Beam-beam effects

(an introduction)

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Why talk about beam-beam effects ?

Colliding beams in storage rings require a long lifetime to produce useful physics

Two principles are important threats to this lifetime:

1. Magnet errors
2. Beam-beam interactions (unavoidable even in principle in colliders)

For beams in collision the beam-beam interactions are the most significant source of nonlinear consequence

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Popular types of colliders:

Particle types:

1. p – p, p – p̄
2. e + e–, e – e– **(in the past ..)**
3. ion – ion, p – ion
4. lepton – hadron
5. photon – photon

Main categories of colliders:

1. **Circular colliders (mainly hadrons, low energy leptons)**
2. **Linear colliders (mainly leptons)**

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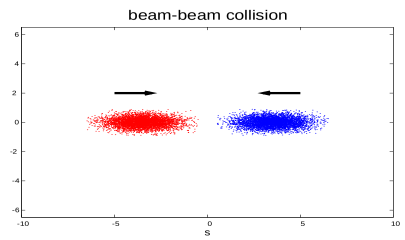
Beam-beam collision



Typically:

- **0.001% (or less) of particles have useful interactions**
- **99.999% (or more) of particles are perturbed**

Some challenges (beam-beam related, incomplete):

Linear colliders:

1. **Luminosity, high energy**
2. **Beams are discarded after the collision (1 beam-beam interaction)**
3. **Strong focusing at collision point, beamstrahlung**
4. **...**

Circular colliders:

1. **One or two rings (exotic: 4 beam collisions)**
2. **Beams are re-used - LHC: $\geq 5 \cdot 10^{10}$ beam-beam interactions per production run (fill) → challenge for the beam dynamics (many different types of beam-beam effects to be understood and controlled)**
3. **...**

Very different problems must be expected, already at early design phase

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We are interested in three main questions:

- **What happens to a single particle ?**
- **What happens to the whole beam?**
- **What happens to the machine?**

Unlike other effects, e.g. space charge or magnetic errors, the beam-beam effects do not become smaller with increasing energy, usually the effects become more important (so far it was always the case)

Unfortunately, despite all progress not all aspects are well understood and a general theory does not exist.

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Beams are always a collection of (a large number of) charges

- Represent electromagnetic potential for other charges, resulting in:

  Forces on itself **(space charge)**

  Forces on opposing beam **(beam-beam effects)**

- Important for high density beams, i.e. high intensity and/or small beams: for **high luminosity !**

Remember (Bruno):

$$L = \frac{N_1 N_2 f n_B}{4\pi\sigma_x\sigma_y} = \frac{N_1 N_2 f n_B}{4\pi \cdot \sigma_x\sigma_y}$$

High luminosity is not good for beam-beam effects ...
Beam-beam effects are not good for high luminosity ...

- Overview: which effects are important for present and future machines (LEP, PEP, Tevatron, RHIC, LHC, FCC, linear colliders, ...)
- Qualitative and physical picture of the effects

Derivations in:
Proceedings, Advanced CAS, Trondheim (2013)
http://cern.ch/Werner.Herr/CAS2011_Chios/bb/bb1.pdf

One has to evaluate three main types of colliding beam machines:
e.g. magnets:

1. Circular hadron colliders
2. Circular lepton colliders
3. Linear lepton colliders

Although 1 and 2 result in very different problems, in the first part the treated is very similar

Early warning: occasionally I shall make assumptions and/or simplifications, but only to keep the formulae readable, no compromise on the physics !

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A beam acts on particles like an electromagnetic lens, but unlike e.g. magnets:

- Very local force
- Very non-linear form of the forces, depending on particle distribution
- Does not represent simple form, i.e. well defined multipoles

Can change distribution as result of interaction (time dependent forces ..)

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Studying beam-beam effects

- Need to know the forces
- Apply concepts of non-linear dynamics
- Apply concepts of multi-particle dynamics
- Analytical models and simulation techniques well developed in the last 20 years (but still a very active field of research)

LHC is a wonderful epitome as it exhibits many of the features revealing beam-beam problems

First step: Fields and Forces

Need fields $\vec{E}$ and $\vec{B}$ of opposing beam with a particle distribution $\rho(x, y, z)$

In rest frame (denoted ′) only electrostatic field: $\vec{E}'$, and $\vec{B}' \equiv 0$

Derive potential $U(x, y, z)$ from ρ and Poisson equation:

$$\Delta U(x, y, z) = -\frac{1}{\epsilon_0}\rho(x, y, z)$$

The electrostatic fields become:

$$\vec{E}' = -\nabla U(x, y, z)$$

Transform into moving frame and calculate Lorentz force $\vec{F}$ on particle with charge $q = Z_2$ e

$$E_{\parallel} = E'_{\parallel}, \quad E_{\perp} = \gamma \cdot E'_{\perp} \ \text{ with : } \ \vec{B} \ = \ \vec{\beta} \times \vec{E}/c$$

$$\vec{F} = q(\vec{E} + \vec{\beta} \times \vec{B})$$

Example Gaussian distribution (not surprising):

$$\rho(x, y, z) = \frac{NZ_1 e}{\sigma_x \sigma_y \sigma_z \sqrt{2\pi}^3} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}\right)$$

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Simple example: Gaussian

For 2D case the potential becomes (see proceedings):

$$U(x, y, \sigma_x, \sigma_y) = \frac{NZ_1 e}{4\pi\epsilon_0} \int_0^{\infty} \frac{\exp(-\frac{x^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q})}{\sqrt{(2\sigma_x^2 + q)(2\sigma_y^2 + q)}} dq$$

Once known, can derive $\vec{E}$ and $\vec{B}$ fields and therefore forces

For arbitrary distribution (non-Gaussian): difficult (or impossible, numerical solution required)

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Force for round Gaussian beams

Assumption 1: $\sigma_x = \sigma_y = \sigma$, $Z_1 = -Z_2 = 1$

Assumption 2: very relativistic → $\beta \approx 1$

- **Only components E_r and B_Φ are non-zero**
- **Force has only radial component, i.e. depends only on distance r from bunch centre where:** $r^2 = x^2 + y^2$

$$F_r(r) = -\frac{Ne^2(1+\beta^2)}{2\pi\epsilon_0 \cdot r}\left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

Compare with other forces (cheating → in 1D):

quadrupole force:

$F(x) \propto x$

sextupole force:

$F(x) \propto x^2$

octupole force:

$F(x) \propto x^3$

beam-beam force:

$$F_r(r) = \pm\frac{Ne^2(1+\beta^2)}{2\pi\epsilon_0 \cdot r}\left[1 - \exp(-\frac{r^2}{2\sigma^2})\right] \quad \text{(remember : } r = \sqrt{x^2 + y^2})$$

... and we still assume $\sigma_x = \sigma_y = \sigma$!!

If not (usually $\sigma_x \neq \sigma_y$ in lepton colliders) ?

$$E_x = \frac{ne}{2\epsilon_0 \sqrt{2\pi(\sigma_x^2 - \sigma_y^2)}} \mathrm{Im} \left[\mathrm{erf}\left(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}} \right) - e^{\left(-\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}\right)} \mathrm{erf}\left(\frac{x\frac{\sigma_y}{\sigma_x} + iy\frac{\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2-\sigma_y^2)}} \right) \right]$$

$$E_y = \frac{ne}{2\epsilon_0 \sqrt{2\pi(\sigma_x^2 - \sigma_y^2)}} \mathrm{Re} \left[\mathrm{erf}\left(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}} \right) - e^{\left(-\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}\right)} \mathrm{erf}\left(\frac{x\frac{\sigma_y}{\sigma_x} + iy\frac{\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2-\sigma_y^2)}} \right) \right]$$

The function erf(t) is the complex error function

$$\mathrm{erf(t)} = e^{-t^2} \left[1 + \frac{2i}{\sqrt{\pi}} \int_0^t e^{z^2}\, dz \right]$$

The magnetic field components follow from:

$$B_y = -\beta_r E_x / c \quad \text{and} \quad B_x = \beta_r E_y / c$$

Assumption/simplification: we shall continue with round beams ...

Fields are similar for Space Charge - what makes the difference ?

For the forces:

- **Space charge dependence on momentum:** $(1-\beta^2)$

  Vanishes at high energy (almost always in lepton machines)

- **beam-beam:** $(1+\beta^2)$ **instead of** $(1-\beta^2)$

  Does not get smaller with increasing energy

Space charge is "smooth", often integrable

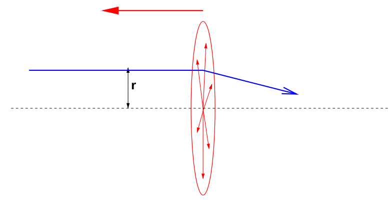
Beam-beam is extremely local, almost never integrable

→ **Totally different problems and effects !**

However: beam-beam only in colliders ..

Beam-beam kick:

- We are careless[a] and use (x, x', y, y') as coordinates
- We need the deflections (kicks $\Delta x'$, $\Delta y'$) of the particles:



Incoming particle (from left) deflected by force from opposite beam (from right)

Deflection depends on the distance r to the centre of the fields

[a] however there is a reason

- Kick ($\Delta r'$): angle by which the particle is deflected during the passage
- Integration of force over the collision, i.e. time of passage Δt (assuming: $m_1 = m_2$ and $Z_1 = -Z_2 = 1$):

$$\text{Newton's law}: \quad \Delta r' = \frac{1}{mc\beta\gamma} \int_{-\frac{\Delta t}{2}}^{+\frac{\Delta t}{2}} F_r(r, s, t) dt$$

with:

$$F_r(r, s, t) = -\frac{Ne^2(1+\beta^2)}{\sqrt{(2\pi)^3}\epsilon_0 r \sigma_s} \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right] \cdot \left[\exp(-\frac{(s+vt)^2}{2\sigma_s^2})\right]$$

➡ **Using the classical particle radius (implies $Z_1 = \pm Z_2$):**

$$r_0 = e^2/4\pi\epsilon_0 mc^2$$

we have (radial kick and in <u>normalized</u> Cartesian coordinates):

$$\Delta r' = -\frac{2Nr_0}{\gamma} \cdot \frac{r}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

$$\Delta x' = -\frac{2Nr_0}{\gamma} \cdot \frac{x}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

$$\Delta y' = -\frac{2Nr_0}{\gamma} \cdot \frac{y}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

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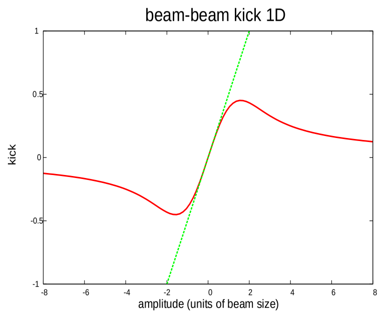
Beam-beam force/kick



- **For small amplitude: linear force (like quadrupole)**
- **For large amplitude: very non-linear force**
- **Particles in a bunch oscillate across the force**

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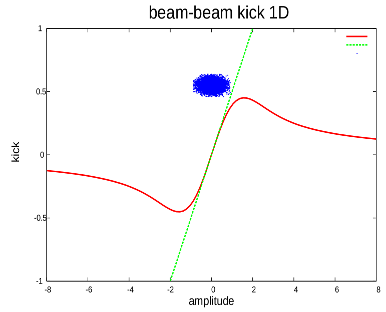
What the particles "see":

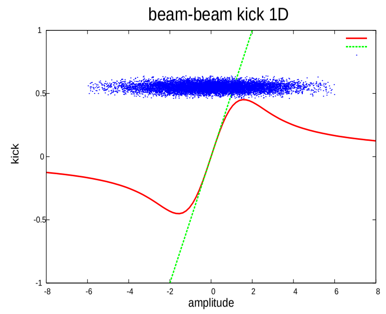


- For small amplitudes: looks like a quadrupole, non-linear part not "seen"

A quadrupole changes the tune, so must beam-beam

Slope defines the "sign" and the "gradient" of the "quadrupole"

What the particles "see":



For large amplitude: particles move across the non-linear part
→ amplitude dependent tune shift

The particles "see" more than one slope (left and right of the peak) ...

Can we quantify the beam-beam strength ?

Tune shift may be a good indicator

- We need the slope of force (kick $\Delta r'$) at zero amplitude (for the time being)
- This defines: beam-beam parameter ξ
- For head-on interactions and round beams ($\beta^* = \beta^*_x = \beta^*_y$):

$$\xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{N \cdot r_o \cdot \beta^*}{4\pi\gamma\sigma^2}$$

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Can we quantify the beam-beam strength ?

- In general for non-round beams ($\beta^*_x \neq \beta^*_y$):

$$\xi_{x,y} = \frac{N \cdot r_o \cdot \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

- Proportional to (linear) tune shift ΔQ_{bb} from beam-beam interaction: $\Delta Q_{bb} \propto \pm \xi$

Good: Reliable measure for strength of beam-beam interaction

Bad: Does not take into account the non-linear part

Ugly: Eventually we have to ..

