Linear Collider Beam Dynamics I, II and III

D. Schulte

Introduction



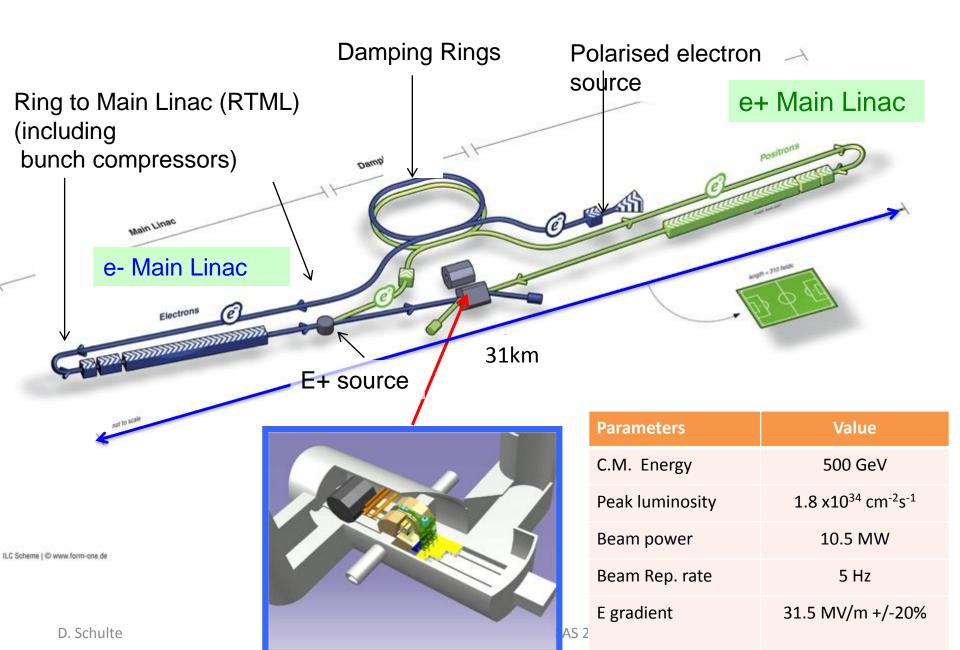
Stepping Stones

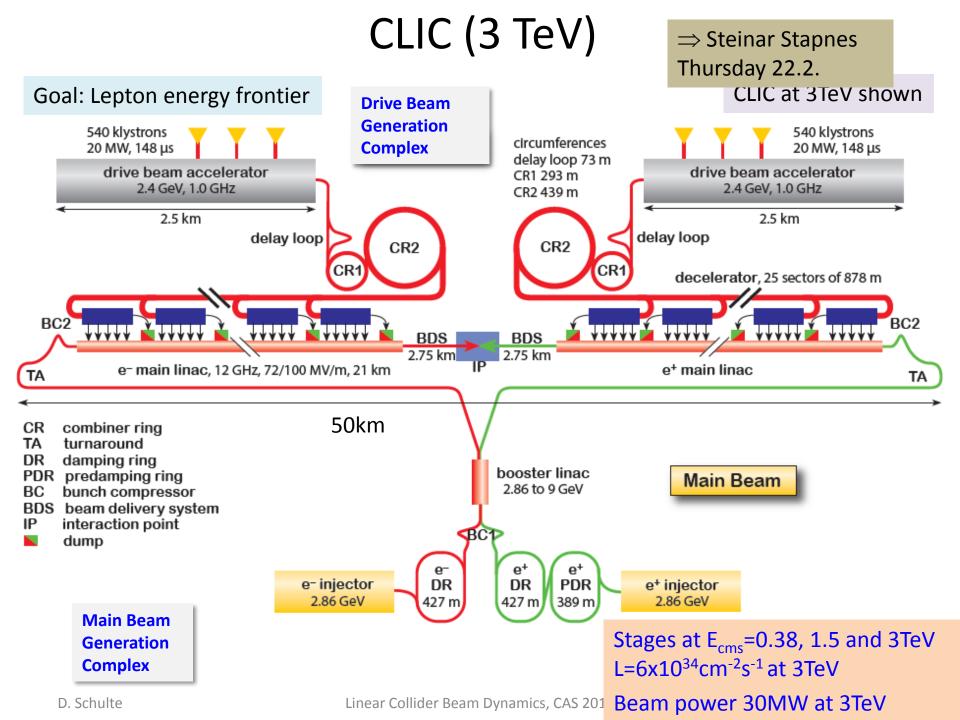
- Beam parameters and main systems
 - Luminosity
 - Main linac and efficiency
- Main linac and beam parameter choice
 - Single bunch energy spread
 - Single bunch beam break-up
 - Multi-bunch beam break-up
- Emittance Preservation
 - Static imperfections
 - Dynamic imperfections

Overall Design and Parameters

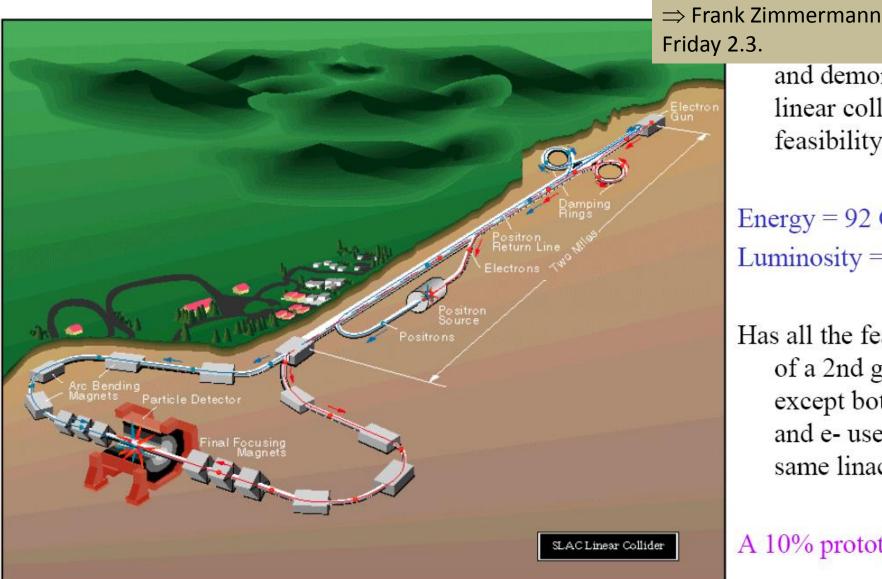


ILC





SLC: The only Linear Collider that existed



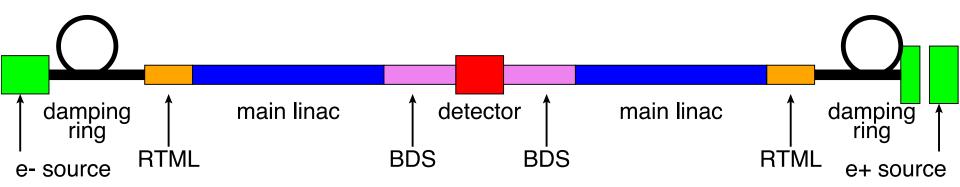
he Z⁰ and demonstrate linear collider feasibility

> Energy = 92 GeVLuminosity = 2e30

Has all the features of a 2nd gen. LC except both e+ and e- used the same linac

A 10% prototype!

Sources



Produce the electron beam

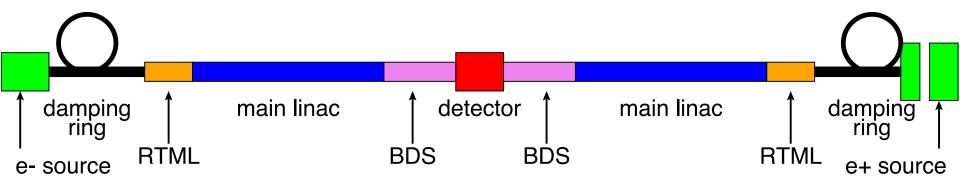
 use a laser to kick electrons out of a cathode

> ⇒ Masao Kuriki Thursday 1.3.

Produce the positron beam

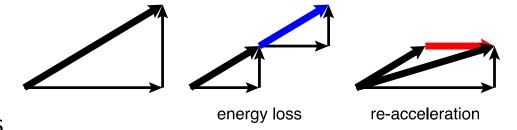
- use an electron beam to produce photons
 - In CLIC in a crystal
 - In ILC in a wiggler
- the photons produce showers in matter
 - harvest the positrons

Damping Rings



Cool the beams

in particular positron beam



Produce flat beams due to physics

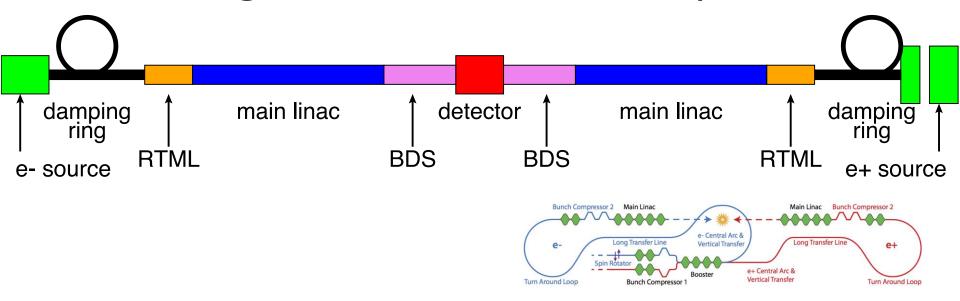
Minimum emittances exist (zero current emittances)

For larger charges they can increase due to collective beam dynamics

⇒ Hermann Schmickler Friday 23.2. ©

⇒ Katsunobu Oide Monday 26.2., Tuesday 27.2.

Ring To Main Linac Transport

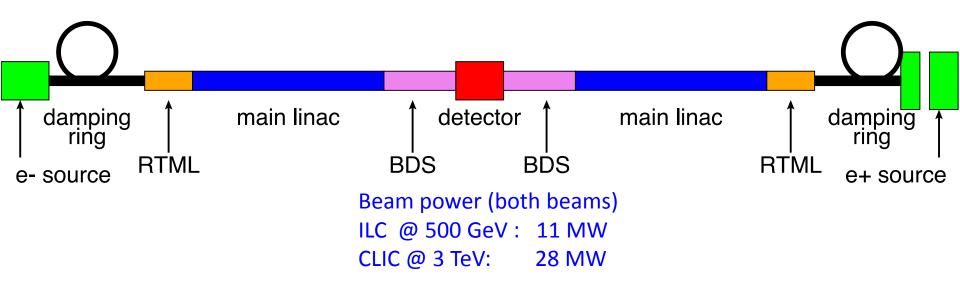


Transports beam from damping ring to main linac

- Compresses bunches from damping ring to main linac (e.g. from 1.6 mm to 70 μ m in CLIC)
- Increase the beam energy to be high enough for transport and main linac



Main Linac



The key for higher energies

The main cost driver

Main power consumer

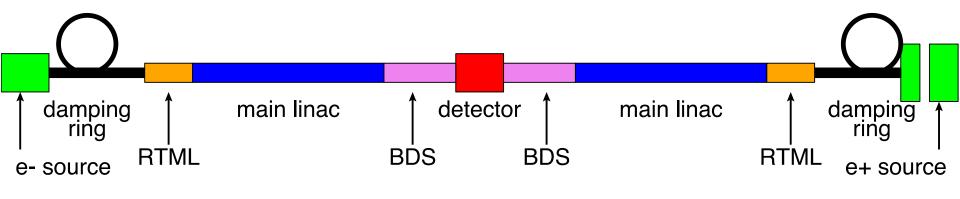
A main ingredient for site choice

The key design driver for other systems

Core of this lecture

Note: 12 hours of main linac lecture in linear collider school only scratches the surface

Beam Delivery System



Cleaning of the beams from tails and halo

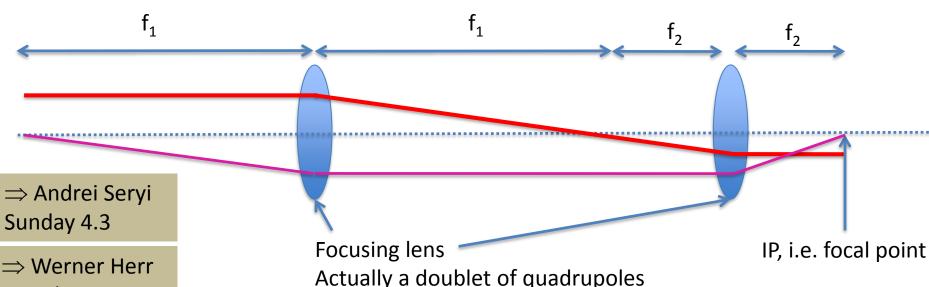
machine protection issue

Monday 26.2.

D. Schulte

Important to avoid background in the detector

Final focus system: Telescope to squeeze the beam to small size, i.e. small beta-function challenging because of the beam energy spread



Linear Collider Beam Dynamics, CAS 2018

Challenges: Energy and Luminosity

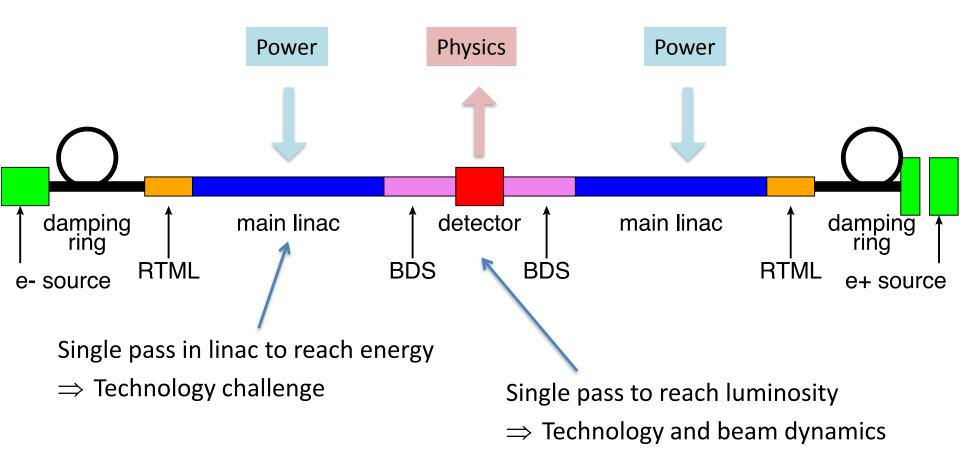


Key Parameters

Parameter	Symbol [unit]	SLC	ILC	CLIC	CLIC
Centre of mass energy	E _{cm} [GeV]	92	500	380	3000
Luminosity	L [10 ³⁴ cm ⁻² s ⁻¹]	0.0003	1.8	1.5	6
Luminosity in peak	L _{0.01} [10 ³⁴ cm ⁻² s ⁻¹]	0.0003	1	0.9	2
Gradient	G [MV/m]	20	31.5	72	100
Particles per bunch	N [10 ⁹]	37	20	5.2	3.72
Bunch length	σ_{z} [μ m]	1000	300	70	44
Collision beam size	$\sigma_{x,y}$ [nm/nm]	1700/600	474/5.9	143/2.9	40/1
Emittance	ε _{x,y} [μm/nm]	~3/3000	10/35	0.95/30	0.66/20
IP beta functions	$\beta_{x,y}$ [mm/mm]	87/10	11/0.48	8/0.1	6/0.07
Bunches per pulse	n _b	1	1312	352	312
Bunch distance	Δz [mm]	-	554	0.5	0.5
Repetition rate	f _r [Hz]	120	5	50	50

Energy and luminosity goals are defined by physics

Main Linear Collider Challenges

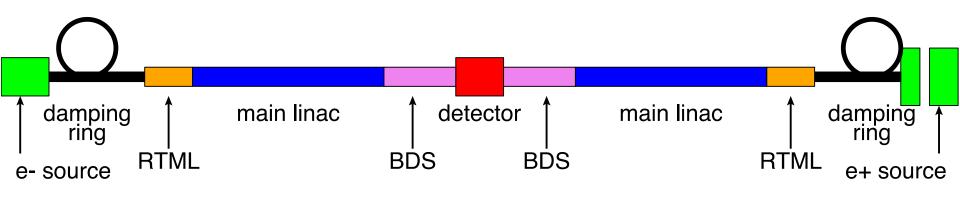


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Note: Emittances are always normalised

Beam Parameters Along the Collider (CLIC 380)



	ε _x [nm]	ε _y [nm]	σ _z [μm]	N [10 ⁹]	E [GeV]
Damping ring exit	700	5	1600	5.2	2.86
End of RTML	850	10	70	5.2	9.0
End of main linac	920	20	70	5.2	190.0
Interaction point	950	30	70	5.2	190.0

Mainly damping ring defines horizontal emittance

Final bunch length defined by main linac

Bunch energy defined by main linac

All systems contribute to vertical emittance

Bunch charge defined by main linac

Energy Drivers

Energy is given by linac length and gradient

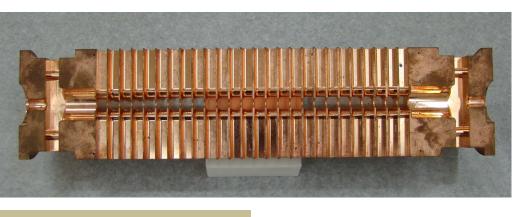
- ⇒ Key technology challenge
- \Rightarrow Bulk of the cost
- \Rightarrow Bulk of the power consumption

Affordability of the project depends on main linac

Also drives beam parameters



⇒ Erk Jensen, Thursday 1.3.



⇒ Walter Wuensch, Sunday/Monday 4/5.3.

ILC:

Superconducting cavity at 31.5 MV/m
The highest field that is deemed possible in mass production

CLIC:

Normal conducting accelerating structures at 100 MV/m for 3 TeV and 72 MV/m for 380 GeV Gradient optimised for cost

Luminosity Drivers

Can re-write normal luminosity formula

$$\mathcal{L} = H_D \frac{N^2}{4\pi\sigma_x \sigma_y} n_b f_r$$

 $\mathcal{L} \propto H_D$ $\frac{N}{\sigma_x}$ $Nn_b f_r$ $\frac{1}{\sigma_y}$ $Nn_b f_r$ $\frac{1}{\sigma_y}$ $Nn_b f_r$ $\frac{1}{\sigma_y}$ $Nn_b f_r$ $Nn_$

H_D: pinch enhancement, typically 1-2
 N: number of particles per bunch
 n_b: number of bunches per train

 f_r : number of trains per second

 $\sigma_{x,v}$: transverse beamsizes

Luminosity Beam Quality spectrum (+bunch length)

How to Produce Luminosity

Note: We use crab crossing Can ignore crossing angle

Squeeze as much out of first part

$$\mathcal{L} = H_D \frac{N^2}{4\pi\sigma_x \sigma_y} n_b f_r$$

Single bunch parameters at IP Determined by upstream accelerator:

$$N, \varepsilon_x, \varepsilon_y, \sigma_z$$

$$\sigma_y = \sqrt{\beta_y \epsilon_y / \gamma}$$

Define the target for the beam delivery system:

Make beta small and create a small waist

How to Produce Luminosity

Note: We use crab crossing Can ignore crossing angle

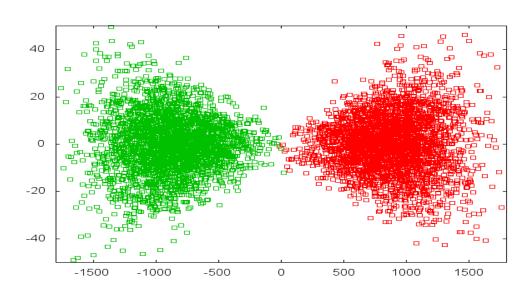
Squeeze as much out of first part

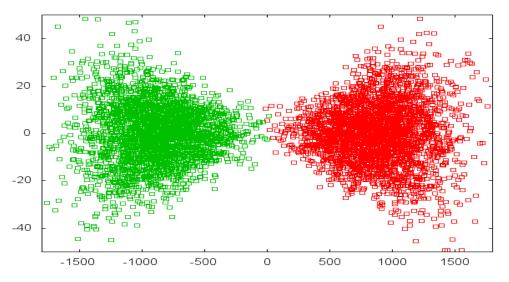
$$\mathcal{L} = H_D \frac{N^2}{4\pi\sigma_x \sigma_y} n_b f_r$$

Single bunch parameters at IP Determined by upstream accelerator:

$$N, \epsilon_x, \epsilon_y, \sigma_z$$

Beams focus each other We get more luminosity





How to Produce Luminosity

Note: We use crab crossing Can ignore crossing angle

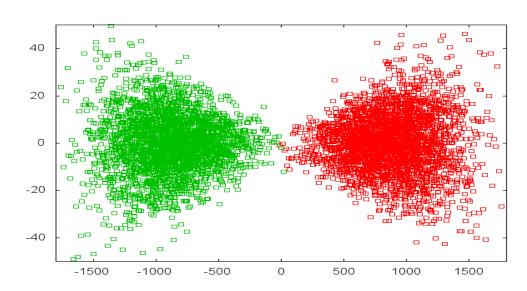
Squeeze as much out of first part

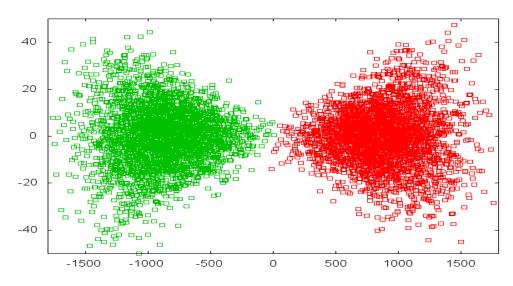
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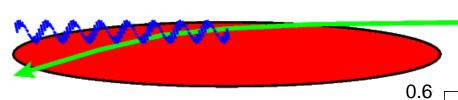
$$N, \epsilon_x, \epsilon_y, \sigma_z$$

Photons are emitted and particles lose energy
The experiments will not like this





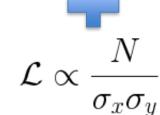
Beamstrahlung



⇒ Werner Herr Monday 26.2.

