

Differences between Electron and Ion Linear Accelerators

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1. A reminder of relativistic mechanics
2. Accelerating structures
3. Beam dynamics
4. Technology
5. Some conclusions

→ Newton mechanics rules our every day world, but can not be extrapolated to velocities close to speed of light c - the standard case for particles in an accelerator!

→ Dynamics equation $F=dp/dt$ (not $F=ma!$) is

$$q[E + (v \times B)] = \frac{d}{dt}(mv)$$

→ When velocity approaches c , the **mass** of the particle increases with the velocity

$$m = m_0 / \sqrt{1 - (v/c)^2} = m_0 \gamma$$

→ Total energy of the particle is equal to rest energy + kinetic energy:

$$E = mc^2 = \gamma m_0 c^2 = m_0 c^2 + m_0 c^2 (\gamma - 1) = m_0 c^2 + T$$

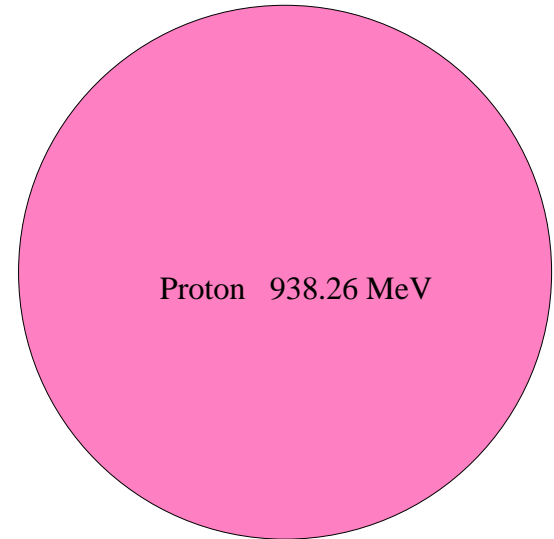
→ Relativistic kinetic energy is:

$$T = m_0 c^2 (\gamma - 1) = m_0 c^2 (1 / \sqrt{1 - (v/c)^2} - 1)$$

Dynamics depends on rest mass:

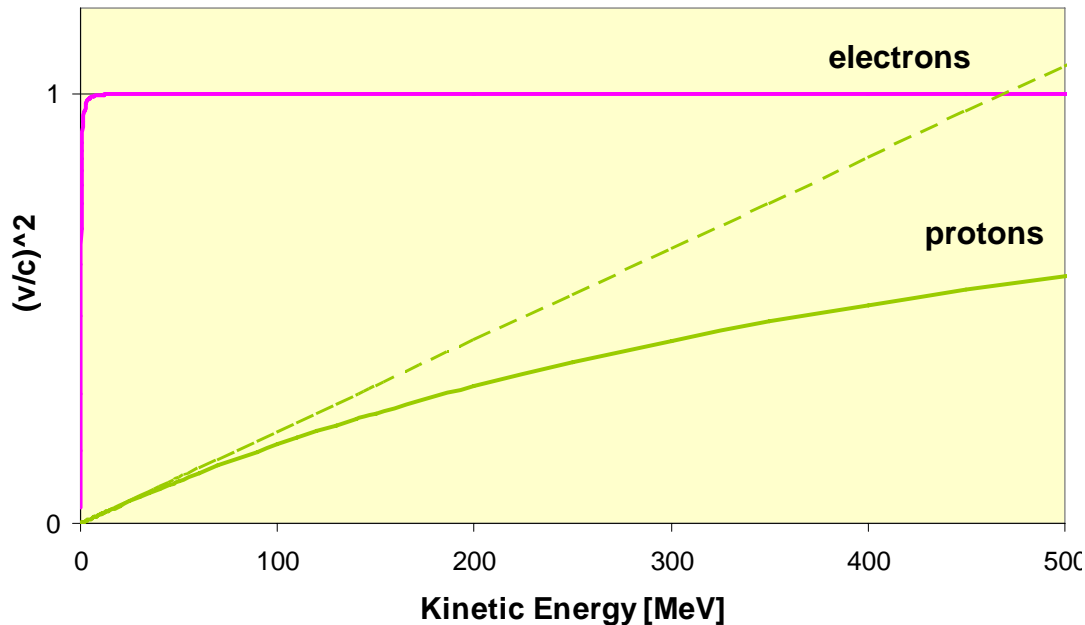


Electron 0.511 MeV



Proton 938.26 MeV

Factor 1'836 in mass between proton and electron



$\beta^2 = (v/c)^2$ as function of kinetic energy T for protons and electrons.

Relativistic (Einstein) relation:

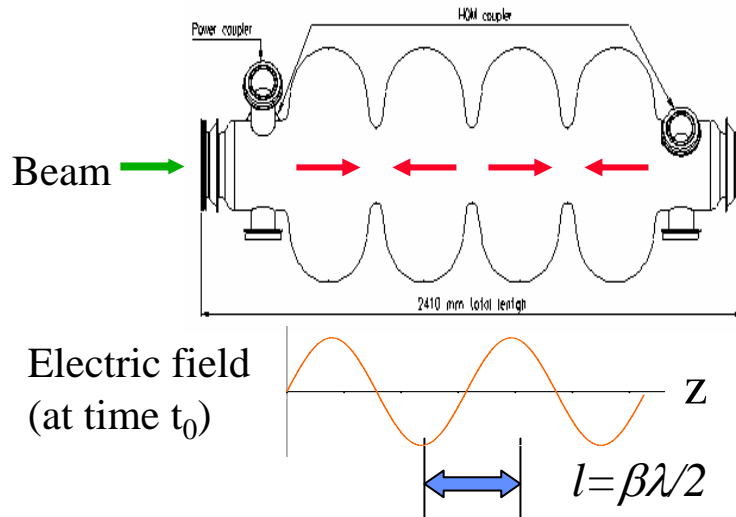
$$T = m_0 c^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) = m_0 c^2 (\gamma - 1)$$

Classic (Newton) relation:

$$T = m_0 \frac{v^2}{2} = m_0 c^2 \left(\frac{1}{2} \frac{v^2}{c^2} \right)$$

- Protons: relativistic from the **MeV range** ($v \sim 0.1c$ at 5 MeV) then increasing velocity up to the **GeV range** ($v \sim 0.95c$ at 2 GeV) → v increasing in all the range of a linac
- Electrons: relativistic from the **keV range** ($v \sim 0.1c$ at 2.5 keV) then increasing velocity up to the **MeV range** ($v \sim 0.95c$ at 1.1 MeV) → $v \sim c$ after few meters of acceleration in a linac (typical gradient 10 MeV/m).

