Concept of Luminosity

(in particle colliders)

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(http://cern.ch/lhc-beam-beam/talks/Zeuthen_luminosity.pdf)

Why colliding beams ?

- **Two beams:**
  $E_1, \vec{p_1}, E_2, \vec{p_2}, m_1 = m_2 = m$
- $E_{cm} = \sqrt{(E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2})^2}$
- **Collider versus fixed target:**
  - **Fixed target:** $\vec{p_2} = 0$
  - $\Longrightarrow \quad E_{cm} = \sqrt{2m^2 + 2E_1 m}$
  - **Collider:** $\vec{p_1} = -\vec{p_2}$ **(usually)**
  - $\Longrightarrow \quad E_{cm} = E_1 + E_2$
- **LHC (pp):**
  14000 GeV versus 118 GeV
- **LEP (e^+e^-):**
  210 GeV versus ?

Colliding beams

Collider performance qualified as:

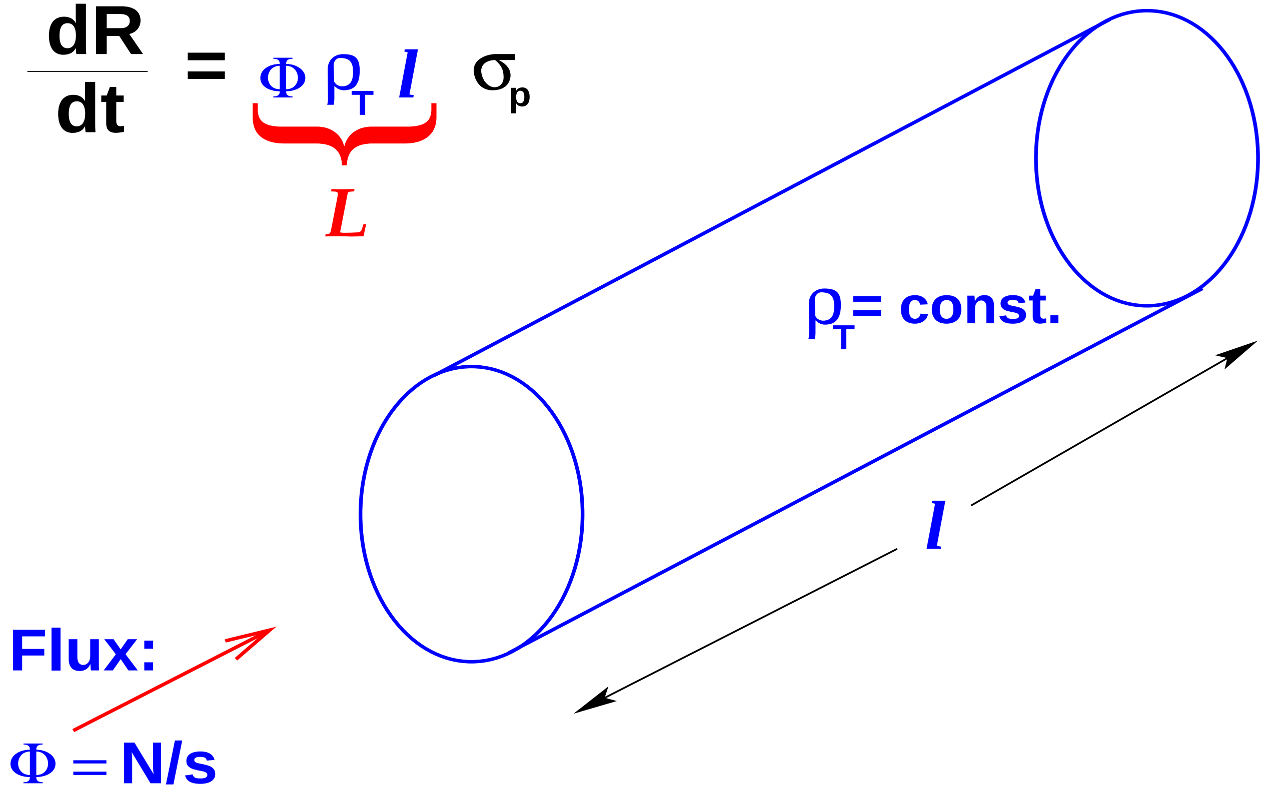
- Available energy
- Number of interactions per second (useful collisions)
- Total number of interactions
- Secondary issues:
  - Time structure of interactions (how often and how many at the same time)
  - Space structure of interactions (size of interaction region)
  - Quality of interactions (background, dead time etc.)

Examples

	Energy (GeV)	$\mathcal{L}$ $cm^{-2}s^{-1}$	rate s^{-1}	$\sigma x/\sigma_y$ $\mu m/\mu m$	Particles per bunch
SPS ($p\bar{p}$)	315x315	$6\ 10^{30}$	$4\ 10^{5}$	60/30	$\approx 10\ 10^{10}$
Tevatron ($p\bar{p}$)	1000x1000	$50\ 10^{30}$	$4\ 10^{6}$	30/30	$\approx 30/8\ 10^{10}$
HERA ($e^{+}p$)	30x920	$40\ 10^{30}$	40	250/50	$\approx 3/7\ 10^{10}$
LHC (pp)	7000x7000	$10000\ 10^{30}$	10^{9}	17/17	$11\ 10^{10}$
LEP ($e^{+}e^{-}$)	105x105	$100\ 10^{30}$	≤ 1	200/2	$\approx 5\ 10^{11}$
PEP ($e^{+}e^{-}$)	9x3	$3000\ 10^{30}$	NA	150/5	$\approx 2/6\ 10^{10}$
KEKB ($e^{+}e^{-}$)	8x3.5	$10000\ 10^{30}$	NA	77/2	$\approx 1.3/1.6\ 10^{10}$

