Lattices for Light Sources

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Contents:

- **Global requirements**: size, brightness, stability
- **Lattice building blocks**: magnets and other devices
- **Emittance**: lattice cells, minimum emittance, vertical emittance
- **Other lattice parameters**: damping, tune, chromaticity etc.
- **Acceptances**: lifetime and injection; physical, dynamic and energy acceptance
- **Lattice errors**: misalignments, multipolar errors

Global requirements

- **Number and length of straight sections** → minimum size
- **Available area** → maximum size
- **Synchrotron radiation properties**
  radiation spectrum, insertion device types → beam energy
  brightness → emittance, coupling, energy spread
- **Stability requirements**
  current stability → beam lifetime → acceptances
  orbit stability → mechanical tolerances, correction schemes
- **Flexibility and future upgrade potential**

- **Budget**

Lattice Design Interfaces

Magnet Design: Technological limits, coil space, multipolar errors

Vacuum: Impedance, pressure, physical apertures, space

Radiofrequency: Energy acceptance, bunchlength, space

Diagnostics: Beam position monitors, space

Alignment: Orbit distortions and correction

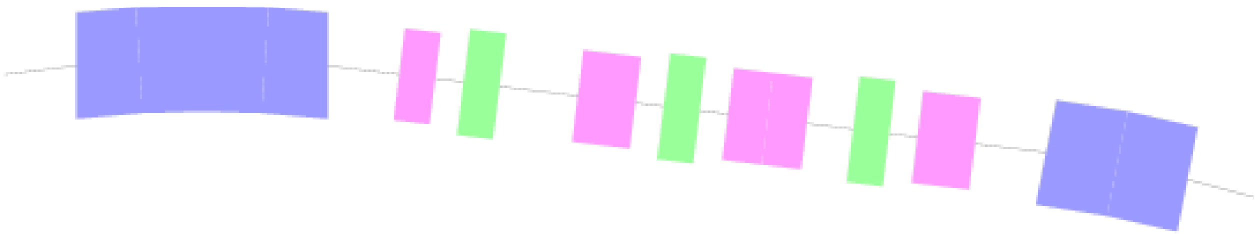
Mechanical engineering: Girders, vibrations

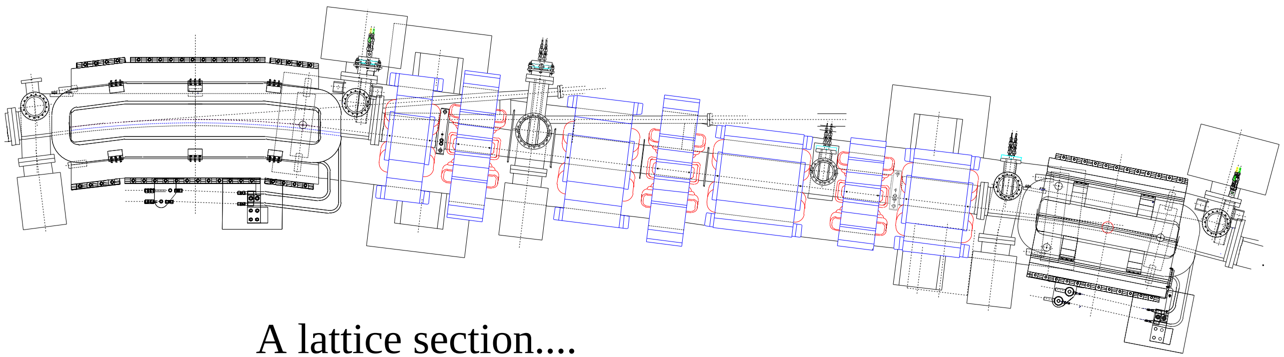
Design engineering: Assembly, feasibility

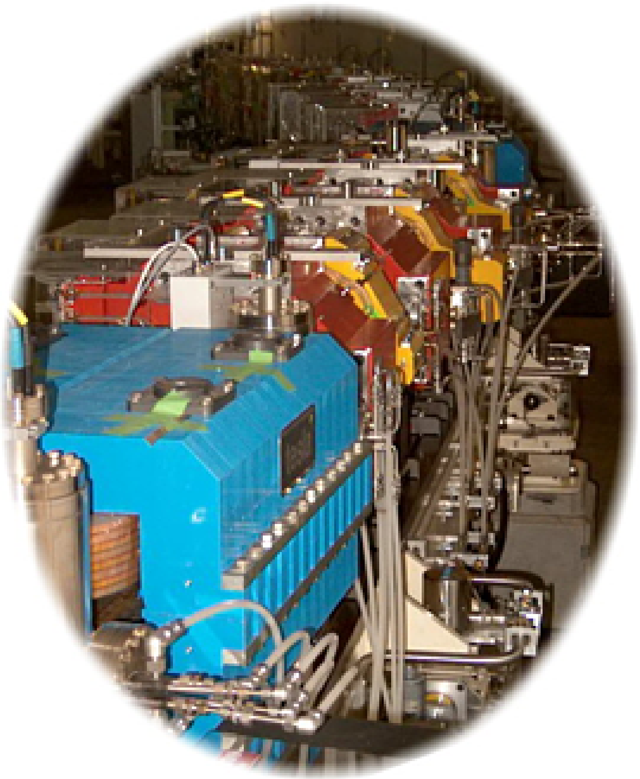
$\Longrightarrow$ Beam current $\longrightarrow$ Vacuum, Radiofrequency

$\Longrightarrow$ Space requirements $\longrightarrow$ Magnet, Vacuum, RF, Diagnostics, Engineering

Space requirements







A lattice section....

