Nonlinearities in light sources

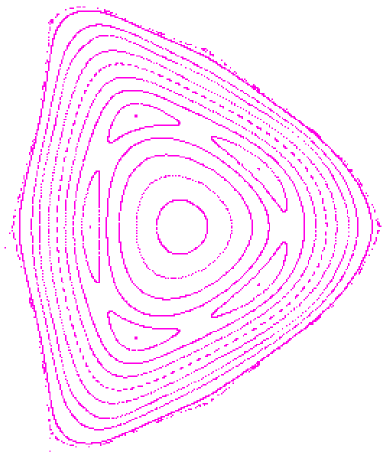
or: *How to correct the chromaticity without destroying the dynamic aperture in a high brightness light source lattice*

Andreas Streun
Swiss Light Source SLS, Paul Scherrer Institute, Villigen, Switzerland

Contents:

- The source of chromaticity
- Chromaticity correction with sextupoles
- Breakdown of dynamic acceptance: investigation of sextupole effects
- First order optimization of the sextupole pattern
- Particular light source problem
- Second order optimization: octupole terms

Nonlinearities in Light Sources



by design:

Sextupoles
for chromaticity correction

The RF bucket

$\gg$

parasitic:

multipolar errors:
$\longrightarrow$ ring magnets
$\longrightarrow$ insertion devices

$\longrightarrow$ **how to deal with?**

$\longrightarrow$ **define tolerances**

Nonlinearities $\longrightarrow$ CHAOS $\longrightarrow$ separatrix for bounded motion (= dynamic acceptance)

Light source problems:

- Transverse acceptance **bad** [i.e. separatrix < beam pipe] $\longrightarrow$ **no injection**
- Lattice energy acceptance (= energy dependant transverse acceptance) **bad** [i.e. < RF energy acceptance] $\longrightarrow$ **low beam lifetime**

Methods for nonlinear optimization

analytical

1^{st} & 2^{nd} order perturbation theory
→ *maps, resonance drive terms, tune shifts with amplitude ...*

- ⊕ quick calculation
- ⊕ interactive optimization (semi-analytic minimization)
- ⊖ no prediction of performance (perturbation!)
  → *"the art of weighting..."*

⟹ **Design**

⟺ **Iterate** ⟺

numerical

Particle tracking
→ *Poincaré plots, dynamic aperture scans, particle spectra...*

- ⊖ slow calculation
- ⊖ difficult to use in minimizer (fractal parameter space!)
- ⊕ valid prediction of performance
  → complete & correct model

⟹ **Proof**

Quadrupole:

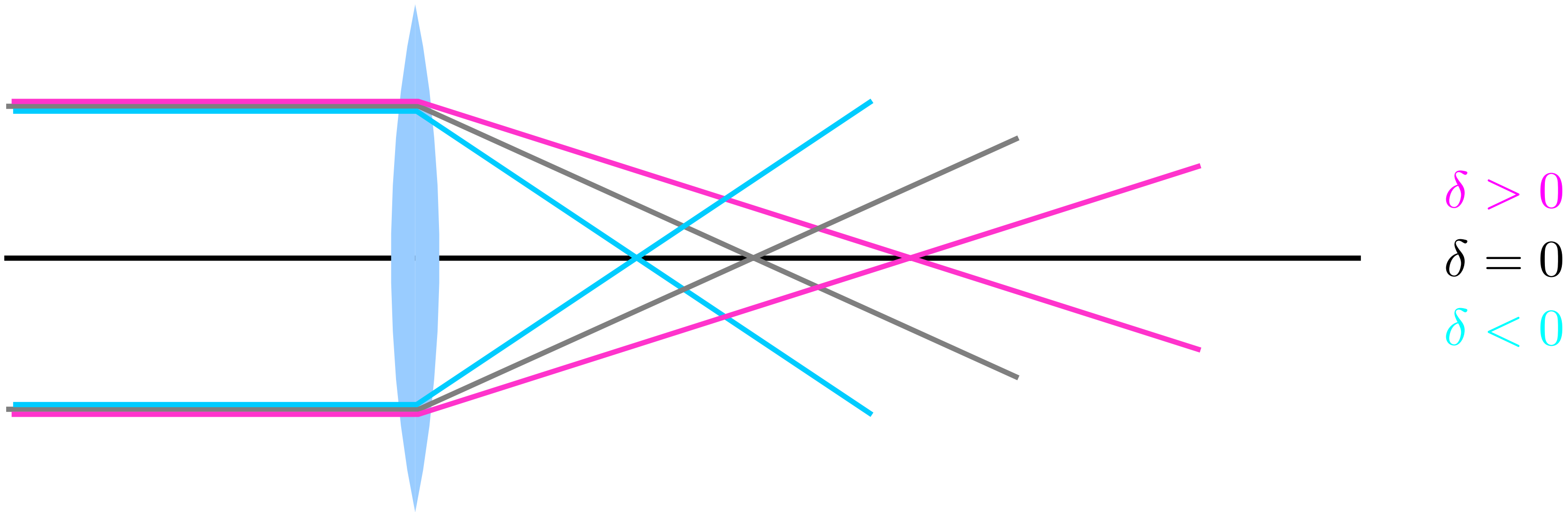
Length L

Strength $b_2 = \frac{1}{(B\,\rho)} \frac{d\,B_y}{d\,x}$

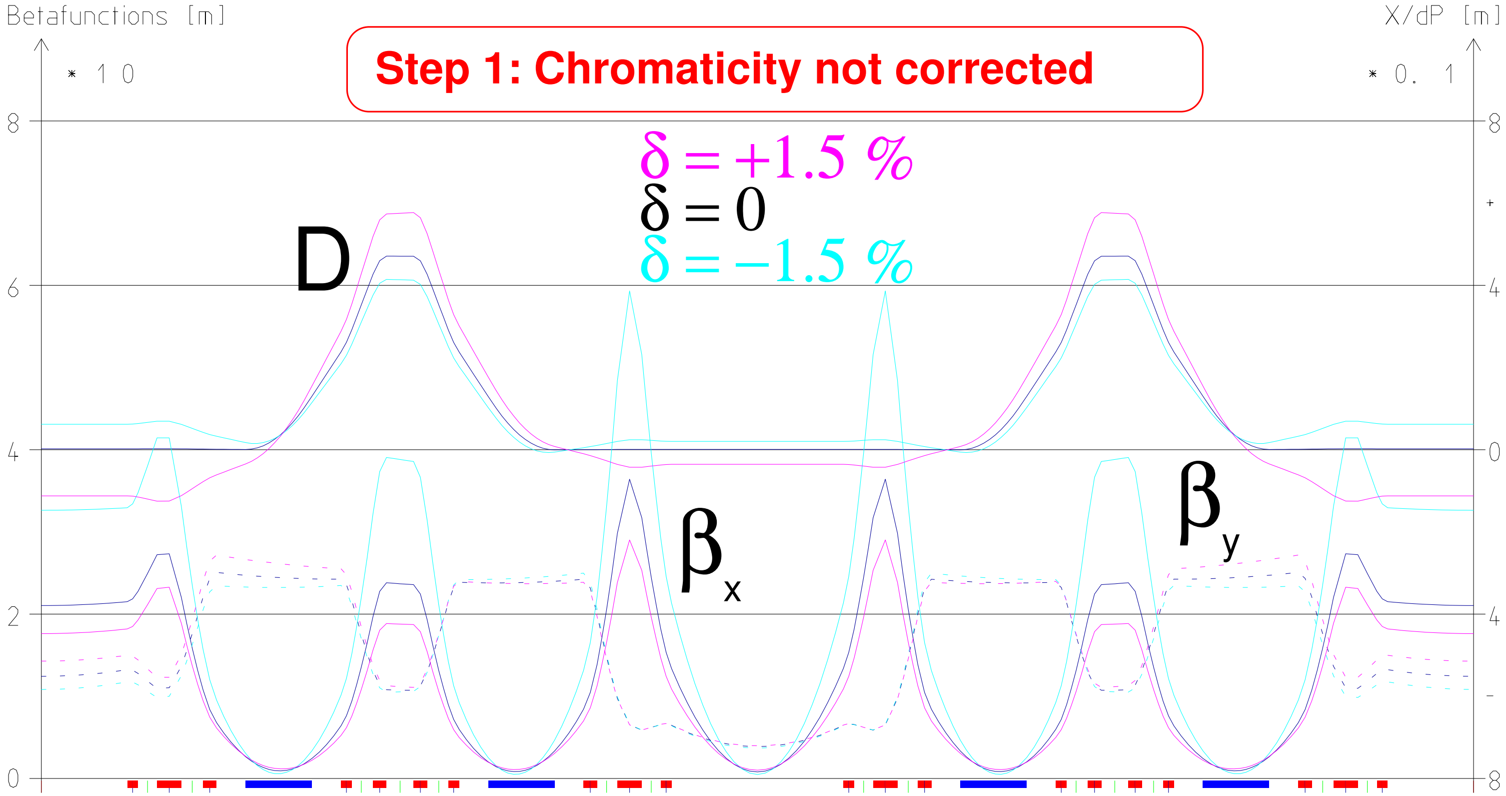
$$(B\,\rho) := \frac{p}{e} = 3.3356\,\mathrm{Tm} \cdot E[\mathrm{GeV}]$$

Kicks on particle: $\Delta x' = -b_2 L x \quad \Delta y' = b_2 L y \qquad (b_2 > 0 \rightarrow \text{horiz.foc.})$

$$\text{Chromatic aberration: } b_2(\delta) = \frac{b_2}{(1+\delta)} \approx b_2\,(1-\delta) \qquad \delta := \frac{\Delta p}{p}$$



Chromaticity



Bends(5.625 deg) Quadrupoles Sextupoles

16*L = 845.99 m (periodic)	$\delta = 0$	Qx = 35.4503	Qz = 11.3976	Ex = 8.17E-0009 @ 6 GeV
		Cx = -96.1574	Cz = -30.4910	Al = 0.00028
dp/p = -1.50 %		Qx = 37.2591	Qz = 11.8640	$\delta = -1.5\ \%$
dp/p = 1.50 %		Qx = 34.1891	Qz = 10.9465	$\delta = +1.5\ \%$

Test lattice: ESRF with original optics for dispersion free straights

Chromaticity

(Gradient error $\Delta b_2\, ds$) $\times$ (one turn matrix $\mathcal{M}$) = (new one turn matrix $\tilde{\mathcal{M}}$)[a]

Gradient error due to chromatic aberration: $\pm\Delta b_2 = \mp b_2 \delta$ (hor./vert.)

$$\begin{pmatrix} 1 & 0 \\ \pm b_2 \delta\, ds & 1 \end{pmatrix} \times \begin{pmatrix} \cos 2\pi Q & \beta \sin 2\pi Q \\ -\frac{\sin 2\pi Q}{\beta} & \cos 2\pi Q \end{pmatrix} = \begin{pmatrix} \cos 2\pi \tilde{Q} & \beta \sin 2\pi \tilde{Q} \\ -\frac{\sin 2\pi \tilde{Q}}{\beta} & \cos 2\pi \tilde{Q} \end{pmatrix}$$

$$\frac{1}{2}\mathrm{Tr}(\tilde{\mathcal{M}}) = \cos 2\pi \tilde{Q} = \cos 2\pi (Q + \Delta Q) = \cos 2\pi Q \pm \frac{1}{2} b_2 \delta\, \beta\, \sin 2\pi Q\, ds$$

$$\Delta Q \ll 1 \longrightarrow \Delta Q = \mp \frac{1}{4\pi} b_2 \delta\, \beta\, ds \qquad \xi = \frac{\Delta Q}{\delta} = \mp \frac{1}{4\pi} \oint_C b_2(s)\, \beta(s)\, ds$$

