Beam lines

Alessandra Lombardi
CERN

Contents

- definition of emittance [see D. Moehl]
- definition of a matched beam
- transfer lines between accelerators and their function
- dynamics in elements of a transfer line , drift, quadrupole, solenoid, bending magnet ....

Emittance-preamble

- Acceleration is a controlled manipulation of an ensemble of charge particles
- Controlled means knowing the law that relates the six (four) coordinates of each particle at any time : position and momentum in each of the three planes (x,px,y,py,z,pz,t)

- 6D Phase space representing a beam of particles contains some 10^6-10^10 points
- Need to define a GLOBAL statistical variable representative of the status of the beam at any time

Particle coordinate definition

Each particle is defined by position and momentum：

$$\vec{x} = (x, p_x, y, p_y, z, p_z)$$

units of meter and eV/c

More convenient is to use position and divergence

$$\vec{x} = (x, x_p = \frac{p_x}{p_z}, y, y_p = \frac{p_y}{p_z}, z, \frac{\Delta p}{p})$$

PARAXIAL APPROXIMATION!!!!

Units of meter and “radians”

Emittance-r.m.s. definition

- Each particle is defined by $x, x_p = px/pz$,

$$E_{rms} = \pi \sqrt{\langle x^2 \rangle \cdot \langle x_p^2 \rangle - \langle x \cdot x_p \rangle^2} \quad meters$$

< > is the average over the beam distribution

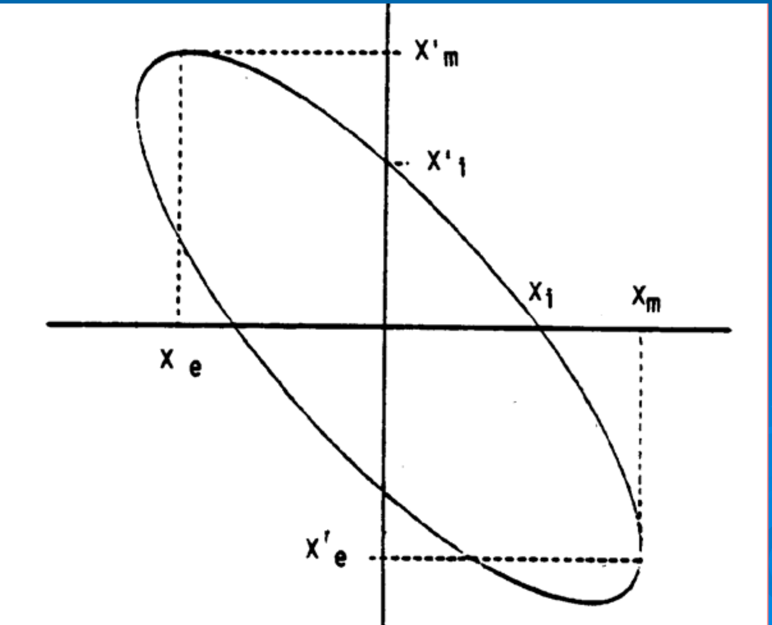
- Statistical propertry of a beam of particles which relates to the volume occupied by this beam in the phase volume

Emittance-Courant Snyder

Under the influence of <u>linear forces</u> the trajectory of a particle in phase space (e.g. x,xp) follows an elliptical path and can be characterized by three parameters (α,β,γ) which follow the relation :

$$\gamma \cdot x^2 + 2 \cdot \alpha \cdot x \cdot x_p + \beta \cdot x_p{}^2 = \frac{E}{\pi} = \varepsilon$$

$$\beta\gamma - \alpha^2 = 1$$

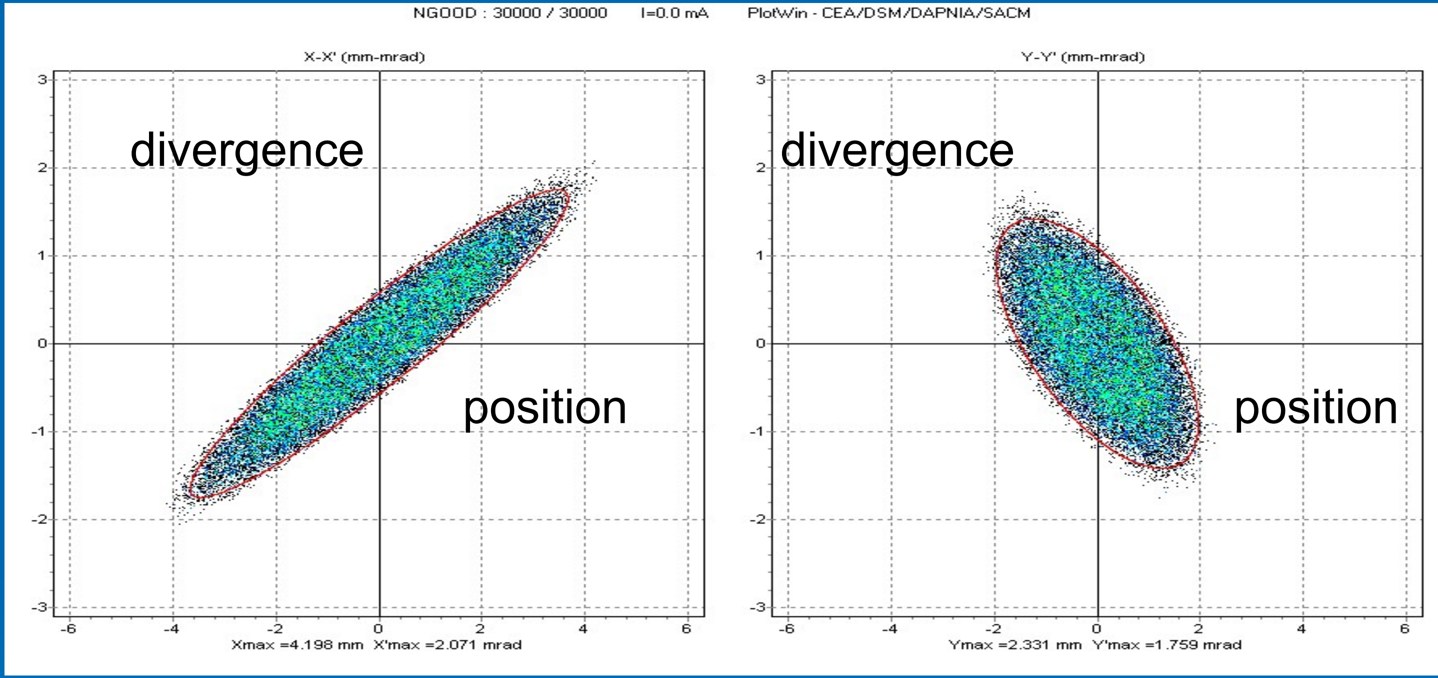


And relate to the beam size and divergence :

$$x_e = -\alpha\sqrt{\varepsilon/\gamma} \quad x_i = \sqrt{\varepsilon/\gamma} \quad x_m = \sqrt{\beta\varepsilon}$$

$$x'_e = -\alpha\sqrt{\varepsilon/\beta} \quad x'_i = \sqrt{\varepsilon/\beta} \quad x'_m = \sqrt{\gamma\varepsilon}$$

