

# Cyclotrons: magnetic design and beam dynamics

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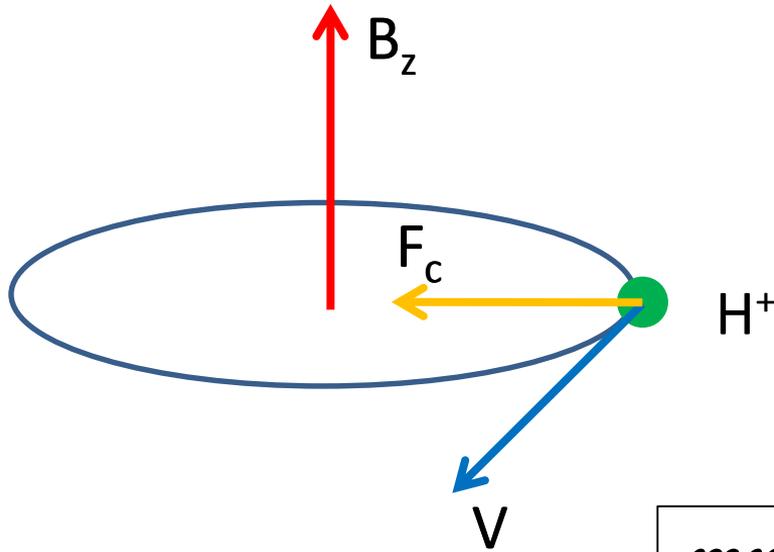
# Outline

- Part I: A short introduction
- Part II: A little bit about focusing and isochronism
- Part III: ..... about injection
- Part IV: ..... about extraction
- Part V: ..... about magnetic design

First lecture: parts 1-3  
Second lecture: parts 3-5

# Part I: A short introduction

# The most basic equation of the cyclotron



- A charged particle in a uniform magnetic field moves on a circle
- The centripetal force is equal to the Lorentz force acting on the particle
- **Thus the rotation frequency of the particle is constant => independent on radius ,velocity , energy or time (in the non-relativistic limit)**

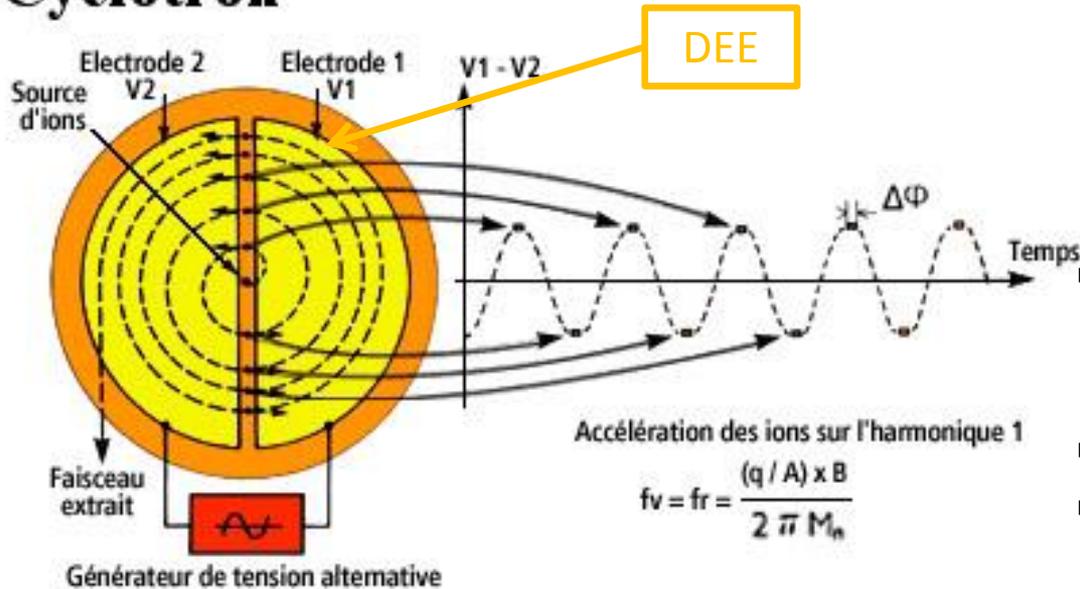
$$\frac{mv^2}{r} = qvB$$

$\Rightarrow$

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

# Consequences of constant cyclotron frequency

## Cyclotron



- Particles can be accelerated with an RF-system operating at constant frequency:

$$F_{RF}(\text{MHz}) = 15.2 h(Z/A) B (\text{Tesla})$$

- The orbit starts in the center (injection) and spirals outward towards the pole radius (extraction)
- The magnet field is constant in time
- RF and magnetic structure are completely integrated => **Same RF structure accelerates many times** => compact and cost-effective
- CW-operation (continuous wave)

Classical cyclotron:  
Lawrence and Livingston, Phys. Rev. 40 (1932) 9

# Classical cyclotron: where is the problem?

- i. In a uniform magnetic field there is no vertical focusing (metastable)
- ii. During acceleration, due to the relativistic mass increase, the revolution frequency decreases in a uniform magnetic field => loss of resonance between RF and the beam => loss of isochronism
- iii. just increasing the magnetic field with radius is not possible => **vertically unstable**

$$\omega = \frac{qB}{m_0} \sqrt{1 - \left(\frac{v}{c}\right)^2}$$

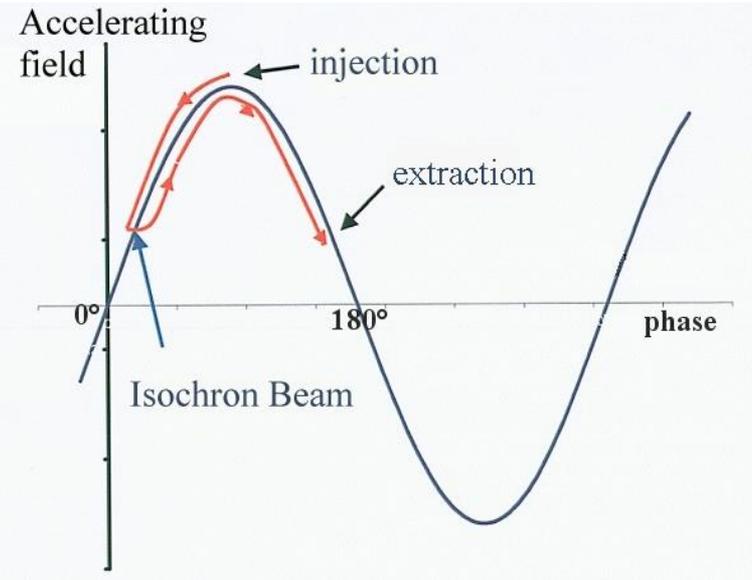
# Can we still use the classical cyclotron? a small side-step

Just accept the problem and see how far you get

The classical cyclotron with a small negative field-gradient is vertically focusing.

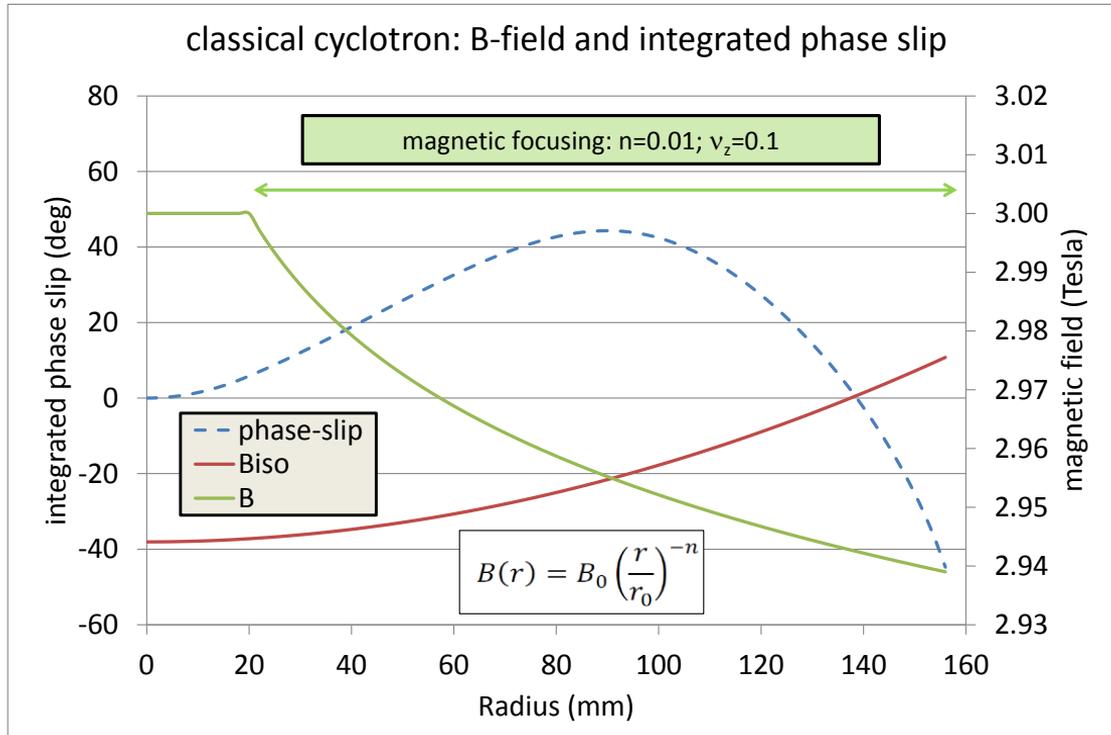
During acceleration the particles will gradually run out of phase with respect to the RF system.

How high energy can we obtain before deceleration sets in?



Courtesy Frédéric Chautard

Make a simple calculation in Excel



Small SC cyclo for PET?  
Could it be competitive?

CIEMAT Madrid

Kleeven/Zaremba CAS-2015: Cyclotrons -  
magnetic design and beam dynamics

## classical cyclotron calculation example

E	10	MeV
B	3	Tesla
$V_{\text{dee}}$	50	kVolt

RF system: 2 dees of  
 $180^\circ$  in push-pull first  
harmonic mode

$N_{\text{gaps}}$	2	
$R_{\text{extr}}$	156	mm
$N_{\text{turns}}$	59	
$\Delta\Phi_{\text{RF-max}}$	45	deg

# Another solution: the **synchro-cyclotron**

- Let the magnetic field gradually decrease with radius in order to obtain **weak vertical focusing** =>

$$v_z = \sqrt{k} \Rightarrow k = -\frac{r}{B} \frac{dB}{dr}$$

- Let the RF frequency gradually decrease with time in order to compensate for the drop of the magnetic field and for the increase of the mass

$$\omega = \frac{qB}{m}$$

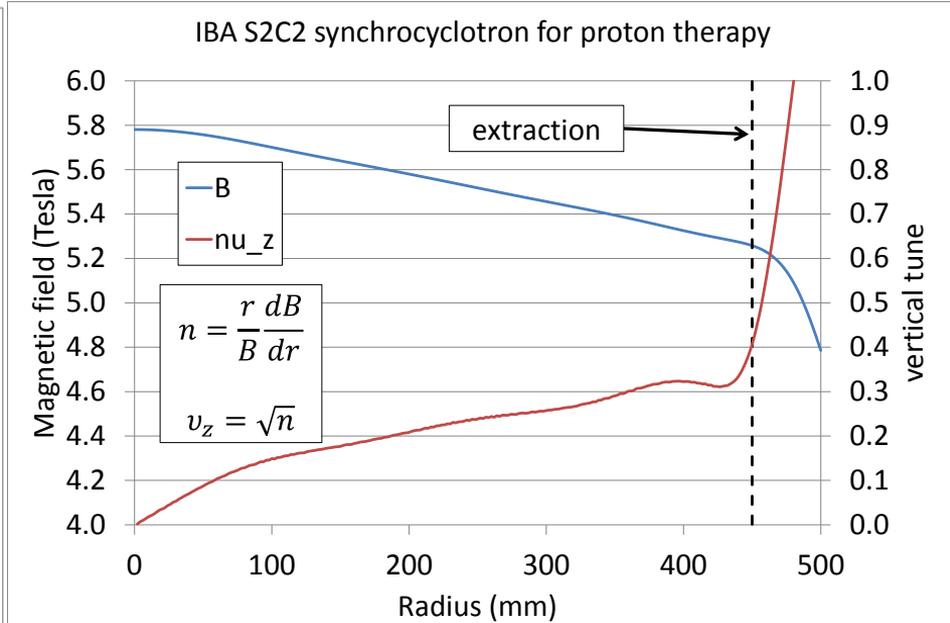
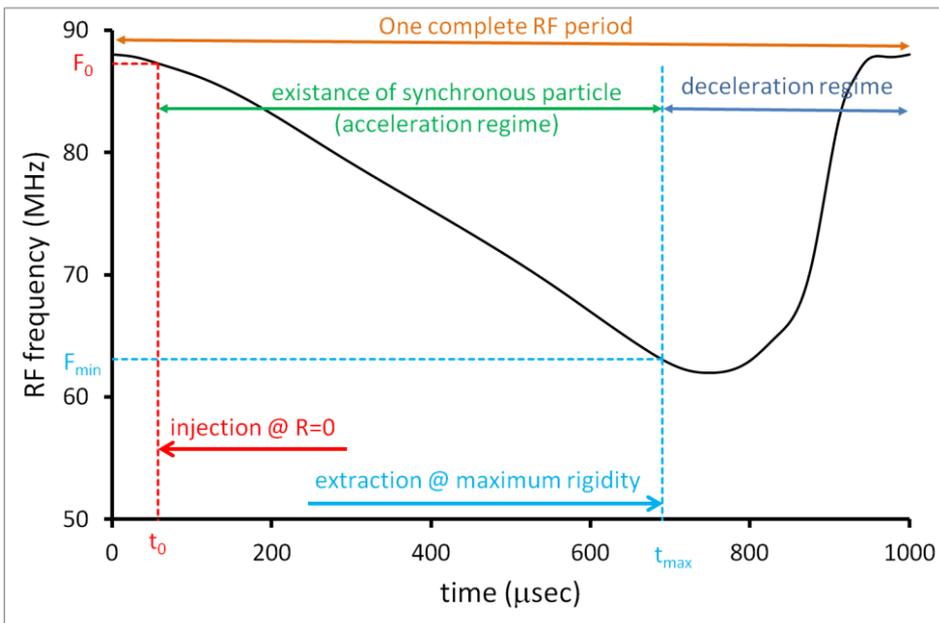
Veksler, J. Phys. USSR **9** (1945)153  
McMillan, Phys. Rev. **68** (1945)143L

Note: synchrotron was  
invented at the same time

# Some consequences

1. The RF is pulsed but the magnetic field is still constant (in time)
2. The beam is no longer CW but modulated in time
3. The mean beam intensity is much lower => OK for proton therapy
4. There is a longitudinal beam dynamics similar to that of the synchrotron
5. Only during a short time-window, beam can be captured in the cyclo-center
6. The timing between RF frequency, RF voltage and ion source need to be well defined and controlled
7. A more complicated RF system because of the required frequency variation
8. The RF frequency can not be varied very fast (rotating capacitor) and therefore the acceleration must be slow => low energy gain per turn => many turns up to extraction => little RF power needed
9. There is only a very small turn-separation. Therefore a special extraction method is needed to get the beam out of the machine (regenerative extraction)

# Example: the IBA S2C2 for proton therapy



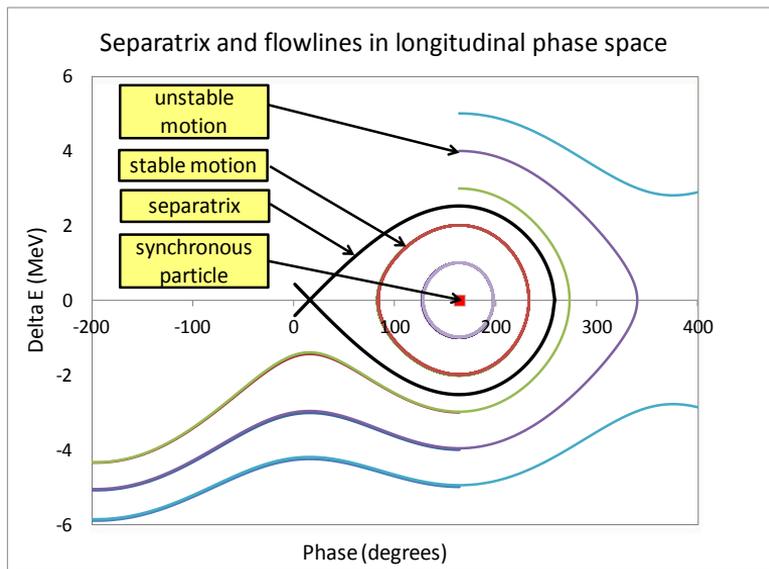
Repetition rate = 1 kHz  
Duty cycle about 100

Superconducting synchro-cyclotron  
Extraction energy 230 MeV

# Longitudinal dynamics in a synchro-cyclotron

- There is a definition of a synchronous particle: everywhere in the synchro-cyclotron, at any moment in time, the revolution frequency of the synchronous particle is equal to RF frequency
- There are oscillations (in energy and phase) of real particles around the synchronous particle
- There is a stability zone for these oscillations defined by a separatrix in the longitudinal phase space
- This separatrix is filled during the beam capture in the synchro-cyclotron center

# Illustration of the longitudinal dynamics



John L. Livingood, Principles of cyclic Particle Accelerators (1961) Chapter 6

## EQUATIONS OF MOTION

$$\frac{d\phi}{dt} = 2\pi F_{RF}(t) \left( 1 - \frac{hF_p}{F_{RF}} \right)$$

$$\frac{d\Delta E}{dt} = F_{RF} \frac{Nq\hat{V}(t)}{h} (\sin \phi - \sin \phi_s)$$

$t$  = time

$\phi$  = RF-phase

$d\Delta E$  = energy deviation

$\phi_s$  = synchronous phase

$F_{RF}$  = RF-frequency

$F_p$  = revolution frequency

$\hat{V}$  = dee voltage

$h$  = harmonic mode

$N$  = number of gaps

$q$  = particle charge

## SYNCHRONOUS PARTICLE

$$\frac{q\hat{V}N \sin \phi_s}{E_s} = - \frac{2\pi}{K\omega_s^2} \frac{d\omega_s}{dt}$$

# Yet another solution: **the isochronous cyclotron**

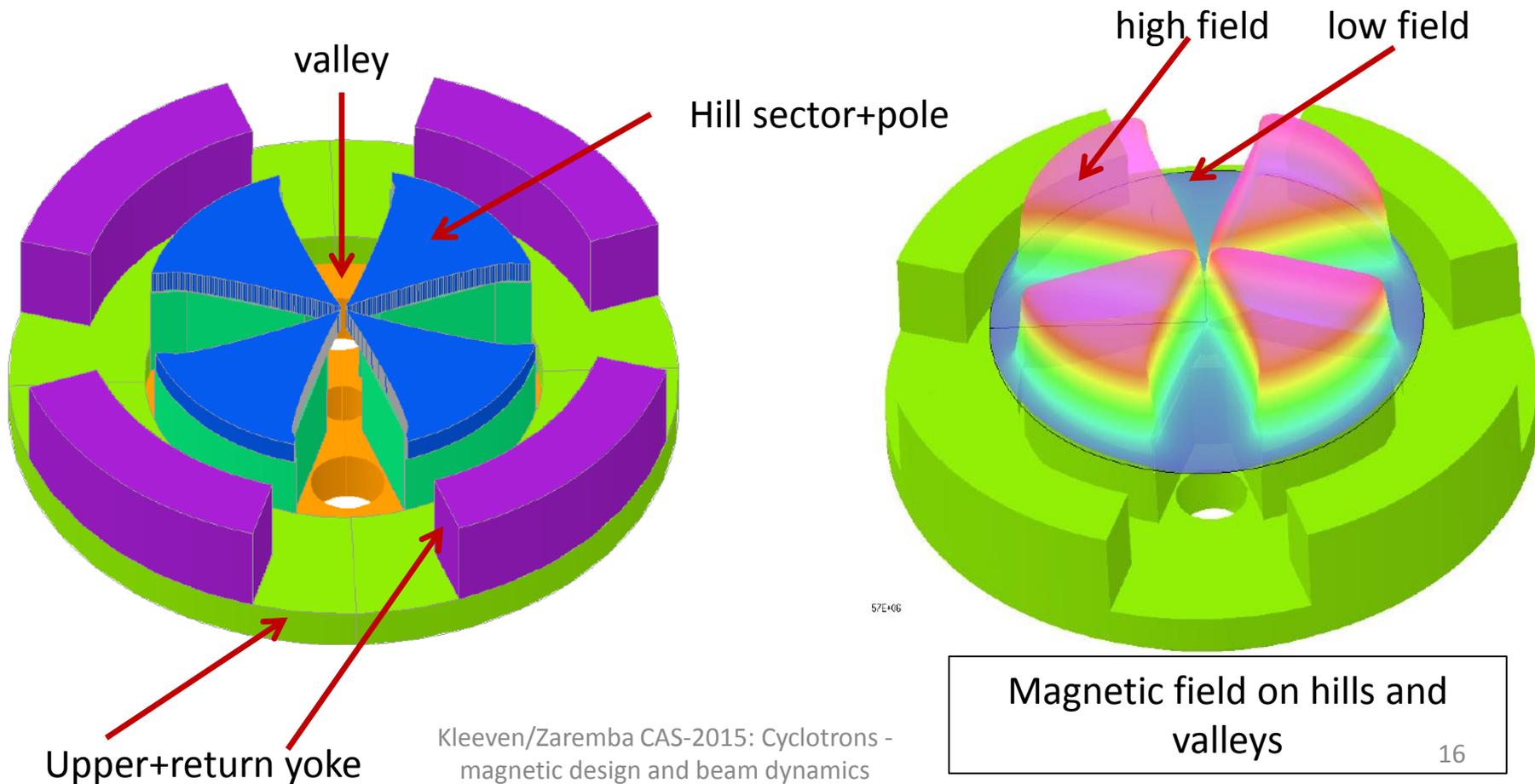
- Two contributions to vertical focusing:

$$F_z = q(\vec{v} \times \vec{B})_z = -q(v_\theta B_r - v_r B_\theta)$$

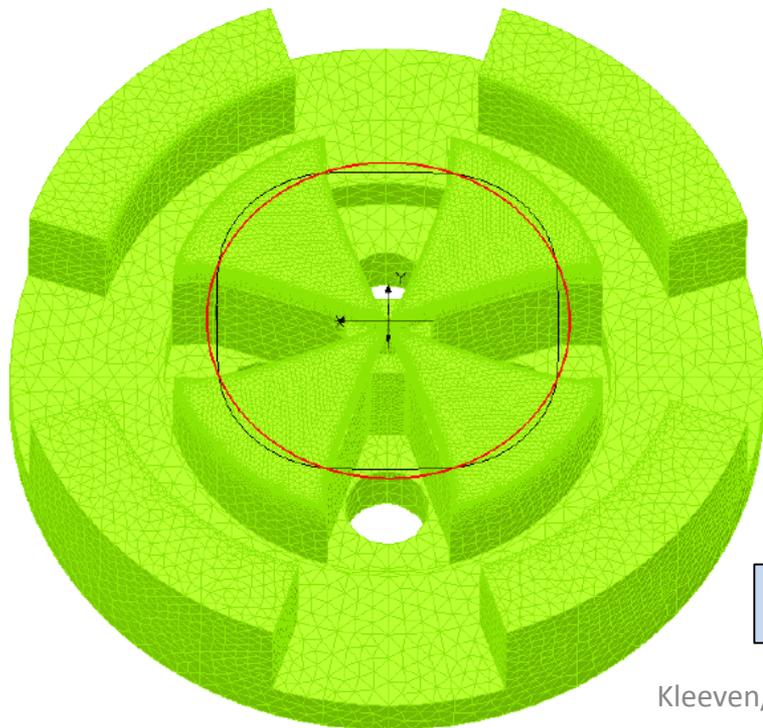
- $v_\theta B_r \Rightarrow$  obtained in the radially decreasing rotationally symmetric magnetic fields as in the classical cyclotron and the synchro-cyclotron
- $v_r B_\theta \Rightarrow$  requires an azimuthal modulation of the magnetic field  $\Rightarrow$  introduce sectors (hills) with high field and valleys with low field  $\Rightarrow$  azimuthally varying field cyclotron  $\Rightarrow$  the field variation creates the non-circular orbit