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some examples: LEP - LHC

	LEP (e^+e^-)	LHC (pp) (2011)
Beam sizes	$\approx$ 200μm · 4μm	$\approx$ 30μm · 30μm
Intensity N	4.0 · 10^{11}/bunch	1.40 · 10^{11}/bunch
Energy	100 GeV	3500 GeV
$\epsilon_x \cdot \epsilon_y$	($\approx$) 20 nm · 0.2 nm	0.5 nm · 0.5 nm
$\beta_x^* \cdot \beta_y^*$	($\approx$) 1.25 m · 0.05 m	1.00 m · 1.00 m
Crossing angle	0.0	240 μrad
Beam-beam parameter(ξ)	0.0700	0.0080 (0.0037)

Back to:

$$\xi_{x,y} = \frac{N \cdot r_o \cdot \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x+\sigma_y)} \propto \frac{\beta^*_{x,y}}{\sigma_{x,y}(\sigma_x+\sigma_y)}$$

i.e. $\xi_x \neq \xi_y$ **for flat beams (mostly leptons) but:**

$$\xi_{x,y} = \frac{\beta_{x,y}}{\sigma_{x,y}} \cdot \frac{1}{(\sigma_x + \sigma_y)} = \frac{\sqrt{\beta_{x,y}}}{\sqrt{\epsilon_{x,y}}} \cdot \frac{1}{(\sigma_x + \sigma_y)}$$

$$\text{if}: \quad \frac{\beta_x}{\beta_y} = \frac{\epsilon_x}{\epsilon_y} \quad \Longrightarrow \quad \xi_x = \xi_y$$

For lepton machines this ratio is sometimes adjusted ...

→ $\xi_x = \xi_y = \xi$ **becomes a minimum**

Still, how big is the tune shift

Take only the linear part

Transformation matrix over the interaction becomes (like thin quadrupole - beam-beam is really thin !) :

$$\begin{pmatrix} 1 & 0 \\ \frac{1}{-f} & 1 \end{pmatrix}$$

For small amplitudes linear force like a quadrupole with focal length f

$$\frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \left[\frac{\xi \cdot 4\pi}{\beta^*}\right]$$

"Full turn matrix" without beam-beam:

$$\begin{pmatrix} \cos(2\pi(Q)) & \beta^* \sin(2\pi(Q)) \\ -\frac{1}{\beta^*} \sin(2\pi(Q)) & \cos(2\pi(Q)) \end{pmatrix}$$

Add "beam-beam thin lens", i.e. the (linear) beam-beam focusing:

$$\begin{pmatrix} \cos(2\pi Q) & \beta_0^* \sin(2\pi Q) \\ -\frac{1}{\beta_0^*} \sin(2\pi Q) & \cos(2\pi Q) \end{pmatrix} \circ \begin{pmatrix} 1 & 0 \\ \frac{1}{-f} & 1 \end{pmatrix}$$

we allow for a change of the tune Q and β :

should become $$= \begin{pmatrix} \cos(2\pi(Q+\Delta Q)) & \beta^* \sin(2\pi(Q+\Delta Q)) \\ -\frac{1}{\beta^*} \sin(2\pi(Q+\Delta Q)) & \cos(2\pi(Q+\Delta Q)) \end{pmatrix}$$

Solving this equation gives us:

$$\cos(2\pi(Q + \Delta Q)) = \cos(2\pi Q) - \frac{\beta_0^*}{2f}\sin(2\pi Q)$$

and

$$\frac{\beta^*}{\beta_0^*} = \sin(2\pi Q) / \sin(2\pi(Q + \Delta Q))$$

$$\rightarrow \quad \left[\frac{\beta^*}{\beta_0^*} = \frac{\sin(2\pi Q)}{\sin(2\pi(Q+\Delta Q))}\right] = \frac{1}{\sqrt{1+4\pi\xi\ \cot(2\pi Q) - 4\pi^2\xi^2}}$$

Both ΔQ and β depend on ξ and tune Q

β can become smaller or larger at interaction point

This is called "Dynamic β"

(for $\xi = 0$ nothing changes)

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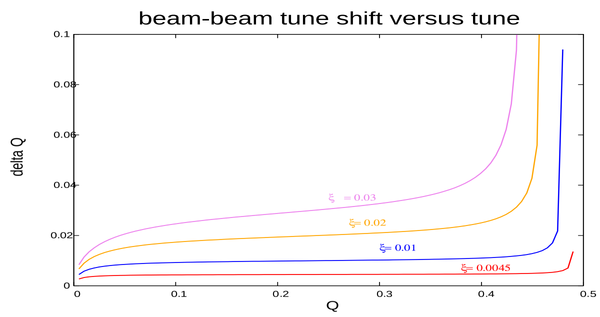

Strong dependence on Q for larger ξ (dynamic β)

$\Delta Q \approx \xi$ only far from integer and half-integer

Is this just a nuisance or can it be useful ??

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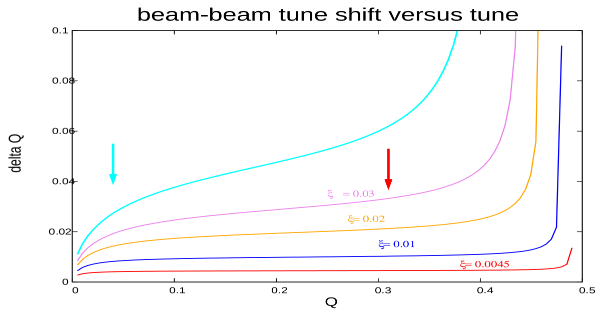


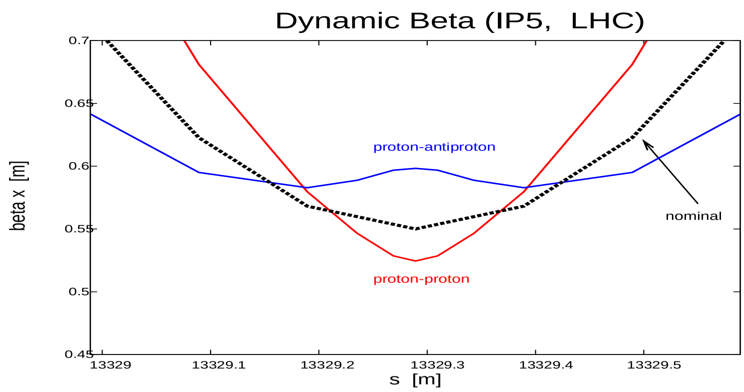
LEP working point (vertical plane):

ΔQ_y **decreased: 0.07** $\Longrightarrow$ $\approx$ **0.05**

β^* **decreased: 5 cm** $\Longrightarrow$ $\approx$ **2.5 - 2.8 cm (Luminosity !)**

LHC working point:

$\Delta Q \approx \xi$, **Weaker effects on** β^* →



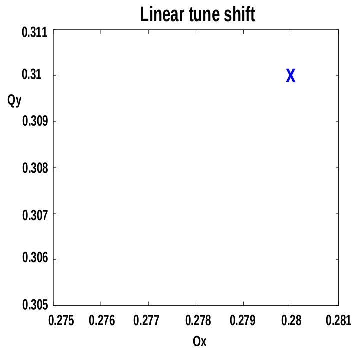
Dynamic β **in LHC, computed for** pp **and** $p\bar{p}$

(with standard LHC parameters)

β^* : **0.55 m** $\Longrightarrow$ **0.52 m**

Linear tune shift - two dimensions

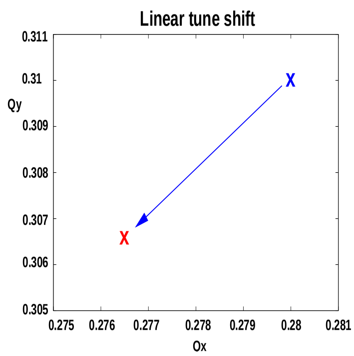
- Start with standard working point



Linear tune shift - two dimensions

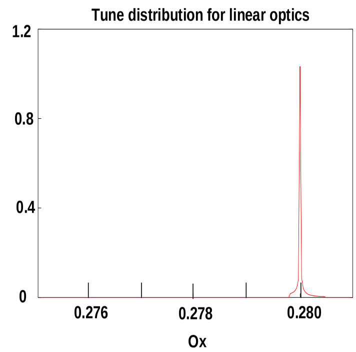
- Start with standard working point
- Tune shift in both planes
- LHC (equally charged beams)

  Tune shift is negative



Tune measurement: linear optics

- Linear force →
- all particles have the same tune
- Only one frequency (tune) visible

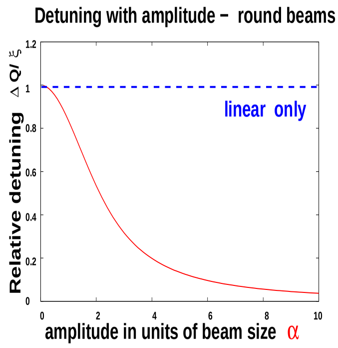


Next: include non-linear part

Non-linear force: Amplitude detuning

- Tune depends on amplitude
- Different particles have different tunes
- Largest effect for small amplitudes

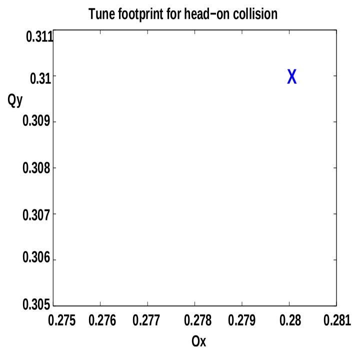
(Calculations in proceedings ...)



→ with $\alpha = \frac{a}{\sigma_*}$ we get: $\Delta Q/\xi = \frac{4}{\alpha^2}\left[1 - I_0(\frac{\alpha^2}{4}) \cdot e^{\frac{-\alpha^2}{4}}\right]$

Non-Linear tune shift - two dimensions

- Start with standard working point



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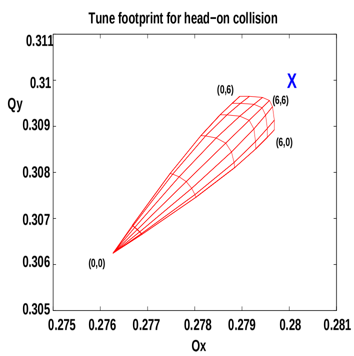
Non-Linear tune shift - two dimensions

- Start with standard working point
- Tunes depend on x and y amplitudes
- No single tune in the beam:

  Tunes are "spread out"

  Point becomes a footprint



Note: the coupling is evident !

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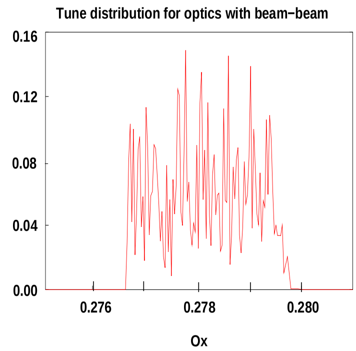
Tune measurement: with beam-beam

- Non-linear force →
- Particles with different amplitudes have different tunes
- We get tune spectra
- Width is about ξ



Slang: weak-strong and strong-strong

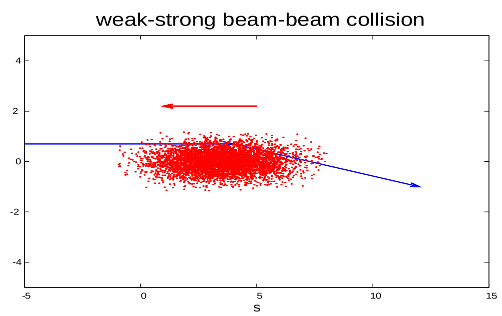
Both beams are very strong (strong-strong):

- Both beam are affected and change during a beam-beam interaction:

  Beam 1 changes beam 2, beam 2 changes beam 1 → beam-beam effects change every time the beams "meet"
- Examples: LHC, LEP, RHIC, ... (FCC ?)
- Evaluation of effects challenging (need to be self-consistent)

One beam much stronger (weak-strong):

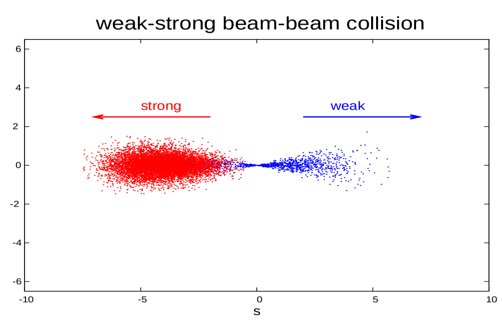
- Only the weak beam is affected and changed due to beam-beam interaction
- Examples: SPS collider, Tevatron, ...



Counter-rotating beam unaffected and treated as a static field

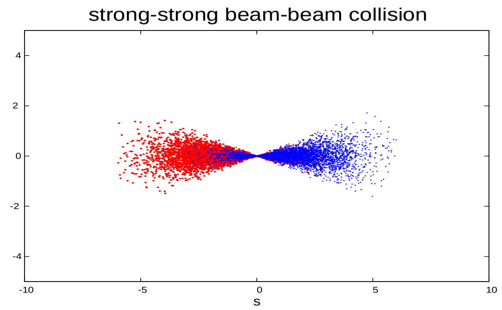
Equivalent to treat single particles, tracking etc.
this is usually done to study single particle stability

(so far only weak-strong was considered)



Counter-rotating beam unaffected and treated as a static field

Weak beam can be strongly perturbed



Both beams are (maybe heavily) distorted

Size, shape, density, (losses ?) ...