Remember the synchronism condition: for multigap acceleration, cell length is proportional to particle velocity



Example: a linac superconducting 4-cell accelerating structure

Synchronism condition:

t (travel between centers of cells) = $T/2$

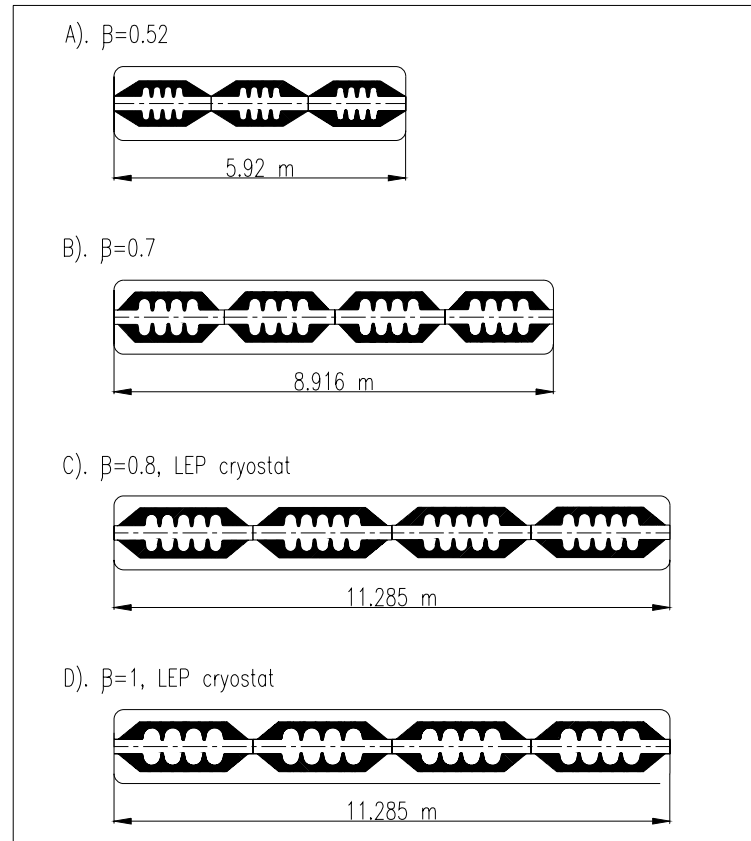
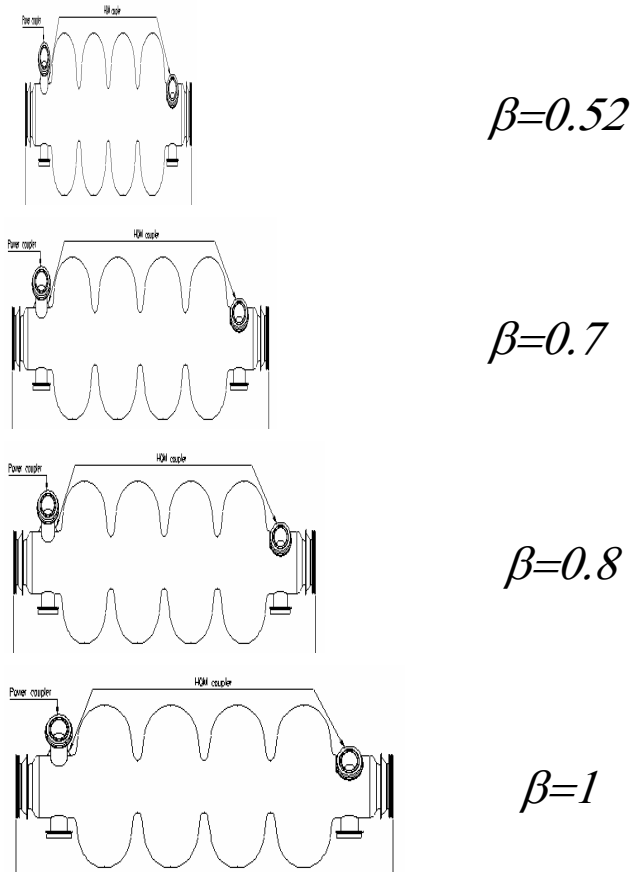
$$\frac{l}{\beta c} = \frac{1}{2f} \quad l = \frac{\beta c}{2f} = \frac{\beta \lambda}{2}$$

consequence of the different velocity profile is that:

1. An electron linac will be made of an **injector** + a **series of identical accelerating structures**, with cells all the same length
2. In an ion linac cell length has to increase (up to a factor 200 !) and the linac will be made of a **sequence of different accelerating structures** (frequency, operating mode, etc.) matched to the ion velocity.

Example: SC linac cavities for different beta

For example, the same Superconducting cavity design can be used for different proton beta's, just changing the cell length accordingly to beta.



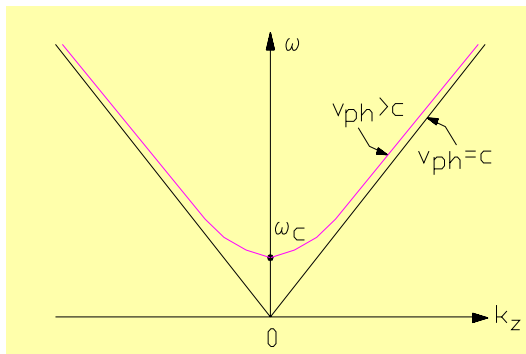
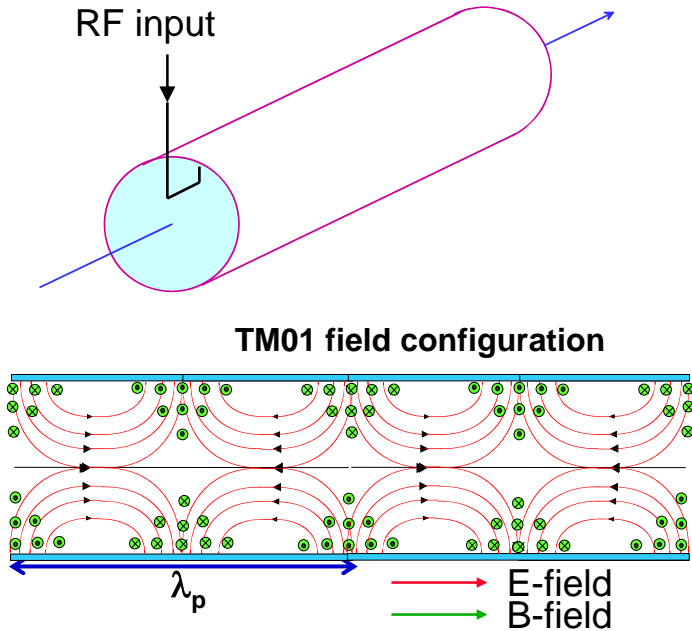


- The forces acting on a particle beam traveling at $v \sim c$ have to be transformed from the laboratory frame to the particle frame via the Lorentz transformations:

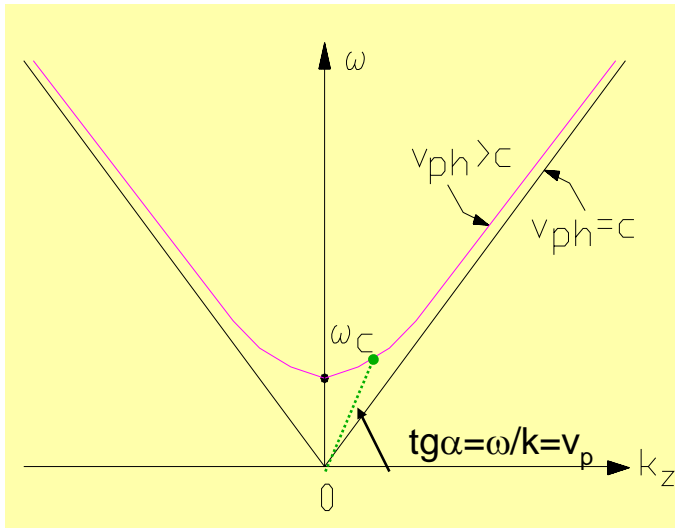
$$\begin{cases} x' = \gamma (x - vt) \\ y' = y \\ z' = z \\ t' = \gamma (t - \beta^2 x/v) \end{cases}$$

- The world seen by a particle moving at the speed of light (an electron) will be much different from the world as seen by a particle moving at $v \ll c$ (an ion).
- Self-forces of the particle beam depend on velocity, and how the particles see the external field depend on velocity.
- Beam dynamics for electrons will be substantially different from beam dynamics for ions.

