RF Linac Structures

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Outline

- Basis
    - o Advantage/disadvantage of linacs
    - o RF Cavities
- Beam dynamics though linacs
    - o Energy gain
    - o Linear motion
    - o RMS matching

Why RF linacs ?

Goal of an accelerator : Accelerate a wanted beam within the lower cost

wanted : particle, energy, emittance, intensity, time structure
cost : construction, operation

Main competitors : RF linacs, Synchrotrons, Cyclotrons...

RF linacs : Particles accelerated on a linear path with RF cavities.

Advantages : High current, high duty-cycle, low synchrotron radiation losses.
Drawbacks : High room & cavities consumption, no synchrotron radiation damping

Main use of linacs : Low energy injectors, high intensity protons beam, high energy lepton colliders.

Linacs main applications

Electrons

- High energy collider : No synchrotron losses
- High-quality e- beam for FEL : Strong focusing
- Medical/Industrial irradiation : Low energy
- Neutron sources : Material study

Protons

- Synchrotron injectors : High intensity, high duty-cycle
- Neutron sources : High Power. Material study, transmutation, nuclear fuel production, irradiation tools, exotic nucleus production

Heavy ions

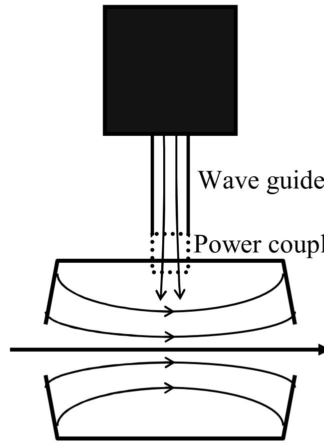
- Nuclear physics research : High intensity, high duty-cycle
- Implantation : Semi-conductors
- Driver for inertial-confinement fusion

RF resonant cavity

Goal : Give kinetic energy to the beam

Basic principle

- Conductor enclosing a close volume,
- Maxwell equations + *Boundary conditions* allow possible electromagnetic field E_n/B_n configurations each oscillating with a given frequency f_n : a ***resonnant mode***. The field is a weighted superposition of these modes.
- The wanted (accelerating) mode is excited at the good frequency and position from a RF power supply through a power coupler,
- The phase of the electric field is adjusted to accelerate the beam.



Elements of mode calculation

Boundary conditions : close to the surface
$$\vec{E}_{//} = \vec{0}$$
$$\vec{B}_{\perp} = \vec{0}$$

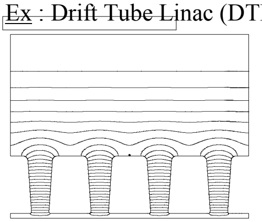
Mode calculation :
$$\vec{\nabla}^2 \vec{E}_n + \frac{\omega_n^2}{c^2} \cdot \vec{E}_n = \vec{0}$$

$\omega_n = 2\pi \cdot f_n$

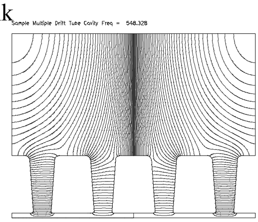
c : speed of light

Electric field :
$$\vec{E}(\vec{r}, t) = \sum e_n(t) \cdot \vec{E}_n(\vec{r})$$

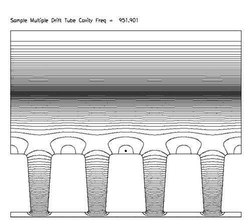
Ex : Drift Tube Linac (DTL) tank



TM_{010} : f=352.2 MHz

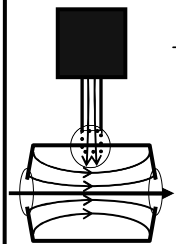


TM_{011} : f=548 MHz



TM_{020} : f=952 MHz

Field time variation



$$\frac{d^2 e_n}{dt^2} + \omega_n^2 \cdot e_n = \boxed{-\frac{\omega_n^2}{\sqrt{\varepsilon\mu}} \cdot \int_S \left(\vec{E} \times \vec{H}_n\right) \cdot \vec{n} \cdot dS \quad \mathbf{1}}$$
$$+ \boxed{\frac{1}{\varepsilon}\frac{d}{dt}\int_{S'} \left(\vec{H} \times \vec{E}_n\right) \cdot \vec{n} \cdot dS' \quad \mathbf{2}} \boxed{-\frac{1}{\varepsilon}\frac{d}{dt}\int_V \vec{J}\left(\vec{r}, t\right) \cdot \vec{E}_n\left(\vec{r}\right) \cdot dV \quad \mathbf{3}}$$

