Free Electron Lasers

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Introduction

Undulator Radiation

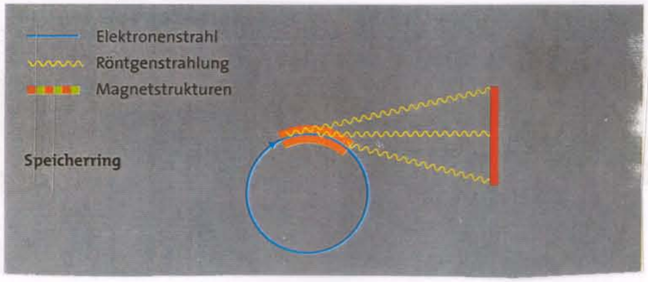
Low-Gain Free Electron Laser

High-Gain FEL, SASE Principle

Experimental Results

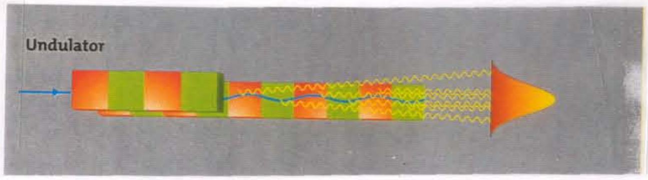
Electron Accelerators as Light Sources

(a)



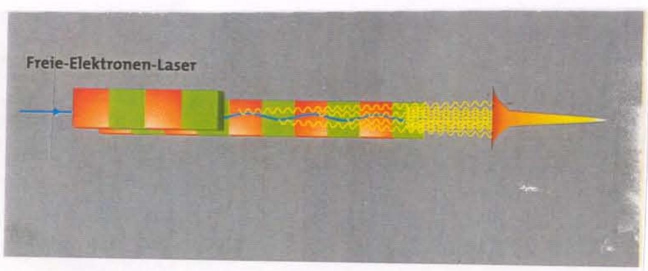
storage ring with bending magnets
continuous spectrum
wide angular distribution

(b)



Undulator radiation
(almost) monochromatic
narrow angular distribution

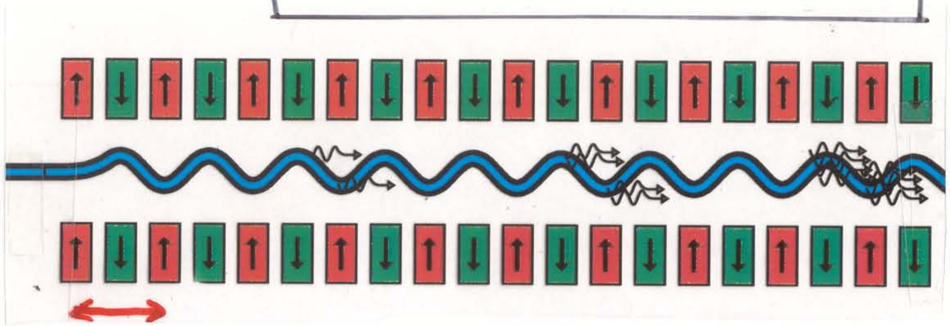
(c)



Free Electron Laser
narrow spectral line
sharp collimation

$$I_N = N^2 \cdot I_1$$ coherent radiation

Undulator Radiation



$\lambda_u \approx 25\,mm$ undulator period

$$B_y(z) = B_0 \cos\left(\frac{2\pi}{\lambda_u}\cdot z\right)$$

$$x(z) = -X_0 \cos\left(\frac{2\pi}{\lambda_u}\cdot z\right)$$

undulator parameter

$$K = \frac{e B_0 \lambda_u}{2\pi m_e c} \approx 1$$

- Moving coordinate system (*)

  $\lambda_u^* = \lambda_u/\gamma$ length contraction

  electron oscillates with $\omega^* = 2\pi \frac{c}{\lambda_u^*} = \gamma \cdot \frac{2\pi c}{\lambda_u}$

- Lorentz transformation of radiation to lab-system

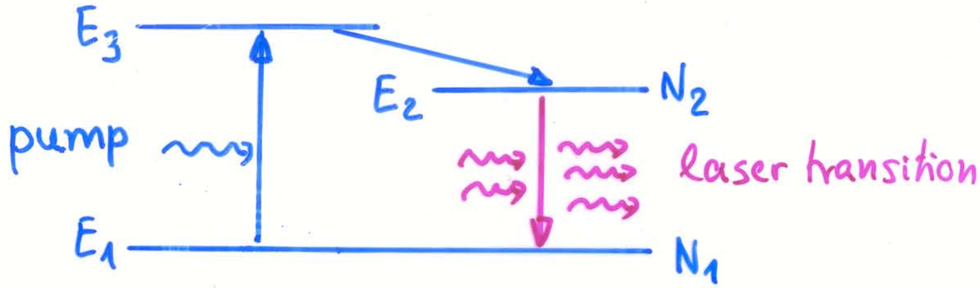
$$\Rightarrow \lambda_e \approx \frac{\lambda_u^*}{\gamma} = \frac{\lambda_u}{\gamma^2}$$

$$\boxed{\lambda_e = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)}$$

monochromatic radiation

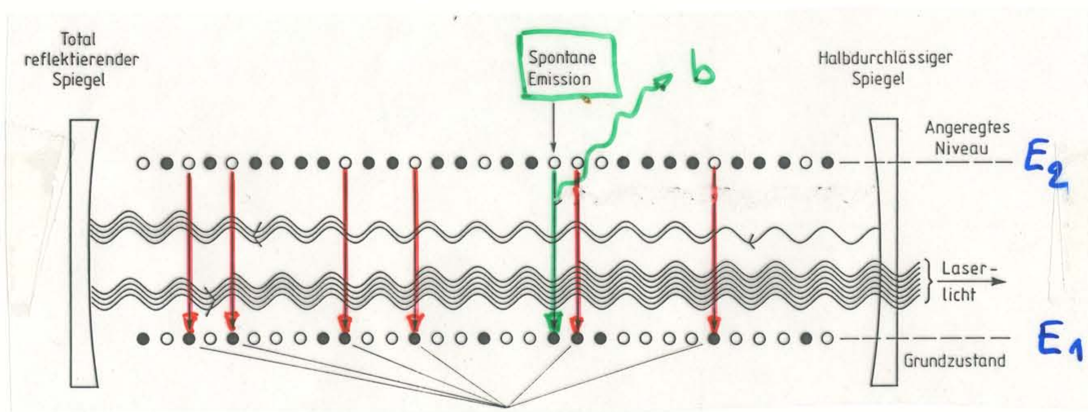
Conventional Lasers

"Bound Electron Lasers"



pumping achieves
population inversion
$N_2 > N_1$

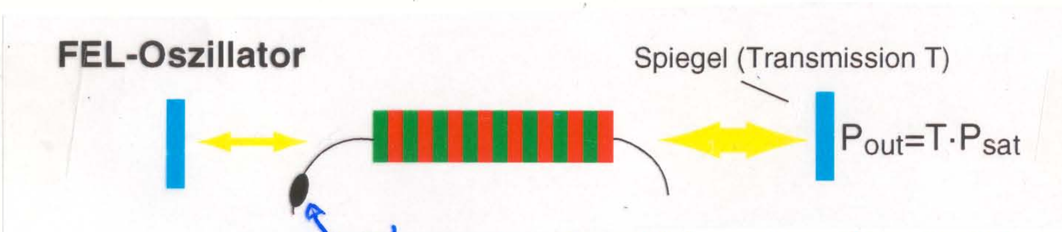
3 main components
(1) active laser medium
(2) energy "pump"
(3) optical resonator



Characteristic feature of FEL:

the roles of active laser medium and of energy pump are both taken over by the relativistic electron bunches

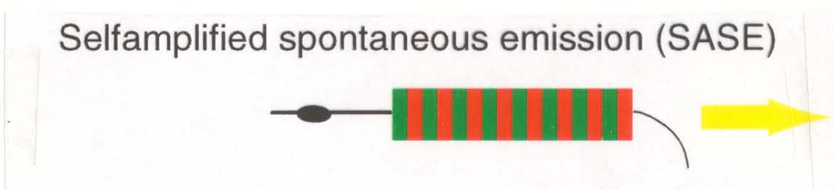
(a) FEL for IR or visible light: equipped with optical resonator



