CERN ACCELERATOR SCHOOL
Intermediate Level Course
Zeuthen 2003

Insertion Devices

Johannes Bahrdt, 25.9.2003

Outline

- Overview
- Insertion Device e-Beam Interaction
- Radiation Properties of Insertion Devices
- Insertion Device Technology

History of Insertion Devices

1947	First discussion of undulator radiation by Ginzburg
1951 / 1953	First production of undulator light in the mm and visible regime by Motz et al.
1976	FEL radiation from a superconducting helical undulator at Stanford: Madey et al.
1979 / 1980	first operation of insertion devices in storage rings (SSRL, LURE, VEPP3)
1980...	first operation of wavelength shifters in storage rings (VEPP3, SRS, VEPP2M)
today	about a dozen of 3rd generation synchrotron radiation light sources; SASE FELs in the visible and UV (80nm) regime
future	SASE-FELs for the energy regime up to 10 keV

Synchrotron Radiation Integrals

$$I_1 = \int \frac{\eta(s)}{\rho} ds$$

$$I_2 = \int \frac{1}{\rho^2} ds$$

$$I_3 = \int \frac{1}{|\rho|^3} ds$$

$$I_{3a} = \int \frac{1}{\rho^3} ds$$

$$I_4 = \int \frac{(1-2n(s))\eta(s)}{\rho^3} ds$$

$$I_5 = \int \frac{H(s)}{|\rho(s)|^3} ds$$

$$H(s) = \frac{1}{\beta}[\eta^2 + <\beta\eta' - 0.5\ \beta'\ \eta>]$$

$$n(s) = \rho^2 \frac{\partial}{\partial x}(1/\rho)$$

Beam Parameter Dependence on SR-Integrals

energy loss per revolution

$$\Delta E = \frac{2}{3} r_e \frac{E^4}{3(mc^2)^3} I_2$$

energy spread

$$(\frac{\sigma_E}{E})^2 = C_q \gamma^2 \frac{I_3}{2I_2 + I_4}$$

emittance

$$\epsilon = C_q \gamma^2 \frac{I_5}{I_2 - I_4}$$

damping times

$$\tau_i = 3T_0/r_0\gamma^3 J_i I_2$$
$$i = x, z, \epsilon$$

damping partition numbers

$$J_x = 1 - I_4/I_2$$
$$J_z = 1$$
$$J_\epsilon = 2 + I_4/I_2$$

polarization time

$$1/\tau_p = \frac{5\sqrt{3}}{8} \frac{\hbar r_e}{m} (\frac{E_0}{mc^2})^5 \frac{I_3}{2\pi R}$$

degree of polarization

$$P_{max} = \frac{8}{5\sqrt{3}} \frac{I_{3a}}{I_3}$$

Fokussing of Insertion Devices

Equation of motions in linear optics

$$\begin{aligned} x''(s) + (1/\rho^2(s) - \kappa(s)) \cdot x(s) &= 0 \\ y''(s) + \kappa(s) \cdot y(s) &= 0 \end{aligned}$$

averaged focussing terms

$$\begin{aligned} < K_x(s) > &= < (1/\rho^2(s) - \kappa(s)) > \\ < K_y(s) > &= < \kappa(s) > \end{aligned}$$

evaluate & along curved trajectory

$$\begin{aligned} \kappa &= \frac{e}{\gamma mc} \frac{\partial B_y}{\partial \xi} \\ &= \frac{e}{\gamma mc} \frac{\partial B_\xi}{\partial y} \\ &= \frac{e}{\gamma mc} \left[\frac{\partial B_y}{\partial x} \cdot \frac{\partial x}{\partial \xi} + \frac{\partial B_y}{\partial s} \cdot \frac{\partial s}{\partial \xi} \right] \\ &\approx \frac{e}{\gamma mc} \left[\frac{\partial B_y}{\partial x} - \frac{\partial B_y}{\partial s} \cdot x' \right] \end{aligned}$$

using Halbach magnetic fields

$$\begin{aligned} B_x &= \frac{k_x}{k_y} B_0 \cdot sinh(k_x x) \cdot sinh(k_y y) \cdot cos(ks) \\ B_y &= B_0 \cdot cosh(k_x x) \cdot cosh(k_y y) \cdot cos(ks) \\ B_s &= -\frac{k}{k_y} B_0 \cdot cosh(k_x x) \cdot sinh(k_y y) \cdot sin(ks) \end{aligned}$$

with $k_x^2 + k_y^2 = k^2$ (Maxwell)

focussing strength:

$$\begin{aligned} < K_x > &= \frac{k_x^2}{2\rho_0^2 k^2} \\ < K_y > &= \frac{k_y^2}{2\rho_0^2 k^2} \end{aligned}$$

Tracking of Particles in Undulator Fields

1: Expand x(s), y(s) and B with respect to initial coordinates and 1/❒ (=x3)

$$
\begin{aligned}
x(s) &= x_i + s \cdot x'_i + \sum_{k,l,m} a_{klm}(x_i, y_i, s) \cdot x_i'^k \cdot y_i'^l \cdot x_3^m \\
y(s) &= y_i + s \cdot y'_i + \sum_{k,l,m} b_{klm}(x_i, y_i, s) \cdot x_i'^k \cdot y_i'^l \cdot x_3^m
\end{aligned}
$$

$$
B(x(s), y(s), s) = B(x_i, y_i, s) + \sum_{kl} \frac{1}{k!l!} \frac{\partial^2 B(x_i, y_i, s)}{\partial^k x \partial^l y} \cdot \Delta x^k \cdot \Delta y^l
$$

2: Insert x(s), y(s) and B into equations of motion and determine a_klm and b_klm

3: change to canonical coordinates and modify transformation

$$
\begin{aligned}
q_x &= x \\
p_x &= A^x/(B\rho) + x'/\sqrt{1 + x'^2 + y'^2} \\
q_y &= y \\
p_y &= A^y/(B\rho) + y'/\sqrt{1 + x'^2 + y'^2}
\end{aligned}
$$

old

$$
(qx_i, px'_i, qy_i, py'_i) \Longrightarrow (qx_f, px'_f, qy_f, py'_f)
$$

new

$$
(qx_i, px'_f, qy_i, py'_f) \Longrightarrow (qx_f, px'_i, qy_f, py'_i)
$$

4: set up generating function and get canonical transformation from derivatives

$$
F = F_{00} + F_{10} \cdot px_f + F_{01} \cdot py_f + F_{20} \cdot px_f^2 + F_{11} \cdot px_f \cdot py_f + F_{02} \cdot py_f^2
$$

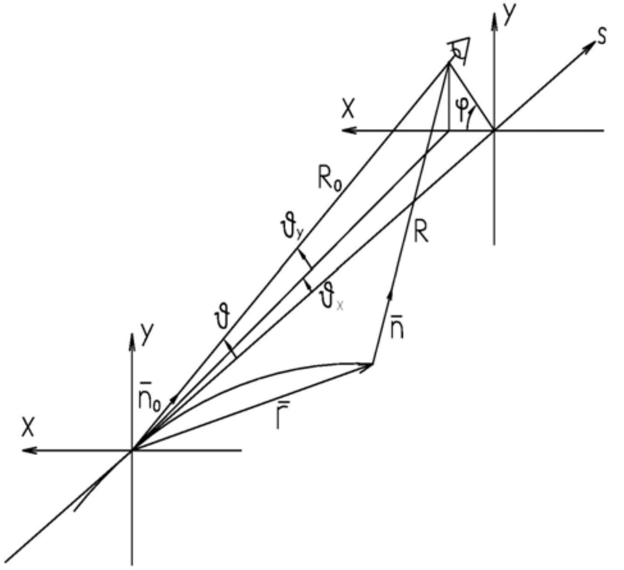
$$
\begin{aligned}
qx_f &= \partial F / \partial px_f \\
px_i &= \partial F / \partial qx_i
\end{aligned}
$$

Radiation Emitted by Accelerated Charged Particles

acceleration fields as derived from
the Lienard Wiechert potentials:

$$E(t) = \frac{e}{4\pi\epsilon_0 c} \cdot \frac{1}{R} \cdot \left[\frac{\vec{n} \times [(\vec{n} - \vec{\beta}) \times \dot{\vec{\beta}}]}{(1 - \vec{\beta} \cdot \vec{n})^3}\right]_{ret}$$

retarded time $t' = t - R(t')/c$



$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{e^2}{16\pi^3\epsilon_0 c}\left|\int_{-\infty}^{\infty}\left[\frac{\vec{n} \times [(\vec{n} - \vec{\beta}) \times \dot{\vec{\beta}}]}{(1 - \vec{\beta} \cdot \vec{n})^3}\right]_{ret} e^{i\omega t} dt\right|^2$$

far field approximation:

$$R(t') \approx R_0(t') - \vec{n}_0 \cdot \vec{r}(t')$$
$$\vec{n} = \vec{n_0}$$

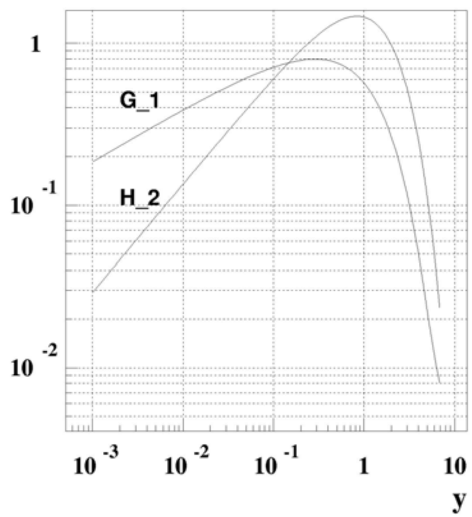
$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{e^2}{16\pi^3\epsilon_0 c}\left|\int_{-\infty}^{\infty}\left[\frac{\vec{n} \times [(\vec{n} - \vec{\beta}) \times \dot{\vec{\beta}}]}{(1 - \vec{\beta} \cdot \vec{n})^2}\right] e^{i\omega(t - \vec{n}\vec{r})} dt\right|^2$$