**Part II:**  
**A little bit about  
vertical focusing and  
isochronism**

# The Azimuthally Varying Field (AVF) cyclotron



Vertical focusing =>  
scaloping of the orbit



0.5 Tesla

$B_\theta$

$B_\theta$  10 mm above  
median plane

-0.5 Tesla

entrance

exit

R coord	650.0	650.0	650.0	650.0	650.0	650.0
Theta coord	0.0	18.0	36.0	54.0	72.0	90.0
Z coord	10.0	10.0	10.0	10.0	10.0	10.0

Component: BT, from buffer: Circle, Integral = -1.02727334719474E-03

$v_r$

$v_r/v_{mod}$  in the  
median plane

0°

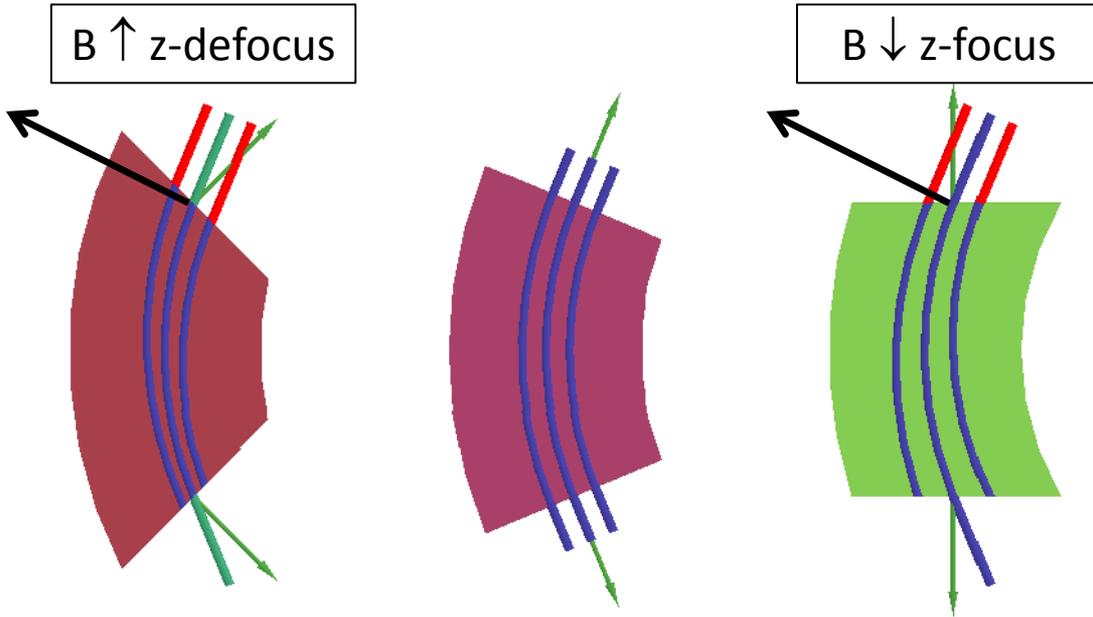
$\theta$

90°

Kleeven/Zaremba CAS-2015  
magnetic design and beam

ATAND(Y/X)

# Cyclotron sector focusing $\cong$ edge focusing

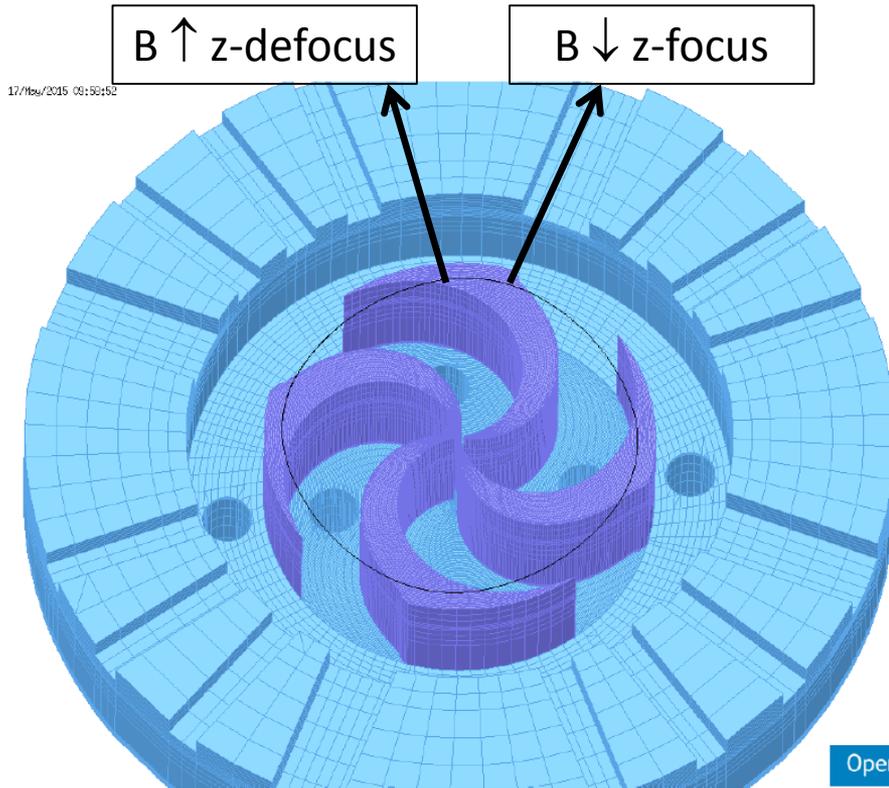


Look how the magnetic field changes when moving outward perpendicular to the orbit:

Increasing  $\Rightarrow$  vertically defocusing

Decreasing  $\Rightarrow$  vertically focusing

# More vertical focusing => pole spiraling



For straight sectors: equal vertical focusing at entrance and exit of sector

Spiraling of the pole changes the focusing strength at the entrance and exit of the sector:

Entrance: strong B-decrease => strong z-focusing

Exit: strong B-increase => strong z-defocus

## **ALTERNATING FOCUSING**

This may give a very large contribution

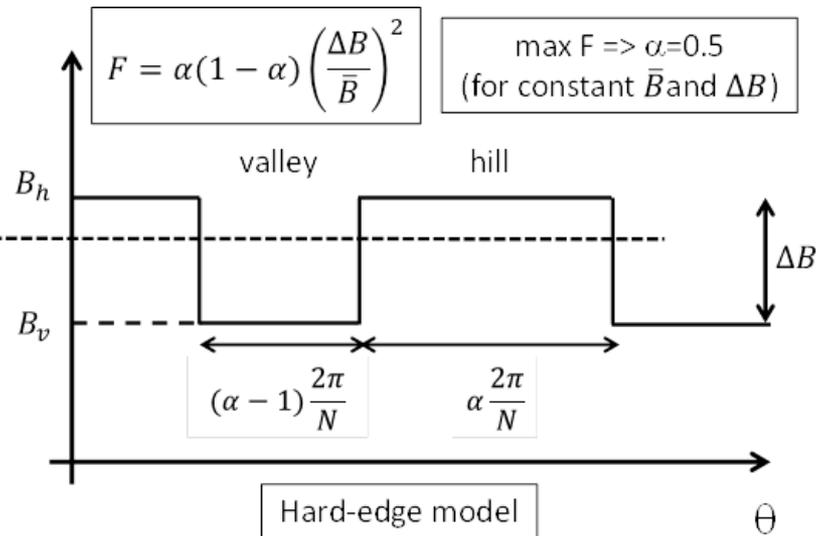
# Distributed focusing in a real cyclotron field

- In a real cyclotron geometry the focusing is distributed along the orbit:
- At pole entrance/exit => edge focusing
- Spiral sectors => alternating (strong) focusing
- In the middle of the hill: often a positive field gradient => vertically defocusing
- In the middle of a valley: often a negative field gradient => vertically focusing

# Flutter: a measure for the azimuthal field variation

Average of the field modulation

$$F(r) = \frac{\overline{B^2} - \bar{B}^2}{\bar{B}^2}$$



$N$  = number of sectors  
 $\alpha$  = 'filling factor'

Fourier harmonic composition  
 of the magnetic field

$$B(r, \theta) = \bar{B}(r) \left\{ 1 + \sum_{n=1}^{\infty} A_n(r) \cos n\theta + B_n(r) \sin n\theta \right\}$$

$$F = \sum \frac{A_n^2 + B_n^2}{2}$$

# Formulas for focusing in an AVF cyclotron

$$v_z^2 = k + \frac{N^2}{N^2 - 1} F(1 + 2 \tan^2 \xi)$$

$$v_r^2 = (1 - k) + \frac{3N^2}{(N^2 - 1)(N^2 - 4)} F(1 + \tan^2 \xi)$$

$k$  = field index =  $-\frac{r}{\bar{B}} \frac{d\bar{B}}{dr}$

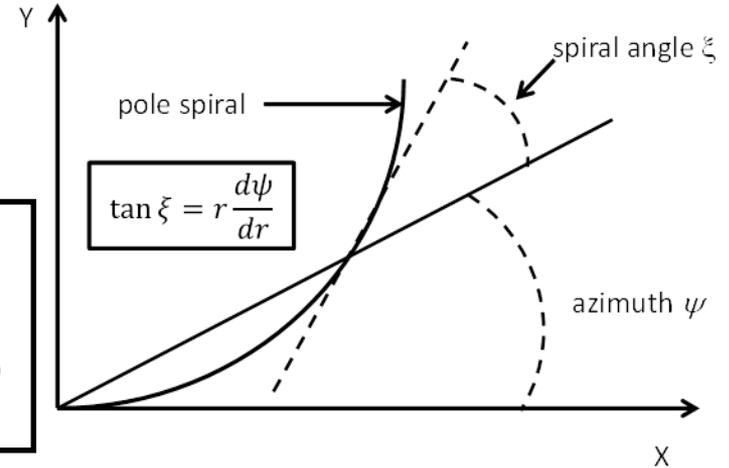
$F$  = flutter

$N$  = number of sectors

$\xi$  = spiral angle

This is an approximation: There is also some dependency on radial gradients of the flutter. See: Hagedoorn and Verster, NIM **18,19** (1962) 201-228

Definition of spiral angle



NOTE: for an isochronous cyclotron:

$$k = 1 - \gamma_{\text{rel}}^2$$

$$v_r \cong \gamma_{\text{rel}}$$



Isochronism => the revolution frequency of the particle is constant everywhere in the cyclotron independent of the energy of the particle

Isochronous cyclotrons have to be isochronized by correct shaping of the average magnetic field as a function of radius

# All cyclotron magnetic fields are mapped in the median plane

Precise mapping and iron pole shimming is needed in order to isochronize the magnetic field



It is not possible to obtain isochronism just from the design => required precision of  $\langle B \rangle \Rightarrow 10^{-4}$  to  $10^{-5}$



- Move Hall-probe or a search coil (S2C2) on a 2D polar grid to obtain a full field-map => automatized and computer controlled system

- Analyse the magnetic field on equilibrium orbits in order to evaluate isochronism
- Shim the hill sectors of the iron in order to improve the isochronism (reduce the RF phase slip)

# Essential information of a cyclotron field map

1. The level of isochronism => integrated RF phase slip
2. The transverse optical stability => tune functions
3. Crossing of dangerous resonances => operating diagram
4. Magnetic field errors
  - First and second harmonic errors => resonance drivers
  - Median plane errors => very difficult to measure
5. ...

# Analysis of a cyclotron field map

1. Static analysis => Acceleration is turned off
  - Computation of the closed orbits and their properties
2. Accelerated orbits => for special problems
  - Central region studies
  - Extraction studies
  - Study of resonance crossings
  - ...

# Closed orbit analysis in a cyclotron

- Closed orbits are obtained by solving the non-accelerated motion. Two types:
  - Equilibrium orbits: have the same N-fold symmetry as the cyclotron. They are obtained in the ideal magnetic field map where errors have been removed
  - Periodic orbits: have a periodicity of  $2\pi$  and are obtained in a real (measured) field map with errors
- Different dedicated programs are available such as CYCLOPS, EOMSU. At IBA we use a home-made program.
- They solve the equations of motion and determine the proper initial conditions such that the orbit closes in itself.

# Information obtained from a closed-orbit analysis

- A family of closed orbits is computed for a full range of energies, covering the full region of acceleration in the cyclotron
- For each orbit the horizontal and vertical tune-functions ( $\nu_r$  and  $\nu_z$ ) and the corresponding resonance diagram of  $\nu_z$  versus  $\nu_r$
- The particle revolution frequency for each energy: from this the isochronism of the field can be evaluated
- The optical functions (Twiss parameters) on each orbit can also be obtained. This may be useful for study of beam extraction.

# Isochronism: integrated RF phase slip

- Closed orbit code gives the RF phase slip per turn
- The integrated (accumulated) phase slip will depend on the number of turns and thus on the energy gain per turn: larger  $V_{dee} \Rightarrow$  less turns  $\Rightarrow$  less slip
- However, energy gain per turn depends on the RF phase slip already accumulated.
- A self-consistent formula is needed:

$$\Phi(E) = \sin^{-1} \left( \frac{2\pi h}{f_{RF}} \int_0^E \frac{\Delta f(E')}{\Delta E_0(E')} dE' \right)$$

$\Phi$  = integrated RF phase slip

$h$  = harmonic mode

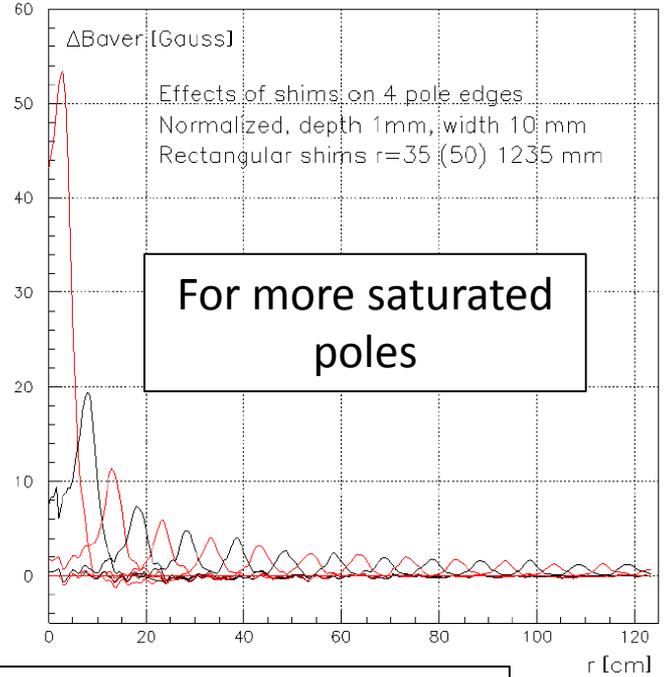
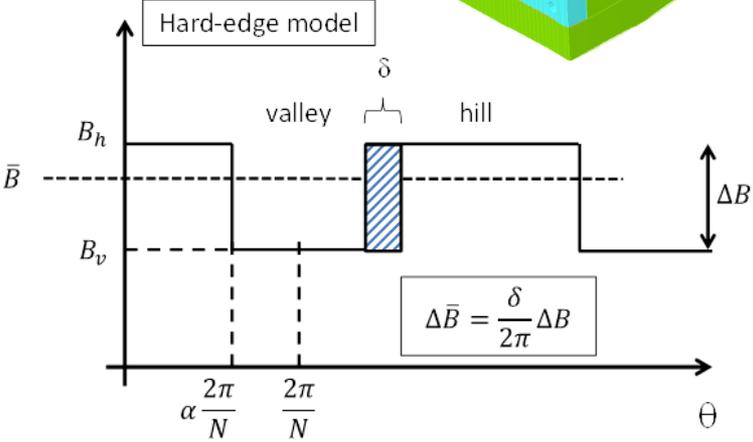
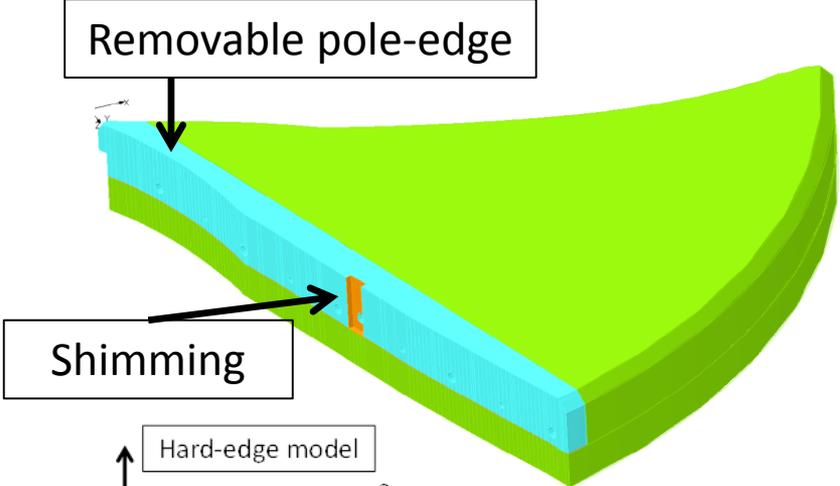
$f_{RF}$  = RF frequency

