



Beamlines and matching to gantries

M. Pullia

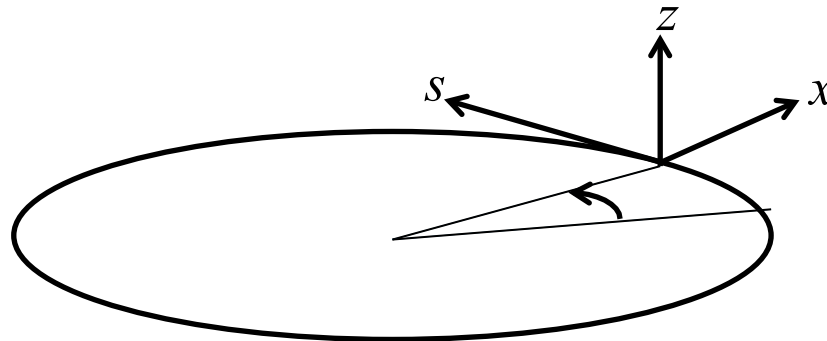
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Beamlines

- A beamline is basically a set of magnets used to **transport the beam** from one position to another, e.g. from the source to the linac or from the main accelerator to the treatment room.
- Besides transporting the beam, a beamline has to give the beam the **right shape**.
- Beamlines include a number of accessory instrumentation and devices, like beam diagnostics, vacuum chamber, vacuum pumps and vacuum gauges.

Nominal orbit and coordinates

- Every accelerator and every beamline is designed along a nominal orbit or a nominal trajectory which is the path of the nominal particle, the one with nominal initial conditions (energy, position and direction).

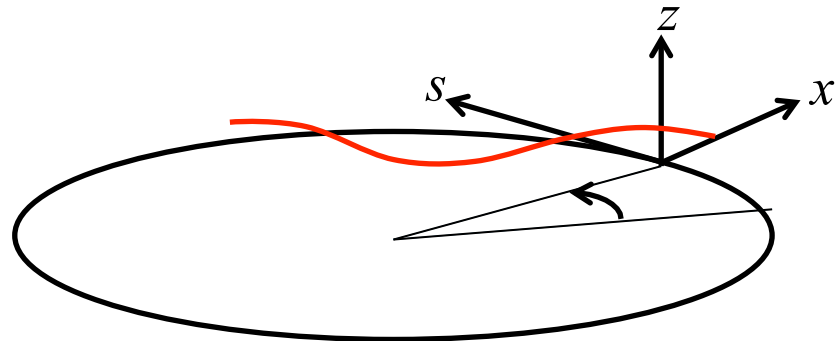


- A coordinate system moving on the reference particle is used to describe small deviations with respect to nominal trajectory.

Betatron oscillations

- With all the possible simplifications and linearizations, the motion of a particle with nominal energy along a magnetic lattice is described by the Hill's equation

$$\frac{d^2 y}{ds^2} + K(s)y = 0 \quad y = x \text{ or } z$$



- Notation:** ' = $\frac{d}{ds}$; e.g. $x' = \frac{dx}{ds}$
- The Hill's equation looks like an harmonic oscillator, but $K(s)$ varies along the lattice depending on the magnetic element at position s . Assume it is constant inside each magnet and varies abruptly when passing from one element to the following (hard edge approximation)
- For each element we can write a **transfer matrix** transporting the initial coordinates to the particle position at the element exit.

Solution of the Hill's equation

- The solution of the Hill's equation can be written

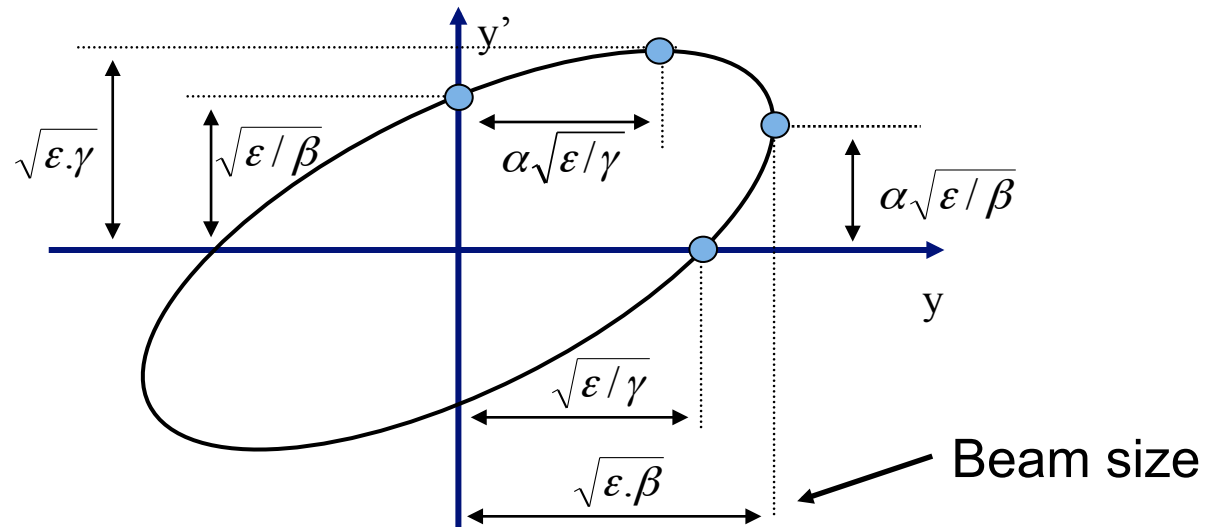
$$y = \sqrt{\varepsilon \beta(s)} \cos(\phi(s) + \phi_0) \implies y' = -\alpha \sqrt{\varepsilon / \beta} \cos(\phi(s) + \phi_0) - \sqrt{\varepsilon / \beta} \sin(\phi(s) + \phi_0)$$

- Where ε and ϕ_0 are initial conditions and $\alpha = -\frac{1}{2} \frac{d\beta}{ds}$. Define $\gamma = \frac{1 + \alpha^2}{\beta}$

$$\gamma y^2 + 2\alpha y y' + \beta y'^2 = \varepsilon$$

Twiss invariant
(better indicated as 2J)
("single particle emittance")

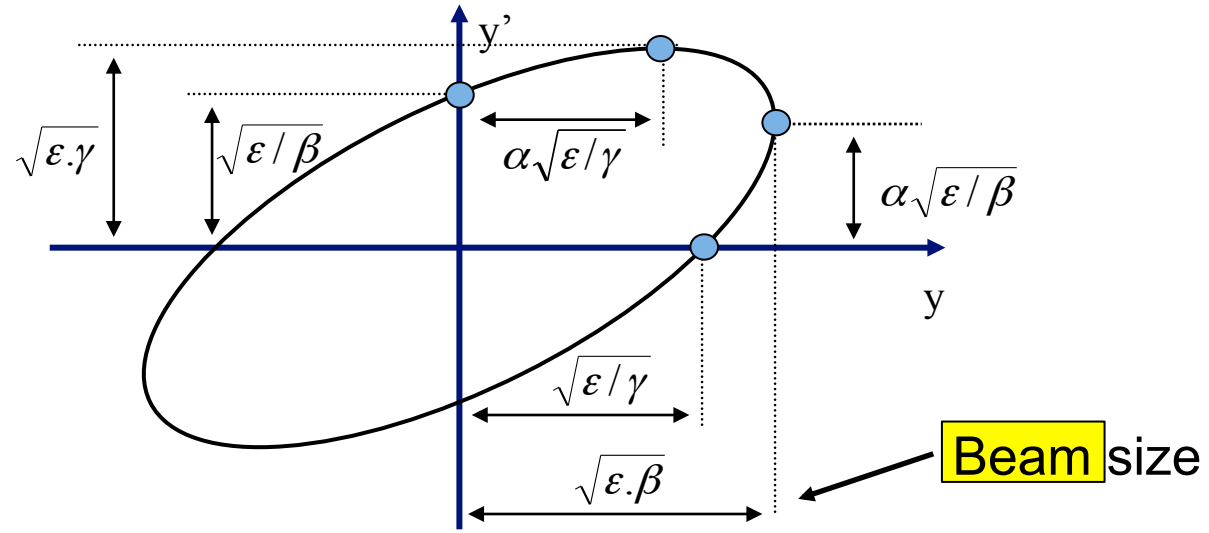
$$\text{Area of ellipse} = \pi \varepsilon$$



$$y^2 + 2\alpha yy' + \beta y'^2 = \varepsilon$$

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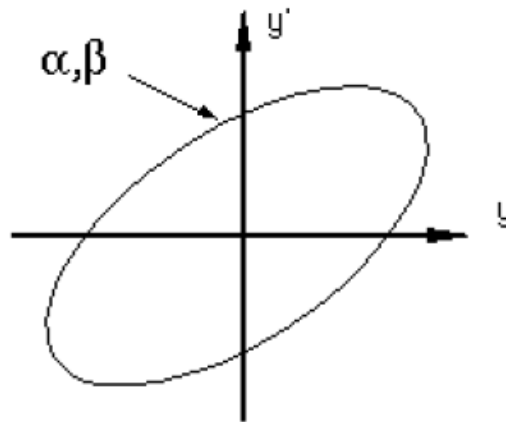
$$\beta = 10 \text{ m}$$

$$\varepsilon = 10 \cdot 10^{-6} \pi \text{ m rad} = 10 \pi \text{ mm mrad}$$

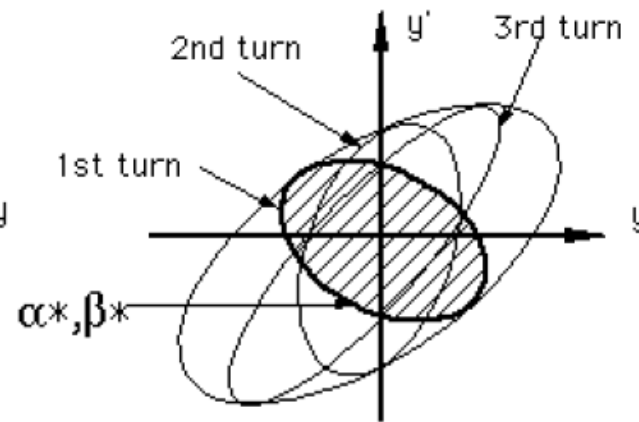
$$\text{BeamSize} = x_{\max} = \sqrt{\varepsilon\beta} = 10 \text{ mm}$$