2 - Accelerating structures for ions and electrons



- In a cylindrical waveguide different **modes** can propagate (=Electromagnetic field configurations, transmitting power and/or information).
- To accelerate particles, we need a mode with longitudinal E-field component on axis: a TM mode (Transverse Magnetic, $B_z=0$). The simplest is TM₀₁.
- We enter RF power at a frequency exciting the TM₀₁ mode: the E-field is periodic on axis, propagation wavelength depends on the frequency and on the cylinder radius. Wave velocity is $v_{ph} = \lambda_p / T = \lambda_p f = \omega / k$
- The relation between excitation frequency and propagation constant $k = 2\pi / \lambda$ is called the **DISPERSION RELATION** (red curve on plot) and represents a fundamental property of waveguides.



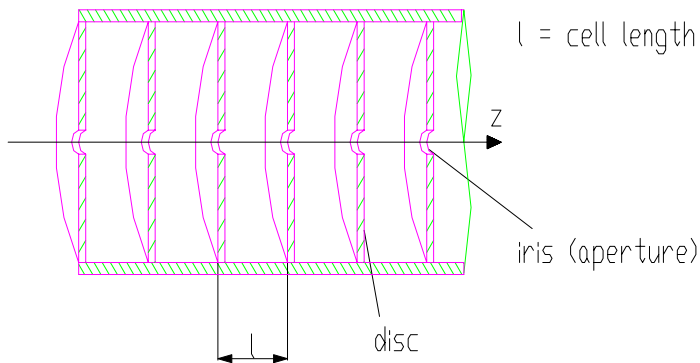
$$\omega^2 = k^2 c^2 + \omega_c^2$$

$$k = 2\pi/\lambda_p$$

$$v_p = \omega/k$$

$$v_g = d\omega/dk$$

- There is a “cut-off frequency”, below which a wave will not propagate. It depends on dimensions ($\lambda_c = 2.61a$ for the cylindrical waveguide).
- At each frequency is associated a phase velocity, the velocity at which a certain phase travels in the waveguide. $v_p = \infty$ at $\omega = \omega_c$ and then decreases towards $v_p = c$ for $\omega \rightarrow \infty$.
- Energy (and information) travel at group velocity, varying with frequency between 0 and c . This velocity has to respect the relativity principle!
- A particle traveling inside our cylinder has to travel at $v = v_{ph}$ to see a constant accelerating E-field → should travel at $v > c$!!!
- We need a “trick” to slow down the wave !



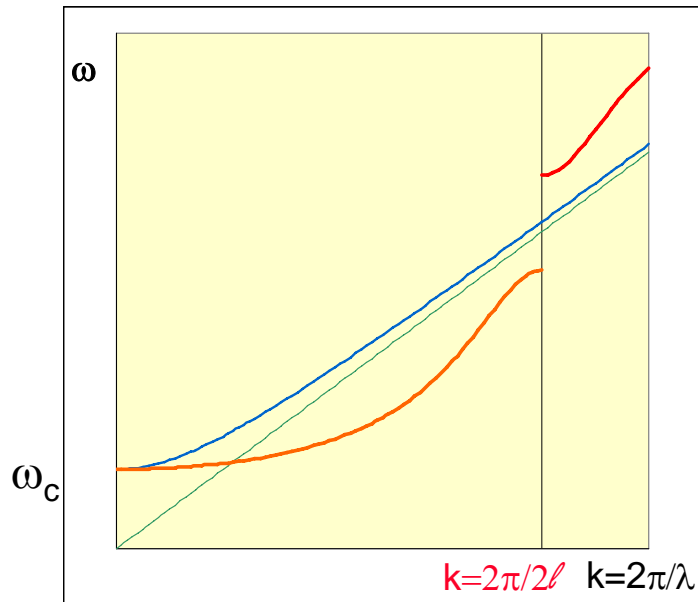
→ Discs inside the cylindrical waveguide, spaced by l , induce multiple reflections between the discs.

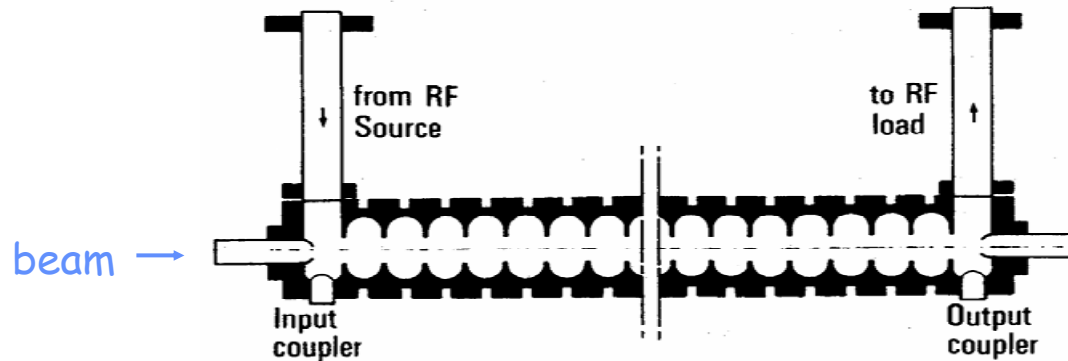
→ Propagation wavelengths $\lambda_p \sim l$ will be most affected by the discs. On the contrary, for $\lambda_p = 0$ and $\lambda_p = \infty$ the wave does not see the discs → the dispersion curve remains that of the empty pipe.

→ At $\lambda_p = l$, the wave will be confined between the discs, and present 2 "polarisations", 2 modes with same wavelength and different frequency → the dispersion curve splits into 2 branches, separated by a stop band.

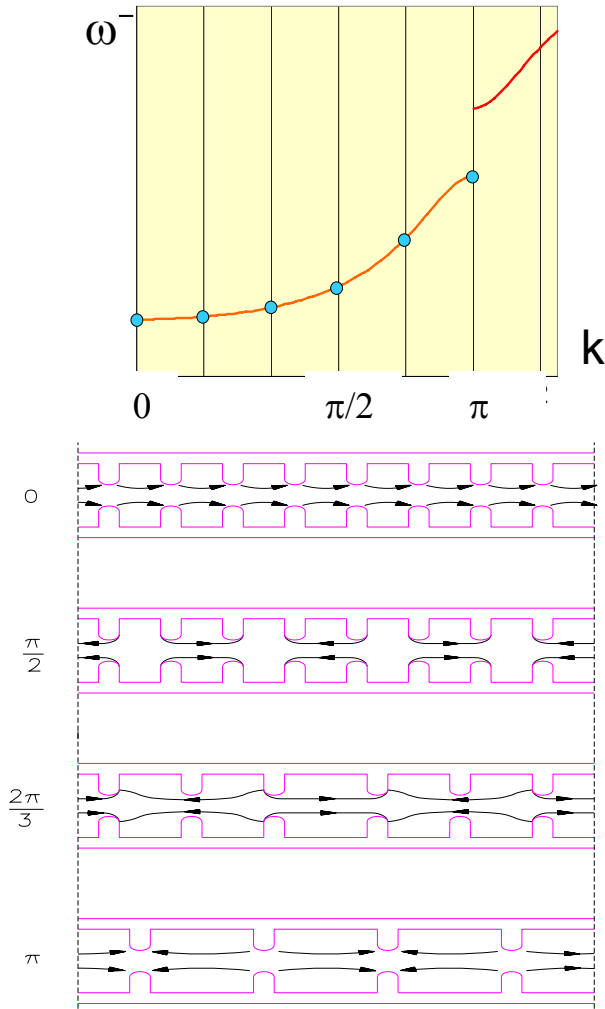
→ In the disc-loaded waveguide, the lower branch of the dispersion curve is now "distorted" in such a way that we can find a frequency such that $v_{ph} = c$ → we can use it to accelerate a particle beam!