$\Delta f$  = closed orbit frequency error

$\Delta E_0$  = nominal energy gain per turn

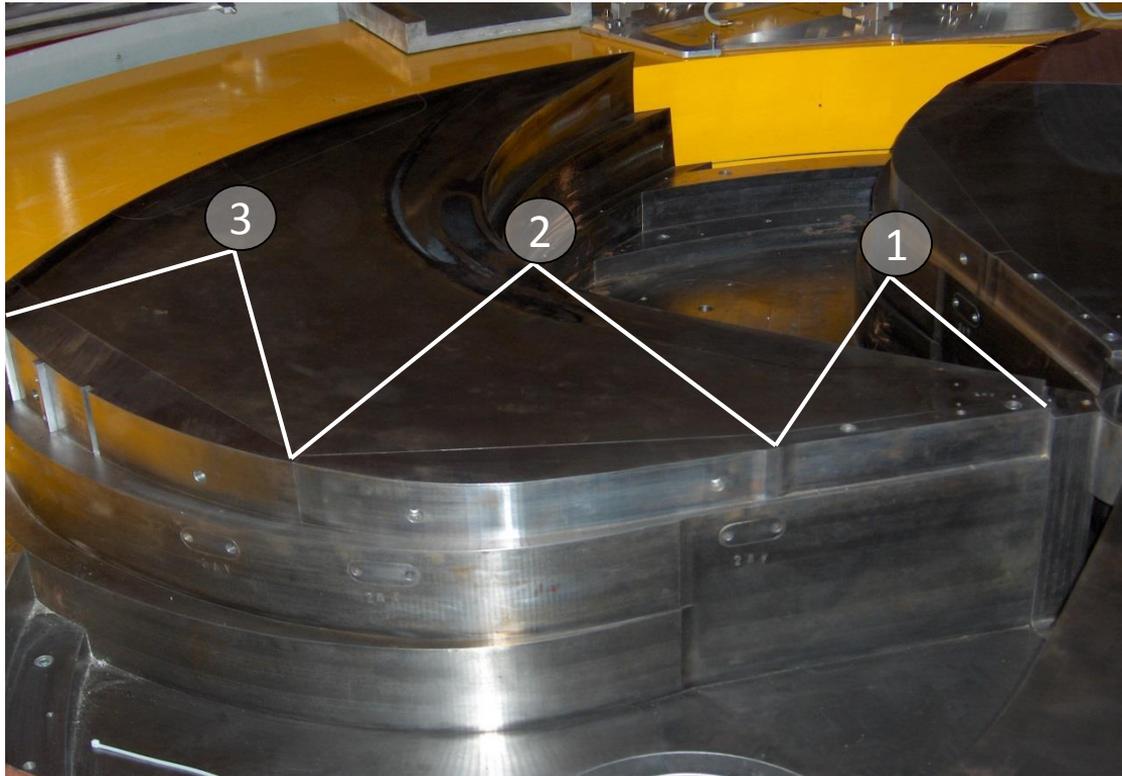
# Isochronization by pole shimming

Calculate shim effect with OPERA3D  
 shimming matrix:  $\text{shim}(r_1) \Rightarrow \Delta B(r_2)$



Simple => hard edge model  
 More advanced => shimming matrix

# Removable pole edges in the C235 cyclotron



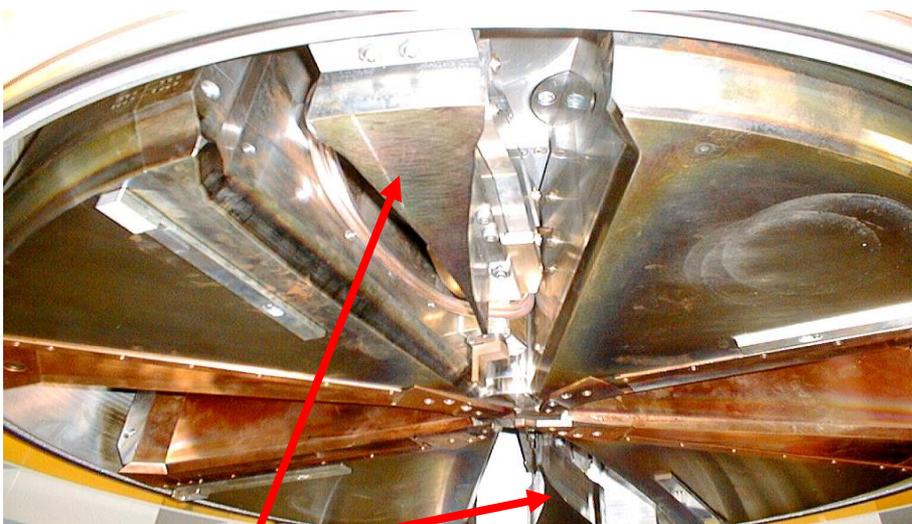
In this isochronous PT cyclotron, there are 3 removable pole edges (per pole) for shimming the average field as needed for isochronism



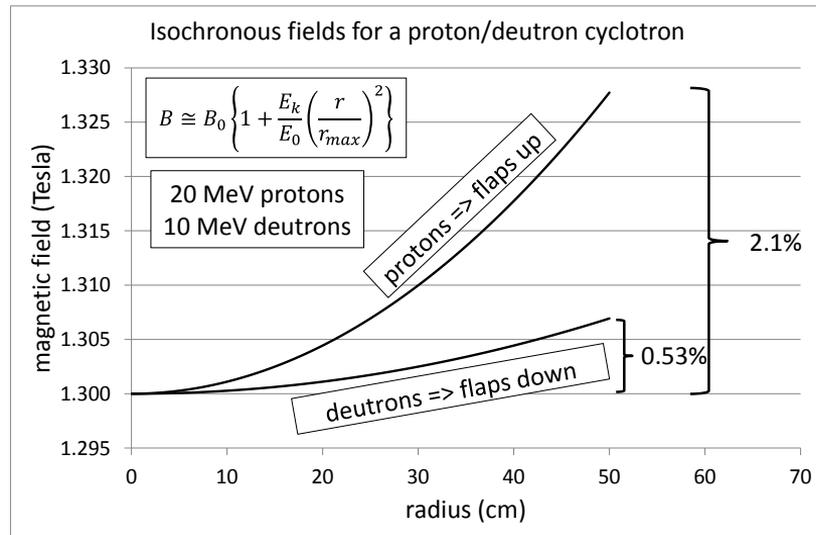
# Isochronization for two different particles

Example: a proton/deuteron isotope production cyclotron

By placing iron shims (flaps) in the valleys which can be moved vertically close to the median plane (protons) or further away from the median plane (deuterons)

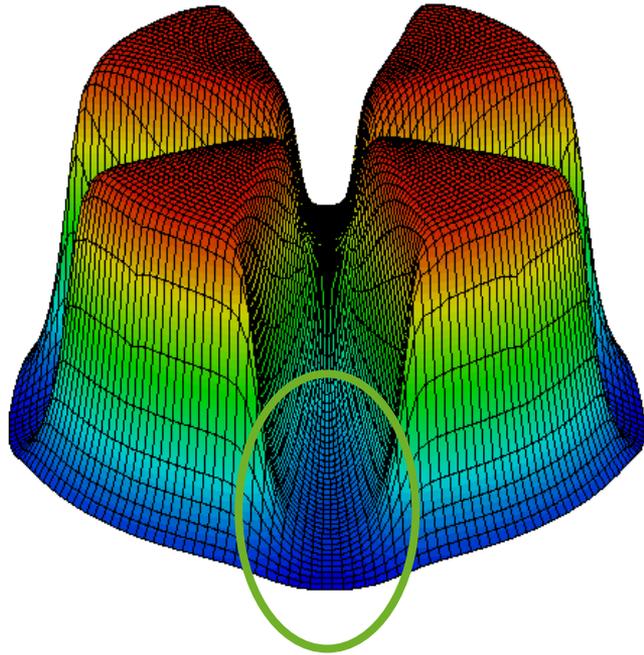


flaps

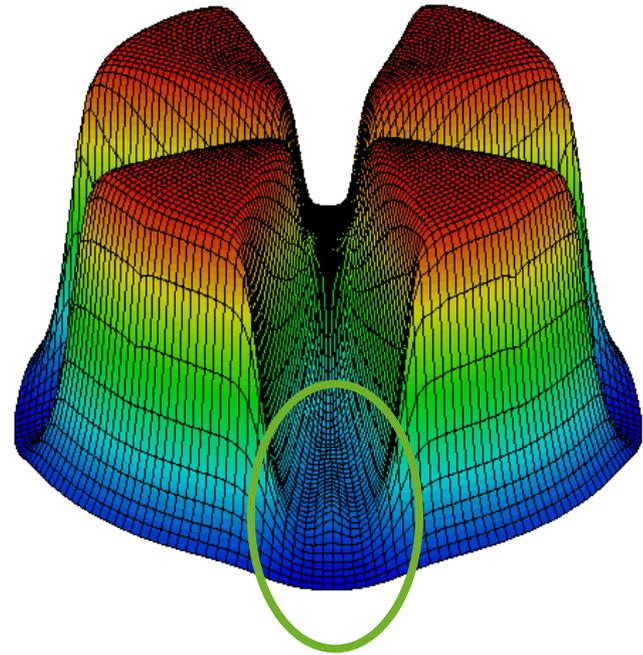


# Movable inserts (flaps) effect

D measured magnetic field

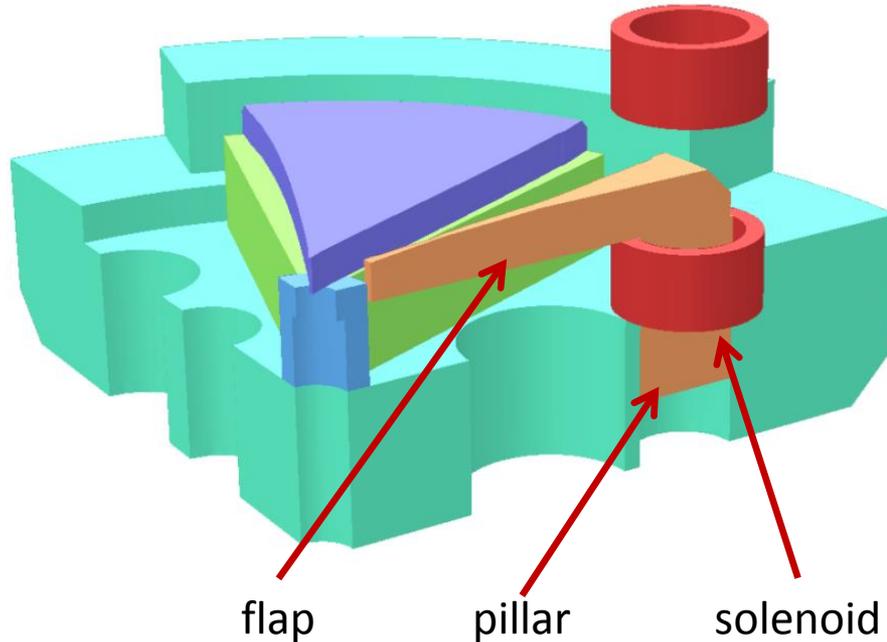


H measured magnetic field



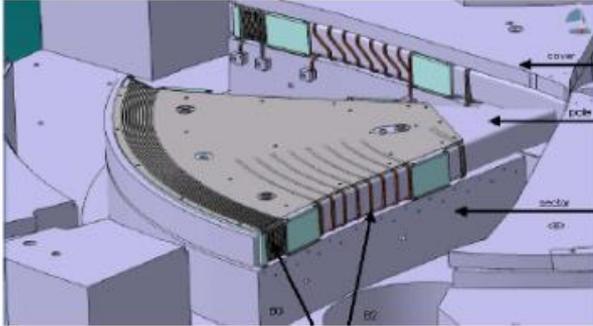
# Isochronization by flaps that do not move

Based on a study for a 70 MeV cyclotron for INFN

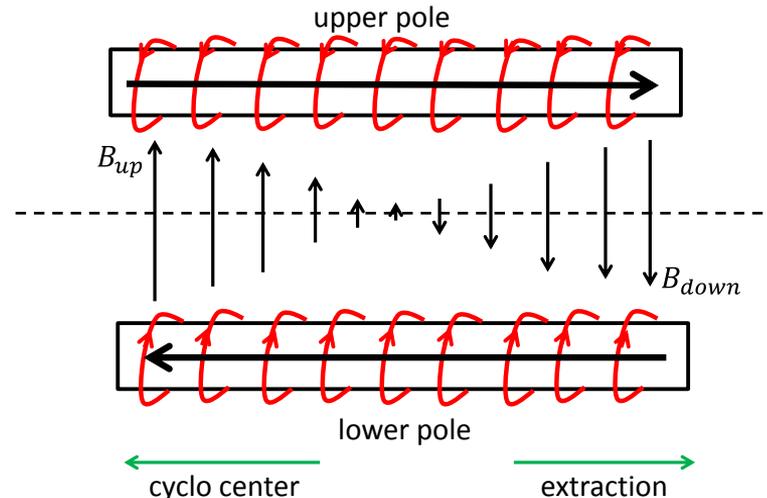


- For higher energies not enough correction can be generated with the flaps
- Flaps are magnetically connected to the yoke by the iron pillar, 'pumping' flux into the flaps
- With the solenoid around the pillar, the amount of flux can be adjusted
- A lot of flux can be 'pumped-up'. Therefore, this method can be applied for higher energy cyclotrons
- For example: 70 MeV protons vs 35 MeV deuterons

# Isochronization of multi-particle C70 cyclotron

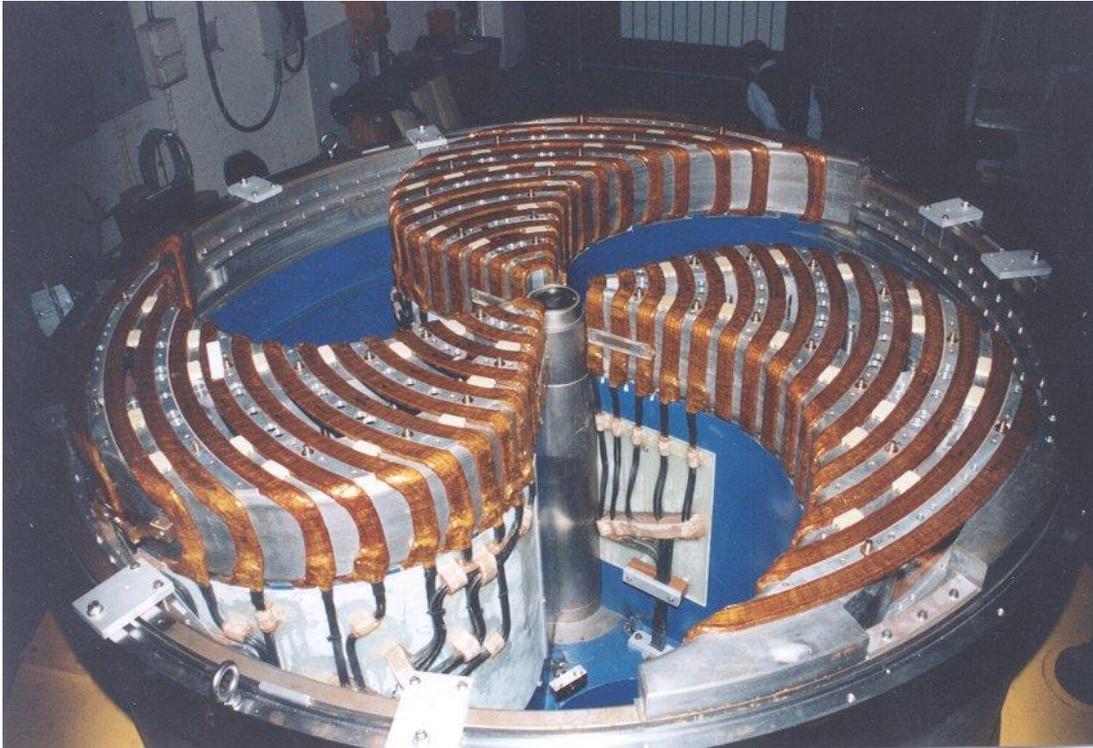


Coils around the pole produce a 'quadrupole-like' field distribution. Field is pushed from low radius to high radius or vice-versa, by changing the sign of the coil current



Panofsky  
quadrupole

# The similar method is applied in the AGOR superconducting cyclotron



Many independent coils  
allow to create a wide  
range of different  
isochronous field maps

# Circular trim coils on the poles

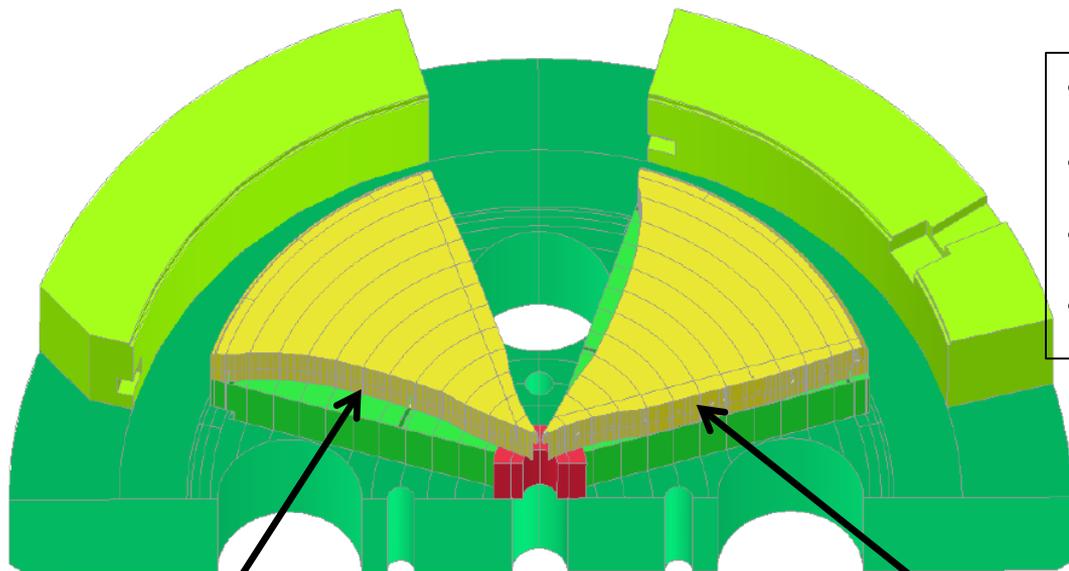
Trim coils of the Berkeley 88-inch cyclotron on a test-bench



Also here several independent but circular coils

For the multi-particle variable energy research-cyclotrons this is probably the most common method

# Example C70: industrial cyclotron for medical isotope production



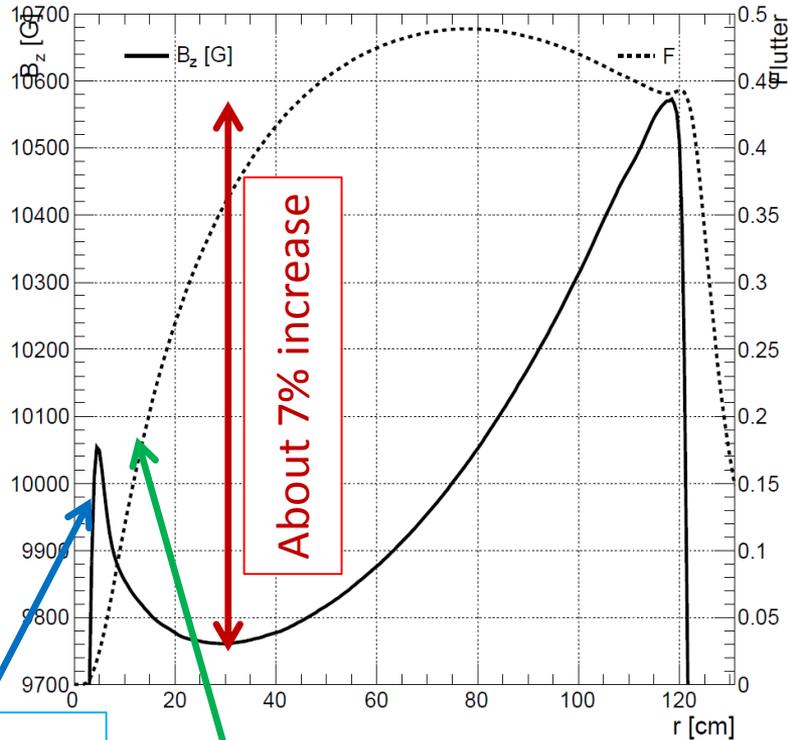
- Currently under commissioning
- 70 MeV  $H^-$ , Intensity 750  $\mu A$
- N=4, axial injection
- Stripping extraction, dual beam

Slight spiraling of poles => fine-tuning of the  $v_z$  curve

Little bit of pole spiral for fine-tuning of  $v_z$

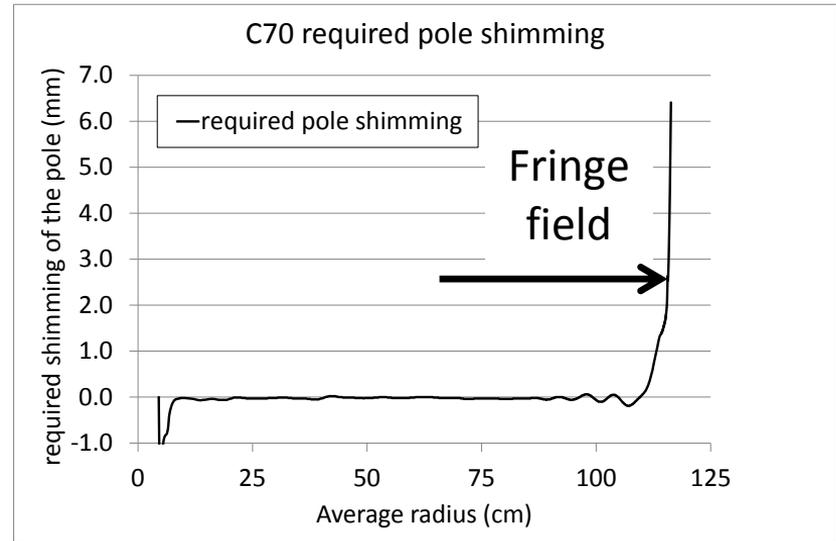
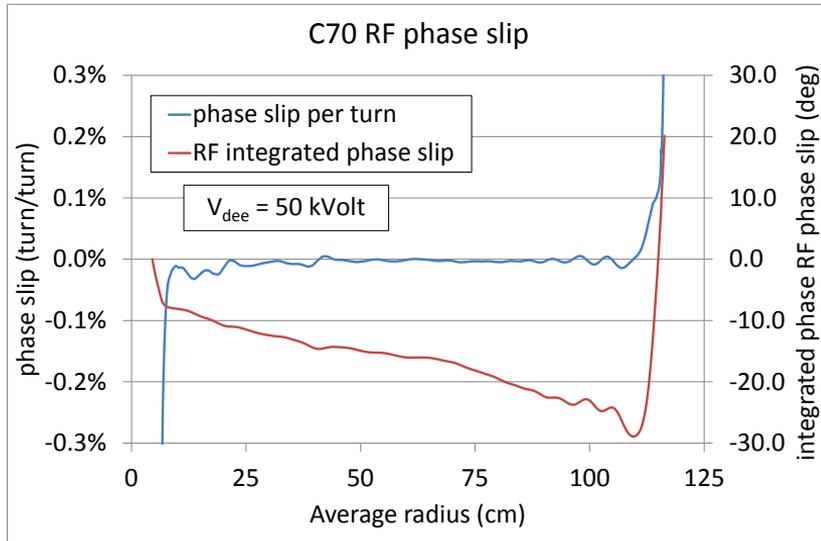
Isochronization by pole shimming

