gamma\beta E_0}{c}$$

$$E = W + E_0 = \gamma E_0$$

$$W = (\gamma - 1)E_0$$

$$F_\xi = |e|\xi$$

$$W = |e| V_{\text{eff}} \cos\varphi$$

Heavy Ions (A, Q)

$$p = \gamma\beta A m_0 c = \frac{\gamma\beta A E_0}{c}$$

$$E = W + A E_0 = \gamma A E_0$$

$$W/A = (\gamma - 1)E_0$$

$$F_\xi = Qe\xi$$

$$W/A = (Q/A)\, V_{\text{eff}} \cos\varphi$$

Photo: Reidar Hahn

Particle velocity vs. kinetic energy

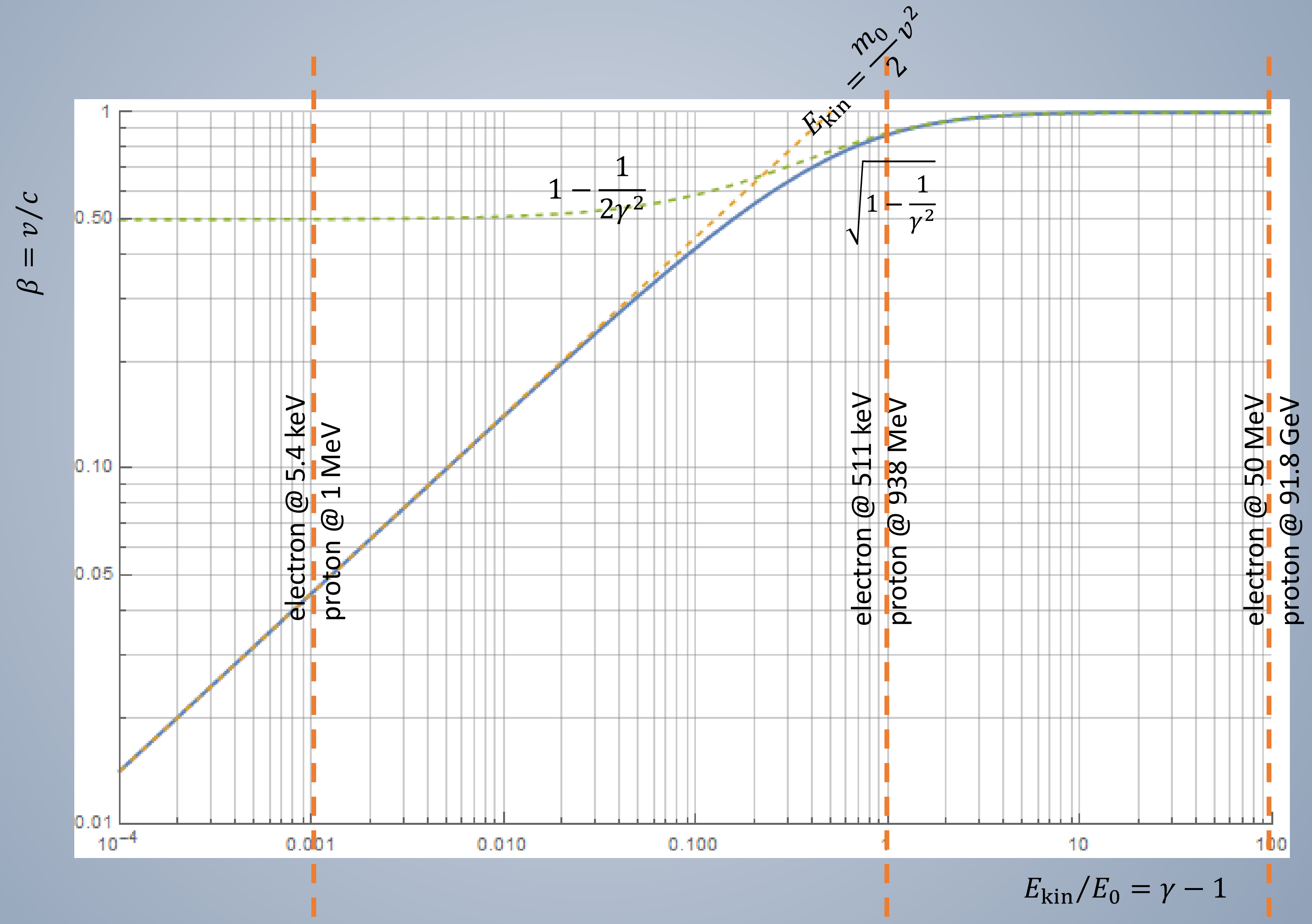



Photo: Reidar Hahn

Accelerating electrons vs. accelerating ions

Example: a 300kV DC bias is enough to get electrons going at a relativistic speed (ie E_o=511keV so γ=1.58, v/c=β=0.78) – for protons a 300kV bias only produces v/c=β=0.025 – for A=30 v/c=β=0.005

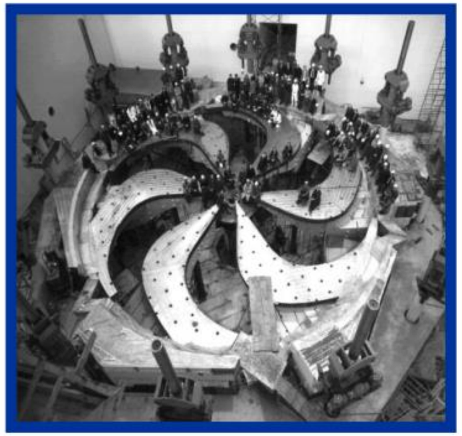
- Electron – 0.511MeV/c^2
  - 300kV - γ=1.58, v/c=β=0.78
  - 550MeV - γ=1011, v/c=β=1

8 gm

ARIEL 300kV e-gun

- Protons – 938 MeV/c^2
  - 300kV - γ=1.003, v/c=β=0.025
  - 550MeV - γ=1.58, v/c=β=0.78

160 kgm



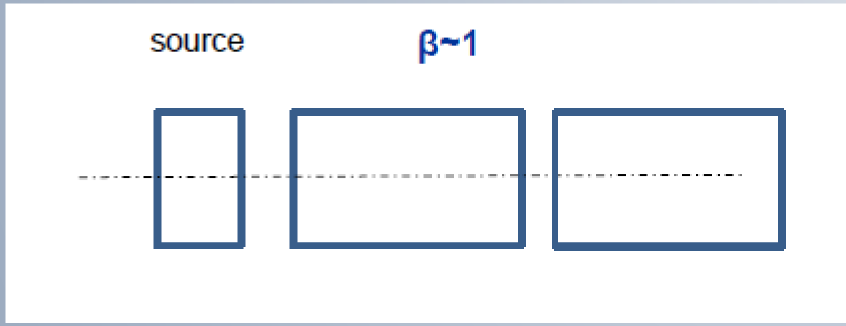
TRIUMF 500MeV cyclotron


Photo: Reidar Hahn

Accelerating electrons vs. accelerating ions

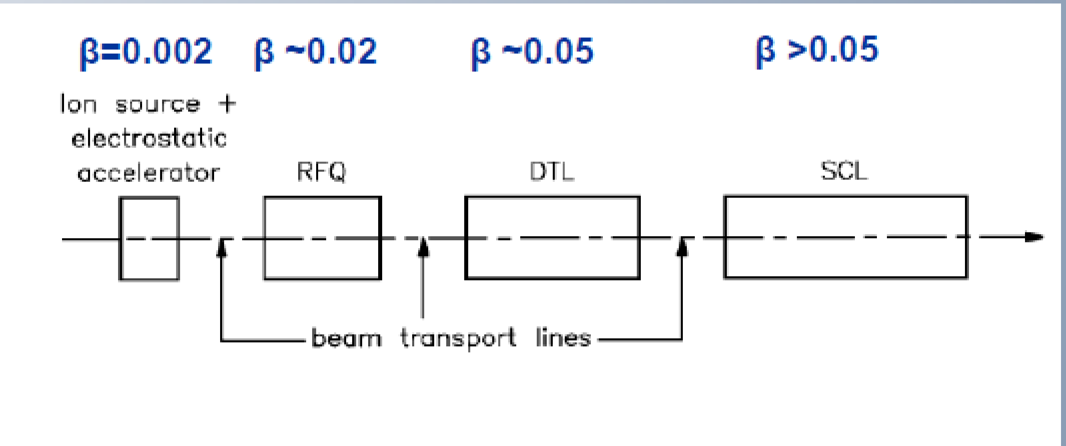
Electrons

Common building blocks – all designed for $\beta = 1$.



Ions

Various building blocks – different technologies, each optimized for a certain velocity range



Limitations of elliptical cavities

- Elliptical cavities have been designed starting at $\beta \geq 0.5$ for CW applications, for $\beta \geq 0.6$ for pulsed (SNS, ESS).
- The π-mode requires cell-to-cell distance of $\beta\lambda/2$, but outer diameter $\approx 0.9\ \lambda$, i.e. at low β the cavity looks more like bellows, sensitive to LFD!

