Methods of Study of Power Converters

for a systematic analysis

by
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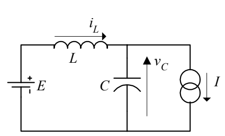
with
the support of LEEI - Toulouse, France

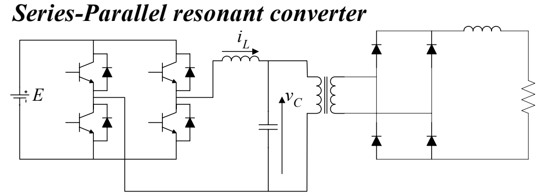
Topics of the presentation – Part I

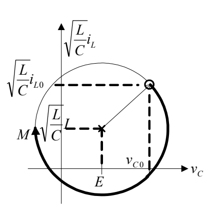
- Part I – The phase plane representation



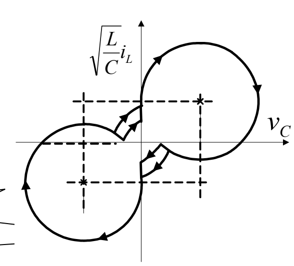
LCEI circuit

Series-Parallel resonant converter





<u>*Time domain waveforms*</u>



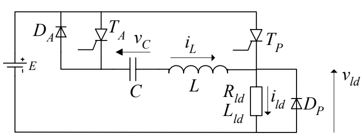
- RECALLS ON THEORY CIRCUIT

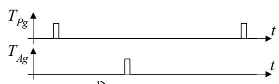
- THEORETICAL BASIS FOR PHASE PLANE CONSTRUCTION

Topics of the presentation – Part II

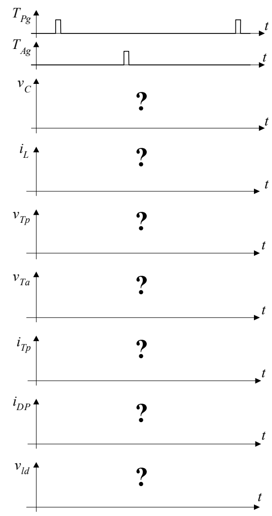
- Part II – Systematic analysis of power converters
  - Use of the phase plane representation

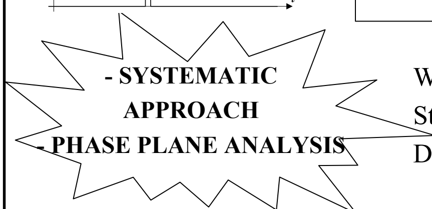
Circuit and firing signals





Time domain waveforms ???



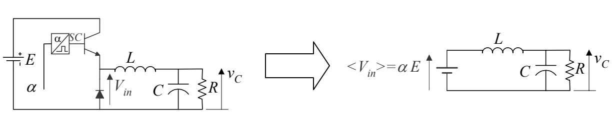


Working principle??
Stability issues ??
Dimensioning??

Topics of the presentation – Part III

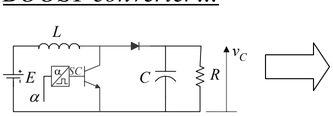
- Part III – Modelling of power converters

BUCK converter…

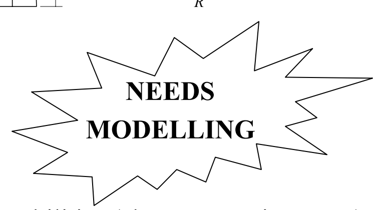


$$\frac{v_C(s)}{\alpha(s)} = \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

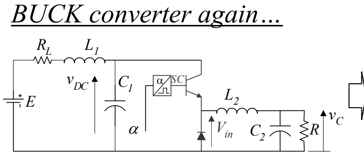
BOOST converter…



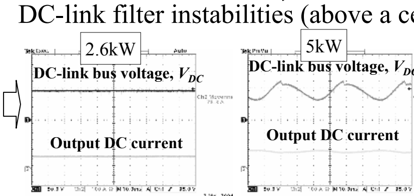
$$\frac{v_C(s)}{\alpha(s)} = ??$$



BUCK converter again…



DC-link filter instabilities (above a certain power)



How??
Why ??
Solutions ??

Contents – Part I

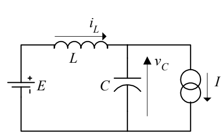
Contents – Part II

Contents – Part III

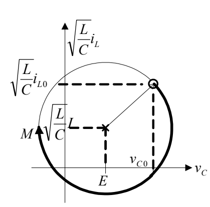
- 5. Modelling of Power Converters
  - 5.1.- Purpose: - control oriented modelling
  - 5.2.-State space models;
  - 5.3.-Equivalent average circuit models

Topics of the presentation – Part I

- Part I – The phase plane representation



LCEI circuit



- RECALLS ON THEORY CIRCUIT
- THEORETICAL BASIS FOR PHASE PLANE CONSTRUCTION

1. Forced state and free state: Recall-1

- 1.1. – State variables

Sate variable definition

- Any linear system behaviour can be described by a set of main variables, whose values define directly its state. Any other quantities of the system can be expressed as a function of these state variables and system inputs.

State variable properties

- State variables cannot change value instantaneously. They have to be continuous functions in time

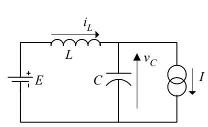
State variables in electrical circuits

- Currents flowing through each independent inductor;
- Voltage across each independent capacitor;

1. Forced state and free state: Recall-2

- 1.2. – Response of a linear system - Free state and forced state

Linear system can be modelled by a set of differential equations…



LCEI circuit

$$\begin{cases} E = L\dfrac{di_L}{dt} + v_C \\ i_L = C\dfrac{dv_C}{dt} + I \end{cases} \Rightarrow E = LC\dfrac{d^2 v_C}{dt^2} + v_C$$

Solution of the differential equation system (time domain)

Full response (t) = Free state response (t) + Forced state response (t)

$R(t) = R_{\Phi}(t) + R_F(t)$

Assumption: Forced state response is finite

Free state response $R_{\Phi}(t)$: response of the circuit without excitation sources:
- voltage sources short circuited;
- current sources in open circuit

Forced state response $R_F(t)$: response of the circuit in steady state
- capacitors behave like open circuits;
- inductors behave like short circuits

1. Forced state and free state: Recall-3

- 1.2. – Response of a linear system – Initial conditions

Initial conditions…

The initial conditions shall be computed once the equation for the free state and forced state responses are added

$$R_\Phi(t) = A\cos(\omega t)$$
$$R_F(t) = Const = K$$

Computation of constant A, from initial conditions:

$$R(t) = R_\Phi(t) + R_F(t) = A\cos(\omega t) + K$$

$$R(0) = R_\Phi(0) + R_F = A + K$$
$$A = R(0) - K$$

Order of the system…

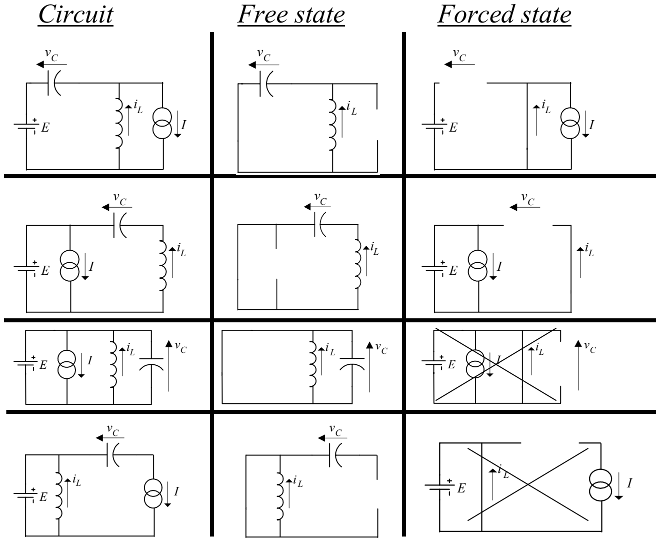
The order of the system is equal to:

the number of state variables = Σ(independent capacitors + independent inductors)