e^--bunch circulating in storage ring

typical gain per passage: few % low-gain FEL

(b) FEL for VUV or X rays: no optical resonator available

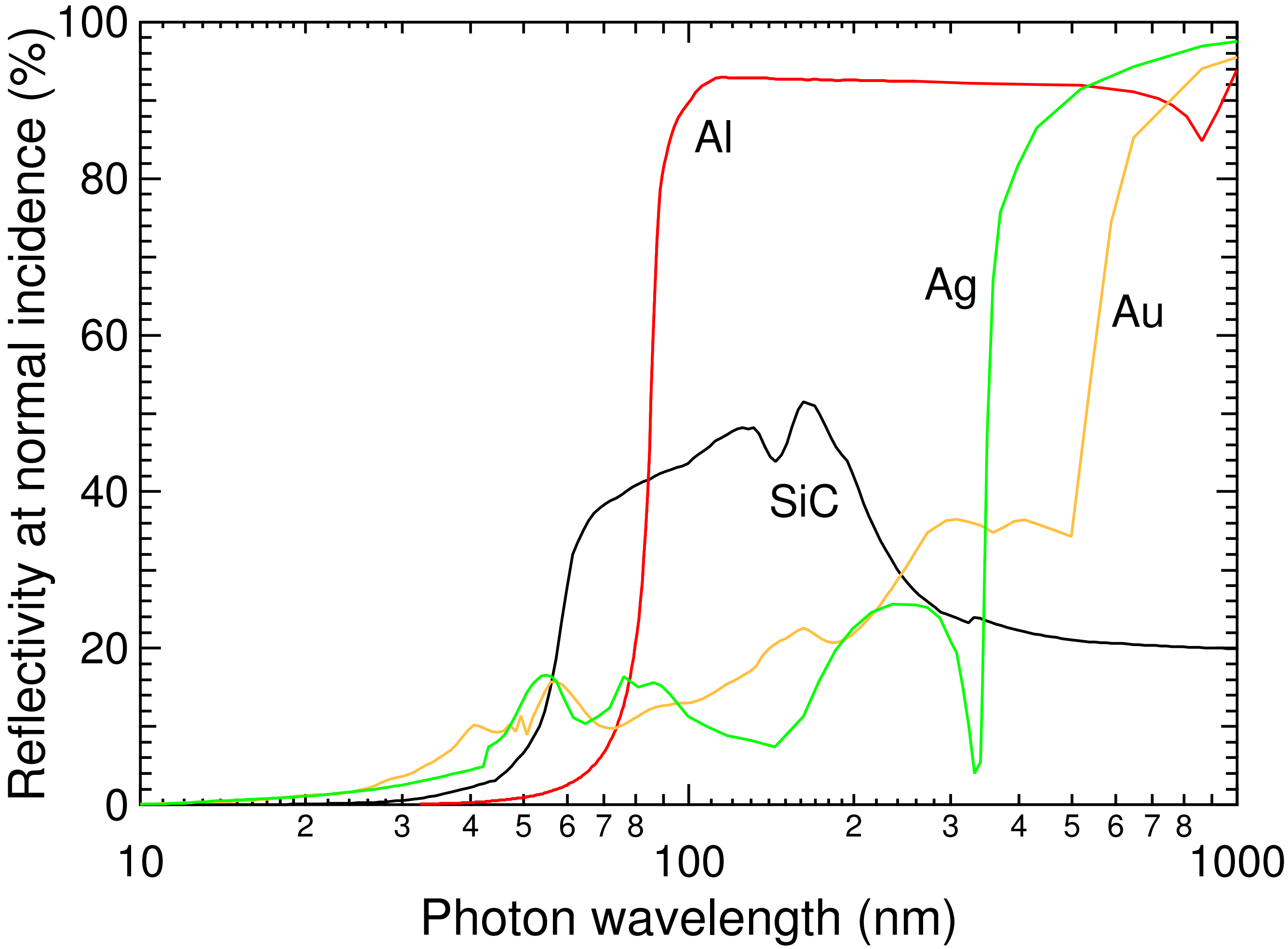


high-gain FEL

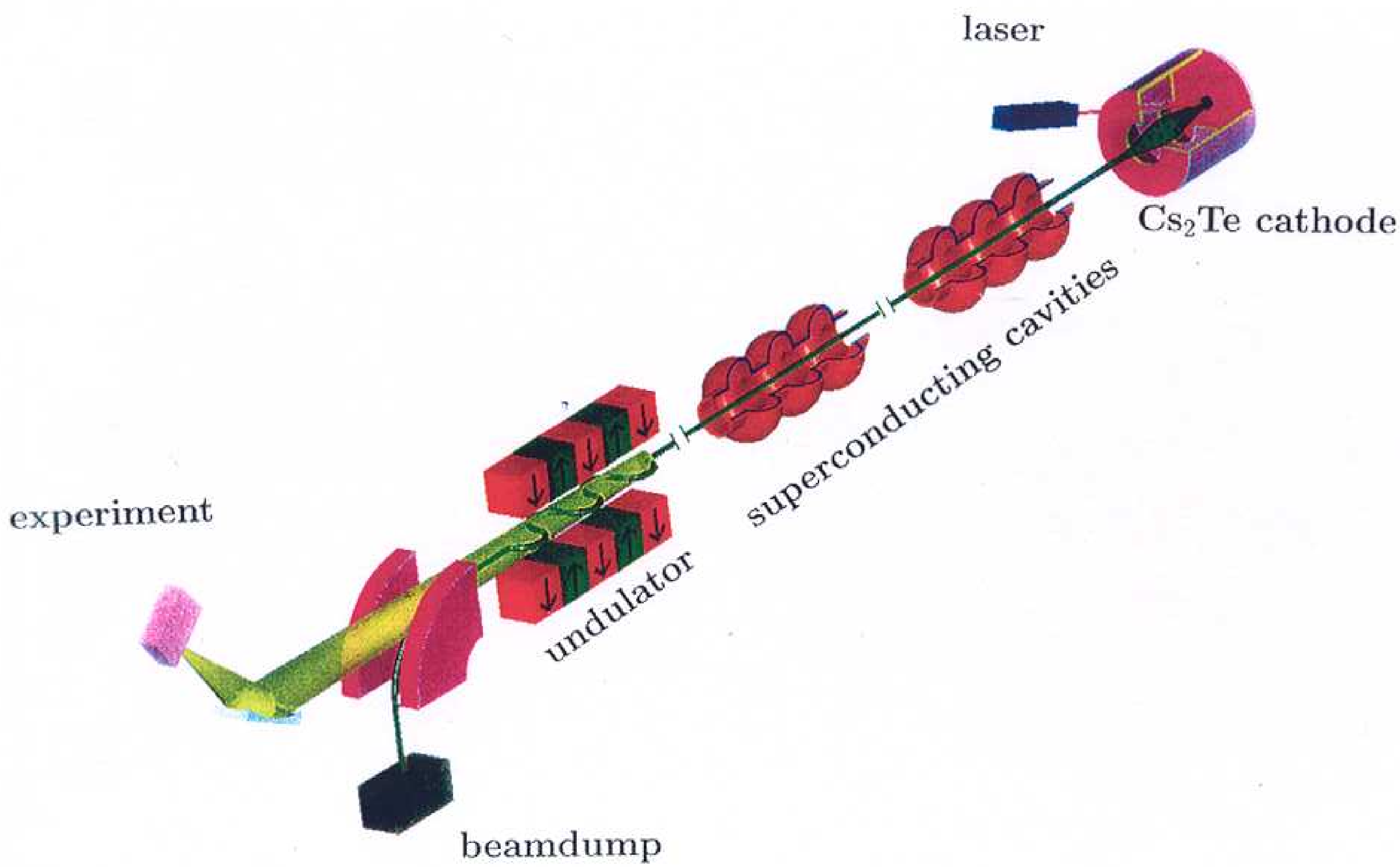
large gain in single passage

Big advantage of FEL: the wavelength is tunable by changing the electron energy.

An optical cavity is no longer possible for wavelengths below 100 nm.



Principle of a Self Amplified Spontaneous Emission (SASE) Free Electron Laser

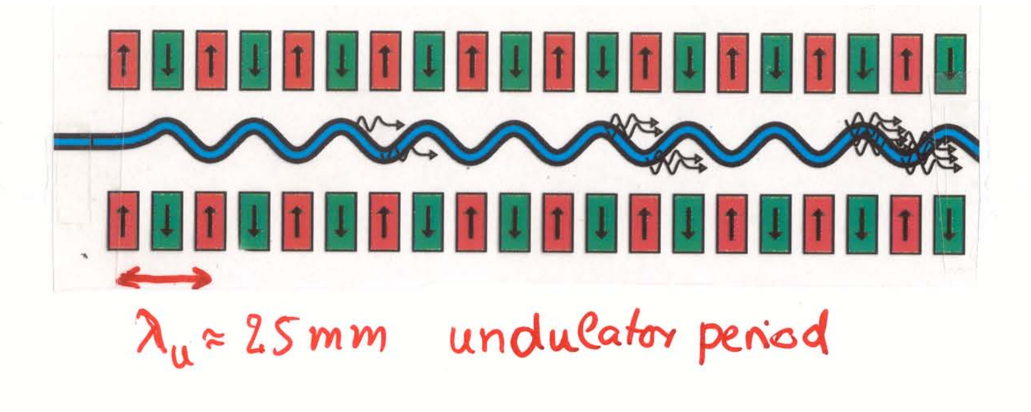


Freie-Elektronen Laser am Linearbeschleuniger (schematisch)

Undulator Radiation

Electron motion in undulator

Schematic view of electron motion in an undulator magnet



Call $W = E_{kin} + m_e c^2$ the total relativistic energy of the electron.
Lorentz factor, normalized velocity: $\gamma = W/(m_e c^2)\,, \quad \beta = \sqrt{1 - 1/\gamma^2}$
The average velocity in z direction is less than βc owing to the sinodoidal trajectory (proof in FEL Course)