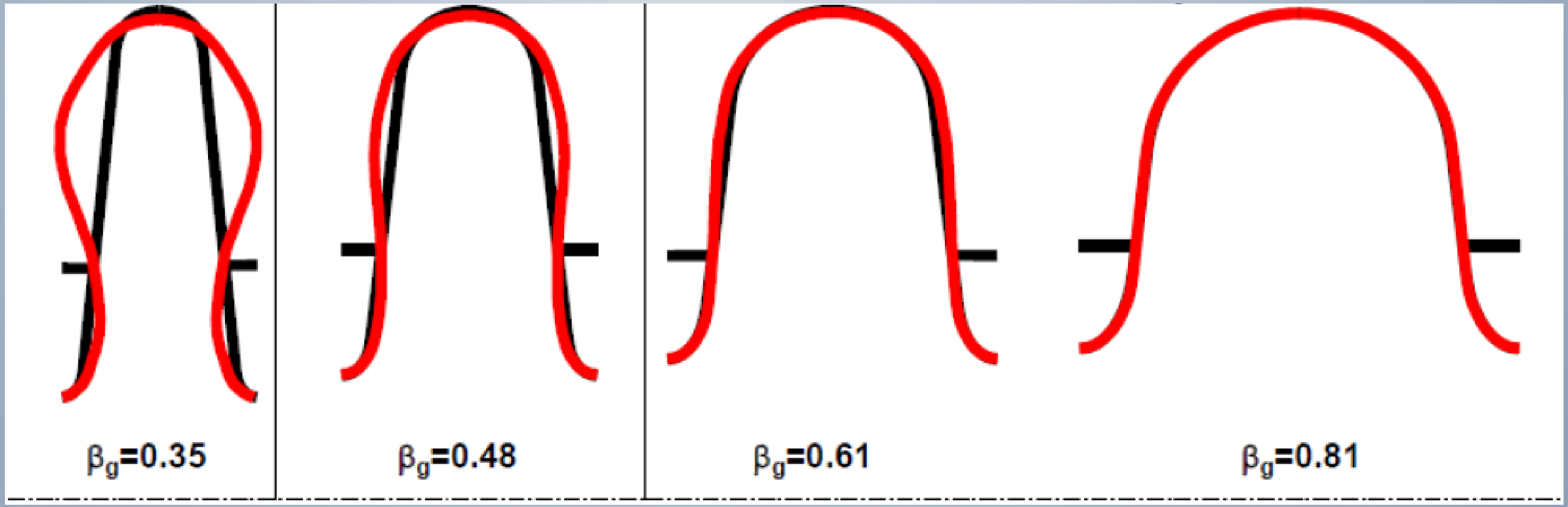
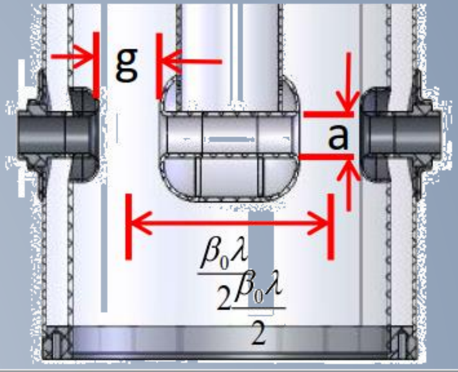
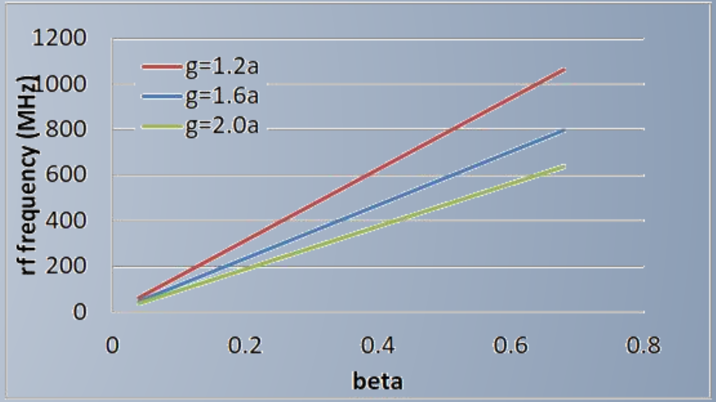


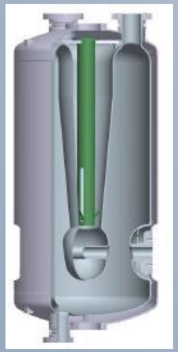

Photo: Reidar Hahn

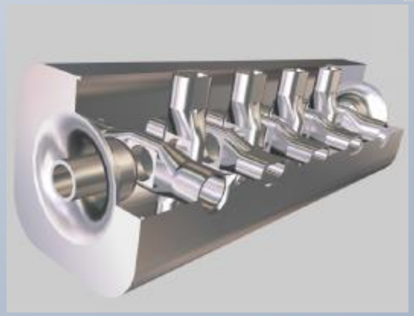
Resonator types for low beta acceleration

- Quarter wave resonator (QWR) $\beta \approx 0.04 \;\ldots 0.2$
- Half wave resonator (HWR) $\beta \approx 0.1 \;\ldots 0.5$
- Single spoke resonator (SSR) $\beta \approx 0.15 \;\ldots 0.7$
- Multi-spoke resonator (MSR) $\beta \approx 0.06 \;\ldots 1$
- For comparison: Elliptical cavities $\beta \approx 0.5 \;\ldots 1$



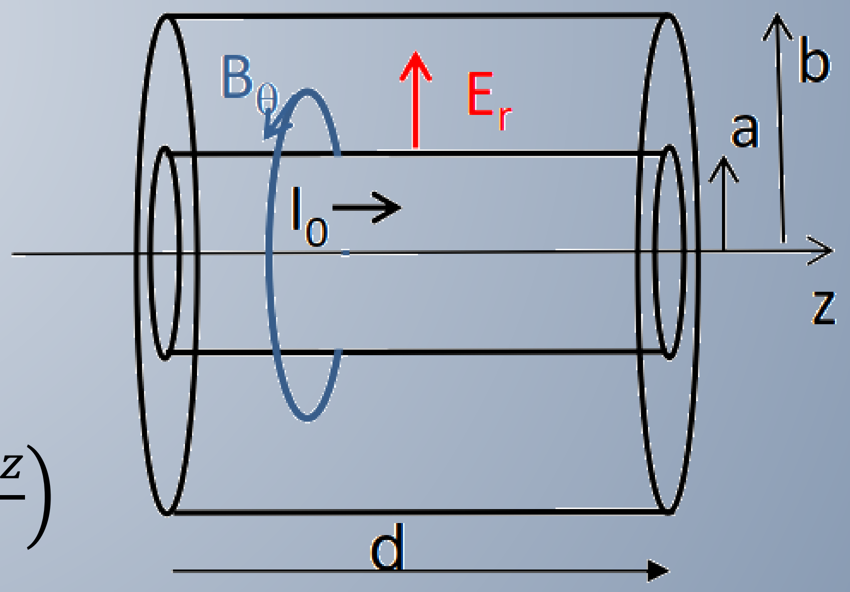






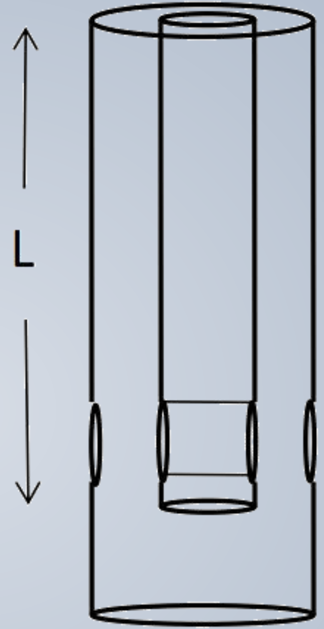
Coaxial resonator

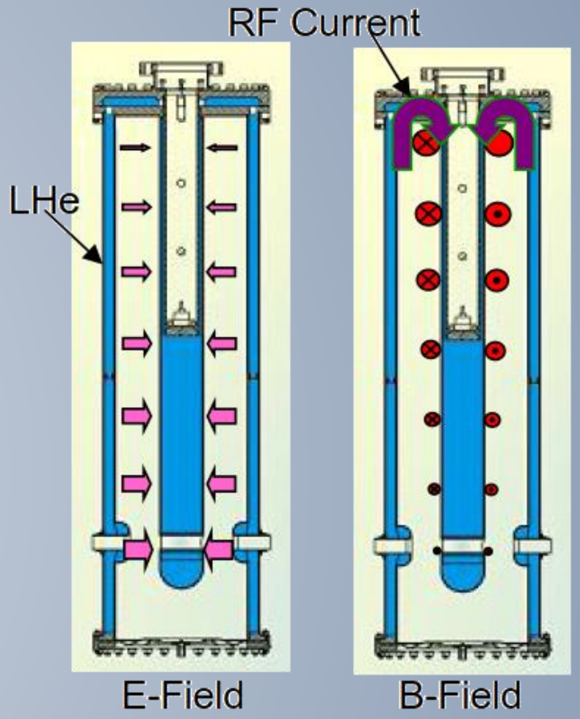
- Consider a coaxial geometry with grounded end plates, an inner conductor with radius a and an outer conductor with radius b.
- A standing wave occurs with E_r vanishing on the end walls at $z = 0$ and $z = d$.
- The remaining non-zero field components are
  $B_\theta = \frac{\mu_0 I_0}{\pi r}\cos\left(\frac{p\pi z}{d}\right)$,
  $E_r = -j2\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{I_0}{2\pi r}\sin\left(\frac{p\pi z}{d}\right)$,
  where $\omega = \frac{p\pi c}{d}$, $p = 1,2,3, \ldots$
- Peak voltage:
  $\hat{V}(z) = \int_a^b E_r(z)\,dr = \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{I_0}{\pi}\ln\frac{b}{a}\sin\left(\frac{p\pi z}{d}\right)$