Number of photons dominates $L_{0.01}/L$

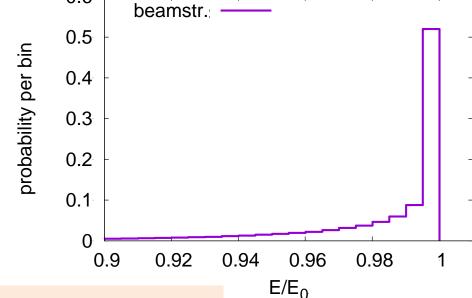
$$n_{\gamma} \propto \frac{N}{\sigma_x + \sigma_y}$$





$$\sigma_x \gg \sigma_y$$

$$\sigma_x + \sigma_y \approx \sigma_x$$



Note:

Somewhat different for 3 TeV But does not change principle

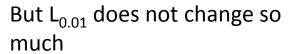


$$\mathcal{L} \propto H_D igg(rac{N}{\sigma_x}igg)$$

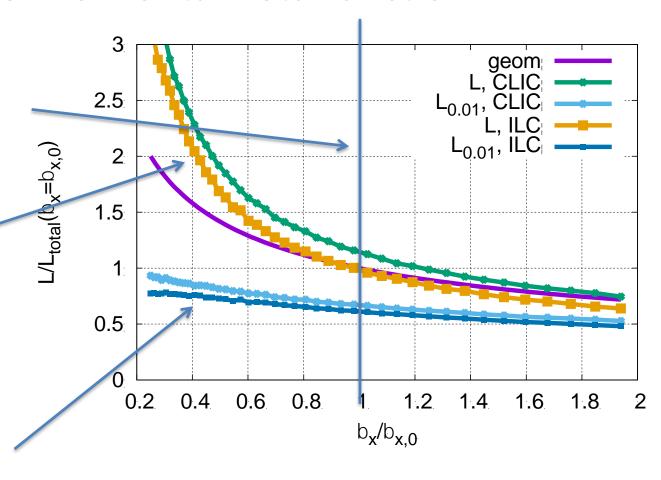
Choice of Horizontal Beta-function

Design value $L_{0.01}/L=60\%$ is good compromise for most physics studies $n_{\gamma}\approx(1)$

The total luminosity L varies strongly with beta-function



Hard to push beta-functions
That low



Hence large β_x (ILC: 11 mm, CLIC 380: 8 mm) And large ϵ_x is acceptable

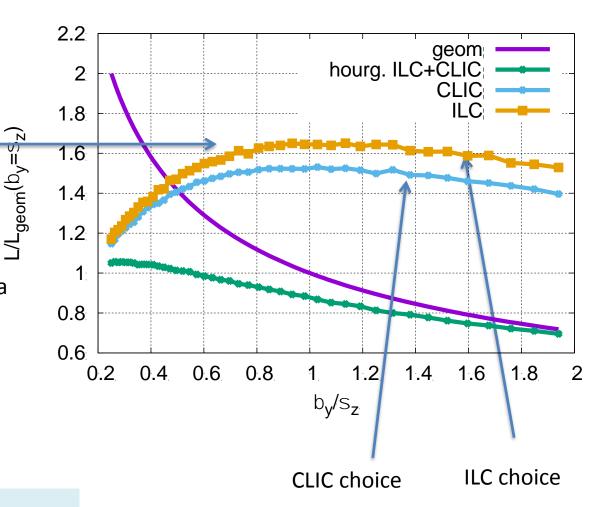
Note both help to make the vertical counterpart small

Choice Vertical Beta-function

Including pinch effect

There is an optimum value for beta

For smaller beta-function the geometric luminosity would increase but the enhancement is reduced



Choose $\beta_v \approx \sigma_z$

Note: at higher energies this becomes increasingly difficult to achieve

More luminosity for smaller vertical emittance ε_v

Additional limits cannot be discussed here

Main Linac and Efficiency



ILC Accelerating Cavity

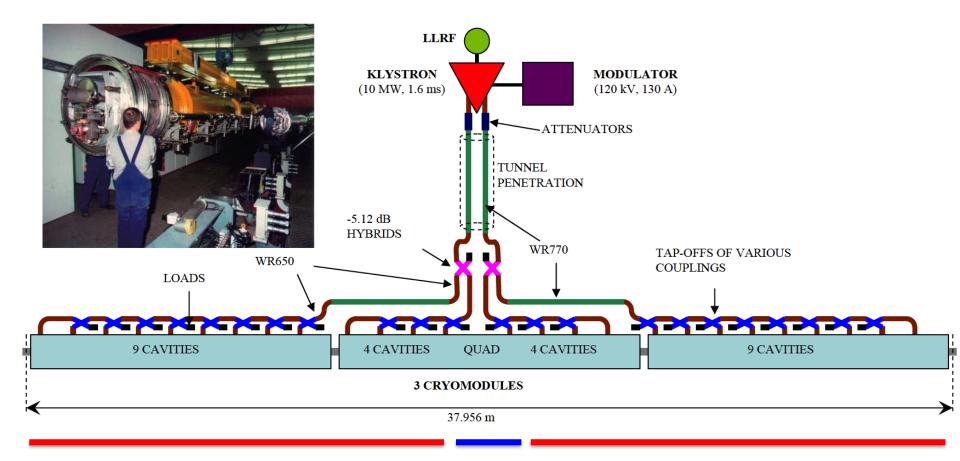


- About 1 m long cavity with 31.5 MV/m,
 - super-conducting
 - 1.3 GHz
 - standing wave
 - constant impedance

 \Rightarrow Erk Jensen,

⇒Thursday 1.3.

Main Linac Unit



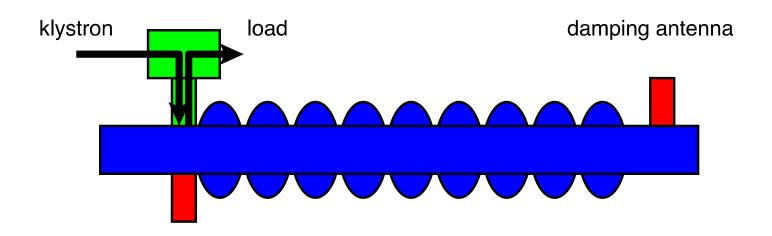
Accelerating cavities O(65%) of linac length

Beam guiding quadrupole
Beam position monitor
Corrector kicker

Accelerating cavities

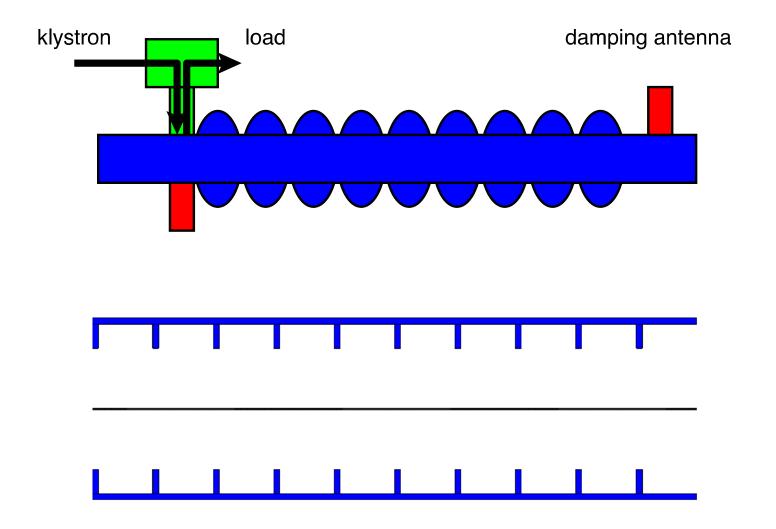
Total length for 500 GeV cms 31 km, some length for beam cleaning and focusing

Standing Wave Cavity

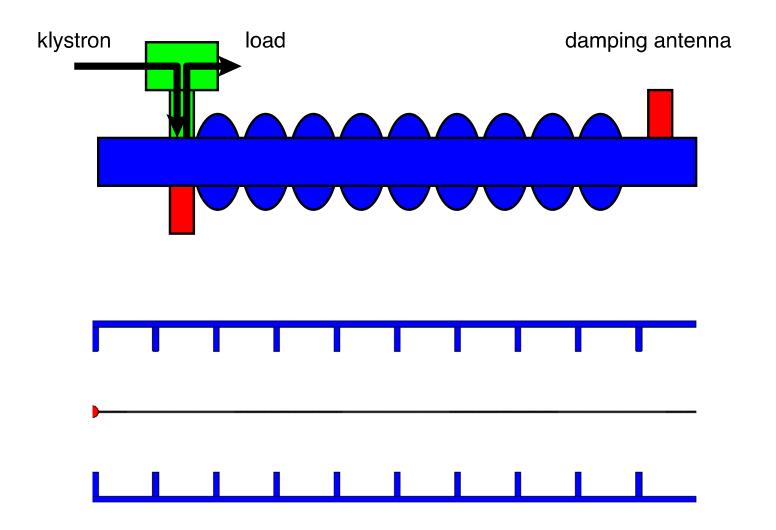


- The power is feed into one end
 - the power is reflected at the coupler
 - as the power in the cavity is increasing, the reflection is reduced
- there is a level when there is no reflection
 - \Rightarrow now switch on the beam

Standing Wave Cavity



Standing Wave Cavity



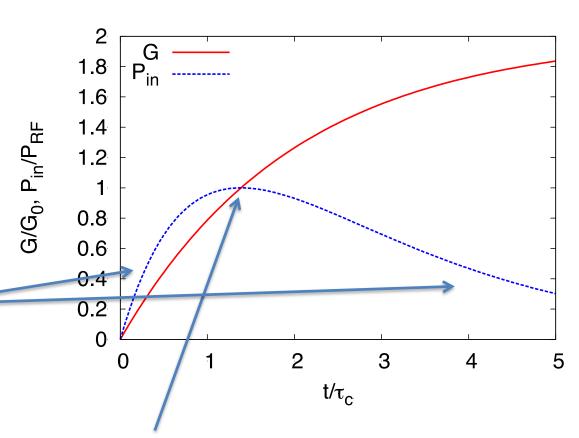
Filling a Standing-wave Cavity

Select the target gradient G₀

Adjust the coupling of the cavity to the RF "external Q"

Only part of RF power flows into cavity

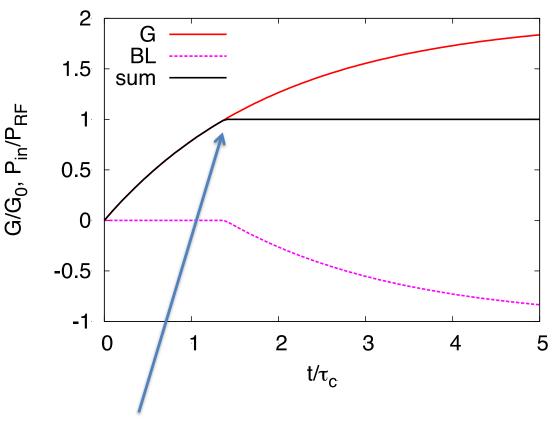
In ILC Filling time is 900 μs Beam time is 720 μs



All the RF power flows into cavity

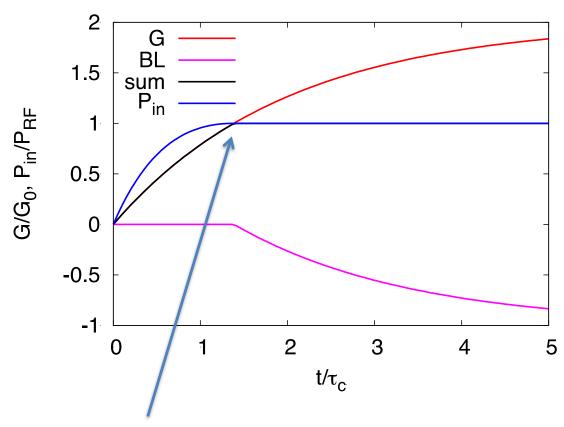
⇒ Erk Jensen, Thursday 1.3.

Filling a Standing-wave Cavity



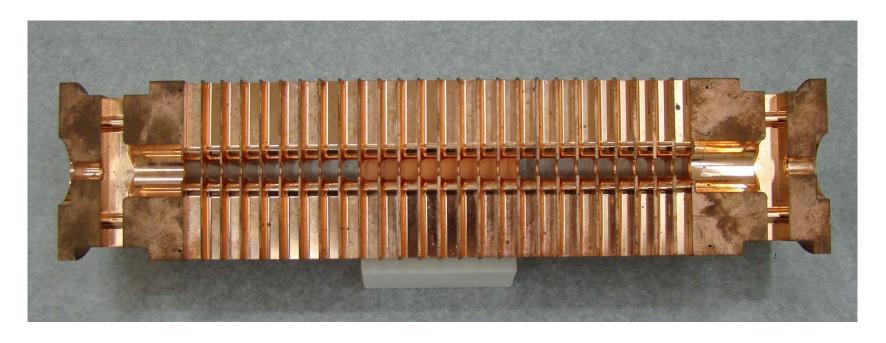
Switch the beam on Takes as much power from cavity as flows in Gradient remains constant All RF power continues to flow in

Filling a Standing-wave Cavity



All RF power continues to flow in

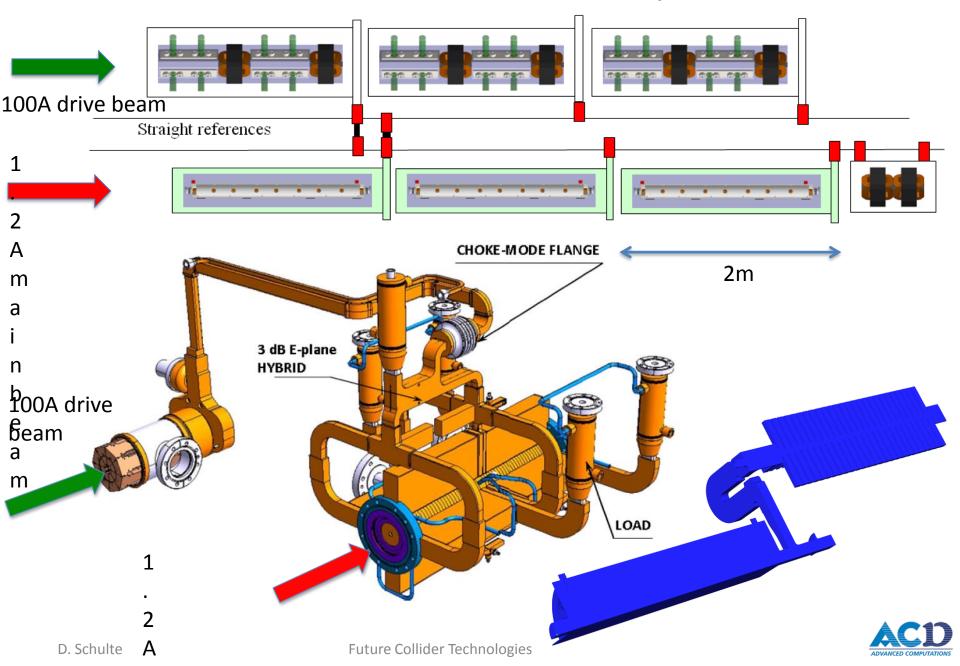
CLIC Accelerating Structure



- About 23 cm long structure with G = 100 MV/m
 - normal-conducting
 - 12 GHz
 - travelling wave
 - constant gradient (almost)

⇒ Walter Wuensch, Sunday/Monday 4/5.3.

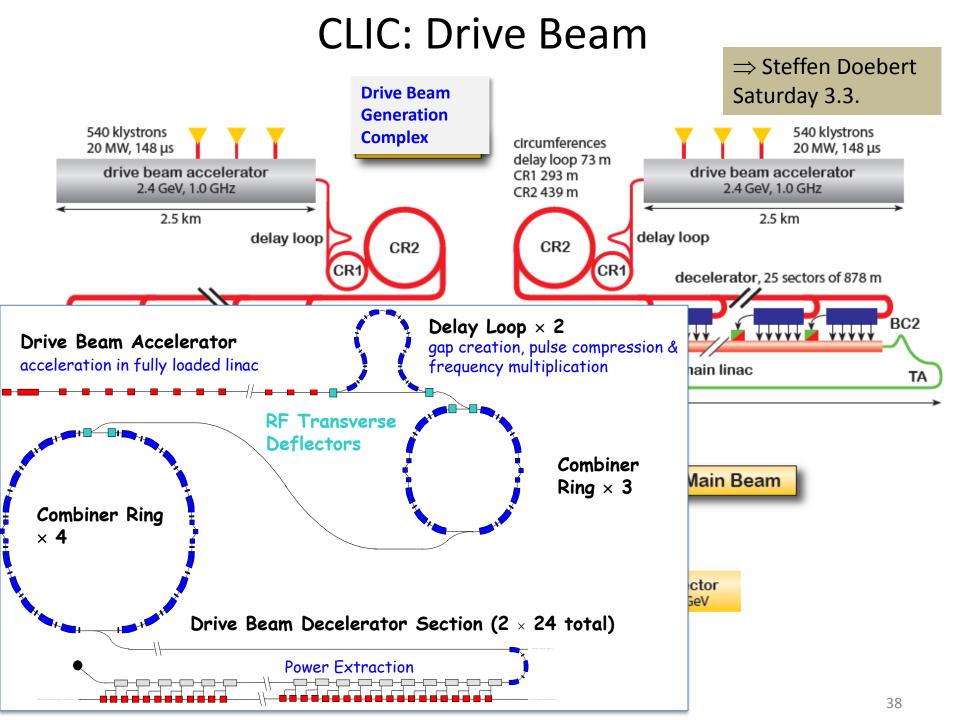
CLIC Two-beam Concept



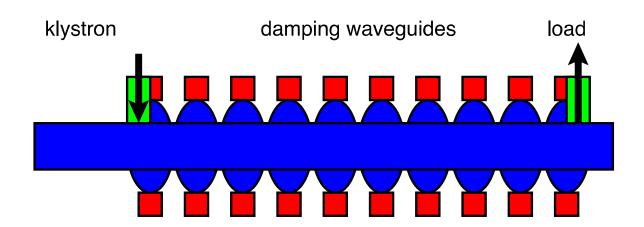
CLIC Two-beam Module

1st module

80 % filling with accelerating structures 11 km for 380 GeV cms 50 km for 3 TeV

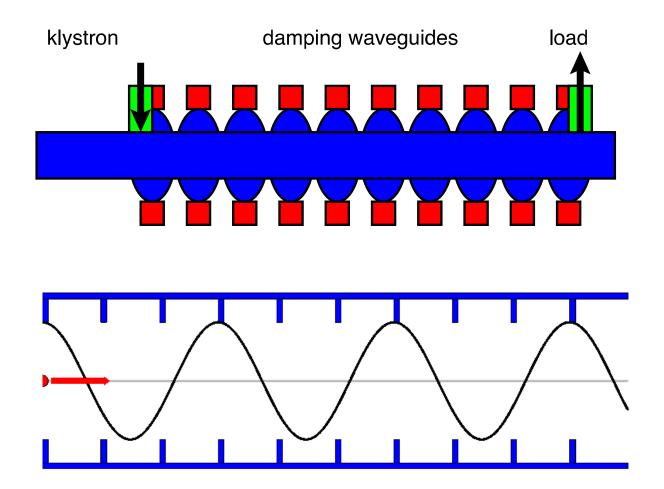


Travelling Wave Structure



- The power is feed into one end
 - no reflection if designed properly
- It slowly moves through the structure
 - group velocity is typically a few percent of the speed of light