Mismatch and filamentation

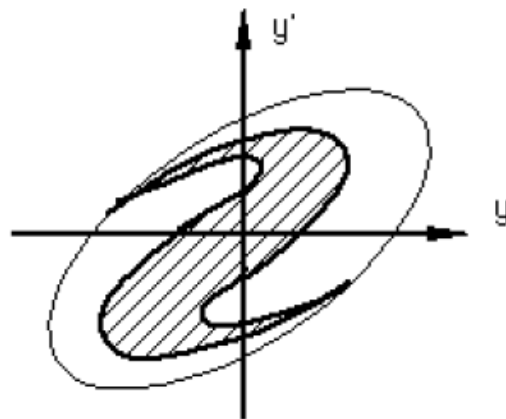
- In the first part of the beam
- E.g. when the beam is not matched to the lattice
- In an ideal case, the beam size remains constant over the length of the periodic structure



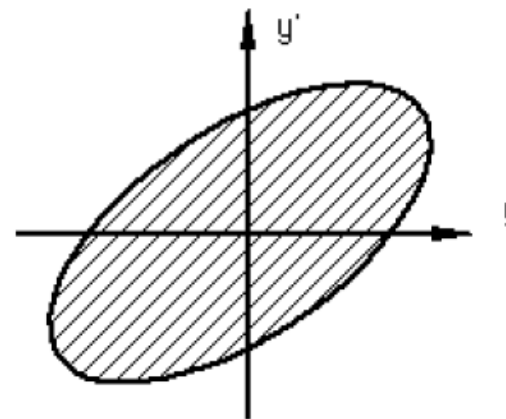
(a) Form of matched ellipse



(b) Unmatched beam



(c) Filamenting beam



(d) Fully filamented beam

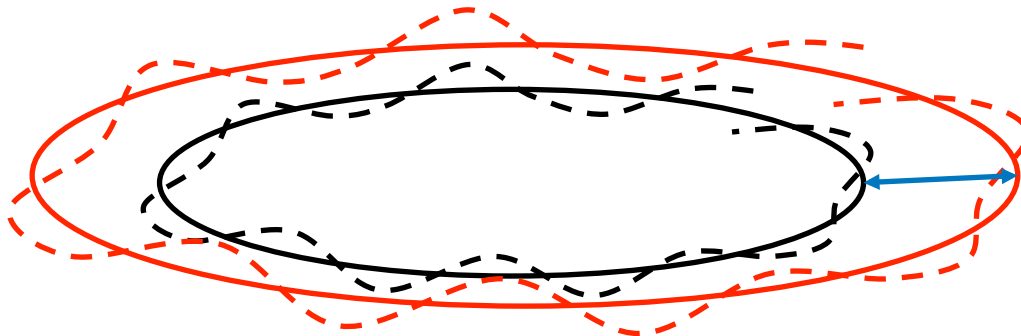
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(Courtesy of P. Bryant)

The dispersion function

- Particles with a (small) momentum deviation are bent differently wrt the nominal particle. Anyway a closed orbit/reference trajectory for particles with the considered momentum can be found and particles not moving on this new path orbit perform betatron oscillations around it.
- The dispersion function expresses the closed orbit variation in terms of $\Delta p/p$



$$\begin{pmatrix} \Delta x(s) \\ \Delta x'(s) \end{pmatrix} = \begin{pmatrix} D_x(s) \\ D'_x(s) \end{pmatrix} \frac{\Delta p}{p_0}$$

- The dispersion function originates from the dipoles and when no dipole is traversed the quantity $\gamma D^2 + 2\alpha D D' + \beta D'^2$ stays constant.

Principal trajectories and beta function

- The solutions of the Hill's equation via principal trajectories (Cos-like and Sin-like, C and S hereafter) and via beta function can be expressed in terms of each other. Thus

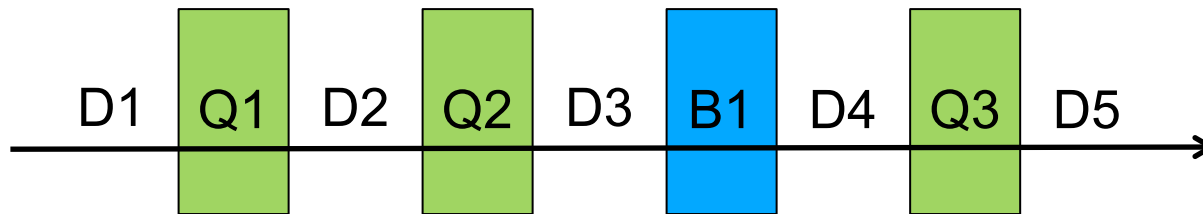
$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = R(s/s_0) \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0} = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$

$$\begin{pmatrix} \beta(s) \\ \alpha(s) \\ \gamma(s) \end{pmatrix} = \begin{pmatrix} C^2 & -2SC & S^2 \\ -CC' & S'C + SC' & -SS' \\ C'^2 & -2S'C' & S'^2 \end{pmatrix} \cdot \begin{pmatrix} \beta(s_0) \\ \alpha(s_0) \\ \gamma(s_0) \end{pmatrix}$$

$$R(s/s_0) = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}} \cdot (\cos \mu + \alpha_0 \cdot \sin \mu) & \sqrt{\beta\beta_0} \cdot \sin \mu \\ \frac{\alpha_0 - \alpha}{\sqrt{\beta\beta_0}} \cdot \cos \mu - \frac{1 + \alpha\alpha_0}{\sqrt{\beta\beta_0}} \cdot \sin \mu & \sqrt{\frac{\beta_0}{\beta}} \cdot (\cos \mu - \alpha \cdot \sin \mu) \end{pmatrix}$$

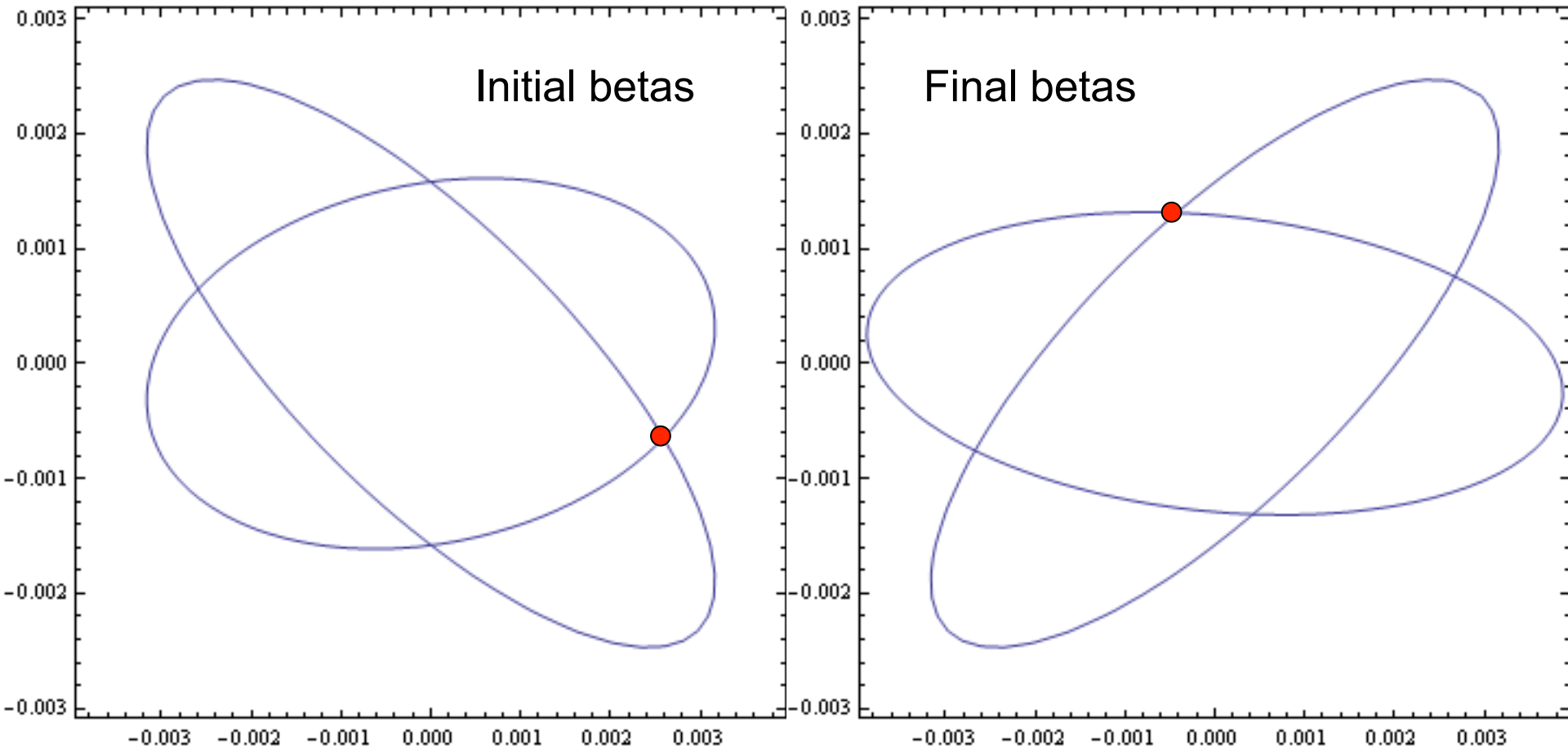
Beam transport

- As we said in slide 1, a beamline is basically **a set of magnets** used to **transport the beam** from one position to another.
- The transport through the beamline is uniquely defined by the matrix multiplication of its components