1. Forced state and free state: Recall-4

- 1.2. – Response of a linear system – Examples

Circuit | *Free state* | *Forced state*



Oscillated circuit

$$R(t) = R_\Phi(t) + R_F(t)$$

Oscillated circuit

$$R(t) = R_\Phi(t) + R_F(t)$$

Non-oscillated circuit

$$i_L(t) = i_{L0} + \frac{E}{L}t \qquad v_C(t) = E$$

Non-oscillated circuit

$$i_L(t) = i_{L0} + \frac{E}{L}t$$

$$v_C(t) = v_{C0} + \frac{I}{C}t$$

2. The phase plane method - 1

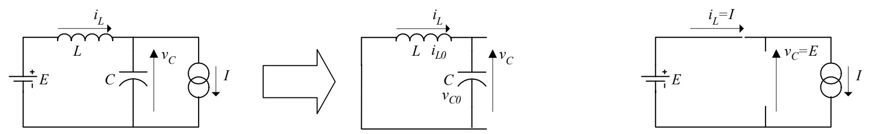
- 2.1. – Theoretical analysis: -state equations

<u>Purpose – study the response of the following LC type circuit with:</u>

- two DC excitation sources, voltage (E) and current (I) sources;

- initial conditions, inductor current (i_{L0}) and capacitor voltage (v_{C0});

- no power losses(*);



LCEI circuit — ***Free state*** — ***Steady state***

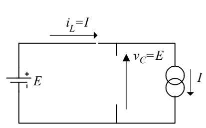
Full circuit response (i_L, v_C) = Free response ($i_{L\Phi}$, $v_{C\Phi}$) + Forced response (i_{LF}, v_{CF})

(*) in practical cases, the losses exist specially in the internal resistance of the inductor. However, the time constant is often much larger than the switching period. If so, the damping effect can be neglected at the switching period scale. Damping effect due to losses will be studied later.

2. The phase plane method - 2

- 2.1. – Theoretical analysis: -state equations

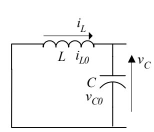
<u>Forced response,</u> (i_{LF}, v_{CF})



Steady state

$$\begin{cases} i_{LF} = I \\ v_{CF} = E \end{cases}$$

<u>Free response,</u> ($i_{L\Phi}$, $v_{C\Phi}$)



Free state

$$\begin{cases} i_{L\phi} = C \dfrac{dv_{C\phi}}{dt} \\ v_{C\phi} = -L \dfrac{di_{L\phi}}{dt} \end{cases}$$

$$i_{L\phi} + LC \frac{d^2 i_{L\phi}}{d^2 t} = 0$$

$$\begin{cases} i_{L\phi}(t) = A\cos(\omega t) + B\sin(\omega t) \\ v_{C\phi}(t) = -BL\omega\cos(\omega t) + AL\omega\sin(\omega t) \end{cases}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

<u>Computation of the full response,</u> (i_L, v_C)

$$\begin{cases} i_L(t) = I + A\cos(\omega t) + B\sin(\omega t) \\ v_C(t) = E - B\omega L\cos(\omega t) + A\omega L\sin(\omega t) \end{cases}$$

<u>Initial conditions</u>

$$\begin{cases} i_L(0) = i_{L0} = I + A \\ v_C(0) = v_{C0} = E - B\omega L \end{cases} \Rightarrow \begin{cases} A = i_{L0} - I \\ B = \dfrac{E - v_{C0}}{\omega L} = \sqrt{\dfrac{C}{L}}(E - v_{C0}) \end{cases}$$

2. The phase plane method - 3

- 2.1. – Theoretical analysis: -state equations

Full response equation (i_L, v_C)

$$\begin{cases} i_L(t) = I + (i_{L0} - I)\cos(\omega t) - \sqrt{\frac{C}{L}}(v_{C0} - E)\sin(\omega t) \\ v_C(t) = E + (v_{C0} - E)\cos(\omega t) + \sqrt{\frac{L}{C}}(i_{L0} - I)\sin(\omega t) \end{cases}$$

Searching for the equation of a circle…

… from the full response equation

$$\frac{L}{C}(i_L(t) - I)^2 + (v_C(t) - E)^2 =$$
$$= \frac{L}{C}(i_{L0} - I)^2\cos^2(\omega t) - 2\sqrt{\frac{L}{C}}(i_{L0} - I)(v_{C0} - E)\cos(\omega t)\sin(\omega t) + (v_{C0} - E)^2\sin^2(\omega t) +$$
$$+ (v_{C0} - E)^2\cos^2(\omega t) + 2\sqrt{\frac{L}{C}}(v_{C0} - E)(i_{L0} - I)\cos(\omega t)\sin(\omega t) + \frac{L}{C}(i_{L0} - I)^2\sin^2(\omega t) =$$
$$= \frac{L}{C}(i_{L0} - I)^2 + (v_{C0} - E)^2$$

Equation of a circle:
- centred in point (x_F, y_F);
- passing in point (x_0, y_0);

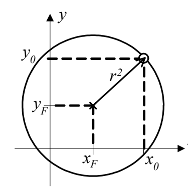
$$\left(\sqrt{\frac{L}{C}}i_L(t) - \sqrt{\frac{L}{C}}I\right)^2 + (v_C(t) - E)^2 = \frac{L}{C}(i_{L0} - I)^2 + (v_{C0} - E)^2$$

$$(y - y_F)^2 + (x - x_F)^2 = (y_0 - y_F)^2 + (x_0 - x_F)^2$$ with

$y = \sqrt{\frac{L}{C}}i_L(t)$; $x = v_C(t)$

$y_F = \sqrt{\frac{L}{C}}I$; $x_F = E$

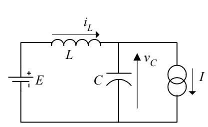
$y_0 = \sqrt{\frac{L}{C}}i_{L0}$; $x_0 = v_{C0}$



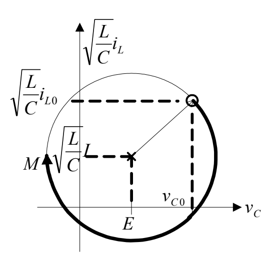
2. The phase plane method - 4

- 2.2. – Graphical representation

Phase plane construction



LCEI circuit



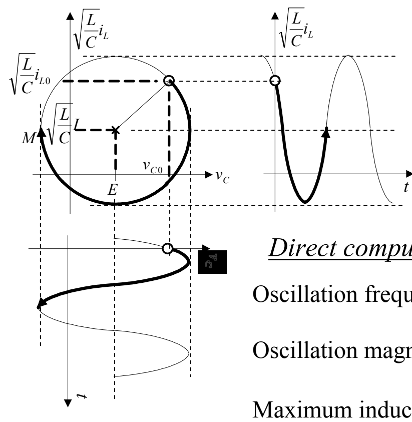
Rule:

Every LCEI type circuit whose free state corresponds to a series LC circuit, will have an oscillant response which may be represented on the phase plane (v_C , $i_L\sqrt{L/C}$) by a circle:
- Centred on the forced state operating point (E, $\sqrt{L/C}\,I$);
- Passing through point (v_{C0} , $i_{L0}\sqrt{L/C}$) , representing the initial conditions;

2. The phase plane method - 5

- 2.2. – Graphical representation

Obtaining the waveforms in the time domain



As the point M moves clockwise around the circle:

- the inductor current corresponds to the points in the ordinates;

- the voltage across the capacitor is the abcissa

Direct computation of waveform quantities

Oscillation frequency $f_{osc} = \frac{1}{2\pi\sqrt{LC}}$

Oscillation magnitude (radius) $r = \sqrt{(v_{C0} - E)^2 + \frac{L}{C}(i_{L0} - I)^2}$

Maximum inductor current $i_{L.\max} = I + \sqrt{\frac{C}{L}}r = I + \sqrt{\frac{C}{L}(v_{C0} - E)^2 + (i_{L0} - I)^2}$

Minimum inductor current $i_{L.\min} = I - \sqrt{\frac{C}{L}}r = I - \sqrt{\frac{C}{L}(v_{C0} - E)^2 + (i_{L0} - I)^2}$