Travelling Wave Structure



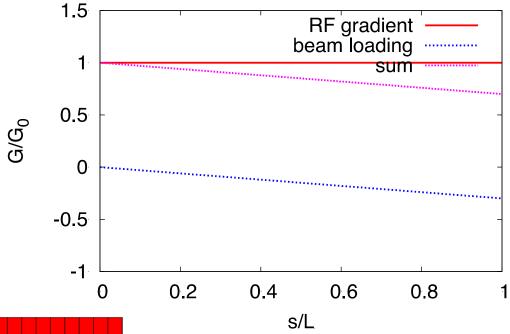
Note: Field should also vary with position, but that exceeds my graphic competences

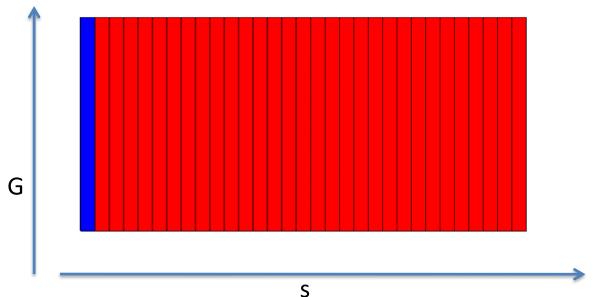
Filling a Travelling Wave Structure

The RF energy is flowing along the structure

Some is given to the beam, some is lost in the wall

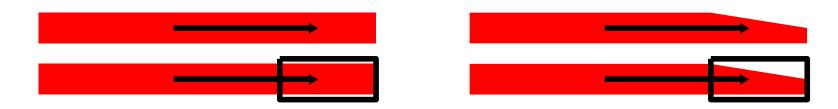
Gradient profile developes



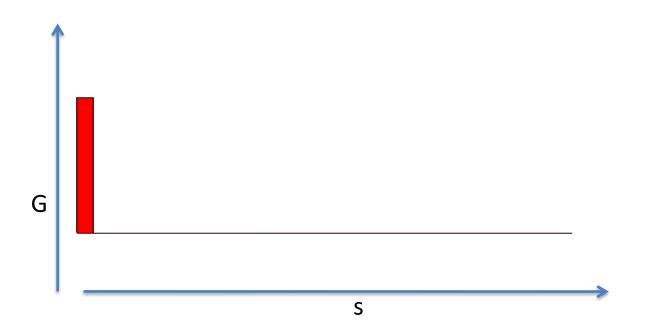


Some power is leaking out at the end

Beam-loading Compensation



In CLIC filling time is O(80 ns) and beam time (O 160 ns) Slightly different for different structures



RF Power to Beam Efficiency

$$\mathcal{L} \propto H_D \, rac{n_{\gamma}}{n_{\gamma}} \, rac{\eta_{RF->beam}}{E_{cm}} rac{P_{RF}}{\sigma_y} \, rac{1}{\sigma_y}$$

Power into beam

$$\eta_{RF \to beam} = \frac{\text{Energy taken by one beam pulse}}{\text{Energy in each RF pulse}}$$

For constant RF pulse power

Per During beam passage
$$\eta_{RF \to beam} = \frac{\tau_{beam}}{\tau_{RF}} \cdot \frac{P_{beam}}{P_{RF}} \qquad \text{RF power during pulse}$$

Simplified

$$\eta_{RF \to beam} = \frac{\tau_{beam}}{\tau_{beam} + \tau_{fill}} \cdot \frac{P_{beam}}{P_{beam} + P_{loss} + P_{out}}$$

Note: what I call τ_{fill} contains several components of which the fill time is the most important; RF experts will learn more

RF to Beam Power Efficiency

$$\eta_{RF \to beam} = \frac{\tau_{beam}}{\tau_{beam} + \tau_{fill}} \cdot \frac{P_{beam}}{P_{beam} + P_{loss} + P_{out}}$$

Super-conducting cavity

- Almost no RF power lost in walls
- But cavity cooling requires power

$$\eta_{RF o beam} = rac{ au_{beam}}{ au_{beam} + au_{fill}}$$

Normal conducting structure

- Important fraction of RF power lost in walls
- Some power draining out of travelling wave structure (usually)

$$\eta_{RF \to beam} = \frac{\tau_{beam}}{\tau_{beam} + \tau_{fill}} \cdot \frac{P_{beam}}{P_{beam} + P_{loss} + P_{out}}$$

Impedance

Energy in the cavity or accelerating cell

Voltage along cavity or cell

$$U = \frac{(GL)^2}{(R/Q)\omega}$$

Impedance, depends on shape of cavity/cell, does not depend on frequency

RF frequency

High R/Q means high wakefields
A bunch extracts the same amount of energy for higher R/Q
But field in cavity must change more since less energy is stored

Important example: smaller apertures yield higher R/Q

Impedance

Energy in the cavity or accelerating cell

Voltage along cavity or cell

$$U = \frac{(GL)^2}{(R/Q)\omega}$$

Impedance, depends on shape

of cavity/cell, does not depend on frequency

Warning

High R/Q mea

A bunch extra

But field in car

Important exa

This definition is in "Linac Ohms"

People also use "Circuit Ohms"

2 "Linac Ohms" = 1 "Circuit Ohm"

RF frequency

/Q

tored

Power Lost in the Structure

Voltage

Power loss

$$P_{loss} = \frac{U}{Q}\omega = \frac{(GL)^2}{R/Q}\frac{1}{Q}$$

Cavity design

Cavity material

$$Q = \frac{\text{Stored energy}}{\text{Ohmic energy loss per radian of RF cycle}} = \frac{U}{P_{loss}} \omega$$

Examples: Q=O(10¹⁰) for superconducting and O(10⁴) for normal conducting

But frequency dependent

Examples

parameter	CLIC	ILC (RDR)	
R'/Q	$\approx 11 \mathrm{k}\Omega/\mathrm{m}$	$1.036\mathrm{k}\Omega/\mathrm{m}$	
Q	≈ 6000	$\approx 10^{10}$	
R'	$\approx 66 \mathrm{M}\Omega/\mathrm{m}$	$\approx 10^7 \mathrm{M}\Omega/\mathrm{m}$	

• ILC:
$$I \approx 5.8 \,\mathrm{mA}$$

• CLIC:
$$I \approx 1.2 \,\mathrm{A}$$

 \Rightarrow

$$\frac{P'_{beam}}{P'_{wall}} \approx 1650$$

$$\frac{P'_{beam}}{P'_{wall}} \approx 0.8$$

• Efficiency is

$$\eta = \frac{\tau_{beam}}{\tau_{beam} + \tau_{fill}} \frac{P_{beam}}{P_{beam} + P_{loss} + P_{out}}$$

Plugging in numbers for ILC

$$\eta \approx \frac{730 \,\mu\text{s}}{730 \,\mu\text{s} + 900 \,\mu\text{s}} \approx 0.45$$

• Plugging in (slightly older) numbers for CLIC

$$\eta = \frac{156 \text{ ns}}{156 \text{ ns} + 83 \text{ ns}} \cdot \frac{27 \text{ MW}}{27 \text{ MW} + 25 \text{ MW} + 12 \text{ MW}} \approx 0.65 \cdot 0.42 \approx 0.277$$

Cryogenics Power (ILC)

Cavities have small losses

$$P_{loss} = const \frac{1}{Q_0} \cdot G^2$$

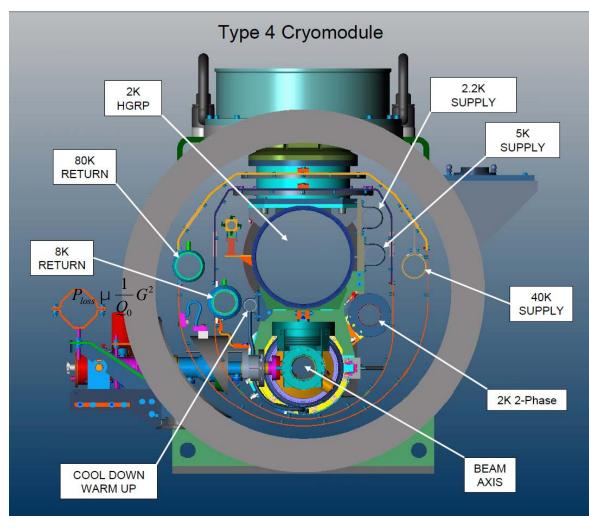
About 1W/m

But cooling costly at low temperatures

Remember Carnot:

$$\begin{split} P_{cryo} &= \frac{1}{h} \frac{T_{room} - T_{source}}{T_{source}} \ \ P_{loss} \\ P_{cryo} & \gg 700 \ \ P_{loss} \end{split}$$

The typical heat load of 1 W/m \Rightarrow about 1 kW/m for cryogenics



Average RF power: 1.6kW/m (3kW/m) Power into beam about 0.7kW/m

ILC Main Linac Pulse Optimisation

Bema current is given by installed RF power

Disclaimer: there have been heated discussions on how that should be optimised...

More RF peak power

Higher beam current Shorter cavity fill time

Higher cost (klystrons and modulators)
Either higher bunch charge or more bunches

Longer RF pulses Higher pulse rate Higher average beam current
Cavity fill time is smaller fraction of pulse (longer pulses)
Higher cost (modulators and klystrons)
More cooling required
Either higher bunch charge or more bunches

Higher average beam power leads to more cost but also more luminosity

CLIC Main Linac Pulse Optimisation

Power to beam

$$P'_{beam} = IG$$

Power lost in structure

$$P'_{loss} = \frac{G^2}{R'}$$

Maximise

$$\frac{P'_{beam}}{P'_{loss}} = \frac{R}{Q}QG$$

Maximise current:

- Maximise bunch charge
- Minimise distance between bunches

Go to the limit! See in the following

High R'/Q (small iris) helps for maximum gradient Less power needed to generate gradient

But high R'/Q is bad for beam stability

Low gradient make machine expensive

Well, it is copper ...

Need to compromise between R'/Q, G and

Note: CLIC Optimisation

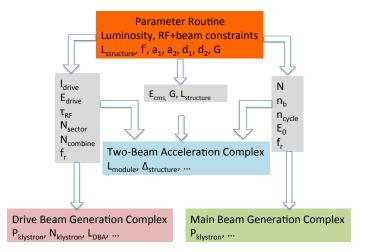
Scan 1.7 billion cases:

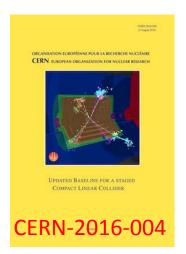
Fix structure design parameters: $a_1, a_2, d_1, d_2, N_c, \phi, G$

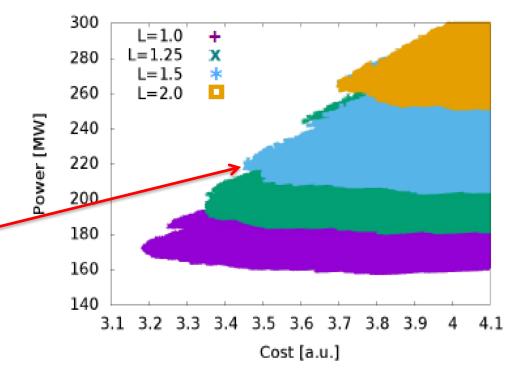
- \Rightarrow key beam parameters
- ⇒ Luminosity, cost and power (including other systems)

Resulting designs:
Colors indicate luminosities

This is the one we picked







Bunch Charge, Length and Spacing Choice



Note: Coordinate System

- We use two frames, the laboratory frame and the beam frame
- The nominal direction of motion of the beam is called s in the laboratory frame,
 the beam moves toward increasing s
- The longitudinal direction is called z in the beam frame, with particles at smaller z moving ahead of particles with larger z
- A particle preserves its longitudinal position within the beam
- The transverse dimensions are x in the horizontal and y in the vertical plane, in both coordinate systems
- People use different systems so find out what they talk about

Main Linac Beam Parameter Choice for CLIC

Want highest beam current for luminosity

- ⇒ Maximise bunch charge
- ⇒ Minimise bunch distance

$$\mathcal{L} \propto H_D \;\; rac{N}{\sigma_x} \;\; N n_b f_r \;\; rac{1}{\sigma_y}$$

Short-range longitudinal wakefields induce energy spread, compensated with RF

 \Rightarrow bunch charge defines bunch length $\sigma_z(N,W_L)$

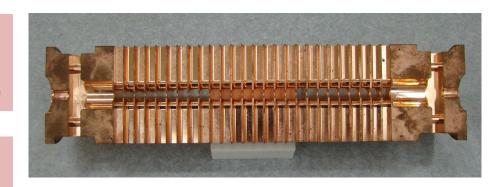
Short-range transverse wakefields can make beam instable

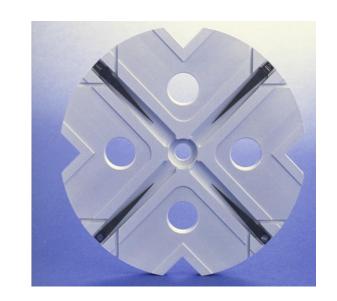
 \Rightarrow limits the bunch charge N (W_T $\sigma_z(N)$)

Transverse long-range wakefield can make the beam instable

⇒ limits the distance between bunches

Beam stability for strongest practical lattice defines beam parameters





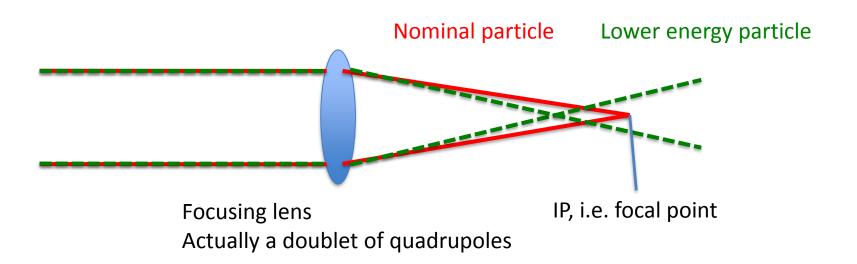
Energy Spread Goal

Energy spread at the end of the linac is critical

The final focus system is limited in energy acceptance Similar problem to camera lens

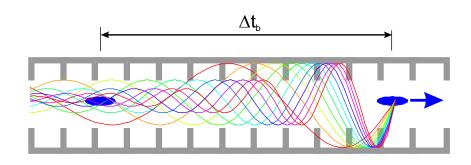
Compensation scheme is used but is also limited in performance

- ⇒ Have to limit the beam energy spread
- ⇒ CLIC goal is 0.35% RMS spread



Wakefields

Particle leaves fields behind that affect subsequent particles: "Wakefields"



Use wakefields to describe the effect of first particle on second one, here relevant are

$$P_z c = NeW_L(z)Le$$

$$P_x c = NeW_{\perp}(z) Le\Delta x$$

$$P_{y}c = NeW_{\perp}(z)Le\Delta y$$

Charge of driving particle

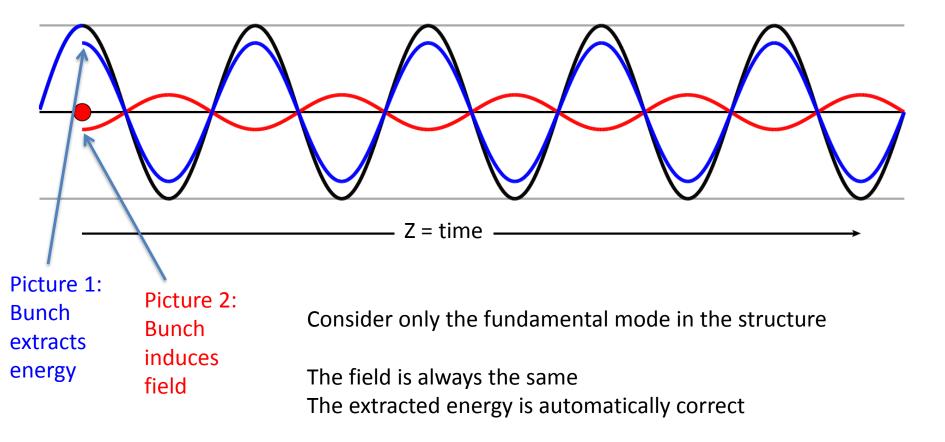
Longitudinal/transverse wakefield

Structure length

Witness particle charge

Transverse offset

Longitudinal Wakefield and Energy



If a bunch extracts a large fraction of the energy in the structure the tail will gain much less energy

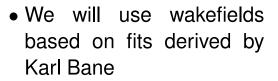
Longitudinal Wakefield (CLIC)

Particle z-position is constant

 \Rightarrow Same wakefield in each accelerating

structure





l length of the cell

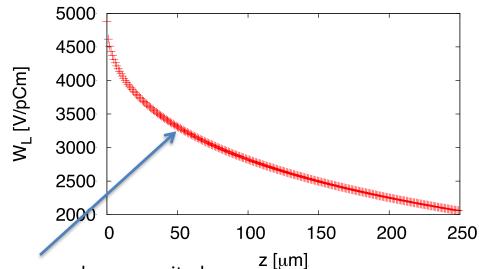
a radius of the iris aperture

g length between irises

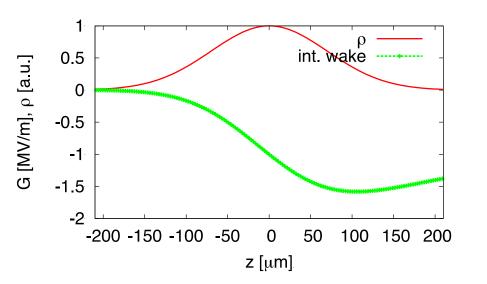
$$z_0 = 0.41a^{1.8}g^{1.6} \left(\frac{1}{l}\right)^{2.4}$$

$$W_L(z) = \frac{Z_0 c}{\pi a^2} \exp\left(-\sqrt{\frac{z}{z_0}}\right)$$

Use CLIC structure parameters



Many high-frequency modes are excited They add because they are cosine-like But they decohere rapidly

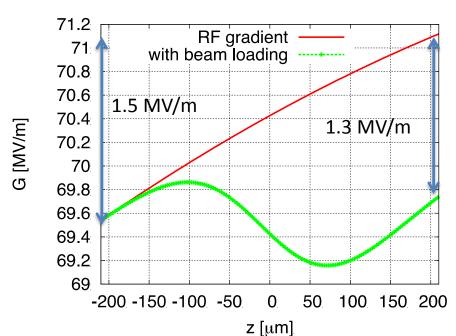


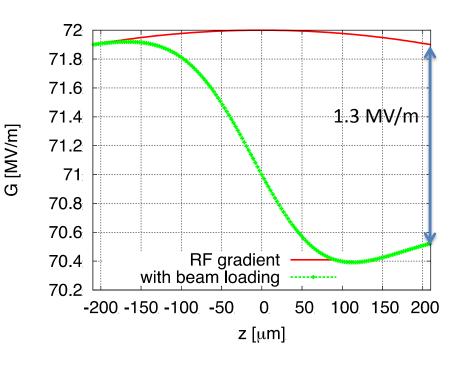
Longitudinal Wakefields and Energy Spread

Loaded gradient along bunch

On-crest acceleration:

- > more than 2% full gradient spread
- > 0.7% RMS energy spread





Off-crest acceleration (12°):

- ➤ 1% full gradient spread
- > 0.35% RMS gradient spread
- ➤ Loose about 2% in gradient

Bunch Length Choice

Fix mean RF phase at 12°

Make the bunch just long enough to reach final RMS energy spread of 0.35%

Hence
$$\sigma_z = \sigma_z(N)$$

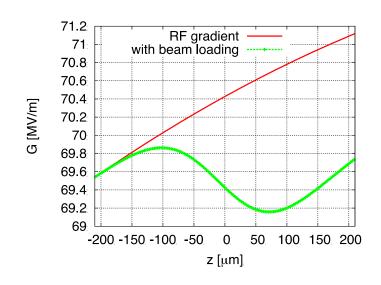
Can chose different RF phases along the linac

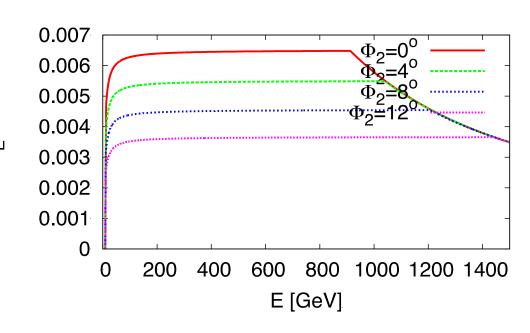
- Small phase in the main part
- and 30° at the end
- To have average phase of 12°

Allows to have larger energy spread in the main part of the linac than at the end

This can help beam stability

⇒ See next section

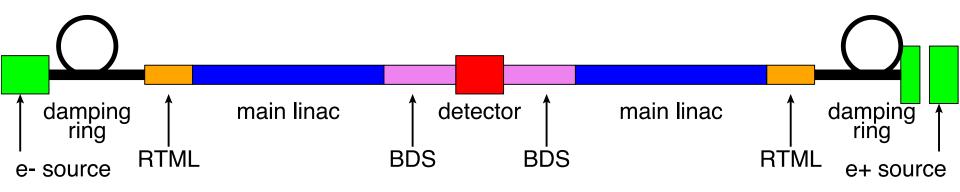




Single-bunch Stability



Beam Parameters Along the Collider (CLIC 380)