Bending Magnets

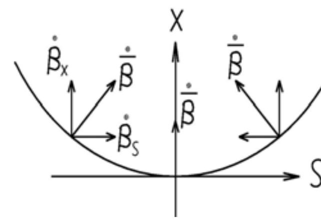
$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{3e^2}{16\pi^3 c\epsilon_0} y^2\gamma^2(1+X^2)^2 \left| K_{2/3}(\xi), i\sqrt{\frac{X^2}{1+X^2}} K_{1/3}(\xi) \right|^2$$

with

$$\begin{aligned} \xi &= \frac{y}{2}(1+(\gamma\theta_y)^2)^{3/2} \\ y &= \omega/\omega_c \\ X &= \gamma\theta_y \\ \omega_c &= (3\gamma^3 c)/(2\rho) \end{aligned}$$



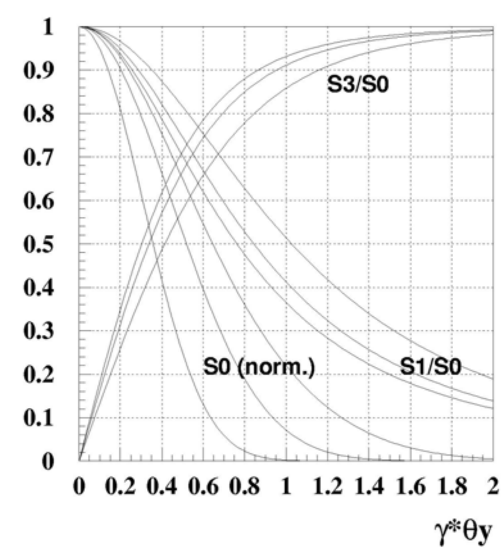
Polarization



$$\begin{aligned} \Delta E_x &\sim -\dot{\beta}_x \cdot (n_y^2 + n_s^2) + \dot{\beta}_s \cdot n_x n_s \\ & \quad (-\beta_x\dot{\beta}_s + \beta_s\dot{\beta}_x) \cdot n_s \\ \Delta E_y &\sim \dot{\beta}_x \cdot n_x n_y + \dot{\beta}_s \cdot n_y n_s \end{aligned}$$

on axis flux density

$$\frac{\partial^2 \tilde{F}}{\partial(\Delta\omega/\omega)\partial\Omega} = 1.327 \cdot 10^{13} \cdot E^2(GeV) \cdot I(A) \cdot H_2$$



integration over vertical angle

$$\frac{\partial^2 \tilde{F}}{\partial(\Delta\omega/\omega)\partial\theta_x} = 2.457 \cdot 10^{13} E(GeV) \cdot I(A) \cdot G_1(y)$$

Undulator Radiation

resonance condition

$$\lambda = \frac{\lambda_0}{2\gamma^2}(1 + K^2/2 + \gamma^2\theta^2)$$

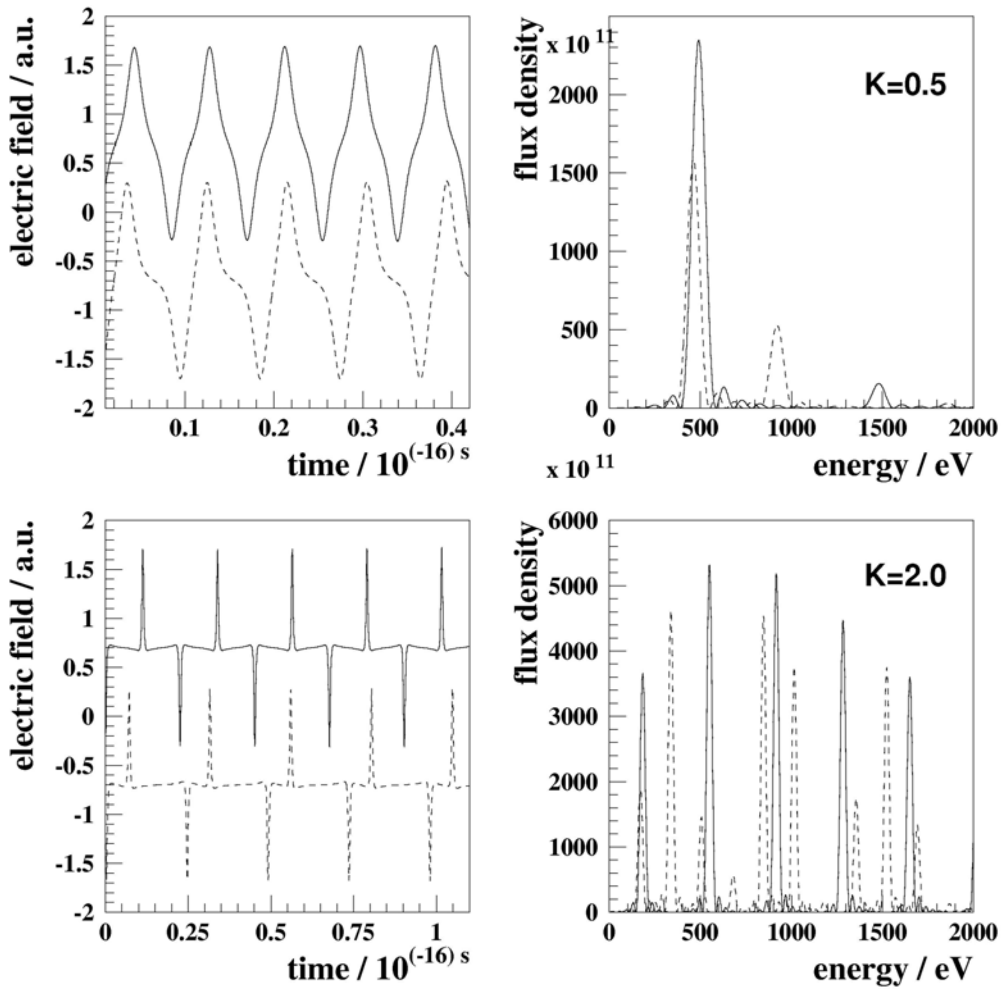
$$K = 93.4 \cdot \lambda_0 \cdot B_0$$

figure 8 motion in moving frame produces higher harmonics

$$x(t) = \frac{Kc}{\gamma\omega_u} sin(\omega_u t)$$

$$s(t) = \bar{\beta}ct - \frac{K^2 c}{8\gamma^2\omega_u} sin(2\omega_u t)$$

electric field at observer   spectrum

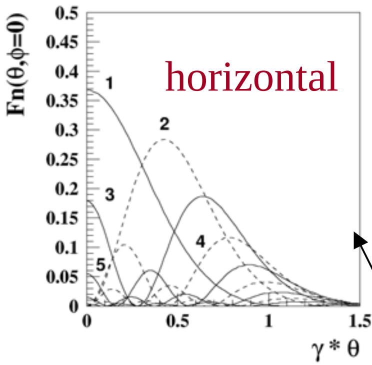


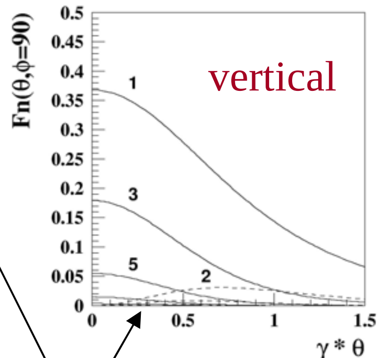
Analytical Approach for Undulator Radiation

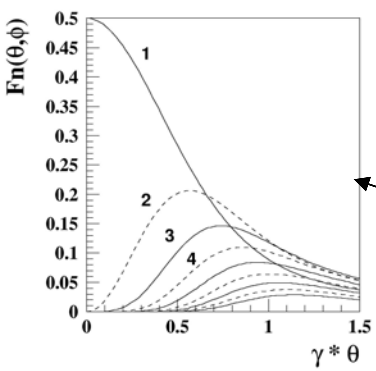
$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{e^2\gamma^2N^2}{4\pi\epsilon_0 c}\cdot F_n(K_x, K_y, \gamma\theta, \phi)\cdot\frac{sin^2(N\pi\frac{\Delta\omega}{\omega_1(\theta)})}{N^2 sin^2(\pi\frac{\Delta\omega}{\omega_1(\theta)})}$$

Fn represents an infinite sum over BESSEL functions.
The last term is called the line shape function and describes the interference effects.

