Introduction to Collective Effects

D. Brandt, CERN

Aim of the lecture:

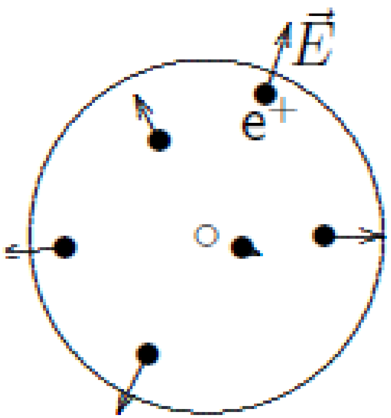
- Introduction to the “jargon”
- Introduction to a few basic concepts
- “Cooking recipes” when working with instabilities
- Details about Collective Effects ➔ G. Rumolo

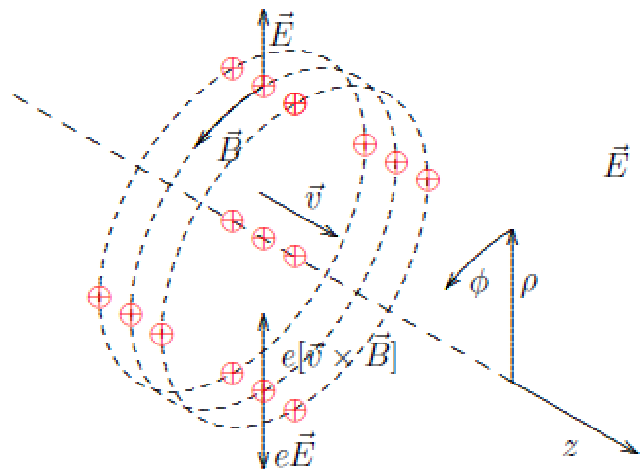
Multi-particle effects:

- Interaction between the charged particles within the bunch (space charge effects)
- Interaction between the bunch and the environment (Impedance – wakes)
- Interaction between the bunches via the impedance (coupled-bunch effects)

Space charge effects:

- We deal with charged particles ($\vec{E}$-field) ➔ repelling effect ➔ reduces the focusing
- Charged particles are moving ➔ $\vec{B}$-field ➔ Lorentz force





Direct space charge:

This force:

$$\vec{F} = e\left(\vec{E} + \left[\vec{v} \times \vec{B}\right]\right)$$

changes the slopes of the individual particles and produces a defocusing effect:

$$\Delta Q_{xy} = -\frac{r_0 R I}{e c E_{xy} \beta^3 \gamma^3}$$

or:

$$\Delta Q_{xy} = -\frac{r_0 N_b}{2\pi E_{xy} \beta^2 \gamma^3}$$

!!!

Direct space charge:

- Direct space charge is a purely INCOHERENT effect

- For bunched beams, the current depends on the position in the bunch (I(s)). This leads to a tune spread and a tune modulation (synchrotron oscillations).

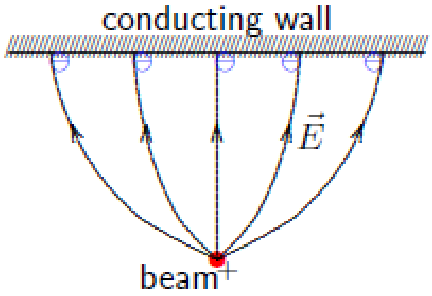
$$\Delta Q_{xy} = -\frac{r_0 R I(s)}{e c E_{xy} \beta^3 \gamma^3}$$

!!!

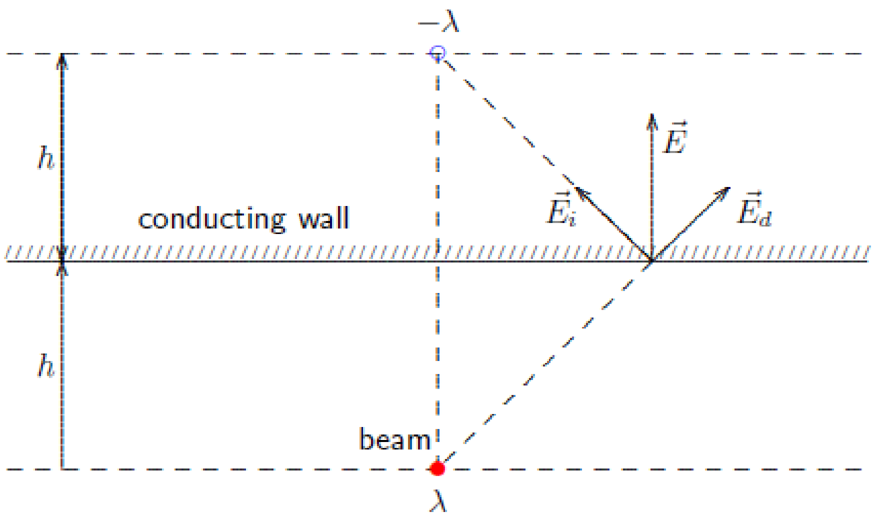
- This tune spread is very important for stability with Landau damping.

Effect of the vacuum chamber:

- A perfectly conducting vacuum chamber imposes a perpendicular electric field as boundary condition on the surface ($E_{//} = 0$).



- In fact, to compute the field, one has to introduce an image charge (-λ) at a distance h from the beam:

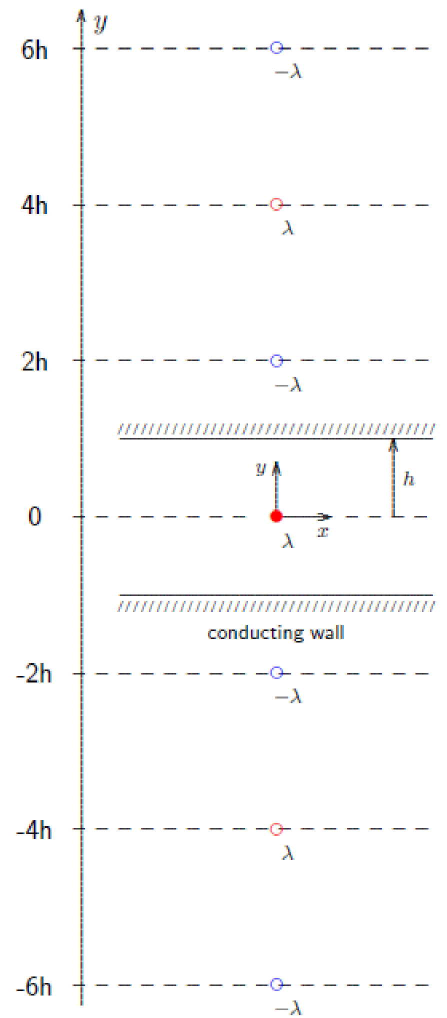


Effect of vacuum chamber:

Indirect space charge

• In fact, the vacuum chamber represents 2 conducting boundatries at h. To satisfy $E_{//}$=0, the procedure is a little bit more complicated:

• Compute the fields E_i due to each line charge, sum the fields, compute the force, new focusing term in equation of motion, compute ΔQ



Total incoherent effect:

The direct and indirect incoherent space charge effects are given by:

$$\Delta Q_{xy} = -\frac{2r_0 IR\langle\beta\rangle_{xy}}{ec\beta^3\gamma}\left\{\frac{1}{2a^2\gamma^2} \mp \frac{\pi^2}{48h^2}\right\}$$

direct

different signs!

indirect

- Indirect space charge still exists at high energy !
- 1/γ effect due to rigidity of the beam at higher energy

Coherent space charge:

- If the beam has a coherent motion, then:

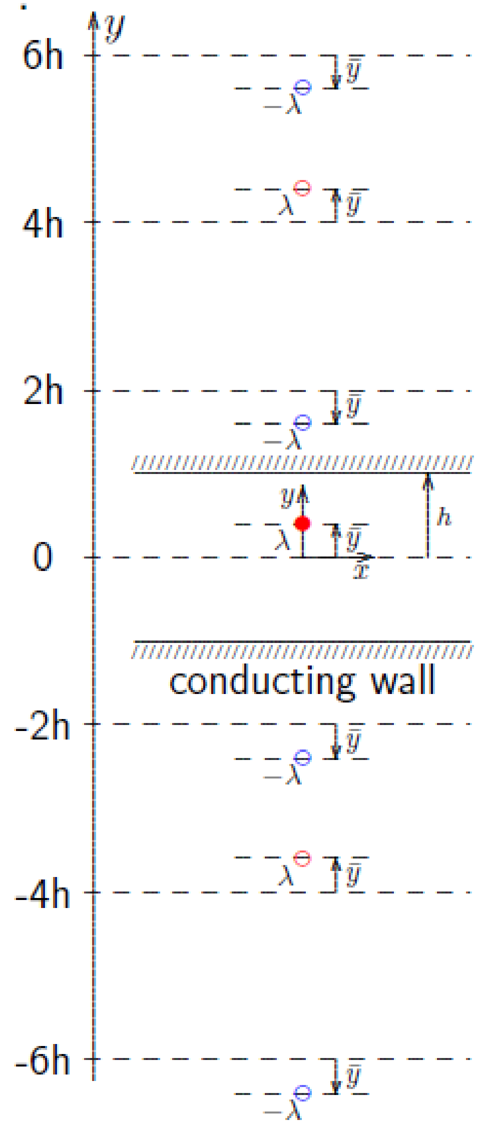
- Direct space charge unaffected, $\Delta Q_{coh} = 0$

- Indirect space charge is modified:

$$(\Delta Q_{xy})_{coh} = -\frac{r_0 I R \langle \beta \rangle_{xy}}{e c \beta^3 \gamma h^2}$$

- Always negative (defocusing) !

- Since $\Delta Q_{coh} \neq \Delta Q_{incoh}$, space charge makes it very difficult to avoid resonances !