	Design limits Δε _y [nm]	Static imperfections Δε _y [nm]	Dynamic imperfections Δε _y [nm]
Damping ring exit	5	0	0
End of RTML	1	2	2
End of main linac	0	5	5
Interaction point	0	5	5
sum	6	12	12

Require 90% probability to meet goal

Average over time

Emittance

The beam particles have different coordinates; they occupy some phase space

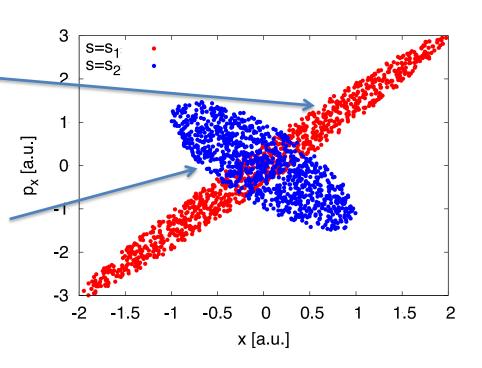
Liouville theorem (from the Liouville equation): the density in phase space around a trajectory remains constant in an unperturbed system, i.e. "the phase space is preserved"

$$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + \sum_{i=1}^{N} \left[\frac{\partial\rho}{\partial q_i} \dot{q}_i + \frac{\partial\rho}{\partial p_i} \dot{p}_i \right] = 0$$

Particle coordinates at one location

Particle coordinates at other location

Area does not change

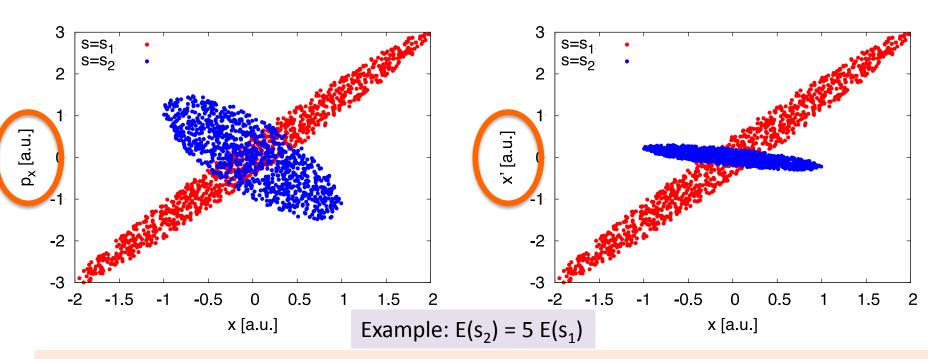


Emittance and Acceleration

When accelerating the phase space remains constant in canonical coordinates

$$(x, y, z, p_x, p_y, p_z)$$

But with the definition used in accelerators emittance shrinks

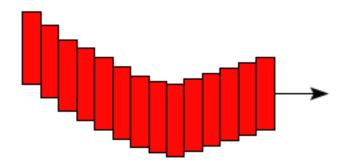


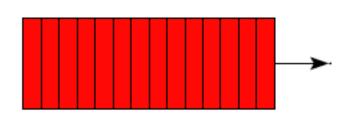
To avoid this linac experts use **normalised emittance** $\varepsilon_N = \gamma \varepsilon$ that does not change I will always do that here but not use the index N

Emittance Definition

Use projected emittance

$$e = \sqrt{\left\langle \left(x - \overline{x}\right)^2 \right\rangle \left\langle \left(x' - \overline{x'}\right)^2 \right\rangle - \left\langle \left(x - \overline{x}\right) \left(x' - \overline{x'}\right) \right\rangle^2}$$





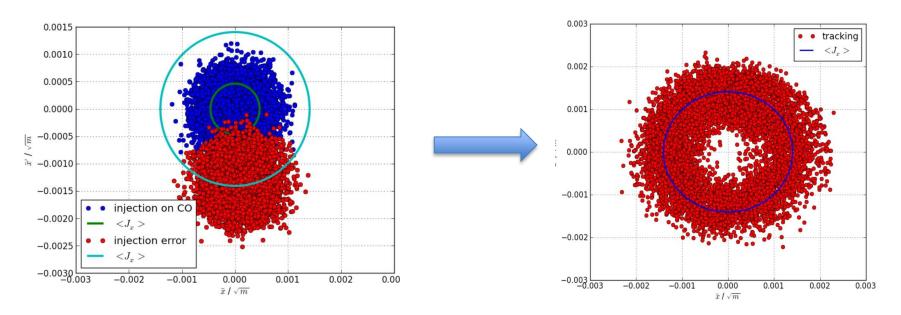
Note: we have to be careful with this, come back at the end

Also ignore coupling between x and y

Offset

Beam jitter leads to luminosity loss

Beam jitter can increase emittance (decoherence), luminosity loss on average remains constant But cannot correct downstream



Can use the "multi-pulse emittance" (combine phase space of subsequent bunches/pulses)

But transverse wakefields can make a jittering beam explode Main linac is huge source of wakefield effects This is why the main linac defines the bunch charge and length Let us have a look

Main Linac Lattice Concept

FODO lattice is used in main linacs

Guiding quadrupoles act like a spring

Particle is comparable to harmonic oscillator (driven with wakes)

$$x_1''(s) + \frac{1}{\beta^2}x_1(s) = 0$$

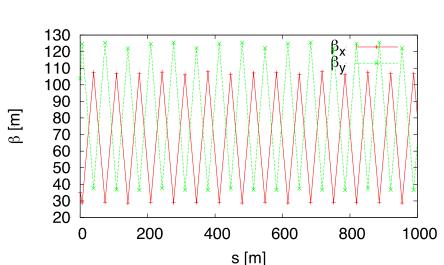
Local wavelength is β Strong focusing means smaller β

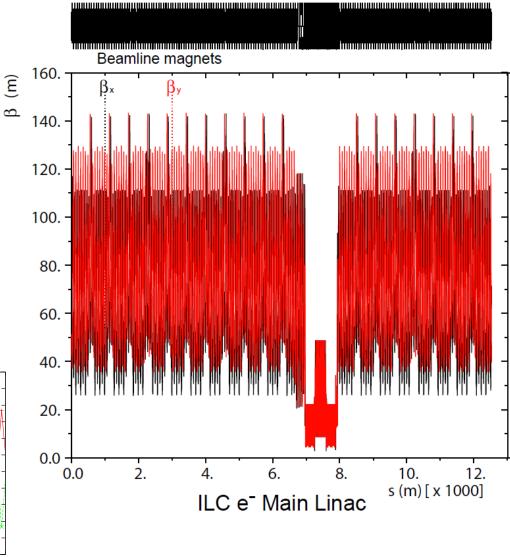
A function of longitudinal position s But equivalent to time dependence t

ILC Lattice Design

Constant quadrupole spacing Constant phase advance

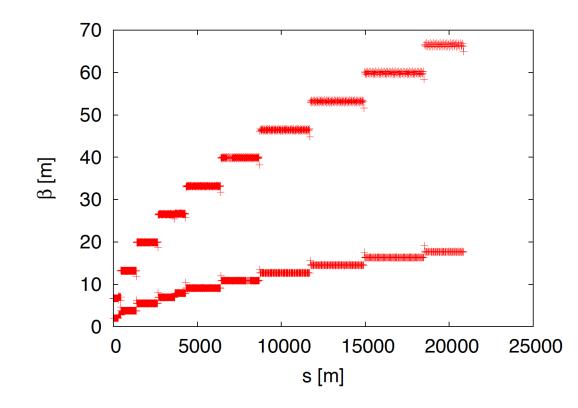
Different phase advance in horizontal and vertical to decouple planes against wakefield effects





CLIC Lattice Design (3 TeV)

- - 10% of linac are quadrupoles
- Used $\mathscr{B} \propto E^{1/2}$, $\Box \sqrt{\ } = \text{const}$
 - Quadrupole spacing and length scale as $E^{1/2}$
 - ⇒ roughly constant fill factor
 - phase advance is chosen to balance between wakefield and ground motion effects
- Total length 20867.6m
 - fill factor 78.6%



- 12 different sectors used
- Matching between sectors using 7 quadrupoles to allow for some energy bandwidth

Note: fill factor = active length/total length

Passage Through the Linac

For simplicity consider constant beta-function Replacing FODO lattice with permanent focusing Great approximation to understand physics

$$x_1''(s) + \frac{1}{\beta^2}x_1(s) = 0$$

$$x_1(0) = x_0 \quad x_1'(0) = 0$$

Solution is well-known

$$x_1(s) = x_0 \cos\left(\frac{s}{\beta}\right)$$

Transverse Wakefields

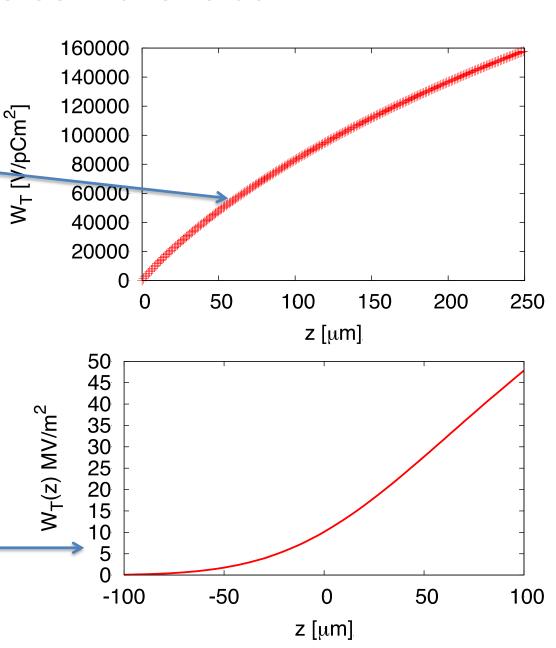
For short distances the wake-field rises linear

Summation of an infinite number of sine-like modes with different frequencies

Parametrisation (K. Bane)

$$W_{\perp}(z) = 4 \frac{Z_0 c z_0}{\pi a^4} \left[1 - \left(1 + \sqrt{\frac{z}{z_0}} \right) \exp\left(-\sqrt{\frac{z}{z_0}} \right) \right]$$
$$z_0 = 0.169 a^{1.79} g^{0.38} \left(\frac{1}{l} \right)^{1.17}$$
$$W_{\perp}(z \ll z_0) \approx 2 \frac{Z_0 c}{\pi a^4} z$$

Coherent bunch offset (worst case)
The tail is deflected to the outside

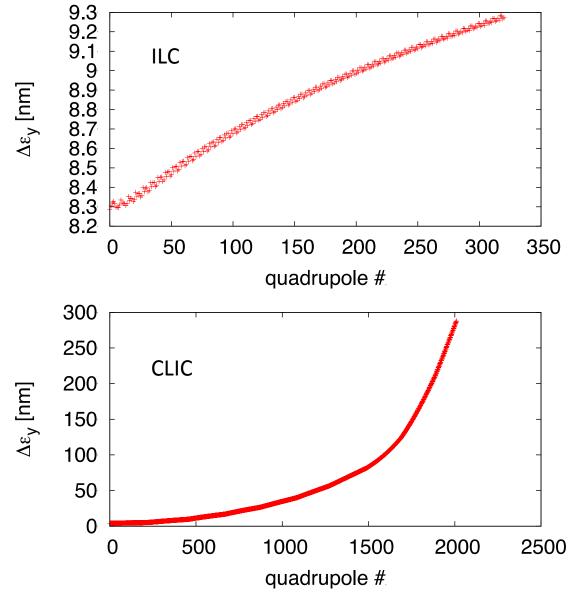


Emittance in Linac

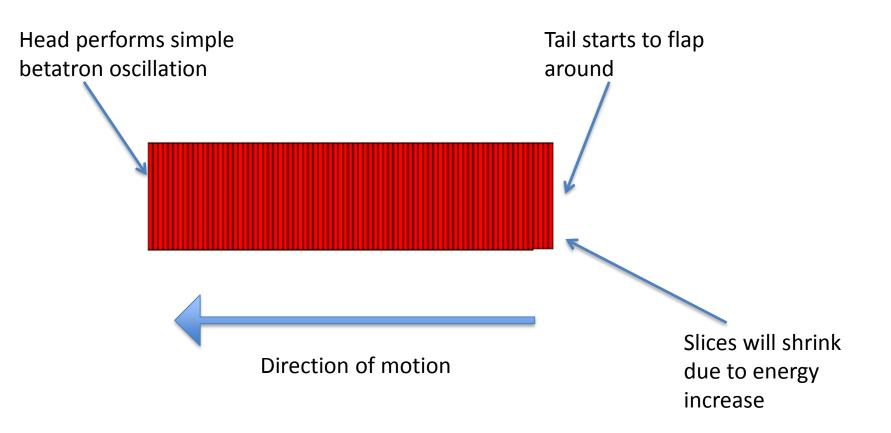
Transverse stability of beam with initial offset σ_v

- No energy spread
- Emittance with respect to beam axis shown

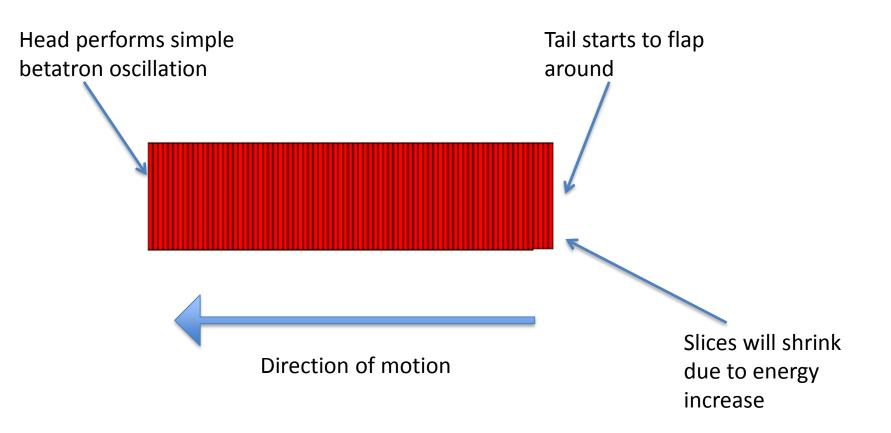
- \Rightarrow Acceptable for ILC
- ⇒ Not acceptable for CLIC



Bunch Transverse Motion (CLIC)



Bunch Transverse Motion (CLIC)



Wakefield Model

Assume bunch can be represented by two particles and constant $K(s) = 1/\beta^2$

- Second particle is kicked by transverse wakefield

$$x_2''(s) + \frac{1}{\beta^2} x_2(s) = \frac{Ne^2 W_{\perp}(\Delta z)}{P_L c} x_1(s)$$

$$x_2(0) = x_0 \quad x_2'(0) = 0$$

$$x_1(s) = x_0 \cos\left(\frac{s}{\beta}\right)$$

Solution is simple with an ansatz

$$x_2(s) = x_0 \cos\left(\frac{s}{\beta}\right) + x_0 \left(Ne^2 W_{\perp}(\Delta z)\right) \left(\frac{\beta s}{2E}\right) \sin\left(\frac{s}{\beta}\right)$$

 \Rightarrow Amplitude of second particle oscillation is growing linearly with s

Discussion

$$x_2(s) = x_0 \cos\left(\frac{s}{\beta}\right) + x_0 \left(Ne^2 W_{\perp}(\Delta z)\right) \left(\frac{\beta s}{2E}\right) \sin\left(\frac{s}{\beta}\right)$$

With proper calculation one finds
$$\qquad \frac{\beta s}{E} \Rightarrow \int \frac{\beta(s)}{E(s)} ds$$

- Factors for the amplitude growth of the second particle
 - β: small beta-function (strong focusing) helps
 - 1/E: high energy helps
 - W_{\perp} : small wakefield helps
 - Shorter bunches
 - N: small bunch charge helps
 - s: shorter linac helps (i.e. higher gradient G)

Discussion

$$x_2(s) = x_0 \cos\left(\frac{s}{\beta}\right) + x_0 \left(Ne^2 W_{\perp}(\Delta z)\right) \left(\frac{\beta s}{2E}\right) \sin\left(\frac{s}{\beta}\right)$$

$$A(z_0) = \tilde{W}_{\perp}(z_0)e \int \frac{\beta(s)}{2E} ds \qquad \begin{array}{c} 20 \\ 18 \\ 16 \\ 14 \\ 12 \\ \hline \end{array}$$

z [μm]

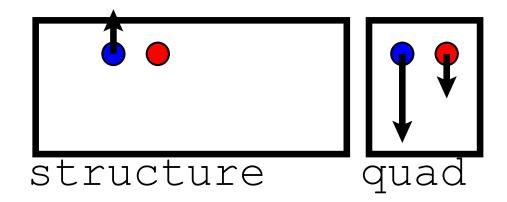
BNS Damping Concept

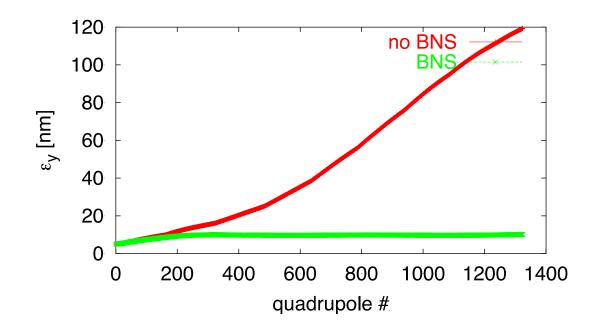
Transverse wakes act as defocusing force on tail

⇒ beam jitter is exponentially amplified

BNS damping (Balakin, Novokhatsky and Smirnov) prevents this growth

- manipulate RF phases to have energy spread
- take spread out at end





BNS Damping

For simplicity assume initial offset but no angle

First particle performs a harmonic oscillation

$$x_1(s) = x_0 \cos\left(\frac{s}{\beta}\right)$$

We want the second particle to perform the same oscillation, i.e.

$$x_2(s) = x_0 \cos\left(\frac{s}{\beta}\right)$$
 $x_1(s)$

Change particle energy for this purpose

$$x_2''(s) + \underbrace{\frac{1}{(1+\delta)\beta^2}} x_2(s) = \underbrace{\frac{Ne^2W_{\perp}(\Delta z)}{E}} x_0 \cos\left(\frac{s}{\beta}\right)$$

Same as changing mass in harmonic oscillator

BNS Damping

$$x_2''(s) + \frac{1}{(1+\delta)\beta^2} x_2(s) = \frac{Ne^2 W_{\perp}(\Delta z)}{E} x_0 \cos\left(\frac{s}{\beta}\right)$$

Plugging in our wanted solution for $x_2(s)$

$$x_2(s) = x_0 \cos\left(\frac{s}{\beta}\right) = x_1(s)$$

we find

$$-\frac{1}{\beta^2}x_0\cos\left(\frac{s}{\beta}\right) + \frac{1}{(1+\delta)\beta^2}x_0\cos\left(\frac{s}{\beta}\right) = x_0\frac{Ne^2W_{\perp}(\Delta z)}{E}\cos\left(\frac{s}{\beta}\right)$$