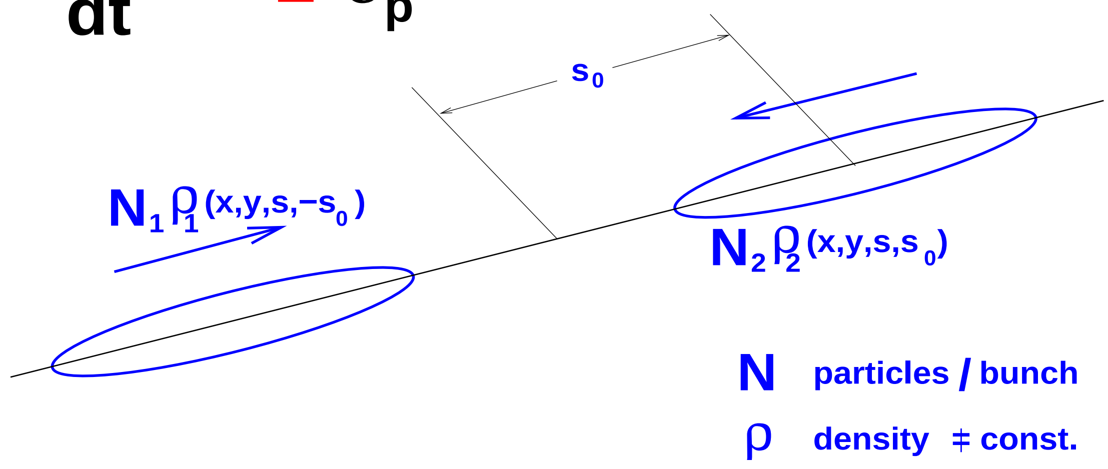
Fixed target luminosity

$$\frac{dR}{dt} = \underbrace{\Phi \, \rho_T \, l}_{L} \; \sigma_p$$



Collider luminosity

$$\frac{dR}{dt} = L \, \sigma_p$$



- $\mathcal{L} \propto K \cdot \int\int\int\int_{-\infty}^{+\infty} \rho_1(x,y,s,-s_0)\rho_2(x,y,s,s_0) dx dy ds ds_0$
- s_0 is "time"-variable: $s_0 = c \cdot t$
- Kinematic factor : $K = \sqrt{(\vec{v_1} - \vec{v_2})^2 - (\vec{v_1} \times \vec{v_2})^2 / c^2}$

Luminosity

- **Assume densities are uncorrelated in all planes**
- **For head-on collisions ($\vec{v_1} = -\vec{v_2}$) we get:**

$$\mathcal{L} = 2 N_1 N_2 f B \int\int\int\int_{-\infty}^{+\infty} dx\,dy\,ds\,ds_0$$

$$\rho_{1x}(x)\rho_{1y}(y)\rho_{1s}(s - s_0)\rho_{2x}(x)\rho_{2y}(y)\rho_{2s}(s + s_0)$$

- **In principle: should know all distributions**
- **Gaussian distributions for analytic calculation**
- **In general: they are a good approximation for most realistic distributions**

Gaussian distribution functions

- $$\rho_{iz}(z) = \frac{1}{\sigma_z\sqrt{2\pi}} \exp\left(-\frac{z^2}{2\sigma_z^2}\right) \qquad i = 1, 2, \quad z = x, y$$

- $$\rho_{is}(s \pm s_0) = \frac{1}{\sigma_s\sqrt{2\pi}} \exp\left(-\frac{(s \pm s_0)^2}{2\sigma_s^2}\right)$$

- **For non-Gaussian profiles not always possible to find analytic form, need a numerical integration**

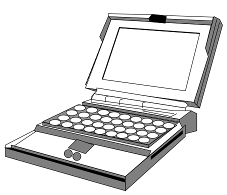
Luminosity for two (equal) beams

- **Simplest case :** $\sigma_{1x} = \sigma_{2x}, \sigma_{1y} = \sigma_{2y}, \sigma_{1s} = \sigma_{2s}$

  or: $\sigma_{1x} \neq \sigma_{2x}, \sigma_{1y} \neq \sigma_{2y}, \sigma_{1s} \approx \sigma_{2s}$

- **Further: no horizontal or vertical dispersion at collision point**

$$\Longrightarrow \quad \mathcal{L} = \frac{N_1 N_2 f B}{4\pi \sigma_x \sigma_y} \left(\mathcal{L} = \frac{N_1 N_2 f B}{2\pi \sqrt{\sigma_{1x}^2 + \sigma_{2x}^2} \sqrt{\sigma_{2y}^2 + \sigma_{2y}^2}} \right)$$



Hints for the integration in the handout

Integration (head-on)

for $\sigma_1 = \sigma_2 \quad \rightarrow \quad \rho_1 \rho_2 \;=\; \rho^2$ **:**

$$\mathcal{L} = \frac{2 \cdot N_1 N_2 f N_b}{(\sqrt{2\pi})^6 \sigma_s^2 \sigma_x^2 \sigma_y^2} \int \int e^{-\frac{x^2}{\sigma_x^2}} e^{-\frac{y^2}{\sigma_y^2}} e^{-\frac{s^2}{\sigma_s^2}} e^{-\frac{s_0^2}{\sigma_s^2}} dx dy ds ds_0$$

integrating over s **and** s_0**, using:**

$$\int_{-\infty}^{+\infty} e^{-at^2} dt \quad = \quad \sqrt{\pi/a}$$

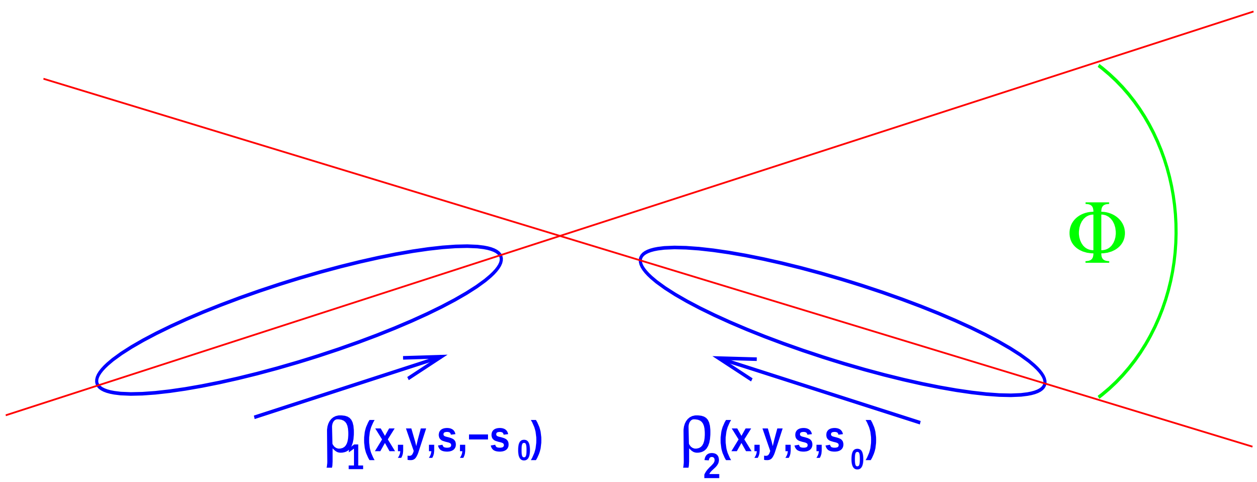
$$\mathcal{L} = \frac{2 \cdot N_1 N_2 f N_b}{8(\sqrt{\pi})^4 \sigma_x^2 \sigma_y^2} \int \int e^{-\frac{x^2}{\sigma_x^2}} e^{-\frac{y^2}{\sigma_y^2}} dx dy$$

finally after integration over x and y: $\Longrightarrow \quad \mathcal{L} = \frac{N_1 N_2 f B}{4\pi \sigma_x \sigma_y}$