Light source: Low emittance: $\epsilon \downarrow \; \rightarrow \; \int b_2 \uparrow \; \rightarrow \; \xi_x \uparrow \qquad (\xi_x \approx -50 \ldots -100)$

$\Longrightarrow$ **Head tail instability**[b]

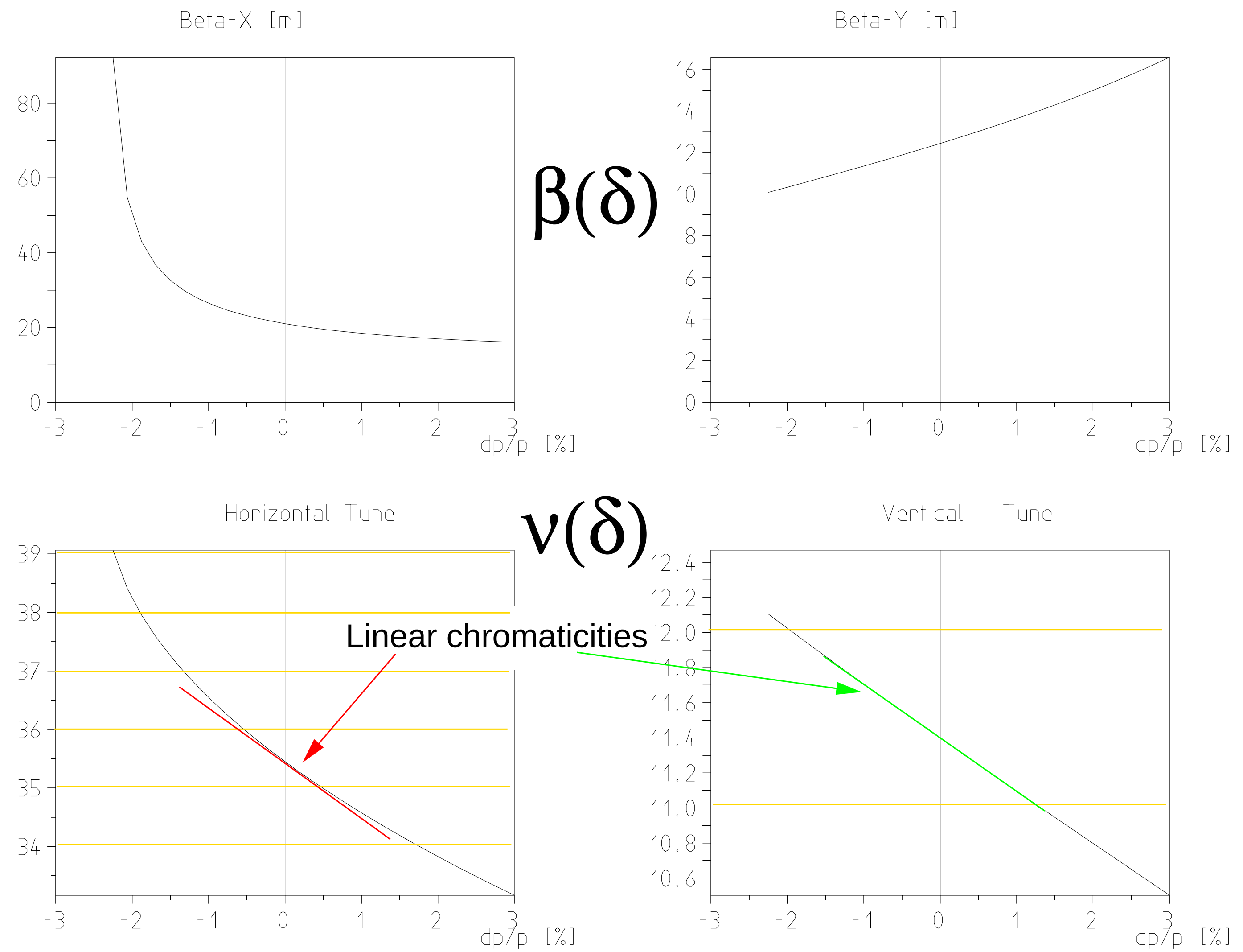
$\Longrightarrow$ **Low energy acceptance**

---

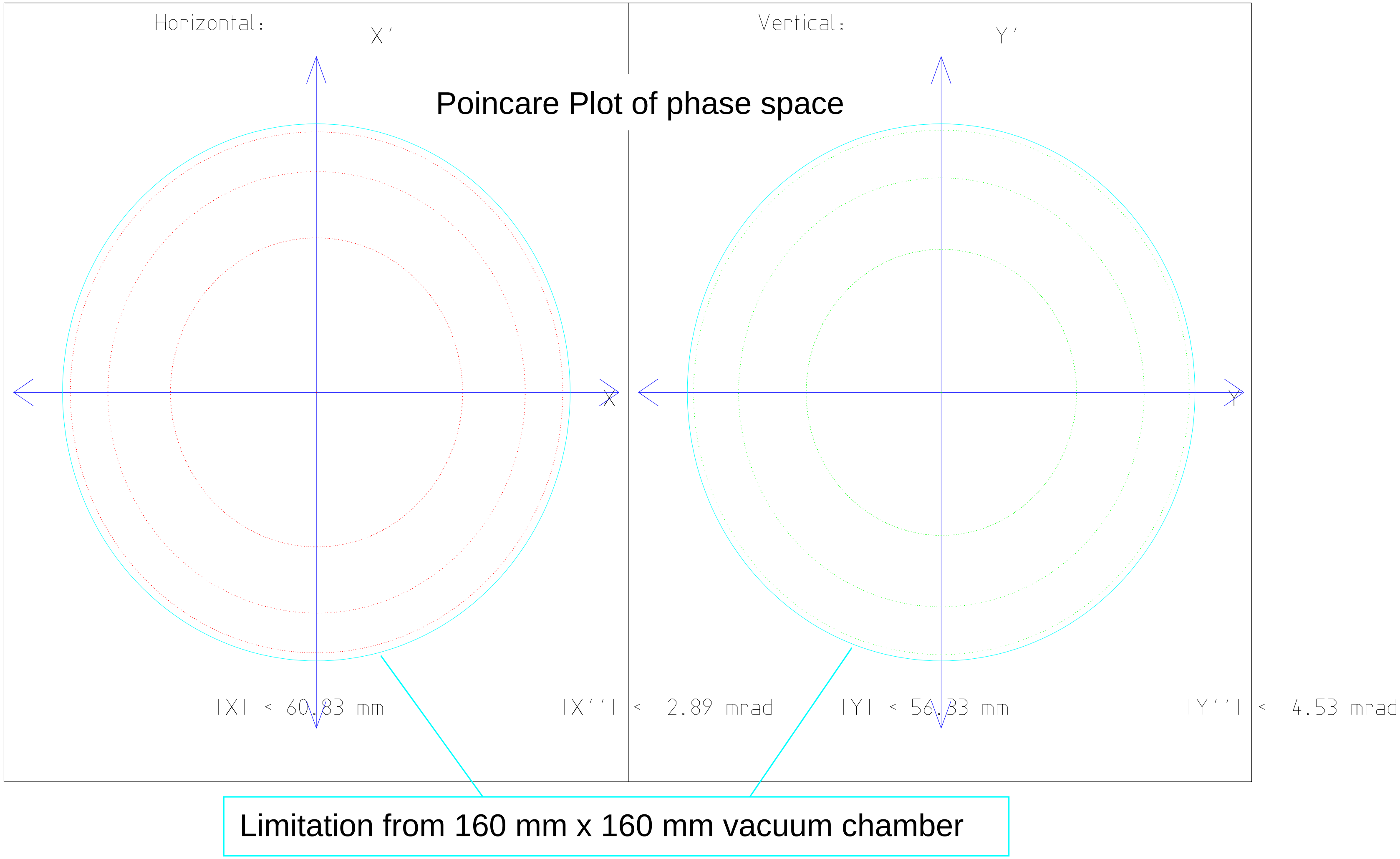
[a] E. D. Courant amd H. S. Snyder, The alternate gradient synchrotron, Ann. Phys. 3

[b] A.W.Chao, Coherent instabilities of a relativistic bunched beam, AIP Conf. Proc. 105 (1982) 353