$$R = M(D5) \cdot M(Q3) \cdot M(D4) \cdot \dots \cdot M(Q1) \cdot M(D1)$$

- This defines the transport, not the beam.



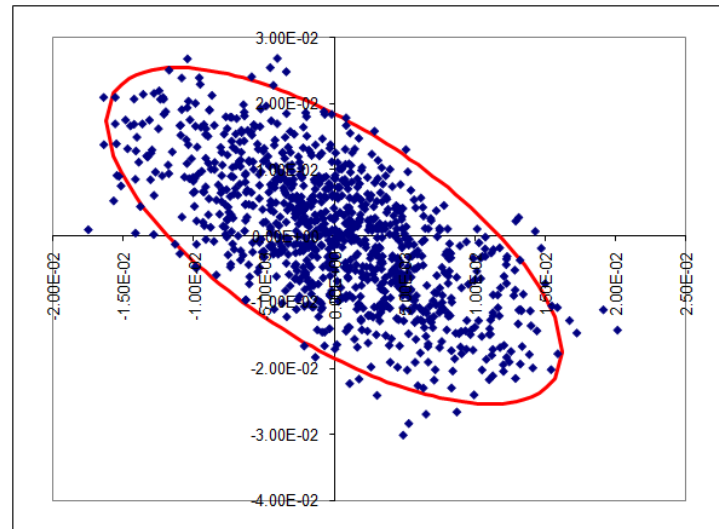
The same particle ends in the same place also with two different initial betas

Circular accelerators and beamlines

- The beta function in a ring is derived considering periodic conditions $K(s+L) = K(s)$. This defines clearly the meaning of the beta function, which describes the accelerator and the beam adapts to it.
- In transfer lines the periodicity condition does not apply. One can choose the initial betas “freely”. Betas are useful if they describe the beam!

Area of ellipse = $\pi\varepsilon$

Now that it is defined by a beam ε is called **beam emittance**



Statistical emittance and Twiss parameters

- Given a particle distribution, how do I choose ε , α and β ?

$$\beta = \frac{\langle x^2 \rangle}{\sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}}$$

$$\alpha = \frac{-\langle xx' \rangle}{\sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}}$$

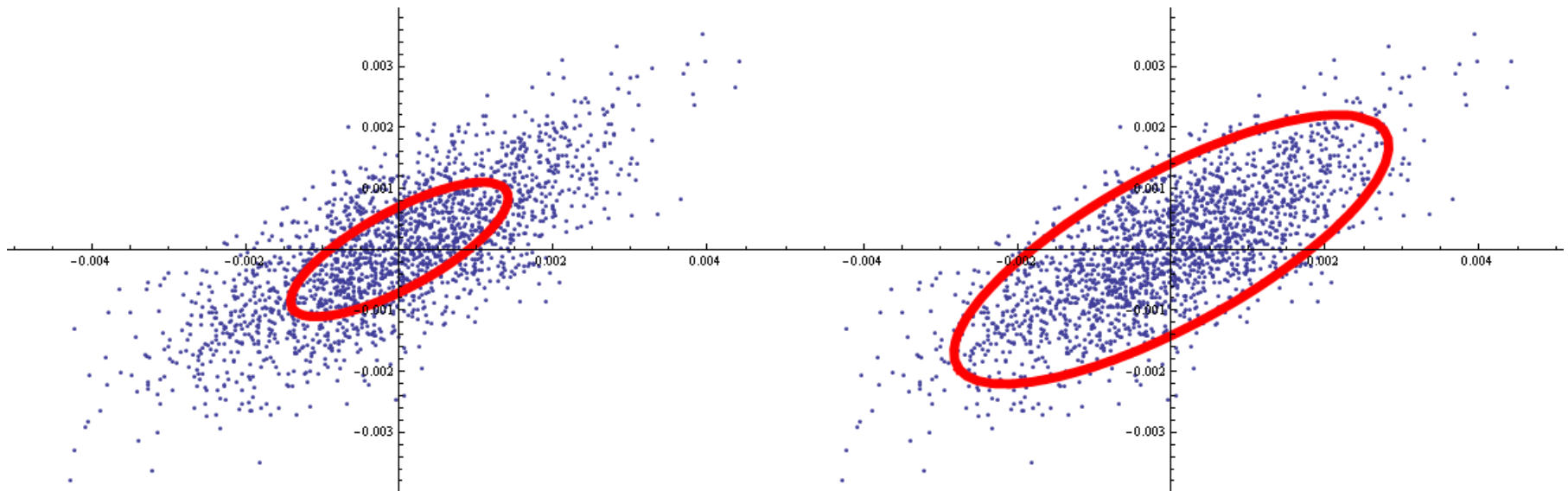
$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

- (proof at the end if we have time)

Statistical emittance and Twiss parameters

Gaussian distribution

$$\frac{1}{2\pi\epsilon_{rms}} \text{Exp}\left(-\frac{\gamma x^2 + 2\alpha x x' + \beta x'^2}{2\epsilon_{rms}}\right)$$

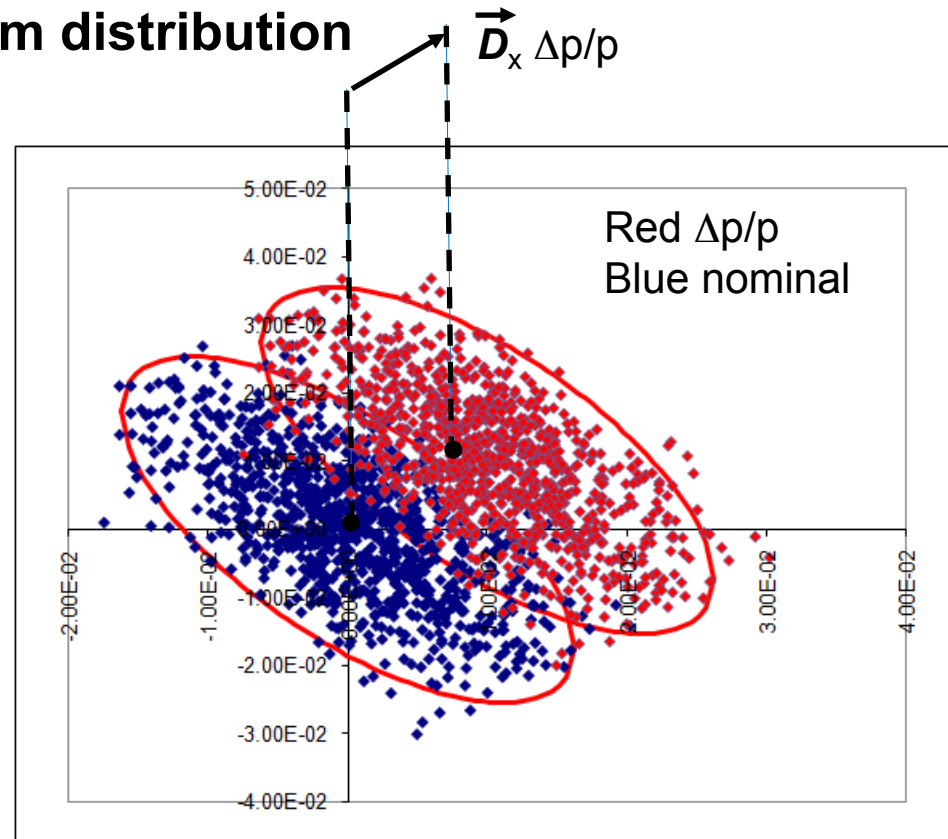


$\epsilon = \epsilon_{rms}; 39\%$

$\epsilon = 4\epsilon_{rms}; 86\%$

Initial dispersion

- As for betas, in a ring the periodic dispersion is clearly defined while in a transfer line the initial value of the dispersion function shall be based on the beam distribution