Complications

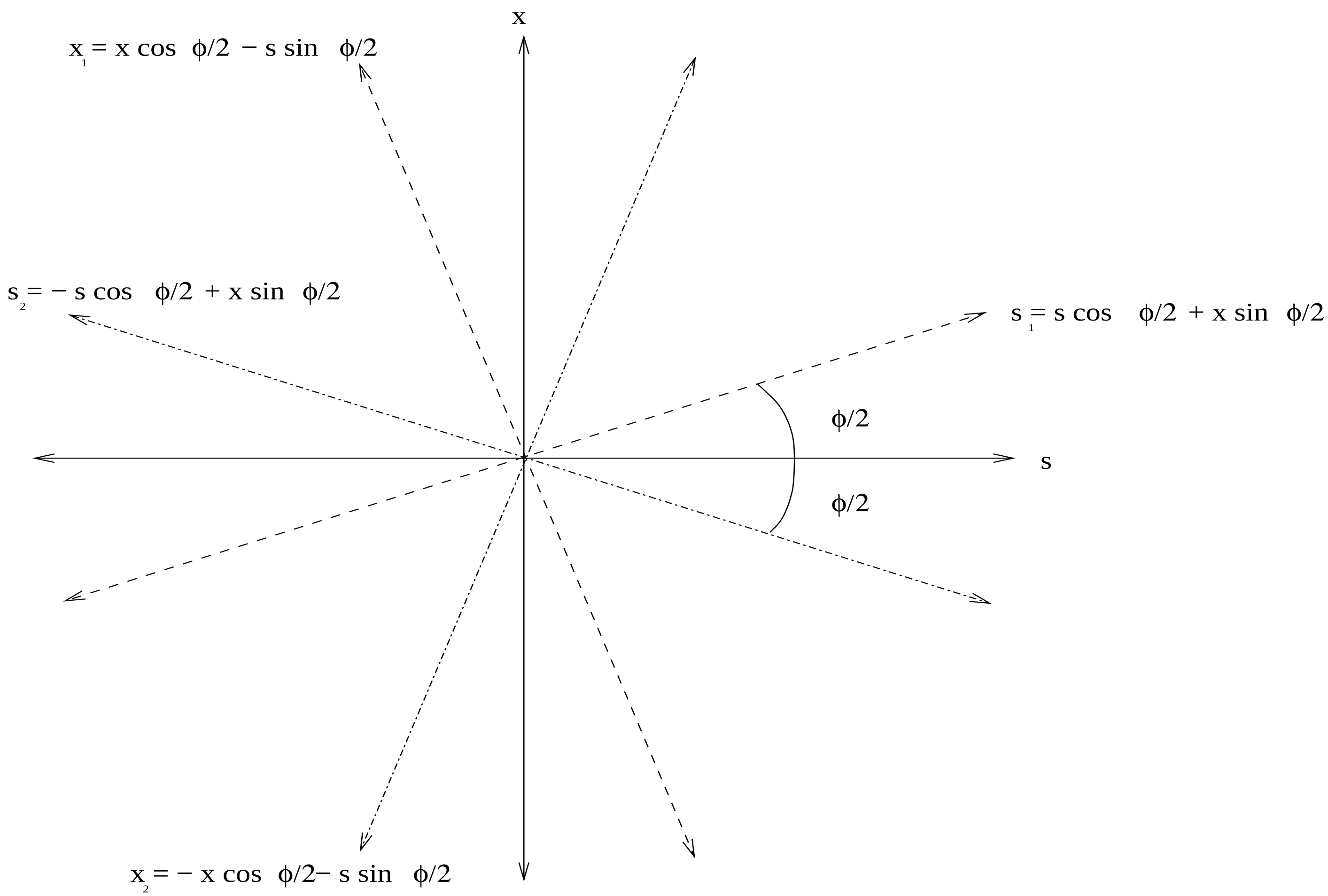
- **Crossing angle**
- **Hour glass effect**
- **Collision offset (wanted or unwanted)**
- **Non-Gaussian profiles**
- **Dispersion at collision point**
- $\delta\beta^{*}/\delta s = \alpha^{*} \neq 0$
- **Strong coupling**
- **etc.**

Collisions at crossing angle



- **Needed to avoid unwanted collisions**
- **For colliders with many bunches: LHC, CESR, KEKB**
- **For colliders with coasting beams**

Collisions angle geometry



Crossing angle

Assume crossing in horizontal plane. Transform to new coordinates:

$$\begin{cases} x_1 = x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, \quad & s_1 = s\cos\frac{\phi}{2} + x\sin\frac{\phi}{2}, \\ x_2 = x\cos\frac{\phi}{2} + s\sin\frac{\phi}{2}, \quad & s_2 = s\cos\frac{\phi}{2} - x\sin\frac{\phi}{2} \end{cases}$$

$$\mathcal{L} = 2\cos^2\frac{\phi}{2} N_1 N_2 f B \int\int\int\int_{-\infty}^{+\infty} dx\,dy\,ds\,ds_0$$
$$\rho_{1x}(x_1)\rho_{1y}(y_1)\rho_{1s}(s_1 - s_0)\rho_{2x}(x_2)\rho_{2y}(y_2)\rho_{2s}(s_2 + s_0)$$

Integration (crossing angle)

use as before:

$$\int_{-\infty}^{+\infty} e^{-at^2} dt \quad = \quad \sqrt{\pi/a}$$

and:

$$\int_{-\infty}^{+\infty} e^{-(at^2+bt+c)} dt \quad = \quad \sqrt{\pi/a} \cdot e^{\frac{b^2-ac}{a}}$$

Further:

- **Since both x and sin($\phi/2$) are small: drop all terms $\sigma_x^k sin^l(\phi/2)$ or $x^k sin^l(\phi/2)$ for all k+l $\geq$ 4**
- **Approximate: sin($\phi/2$) $\approx$ tan($\phi/2$)$\approx$ $\phi/2$**

Crossing angle

- **Crossing Angle** $\Rightarrow \mathcal{L} = \frac{N_1 N_2 f B}{4\pi\sigma_x\sigma_y} S$
- S **is the reduction factor**
- $S = \frac{1}{\sqrt{1 + (\frac{\sigma_x}{\sigma_s}\tan\frac{\phi}{2})^2}} \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}}$
- **For small crossing angles and** $\sigma_s \gg \sigma_{x,y}$
- $\Rightarrow S = \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}} \approx \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}}$

Example LHC:

Φ = **285** μ**rad,** σ_s = **7.5 cm, S = 0.81**

Crossing angle

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Example LHC:

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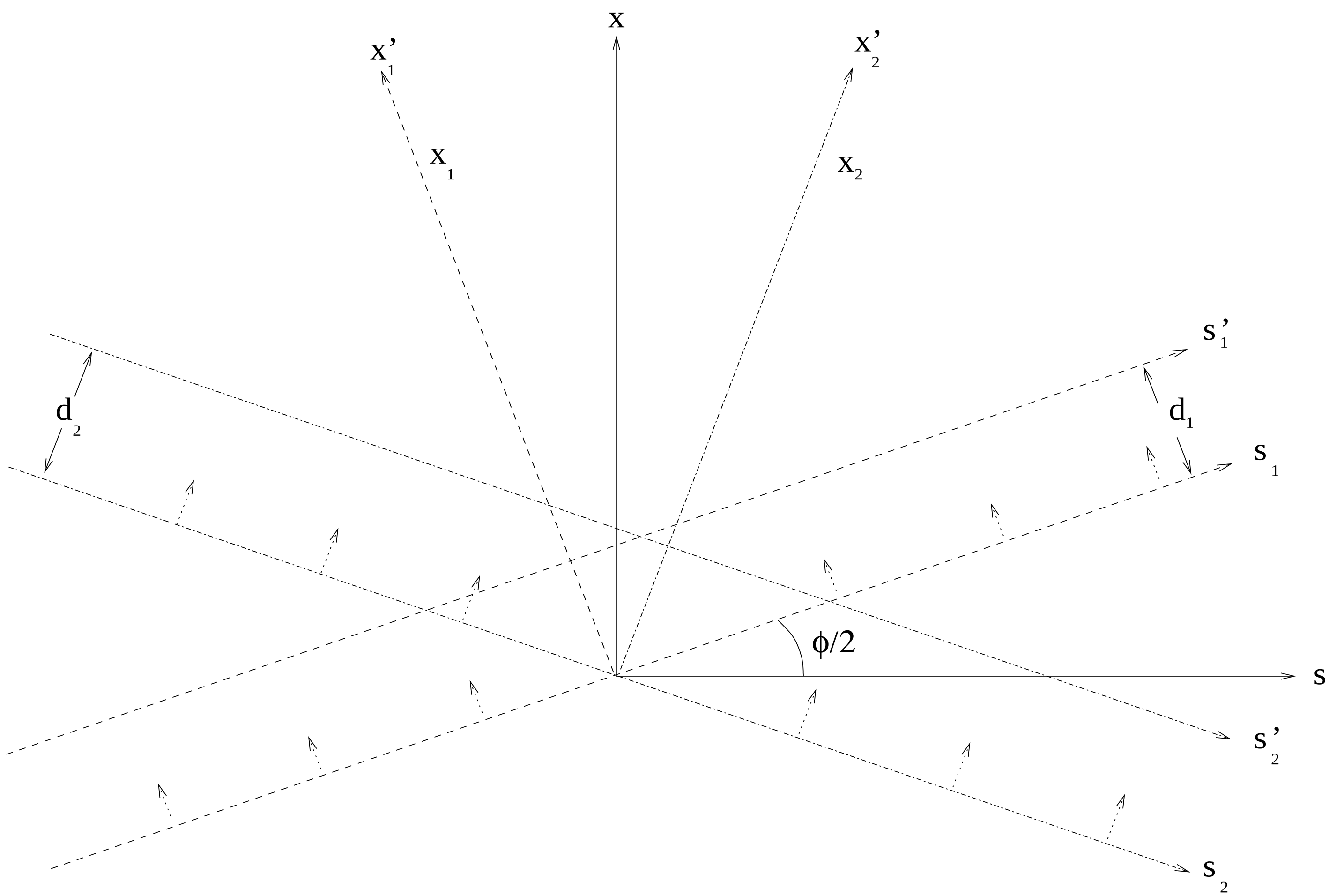
Offset and crossing angle



Offset and crossing angle

- **Offsets in crossing plane: d_1 and d_2**
- **Transformations slightly changed:**

$$\begin{cases} x_1 = d_1 + x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, & s_1 = s\cos\frac{\phi}{2} + x\sin\frac{\phi}{2}, \\ x_2 = d_2 + x\cos\frac{\phi}{2} + s\sin\frac{\phi}{2}, & s_2 = s\cos\frac{\phi}{2} - x\sin\frac{\phi}{2} \end{cases}$$

- **Gives after integration over y and s_0:**

$$\mathcal{L} = \frac{N_1 N_2 f N_b}{8\pi^2 \sigma_s \sigma_x^2 \sigma_y} 2\cos^2\frac{\phi}{2} \int\int e^{-\frac{x^2\cos^2(\phi/2)+s^2\sin^2(\phi/2)}{\sigma_x^2}} e^{-\frac{x^2\sin^2(\phi/2)+s^2\cos^2(\phi/2)}{\sigma_s^2}}$$

$$\times e^{-\frac{d_1^2+d_2^2+2(d_1+d_2)x\cos(\phi/2)-2(d_2-d_1)s\sin(\phi/2)}{2\sigma_x^2}} dxds.$$

Offset and crossing angle

- **After integration over x:**

$$\mathcal{L} = \frac{N_1 N_2 f N_b}{8\pi^{\frac{3}{2}}\sigma_s} 2\cos\frac{\phi}{2}\int_{-\infty}^{+\infty} W \frac{e^{-(As^2+2Bs)}}{\sigma_x \sigma_y} ds$$

- **with:**

$$A = \frac{\sin^2\frac{\phi}{2}}{\sigma_x^2} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2} \quad B = \frac{(d_2 - d_1)\sin(\phi/2)}{2\sigma_x^2}$$