1) *Joule losses in conductor* $= -\frac{\omega_{RF}}{Q_{0n}} \cdot \dot{e}_n$ S : conductor surface

2) *Energy exchange with outside* S' : open surface

$$= \underbrace{-\frac{\omega_{RF}}{Q_{exn}} \cdot \dot{e}_n}_{\text{losses}} + \underbrace{S_n(t) \cdot e^{j(\omega_{RF} t + \varphi_0)}}_{\text{feed}}$$

3) *Energy exchange with beam : Beam loading* V : enclosed volume

$$= k_n \cdot \underline{I}(t)$$

Field time variation (cont)

The last equation can be modelled by :

$$\frac{d^2 e_n}{dt^2} + \frac{\omega_{RF}}{Q_n} \cdot \frac{de_n}{dt} + \omega_n^2 \cdot e_n = S_n \cdot e^{j(\omega_{RF} t + \varphi_0)} + k_n \cdot \underline{I}(t)$$

Which is a damped harmonic oscillator in a forced regime

With : $\frac{1}{Q_n} = \frac{1}{Q_{0n}} + \frac{1}{Q_{exn}}$ the quality factor of the cavity

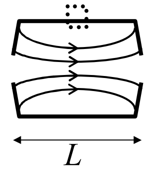
$\tau = 2 \cdot \frac{Q_n}{\omega_{RF}}$ is the filling time of the cavity

$S_n \cdot e^{j(\omega_{RF} t + \varphi_0)}$ is the RF source

$k_n \cdot \underline{I}(t)$ is the beam loading

RF definitions and properties

Per cavity



Cavity length : ***L***

Cavity voltage $\boldsymbol{V_0}$: $$V_0 = \int \hat{E}_z(z) \cdot dz$$

Dissipated power $\boldsymbol{P_d}$: Mean power dissipated in conductor over one RF period

Shunt impedance ***R*** : $$R = \frac{V_0^2}{2 \cdot P_d} \qquad \boxed{P_d = \frac{1}{2} R \cdot V_0^2}$$

Transit time factor ***T*** (calculated latter) : $$\Delta W_{\max} = qV_0 \cdot T$$

ΔW_{max} : Maximum energy that can be gained by a particle in the cavity

Effective shunt impedance : $$\boxed{RT^2 = \frac{\Delta W_{\max}^2}{2P_d}}$$

RF definitions and properties

Per unit length

Cavity mean electric field $\boldsymbol{E_0}$: $$E_0 = \frac{V_0}{L} = \frac{1}{L} \cdot \int \hat{E}_z(z) \cdot dz$$

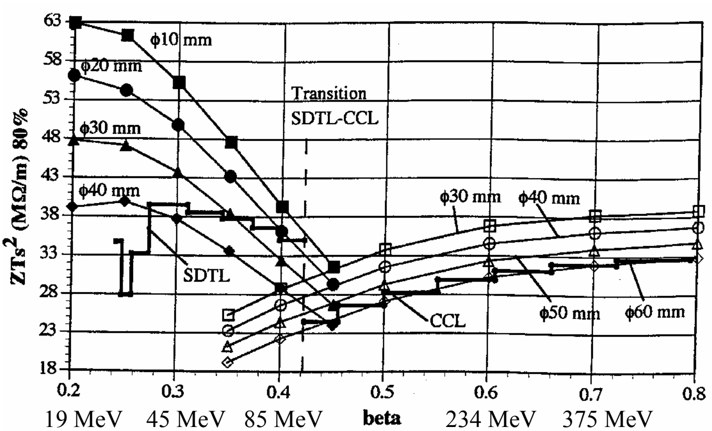
Shunt impedance per unit length ***Z*** : $$Z = \frac{E_0^2}{2 \cdot P'_d} = \frac{R}{L} \qquad \boxed{P'_d = \frac{1}{2} Z \cdot E_0^2}$$