→ We have built a linac for $v \sim c$ → a **TRAVELING WAVE** (TW) ELECTRON LINAC



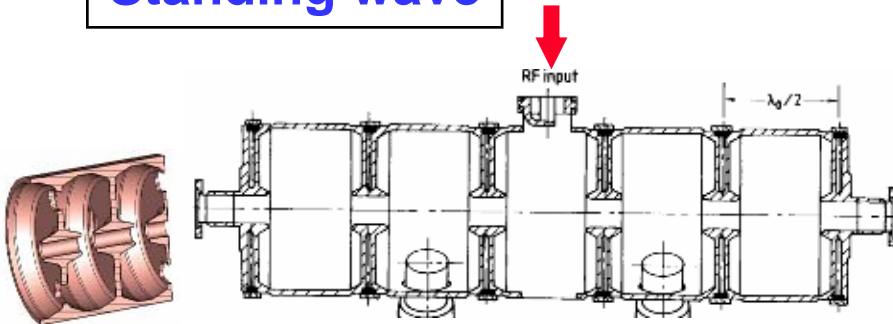


- Disc-loaded waveguide designed for $v_{ph}=c$ at a given frequency, equipped with an input and an output coupler.
- RF power is introduced via the input coupler. Part of RF power is dissipated in the structure, part is taken by the beam (beam loading) and the rest is absorbed in a matched load at the end of the structure. Usually, structure length is such that ~30% of power goes to the load.
- This "traveling wave" structure is the standard linac for **electrons from $\beta \sim 1$** .
- Can not be used for ions at $v < c$: constant cell length does not allow synchronism long structures, no space for transverse focusing



- To obtain an accelerating structure for ions we close our disc-loaded structure at both ends with metallic walls to induce multiple reflections of the waves. Only modes that have right phase at the covers are allowed → only some frequencies on the dispersion curve are allowed.
- These **STANDING WAVE MODES** are generated by the sum of 2 traveling waves in opposite directions, adding always in the same way in the different cells.
- The particles must be in phase with the E-field on axis. Synchronism condition (π mode) for cell length $\ell = \beta\lambda/2$.
- Standing wave structures can be used for **any β** (→ ions and electrons) and can follow the **increase in β** of the ions.

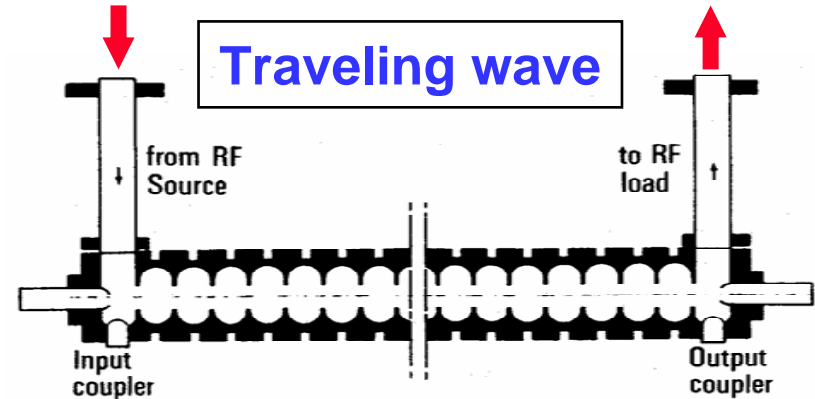
Standing wave



Chain of coupled cells in SW mode.
Coupling (bw. cells) by slots (or open). On-axis aperture reduced, higher E-field on axis and power efficiency.
RF power from a coupling port, dissipated in the structure (ohmic loss on walls).
Long pulses. Gradients 2-5 MeV/m

Used for Ions and electrons at all energies

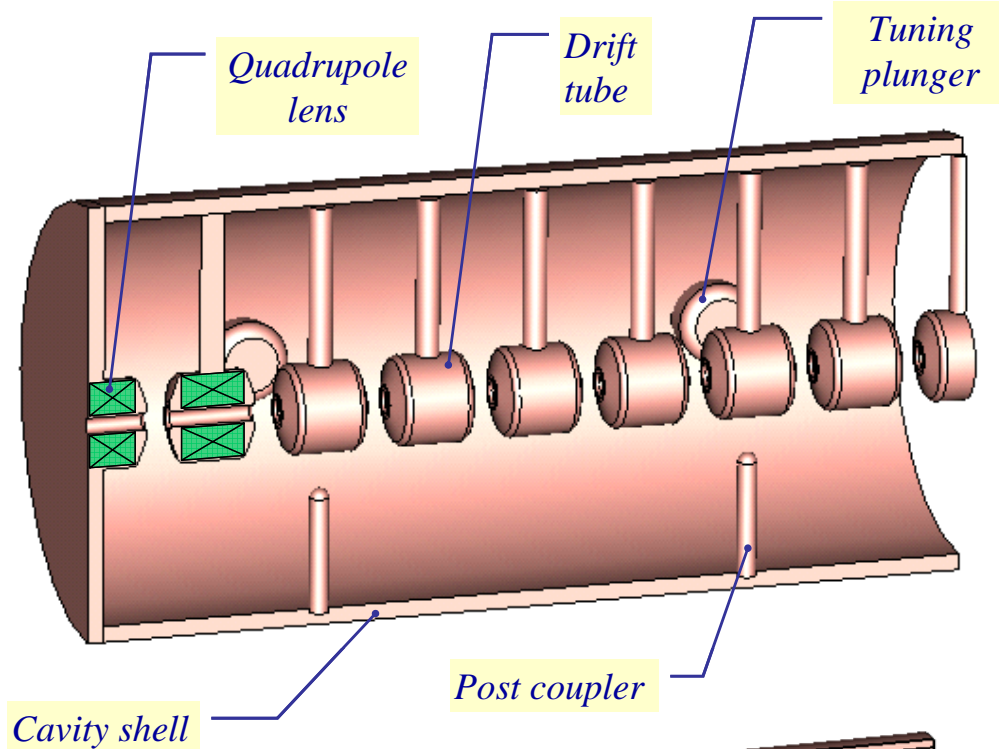
Traveling wave



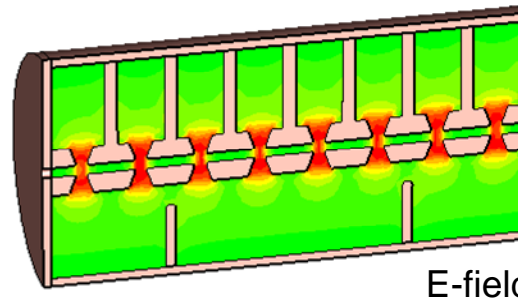
Chain of coupled cells in TW mode
Coupling bw. cells from on-axis aperture.
RF power from input coupler at one end, dissipated in the structure and on a load.
Short pulses, High frequency.
Gradients 10-20 MeV/m

Used for Electrons at $v \sim c$

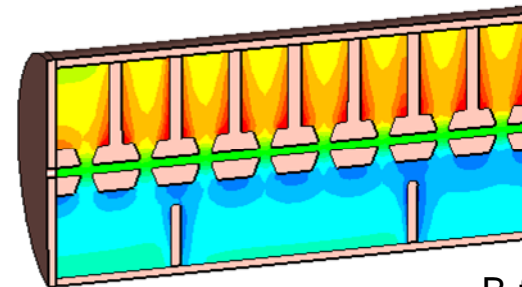
Comparable RF efficiencies



Standing wave linac structure for protons, $\beta=0.1-0.5$
Chain of coupled cells, completely open (no walls), maximum coupling.
Operating in 0-mode, cell length $\beta\lambda$.