A space charge limited accelerator:

CERN PS Booster Synchrotron

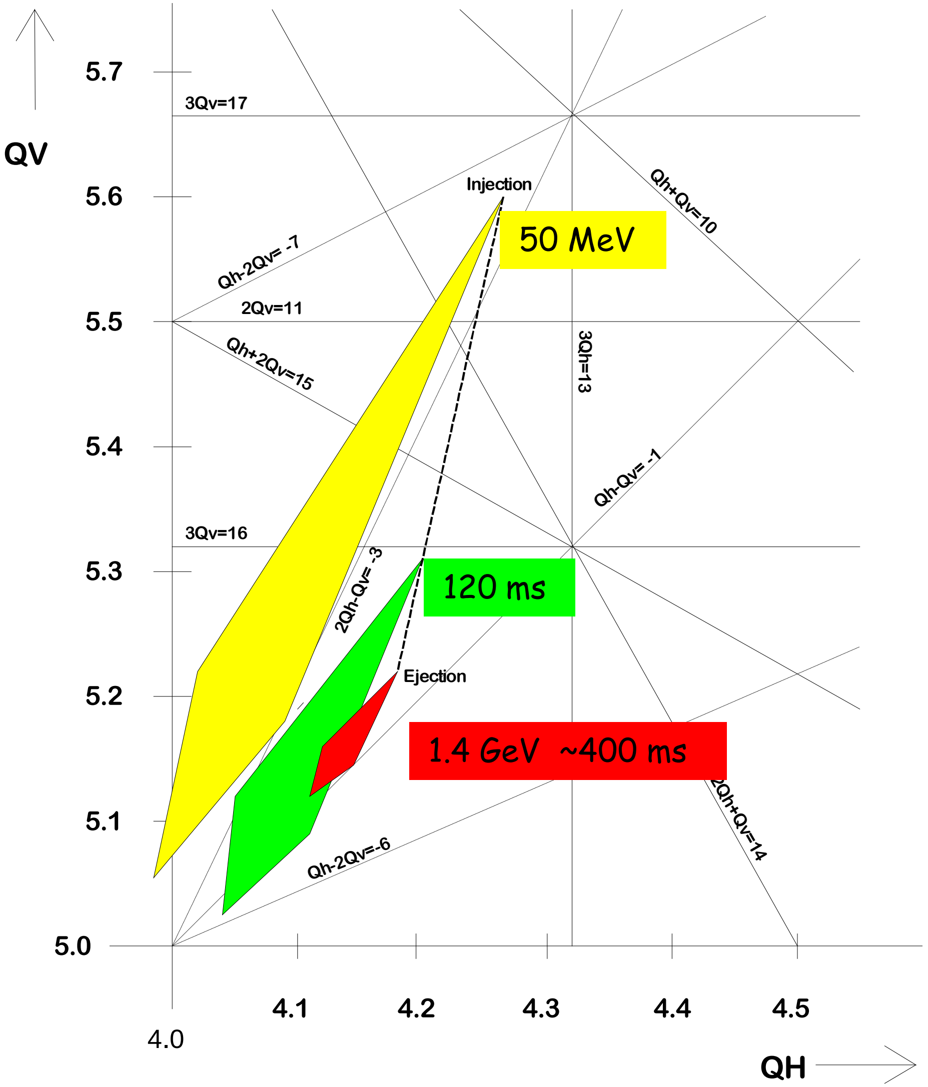
N = 10^{13} protons

$E_x^* = 80\ \mu$rad m $[4\ \beta\gamma\ \sigma_x^2/\beta_x]$ hor. emittance

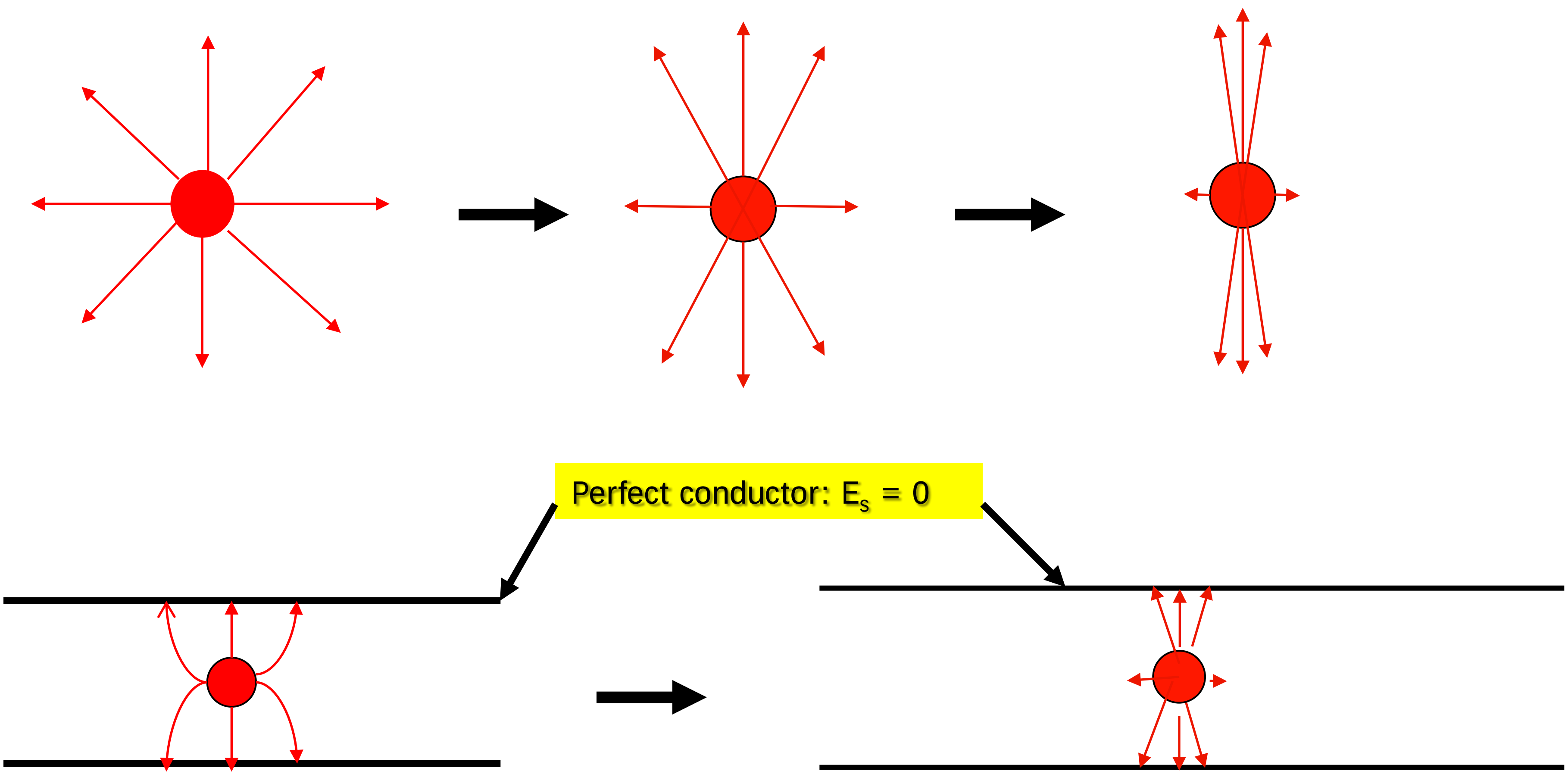
$E_y^* = 27\ \mu$rad m vertical emittance

- Direct space charge tune spread **~0.55 at injection**, covering 2nd and 3rd order stop-bands
- **"necktie"-shaped tune spread shrinks rapidly** due to the $1/\beta^2\gamma^3$ dependence
- Enables the working point to be moved **rapidly** to an area clear of strong stop-bands

K. Schindl, CAS Baden/Austria, 2004



Interaction beam – vacuum chamber:



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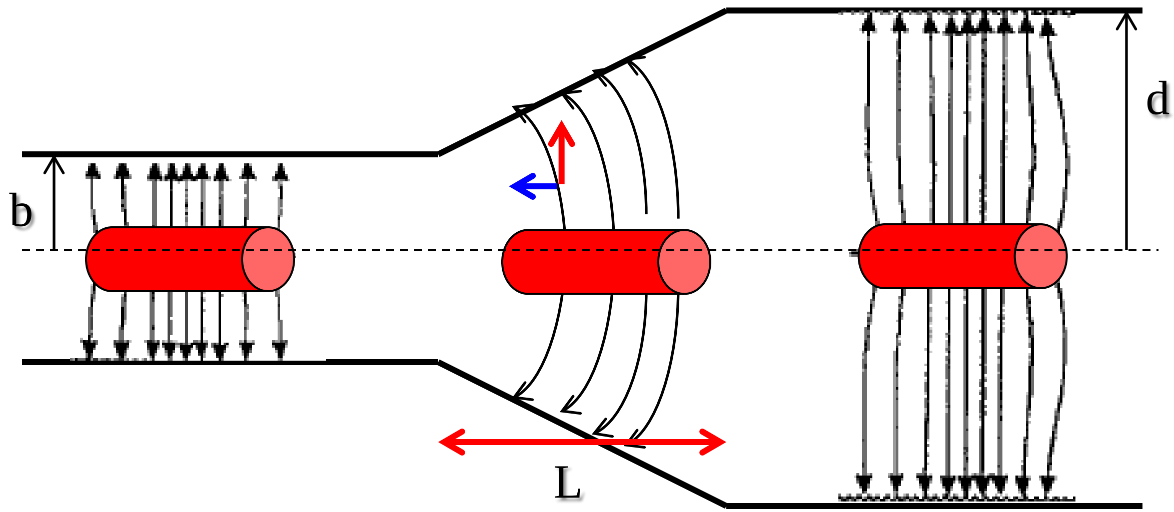
Change in the cross-section

- If conductor is not perfect, or, even worse, if $b \neq$ const.



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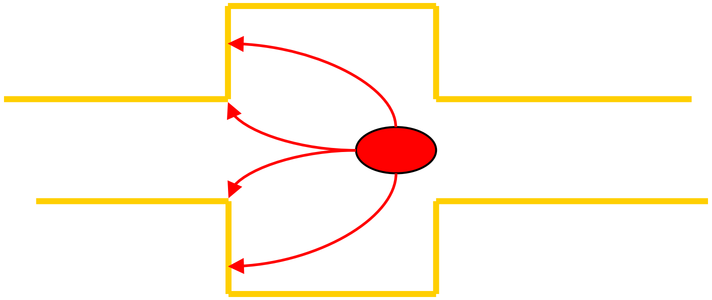
Es ≠ 0 => there is an interaction between the beam and the wall!

Abrupt changes:

Worst case: abrupt changes in the cross-section of the pipe:



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The beam looses energy (heating problems), but the induced fields can act back on the bunch or on the following bunches:

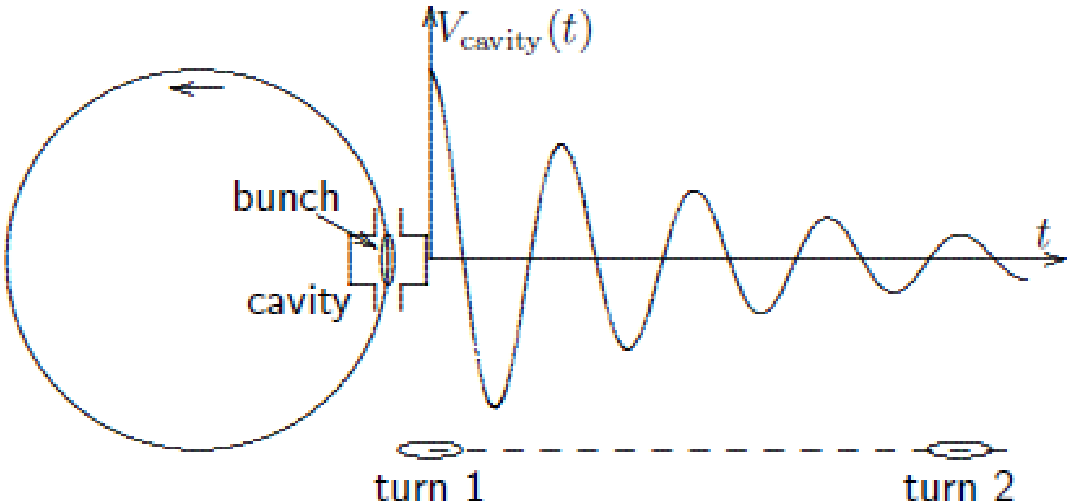
Interaction beam – vacuum chamber

- There are many different types of interactions
- Here we shall focus on two of them, namely:

- Abrupt changes of the vacuum chamber cross-section, i.e. the beam traverses cavity-like objects

- The conductor is not perfect (resistive wall)

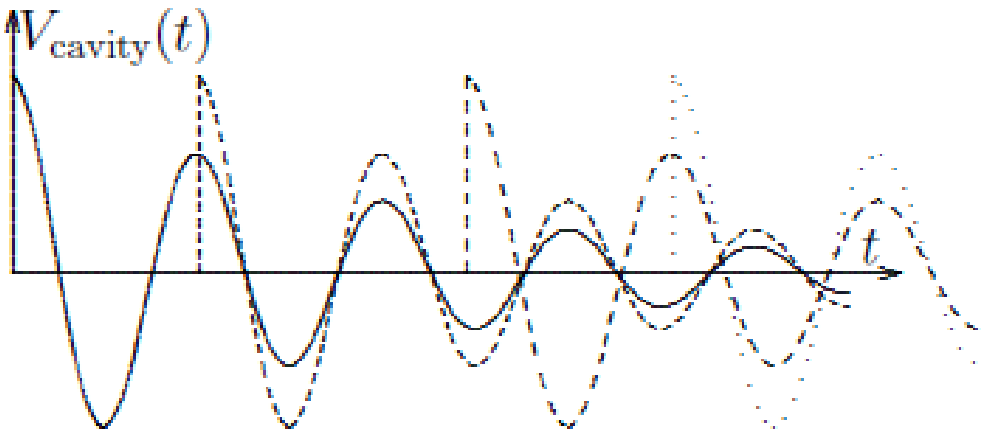
Bunch traversal of a cavity-like object



- The bunch induces a voltage V(t) oscillating in the cavity

What could happen ?

• Let us consider a single bunch coming back through the cavity or different bunches crossing the cavity:

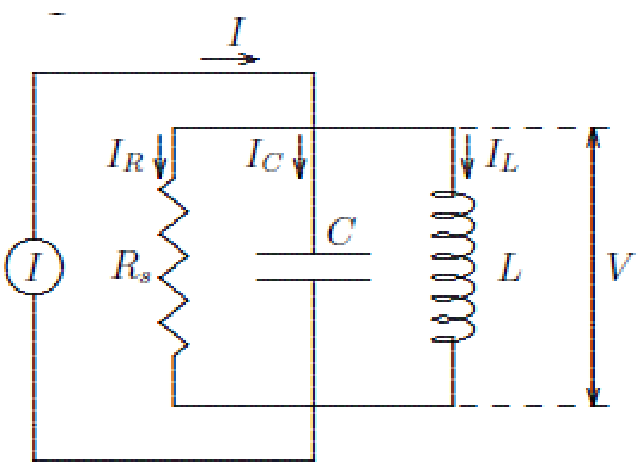


• It is intuitively clear that the voltages induced during the different passages can add or compensate each other.

• This can lead to growing oscillations of the particle motion and result in an instability (or damping).

The concept of wake-impedance

- Each cavity-like object has narrow-band oscillation modes which are interpreted as RLC-circuits:



$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$Q = R_s C \omega_r$$

$$\alpha = \frac{\omega_r}{2Q}$$

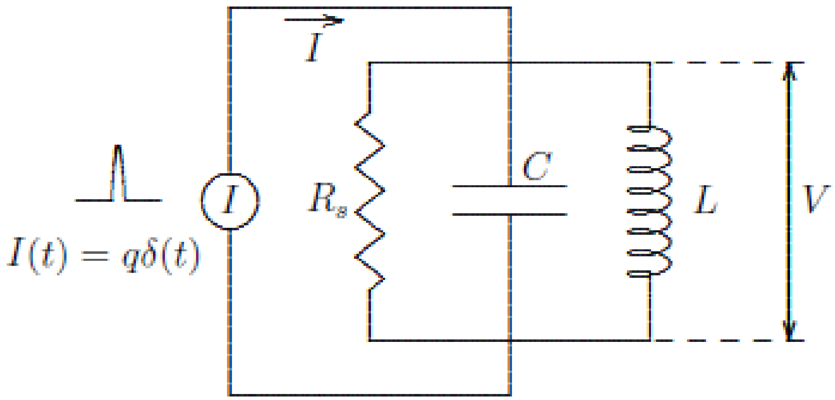
$$L = \frac{R_s}{Q\omega_r}$$

$$C = \frac{Q}{R_s \omega_r}$$

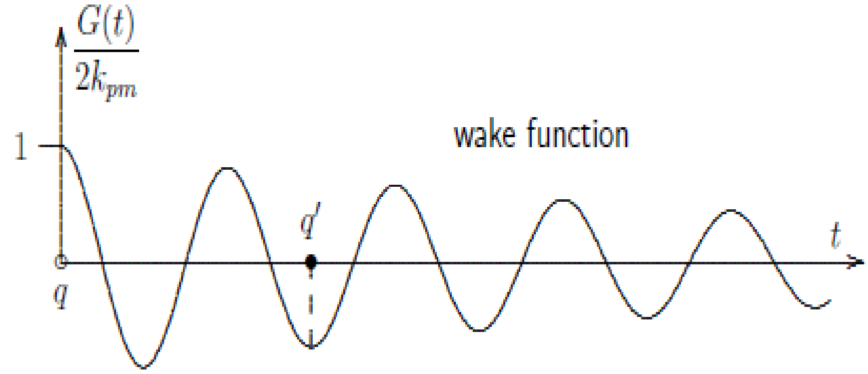
- Driving this circuit with a current I gives the voltages and currents across the elements

The Green's function – Wake function

G(t)=V(t)/q= Green's or Wake function



A voltage induced by a charge q at t=0 changes the energy of a second charge q' traversing the cavity at t by U=-q'V(t)=-qq'G(t)



Note:

G(t<0)=0

Fundamental relation:

$$Z(\omega) = \int_{-\infty}^{\infty} G(t) e^{-j\omega t} dt$$

G(t<0)=0

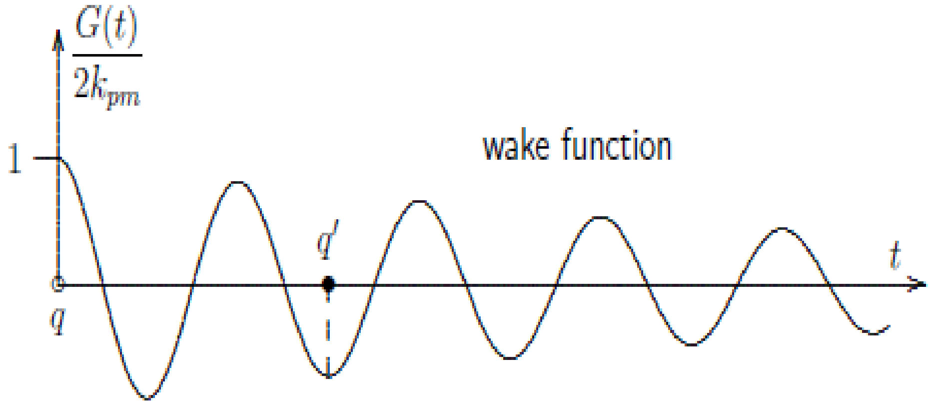
The impedance Z(ω) is the Fourier transform of the Green's Function G(t) !

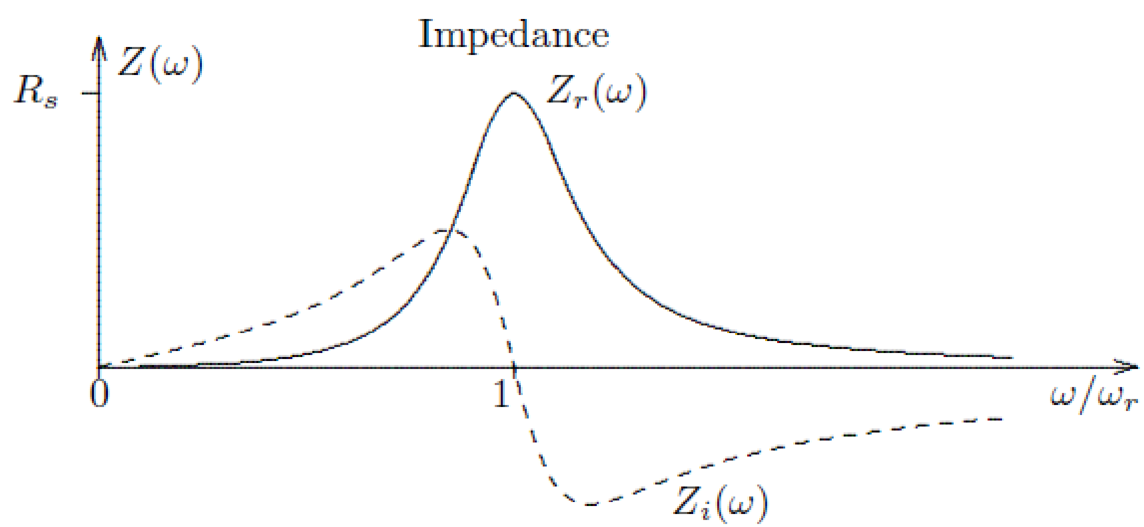
As a consequence, for collective effects, it is completely equivalent to work in the time domain (wake) or in the frequency domain (impedance) !

$$Z_R(\omega) = R_s \frac{1}{1 + Q^2\left(\frac{\omega_r^2 - \omega^2}{\omega_r \omega}\right)^2}$$

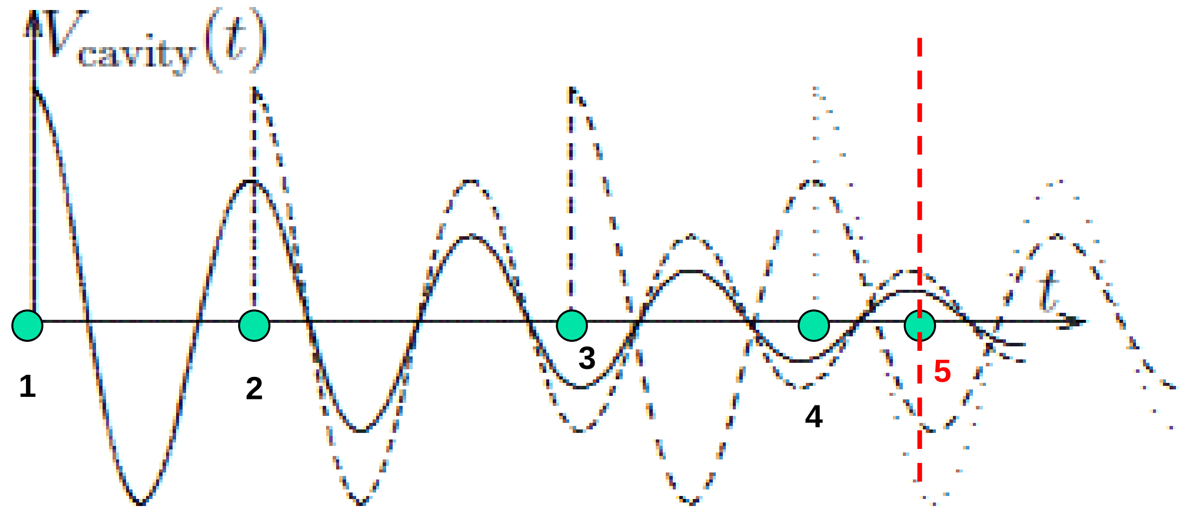
$$Z_I(\omega) = -R_s \frac{Q \frac{\omega^2 - \omega_r^2}{\omega_r \omega}}{1 + Q^2\left(\frac{\omega^2 - \omega_r^2}{\omega_r \omega}\right)^2}$$

