CERN Accelerator School

Beam Cooling

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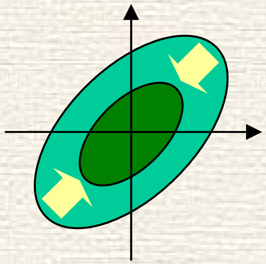
Beam Cooling, and Why?

The momentum spread, transverse and longitudinal, of particles in a beam can be seen as a thermal motion, and cooling is to reduce the momentum spread. Reducing the momentum spread also means increasing the phase-space density.



Beam cooling can be used for many purposes:

- Accumulation of antiprotons or other particles to increase beam current
- Increase of luminosity in colliders
- Counteraction of heating in internal targets
- Improved precision in measurements
- Emittance control at deceleration
- ...

Methods for Beam Cooling

Different cooling methods are used for different kinds of beams, but only three kinds of active cooling have been implemented so far:

- Electrons cool themselves by emitting radiation, this is for another lecture
- Electron cooling for low/medium-energy protons, antiprotons and ions*
- Stochastic cooling for medium/high-energy protons, antiprotons and ions
- Laser cooling for a few kinds of atomic ions
- Ionization cooling, soon, for muons

*) all cooled particles will be referred to as ions

Gersh Budker, 1966

Эффективный метод демпфирования колебаний частиц в протонных и аити-протонных накопителях

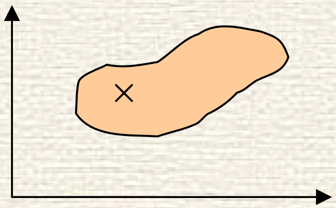
Simon van der Meer, 1972

Stochastic damping of betatron oscillations in the ISR

Liouville's Theorem

For a system of N particles, Liouville's theorem can be formulated:

- in μ space, which is the 6-dimensional phase space where each particle is represented by a point (6 coordinates because each particle has x, y, z, p_x, p_y and p_z)
- in Γ space, which is the 6N-dimensional phase space where the whole system (beam) is represented by a single point

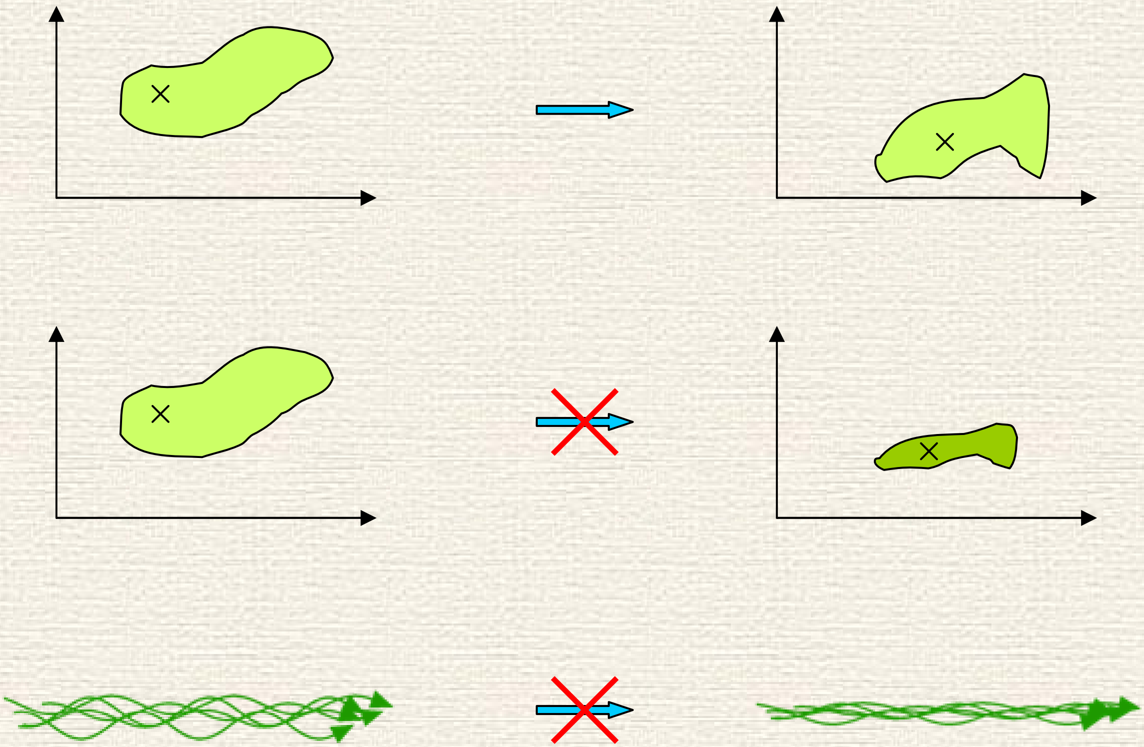


In either case, the theorem says that the phase-space density, measured along the trajectory of a given point, does not change in time.

Fine print: Liouville's theorem in μ space requires non-interacting particles in a conservative dynamical system, evolving according to Hamilton's equations. In Γ space the theorem is valid also for interacting particles. In practice it often works in μ space even if particles interact, as long as hard collisions can be neglected.