Transverse phase space and focusing



Bet a= 6.37 mm/Pi.mrad
Alpha = -2.88

DEFOCUSED

Beta = 1.80 mm/Pi.mrad
Alpha = 0.83

FOCUSED

Matching a beam

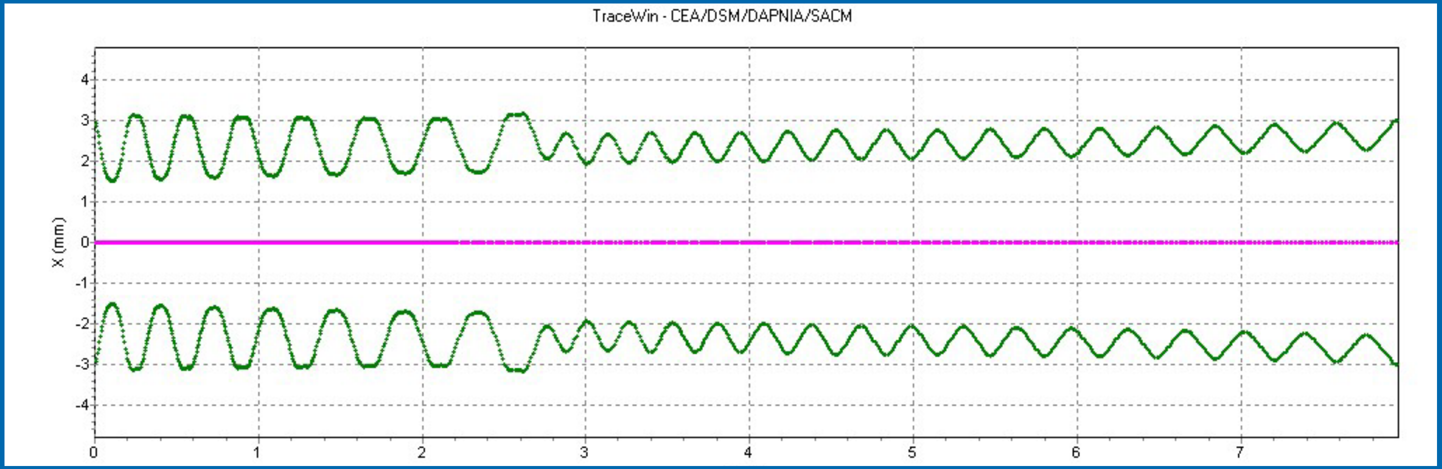
- Preparing the beam to the focusing structure of the accelerator
- [You will learn later that] in an accelerator one tries to give energy to the beam while keeping the both the transverse dimension confined. From the transverse point of view the accelerator is build up as a (super) periodic structure where the envelope of the beam oscillates periodically and the normalised phase space portrait is identical after each period.

- There is only one orientation of the input phase space for which the above conditions are met.

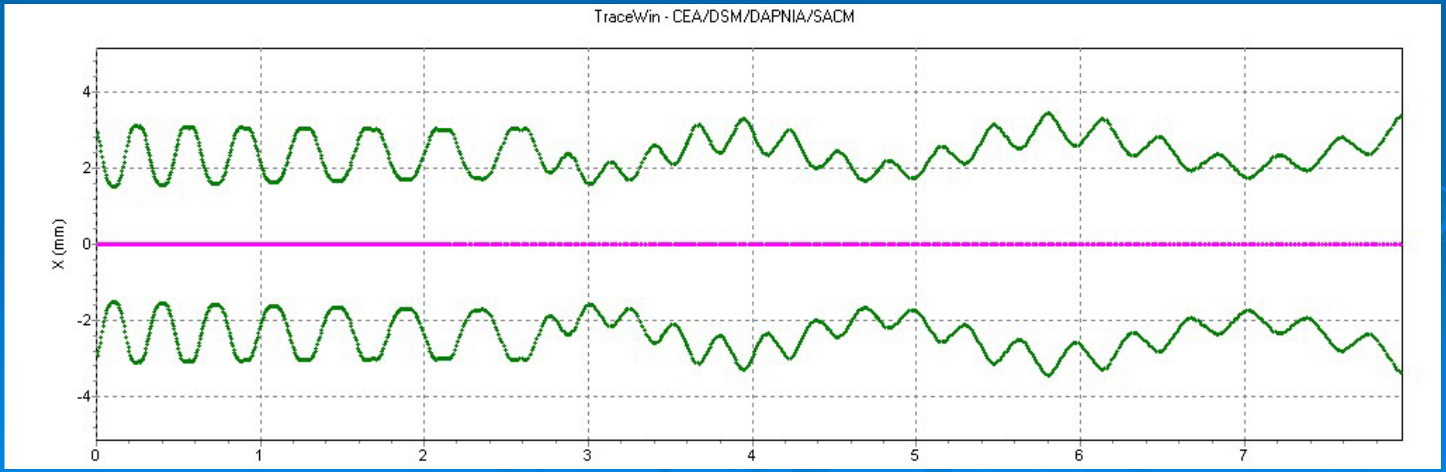
- Bringing the beam at the input of the accelerator with the correct α and β in each plane is called "matching the beam" and its done with magnetic elements in a beam line