Step 1: Chromaticity not corrected



Step 1: No chromaticity correction: Linear betatron motion

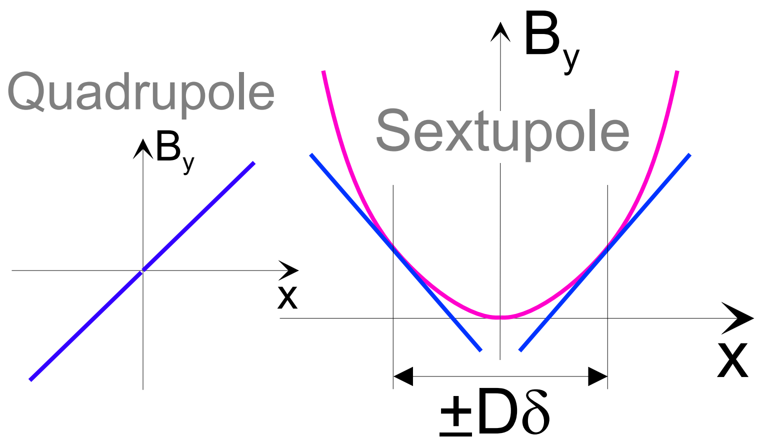


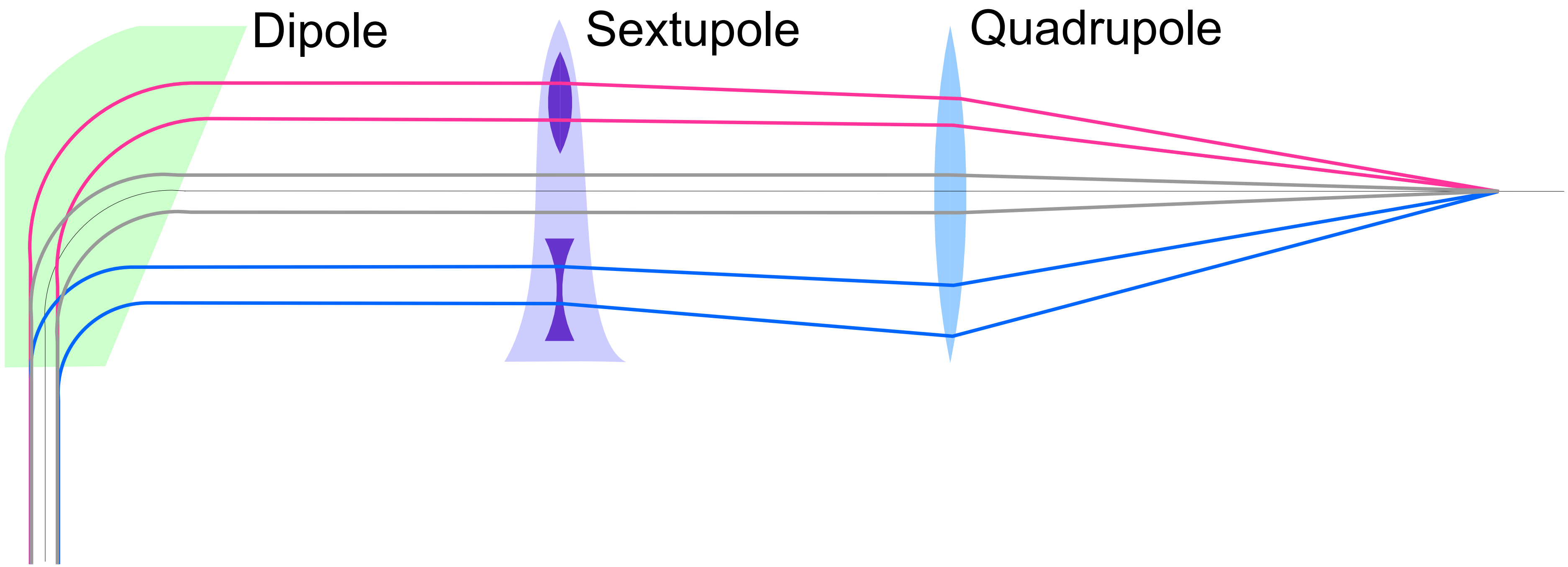
Chromaticity correction

Sextupole: $B_y(x) = \frac{1}{2} B'' x^2$

local gradient: $B'_y(x) = B'' x$

"Order" by momentum: $x(\delta) = D\delta$





Quadrupole: $b_2 = \frac{1}{(B\,\rho)}\frac{d\,B_y}{d\,x}$

$$\Delta x' = -b_2 L x$$
$$\Delta y' = b_2 L y$$

Sextupole: $b_3 = \frac{1}{2}\frac{1}{(B\,\rho)}\frac{d^2\,B_y}{d\,x^2}$

$$\Delta x' = -b_3 L (x^2 - y^2)$$
$$\Delta y' = 2 b_3 L x y$$

Chromatic aberrations: $b_n(\delta) = b_n/(1+\delta) \approx b_n(1-\delta)$

Sextupoles in dispersive regions: $x \rightarrow D\delta + x \quad y \rightarrow y$

Kicks on a particle (keep up to second order in products of x, y, δ) :

Quadrupole: $\Delta x' = -b_2 L x + [b_2 L]\,\delta\, x \qquad \Delta y' = +b_2 L y - [b_2 L]\,\delta\, y$

Sextupole: $\Delta x' = -[2 b_3 L D]\,\delta\, x - b_3 L (x^2 - y^2) - b_3 L D^2 \delta^2$

$\Delta y' = +[2 b_3 L D]\,\delta\, y + 2 b_3 L\, x y$

$\Longrightarrow$ ☺ Chromaticity correction for $(2 b_3 L D \stackrel{!}{=} b_2 L)$:

$\Longrightarrow$ ☹ nonlinear kicks...

$$\xi_{x/y} = \pm\frac{1}{4\pi}\oint_C [2b_3(s)D(s) - b_2(s)]\,\beta_{x/y}(s)\,ds$$

$$= \frac{1}{4\pi}\Big(\pm\sum_{\text{sext}} 2(b_3L)_n\,\beta_{(x/y)n}\,D_n \mp \sum_{\text{quad}}(b_2L)_n\,\beta_{(x/y)n}\Big) \overset{!}{=} 0\ \ [+\Delta\xi_{x/y}]$$

Linear system: **2** *families* of sextupoles **SF, SD**

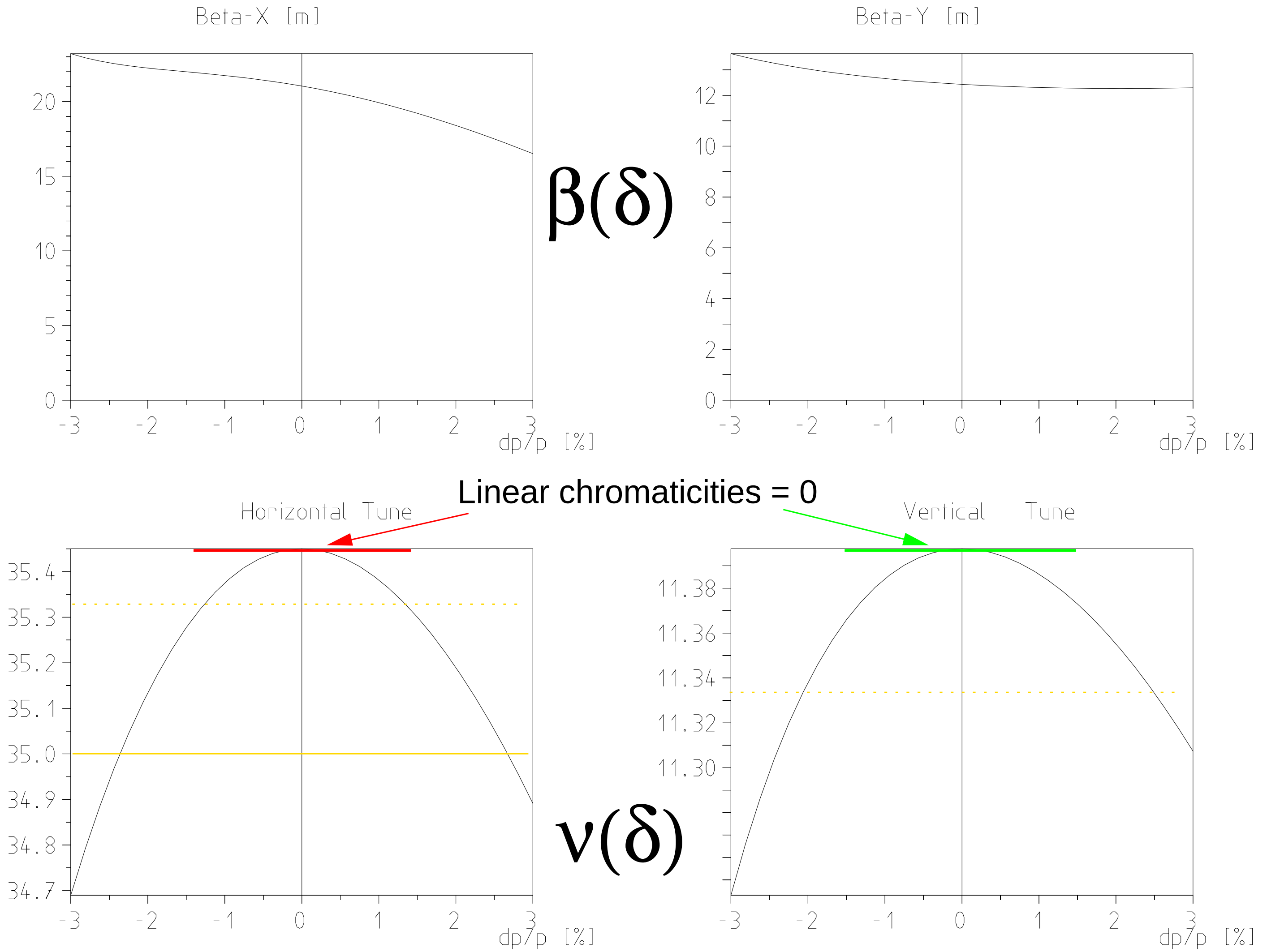
$$\frac{1}{2\pi}\begin{pmatrix} +\sum_{n\in\mathrm{SF}}\beta_{xn}D_n & +\sum_{n\in\mathrm{SD}}\beta_{xn}D_n \\ -\sum_{n\in\mathrm{SF}}\beta_{yn}D_n & -\sum_{n\in\mathrm{SD}}\beta_{yn}D_n \end{pmatrix}_{2\times 2} \times \begin{pmatrix} (b_3L)_{\mathrm{SF}} \\ (b_3L)_{\mathrm{SD}} \end{pmatrix}_{1\times 2}$$

$$= \frac{1}{4\pi}\begin{pmatrix} +\sum_{\mathrm{Quad}}(b_2L)\,\beta_{xn} \\ -\sum_{\mathrm{Quad}}(b_2L)\,\beta_{yn} \end{pmatrix}_{1\times 2} \left[+\begin{matrix}\Delta\xi_x \\ \Delta\xi_y\end{matrix}\right] \Longrightarrow (b_3L)_{\mathrm{SF}},\ (b_3L)_{\mathrm{SD}}$$

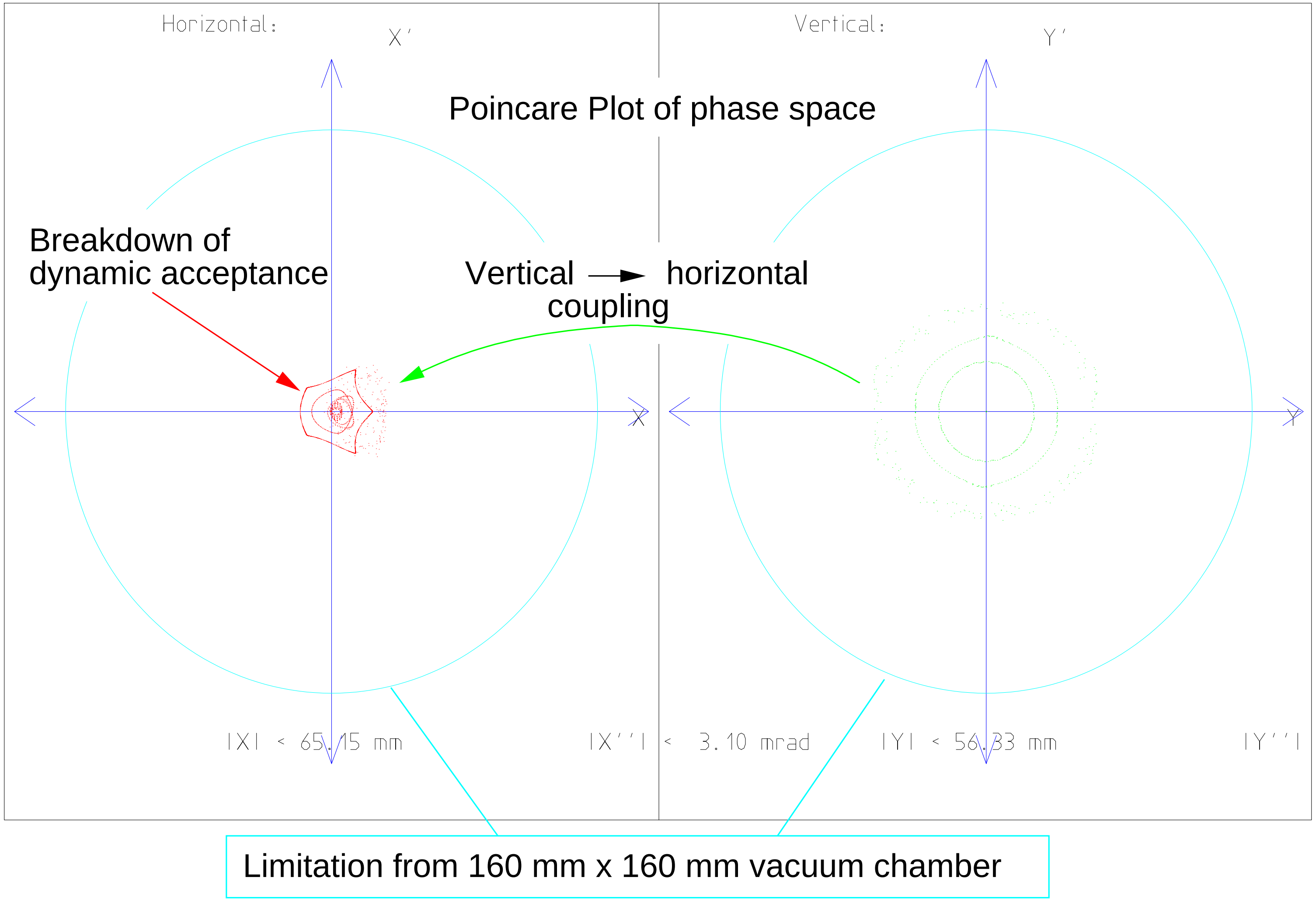
Decoupling:

$\longrightarrow$ SF locations: $D\uparrow$, $\beta_x\uparrow$, $\beta_y\downarrow$ $\qquad\longrightarrow$ SD locations: $D\uparrow$, $\beta_x\downarrow$, $\beta_y\uparrow$

Step 2: Chromaticity correction with 2 sextupole families



Step 2: Chromaticity correction with 2 sextupole families



The Hamiltonian

$$H(s) = \overbrace{\frac{p_x^2 + p_y^2}{2(1+\delta)}}^{\text{kinetic}} \overbrace{- \underbrace{b_1\, x\, \delta}_{\text{dispersive}} + \underbrace{\frac{b_1^2}{2}\, x^2}_{\text{focussing}}}^{\text{Dipoles}} + \underbrace{\overbrace{\frac{b_2}{2}\,(x^2 - y^2)}^{\text{Quadrupoles}}}_{H_2(s)} + \underbrace{\overbrace{\frac{b_3}{3}\,(x^3 - 3xy^2)}^{\text{Sextupoles}}}_{H_3(s)} + \ldots$$

Hamiltonian equations of motion (local):

$x'(s) = \frac{\partial H}{\partial p_x} = \frac{p_x}{1+\delta}$ $\qquad p_x'(s) = -\frac{\partial H}{\partial x} = b_1\delta - (b_1^2 + b_2)x + b_2 y - b_3(x^2 - y^2)$

$\rightarrow$ e.g. kick from quad. (length L): $\quad \Delta x' = \int_Q x''\, ds = \int_Q \frac{p_x'}{1+\delta}\, ds = -\frac{b_2}{1+\delta}\, L\, x$

Goal: Global Quad. & Sextupole contribution: $\int_{\text{cell}} [H_2(s) + H_3(s)]\, ds$

$\rightarrow$ independant of δ: chromaticity corrected

$\rightarrow$ linear and uncoupled ($\sim x^2,\ y^2$): cancellation of nonlinear kicks

Insert betatron oscillations (global, linear, flat lattice):

$$x(s) = \sqrt{2J_x\,\beta_x(s)}\cos\phi(s) + D(s)\,\delta \qquad y(s) = \sqrt{2J_x\,\beta_x(s)}\cos\phi(s)$$

$$\Longrightarrow \quad \int_{\text{cell}} [H_2(s) + H_3(s)]\,ds = \sum h_{jklmp} \text{ with}$$

$$h_{jklmp} \propto \sum_n^{N_\text{sext}} (b_3\,L)_n\,\beta_{xn}^{\frac{j+k}{2}}\;\beta_{yn}^{\frac{l+m}{2}}\;D_n^p\,e^{i\{(j-k)\phi_{xn}+(l-m)\phi_{yn}\}} - \left[\sum_n^{N_\text{quad}} (b_2\,L)_n\,\beta_{xn}^{\frac{j+k}{2}}\;\beta_{yn}^{\frac{l+m}{2}}\;e^{i\{(j-k)\phi_{xn}+(l-m)\phi_{yn}\}}\right]_{p\neq 0}$$

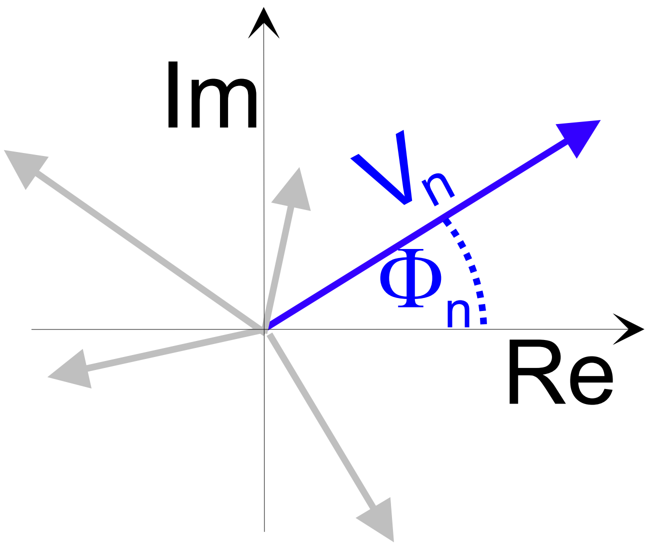
$$h = \sum_n^{N_\text{sext}} V_n e^{i\Phi_n}\ [\,+\ldots \text{quads for } p \neq 0 \ldots]$$

Sextupole$_n$ $\leftrightarrow$ complex vector:

Length $V_n = V_n\,(b_3, L, \beta_x, \beta_y, D)$

Angle $\Phi_n = \Phi_n\,(\phi_x + \phi_y)$

- $\Phi_n = 0\ \forall\ n$ $\rightarrow$ tune shifts
- $\Phi_n \neq 0$ $\rightarrow$ resonances



First order sextupole [+quadrupole] Hamiltonian

- **2 phase independant terms → chromaticities:**

$$h_{11001} = +J_x\delta\left[\sum_n^{N_{sext}}(2b_3\,L)_n\beta_{xn}D_n - \sum_n^{N_{quad}}(b_2\,L)_n\beta_{xn}\right] \quad \rightarrow \quad \xi_x$$

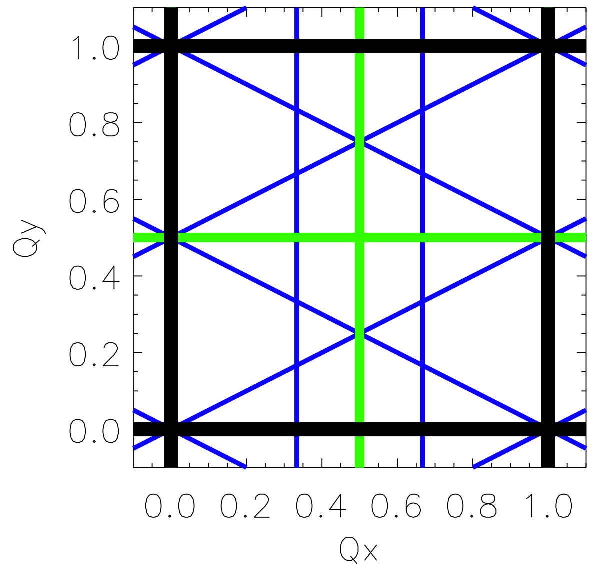
$$h_{00111} = -J_y\delta\left[\sum_n^{N_{sext}}(2b_3\,L)_n\beta_{yn}D_n - \sum_n^{N_{quad}}(b_2\,L)_n\beta_{yn}\right] \quad \rightarrow \quad \xi_y$$

- **7 phase dependant terms → resonances:** $h^N := h$ for N cells, $N \rightarrow \infty \quad \Longrightarrow$

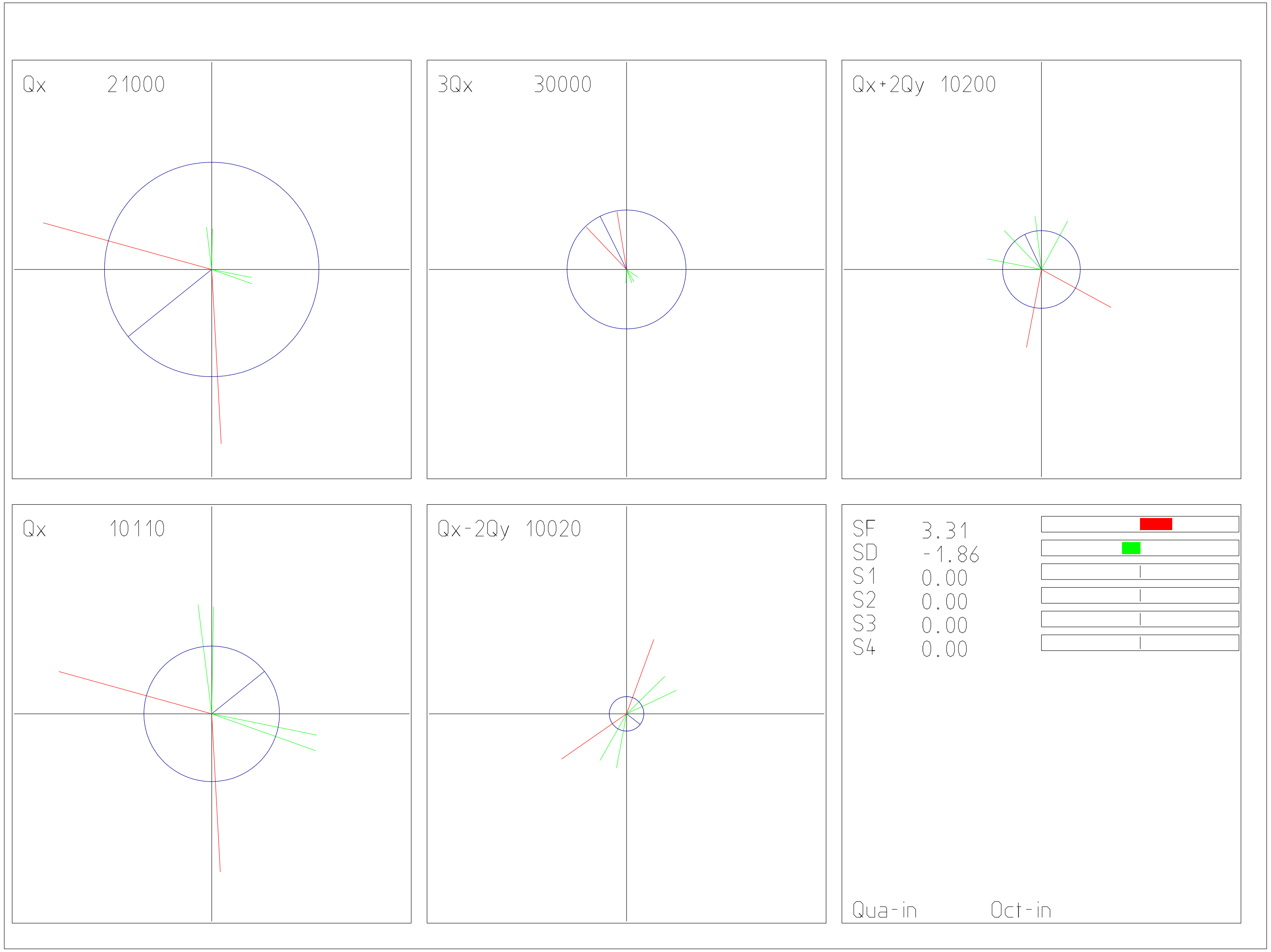
$$|h^{\infty}_{jklmp}| = \frac{|h_{jklmp}|}{2\sin\pi[a_xQ_x^{\text{cell}} + a_yQ_y^{\text{cell}}]}$$