Liouville's Theorem, in Simple Words

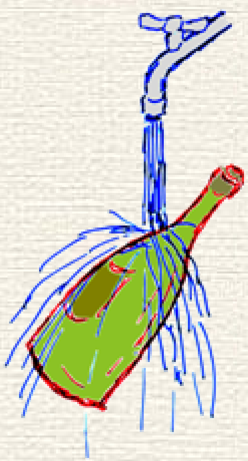
No fancy scheme of magnets or electrostatic fields can shrink the beam emittance



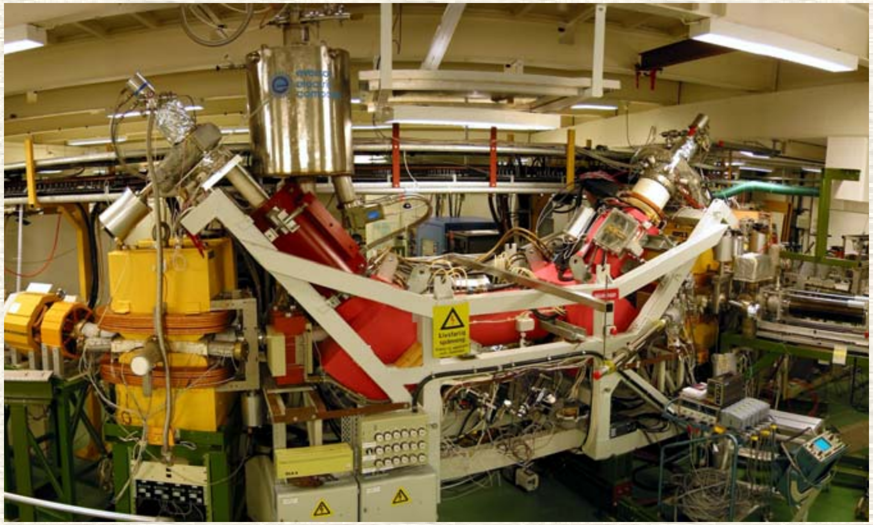
Electron Cooling

Electron cooling is like pouring cold electrons on the warm ions. Heat is transferred from the circulating ions to fresh, cold electrons through Coulomb interactions.