Fn



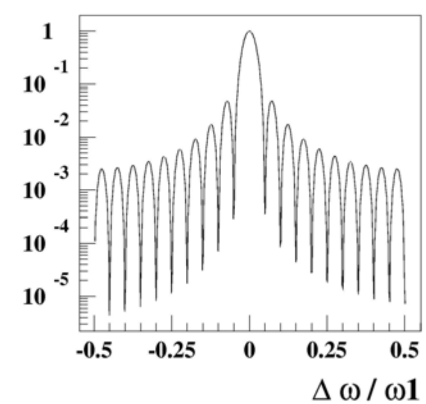


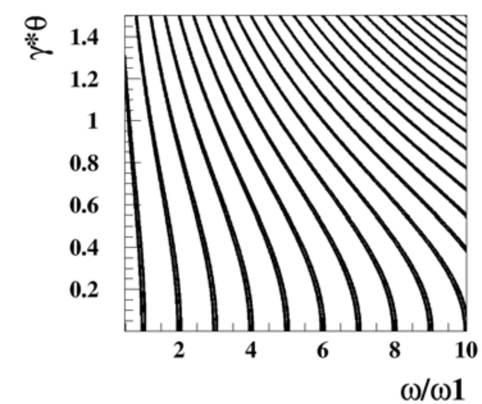


planar device
Ky=1

helical device
Kx=Ky=1

line shape function





The angular divergence and the spectral width can be derived from the line shape function

divergence $\sigma_{r'} = \sqrt{\lambda/2L}$

spectral width $\frac{\Delta\omega}{\omega_n} = \frac{1}{nN}$

Useful Equations in Practical Units

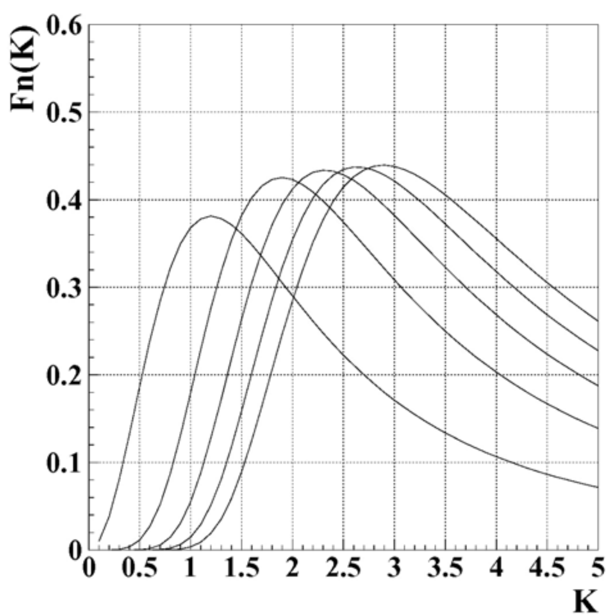
on axis flux density

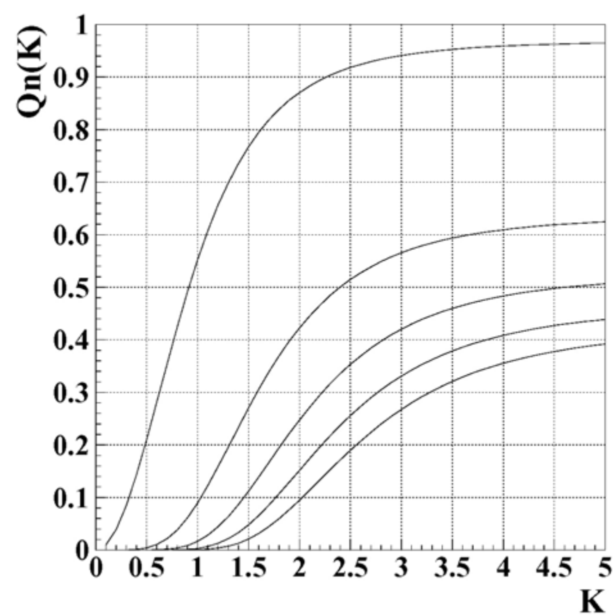
$$\frac{\partial^2 \tilde{F}}{\partial(\Delta\omega/\omega)\partial\Omega} = 1.744 \cdot 10^{14} \cdot N^2 \cdot E^2(GeV) \cdot I(A) \cdot F_n(K)$$

flux over the central cone

$$\frac{\partial \tilde{F}}{\partial(\Delta\omega/\omega)} = 1.431 \cdot 10^{14} \cdot N \cdot Q_n \cdot I(A)$$

$$Q_n = (1 + K^2/2) \cdot F_n/n$$

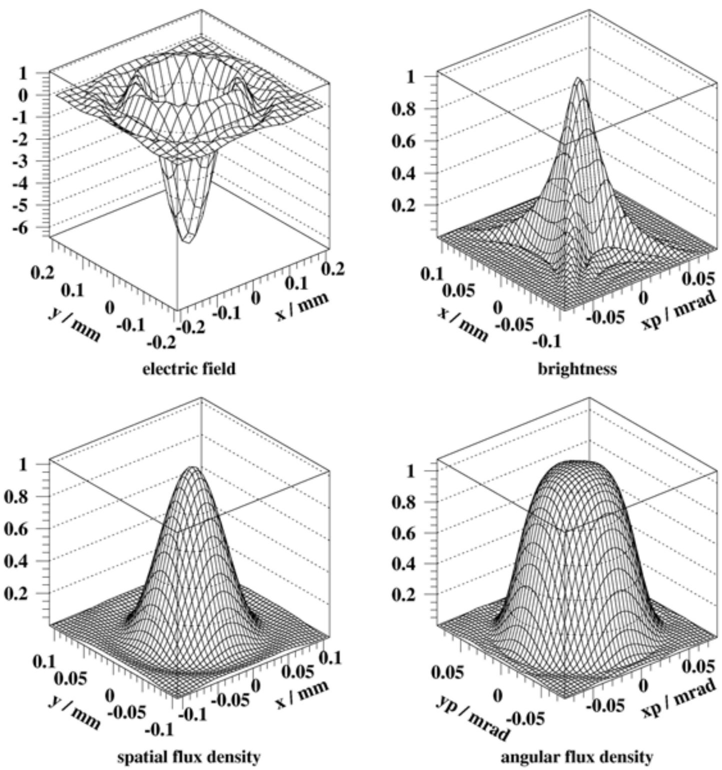




Brightness (Wigner, K.-J. Kim)

$$
\begin{aligned}
B_0(\vec{x},\vec{\Phi}) &= c\cdot\int d^2\xi\cdot A(\vec{x},\vec{\xi})\cdot exp(i\cdot\frac{2\pi}{\lambda}\cdot\vec{\Phi}\cdot\vec{\xi}),\\
A(\vec{x},\vec{\xi}) &= E_y^*(\vec{x}+\vec{\xi}/2)\cdot E_y(\vec{x}-\vec{\xi}/2)+E_z^*(\vec{x}+\vec{\xi}/2)\cdot E_z(\vec{x}-\vec{\xi}/2).
\end{aligned}
$$

The brightness is not positive definite.
Physical quantities are the angular or spatial flux density, which are derived via integration of the brightness in space or solid angle.
The electron beam emittance can be convoluted with the 4D-brightness.



Assuming a angular and spatial Gaussian distribution of the photon beam the brightness can be evaluated from:

$$
\frac{\partial^3 \tilde{F}_e}{\partial(\Delta\omega/\omega)\partial\vec{x}\partial\Omega} = \frac{\frac{\partial^2\tilde{F}}{\partial(\Delta\omega/\omega)\partial\Omega}\sigma_{r'}^2}{\sqrt{\sigma_r^2+\sigma_x^2}\sqrt{\sigma_r^2+\sigma_y^2}\sqrt{\sigma_{r'}^2+\sigma_{x'}^2}\sqrt{\sigma_{r'}^2+\sigma_{y'}^2}}
$$

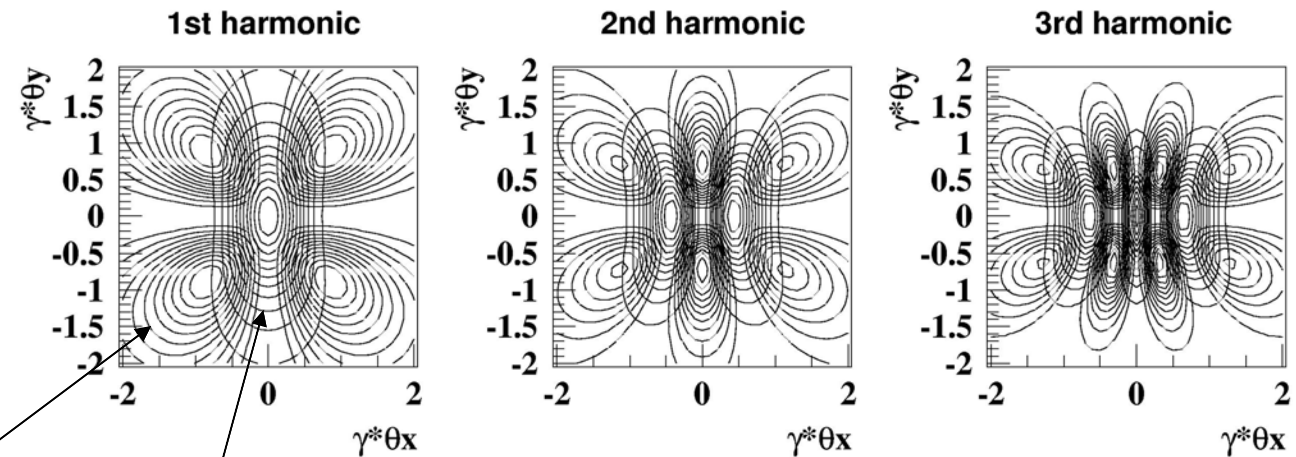
The beam size can be approximated with:

$$
\sigma_r = \frac{1}{\pi\sqrt{2}}\sqrt{\lambda L}
$$

Polarization

$$F_n(K_x, K_y, \gamma\theta, \gamma\phi) \;=\; \frac{n^2}{(K_x^2/2 + K_y^2/2 + (\gamma\theta^2))^2}\,|A_x, A_y|^2 = \;a \cdot (A_x^2 + A_y^2)$$