$a_x = (j-k) \quad a_y = (l-m)$

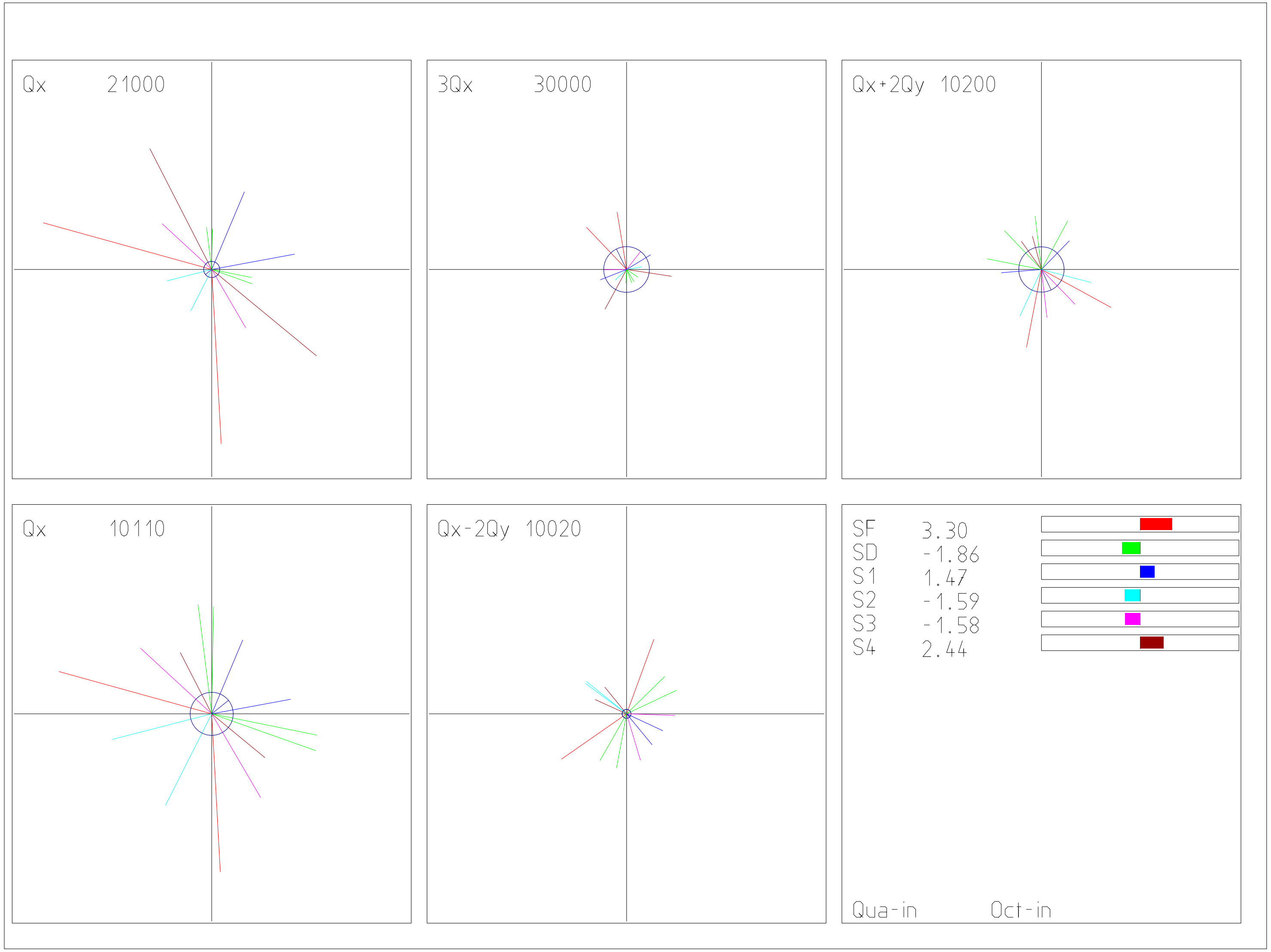
$$h_{21000} = h^*_{12000} \longrightarrow \mathbf{Q_x}$$
$$h_{30000} = h^*_{03000} \longrightarrow 3\,Q_x$$
$$h_{10110} = h^*_{01110} \longrightarrow \mathbf{Q_x}$$
$$h_{10200} = h^*_{01020} \longrightarrow Q_x + 2\,Q_y$$
$$h_{10020} = h^*_{01200} \longrightarrow Q_x - 2\,Q_y$$
$$h_{20001} = h^*_{02001} \longrightarrow 2\,Q_x$$
$$h_{00201} = h^*_{00021} \longrightarrow 2\,Q_y$$



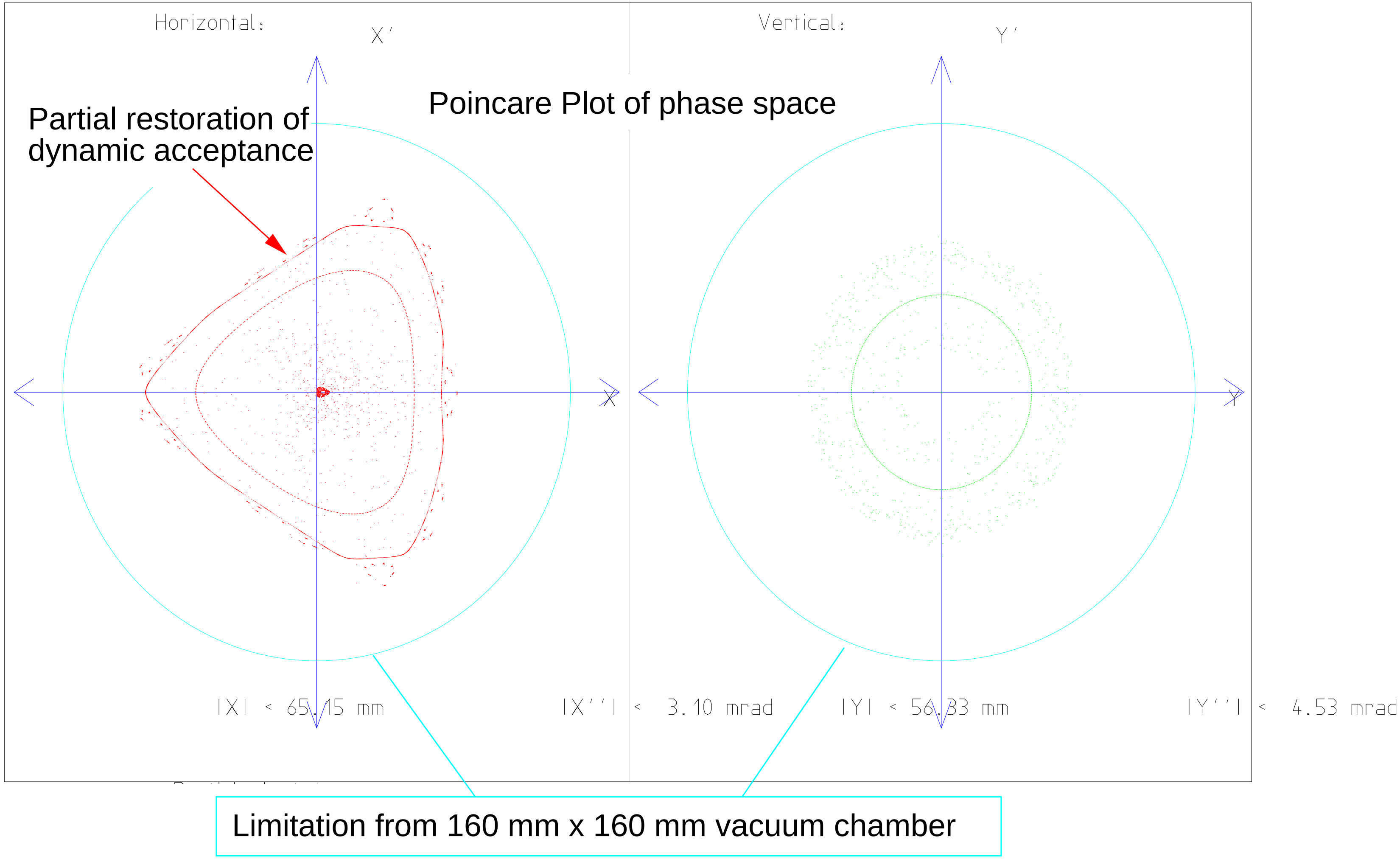
2 Sextupole families (ESRF standard cell)