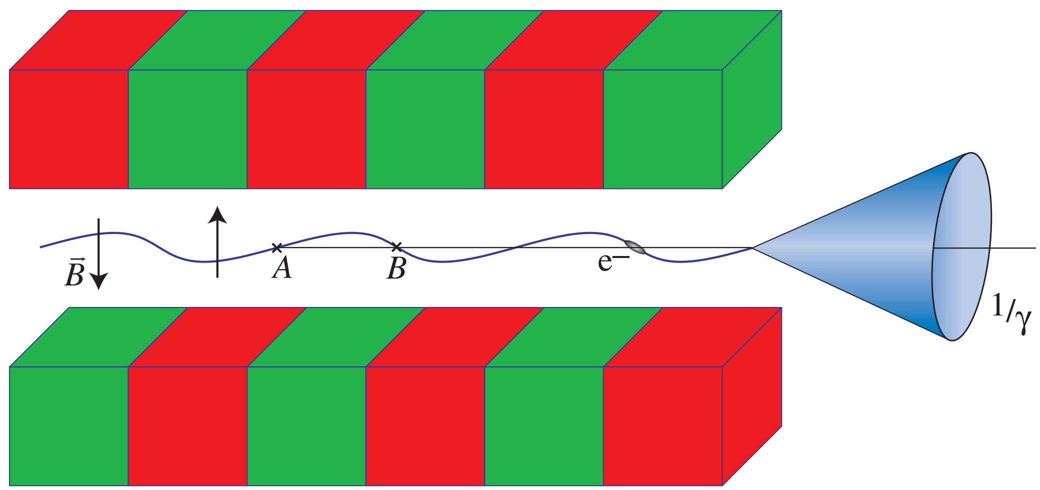
$$\bar{v}_z \equiv \bar{\beta}\, c = \left(1 - \frac{1}{2\gamma^2}\,(1 + K^2/2)\right)\, c \quad \text{with} \quad K = \frac{eB_0\lambda_u}{2\pi m_e c}$$

K is called the **undulator parameter**

Synchrotron radiation is emitted inside a cone with opening angle $1/\gamma$ (see lecture by Rivkin)

Undulator: $K \leq 1$, the electron trajectory is inside the radiation cone. Therefore, the photons emitted by a given electron at various positions along the undulator interfere with each other. This has the important consequence that the radiation is monochromatic, in contrast to synchrotron radiation in bending magnets.

Note: different electrons radiate independently and do not interfere.



Lorentz transformation to moving coordinate system

Consider a coordinate system (x^*, y^*, z^*) moving with the average velocity $\bar{v}_z = \bar{\beta} c$ of the electron. The undulator period appears shortened due to the relativistic length contraction

$$\lambda_u^* = \lambda_u / \gamma$$

In the moving system the electron carries out a harmonic oscillation in x direction with the frequency

$$\omega^* = \gamma \, c \, 2\pi / \lambda_u$$

(Superimposed is a small longitudinal oscillation, which will be ignored here, it leads to higher harmonics in the radiation).
In the moving system the electron emits dipole radiation with a frequency $\omega^* = \gamma \, \omega_u$ (with $\omega_u = c \, 2\pi / \lambda_u$) and a wavelength $\lambda^* = \lambda_u / \gamma$

Remember: λ_u is the undulator period, i.e. the distance between two equal poles.
Take typical values: $\lambda_u = 25$ mm, $\gamma = 1000 \quad \Rightarrow \quad \lambda^* = 25\ \mu$m.

Transformation of radiation into laboratory system

We are interested in the wavelength of the light emitted in forward direction. The Lorentz transformation of the photon energy (this can also be considered as the relativistic Doppler shift) reads

$$\hbar\omega^* = \gamma\hbar\omega_\ell$$

After a little algebra we get for the wavelength of the undulator light (see FEL Course)

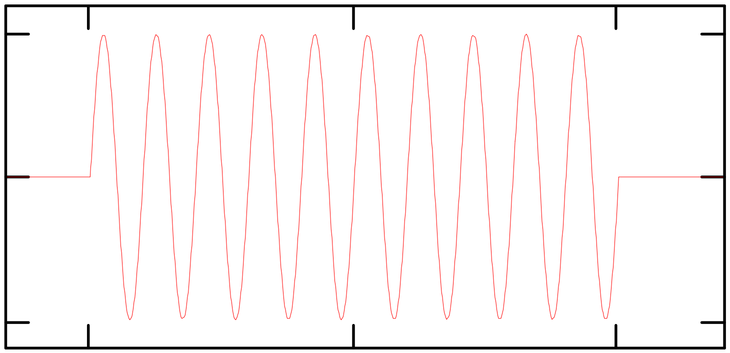
$$\boxed{\lambda_\ell = \frac{\lambda_u}{2\gamma^2}\left(1 + K^2/2\right)}$$

$\gamma = 9$

$\gamma = 99$

Properties of undulator radiation

An electron passing an undulator with N_u periods produces a wavetrain with N_u oscillations.

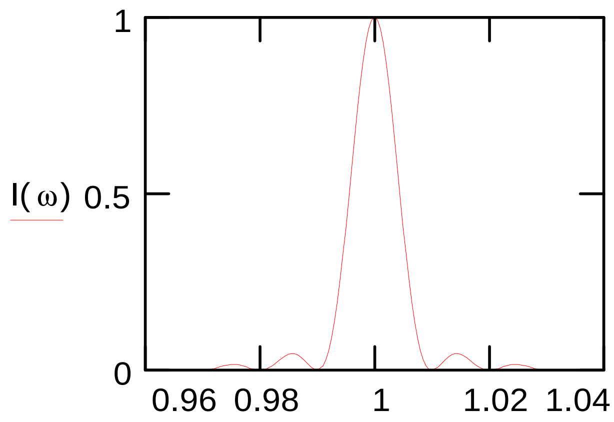


Finite wave train
(here with 10 periods)

The time duration of the wave train is $T = N_u \lambda_\ell / c$. Due to the finite duration the radiation is not monochromatic but contains a frequency spectrum which is obtained by Fourier transformation (see FEL Course).
The spectral intensity is

$$I(\omega) \propto \left(\frac{\sin \xi}{\xi} \right)^2 \quad \text{with} \quad \xi = \frac{\Delta\omega\, T}{2} = \pi\, N_u \, \frac{\omega - \omega_\ell}{\omega_\ell}$$

It has a maximum at $\omega = \omega_\ell$ and a width proportional to $1/N_u$.



Spectral intensity for a wave train with N_u = 100 periods

In the figure, the normalized intensity is plotted as a function of ω/ω_ℓ. The total radiation power (integrated over frequency) is the same as in a bending magnet (see Rivkin):

$$P_{rad} = \frac{2\alpha\hbar c^2\gamma^4}{3\rho^2}$$

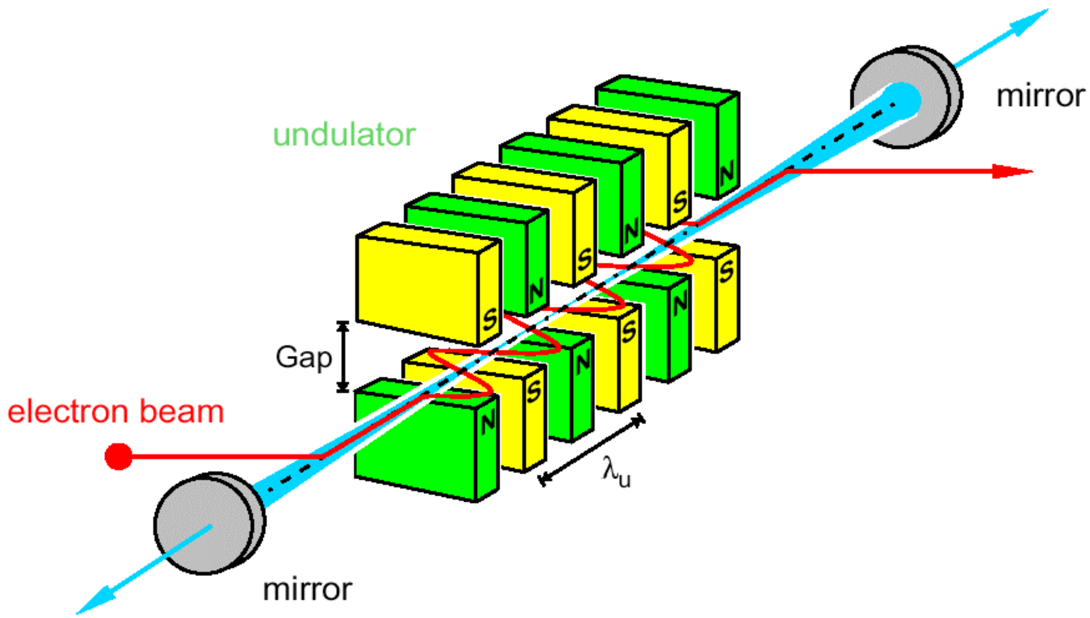
Main differences to synchrotron radiation in bending magnets: (a) undulator radiation is confined to a narrow spectral line, (b) the radiation is well collimated. Note, however, that different electrons radiate indepedently both in bending magnets and in undulators, hence the intensity depends linearly on the number N of electrons per bunch:

$$P_N = N \cdot P_1$$

Low-Gain FEL

Energy transfer from electron to light wave

Consider "seeding" by an external light source with wavelength λ_ℓ



The light wave is co-propagating with the relativistic electron beam. It is described by a plane electromagnetic wave

$$E_x(z,t) = E_0 \cos(k_\ell z - \omega_\ell t) \quad \text{with} \quad k_\ell = \omega_\ell / c$$