# Example C70: average field and flutter



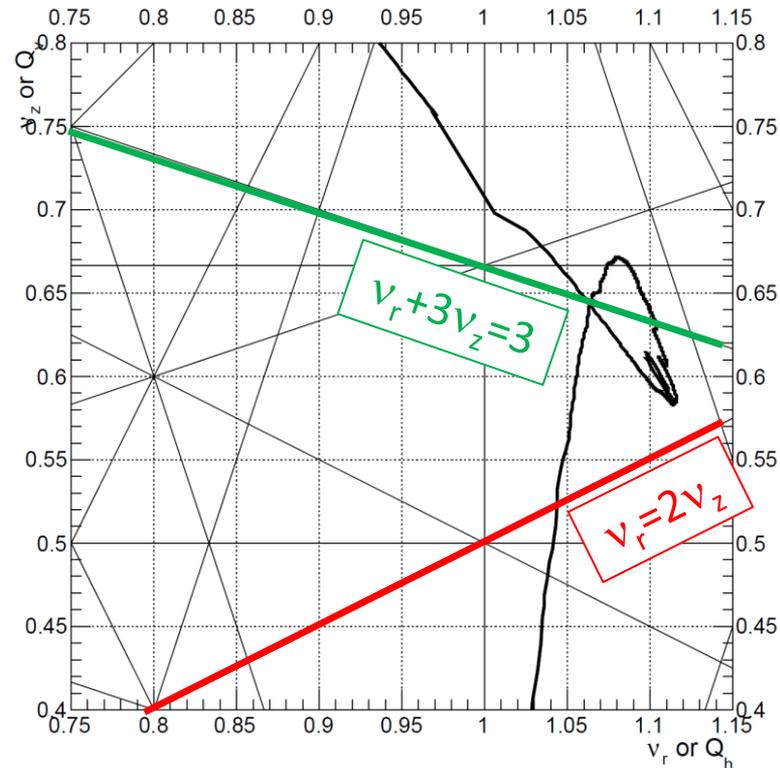
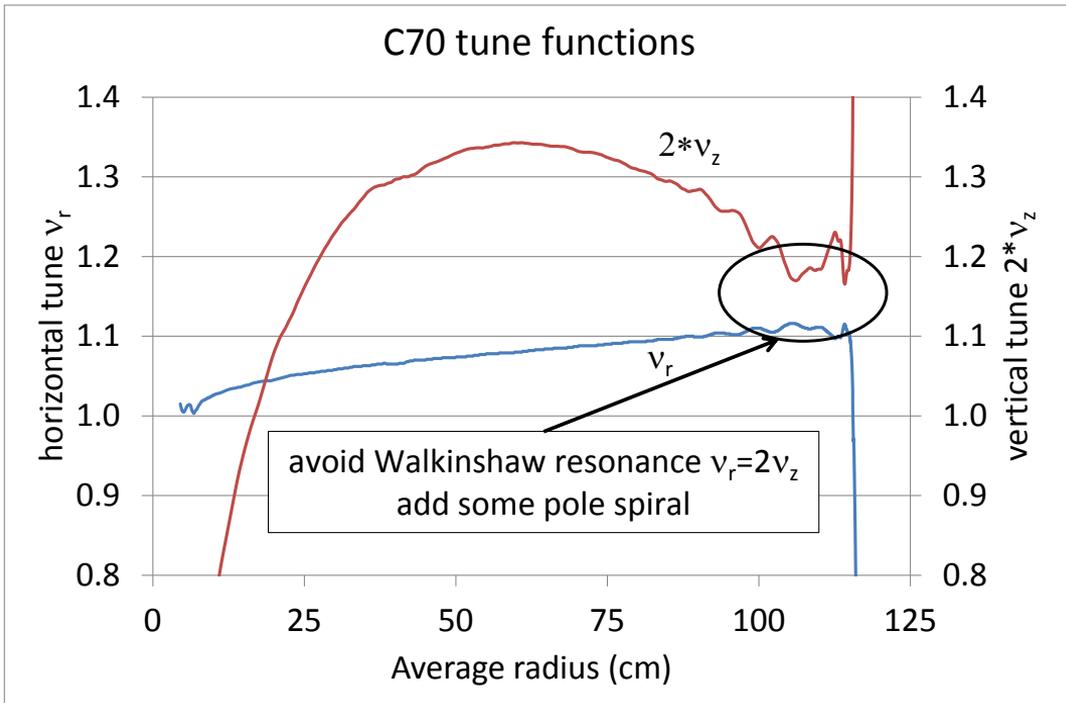
- Average field increases with roughly 1% per 10 MeV
- No flutter in the cyclotron center; what about focusing in the center?
  - Local field bump provides some weak focusing
  - RF electric field will provide some vertical focusing
- Sharp field drop in the center due to the axial hole for injection

# Example C70: isochronism



Closed orbits are found up to 71.4 MeV. The most outer orbits enter into the radial fringe area of the pole. These orbits can no longer be corrected. The maximum phase slip of  $30^\circ$  is considered acceptable. The actual shift will be smaller because particles are extracted up to 70 MeV

# Example C70: tune functions and operating diagram



$v_r=2v_z \Rightarrow$  a structural resonance; may be dangerous, better to avoid

$v_r+3v_z=3 \Rightarrow$  non-structural, driven by harmonic 3; considered as non-dangerous

# Harmonic field errors in the map (1)

- A first harmonic field error will de-center the closed orbit. This effect becomes big when  $v_r \cong 1$ . This happens in the cyclotron center and in the radial fringe region of the pole. An off-centered orbit may become more sensitive to other resonances. Too high harmonic errors must be avoided.
- A localised first harmonic field bump may be used to create a coherent beam oscillation giving extraction of the beam from the cyclotron => **precessional extraction**
- The gradient of the first harmonic can drive the  $2v_z=1$  resonance. In the stop band of this resonance, the motion becomes vertically unstable. This may lead to amplitude growth and emittance growth. The stop-band of this resonance is given by:

$$4v_z \left| v_z - \frac{1}{2} \right| < \sqrt{r \frac{dC_1}{dr}}$$

$$C_n = \sqrt{A_n^2 + B_n^2}$$

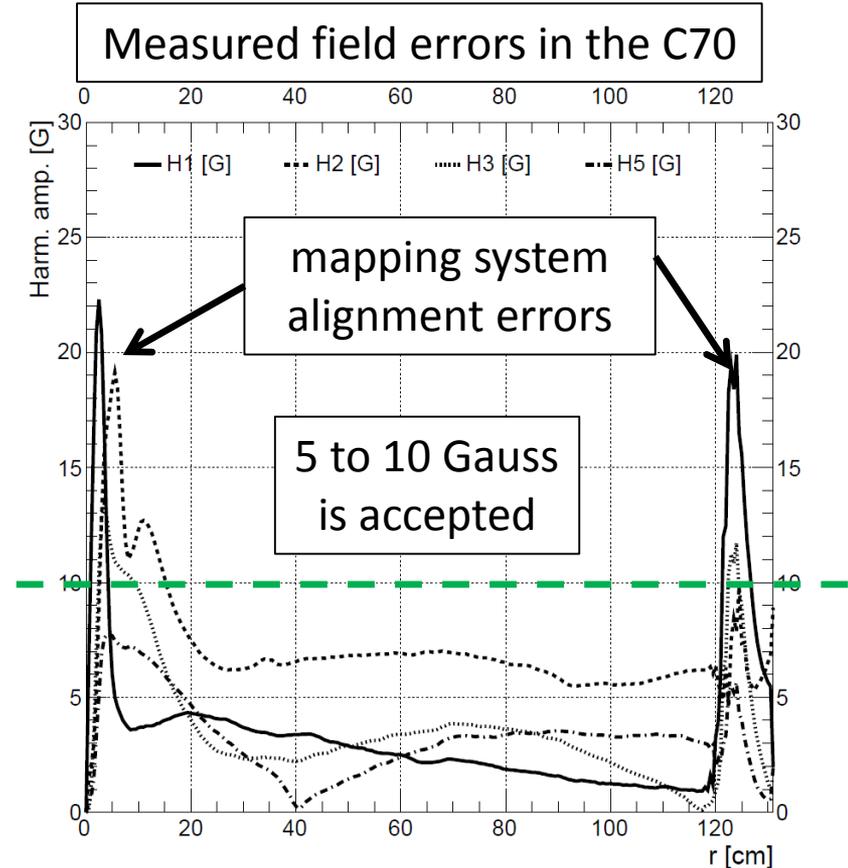
# Harmonic field errors in the map (2)

- The gradient of the second harmonic can drive the  $2\nu_r=2$  resonance. In the stop band of this resonance, the horizontal motion becomes unstable. This problem may occur when movable 2<sup>nd</sup> harmonic iron shims are used to isochronize a dual-particle (proton/deuteron) cyclotron. The stop-band of this resonance is given by:

$$4|\nu_r - 1| < |2C_2 + r \frac{dC_2}{dr}|$$

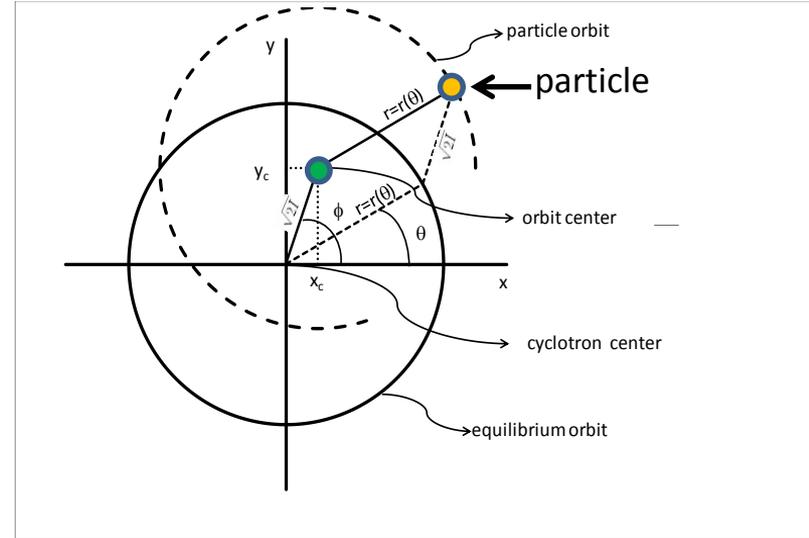
See also: Kleeven, Hagedoorn et.al., Cyclotron Conference Vancouver (1992) 380-383

- The same  $2\nu_r=2$  resonance is used in the synchro-cyclotron for extraction of the beam



# The notion of orbit centre in a cyclotron

- Betatron oscillations in a cyclotron can be represented by amplitude and phase, but also by the coordinates of the orbit centre.
- The latter can be more convenient because the orbit centre oscillates slowly (frequency  $\nu_r - 1$ ) as compared to the betatron oscillation itself ( $\nu_r$ )
- In the orbit centre representation, the equations of motion can be simplified using approximations that make use of the slowly varying character of this motion and the integration can be done much faster
- This may be especially useful in a synchro-cyclotron where the particle makes many turns (50000) and full orbit integration from source to extraction is almost impossible



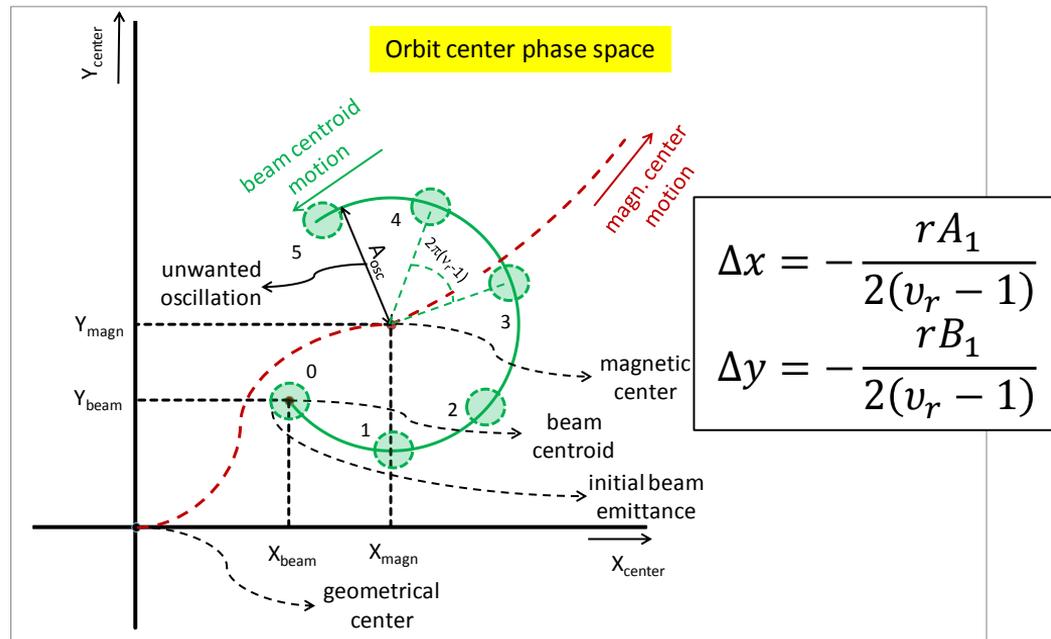
Radial betatron oscillation around the equilibrium orbit in terms of the coordinates of the orbit center. The real orbit can be reconstructed from the orbit center coordinates and the equilibrium orbit radius  $r(\theta)$ . A Hamiltonian description is used for the dynamics of the orbit center. In this illustration an equilibrium orbit with circular shape (synchro-cyclotron) is shown. For AVF cyclotrons this will be a scalloped orbit.

# The notion of magnetic centre in a cyclotron

Particles execute a betatron-oscillation around the magnetic center. A first harmonic field error displaces the magnetic center of the cyclotron relative to the geometrical center. When there is acceleration, the magnetic center itself is also moving and the total motion is a superposition of the two separate motions. Beam quality degrades when the beam centroid is not following the magnetic center. This may occur in two ways:

- i. a beam centering error at injection
- ii. accelerating through a region where the gradient of the 1st harmonic is large (non-adiabatic effect  $\neq$  synchro-cyclo).

$A_{osc}$  is the amplitude of the betatron oscillation and is a good measure for the harmful effect of the centering. Numbers indicate subsequent turns.



$$\Delta x = -\frac{rA_1}{2(v_r - 1)}$$

$$\Delta y = -\frac{rB_1}{2(v_r - 1)}$$

Hagedoorn and Verster (NIM **18,19** (1962) 201-228) have derived the Hamiltonian for the orbit center motion. The theory includes the linear motion, nonlinear motion (separatrix) and the influence of field errors

# Part III:

## A little bit about injection

# Injection into a cyclotron

Transfer of the beam from the ion source onto the equilibrium orbit in the center of the cyclotron, two approaches:

## 1. Internal Ion Source:

- Ion source placed in the center of the cyclotron
- Source is ‘integrated part’ of the accelerating structure
- Is used in proton therapy cyclotrons as well as isotope production cyclotrons

## 2. External Ion Source:

- Ion source placed outside of the machine
- An injection line with magnets and electrostatic inflector is needed
- Is used in high intensity isotope production cyclotrons (and in IBA C400)

# Injection: some important design goals

1. Centering of the beam with respect to the cyclotron magnetic center. Equivalent to placing of the beam on the correct equilibrium orbit given by the injection energy
2. Vertical centering with respect to the median plane
3. Longitudinal matching => bunching => compressing the DC beam from the ion source into shorter packages at the frequency of the RF
4. Matching of the beam phase space into the cyclotron acceptance or eigenellipse (if possible)
5. Preserve as well as possible the beam quality with minimum losses between the ion source and the cyclotron center

# There are many constraints in the design of a new central region

1. Magnetic structure
  - Magnetic field value and shape in the center
  - Geometrical space available for the central region, inflector, ion source etc
2. Accelerating structure
  - The number of accelerating dees (one, two, three or four)
  - The dee-voltage
  - The RF harmonic mode
3. Injected particle
  - Charge and mass of the particle(s)
  - Number of ion sources to be placed (one or two)
  - Injection energy
4. ....

# Injection: internal ion source

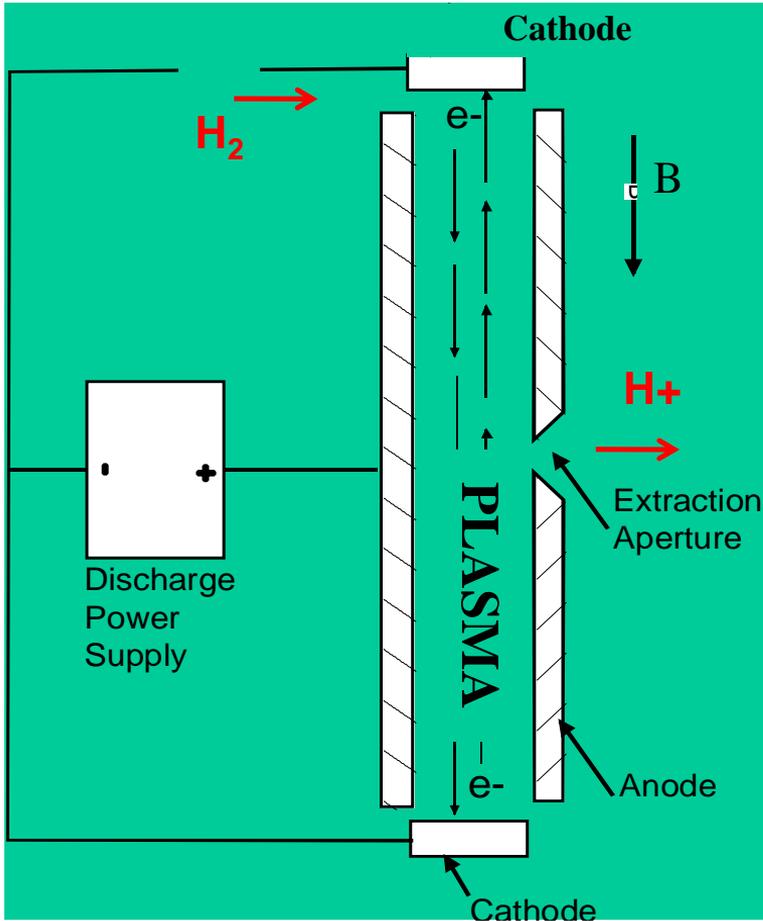
## Some advantages

- Simple and cost-effective: simple ion source; no injection line needed
- Compact:
  - two ion sources can be placed simultaneously
  - Can be used in the high-field (6 to 9 Tesla) superconducting cyclotrons

## Some disadvantages/limitation

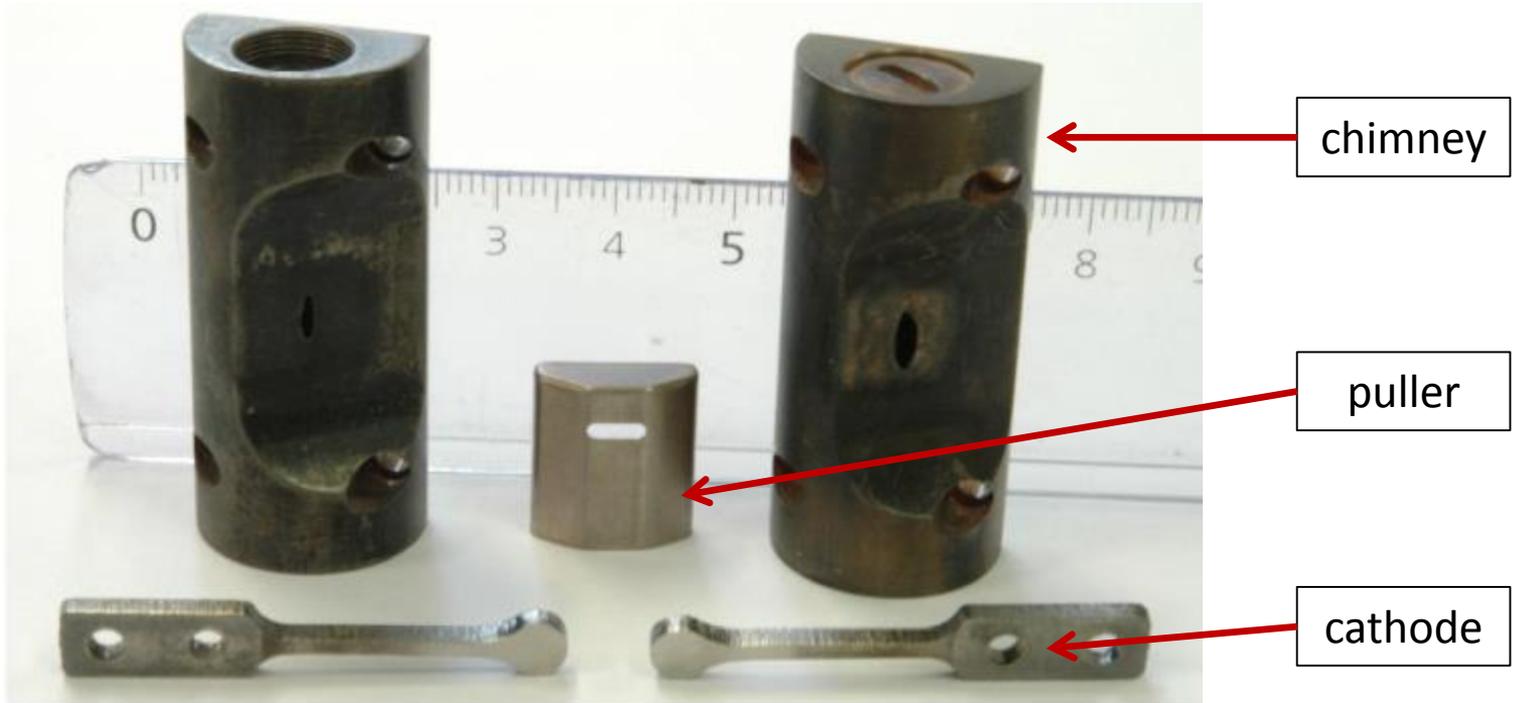
- Low to moderate beam intensities
- Simple ion species ( $H^+$ ,  $H^-$ , deuterons, He-3, He-4)
- Beam matching/bunching/manipulation not possible
- Gas-leak directly into the cyclotron (bad for negative ions)
- Machine has to be stopped for ion source maintenance