6 Sextupole families (4 *harmonic* families in straight sections)

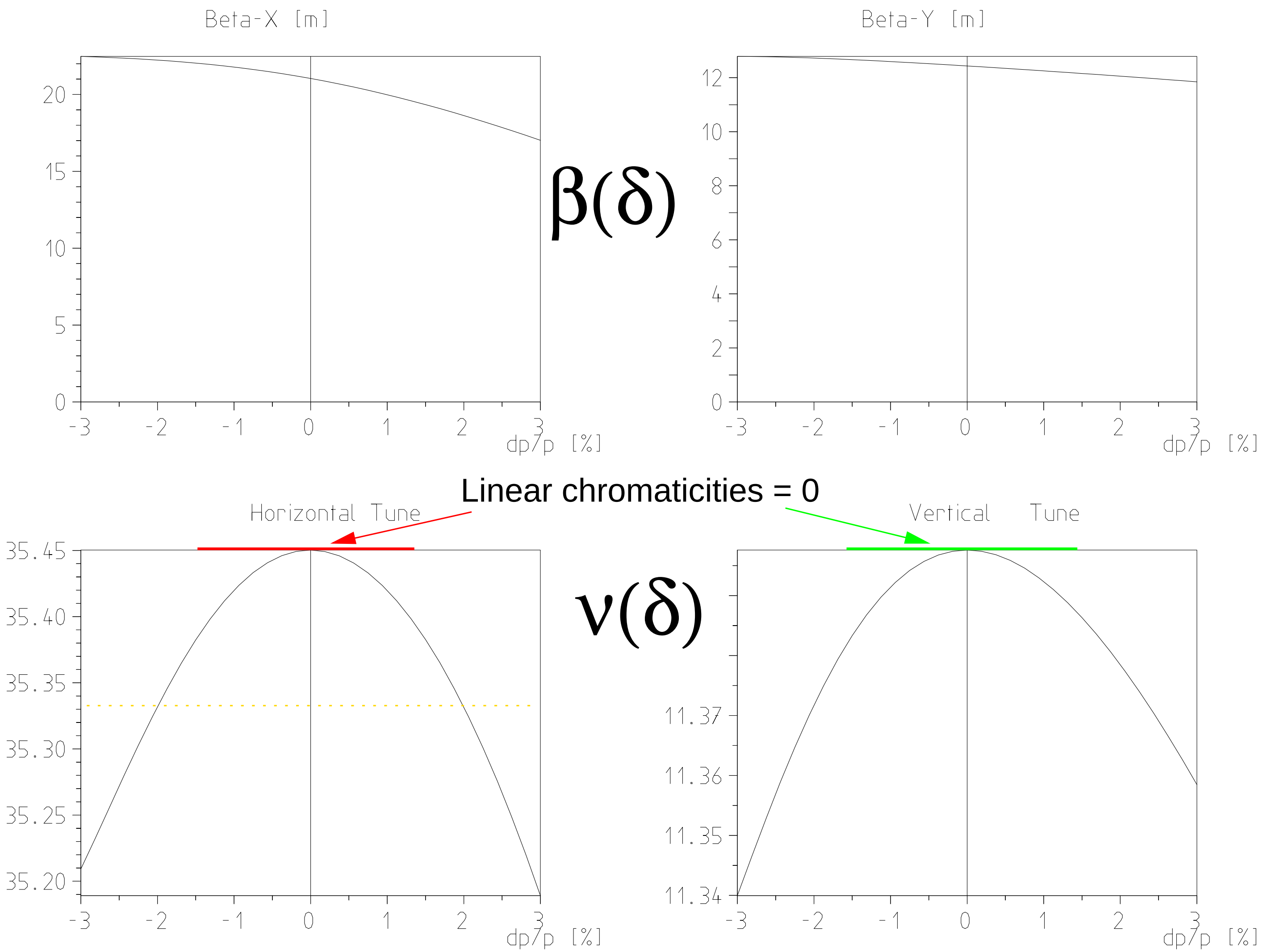


Step 3: First order sextupole optimization with 6 families



First order sextupole optimization

Step 3: First order optimization with 6 sextupole families



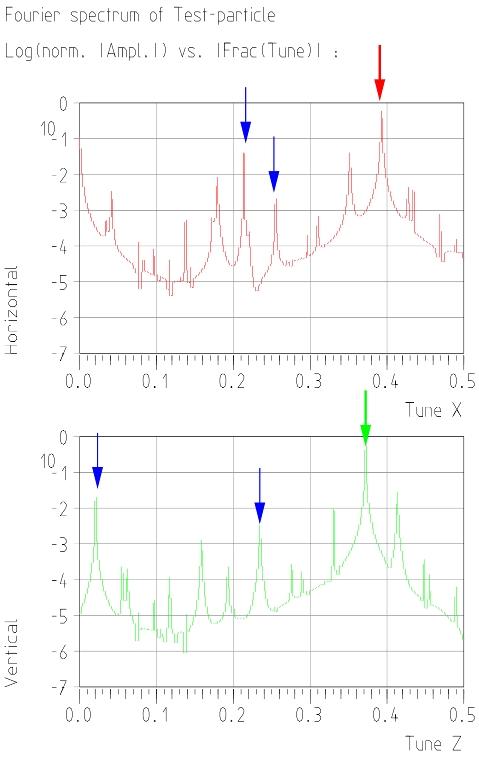
Fourier spectrum

Suppression of sextupolar resonances

2 sextupole families | **6** sextupole families

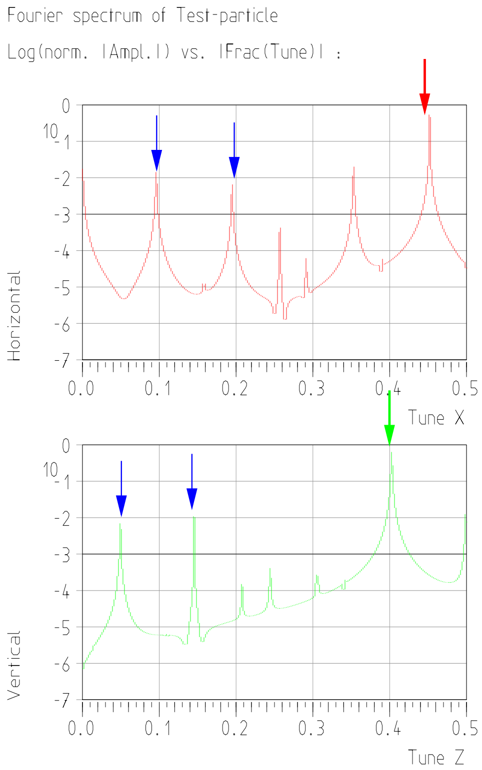
Tracking of test particle ($x_o = y_o = 4$ mm, $x'_o = y'_o = 0,\ \delta = 0$) and FFT:

2 sextupole families



Tune		Amp. [mm]	Guess
35.4503:			
0.3931:	Qx	3.5591	-
-0.0003:	-	0.6327	-
0.2138:	2Qx	0.2809	3Qx=112
0.3517:	Qx-2Qz	0.2256	2Qx-2Qz=48
0.1793:	3Qx	0.0491	4Qx=144
0.4275:	?	0.0251	-
0.4345:	?	0.0248	-
0.0415:	?	0.0201	-
0.2552:	2Qz	0.0128	Qx-2Qz=16
0.4687:	?	0.0043	-
11.3976:			
0.3724:	Qz	3.9946	-
0.4138:	2Qx-Qz	0.1829	2Qx-2Qz=48
0.0207:	Qx-Qz	0.1454	Qx-2Qz=16
0.3309:	?	0.0719	-
0.2345:	Qx+Qz	0.0217	Qx+2Qz=64
0.1586:	2Qx+Qz	0.0082	2Qx+2Qz=96

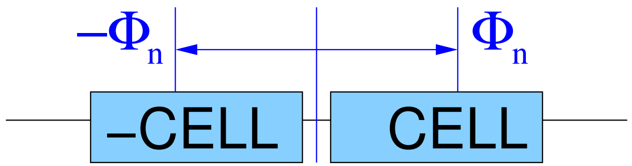
6 sextupole families



Tune		Amp. [mm]	Guess
35.4503:			
0.4520:	Qx	4.1865	-
0.3531:	Qx-2Qz	0.1378	2Qx-2Qz=48
-0.0003:	-	0.1166	-
0.0960:	2Qx	0.0955	3Qx=112
0.1949:	2Qz	0.0445	Qx-2Qz=16
11.3976:			
0.4025:	Qz	3.9480	-
0.4986:	2Qx-Qz	0.0818	2Qx-2Qz=48
0.1455:	Qx+Qz	0.0830	Qx+2Qz=64
0.0495:	Qx-Qz	0.0472	Qx-2Qz=16

Systematic first order optimization:†

9 terms h_{jklmp} (7 complex, 2 real) $\Longrightarrow$ Symmetry:



$\rightarrow$ **16** sextupole families $\quad Im(h_{jklmp}) = 0 \rightarrow$ **9** sextupole families.

Linear system for M families of sextupoles:

$$\left\{ \sum_{n \in \{\mathrm{S}m\}} \beta_n^{(\ldots)} D_n^{(\ldots)} e^{i\{(\ldots)\phi_n\}} \ldots \right\}_{9 \times M} \times \left\{ (b_3 L)_m \right\}_{M \times 1} = \left\{ \sum_{\mathrm{Quad}} (b_2 L) \ldots \right\}_{1 \times 9}$$

Light source problem: $\epsilon \downarrow \Longrightarrow \Delta\phi_x^{\mathrm{cell}} \rightarrow 180° \Longrightarrow e^{i2\phi_x} \approx 1$

$2Q_x$ resonance driving term h_{20001} proportional to chromaticity $\xi_x \propto h_{11001}$

$$2Q_x \rightarrow \frac{\partial \beta_x}{\partial \delta} \rightarrow \xi_x^{(2)} = \frac{\partial^2 Q_x}{\partial \delta^2} \rightarrow \text{energy acceptance} \downarrow$$

No solution for $\{(b_3 L)_m\} \Longrightarrow$ suppression by $\Delta\phi_x^{\mathrm{straight}}$

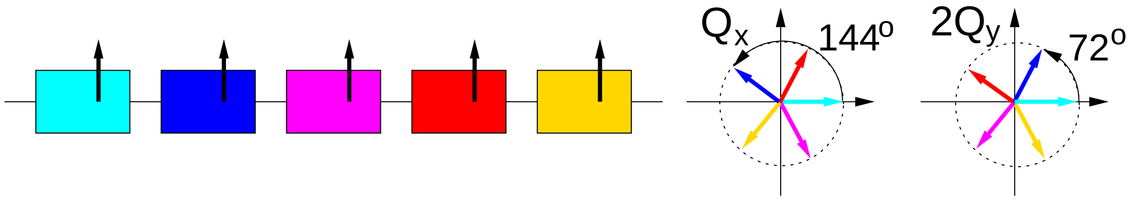
† J.Bengtsson et al., Increasing the energy acceptance of synchrotron light storage rings, NIM A 404 (1998) 237

Phase cancellation schemes

Periodicity: N cells

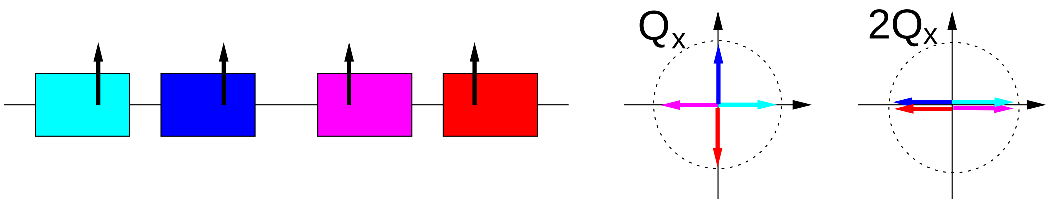
$$N\Delta Q_x^{\rm cell}, \quad [3N\Delta Q_x^{\rm cell}, \quad 2N\Delta Q_x^{\rm cell}], \quad 2N\Delta Q_y^{\rm cell} \quad \longrightarrow \text{integer!}$$

e.g. $N = 5,\ \Delta Q_x^{\rm cell} = 0.4\,(=144°),\ \Delta Q_y^{\rm cell} = 0.1\,(=36°)$



Symmetry: Lattice section vs. mirror image

$$\Delta Q_x^{\rm cell} = \frac{2n_x+1}{4}, \quad \Delta Q_y^{\rm cell} = \frac{2n_y+1}{4} \quad (n_x, n_y \text{ integers}):$$



$\rightarrow$ phase advances over straight sections $\rightarrow \beta_x, \beta_y$ in straights.