Quarter-wave resonator (QWR)

- The most popular coaxial TEM mode cavity is the quarter wave resonator – capacitively loaded $\lambda/4$ transmission line
- The inner conductor is open at one end with a resonant length of $(1+2p)\,\lambda/4,\ p=0,1,2,\ldots$
- For acceleration, $p=0$ is chosen.
- The maximum voltage builds up on the open tip – the maximum current at the root.
- A beam tube is arranged near the end of the tip.





Half-wave resonator (HWR)

- In the HWR the beam port is at the centre of the inner conductor of a coaxial resonator, coincident with the maximum voltage for $p = 1$.
- Magnetic fields loop around the inner conductor with peak fields at the shorted ends.
- For acceleration, $p = 1$ is chosen.

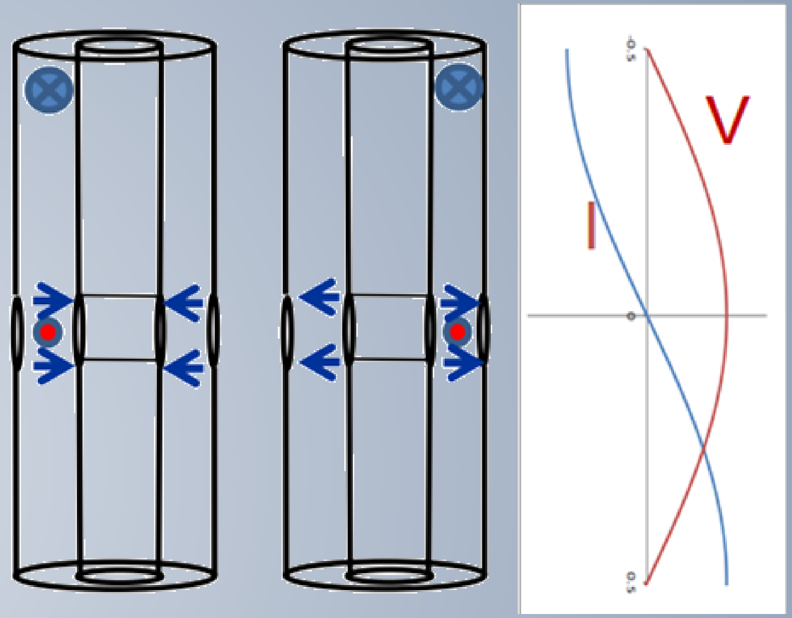
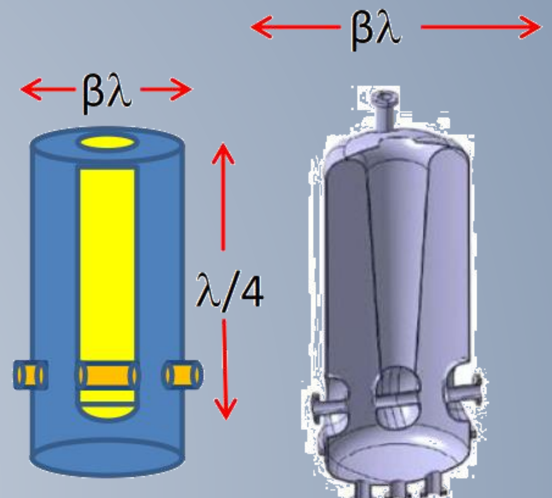


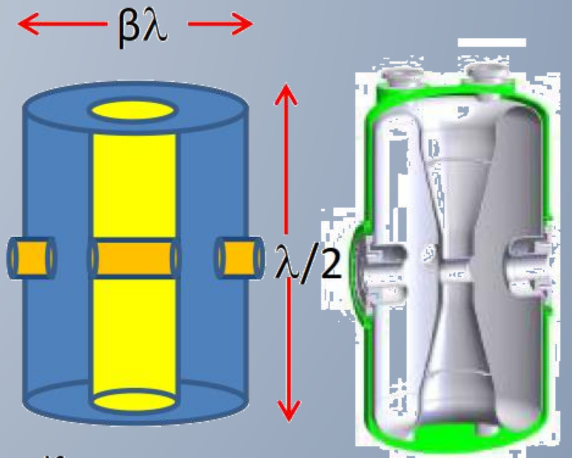
Photo:
Reidar Hahn

QWR vs. HWR

- QWR is the cavity of choice for low beta applications where a low frequency is needed
  - requires ~50% less structure compared to HWR for the same frequency – rf power loss is ~50% of HWR for same frequency and β_0.
  - allows low frequency choice giving larger longitudinal acceptance.
  - R/Q twice that of HWR.
  - Asymmetric field pattern introduces vertical steering especially for light ions that increases with velocity – avoid use for $\beta_0 > 0.2$.
  - Less mechanically stable than HWR due to unsupported end (microphonics).
- HWR is chosen in mid velocity range ($\beta_0 > 0.2$) or where steering must be eliminated (i.e. high intensity light ion applications)
  - produces twice rf losses for the same β_0 and λ.
  - is 2x longer for the same frequency.
  - Pluses are the symmetric field pattern and increased mechanical rigidity.



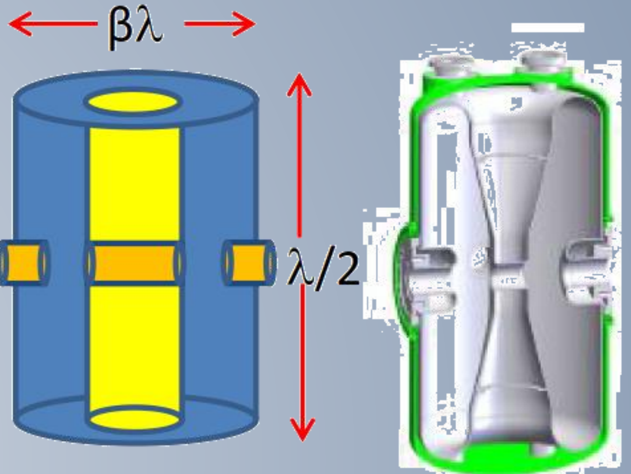
Quarter Wave - QWR



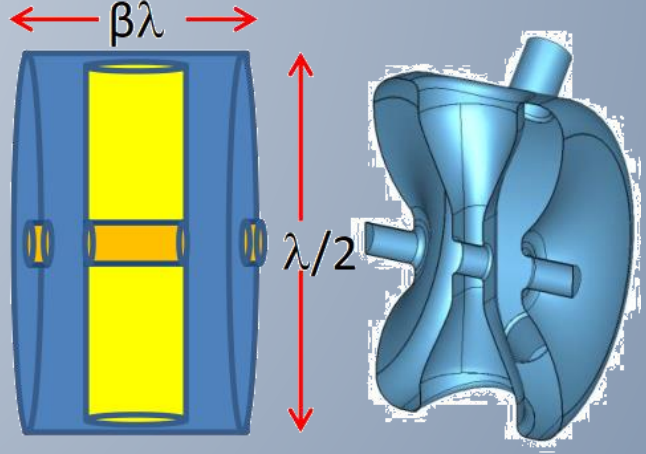
Half Wave - HWR

HWR vs. Single Spoke Resonator (SSR)

- A single spoke resonator (SSR) is another variant of the half-wave TEM mode cavity class.
- In HWR the outer conductor is coaxial with the inner conductor (with diameter $\beta_0\lambda$) while in the spoke cavities the outer cylinder is co-axial with the beam tube with diameter $\lambda/2$. It means that for $\beta_0 < 0.5$ the SSR has a larger overall physical envelop than the HWR for the same frequency.
- Thus for low beta applications $(0.1 < \beta < 0.25)$ HWRs are chosen at $\approx 160\ \text{MHz}$, while SSRs are preferred at $\approx 320\ \text{MHz}$.
- The spoke geometry allows an extension along the beam path to provide multiple spokes in a single resonator giving higher effective voltage, but with a narrower transit time acceptance.