# Injection: cold cathode PIG ion source



- Electron emission due to electrical potential on the cathodes
- Electron confinement due to the magnetic field along the anode axis
- Electrons produced by thermionic emission and ionic bombardment
  - Start-up: 3 kV to strike an arc
  - At the operating point : 100 V
- cathodes heated by the plasma (100 V is enough to pull an outer  $e^-$  off the gas atoms)
- Hot cathode PIG => heated with filament

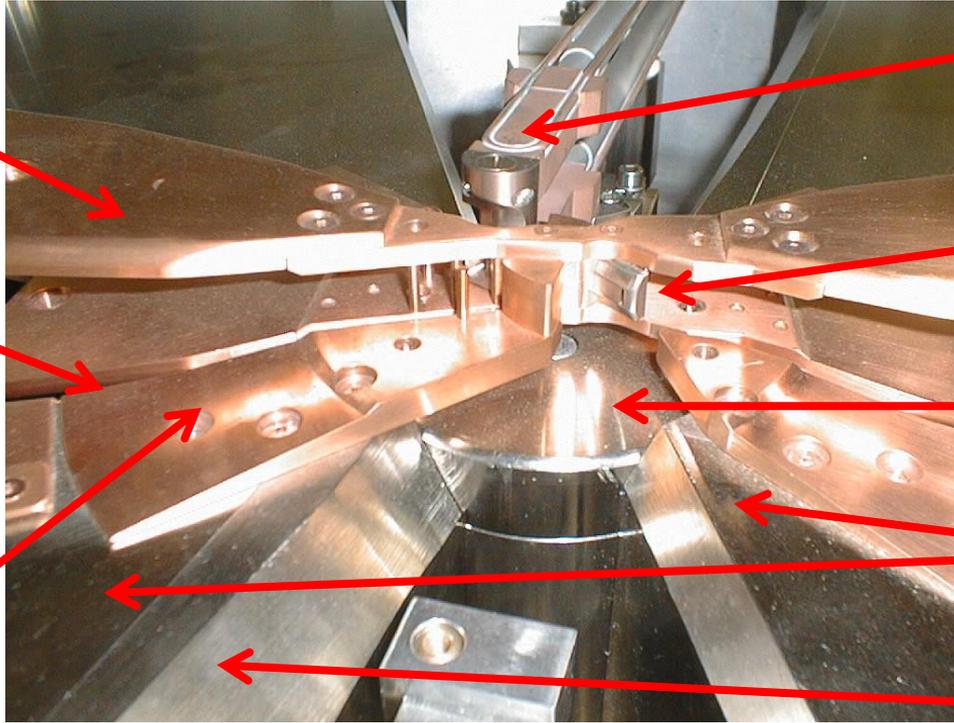
# Chimney, cathodes and puller



Chimney: copper-tungsten  $\Rightarrow$  good heat properties; machinable

Cathodes: tantalum  $\Rightarrow$  high electron emission; shaped to reduce heat conduction

# Example: central region of a compact cyclotron



2 Dees at  $V_{dee}$

4 accelerating gaps

dummy dees at ground

Small gap ( $\cong 1.5$  mm) between chimney and puller

2 ion sources ( $H^-$  and  $D^-$ )

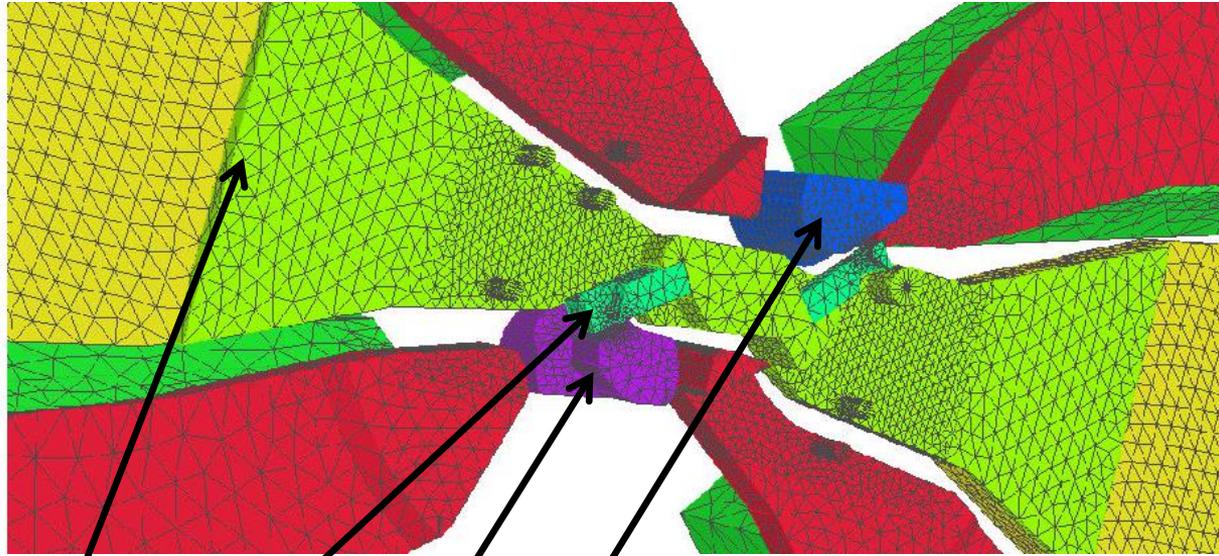
Puller at  $V_{dee}$

Central plug to adjust field in the center

4 poles

4 removable pole edges for shimming of isochronism

# OPERA3D finite element model of a central region



Dee

puller

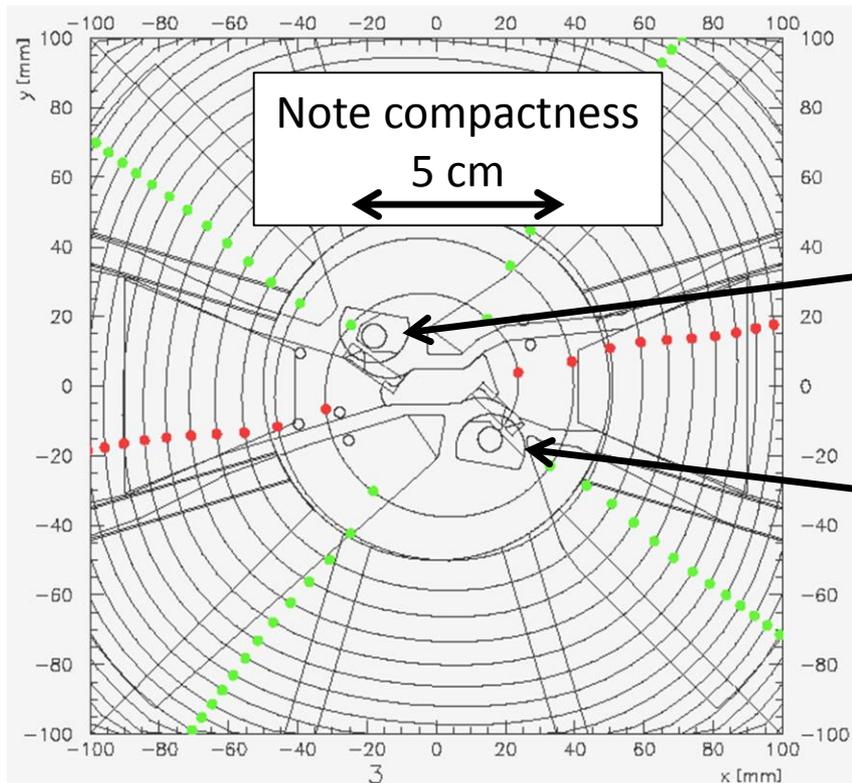
H<sup>-</sup> source

D<sup>-</sup> source

- Goal: compute a 3D electric potential map that serves as input for an orbit tracking code.
- Electrostatic  $\Rightarrow \lambda_{RF} \gg$  structure size
- Optimize beam centering, focusing, transmission etc.

- Fine meshing where needed  $\Rightarrow$  source puller gap
- Modeling of complete accelerating structure
- Orbit tracking from source to extraction
- Parametrize for easy modification and optimization

# Orbit tracking (C18/9 isotope production cyclotron)



E-fields => from Opera  
B-fields => measured or Opera

D<sup>-</sup> source;  
h=4

H<sup>-</sup> source  
h=2

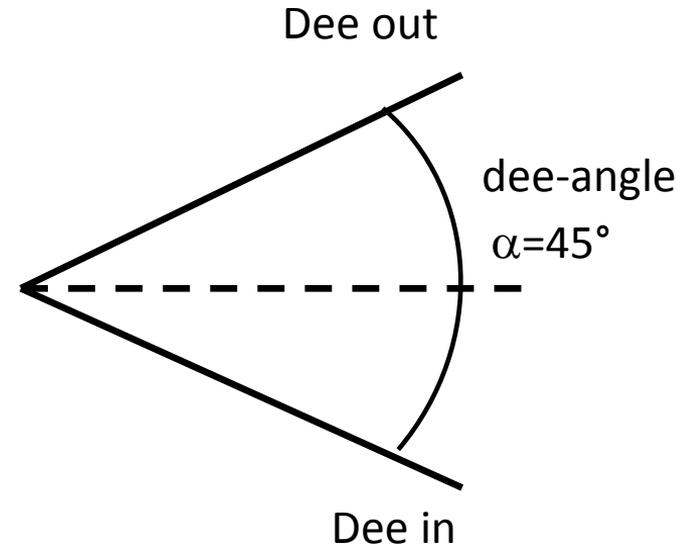
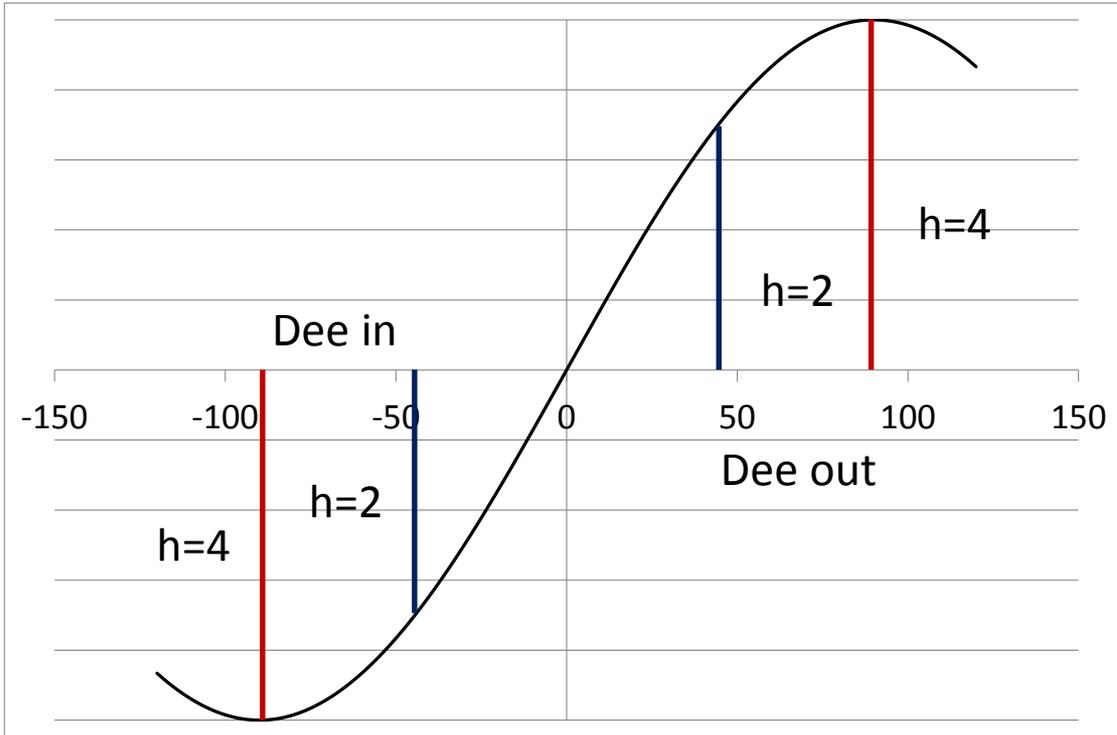
D<sup>-</sup> source is placed further out because of larger orbit

Cut D<sup>-</sup> chimney for H<sup>-</sup> passage

**Red dots:** position of particle when  $V_{dee} = 0$

**Green dots:** position of particle when  $V_{dee} = V_{max}$

# Energy gain per turn



$$\Delta E_k = qV_{dee}N \sin\left(\frac{h\alpha}{2}\right) \cos \Phi_{RF}$$

$h=2 \Rightarrow 71\%$

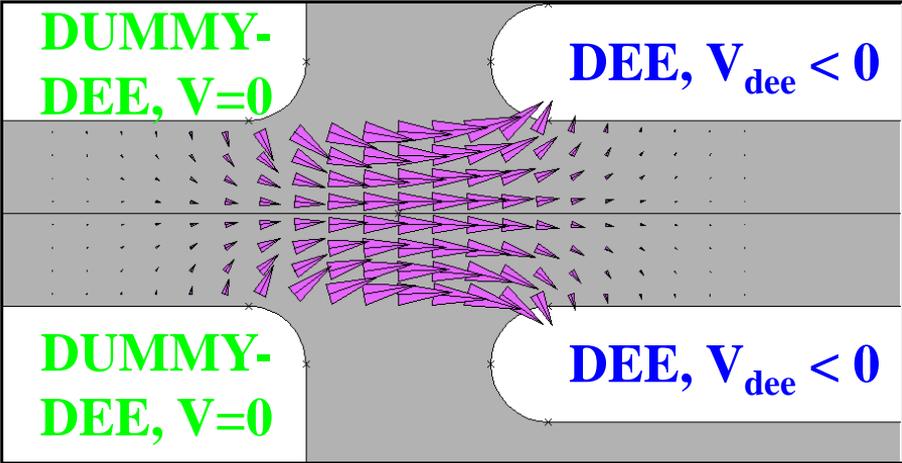
$h=4 \Rightarrow 100\%$

# Vertical focusing in the center

- Azimuthal Field Variation (AVF) goes to zero in the cyclotron center  $\Rightarrow$  magnetic vertical focusing disappears
- Two remedies
  - Add a magnetic field bump in the center  $\Rightarrow$  negative field gradient creates vertical focusing: field bump of a few hundred Gauss  $\Rightarrow$  central plug
  - The first few accelerating gaps provide electrical focusing  $\Rightarrow$  proper positioning of accelerating gaps during the design to get some phase focusing

# Vertical Electrical Focusing in accelerating gap: two contributions

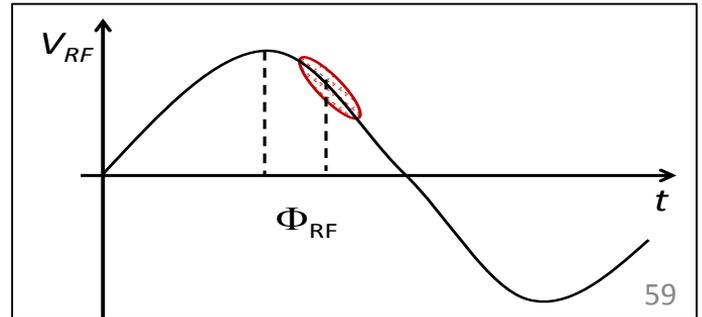
Vertical cross section

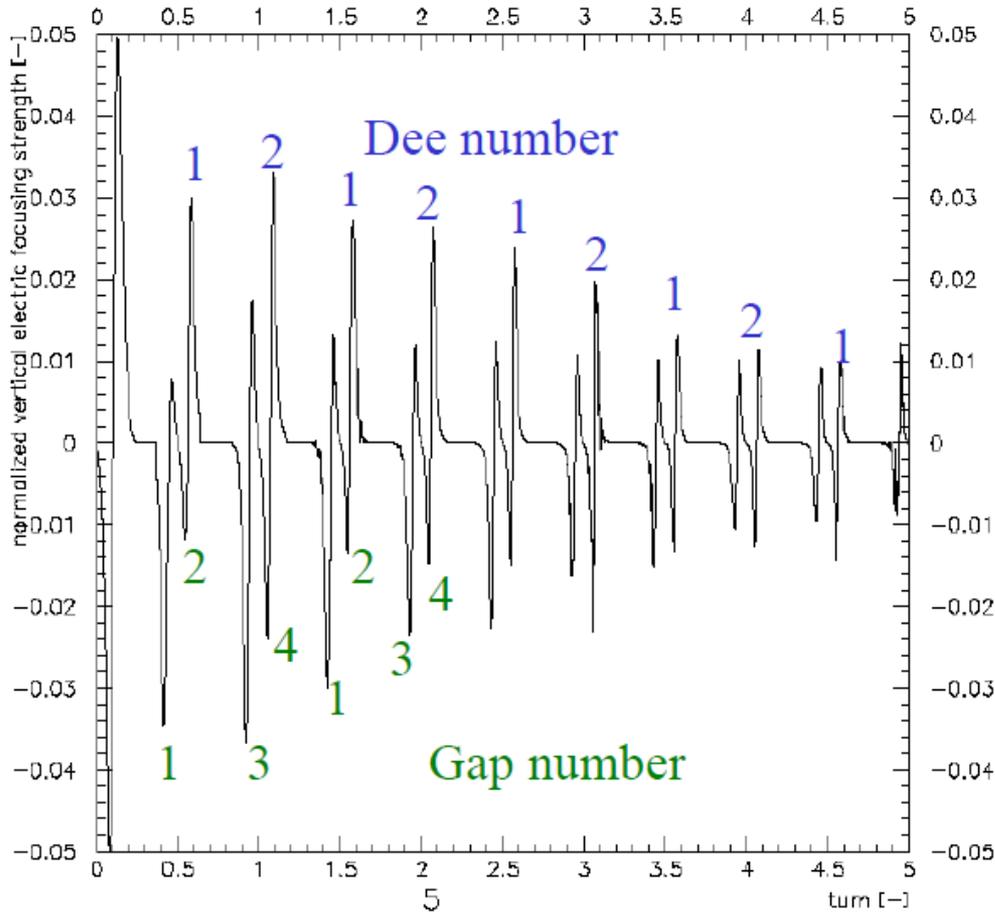


1<sup>st</sup> half => focusing      2<sup>nd</sup> half => defocusing

Falling slope of RF wave => net focusing (phase focusing)

- i. Due to the shape of electric field lines in the gap: first half is focusing and second half is defocusing => total effect is focusing => comparable to **Einzel lens**
- ii. Due to RF effect: If E-field is decreasing in time at moment of acceleration => falling slope of RF sine wave => second defocusing half is less important => net focusing (phase focusing)





Kleeven/Zaremba CAS-2015: Cyclotrons - magnetic design and beam dynamics

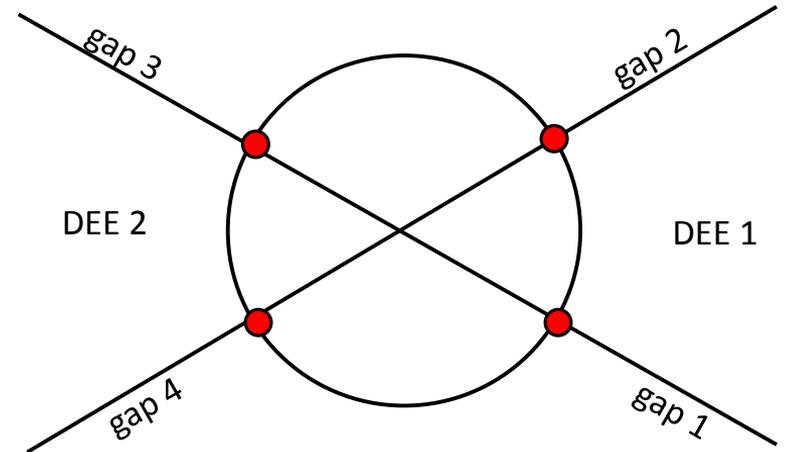
# Vertical electrical focusing forces

Particle tracking (5 turns)

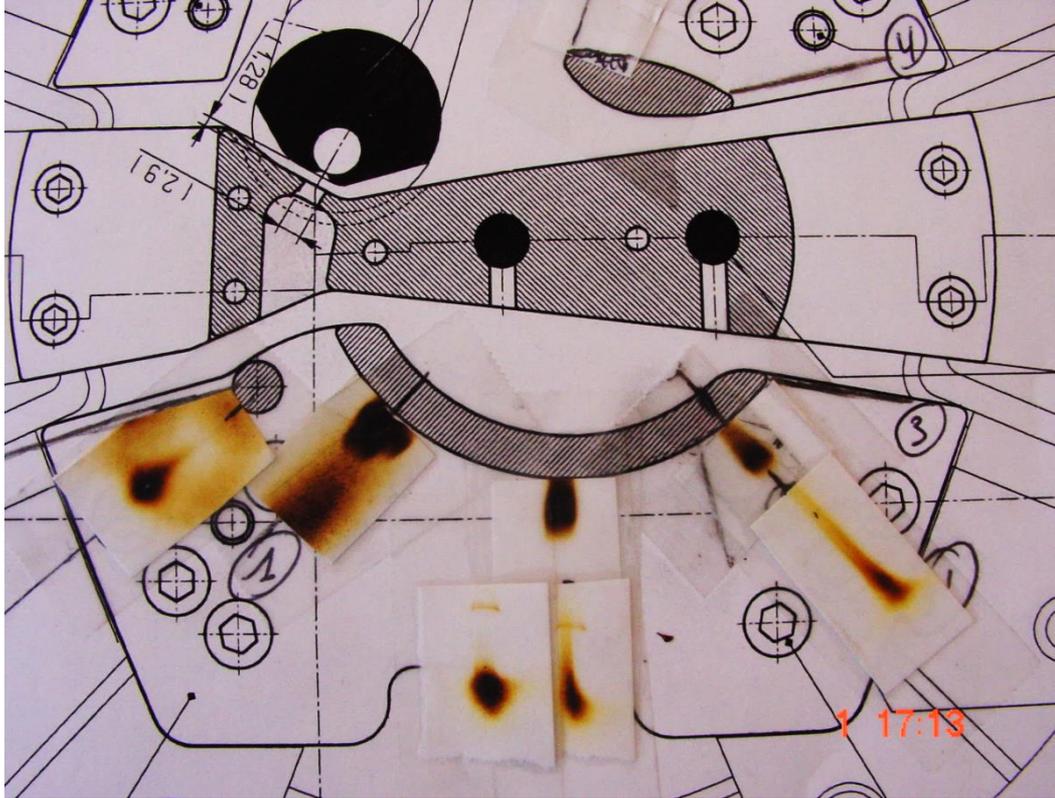
2-dee system (4 gaps)

Minus sign  $\Rightarrow$  focusing

**Focusing quickly weakens after a few turns**



# Finding the beam in the cyclotron center



Burning paper with the beam in order to find its position in the cyclotron center.