Dissipated power per unit length $\boldsymbol{P_d}$*'* over one RF period

Maximum energy that can be gained per unit length by a particle with charge q in the cavity: $$\Delta W'_{\max} = qE_0 \cdot T$$

Effective shunt impedance per unit length : $$\boxed{ZT^2 = \frac{\Delta W'^2_{\max}}{2P'_d}}$$

Example of use of effective shunt impedance ZT^2



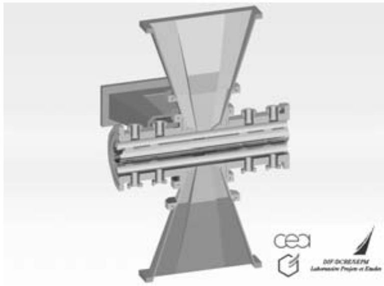
The effective shunt impedance of the structures has been chosen to set the transition energy between sections for TRISPAL project (*C. Bourra, Thomson*).

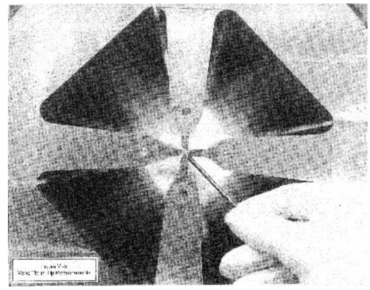
Designing a cavity consists in :

- Fitting the accelerating mode frequency with the RF frequency,
- Maximise the shunt impedance of the cavity
- Rejecting the unwanted modes frequencies far from the RF frequency,
- Calculating the tuning system,
- Increasing the Q_0 of the accelerating mode,
- Calculating the energy deposition geometry to define the cooling system (~20W/cm^2 max), the temperature increase and the associated frequency shift,
- Matching the coupler to the accelerating mode,
- Damping the High Order Modes (HOM) considered as dangerous (having a frequency close to a multiple of the RF frequency), mainly excited by the beam itself and responsible of power losses or beam dynamics perturbations,
- Increasing the beam aperture to reduce beam losses,
- Reducing the peak electric field to reduce electron field emission,
- Reducing the peak magnetic field to avoid quenches (th. Nb max at 2K : 200 mT),
- Adjusting the cavity geometry to reduce the multipactor probabilities,
- Calculating and minimising the cavity deformation and the associated frequency shift through the actions of electromagnetic forces pressure (Lorentz forces),
- Introducing RF peak-up necessary for the field phase and amplitude control,
- Transmitting all these data to the guy calculating the low level RF control,
- ...

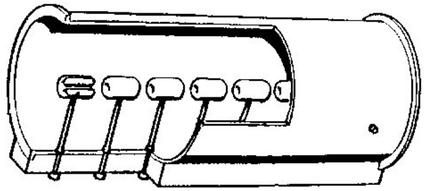
Various types of cavity : Tank

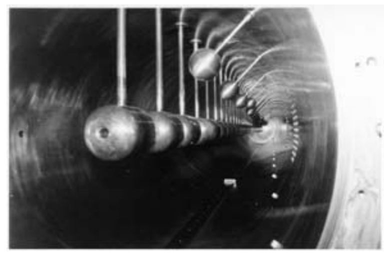
RFQ (low energy ~50 keV-7 MeV)





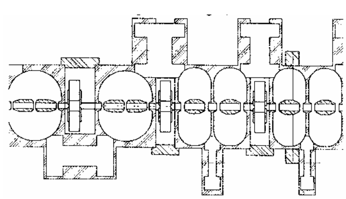
DTL (medium energy ~5-100 MeV)