Matching dispersion to zero and achromatic optics

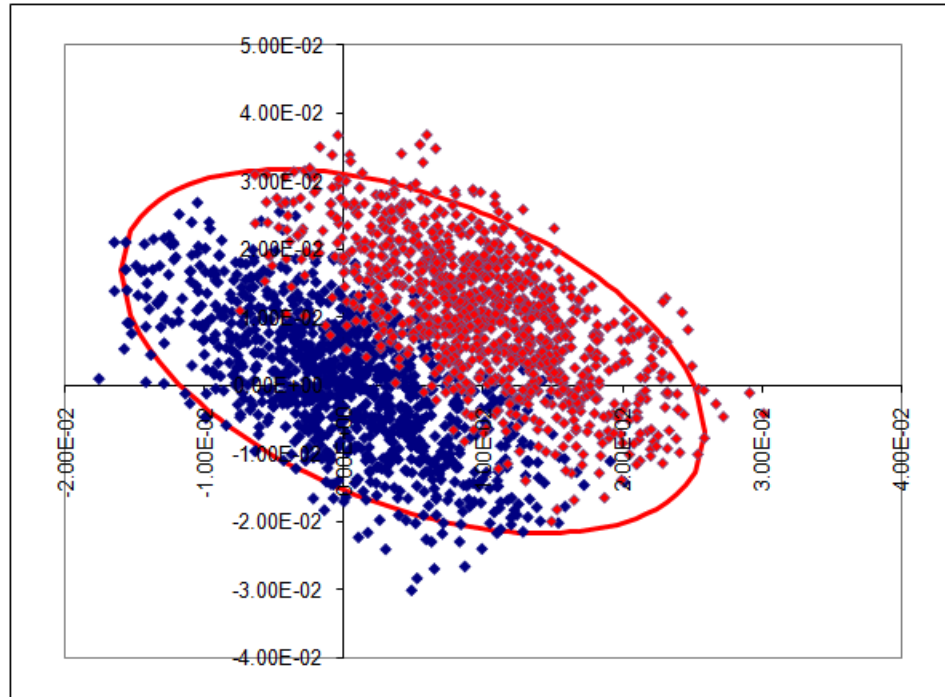
- Measuring beam dispersion is not easy. Can I mix up dispersion and emittance and have an optics that starting with $(D_y, D_y') = (0, 0)$ yields at the end of the line $(D_y, D_y') = (0, 0)$ [achromatic transport]?
- Yes, if for the given $\Delta p/p$ the correlation is acceptable:

E.g. for 230 MeV protons,

$\Delta p/p = 0.001 \iff 1 \text{ mm}$

of water in range.

- Emittance may seem to diminish along the line



Defining the problem

- to **transport the beam** from one position to another
- to give the beam the **right shape**.
- **We have now defined the initial conditions**
- **We have to define what we want to obtain**
 - If the line is bringing the beam to an accelerator, the beam is defined by the injection
 - If the line transports the beam to an experimental cave the beam has to satisfy the user needs

What do you want? (e.g. at the isocenter)

- Of course what you want at the isocenter also depends on the user...
- As an example, let's make a few simplifying assumptions:
 - We neglect the vacuum window and the air (patient in vacuum...)
 - We assume a **gaussian** beam distribution
 - We want $(D_y, D_y') = (0, 0)$, such that the beam is minimized wrt $\Delta p/p$, that the beam does not move if the particle momentum slightly changes and that the particle remains in the spot.
 - We want a **FW** of a **parallel** divergent or convergent beam ($\alpha = 0$)
 - The beam has a **radius of 1 π mm mrad**
- Then $\text{HWHM} = 1.16\sigma = 5 \text{ mm} \Rightarrow (\epsilon\beta)^{0.5} = 4.24 \text{ mm} \Rightarrow \beta = 18 \text{ m}$

These numbers are just an example to give a feeling of the orders of magnitude!

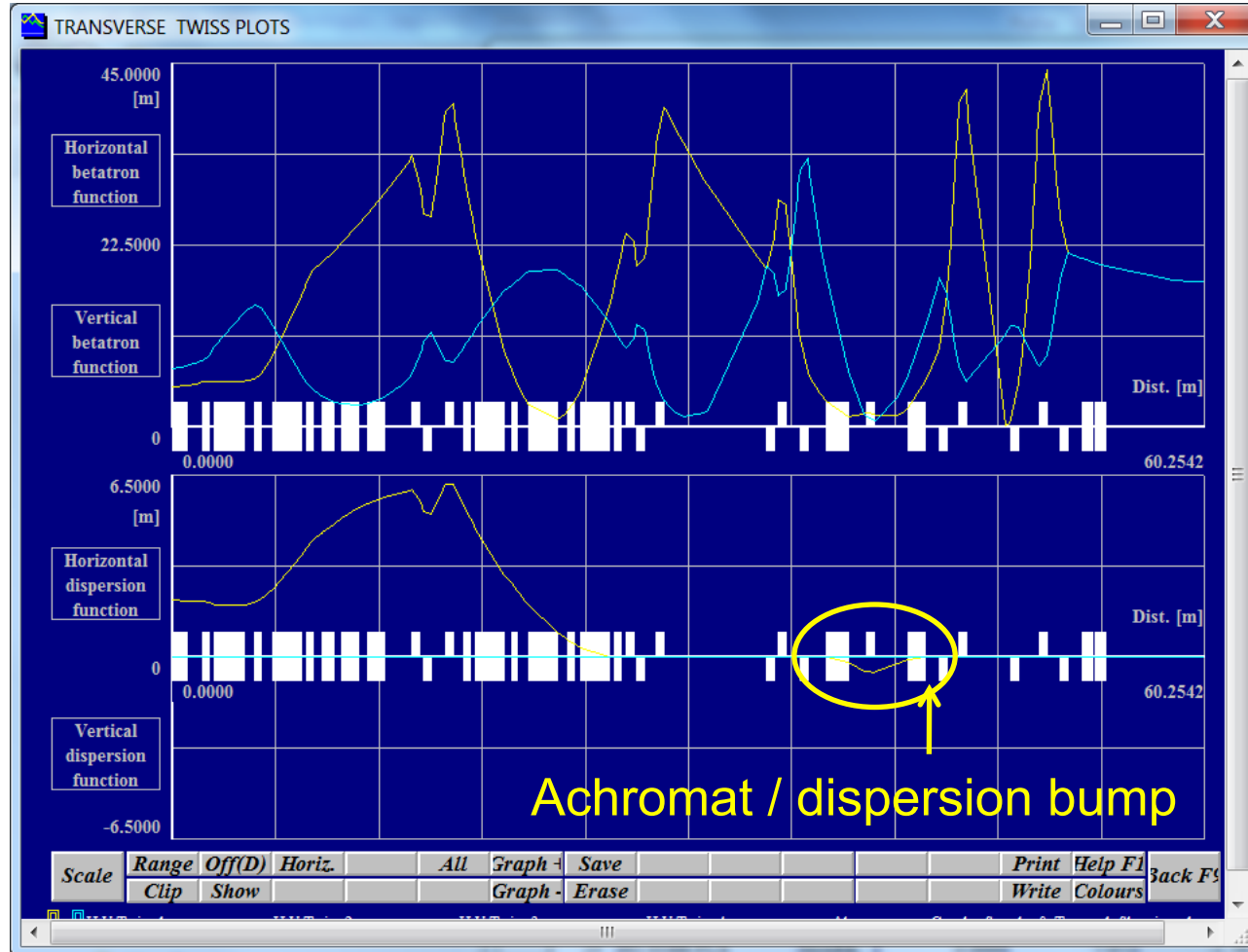
- Ideally long transfer lines consist of a **regular cell structure** over the majority of their length with **matching sections** at either end to match them with their injector and user machines.

- In short lines the “regular cell structure over the majority of the length” often does not exist

Matching the beam

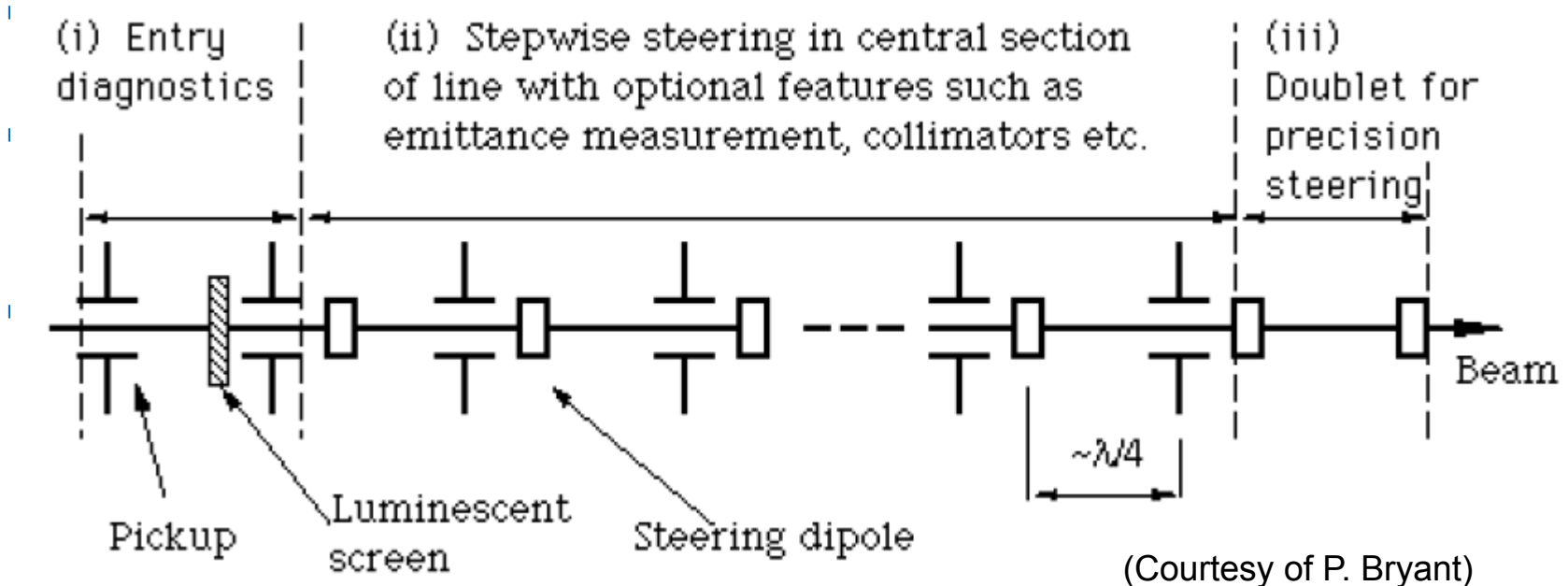
- Optics codes evaluate the betatron function along the beamline as a function of the magnets characteristics assumed. They always foresee minimization/**matching routines** that help the physicist in obtaining the desired values for the “betas” at the isocenter, where “betas” includes $\beta_x, \alpha_x, \beta_z, \alpha_z, D_x, D_x'$ (and D_z, D_z').
- The usable variables are quadrupoles (both during the design and during the operation phase), dipole shape and drift length (only during the design phase).
- Generally in a line more than 6 quadrupoles are available and thus in principle many solutions are available. Unluckily often the solution given by the program requires unfeasible hardware, the beam size along the transfer line is too large, a (local) minimum found does not correspond to what you want, etc... and finding the optical solution may require some work.