(top) .....as seen by the lattice designer

(bottom) ..... as seen by the design engineer

(right) ..... and how it looks in reality

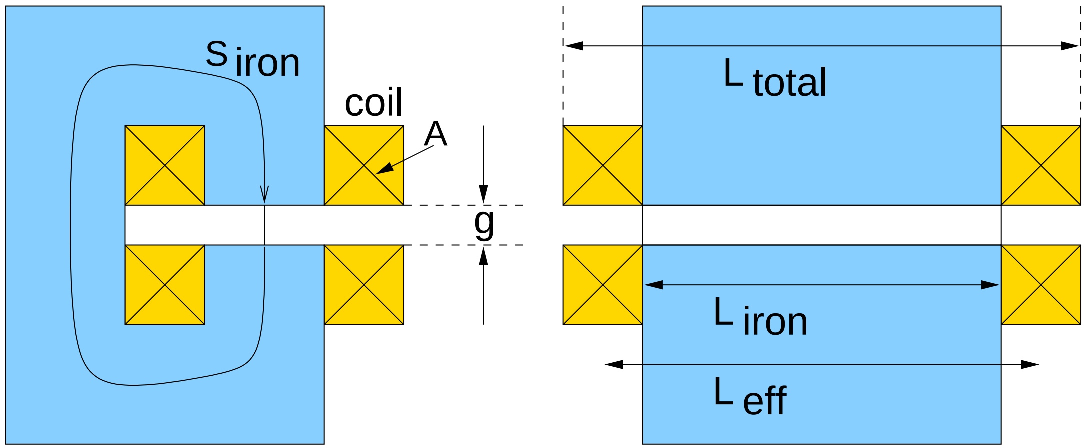
Lattice components

	Parameters	Purpose
Bending magnet	$b_1 \overset{!}{=} \rho^{-1}$, ϕ, b_2, $\zeta_{\mathrm{in,out}} \ldots$	form a ring
Quadrupole	b_2, L	focussing
Sextupole	b_3, L	chromaticity correction
RF cavity	λ_{rf}, V_{rf}	acceleration, long. focussing
Septum magnet	position & width	injection
Kicker magnet	$\int b_1(t)dl$	injection
Correctors	$\int b_1 dl$, $\int a_1 dl$	orbit correction
BPM	passiv	orbit measurement
Skew Quadrupole	$\int a_2 dl$	coupling correction
Undulator	λ_u, N_u, B_u, g	$\rightarrow$ synchrotron light

Iron dominated dipole magnet

$$\oint H ds = \int\int j da \qquad \Longrightarrow \qquad \text{Coil cross section } A$$

$$A = \frac{B}{2j_c}\left(\frac{S_{\text{iron}}}{\mu_o \mu_r} + \frac{g}{\mu_o}\right) \xrightarrow{\mu_r \gg 1} A \approx \frac{Bg}{2j_c \mu_o}$$



Magnet poletip fields and apertures

	coil width $\frac{L_{\mathrm{tot}}-L_{\mathrm{eff}}}{2}$ [mm]	poletip field B_{pt} [T]	aperture R [mm]
Bending magnets:	$65\ldots150$	1.5	$20\ldots35$ (=g/2)
Quadrupoles:	$40\ldots70$	0.75	$30\ldots43$
Sextupoles:	$40\ldots80$	0.6	$30\ldots50$

(data from various light sources)

$$\boxed{\text{Magnet power} \propto R^n}$$

$\Longrightarrow$ **Apertures:** As small as possible
As large as necessary $\longrightarrow$ acceptance

Lattice Design Code

Model: complete set of elements, correct methods for tracking and concatenation, well documented approximations

Elementary functions: beta functions and dispersions, periodic solutions, closed orbit finder, energy variations, tracking, matching

Toolbox: Fourier transforms of particle data ($\longrightarrow$ resonance analysis), minimizaton routines ($\longrightarrow$ dynamic aperture optimization, coupling suppression) , linear algebra package ($\longrightarrow$ orbit correction)

User convenience: editor functions, graphical user interface, editable text files

Extended functions: RF dimensioning, geometry plots, lifetime calculations, injection design, alignment errors, multipolar errors, ground vibrations

Connectivity: database access, control system access ($\longrightarrow$ real machine)

Natural horizontal emittance

Flat lattice:

$$\epsilon_{xo}[\text{nm}\cdot\text{rad}] = \underbrace{\frac{55\,\hbar c}{32\sqrt{3}\,m_e c^2}}_{3.83\cdot 10^{-13}\ \text{m}} \gamma^2 \frac{I_5}{J_x I_2} = 1470\,(E[\text{GeV}])^2 \frac{\langle \mathcal{H}/\rho^3 \rangle}{J_x \langle 1/\rho^2 \rangle}$$

Lattice invariant (or “dispersion’s emittance”):

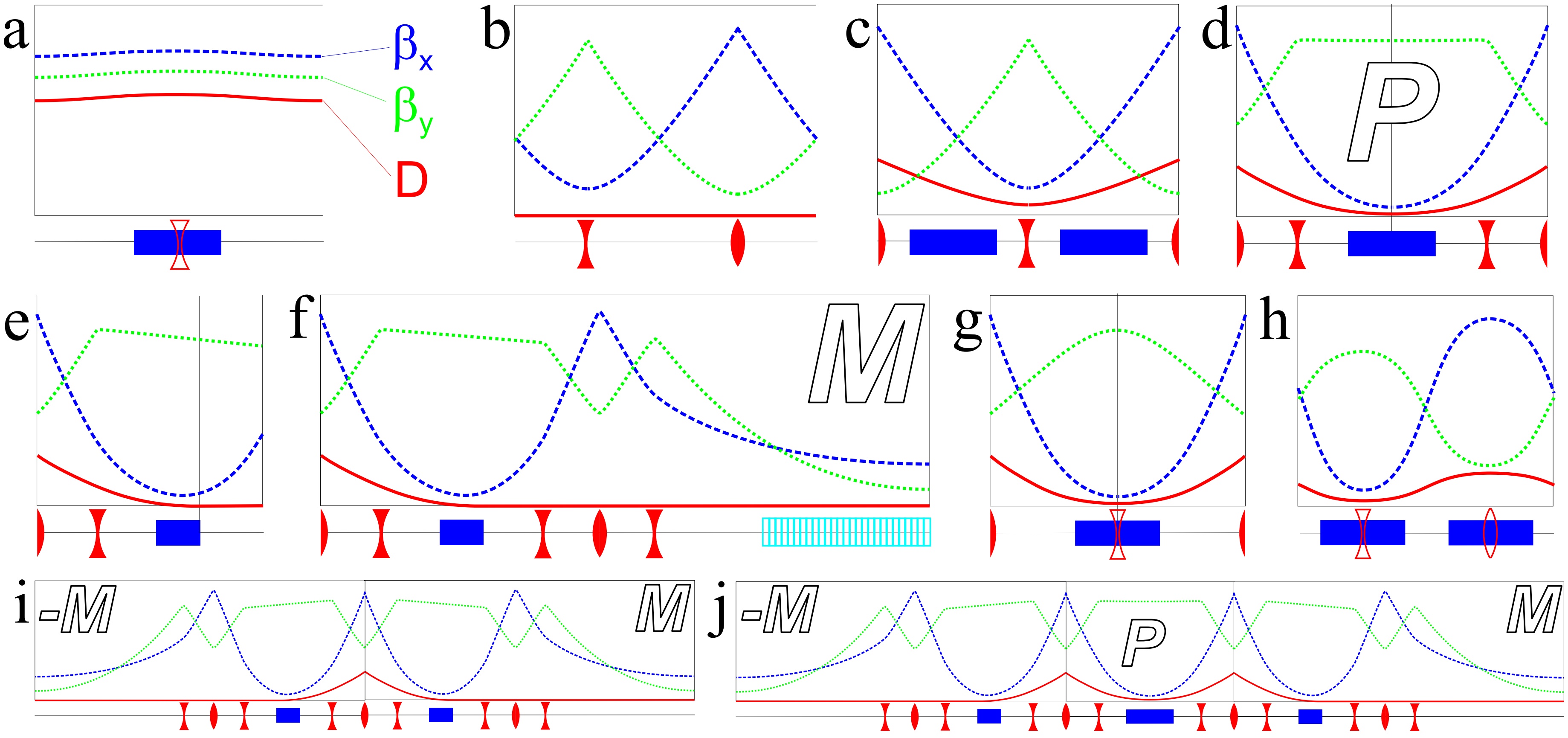
$$\mathcal{H}(s) = \gamma_x(s) D(s)^2 + 2\alpha_x(s) D(s) D'(s) + \beta_x(s) D'(s)^2$$