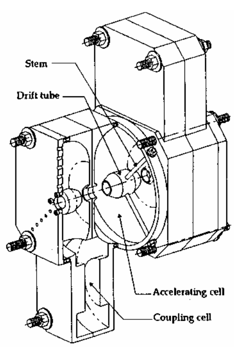




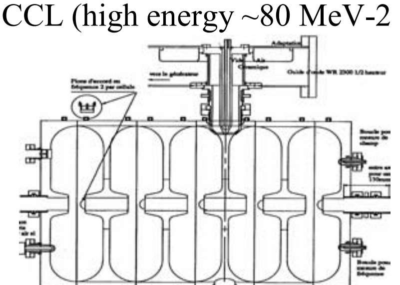
Various types of cavity : Coupled cavity

CCDTL (medium energy ~5-100 MeV)





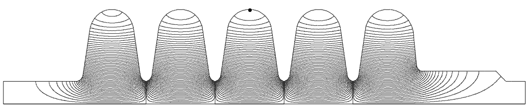
CCL (high energy ~80 MeV-2 GeV)

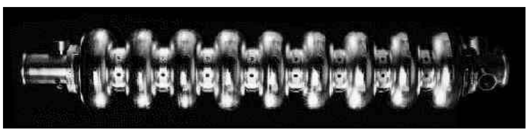


Various types of cavity : Superconducting

Spoke (medium energy ~20-100 MeV)

Elliptical (high energy ~100MeV- 2 GeV)





One word on travelling wave cavity

These cavities are essentially used for acceleration of ultra-relativistic particles

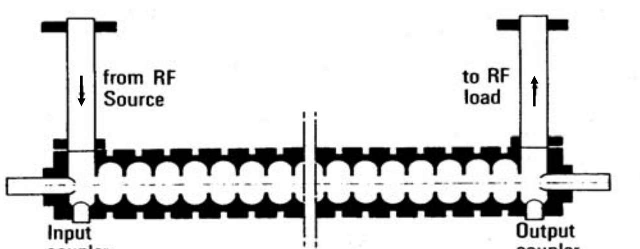
The longitudinal field component is :

$$E_z(r,z,t) = \sum_n E_n(r) \cdot e^{j(\omega t - k_n z)}$$

$E_n(r) \cdot e^{j(\omega t - k_n z)}$ is a space harmonic of the field, given by the cavity periodicity

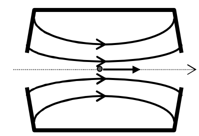
Particle whose velocity is close to the phase velocity of the space harmonic exchanges energy with it. Otherwise, the mean effect is null.

$$v_p \approx v_{\varphi n} = \frac{\omega}{k_n}$$



The transit time factor and the particle phase

Energy gained by a particle in a cavity of length ***L*** :



$$\Delta W = \int qEz(s)\cdot \cos(\phi(s))\cdot ds$$

with : $\phi(s) = \phi_0 + \omega \cdot t = \phi_0 + \frac{\omega}{c}\int_{s_0}^{s}\frac{ds}{\beta_z(s)}$

Assuming a *constant velocity* : $\overline{\beta}$

$$\Delta W = \int qEz(s)\cdot cos\left(\phi_0 + \frac{\omega}{\overline{\beta}c}(s - s_0)\right)\cdot ds \quad \Rightarrow \quad \boxed{\Delta W = qV_0 \cdot T(\overline{\beta})\cdot \cos\phi_p}$$

with : $V_0 = \int |Ez(s)|\cdot ds$ Cavity Voltage

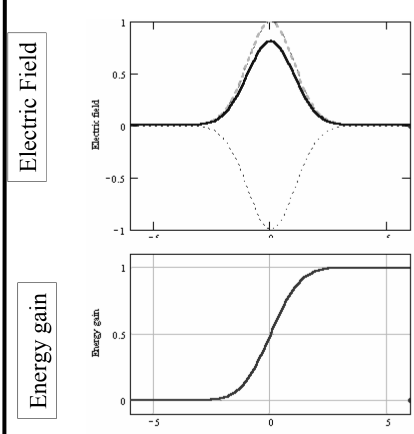
$\phi_p = \arctan\left(\frac{\int Ez(s)\cdot \sin(\phi(s))\cdot ds}{\int Ez(s)\cdot \cos(\phi(s))\cdot ds}\right)$ Synchronous phase