Planar Undulator, K=1



$a \cdot A_y^2$ $\qquad$ $a \cdot A_x^2$

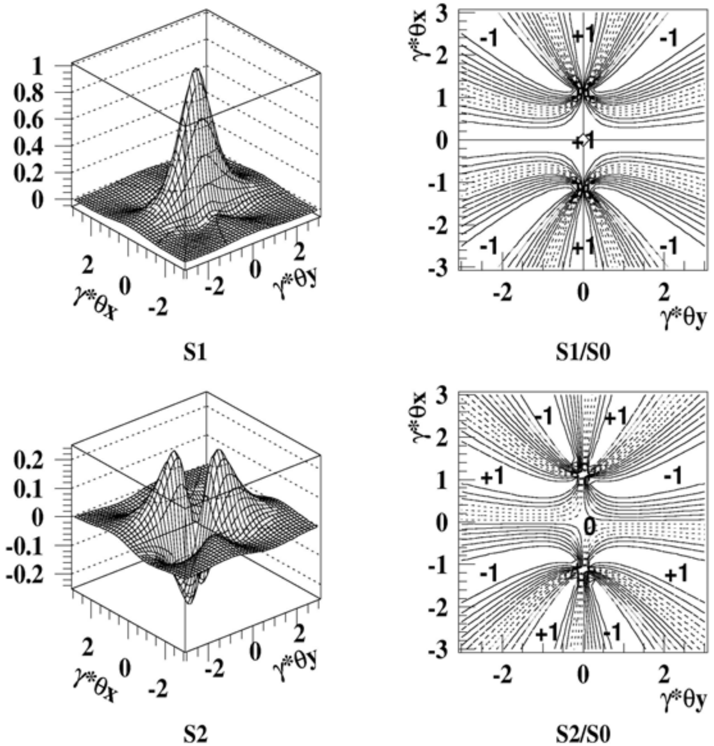
Definition of Stokes Parameters

$$\begin{aligned} S_0 &= E_x^2 + E_y^2 \\ S_1 &= E_x^2 - E_y^2 \\ S_2 &= 2 \cdot E_x E_y cos(\delta) \\ S_3 &= 2 \cdot E_x E_y sin(\delta) \end{aligned}$$

$$S_1^2 + S_2^2 + S_3^2 \;\leq\; S_0^2$$

Polarization Properties

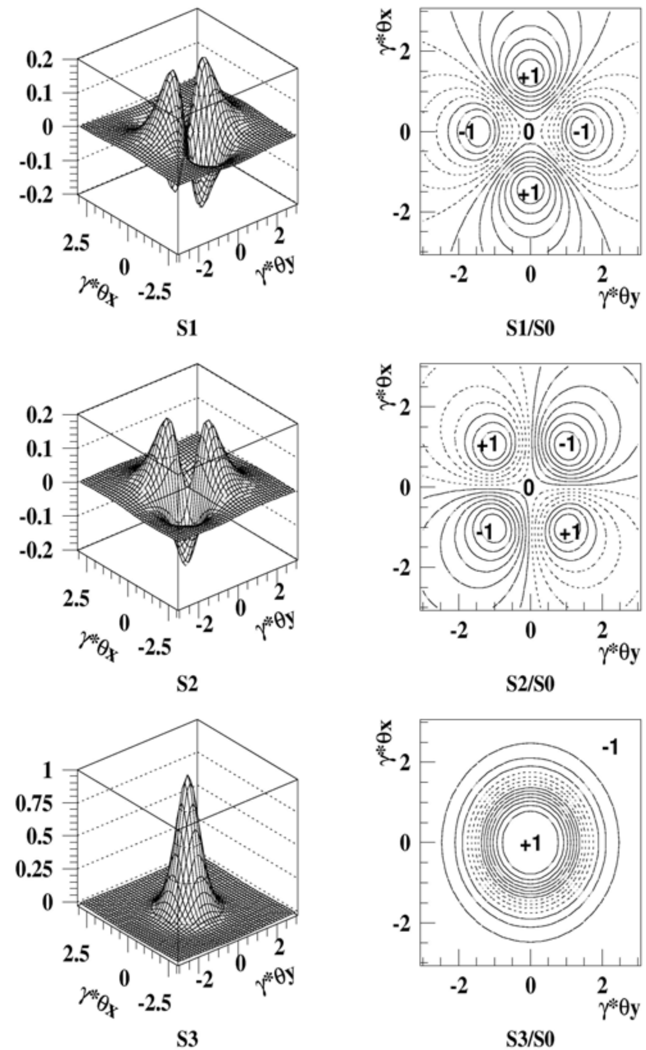
Planar Devices



rel. polarized flux

degree of polarization

Helical Devices

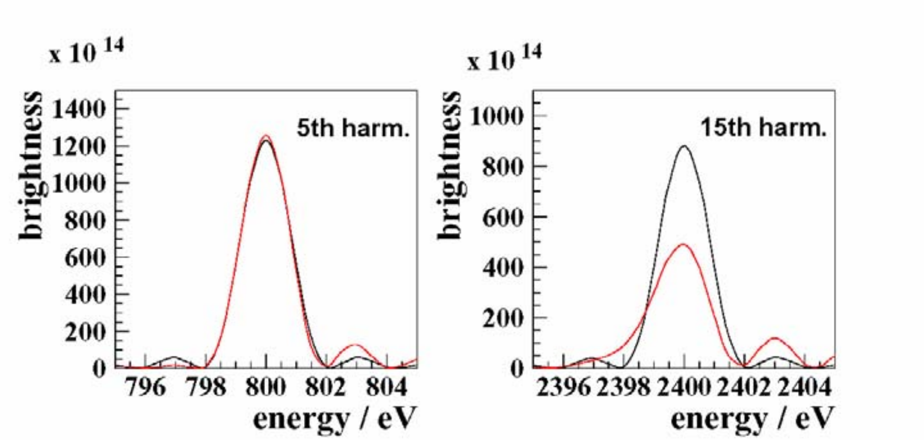


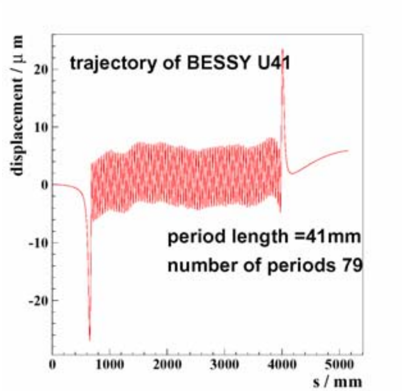
rel. polarized flux

degree of pol.

Sources of Brightness Degradation

Undulator errors





Phase error U41:
σ_ϕ=2.6°

reduction of on axis
flux density (Walker):

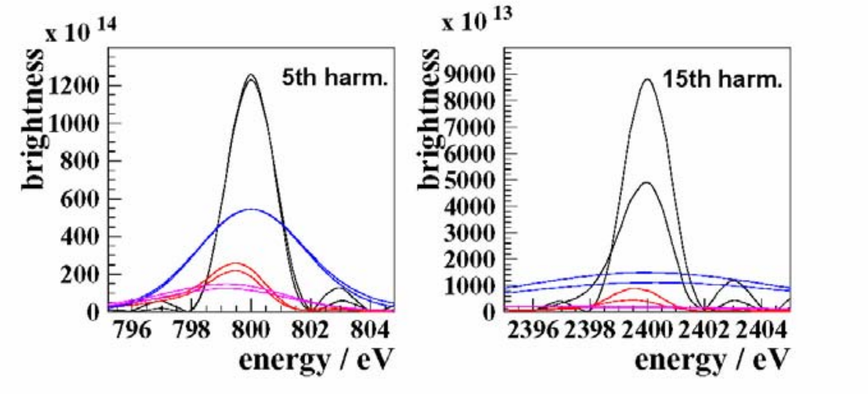
$$R = \frac{1 - e^{-\sigma_\phi^2 N^2}}{N^2}$$

$$\Delta\Phi = \frac{2\pi}{\beta\lambda(B\rho)^2} \cdot \int_0^z \left[\int_0^{z'} B_y^{fit} dz'' \cdot \int_0^{z'} B_y^{res} dz'' \right] dz' + \frac{1}{\beta\lambda(B\rho)^2} \cdot 0.5 \cdot \int_0^z \left[\int_0^{z'} B_y^{res} dz'' \cdot \int_0^{z'} B_y^{res} dz'' \right] dz'$$

Beam parameters

beam emittance: 6.e-9π m rad
β_x = 0.94 m
β_y = 2.1 m

energy spread: 1.e-3



black: without emittance, energy spread
red: emittance included
blue: energy spread included
magenta: emittance and energy spread incl.