Equivalent representations:





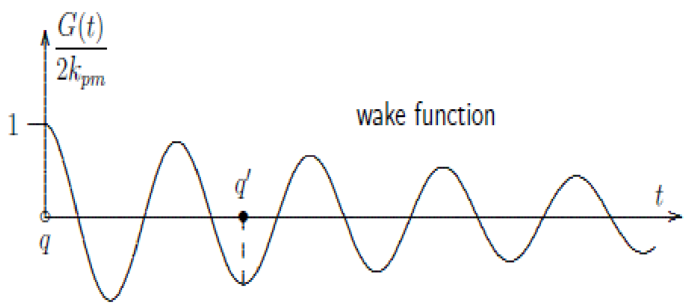
From Wake (Green's) to Potential

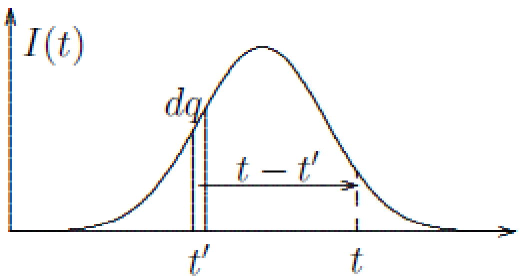


• Effect on particle 5 is the sum of the wakes (Green's) produced by all the particles in front of it.

• Performing this sum (integral) for all the particles in the bunch yields the wake potential !

The wake potential





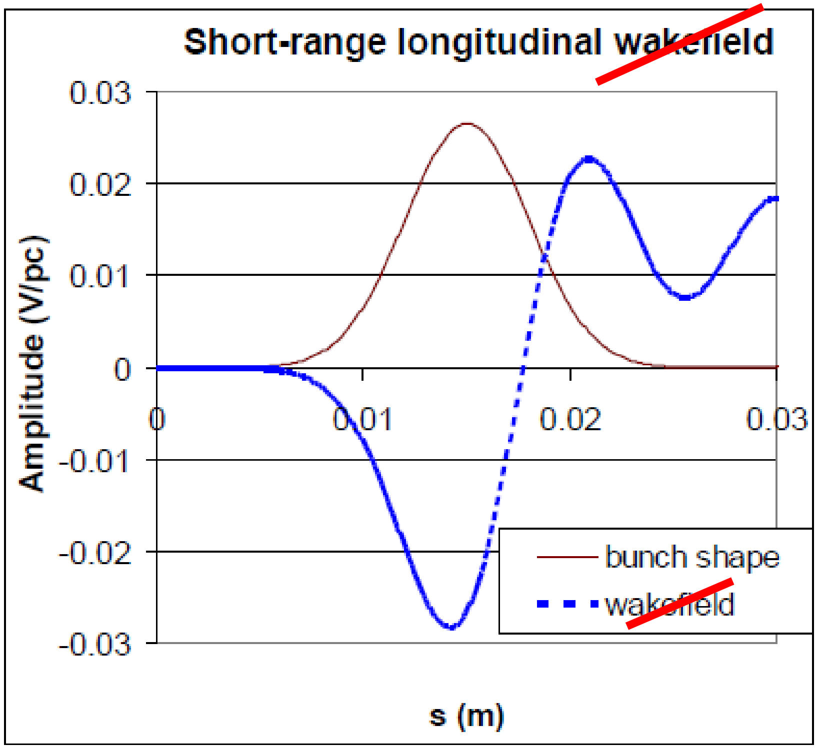
The wake potential (what we obtain from time-domain codes - V/pC):

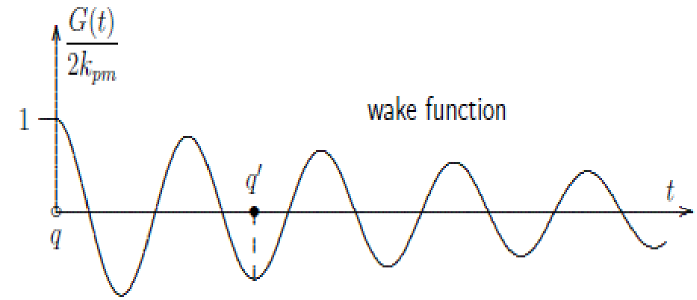
$$W(t) = V(t) / q = \int_{-\infty}^{t} I(t')G(t')dt'$$

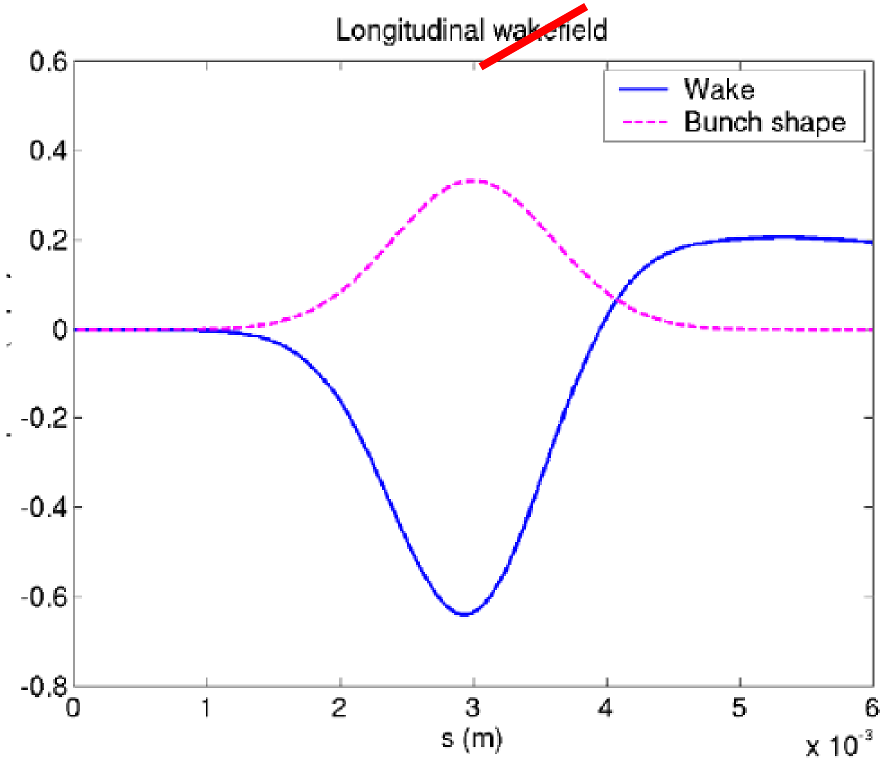
The loss factor k(σ) in V/pC:

$$k(\sigma) = \int_{-\infty}^{+\infty} I(t')W(t')dt'$$

Examples of wake potentials:







In practice…

In practice, for a given object, it is not possible to obtain the Green's function:

Frequency domain:
(HFSS, URMEL,…)

• Compute all the resonances up to the highest possible frequency

• Build the appropriate sum and use this as a pseudo Green's function

Time domain:
(Particle Studio, MAFIA,…)

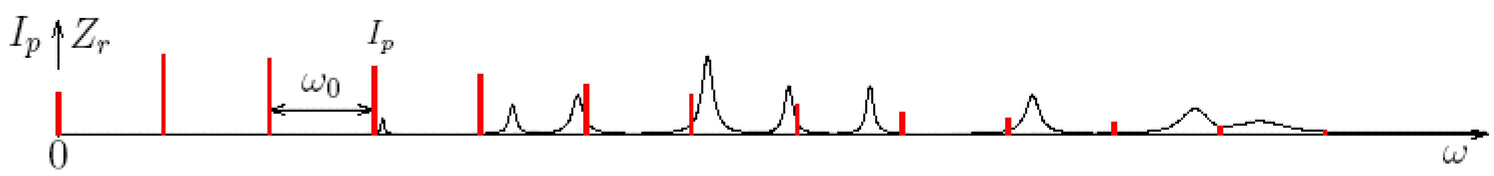
• Evaluate the wake potential for the smallest possible bunch length
• Assume this wake potential is the Green's function (wake field !)
• Perform a Fourier transform to get the impedance Z(ω)

General rules (1):

• If you deal with high-Q resonators (long memory), then you have to deal with each resonator separately.

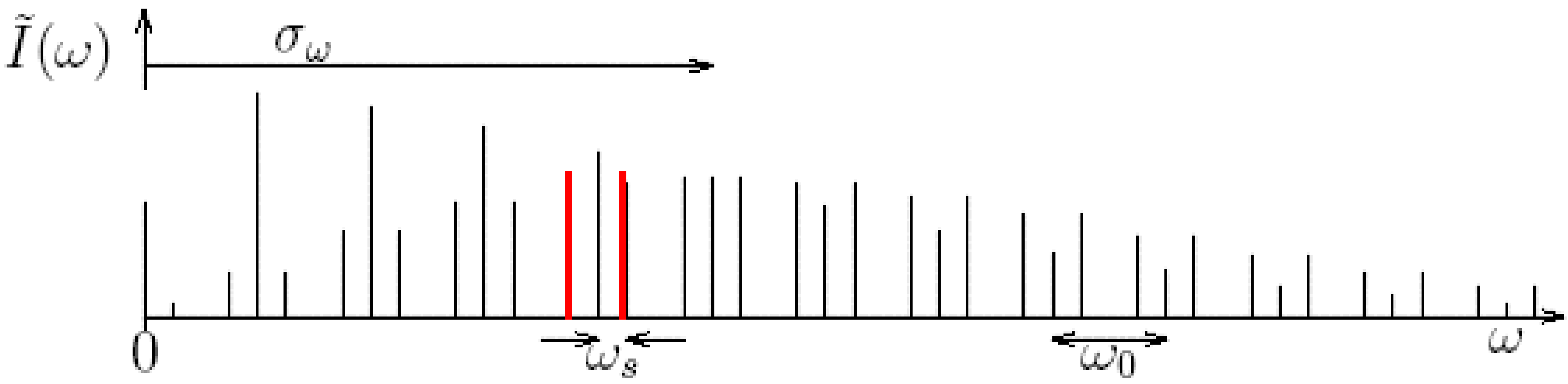
➔ For each resonance, study the interaction with the beam

➔ Study the interactions where the frequencies seen by the bunch and those of the impedance do overlap !