Always treat both <u>beams</u> - not particles (self-consistently)

How to get it self-consistent, some examples:

1. **Coupled Vlasov equations**
   - **Usually difficult to solve analytically, need perturbative treatment, but still ...**
   - **Numerical solver**
2. **Simulation**
   - **Multi-particle tracking**
   - **Gives desired results, but requires computing resources and <u>very</u> careful analysis (numerical problems, intelligent choice of field solver (!), ...)**

Single particle (very weak !) - incoherent effects

Single particle dynamics: treat as a particle through a static electromagnetic lens

Basically non-linear dynamics

All bad single particle effects observed:

- Unstable and/or irregular motion
- Enhanced diffusion
- Beam blow up
- Bad lifetime, particle loss
- Resonances

Observations hadrons

Non-linear motion can become chaotic

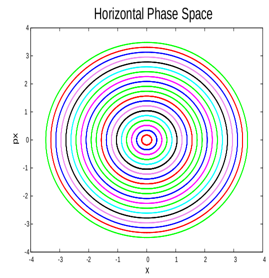
- reduction of "dynamic aperture"
- particle loss and bad lifetime

Strong effects in the presence of noise or ripple

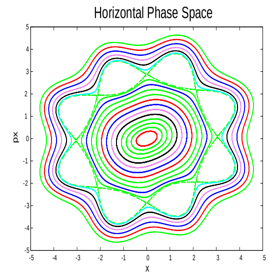
Evaluation is done by simulation

→ resonances ...

- Without beam-beam
- All particle have the same tune (all on circles)



- With (head-on) beam-beam
- Tune depends on amplitude
- For some amplitudes they are on resonances



Observations leptons

Remember $\Longrightarrow \mathcal{L} = \frac{N_1 N_2 f n_B}{4\pi\sigma_x\sigma_y}$

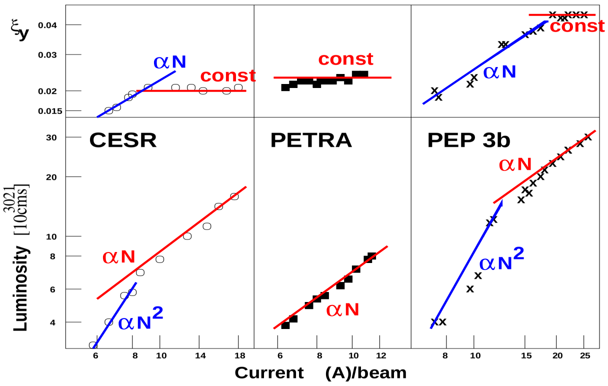
Luminosity should increase $\propto N_1 N_2$

➡ for: $N_1 = N_2 = N$ ➡ $\propto N^2$

Beam-beam parameter should increase $\propto N$

What is observed ?

Beam-beam observations: lepton colliders



Does not show the expected behaviour $(L \propto N^2)$

Have to introduce the concept of "beam-beam limit"

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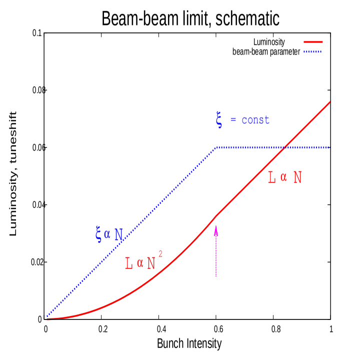
Beam-beam limit (schematic)

- Beam-beam parameter increases linearly with intensity $\xi \propto N$

  but saturation above some intensity, $\xi = const.$ no more increase
- Luminosity increases with intensity squared $L \propto N^2$

  but above this intensity, $L \propto N$ linear increase
- This intensity: **beam-beam limit**



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What is happening ?

we have
$$\xi_y = \frac{N r_0 \beta_y}{2\pi\gamma\sigma_y(\sigma_x + \sigma_y)} \stackrel{(\sigma_x \gg \sigma_y)}{\approx} \frac{r_0\beta_y}{2\pi\gamma(\sigma_x)} \cdot \frac{N}{\sigma_y}$$

and
$$\mathcal{L} = \frac{N^2 f n_B}{4\pi\sigma_x\sigma_y} = \frac{N f n_B}{4\pi\sigma_x} \cdot \frac{N}{\sigma_y}$$

- **Above beam-beam limit: σ_y increases when N increases to keep ξ constant → equilibrium emittance !**
- **Therefore: $\mathcal{L} \propto N$ and $\xi \approx$ constant**
  - **ξ_{limit} is NOT a universal constant !**
  - **Difficult to predict, but hard to exceed 0.05**

What is happening ?

- **Where does it come from ?**
  - **From synchrotron radiation: vertical plane damped, horizontal plane excited**
  - **Horizontal beam size usually (much) larger**
  - **Vertical beam-beam effect depends on horizontal (large) amplitude**
  - **Coupling from horizontal to vertical plane**
- **Equilibrium between this excitation and damping determines the beam-beam limit ξ_{limit}**

Lesson: Keep the coupling small !

Problems with hadron machines

- Difference: hadron (e.g. protons) machines have no or very little damping, beams usually round

  No equilibrium emittance - no hard beam-beam limit just gets worse and worse ...
- Very hard to exceed 0.01
- Losses or lifetime extremely hard to predict, a prediction within a factor 2 is pretty good ..

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The next problem

Remember:

$$\Longrightarrow \quad \mathcal{L} = \frac{N_1 N_2 f \cdot n_B}{4\pi\sigma_x\sigma_y}$$

How to collide many bunches (for high $\mathcal{L}$) ??

Must avoid unwanted collisions !!

Separation of the beams:

- → Pretzel scheme (SPS,LEP,Tevatron)
- → Bunch trains (LEP,PEP)
- → Crossing angle (LHC)   FCC ?

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Separation: LHC

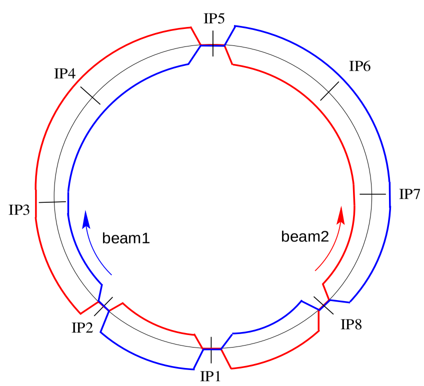
Many equidistant bunches (2808 per beam)

Two beams already separated in two separate beam pipes except:

- The two beams have to exchange between inner and outer beam pipes
- Four experimental areas
- Need local separation

Two horizontal and two vertical crossing angles

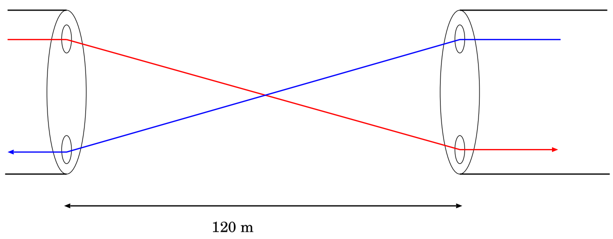
Layout of LHC



Symmetric crossing in experimental areas

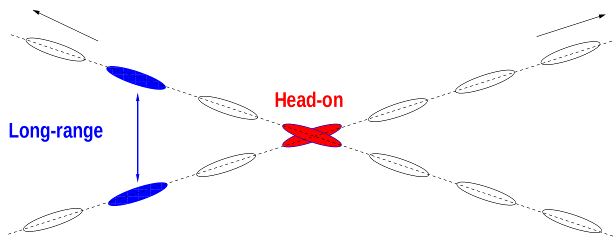
Two beams, 2808 bunches each, every 25 ns

In common chamber around experiments



Over 120 m: about 30 parasitic interactions

Crossing angles (example LHC)



Particles experience distant (weak) forces

Separation typically 6 - 12 σ

→ We get so-called **long range interactions**

What is special about them ?

In addition to the head-on effects:

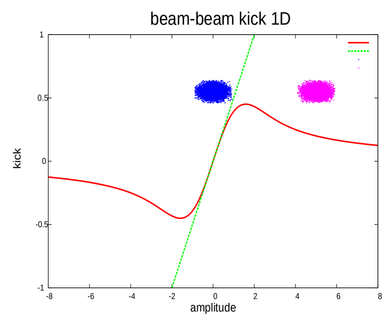
- Break symmetry between planes, stronger resonance excitation
- Mostly affect particles at large amplitudes
- Cause effects on closed orbit, tune, chromaticity, ..
- Special case: PACMAN effects
- Tune shift has opposite sign in plane of separation

  E.g. case with a horizontal crossing angle:

  Horizontal tune shift positive, vertical tune shift negative

Why opposite tuneshift ???

What do the particles "see":



- Local slope has opposite sign for large separation
- Opposite sign for focusing !

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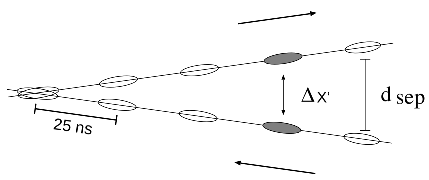

→ **Modified "kick" for horizontal separation d:**

$$\Delta x'(x+d, y, r) = -\frac{2Nr_0}{\gamma} \cdot \frac{(x+d)}{r^2}\left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

(with: $r^2 = (x+d)^2 + y^2$**)**

Red flag: to use this expression (e.g. in a simulation) there is a small complication (if interested ask offline, or discussion session)

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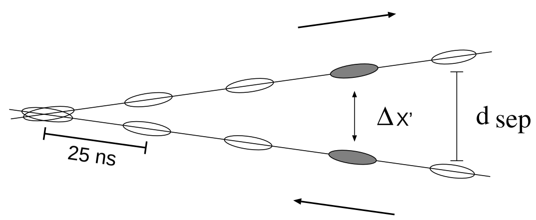

- **Number of long range interactions depends on spacing and length of common part**
- **In LHC 15 collisions on each side, 120 in total !**
- **Effects depend on separation:**

$\Delta Q \propto -\frac{N}{d^2}$ **(for large enough d !)** **footprints ??**

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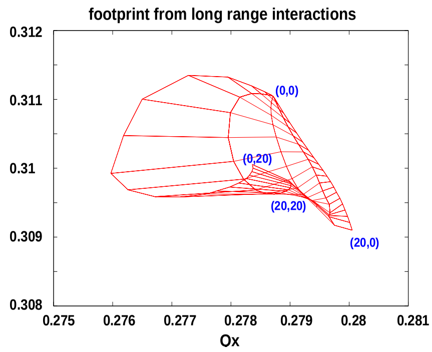
- Tuneshift large for largest amplitudes (where non-linearities are strong)
- We should expect problems at small separation
- Footprint is very asymmetric
- Size proportional to $\frac{1}{d^2}$



One observes a "folding"
(may also appear for head-on + octupoles)

For small separation, the size of the footprint can be large → particle losses

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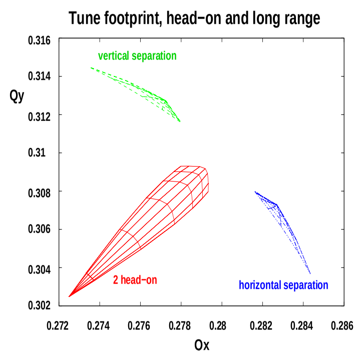
- Compare foot print for different contributions
- Seem to be totally separated
- For horizontal and vertical separation go opposite directions

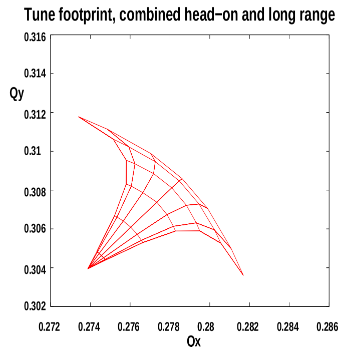


Can we take advantage of that ??

(Note: a second head-on collision just doubles the size of the footprint)

Two interaction points:

Two head-on footprints
+ One horizontal long range
+ One vertical long range

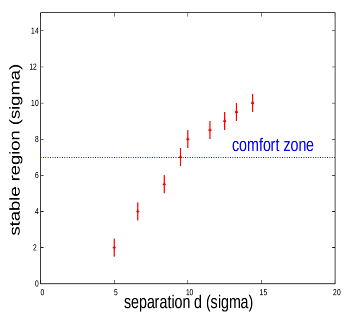


- **Alternating (i.e. one vertical and horizontal each), implemented in the LHC**
- **Seems to get some compensation, i.e. overall footprint decreased and symmetric**
- **Is that all ? (see later)**

Small crossing angle $\Longleftrightarrow$ small separation

Stable region versus separation in units of beam size σ

Minimum separation for LHC:
$\approx$ 10 σ



For too small separation: particles may be lost and/or bad liftime

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A "little" headache:

Beam separation d (in units of σ) depends on:

$$d \;\propto\; \sqrt{\beta^*}$$

High luminosity → small β^* → small separation → big problems

since footprint size (and other effects) : $\frac{1}{d^2}$

a very small β^* may be difficult to reach (also for other reasons).