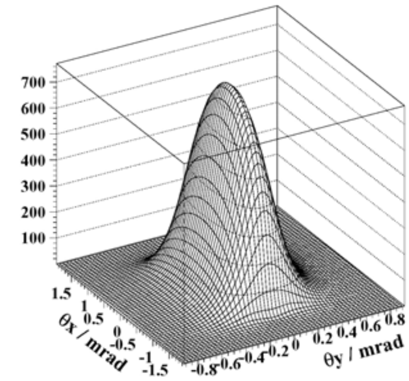
Angular Flux Density of Insertion Devices

$$\frac{\partial P}{\partial \Omega}(W/mrad^2) = 0.01344 \cdot E(GeV)^2 \cdot I(A) \cdot N \cdot \int_{-\lambda_0/2}^{\lambda_0/2} \left[\frac{v_x'^2 + v_y'^2}{D^3} - \frac{((v_x^2)' + (v_y^2)')^2}{D^5} \right] ds$$

$$D = 1 + v_x^2 + v_y^2$$

$$v_{x/y} = \gamma(\beta_{x/y} - \theta_{x/y})$$

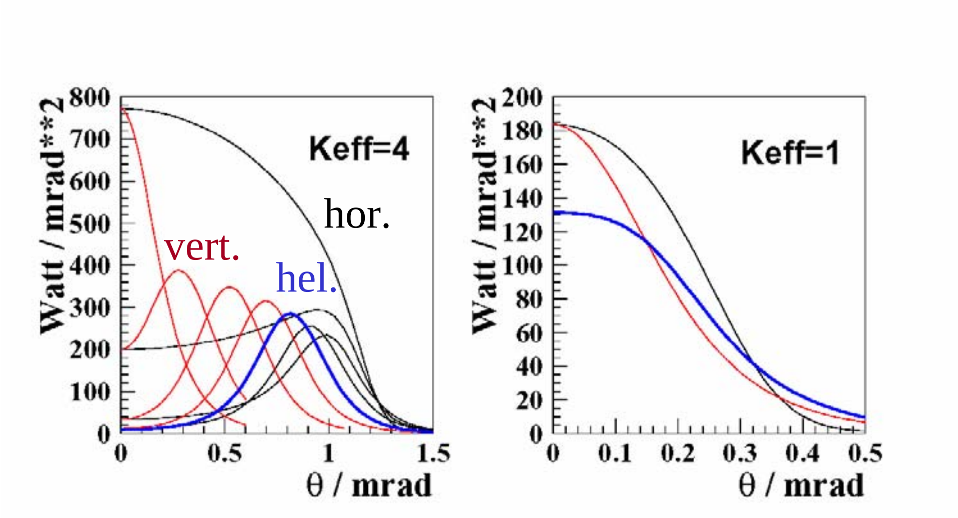
planar device, K=4



energy=1.7GeV, current=0.1A, N=100, ●=50mm

Devices with low on axis power:

angular flux density
Kx/Ky=0, 0.25, 0.5, 0.75, 1.0



helical device, Keff=4

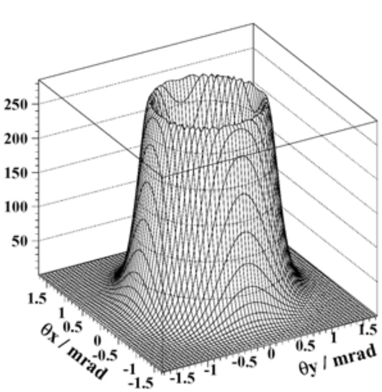
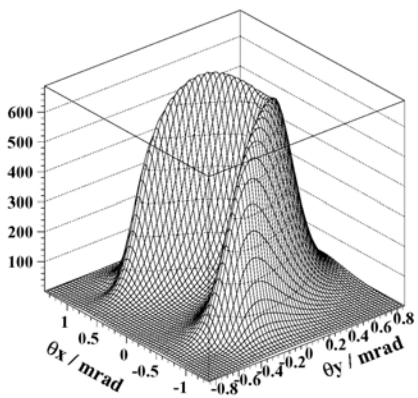
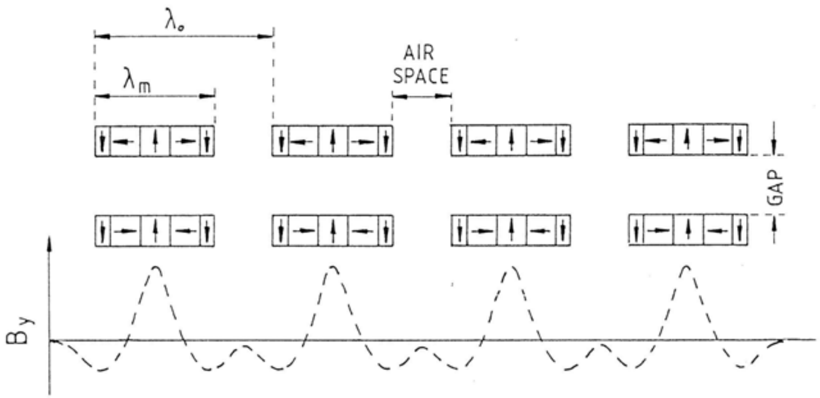
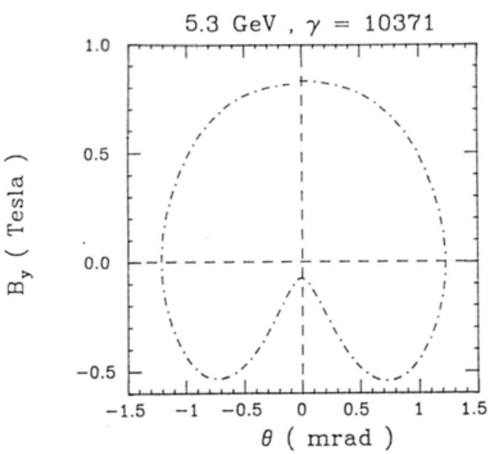


figure-8 undulator



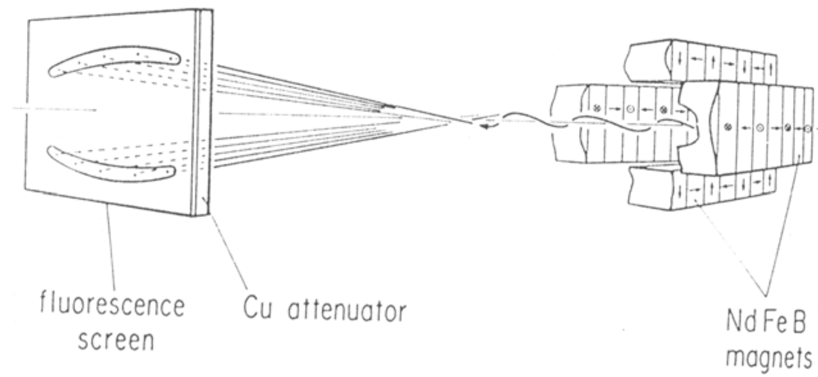
Asymmetric Wiggler



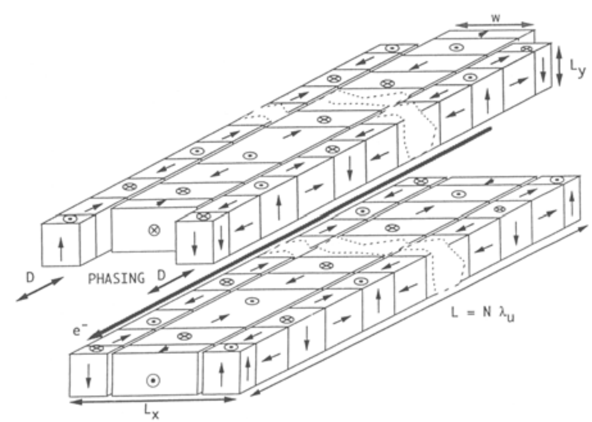


J. Pflüger, G. Heintze, Nucl. Instr. and Meth. 289 (1990) 300-306
J. Goulon et al. Nucl. Instr. and. Meth. 254 (1987) 192-201

Elliptical Wiggler



S. Yamamoto et al., Phys. Rev. Lett., 62 (1989) 2672-2675



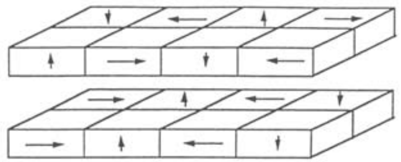
X. M. Marechal et al., Rev. Sci. Instr. 66 (1995) 1937-1939