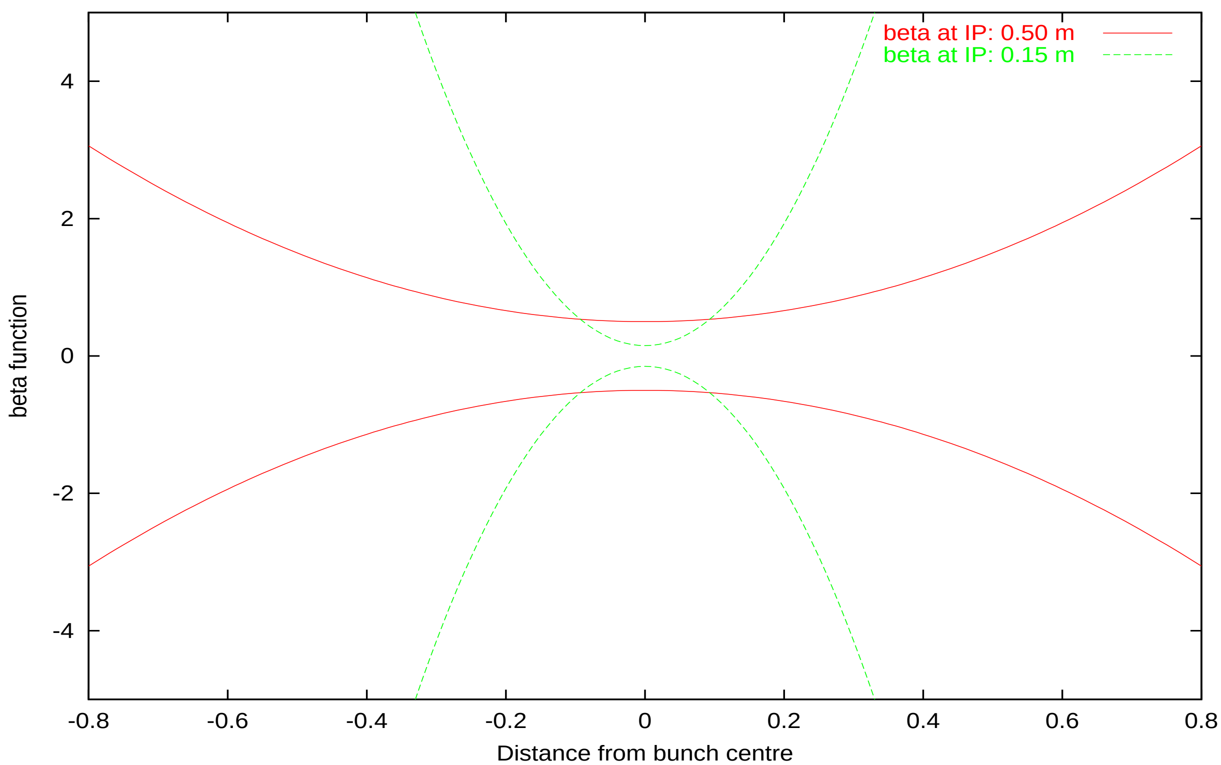
- **and**

$$W = e^{-\frac{1}{4\sigma_x^2}(d_2-d_1)^2}$$

$\Longrightarrow$ **Luminosity with correction factors:**

$$\mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S$$

Hour glass effect



- **β-functions depends on position s**
- **Usually:** $\beta(s) = \beta^{*}(1 + \left(\frac{s}{\beta *}\right)^{2})$
  i.e. $\sigma \implies \sigma(s) \neq$ **const.**
- **Important when** β^{*} **comparable to bunch length** σ_{s}

Hour glass effect

- **Replace σ by σ(s) in formulae**
- $$\mathcal{L} = \left(\frac{N_1 N_2 f B}{8\pi\sigma_x^*\sigma_y^*}\right) \frac{2\cos\frac{\phi}{2}}{\sqrt{\pi}\sigma_s} \int_{-\infty}^{+\infty} \frac{e^{-s^2 A}}{1+(\frac{s}{\beta^*})^2} ds$$
- $$A = \frac{\sin^2\frac{\phi}{2}}{(\sigma_x^*)^2[1+(\frac{s}{\beta^*})^2]} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2}$$
- $\rightarrow$ **Numerical Integration**

Calculations for the LHC

- $N_1 = N_2 = 1.15 \times 10^{11}$ **particles/bunch**
- 2808 **bunches/beam**
- $f = 11.2455$ kHz, $\phi = 285$ μ**rad**
- $\beta_x^* = \beta_y^* = 0.55$ m
- $\sigma_x^* = \sigma_y^* = 16.6$ μm, $\sigma_s = 7.7$ cm

- **Simplest case (Head on collision):**

$$\mathcal{L} = 1.200 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

- **Effect of crossing angle:**

$$\mathcal{L} = 0.973 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

- **Effect of crossing angle & Hourglass:**

$$\mathcal{L} = 0.969 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

If the beams are not Gaussian ??

Exercise:

- **Assume flat distributions:** **(normalized to 1)** $\rho_1 = \rho_2 = \frac{1}{2a}$, **for** $[-a \leq z \leq a]$, $z = x, y$
- **Calculate r.m.s. in x and y:** $< (x,y)^2 >= \int_{-\infty}^{+\infty} (x,y)^2 \cdot \rho(x,y)\mathbf{dxdy}$
- **and** $\mathcal{L} = \int_{-\infty}^{+\infty} \rho_1(x,y)\rho_2(x,y)$ **dxdy**
- **Compute:** $\mathcal{L}\sqrt{< x^2 >< y^2 >}$
- **Repeat for Gaussian distribution and compare**
- **Try other distributions (parabolic, triangular, cosinelike etc.)**

Integrated luminosity

- $$\mathcal{L}_{\rm int}(s) = \int_0^T \int_{-s}^{+s} \mathcal{L}(s', t) ds' dt$$
- **The real figure of merit:**
  $\mathcal{L}_{\rm int}(s) \cdot \sigma_p$ **= number of events**
- **During data taking: continuous recording of** $\mathcal{L}$
- **For studies: assume some life time behaviour. E.g.**
  $\mathcal{L}(t) \longrightarrow \mathcal{L}_0 \exp\left(-\frac{t}{\tau}\right)$
- **Contributions to life time from: intensity decay, emittance growth, bunch length increase etc.**