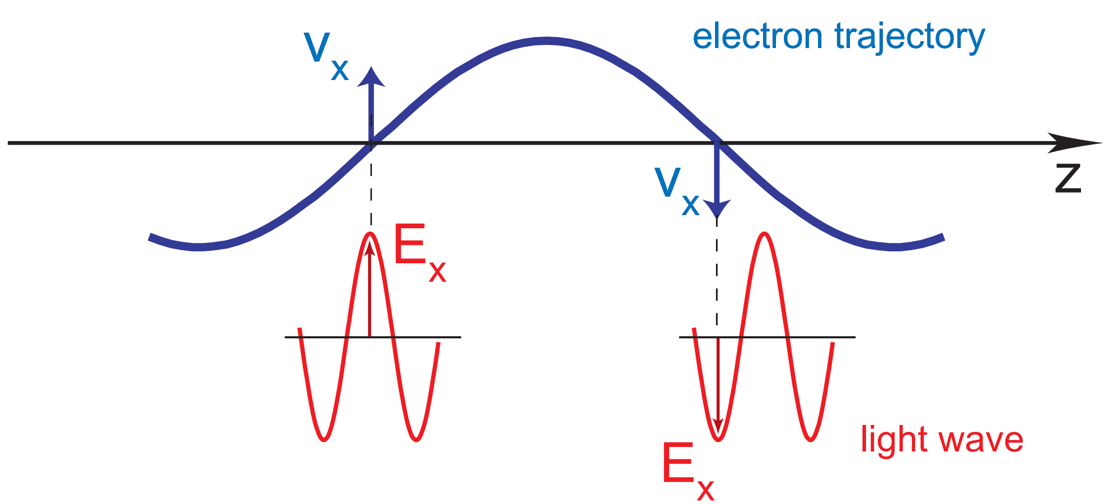
Question: can there be a continuous energy transfer from electron beam to light wave?
The electron energy $W = \gamma m_e c^2$ changes in time dt by

$$dW = \vec{v} \cdot \vec{F} = -e v_x(t) E_x(t) dt$$

The average electron speed in z direction is $\bar{v}_z = c\left(1 - \frac{1}{2\gamma^2}\left(1 + K^2/2\right)\right) < c$
To determine the condition for resonant energy transfer we compute the electron and light travel times for a half period of the undulator:

$$t_{el} = \lambda_u/(2\bar{v}_z), \quad t_{light} = \lambda_u/(2c)$$

Continuous energy transfer happens if $\omega_\ell(t_{el} - t_{light}) = \pi$
(Remark: also $3\pi, 5\pi \ldots$ are possible, leading to higher harmonics of the radiation)



From this condition we compute the light wavelength (see FEL Course)

$$\boxed{\lambda_\ell = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)}$$

This wavelength is identical with the undulator radition wavelength (in forward direction).

The quantitative treatment of the energy transfer from the electron to the light wave is presented in the FEL Course. Here I quote the results. Introducing so so-called ponderomotive phase:

$$\psi \equiv (k_\ell + k_u)z - \omega_\ell t$$

one can show that the time-variation of the electron γ factor and of the phase are

$$\frac{d\gamma}{dt} = -\frac{eE_0K}{2m_e c\gamma_r^2}\sin\psi \qquad \frac{d\psi}{dt} = 2k_u c\frac{\gamma - \gamma_r}{\gamma_r}$$

where the "resonant" gamma-factor is defined by the condition

$$\lambda_\ell = \frac{\lambda_u}{2\gamma_r^2}\left(1 + \frac{K^2}{2}\right)$$

Note: if the electron energy is equal to $E_r = \gamma_r m_e c^2$ then the undulator radiation produced by the electron beam has exactly the wavelength of the seed laser. In an FEL, however, one has to run the e-beam at a slightly higher energy $E = \gamma m_e c^2 > E_r$ in order to amplify the light wave.
The combination of the two first order equations yields the "Pendulum Equation" of the low-gain FEL

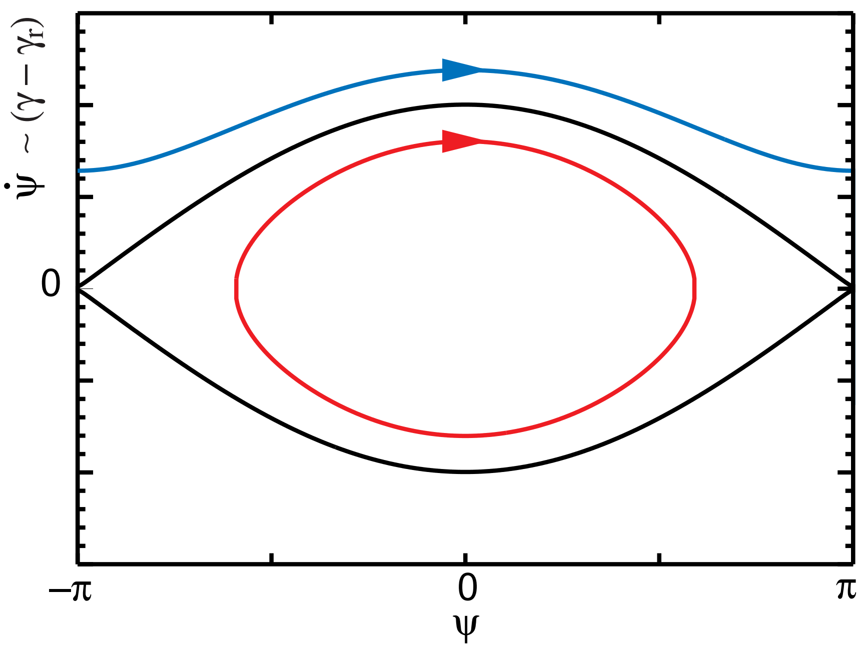
$$\boxed{\ddot{\psi} + \Omega^2 \sin\psi = 0} \quad \text{with} \quad \Omega^2 = \frac{e E_0 K k_u}{m_e \gamma_r^2}$$

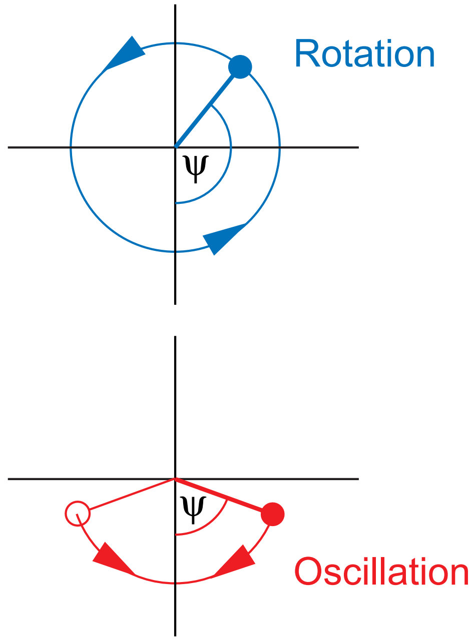
Phase space representation

The two equations

$$\frac{d\gamma}{dt} = -\frac{eE_0K}{2m_ec\gamma_r^2}\sin\psi \qquad \frac{d\psi}{dt} = 2k_uc\frac{\gamma-\gamma_r}{\gamma_r}$$

can be used to plot the trajectories in the (ψ, γ) phase space. There is a close analogy with the motion of a mathematical pendulum. At small amplitude we get a harmonic oscillation. With increasing angular momentum the motion becomes unharmonic. At very large angular momentum one gets a rotation (unbounded motion).

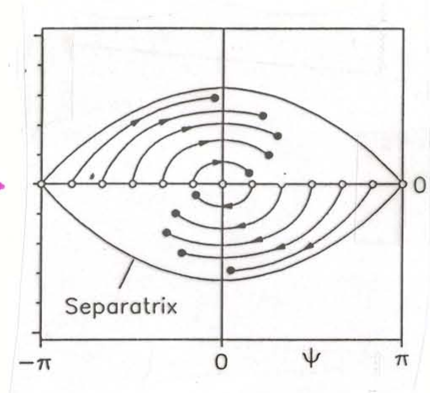




Phase space trajectories for many electrons

a) e^- beam "on resonance" with λ_e of seed laser

$\gamma = \gamma_r$ →



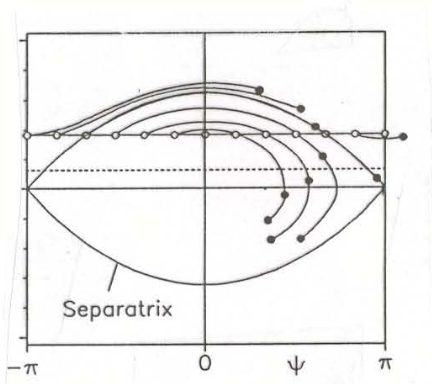
$\gamma = \gamma_r \qquad \lambda_e = \frac{\lambda_u}{2\gamma_r^2}\left(1 + \frac{K^2}{2}\right)$

half of the e^- take energy from light wave
half " " e^- give " to " "