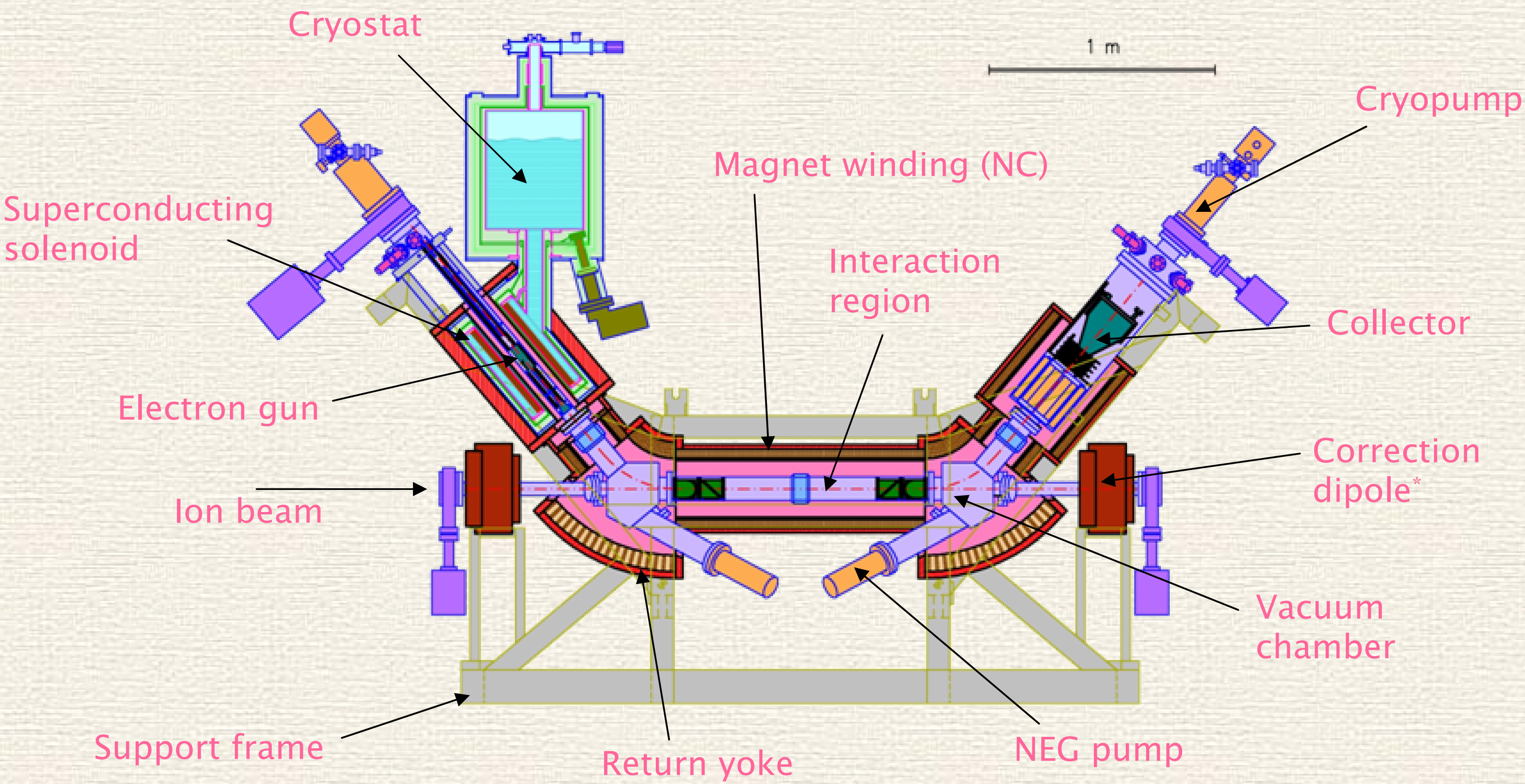
Principle



Practice



CRYRING Electron Cooler



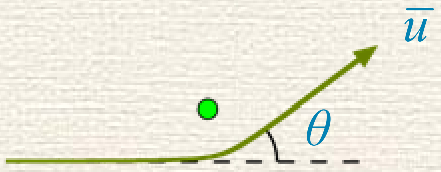
*) The electrons are deflected vertically in the toroids, but the ions deflect horizontally. Why?

Electron Cooling Force

In a microscopic picture, the ions are slowed down through collisions with the electrons, experiencing a friction. How large is the friction? This can be estimated by studying binary encounters between ions and electrons.

A starting point is the Rutherford cross section:

$$\sigma(\theta) = \frac{Z^2 q^4}{4(4\pi\varepsilon_0)^2 \mu^2 u^4} \frac{1}{\sin^4(\theta/2)}.$$



Here, Z is the ion charge state and μ is the reduced mass $m_i m_e/(m_i+m_e)$. From the figure it is seen that

$$\Delta u_z = u(1-\cos\theta) = 2u\sin^2(\theta/2),$$

$$\langle \Delta u_z \rangle = \int_\Omega \Delta u_z \sigma(\theta) u \,\mathrm{d}\Omega = 2\pi \int_{\theta_{min}}^{\theta_{max}} \Delta u_z \sigma(\theta) u \sin\theta \,\mathrm{d}\theta.$$

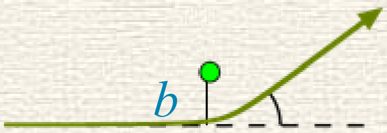
Electron Cooling Force

The integral is simple to solve, and the result is

$$\langle \Delta u_{\mathrm{z}} \rangle = -4\pi \left(\frac{Zq^2}{4\pi\varepsilon_0} \right)^2 \frac{L_{\mathrm{C}}}{\mu^2 u^2}, \qquad \text{(and } \langle \Delta u_{\mathrm{x}} \rangle = \langle \Delta u_{\mathrm{y}} \rangle = 0 \text{)}$$

where we have introduced the Coulomb logarithm

$$L_{\mathrm{C}} = \ln \frac{\sin(\theta_{\max}/2)}{\sin(\theta_{\min}/2)} \approx \ln \frac{\sin\theta_{\max}}{\sin\theta_{\min}} \approx \ln \frac{b_{\max}}{b_{\min}}.$$



We are more interested in the change in ion velocity v_{i} than in the relative velocity u, and we use the fact that ions are much heavier than electrons:

$$\langle \Delta \bar{v}_{\mathrm{i}} \rangle = -4\pi \left(\frac{Zq^2}{4\pi\varepsilon_0} \right)^2 \frac{L_{\mathrm{C}}}{m_{\mathrm{i}} m_{\mathrm{e}}} \frac{\bar{v}_{\mathrm{i}} - \bar{v}_{\mathrm{e}}}{\left| \bar{v}_{\mathrm{i}} - \bar{v}_{\mathrm{e}} \right|^3}.$$

Electron Cooling Force

This corresponds to a force acting on the ions

$$\overline{F}_\mathrm{i} = -4\pi\left(\frac{Zq^2}{4\pi\varepsilon_0}\right)^2 \frac{L_\mathrm{C}}{m_\mathrm{e}} \frac{\overline{v}_\mathrm{i} - \overline{v}_\mathrm{e}}{\left|\overline{v}_\mathrm{i} - \overline{v}_\mathrm{e}\right|^3}.$$

The force has to be integrated over the (flattened Maxwellian) velocity distribution of the electrons, giving