Possible solutions:
- large crossing angle - but other detrimental effects
- flat beams (i.e. $\beta^*_x \neq \beta^*_y$)
- partial compensation (e.g. octupoles, wires, see later)

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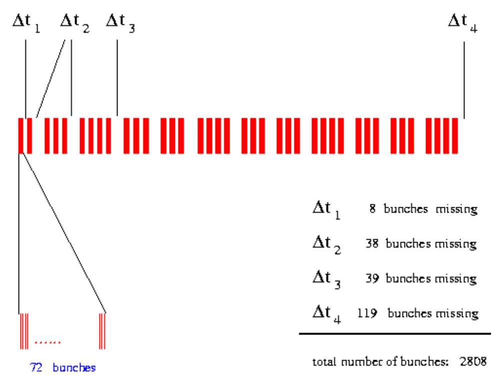
More jargon: PACMAN bunches



- **Trains not continuous: gaps for injection, extraction, dump ..**
- **2808 of 3564 possible bunches**

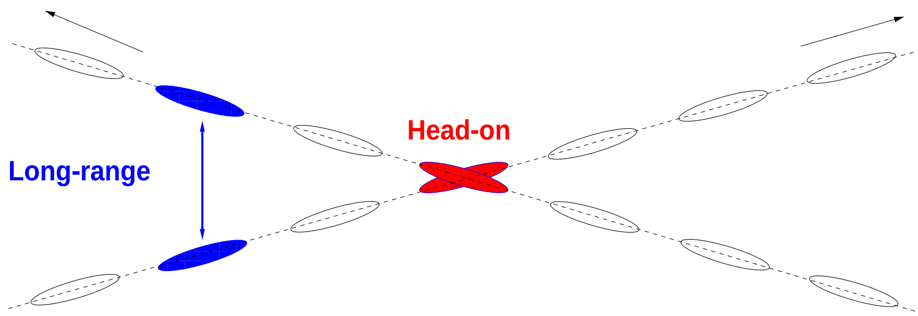
Trains for beam 1 and beam 2 are symmetric:

At interaction point:

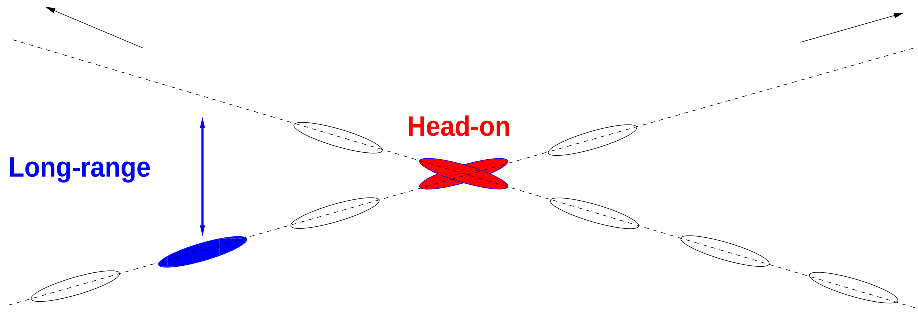
bunch 1 meets bunch 1,   bunch 2 meets bunch 2, etc.

hole 1 meets hole 1,   hole 2 meets hole 2, etc.

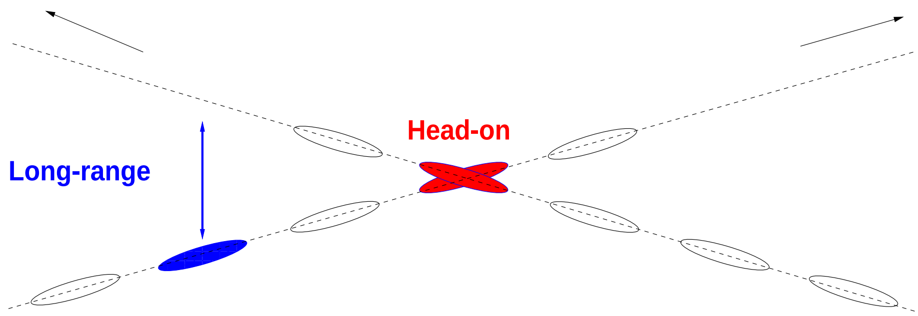
But not always ...



What we want ...



What we get ...



- Some Bunches meet holes (at beginning and end of batch)
- Cannot be avoided
- Worst case: less than half of long range collisions (depends on collision scheme and gaps)

When a bunch meets a "hole":

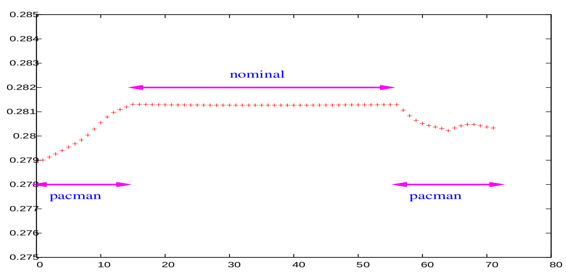
- Miss some long range interactions → PACMAN bunches
- They see fewer unwanted interactions in total
- Different integrated beam-beam effect
- Long range effects for different bunches will be different:

→ Different tune, chromaticity, orbit ...

- May be more difficult to optimize

Note: this case is specific LHC, but something similar happens in other machines in some form ..

Example: tune along the train, two horizontal (H + H) crossings

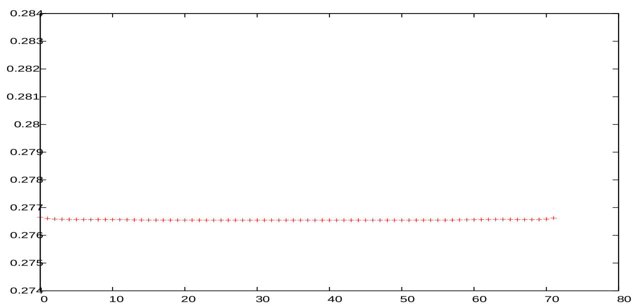


Horizontal tune along bunch train (computed)

Tune spread between bunches rather large ($\approx$ 0.0025), effects add up for several crossings

Too large ($\Delta Q \approx 0.01$) for 4 IPs

Example: tune along the train, alternating (H + V) crossings

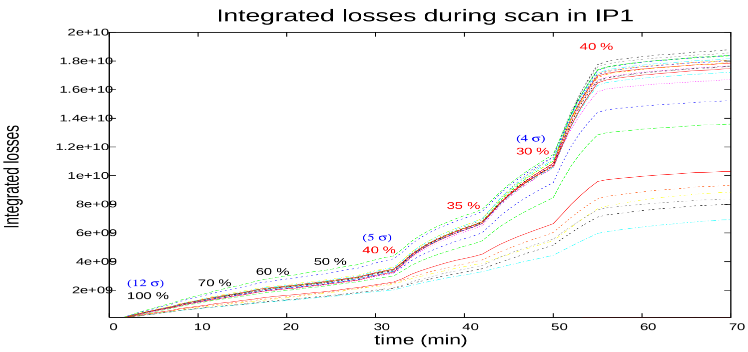


Horizontal tune along bunch train (computed)

Tune spread has disappeared due to compensation by alternating crossings (1 horizontal and 1 vertical)

This is the <u>real</u> reason for alternating crossings in the LHC !

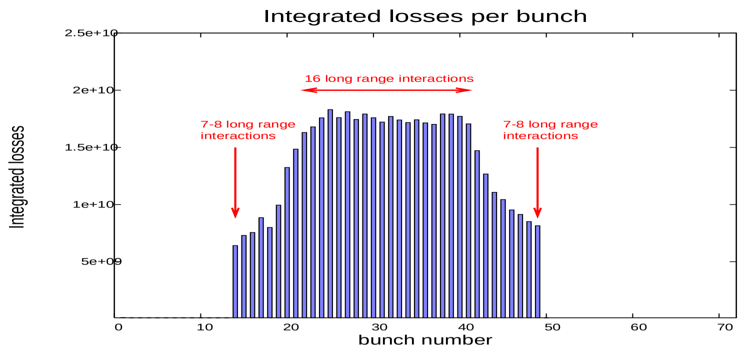
Can be measured (without compensation):



Separation (crossing angle) slowly decreased from 12 σ to 4 σ

Bunches with <u>more</u> long range collisions suffer first

... along the train:



Losses follow exactly the number of long range interactions

→ in LHC losses completely dominated by long range effects

Interlude:

Both, head-on and long range contribute to chromaticity

Head-on (modulation of β-function):

$$C^{HO}_{x,y} = \Delta Q_{x,y} \cdot W_{x,y}, \quad W_{x,y} = \frac{1}{\beta_{x,y}} \frac{\partial \beta_{x,y}}{\partial \delta_p}$$

Long range (modulation of separation):

$$C^{LR}_{x,y} = \left(D_x \frac{\partial}{\partial d_x} + D_y \frac{\partial}{\partial d_y} \right) \cdot \Delta Q^{LR}_{x,y}$$

Result: different bunches, different chromaticity

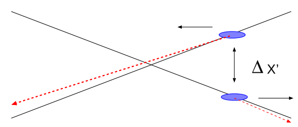
Closed orbit effects

Starting from the kick $\Delta x'$ for long range interactions:

$$\Delta x'(x+d, y, r) = -\frac{2Nr_0}{\gamma} \cdot \frac{(x+d)}{r^2} \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

For well separated beams ($d \gg \sigma$) the force (kick) has an amplitude independent contribution:[a]

$$\Delta x' = \frac{const.}{d} \cdot \left[1 - \frac{x}{d} + O\left(\frac{x^2}{d^2}\right) + ... \right]$$



[a] THIS is the complication mentioned before ...

This constant and amplitude independent kick changes the orbit !

Has been observed in LEP with bunch trains (and was bad)

So should be evaluated by computation, however:

Change of orbit → change of separation → change of orbit ...
Change of tune → change of separation → change of tune ...

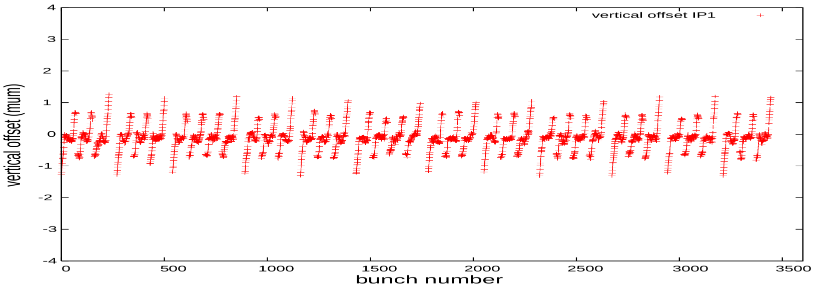
All PACMAN bunches will be on different orbits

In LEP: 8 bunches, in LHC: 2808 bunches

Requires the self-consistent computation of 5616 orbit !

(first time done in 2001, calculation took 45 minutes)

PACMAN Orbit effects: calculation



- Vertical offset expected at collision points, sizeable with respect to beam size
- Predicted orbits from self-consistent computation

  Does it have anything to do with reality ?
- Cannot be resolved with beam position measurement, but ..

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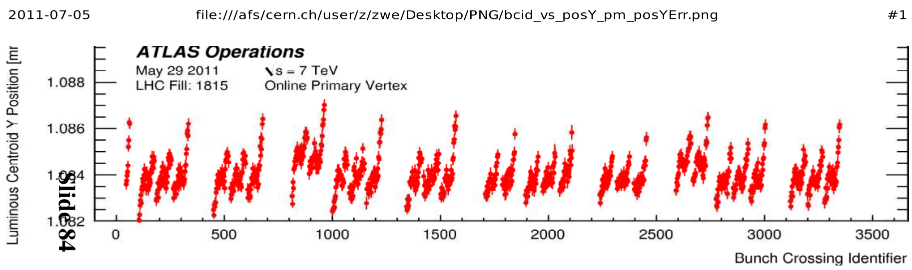
PACMAN Orbit effects: measurement



- Measured vertex centroid in ATLAS detector
- Very good agreement with computation

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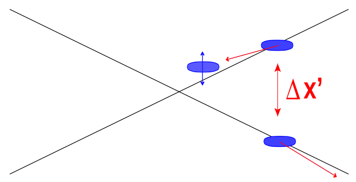
Coherent beam-beam effect



When bunches are well separated:

All particles in a bunch "see" the same kick

Whole bunch sees a kick as an entity (coherent kick)

The coherent kick of separated beams can excite coherent dipole oscillations

All bunches couple because each bunch "sees" many opposing bunches: many coherent modes possible !

When bunches are separated much less than one σ (quasi head-on interactions):

Remember orbit kick ➡ all particles "see" the same kick

$$\Delta x' = \frac{const.}{d} \cdot \left[1 - \overbrace{\frac{x}{d} + O\left(\frac{x^2}{d^2}\right)}^{other\ parts} + \ldots \right]$$

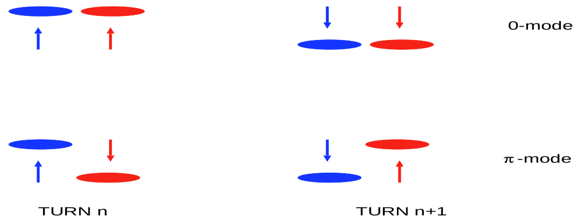
There is "one part" of the kick that is the same for all particles

This part also excites dipolar oscillations

There are "other parts" which are different for the particles

These parts can change the particle distribution

Simplest case - one bunch per beam:



Coherent modes: two bunches are "locked" in a coherent oscillation, can be either:

Two bunches oscillate "in phase": 0-mode

Two bunches oscillate "out of phase": π-mode

0-mode has no tune shift and is stable

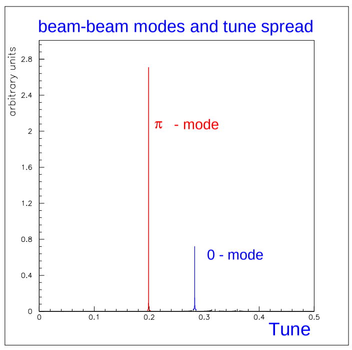
π-mode has large tune shift and can be unstable

Tune of the modes (schematic):

Two separate modes visible

0-mode is at unperturbed tune

π-mode is shifted by $Y \cdot \xi$



Y is called: "Yokoya factor"

Typically: $1 \leq Y \leq 1.35$

This is due to the "other parts"[a]

(otherwise $Y = 1$)

[a] say goodbye to Gaussian distributions

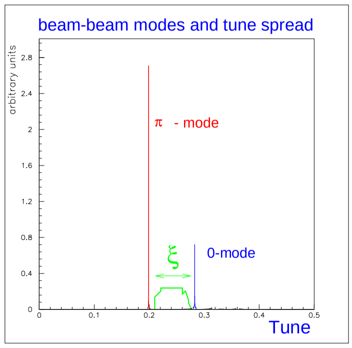
Q: what about the tune spread $\Delta Q \approx \xi$?