⇒ overall gain is zero

b) e^- beam has slightly higher energy $\gamma > \gamma_r$

$\gamma > \gamma_r$ →



few electrons take energy from light
more electrons give energy to light

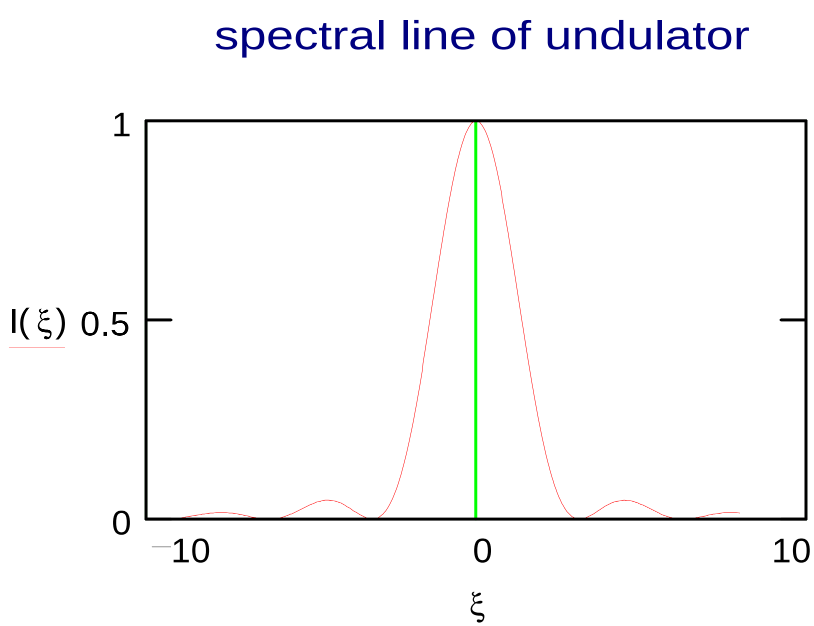
⇒ overall gain is positive

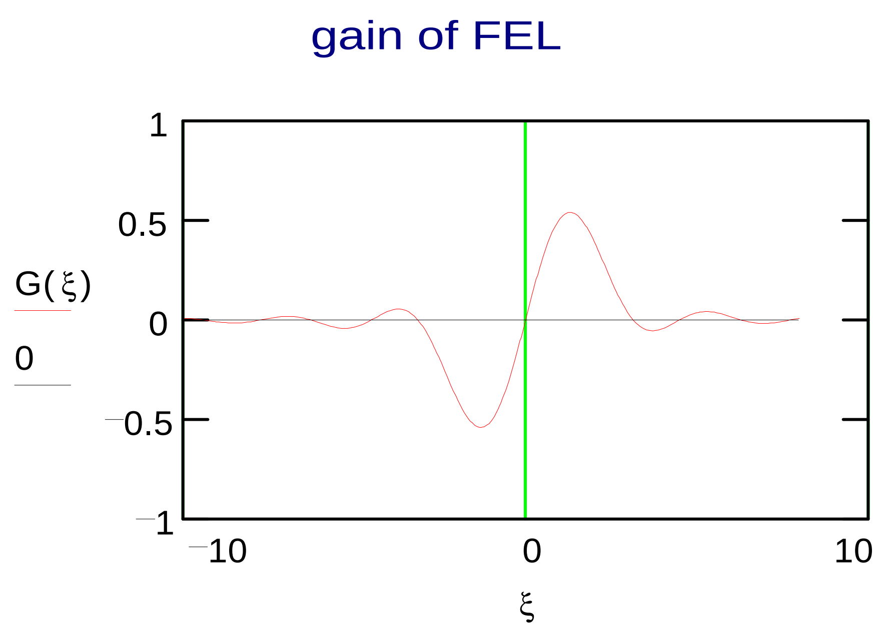
The mathematical treatment of the energy transfer between electron and light wave is quite involved, see the FEL Course. The essential results is that the FEL-gain is given by

$$G(\xi) = -\frac{\pi\, e^2 K^2 N_u^3 \lambda_u^2\, n_e}{4\varepsilon_0 m_e c^2 \gamma_r^3} \cdot \frac{d}{d\xi}\left(\frac{\sin^2\xi}{\xi^2}\right) \quad \text{with} \quad \xi = \pi\, N_u \frac{\omega - \omega_\ell}{\omega_\ell}$$

Madey Theorem

The FEL gain curve is obtained by taking the negative derivative of the line-shape curve of undulator radiation.



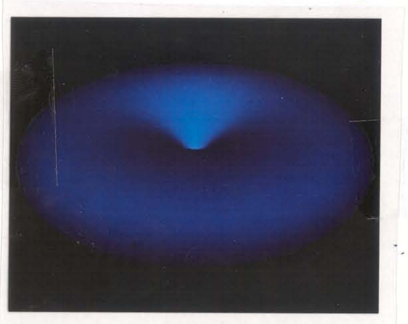


High-Gain FEL

The essential feature of the high-gain FEL is that a large number of electrons radiate coherently. In that case, the intensity of the radiation field grows quadratically with the number of particles: $I_N = N^2 \, I_1$.

Consider "point-like" bunch, charge $Q = -Ne$

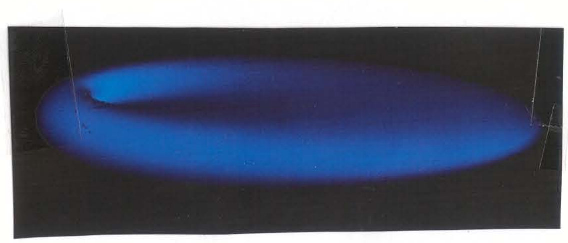
Let bunch carry out harmonic oscillation $x(t) = x_0 \cos \omega_0 t$



radiated power

$$P = N^2 \cdot \frac{e^2 x_0^2}{6\pi \varepsilon_0 c^3} \cdot \omega_0^4$$

power proportional to N^2



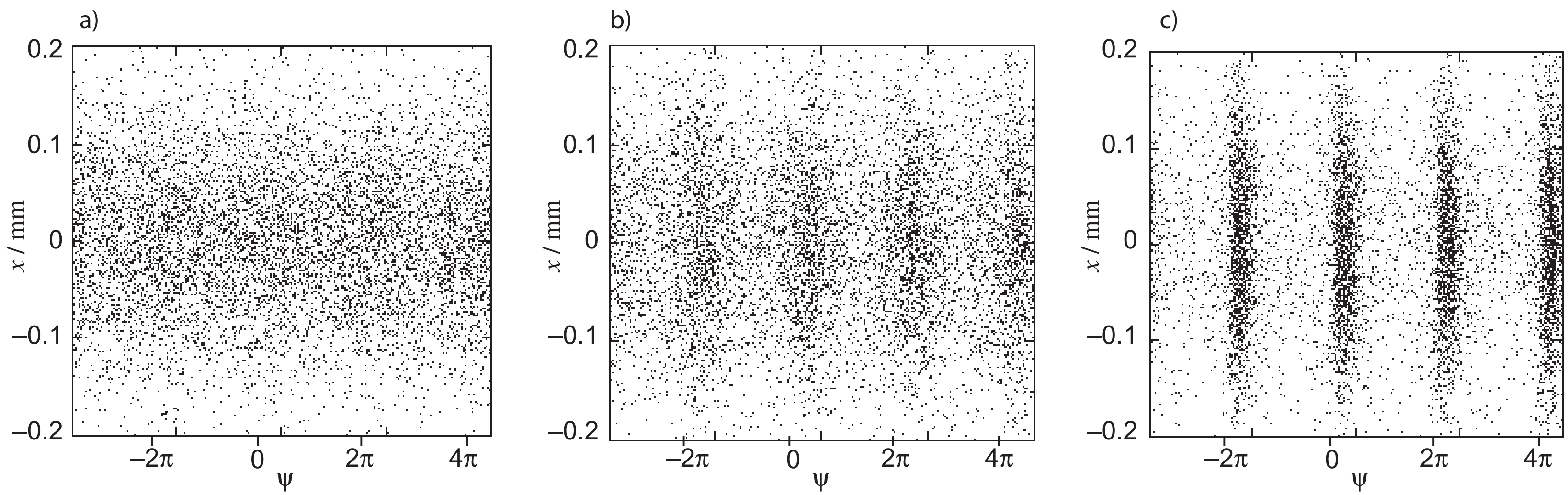
dipole moving with $v \to c$

strong forward collimation of power

$\omega = \gamma \cdot \omega_0$ Doppler effect

but: total radiated power remains invariant

Big problem: the particle bunches are much longer than the FEL wavelength, it appears impossible to produce intense electron bunches with a length $\ll \lambda_\ell$.
The way out of this dilemma is given by the process of **microbunching:**
Electrons which lose energy to the light wave travel a longer path in the undulator, electrons which gain energy from the light wave travel a shorter path. The result is a modulation of the longitudinal velocity. This velocity modulation leads eventually to a concentration of the electrons in slices which are much shorter than λ_ℓ .