In the end you match your line



Beam monitoring and steering

■ The general approach is



- Two monitors are needed at the exit of the line as input for the two steerers. If they are not part of the following line/accelerator, they have to be part of the line under consideration.

Telescopes

- **Recall**

$$R(s/s_0) = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}} \cdot (\cos \mu + \alpha_0 \cdot \sin \mu) & \sqrt{\beta\beta_0} \cdot \sin \mu \\ \frac{\alpha_0 - \alpha}{\sqrt{\beta\beta_0}} \cdot \cos \mu - \frac{1 + \alpha\alpha_0}{\sqrt{\beta\beta_0}} \cdot \sin \mu & \sqrt{\frac{\beta_0}{\beta}} \cdot (\cos \mu - \alpha \cdot \sin \mu) \end{pmatrix}$$

- **If $\mu = n\pi$, $\sin \mu = 0$ and $S = 0$.
Since the R matrix is independent of the Twiss parameters:**

$$R(s/s_0) = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}} & 0 \\ \frac{\alpha_0 - \alpha}{\sqrt{\beta\beta_0}} & \sqrt{\frac{\beta_0}{\beta}} \end{pmatrix}$$

- ***A lattice with integer- π phase advance in one plane, has the same phase advance for any incoming lattice functions in that plane.***

Telescopes

- If the lattice is matched to have $\alpha = \alpha_0$ for one set of Twiss parameters, then $C'=0$.
- Then $C' = 0$ for every incoming set of Twiss parameters and

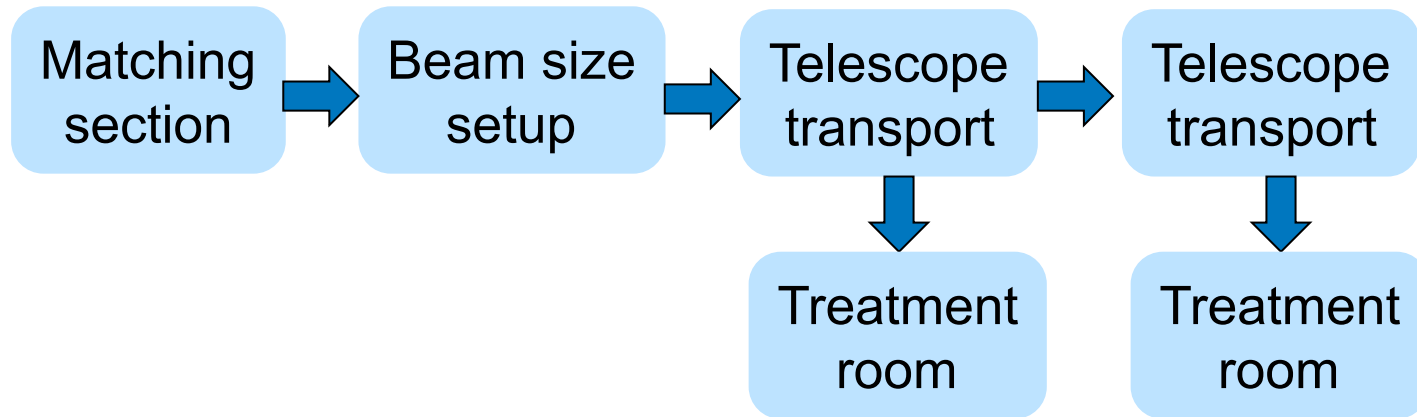
$$R(s/s_0) = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}} & 0 \\ \frac{\alpha_0 - \alpha}{\sqrt{\beta\beta_0}} & \sqrt{\frac{\beta_0}{\beta}} \end{pmatrix}$$

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} C^2 & 0 & 0 \\ 0 & CS' = 1 & 0 \\ 0 & 0 & S'^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix}$$

$$R(s/s_0) = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}} & 0 \\ 0 & \sqrt{\frac{\beta_0}{\beta}} \end{pmatrix}$$

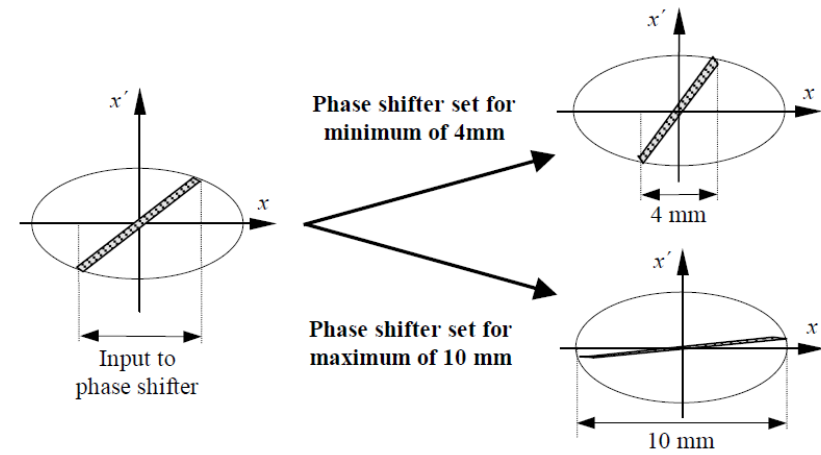
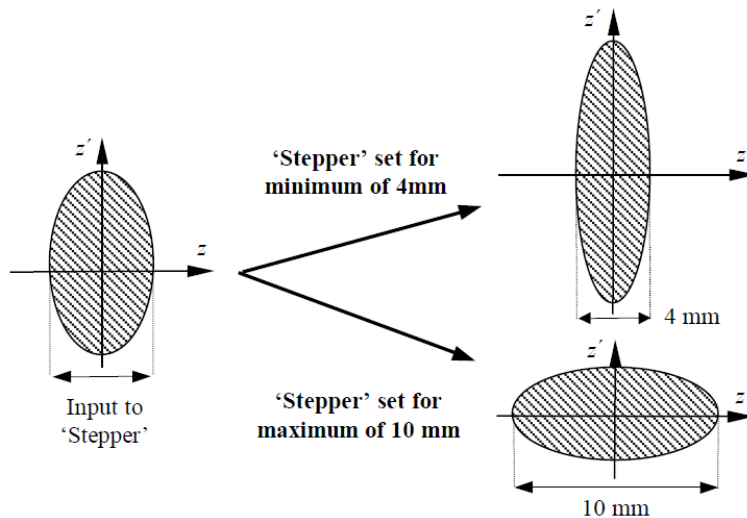
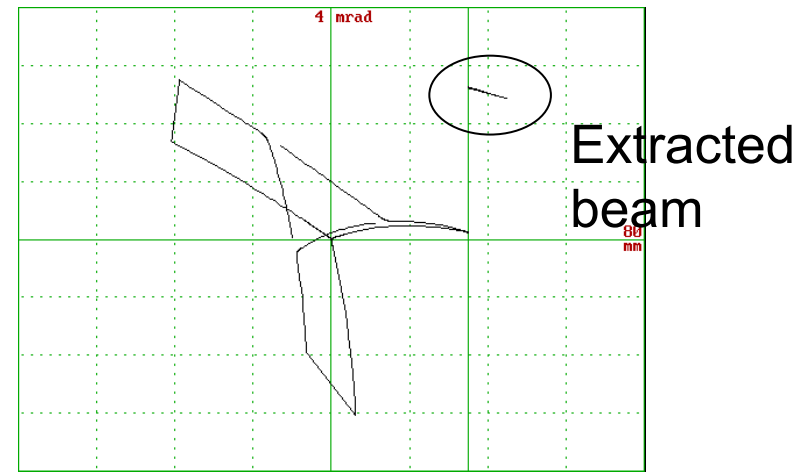
- **A lattice matched for $\mu = n\pi$ and $\alpha = \alpha_0$ will provide a constant magnification of β and leave α untouched for any set of incoming Twiss parameters.**