2. The phase plane method - 6

- 2.3. – Response of a LC circuit with damping effect

Differential equation: free state with damping (i_L, v_C)

$$i_{L\phi} + RC\frac{di_{L\phi}}{dt} + LC\frac{d^2 i_{L\phi}}{d^2 t} = 0$$

Full response with LIGHT damping effect (i_L, v_C)

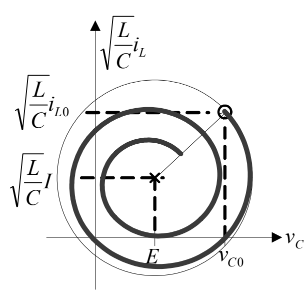
$$\begin{cases} i_L(t) = I + e^{-\alpha t}\left(C_1\cos(\omega t) + C_2\sin(\omega t)\right) \\ v_C(t) = E + e^{-\alpha t}\left(C_3\cos(\omega t) + C_4\sin(\omega t)\right) \end{cases}$$

Spiral:

-Starting in the initial conditions point;

-Evolving clockwise, initially along the un-damped circle;

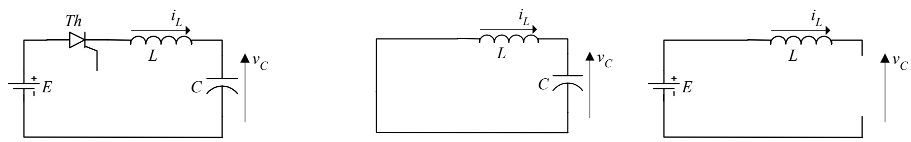
-Converging towards the steady state operating point



2. The phase plane method - 7

- 2.4. – Practical examples

2.4.1. - Charging a capacitor from a DC voltage source



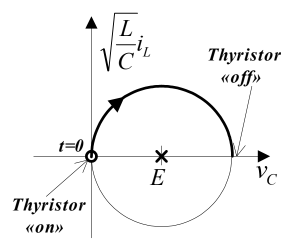
Capacitor charger circuit · *Free state* · *Steady state*

Half-circle:

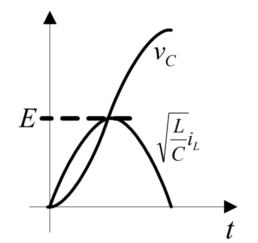
-Thyristor is switched "on" with zero current/zero voltage;

-Thyristor switches "off" when the current reaches zero;

-The capacitor is charged at twice the DC voltage source, E



Phase plane

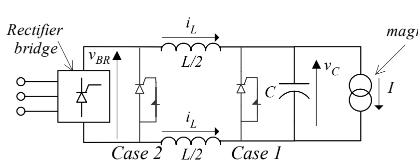


Time domain waveforms

2. The phase plane method - 8

- 2.4. – Practical examples

2.4.2. - Thyristor rectifier with Free-Wheel Thyristor (FWT) CROWBAR



-*BOD*: - Break-Over Diode

-*FWT* turns "*on*" if $V_{FWT} > V_{BOD}$;

-*FWT* turns "*off*" if $I_{FWT} = 0$;

Experimental observation at SPS main supplies, following "mains shutdown"

-In *case 1*: *FWT* switches "on" only once and assures full magnet discharge;
However, important peak-current on discharge of capacitor *C;*

-In *case 2*: *FWT* switches "on/off" continuously (BANG-BANG mode) in "***some cases***"

⇒ *Explanations ???*

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2. The phase plane method - 9

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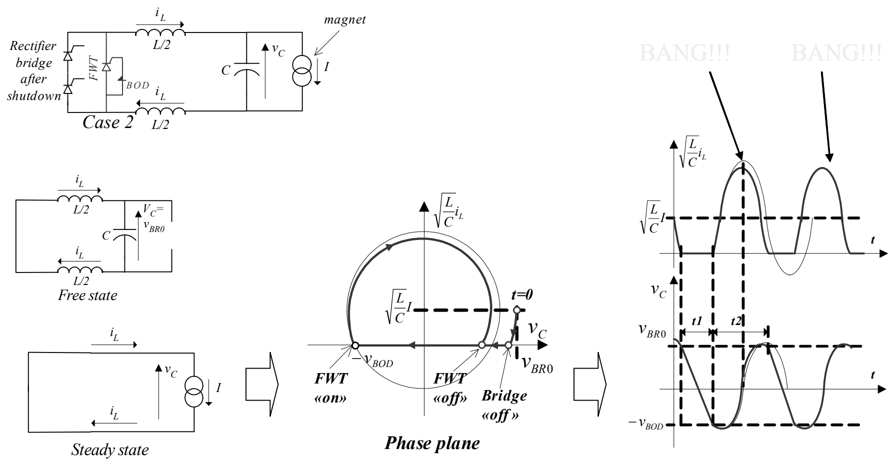
- 2.4. – Practical examples

2.4.2. - Thyristor rectifier with Free-Wheel Thyristor (FWT) CROWBAR
Behavioural study following mains shutdown



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2. The phase plane method - 10

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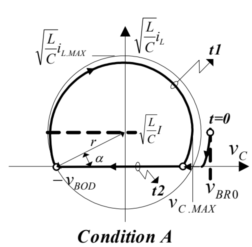
- 2.4. – Practical examples

2.4.2. - Thyristor rectifier with Free-Wheel Thyristor (FWT) CROWBAR



Condition A ⇒ $v_{BR0} > \sqrt{\frac{L}{C}} I$

- Rectifier bridge turns "off";
- *FWT* operates in BANG-BANG mode

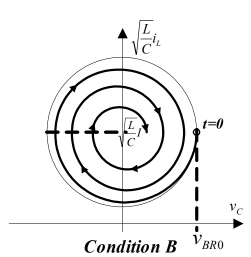
$$i_{L.MAX} \approx I + \sqrt{I^2 + \frac{C}{L} v_{BOD}^2}$$

$$v_{C.MAX} \approx \sqrt{\frac{L}{C} I^2 + v_{BOD}^2}$$

$$t1 \approx \frac{2Cv_{BOD}}{I}$$

$$t2 \approx \left(\pi + 2\,atan\left(\frac{I\sqrt{L/C}}{v_{BOD}}\right)\right)\sqrt{LC}$$

$$f_{OSC} = \frac{1}{t1 + t2}$$



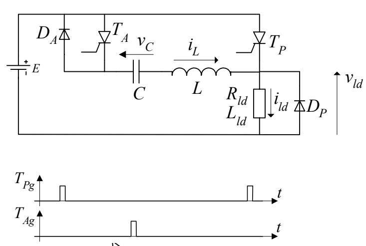
Condition B ⇒ $v_{BR0} < \sqrt{\frac{L}{C}} I$

- Rectifier bridge keeps switched "on";
- *FWT* never switches "on" (magnet discharge is made trough the rectifier bridge)

Topics of the presentation – Part II

- Part II – Systematic analysis of power converters
  - Use of the phase plane representation

Circuit and firing signals



Working principle??
Stability issues ??
Dimensioning??

- SYSTEMATICAL APPROACH
- PHASE PLANE ANALYSIS

- Classification of methods of study

- Flowchart for a systematic analysis

3. Classification of Methods of Study

- 3. Classification of Methods of Study

  3.1.-Analytical methods: - Based on equation derivations and time-domain waveforms;

  3.2.-Graphical representations: - Look-up-tables; Graphs in P.U. units;

  3.3.-Graphical/analytical methods:- Analytical methods + Phase plane;

  3.4.-Simulation methods:- Based on dedicated computer software and CAD tools

  3.4.1.-Functional based methods: - Power converter is considered as a "black box". Power converter as an input/output block diagram. No internal details are considered. Not valid for discontinuous operation

  ✓ ***3.4.2.-Sequential analytical methods***:- The converter operation is decomposed in several sequences. In each sequence there is no switching

  Without "a priori" knowledge: - all the sequences have to be analysed;

  With "a priori" knowledge: - some inexistent sequences are eliminated at the beginning of the study

4. Sequential analytical methods - 1

- 4.1. - The principle.