BNS Damping

$$-\frac{1}{\beta^2}x_0\cos\left(\frac{s}{\beta}\right) + \frac{1}{(1+\delta)\beta^2}x_0\cos\left(\frac{s}{\beta}\right) = x_0\frac{Ne^2W_{\perp}(\Delta z)}{E}\cos\left(\frac{s}{\beta}\right)$$

which is fulfilled for

$$\frac{1}{(1+\delta)\beta^2} - \frac{1}{\beta^2} = \frac{Ne^2W_{\perp}(\Delta z)}{P_L c}$$

$$\delta \approx -\frac{\beta^2}{E} N e^2 W_{\perp}(\Delta z)$$

Small beta-function

Small bunch charge

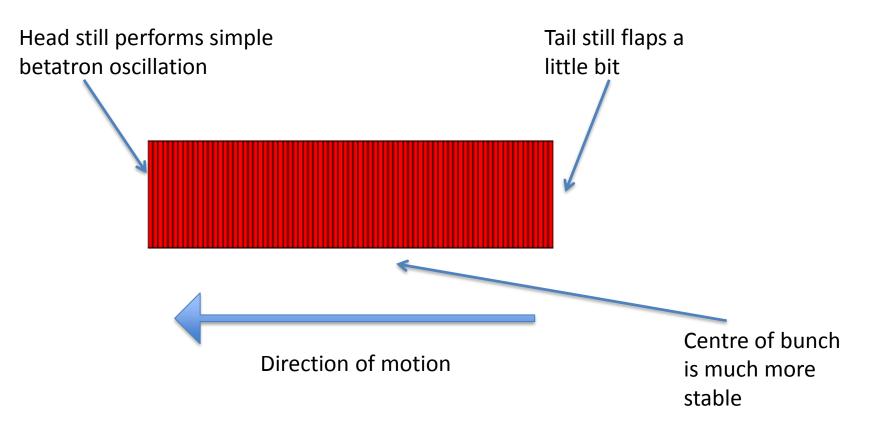
CLIC choice

$$\beta(s) \propto \sqrt{E(s)}$$

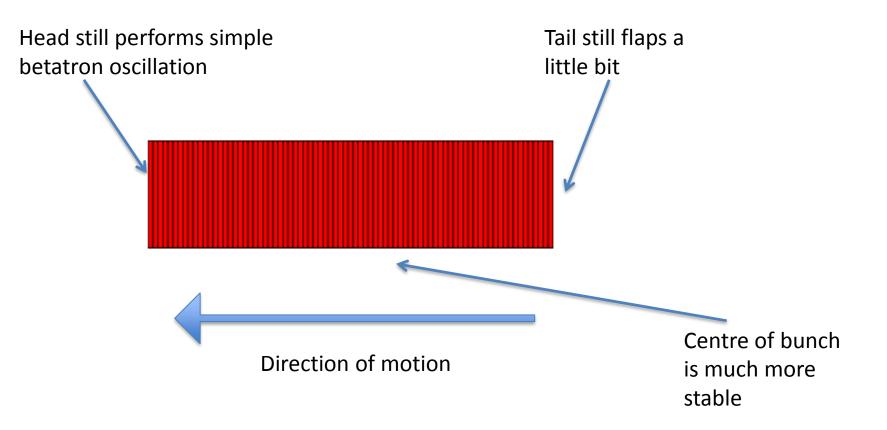
Small wakefields

Allows $\delta = \mathrm{const}$

Bunch in Main Linac



Bunch in Main Linac



Energy Spread in the Linac

Cannot exactly match energy profile and wakefield

⇒ Shapes of energy spread and integrated wake differ

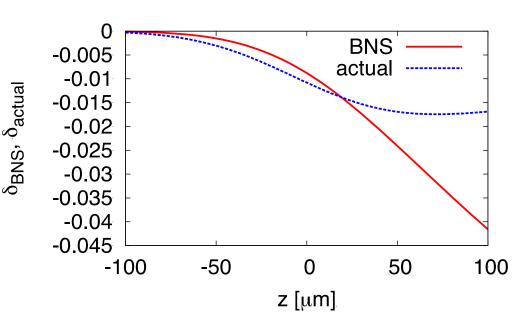
Only cure coherent offset

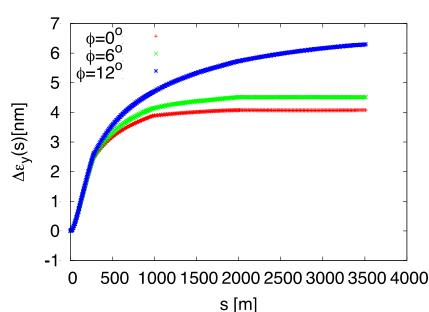
⇒ Slope along bunch still has an effect

Energy spread also helps detuning

In summary

- ⇒ Can only obtain some correction
- ⇒ Broad acceptable range
- ⇒ Different RF phases in linac are OK



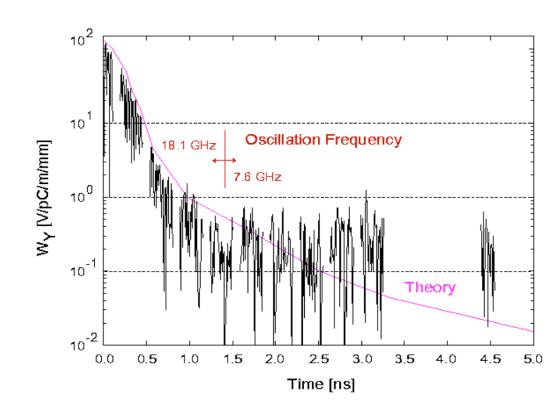


Multi-bunch Stability



Multi-bunch Wakefields

- Long-range transverse wakefield determines how close one can put the bunches in the linac
- Longrange transverse wakefields are sine-like
- They can be reduced by
 - Damping
 - Detuning

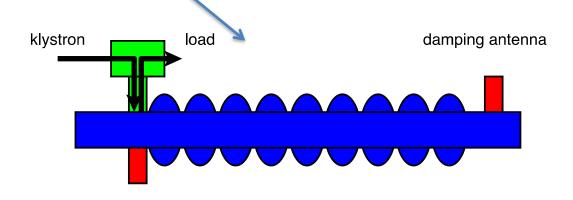


$$W_{\perp}(z) = \sum_{i=1}^{\infty} 2k_i \sin\left(2\pi \frac{z}{\lambda_i}\right) \exp\left(-\frac{\pi z}{\lambda_i Q_i}\right)$$

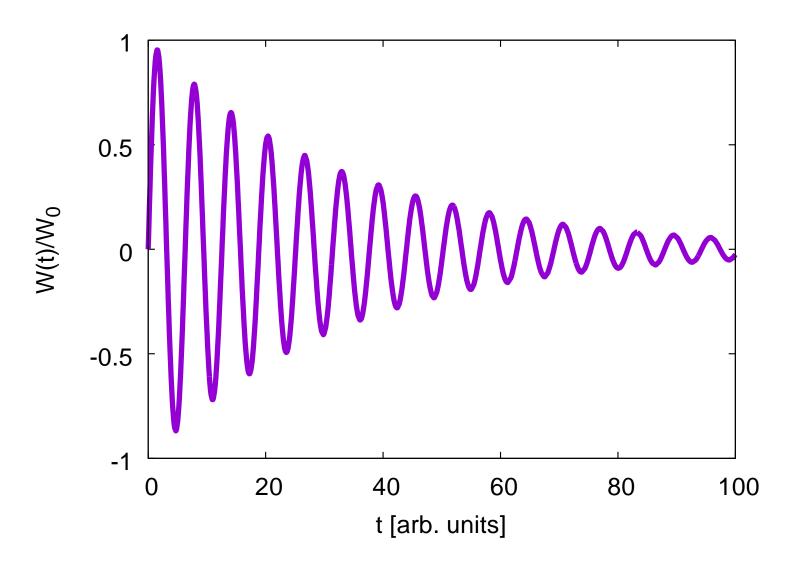
Damping

- Damping = extract power of transverse modes
- In CLIC, each cell has waveguides
 - Fundamental mode cannot escape
 - Strong damping, Q=O(10)
- ILC has antennas at the end
 - Weaker damping, Q=O(10⁴)





Effect of Damping



Detuning

Introducing a spread in wakefield frequencies helps:

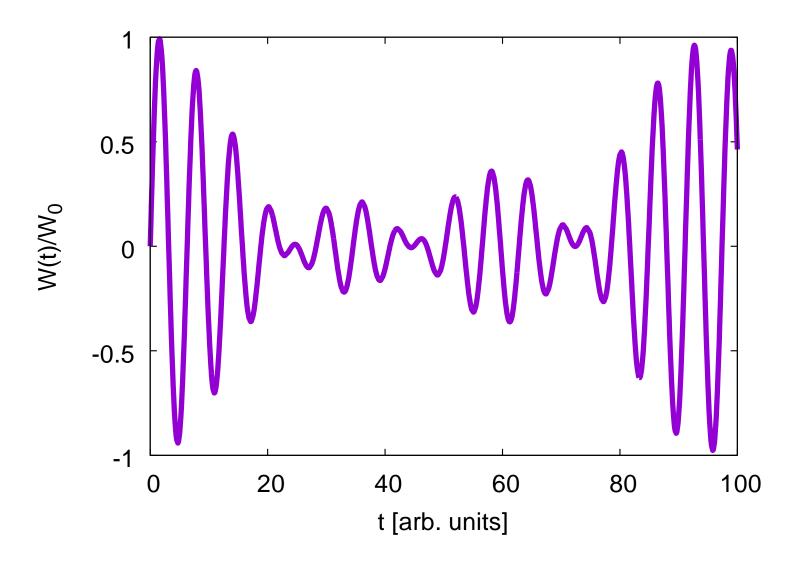
Example for two modes

$$W(z) = W_0 \frac{\sin((k+\Delta)z) + \sin((k-\Delta)z)}{2}$$

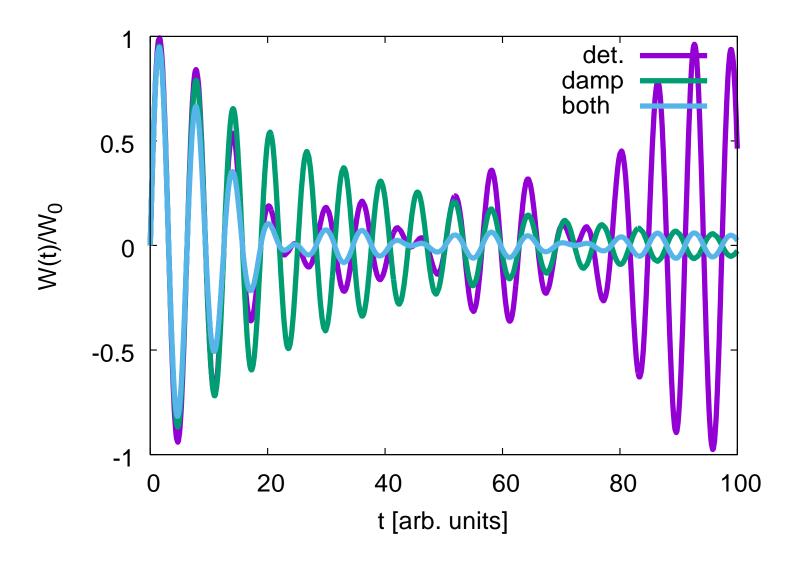
$$W(z) = W_0 \sin(kz) \cos(\Delta z)$$

In CLIC structure each cell is different, has a different transverse mode

Illustration of Detuning



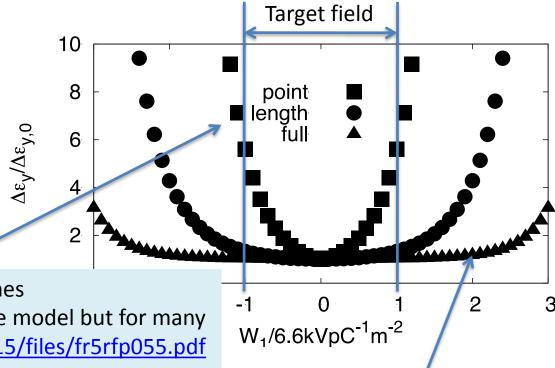
Combined Effect



Multi-bunch Effect in CLIC

Wakefield amplitudes are large Strong damping (Q=O(10)) Detuning (each cell is different)

Each bunch mainly kicks the immediately following one



Analytic estimate: point-like bunches
Using model similar to two-particle model but for many
https://cds.cern.ch/record/1227215/files/fr5rfp055.pdf

Luminosity loss is amplified by factor 4.9, acceptable

Chose smallest spacing consistent with maximum wakefield

6 buckets, i.e. 0.5ns

Fully real simulation: Energy spread stabilises, very acceptable

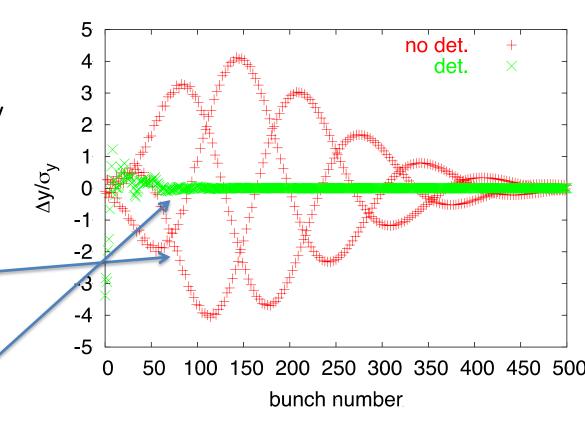
Multi-bunch effect in ILC

Small wakefield amplitudes

- little damping (Q=O(10⁴))
- random detuning cavity to cavity $(O(10^{-3}))$

Cavity misalignment simulated

No detuning is not acceptable



Residual bunch-to-bunch offsets with detuning But should be acceptable

Imperfections



Reminder

Imperfections are the main source of final vertical emittance

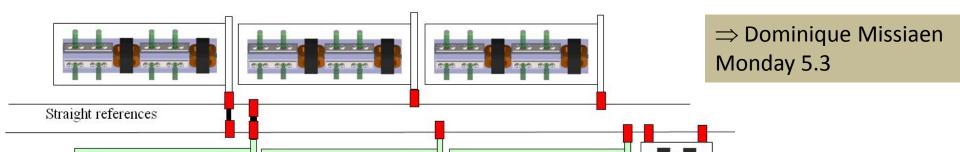
	Design limits Δε _y [nm]	Static imperfection s Δε _y [nm]	Dynamic imperfections Δε _y [nm]
Damping ring exit	5	0	0
End of RTML	1	2	2
End of main linac	0	5	5
Interaction point	0	5	5
sum	6	12	12

Discuss them for the main linac as an example

CLIC Pre-alignment Procedures

200 m

Establish reference system with overlapping wires, has some error but is not critical



Modules have sensors for wire position and can be moved remotely

The error for this is most critical, required accuracy of reference points is $O(10\mu m)$



Pre-alignment Performance

Element	error	with respect to	alignment	
			ILC	CLIC
Structure	offset	Girder	$300\mu\mathrm{m}$	5 μm
Structure	tilts	Girder	300μ radian	200(<i>*</i>)μm
Girder Girder	offset	survey line	$200\mu{ m m}$	$9.4\mu\mathrm{m}$
Quadrupole	tilt	survey line	20 μ radian	9.4 μ radian
	offset	girder/survey line	$300\mu{ m m}$	17 μm
Quadrupole	roll	survey line	300μ radian	≤ 100 µradian
BPM	offset	girder/survey line	$300\mu{ m m}$	14 <i>μ</i> m
BPM	resolution	BPM	≈1 <i>µ</i> m	$0.1 \mu m$
Wakefield mon.	offset	center wake center	_	$3.5\mu\mathrm{m}$

Difficult to pre-align components in superconducting module

Important R&D development has been carried out for CLIC

* This is mainly bookshelfing

Emittance Growth (ILC)

Error	with respect to	value	$\Delta \varepsilon_y$ [nm]
Cavity offset	module	$300~\mu \mathrm{m}$	3.5
Cavity tilt	module	300 μ radian	2600
BPM offset	module	$300~\mu \mathrm{m}$	0
Quadrupole offset	module	$300~\mu \mathrm{m}$	700000
Quadrupole roll	module	300 μ radian	2.2
Module offset	perfect line	200 μ m	250000
Module tilt	perfect line	20 μ radian	1 880

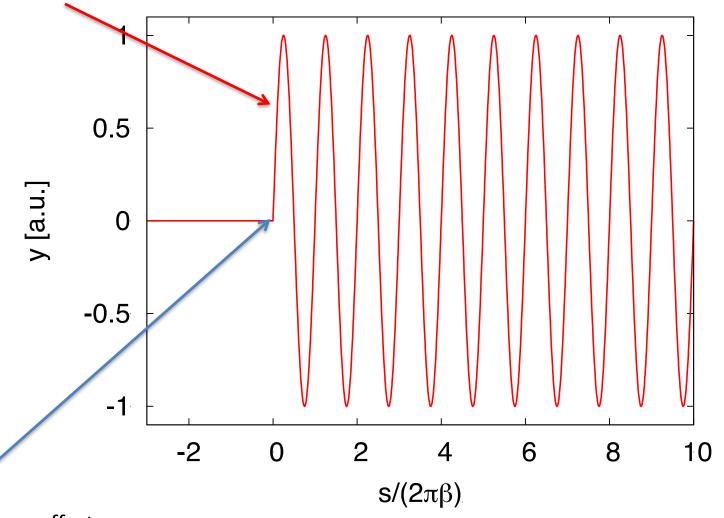
Cavity tilts are important
Beam is kicked by accelerating field

Module angles lead to cavity tilts

Module offset offsets quadrupole

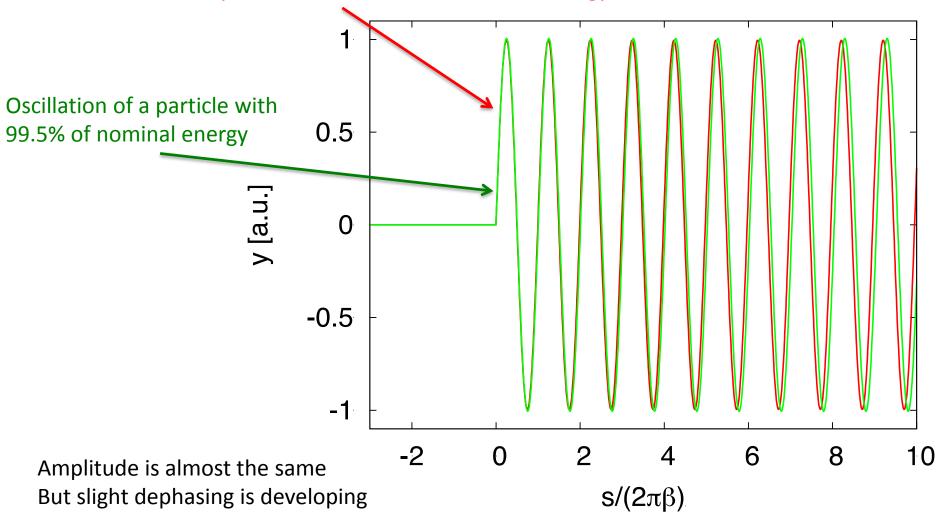
Largest problem quadrupole offsets

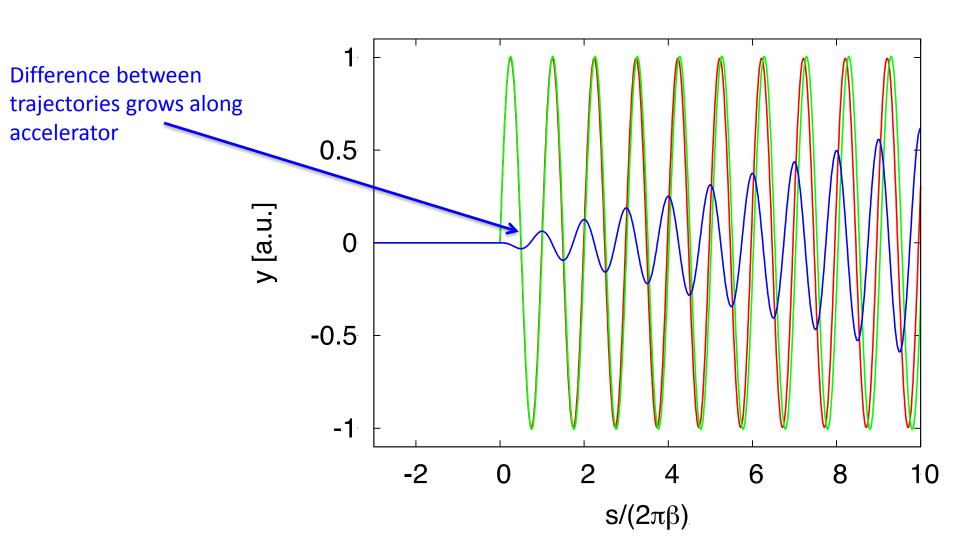
Oscillation of a particle with nominal energy