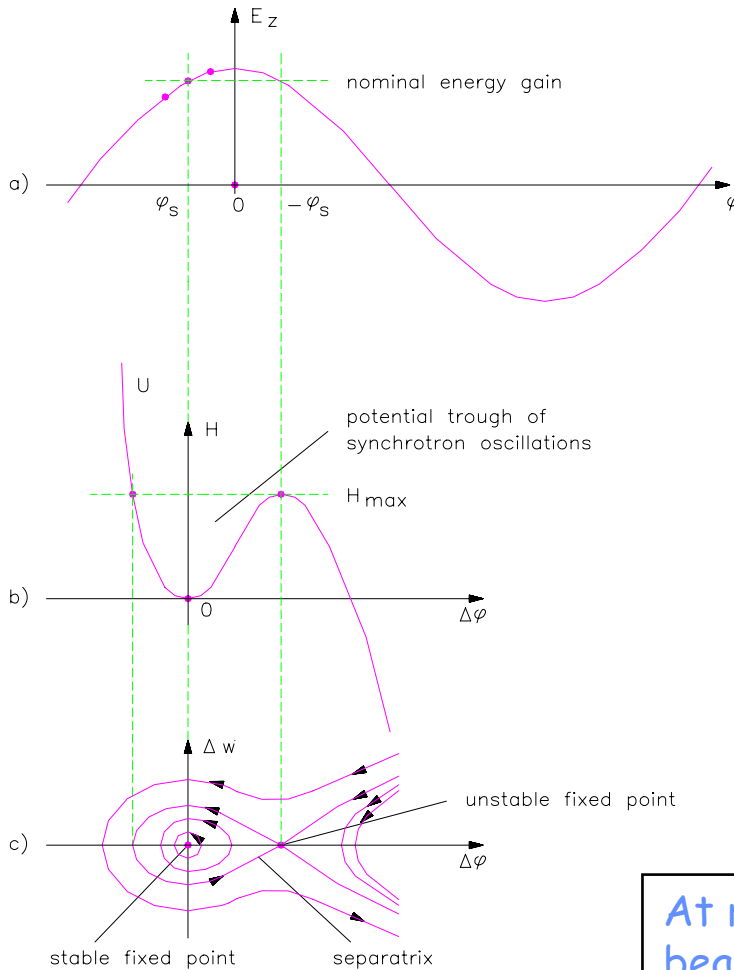


E-field



B-field

3 - Beam Dynamics in Ion and Electron Linacs



- Ions are accelerated around a (negative) synchronous phase.
- Particles around the synchronous one perform oscillations in the longitudinal phase space.
- Frequency of small oscillations:

$$\omega_l^2 = \omega_0^2 \frac{qE_0 T \sin(-\varphi)\lambda}{2\pi mc^2 \beta\gamma^3}$$

- Tends to zero for relativistic particles $\gamma \gg 1$.
- Note phase damping of oscillations:

$$\Delta\varphi = \frac{\text{const}}{(\beta\gamma)^{3/4}} \quad \Delta W = \text{const} \times (\beta\gamma)^{3/4}$$

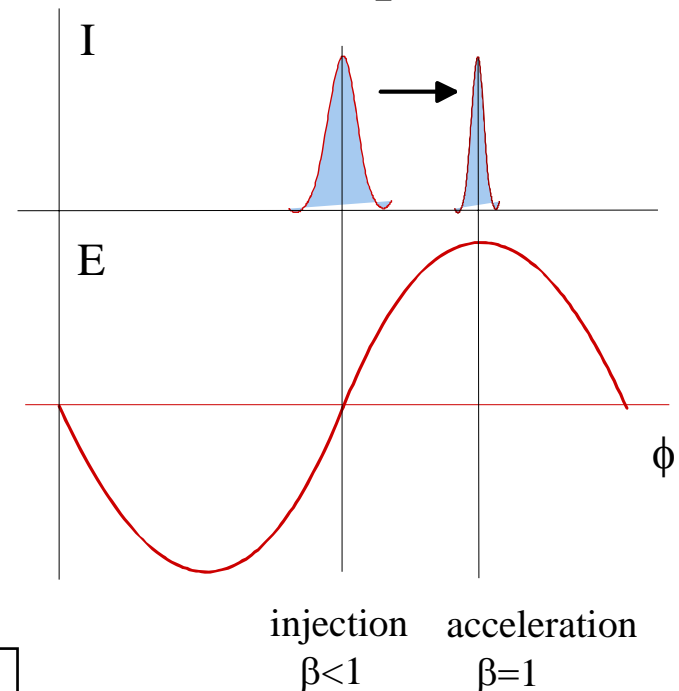
At relativistic velocities phase oscillations stop, the beam is compressed in phase around the initial phase. The crest of the wave can be used for acceleration.

- Electrons at $v=c$ remain at the injection phase.
- Electrons at $v < c$ injected into a TW structure will move from injection phase φ_0 to an asymptotic phase φ , which depends only on gradient and β_0 at injection.
- The beam can be injected with an offset in phase, to reach the crest of the wave at $\beta=1$
- Capture condition, relating E_0 and β_0 :

$$\frac{2\pi}{\lambda_g} \frac{mc^2}{qE_0} \left[\sqrt{\frac{1-\beta_0}{1+\beta_0}} \right] = 1$$

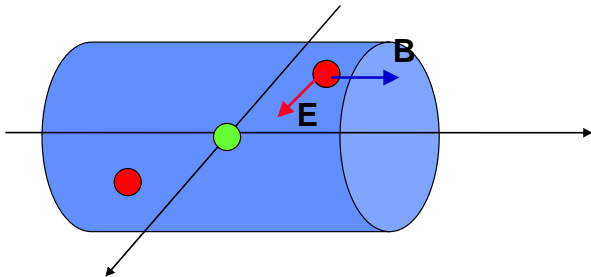
Example: $\lambda=10\text{cm}$, $W_{\text{in}}=150\text{ keV}$ and $E_0=8\text{ MV/m}$.

$$\sin \varphi = \sin \varphi_0 + \frac{2\pi}{\lambda_g} \frac{mc^2}{qE_0} \left[\sqrt{\frac{1-\beta_0}{1+\beta_0}} - \sqrt{\frac{1-\beta}{1+\beta}} \right]$$



In high current linacs, a bunching and pre-acceleration sections up to 4-10 MeV prepares the injection in the TW structure (that occurs already on the crest)

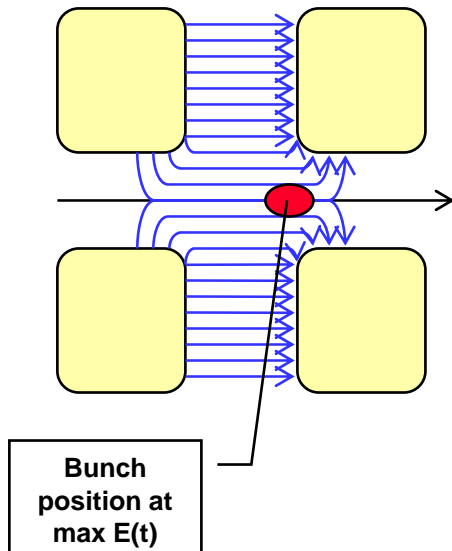
- Large numbers of particles per bunch
($<10^{10}$ in a proton linac, $<10^{11}$ in a standard electron linac).
- Coulomb repulsion between particles (space charge) plays an important role.
- But **space charge forces $\sim 1/\gamma^2$ disappear at relativistic velocity**