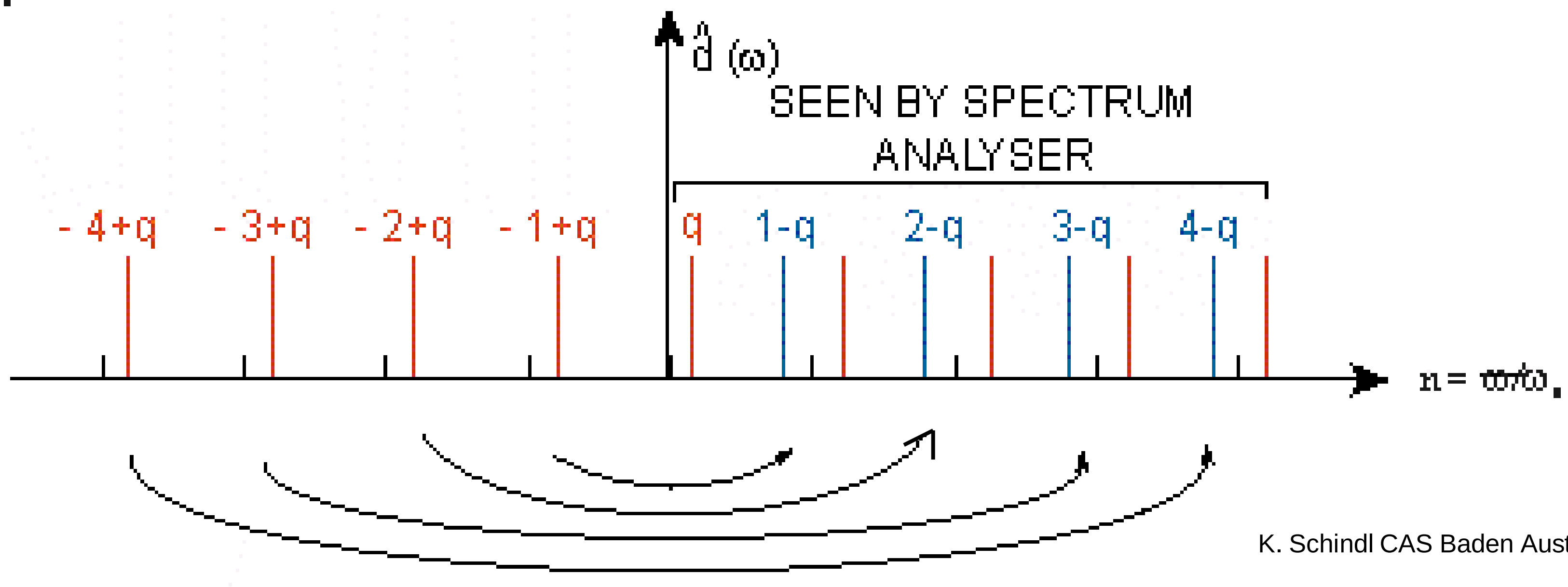
• Bunch executes synchrotron oscillations ➔ modulation of the signals ➔ sidebands in the spectrum, Q_s apart from the carriers

Oscillating bunch (longitudinal):



• Picture is similar in the transverse planes, but sidebands separated by the non-integer part of the betatron tune (q)

Transverse plane (q=0.2)



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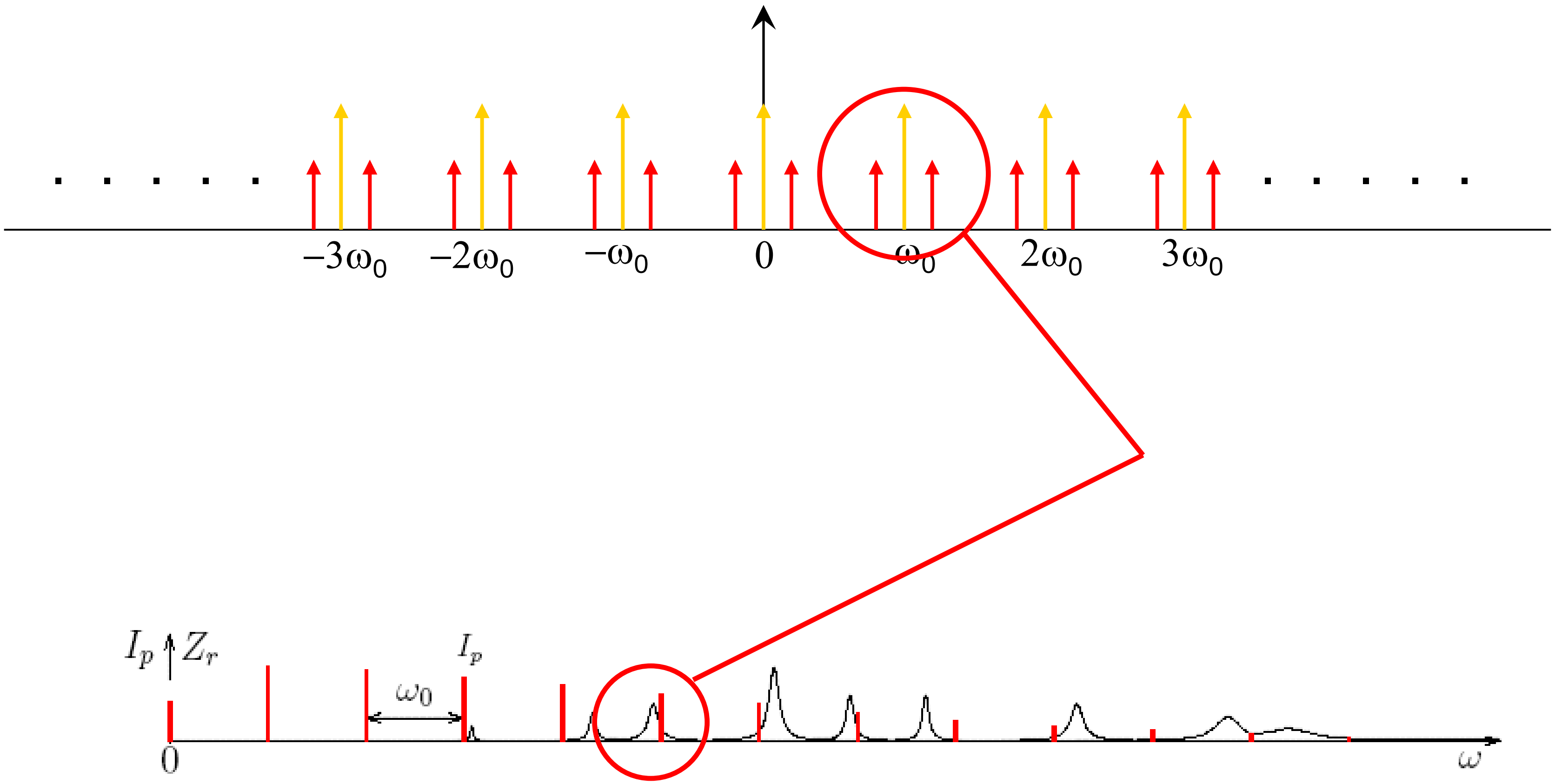
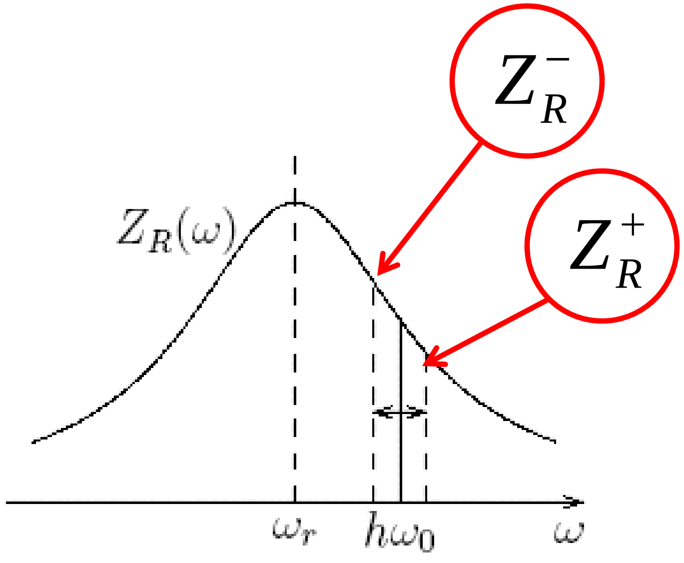
- On the spectrum analyser, we observe the « upper side bands » (USB) and the « lower side bands » (LSB (blue)).

Working with side bands…
and Impedances

Stability with Side Bands:



Intuitive picture:



Above transition:

- ω_0 smaller when energy is high

- if $\omega_r < h\omega_0$, bunch sees more impedance if it has an energy excess (more losses).

- Less losses if it has a lack of energy

➔ damping !

- Situation reversed below transition !

First recepy (Golden Rule 1):

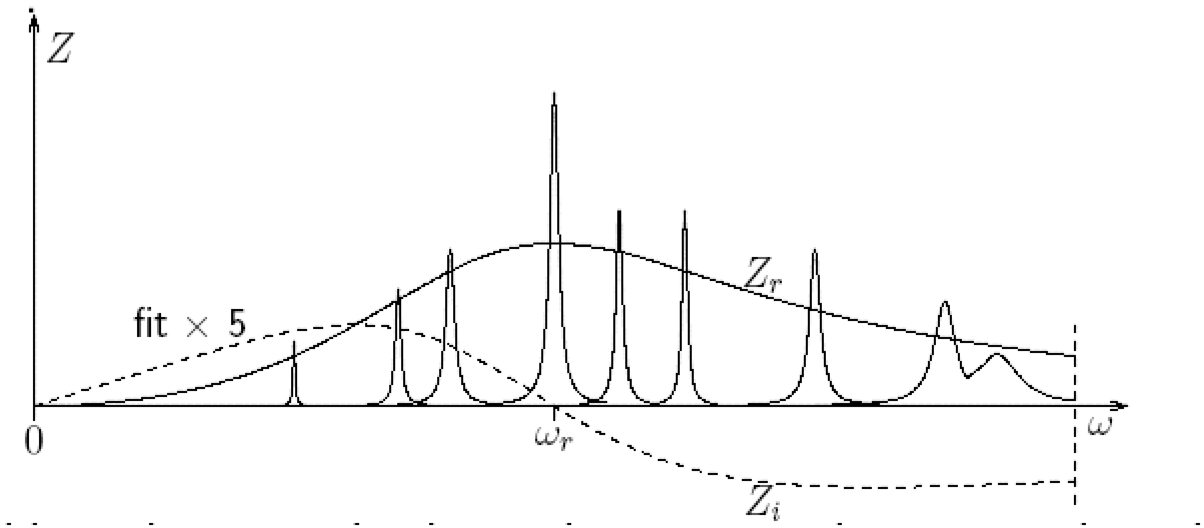
Conditions for stability (damping) SB:

	Below transition	**Above transition**
Longitudinal	$Z_R^+ > Z_R^-$	$Z_R^+ < Z_R^-$
Transverse	$Z_{RT}^+ > Z_{RT}^-$	$Z_{RT}^+ > Z_{RT}^-$
Chromaticity	Q’ < 0	Q’ > 0