0-mode is at the unperturbed tune $\Longrightarrow \Delta Q_0 = 0$

π-mode is shifted by $Y \cdot \xi$

$\Longrightarrow \Delta Q_\pi \approx 1.35 \cdot \xi$

Incoherent spread between $[0.0, 1.0] \cdot \xi \Longrightarrow \Delta Q_i \approx 1.00 \cdot \xi$

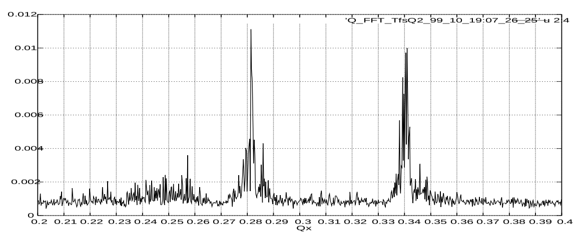


This is bad news: $\Delta Q_\pi > \Delta Q_i$

Strong-strong case: π-mode is shifted outside the tune spread

No Landau damping possible for this mode, requires: $\Delta Q_\pi \leq \Delta Q_i$

What was measured: LEP

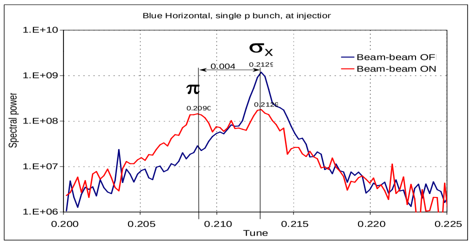


$\Uparrow$ $0 : 0.28$ $\quad$ $\Uparrow$ $\pi : 0.34$

Both modes clearly visible

They are real !

What was measured: RHIC



Courtesy W. Fischer (BNL)

Spectral power as function of the tune, with and without beam-beam interaction

With beam-beam interaction the two modes visible

First we consider head-on collision of one bunch per beam a and b

Particle distributions ψ^a and ψ^b mutually changed by interaction (by the "other parts")

Interaction depends on particle distributions

- **Beam ψ^a solution depends on beam ψ^b**
- **Beam ψ^b solution depends on beam ψ^a**

Can one find a self-consistent solution ?

What is the equation of motion ?

→ **For distribution function: Vlasov equation** for beam a:

$$\frac{\partial \psi^a}{\partial t} = -q_x p_x \frac{\partial \psi^a}{\partial x} + \left(\frac{\partial p_x}{\partial t}\right)\frac{\partial \psi^a}{\partial p_x}$$

$$\frac{\partial \psi^a}{\partial t} = -q_x p_x \frac{\partial \psi^a}{\partial x} + \underbrace{\left(q_x x + \delta_p(t)\cdot 4\cdot \pi\xi_x \textbf{ p.v.} \int_{-\infty}^{+\infty} \frac{\rho^b(x';t)}{x-x'}dx'\right)}_{\text{force from beam b on beam a}} \frac{\partial \psi^a}{\partial p_x}$$

$$\rho^b(x;t) = \int_{-\infty}^{\infty} \psi^b(x,p_x;t)dp_x$$

The same thing for beam b, two coupled differential equations for beam distributions $\psi^a(x,p_x)$ and $\psi^b(x,p_x)$

Normally cannot find exact solutions

→ **Need numerical solutions**

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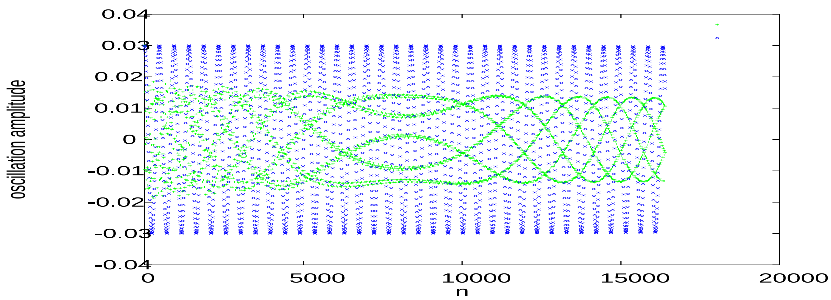
just the result (if interested, ask a lecturer):



What is shown: difference and sum of bunch centres

$< \psi^a > - < \psi^b >$ π **- mode**

$< \psi^a > + < \psi^b >$ 0 **- mode**

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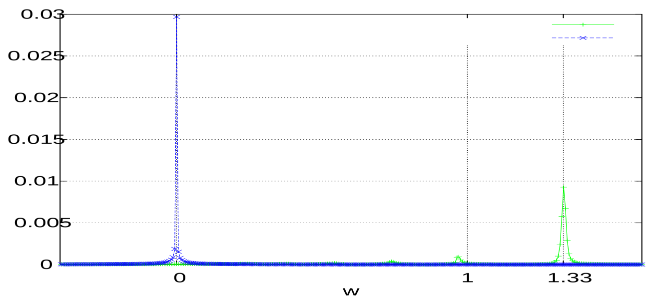
Does it make any sense ?



A Fast Fourier Transform of this motion:

Tune shifts $0.0 \cdot \xi$ and $1.33 \cdot \xi$

Get the same frequencies as the simulation (but 10^5 times faster ...)

How to deal with the problems ?

Every "Coherent Motion" requires 'organized' motion of many/all particles

Requires a "high degree of symmetry" (between the beams)

"High degree of symmetry" → large Yokoya factor Y

Possible countermeasure break the symmetry by:

- Different bunch intensity
  (results in different tune shifts for the two beams)
- Different nominal tunes of the two beams

➤ Y becomes smaller and smaller

➤ π-mode should move into the tune spread (and is damped)

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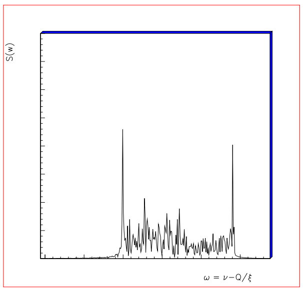
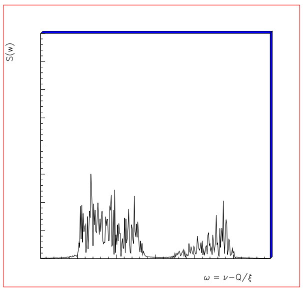
Beams with unequal tunes

$Q_1 = Q_2$ | $Q_1 \neq Q_2$





Bunches with different tunes cannot maintain coherent motion

$Q_2 - Q_1 = 0.002$

Landau damping restored, no π-mode visible

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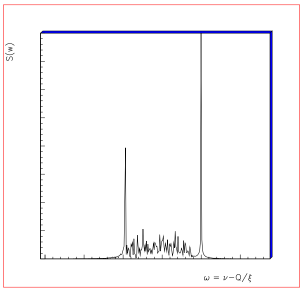
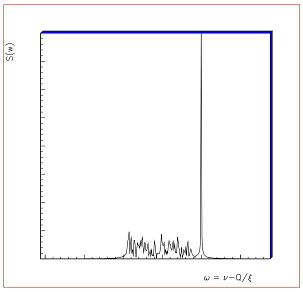
Beams with unequal intensities

$I_1 = I_2$ | $I_1 \neq I_2$





Bunches with different intensities cannot maintain coherent motion

$I_2 \approx 0.5 \cdot I_1 \implies \Delta Q \approx 0.5 \cdot \xi = 0.00187$ **(just enough !)**

Landau damping restored, no π-mode visible

These were coherent modes due to head-on interactions

What about coherent modes due to long range interactions ?

Separation now much more than σ, in LHC more than $10 \cdot \sigma$!

- Do they exist ? yes ...
- Do we understand them ? more or less ...
- Are they more complicated ? YES[a] - many more modes ...
- Key problems: they are not damped by head-on incoherent spectrum[b] (even if they sit inside the incoherent spectrum !!!)

---

[a] Deserves a dedicated lecture - offline discussions more appropriate
[b] Deserves another dedicated lecture

Can one compensate or reduce beam-beam effects ?

Find 'lenses' to correct beam-beam effects

Head on effects:

- Non-linear "electron lens" to reduce spread
- Tests in progress

Long range effects:

- At very large distance: force is 1/r
- Same force as a wire !
- Tests in the LHC

So far: mixed success with active compensation, passive compensation (better: reduction) more practical and (often) more beneficial

Others: Möbius lattice - flat beams

Idea:

- Reduced beam-beam problems for round beams

Principle:

- Interchange horizontal and vertical plane each turn

Effects:

- Round beams for leptons
- Some compensation effects for beam-beam interaction
- First test at CESR at Cornell

Interlude ...

Some final words (circular hadron colliders):

Is beam-beam always bad ?

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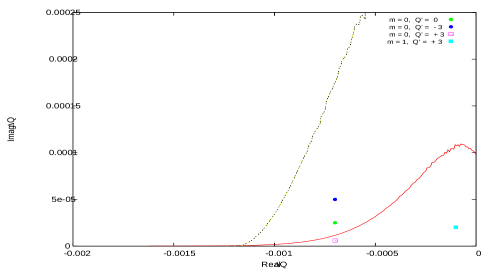
Stability diagram and head-tail modes



- Stabilization with octupoles or colliding beams
- (Head-on) collisions much more efficient than octupoles for Landau damping, in particular for high energies[a]

→ Beam-beam collisions save the day (e.g. LHC)

[a] see e.g. Proceedings CAS Trondheim (2013), Some PhD theses at EPFL

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Is beam-beam always bad ?

No, but requires a careful compromise between:

- Detrimental beam-beam effects (dynamic aperture, losses, instabilities, ...)
- Effects on optics (plenty ..)
- Effects on luminosity
- Effects on Landau Damping properties and coherent instabilities
- Requirements from particle physics experiments and machine protection
- A few more ...

→ Now for something completely different ..

Beam-beam effects in linear colliders

- Mainly (only ?) $e+e-$ colliders
- Past collider: SLC (SLAC)
- Under consideration: CLIC, ILC
- Under construction:
- Special issues:
  - Particles collide only once (dynamics) !
  - Can accept to spoil the beam during collision
  - Beam-beam considerations very different
  - Particles radiate photons crossing the other beam, leading to an energy spread. Have to introduce "Differential Luminosity"

Main effects to look at (here):

- Disruption (modification of beam size, mainly classical)
- Beamstrahlung (spread of CM energy, requires quantum treatment)
- High energy photons interact with EM fields

Assumption for classical treatment:

→ Basically same arguments like beam-beam:

- No acceleration by longitudinal fields
- Lorentz contraction: beam-beam → longitudinal positions must overlap
- High energy: interaction within bunch can be ignored: $O(\frac{E}{\gamma^2})$
- Only $\vec{E}$ fields needed: $\vec{E} + \vec{v} \times \vec{B} \approx 2\vec{E}$ (remember lectures on relativity)

Luminosity in linear colliders

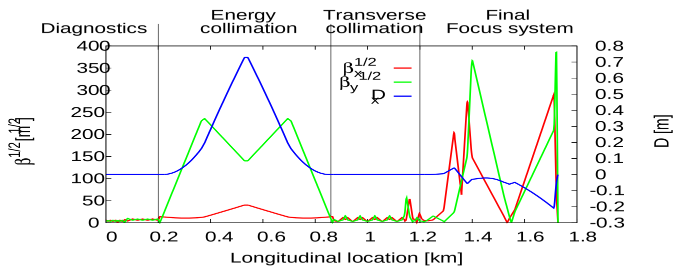
- Basic formula:

$$\text{From}: \quad \mathcal{L} = \frac{N^2\, f\, n_b}{4\pi\sigma_x\sigma_y} \quad \text{to}: \quad \mathcal{L} = \frac{N^2\, f_{rep}\, n_b}{4\pi\sigma_x\sigma_y}$$

- Replace Revolution Frequency f by Repetition Rate f_{rep}.
- And introduce effective beam sizes $\overline{\sigma_x}, \overline{\sigma_y}$:

$$\mathcal{L} = \frac{N^2\, f_{rep}\, n_b}{4\pi\overline{\sigma_x}\,\overline{\sigma_y}}$$

Final focusing in linear colliders



(Courtesy R. Tomas)

- "Final Focus" and "Beam delivery System"
- At the end of the beam line only !
- Smaller β^* in linear colliders (10 mm $\times$ 0.2 mm !!)

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Why "effective beam sizes ?