$\Longrightarrow$ Iterate: linear $\Longleftrightarrow$ nonlinear lattice design

Second order sextupole [+first order octupole] Hamiltonian

$$\sum_n \sum_m (b_3L)_n (b_3L)_m \times (\beta_n,\ \phi_n\ \beta_m,\ \phi_m\ \ldots) + \left[\sum_q (b_4L)_q \times (\beta_q,\ \phi_q\ \ldots)\right]$$

- **3 phase independant terms → amplitude dependant tune shifts:**

$$\frac{\partial Q_x}{\partial J_x} \quad \frac{\partial Q_x}{\partial J_y} = \frac{\partial Q_y}{\partial J_x} \quad \frac{\partial Q_y}{\partial J_y}$$

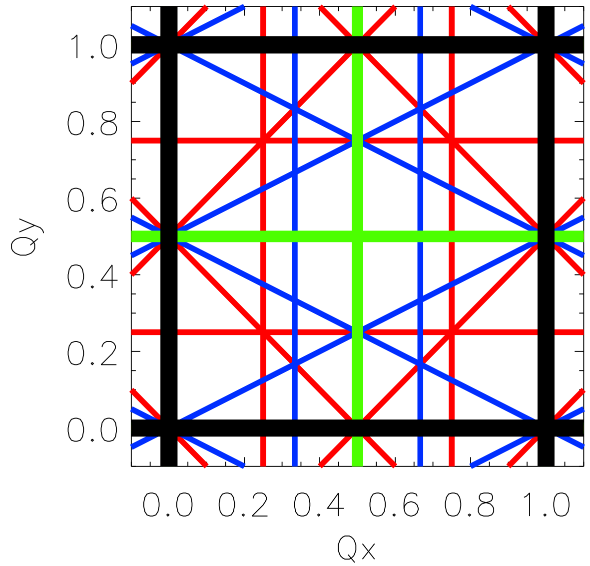
- **2 phase independant off-momentum terms → second order chromaticities:**

$$\xi^{(2)}_{x/y} = \frac{\partial^2 Q_{x/y}}{\partial \delta^2}$$

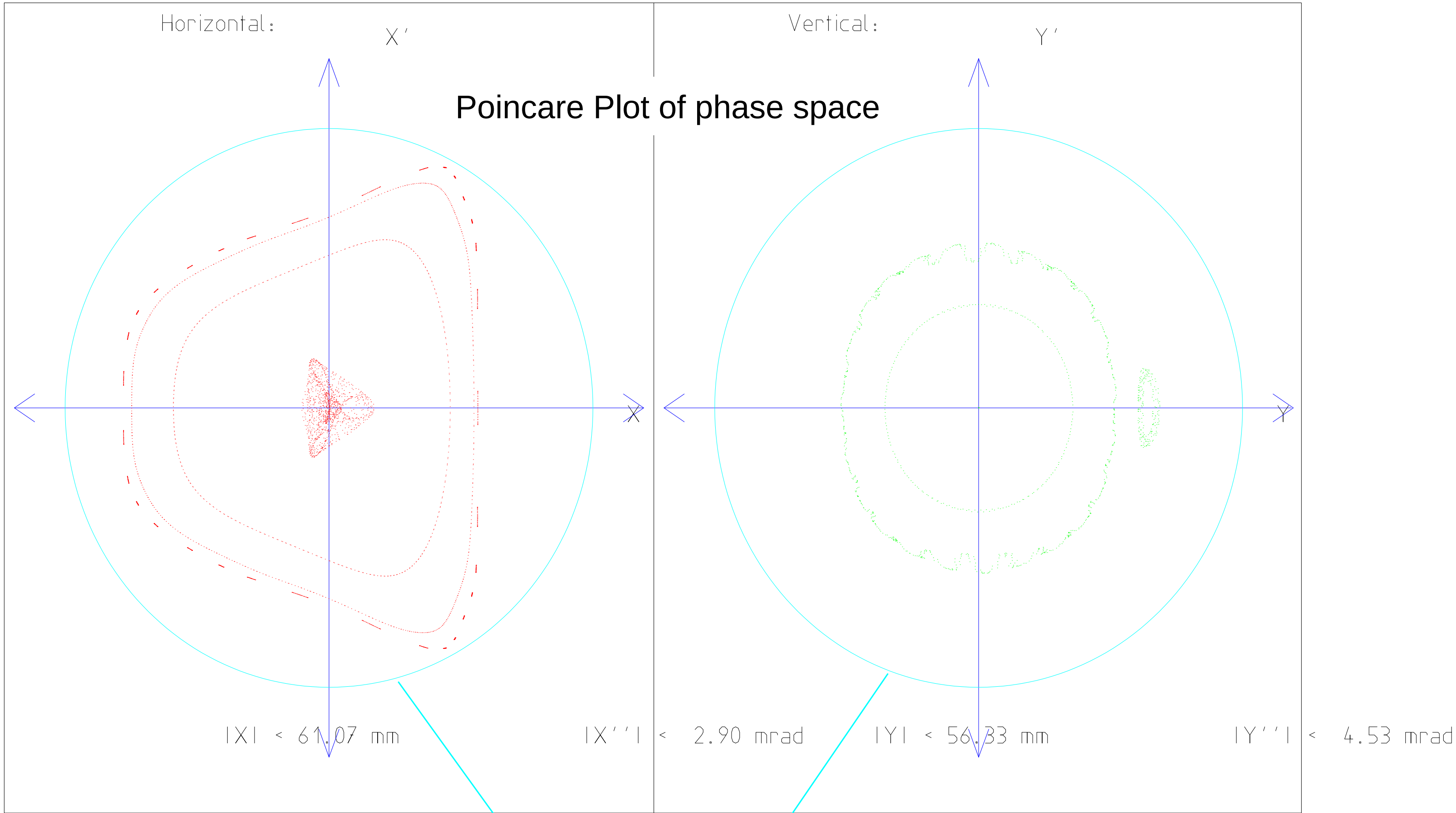
- **8 phase dependant terms**

  → octupolar resonances:

$$h_{40000} \to 4Q_x \qquad h_{31000} \to 2\,Q_x$$
$$h_{00400} \to 4\,Q_y \qquad h_{20110} \to 2\,Q_x$$
$$h_{20200} \to 2Q_x + 2\,Q_y \qquad h_{00310} \to 2\,Q_y$$
$$h_{20020} \to 2Q_x - 2\,Q_y \qquad h_{01110} \to 2\,Q_y$$



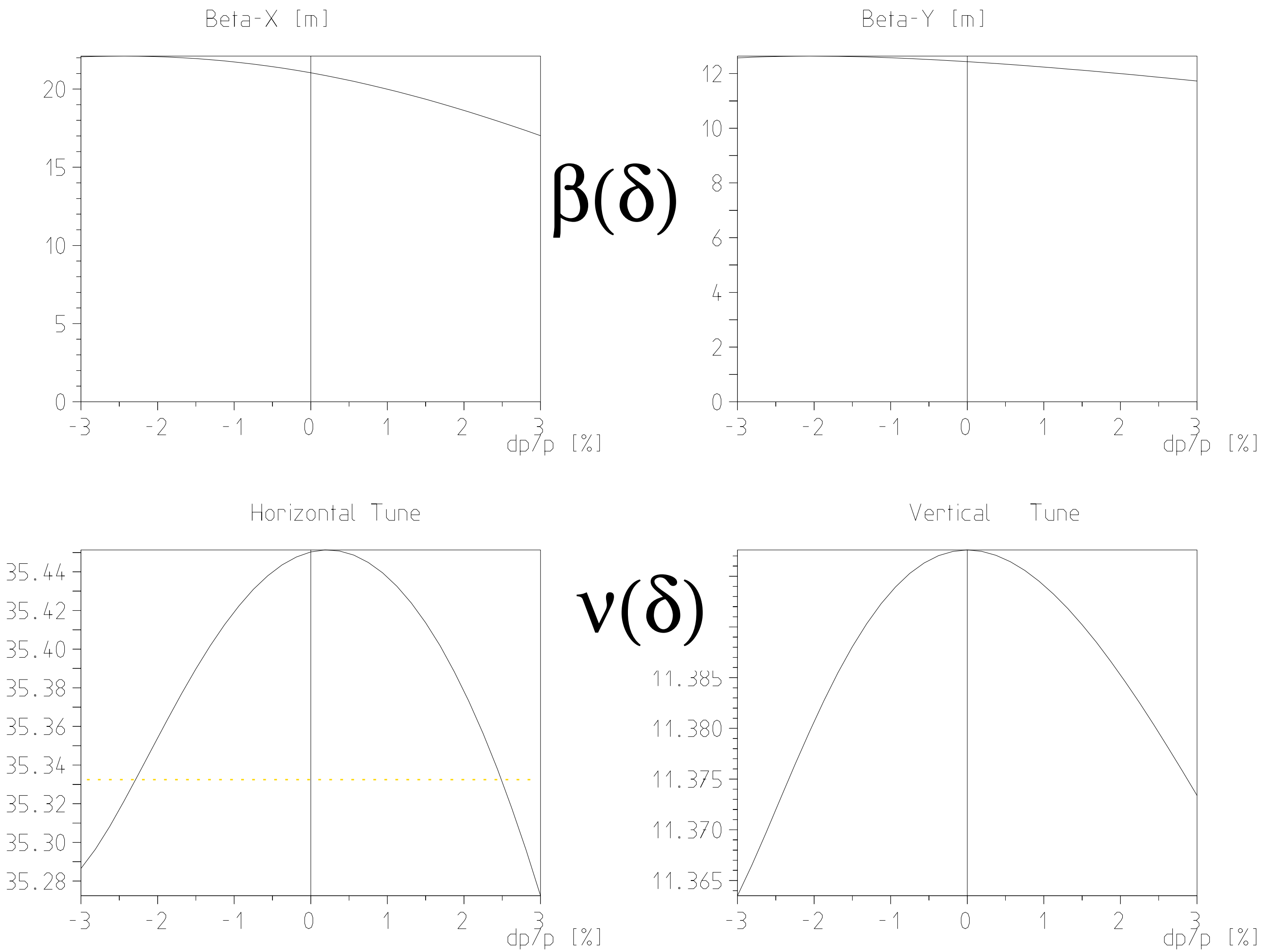
Step 4: Second order sextupole optimization with 6 families