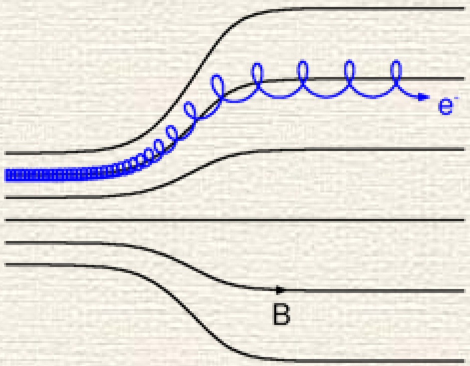
$$\overline{F}_\mathrm{i} = -4\pi\left(\frac{Zq^2}{4\pi\varepsilon_0}\right)^2 \frac{n_\mathrm{e} L_\mathrm{C}}{m_\mathrm{e}} \int f(\overline{v}_\mathrm{e}) \frac{\overline{v}_\mathrm{i} - \overline{v}_\mathrm{e}}{\left|\overline{v}_\mathrm{i} - \overline{v}_\mathrm{e}\right|^3} \,\mathrm{d}^3 v_\mathrm{e},$$

where n_e is the electron density, L_C is treated as a constant (around 10) and

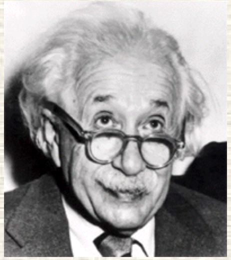
$$f(\overline{v}_\mathrm{e}) = \frac{m_\mathrm{e}}{2\pi k T_\perp}\left(\frac{m_\mathrm{e}}{2\pi k T_\parallel}\right)^{1/2} \exp\left(-\frac{m_\mathrm{e}}{2kT_\perp} v_{\mathrm{e}\perp}^2 - \frac{m_\mathrm{e}}{2kT_\parallel} v_{\mathrm{e}\parallel}^2\right).$$

Electron Cooling Force

The electrons are emitted from a cathode at about 900 °C ($kT \approx 100$ meV). When they are accelerated, the longitudinal energy spread in the beam frame of reference shrinks (show this!) down to $kT_{||} \approx 0.01$ meV. The transverse electron temperature can be reduced down to $kT_{\perp} \approx 1$ meV by expanding the electron beam between the gun and the interaction region.



Colder electrons in principle cool better, but the magnetic field also can make cooling stronger. Effects of the magnetic field enter into the Coulomb logarithm, and this is where the theory becomes difficult!

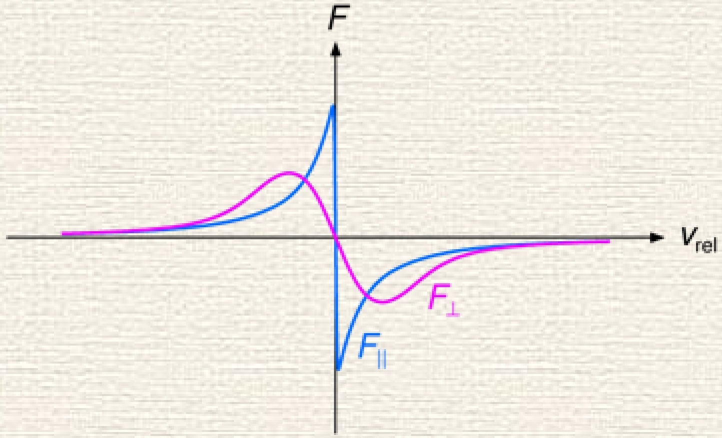


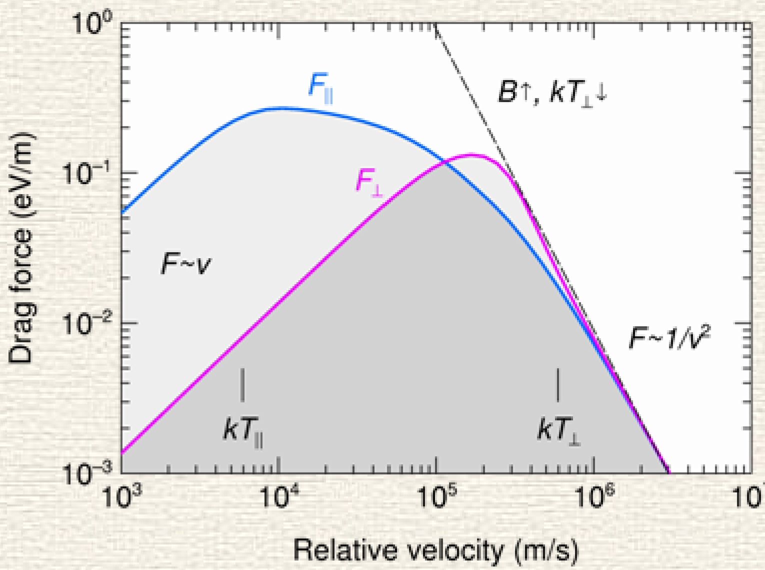
Relativity makes electron cooling slower: For example, the longitudinal drag rate is $\sim 1/\gamma$, and the transverse cooling time is $\sim\gamma^5$. That is one reason why electron cooling is (so far!) not used for relativistic beams.

Electron Cooling Force

The drag force is proportional to the relative velocity v for small v and to $1/v^2$ for v large compared to the thermal electron velocities.