- Using the enhancement factor H_D:

$$\mathcal{L} = \frac{N^2\, f_{rep}\, n_b}{4\pi\overline{\sigma_x}\,\overline{\sigma_y}} \rightarrow \mathcal{L} = \frac{H_D \cdot N^2\, f_{rep}\, n_b}{4\pi\sigma_x\,\sigma_y}$$

- Enhancement factor H_D takes into account reduction of nominal beam size by the disruptive field (pinch effect)
- Related to so-called disruption parameter $\mathcal{D}$:

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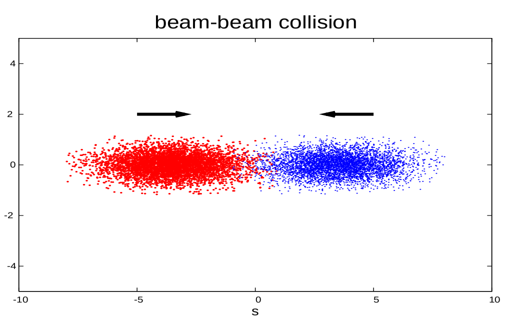
For weak disruption $\mathcal{D} \ll 1$ and round beams:

$$H_D = 1 + \frac{2}{3\sqrt{\pi}}\mathcal{D} + \mathcal{O}(\mathcal{D}^2)$$

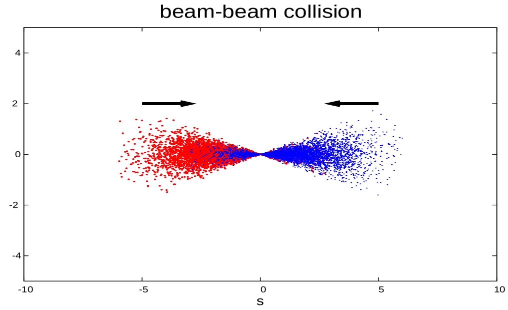
For strong disruption and flat beams: computer simulation necessary, maybe can get some scaling

Disruption plays a key role !

Pinch effect - disruption

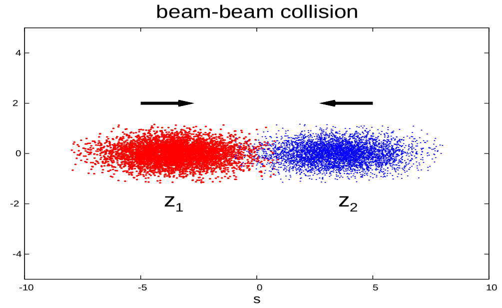


Pinch effect - disruption



- Additional focusing by opposing beams

Equation of motion (e.g. of the electrons):



Coordinates of bunches (overlap of bunch centres at $t = 0$)

$$z_2 = -z_1 + 2t$$

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Assuming N the number of particles in a beam, γ the "energy" (in terms of electron rest mass) and

$$\rho_l(z) = \int \rho(x, y, z, t)\mathrm{d}x\mathrm{d}y$$

the longitudinal density, the equation of motion for an electron is

$$\frac{d^2x}{dt^2} + \frac{4Nr_e}{\gamma} \cdot \rho_l(z_2) \cdot \frac{\partial \Phi}{\partial x} = 0 \qquad \left(\frac{1}{f_{bb}} = \frac{\Delta x'}{x} = \frac{Nr_e}{\gamma \sigma^2} \right)$$

$\Delta \Phi = 2\pi \cdot \rho_t(x, y, z_2, t)$ **is the Poisson equation**

and $\rho_t(x, y, z, t)$ is the <u>transverse</u> distribution which is obviously

$$\rho_t(x, y, z, t) = \frac{\rho(x, y)}{\rho_l(z)}$$

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For a round beam $\sigma = \sigma_x = \sigma_y$ a solution is

$$\Phi(x, y) = \int_0^r \frac{1 - e^{-r^2/2\sigma^2}}{r} dr \quad \text{with : } r^2 = x^2 + y^2$$

The solutions for "flat" or "elliptic" beams can be found in the literature
For the latter and near the centre of a Gaussian (what else) beam, we get

$$\Phi(x, y) = \frac{x^2}{2\sigma_x (\sigma_x + \sigma_y)} + \frac{y^2}{2\sigma_y (\sigma_x + \sigma_y)}$$

then, (see above)

$$\frac{d^2x}{dt^2} + \frac{4Nr_e}{\gamma} \cdot \rho_l(z) \cdot \frac{x}{\sigma_x (\sigma_x + \sigma_y)} = 0$$

giving the beam-beam deflection angle (and focal length)

$$x' \text{ (rather) } \dot{x} = \frac{2Nr_e}{\gamma} \cdot \frac{x}{\sigma_x (\sigma_x + \sigma_y)} = \frac{1}{f_{bb}}$$

Disruption parameter

Using the bunch length σ_z and the focal length due to the beam-beam interaction, one can define and write

$$D_{x,y} = \frac{2Nr_e}{\gamma} \cdot \frac{\sigma_z}{\sigma_{x,y}(\sigma_x + \sigma_y)} \qquad \left(\frac{1}{f_{bb}} = \frac{\Delta x'}{x} = \frac{Nr_e}{\gamma\sigma^2}\right)$$

The disruption parameter is defined as the ratio of the r.m.s. bunch length to the focal length of the interaction

Equation of motion:

Using this definition and for a uniform longitudinal density the final equation of motion becomes (near the axis, as said above)

$$\frac{d^2x}{dt^2} + \underbrace{\frac{D}{\sqrt{3}\cdot\sigma_z^2}}_{\omega_0^2} x = 0$$

only valid in the range:

$$\frac{-\sqrt{3}\sigma_z}{2} < t < \frac{\sqrt{3}\sigma_z}{2}$$

This gives rise to sinusoidal oscillations and for the number of oscillations in this range we get:

$$\frac{\sqrt{\sqrt{3}D}}{2\pi}$$

Kink instability ...

- Observed and relevant in (plasmas):
  - Fusion reactors
  - Astrophysics (sun, jets, pulsars, ..)

→ Treat as (relativistic) fluid ...

Assume again the beam as an elliptical cylinder[a] ($\sigma_y \ll \sigma_x$)

When the beams collide with an offset, e.g. $\Delta_y = (y_1 - y_2)$ the equations of motion are (without derivation, see books on hydrodynamics):

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial s}\right)^2 y_1 = -\omega_0^2 (y_1 - y_2)$$
$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial s}\right)^2 y_2 = +\omega_0^2 (y_1 - y_2)$$

Of course the right hand side of the equations represent the forces

[a] warning: there are many different models

The solution is an oscillation $y_{1,2} \ = \ a_{1,2} \cdot e^{(ks \ - \ \omega t)}$ **where**

$$\begin{pmatrix} (\omega - k)^2 - \omega_0^2 & \omega_0^2 \\ \omega_0^2 & (\omega + k)^2 - \omega_0^2 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0$$

then $\omega^2 \ = \ k^2 \ + \ \omega_0^2 \ \pm \ \sqrt{4\omega_0^2 k^2 \ + \ \omega_0^4}$

The exponential growth if $| \, k \, | \ < \ \sqrt{2}\omega_0$ **is the so-called "kink instability":**

In this mode the bunch (cylinder) wobbles along the longitudinal direction

Side note: when instead the diameter changes, it is called "sausage" mode

However:

If the offset is much larger than the vertical beam size, i.e. $\sigma_y \ \ll \ \Delta_y \ \ll \ \sigma_x$ **then we can approximate the equation of motion by:**

$$\frac{d^2 y}{dt^2} \ + \ \frac{\pi}{6} \frac{\sigma_y \, D_y}{\sigma_z^2} \ = \ 0$$

The solution is obviously a parabola and the beam centres can overlap in time if

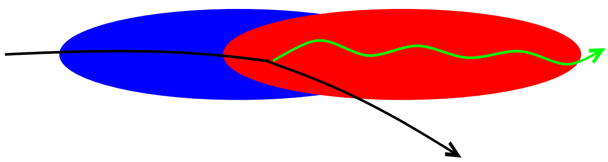
$$\Delta_y \ \le \ \frac{\pi}{4} \sigma_y D_y$$

i.e. for large disruption parameters the is no large loss of luminosity ...

Beamstrahlung

- Disruption at interaction point is basically a strong "bending"
- Results in strong synchrotron radiation: beamstrahlung
- This causes (unwanted):
  - Spread of centre-of-mass energy
  - Pair creation and detector background
- Again: luminosity is not the only important parameter

Beamstrahlung ...



Bremsstrahlung/synchrotron radiation in the electro-magnetic fields of opposing bunch leads to emission of photons

Fields are extremely strong: can be in kT range ...

Special: photons can produce pairs of leptons (and quarks !), i.e. QED and QCD background

Beamstrahlung ...

For a relativistic flat beam (see before !)

$$E_y(y,z) \;=\; \frac{q \cdot N}{2\sqrt{2\pi}\epsilon_0 \cdot \sigma_x} \cdot erf\left(\frac{y}{\sqrt{2}\sigma_y}\right)\rho(z)$$

assuming a Gaussian longitudinal profile $\rho(z)$:

$$E_y \;=\; \frac{q \cdot N}{4\pi\epsilon_0 \cdot \sigma_x\sigma_z}$$ very small → high fields

for the magnetic field we get:

$$B \longrightarrow \frac{E_y}{c}$$

Some numbers:

N = $2 \cdot 10^9$ **e/b**

σ_x = **70 nm**

σ_y = **0.7 nm**

σ_z = **35 μm**

$E_y \geq 10^{12}\,\frac{V}{m}$ → **B > 3 kT**

Critical field B_c:

Schwinger limit: for higher fields the field becomes non-linear:

→ High enough energy: pair production from $\gamma\gamma$ collisions

The critical electric field is

$$E_c = \frac{m_e^2 \, c^3 \, 2\pi}{q \cdot h} \approx 1.3 \cdot 10^{18} \text{ V/m}$$

and the corresponding critical magnetic field:

$$B_c = \frac{E}{c} = 4.4 \cdot 10^{9} \text{ T}$$

→ B_c means onset of quantum regime ...

Beamstrahlung Parameter Y

The parameter Y is defined as:

$$Y \equiv \frac{2}{3} \frac{E_c}{E_0} = \frac{2}{3} \frac{\omega_c \, \hbar}{E_0}$$

where E_0 is the electron energy before radiating and E_c the photon energy corresponding to a bending radius ρ:

$$E_c = \frac{2}{3} \frac{\hbar c \gamma^3}{\rho}$$

The bending radius ρ of the trajectory crossing a Gaussian bunch is not constant.

Beamstrahlung Parameter Y

Averaging E_c over the bunch distribution one obtains (ignoring the pinching):

$$Y = \gamma \cdot \frac{< E + B >}{B_c} \approx \frac{5}{6} \; \frac{r_e^2 \, \gamma \, N}{\alpha \, \sigma_z \, (\sigma_x \; + \; \sigma_y)}$$

with:

$$\alpha \;\; = \;\; \text{fine structure constant} \;\; = \;\; \frac{1}{137}$$

Important numbers (again: Y is not constant during the collision):

$$Y_{max} \;\; \approx \;\; \frac{2 \, r_e^2 \, \gamma \, N}{\alpha \, \sigma_z \, (\sigma_x \; + \; \sigma_y)} \qquad\qquad < Y > \;\; \approx \;\; \frac{5}{12} \, Y_{max}$$

Note: bunch length σ_z becomes a very important parameter !

What about Y ?

Since B_c relevant for quantum regime:

<u>Low field, small Y: classical regime</u>

Typical storage ring (e.g. LEP):
fields smaller than 1T (LEP $\approx$ 0.1 T) → Y $\ll$ 1

<u>High field, large Y: quantum regime</u>

Beamstrahlung: for large B (from collision) and high γ
→ Y $\gg$ 1 (deep quantum regime)

Number of photons and Energy Loss

Average number of γ per electron (relevant for energy loss/spread):

$$N_\gamma \;=\; 2.54 \cdot \frac{\alpha \sigma_z Y}{\lambda_c \gamma} \cdot U_0(Y)$$

where

$$U_0(Y) \;\approx\; \frac{1}{(1 \;+\; Y^{2/3})^{1/2}}$$

Note: λ_c **is the (reduced) Compton wavelength:**

$$\lambda_c \;=\; \frac{\hbar}{m_e\, c} \;=\; \textbf{0.386}\; 10^{-12}\ \textbf{m)}$$

Number of photons and Energy Loss

Average energy loss per electron:

$$\delta_E = \left\langle -\frac{\Delta E}{E} \right\rangle \;\approx\; 1.24 \frac{\alpha \sigma_z < Y >}{\lambda_C \gamma} \;< Y > \cdot U_1(< Y >)$$

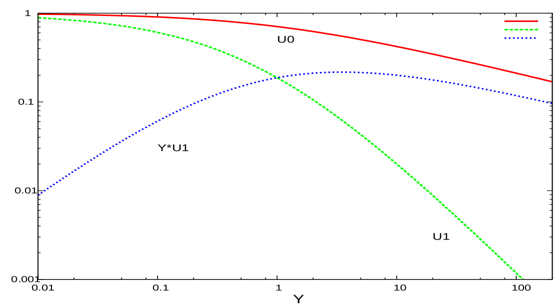
where

$$U_1(Y) \;\approx\; \frac{1}{(1 \;+\; (1.5Y)^{2/3})^2}$$

δ_E **characterizes the spread in the centre-of-mass energy of the electrons/positrons**

Note: for $Y \;\geq$ the fractional energy loss can exceed 1 and the quantum correction $U_1(Y)$ is needed

Functions U_0, U_1 and $Y \cdot U_1$



Note:

U_0 **is approximately 1.0 for not too large** Y

$Y \cdot U_1$ **approximately constant for large** Y ➡ δE **!**

Some comments:

You may have seen these formulae in a different form, just remember:

$r_e = 2.818\ 10^{-15}$ **m**

$\epsilon_0 = 8.854\ 10^{-12}$ **Farad m^{-1}**

$c \approx 3 \cdot 10^{8}$ **m**

$\alpha = \frac{1}{137}$

Differential Luminosity

Beamstrahlung changes the energy spectrum of electrons (positrons), in particular because they can emit more than one photon: $n_\gamma > 1$.