PIMMS modular approach



Phase shifter – stepper

- The beam size setup is made in PIMMS with a “phase shifter” (in the horizontal plane) and a “stepper” in the vertical plane (eventually combined in a single module)



Tomoscope

- The bar of charge rotation can be used to reconstruct a beam “tomography”. Recently demonstrated at MedAustron

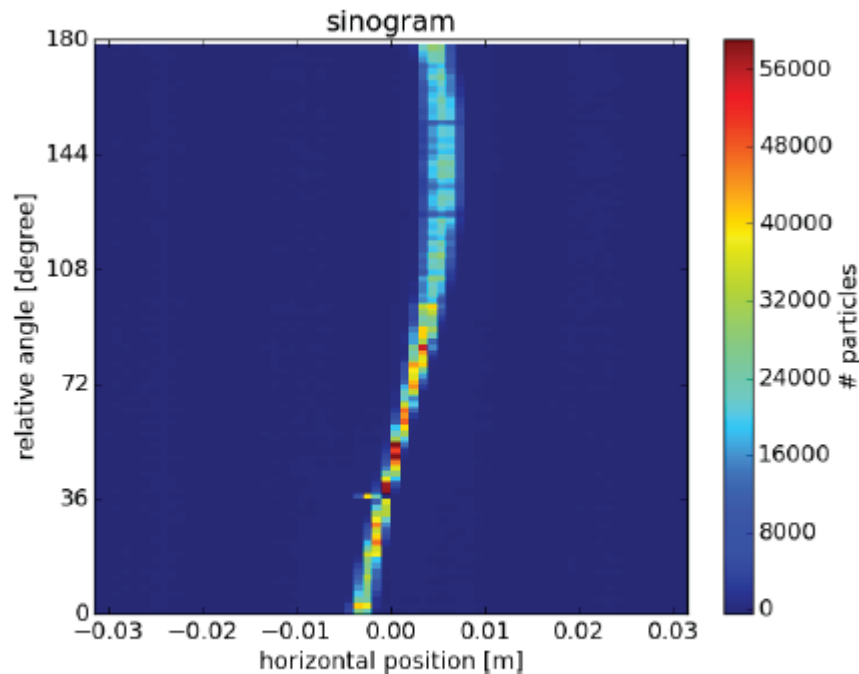


Figure 7: Sinogram of the measured projections

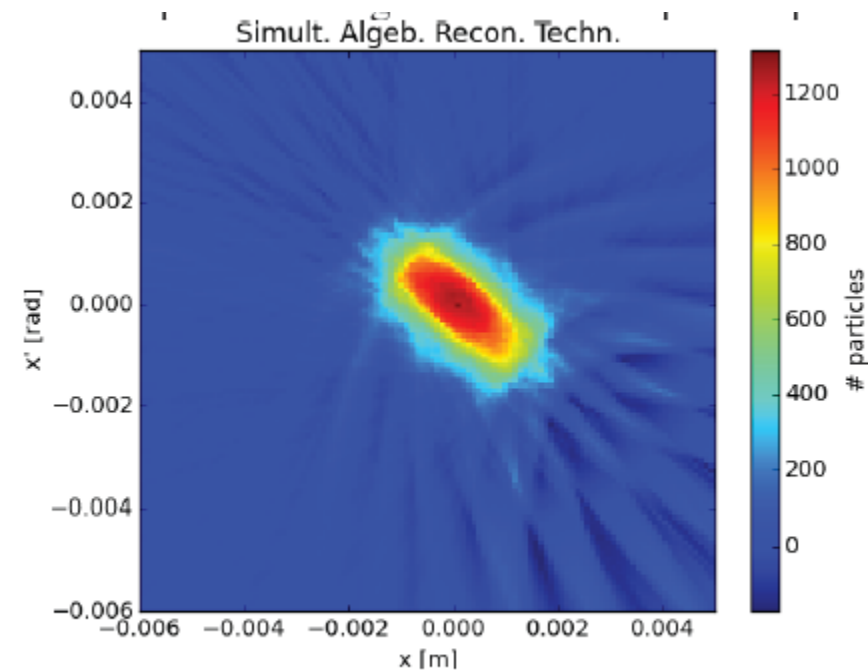
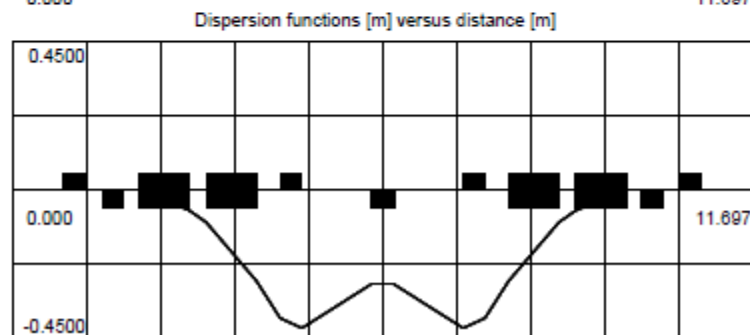
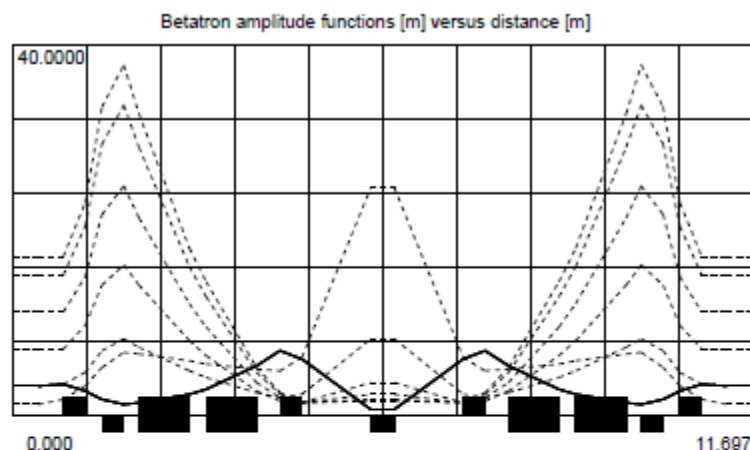
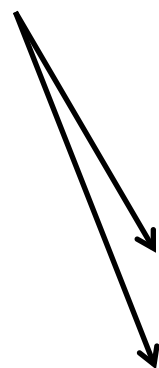


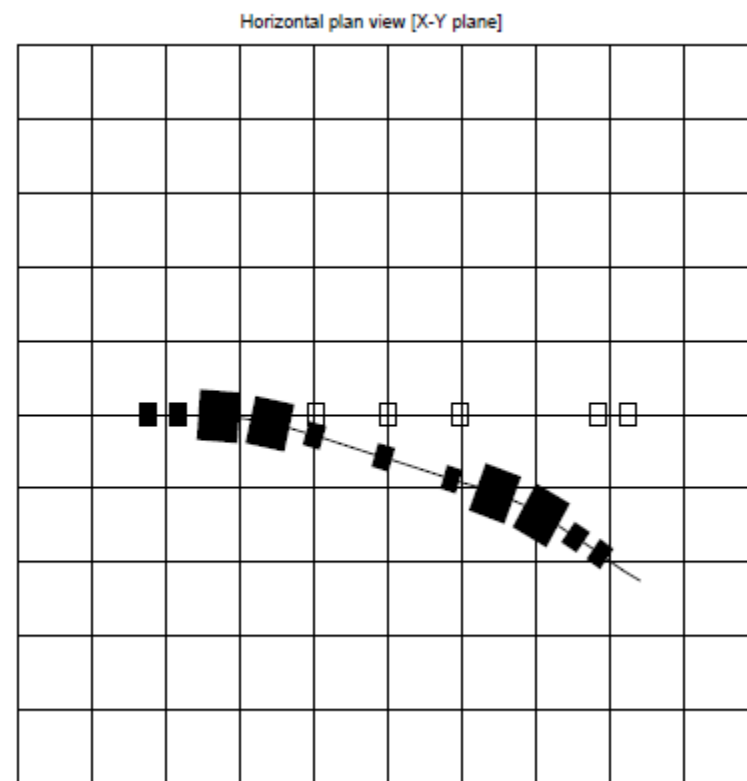
Figure 9: Reconstruction using SART
(Courtesy of A. Wastl)

Example telescope

- Achromatic bending towards a treatment room
- Verify that all the sttings go through without problems



Horizontal ——— Vertical - - - - -



Matching to gantries

■ Symmetric beam method with zero dispersion (exact)

- The beam at the entrance to the gantry must have zero dispersion and must be **rotationally symmetric** i.e. the same distribution (gaussian or KV) with equal Twiss functions and equal emittances in both planes at the entry to the gantry.
- The gantry must be designed to be a closed dispersion bump in the plane of bending (achromatic transport)

■ Round beam method with zero dispersion (partial)

- The beam must have zero dispersion, the **same distribution (gaussian or KV)** in both planes with the condition $\varepsilon_x \beta_x = \varepsilon_z \beta_z$ at the entry to the gantry. It would also be desirable but not absolutely necessary to have $\alpha_x = \alpha_z = 0$
- The gantry must be designed with phase advances of multiples of π in both planes, same magnification in the two planes and a closed dispersion bump in the plane of bending
- The drawback in this case is that the optics inside the gantry changes with the rotation angle.

