Accelerator Basics

D. Brandt, CERN

Accelerators in the world (2002)

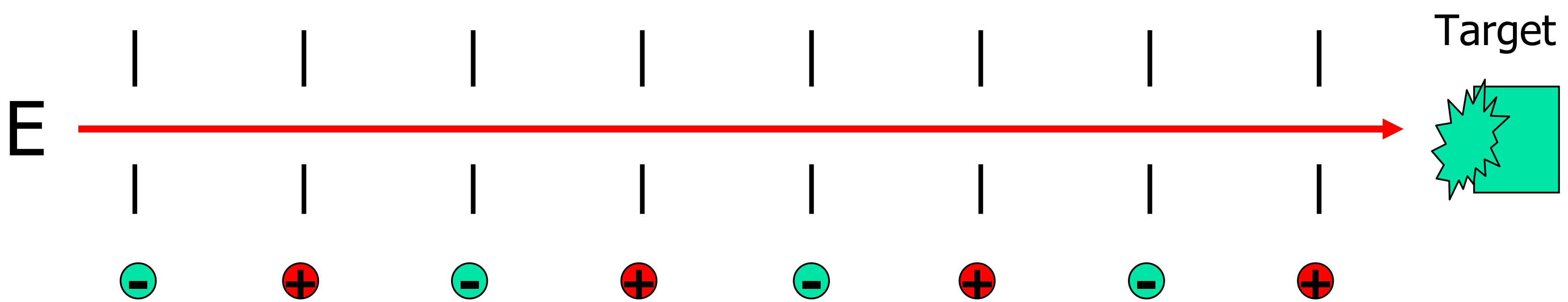
Basic and Applied Research		Medicine	
High-energy phys.	120	Radiotherapy	7500
S.R. sources	50	Isotope Product.	200
Non-nuclear Res.	1000	Hadron Therapy	20
Industry			
Ion Implanters	7000		
Industrial e- Accel.	1500	Total:	17390

Courtesy: W. Mondelaers JUAS 2004

Common to all these….

- Some kind of magnets have to be present…
- Most magnets will require electrical powering…
- Their performance will depend on the stability of the power source…

Ideal linear machines (linacs)



Available Energy : $E_{c.m.} = m \cdot (2\gamma+2)^{1/2}$

with $\gamma = E/E_0$

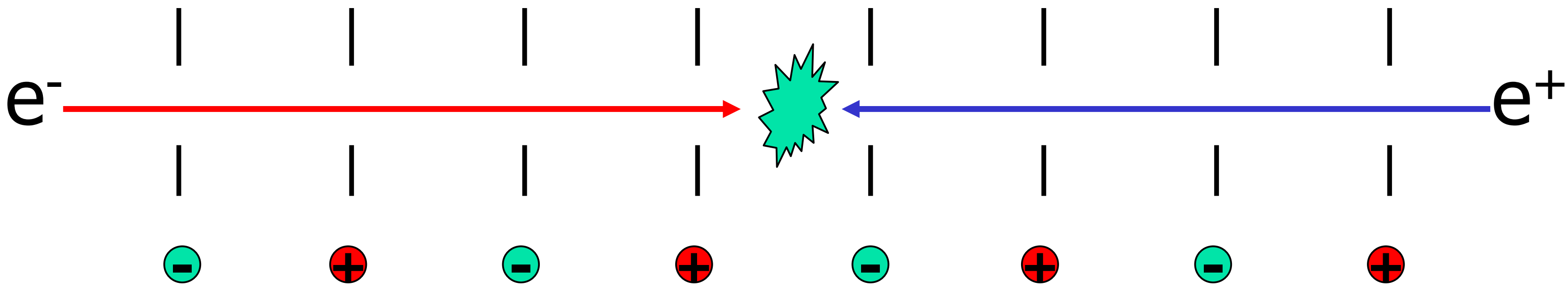
<u>Advantages</u>: Single pass

High intensity

<u>Drawbacks</u>: Single pass

Available Energy

Improved solution for $E_{c.m.}$



Available Energy : $E_{c.m.} = 2m\gamma = 2E$

with $\gamma = E/E_0$

Advantages: High intensity

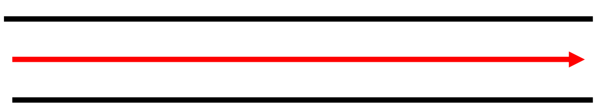
Drawbacks: Single pass

Space required

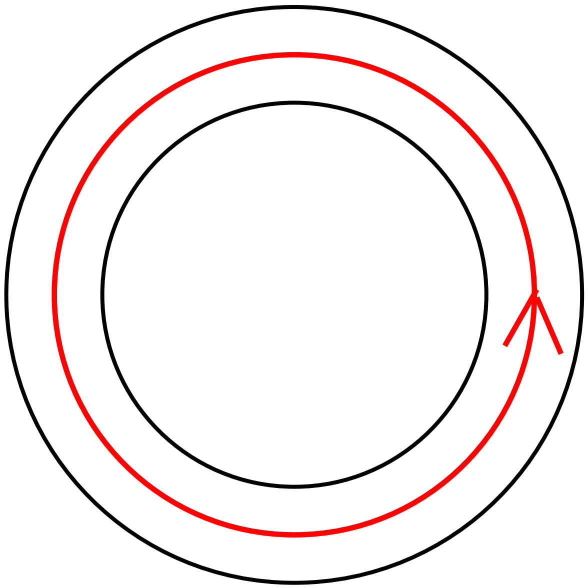
Keep particles: circular machines

Basic idea is to re-use the particles or keep them in the machine.

<u>Move from the linear design</u>



<u>To a circular one:</u>

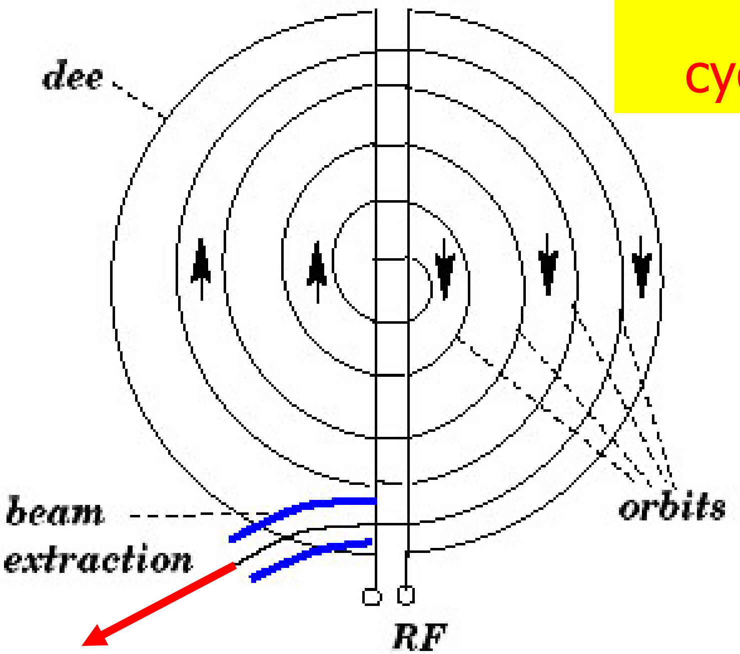


- Need Bending
- Need Dipoles!

Circular machines ($E_{c.m.} \sim (\gamma)^{1/2}$)

fixed target:

cyclotron



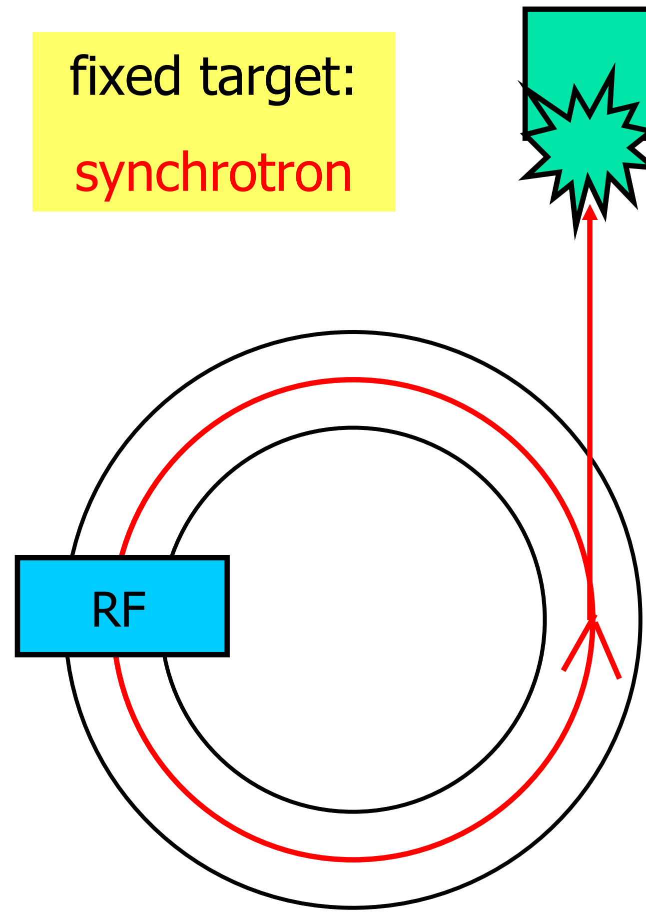
huge dipole, compact design,

B = constant

low energy, single pass.