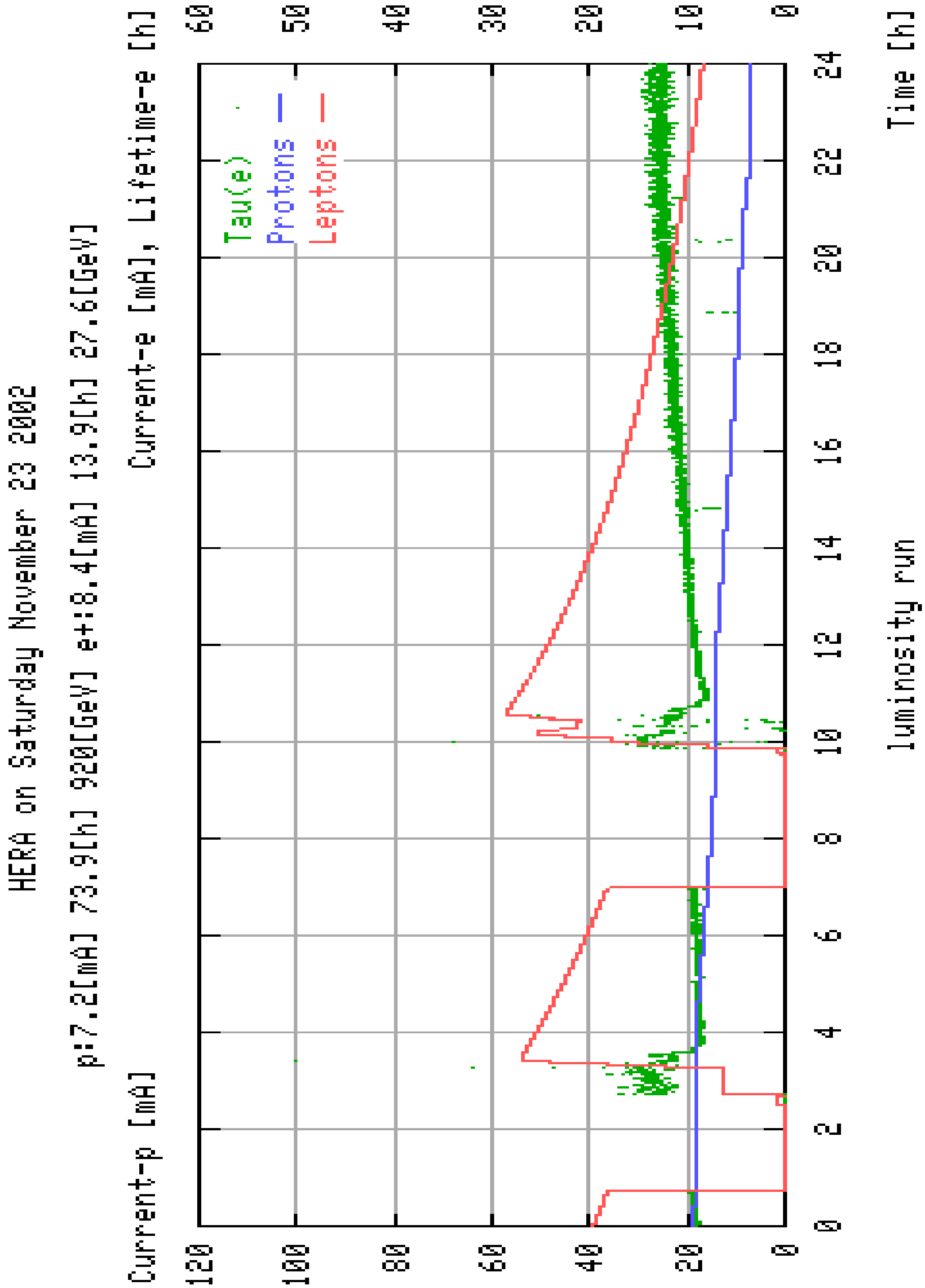
Integrated luminosity

- **Knowledge of preparation time allows optimization of $\mathcal{L}_{\text{int}}$**

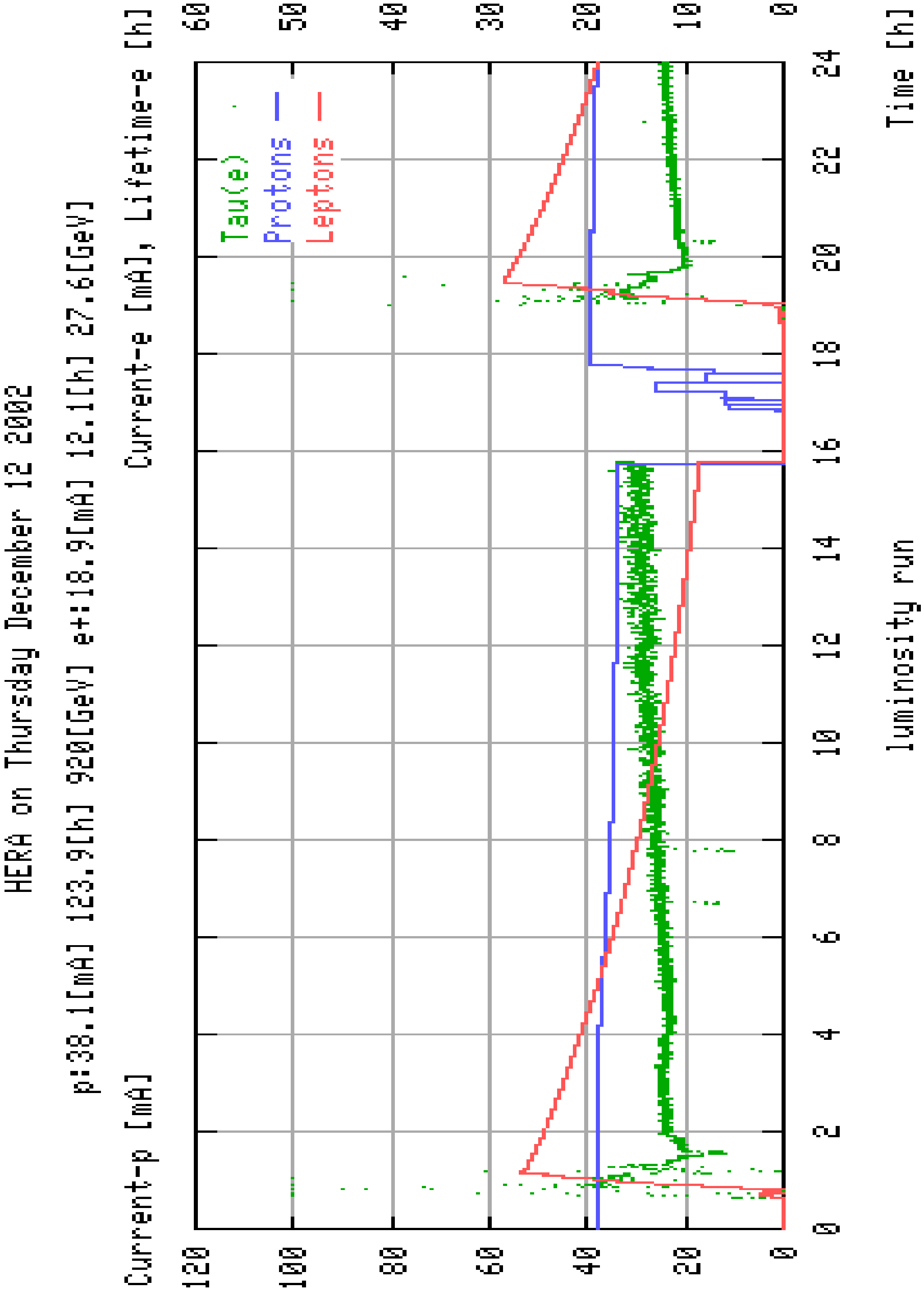
- **Typical run times LEP:**
  $t_r \approx$ 8 - 10 hours
- **For LHC long preparation time t_p expected**

Maximising Integrated luminosity

- **Assume exponential decay of luminosity** $\mathcal{L}(t) = \mathcal{L}_0 \cdot e^{t/\tau}$
- **Average luminosity** $< \mathcal{L} >$
  $< \mathcal{L} > = \frac{\int_0^{t_r} dt \mathcal{L}(t)}{t_r + t_p} = \mathcal{L}_0 \cdot \tau \cdot \frac{1 - e^{-t_r/\tau}}{t_r + t_p}$
- t_r **is a "free" parameter, i.e. can be chosen**
- **(Theoretical) maximum for:**
  $t_r \approx \tau \cdot \ln(1 + \sqrt{2t_p/\tau} + t_p/\tau)$
- **Example LHC (OB):** $t_p \approx$ **10h,** $\tau \approx$ **15h,** $\Rightarrow t_r \approx$ **15h**
- **Exercise: Would you try to improve** τ **or** t_p **???**