Elliptical / Helical Undulators

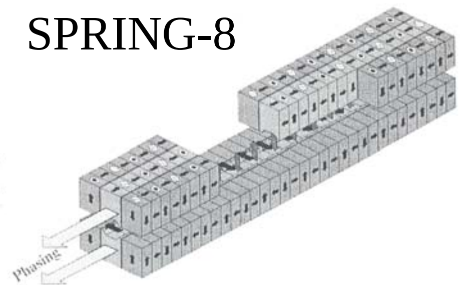
Permanent magnet devices

Various types of planar helical devices

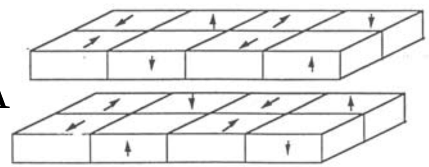
ESRF



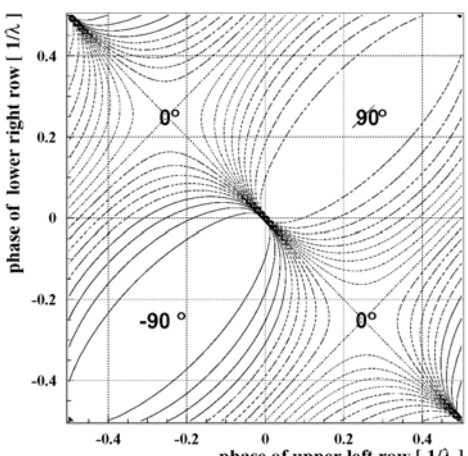
SPRING-8

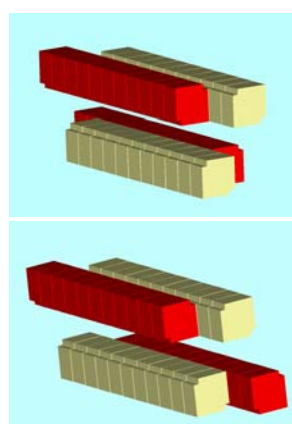


ELETTRA



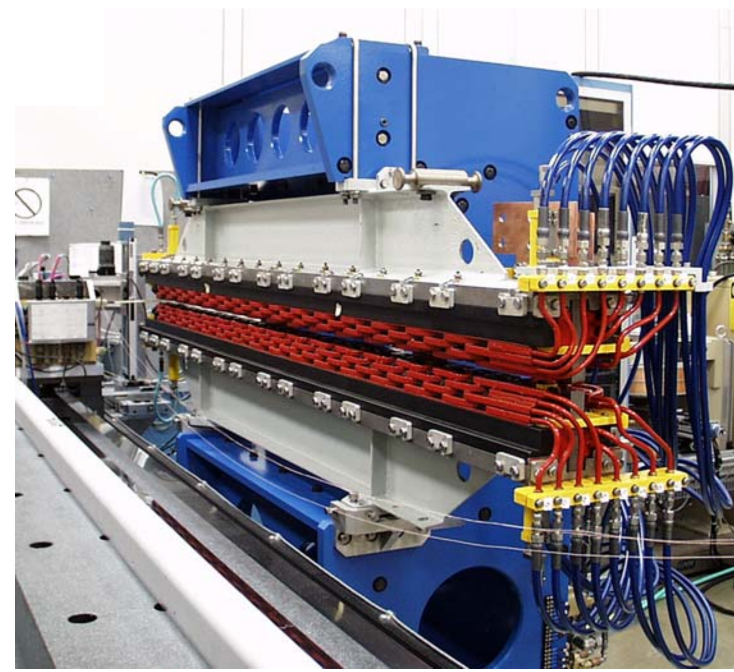
APPLE II provides highest fields





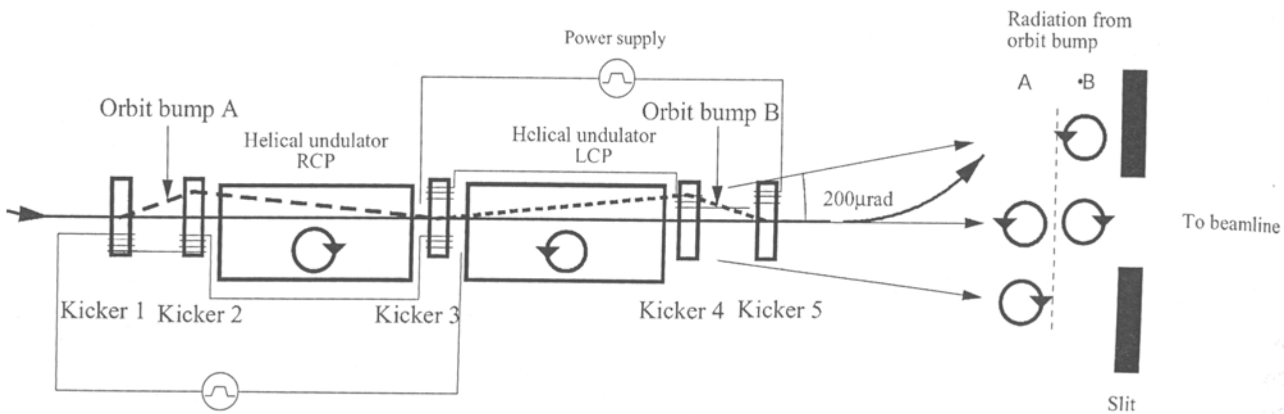
electromagnetic devices

+ fast helicity switching
+ mechanically simple
- limited to long periods
- weak fields

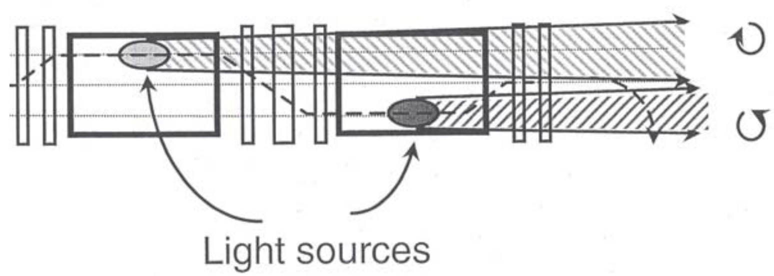


Advanced Photon Source

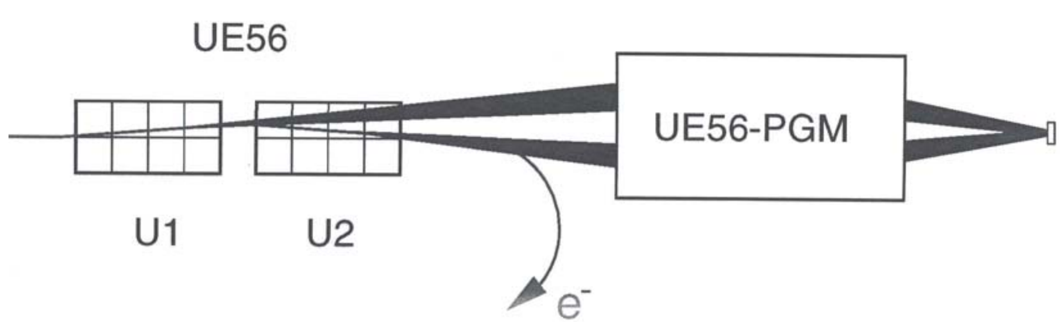
Fast Helicity Switching with Double Undulators