Matching a beam

BEAM IS MATCHED



BEAM IS MIS-MATCHED

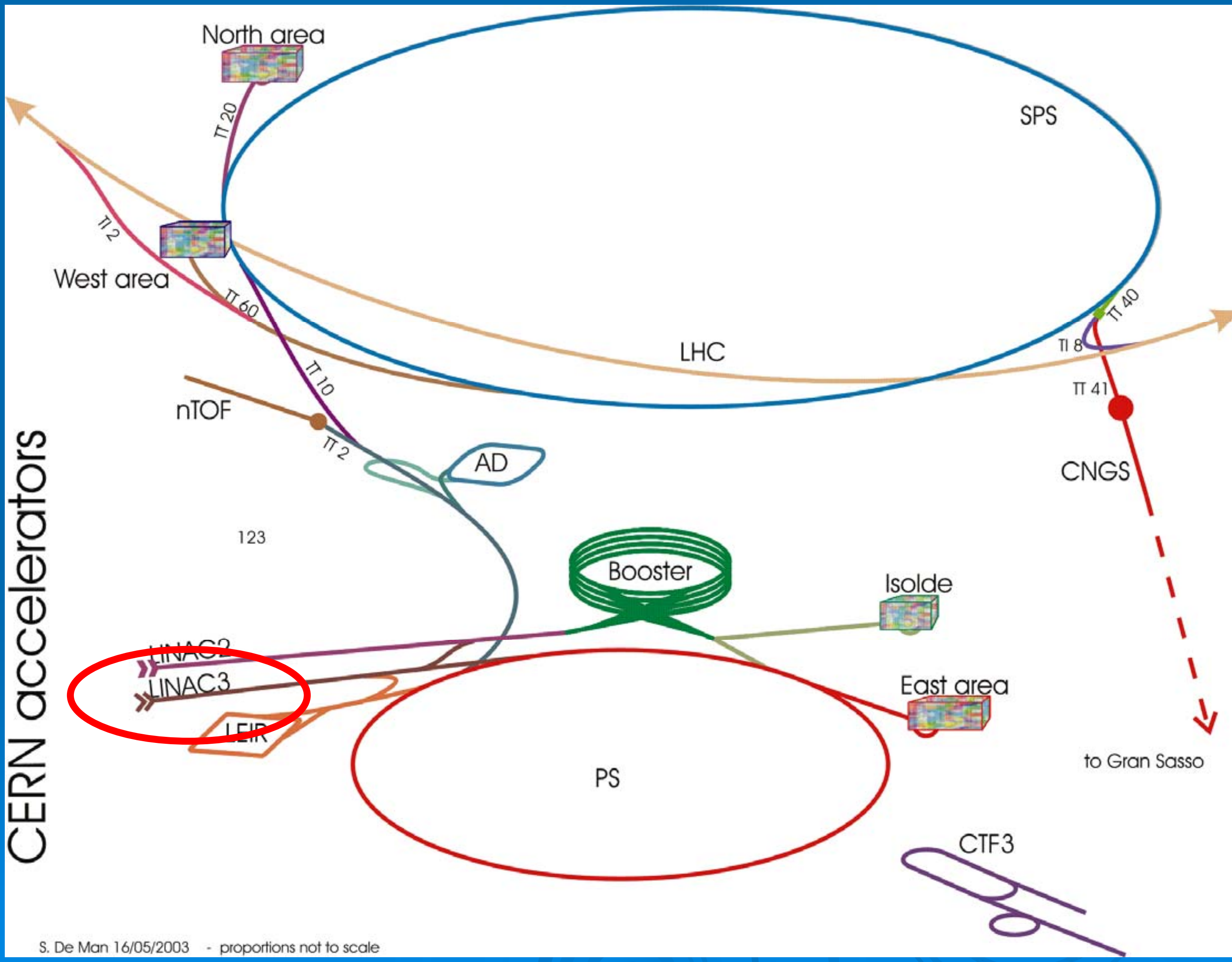


Program TRACEWIN –CEASaclay-D.Uriot,R.Duperrier,N.Pichoff

Transfer line-need for a

- Adapt the beam parameters from one stage of acceleration to the other (often accelerator systems have been built in stages)
- Leave space for diagnostics, safety elements.
- Delivering a beam to a lower than final energy
- Get rid of unwanted particles in dedicated spots, to avoid activation of downstream accelerators
- Correct for errors
- ………

Typical layout of an accelerator facility



CERN-Lead ion linac

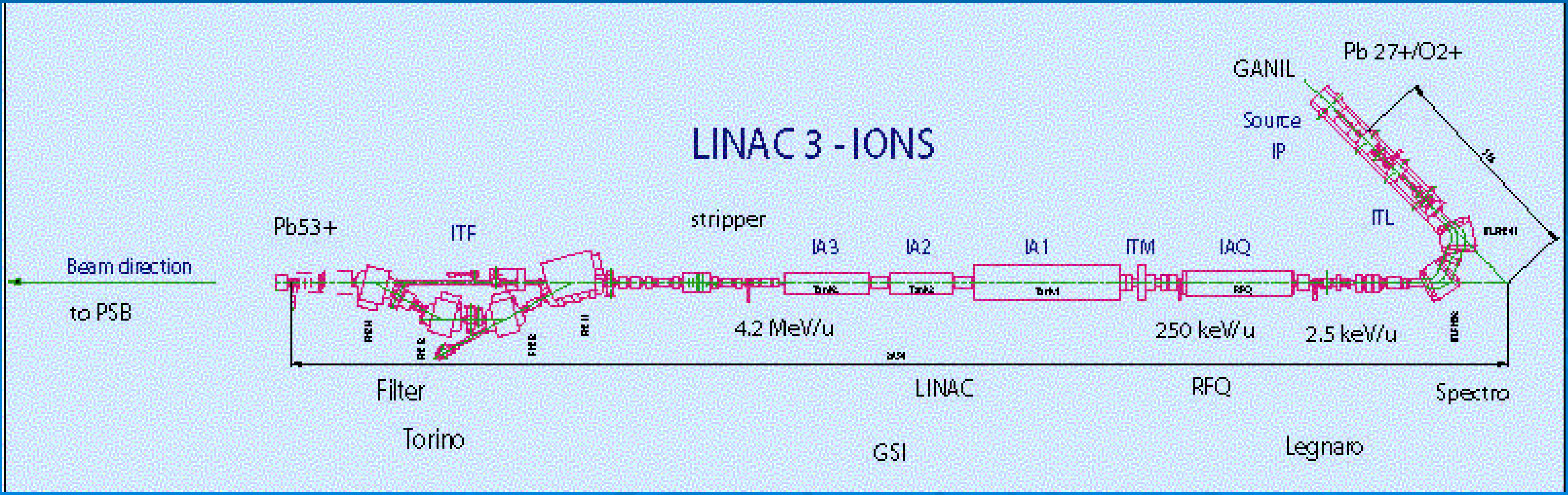
•SOURCE : produces 0.8 mA of lead ions composed of 10 charge states around 25+

•LEBT : selects one charge states (nominal : 25+)

•RFQ (100MHz) + 3 IH tanks (100 and 200MHz) : increase the energy from 0.0025 to 4.2 MeV/u

•STRIPPER : converts lead 25+ in lead 54+ (and 4 adjacent charge states)