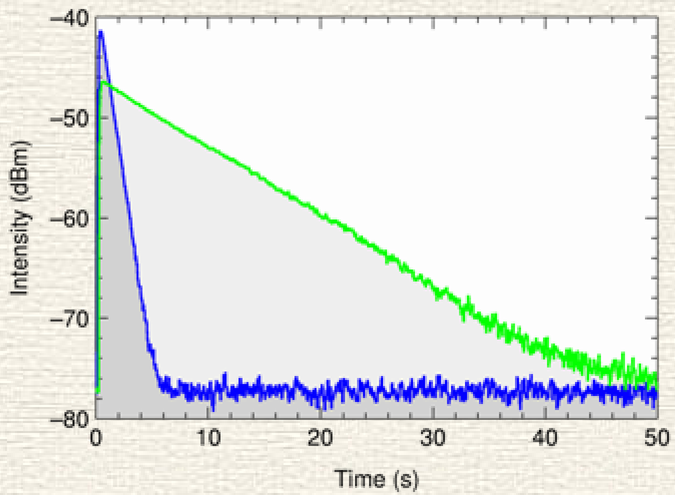
Electron cooling is thus fast and characterized by a time constant or rate for small v, but it is slower for large v.





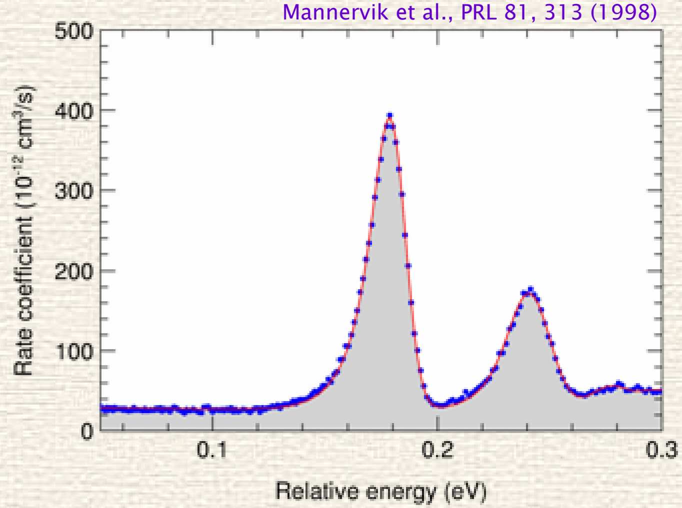
Transverse and longitudinal drag force as functions of the corresponding relative velocity component, calculated with $kT_\perp = 100$ meV, $kT_{||} = 0.1$ meV and without effects of the magnetic field. For singly charged ions and $n_e = 1\times10^{14}\ m^{-3}$.

Recombination



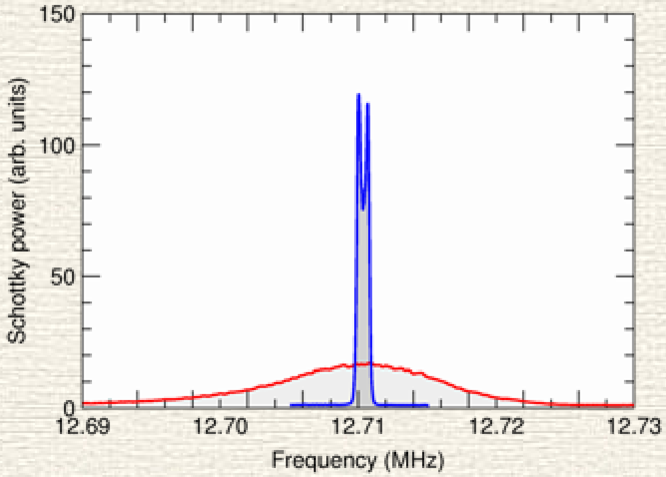
Recombination between highly charged ions and electrons can reduce the beam lifetime. An extreme example is Pb^{53+}, where a 53 mA electron beam reduced the lifetime in CRYRING from 12.8 to 1.2 s due to a dielectronic resonance near 0 eV.

Mannervik et al., PRL 81, 313 (1998)



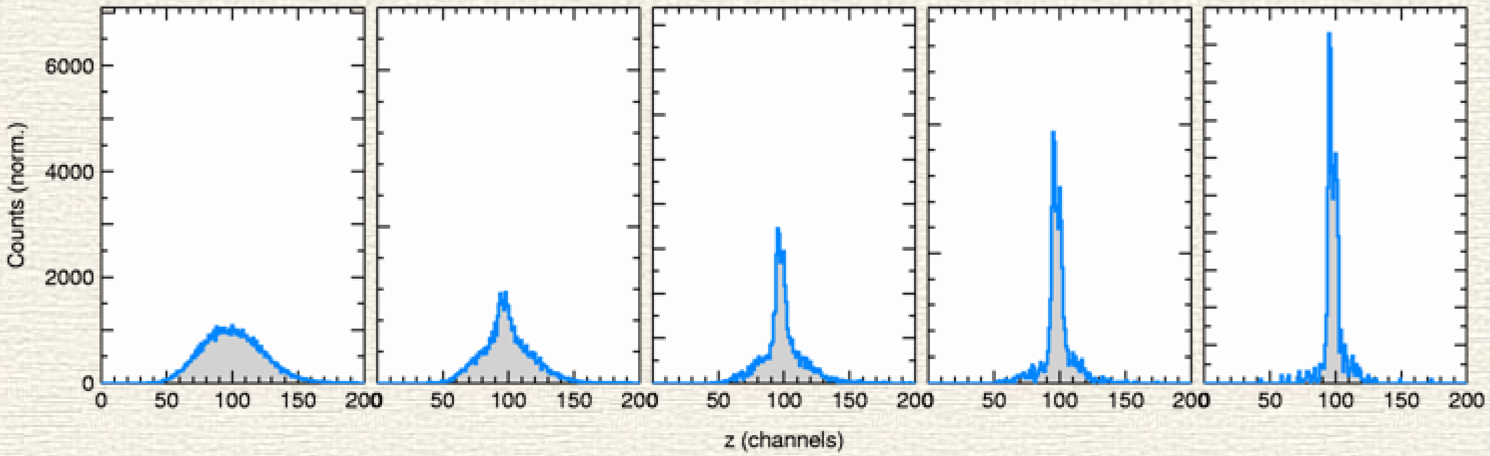
The peak shape of dielectronic recombination resonances can be used to determine the electron temperature very accurately. Here, the good fit between experiment (blue) with C^{3+} ions and theory (red) shows that the electron temperature is $kT_{\perp} = 9.4$ meV and $kT_{||} = 0.08$ meV.