fixed target:

synchrotron



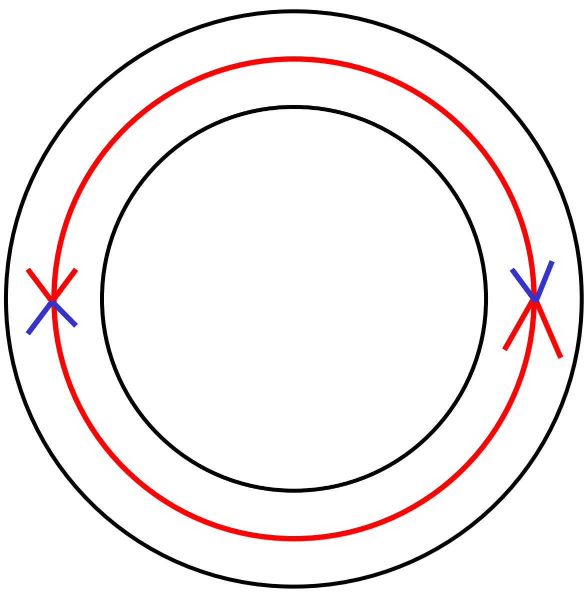
varying B, small magnets, high energy

Colliders ($E_{c.m.}$=2E)

<u>Colliders:</u>

electron – positron

proton - antiproton



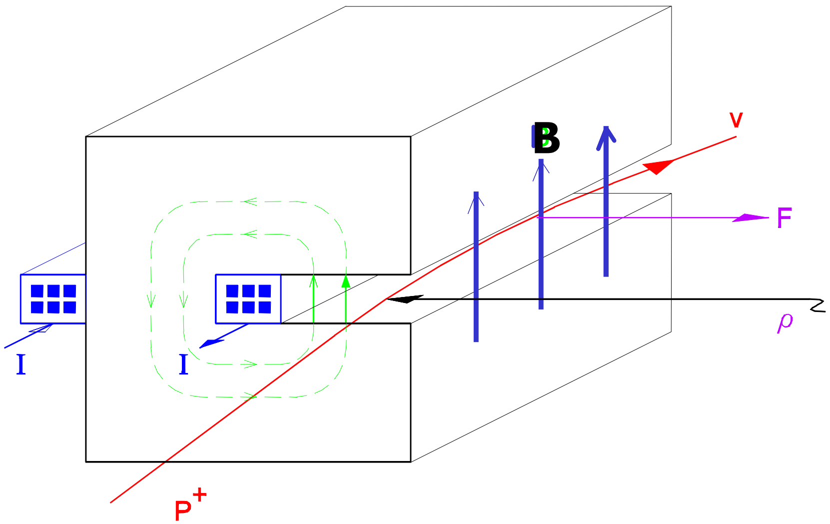
Colliders with the same type of particles (e.g. p-p) require two separate chambers. The beam are brought into a common chamber around the interaction regions

Ex: LHC

8 possible interaction regions

4 experiments collecting data

Circular machines: Dipoles



Classical mechanics:

Equilibrium between two forces

Lorentz force: $F = e.(\underline{v} \times \underline{B})$

Centripetal force: $F = mv^2/\rho$

$evB = mv^2/\rho$

Magnetic rigidity:

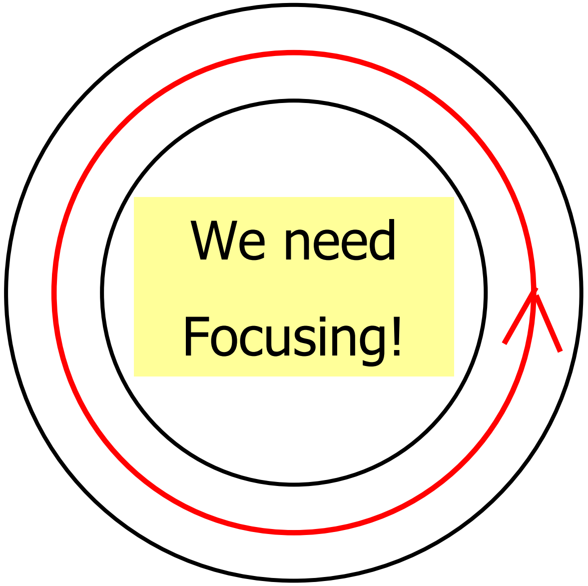
$B\rho = mv/e = p/e$

Relation also holds for relativistic case provided the classical momentum mv is replaced by the relativistic momentum p

Ideal circular machine:

- Neglecting gravitation
- Neglecting radiation losses in the dipoles

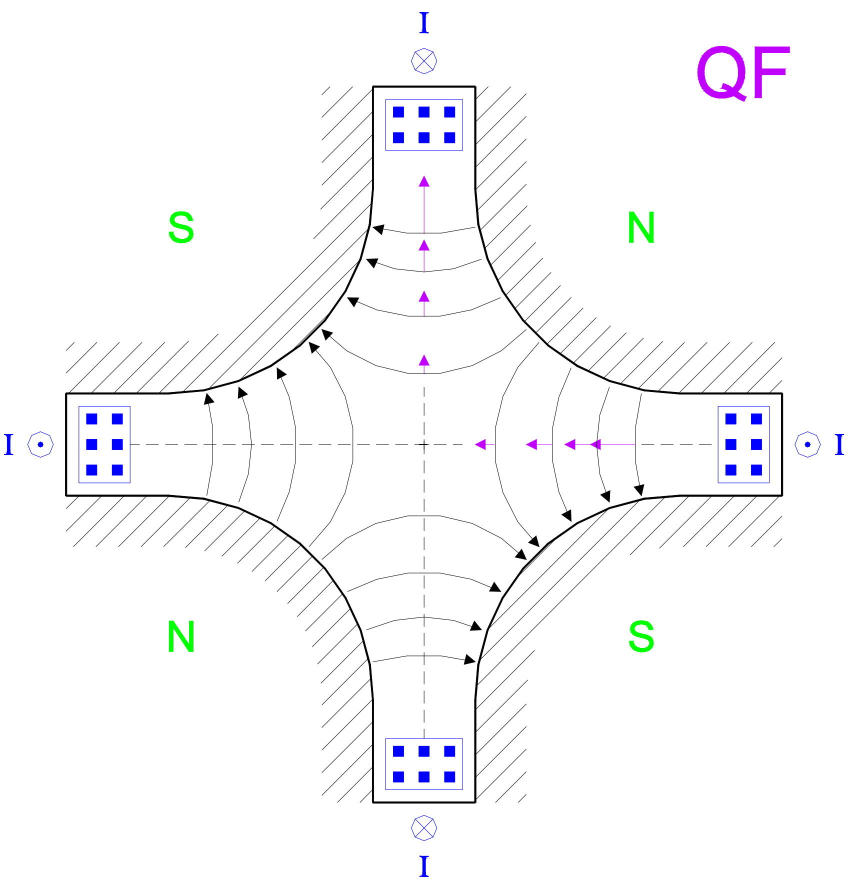
ideal particle would happily circulate on axis in the machine for ever!



Unfortunately: real life is different!

Gravitation: Δy = 20 mm in 64 msec!	
Alignment of the machine	Limited physical aperture
Ground motion	Field imperfections
Energy error of particles and/or $(x, x')_{inj} \neq (x, x')_{nominal}$	
Error in magnet strength (power supplies and calibration)	

Focusing with quadrupoles



$$F_x = -g.x$$

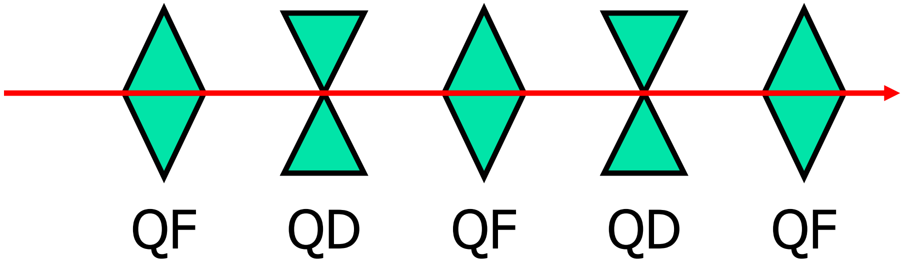
$$F_y = g.y$$

Force increases linearly with displacement.

Unfortunately, effect is opposite in the two planes (H and V).

Basic new idea:

Alternate QF and QD



Particle dynamics

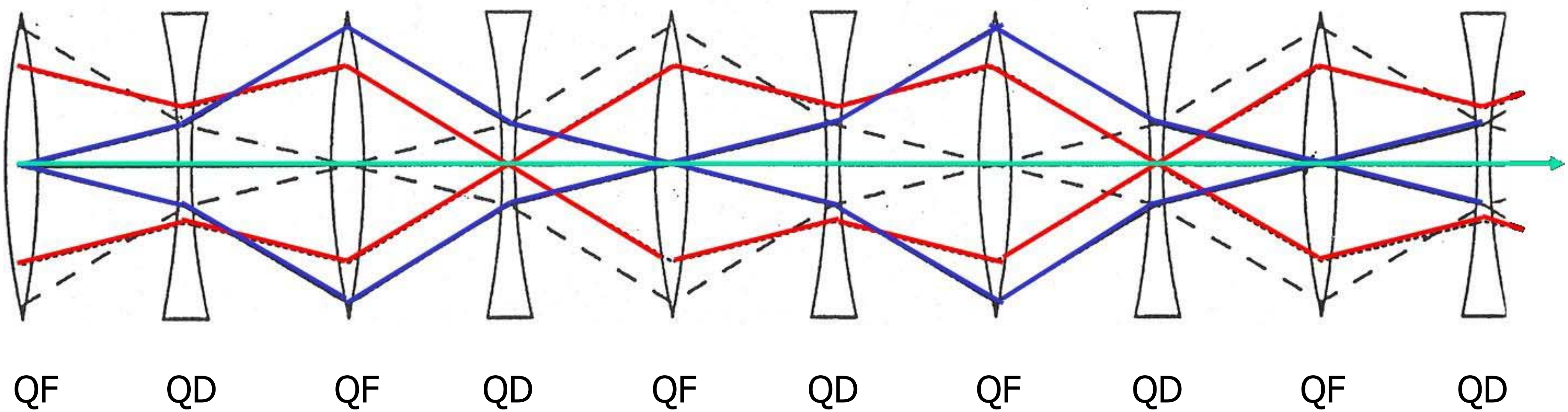
- Particles are characterised by:
  - Their position $x(s)$ or $y(s)$ along the accelerator
  - Their slope $x'(s)$ or $y'(s)$ along the accelerator
  - Their energy or momentum deviation $\Delta p/p$

Let us first consider a « non-ideal » injection in position and slope

(x, x') or (y, y')

Alternating gradient focusing

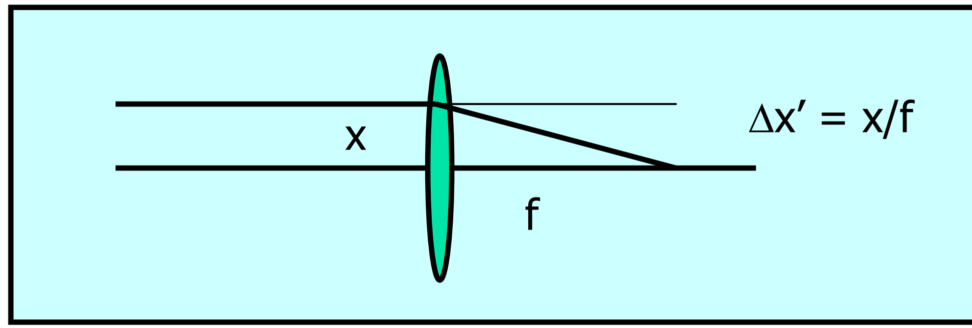
It can be shown that a section composed of alternating focusing and defocusing elements has a **net focusing effect**, provided the quadrupoles are correctly placed. What happens to « non-ideal » particles?