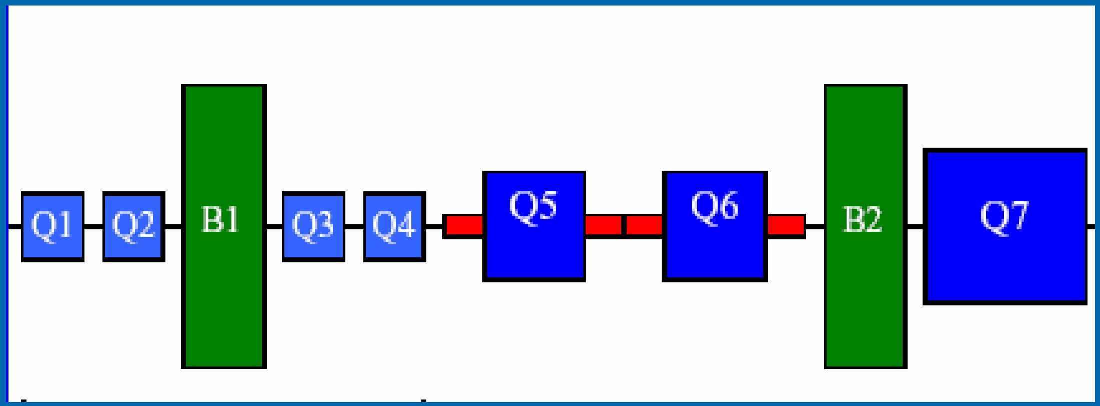
•FILTER LINE : selects one charge states and delivers 25 µA of lead 54+



Transfer line

- From the particle source to the first RF accelerator
- From linear accelerator to linear accelerator
- Injection from a linear accelerator in a circular accelerator [see Jongen]

Calculation of the dynamics in a transfer line



1-Input 2-output

$$\overrightarrow{x_2} = R \cdot \overrightarrow{x_1}$$

$$R = R_7 R_{67} R_6 R_{56} \ldots\ldots\ldots$$

Under the influence of linear force the coordinates at a point 2 are a linear combination of the coordinates at point1

The matrix of the system is the multiplication of the matrix representing each element

dynamics in a transfer line

$$\overrightarrow{x_2} = R \cdot \overrightarrow{x_1}$$

- Given $\vec{x}_1$ can calculate $\vec{x}_2$
- Chosen $\vec{x}_2$ can calculate $\vec{x}_1$
- Modify the element of the R-matrix , i.e. the settings of the line, so that

for a given $\vec{x}_1$ I can obtain a wanted $\vec{x}_2$

MATCHING

R-matrix

$$R = \begin{matrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{matrix}$$

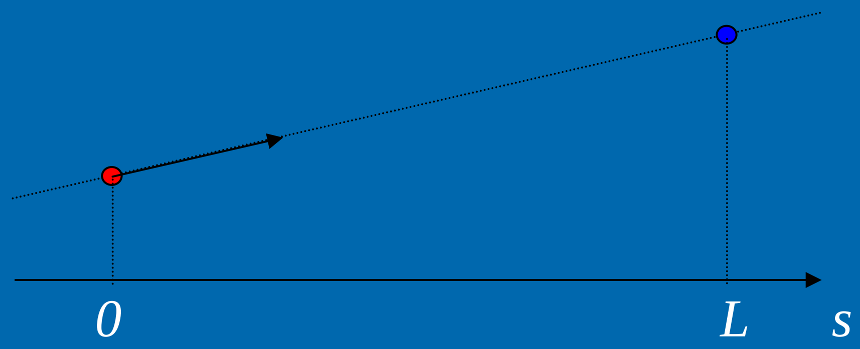
$$\begin{bmatrix} 1 & m & 1 & m & 1 & m \\ m^{-1} & 1 & m^{-1} & 1 & m^{-1} & 1 \\ 1 & m & 1 & m & 1 & m \\ m^{-1} & 1 & m^{-1} & 1 & m^{-1} & 1 \\ 1 & m & 1 & m & 1 & m \\ m^{-1} & 1 & m^{-1} & 1 & m^{-1} & 1 \end{bmatrix}$$

Transfer matrix of typical elements of a transfer line

- Drift
- quadrupoles
- Solenoids
- Trajectory control (steerers,bending)