The particles within a **micro-bunch** radiate coherently. The resulting strong radiation field enhances the micro-bunching even further.
Result: “collective instability”, exponential growth of radiation power

The ultimate power is $P \propto N_c^2$ where N_c is the number of particles in a coherence region
typical value $N_c \approx 10^6 \quad \Rightarrow \quad P_{FEL} = 10^6 P_{undulator}$

Coherent action is what counts:

An approximate analytic description of the high-gain FEL requires the self-consistent solution of the coupled pendulum equations and the inhomogeneous wave equation for the electromagnetic field of the light wave.

In the **1D-FEL theory** the dependencies on the transverse coordinates x, y are disregarded.
The wave equation for the radiation field E_x reads

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2} = \mu_0 \frac{\partial j_x}{\partial t}$$

where the current density $\vec{j}$ is generated by the electron bunch moving on its cosine-like trajectory.
In addition, one has to consider the longitudinal space charge field E_z which is generated by the gradually evolving periodic charge density modulation. After a lot of tedious mathematical steps and several simplifying assumptions one arrives at a third-order differential equation for the "slowly varying amplitude" of the electric field of the light wave:

$$\frac{d^3\tilde{E}_x}{dz^3} - 4ik_u\eta\frac{d^2\tilde{E}_x}{dz^2} - 4k_u^2\eta^2\frac{d\tilde{E}_x}{dz} - i\Gamma^3\tilde{E}_x(z) = 0$$

Here we have introduced the gain parameter Γ and the relative energy deviation η

$$\Gamma = \left(\frac{\mu_0 K^2 e^2 k_u n_e}{4\gamma^3 m_e} \right)^{\frac{1}{3}} \qquad \eta = \frac{\gamma - \gamma_r}{\gamma_r}$$

and assumed that the electron beam has negligible energy spread.

This third-order differential equation can be solved analytically. For the case $\gamma = \gamma_r$ one obtains

$$\tilde{E}_x(z) = A_1 \exp\left(-i\Gamma z\right) + A_2 \exp\left(\frac{i + \sqrt{3}}{2} \Gamma z \right) + A_3 \exp\left(\frac{i - \sqrt{3}}{2} \Gamma z \right)$$

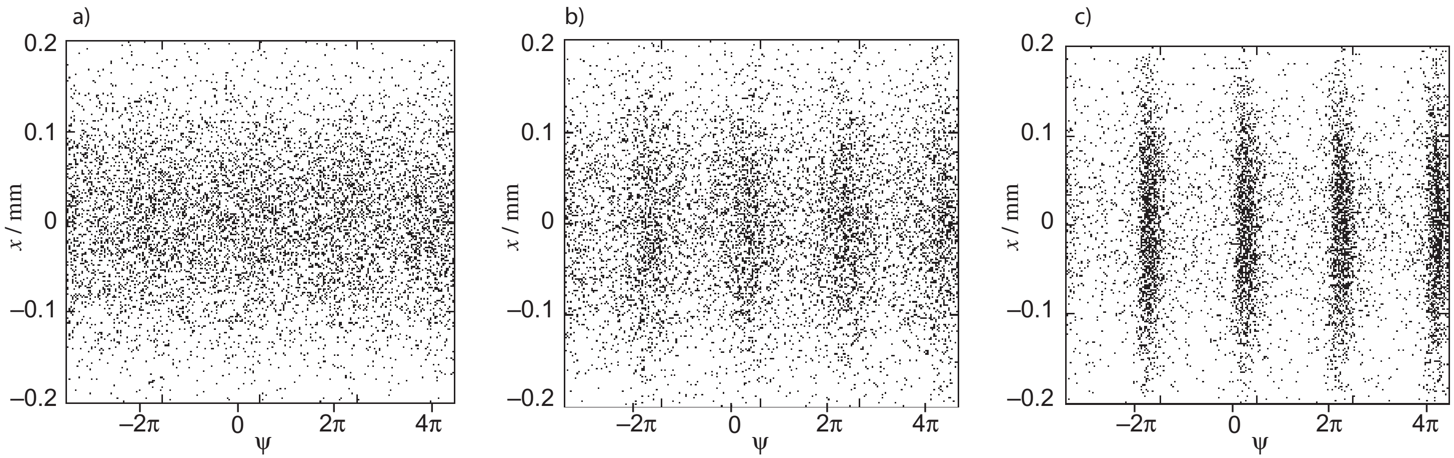
The second term exhibits exponential growth as a function of the position z in the undulator. The electric field grows exponentially as $\exp(\frac{\sqrt{3}}{2}\Gamma z)$, the power grows as $\exp(\sqrt{3}\Gamma z)$.

The gain parameter Γ is related to two parameters which are in widespread use: the Pierce parameter and the power gain length

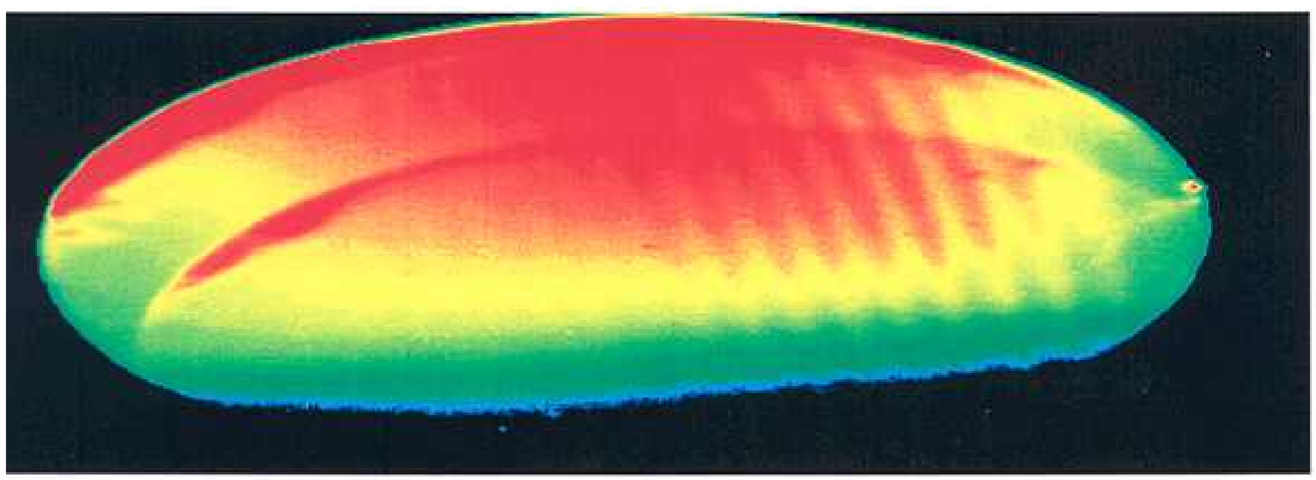
$$\rho_{pierce} = \frac{\lambda_u \Gamma}{4\pi} \qquad L_g = \frac{1}{\sqrt{3}\,\Gamma}$$

The above calculations, which have been sketched only very briefly, indicate that there is an onset of an "instability", leading to a progressing microbunching and an exponential increase in radiation power along the undulator. A quantitave treatment requires elaborate numerical simulations.