The « non-ideal » particles perform an oscillation around the « ideal » trajectory.

Thin lens analogy of AG focusing

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$

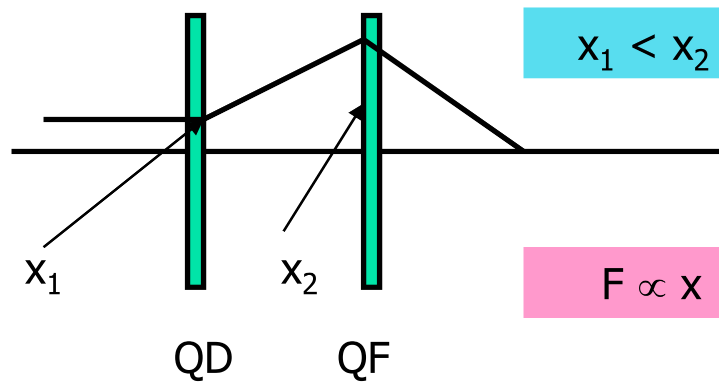


$X_{out} = x_{in} + 0.x'_{in}$
$x'_{out} = (-1/f).x_{in} + x'_{in}$

$$\text{Drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

More intuitively:

$$\text{QF-Drift-QD} = \begin{pmatrix} 1-L/f & L \\ -L/f^2 & 1+L/f \end{pmatrix}$$

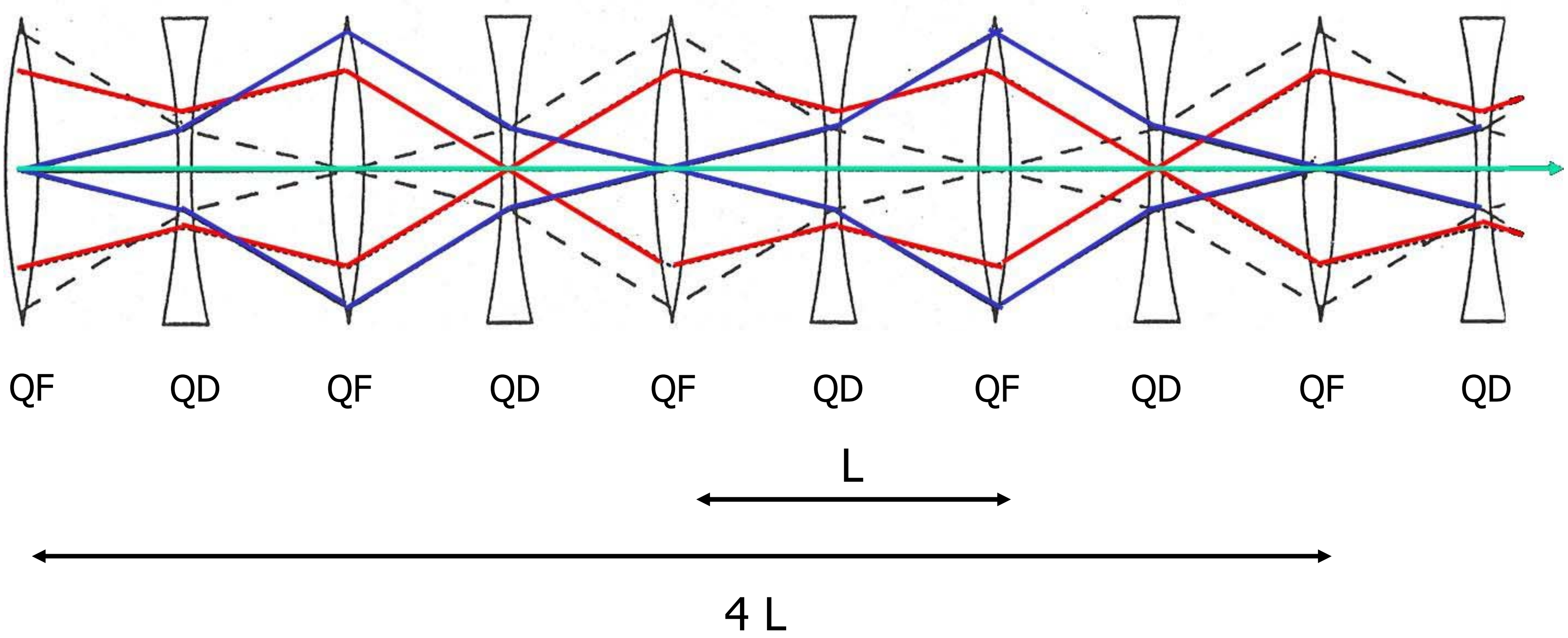


$x_1 < x_2$

$F \propto x$

Initial: $x = x_0$ and $L < f$
$x' = 0$

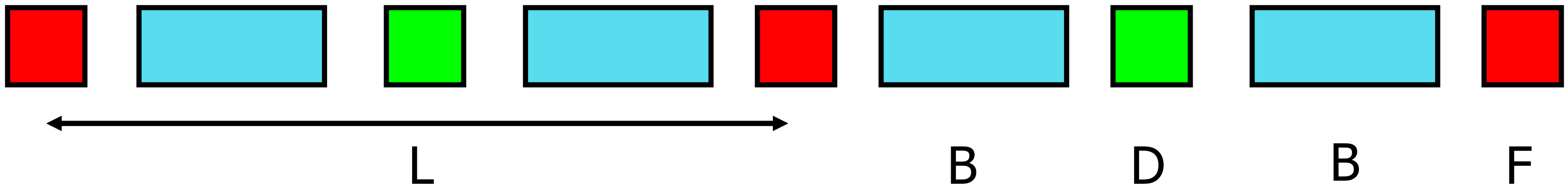
The FODO cell:



One complete oscillation in 4 cells ⇒ 90°/ cell ⇒ $\mu = 90°$

Real circular machines (no errors!)

The accelerator is composed of a periodic repetition of FODO cells:



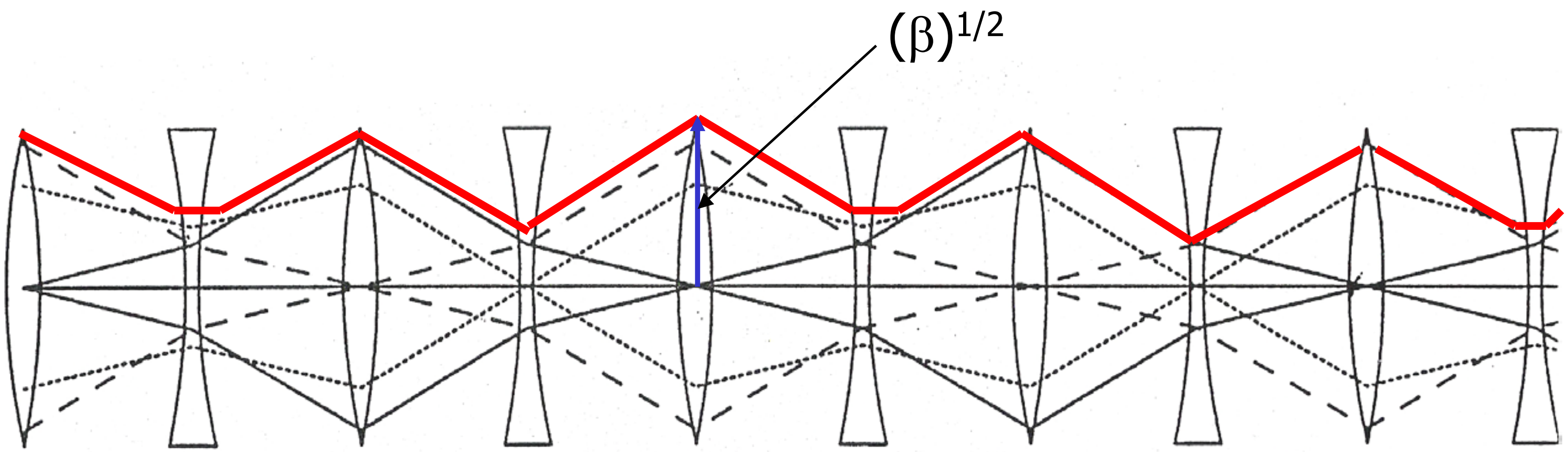
- The phase advance per cell μ can be modified, in each plane, by varying the strength of the quadrupoles.

- The ideal particle will follow a particular trajectory, which closes on itself after one revolution: the closed orbit.

- The real particles will perform oscillations around the closed orbit.