Limitation from 160 mm x 160 mm vacuum chamber

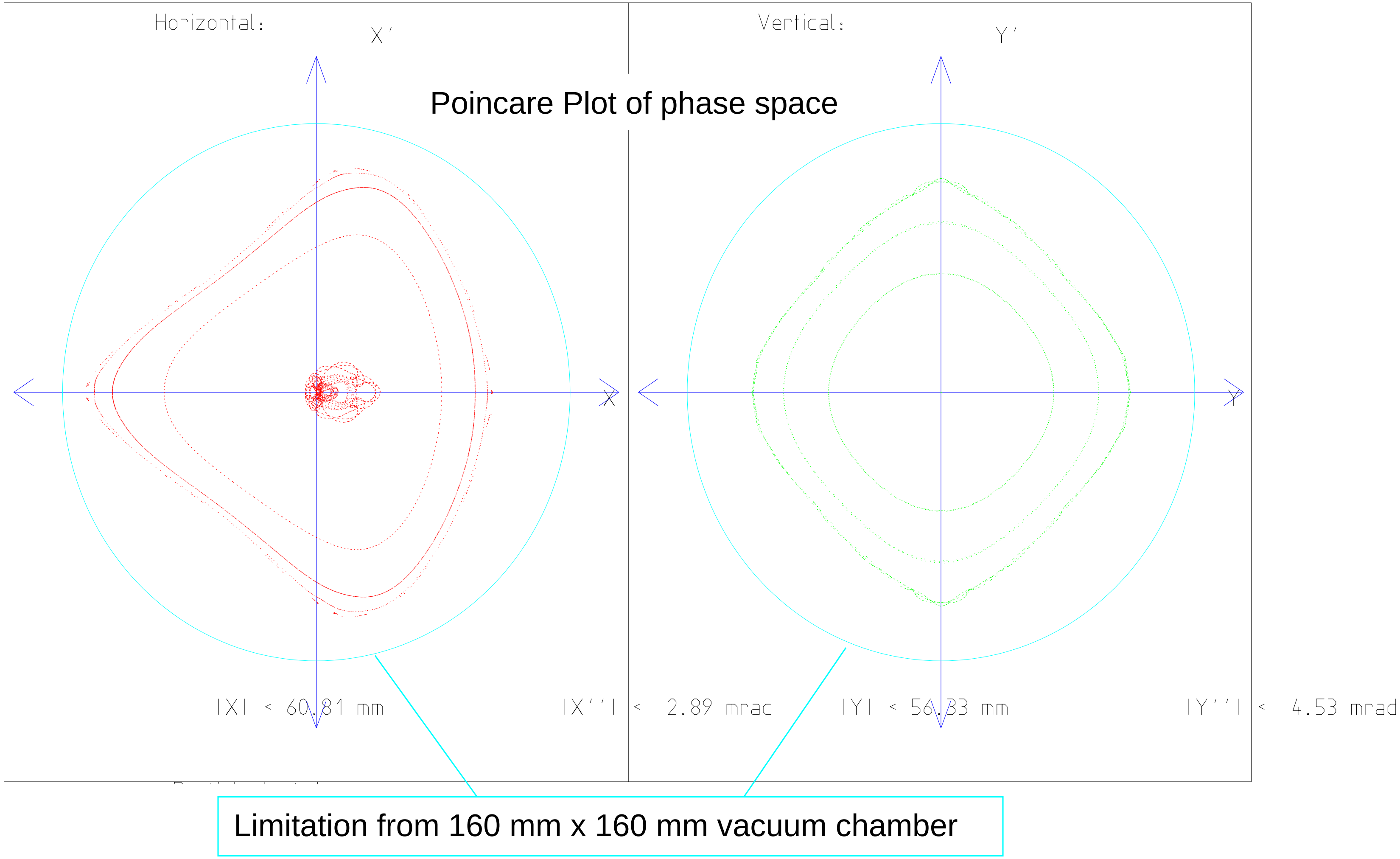
Second order sextupole optimization

Step 4: Second order optimization with 6 sextupole families



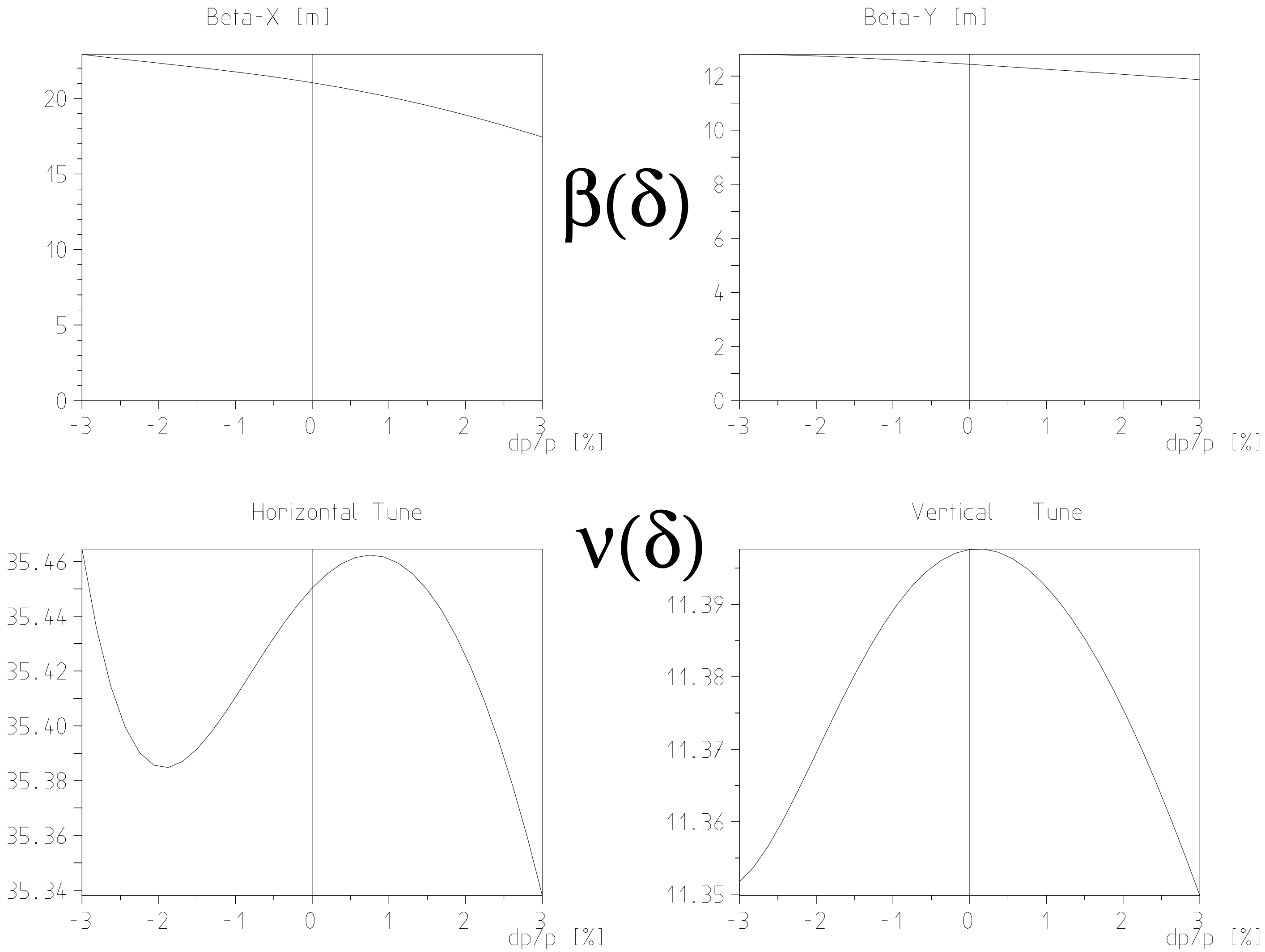
Nonlinearities in Light Sources

[Step 5: Further optimization by adding 8 octupole families]



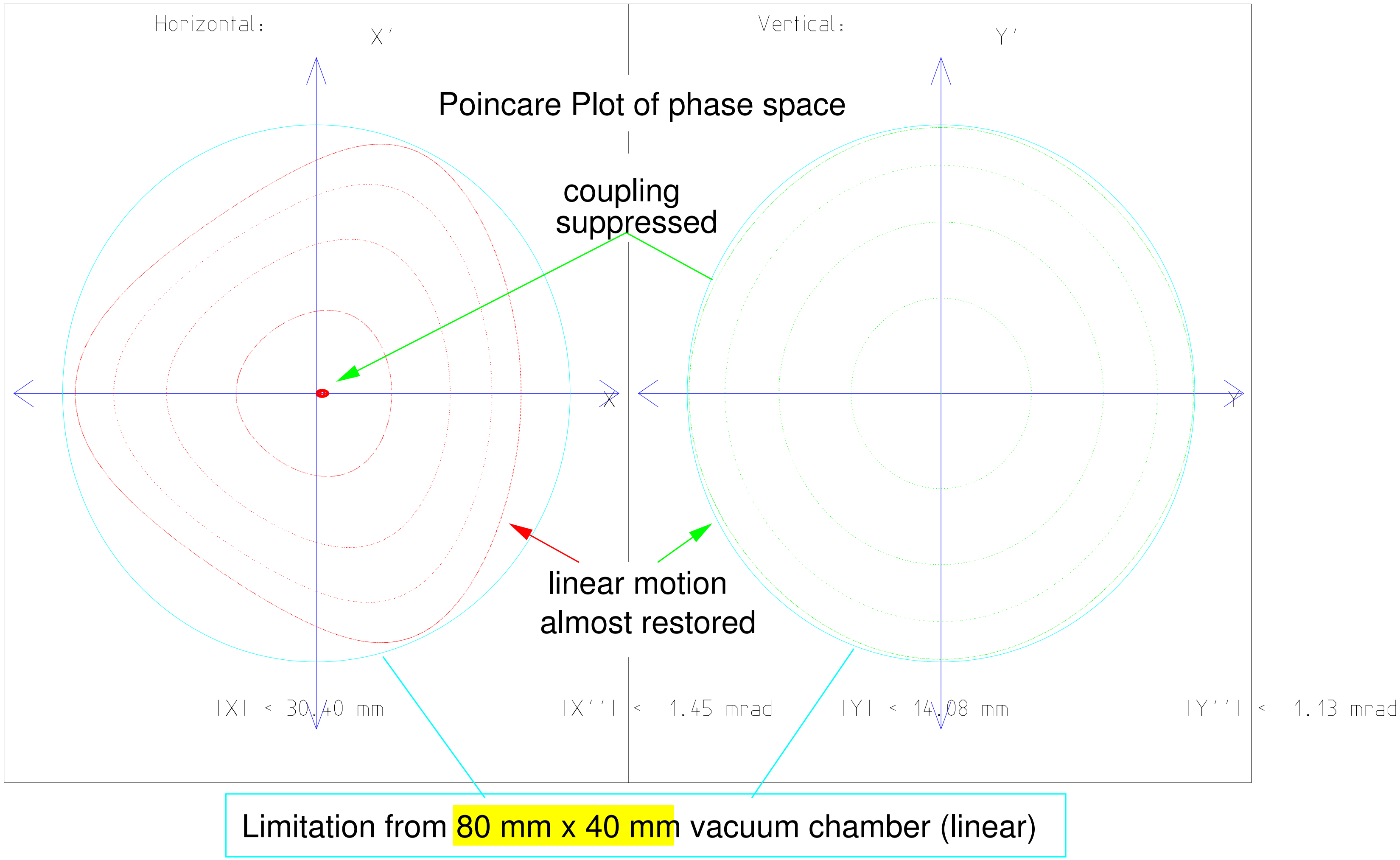
Second order sextupole optimization

[Step 5: Octupoles for suppression of 2nd order chromaticities]



Second order sextupole optimization

Including real beam pipe limitations (+/–40 mm wide, +/–20 mm high)



Sextupole pattern optimization strategy

- **decouple chromatic sextupoles**
  $\rightarrow$ careful placement of SD, SF families
- **exploit symmetry and periodicity**
  $\rightarrow$ back to *linear* lattice design and machine layout
- **install "harmonic" sextupoles**
  $\rightarrow \approx 4 \ldots 8$ additional families [in straight sections]
- **minimize all 1^{st} order and some 2^{nd} order terms**
  $\rightarrow$ sextupolar resonances, tune shifts with amplitude and 2^{nd} order chromaticities
- **check by tracking**
  $\rightarrow$ dynamic aperture scans – also include misalignments and multipolar errors
- **iterate…**