Cooled Beams



Longitudinal cooling of deuterons in CRYRING.

Transverse cooling of H^- in CRYRING, Δt = 300 ms. Note faster cooling of core.



Electron Cooling, Summarized

In a macroscopic picture, heat is transferred from a hot ion beam to a cold electron beam

Microscopic picture includes Coulomb interactions and binary collisions

Friction-like force on ions

Time scale from milliseconds to hours

Difficult for high energies and hot ion beams

Hardware is mainly magnets (high-quality field), electron optics with gun and collector, and vacuum

Now in operation at CERN, GSI, Heidelberg, Jülich, Århus, Stockholm, Uppsala, Tokyo, Chiba, and soon Fermilab and Lanzhou

Stochastic Cooling

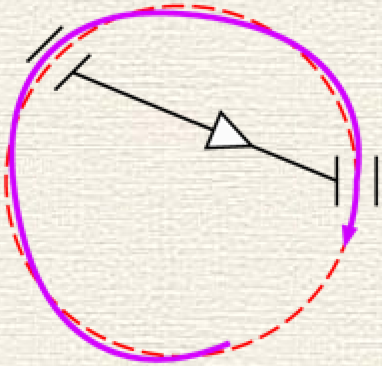
Liouville's theorem of course doesn't hold if an external device detects particle positions and moves the particles onto better orbits. Stochastic cooling uses beam pickups and kickers to perform this.

Principle

Practice

R. Pasquinelli, COOL03

Transverse Stochastic Cooling



Assume that we have a pickup, amplifier, etc., with a bandwidth W. According to the Nyquist theorem, it samples the beam with a time resolution $T_s=1/(2W)$. The number of particles passing through the detector during that time is $N_s=N\,T_s/T=N/(2WT)$ if N is the total number of particles in the ring and T their revolution time.

In a simple model of transverse (or betatron) cooling, a difference pickup measures the centre of gravity of the sample which has a displacement

$$\langle x \rangle = \frac{1}{N_s}\sum_{i=1}^{N_s} x_i,$$

and a kicker corrects the position of the sample, moving it by $\Delta x = -\langle x \rangle$, such that its centre of gravity ends up on the nominal orbit. (The kicker actually changes the direction of motion of the particles, of course.)

Stochastic Cooling Rate

Let's separate a test particle from all other particles and write

$$\langle x\rangle = \frac{1}{N_s}\sum_i x_i = \frac{x_t}{N_s} + \frac{1}{N_s}\sum_{i\neq t} x_i \equiv \frac{x_t}{N_s} + \langle x\rangle^*.$$

The test particle, like all other particles, is displaced by $\Delta x = -\langle x\rangle$. We can thus be write

$$\Delta x_t = -\langle x\rangle = -\frac{x_t}{N_s} - \langle x\rangle^*.$$

Clearly, the displacement of the test particle depends on its own position but also on all the other particles in the beam. If we take the (drastic) step of neglecting the influence of all other particles, i.e., the last term, we find a cooling rate $\Delta x_t / x_t = 1/N_s$ per turn. After multiplication with the revolution frequency we end up with the cooling rate per second:

$$\frac{1}{\tau} = \frac{1}{TN_s} = \frac{2W}{N}.$$

Stochastic Cooling Rate

Surprisingly, this simple derivation overestimates the actual, optimal cooling rate by only a factor of 2. A more rigorous derivation gives a rate

$$\frac{1}{\tau} = \frac{W}{N}\left[2g - g^2(M+U)\right].$$

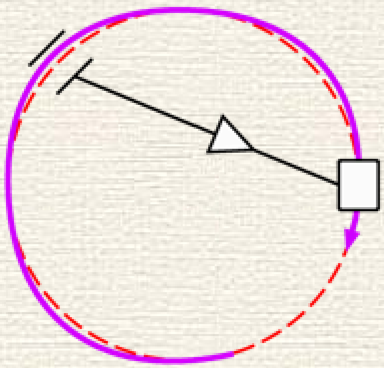
This is the cooling rate for the amplitude x. The cooling rate for the emittance, proportional to x^2, is twice as big. We have now also intro-duced the gain g, the mixing factor M and the noise-to-signal ratio U.