- The number of oscillations for a complete revolution is called the Tune Q of the machine (Qx and Qy).

The beta function β(s)



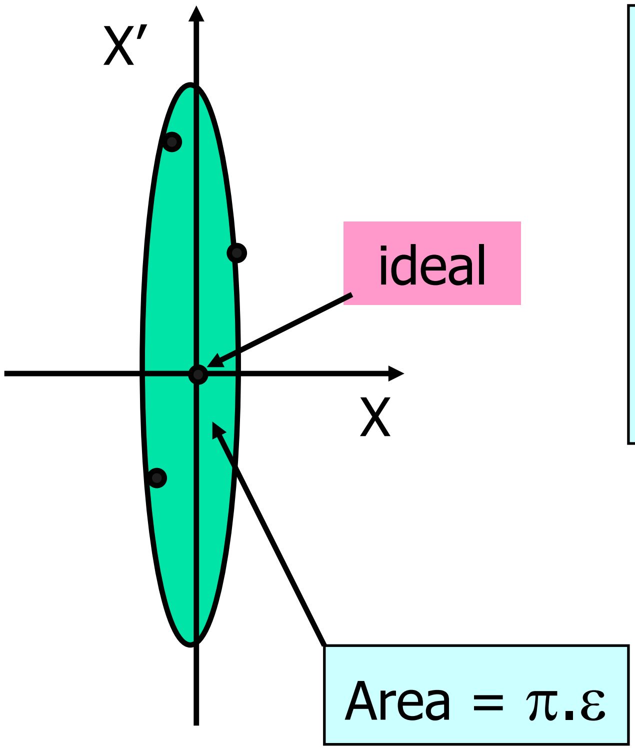
The β-function is the envelope around all the trajectories of the particles circulating in the FODO lattice.

The β-function has a minimum at the QD and a maximum at the QF, ensuring the net focusing effect of the lattice.

It is a periodic function in the FODO lattice. The oscillations of the particles are called betatron motion or betatron oscillations.

Phase space

- Select the particle in the beam with the largest betatron motion and plot its position vs. its phase (x vs. x′) at some location in the machine for many turns.

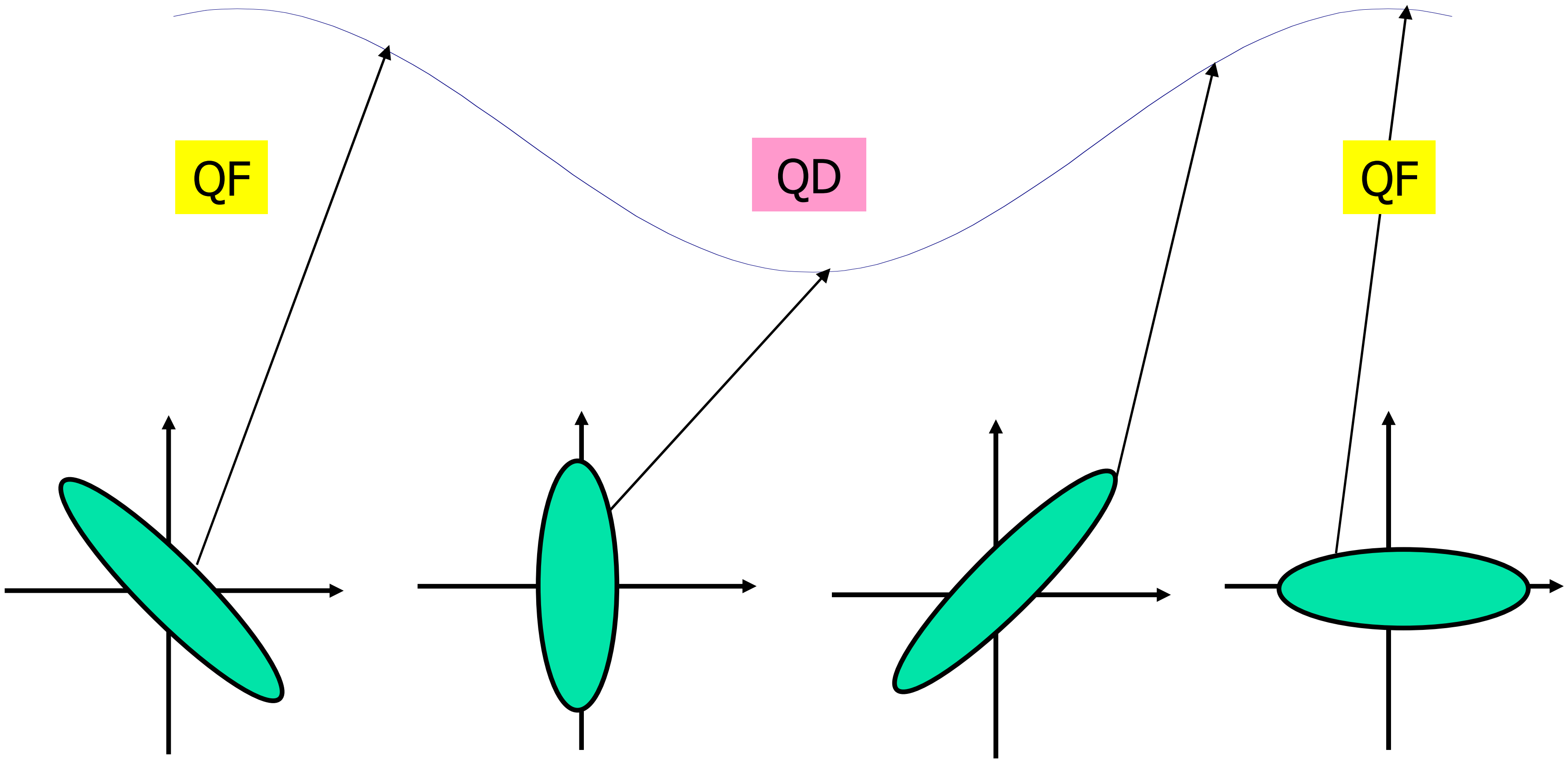


- ε Is the emittance of the beam [π mm mrad]
- ε is a property of the beam (quality)
- Measure of how much particle depart from ideal trajectory.
- β is a property of the machine (quadrupoles).

Beam size [m]

$$\sigma = (\varepsilon.\beta)^{1/2}$$

Emittance conservation



The shape of the ellipse varies along the machine, but its area (the emittance ε) remains constant.

Recapitulation (1)

- The fraction of the oscillation performed in a FODO cell is called the phase advance μ per cell.
- The total number of oscillations over one full turn of the machine is called the betatron tune Q.
- The envelope of the betatron oscillations is characterised by the beta function $\beta(s)$. This is a property of the quadrupole settings.
- The quality of the (injected) beam is characterised by the emittance ε. This is a property of the beam and is invariant around the machine.
- The r.m.s. beam size (measurable quantity) is $\sigma = (\beta.\varepsilon)^{1/2}$.

What about a « non-ideal » injection energy $\Delta p/p \neq 0$?

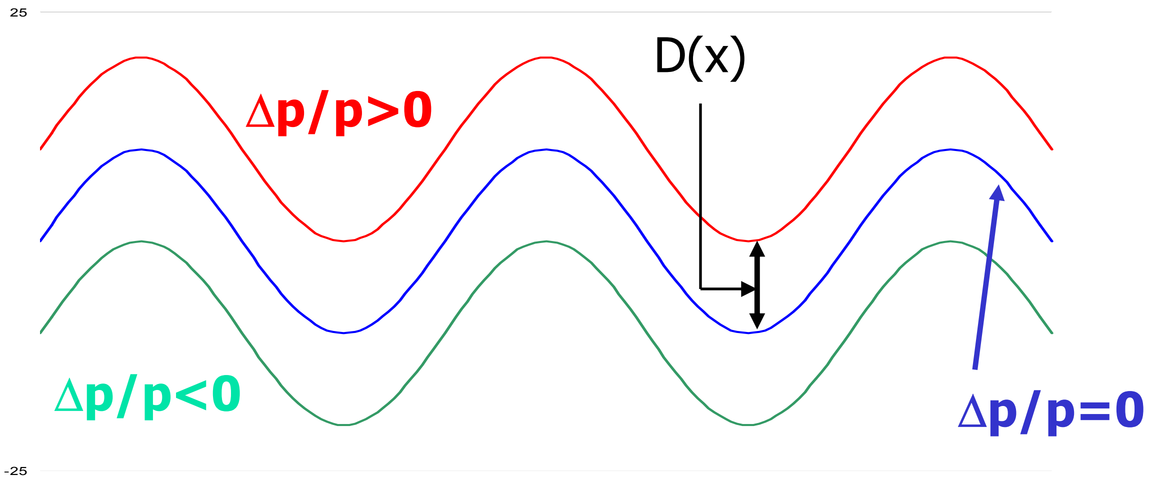
Off momentum particles ($\Delta p/p \neq 0$)

Effect from Dipoles

- If $\Delta p/p > 0$, particles are less bent in the dipoles ➨ should spiral out !
- If $\Delta p/p < 0$, particles are more bent in the dipoles ➨ should spiral in !

No!