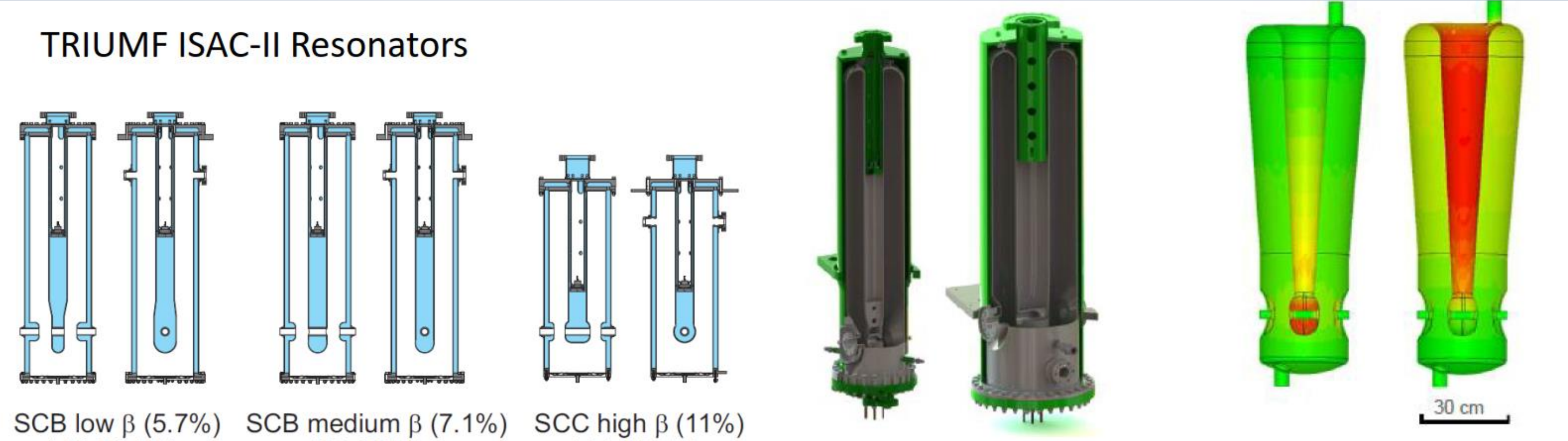


Half Wave - HWR

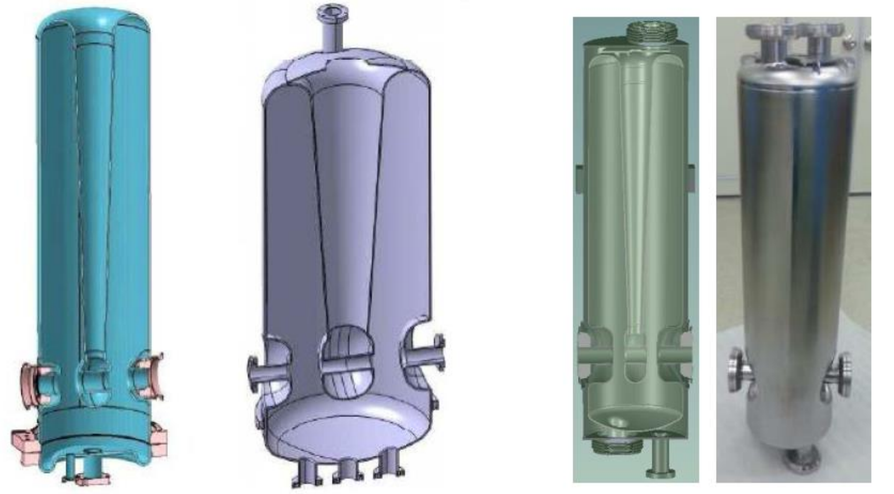


Single spoke - SSR

Cavity types – QWRs



SCB low β (5.7%) 106.08 MHz — SCB medium β (7.1%) 106.08 MHz — SCC high β (11%) 141.44 MHz — FRIB β=0.041, 0.085 80.5MHz — ANL β=0.077, 0.085 72.5MHz

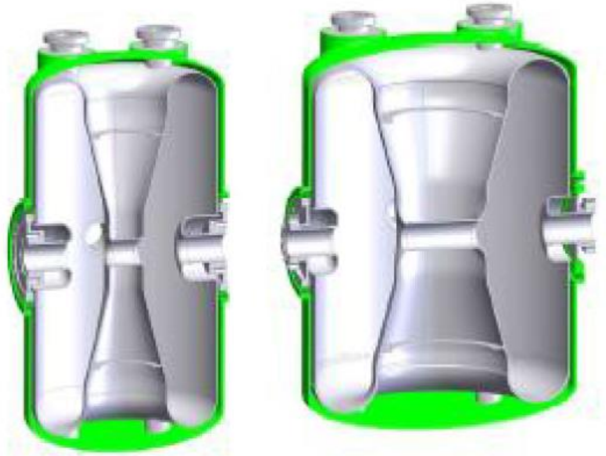


Spiral-2 β=0.007, 0.12 88.05MHz — RAON β=0.047, 81.25 MHz

Typical range:
$0.04 < \beta < 0.2$
$50\ \text{MHz} \leq f \leq 160\ \text{MHz}$

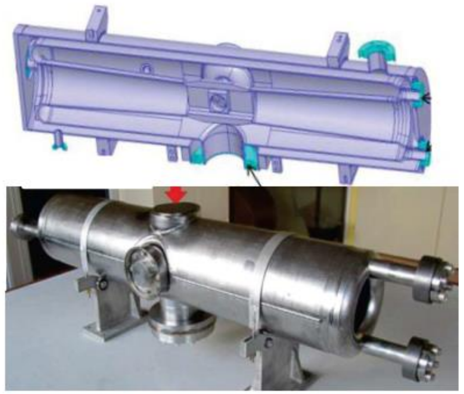
Cavity types – HWRs

IMP β=0.10, f=162.5MHz

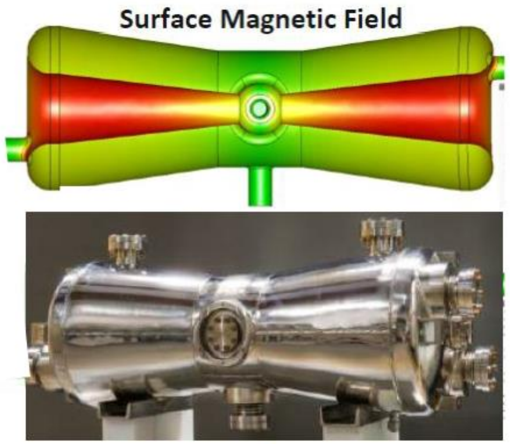


FRIB β=0.29, 0.53 f=322MHz

FRIB β=0.29, 0.53 f=322MHz



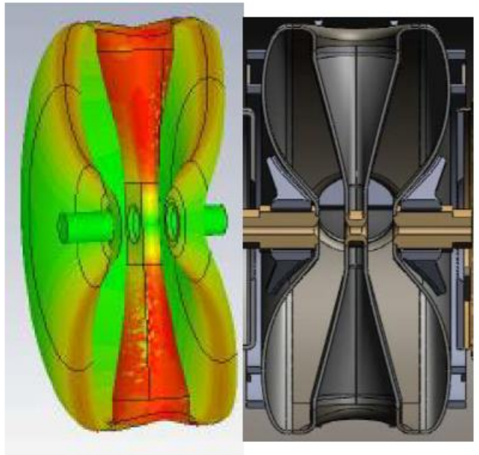
IFMIF β=0.11, f=175MHz



ANL β=0.112, f=162.5MHz

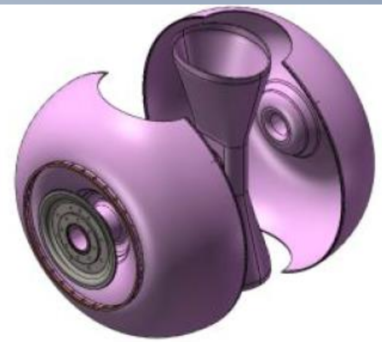
Typical range:
$0.1 < \beta < 0.5$
$140\text{ MHz} \leq f \leq 325\text{ MHz}$

Cavity types – SSRs

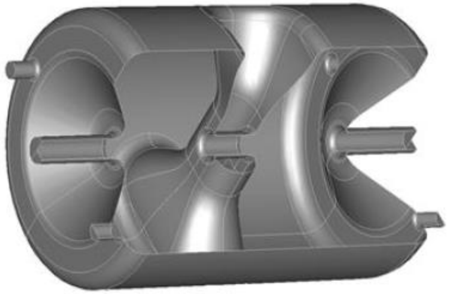


IHEP β=0.12, f=325MHz

FNAL β=0.215, f=325MHz



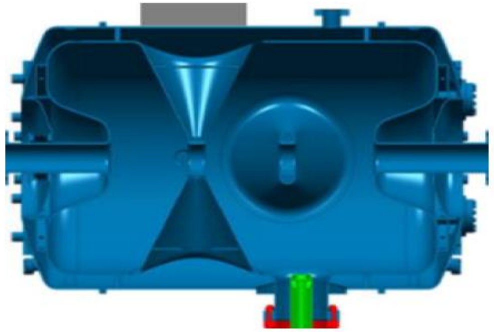
TRIUMF/RISP β=0.3, f=325MHz



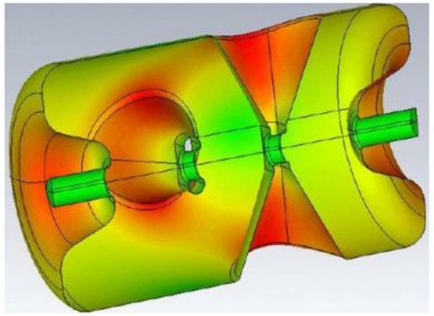
325 MHz, $\beta_0 = 0.82$
Single-Spoke Cavity