Gives also a rough idea of beam height, beam width and turn-separation

# Axial Injection

Axial injection  $\Rightarrow$  most relevant for compact cyclotrons

- Along the vertical symmetry axis of the cyclotron
- In the center, the beam is bent by  $90^\circ$  into the median plane
- For this an electrostatic inflector device is used

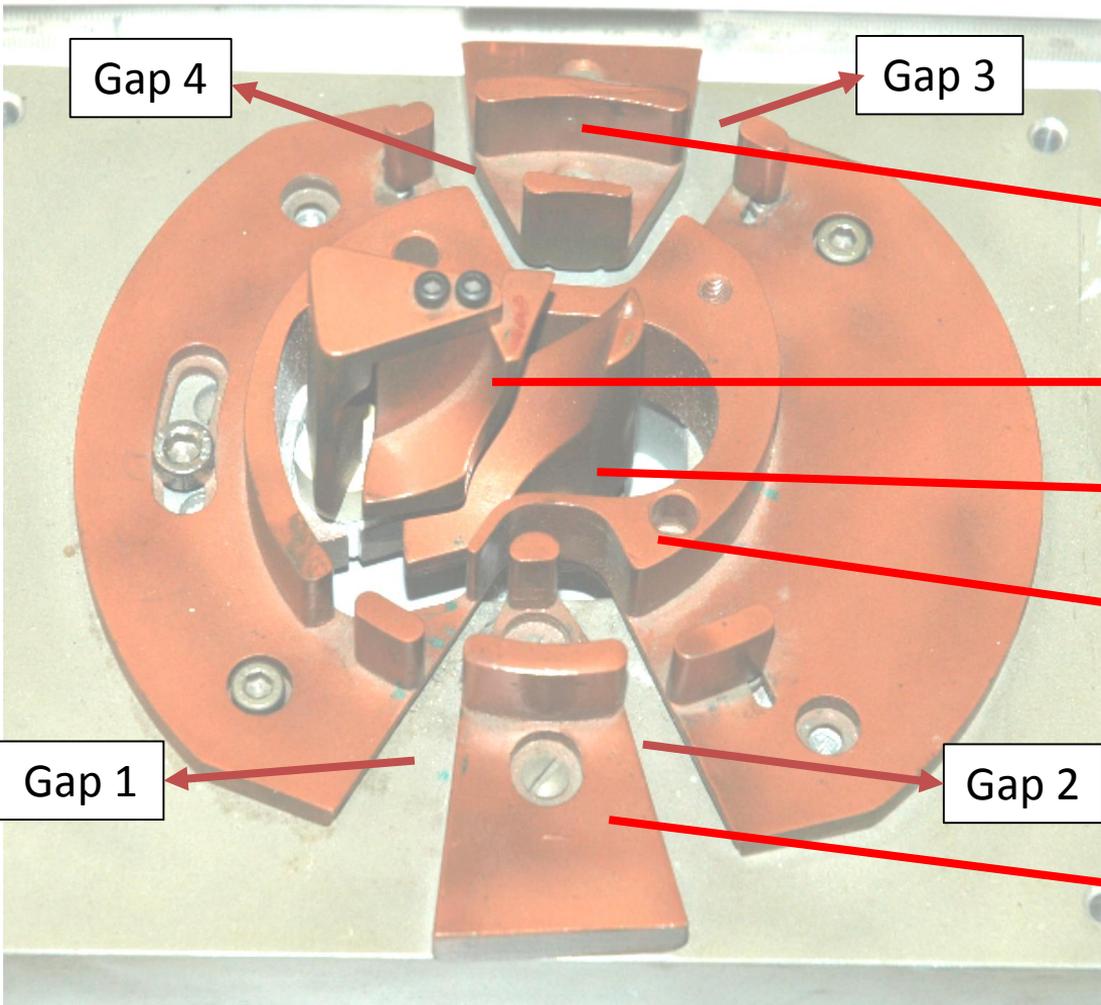
# Spiral inflector for Axial Injection

- The E-field between 2 electrodes bends the beam 90° from vertical to horizontal. The presence of the cyclotron B-field creates a 3D orbit
- The spiral inflector is basically a cylindrical capacitor which is gradually twisted in order to take into account the spiraling of the trajectory induced by the vertical magnetic field
- E-field always perpendicular to velocity  $\Rightarrow$  orbit on equipotential  $\Rightarrow$  this allows for low electrode voltage

$$\boxed{\frac{qV}{E} = \frac{2d}{A}}$$

- Two free design parameters available to obtain orbit centering
  1. Electric radius A (equivalent to height of inflector)
  2. Tilt parameter  $k'$  (equivalent to a change of magnetic field)
- Very compact geometry
- Complicated electrode structure needs a 5 axis milling machine

# spiral inflector scale 1:1 model



Gap 4

Gap 3

Gap 1

Gap 2

Left dee tip

upper electrode

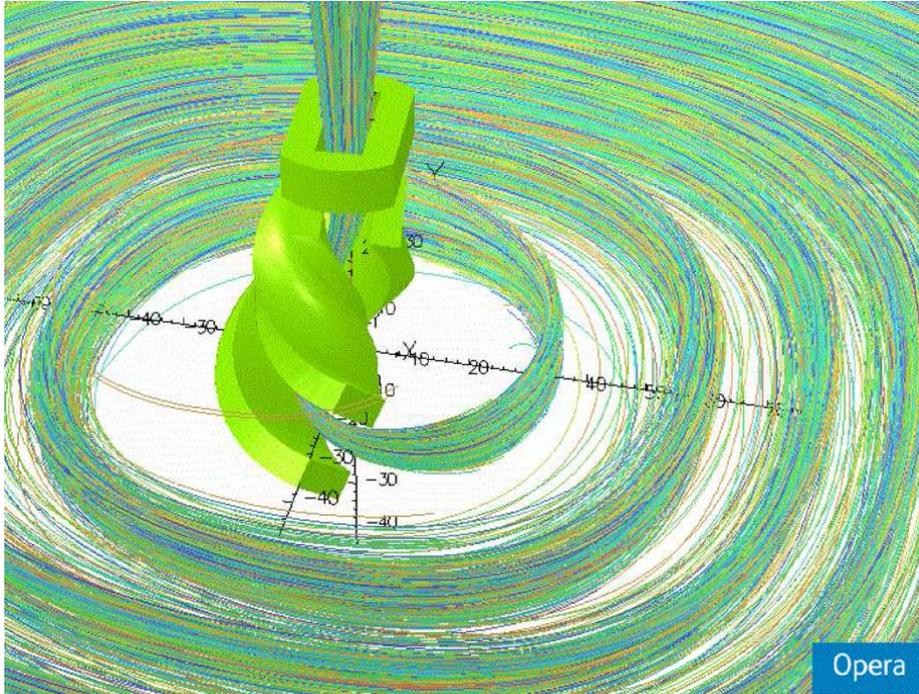
lower electrode

housing

right dee tip

# Inflector simulations

Calculated orbits imported in Opera3D



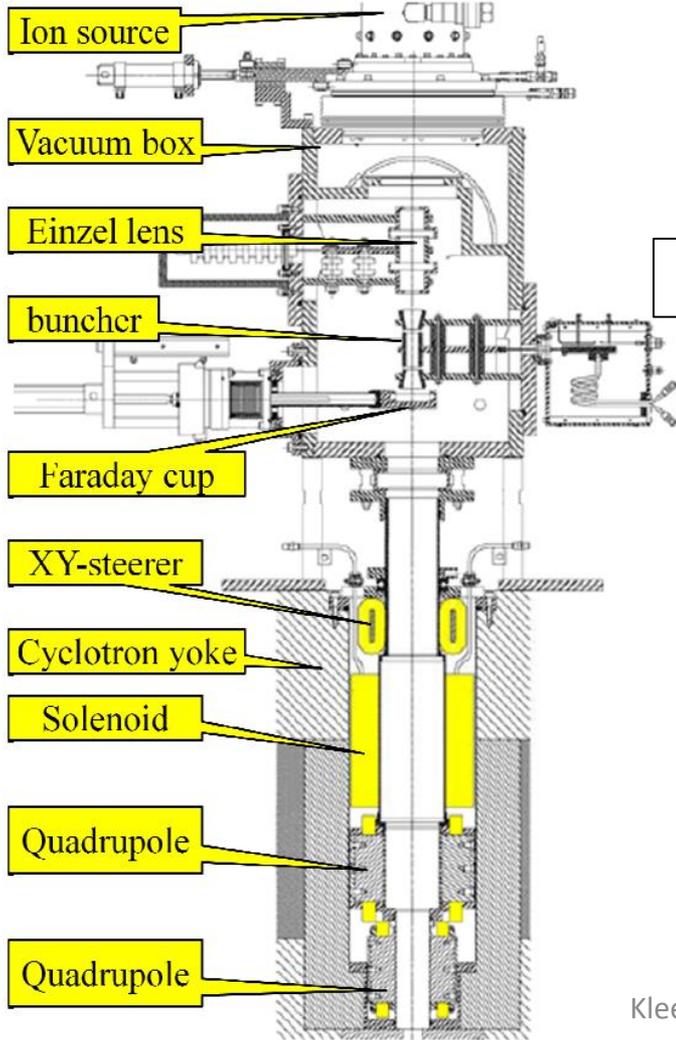
Kleeven/Zaremba CAS-2015: Cyclotrons - magnetic design and beam dynamics

- Spiral inflector is a complex 3D problem
- 3D fields (B,E) are needed => Opera3d
- In house developed tracking code
- Calculated orbits are imported in Opera3d post-processor
- Tilt is seen as the electrode-rotation at the exit

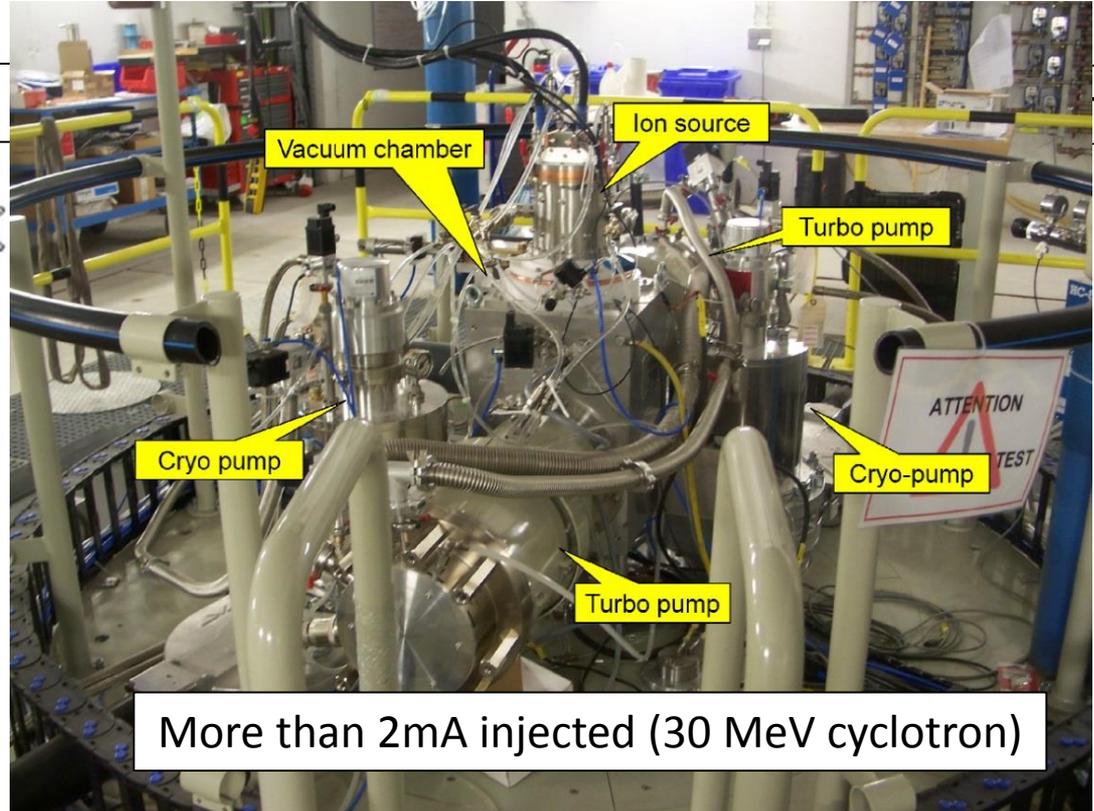
C70-example



An additional horizontal deflector is needed for multi-particle cyclotron



# Injection line



More than 2mA injected (30 MeV cyclotron)

# Ion source and central region of the S2C2

Central region size with a very compact cold cathode PIG source

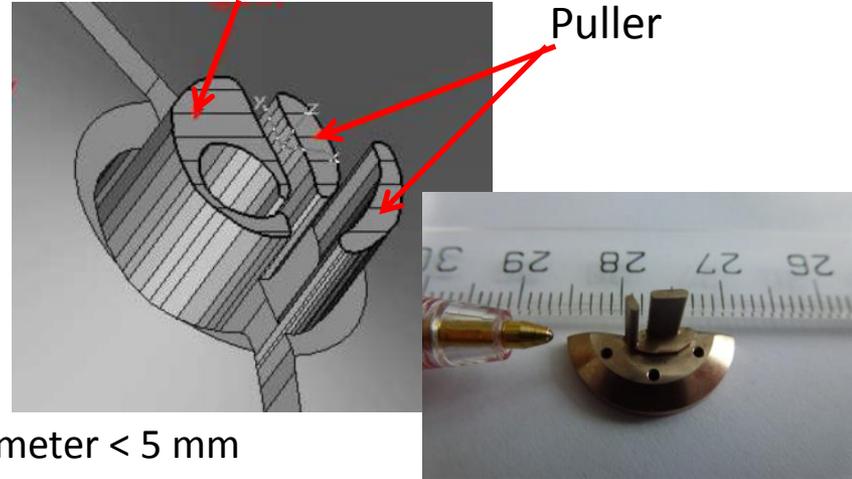
Due to the high magnetic field (5,74T) and the low dee voltage (11kV), the source has to be extremely compact:

Chimney

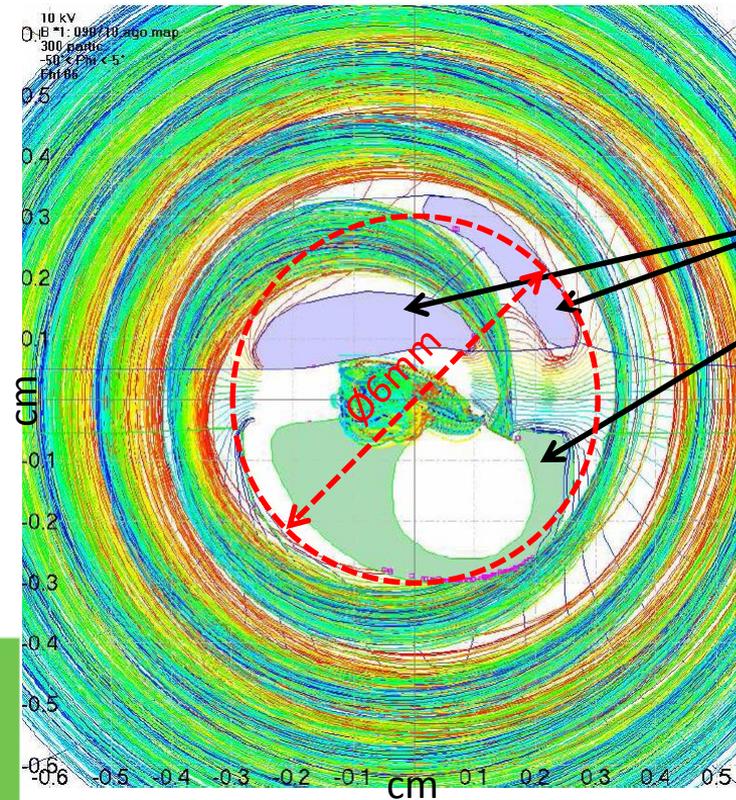
Puller

Puller

Chimney

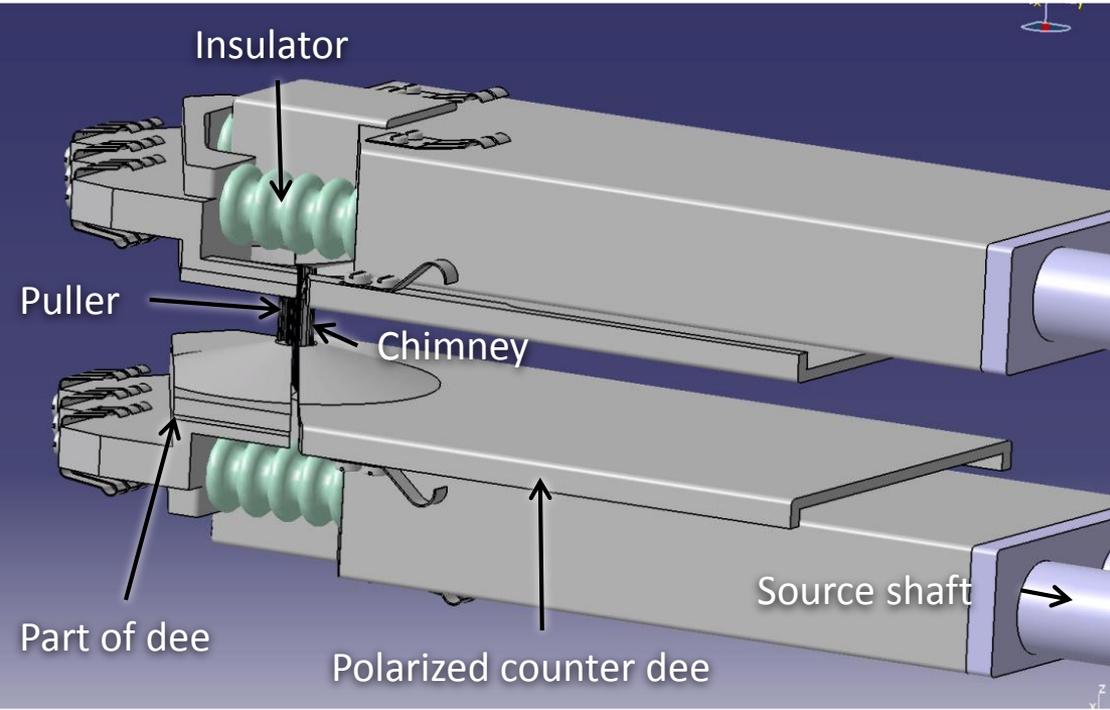


1. Source diameter < 5 mm
2. Vertical gap in the center 6 mm
3. First 100 turns within a radius of 3 cm

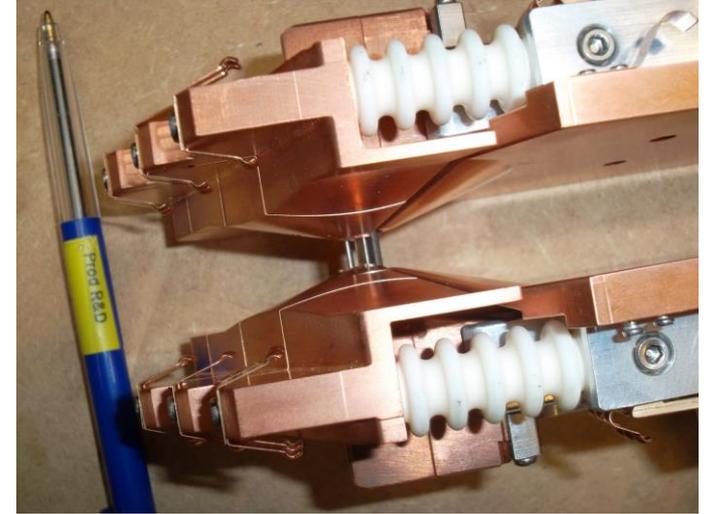


# Ion source and central region of the S2C2

The Ion Source and the central region, can be extracted as one assembly for easy maintenance and precise repositioning, without turning down the magnetic field.