Horizontal damping partition $J_x \approx 1$ $\quad$ $\langle \ldots \rangle$ lattice average $\quad$ $\langle \ldots \rangle_{\text{mag}}$ magnets average

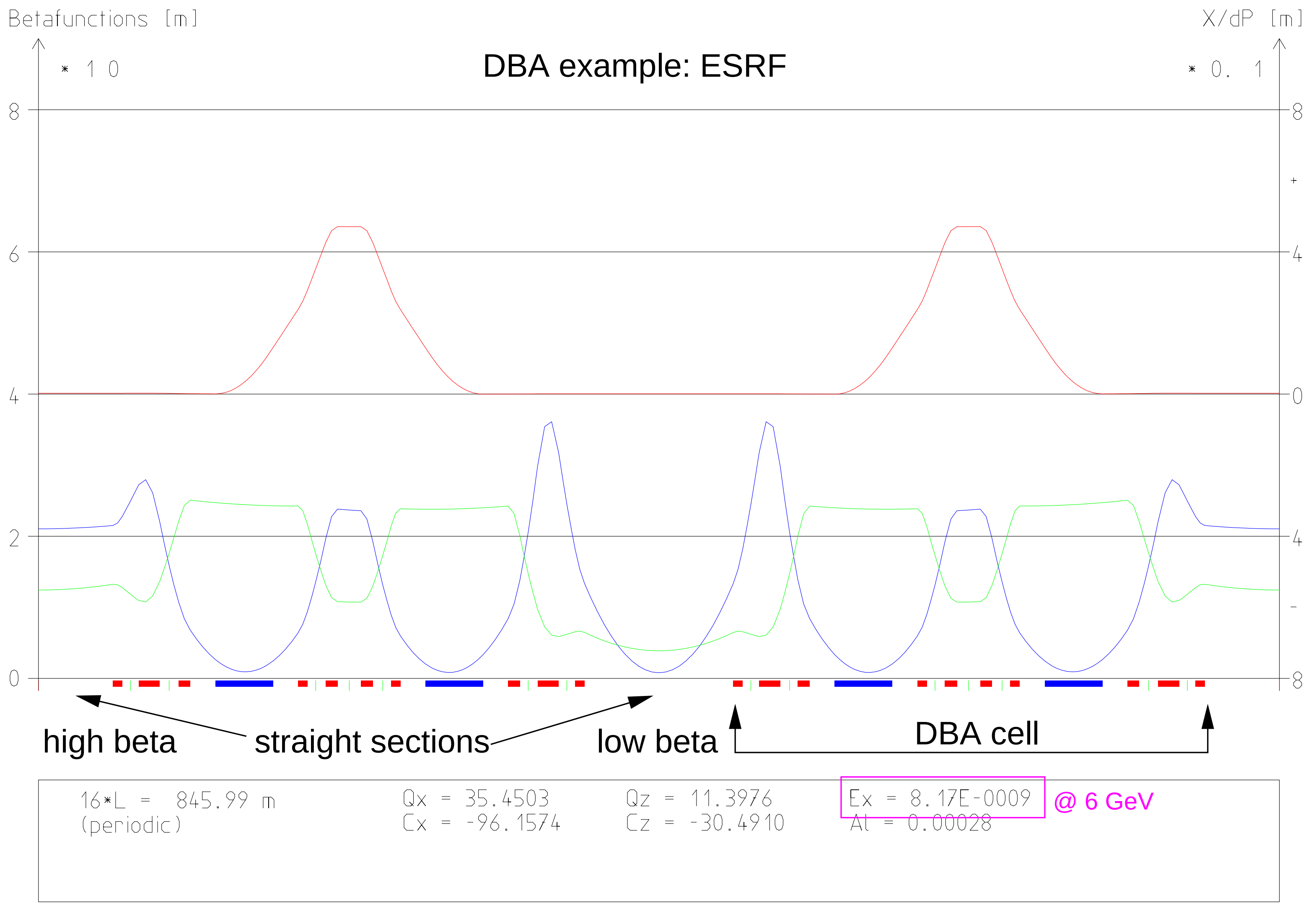
Simplification for isomagnetic lattice:

$$\epsilon_{xo}[\text{nm}\cdot\text{rad}] = 1470\,(E[\text{GeV}])^2 \frac{\langle \mathcal{H} \rangle_{\text{mag}}}{\rho J_x}$$

Building low emittance lattices . . .