Energy spectra for the two beams are $\Psi(x_1, t)$ and $\Psi(x_2, t)$. We define $t = 0$ when the bunches first meet and a longitudinal variable $z = 0$ at the front of each bunch.

One writes for a "differential luminosity" the convolution integral:

$$\frac{d^3L(x_1, x_2, 0)}{dx_1\, dx_2\, dz} = \frac{2}{l}\int_0^{l/2} dt\psi(x_1, t)\, \psi(x_2, 0)$$

where l is the total bunch length and $\frac{l}{2}$ the total collision "time

Note: the first "z" slice of beam 1 always sees an unperturbed (fresh) beam 2

As time (and z) evolve, a slice in z of beam 1 sees at a time $t = z/2$:

$$\frac{d^3L(x_1, x_2, z)}{dx_1\, dx_2\, dz} = \frac{2}{l}\int_0^{l/2} dt\psi(x_1, t)\, \psi(x_2, z/2)$$

after adding the z-slices for beam 1:

$$\frac{d^2L(x_1, x_2)}{dx_1\, dx_2} = \frac{4}{l}\int_0^{l/2} \psi(x_1, t)\, dt \int_0^{l/2} \psi(x_2, z)\, dz$$

Effective particle energy

The centre-of-mass energy for two colliding particles is always expressed as s, the square of the energy: $s = (E_1 + E_2)^2$, i.e. in the usual case $s = 4E_0$ and normalized to the reference energy E_0 it becomes $s = 4$

It is not difficult to show that for the effective energy one has:

$$s_{eff} \equiv x_1 x_2$$

Provided a model for ψ for the two beams is available, one can compute (probably numerically) the differential luminosity as a function of s, i.e. of the effective centre-of-mass energy:

$$\frac{dL(s)}{ds} = L_0 \int_s^1 \int_0^1 \delta(s - x_1 x_2) \cdot \psi(x_1)\, \psi(x_2)\, dx_1\, dx_2$$

The differential luminosity

$$\frac{d^2 L(x_1, x_2)}{dx_1\, dx_2} = \frac{4}{l} \int_0^{l/2} \psi(x_1, t)\, dt \int_0^{l/2} \psi(x_2, z)\, dz$$

will normally have a sharp peak at the nominal energies and strong contributions from particles with full (or almost full) energy and particles with energies degraded by the beamstrahlung.

How to keep beamstrahlung and side effects small ?

Some options:

- Shorter bunches ...
- Flat beams ...

  For flat bunches with a ratio $R = \sigma_x/\sigma_y$ we can reduce $\delta_e \propto R/(1 + R)^2$. For a fixed area $\sigma_x\sigma_y$ the luminosity would not be reduced

Not mentioned:

- "Flip-flop", special feature for strong-strong interactions
- Asymmetric beams
- Coasting beams
- Synchrobetatron coupling
- Monochromatization
- ... and many more

BACKUP SLIDES 1

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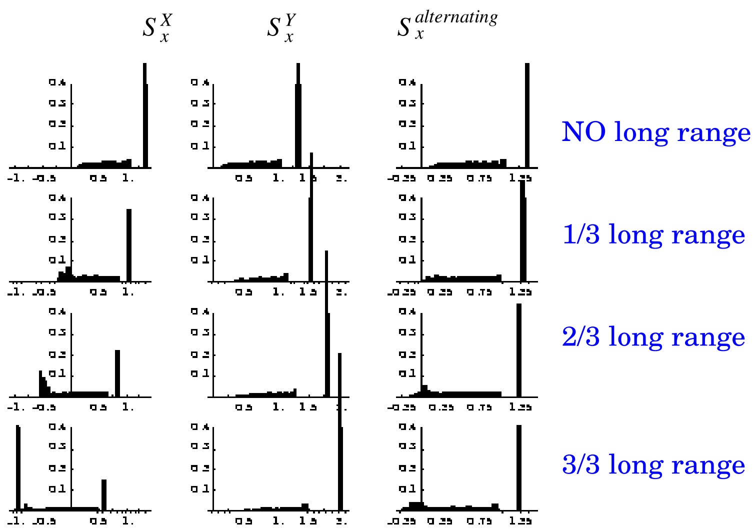

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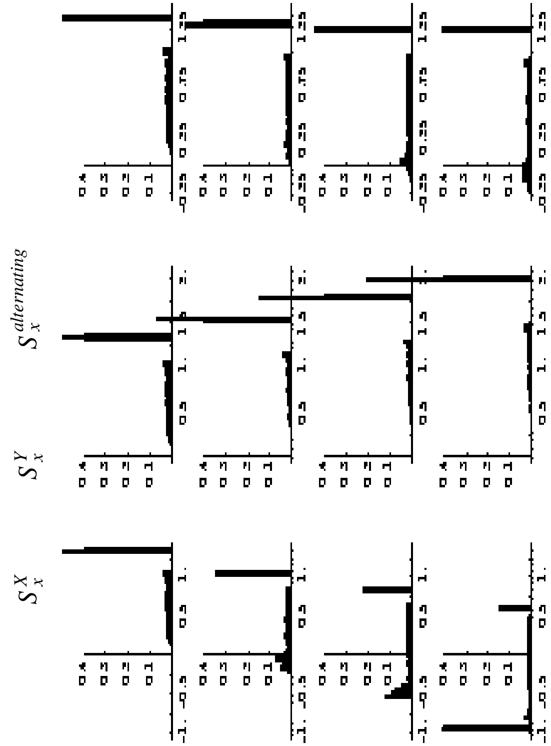

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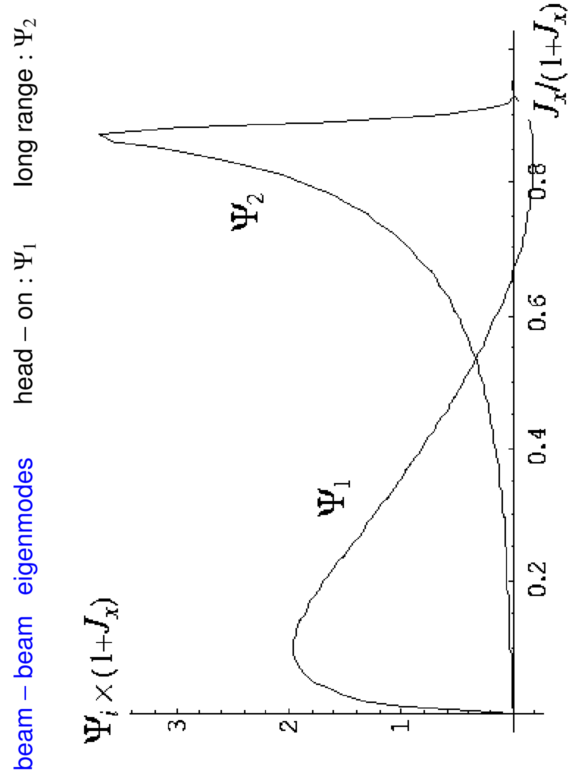

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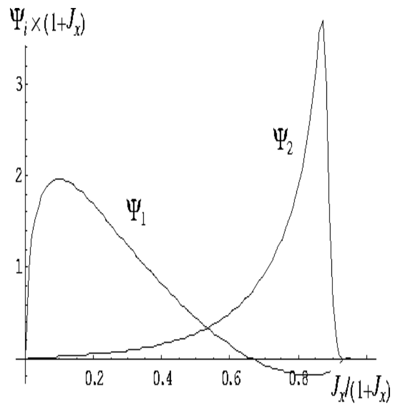

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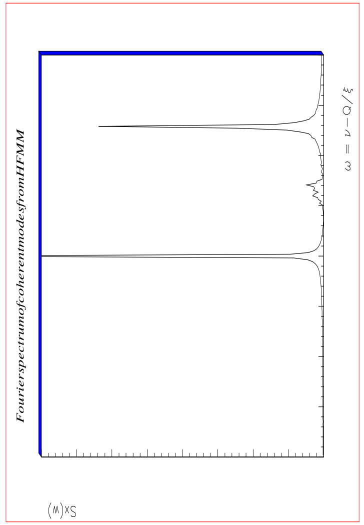
Long range modes, 10 σ_x



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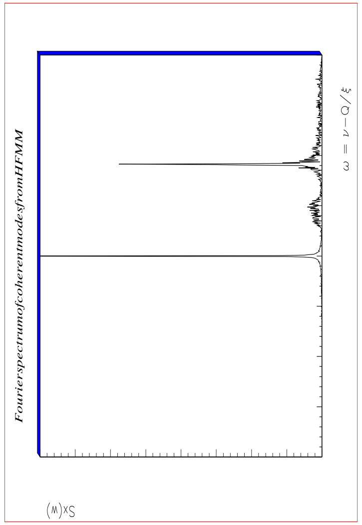
Long range modes, 6 σ_x



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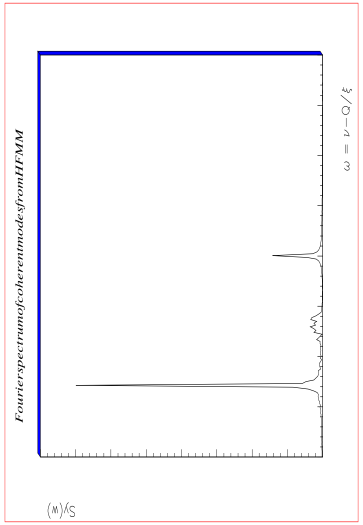
Long range modes, 10 σ_x



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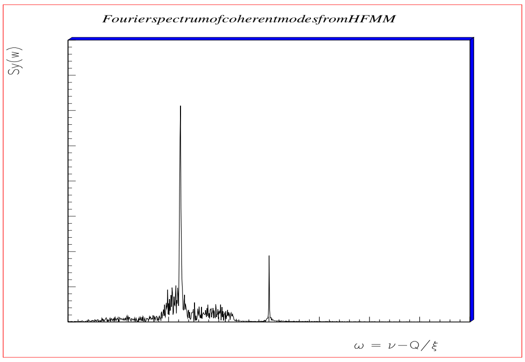
Long range modes, 6 σ_x



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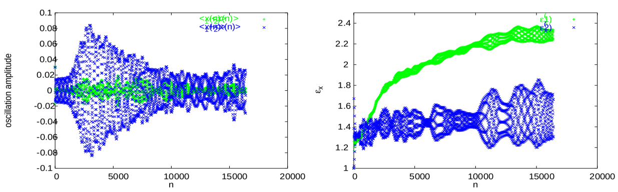
Results:



- Two beams have slightly different tunes
- Modes are damped, but emittance grows

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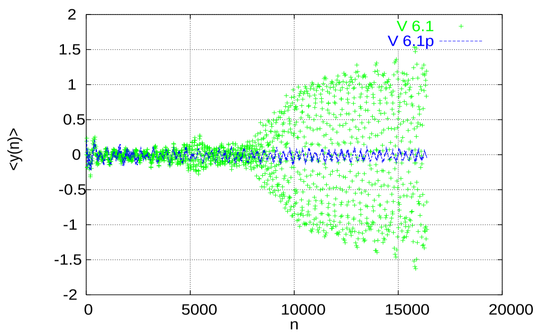
Multiple interaction points:

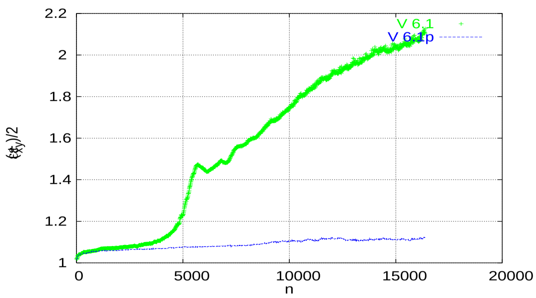
- Important: phase advance between interactions
- LHC has two independent rings
- Options to cancel coherent interaction at IPs:
  - Different working points $\Longrightarrow$
  - Phase advance redistribution (precision !)
  - Integer tunesplit
  - Small tune split and same working point

Phase advance redistribution



- Local phase change of 0.02

Phase advance redistribution



- Local phase change of 0.02

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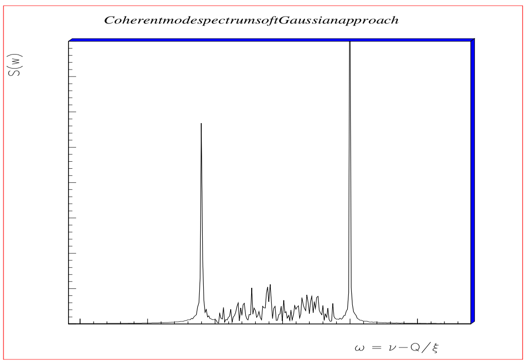

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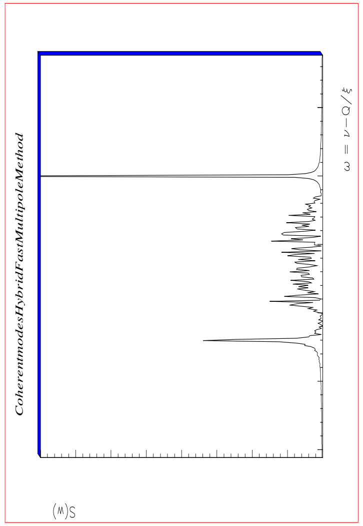

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BACKUP SLIDES 2

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Tools and methods:

Analysis and purpose:

- **Evaluate stability of solution**
- **Calculate frequency spectra of oscillations**
- **Identify discrete spectral lines of oscillations**

Tools:

- **Numerical integration of Vlasov equation**
- **Multi-particle and multi-bunch tracking**
- **Perturbation theory**