Drift

- Empty space of length L

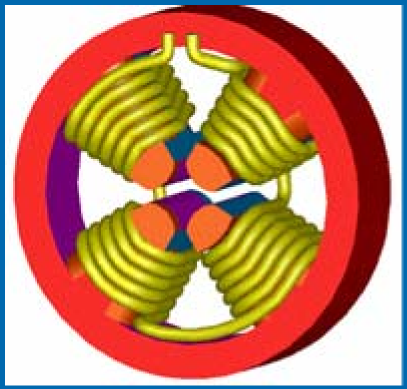


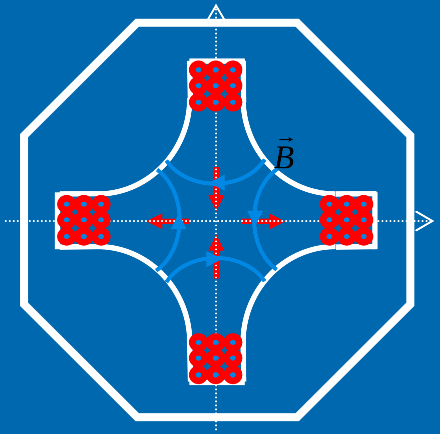
$$x_2 = x_1 + x_{p1} \cdot L$$

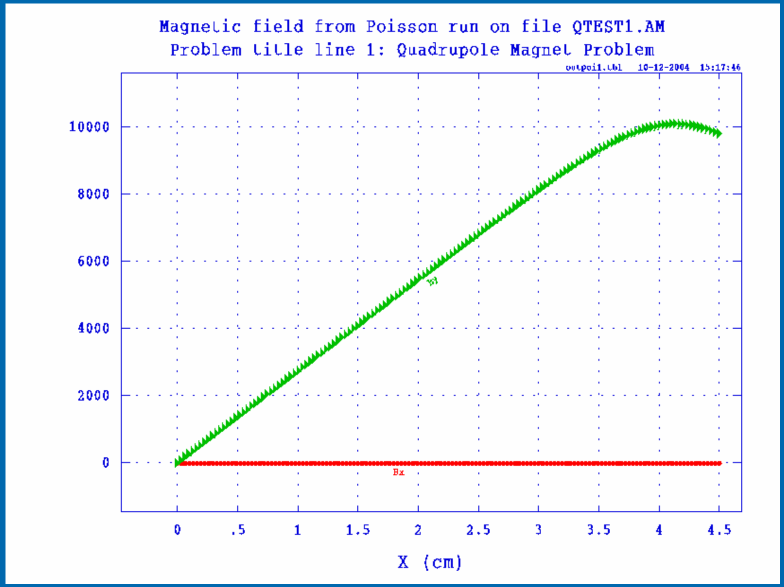
$$x_{p2} = x_{p1}$$

$$R = \begin{pmatrix} 1 & L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

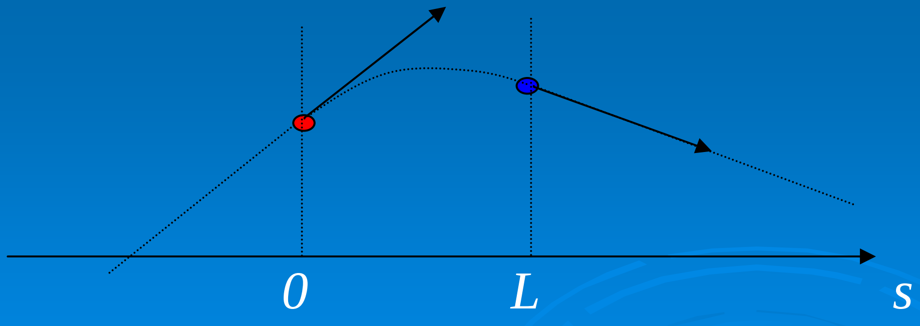
Quadrupole

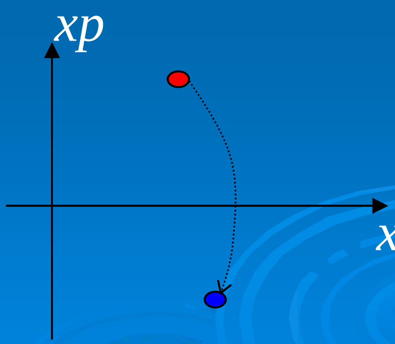






Length L ; gradient B/a ;





Quadrupole

$$B\rho = \frac{m_o c\beta\gamma}{q} = \frac{momentum}{charge}$$

MAGNETIC RIGIDITY in Tesla x meter

$$k = \left[\left| \frac{B'}{B_\rho} \right| \right]^{1/2}$$

K^2 is the STRENGHT or NORMALISED GRADIENT in 1/meter

$$R_{xx} = \begin{pmatrix} \cos(k \cdot L) & \frac{1}{k} \cdot \sin(k \cdot L) \\ -k \cdot \sin(k \cdot L) & \cos(k \cdot L) \end{pmatrix}$$

FOCUSING PLANE

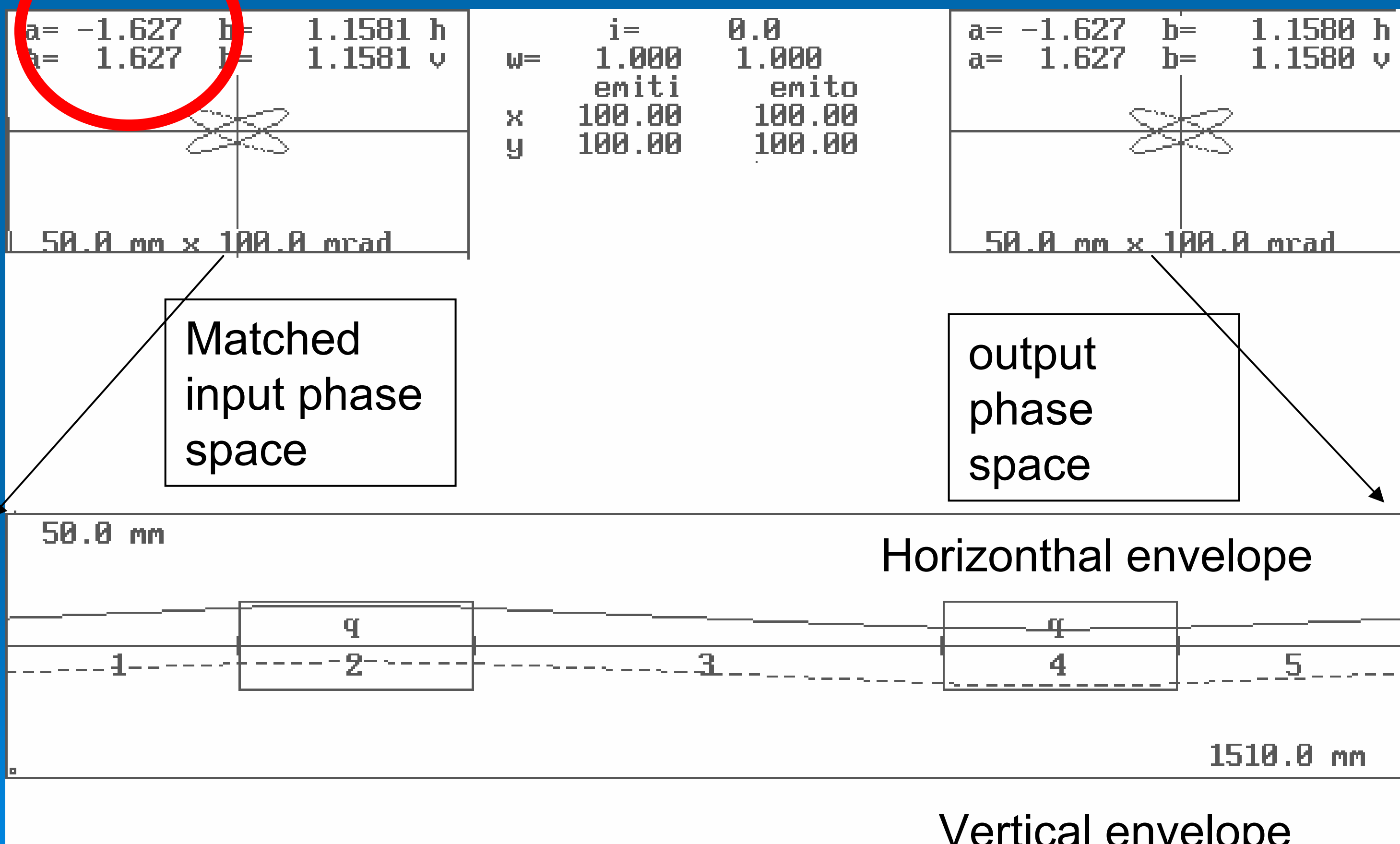
$$R_{yy} = \begin{pmatrix} \cosh(k \cdot L) & \frac{1}{k} \cdot \sinh(k \cdot L) \\ k \cdot \sinh(k \cdot L) & \cosh(k \cdot L) \end{pmatrix}$$