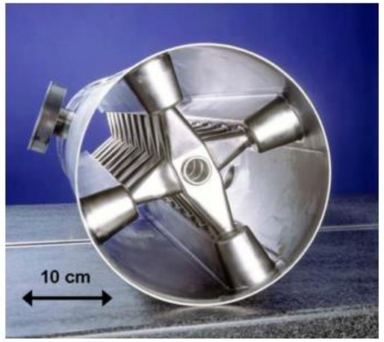
Typical range:
$0.15 < \beta < 0.7$
$320 \text{ MHz} \leq f \leq 700 \text{ MHz}$

Cavity types – multi-cell

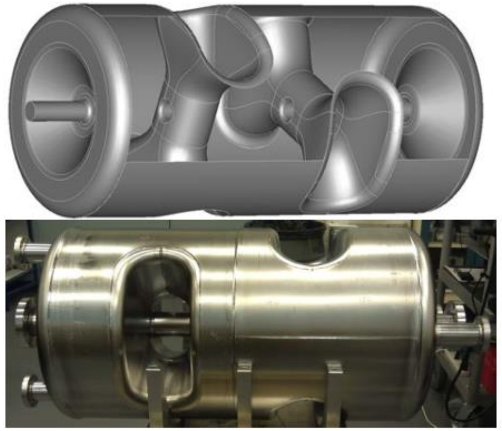


ESS/IPN β=0.50, f=352MHz

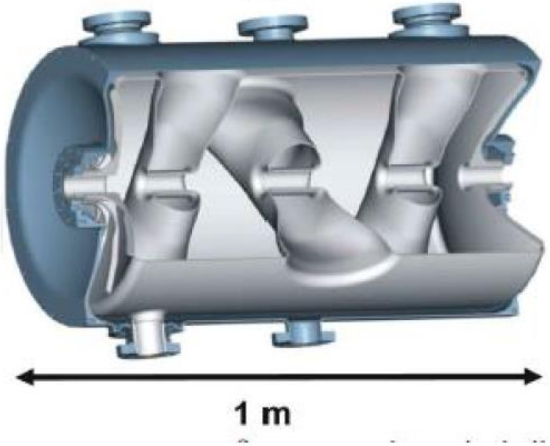




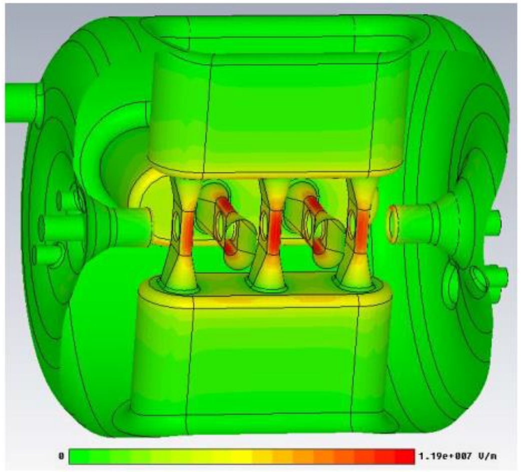
IAP 360 MHz, β_0~0.1
19 gap CH resonator



500 MHz, $\beta_0 = 1$
Double-Spoke Cavity



ANL β=0.63, f=345MHz



IMP CH β=0.067, f=162.5MHz

CH: Crossbar H-mode

Accelerating cavity velocity/frequency chart

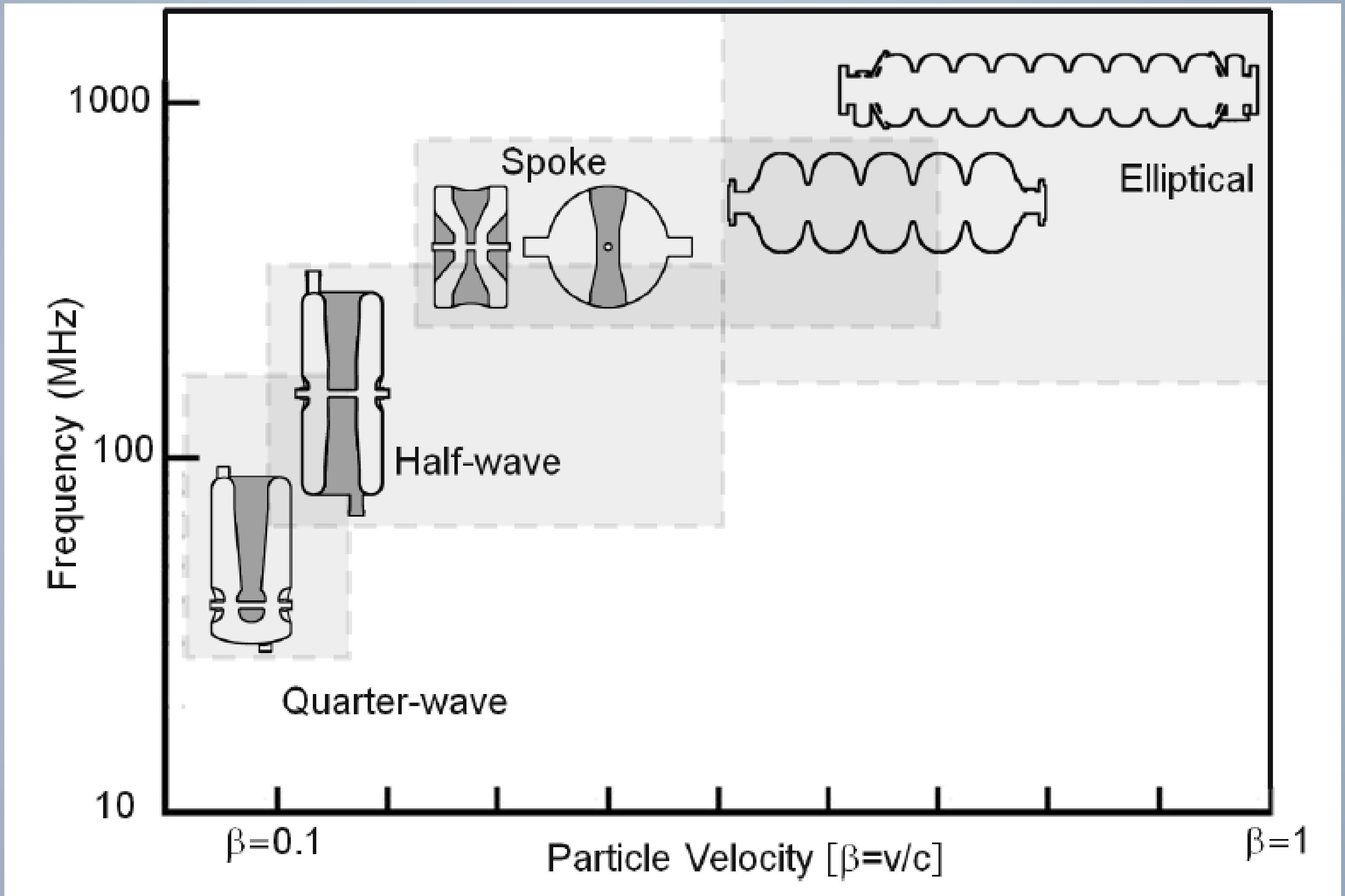
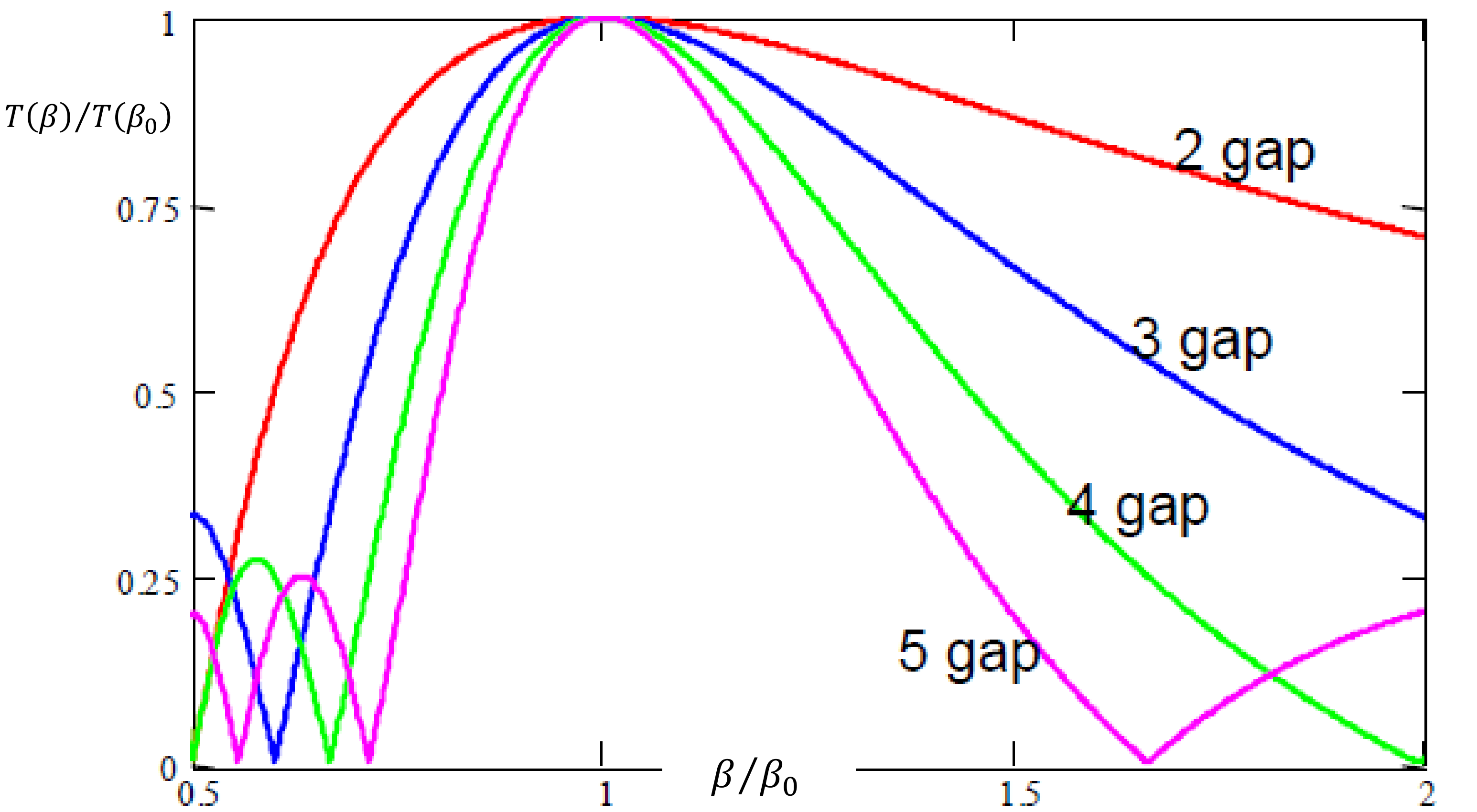