There is an equilibrium with the restoring force of the quadrupoles

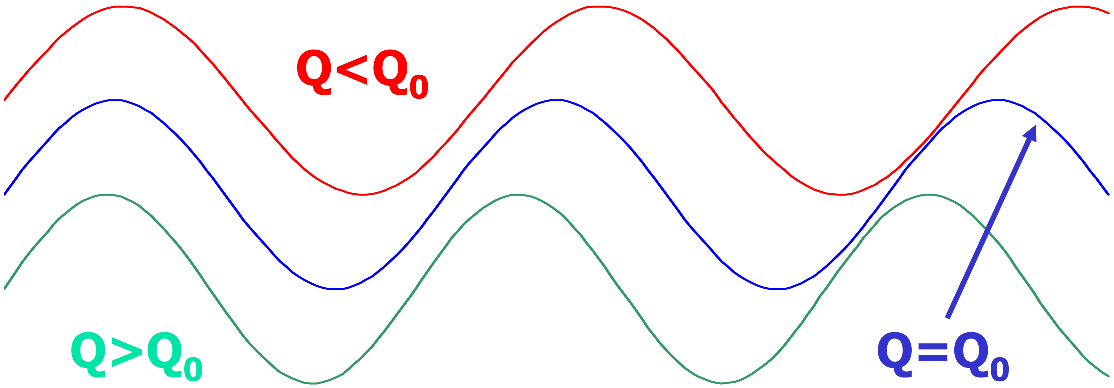


Off momentum particles ($\Delta p/p \neq 0$)

Effect from Quadrupoles

- If $\Delta p/p > 0$, particles are less focused in the quadrupoles ➨ **lower Q !**
- If $\Delta p/p < 0$, particles are more focused in the quadrupoles ➨ **higher Q !**

Particles with different momenta would have a different betatron tune $Q=f(\Delta p/p)$!



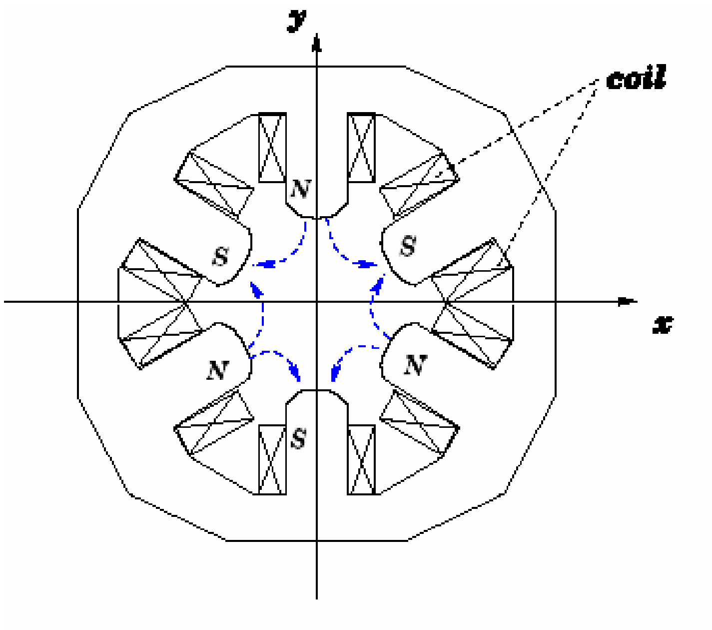
The chromaticity Q′

- The tune dependence on momentum is of fundamental importance for the stability of the machine. It is described by the chromaticity of the machine Q′:

$$Q' = \Delta Q / (\Delta p/p)$$

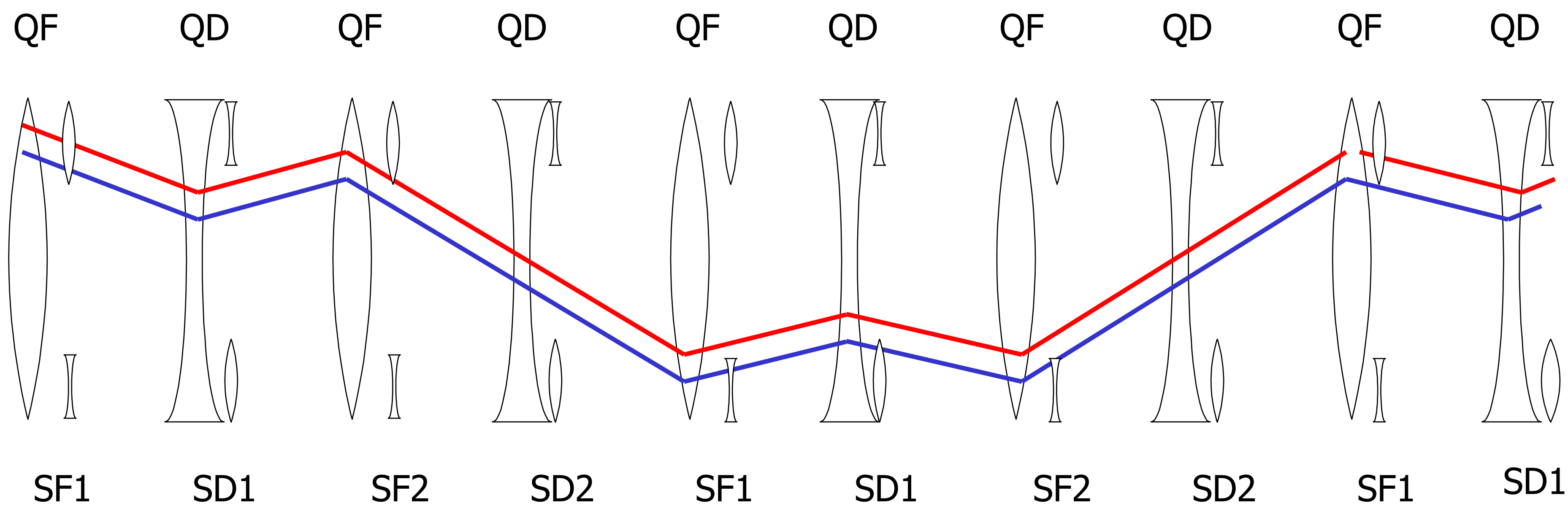
- For relativistic particles, the chromaticity has to be positive (stability)!
- The natural chromaticity of the machine is negative!
- The chromaticity has to be corrected and kept under control.
- This is achieved by means of sextupoles

The sextupoles (SF and SD)



- $\Delta x' \propto x^2$
- A SF sextupole basically « adds » focusing for the particles with $\Delta p/p > 0$, and « reduces » it for $\Delta p/p < 0$.
- The chromaticity is corrected by adding a sextupole after each quadrupole of the FODO latice.

Chromaticity correction



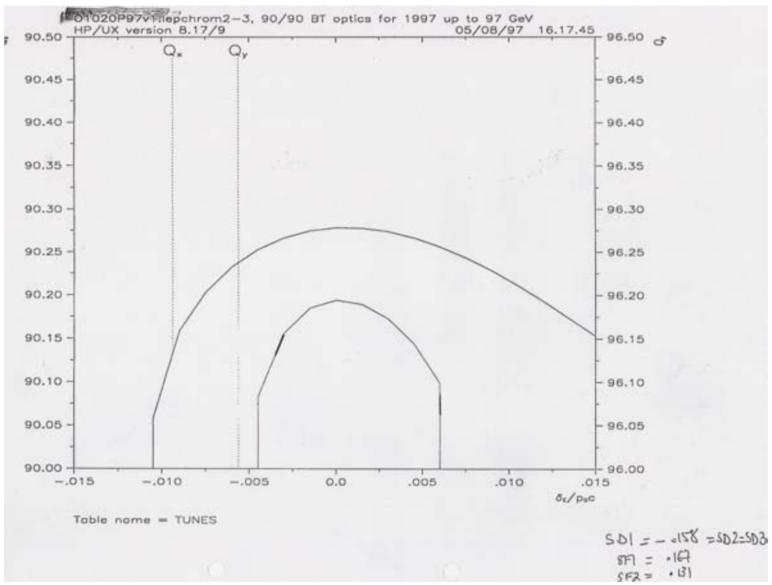
The undesired effect of sextupoles on particles with the nominal energy can be avoided by grouping the sextupoles into « families ».

Nr. of families:

$N = (k \cdot 180\,°/\mu) = \text{Integer}$

e.g. $180\,°/\,90\,° = 2$

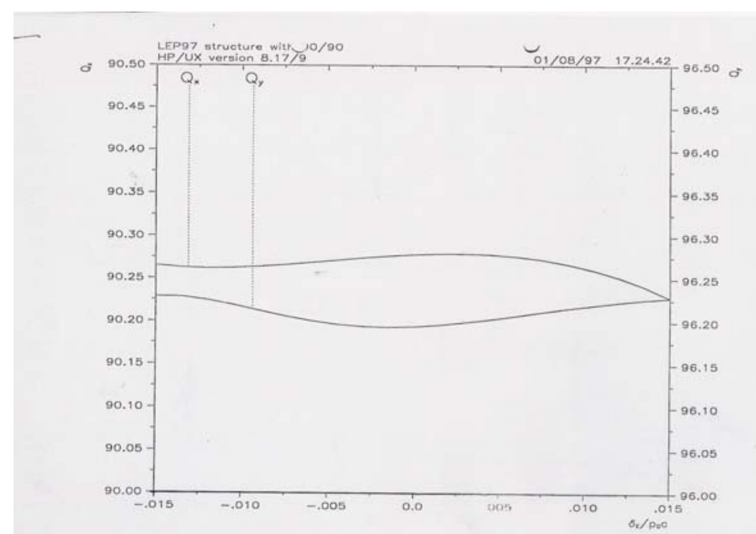
Tune vs. momentum



Correction with 1 sextupole family:

Bad!

Off momentum particles rapidly cross the integer (Qy!).



Correction with 2 sextupole families:

Excellent!

Tunes remain almost constant over the whole range of momentum!

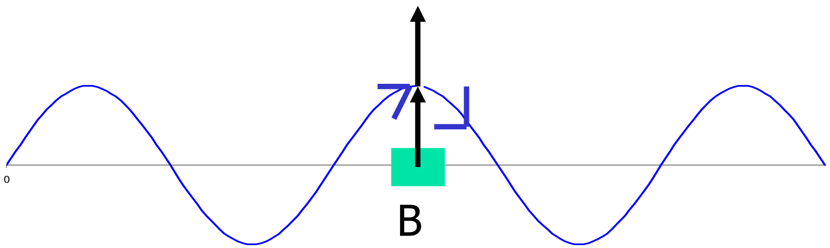
Forbidden values for Q

- An error in a dipole gives a kick which has always the same sign!

Integer Tune Q = N

Forbidden!