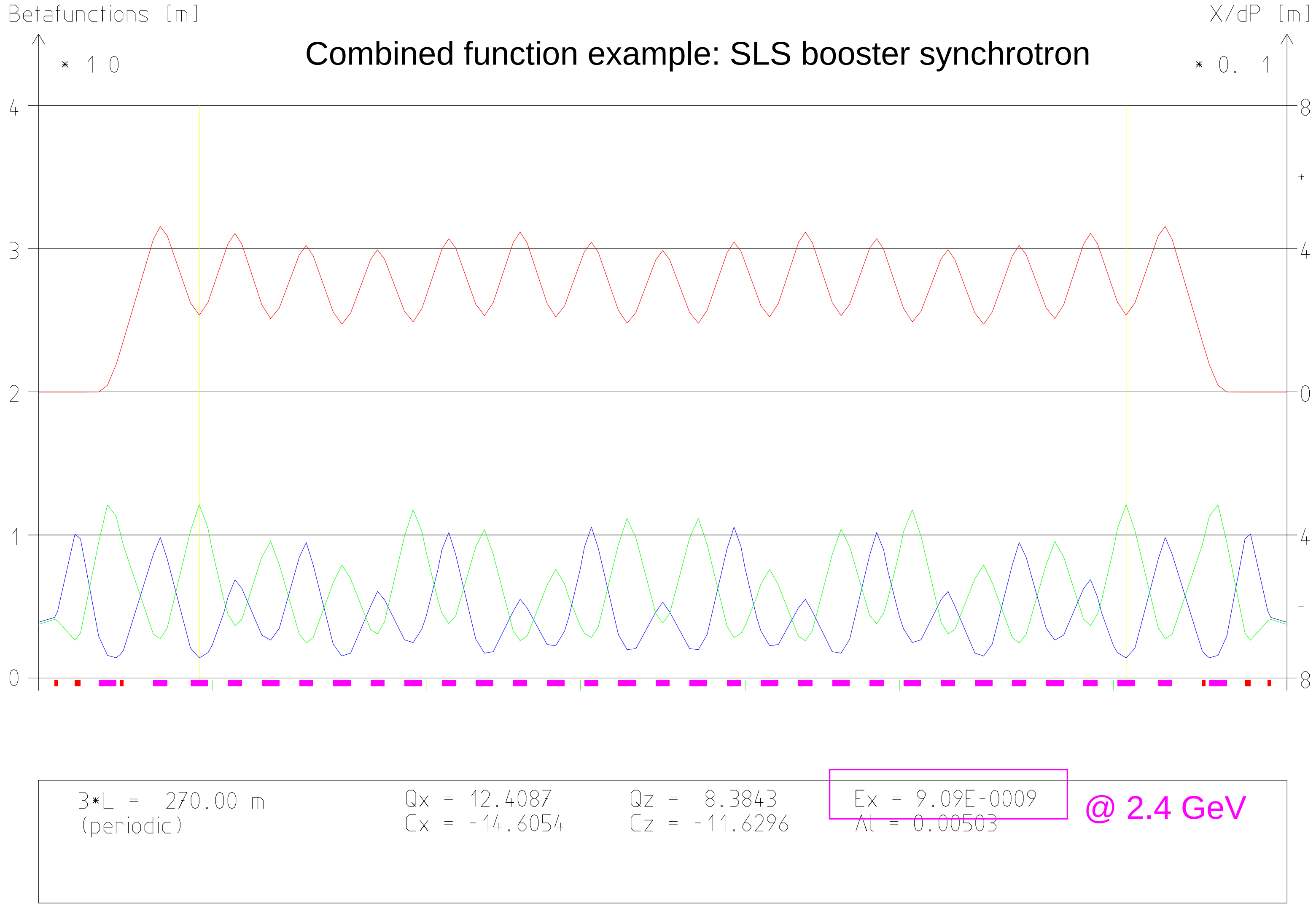
Lattice Cells



Lattice Cells

Lattices for Light Sources

Lattice Cells

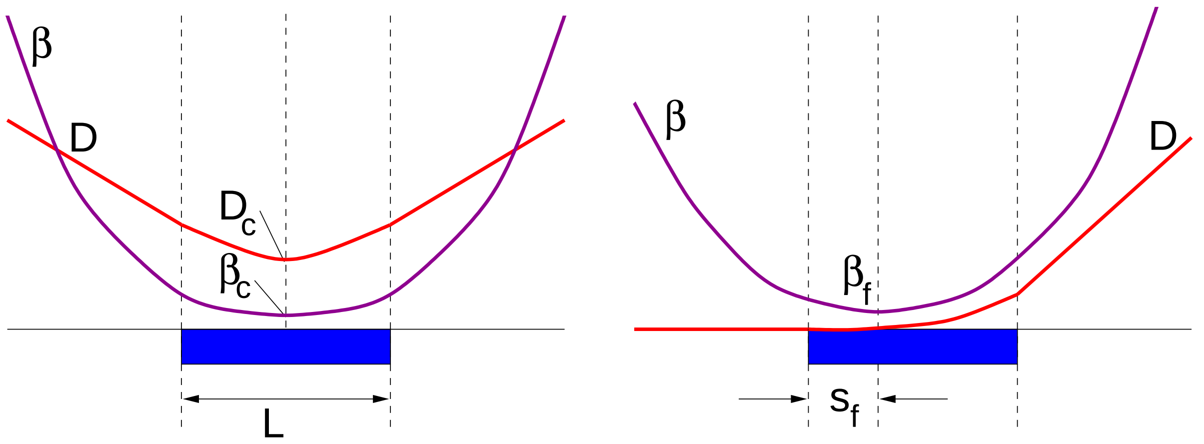


Minimum Emittance

$d\langle\mathcal{H}(\alpha_{xc},\beta_{xc},D_c,D_c')\rangle_{\text{mag}} = 0 \implies$ Minimum emittance:

$$\epsilon_{xo}[\text{nm}\cdot\text{rad}] = 1470\,\frac{(E[\text{GeV}])^2}{J_x}\,\frac{\Phi^3 F}{12\sqrt{15}}$$

Φ [rad] magnet deflection angle ($\Phi/2 \ll 1$)



$F = 1$

$\beta_{xc} = \frac{1}{2\sqrt{15}}L \quad D_c = \frac{1}{24\rho}L^2$

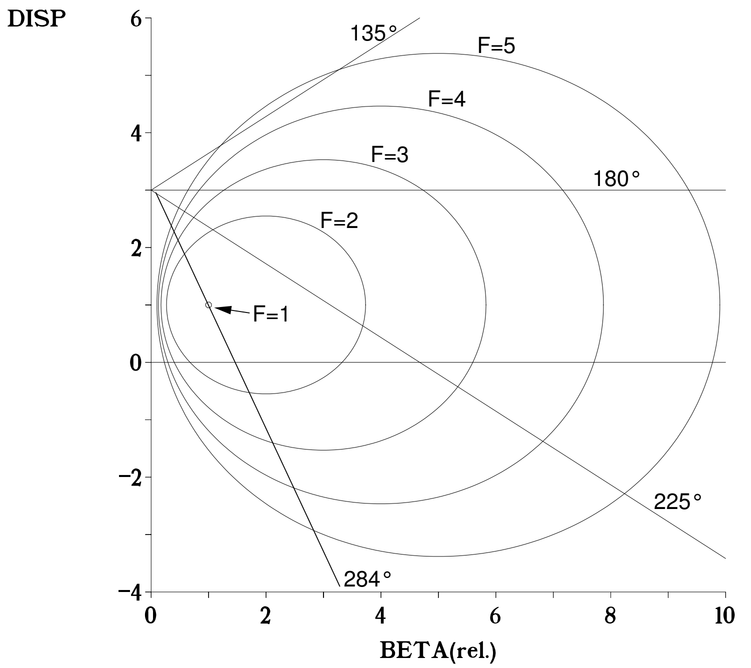
$F = 3$

$s_f = \frac{3}{8}L \quad \beta_{xf} = \sqrt{\frac{3}{320}}L$

Minimum emittance 2

Deviations from minimum:

$$b = \frac{\beta_{xc}}{\beta_{xc,\min}} \qquad F = \frac{\epsilon_{xo}}{\epsilon_{xo,min}}$$
$$d = \frac{D_c}{D_{c,\min}}$$



Relative emittance F:

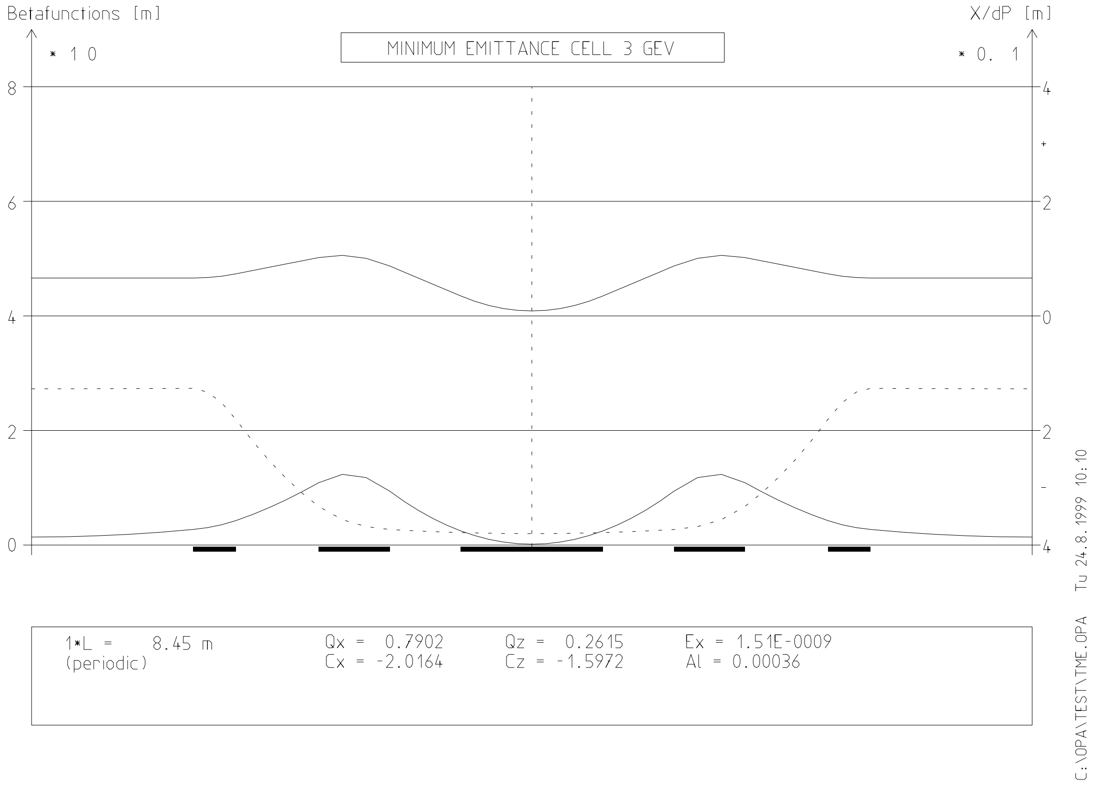
$$\frac{5}{4}(d-1)^2 + (b-F)^2 = F^2$$

Phase advance in cell:

$$\Psi = 2\arctan\left(\frac{6}{\sqrt{15}}\frac{b}{(d-3)}\right)$$

$$F = 1 \implies \Psi = 284.5°$$

Minimum emittance cell



10° gradient free sector bend, b=d=1, $E = 3$ GeV

$\Longrightarrow F = 1$: Theoretical minimum emittance (`Ex`)= 1.5 nm·rad

Tune advance (`Qx`)=0.7902 $\Longleftrightarrow$ Ideal phase advance $\Psi = 284.5^\circ$.

Damping times

$$\tau_i = 6.67\ \text{ms}\ \frac{C\ [\text{m}]\ E\ [\text{GeV}]}{J_i\ U\ [\text{keV}]} \qquad J_x = 1 - \mathcal{D} \quad J_y = 1 \quad J_s = 2 + \mathcal{D}$$

$$\mathcal{D} = \frac{1}{2\pi} \int_{\text{mag}} D(s) \left[b_1(s)^2 + 2 b_2(s) \right] ds$$

Energy loss per turn: $U\ [\text{keV}] = 26.5\,(E[\text{GeV}])^{\mathbf{3}}\ B[\text{T}]$

Stability requirement: $-2 < \mathcal{D} < 1$

Separate function bends: $\mathcal{D} \ll 1$ in light sources.

Combined function bending magnets: Adjust gradients!
Option: Vertical focusing in bending magnet: $b_2 < 0 \rightarrow J_x \rightarrow 2$: half emittance!

Energy spread and Beam size

r.m.s. natural energy spread:

$$\sigma_e = 6.64 \cdot 10^{-4} \cdot \sqrt{\frac{B[T]\, E[GeV]}{J_s}} \qquad J_s \approx 2$$

Beam size and effective emittance:

$$\sigma_x(s) = \sqrt{\epsilon_x\, \beta_x(s) + (\sigma_e\, D(s))^2} \qquad \sigma_y(s) = \sqrt{\epsilon_y\, \beta_y(s)}$$

$$\epsilon_{x,\text{eff}}(s) = \sqrt{\epsilon_{xo}^2 + \epsilon_{xo}\mathcal{H}(s)\sigma_e^2}$$

Vertical emittance

Ideal flat Lattice: $\mathcal{H}_y \equiv 0 \quad \longrightarrow \quad \epsilon_y = 0$

Real Lattice: Errors as sources of vertical emittance ϵ_y

Vertical dipoles (a_1):	Skew quadrupoles (a_2):	
Dipole rolls	Quadrupole rolls	roll $= s$-rotation
Quadrupole heaves	Sextupole heaves	heave $= \Delta y$ displacement
Vertical dispersion (D_y)	Linear coupling (κ)	
$\rightarrow$ orbit correction	$\rightarrow$ skew quadrupoles	for suppression

Emittance ratio $g = \frac{\epsilon_y}{\epsilon_x} \quad \rightarrow \quad \epsilon_x = \frac{1}{1+g}\epsilon_{xo} \quad \epsilon_y = \frac{g}{1+g}\epsilon_{xo}$

Coupling corrected lattices: $g \approx 10^{-3}$

BUT: Diffraction limitation $\rightarrow$ Brightness $\sim 1/g$ only for hard X-rays
Touschek lifetime $\sim$ (bunch volume) $\sim \sqrt{g}$

Circumference and periodicity

Circumference C

- Area $\rightarrow$ minimize
- Optics $\rightarrow$ relax
- Spaces $\rightarrow$ reserve
- RF harmonic number
  $\rightarrow C = h\lambda_{rf}$
  $\rightarrow h = h_1 \cdot h_2 \cdot h_3 \ldots$

Ritsumeikan PSR	$C = 98$ cm
LEP	$C = 27$ km

Periodicity N_{per}

Advantages of large periodicity:

- simplicity: design & operation
- stability: resonances
- cost efficiency: few types

DORIS:	$N_{\text{per}} = 1$
APS:	$N_{\text{per}} = 40$

Working point

Betatron resonances:

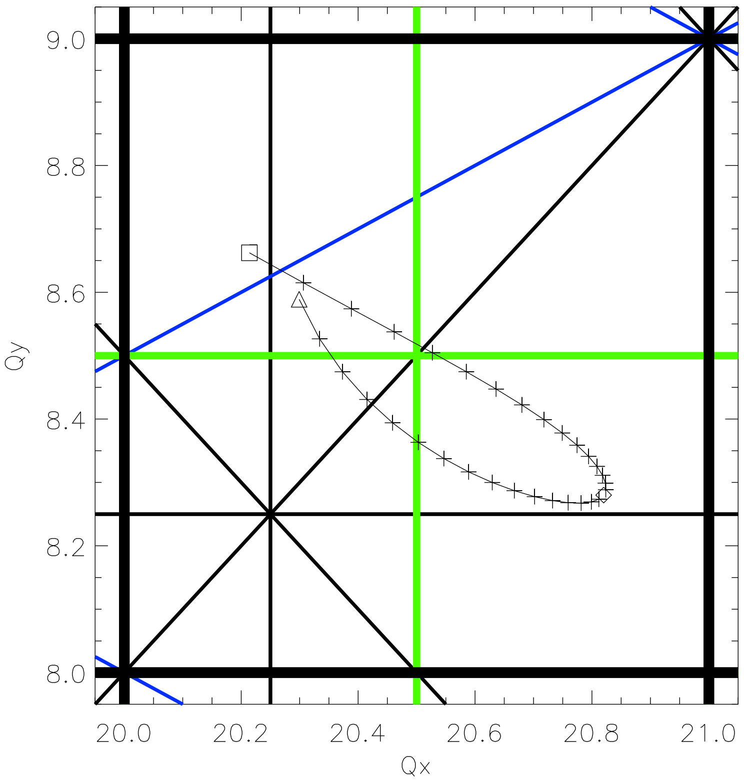
$$\boxed{aQ_x + bQ_y = p}$$

order: $n = |a| + |b|$
systematic: N_{per}/p = integer
regular: b even, skew: b odd
(a, b, k, n, N_{per}, p integers)

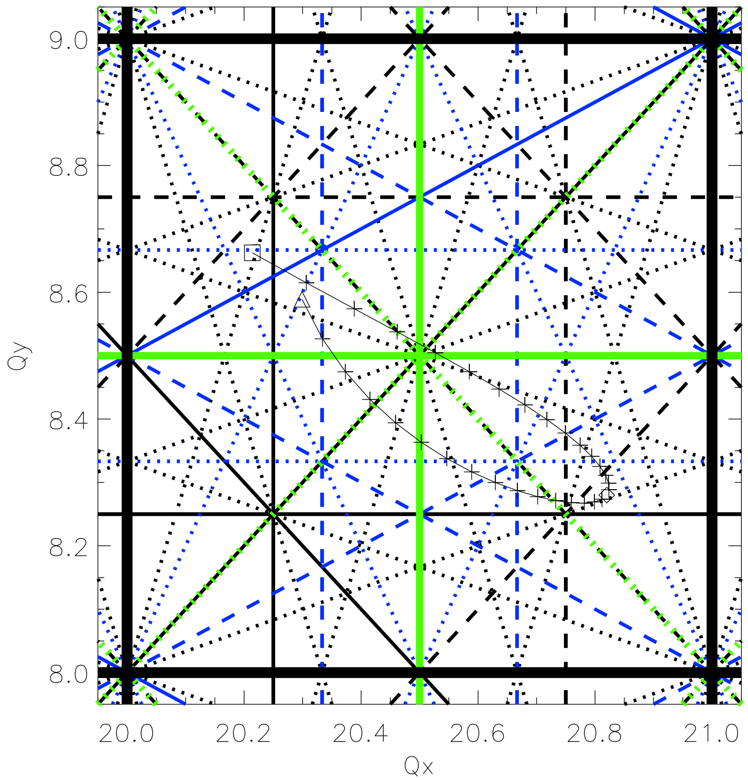
Tune constraints:

- NO integer — $Q_{x;y} = k \longrightarrow$ dipolar errors
- NO half integer — $Q_{x;y} = (2k+1)/2 \longrightarrow$ gradient errors
- NO sum resonance — $Q_x + Q_y = p \longrightarrow$ coupling
- NO sextupole resonances — $Q_x = p$, $3Q_x = p$, $Q_x \pm 2Q_y = p$ $\longrightarrow$ dynamic acceptance
- Multiturn injection: — $|\text{frac}(Q)| \geq 0.2 \longrightarrow$ septum
- and more…

Working point: Example



Ideal lattice



Real Lattice

Acceptance

Acceptance: 6D volume of stable particles $\longrightarrow$ decoupling:
horizontal, vertical and longitudinal 2D-acceptances

Physical acceptance Linear lattice $\longrightarrow$ vacuum chamber $\longrightarrow$ "known"

Dynamic acceptance Nonlinear lattice $\longrightarrow$ separatrix $\longrightarrow$ "unknown"

Longitudinal acceptance

- RF energy acceptance (bucket height)
- Lattice energy acceptance = δ-dependant horizontal acceptance

Dynamic aperture = *local* projection of dynamic acceptance
acceptance [mm·mrad] $\longleftrightarrow$ aperture [mm]

Design criterion: Dynamic acceptance $>$ physical acceptance

Acceptance → Lifetime

Single particle processes

→ exponential decay

Interaction with residual gas nuclei:

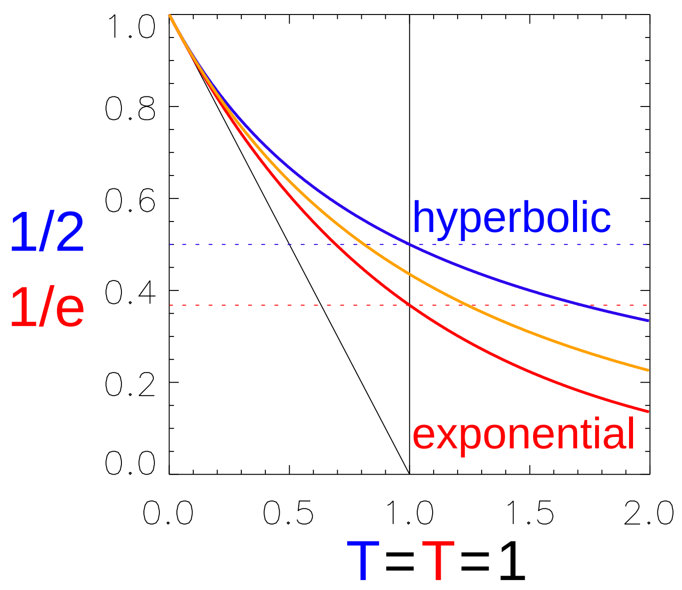
elastic scattering & bremsstrahlung

Two particle processes (= space charge effects)

→ hyperbolic decay

Touschek effect (= intrabeam scattering)