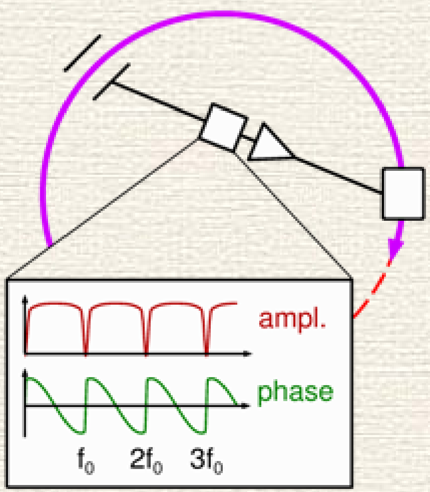
The gain is proportional to the electronic gain, and it is defined as the ratio $-\Delta x/\langle x \rangle$. The mixing factor tells how many turns it takes for the samples to get randomized after a correction. U depends on both the beam and the pickup (signal strength) and on the electronics (noise). The so-called bad mixing is neglected here. Optimal cooling rate is obtained when $g=1/(M+U)$ and is equal to

$$\frac{1}{\tau} = \frac{W}{N}\frac{1}{M+U}.$$

Longitudinal Stochastic Cooling

There are two basic types of momentum cooling: One technique, Palmer cooling, uses the corre- lation between momentum and position in regions of high dispersion. A transverse difference pickup connected to a longitudinal kicker, or rf gap, then can provide cooling. The expression for cooling rate is the same as for the transverse case.





The other technique, filter cooling, uses notch filters that have a period equal to the desired revolution frequency and a gain that changes sign at the centre of each Schottky band. A signal from a sum pickup above the centre frequency gives a decelerating field in the kicker and vice versa.

Stochastic Cooling, Summarized

The best would be a “Maxwell’s demon” that puts individual particles onto correct orbits

But it is enough to take samples of a larger number of particles and put their average positions right, if the process is repeated many times.

Hardware is mainly pickups, amplifiers, filters and kickers

Important things are band-width, power, signal strength and noise

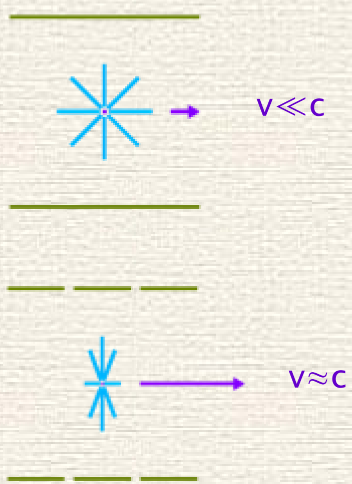
Extension to optical frequencies?

Now in operation at CERN, Fermilab, GSI, FZ Jülich

Stochastic Cooling vs. Electron Cooling

Stochastic cooling works best with relativistic particles because one can then reach higher bandwidths, up to 4–8 GHz. This is because the particles move rapidly past the detectors and kickers and because their charge distribution is Lorentz contracted.

In a "small accelerator" with, e.g., $v=0.1c$ and pickup length $l=0.1$ m, the maximum bandwidth will be in the order of v/l or 300 MHz. In this sense, stochastic cooling is complementary to electron cooling.



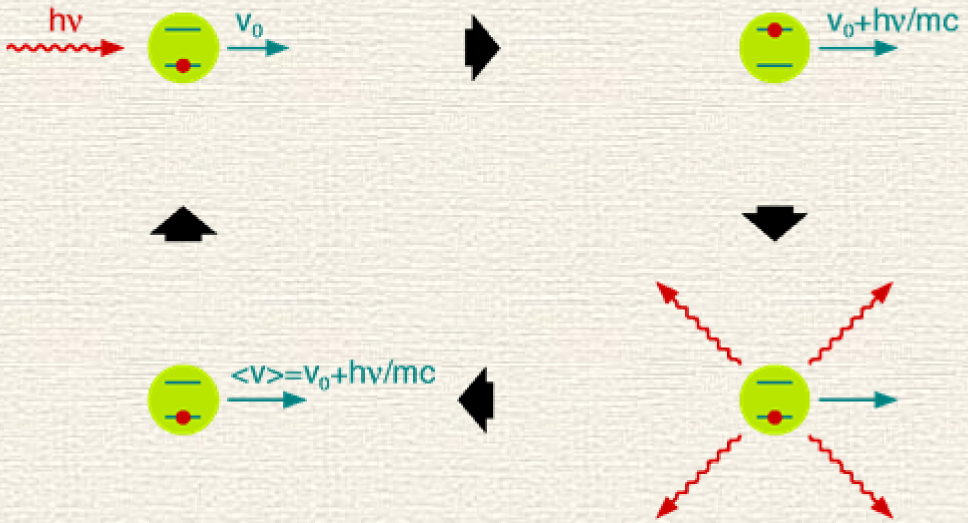
Other differences and similarities

- Stochastic cooling is faster for weak beams than for intense beams, while electron cooling is independent of beam intensity.
- Stochastic cooling is better for hot beams, because of better signal, while electron cooling gets slower for hot beams.
- With both methods, highly charged ions cool faster. In stochastic cooling because of better signal-to-noise ratio.
- Electron cooling is easier in case of variable beam velocities.