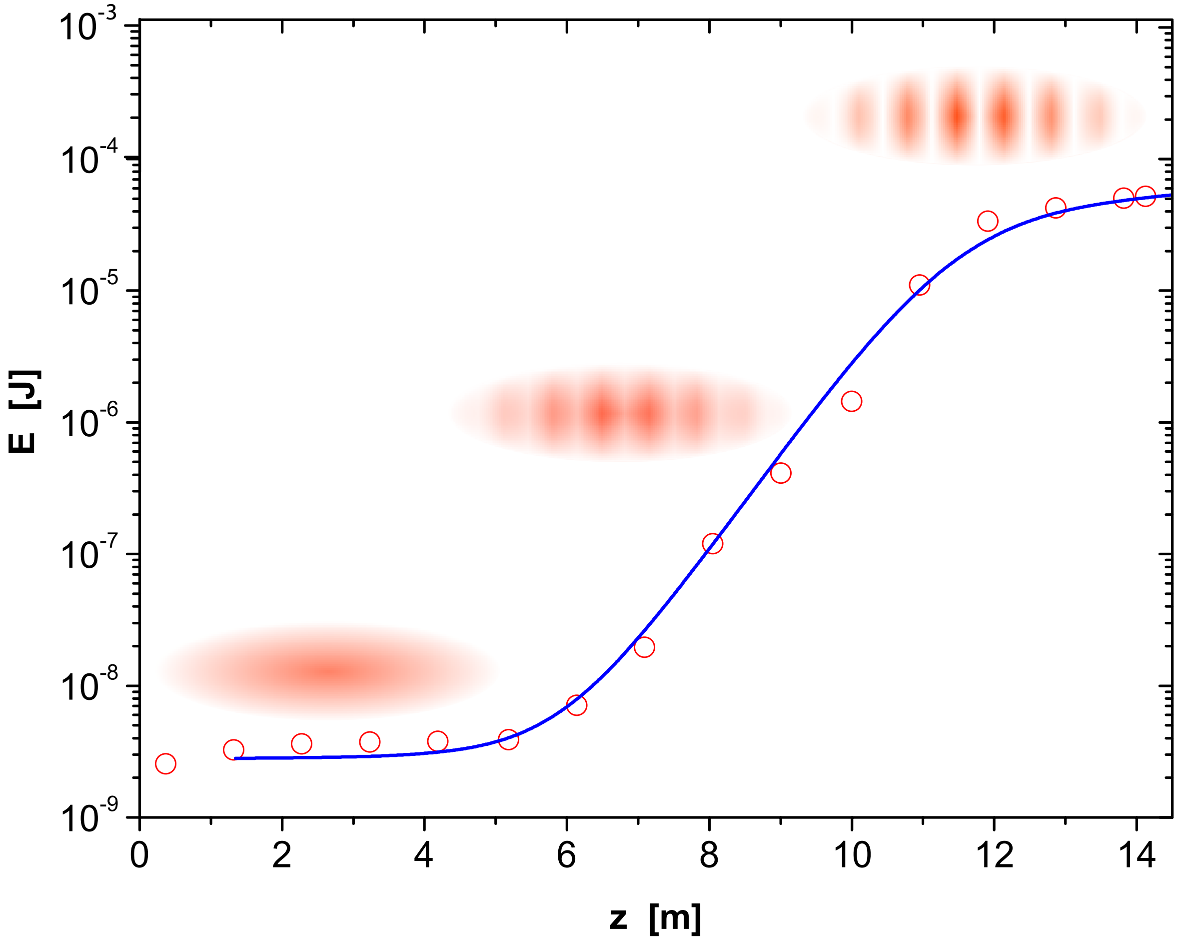
Simulation of microbunching



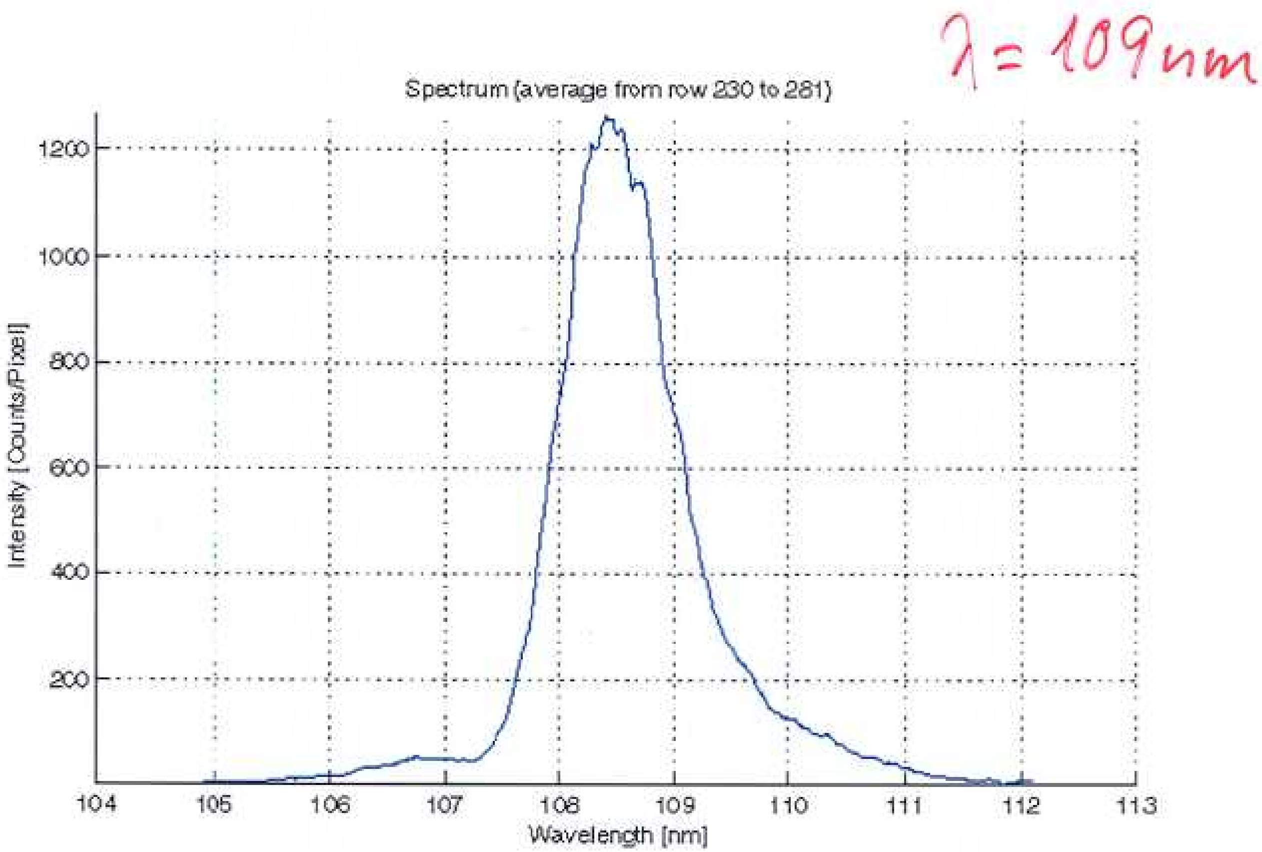
Experimental observation of microbunching at the $60\ \mu$m FEL Firefly, Stanford



Exponential growth of radiation power and progressing microbunching in a long undulator

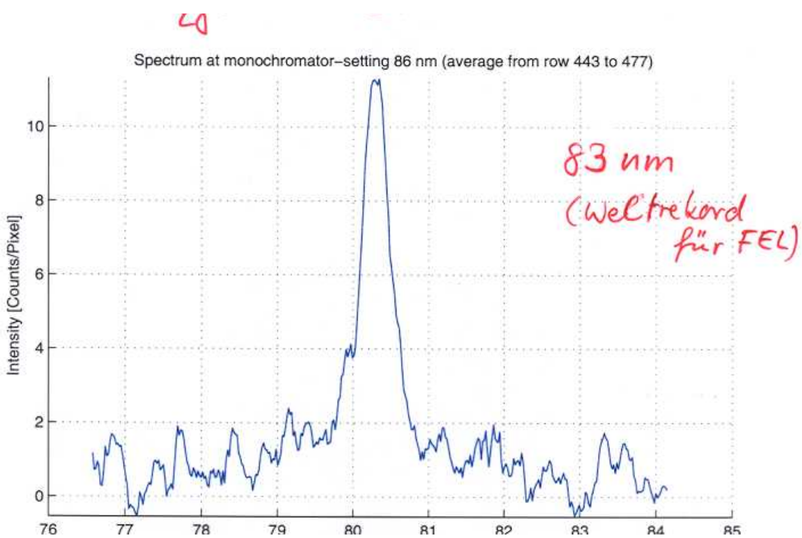


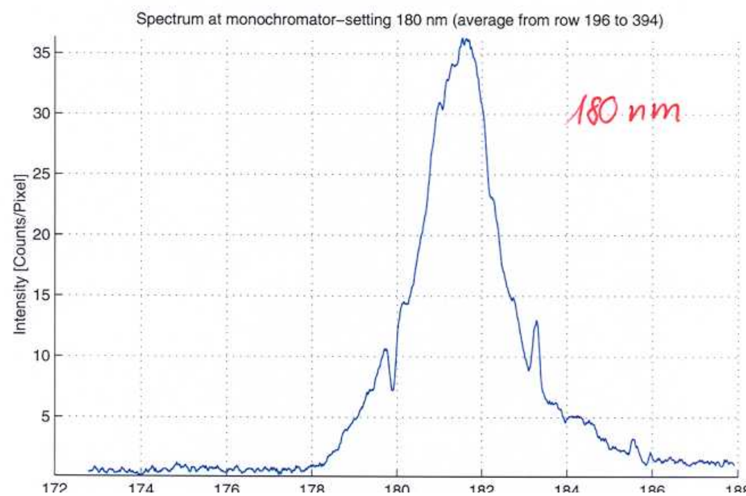
First lasing at the TESLA Test Facility (TTF) Free Electron Laser
Bjorn Wiik Price 2000 for Evgeni Saldin, Evgeni Schneidmiller, Mikhail Yurkov



The FEL as a wavelength tunable laser:

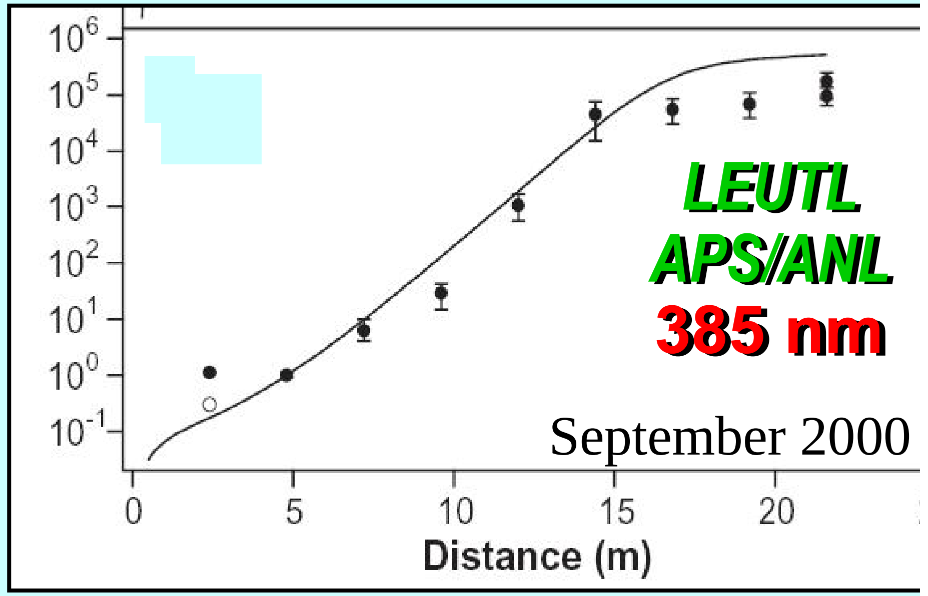
$$\lambda = \frac{\lambda_u}{2\gamma^2}\,(1 + K^2/2)$$