- Description of the power converter operation on a ***"sequence-by-sequence"*** basis;

- Analyse: a)- Conditions for sequence transitions;
  b)- State evolutions during each sequence

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4. Sequential analytical methods - 2

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- 4.1. - The principle.

a)- Sequence transition ⇒ switching of one or several semiconductors

ex. conditions for switching on: - diode: $V_T > 0$
- thyristor: $V_T > 0$ & "gate signal *on*"
- transistor: "gate signal *on*"

ex. conditions for switching off: - diode & thyristor: $I_T = 0$
- transistor: "gate signal *off*"

4. Sequential analytical methods - 4

- 4.1. - The principle.

b)- State evolutions during each sequence

For a given sequence, compute the state variables:

- Voltages across the capacitors
- Currents through the inductors

⇩

Compute the expressions of the currents and voltages across the switches:

⇩

Go back to point a) - Check for conditions for a new sequence transition

4. Sequential analytical methods - 5

- 4.2. – Flowchart for a systematic analysis.

For a given sequence…

1. *Search for the order of the system*

2. Compute expressions for(remark2)

- Voltage across open semiconductors,
- Current across closed semiconductors

} versus sources values, state variables

3. Test of compatibility: - Check for the existence of a given sequence

- For all switches except the last one changing state:

  all voltage across open diodes must be <0,

  all currents through closed switches must be >0

  …

4. Sequential analytical methods - 6

- 4.2. – Flowchart for a systematic analysis

For a given sequence…

4. Compute the expressions for state variables

- solution of differential equations system in forced and free states, (initial conditions = final state of the former sequence)
- phase plane representation

5. Check for the events that may generate a switching (*remark5)*

6. Selection of the event leading to a sequence transition

4. Sequential analytical methods - 7

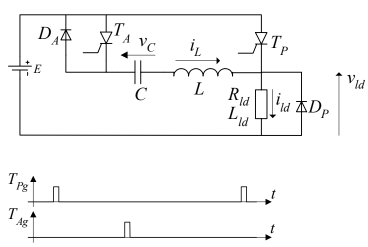
- 4.3. – Choice of the first sequence

- In some cases, a bad choice of the first sequence may lead to a “divergent” result in the circuit analysis (turn-around);
- This is namely the case of circuits where initial conditions for a steady state cycle are not obvious to determine (they depend on the way transient operation behaves, pre-charging procedures, etc.);
- Practical rules for choosing the first sequence yielding analysis consistency and “convergence”:
  a) - chose a sequence where the load is connected to the source (active phase);
  b) - chose a sequence corresponding to a free-wheeling state;
  c) - chose a sequence corresponding to a discontinuous conduction;

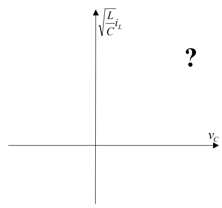
4. Sequential analytical methods - 8

- 4.4.-Example:- study of a thyristor chopper

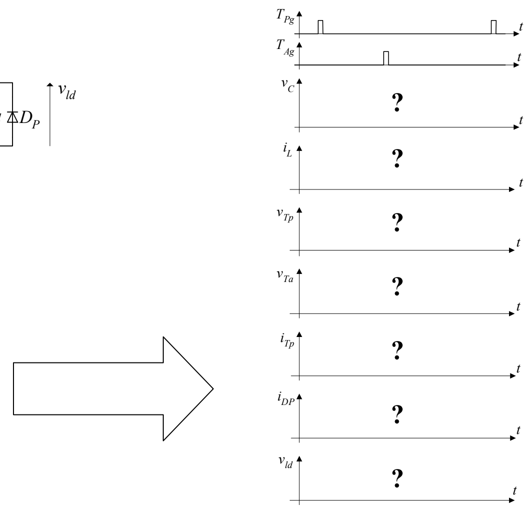
Circuit and firing signals



Phase plane



Time domain waveforms

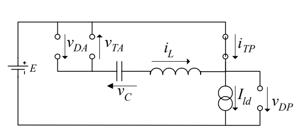


4. Sequential analytical methods - 9

- 4.4.-Example:- study of a thyristor chopper

Sequence n° 1

(active phase–"guess")



Supposition: $v_C = v_{C0} < 0$

1)- Search for the order of the system

LC circuit is open ⇒ No evolution of the state variables

2)- Voltages & Currents on semiconductors

$v_{TA} = -v_C$ $v_{DP} = -E$

$v_{DA} = +v_C$ $i_{TP} = I_{ld}$

3)- Test of compatibility

$v_{DA} < 0$; $v_{DP} < 0$; $i_{TP} > 0$; $v_{TA} > 0$ *(no gate signal)*

Seq. OK

4)- Evolution of state variables

No evolutions

5)- Events that may generate a switching

$v_C < 0 \Rightarrow v_{TA} > 0$  TA switches "on" if gate signal

6)- The event leading to a sequence transition

TA switches on when gate signal

Final cond: $v_C = v_{C0}$; $i_L = 0$

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4. Sequential analytical methods - 10

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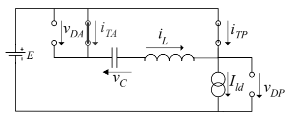
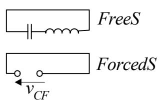
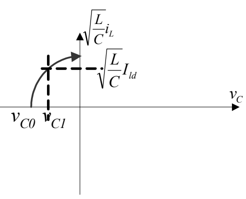
- 4.4.-Example:- study of a thyristor chopper

Sequence n° 2

TA has switched on







1)- Search for the order of the system

Free state circuit of 2nd order ⇒ Circle in the phase plane

2)- Voltages & Currents on semiconductors

$i_{TA} = i_L$ $v_{DP} = -E$

$v_{DA} = -\Delta$ $i_{TP} = I_{ld} - i_L$

3)- Test of compatibility

$v_{DA} < 0$; $v_{DP} < 0$; $i_{TP} > 0$ **Seq. OK**

4)- Evolution of state variables

Circle: $(v_{CF}, i_{LF})=(0, 0)$; $(v_{C0}; i_{L0})=(v_{C0}, 0)$

5)- Events that may generate a switching

TA may switch off if $i_{TA}=0 \Rightarrow i_L=0$; DA and DP keep off;
TP may switch off if $i_{TP}=0 \Rightarrow i_L=I_{ld}$

6)- The event leading to a sequence transition

TP switches off before TA
Final cond: $v_C = v_{C1}$; $i_L = I_{ld}$

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4. Sequential analytical methods - 11

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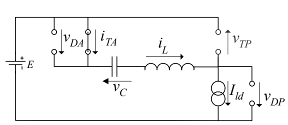
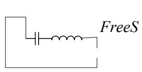
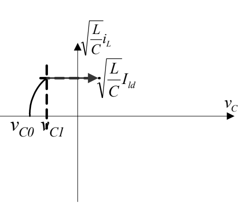
- 4.4.-Example:- study of a thyristor chopper

Sequence n° 3

TP has switched off







1)- Search for the order of the system

Free state circuit in open loop ⇒ straight line in phase plane

2)- Voltages & Currents on semiconductors

$i_{TA} = i_L = I_{ld}$ $v_{DP} = -E + v_C + v_L = -E + v_C$

$v_{DA} = -\Delta$ $v_{TP} = v_C + v_L = v_C$

3)- Test of compatibility

$v_{DA} < 0$; $v_{DP} = -E + v_{C1} < 0$; $i_{TA} = i_L = I_{ld} > 0$ **Seq. OK**

4)- Evolution of state variables

Straight line: $i_L=I_{ld}=Cst$; $v_C(t)=I_{ld}/C*t+v_{C1}$

5)- Events that may generate a switching

TA keeps on; DA keeps off; DP switches on if $v_C>E$
TP switches on if $v_C>0$ and gating signal applied

6)- The event leading to a sequence transition

TP must not switch on (lost of control);**DP switches on** when $v_C>E$
Final cond: $v_C = E$; $i_L = I_{ld}$

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4. Sequential analytical methods - 12

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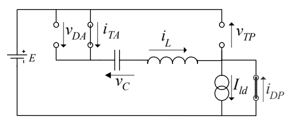
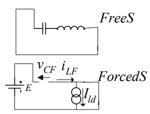
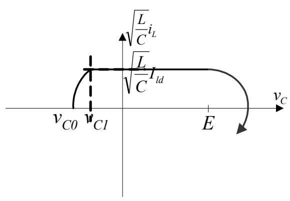
- 4.4.-Example:- study of a thyristor chopper