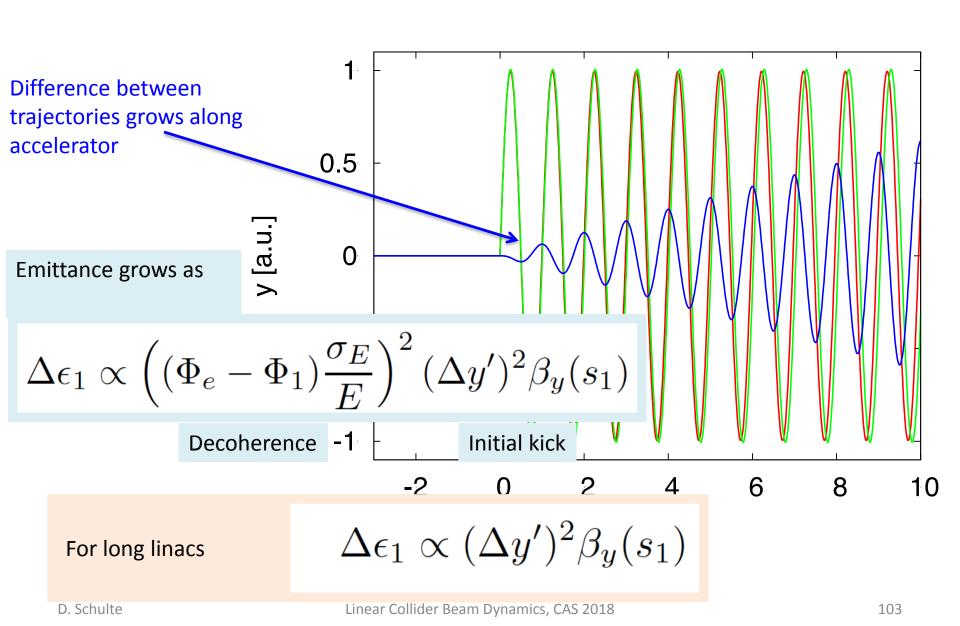


Here a kick is applied, e.g. a quadrupole with an offset

Oscillation of a particle with 100.5 % of nominal energy







Beam-based Alignment and Tuning

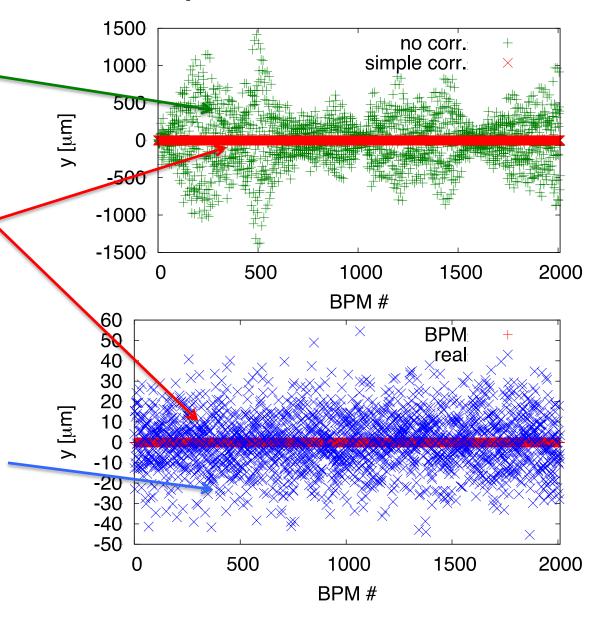
- Make beam pass linac by aligning quadrupoles
 - one-to-one correction
- Remove dispersion by aligning BPMs and quadrupoles
 - dispersion free steering
 - Ballistic alignment
 - kick minimisation
- Remove wakefields locally (CLIC only)
 - RF alignment
- Remove dispersive and wakefield effects globally
 - Emittance tuning bumps
 - Luminosity tuning bumps

Trajectory with Simple Correction

BPM readings if no beam-based correction is applied

After one-to-one correction all BPMs read zero

But beam still is offset, because BPMs have offsets

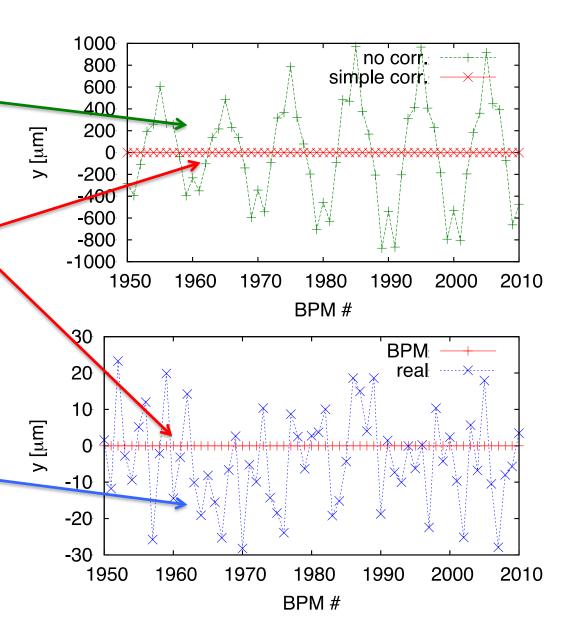


Trajectory at the End of the Linac

With no correction, at the end of the linac beam performs betatron oscillation

After one-to-one correction all BPMs read zero

No betatron oscillation has been build-up if we use one-to-one correction

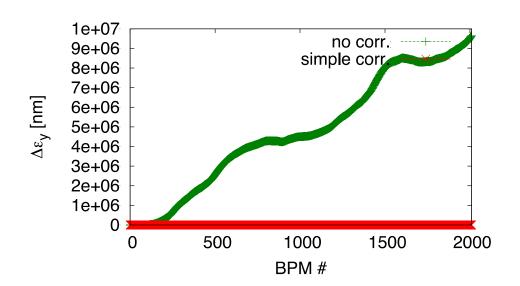


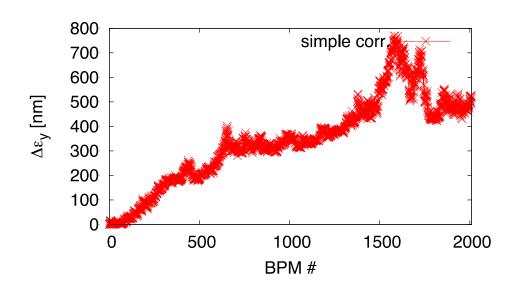
Emittance Growth

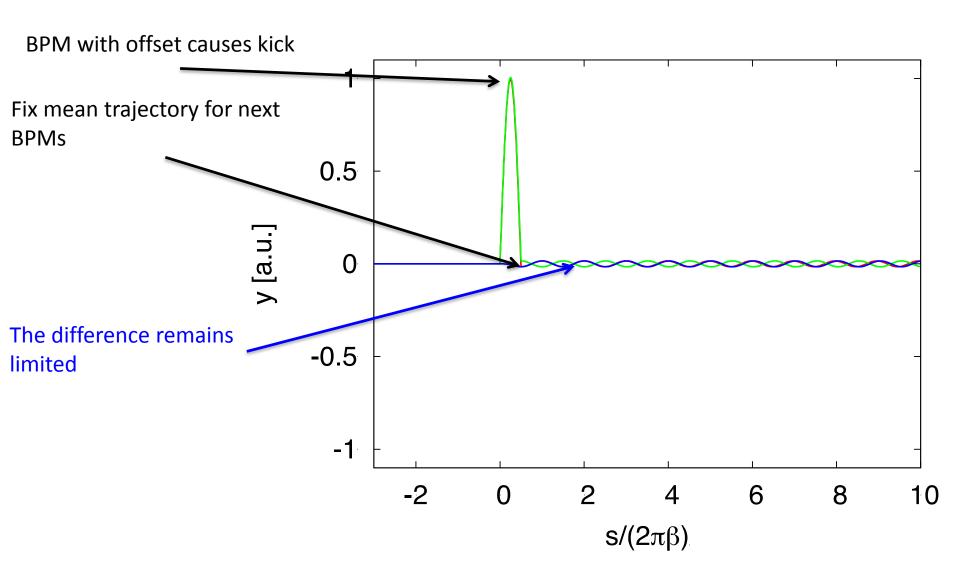
The emittance growth with no correction is very large

The simple steering yields many orders of magnitude improvement

But still the emittance growth is far above the target







Dispersion and Emittance Growth

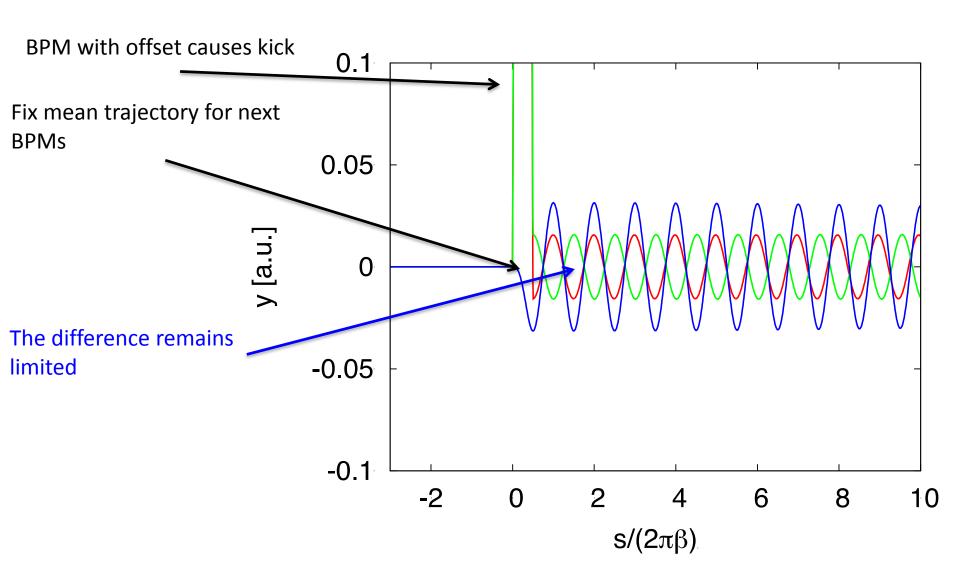
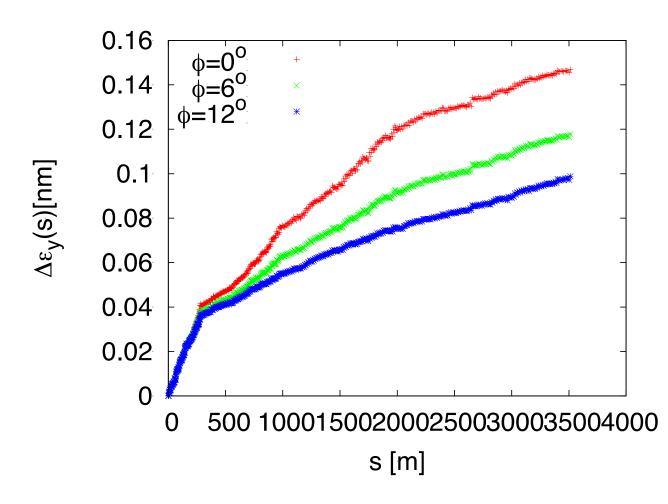


Illustration: BPM Misalignment (CLIC 380 GeV)

Larger energy spread makes us more sensitive to BPM misalignments



Emittance Growth after One-to-one Steering (ILC)

Error	with respect to	value	$\Delta \varepsilon_y$ [nm]	$\Delta \mathcal{E}_{y,121}$ [nm]
Cavity offset	module	$300~\mu m$	3.5	0.2
Cavity tilt	module	300 μ radian	2600	< 0.1
BPM offset	module	300 μ m	0	360
Quadrupole offset	module	$300~\mu m$	700000	0
Quadrupole roll	module	300 μ radian	2.2	2.2
Module offset	perfect line	200 μ m	250000	155
Module tilt	perfect line	20 μ radian	880	1.7

Quadrupole issue solved

BPM issue created Module offset leads to BPM offset

Still much better than before

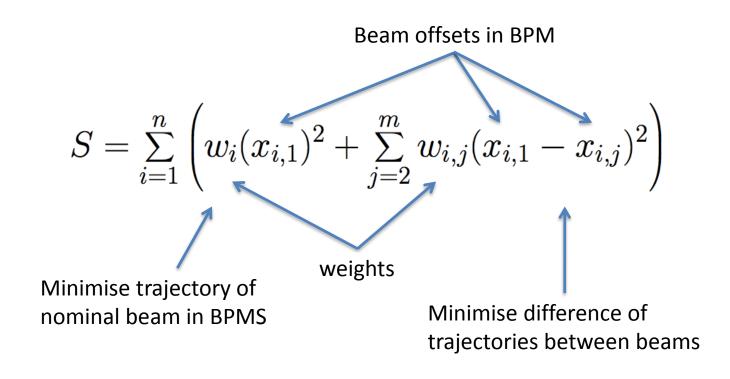
Note:

Emittance should scale as

$$\Delta\epsilon \propto (\Delta y)^2$$

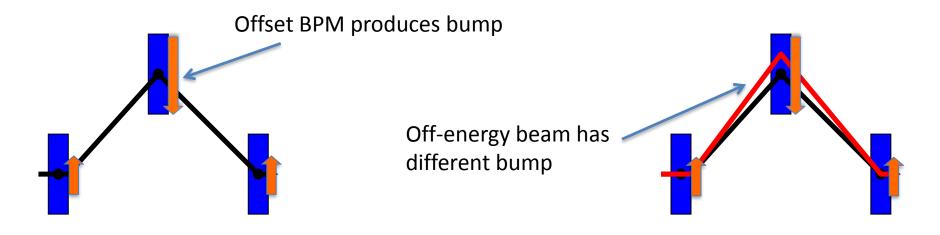
Dispersion Free Steering

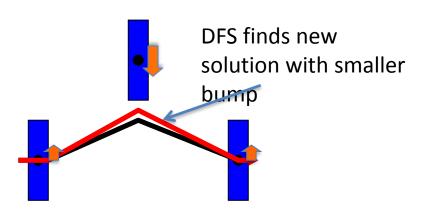
- Basic idea: use different beam energies
- Accelerate beams with different gradient and initial energy
- Optimise trajectories for different energies together

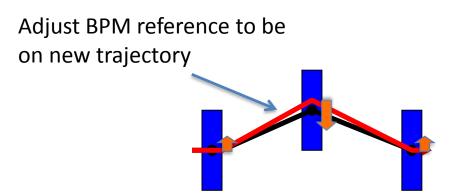


More weight on this

Dispersion Free Illustration





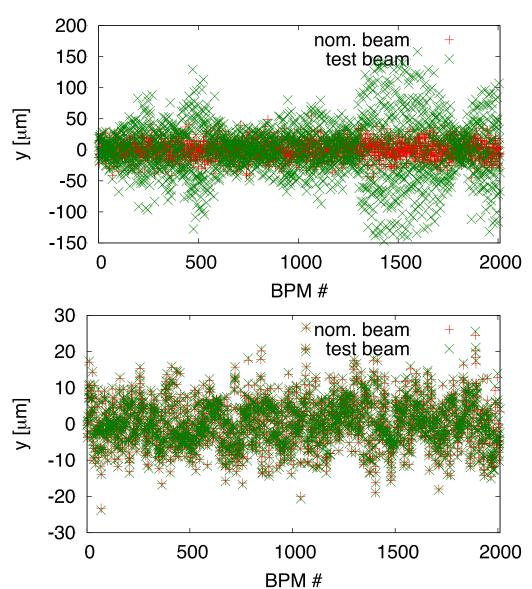


Dispersion Free Steering BPM Readings

A beam that has a different energy has a bad trajectory

The cancellation of different corrector kicks does not work very well because the phase advance is different for different energies

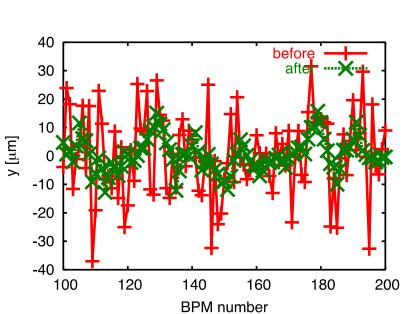
After dispersion free steering both beams take almost the same path

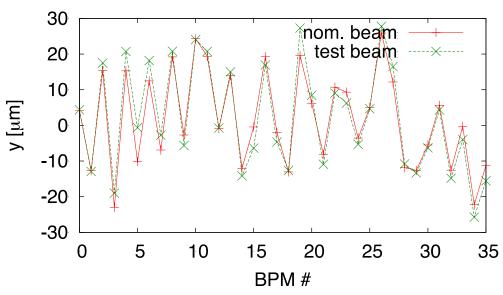


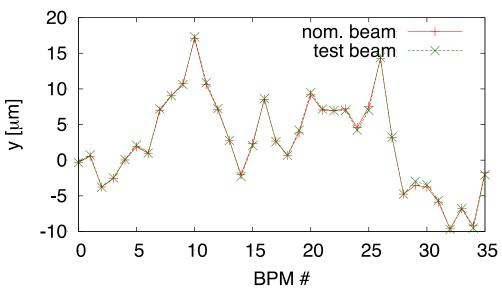
At the Beginning of the Linac

A small difference in trajectories starts between the two beams

The dispersion free steering almost completely removes this difference







Resulting Emittance Growth (ILC)

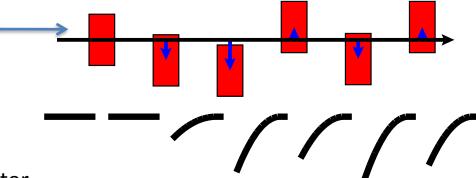
Error	with respect	value	$\Delta \varepsilon_y$ [nm]	$\Delta \varepsilon_{y,121}$ [nm]	Δε _{y,dfs} [nm]
	to				
Cavity offset	module	300 μm	3.5	0.2	0.2(0.2)
Cavity tilt	module	300 μ radian	2600	< 0.1	1.8(8)
BPM offset	module	$300~\mu \mathrm{m}$	0	360	4(2)
Quadrupole offset	module	$300~\mu \mathrm{m}$	700000	0	0(0)
Quadrupole roll	module	300 μ radian	2.2	2.2	2.2(2.2)
Module offset	perfect line	$200~\mu \mathrm{m}$	250000	155	2(1.2)
Module tilt	perfect line	20 μ radian	880	1.7	

Dispersion free steering largely cures the BPM offset issue

RF Structure Alignment

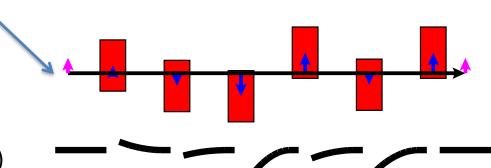
Structures are scattered on the girder

⇒ Wakefield kick



Measure beam offset with wakefield monitor Move girder to remove mean offset

⇒ No net wakefield kick

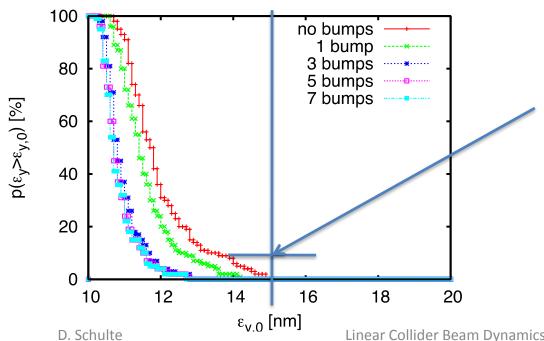


Limit mainly from

- accuracy of wakefield monitors (3.5 μm)
- reproducibility of wakefield
- tiny variation of betatron phase along girder

Final Emittance Growth (CLIC 3 TeV)

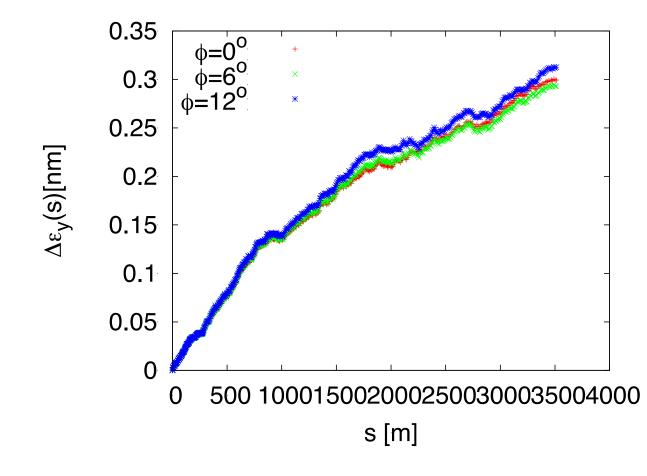
imperfection	with respect to	symbol	value	emitt. growth
BPM offset	wire reference	$\sigma_{\scriptscriptstyle BPM}$	14 <i>μ</i> m	0.367 nm
BPM resolution		σ_{res}	$0.1 \mu \mathrm{m}$	0.04 nm
accelerating structure offset	girder axis	σ_4	$10\mu{ m m}$	0.03 nm
accelerating structure tilt	girder axis	σ_t	200 μ radian	0.38 nm
articulation point offset	wire reference	σ_5	12 μm	0.1 nm
girder end point	articulation point	σ_6	5 μm	0.02 nm
wake monitor	structure centre	σ_7	$3.5\mu\mathrm{m}$	0.54 nm
quadrupole roll	longitudinal axis	σ_r	100 μ radian	≈ 0.12 nm



Goal: less than 10% above 15 nm

Further improvement using tuning bumps

Illustration: Structure Misalignment (CLIC 380 GeV)



Almost not sensitive to energy spread

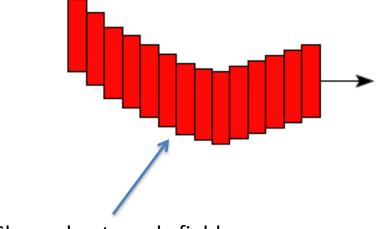
Tuning Bumps

Compensate an effect globally

Minimise beam size/emittance or maximise luminosity

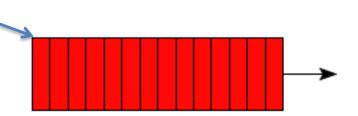
Remove a correlation between particles e.g. average wakefield kick can be compensated in one location