$T = \frac{1}{V_0}\int Ez(s)\cdot \cos(\phi(s) - \phi_p)\cdot ds$ Transit-time factor : $\boxed{0 < T < 1}$

$= \frac{1}{V_0}\left|\int Ez(s)\cdot e^{j\phi(s)}\cdot ds\right|$

Example 1 : The transit time factor in a one-cell cavity

Fast particle : $T \cong 1$



··· Field amplitude

— Field in the cavity with time

··· Field seen by a non synchronous particle

— Energy gain

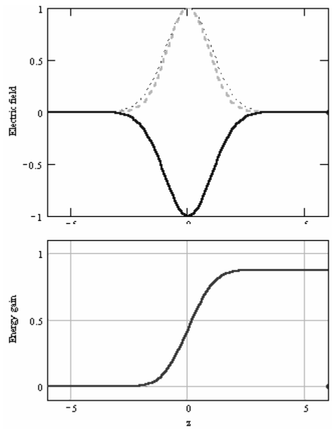
$$\Delta W = qV_0 \cdot T(\overline{\beta})$$

Example 1 : The transit time factor in a one-cell cavity

Medium fast particle : $T \cong 0.85$

Electric Field

Energy gain



··· Field amplitude

— Field in the cavity with time

··· Field seen by a non synchronous particle

— Energy gain

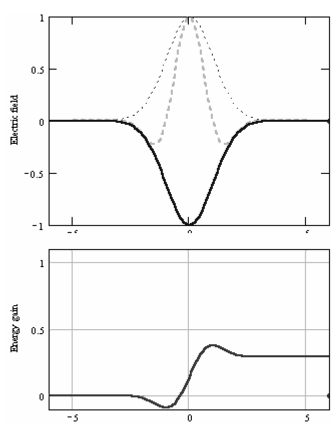
$$\Delta W = qV_0 \cdot T\left(\overline{\beta}\right)$$

Example 1 : The transit time factor in a one-cell cavity

Slow particle : $T \cong 0.3$

Electric Field

Energy gain



··· Field amplitude

— Field in the cavity with time

··· Field seen by a non synchronous particle

— Energy gain

$$\Delta W = qV_0 \cdot T\left(\overline{\beta}\right)$$

The synchronous particle - Linac design

The linac is designed with a hypothetical *synchronous particle*. Its phase in a cavity is called here the *synchronous phase* :

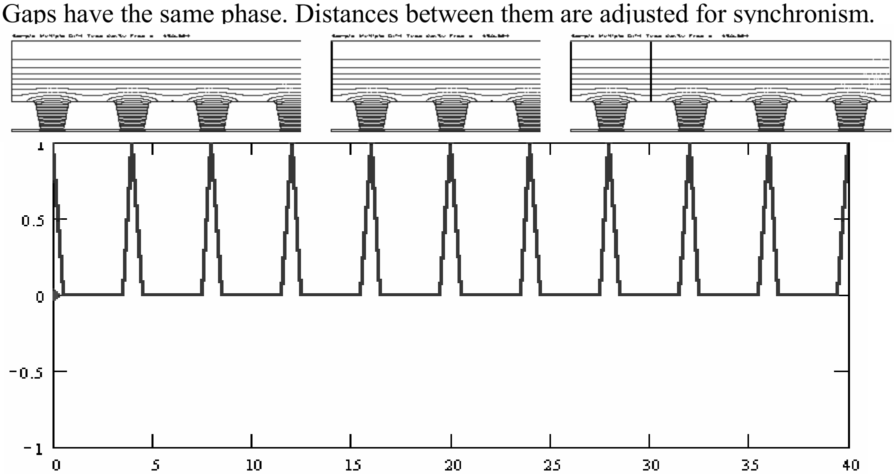
- The absolute phase φ_i and the velocity β_{i-1} of this particle being known at the entrance of cavity *i*, its RF phase ϕ_i is calculated to get the wanted synchronous phase ϕ_{si}, $\boxed{\phi_i = \varphi_i - \phi_{si}}$
- the new velocity β_i of the particle can be calculated from, $\boxed{\Delta W_i = qV_0T \cdot \cos\phi_{si}}$
  - ① if the phase difference between cavities *i* and *i+1* is given, the distance D_i between them is adjusted to get the wanted synchronous phase ϕ_{si+1} in cavity *i+1*.
  - ② if the distance D_i between cavities *i* and *i+1* is set, the RF phase ϕ_i of cavity *i+1* is calculated to get the wanted synchronous phase ϕ_{si+1} in it.