Beam at entrance to a rotated gantry

$$\sigma_{n.c.} = \begin{pmatrix} \varepsilon_x \beta_x & -\varepsilon_x \alpha_x & 0 & 0 \\ -\varepsilon_x \alpha_x & \varepsilon_x \gamma_x & 0 & 0 \\ 0 & 0 & \varepsilon_y \beta_y & -\varepsilon_y \alpha_y \\ 0 & 0 & -\varepsilon_y \alpha_y & \varepsilon_y \gamma_y \end{pmatrix}$$

$$M_R(\alpha) \cdot \sigma_{n.c.} \cdot M_R^T(\alpha) =$$

$$= \begin{pmatrix} \cos^2(\alpha)\varepsilon_x\beta_x + \sin^2(\alpha)\varepsilon_y\beta_y & -\cos^2(\alpha)\varepsilon_x\alpha_x - \sin^2(\alpha)\varepsilon_y\alpha_y & \cos(\alpha)\sin(\alpha)(-\varepsilon_x\beta_x + \varepsilon_y\beta_y) & \cos(\alpha)\sin(\alpha)(\varepsilon_x\alpha_x - \varepsilon_y\alpha_y) \\ -\cos^2(\alpha)\varepsilon_x\alpha_x - \sin^2(\alpha)\varepsilon_y\alpha_y & \cos^2(\alpha)\varepsilon_x\gamma_x + \sin^2(\alpha)\varepsilon_y\gamma_y & \cos(\alpha)\sin(\alpha)(\varepsilon_x\alpha_x - \varepsilon_y\alpha_y) & \cos(\alpha)\sin(\alpha)(-\varepsilon_x\gamma_x + \varepsilon_y\gamma_y) \\ \cos(\alpha)\sin(\alpha)(-\varepsilon_x\beta_x + \varepsilon_y\beta_y) & \cos(\alpha)\sin(\alpha)(-\varepsilon_x\alpha_x + \varepsilon_y\alpha_y) & \sin^2(\alpha)\varepsilon_x\beta_x + \cos^2(\alpha)\varepsilon_y\beta_y & -\sin^2(\alpha)\varepsilon_x\alpha_x - \cos^2(\alpha)\varepsilon_y\alpha_y \\ \cos(\alpha)\sin(\alpha)(-\varepsilon_x\alpha_x + \varepsilon_y\alpha_y) & \cos(\alpha)\sin(\alpha)(-\varepsilon_x\gamma_x + \varepsilon_y\gamma_y) & -\sin^2(\alpha)\varepsilon_x\alpha_x - \cos^2(\alpha)\varepsilon_y\alpha_y & \sin^2(\alpha)\varepsilon_x\gamma_x + \cos^2(\alpha)\varepsilon_y\gamma_y \end{pmatrix}$$

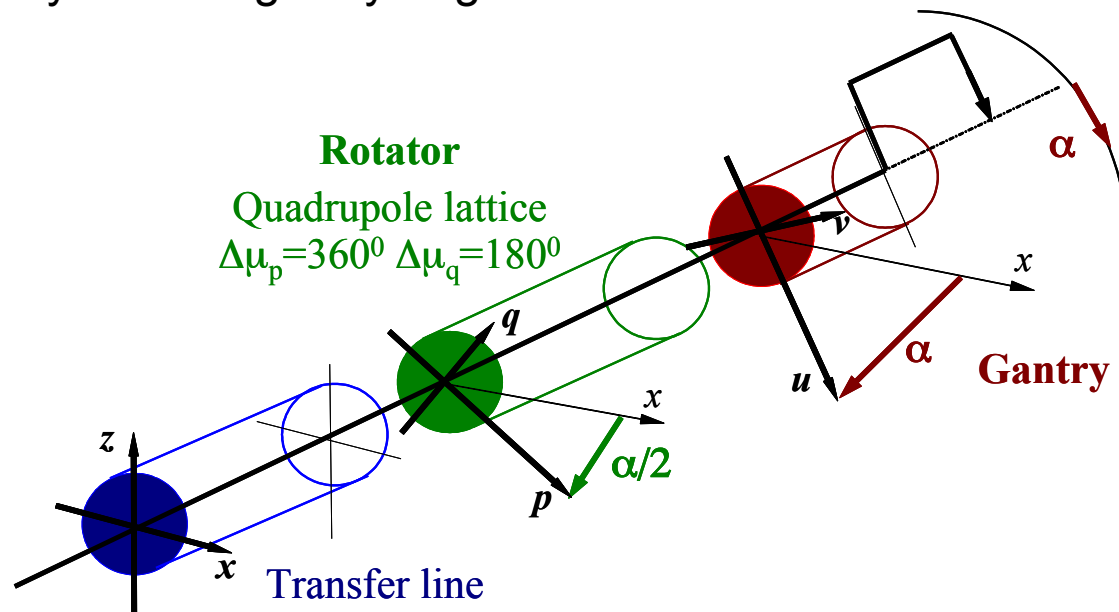
Matching to gantries

▪ Rotator method (exact) [Lee Teng, Fermilab]

- The rotator is a lattice with transfer function

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- It is rotated by half the gantry angle



Matching to gantries

■ Rotator method (cont)

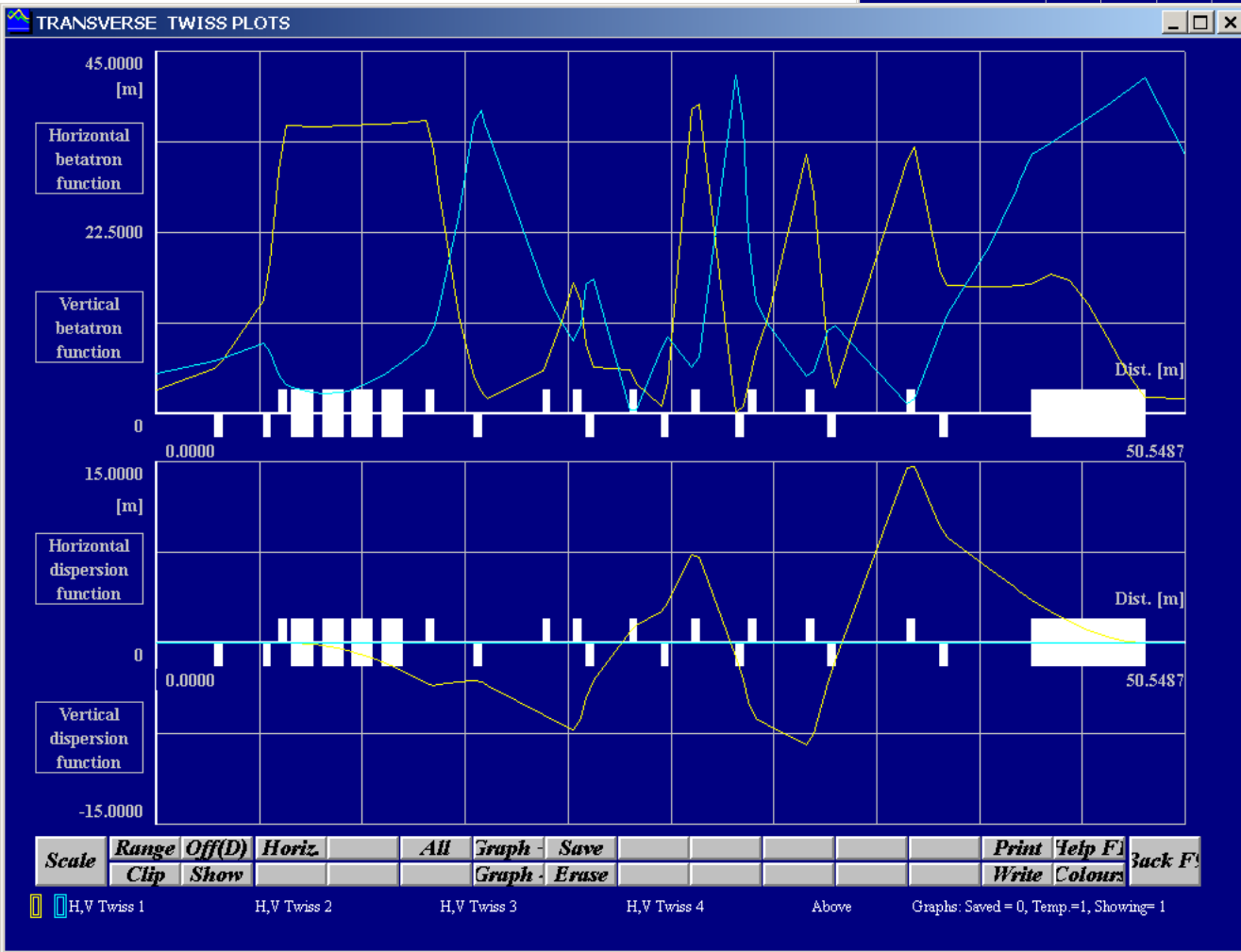
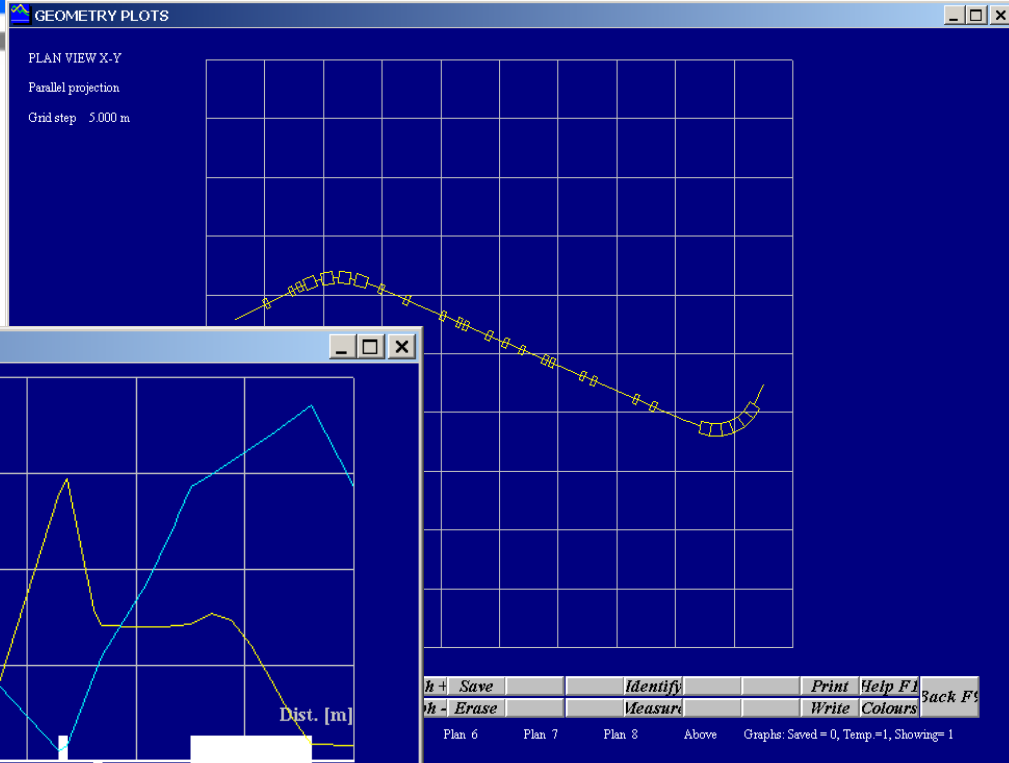
- The overall transfer matrix between fixed line and gantry is