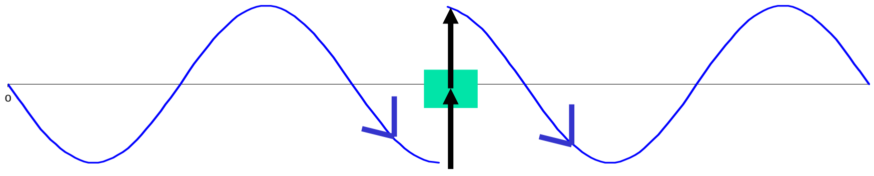
The perturbation adds up!



Half-integer Tune Q = N + 0.5

O.K. for an error in a dipole!

The perturbation cancels after each turn!



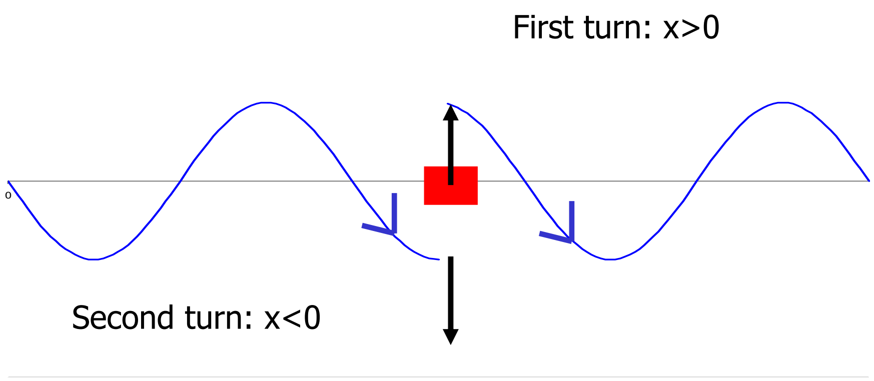
Forbidden values for Q

- An error in a quadrupole gives a kick whose sign depends on x

Half-integer Tune Q = N + 0.5

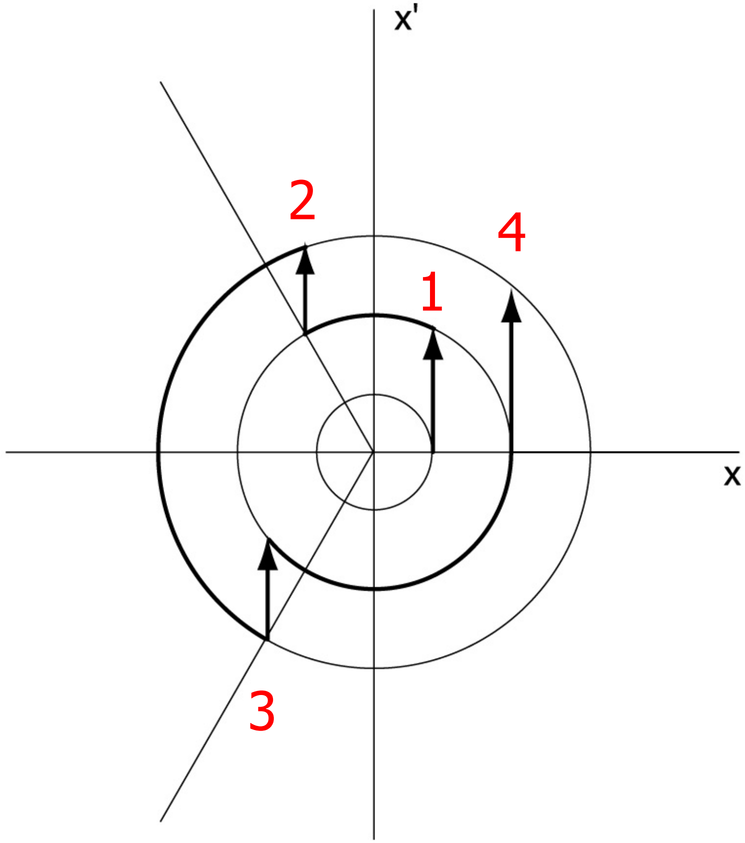
Forbidden !

The amplitude of the oscillation is steadily increasing!



What about a 1/3-integer Tune?

1/3 integer Tune



Tune Q = N + 0.33

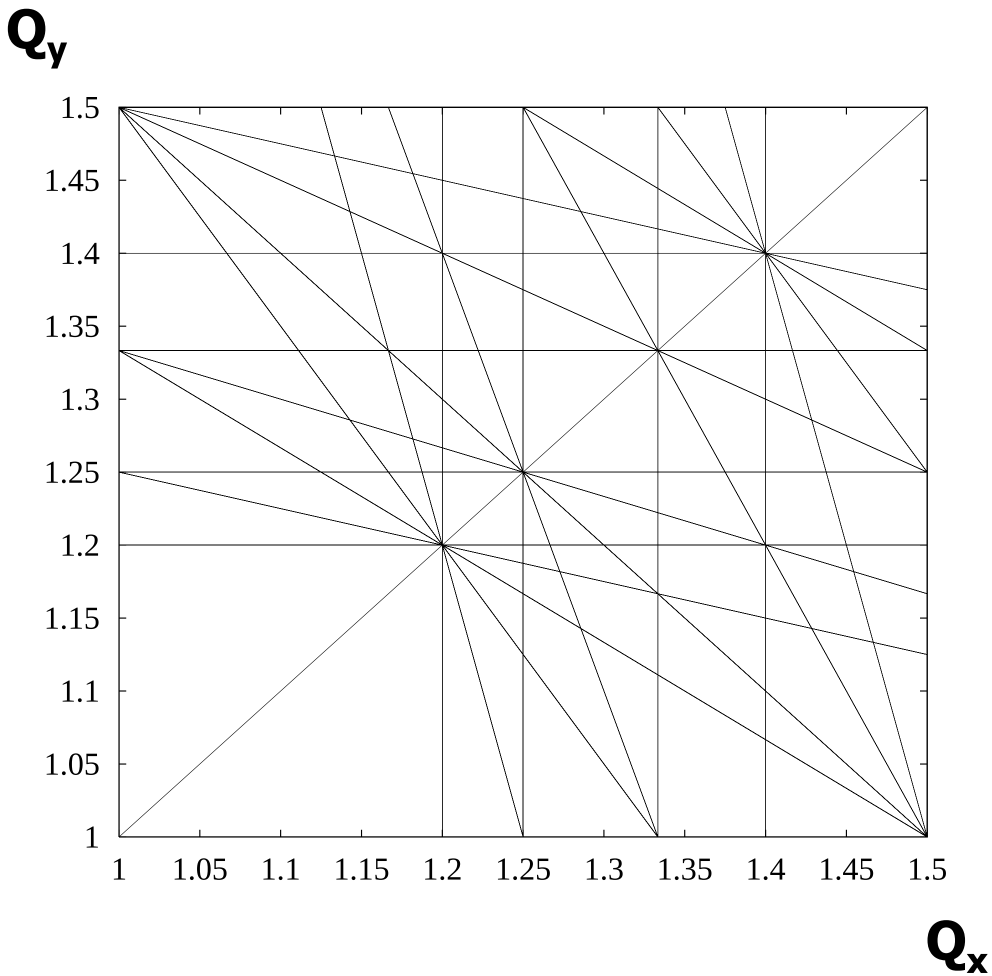
Forbidden !

The amplitude of the oscillation increases every third turn!

One would come to similar conclusions for

Q = 1/4 , 1/5, …

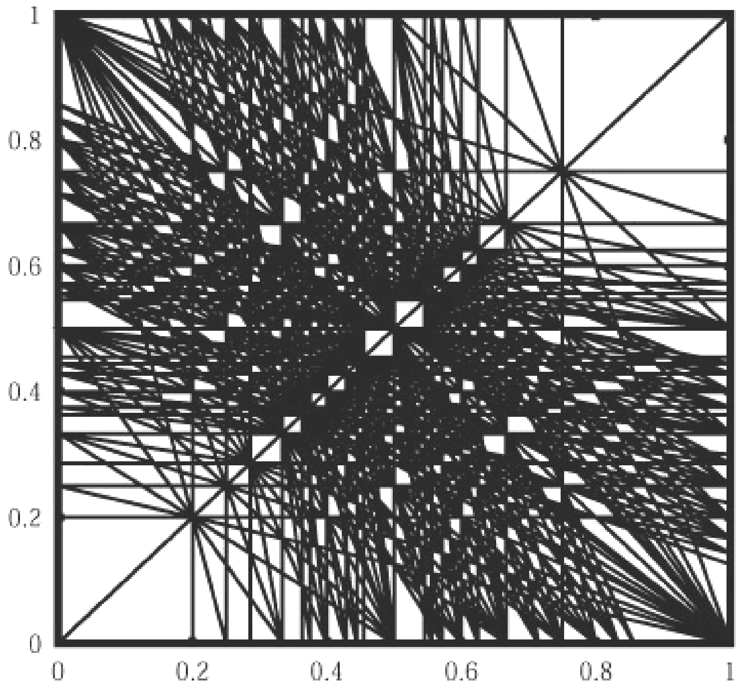
Tune diagram for leptons



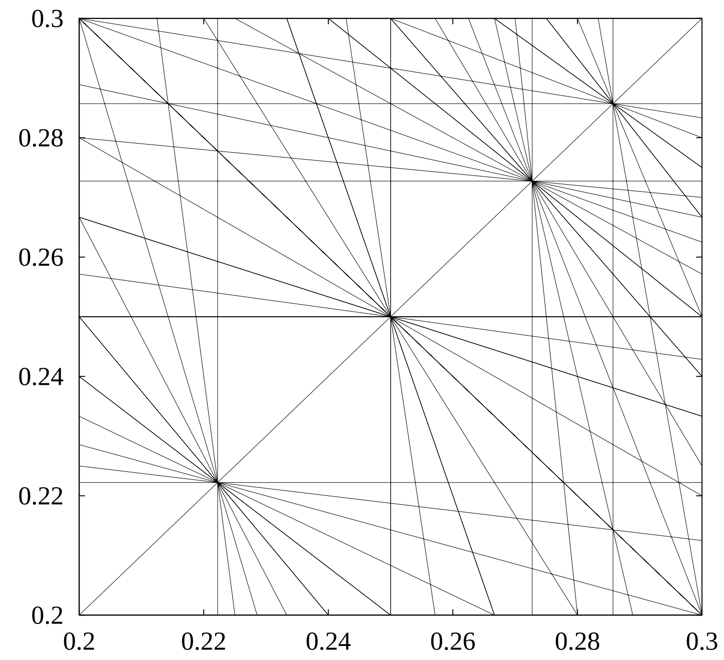
Tune values (Qx and/or Qy) which are forbidden in order to avoid resonances

The lowest the order of the resonance, the most dangerous it is.