Laser Cooling

In laser cooling, a velocity-dependent (non-conservative!) force is obtained when ions absorb photons, and thus momentum, depending on their velocity through the Doppler shift.

Momentum is absorbed from one direction but re-emitted isotropically. A net momentum transfer results.



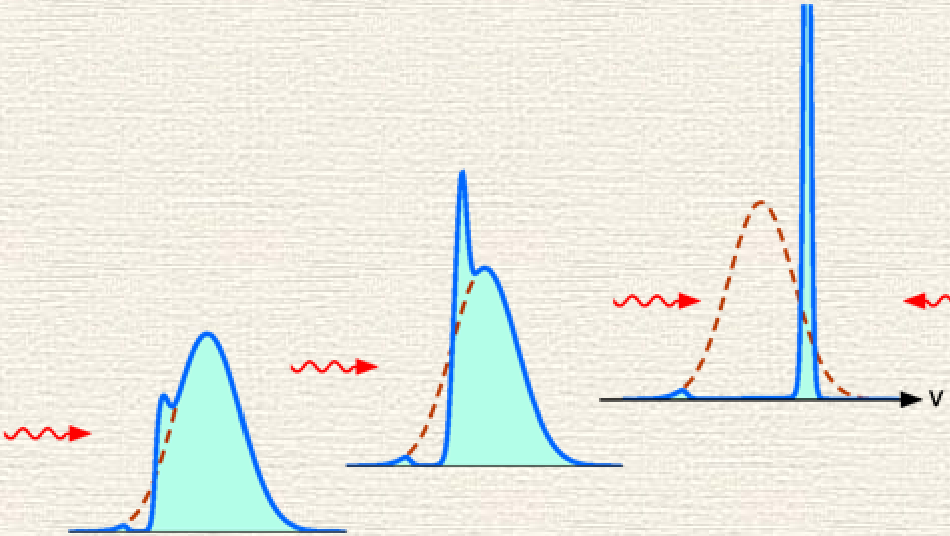
Laser cooling works with ions having closed, two-state transitions at optical wavelengths (or transitions that are Doppler shifted to optical wavelengths).

Laser cooling requires specific ions, is difficult transversally, but is strong and faster than electron or stochastic cooling.

Laser Cooling

The range of the laser force is short, so one needs to sweep the laser frequency, or change the ion velocity, to cover the whole ion velocity distribution.

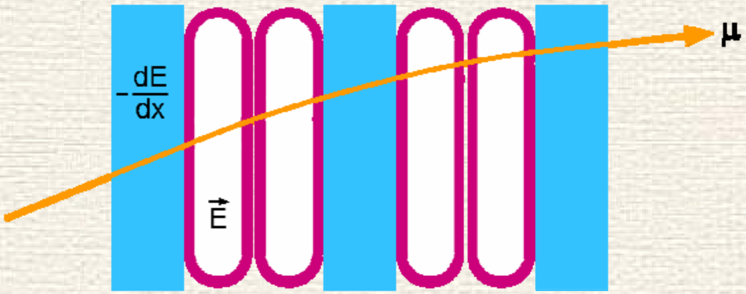
To reach a cold equilibrium one also needs a restoring force. This can be a achieved with, e.g., a counter-propagating laser, the rf or an induction accelerator.



So far, laser cooling has only been applied to $^{7}Li^{+}$, $^{9}Be^{+}$, $^{24}Mg^{+}$ and, recently, C^{3+} where the transition is Doppler-shifted into the optical wavelength range.

Ionization (Muon) Cooling

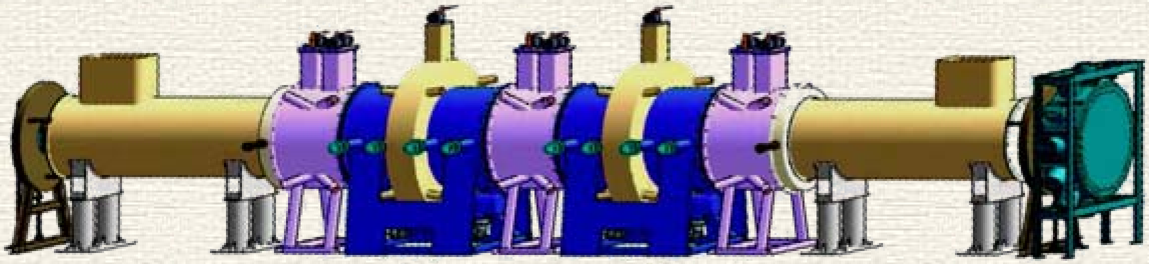
With ionization cooling, all momentum components are reduced when the particles pass through matter, but longitudinal momentum only is restored in rf cavities. The net result is a reduction of transverse momentum.



Ionization cooling only works with muons. Hadrons make nuclear reactions and electrons produce bremsstrahlung. Also, emittance increase due to scattering must not dominate over the cooling effect.

Longitudinal cooling through dispersion and wedge absorbers.

MICE, Muon Ionization Cooling Experiment



The End