RF phase	ϕ_{i-1}		ϕ_i		ϕ_{i+1}
Particle velocity		β_{si-1}		β_{si}	
Distances		D_{i-1}		D_i	
Synchronous phase	ϕ_{si-1}		ϕ_{si}		ϕ_{si+1}
Cavity number	*i-1*		*i*		*i+1*

Synchronism condition : $\boxed{\phi_{si+1} - \phi_{si} = \omega \cdot \dfrac{D_i}{\beta_{si}\, c} + \phi_{i+1} - \phi_i + 2\pi n}$

① Linac with coupled cavities (DTL)

Gaps have the same phase. Distances between them are adjusted for synchronism.



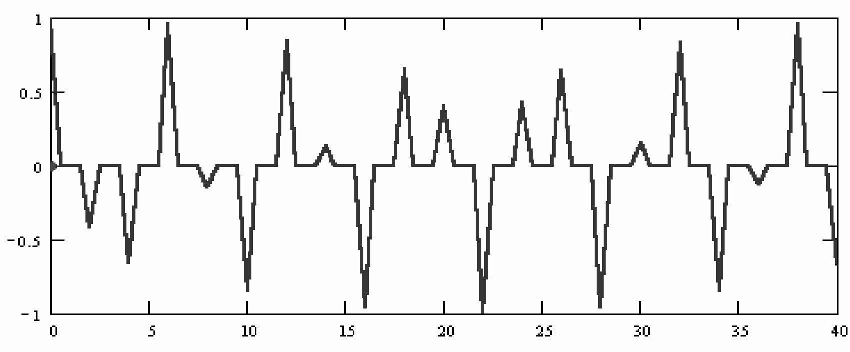
— Field in cavities

● Particle synchronous with the field — Its energy gain

● Particle not synchronous with the field — Its energy gain

② Linac with independently phased cavities (SCL)

The distance between the cavities is given. Cavities are phased to accelerate a given particle.



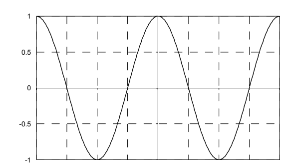
— Field in cavities

● Particle synchronous with the field — Its energy gain

● Particle not synchronous with the field — Its energy gain

Choice of the synchronous phase

Acceleration condition : The field should accelerate the particle



$$\Delta W > 0 \Rightarrow \quad qV_0 T \cdot \cos\phi_p > 0$$

$qV_0 > 0: \ \phi_p \in [-90°, 90°]$

$qV_0 < 0: \ \phi_p \in [90°, 270°]$

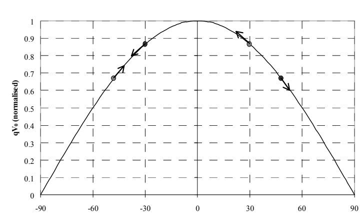
Stability condition : Late particles should gain more energy that early ones

$$\frac{d\Delta W}{d\phi_p} > 0 \Rightarrow \quad qV_0 T \cdot \sin\phi_p < 0$$

$qV_0 > 0: \ \phi_p \in [-180°, 0°]$

$qV_0 < 0: \ \phi_p \in [0°, 180°]$

$$\boxed{\begin{array}{l} qV_0 > 0: \ \phi_p \in [-90°, 0°] \\ qV_0 < 0: \ \phi_p \in [90°, 180°] \end{array}}$$