Force on a particle inside a long bunch
with density $n(r)$ traveling at velocity v :

$$E_r = \frac{e}{r} \int_0^r n(r) r dr \quad B_\phi = \frac{ev}{r} \int_0^r n(r) r dr$$

$$F = e(E_r - vB_\phi) = eE_r \left(1 - \frac{v^2}{c^2}\right) = eE_r (1 - \beta^2) = \frac{eE_r}{\gamma^2}$$



- RF defocusing experienced by particles crossing a gap on a longitudinally stable phase.
- In the rest frame of the particle, only electrostatic forces → no stable points (maximum or minimum) → radial defocusing.
- Lorentz transformation and calculation of radial momentum impulse per period (from electric and magnetic field contribution in the laboratory frame):

$$\Delta p_r = -\frac{\pi e E_0 T L r \sin \varphi}{c \beta^2 \gamma^2 \lambda}$$

- **Transverse defocusing $\sim 1/\gamma^2$ disappears at relativistic velocity** (transverse magnetic force cancels the transverse RF electric force).
- **Important consequence: in an electron linac, transverse and longitudinal dynamics are decoupled !**

Transverse Beam Dynamics in a linac: the equilibrium between external focusing force and internal defocusing forces, which determines the phase advance of beam oscillations σ_z (here expressed per unit length):

Ph. advance = Ext. quad focusing - RF defocusing - space charge - Instabilities

$$\left(\frac{\sigma_t}{N\beta\lambda}\right)^2 = \left(\frac{qGl}{2mc\beta\gamma}\right)^2 - \frac{\pi q E_0 T \sin(-\varphi)}{mc^2 \lambda \beta^3 \gamma^3} - \frac{3qI\lambda(1-f)}{8\pi r_0^3 mc^3 \beta^2 \gamma^3} - \dots$$

Electron Linac:

Ph. advance = Ext. focusing + ~~RF defocusing~~ + ~~space charge~~ + Instabilities

For $\gamma \gg 1$ (electron linac): RF defocusing and space charge disappear, *phase advance* $\rightarrow 0$. External focusing is required only to control the emittance and to stabilize the beam against instabilities (as *wakefields* and *beam breakup*).

Focusing provided by quadrupoles (but solenoids for low β !).

Different **distance between focusing elements** (=1/2 length of a FODO focusing period)! For the main linac accelerating structure (after injector):

Protons, (high beam current and high space charge) require short distances:
 $\beta\lambda$ in the main linac, from $\sim 70\text{mm}$ (3 MeV, 352 MHz) to $\sim 250\text{mm}$ (40 MeV),
can be increased to $4-10\beta\lambda$ at higher energy (>40 MeV).

Heavy ions (low current, no space charge):
 $2-10 \beta\lambda$ in the main linac ($>\sim 150\text{mm}$).

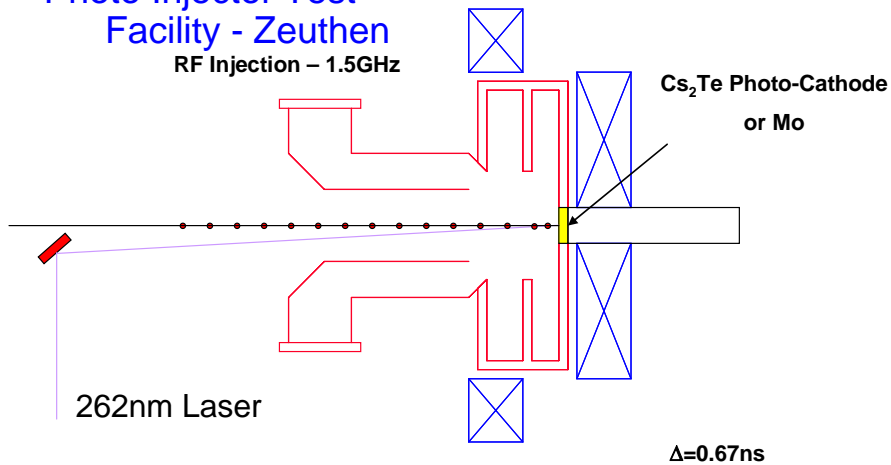
Electrons (no space charge, no RF defocusing):
up to several meters, depending on the required beam conditions. Focusing is
mainly required to control the emittance.

4. Technologies

The technology for particle production is completely different for electrons and ions.

Photo Injector Test Facility - Zeuthen

RF Injection - 1.5GHz



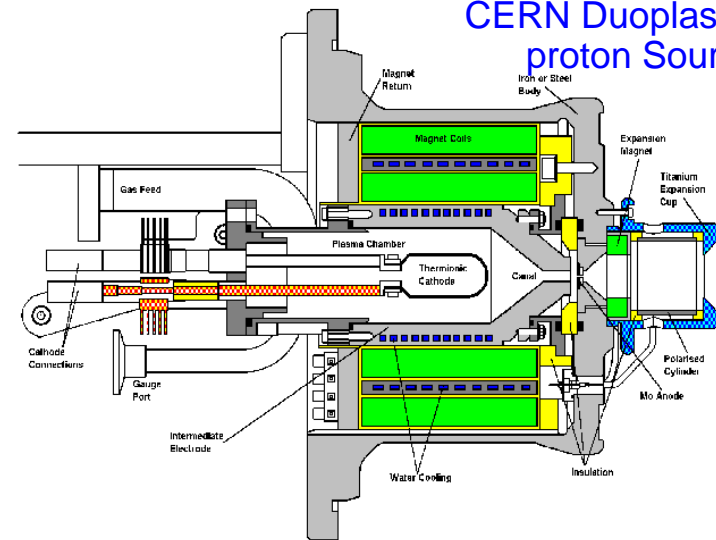
Electron sources:

give energy to the free electrons inside a metal to overcome the potential barrier at the boundary.

Used for electron production:

- thermoionic effect
- laser pulses
- surface plasma

CERN Duoplasmatron proton Source



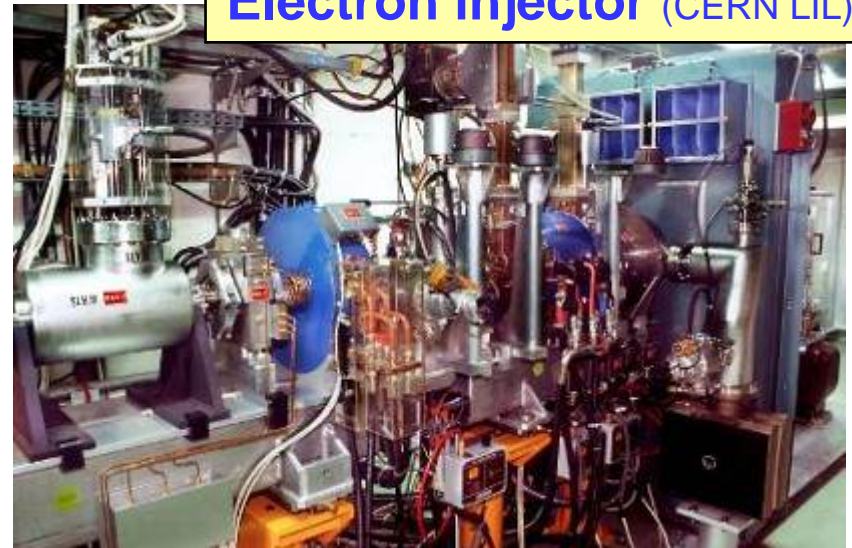
Ion sources:

create a plasma and optimise its conditions (heating, confinement and loss mechanisms) to produce the desired ion type. Remove ions from the plasma via an aperture and a strong electric field.