$$M_0 = \begin{pmatrix} \cos\frac{\alpha}{2} & 0 & \sin\frac{\alpha}{2} & 0 \\ 0 & \cos\frac{\alpha}{2} & 0 & \sin\frac{\alpha}{2} \\ -\sin\frac{\alpha}{2} & 0 & \cos\frac{\alpha}{2} & 0 \\ 0 & -\sin\frac{\alpha}{2} & 0 & \cos\frac{\alpha}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \cos\frac{\alpha}{2} & 0 & \sin\frac{\alpha}{2} & 0 \\ 0 & \cos\frac{\alpha}{2} & 0 & \sin\frac{\alpha}{2} \\ -\sin\frac{\alpha}{2} & 0 & \cos\frac{\alpha}{2} & 0 \\ 0 & -\sin\frac{\alpha}{2} & 0 & \cos\frac{\alpha}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

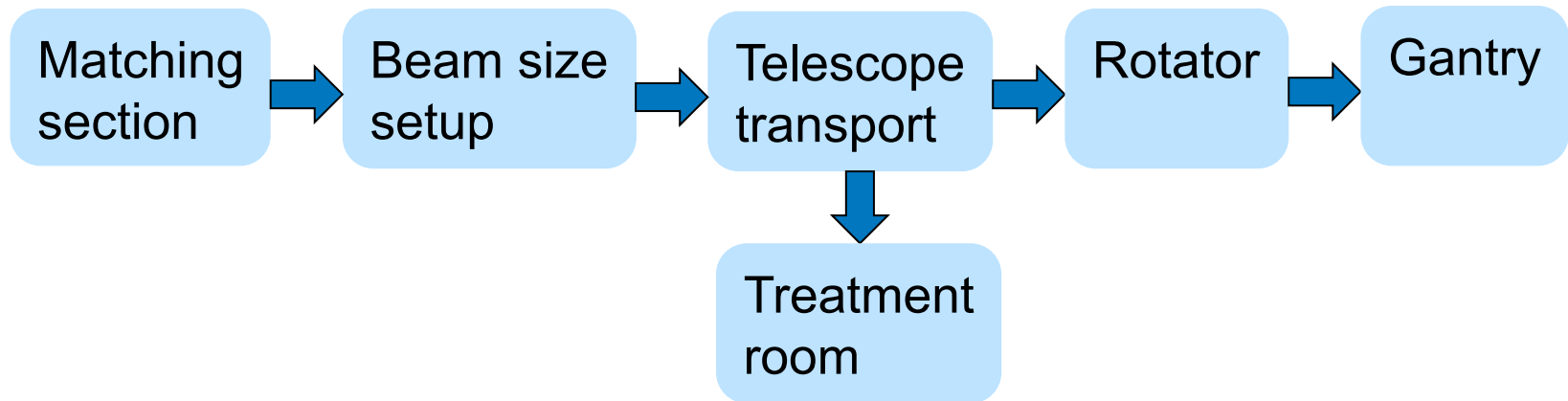
Rotation between rotator and gantry

Rotation between line and rotator

- **Maps the beam 1:1 to the gantry independent of the angle**
- **It also rotates the dispersion, which can be closed in the fixed line!**
- **Beams from synchrotrons are strongly asymmetric, and clearly benefit of the rotator approach**



PIMMS modular approach



Statistical emittance and Twiss parameters (cont)

- Let f be the probability density in phase space: $\iint f(x, x') dx dx' = 1$
- To be consistent with the description of the beam based on the beta function, let's assume that the "iso-density" lines in phase space are ellipses

$$f(x, x') = f(\gamma x^2 + 2\alpha x x' + \beta x'^2)$$

- Let be

$$\langle x^2 \rangle = \iint x^2 f(x, x') dx dx' \quad \langle x'^2 \rangle = \iint x'^2 f(x, x') dx dx' \quad \langle x x' \rangle = \iint x x' f(x, x') dx dx'$$

- Define the normalised coordinates

$$\begin{pmatrix} X \\ X' \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{\beta}} & 0 \\ \alpha & \sqrt{\beta} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}$$

- **The ellipse in real coordinates transforms in a circle in normalised phase space. A circle with the same area (emittance) as the ellipse!**

$$\begin{aligned} \gamma x^2 + 2\alpha x x' + \beta x'^2 &= \gamma (\sqrt{\beta} X)^2 + 2\alpha (\sqrt{\beta} X) \left(\frac{-\alpha}{\sqrt{\beta}} X + \frac{1}{\sqrt{\beta}} X' \right) + \beta \left(\frac{-\alpha}{\sqrt{\beta}} X + \frac{1}{\sqrt{\beta}} X' \right)^2 \\ &= X^2 + X'^2 \end{aligned}$$

$$\begin{aligned}\langle x^2 \rangle &= \iint x^2 f(x, x') dx dx' = \iint (\sqrt{\beta} X)^2 f(X^2 + X'^2) dX dX' \\ &= \beta M_{X^2}\end{aligned}$$

$$\begin{aligned}\langle x'^2 \rangle &= \iint x'^2 f(x, x') dx dx' = \iint \left(\frac{-\alpha}{\sqrt{\beta}} X + \frac{1}{\sqrt{\beta}} X' \right)^2 f(X^2 + X'^2) dX dX' \\ &= \frac{\alpha^2}{\beta} M_{X^2} + \frac{1}{\beta} M_{X'^2} - \frac{2\alpha}{\beta} M_{XX'}\end{aligned}$$

$$\begin{aligned}\langle xx' \rangle &= \iint xx' f(x, x') dx dx' = \iint \sqrt{\beta} X \left(\frac{-\alpha}{\sqrt{\beta}} X + \frac{1}{\sqrt{\beta}} X' \right)^2 f(X^2 + X'^2) dX dX' \\ &= -\alpha M_{X^2} + M_{XX'}\end{aligned}$$

- **Where** $M_{X^2} = \iint X^2 f(X^2 + X'^2) dX dX'$, $M_{XX'}$, and $M_{X'^2}$ depend only on f and not on the Twiss parameters

- Let's evaluate

$$\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2 = \dots = M_{X^2} M_{X'^2} = M_{X^2}^2$$

- Then by simple inspection of

$$\begin{aligned} \langle x^2 \rangle &= \iint x^2 f(x, x') dx dx' = \iint (\sqrt{\beta} X)^2 f(X^2 + X'^2) dX dX' \\ &= \beta M_{X^2} \end{aligned}$$

- One obtains

$$\beta = \frac{\langle x^2 \rangle}{\sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}}$$

- and similarly for α

Winagile

- You can download WinAgile at the following link(s):
- Main Program:
- <https://dl.dropboxusercontent.com/u/4851817/Winagile4-11A.exe>
- On-line Help File:
- <https://dl.dropboxusercontent.com/u/4851817/Agilehelp.pdf>
- Quick Guide:
- <https://dl.dropboxusercontent.com/u/4851817/QuickGuide.pdf>
- User Guide:
- <https://dl.dropboxusercontent.com/u/4851817/UserGuide.pdf>
- Ex. Lattices:
- <https://dl.dropboxusercontent.com/u/4851817/ExampleLattices.zip>
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Thank you for your attention

“Anything one man can imagine, other men can make real.”

Jules Verne