General equations of motion

$$\frac{d\vec{p}}{dt} = q \cdot \left(\vec{v} \times \vec{B} + \vec{E}\right) = \vec{F}$$

$$+ \quad p_w = \beta_w \gamma \cdot mc \qquad \Rightarrow \qquad \begin{cases} \dfrac{d\gamma\beta_x}{ds} = \dfrac{F_x}{mc^2\beta_z} = \dfrac{d\gamma\beta_z x'}{ds} \\ \dfrac{d\gamma\beta_y}{ds} = \dfrac{F_y}{mc^2\beta_z} = \dfrac{d\gamma\beta_z y'}{ds} \\ \dfrac{d\gamma\beta_z}{ds} = \dfrac{F_z}{mc^2\beta_z} \end{cases}$$

$$+ \quad dt = ds / \beta_z c$$

$\beta_w c$ is the particle velocity along w direction
γ is the particle reduced energy,
q and m its charge and rest mass.
x and y are transverse directions,
s is the abscissa along longitudinal direction z,
x' and y' are called the particle slopes.

$$\boxed{x'' + \frac{d\gamma\beta_z / ds}{\gamma\beta_z} x' = \frac{F_x}{mc^2\gamma\beta_z^2}}$$

These equation are <u>non linear</u>, <u>coupled</u> and <u>damped</u>.
Each element (cavity, quadrupole …) contributes to the force.

Linear force

In the highest simplification level, the external force along direction w (x, y or φ) can be considered <u>periodic</u>, <u>linear</u>, <u>uncoupled</u> and <u>undamped over one period</u> :

Hill equation : $\dfrac{d^2 w}{ds^2} + k_w(s) \cdot w = 0 \qquad k_w(s+S) = k_w(s)$

Giving : $w(s) = \sqrt{\varepsilon_0 \beta_{wm}(s)} \cdot \cos(\mu(s) + \mu_0)$

with : μ_0 and ε_0 constant, $\mu(s) = \mu_0 + \displaystyle\int_{s_0}^{s} \frac{ds}{\beta_{wm}(s)}$, and : $\beta_{wm}(s+S) = \beta_{wm}(s)$

In the (w, w') phase-space, the particle is moving on an ellipse of equation :

$$\gamma_{wm}(s) \cdot w^2 + 2\alpha_{wm}(s) \cdot ww' + \beta_{wm}(s) \cdot w'^2 = \varepsilon_0$$

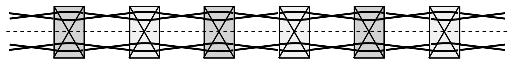
with : $\alpha_{wm}(s) = -\dfrac{1}{2}\dfrac{d\beta_{wm}}{ds}$ and $\gamma_{wm}(s) = \dfrac{1+\alpha_{wm}(s)^2}{\beta_{wm}(s)}$ <u>Courant-Snyder</u> parameters.

The *phase advance* of the particle in a lattice is then : $\sigma = \mu(s+S) - \mu(s)$

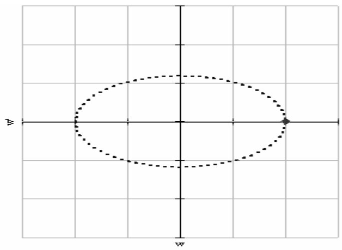
Particle motion in a FODO Lattice

● Particle

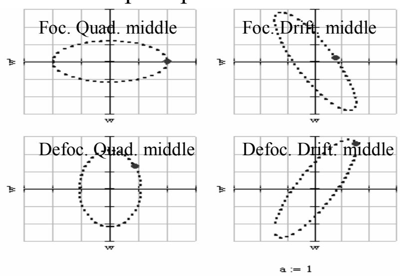
······ Particle ellipse

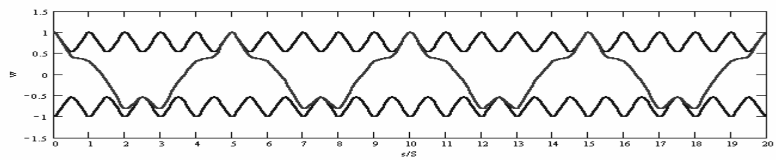


Phase-space trajectory



Phase-space periodic looks





▬ Particle trajectory ▬ Particle ellipse maximum size

RMS dimensions and Beam Twiss parameters

The rms dimensions of the beam are defined statistically as followed :