Working with impedances…
and bunch spectrum

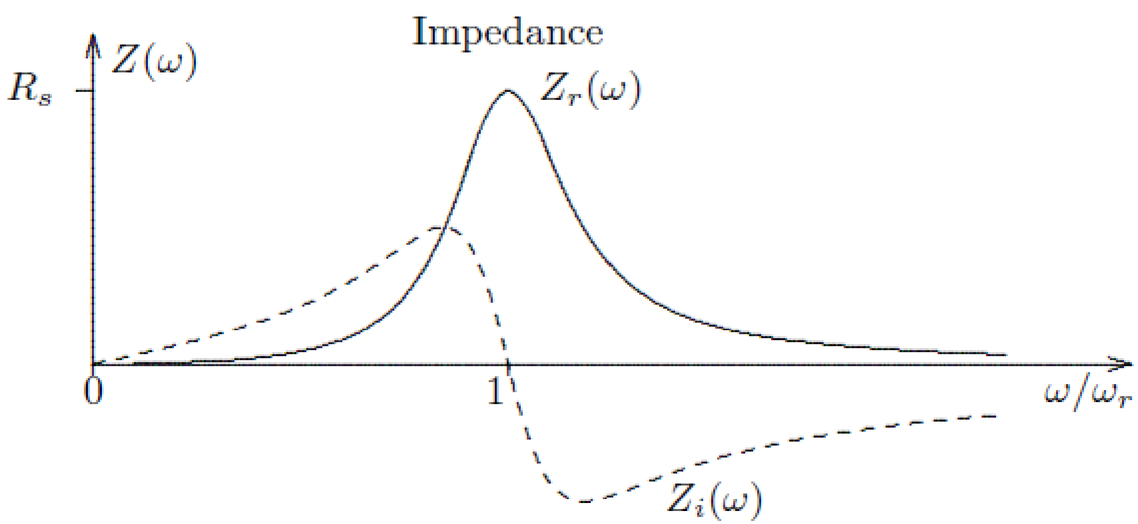
General rules (2):

• If you are interested in single traversal stability, it is very useful and practical to move to the Broad Band Resonator Model (BBR). Only 3 parameters to be defined to obtain $Z_R(\omega)$, $Z_I(\omega)$ and $G(t)$. Usually , one takes a value of Q = 1.



BBR model:

• With the BBR model, you treat the whole machine as a single resonator, for which the expressions for the impedance (wake field) are known:



Longitudinal impedance: |Z/n|

- Usually, the longitudinal impedance of a machine is rather characterised by the value |Z/n| (sum of the inductive parts at low frequencies divided by $n=\omega/\omega_0$). $|Z/n| = L\omega_0$

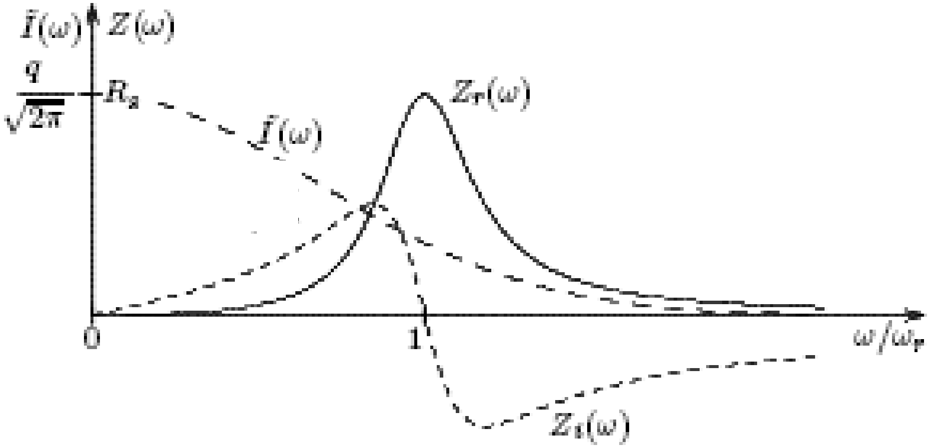
- This has the advantage that the resulting value becomes independent of the size of the machine and allows therefore for an easy comparison between different machines.

- In addition, by plotting |Z/n| rather than $Z(\omega)$, it has the advantage that the plots for longitudinal and transverse impedances exhibit a very similar behaviour.

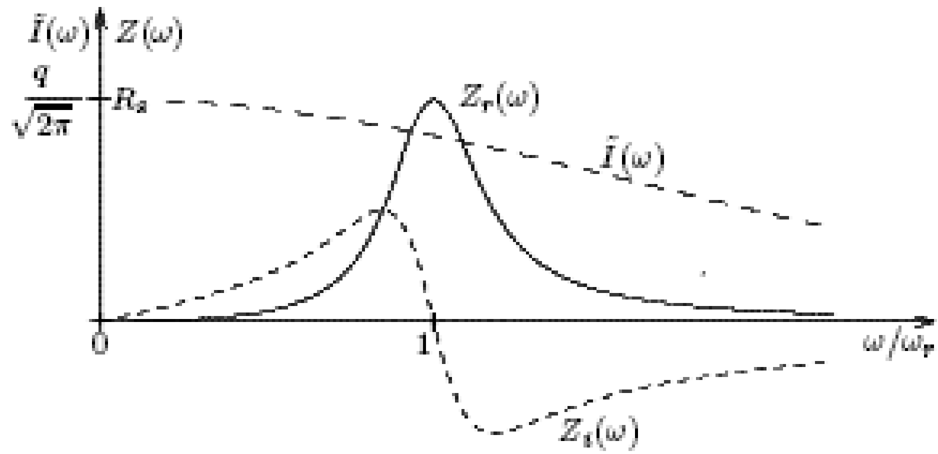
|Z/n| as a function of time:

Machine	|Z/n| [Ω]
PS (~ 1960)	> 50
SPS (~ 1970)	~ 20
LEP (~ 1990)	~ 0.25 (1.0)
LHC (~ 2009)	~ 0.10 (0.25)

Effect of the bunch length:



- Long bunch



- Short bunch

Second recepy (longitudinal):

Golden Rule 2

The convolution between the bunch spectrum and the real part of the longitudinal impedance yields the energy loss of the bunch interacting with the impedance

The convolution between the bunch spectrum and the imaginary part of the longitudinal impedance yields the tune shift ΔQ resulting from the interaction

$$k_{pm} \approx \frac{1}{q^2} \int Z_R(\omega) \left| \tilde{I}(\omega) \right|^2 d\omega$$

$$\Delta Q \propto \int Z_I(\omega) \left| \tilde{I}(\omega) \right|^2 d\omega$$

Transverse impedance:

For the BBR-model, there exists a very handy relation for the transverse impedance $Z_T(\omega)$, namely:

$$Z_T(\omega) = \frac{2R}{b^2}\left|\frac{Z_L(\omega)}{n}\right|$$

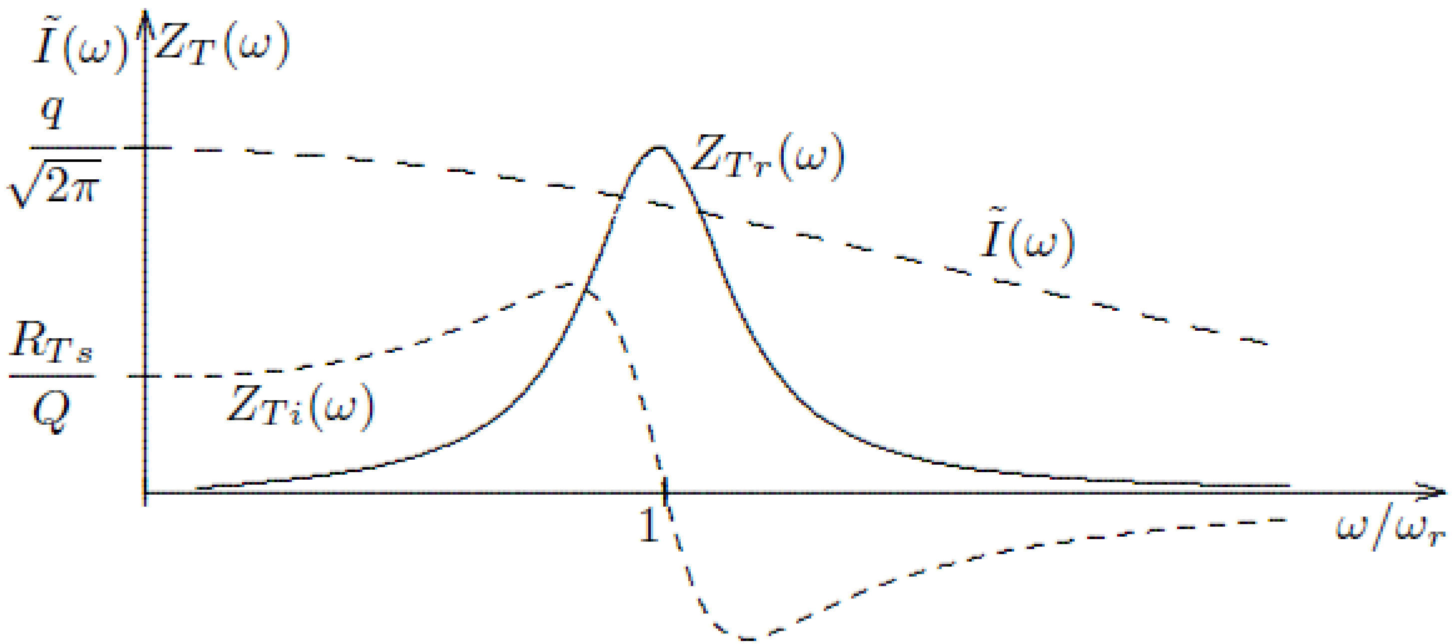
$$n = \frac{\omega}{\omega_0}$$

$$Z_T = \frac{\Omega}{m}$$

Watch out !!!

$$Z_L(\omega) \propto \frac{1}{b} \quad \Rightarrow \quad Z_T(\omega) \propto \frac{1}{b^3}$$

Transverse impedance (Ω/m)



Here again, perform convolution between bunch spectrum and impedance

Third recepy (transverse):

Golden Rule 3

The convolution between the bunch spectrum and the real part of the transverse impedance yields the growth time of the instability

The convolution between the bunch spectrum and the imaginary part of the transverse impedance yields the tune shift ΔQ resulting from the interaction

- By changing the chromaticity Q', the spectrum of the bunch is shifted in frequency ➔ the convolutions are modified !