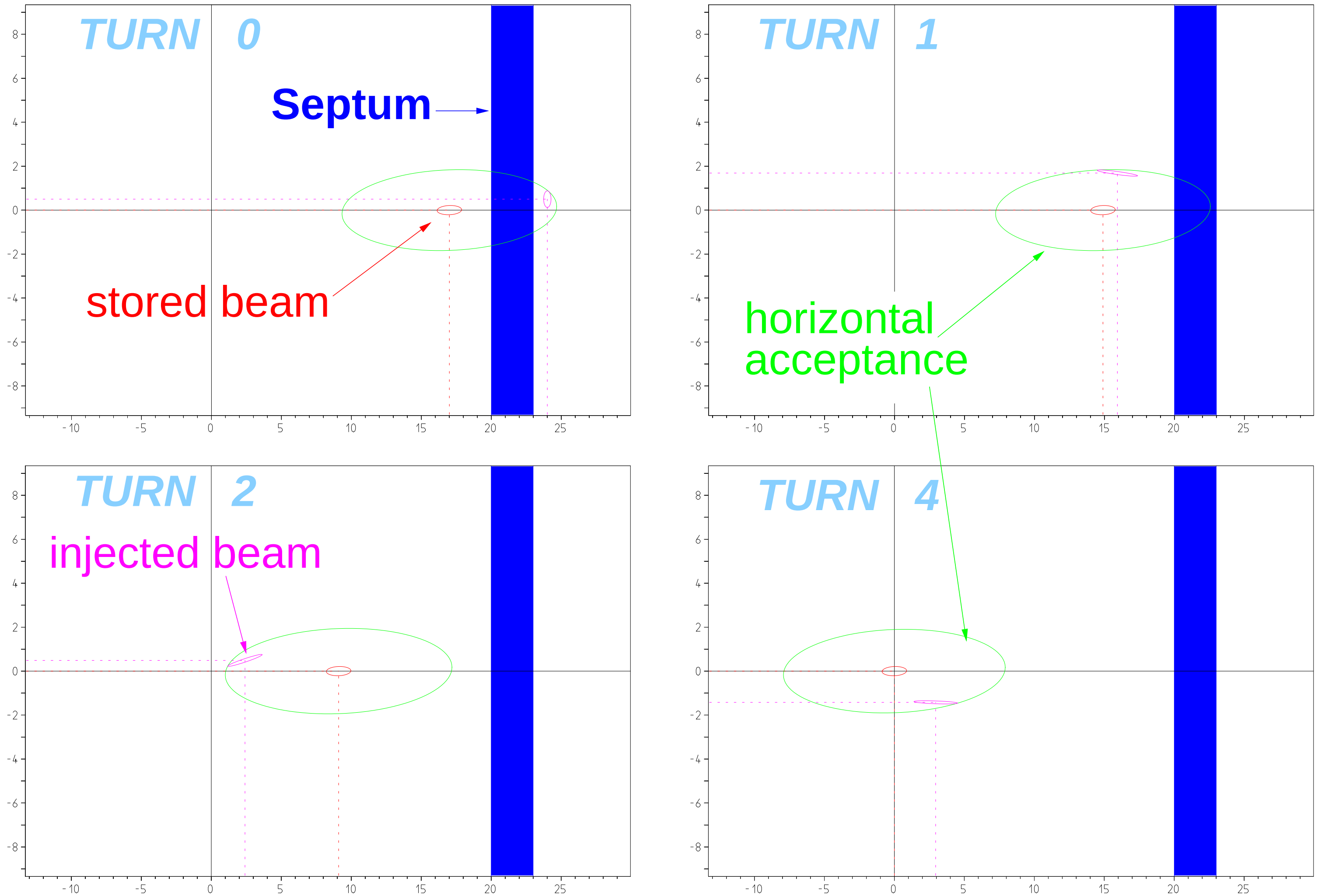
(Colliders: beam beam bremsstrahlung)



$$T_{\mathrm{el}} \sim \gamma^2 \frac{A_y}{P} \qquad T_{\mathrm{bs}} \sim \frac{\delta_{\mathrm{acc}}^{\sim 0.2}}{P} \qquad T_{\mathrm{t}} \sim \frac{\gamma^3 \sigma_s}{I_{\mathrm{sb}}} \, \epsilon_{xo} \sqrt{g} \, \langle [\delta_{\mathrm{acc}}(s)]^{2\ldots3} \beta(s) \ldots \rangle_c$$

A_y = **vertical acceptance**, P = pressure, σ_s = bunch length, I_{sb} = bunch current
δ_{acc} = **energy acceptance**, ϵ_{xo} = natural emittance, g = emittance ratio

Acceptance → Injection



Physical acceptance

Linear lattice (quads and bends only): “infinite” dynamic acceptance

Particle at acceptance limit A_x:

$$x(s) = \sqrt{A_x \cdot \beta_x(s)}\cos(\phi(s)) + D(s) \cdot \delta$$

Particle loss: $|x(s)| \geq a_x(s)$ somewhere.

Acceptance

$$A_x = \min\left(\frac{(a_x(s) - |D(s) \cdot \delta|)^2}{\beta_x(s)}\right) \qquad A_y = \min\left(\frac{a_x(s)^2}{\beta_x(s)}\right)$$

A_x invariant of betatron motion. $\rightarrow$ Projection:

$$x_{\max}(s) = \pm\sqrt{A_x \cdot \beta_x(s)} + D(s) \cdot \delta \qquad y_{\max}(s) = \pm\sqrt{A_y \cdot \beta_y(s)}$$

Dynamic acceptance

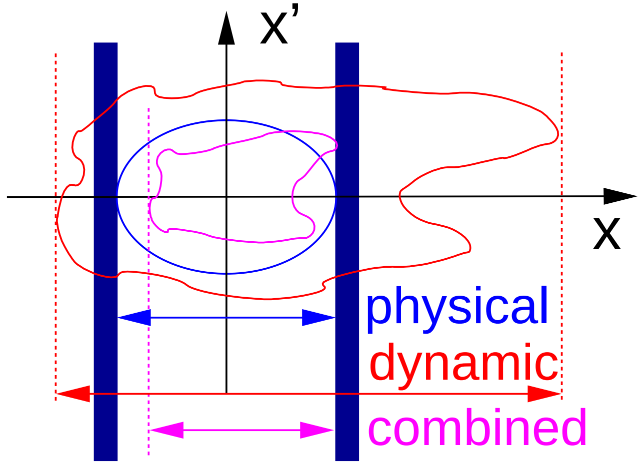
Separatrix for “stable” motion (no physical limitations)

stable $\longrightarrow$ test particle not lost in tracking

$\longrightarrow$ sufficient number of turns: ($\sim$1 damping time, $\sim$10 synchrotron oscillations)