Photo: Reidar Hahn

Transit Time factor vs. β for multiple gaps



Normalized transit time factor curves vs. normalized velocity, for cavities with different number of gap

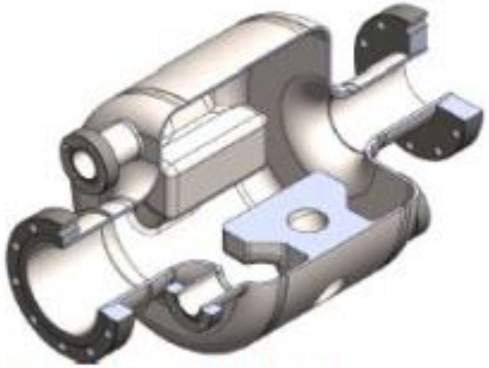
High-β spoke cavities

- High velocity spoke cavities with $\beta > 0.8$ are being designed as alternative to elliptical cavities
- Features:
  - relatively compact
    - between 20% and 50% smaller (radially) for low-β cavities
    - for high β diameter close to TM counterparts
  - allows low frequency at reasonable size
  - mechanically stable – high shunt impedance

325 MHz β=0.82 Single Spoke Cavity

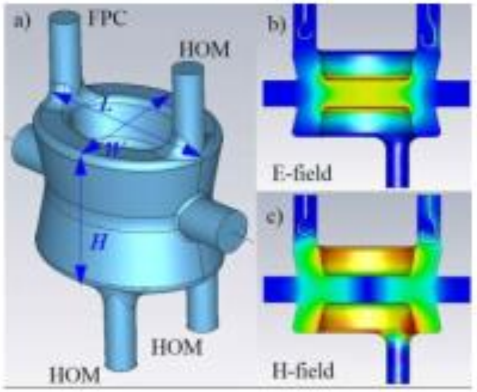
500 MHz β=1.0 Double Spoke Cavity

Deflecting mode cavities

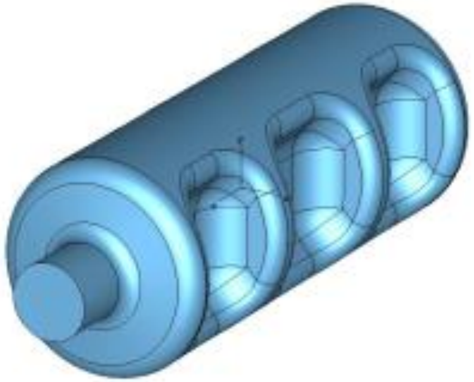


TRIUMF 650MHz

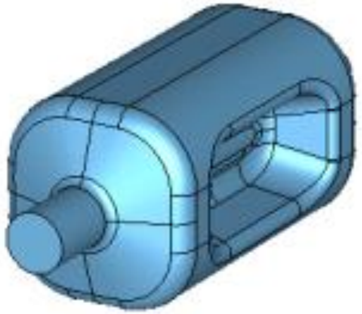
double quarter wave (DQW) – 400MHz – BNL/CERN



RFD – multi-cell – 953MHz – ODU

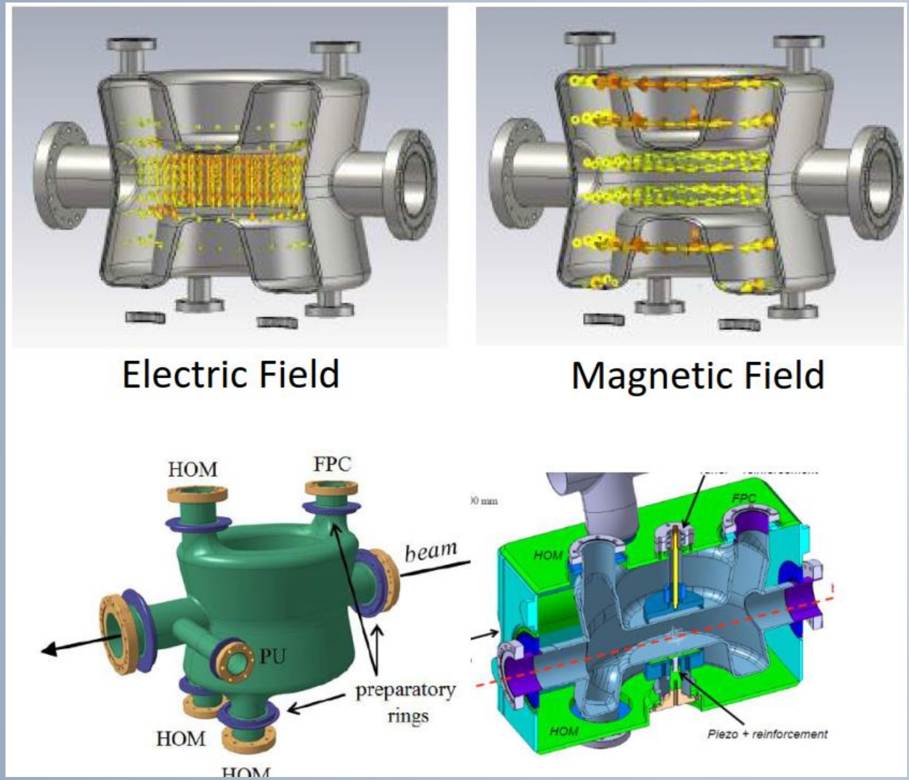


RF Dipole (RFD) – 400MHz – ODU/CERN



Prototype HL-LHC Crab Cavity prototypes

“DQW” (vertical deflection)



“RFD” (vertical deflection)