Ion injector (CERN Linac1)



Electron injector (CERN LIL)



3 common problems for protons and electrons after the source, up to ~ 1 MeV energy:

1. large space charge defocusing
2. particle velocity rapidly increasing
3. need to form the bunches

Solved by a special **injector**: - (note that focusing by solenoids is used in both cases!)

RFQ bunching, focusing and accelerating structure for ions.

Standing wave bunching and pre-accelerating section for electrons.

☞ For all particles, the injector is where the emittance is created!

approximate scaling laws for linear accelerators:

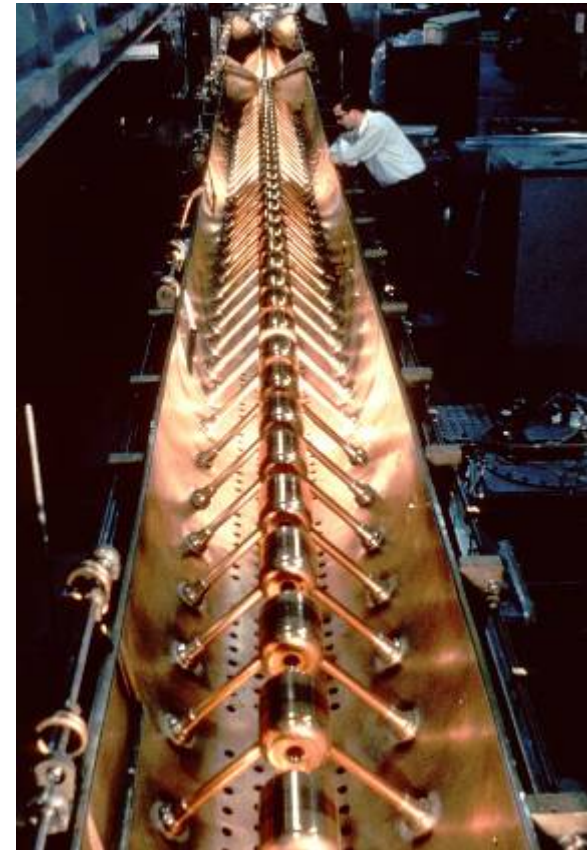
⇒ RF defocusing (ion linacs)	~ frequency
⇒ Cell length ($=\beta\lambda/2$)	~ (frequency) ⁻¹
⇒ Peak electric field	~ (frequency) ^{1/2}
⇒ Shunt impedance (power efficiency)	~ (frequency) ^{1/2}
⇒ Accelerating structure dimensions	~ (frequency) ⁻¹
⇒ Machining tolerances	~ (frequency) ⁻¹

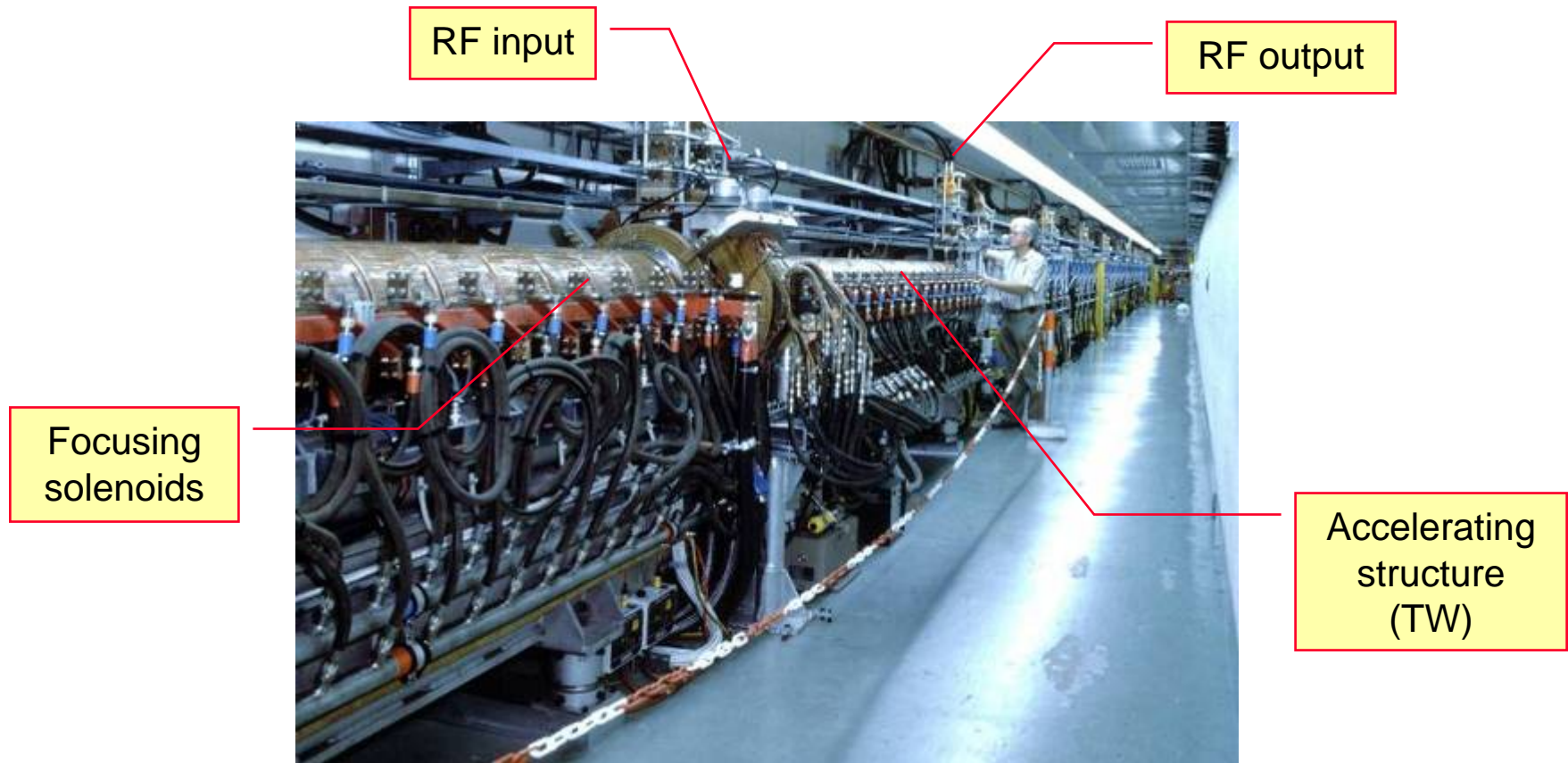
- Higher frequencies are economically convenient (shorter, less RF power, higher gradients possible) but the limitation comes from mechanical precision required in construction (tight tolerances are expensive!) and beam dynamics for ion linacs.
- Electron linacs tend to use higher frequencies than ion linacs (0.5-30 GHz), usual frequency 3 GHz (10 cm wavelength). No limitations from beam dynamics, iris in TW structure requires less accurate machining than nose in SW structure.
- Proton linacs use lower frequencies (100-800 MHz), increasing with energy (ex.: 350 - 700 MHz): compromise between focusing, cost and size.
- Heavy ion linacs tend to use even lower frequencies (30-200 MHz), dominated by the low beta in the first sections (CERN RFQ at 100MHz, 25 keV/u: $\beta\lambda/2=3.5\text{mm}$!)