DEFOCUSING PLANE

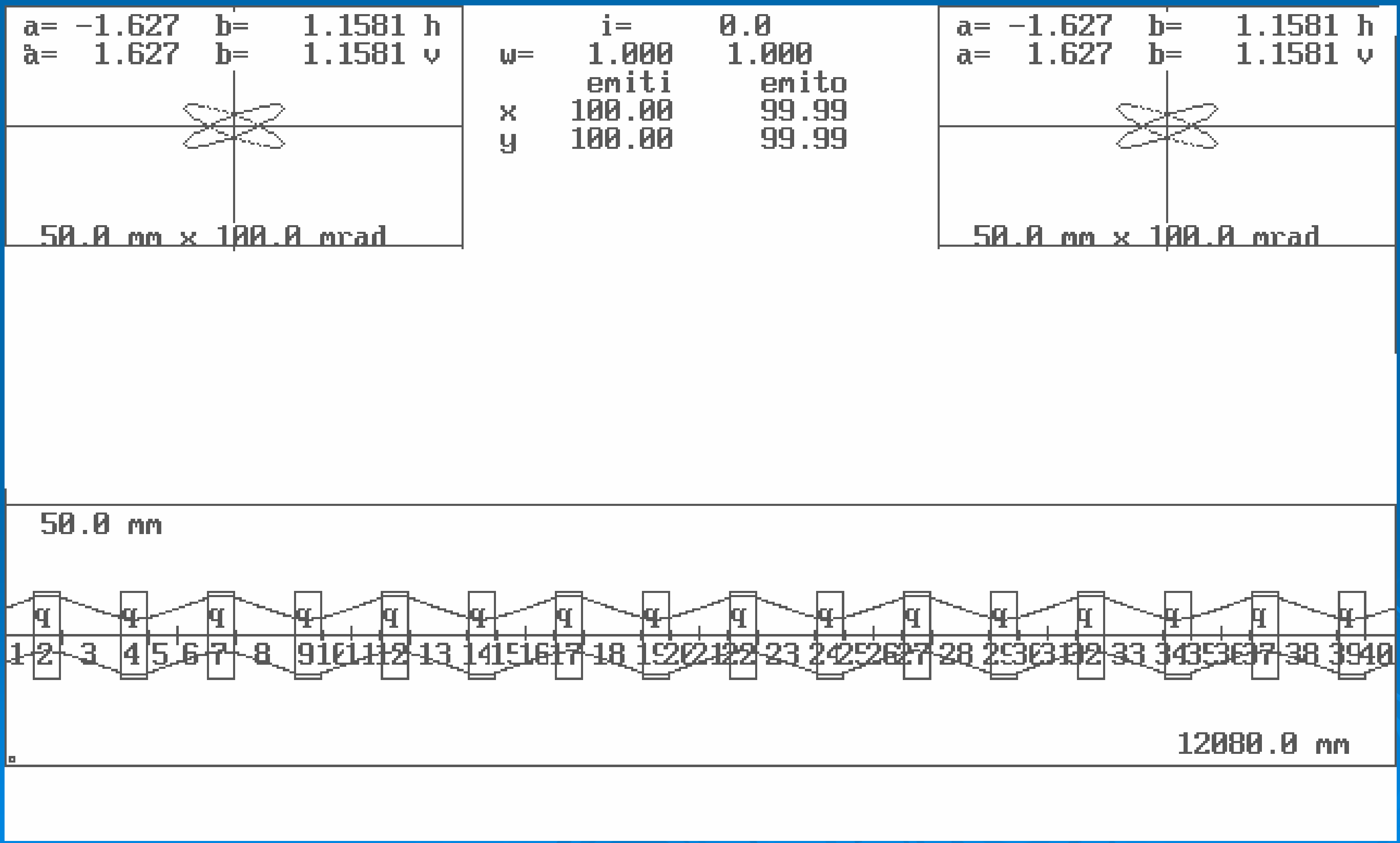
FODO

OPPOSITE SIGN ALPHA

Sequence of a Focusing and defocusing quadrupole



FODO CHANNEL



FODO

- periodic focusing channel : the beam 4D phase space is identical after each period
- Equation of motion in a periodic channel (Hill's equation) has periodic solution :