$\longrightarrow$ machine model for tracking: correct & complete & realistic

Available aperture =
dynamic aperture with physical limitations



$\longrightarrow$ Dynamic aperture should

- be wider than beampipe
- have little distortions

$\Longrightarrow$ *careful* balancing of sextupoles

$\Longrightarrow$ set tolerances for magnets and IDs alignment and multipolar errors

Energy acceptance

Horizontal acceptance $A_x = 0$ for $|\delta| > \min(a_x(s)/|D(s)|)$

BUT:

Scattering processes $\rightarrow$ energy change of core particles:
$\vec{X} = (\approx 0, \approx 0, \approx 0, \approx 0, \delta, 0)$

Betatron oscillation around dispersive orbit with amplitude A_x

$$A_x = \gamma_{xo}(D_o\delta)^2 + 2\alpha_{xo}(D_o\delta)(D'_o\delta) + \beta_{xo}(D'_o\delta)^2 = \mathcal{H}_o\delta^2$$

$\beta_{xo} := \beta_x(s_o)$ etc., s_o = location of scattering event!

Maximum value of betatron oscillation:

$$x(s) = \sqrt{A_x\beta_x(s)} + |D(s)\delta| = \left(\sqrt{\mathcal{H}_o\beta_x(s)} + |D(s)|\right) \cdot |\delta|$$

Local energy acceptance:

$$\delta_{\rm acc}(s_o) = \pm \min\left(\frac{a_x(s)}{\sqrt{\mathcal{H}_o \beta_x(s)} + |D(s)|}\right)$$

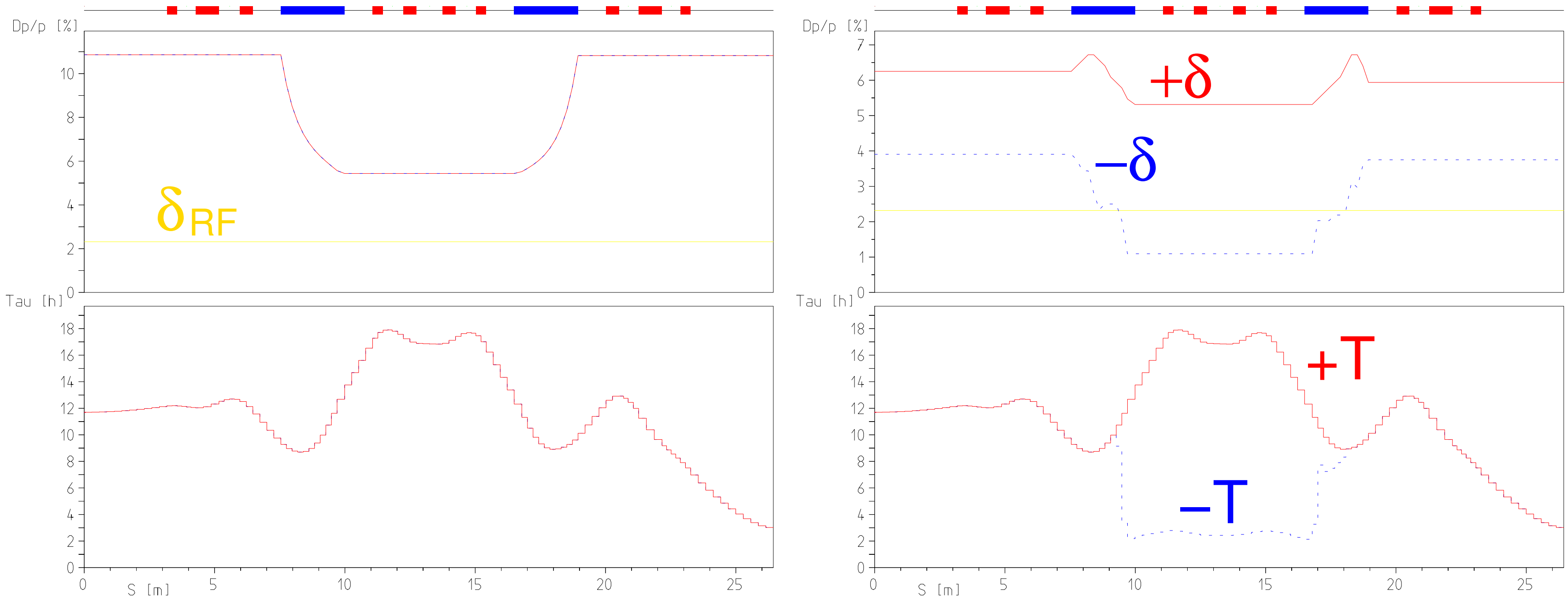
Energy acceptance for different lattice locations ($a_x(s) = a_x$):

In dispersionfree section:
$\mathcal{H}_o = 0 \quad \longrightarrow \quad \delta_{\rm acc} = \pm a_x / D_{\rm max}$

At location of maximum dispersion:
$\mathcal{H}_o = \gamma_o D^2_{\rm max} \quad \longrightarrow \quad \delta_{\rm acc} = \pm a_x / (2D_{\rm max})$

Local energy acceptance and Touschek lifetime

linear | nonlinear (2 Sextupole Families)



Test Lattice: ESRF standard cell (dispersionfree straight sections)

$$\delta_{\text{acc}} = \min\{+\delta_{\text{acc}}^{\text{Lattice}}\,,\; -\delta_{\text{acc}}^{\text{Lattice}}\,,\; \delta_{\text{acc}}^{\text{RF}}\} \rightarrow T_{\text{t}}$$

Lattice Imperfections

- Magnet misalignments
  - Closed Orbit distortion and correction $\longrightarrow$ BPMs and correctors
  - Correlated misalignments: magnet girders and dynamic alignment concepts
  - Ground waves and vibrations: orbit feedback
  - Beam rotation and coupling control
- Multipolar errors (Magnets and Undulators): Dynamic acceptance
- Gradient errors: Beta-beat $\longrightarrow$ quadrupole current control

$\Longrightarrow$ Stability requirements (photon beam on sample) $< 1\ \mu$m

Trends in lattice design

- **reduce magnet gap**
  Mini-gap undulators define acceptance anyway
  Example: 5 mm gap $\times$ 2 m length, $\beta_{y,\max} \approx 25$ m
  dipole chamber wall ≈ 3 mm $\rightarrow$ dipole gap 25 mm (instead of > 40 mm).

- **relax on flexibility**
  progress in computing, engineering and manufacturing
  $\rightarrow$ calculated optics *will* become reality!

- **exploit $J_x \rightarrow 2$**
  $\epsilon \rightarrow \frac{1}{2}\epsilon$, $\sigma_e \rightarrow \sqrt{2}\,\sigma_e$ $\rightarrow D = 0$ in straights, vertical focussing in bends.

- **try octupoles**
  1^{st} order attack of 2^{nd} order sextupole terms