Tune diagram for protons



The particles have a certain tune spread, the bunch thus represents a small area rather than a point in the tune diagram.

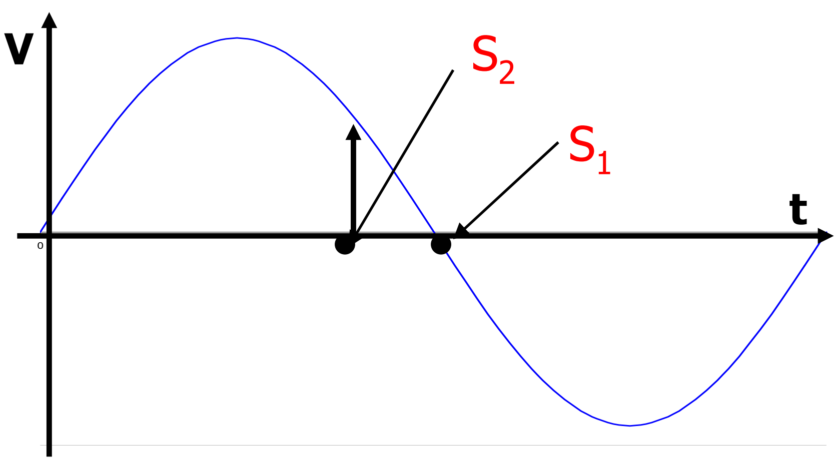


Summary for transverse planes

- A particle is described by its position and its slope (x, x′) and (y, y′)
- The circular trajectory is obtained with the dipoles.
- The particles are kept together with the quadrupoles.
- The particles perform betatron oscillations around the closed orbit.
- The number of oscillations per turn (the tune Q) has to be chosen very carefully in each plane to avoid resonances.
- The phase advance per cell (μ) can be varied with the quadrupoles.
- The natural chromaticity of the machine (<0) is corrected with sextupoles.

Longitudinal dynamics

- We have to provide energy to the particles either to accelerate them or to compensate for the losses they experienced during one turn.
- The energy is not provided by electro-static plates, but with RF cavities.
- The ideal particle should arrive exactly at the same time at the cavity after each revolution (synchronous particle).



Equilibrium: $f_{RF} = h \cdot f_{rev}$

$f_{rev} = (1/2\pi) \cdot (q/m\gamma) \cdot B$

Energy depends on magnetic field

Radiation losses U_0

- Charged particles bent in a magnetic field emit synchrotron radiation!

with $\gamma = E/E_0 = m/m_0$ and m_0 is the rest mass

Energy loss:

$$eU_0 = A \cdot \gamma^4/\rho$$

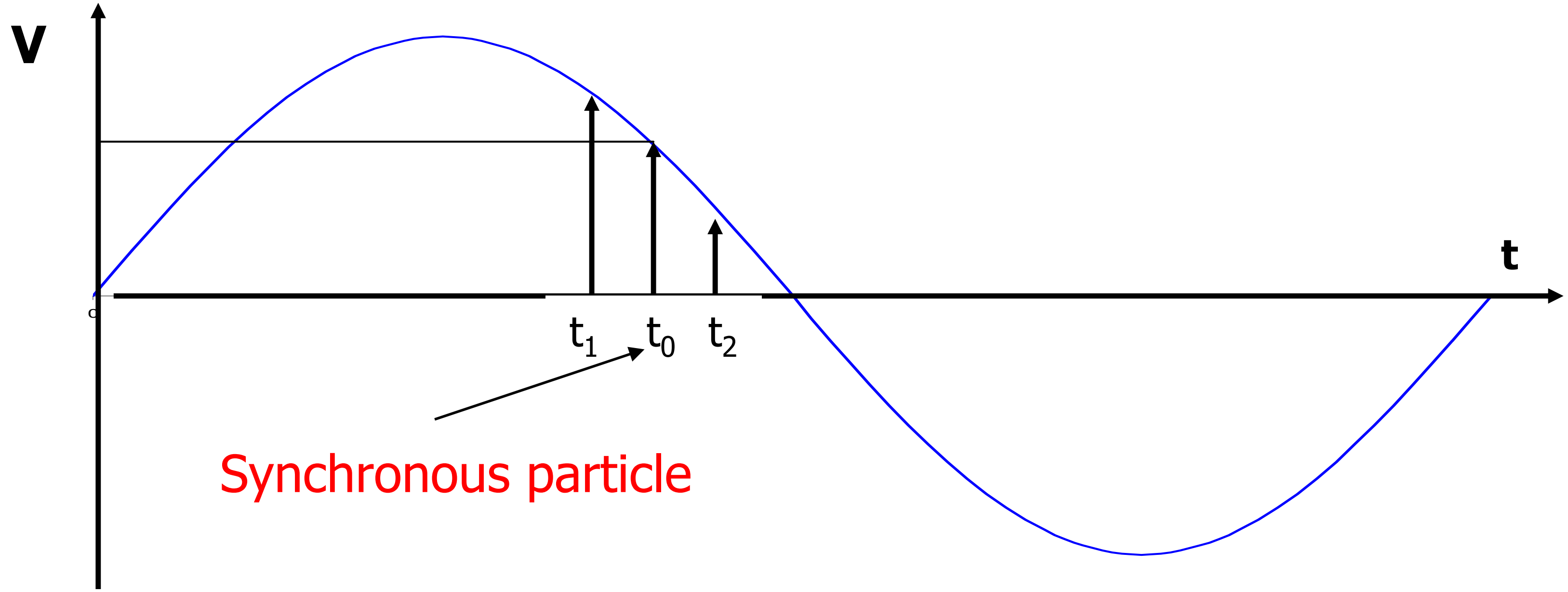
m_0 proton = 0.938 GeV/c^2

m_0 electron = 0.511 MeV/c^2

$(m_{o\text{-}p}/m_{o\text{-}e})^4 = (1836)^4 \cong 10^{13}$

Collider	B (T)	E/beam (GeV)	γ	eU_0 (GeV)
LEP ($e^+ e^-$)	0.12	100	196000	2.92
LHC (p-p)	8.3	7000	7500	0.00001

Off momentum particles

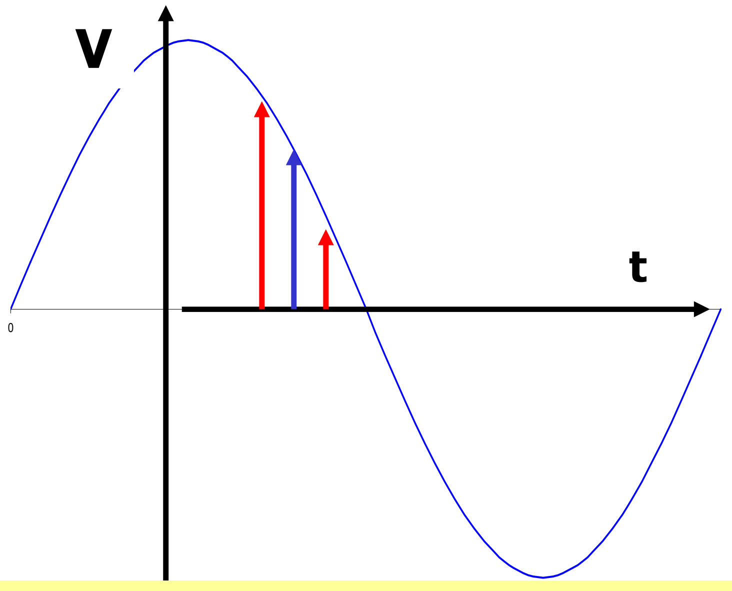


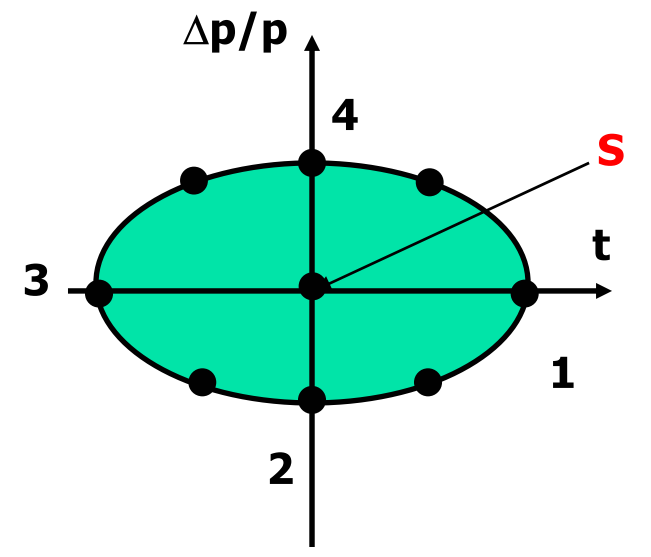
On momentum particle arrives at t_0 ➔ $V = V_0$ ➔ o.k.

$\Delta p/p > 0$ have a longer path ➔ arrive late, e.g. t_2 ➔ $V_2 < V_0$

$\Delta p/p < 0$ have a shorter path ➔ arrive early, e.g. t_1 ➔ $V_1 > V_0$

Synchrotron oscillations





1) Correct energy but late, not enough voltage ➨ will loose energy.

2) On time, correct voltage, on short orbit ➨ will gain energy.