Numerical integration

- **Vlasov equation is Partial Differential Equation**
- **Aim: find distribution and its time evolution**
- **Use: numerical integration with Finite Difference Methods**
- **Basic concept:**
  - **Replace derivative by finite differences**
  - **Represent continuous function $\psi(x, t)$ by two-dimensional grid u_j^n ($t \rightarrow n$, $x \rightarrow j$)**

Example 1:

$$\frac{\delta u}{\delta t} = \lambda \frac{\delta^2 u}{\delta x^2}, \qquad u(x, t=0) = u_j^0 = f(x) \qquad \textbf{becomes :}$$

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \lambda \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2}$$

$$\Longrightarrow \qquad u_j^{n+1} = u_j^n + \frac{\lambda \Delta t}{\Delta x^2}(u_{j+1}^n - 2u_j^n + u_{j-1}^n)$$

spacial step Δx: $x_j = j \cdot \Delta x$

time step Δt for t: $t_n = n \cdot \Delta t$

initial distribution: $u_j^0 = f(x_j)$

Example 2:

$$\frac{\delta^2 u}{\delta t^2} = \lambda^2 \frac{\delta^2 u}{\delta x^2}, \qquad u(x, t=0) = u_j^0 = f(x) \qquad \textbf{becomes :}$$

$$\frac{u_j^{n+1} - 2u_j^n + u_{j-1}^n}{\Delta t^2} = \lambda^2 \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2}$$

$$\Longrightarrow \qquad u_j^{n+1} = 2(1 - \left(\frac{\lambda \Delta t}{\Delta x}\right)^2) u_j^n + \left(\frac{\lambda \Delta t}{\Delta x}\right)^2 (u_{j+1}^n + u_{j-1}^n) - u_j^{n-1}$$

Exercise $\rightarrow$ try to solve:

$$\frac{\delta^2 u}{\delta t^2} = 4 \frac{\delta^2 u}{\delta x^2}, \qquad 0 < x < 1, \qquad u(x, 0) = \sin(\pi x)$$

Strategy for numerical integration

- **Use finite difference scheme**
- **Back to our problem which looks like (per beam):**

$$\frac{\partial\psi(x,p_x;t)}{\partial t} = -A(x;t)\frac{\partial\psi(x,p_x;t)}{\partial p_x} - B(p_x)\frac{\partial\psi(x,p_x;t)}{\partial x},$$

- **In each substep integrate in one direction (operator splitting):**

$$\frac{\partial\psi(x,p_x;t)}{\partial t} = -A(x;t)\frac{\partial\psi(x,p_x;t)}{\partial p_x}$$
$$\frac{\partial\psi(x,p_x;t)}{\partial t} = -B(p_x)\frac{\partial\psi(x,p_x;t)}{\partial x}$$

- **Discretise $\psi(x,p_x;t)$ on the grid (i,j,n): U_{ij}^n (n = t/Δt)**

Grid calculations:

Each substep equation is of the type: $\frac{\delta u}{\delta t} = \lambda\frac{\delta f(u)}{\delta p_x}$
with: $f(u) = A(x)\cdot u$

- u_{ij}^n **is the discretisation at time** $t = n\Delta t$ **of the density** $\psi(x,p_x;t)$ **for** $x = i\Delta x$ **and** $y = j\Delta p_x$
- **Use the Lax-Wendroff scheme which looks like (e.g. first half-step in j-direction):**

$$u_{ij}^{n+1/2} = u_{ij}^n - \frac{A(x;t)}{2}\frac{\Delta t}{\Delta p_x}(u_{ij+1}^n - u_{ij-1}^n) + \frac{1}{2}\left(A(x;t)\frac{\Delta t}{\Delta p_x}\right)^2(u_{ij+1}^n - 2u_{ij}^n + u_{ij-1}^n)$$

Exercise → try to derive (hint: Taylor expansion of u in t)

Grid calculations:

- Putting it all together with $f = A(x)u$ and a similar half-step for $g = B(p_x)u$ (now going the i-direction) one gets
  $u_{ij}^{n} \rightarrow u_{ij}^{n+\frac{1}{2}} \rightarrow {u_{ij}}^{n+1}$
- Small complication: in presence of discontinuities this method may generate oscillations
- Remedy: introduce artificial 'viscosity'

Strategy for simulations

- Represent bunches by macro particles (10^4)
- Track each particle individually around machine
- At interaction points evaluate force from other beam on each particle (that is where space charge effects are treated similar)
- In principle: for each particle in beam a calculate the integral over $\rho^b(x', t)$
- In practice not possible, unless one makes assumptions (e.g. Gaussian beams etc.), need other techniques

FIELD COMPUTATION

Solve Poisson equation for potential $\Phi(x,y)$ with charge distribution $\rho(x,y)$:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Phi(x,y) = -2\pi\rho(x,y)$$

formally possible with Green's function:

$$\Phi(x,y) = \int G(x-x', y-y')\rho(x,y)dx'dy'$$

and (for open boundary):

$$G(x-x', y-y') = -\frac{1}{2}\ln[(x-x')^2 + (y-y')^2]$$

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Techniques for field (force) calculation

- **Soft Gaussian approximation: Assume Gaussian distribution with varying centre and width (fast but not precise, but o.k. for incoherent studies)**
- **Particle-particle methods: (precise but slow, typical: N = 10^4)**
- **Particle-mesh methods: evaluate field on a finite mesh (precise, but slow for separated beams)**
- **Hybrid Fast Multipole Methods (HFMM): recent method, precise and much better for separated beams and beam halos**

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PARTICLE-PARTICLE METHODS (PP)

- Simple: accumulate forces by finding the force F(i,j) between particle i and particle j
- Problem: computational cost is O(N_p^2)
- For our problems typically $N_p \geq 10^4$
- Used sometimes in astrophysics
- For $N_p < 10^3$ and for close range dynamics good

PARTICLE-MESH METHODS (PM)

- Approximate force as field quantity on a mesh.
- Differential operators are replaced by finite difference approximations.
- Particles (i.e. charges) are assigned to nearby mesh points (various methods).
- Problems:
  - Computational cost is O$(N_g \ln(N_g))$
  - Bad to study close encounters
  - Not ideal when mesh is largely empty (e.g. long range interactions)

PARTICLE-MESH METHODS

- Main steps:
  - Assign charges to mesh points (NG, TSC, CIC)
  - Solve field equation on the mesh (many variants)
  - Calculate force from mesh defined potential
  - Interpolate force on grid ($N_g \cdot N_g$) to find force on particle

PARTICLE ASSIGNMENT METHODS

- NGP: (Nearest Grid Point), densities at mesh points are assigned by the total amount of charge surrounding the grid point, divided by the cell volume. Drawback are discontinuous forces.
- CIC: (Cloud in cell), involve 2^K nearest neighbours, (K = dimension of the problem), give continuous forces.
- TSC: (Triangular Shaped Cloud), use assignment interpolation function that is piecewise quadratic.

IN PRACTICE ...

- Must look at:
  - Stability
  - Noise reduction (short scale fluctuations due to granularity)
  - Number of particles
  - Size of grid cells
  - . . .

A MORE RECENT APPROACH

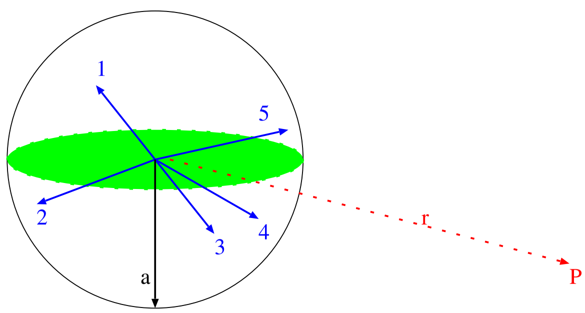
- Fast Multipole Method (FMM)
- Derived from particle-particle methods, i.e. particles are not on a grid
- Tree code: treat far-field and short-field effects separately
- Relies on composing multipole expansions
- Computing cost: between O(N_p) and O$(N_p \ln(N_p))$

Fast Multipole Method

- Well-known multipole expansion at point P for k point charges q_i:

$$\Phi(\vec{r}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{M_l^m}{r^{l+1}} Y_l^m(\theta, \phi)$$

$$\textbf{with :} \quad M_l^m = \sum_{i=1}^{k} q_i a_i^l Y_l^{*m}(\alpha_i, \beta_i)$$

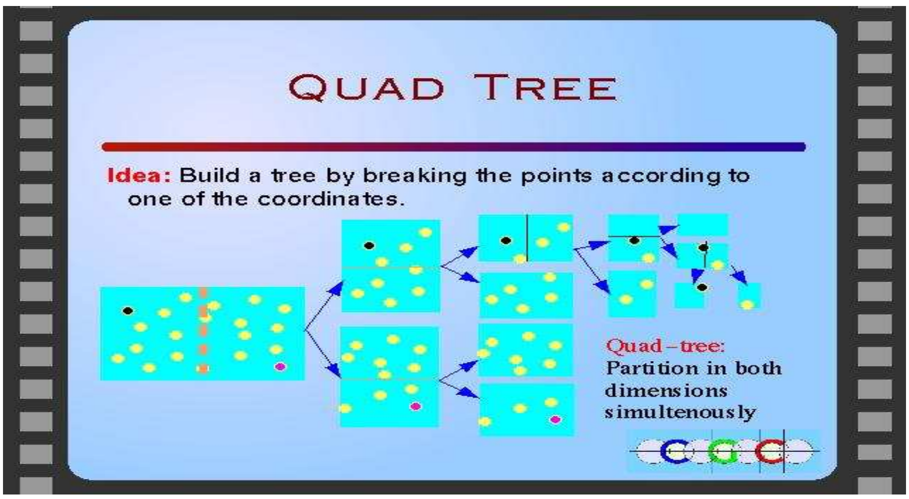


- Order determined by desired accuracy

FMM PROCEDURE

- Hierarchical spatial decomposition into small cells and sub-cells (e.g. quad-tree)
- Multipole expansion for each sub-cell
- Expansions in cells are combined to represent effect of larger and larger groups of particles
- A ’calculus’ is defined to relocate and combine multipole expansion
- Far-field effects are combined with near-field effects to give potential (and field) at every particle

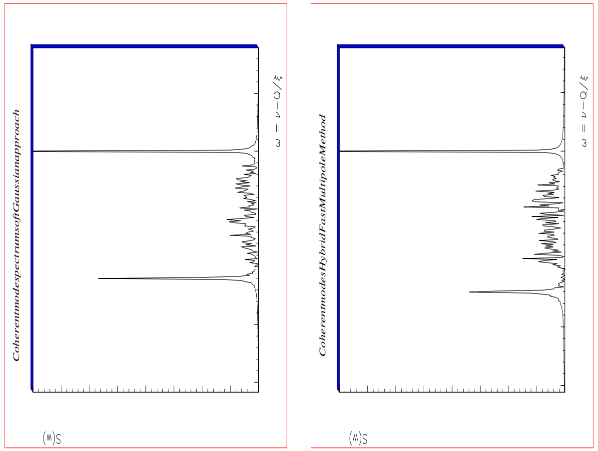
QUAD-TREE DIVISION



A VARIATION: HFMM

- **H**ybrid **F**ast **M**ultipole **M**ethod: FMM with a grid
- Assign particles on a grid, use FMM to calculate fields at grid points
- Particles may or may not be assigned to a grid
- Particle outside the grid are treated with standard FMM
- Precision is excellent and O(N_g) when all particles are on the grid
- Ideal for separated beams

Is a Gaussian good enough ??
(or: why all this effort ?)



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- Factor is 1.1 (and not 1.214)

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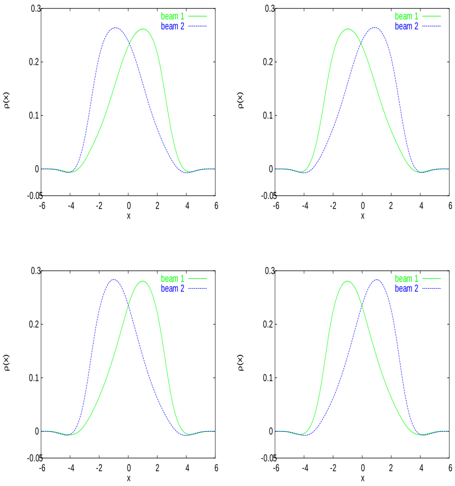
Why not ??



- "Skewness" important !
- Mostly core participates in oscillation
- Exercise: why does that change the frequency ?

Strategy of perturbation theory

- $\psi^a(x, p_x; t) \;=\; \psi^a_s \;+\; \psi^a_o(x, p_x; t)$
- **Go to I_x and ϕ_x (action and angle)**
- **Fourier expansion:**
  $\psi^a_o(I_x, \phi_x; t) \;=\; \sum_m \; exp(im\,\phi_x \;-\; \nu\,t) \;\cdot\; e^{(-I_x/2)} \;\cdot\; f^a_m(I_x)$
- **In Vlasov equation:** $i\frac{\partial}{\partial t}\vec{f} \;=\; \xi \cdot A \cdot \vec{f}$
- **With** $\vec{f} \;=\; (f^a, f^b)$ **and** $f \;\sim\; exp(-\xi\,\lambda\,t)$
- **Eigenvalues of** $\lambda\,\vec{f} \;=\; A\,\vec{f}$ **related to mode frequencies**
- **Can obtain eigenmodes** $\psi^a_{o,\lambda}(I_x, \phi_x; t)$