Resistive wall (longitudinal)

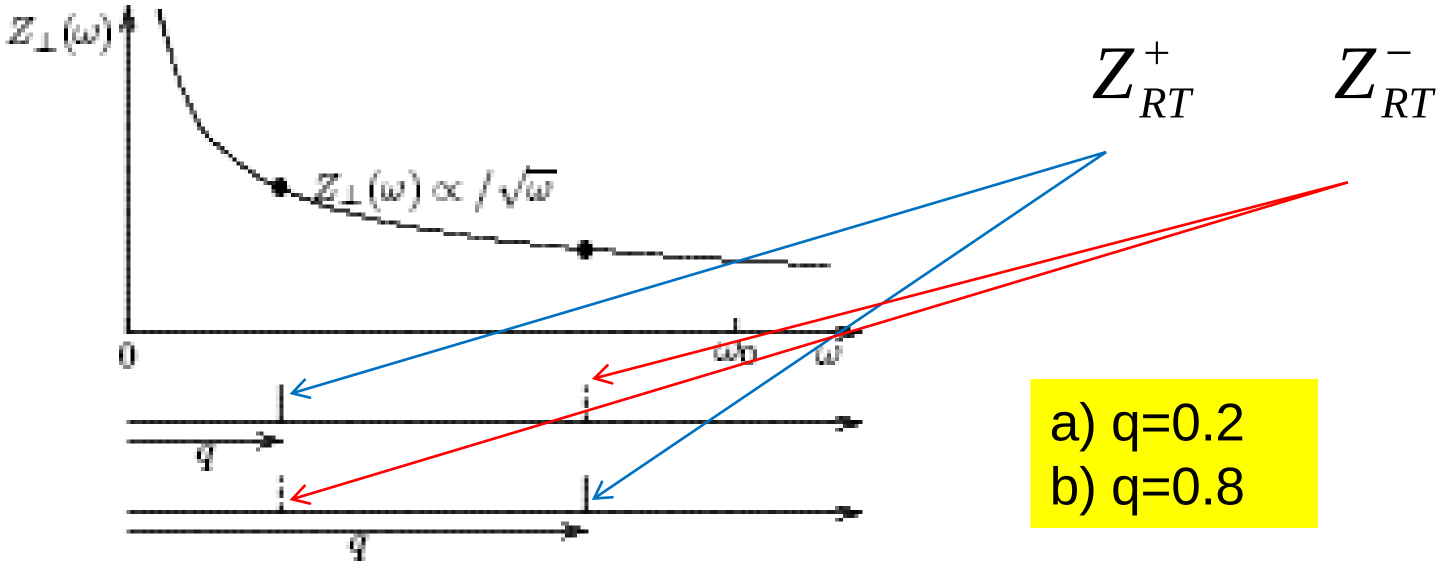
$$Z(\omega) = \frac{(1+j)R}{b}\sqrt{\frac{\mu_0\omega}{2\sigma_c}}$$

- Resistive and Inductive parts have equal magnitude
- $Z(\omega)$ is a steadily growing function ($\omega^{1/2}$)
- $Z^{+}_{R}(\omega) > Z^{-}_{R}(\omega)$ ➔ no damping above transition (Rule 1) !

➔ Potential source of unstability !

… and what about the transverse plane ? $Z_T \propto Z_L/\omega$!!!

Resistive wall (transverse)



a) q=0.2
b) q=0.8

- Transverse stability: $Z^{+}_{RT} > Z^{-}_{RT}$!

➔ Select a tune q below the half integer (q<0.5) for stability !

Cures from a general point of view:

• Damp Higher Order modes (HOM = resonances) in the cavities by HOM Dampers (unwanted mode picked up by an antenna and sent to a damping resistor).

• Protect from unwanted « cavities » in the beam pipe with adequate RF-shielding (mimick a smooth beam pipe).

• Avoid any abrupt changes in the beam pipe cross section.

• Use highly conductive materials whenever possible.

• Use feedback systems (both for longitudinal and transverse planes).

Coupled-bunch effects

• With N bunches in the machine, there exist N different possible modes of oscillations (n=0, 1, 2…N-1).

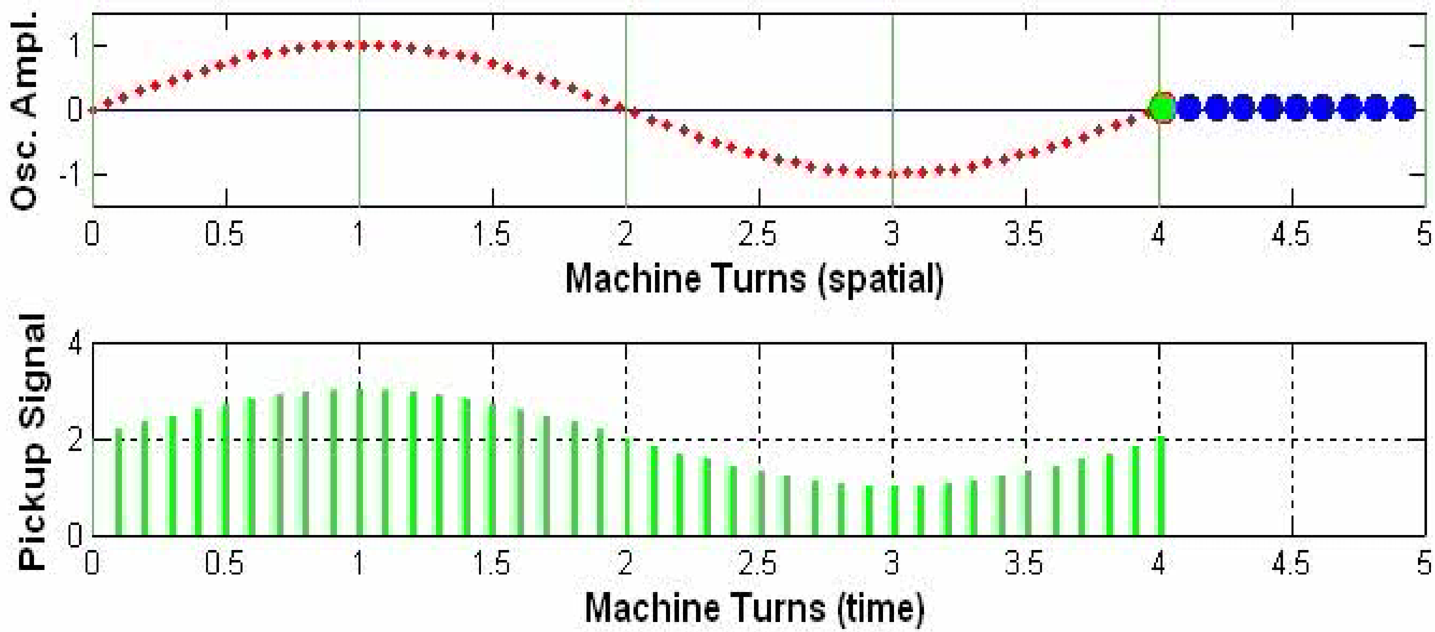
• If the bunches are stable, then there exist only lines at the multiple of $N\omega_0$.

• If one mode (e.g. k) is unstable, then lines at $k\omega_0$ will appear with both USB and LSB.

• Usually there are lines (USB and LSB) at most of the multiple of the revolution frequency.

Coupled-bunch oscillations modes n

n=0



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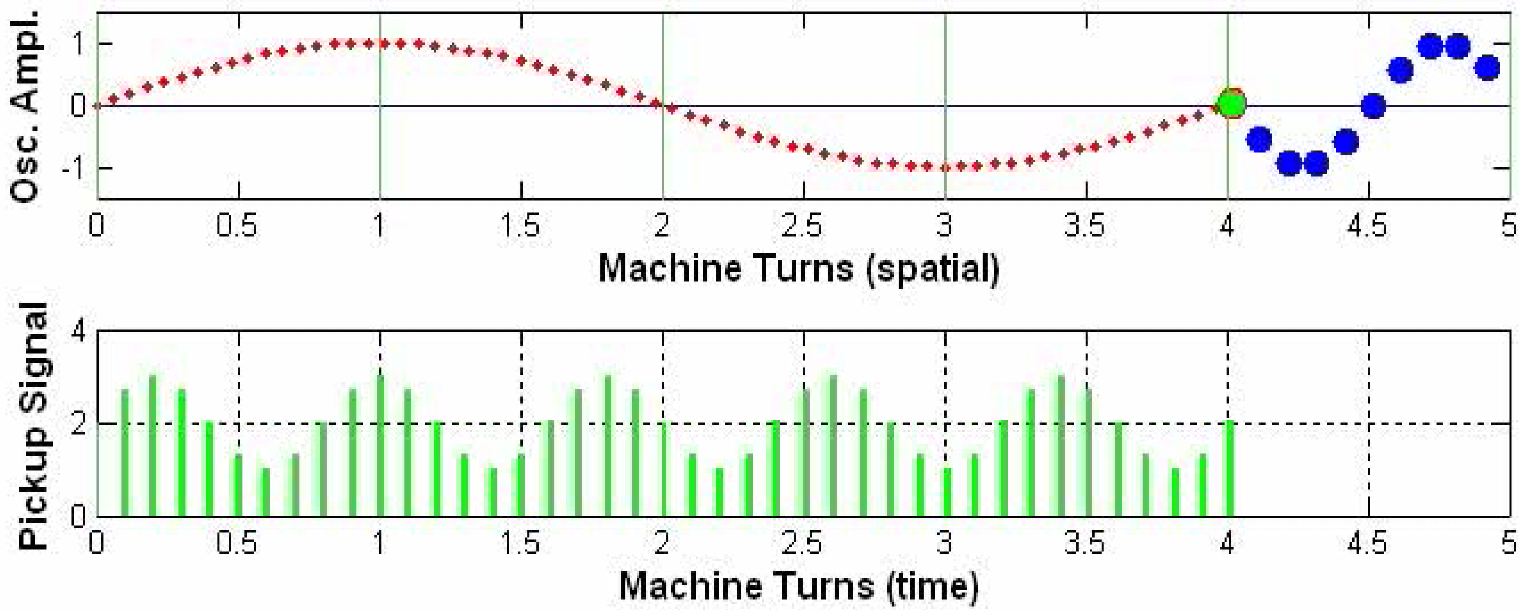
n = 1



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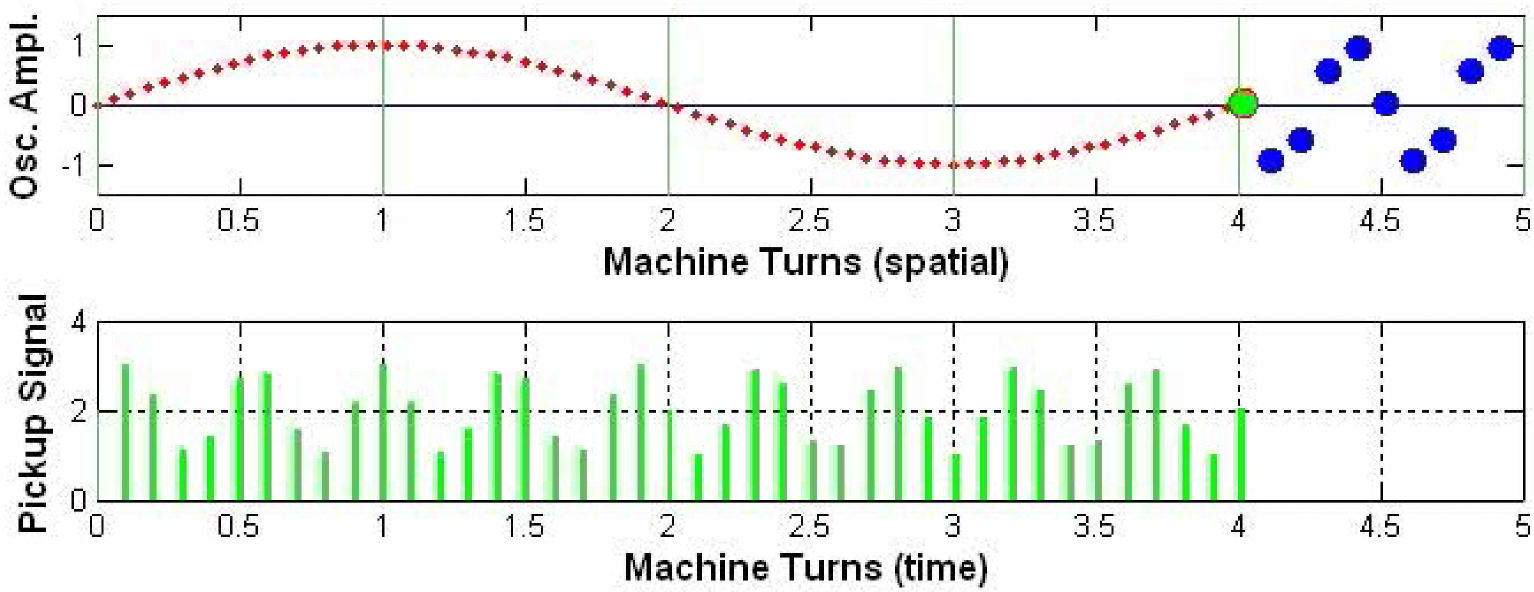
n = 2