Sequence n° 4

DP has switched on







1)- Search for the order of the system

Free state circuit of 2nd order ⇒ Circle in the phase plane

2)- Voltages & Currents on semiconductors

$i_{TA} = i_L$ $\quad$ $i_{DP} = I_{ld} - i_L$

$v_{DA} = -\Delta$ $\quad$ $v_{TP} = E$

3)- Test of compatibility

$v_{DA} < 0 \; ; \; i_{TA} = i_L > 0 \; ; \; v_{TP} = E > 0 (no\ gate\ signal)$

Seq. OK

4)- Evolution of state variables

Circle: $(v_{CF}, i_{LF}) = (E, 0)$; $(v_{C0}; i_{L0}) = (E, I_{ld})$

5)- Events that may generate a switching

TA switches off if i_L=0; DA keeps off; DP switches off if $i_L = I_{ld}$
TP switches on if gating signal applied

6)- The event leading to a sequence transition

TP must not switch on (lost of control); **TA switches off** before D

Final cond: $v_C = E + I_{ld}\sqrt{L/C} \; ; \; i_L = 0$

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4. Sequential analytical methods - 13

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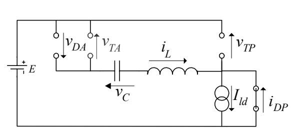
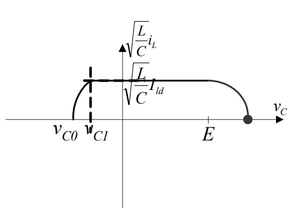
- 4.4.-Example:- study of a thyristor chopper

Sequence n° 4-bis

TA has switched off





1)- Search for the order of the system

LC circuit is open ⇒ No evolution of the state variables

2)- Voltages & Currents on semiconductors

$v_{TA} = E - v_C$ $\quad$ $i_{DP} = I_{ld}$

$v_{DA} = v_C - E$ $\quad$ $v_{TP} = E$

3)- Test of compatibility

$v_{DA} = v_C - E > 0 \; ; \; v_{TP} = E > 0 (no\ gate\ signal) \; ; \; i_{DP} > 0$

Seq. not OK

Voltage across diode DA is positive

⇒DA switches on immediately

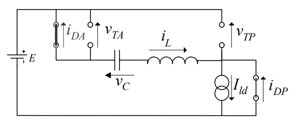
This sequence has no physical existence
(DA switches on at the same time TA switches off)

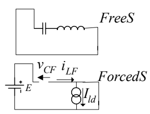
4. Sequential analytical methods - 14

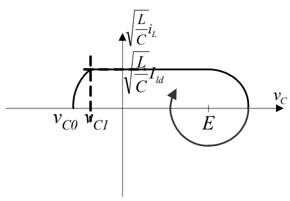
- 4.4.-Example:- study of a thyristor chopper

Sequence n° 5

DA has switched on







1)- Search for the order of the system

Free state circuit of 2nd order ⇒ Circle in the phase plane

2)- Voltages & Currents on semiconductors

$v_{TA} = -\Delta$; $i_{DP} = I_{ld} - i_L$

$i_{DA} = -i_L$; $v_{TP} = E$

3)- Test of compatibility

$v_{TA} < 0$; $i_{DP} > 0$; $v_{TP} > 0$(*no gate signal)* **Seq. OK**

4)- Evolution of state variables

Circle: $(v_{CF}, i_{LF}) = (E, 0)$; $(v_{C0}; i_{L0}) = (E + I_{ld}\sqrt{L/C}, 0)$

5)- Events that may generate a switching

TA keeps off; DA switches off if $i_L=0$; DP switches off if $i_L=I_{ld}$
TP switches on if gating signal applied

6)- The event leading to a sequence transition

TP must not switch on (lost of control);**DA switches off** before DP

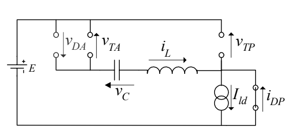
Final cond: $v_C = E - I_{ld}\sqrt{L/C}$; $i_L = 0$

4. Sequential analytical methods - 15

- 4.4.-Example:- study of a thyristor chopper

Sequence n° 6

DA has switched off



1)- Search for the order of the system

LC circuit is open ⇒ No evolution of the state variables

2)- Voltages & Currents on semiconductors

$v_{TA} = E - v_C$; $i_{DP} = I_{ld}$

$v_{DA} = v_C - E$; $v_{TP} = E$

3)- Test of compatibility

$v_{TA} > 0$(*no gate signal); $i_{DP} > 0$; $v_{TP} > 0$(*no gate signal)*

Seq. OK

4)- Evolution of state variables

No evolutions

5)- Events that may generate a switching

TA switches on if gate signal; DA keeps off; DP keeps on;
TP switches on if gating signal applied

6)- The event leading to a sequence transition

TA must not switch on (lost of control); **TP switches on** with gate signal

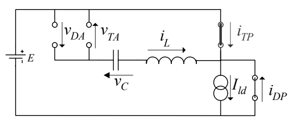
Final cond: $v_C = E - I_{ld}\sqrt{L/C}$; $i_L = 0$

4. Sequential analytical methods - 16

- 4.4.-Example:- study of a thyristor chopper

Sequence n° 6 - bis

TP has switched on



1)- Search for the order of the system

LC circuit is open ⇒ No evolution of the state variables

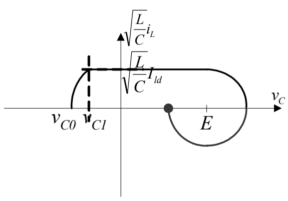
2)- Voltages & Currents on semiconductors

$v_{TA} = -v_C$ $\quad$ $i_{DP} = I_{ld} - I_{CC}$

$v_{DA} = v_C$ $\quad$ $i_{TP} = I_{CC}$ $\quad$ I_{CC} = *short circuit current of E*

3)- Test of compatibility

$v_{DA} > 0\ ;\ v_{TA} < 0\ ;\ i_{DP} < 0$ $\quad$ **Seq. not OK**



Voltage across diode DA is positive

⇒DA switches on immediately

Current on diode DP is negative

⇒DP switches off immediately

This sequence has no physical existence

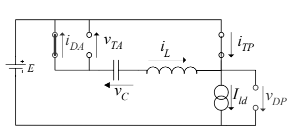
(Once TP switches on; DP switches off and DA switches on)

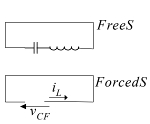
4. Sequential analytical methods - 17

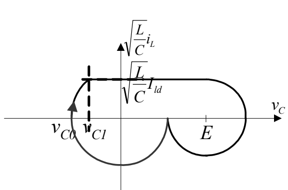
- 4.4.-Example:- study of a thyristor chopper

Sequence n° 7

DP has switched off,
DA has switched on







1)- Search for the order of the system

Free state circuit of 2nd order ⇒ Circle in the phase plane

2)- Voltages & Currents on semiconductors

$v_{TA} = -\Delta$ $\quad$ $v_{DP} = -E$

$i_{DA} = -i_L$ $\quad$ $i_{TP} = I_{ld} - i_L$

3)- Test of compatibility

$v_{TA} < 0\ ; v_{DP} < 0$ $\quad$ **Seq. OK**

4)- Evolution of state variables

Circle: $(v_{CF}, i_{LF})=(0, 0)$; $(v_{C0}; i_{L0})=(E-I_{ld}\sqrt{L/C}, 0)$

5)- Events that may generate a switching

TA keeps off; DA switches off if i_L=0; DP keeps off
TP switches off if $i_L=I_{ld}$