$$x(z) = \sqrt{\varepsilon_0 \beta(z)} \cdot \cos(\sigma(z))$$

emittance

beta function , has the periodicity of the focusing period

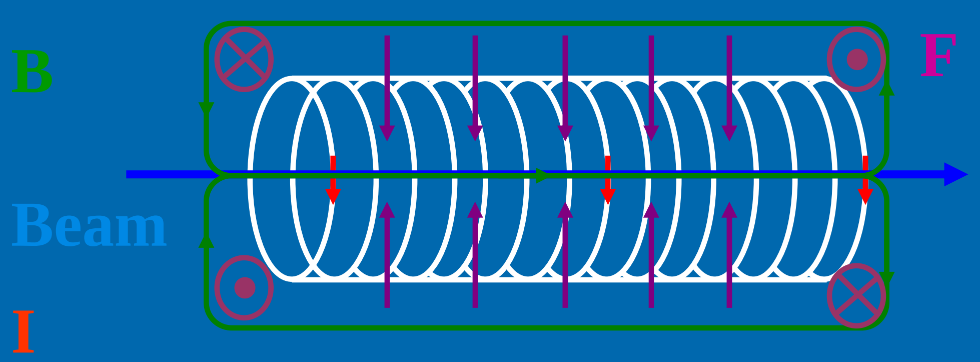
transverse phase advance

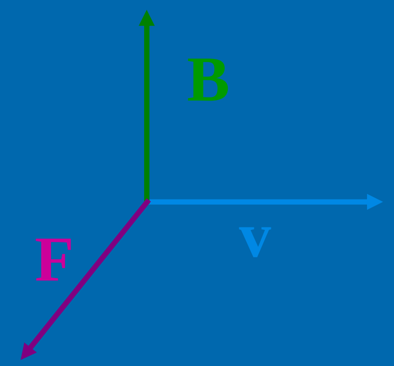
$$\beta(z+l) = \beta(z)$$

$$\sigma(z) = \int_0^z \frac{dz}{\beta(z)}$$

review N. Pichoff course

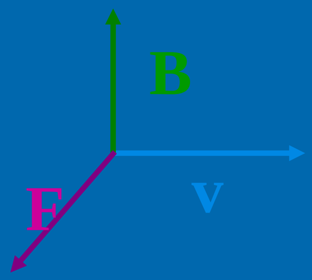
Solenoid





Input : $B = B_\perp$

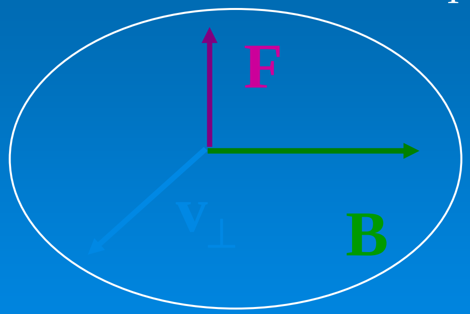
$F \propto v \cdot B$



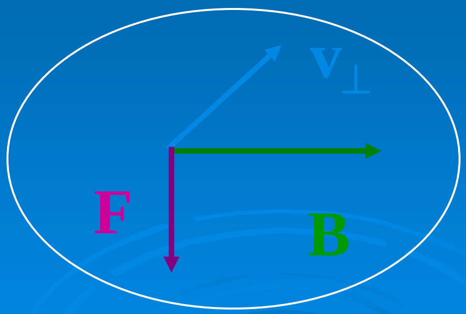
Beam transverse rotation :