Energy spread and phase advance give limits

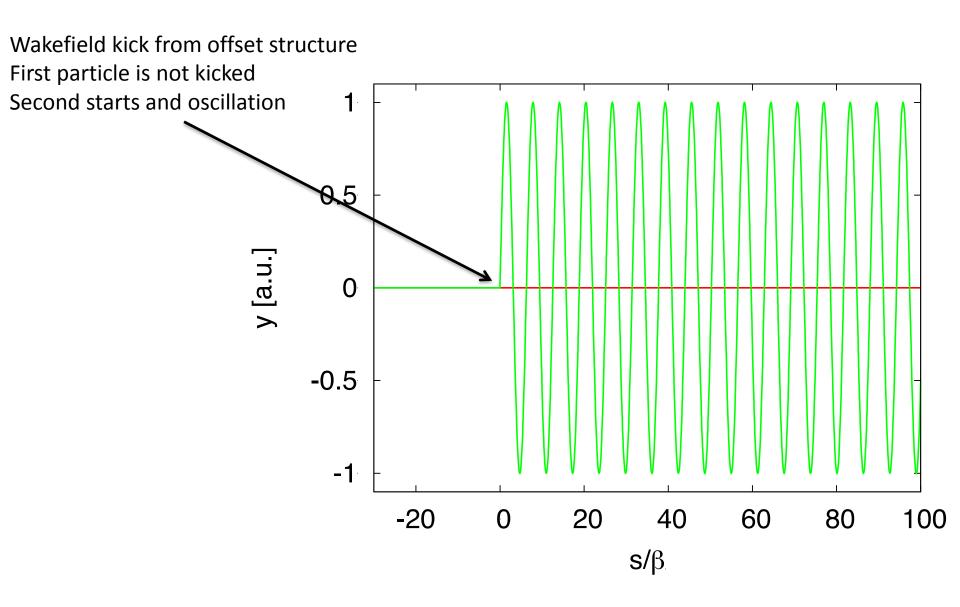


Shape due to wakefields

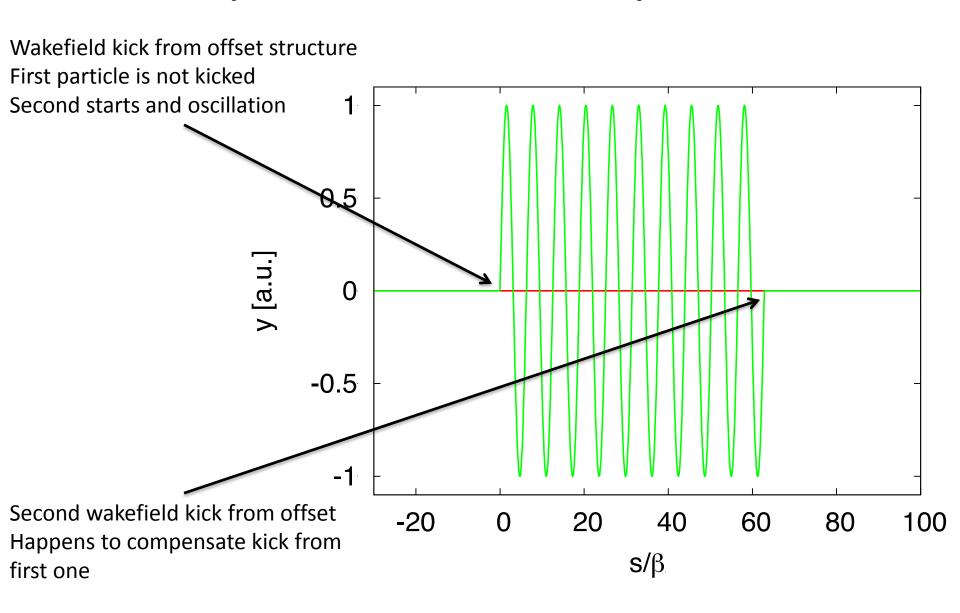
Apply wakefield kick to make bunch straight again



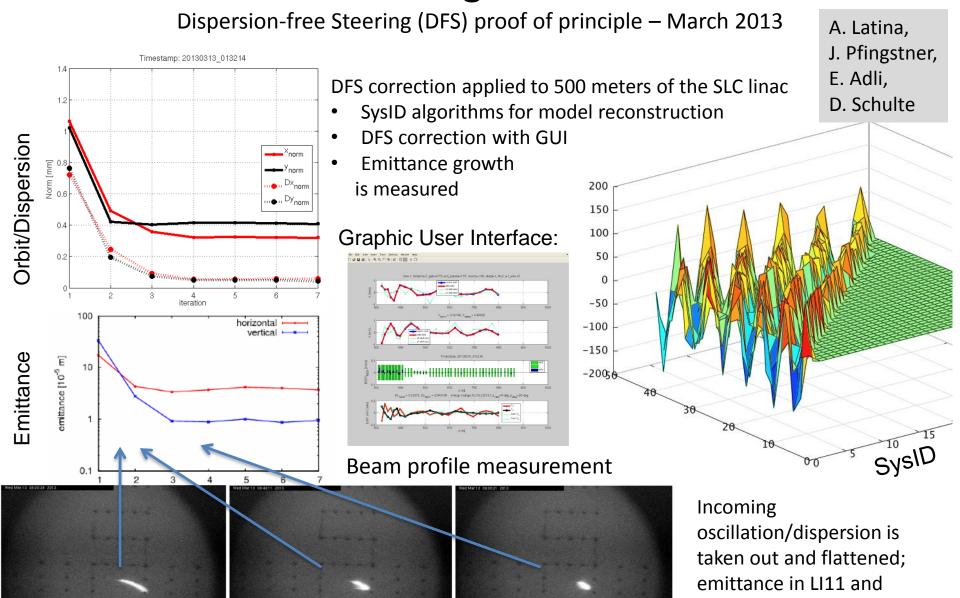
Simple Wakefield Model



Simple Wakefield Bump Model



CLIC Beam-Based Alignment Tests at FACET



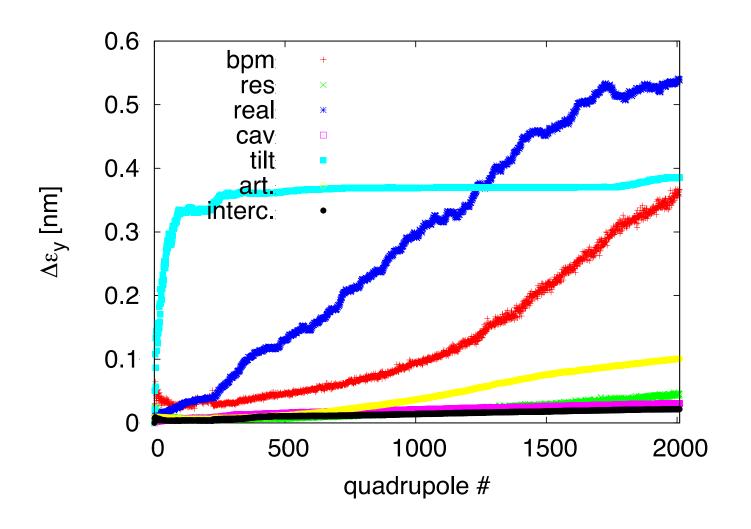
Before correction

After 1 iteration follider Beam Dy After 3 Attendations

emittance growth

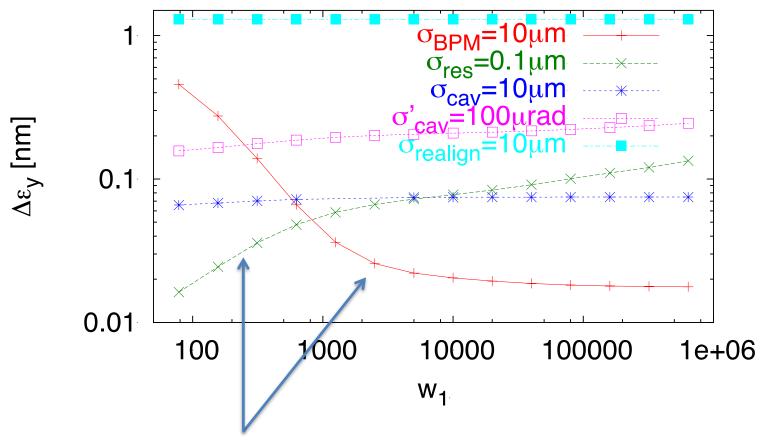
significantly reduced.

Note: Emittance Along Linac (CLIC 3 TeV)



Note: Choice of Weights

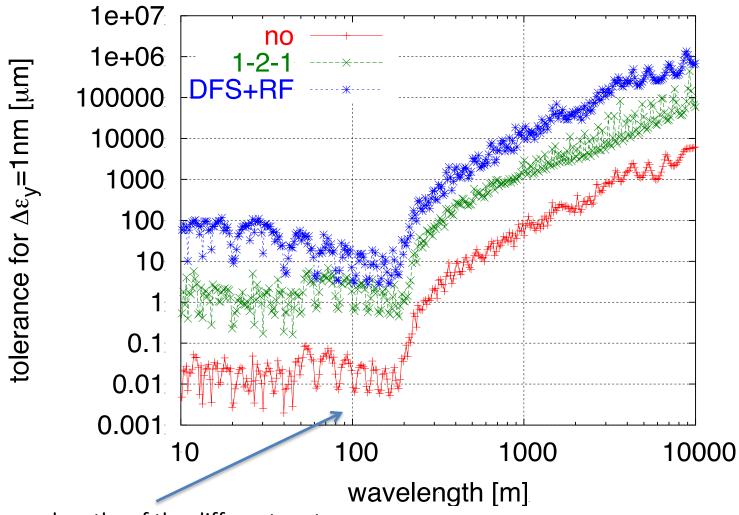
For some old CLIC design



Main trade-off is for BPM offset and resolution

Note: Emittance and Wavelength

Reference line error with given wavelength



Betatron wavelengths of the different sectors

Dynamic Imperfections



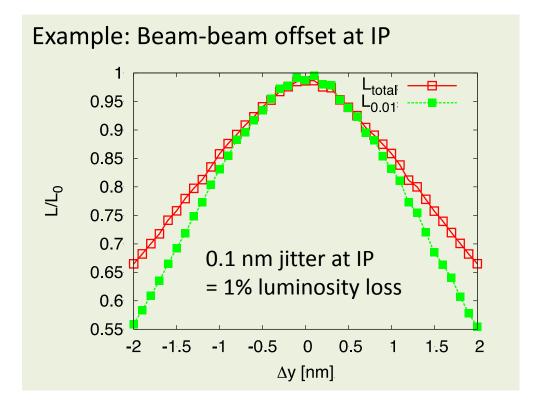
Dynamic Imperfections

Many imperfections, e.g.

- Ground motion
- RF amplitude and phase jitter
- Magnet power supply ripple
- ...

Impact on luminosity

- Direct loss
 - -(trajectory jitter, emittance growth)
- Luminosity fluctuations
 - can impact tuning
- Trajectory jitter
 - can impact beam-based alignment



Mitigation methods

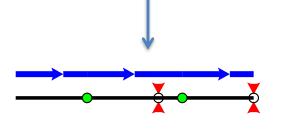
- Beam-based feedback
- Stable hardware
- Specific systems

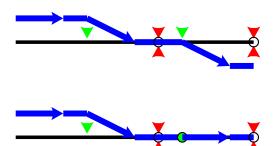
Feedback Design

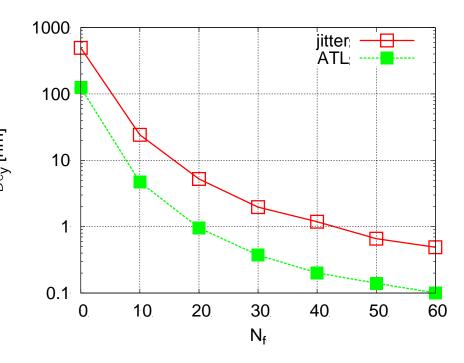
Local feedback

- E.g. fix trajectory at collision point
- But does not cure emittance growth

Several trajectory feedback points help
But lead to overcorrection if independent







Use MIMO (Multiple Input Multiple Output)

- Take all information and correct globally
- One feedback loop

Feedback Design and Speed

Local feedback

- Can be used within beam pulse
 - Marginal for CLIC (170 ns pulse, but beam-beam feedback)
 - Possible for ILC (554 ns between bunches), bunch-to-bunch noise amplified along the machine

MIMO feedback

- Used from pulse to pulse
 - Hard within ILC beam pulse (720 μs vs. 100 μs roundtrip for linac)
 - Impossible for CLIC
- Important basis of the feedback systems, e.g. trajectory feedback
- \Rightarrow But cannot correct faster than 20 μs (CLIC) and 200 μs (ILC)

⇒ Use additional feedback systems for fast effects, independent of the beam

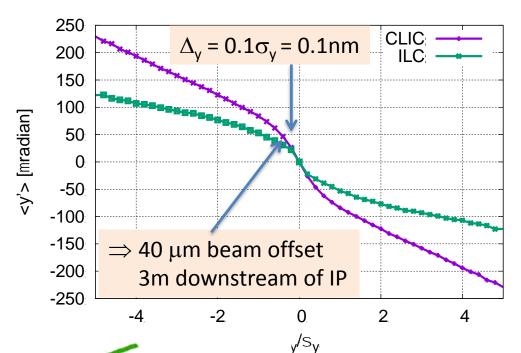
CLIC Beam-beam Feedback System

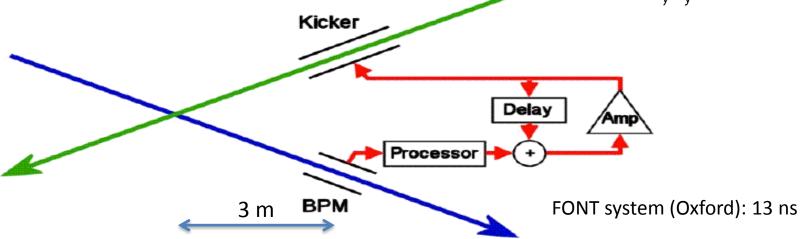
Strong deflection allows to easily measure and correct offset

10 ns from IP to BPM
13 ns to apply correction kick

10 ns from kicker to IP

33 ns latency vs 170 ns beam pulse





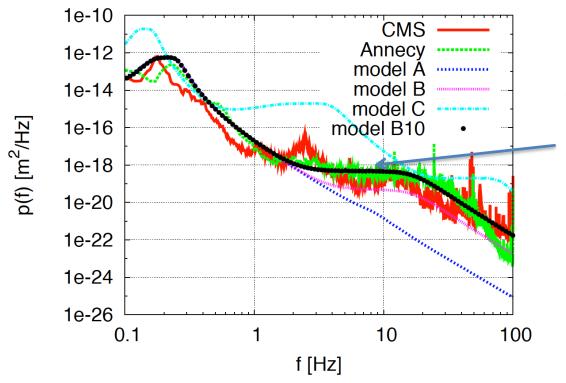
Example: Ground Motion

In CLIC can reduce dynamic effects at frequencies lower than a few Hz

⇒ Andrei Seryi Friday 2.3.

In ILC can use a bunch-bunch feedback system

- But be careful, bunch-to-bunch noise will be amplified
- e.g. the damping ring extraction kicker kicks each bunch separately, so it will induce noise

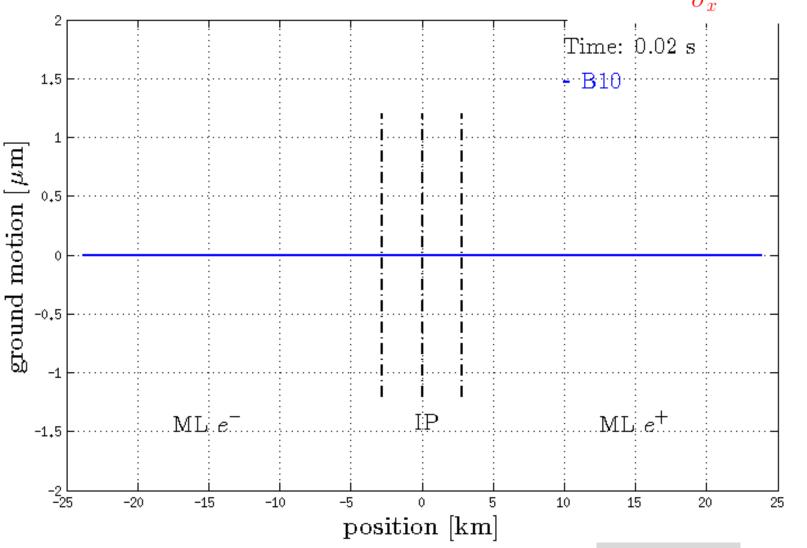


We spot a problem:

Frequencies cannot be mitigated by beam feedback

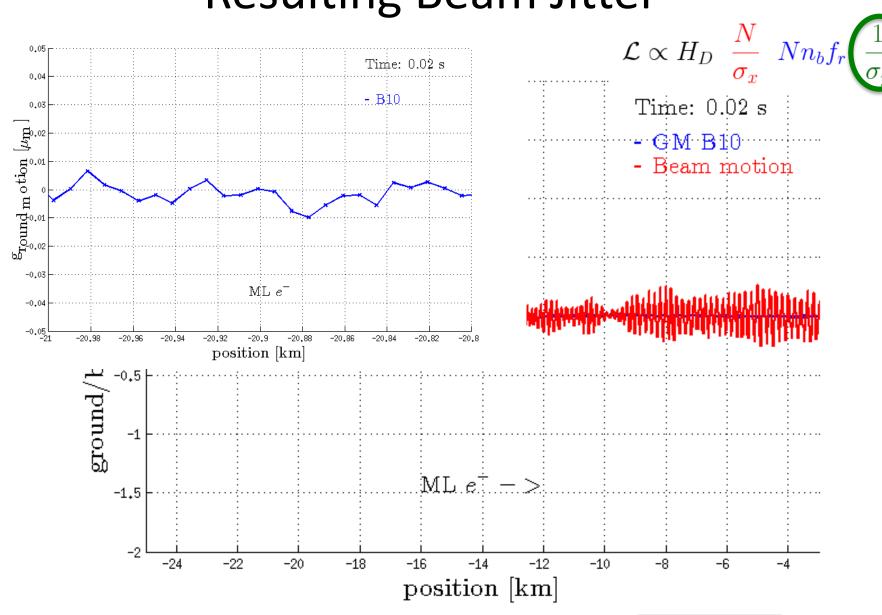
Example Issue: Ground Motion





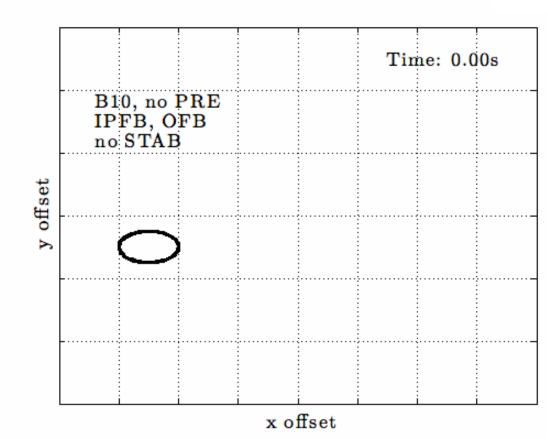
J. Pfingstner

Resulting Beam Jitter

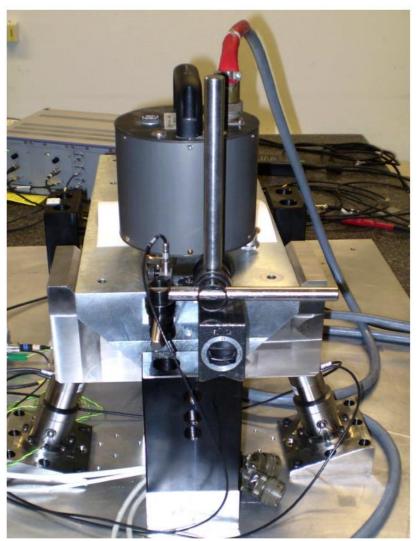


Beams at Collision

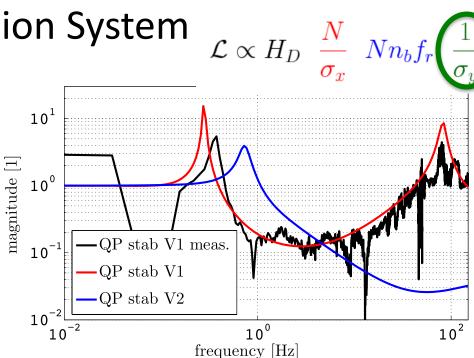


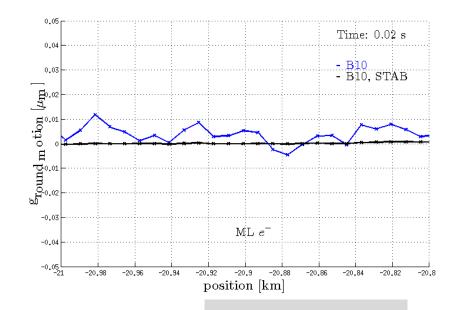


Stabilisation System



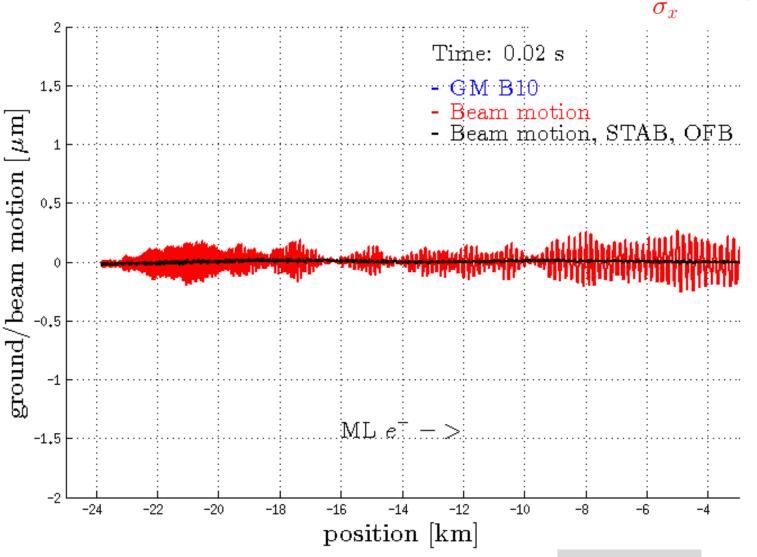
K. Artoos et al.