**CERN Linac2 Drift Tube Linac accelerating tank 1 (200 MHz).
The tank is 7m long (diameter 1m) and provides an energy
gain of 10 MeV.**

**Focusing is provided by (small) quadrupoles inside drift
tubes (right). Length of drift tubes (cell length) increases
with proton velocity.**



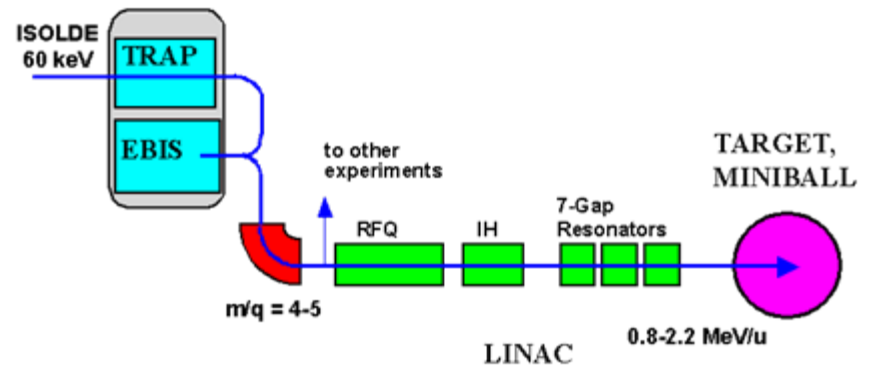
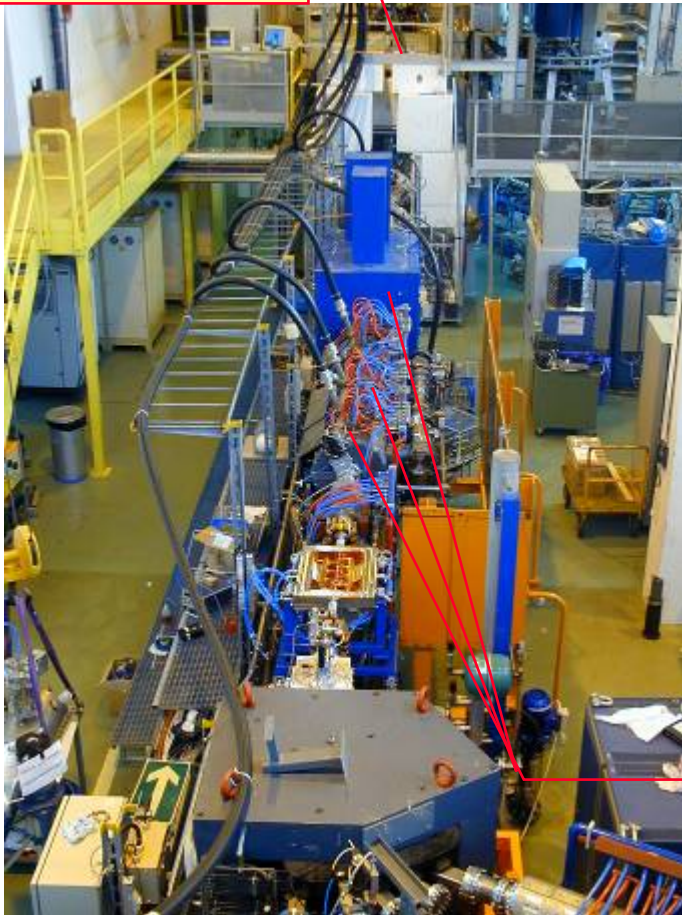


The old CERN LIL (LEP Injector Linac) accelerating structures (3 GHz). The TW structure is surrounded by focusing solenoids, required for the positrons.



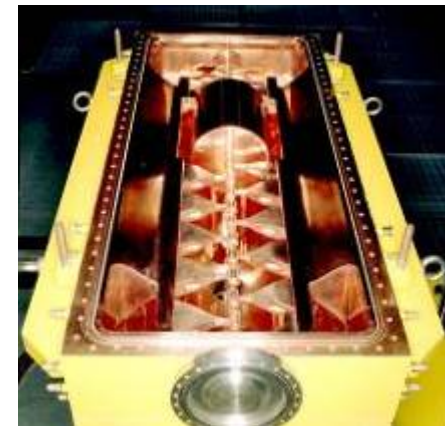
A 3 GHz LIL accelerating structure used for CTF3. It is 4.5 meters long and provides an energy gain of 45 MeV. One can see 3 quadrupoles around the RF structure.

Particle source



The REX heavy-ion post accelerators at CERN. It is made of 5 short standing wave accelerating structures at 100 MHz, spaced by focusing elements.

Accelerating structures



- Type of **RF power source** depend on frequency:
 - ☞ Klystrons (>350 MHz) are used for electron linacs and modern proton linacs. RF distribution via waveguides.
 - ☞ RF tube amplifiers (<400 MHz) are used for proton and heavy ion linacs. RF distribution via coaxial lines.
- **Construction technology** depends on dimensions (→on frequency):
 - ☞ brazed copper parts (>500 MHz) are commonly used for electron linacs.
 - ☞ copper or copper plated welded/bolted parts are commonly used for ion linacs (<500 MHz).



**3 GHz klystron
(CERN LPI)**

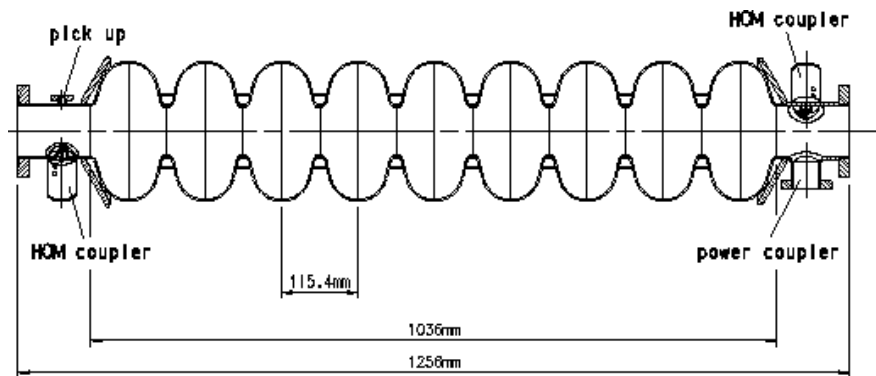
**200 MHz triode amplifier
(CERN Linac3)**

What is new (& hot) in the field of linacs?

1. Frequencies are going up for both proton and electron linacs (←less expensive precision machining). Modern proton linacs start at 350-400 MHz, end at 800-1300 MHz. Modern electron linacs in the range 3-30 GHz.
2. Superconductivity is progressing fast, and is being presently used for both electron and ion linacs → standing wave structures in the frequency range from ~100 MHz to 1300 MHz.

Superconductivity is now **bridging the gap between electron and ion linacs**.

The 9-cell TESLA SC cavities at 1.3 GHz for electron linear colliders, are now proposed for High Power Proton Accelerators...



Trying to compare the incomparable !

Characteristics of 3 linear accelerators, for protons, heavy ions and electrons, operating at CERN as injectors for synchrotrons:

	Linac 2, 1978	Linac 3, 1994	LIL, 1986	
	Protons	Pb ²⁷⁺ ions	Electrons	
Energy	50	4.2 /u	750	MeV
RF Frequency	202	101-202	3000	MHz
Beam current	180	0.08	60	mA
Repetition freq.	2	10	100	Hz
Pulse length	120	1000	0.01	μs
Linac length	35	11	101	
Acc. gradient	2	~5	12	MV/m
Real estate grad.	1.4	3.8	7.5	MeV/m.u
Norm. tr. emittance	1.2	1	80	π mm mrad, rms