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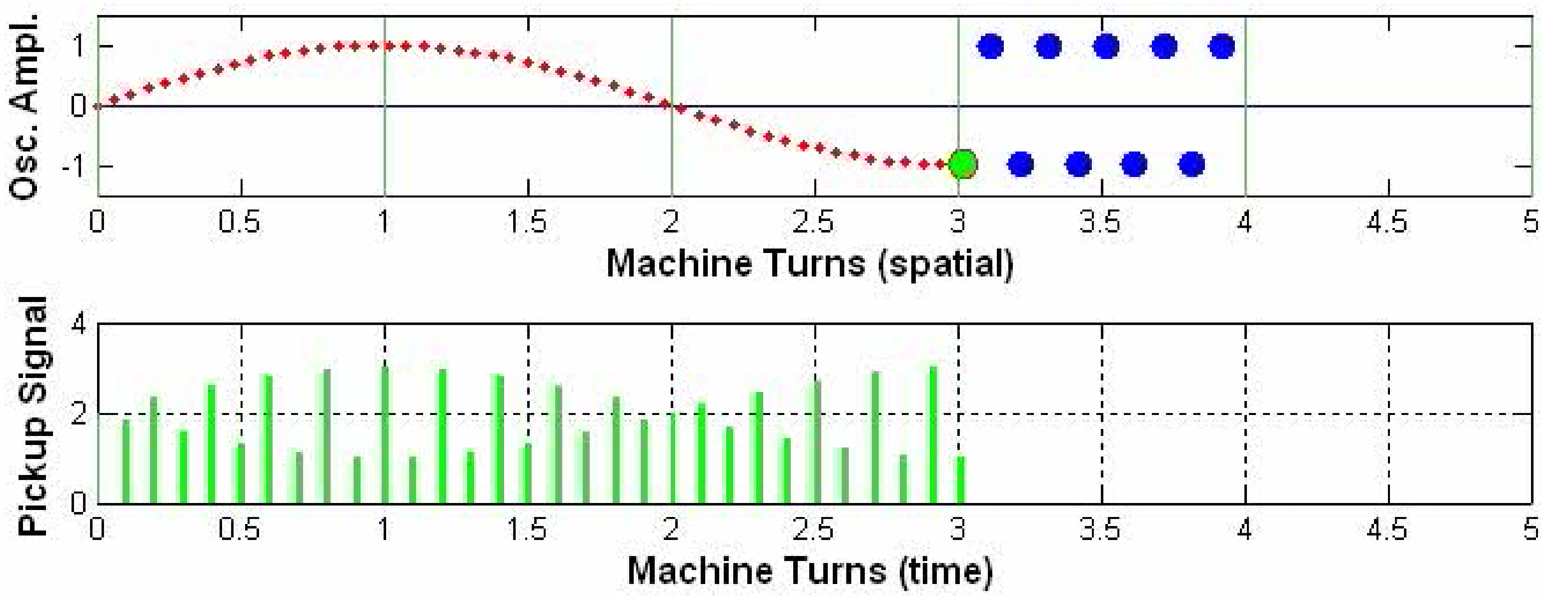
… n = 5



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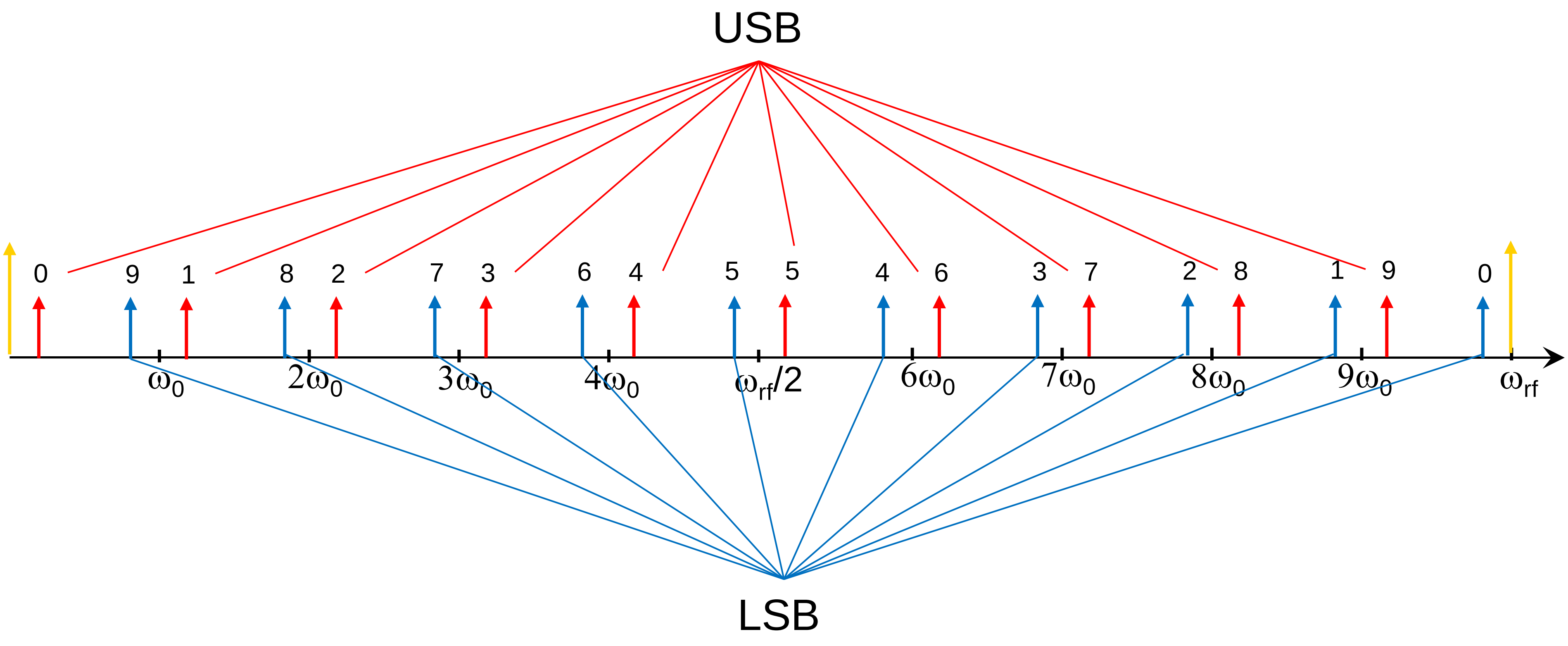
Identification of modes n:

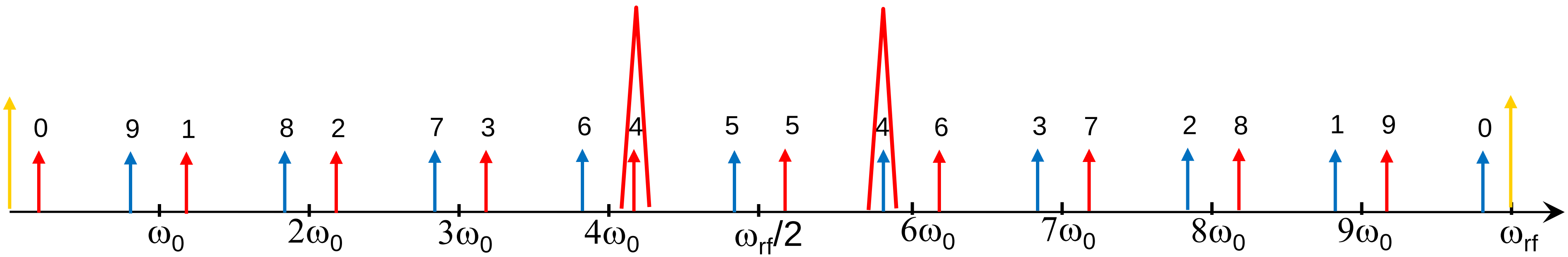


Fourth recepy (Golden Rule 4):

Stability with side bands MB:

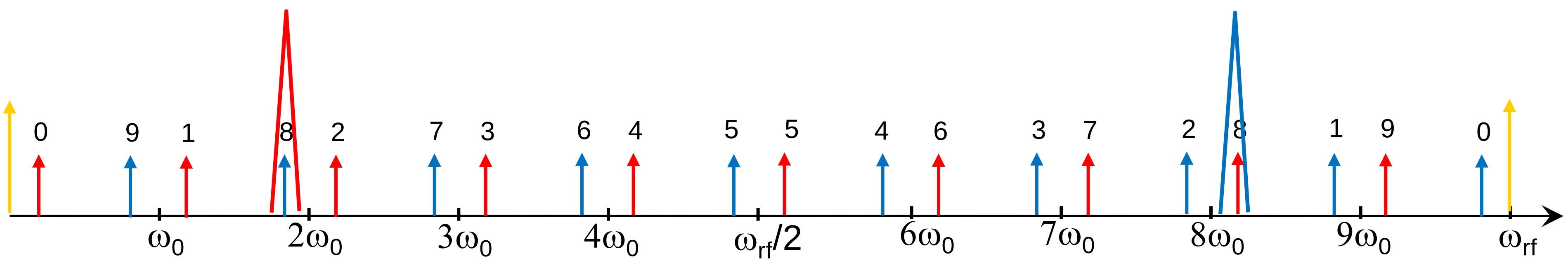
	Below transition	Above transition
Longitudinal	Lower SB	Upper SB
Transverse	Upper SB	Upper SB

What about…



- No problem, on the USB ➔ stabilizing !
- Bad, on the LSB ➔ unstable !

What could we do…?



- Bad, on the LSB of the mode n=8 !

- Add an impedance (e.g. a cavity) tuned on the USB of n=8 which will damp the instability !

All the material presented in this lecture has been directly taken from the numerous lectures given at CAS by **Professor Albert Hofmann**.

I would like to sincerely thank Albert for all his efforts in producing such highly professional and pedagogical lectures for CAS !