$v_\perp \propto v \cdot B \cdot r$

Middle : $B = B_l$



$x < 0$



$x > 0$

$F \propto v_\perp \cdot B \propto v \cdot B^2 \cdot r$

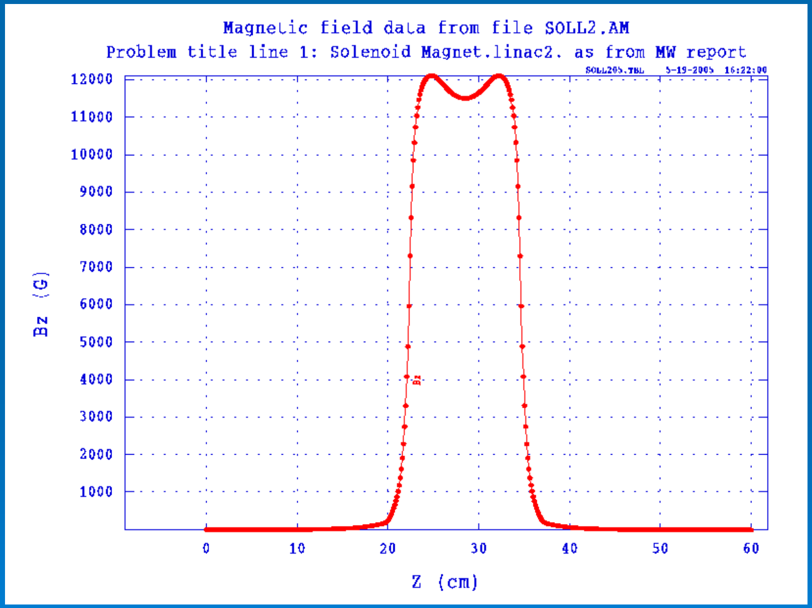
Beam linear focusing

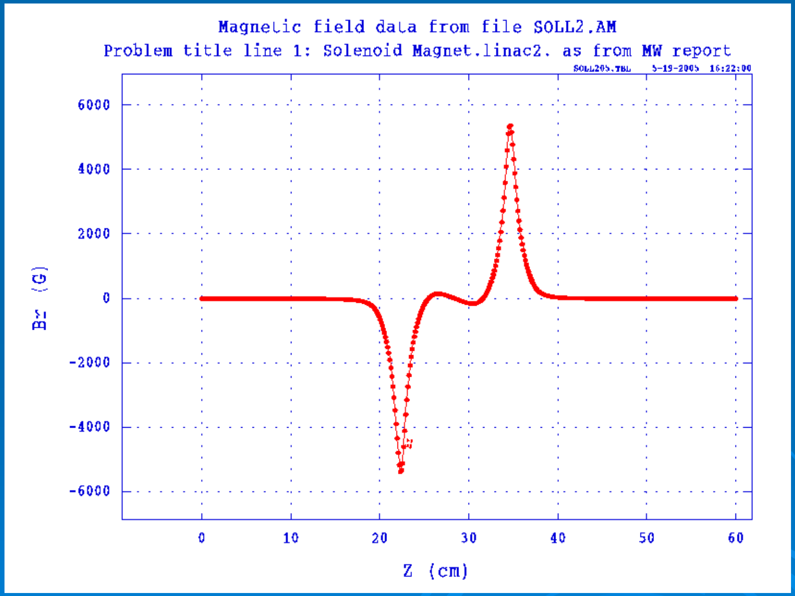
Solenoid

Longitudinal and transverse magnetic field off axis

Rotation

Radial focusing





solenoid

solenoid

Solenoid

$$k = \frac{B}{2B\rho} \quad S = \sin(kL) \quad C = \cos(kL)$$

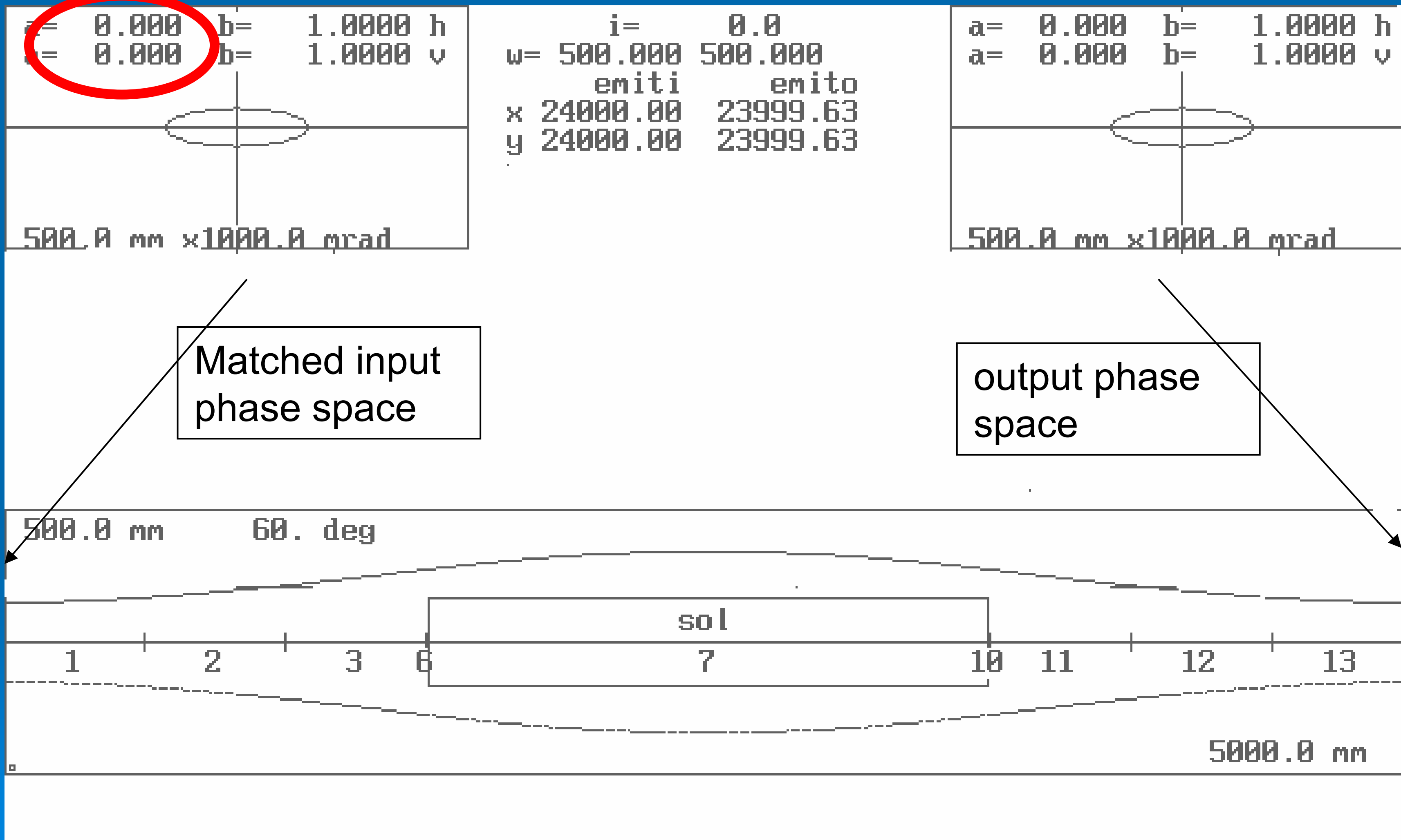
$$R_{xx} = R_{yy} = \begin{pmatrix} C^2 & \frac{1}{k} \cdot SC \\ -k \cdot SC & C^2 \end{pmatrix} \qquad R_{xy} = -R_{yx} = \begin{pmatrix} SC & \frac{1}{k} \cdot S^2 \\ -k \cdot S^2 & SC \end{pmatrix}$$

Focusing in both planes simultaneously

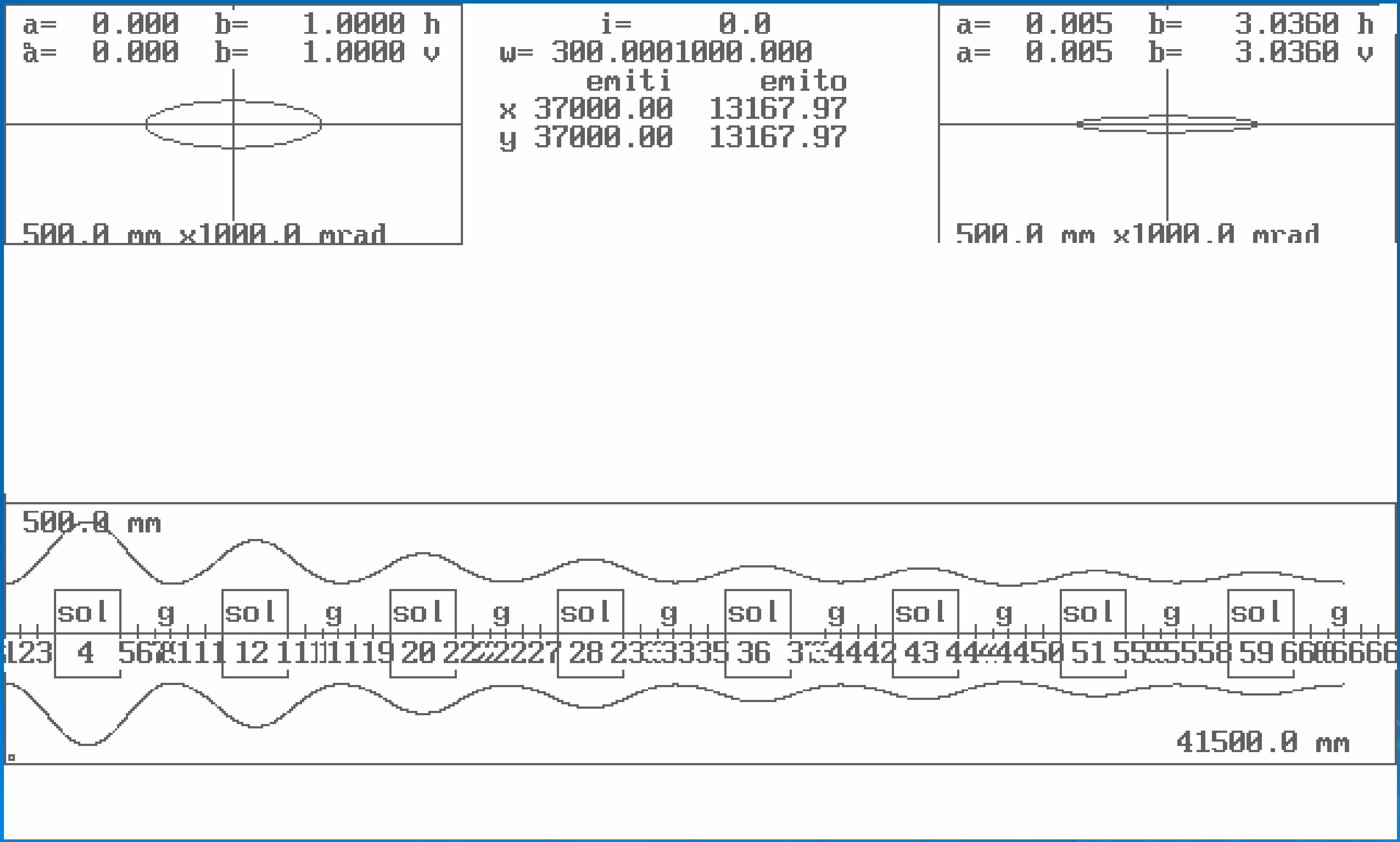
Coupling between x and y planes

FOFO

IDENTICAL x and y



FOFO channel



Summary

- Definition of emittance and matched beam [more by D. Moehl and ….]
- Importance / necessity of the transfer line
- Linear transport in a transfer line for the transverse plane

Further reading

- Most of the material in this lecture follows the convention of TRACE3d (envelope tracking code) and the formulas can be found in the user guide

- M. Reiser ,"Theory and design of charged particle beams", Wiley & Sons