3) Correct energy but early, too large voltage ➨ will gain energy.

4) On time, correct voltage, on long orbit ➨ will loose energy.

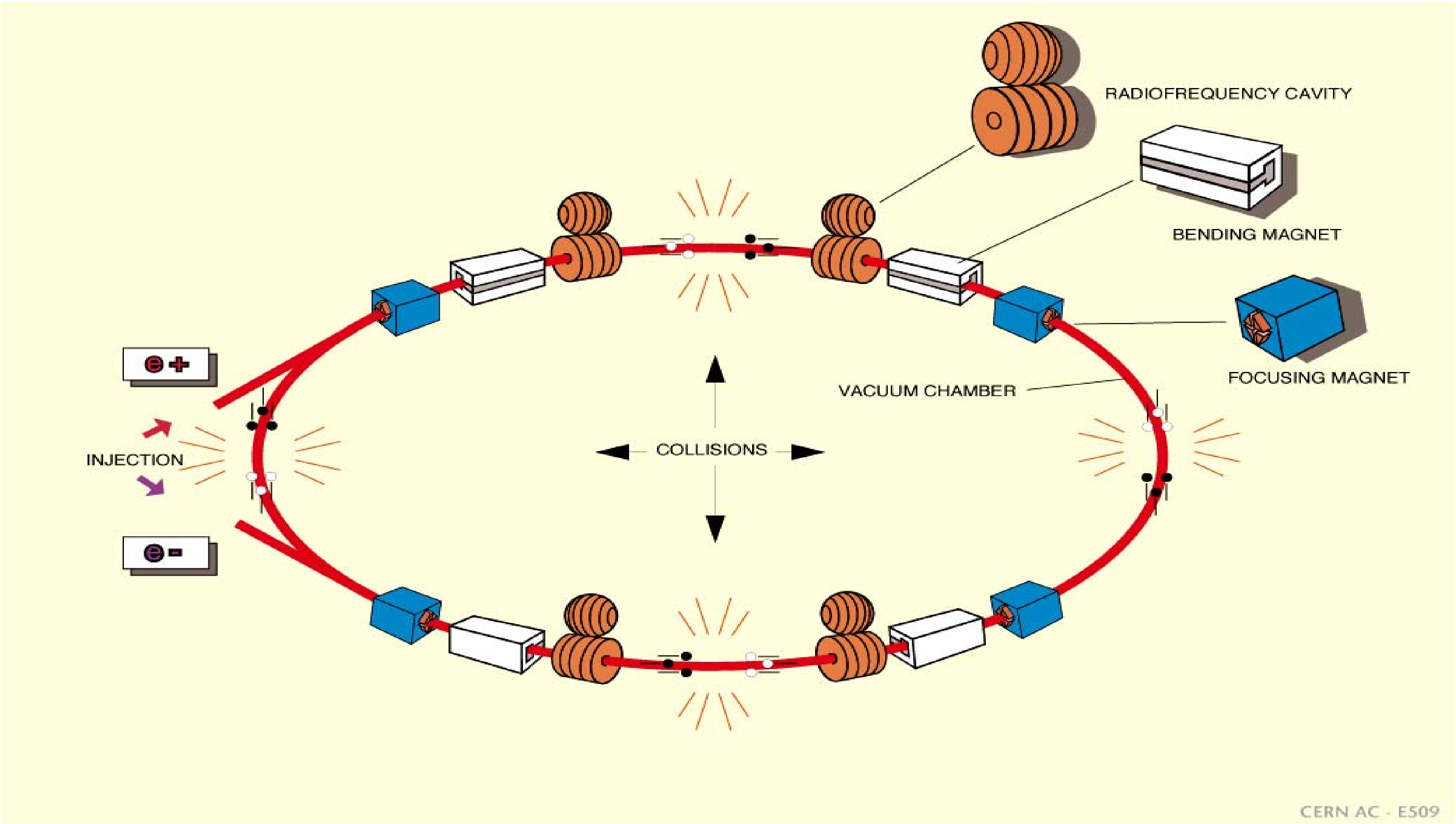
Synchrotron oscillations

- In the longitudinal plane, particles also perform oscillations, the synchrotron oscillations.
- These oscillations are characterised by the synchrotron tune Q_s.
- The frequency of the synchrotron oscillations is very different from that of the betatron oscillations: $Q_\beta > 1$ $Q_s << 1$

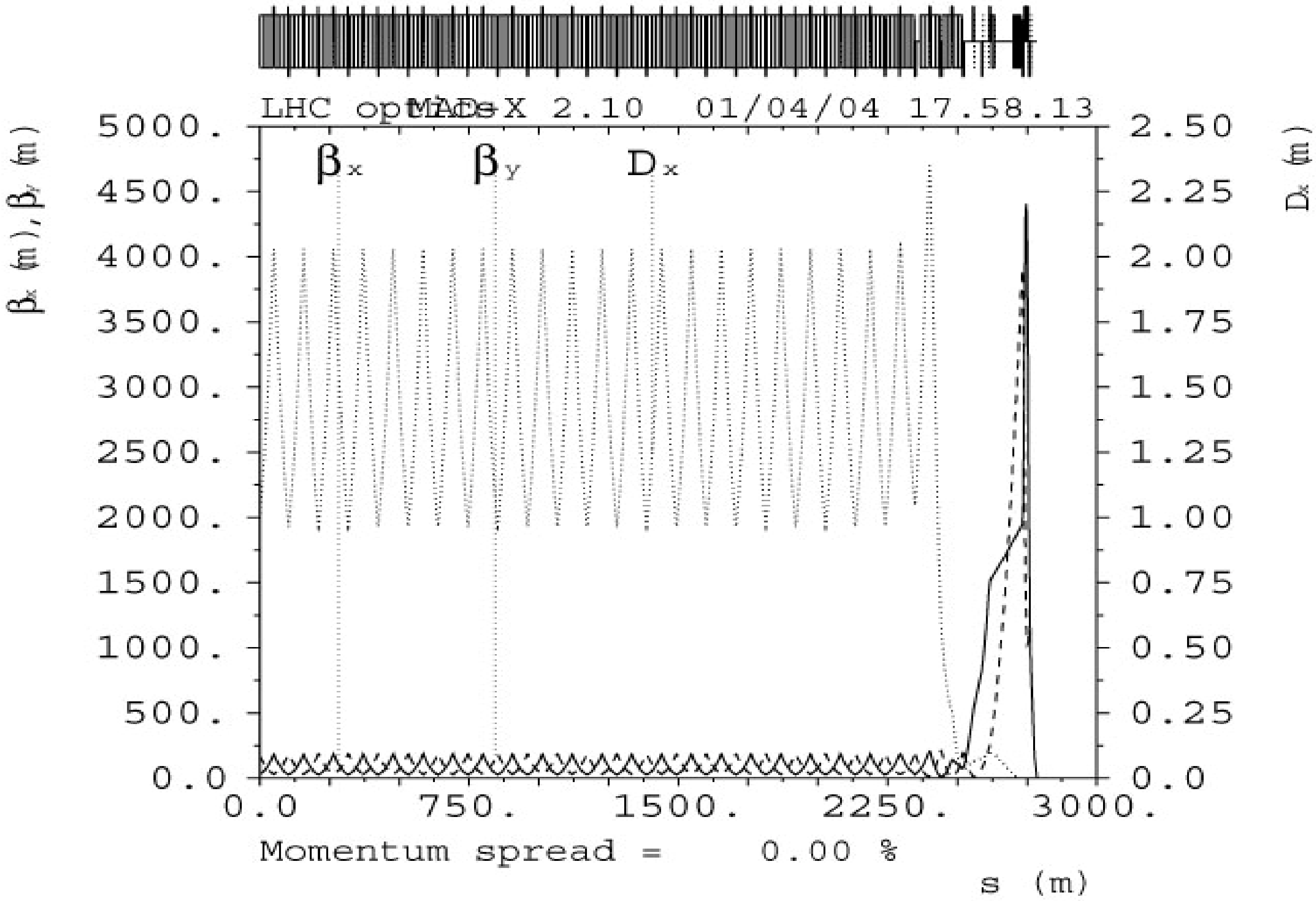
The RF system imposes limits on t (i.e. t_1 and t_2) and $\Delta p/p$ for which the particles are stable and perform synchrotron oscillations within the bunch. Outside these limits the particles are lost.

The RF cavities restore energy losses, ensure correct energy of the beam(s) and maintain particles grouped into bunches longitudinally.

Real high energy collider



Beta function in a real machine



Back to Power converters

How do the stability and the accuracy of the Power Converters affect the accelerator?

- Accuracy of the energy of the beam(s) : dipoles

- Modify the tunes of the machine : quadrupoles

- Perturb the closed orbit of the machine : field errors/fluctuations

Closed orbit

- Any imperfection or perturbation of the guide field will distort the closed orbit, which, so far was the theoretical axis of the machine.
- The ideal particle will no longer go straight down the centre of the vacuum chamber, but will follow a perturbed closed orbit (still closing on itself).
- The betatron oscillations of the particles will be superimposed to this distorted closed orbit ➨ Aperture and non-linearities.

$$x(s) = (\beta_i \, \beta(s))^{1/2} / (2\sin(\pi Q)) \; . \; \theta_i \; . \; \sin(\phi(s) - \phi_i)$$

$$x'(s) = (\beta_i \, / \beta(s))^{1/2} / (2\sin(\pi Q)) \; . \; \theta_i \; . \; \cos(\phi(s) - \phi_i)$$

Closed orbit and Power Converters

- In modern machines (aperture is very expensive), it is therefore essential to control the closed orbit with great care

LHC: $\Delta x, \Delta y < 4$ mm and r.m.s. < 0.5 mm

- In low-beta insertions (very large beta values before and after the I.P.), imperfections or perturbations of the guide field can have dramatic consequences (vacuum chamber, non-linear fields).

The stability and the accuracy of the power converters is one of the key ingredients to ensure the expected performance of the machine !