Dee and counter dee are biased at 1 kV DC, to suppress multi-pactor

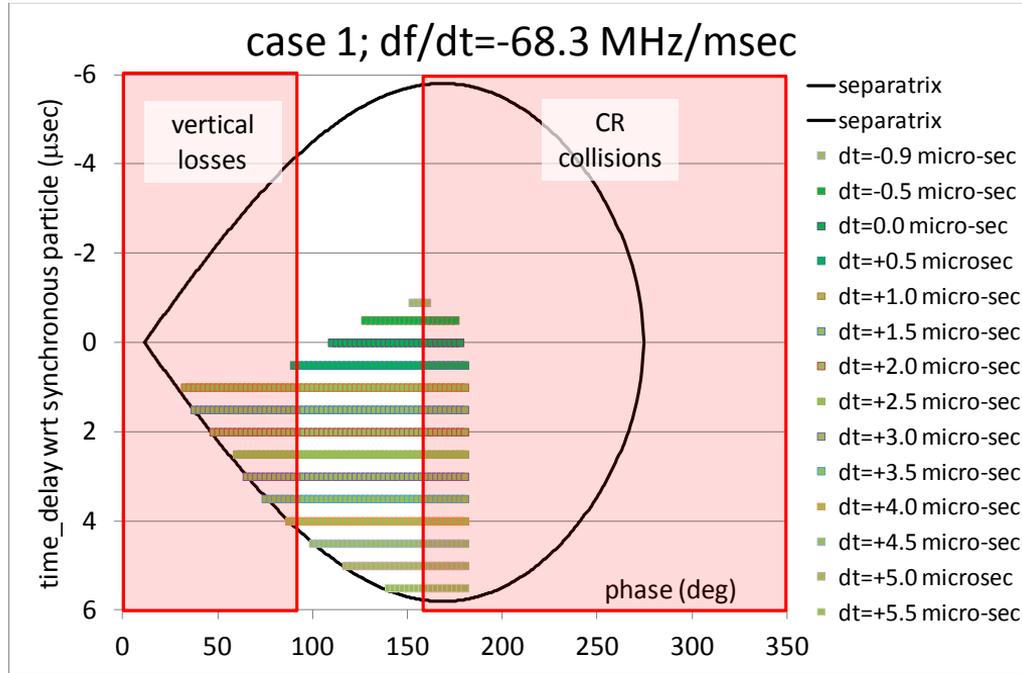


# By the way: why a SC synchrocyclotron for PT

- An isochronous cyclotron needs flutter
- Flutter can only be created by the iron (not by the coil)
- Maximum achievable field modulation about 2 Tesla
- If average field is pushed too far up (using a SC coil) than no longer enough flutter => not enough vertical focusing
- In a synchro-cyclotron this problem does not occur

**In a synchrocyclotron you can fully exploit the potential offered by superconductivity**

# Simulation of beam capture in the S2C2



Bohm and Foldy, The Physical Review 72 (1947) 649-661

A combined study of cyclotron central region and subsequent acceleration

Particles are started at the ion source at different time-moments and at different RF phases.

Only a subset is captured

In the central region there are additional transverse (horizontal/vertical) losses due to collisions with the geometry

# Part IV: A little bit about extraction

# Extraction from a cyclotron

- Extraction: transfer of the beam from an internal orbit to the application outside of the magnetic field
- Often a difficult process. Why?
  1. The magnetic field is a trap: When the particle enters into the radial fringe field of the pole, it runs out of RF phase and will be decelerated  $\Rightarrow$  particle is « reflected » inwards (if nothing is done to prevent this)
  2. The orbits pile up at high radii  $\Rightarrow$  smaller and smaller turn-separation

$$R \propto \sqrt{E}$$

3. The beam quality is quickly destroyed in the non-linear fringe field

# Different ways of extraction

1. No extraction at all => place an internal target
  - Can be done for isotope production (a little bit dirty)
2. Stripping extraction ( $H^-$  cyclotrons; or  $H_2^+$ )
  - Isotope production cyclotrons
3. Extraction with an electrostatic deflector (ESD)
  - Proton therapy cyclotrons (Varian, IBA, SHI)
4. Regenerative extraction => synchrocyclotron
  - Proton therapy cyclotrons (Mevion, IBA)
5. Self-extraction => suitable shaping of the magnetic field
  - One IBA prototype cyclotron but needs further improvement

Cases 3 and 4  
require some  
way to increase  
the turn  
separation  
before extraction

# Stripping Extraction (1)

Beam passes through a thin foil to remove electrons and suddenly change of the orbit curvature

$$\rho_f = \frac{Z_i M_f}{Z_f M_i} \rho_i$$

- Example H-minus,  $H^- \Rightarrow H^+ + 2 e^-$  (IBA C18/9, C30, ACS TR30, GE)

- => Radius of curvature changes sign

$$\rho_f = -\rho_i$$

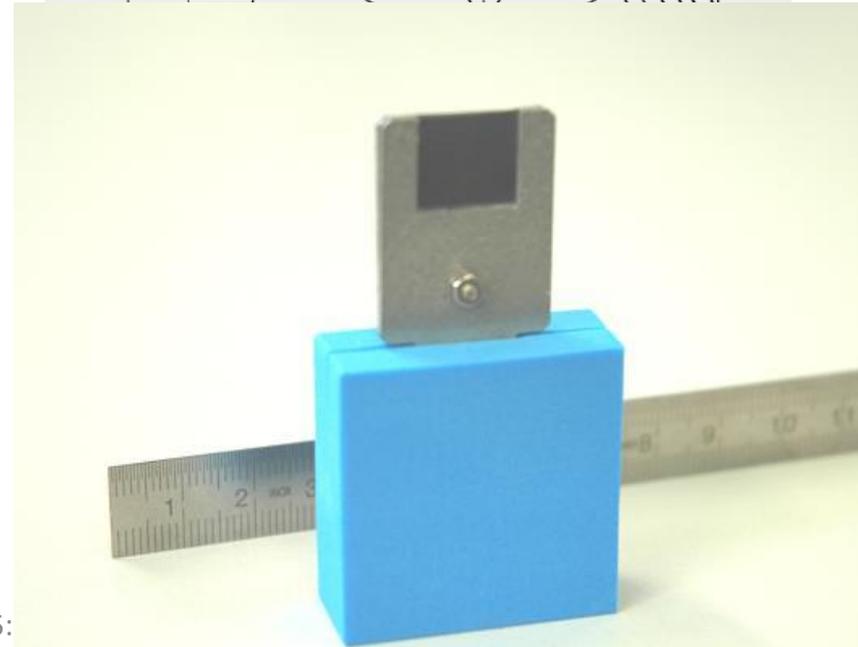
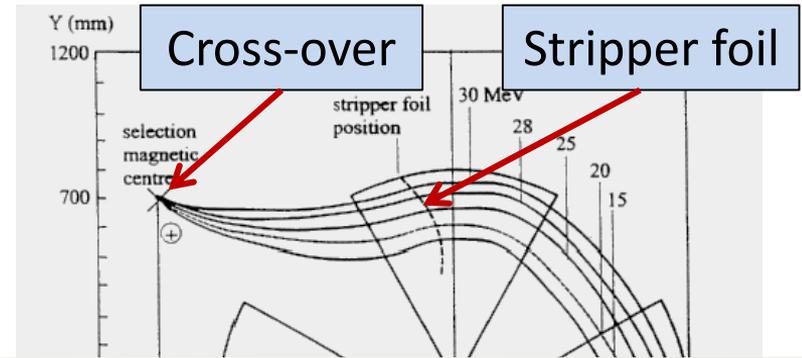
- Example  $H_2^+ \Rightarrow 2 H^+ + e^-$

- Requires a much larger machine, because the extracted energy reduces with a factor 4 compared to protons
- Only works when there is enough flutter

$$\rho_f = \frac{\rho_i}{2}$$

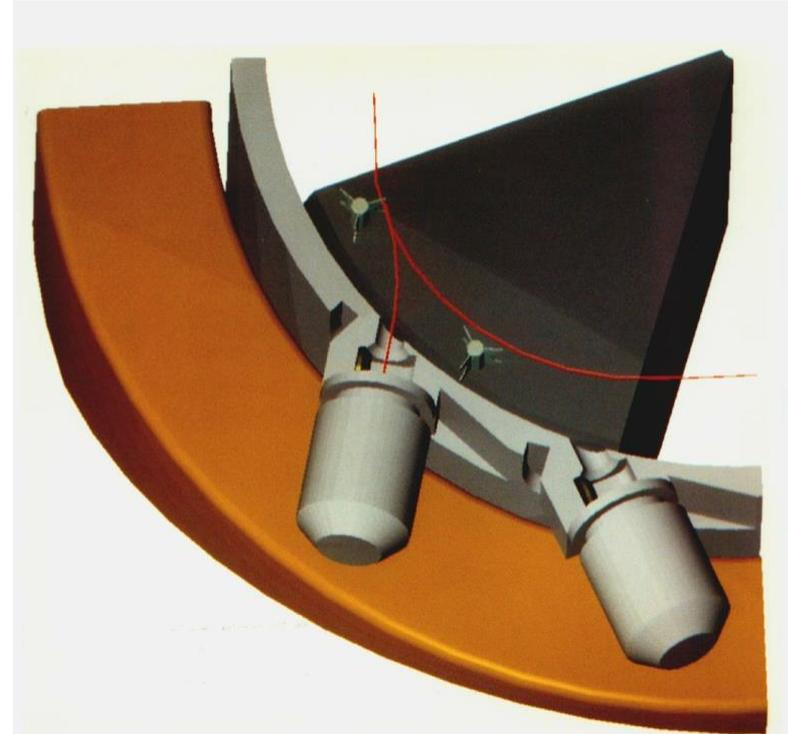
## H<sup>-</sup> stripping extraction (2)

- Stripper foil removes the two electrons of the H<sup>-</sup> ion and orbit curvature changes sign
- Energy variation by moving stripper position
- All energies go to one crossover point by proper foil azimuthal position
- Place combination magnet at crossover
- Ideal solution for industrial cyclotrons



# Stripping Extraction (3)

- Other advantages
  - Simple and 100 % extraction efficiency
  - Multiple targets around the machine
  - Dual beam extraction
  - Good extracted beam optics
- Limitations due to stripping losses
  - Low B-field  $\Rightarrow$  large magnet (Triumf 500 MeV/3 kG)
  - Good vacuum required (expensive)
  - OK for isotope production but not for proton therapy



# A side step: why cyclotrons for isotope production?

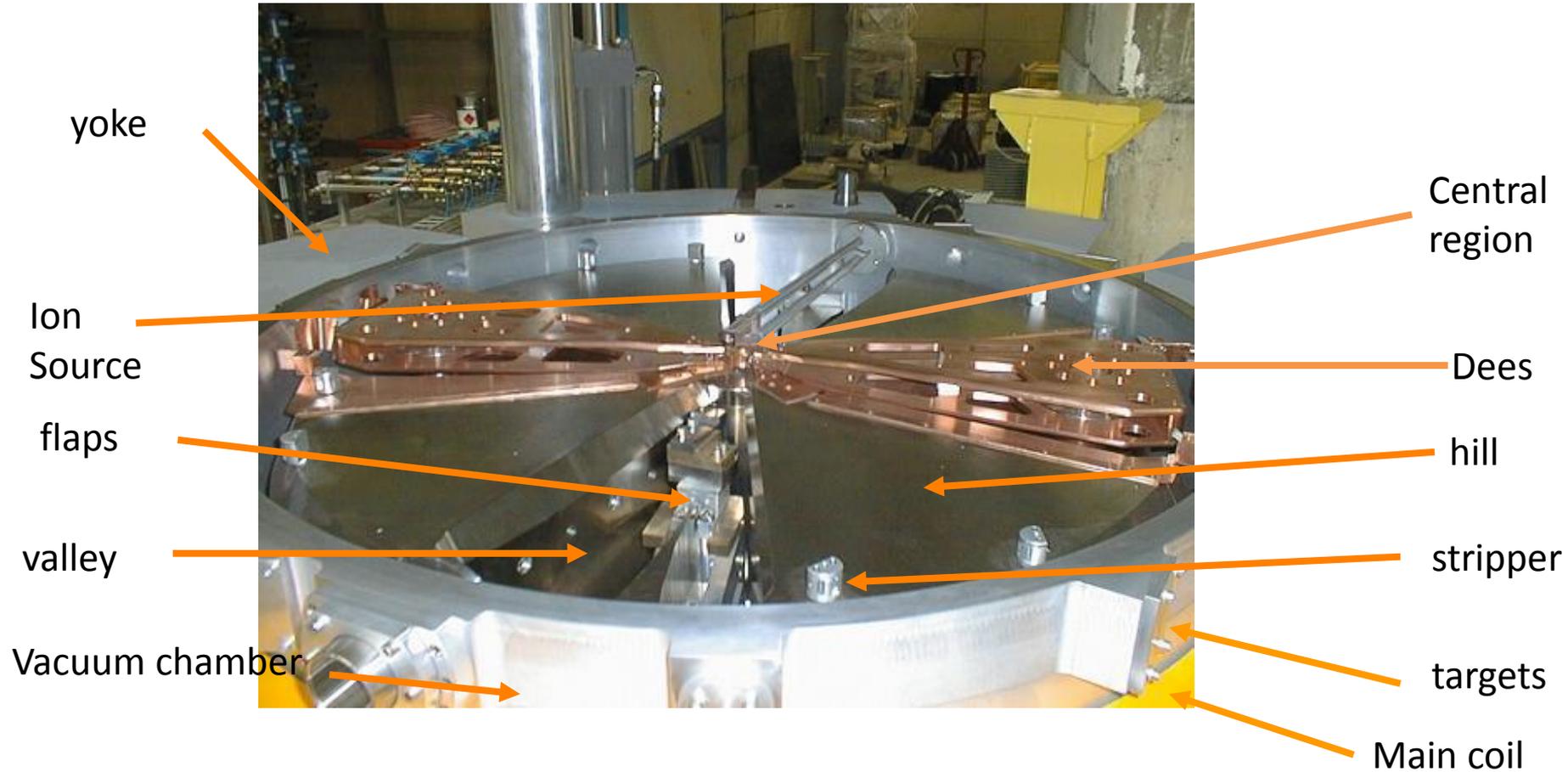
- Cost-effective machines for achieving:
  - required energies (<100 MeV) and
  - high currents (upto 1 to 2 mA)
- Efficient use of RF power => same accelerating structure used multiple times
- Compact =>
  - magnet and RF integrated into one system
  - Single stage => no injector accelerator needed
- Moderate magnetic fields: 1 to 2 Tesla
- Simple RF system:
  - Constant RF-frequency (10-100 MHz) => CW operation
  - Moderate voltages (10-100 kV)
- Relative easy injection (internal ion source or axial injection)
- Simple extraction (stripping for H<sup>-</sup> ions)

IBA was founded in 1986.  
Since then **more than 300**  
**isotope production cyclotrons**  
have been sold by IBA  
Many more by competitors

# IBA isotope production cyclotrons: some general features

- Deep-valley magnetic structure
  - Strong azimuthal variation of  $B \Rightarrow$  Strong focussing
  - Small gap requiring low power dissipation
- Acceleration of negative ions ( $H^-$  or  $D^-$ )  $\Rightarrow$ 
  - Stripping  $\Rightarrow$  very easy using thin carbon foil
  - 100% extraction efficiency
- 4-fold symmetry
  - Two accelerating structures (dees) in two valleys  $\Rightarrow$
  - Very compact; two other valleys for pumping, ESD....
- Injection from internal PIG-source (PET-isotopes) or with a spiral inflector (SPECT  $\Rightarrow$  cyclone 30)

# Compact Deep-valley Cyclotron Design



# Some commercial cyclotron vendors/manufacturers

**SIEMENS**

Germany (RP)

**VARIAN**  
medical systems

USA (PT)



GE, USA (RP)

**MEVION**  
medical systems

USA (PT)

**Best**

Canada (RP)

**SUMITOMO**

Japan (RP+PT)

**A** ADVANCED  
CYCLOTRON SYSTEMS, INC.

Canada (RP)

**Iba**  
Particle  
Therapy

↕ C235

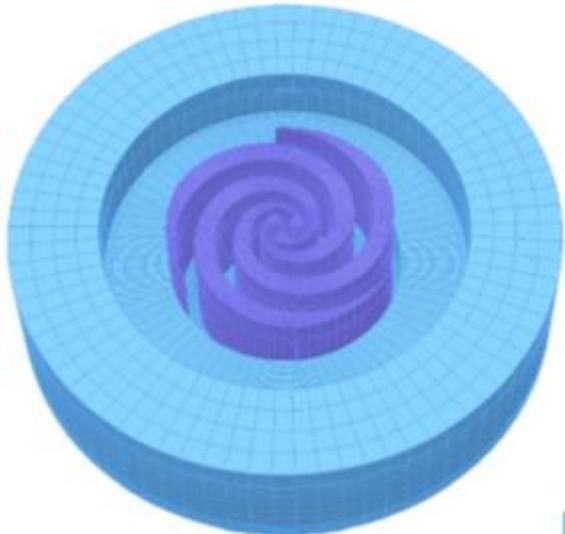
Belgium (RP+PT)

# 3-fold symmetry cyclotron for proton therapy?

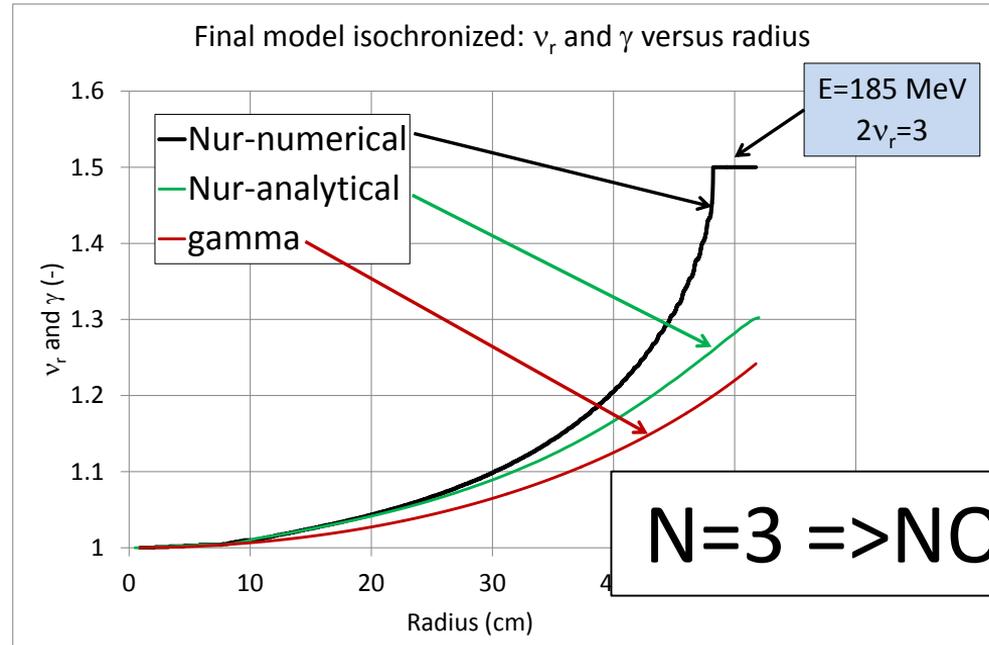
another small side-step

The simple formula  $v_r = \gamma$  is not valid when approaching the structural resonance  $2v_r = 3$ . This resonance may occur when  $N=3$ .