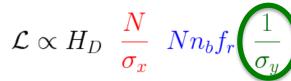


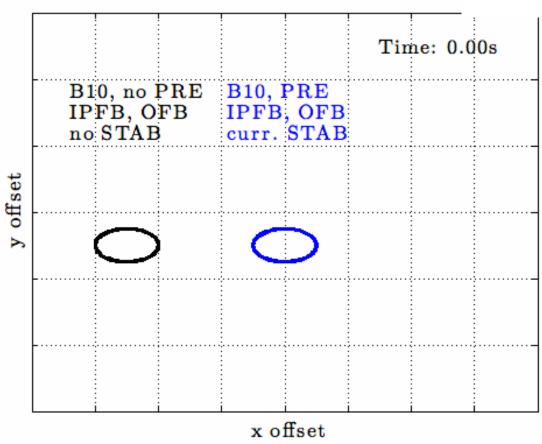
Impact of Stabilisation on Beam





Beam at Collision

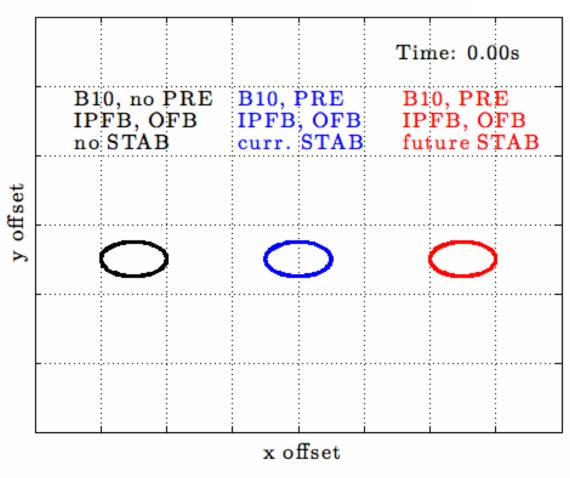




J. Pfingstner

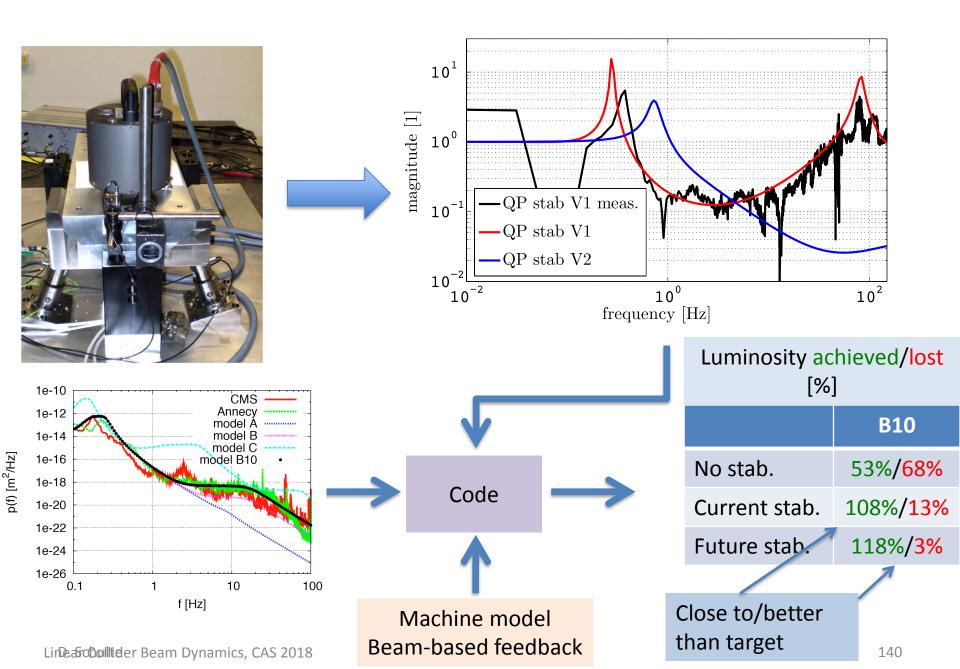
Beam at Collision





J. Pfingstner

Active Stabilisation Results



Tools and Warnings



Tools

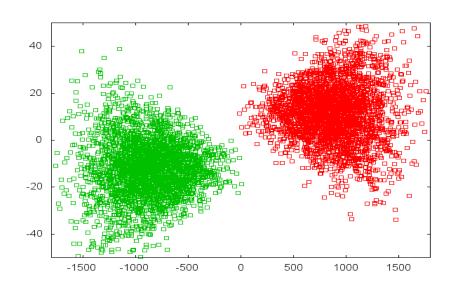
Analytic calculations to understand general physics

But need simulation tools for quantitative predictions

Beam tracking (e.g. PLACET, several other codes)



Beam-beam interaction (e.g. GUINEA-PIG, other code CAIN)

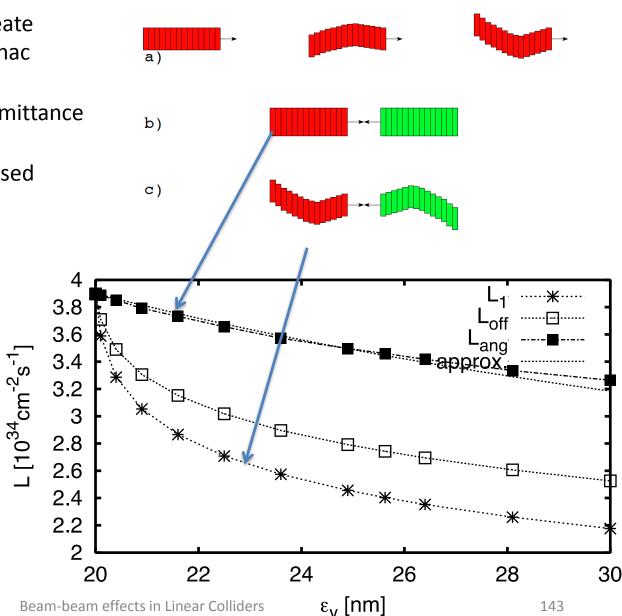


Note: The Banana Effect

- a) Wakefields+dispersion can create banana-shaped bunch in main linac
- b) Often model with projected emittance
- c) The correct shape should be used

For large disruption (ILC) banana can reduce luminosity

Study done for TESLA Similar disruption as ILC



Note: More Information

CAS on Intensity Limitations 2015 (https://indico.cern.ch/event/362960/):

Main Linac:

https://indico.cern.ch/event/362960/contributions/1776181/attachments/1181326/17104

23/Ferrario Lectrure PWA final.pdf

Beam-beam (linear collider):

https://indico.cern.ch/event/362960/contributions/1776145/attachments/1183340/17144

76/Beam-beamx.pptx

Linear Collider School 2016 (https://agenda.linearcollider.org/event/7333/)

Linac:

AB_basic https://agenda.linearcollider.org/event/7333/contributions/38078/

A1_1 https://agenda.linearcollider.org/event/7333/contributions/38078/

A1_2 https://agenda.linearcollider.org/event/7333/contributions/38084/

A1_3 https://agenda.linearcollider.org/event/7333/contributions/38090/

And much more ...

DFS steering:

SLAC-PUB-5222

Some linac formulae:

https://journals.aps.org/prab/pdf/10.1103/PhysRevSTAB.3.121002

Conclusion



Many thanks for listening

To the linar collider collaborations for the work presented

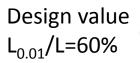
and to the people from whom I stole figures: E. Adli, K. Artoos, A. Grudiev, E. Jensen, A. Latina, H.

Mainaud Durand, J. Pfingstner, J. Snuverink, I. Syratchev, W. Wuensch

Reserve

Beamstrahlung

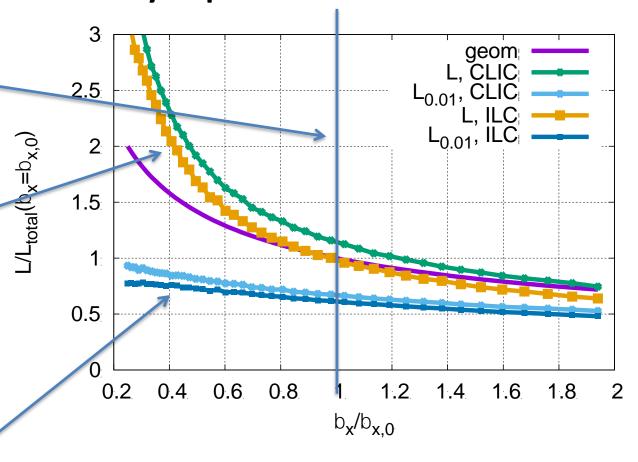
Luminosity Spectrum



The total luminosity L varies strongly with beta-function

But L_{0.01} does not change so much

Hard to push beta-functions
That low



So tend to use $L_{0.01}/L=60\%$ as criterion

Reasonable compromise for most physics studies

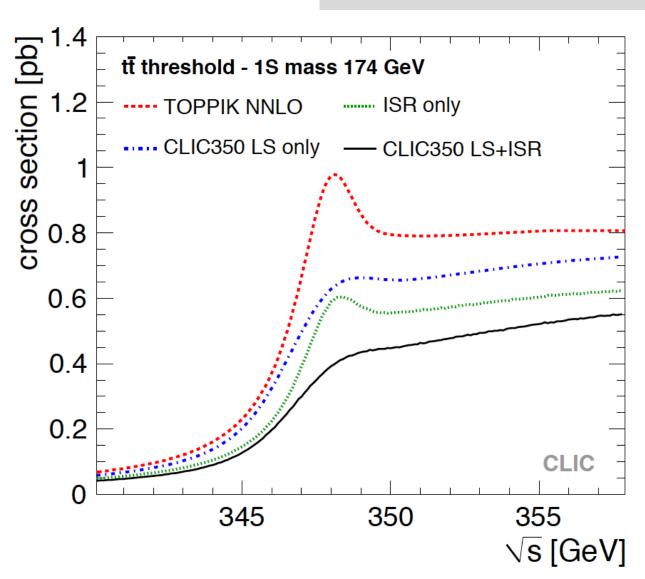
Top Production at Threshold

K. Seidel et al. arXiv:1303.3758

Top production at threshold is strongly affected by beam energy spread and beamstrahlung

For $L_{0.01} > 0.6$ L impact of beamstrahlung is comparable to ISR

But depends on physics



Note: Luminosity Drivers

In the classical regime

$$\mathcal{L} \propto H_D \; n_{\gamma} \; \eta_{RF->beam} rac{P_{RF}}{E_{cm}} \; rac{1}{\sigma_y}$$

In the quantum regime

$$\mathcal{L} \propto H_D \; rac{n_{\gamma}^{3/2}}{\sqrt{\sigma_z}} \; \eta_{RF->beam} rac{P_{RF}}{E_{cm}} \; rac{1}{\sigma_y}$$

$$\sigma_y = \sqrt{\beta_y \epsilon_y / \gamma}$$

Beamstrahlung Optimisation

For low energies (classical regime) number of emitted photons

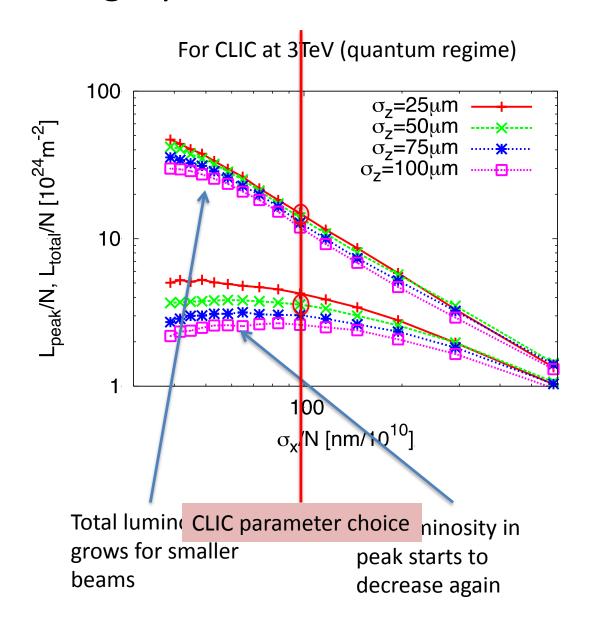
$$n_{\gamma} \propto E_{\gamma} \propto rac{N}{\sigma_x + \sigma_y}$$
 $\mathcal{L} \propto rac{N}{\sigma_x \sigma_y}$

Hence use

$$\sigma_x \gg \sigma_y$$

$$\sigma_x + \sigma_y \approx \sigma_x$$

$$\mathcal{L} \propto H_D \left(\frac{N}{\sigma_x} \right) N n_b f_r \;\; rac{1}{\sigma_y}$$



CLIC Parameter Development

How CLIC Parameters Were Done

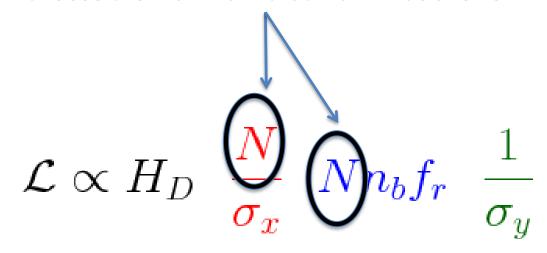
For each structure determine bunch charge and length

Determine emittance from damping ring

Add emittance growth based on detailed studies

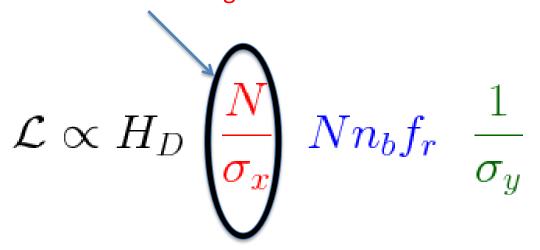
$$\mathcal{L} \propto H_D \ \frac{N}{\sigma_x} \ N n_b f_r \ \frac{1}{\sigma_y}$$

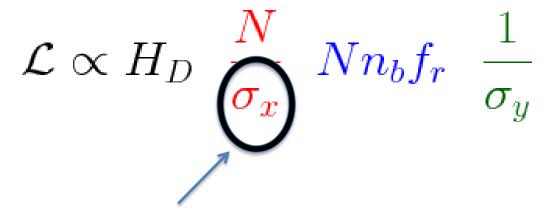
Choose the maximum that main linac allows



Note: Choosing a different structure gives a different limit

Nature defines minimum beam size as function of bunch charge



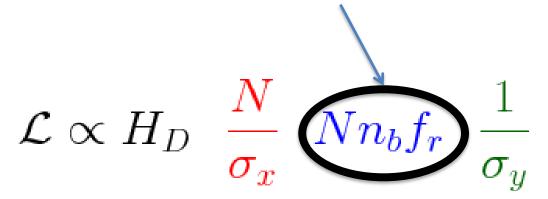


Damping ring and BDS define minimum beam size Can achieve our target but not go much below

Note: A bit of self-fulfilling prophecy

Minimum beam size is know in optimisation Structures that require small N are removed because of low luminosity

Higher efficiency in main linac would allow for more beam current



Note: A bit of self-fulfilling prophecy
Structures that allow only small N are good for RF
But the luminosity is low so the are not liked by the optimisation

$$\mathcal{L} \propto H_D \frac{N}{\sigma_x} \frac{N n_b f_r}{\sigma_y}$$

If we could do better, we would

All system contribute to vertical emittance Mainly due to imperfections

BDS defines beta-function limit (some contribution from main linac)

Equivalence of Power and Field Picture

Note: Equivalence of Wakefield and Power Picture

Why can a wakefield model be used for the beam loading?
i.e.

$$\Delta G(q) = \text{const } q$$

• The energy stored per unit length in the accelerating structure is

$$E'(s) = \frac{G(s)^2}{(R'/Q)(s)\omega}$$

- The reduction of acclerating field due to the passing charge q is $-\Delta G(s)$
- This yields for the energy lost by the structure

$$\Delta E'_{lost}(s) = \frac{G^2(s) - (G(s) - \Delta G(s))^2}{(R'/Q)(s)\omega} \Rightarrow \Delta E'_{lost}(s) = \frac{2G(s)\Delta G(s) - (\Delta G(s))^2}{(R'/Q)(s)\omega}$$

The beam extracts an energy

$$\Delta E'_{beam}(s) = q \left(G(s) - \frac{1}{2} \Delta G(s) \right)$$

hence

$$\begin{split} q\left(G(s) - \frac{1}{2}\Delta G(s)\right) &= \frac{2G(s)\Delta G(s) - (\Delta G(s))^2}{(R'/Q)(s)\omega} \\ \Rightarrow \Delta G(s) &= \frac{(R'/Q)(s)\omega}{2}q \end{split}$$

- ⇒ The gradient change depends only on the charge not the initial gradient, as expected
 - Note: I simplified a bit (sorry, but this is easier with cheating)

Single Bunch Energy Spread Simple Model

Simplified Treatment of Single Bunch Energy Spread

Assume

- $W_z(s) = W_z = \text{const}$
- ullet uniform bunch with length $L\ll\lambda$
- and use linear approximation

Field seen by first particle

$$G_H = G\cos\left(\phi - \frac{L}{2}\frac{2\pi}{\lambda}\right) \approx G\left(\cos(\phi) - \frac{L}{2}\frac{2\pi}{\lambda}\sin(\phi)\right)$$

Field seen by last particle

$$G_T = G\cos\left(\phi + \frac{L}{2}\frac{2\pi}{\lambda}\right) \approx G\left(\cos(\phi) + \frac{L}{2}\frac{2\pi}{\lambda}\sin(\phi)\right) - NeW_z$$

We require (this automatically solves the equation for all other particles)

$$G_H = G_T$$

which leads to

$$L = \frac{NeW_z}{G} \frac{\lambda}{2\pi \sin(\phi)}$$

Speed of Deconeherence in Simple Model

Beam Offset and Decoherence

Solve equation of motion for particle with different energy

$$x_2(0) = x_0 \quad x_2'(0) = 0$$
$$x_2''(s) + \frac{1}{\beta_2^2} x_2(s) = x_2''(s) + \frac{1}{(1+\delta)\beta^2} x_2(s) = 0$$

Solution has the same amplitude (small difference in maximum angle) but different frequency Particle will decohere from the nominal energy

$$x_2(s) = x_0 \cos\left(\frac{s}{\beta_2}\right) = x_0 \cos\left(\frac{s}{\sqrt{1+\delta\beta}}\right) \approx x_0 \cos\left(\frac{s}{(1+0.5\delta)\beta}\right)$$

With energy spread beam offset will turn into beam size growth

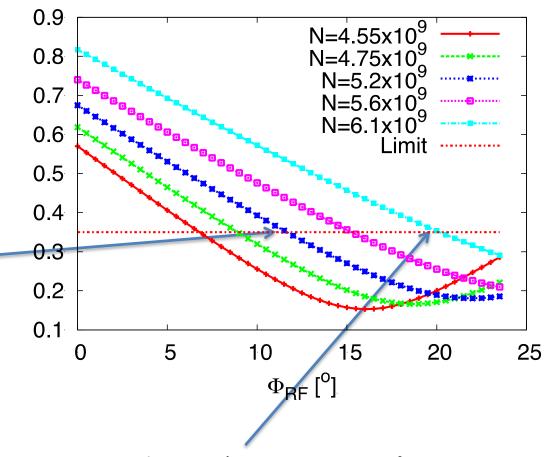
Choice of Mean RF Phase in CLIC

Note: Choice of Average RF Phase

Examples of beams with the same transverse wakefield effects i.e. larger N means shorter bunch

CLIC bunch at 380 GeV Running at 12° 2% gradient loss

12° is a good compromise



17% more charge requires 20°6% gradient lossMore sensitive to phase jitter

Emittance Bumps in ILC

Some Old Example for ILC