rms size : $\tilde{w} = \sqrt{\left\langle \left(w - \langle w \rangle\right)^2 \right\rangle}$

rms slope : $\tilde{w}' = \sqrt{\left\langle \left(w' - \langle w' \rangle\right)^2 \right\rangle}$

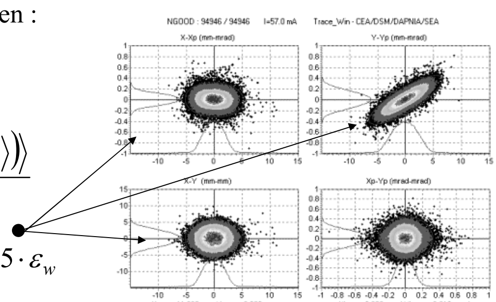
rms emittance : $\tilde{\varepsilon}_w = \sqrt{\tilde{w}^2 \cdot \tilde{w}'^2 - \left\langle \left(w - \langle w \rangle\right) \cdot \left(w' - \langle w' \rangle\right) \right\rangle^2}$

The beam Twiss parameters are then :

$$\beta_w = \frac{\tilde{w}^2}{\tilde{\varepsilon}_w} \qquad \gamma_w = \frac{\tilde{w}'^2}{\tilde{\varepsilon}_w}$$

$$\alpha_w = -\frac{\left\langle \left(w - \langle w \rangle\right) \cdot \left(w' - \langle w' \rangle\right) \right\rangle}{\tilde{\varepsilon}_w}$$

$$\gamma_w \cdot w^2 + 2\alpha_w \cdot ww' + \beta_w \cdot w'^2 = 5 \cdot \varepsilon_w$$



RMS matched beam

The beam is matched when :

$$\beta_w = \beta_{wm}$$

$$\alpha_w = \alpha_{wm}$$

$$\gamma_w = \gamma_{wm}$$

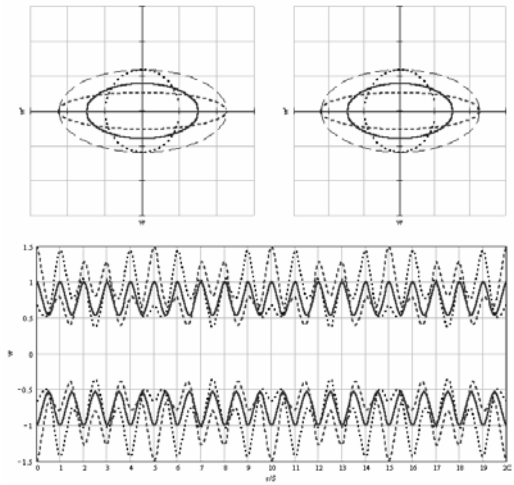
— Matched beam

···· Bigger input beam

····· Smaller input beam

– - Phase-space scanned by the mismatched beams

Phase-space trajectory   Phase-space periodic looks



50% mismatched beam

Summary

• Linacs are competitive for low energy, high current, high duty cycle beams or very high energy light-particles (e^+-e^-) colliders.

• Acceleration is generally done with RF resonant cavities, confinement with quadrupoles (except at very low energy).

• Cavities are pieces of metal (Cu or Nb) whose shape is optimised to accelerate the particles at the RF frequency with the higher efficiency (ZT^2 as high as possible) and the lower cost. The choice of the accelerating electric field E is a compromise between the linac length reduction ($\rightarrow E\nearrow$) and the power dissipation ($\rightarrow E\searrow$) .

• RF phases in cavities are adjusted with respect to a synchronous particle to accelerate the beam and keep it bunched (synchronous phase choice).

• Forces are linearised to calculate the beam matching to the structure.