6)- The event leading to a sequence transition

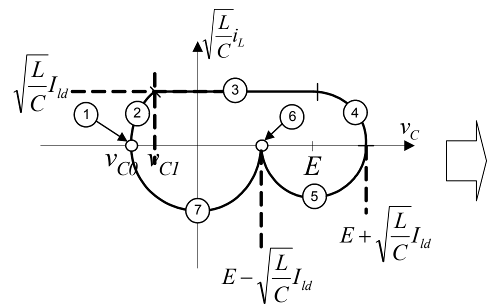
DA switches off before TP

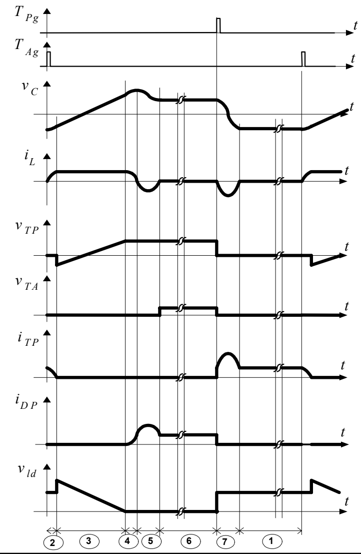
Final cond: $v_C = -(E - I_{ld}\sqrt{L/C})\ ;\ i_L = 0$

4. Sequential analytical methods - 18

- 4.4.-Example:- study of a thyristor chopper

Complete phase plane and waveforms

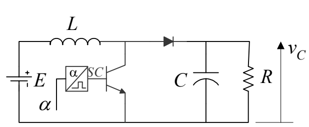




Topics of the presentation – Part III

- Part III – Modelling of power converters

BOOST converter…



$$\frac{v_C(s)}{\alpha(s)} = ??$$

NEEDS MODELLING

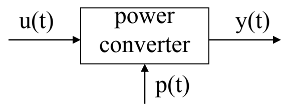
Two methods:

- State space average models;
- Equivalent average circuit models

5. Modelling of power converters - 1

- 5.1. – Purpose:- Control oriented modelling.

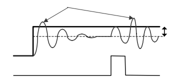
Open loop system



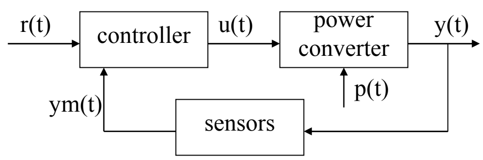
u(t)-input: duty-cycle, firing angle, …

y(t)-output: delivered voltage, …

p(t)-perturbation: mains disturbances,…

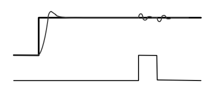


Closed loop system



r(t)-reference: command level

ym(t)-measure: voltage sensor, DCCTs…



- Power converter model: - mathematical representation of the system's dynamic behaviour (set of differential equations);

- Models are used within control theory in order to improve dynamic performance and perturbation immunity

5. Modelling of power converters - 2

- 5.2. – State space models.

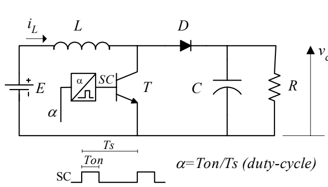
$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

u – reference vector

x – state vector

A,B,C,D – constant matrices

Support example – the BOOST DC/DC converter (continuous mode, $i_L(t)>0$)



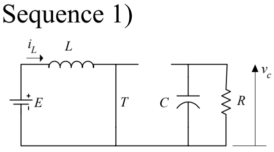
Goal: Compute the transfer function between α and v_C: $v_C(s)=func(\alpha(s))$

2 sequences

1) T closed, duration αTs

2) T open, duration $(1-\alpha)Ts$

Sequence 1)

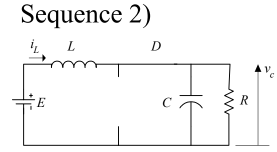


$$E = L\frac{di_L}{dt};\; v_C = -RC\frac{dv_C}{dt}$$

$$\underbrace{\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}}_{A_1} \underbrace{\begin{bmatrix} i_L \\ v_C \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 1/L \\ 0 \end{bmatrix}}_{B_1} \underbrace{E}_{u}$$

$$\begin{cases} \dot{x} = A_1 x + B_1 u \\ y = C_1 x + D_1 u \end{cases}$$

Sequence 2)



$$E = L\frac{di_L}{dt} + v_C;\; i_L = C\frac{dv_C}{dt} + \frac{v_C}{R}$$

$$\underbrace{\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & -1/L \\ 1/C & -\frac{1}{RC} \end{bmatrix}}_{A_2} \underbrace{\begin{bmatrix} i_L \\ v_C \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 1/L \\ 0 \end{bmatrix}}_{B_2} \underbrace{E}_{u}$$

$$\begin{cases} \dot{x} = A_2 x + B_2 u \\ y = C_2 x + D_2 u \end{cases}$$

$$y = v_C \Rightarrow C_1 = C_2 = [0\;\;1];\; D_1 = D_2 = 0$$

5. Modelling of power converters - 3

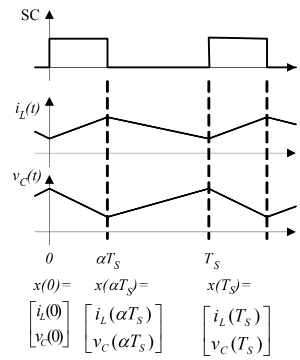
- 5.2. – State space models.

Integration of the state differential equation

$$u(t) = constant \quad = E$$

$$\dot{x} = Ax + Bu \Rightarrow x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}B.Ed\tau$$

$$\Rightarrow x(t) = e^{At}x(0) + A^{-1}(e^{At} - I)B.E$$



Application to the present example

$$\begin{cases} x(\alpha T_S) = e^{A_1\alpha T_S}x(0) + A_1^{-1}(e^{A_1\alpha T_S} - I)B_1E \\ x(T_S) = e^{A_2(1-\alpha)T_S}x(\alpha T_S) + A_2^{-1}(e^{A_2(1-\alpha)T_S} - I)B_2E \end{cases}$$

$$\begin{cases} x(\alpha T_S) = F_1x(0) + G_1E \\ x(T_S) = F_2x(\alpha T_S) + G_2E \end{cases}$$

with

$$F_1 = e^{A_1\alpha T_S};\ G_1 = A_1^{-1}(e^{A_1\alpha T_S} - I)B_1E$$
$$F_2 = e^{A_2(1-\alpha)T_S};\ G_2 = A_2^{-1}(e^{A_2(1-\alpha)T_S} - I)B_2E$$

5. Modelling of power converters - 4

- 5.2. – State space models.

Computation of a unique equation

$$x(T_S) = F_2(F_1x(0) + G_1E) + G_2E$$
$$= F_2F_1x(0) + (F_2G_1 + G_2)E$$
$$x(T_S) = Fx(0) + GE \quad with \quad F = F_2F_1,\ G = F_2G_1 + G_2$$

Approximation of the exponential to a 1st order

$$e^{A_1\alpha T_S} \approx I + A_1\alpha T_S + \frac{(A_1\alpha T_S)^2}{2!} + ...$$

$$e^{A_2(1-\alpha)T_S} \approx I + A_2(1-\alpha)T_S + \frac{(A_2(1-\alpha)T_S)^2}{2!} + ...$$

Simplification of the matrices F and G

$$F \approx (I + A_2(1-\alpha)T_S).(I + A_1\alpha T_S) = I + A_1\alpha T_S + A_2(1-\alpha)T_S + A_2A_1\alpha(1-\alpha)T_S^2$$

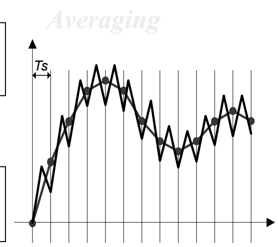
$$G \approx (I + A_2(1-\alpha)T_S).(A_1^{-1}(I + A_1\alpha T_S - I).B_1) + A_2^{-1}(I + A_2(1-\alpha)T_S - I).B_2$$
$$= \alpha T_SB_1E + (1-\alpha)T_SB_2E + A_2(1-\alpha)T_S\alpha T_S$$

5. Modelling of power converters - 5

- 5.2. – State space models.