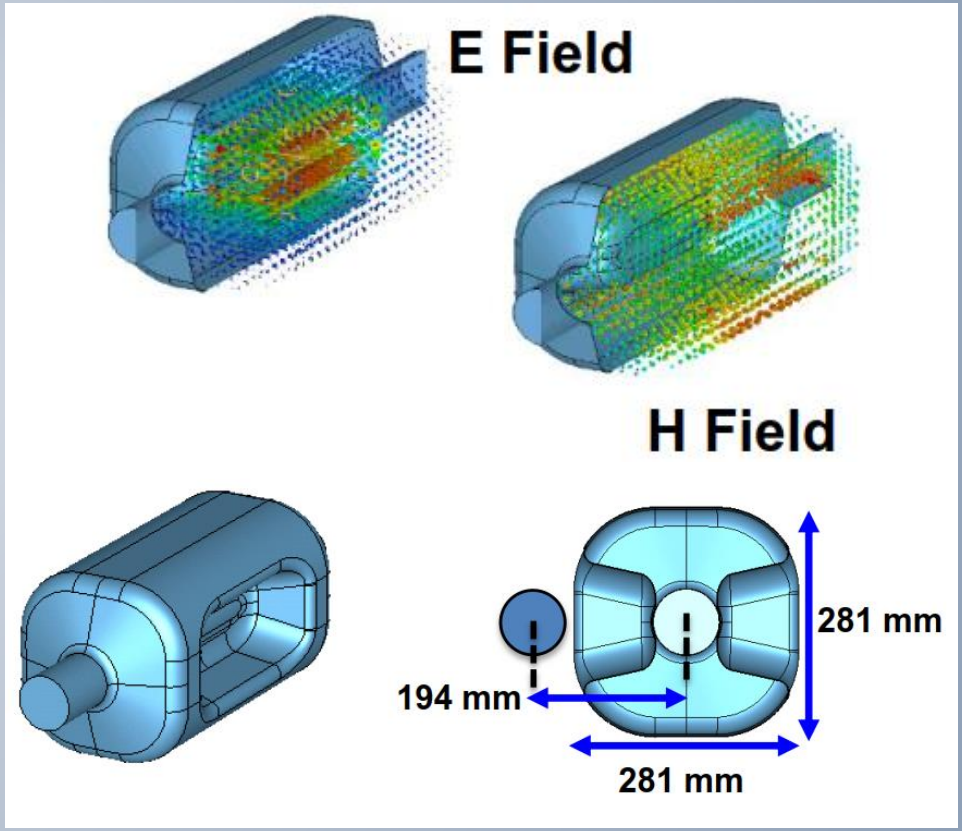
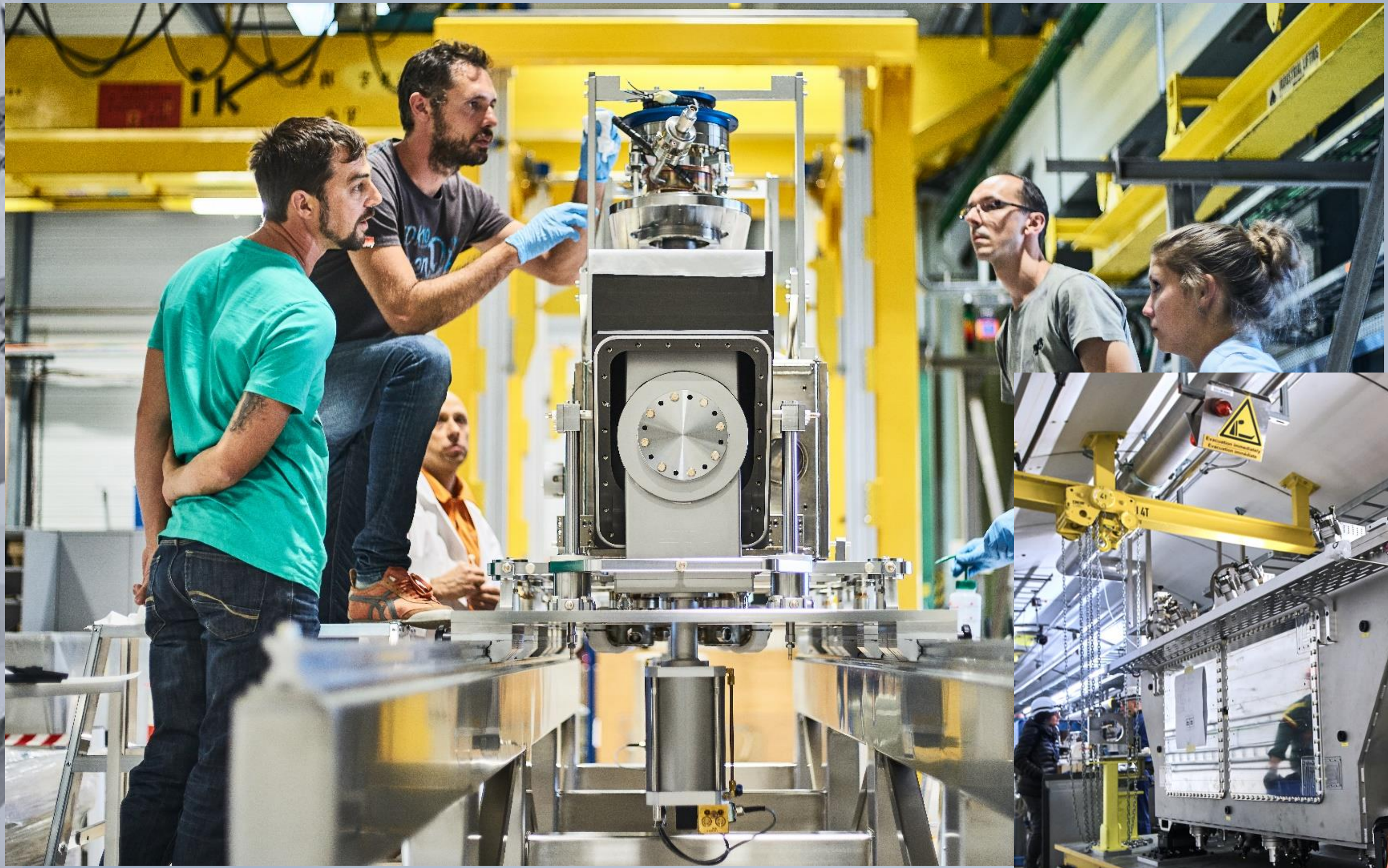



Photo:
Reidar Hahn

The real thing (HL-LHC Crab Cavity)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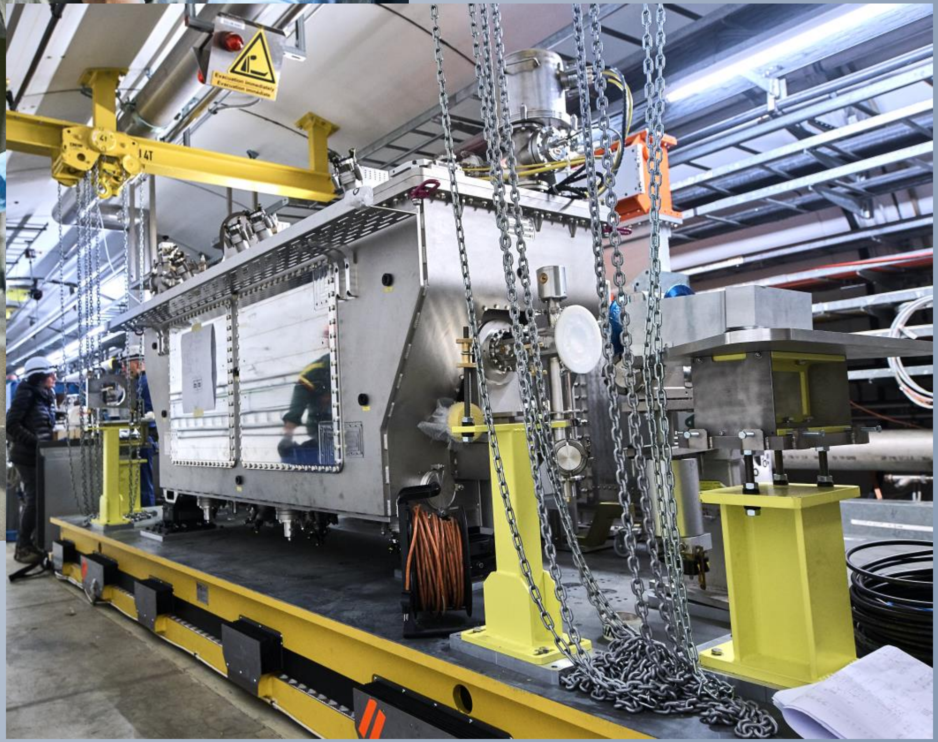
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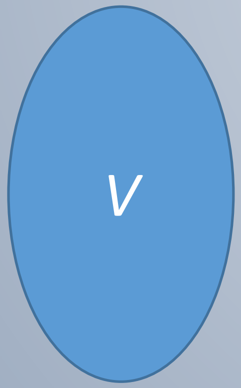
Tuners

Small boundary perturbation

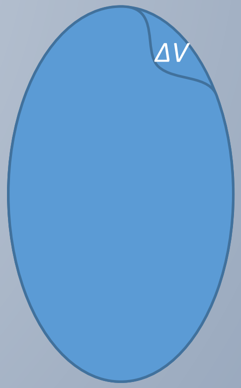
- Perturbation calculation is used to understand the basics for cavity tuning – it is used to analyse the sensitivity to (small) surface geometry perturbations.
  - This is relevant to understand the effect of fabrication tolerances.
  - Intentional surface deformation or introduced obstacles can be used to tune the cavity.
- The basic idea of the perturbation theory is use a known solution (in this case the unperturbed cavity) and assume that the deviation from it is only small. We just used this to calculate the losses (assuming H_t would be that without losses).
- The result of this calculation leads to a convenient expression for the (de)tuning:



unperturbed: ω_0

$$\frac{\omega - \omega_0}{\omega} = \frac{\iiint_{\Delta V} \left(\mu_0 |H_0|^2 - \varepsilon |E_0|^2\right) dV}{\iiint_V \left(\mu_0 |H_0|^2 + \varepsilon |E_0|^2\right) dV}$$