SPRING-8
dynamic electron orbit bump, angular separation

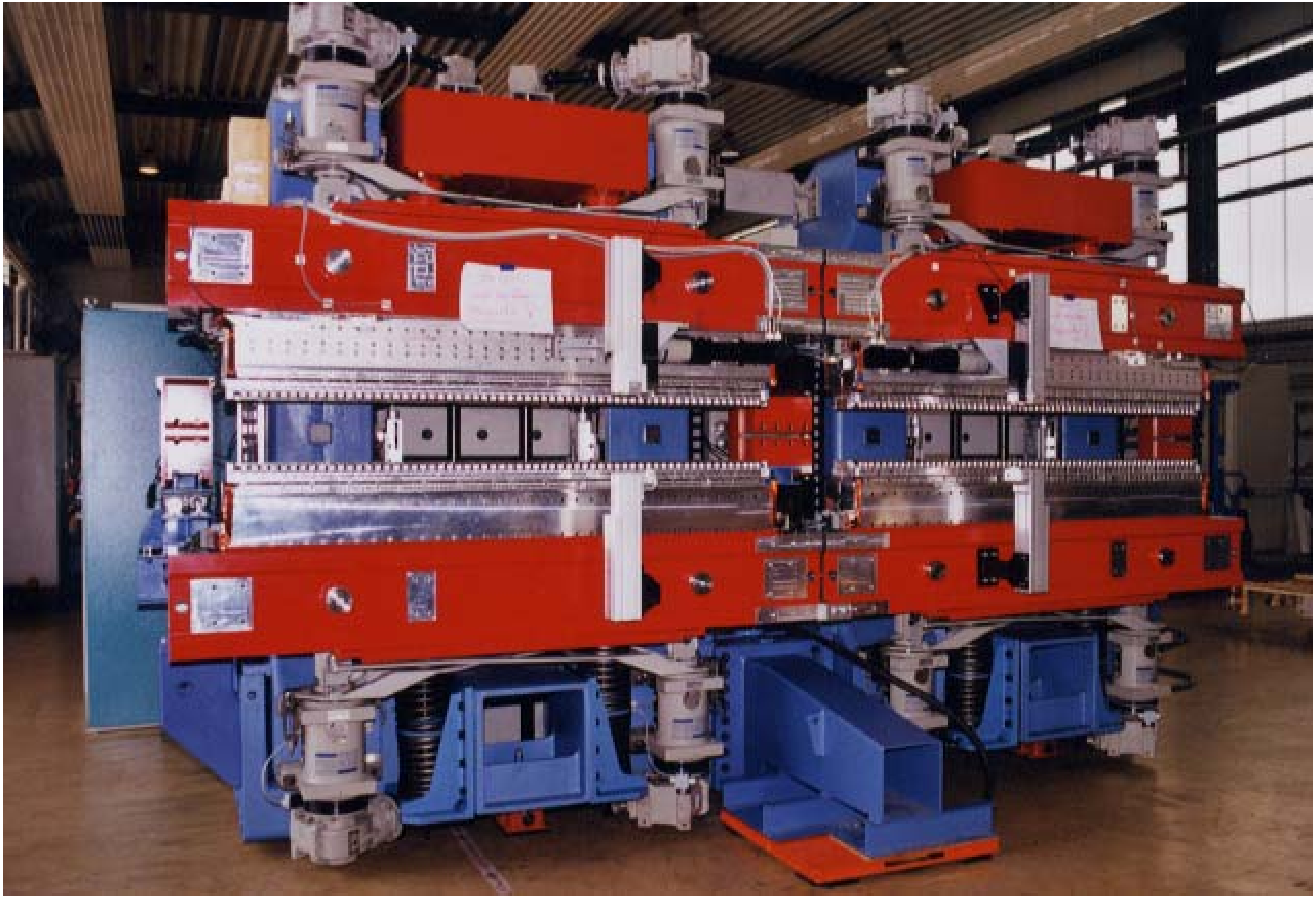


SLS
static displacement, separation in focal plane



BESSY
static angle, angular separation

BESSY UE56 Double Undulator for fast Helicity Switching

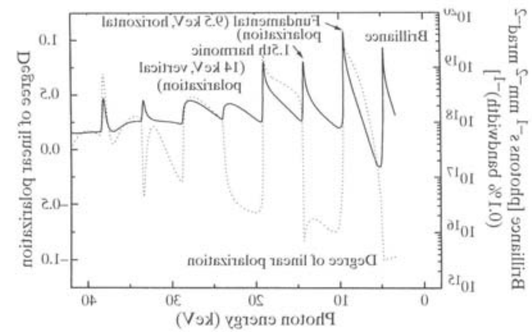


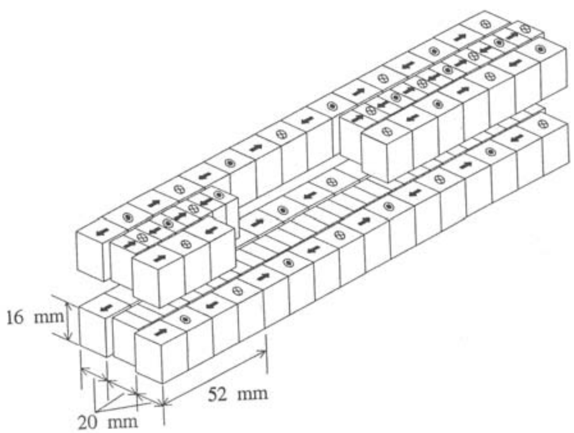
IDs with different states of polarization at different harmonics

+ polarization switching without complicated mechanics
+ suitabe for in vacuum applications
- less flux
- slow switching frequency

figure-8 undulator

alternatively horizontal and vertical polarization at successive harmonics





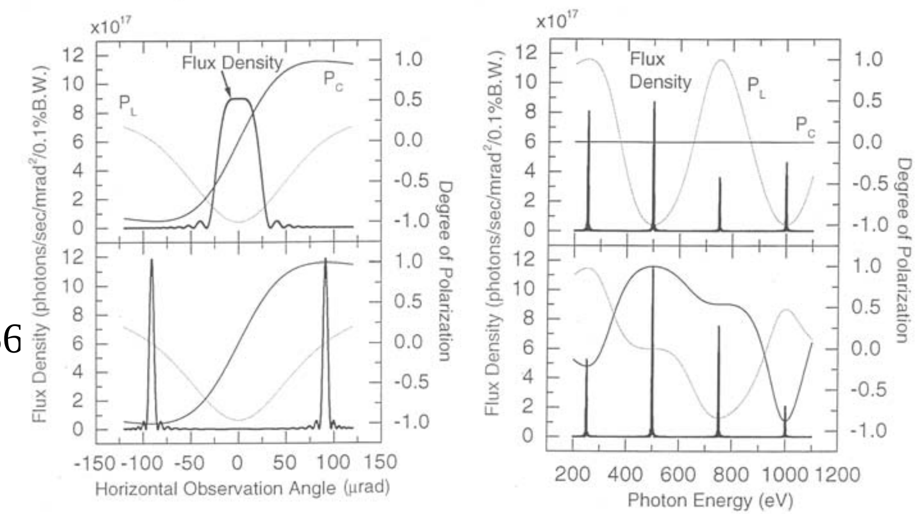
asymmetric figure-8 undulator

up to 80% circular polarization in certain harmonics

parabolic undulator

off axis circularly polarized light

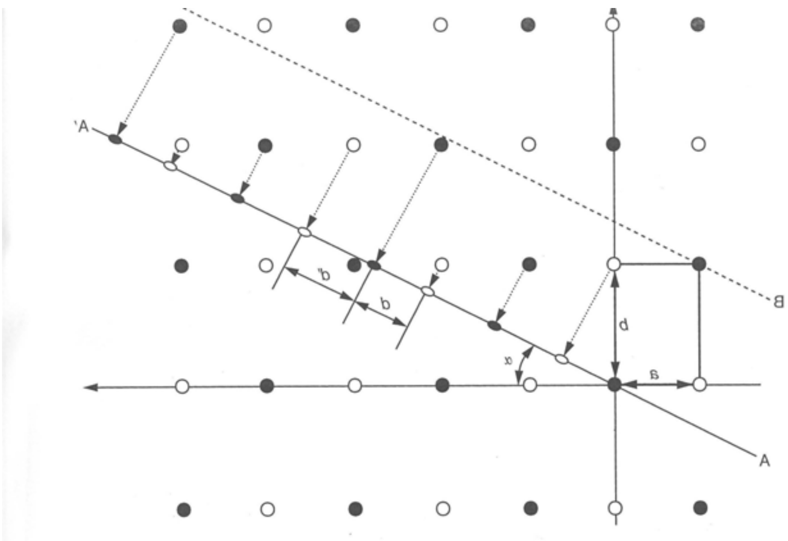
period length of vert. field half the value of hor. field
relative phase = 0 deg. ❾ parabolic undulator
relative phase = 90 deg. ❾ figure-8 undulator
asymetric figure-8 und.



T. Tanaka, H. Kitamura, NIM 467-468 (2001) 153-156
T. Tanaka, H. Kitamura, NIM 364 (1995) 368-373

Quasiperiodic Undulators

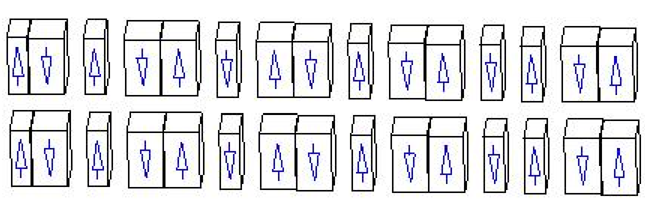
generate 1D-quasiperiodic lattice



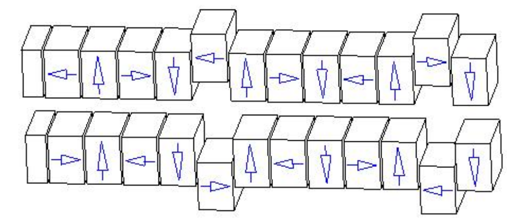
$$z_m = \frac{d}{r \cdot tan(\alpha)} \cdot \left(m + (r \cdot tan(\alpha) - 1)\left[\frac{tan(\alpha)}{r + tan(\alpha)} m + 1\right]\right)$$