Discrete-time state equation

$$x(T_S) = Fx(0) + GE = \\ = (I + A_1\alpha T_S + A_2(1-\alpha)T_S)x(0) + (B_1\alpha T_S + B_2(1-\alpha)T_S)E$$



Continuous-time state equation (linear interpolation)

$$\dot{x} = \frac{x(T_S) - x(0)}{T_S} = \underbrace{\left[A_1\alpha + A_2(1-\alpha)\right]}_{A}x(0) + \underbrace{\left[B_1\alpha + B_2(1-\alpha)T_S\right]}_{B}E$$

State space model (generic power converter, 2 sequences)

$$\dot{x} = Ax + B.E \quad with \begin{cases} A = A_1\alpha + A_2(1-\alpha) \\ B = B_1\alpha + B_2(1-\alpha) \end{cases}$$

Matrices A and B are the sum of A_1,A_2 and B_1,B_2, respectively, pondered by their « existence duration »

$$y = Cx + D.E \quad with \begin{cases} C = C_1\alpha + C_2(1-\alpha) \\ D = D_1\alpha + D_2(1-\alpha) \end{cases}$$

Same applies for matrices C and D (in a generic case)

5. Modelling of power converters - 6

- 5.2. – State space models.

... Coming back to the former BOOST DC/DC converter

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} \quad B_1 = \begin{bmatrix} 1/L \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & -1/L \\ 1/C & -\frac{1}{RC} \end{bmatrix} \quad B_2 = \begin{bmatrix} 1/L \\ 0 \end{bmatrix}$$

$$A = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}}_{A_1}\alpha + \underbrace{\begin{bmatrix} 0 & -1/L \\ 1/C & -\frac{1}{RC} \end{bmatrix}}_{A_2}(1-\alpha)$$

$$B = \underbrace{\begin{bmatrix} 1/L \\ 0 \end{bmatrix}}_{B_1 = B_2}$$

State space model for the BOOST DC/DC converter

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & -\frac{1}{L}(1-\alpha) \\ \frac{1-\alpha}{C} & -\frac{1}{RC} \end{bmatrix}}_{A}x + \underbrace{\begin{bmatrix} 1/L \\ 0 \end{bmatrix}}_{B}E$$

Matrix A is dependent on α

-> **non linear model**

(common for most of the power converter structures)

(back)

5. Modelling of power converters - 7

- 5.2. – State space models.

Small signal linear model

Purpose: - Establish a small signal state model around a predefined operating point

Assumptions

E is constant $(E(s) = E)$

$x = x_0 + \hat{x}$; $\alpha = \alpha_0 + \hat{\alpha}$

Computation of the DC operating point

$$\dot{x} = A(\alpha)x + B(\alpha)E$$

$$\dot{x} = 0 \Rightarrow x_0 = -A(\alpha_0)^{-1}.B(\alpha_0).E$$

Linearisation

$$\dot{x} = f(x, \alpha, E) \qquad E = const$$

$$\dot{\hat{x}} = \left.\frac{\partial f}{\partial x}\right|_{(x_0,\alpha_0)} \hat{x} + \left.\frac{\partial f}{\partial \alpha}\right|_{(x_0,\alpha_0)} \hat{\alpha}$$

from the former state space model

$$\dot{x} = \underbrace{Ax + B.E}_{f(x,\alpha,E)} \quad with \begin{cases} A = A_1\alpha + A_2(1-\alpha) \\ B = B_1\alpha + B_2(1-\alpha) \end{cases}$$

$$y = Cx + D.E \quad with \begin{cases} C = C_1\alpha + C_2(1-\alpha) \\ D = D_1\alpha + D_2(1-\alpha) \end{cases}$$

$$\dot{\hat{x}} = \underbrace{A(\alpha_0)}_{A_0}\hat{x} + \underbrace{\left[(A_1 - A_2)x_0 + (B_1 - B_2)E\right]}_{B_0}\hat{\alpha}$$

$$\hat{y} = \underbrace{C(\alpha_0)}_{C_0}\hat{x} + \underbrace{\left[(C_1 - C_2)x_0 + (D_1 - D_2)E\right]}_{D_0}\hat{\alpha}$$

5. Modelling of power converters - 8

- 5.2. – State space models.

Small signal state space model

$$\begin{cases} \dot{\hat{x}} = A_0\hat{x} + B_0\hat{\alpha} \\ \hat{y} = C_0\hat{x} + D_0\hat{\alpha} \end{cases} \quad with \quad \begin{cases} A_0 = A(\alpha_0); \quad B_0 = (A_1 - A_2)x_0 + (B_1 - B_2)E \\ C_0 = C(\alpha_0); \quad D_0 = (C_1 - C_2)x_0 + (D_1 - D_2)E \end{cases}$$

Laplace transformation

$$s\hat{x}(s) = A_0\hat{x}(s) + B_0\hat{\alpha}(s)$$

$$\begin{cases} \dfrac{\hat{x}(s)}{\hat{\alpha}(s)} = \left[sI - A_0\right]^{-1}.B_0 \\ \dfrac{\hat{y}(s)}{\hat{\alpha}(s)} = C_0(sI - A_0)^{-1}B_0 + D_0 \end{cases}$$

- 5.2. – State space models.

... Coming back to the former BOOST DC/DC converter

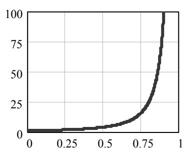
DC operating point

$$x_0 = -A(\alpha_0)^{-1}.B(\alpha_0).E = \frac{LC}{(1-\alpha_0)^2}\begin{bmatrix} -\frac{1}{RC} & \frac{1}{L}(1-\alpha_0) \\ -\frac{1}{C}(1-\alpha_0) & 0 \end{bmatrix}.\begin{bmatrix} 1/L \\ 0 \end{bmatrix}.E = E\begin{bmatrix} \frac{1}{R(1-\alpha_0)^2} \\ \frac{1}{1-\alpha_0} \end{bmatrix}$$

$$i_{L0} = \frac{E}{R}\frac{1}{(1-\alpha_0)^2}$$

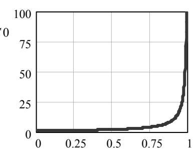
$$v_{C0} = \frac{1}{1-\alpha_0}E$$

$\frac{R}{E}i_{L0}$



α_0

$\frac{1}{E}v_{C0}$



α_0

From the small signal state space model…

$$\dot{\hat{x}} = A_0\hat{x} + B_0\hat{\alpha} \quad with \quad A_0 = \begin{bmatrix} 0 & -\frac{1}{L}(1-\alpha_0) \\ \frac{1}{C}(1-\alpha_0) & -\frac{1}{RC} \end{bmatrix} \quad ; \quad B_0 = \begin{bmatrix} \frac{E}{L}\frac{1}{1-\alpha_0} \\ -\frac{E}{RC}\frac{1}{(1-\alpha_0)^2} \end{bmatrix}$$

- 5.2. – State space models.

... Coming back to the former BOOST DC/DC converter

Laplace transformation of the state equation

$$\frac{\hat{x}(s)}{\hat{\alpha}(s)} = [sI - A_0]^{-1}.B_0 = \frac{1}{s^2 + s\frac{1}{RC} + \frac{1}{LC}(1-\alpha_0)^2}\begin{bmatrix} s\left(\frac{E}{L}\frac{1}{(1-\alpha_0)}\right) + \frac{2E}{RCL}\frac{1}{(1-\alpha_0)} \\ \frac{E}{LC}\left(1 - s\frac{L}{R}\frac{1}{(1-\alpha_0)^2}\right) \end{bmatrix}$$

Transfer functions $\hat{i}_L(s)/\hat{\alpha}(s)$ and $\hat{v}_C(s)/\hat{\alpha}(s)$

$$\frac{\hat{i}_L(s)}{\hat{\alpha}(s)} = \frac{s\left(\frac{E}{L}\frac{1}{(1-\alpha_0)}\right) + \frac{2E}{RCL}\frac{1}{(1-\alpha_0)}}{s^2 + s\frac{1}{RC} + \frac{1}{LC}(1-\alpha_0)^2}$$

$$\frac{\hat{v}_C(s)}{\hat{\alpha}(s)} = \frac{\frac{E}{LC}\left(1 - s\frac{L}{R}\frac{1}{(1-\alpha_0)^2}\right)}{s^2 + s\frac{1}{RC} + \frac{1}{LC}(1-\alpha_0)^2}$$

5. Modelling of power converters - 11

- 5.3. – Equivalent average circuit models.