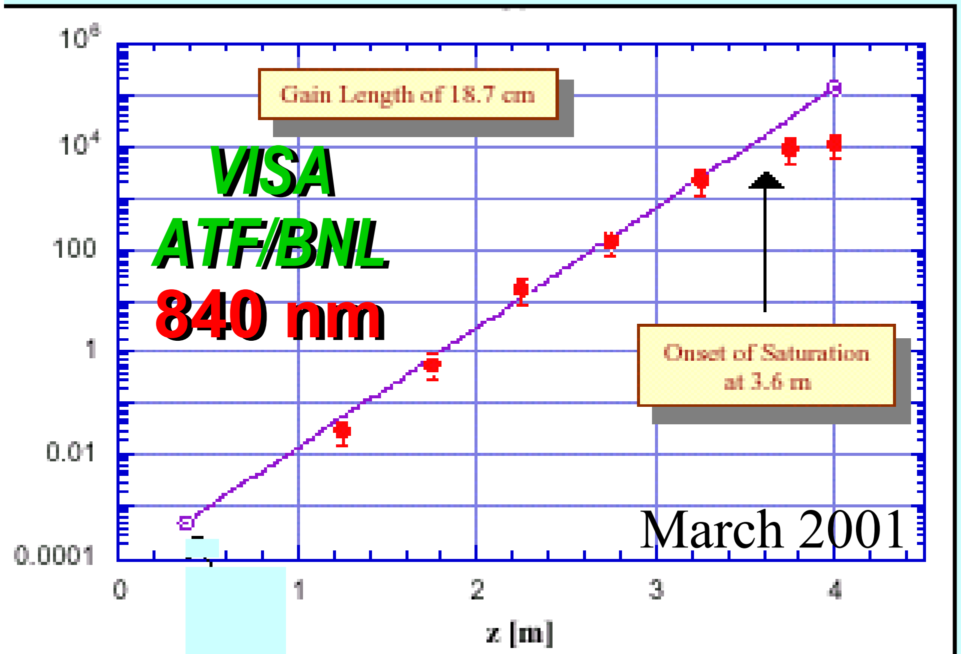


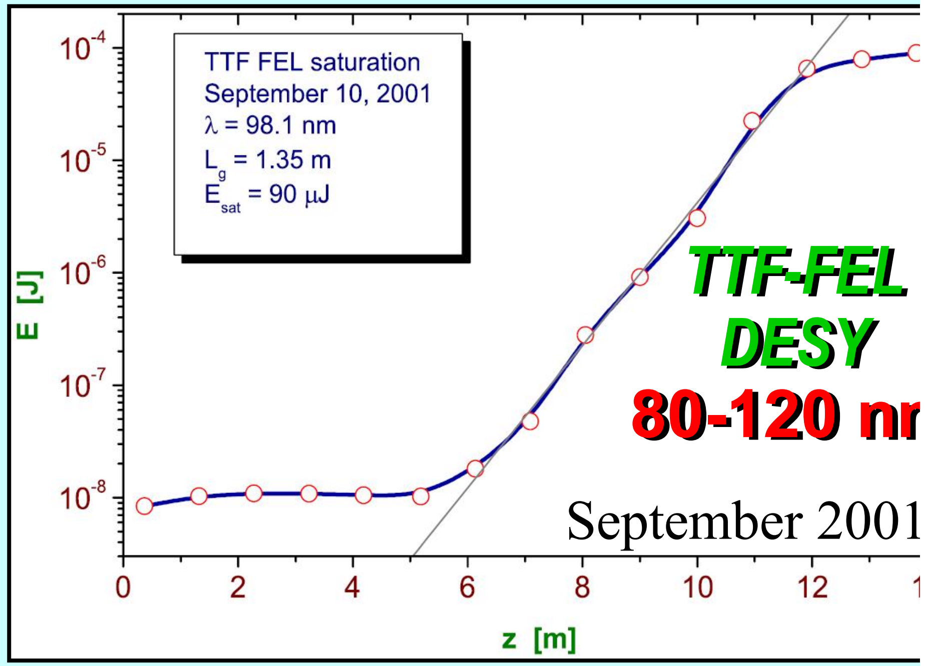


SASE FELs: State of the art

All observations agree with theor. expectations/ computer models







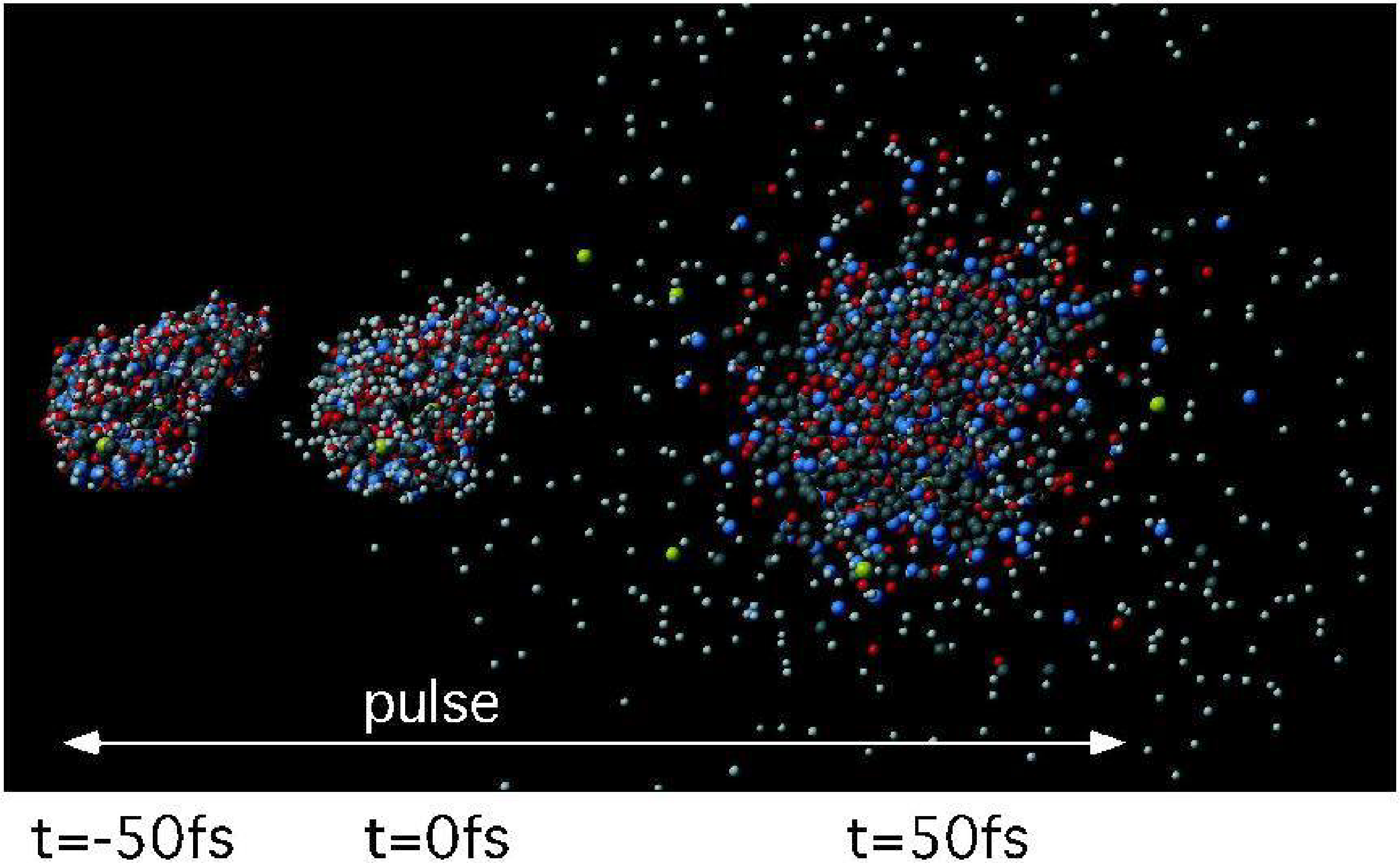


Figure 3.7.5.: *Coulomb explosion of a T4 lysozyme molecule (H: white; C: grey; N: blue; O: red; S: yellow) induced by the radiation damage caused by a* 3×10^{12} *photon per* $(0.1\,\mu m)^2$ *pulse of 12.4 keV energy. The FWHM of the pulse was 50 fs. The molecule is shown at the beginning, in the middle and after the pulse. Even after half of the pulse has passed, the distortions are small. After the pulse the Coulomb explosion is under way [7]. The distortion of the molecule during the time of the pulse is considerably smaller for lower flux densities during the pulse (see also Fig. 3.7.4).*