r = b / a

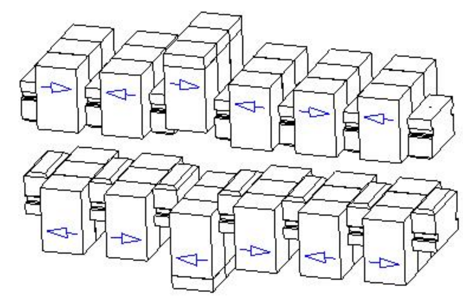
original design

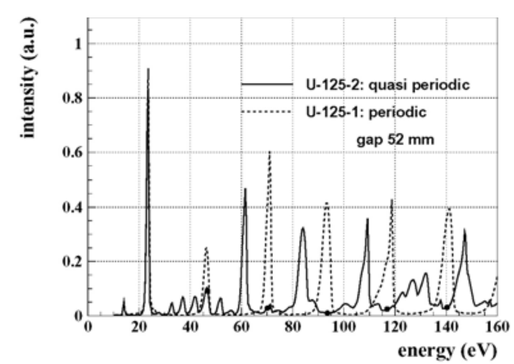


ESRF / ELETTRA design



BESSY design





spectra derived from measured magn. fields

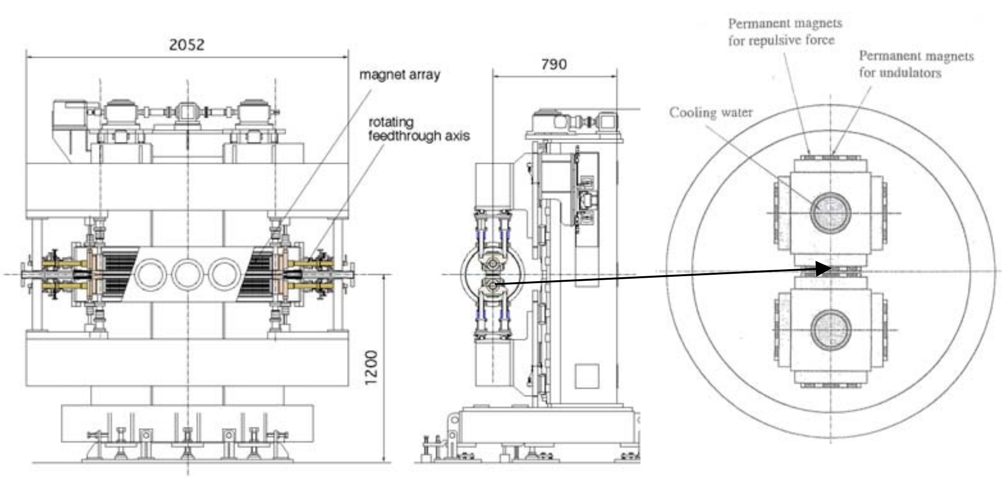
Small Period Devices

in vacuum undulators

Complicated but mature technique

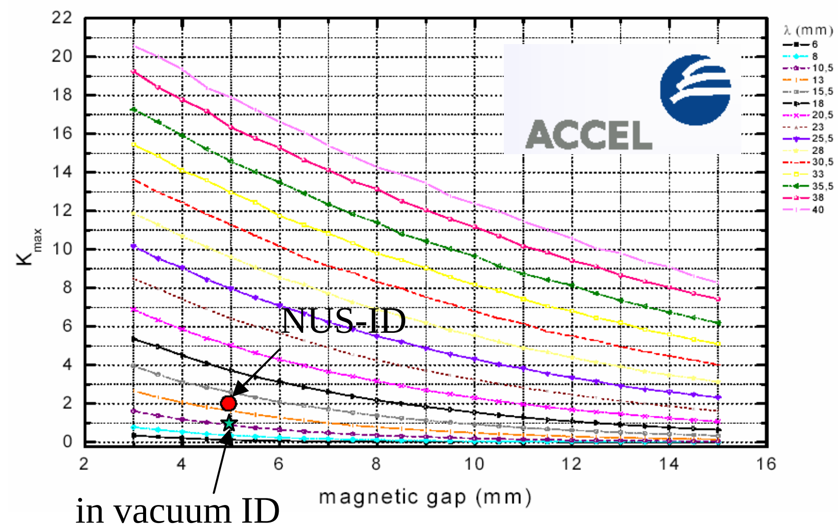
- Coating of magnets to reduce outgassing
  Ti+TiN ion plating of NdFeB magnets (SPRING8)
- high coercive magnetic material (bakeout at 125°)
- thin metal sheet to reduce image current heating
  (50❍m Ni + 10❍m Cu)
- water cooled RF-fingers
- special shimming techniques

in vacuum revolver (SPRING 8)



superconducting undulators

under development



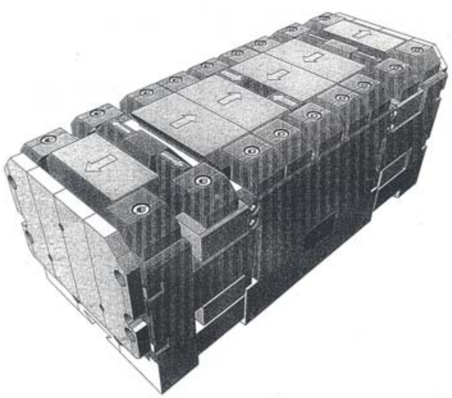
NUS-ID
B = 1.3 Tesla
● = 14mm
gap = 5mm
50 periods
🖎☞ = 5.7°
(nach Dipolkorr.)

High Field Devices

non superconducting

Hybrid wiggler
B ” 2 Tesla
(many SR-facilities)

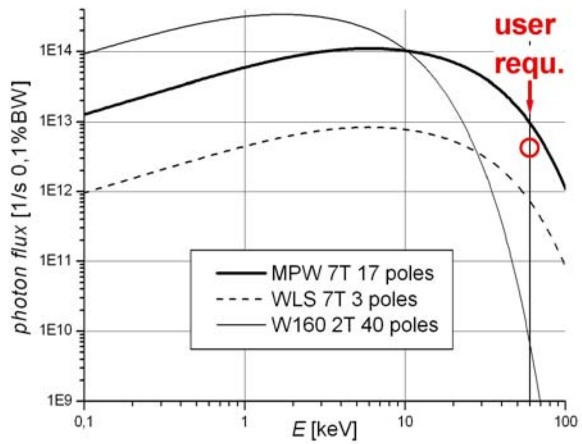
Asymmetric wiggler
3.1 Tesla
11m gap, ●=378mm
(ESRF)



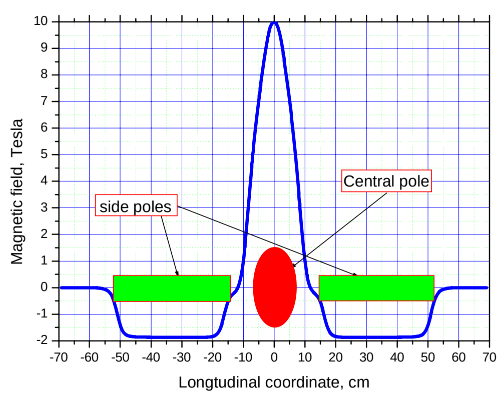
5 T WLS
Perm. magn. + coils
(Budker Institute)

superconducting

HMI Multipole wiggler (BESSY)



10 Tesla WLS (SPRING8)



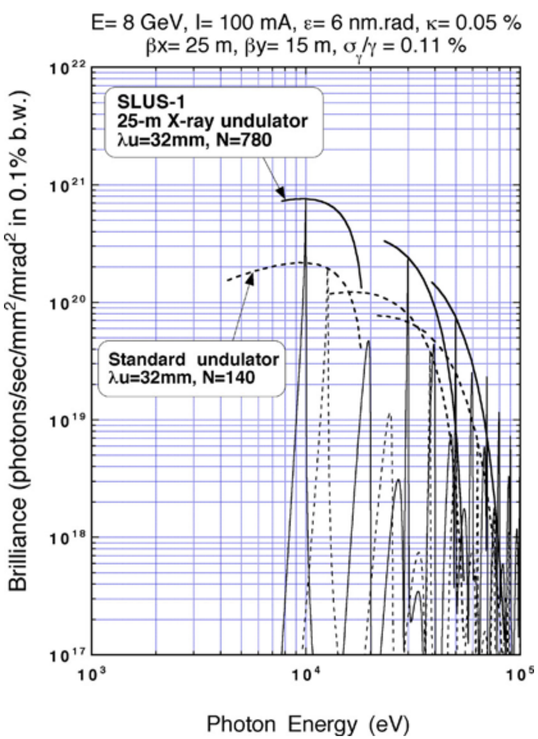
3.5 Tesla wiggler
46 poles, ●=61mm,
gap=10.2mm
(MAX-Lab, ELETTRA)

Superbends
ALS,
2 years of operation

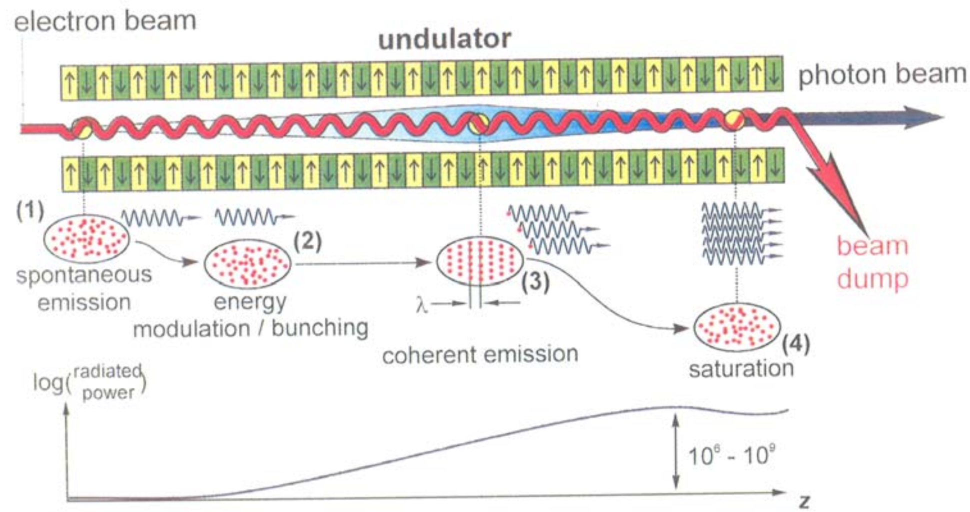
Long Undulators

spontaneous emission

SPRING-8
25 m ID
in vacuum



stimulated emission (SASE)



saturation demonstrated:

VISA	800 nm
LEUTL	300 nm
TTF	80nm

projects:

TESLA	0.1 nm
LCLC	0.15 nm
SCSS SPRING8	3.6 nm
BESSY	1.2 nm