HERA on Saturday November 23 2002
p:7.2[mA] 73.9[h] 920[GeV] e+:8.4[mA] 13.9[h] 27.6[GeV]
Current-p [mA]
Current-e [mA], Lifetime-e [h]
Tau(e)
Protons
Leptons
luminosity run
Time [h]


HERA on Thursday December 12 2002
p:38.1[mA] 123.9[h] 920[GeV] e+:18.9[mA] 12.1[h] 27.6[GeV]
Current-p [mA]
Current-e [mA], Lifetime-e [h]
Tau(e)
Protons
Leptons
luminosity run
Time [h]

Luminous region

- **Density distribution of interaction vertices**
- **OR: Which fraction of collisions occur $\pm$ s from the IP ?**
- **Important for experiments !**
- **Depends on σ_x, σ_y, and σ_s**
- **But also on crossing angle ϕ**

$$\mathcal{L}_0 = \int_{-\infty}^{+\infty} \mathcal{L}(s')ds' \longrightarrow \mathcal{L}(s) = \int_{-s}^{+s} \mathcal{L}(s')ds'$$

$$\mathcal{L}(s) = \left(\frac{N_1 N_2 f B}{8\pi\sigma_x^* \sigma_y^*} \right) \frac{2\cos\frac{\phi}{2}}{\sqrt{\pi}\sigma_s} \sqrt{\frac{\pi}{A}} \ \mathrm{erf}\left(\sqrt{A} \ \ s \right)$$

Some results for LHC

- $\sigma_s = 7.7$ cm, $\beta^* = 0.55$ m, $\phi = 285\ \mu$rad:
- 100% **lumi** $\rightarrow$ $s = \pm\ 12$ cm $\longrightarrow$ $s = \pm\ 12$ cm
- 95% **lumi** $\rightarrow$ $s = \pm\ 8$ cm $\longrightarrow$ $s = \pm\ 9$ cm
- 90% **lumi** $\rightarrow$ $s = \pm\ 7$ cm $\longrightarrow$ $s = \pm\ 8$ cm
- 85% **lumi** $\rightarrow$ $s = \pm\ 6$ cm $\longrightarrow$ $s = \pm\ 6.5$ cm
- 80% **lumi** $\rightarrow$ $s = \pm\ 5.5$ cm $\longrightarrow$ $s = \pm\ 6$ cm

Interactions per crossing

- **Luminosity $\propto$ B $N_1 N_2$**
- **In LHC: crossing every 25 ns**
- **Per crossing approximately 20 interactions**
- **May be undesirable (pile up in detector)**
- **$\Longrightarrow$ more bunches B, or smaller N ??**

Luminosity measurement

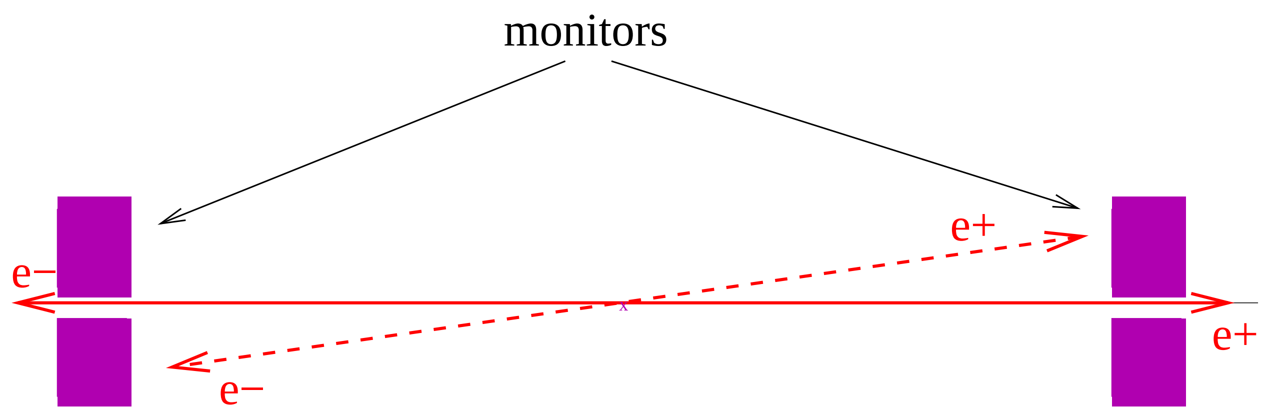
- **Provide a signal proportional to interaction rate**
- **Large dynamic range:**
  10^{27} cm^{-2}s^{-1} to 10^{34} cm^{-2}s^{-1}
- **Very fast, if possible for individual bunches**
- **Used for optimization**
- **For absolute luminosity need calibration method**

Luminosity calibration

(e^+e^-)

- **Use well known and calculable process**
- $e^+e^- \rightarrow e^+e^-$ **elastic scattering (Bhabha scattering)**
- **Have to go to small angles** $(\sigma_{el} \propto \Theta^{-3})$
- **Small rates at high energy** $(\sigma_{el} \propto \frac{1}{E^2})$

Luminosity calibration



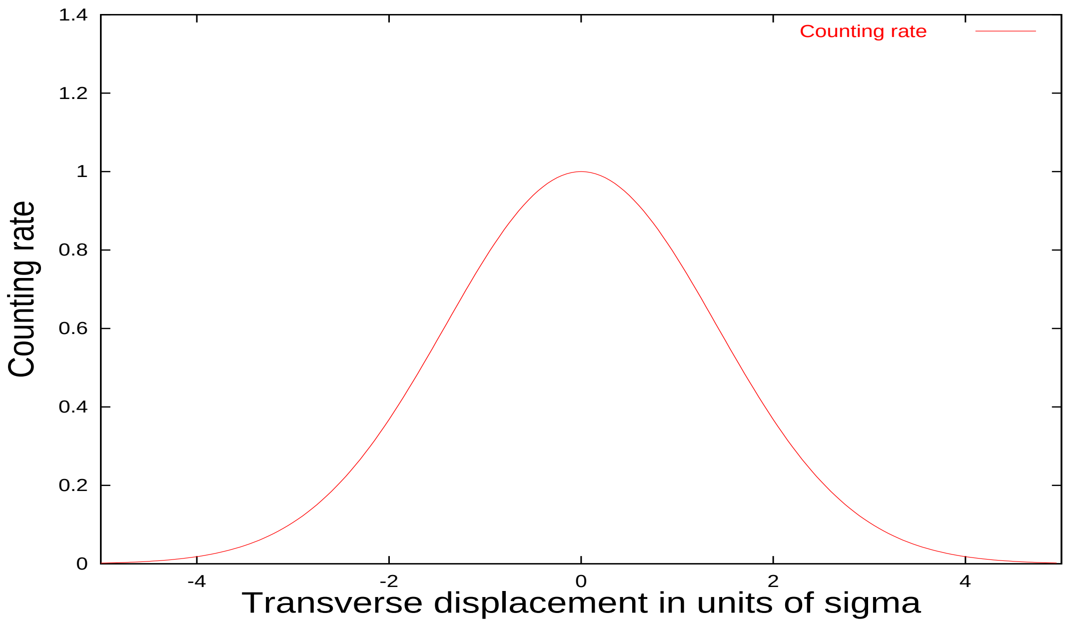
- Measure coincidence at small angles
- Low counting rates (LEP: 25 Hz for $\mathcal{L} = 10^{30}\text{cm}^{-2}\text{s}^{-1}$)
- Background may be problematic