Validity hypothesis: - Time constants >> sampling period

Principle: - replacing the switches by equivalent voltage or current sources in order to obtain a linear/time-continuous equivalent circuit;

- The equivalent circuit is valid for average values at the switching frequency scale

"Recipe" to obtain the equivalent circuit

1. Select any sequence whose duration is known;

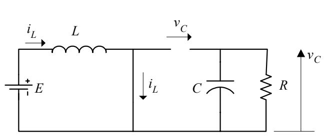
for that sequence:

2. Compute:
   2.1. - the current through all closed switches as a function of state variables and/or source values;
   2.2. - the voltages across all open switches as a function of state variables and/or source values;
3. Draw an "equivalent" electrical circuit, by replacing:
   3.1. – the closed switches by the related current pondered by their existence duration;
   3.2. – the open switches by the related voltage sources pondered by their existence duration;

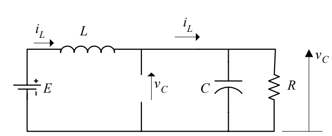
5. Modelling of power converters - 12

- 5.3. – Equivalent average circuit models.

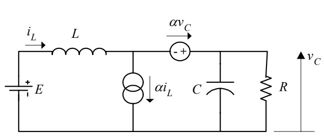
Exemplification on the BOOST converter



Sequence 1: - durationα

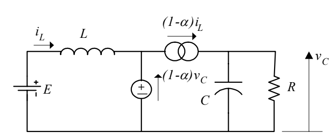


Sequence 2: - duration$(1-\alpha)$



Sequence 1: - durationα

Equal to the ***state space model*** on slide *"Mod Power Conv-6"*



Sequence 2: - duration$(1-\alpha)$

$$\begin{cases} E = L\dfrac{di_L}{dt} - \alpha v_C + v_C \\ i_L - \alpha i_L = C\dfrac{dv_C}{dt} + \dfrac{v_C}{R} \end{cases}$$

$$\underbrace{\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & -\dfrac{1-\alpha}{L} \\ \dfrac{1-\alpha}{C} & -\dfrac{1}{RC} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_L \\ v_C \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 1/L \\ 0 \end{bmatrix}}_{B} \underbrace{E}_{u}$$

$$\begin{cases} E = L\dfrac{di_L}{dt} + (1-\alpha)v_C \\ (1-\alpha)i_L = C\dfrac{dv_C}{dt} + \dfrac{v_C}{R} \end{cases}$$

5. Modelling of power converters - 13

- 5.3. – Equivalent average circuit models.

Small signal linearisation

$$\dot{x} = Ax + Bu$$

DC operating point

$$x_0 = -A(\alpha_0)^{-1}.B(\alpha_0).E \Rightarrow x_0 = \begin{bmatrix} i_{L0} \\ v_{C0} \end{bmatrix} = \begin{bmatrix} \frac{E}{R}\frac{1}{(1-\alpha_0)^2} \\ \frac{1}{1-\alpha_0}E \end{bmatrix}$$

Linearisation

$$\begin{array}{l} \dot{x} = f(x,\alpha,E) \\ f = Ax + Bu \\ E = const \end{array} \Rightarrow \dot{\hat{x}} = \left.\frac{\partial f}{\partial x}\right|_{(x_0,\alpha_0)}\hat{x} + \left.\frac{\partial f}{\partial \alpha}\right|_{(x_0,\alpha_0)}\hat{\alpha} \Rightarrow \dot{\hat{x}} = A(\alpha_0)\hat{x} + \left.\frac{\partial A}{\partial \alpha}\right|_{(\alpha_0)} x_0\hat{\alpha}$$

$$A_0 = \begin{bmatrix} 0 & -\frac{1}{L}(1-\alpha_0) \\ \frac{1}{C}(1-\alpha_0) & -\frac{1}{RC} \end{bmatrix}$$

$$B_0 = \left.\frac{\partial A}{\partial \alpha}\right|_{(\alpha_0)} x_0 = \frac{\partial}{\partial \alpha}\left.\begin{bmatrix} 0 & -\frac{1-\alpha}{L} \\ \frac{1-\alpha}{C} & -\frac{1}{RC} \end{bmatrix}\right|_{\alpha_0} x_0 = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & 0 \end{bmatrix}\begin{bmatrix} \frac{E}{R}\frac{1}{(1-\alpha_0)^2} \\ \frac{1}{1-\alpha_0}E \end{bmatrix}$$

$$\dot{\hat{x}} = \underbrace{\begin{bmatrix} 0 & -\frac{1}{L}(1-\alpha_0) \\ \frac{1}{C}(1-\alpha_0) & -\frac{1}{RC} \end{bmatrix}}_{A_0}\hat{x} + \underbrace{\begin{bmatrix} \frac{1}{(1-\alpha_0)}\frac{E}{L} \\ -\frac{E}{RC}\frac{1}{(1-\alpha_0)^2} \end{bmatrix}}_{B_0}\hat{\alpha}$$

Laplace transformation

$$\frac{\hat{x}(s)}{\hat{\alpha}(s)} = [sI - A_0]^{-1}.B_0 \Rightarrow \frac{\hat{i}_L(s)}{\hat{\alpha}(s)} = \frac{s\left(\frac{E}{L}\frac{1}{(1-\alpha_0)}\right) + \frac{2E}{RCL}\frac{1}{(1-\alpha_0)}}{s^2 + s\frac{1}{RC} + \frac{1}{LC}(1-\alpha_0)^2} \qquad \frac{\hat{v}_C(s)}{\hat{\alpha}(s)} = \frac{\frac{E}{LC}\left(1 - s\frac{L}{R}\frac{1}{(1-\alpha_0)^2}\right)}{s^2 + s\frac{1}{RC} + \frac{1}{LC}(1-\alpha_0)^2}$$

The END

THE END

REMARKS ON POINT 4.2

FLOWCHART FOR A SYSTEMATIC ANALYSIS

- 4.2. – Flowchart for a systematic analysis.

1. Search for the order of the system (back)

Analyse the equivalent circuit corresponding to the free state:

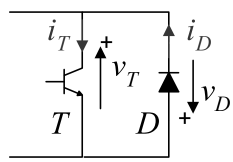
- The order of the system correspond to the number of independent state variables, i.e. = Σ (independent inductors + independent capacitors)

Remark2. Expressions for voltages and current on the switches

- 4.2. – Flowchart for a systematic analysis.

2. Compute expressions for voltage and current on semiconductors (back)

- The current through each semiconductor is taken as positive when it flows in the sense of the conduction;
- The voltage across each semiconductor is taken in the opposite sense of the current flow (reception convention);
- Each semiconductor at closed state is considered as a voltage source, V_D=Δ (pn junction directly polarised)

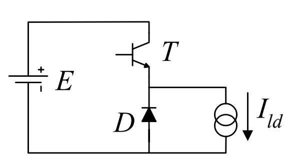


Remark3. Test of compatibility

- 4.2. – Flowchart for a systematic analysis.

3. Test of compatibility (back)

- A state change of a semiconductor may induce an instantaneous change on another semiconductor, leading to several switching events at the same instant;
- This new switching event produces another sequence;
- A new test of compatibility has to be performed in order to find the **stable sequence**



T on => D off ; instantaneously
T off => D on ; instantaneously
T on and D on: inexistent sequence

Remark4. Expressions for state variables

- 4.2. – Flowchart for a systematic analysis.

4. Compute the expressions for state variables (back)

- By knowing the free state and forced state circuit responses, the evolution of the state variables has to be drawn:
  - Solving the differential equations system in time domain;
  - Use of the phase plane method (suitable for most of the practical cases)

Remark5. Events leading to a switching

- 4.2. – Flowchart for a systematic analysis.

5. Check for the events that may generate a switching (back)

Types of events leading to a possible sequence transition

Switches with natural switching

- natural turning off: current on the switch at closed state = 0;
- natural turning on: voltage across the switch at open state >= 0;

Switches with forced switching

- forced turning off: current on the switch at closed state >0 & gate signal *off*;
- forced turning on: voltage across the switch at open state > 0 & gate signal *on*;

Remark6. – The event that leads to a sequence transition

- 4.2. – Flowchart for a systematic analysis.

6. Selection of the event leading to a sequence transition (back)

- Find the event, among those counted at point 5, occurring earlier in time;
- This task demands often deep reflection and knowledge of the system operating conditions