Slater's Theorem

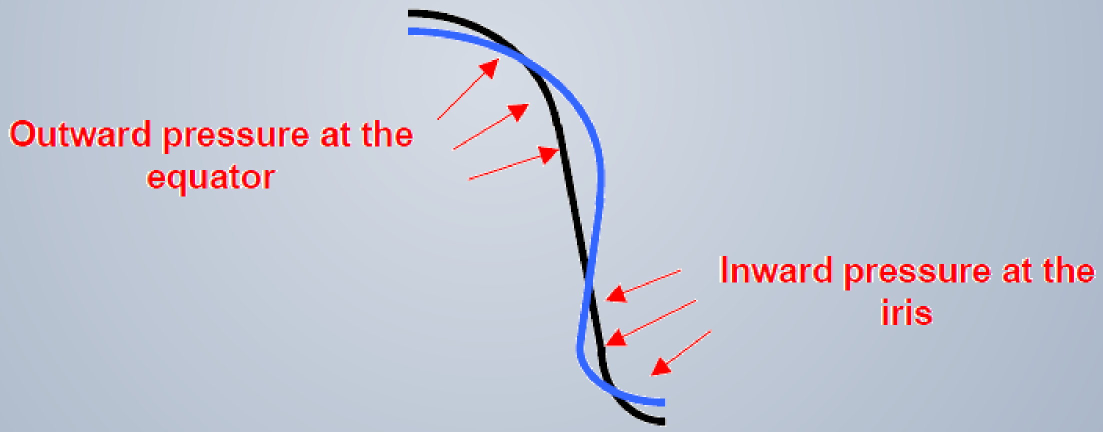


perturbed: ω

John C. Slater
1900 – 1976

Lorentz force detuning (“*LFD*”)

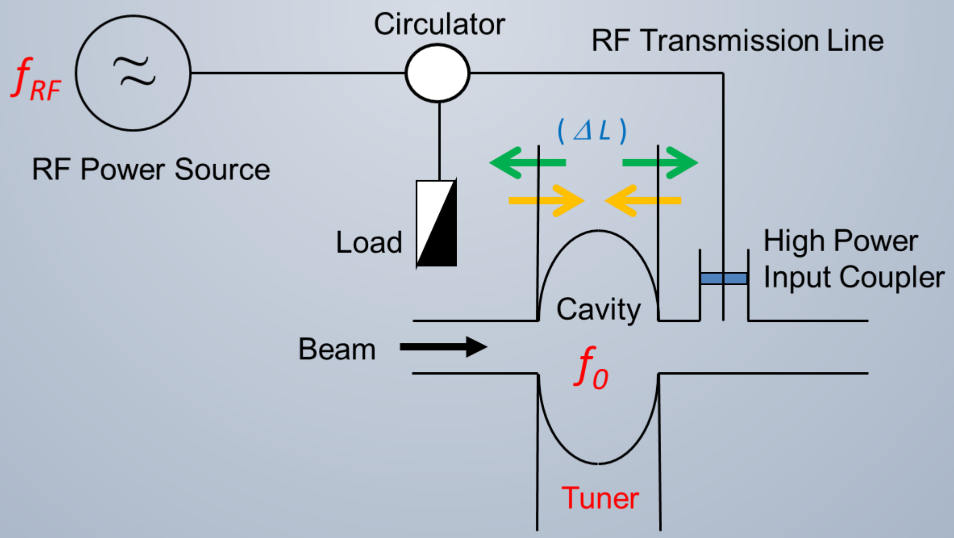
- The presence of electromagnetic fields inside the cavity lead to a mechanical pressure on the cavity.
- Radiation pressure: $P = \frac{1}{4}(\mu_0|H|^2 - \varepsilon_0|E|^2)$
- Deformation of the cavity shape:



- Frequency shift: $\Delta f = K\ L\ |E_{acc}|^2$; typical: $K\ L \approx -(1 \dots 10)\mathrm{Hz}/\left(\frac{\mathrm{MV}}{\mathrm{m}}\right)^2$
- This requires good stiffness – and the possibility to tune rapidly!

Tuner principle

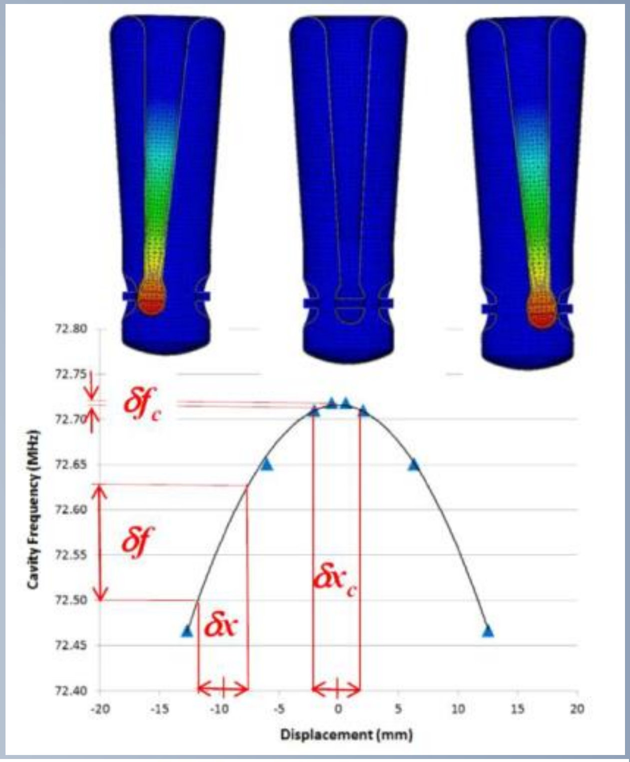
- Slow tuners:
  - compensate for mechanical tolerances,
  - realized with stepper motor drives
- Fast tuners:
  - compensate Lorentz-force detuning and reactive beam loading
  - realized with piezo crystal (lead zirconate titanate – PZT)
- Tuning of SC cavities is often realized by deforming the cavity:

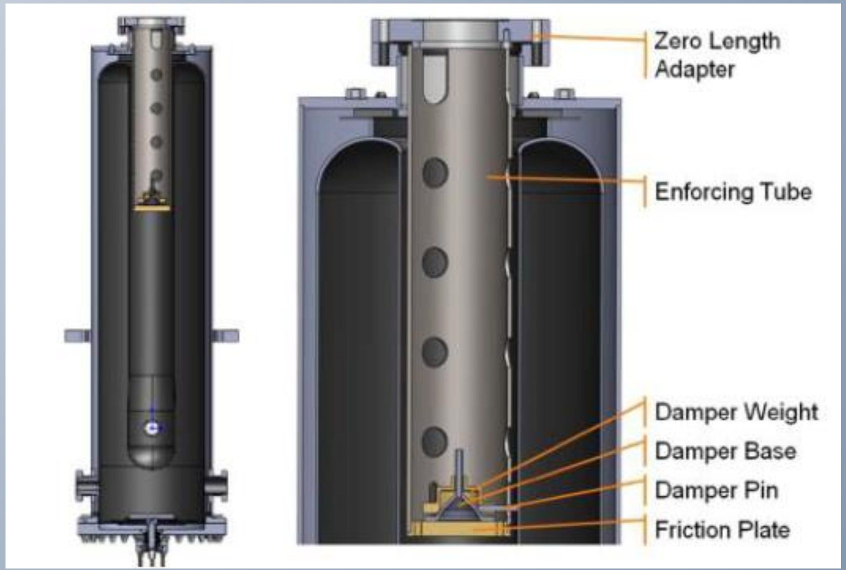


courtesy: Eiji Kako/KEK

Microphonics

- Driven by mechanical vibration in the environment.
- QWRs are particularly problematic due to the pendulum action of the inner conductor, which can have very low mechanical frequencies $((50 \ldots 100)\ \mathrm{Hz})$
  - need to reduce the RMS detuning to $\ll 10\%$ of the available BW to avoid nuisance
  - the other option is to increase the BW (lower Q_L, costs power)
- Mitigation:
  - stiffening during design/manufacture
  - centering the inner conductor by plastic deformation so that $df/dx = 0$.
  - adding passibe dampers
  - reduce environmental noise





Frequency compensation/tuning

- For QWRs a tuning plate at the open end near the beam tube is generally used.
- For QWRs with removable tuning plate, a Nb puck can be welded to it – this reduces the cavity f_0 by increasing the equivalent C.
- This puck can be trimmed after final fabrication