Luminosity calibration

(hadrons, e.g. pp or $p\bar{p}$)

- Have to measure beam current and beam sizes
- Beam size measurement:
  - wire scanner or synchrotron light monitors
  - measurement with beam ... $\rightarrow$ remember luminosity with offset
  - move the two beams against each other in transverse planes (van der Meer scan)
  - already used at Intersecting Storage Rings (ISR)

Luminosity optimization

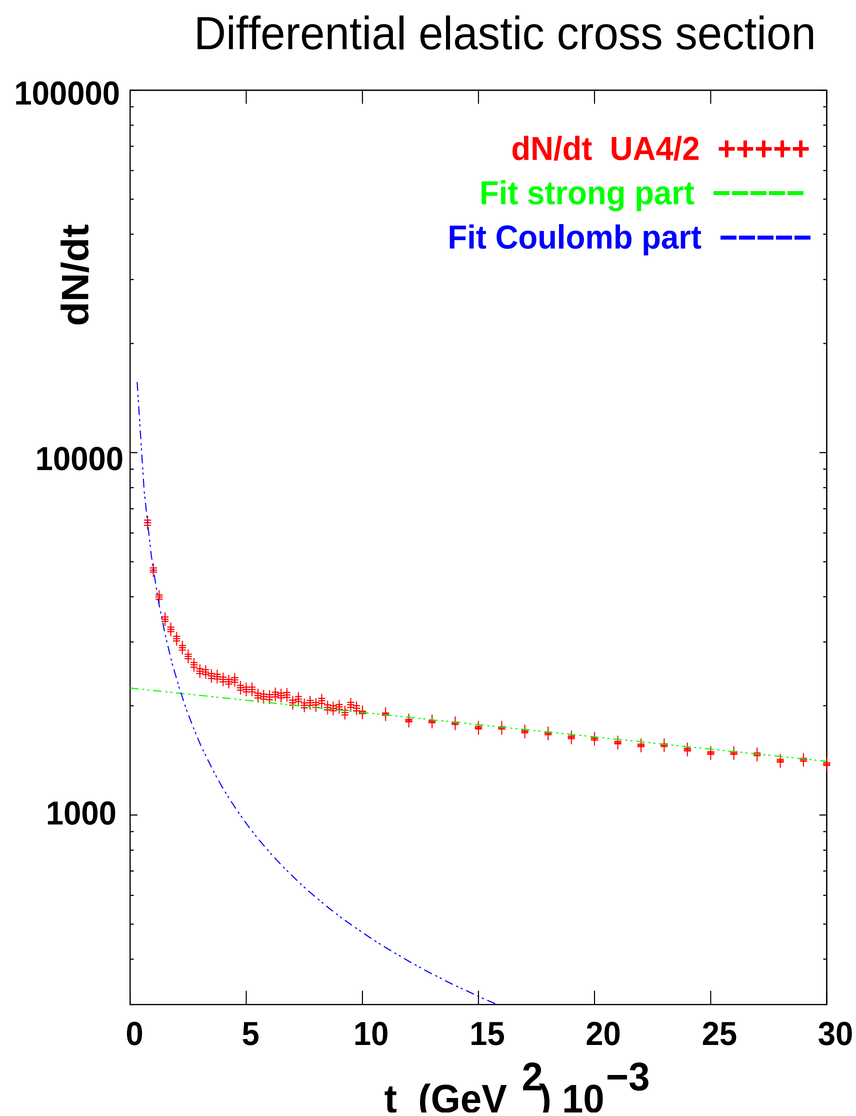


- **Remember:** $W = e^{-\frac{1}{4\sigma^2}(d_2 - d_1)^2}$
- $\rightarrow$ **ratio of luminosity** $\mathcal{L}(\mathrm{d})/\mathcal{L}_0$
- **Determines** σ
- **... and centres the beams !**
- **"beam-beam deflection scans ..."**

$\Longrightarrow$

Absolute value of $\mathcal{L}$ (pp or $p\bar{p}$)

- **By total rate and optical theorem (luminosity independent determination of σ_{tot}):**
  - $\sigma_{tot} \cdot \mathcal{L} = N_{inel} + N_{el}$ **(Total counting rate)**
  - $\lim_{t \to 0} \frac{d\sigma_{el}}{dt} = (1 + \rho^2) \frac{\sigma_{tot}^2}{16\pi} = \frac{1}{\mathcal{L}} \frac{dN_{el}}{dt}|_{t=0}$
- $\mathcal{L} = \frac{(1+\rho^2)}{16\pi} \frac{(N_{inel}+N_{el})^2}{(dN_{el}/dt)_{t=0}}$
- **Luminosity determined from experimental rates**



- Measure dN/dt at small t (0.01 < GeV) and extrapolate to t = 0.0
- Needs special optics to allow measurement at very small t
- Measure total counting rate N_{el} + N_{inel}

  Needs good detector coverage
- Often use slightly modified method, precision 1 − 2 %

Absolute value of $\mathcal{L}$ (pp or $p\bar{p}$)

- **By Coulomb normalization:**
  - **Coulomb amplitude exactly calculable:**
  - $$\lim_{t \to 0} \frac{d\sigma_{el}}{dt} = \frac{1}{\mathcal{L}} \frac{dN_{el}}{dt}\Big|_{t=0} = \pi \mid f_c + f_N \mid^2$$
    $$\simeq \pi \mid \frac{2\alpha_{em}}{-t} + \frac{\sigma_{tot}}{4\pi}(\rho + i) e^{B\frac{t}{2}} \mid^2 \simeq \frac{4\pi\alpha_{em}^2}{t^2} \Big|_{|t| \to 0}$$
  - **Fit gives:** σ_{tot}, ρ, B **and** $\mathcal{L}$
- **Can be done measuring only elastic scattering (No N_{inel} needed !)**
- $\Longrightarrow$ **Roman Pots**

Not treated :

- Coasting beams (e.g. ISR)
- Asymmetric colliders (e.g. PEP, HERA)
- Linear colliders (SLC, TESLA etc.)

How to cook high Luminosity ?

- **Get high intensity**
- **Get small beam sizes (small ϵ and β^*)**
- **Get short bunches**
- **Get many bunches**
- **Get exact head-on collisions**