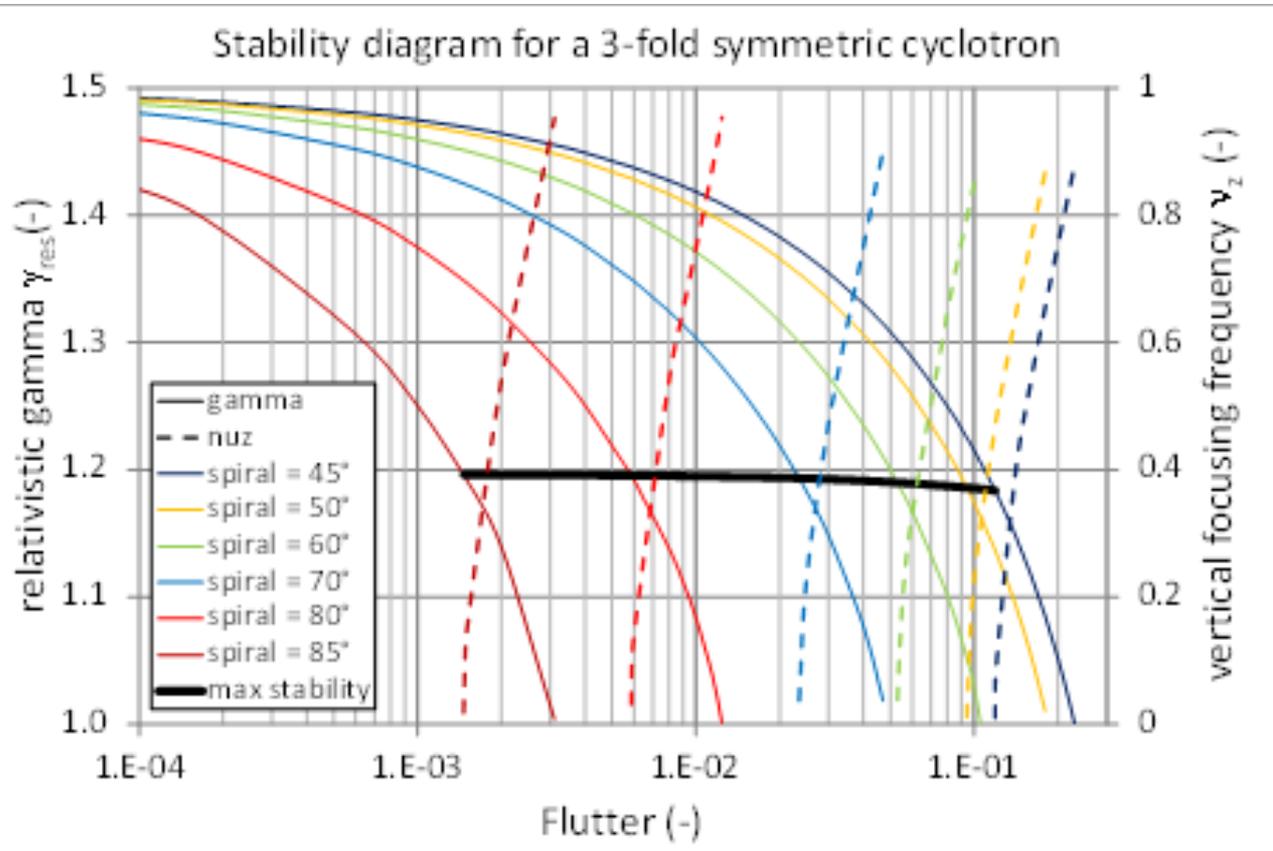
Consider PT-cyclotron of 230 MeV  $\Rightarrow \gamma = 1.25$  far from resonance  $v_r = 1.5$ ? => Not far enough !



A compact  $H_2^+$  cyclotron ( $N=3$ ) previously studied at IBA to see about variable energy extraction



# Stability diagram for N=3 cyclotron



**Analytical model for  $2\nu_r=3$  resonance**

Look at full parameter space of flutter and spiral angle

Taking into account:

- Vertical stability
- Stay away from  $2\nu_r=3$  stop band

**Maximum  $\gamma_{rel}$  always below 1.2  $\Rightarrow$  185 MeV**

# Extraction continued: turn-separation in a cyclotron

A Coherent beam oscillation  
is an oscillation  
around the equilibrium orbit



$$r(\theta) = \overbrace{r_0(\theta)}^{\text{EO}} + \underbrace{x(\theta)}_{\text{amplitude}} \sin(\underbrace{\nu_r \theta + \theta_0}_{\text{betatron oscillation phase}})$$



There are three different  
mechanisms to create turn  
separation



$$\Delta r(\theta_i) = \overbrace{\Delta r_0(\theta_i)}^{\text{acceleration}} + \underbrace{\Delta x \sin(2\pi n(\nu_r - 1) + \theta_0)}_{\text{precession}} \quad \overbrace{\phantom{\Delta x \sin(2\pi n(\nu_r - 1) + \theta_0)}}^{\text{resonance}}$$

$$+ \underbrace{2\pi(\nu_r - 1)x \cos(2\pi n(\nu_r - 1) + \theta_0)}_{\text{precession}}$$

# How can turn-separation be used for extraction

- I. By acceleration  $\Rightarrow$  high dee-voltage  $\Rightarrow$  IBA/SHI C235
- II. By resonances (coherent beam oscillations)
  - Precessional extraction (more subtle)  $\Rightarrow$  Varian SC cyclotron
    - Create oscillation amplitude with 1st harmonic or beam off-centering
    - Accelerate into fringe field where  $\nu_r \sim 0.7$
    - Turn separation obtained from betatron phase advance
  - Regenerative extraction (even more subtle) (IBA S2C2, Mevion Monarch)
    - Second harmonic gradient bump:  $2\nu_r=2$ ;  $\nu_r$  is locked to 1 in the stopband
    - Exponential growth of betatron amplitude

# Deflecting and guiding the beam out

A generic method of precessional extraction in a few steps

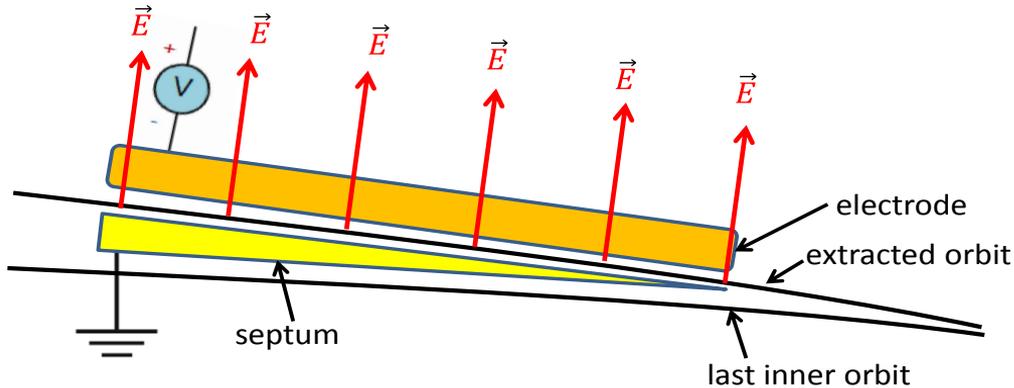
- i. Create an oscillation amplitude  $\Rightarrow$  by harmonic coils, trim rods or initial beam off-centering (at the ion source)
  - Obtain turn-separation by precession
- ii. Provide an initial radial kick  
 $\Rightarrow$  Electrostatic deflector ESD (peel off last turn)
- iii. Reduce B-field and minimize optical damage when passing the fringe field  $\Rightarrow$  Gradient corrector channels
- iv. Re-focus the beam as quickly as possible to handle beam divergencies created in the fringe field  
 $\Rightarrow$  First quadrupole doublet (in return yoke)

Non-adiabatic effect needed  $\Rightarrow$

Example of a  
harmonic coil



- DC radial E-field creates initial angular kick to deflect beam
- Inner electrode (septum) on ground potential
  - No disturbance on inner orbits
  - Knife thin (0.1 mm) and
  - V-shape at entrance (distribute heat)
  - Water cooled  $\Rightarrow$  limitation for maximum beam intensity
- Outer electrode on negative potential
- Electrode shape = orbit shape

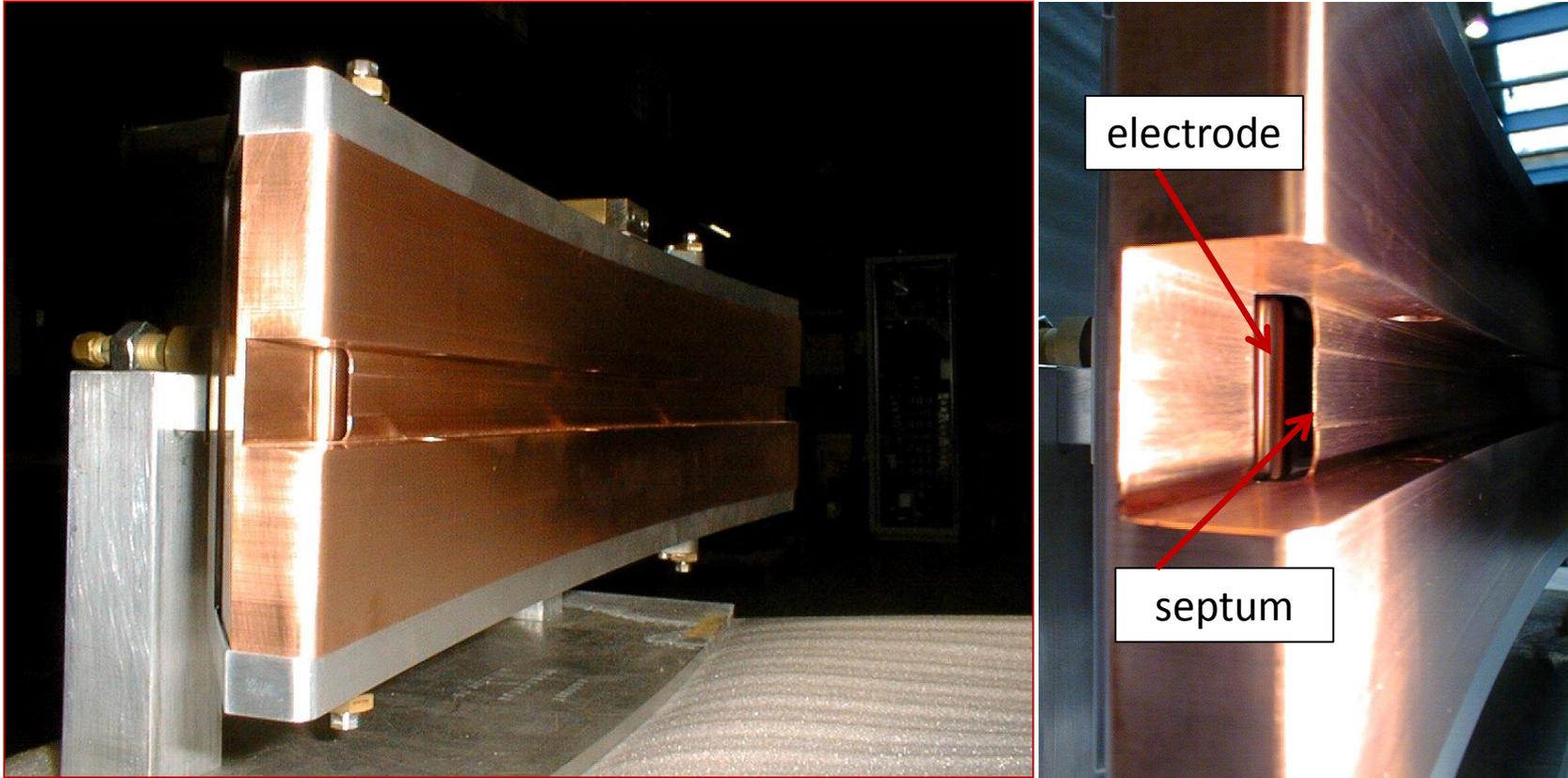


# Electrostatic Deflector

ESD for IBA C235

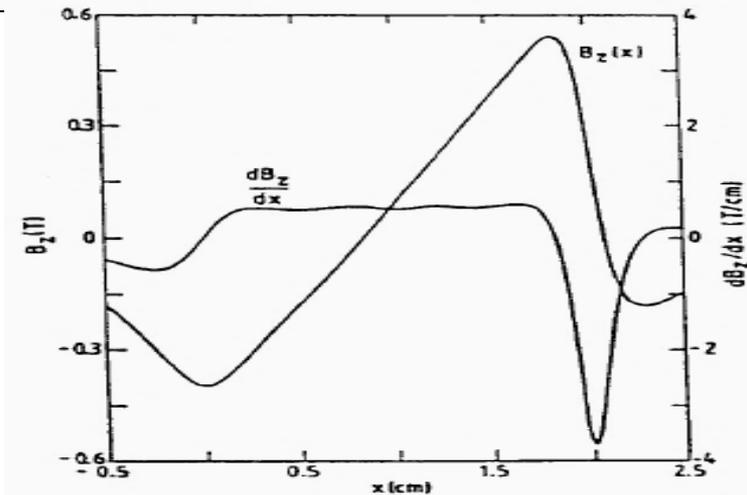
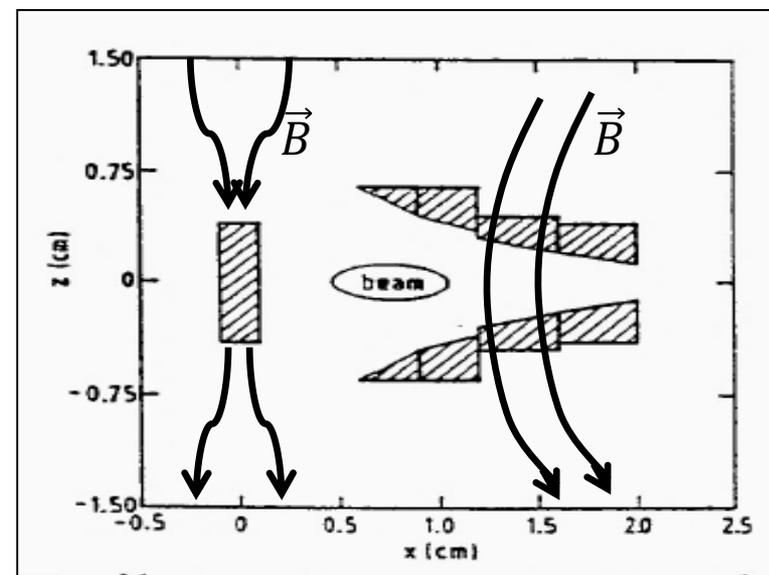


# C235 Electrostatic deflector



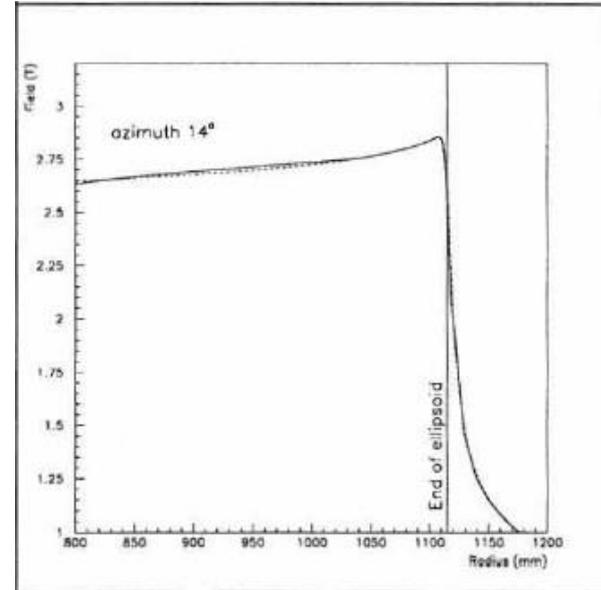
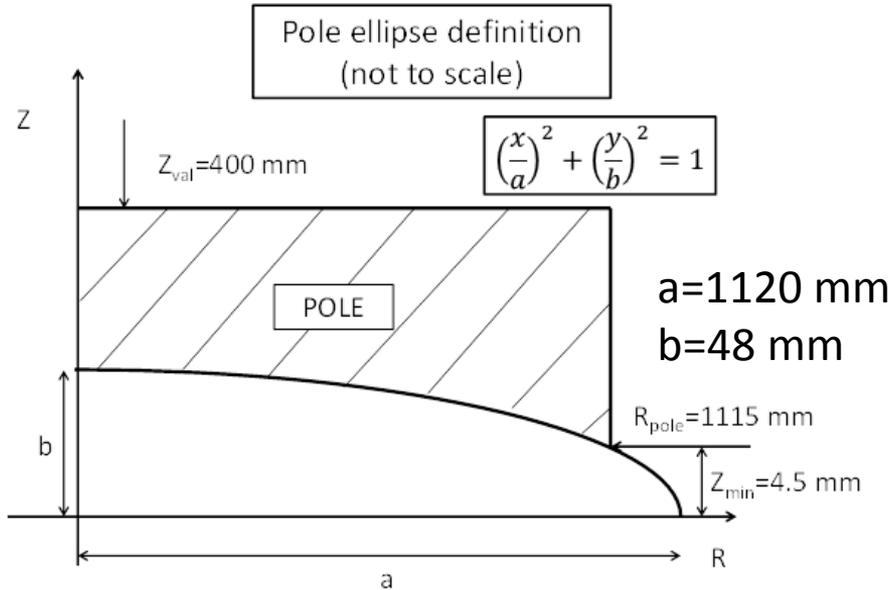
# Gradient Corrector focusing Channel

- Goal:
  - Guide the beam through the fringe field
  - Lower magnetic field on extraction path
  - reduce vertical/increase radial focusing through fringe field
- Different types
  - Passive: soft iron magnetized by the main field
  - Active:
    - Using permanent magnets
    - Using coils
- Designed in such a way as to minimize adverse effects on internal orbits



# Extraction in the IBA C235

A very sharp transition from stable to unstable



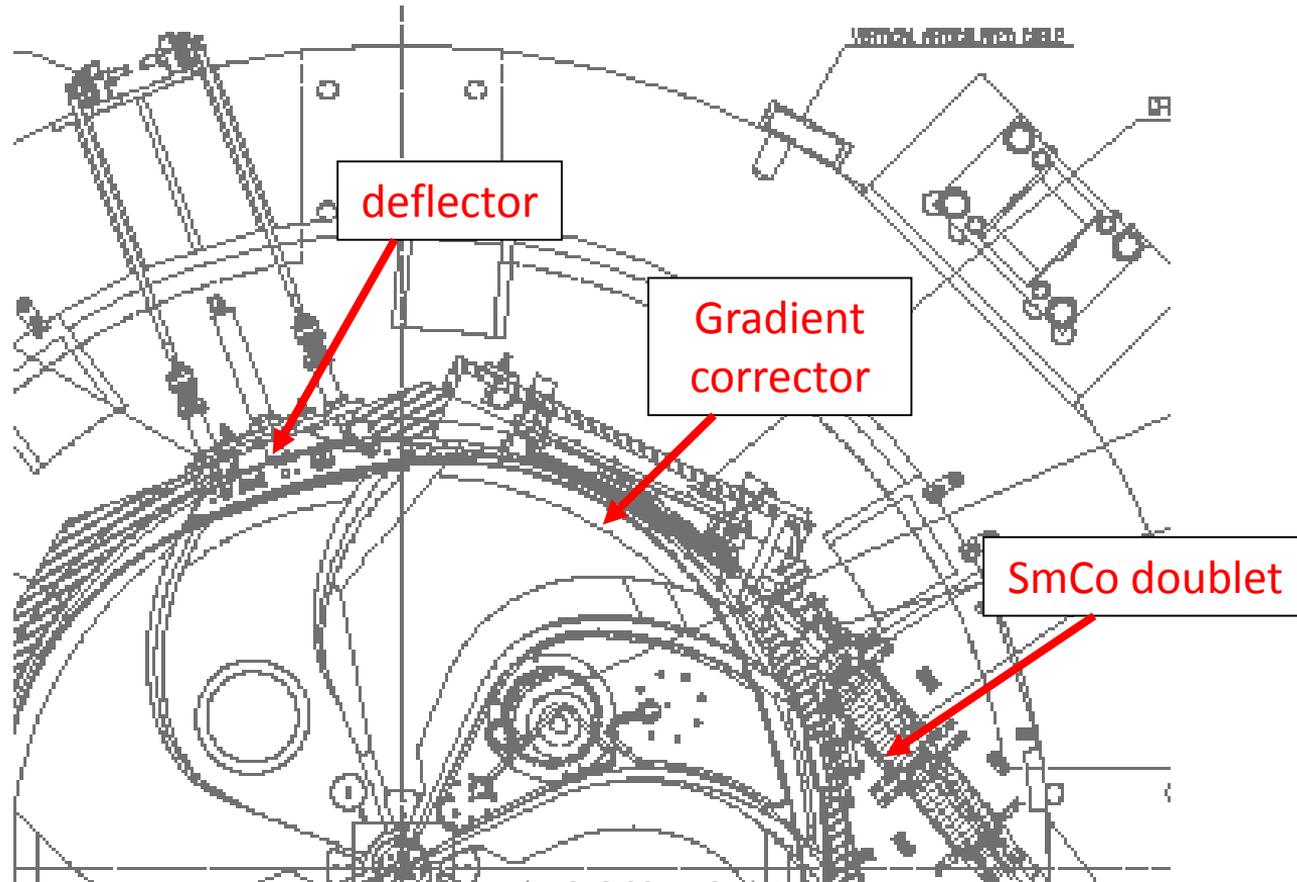
The pole gap in the C235 has an elliptical form.

This allows to obtain a good field region very close to the radius of the pole

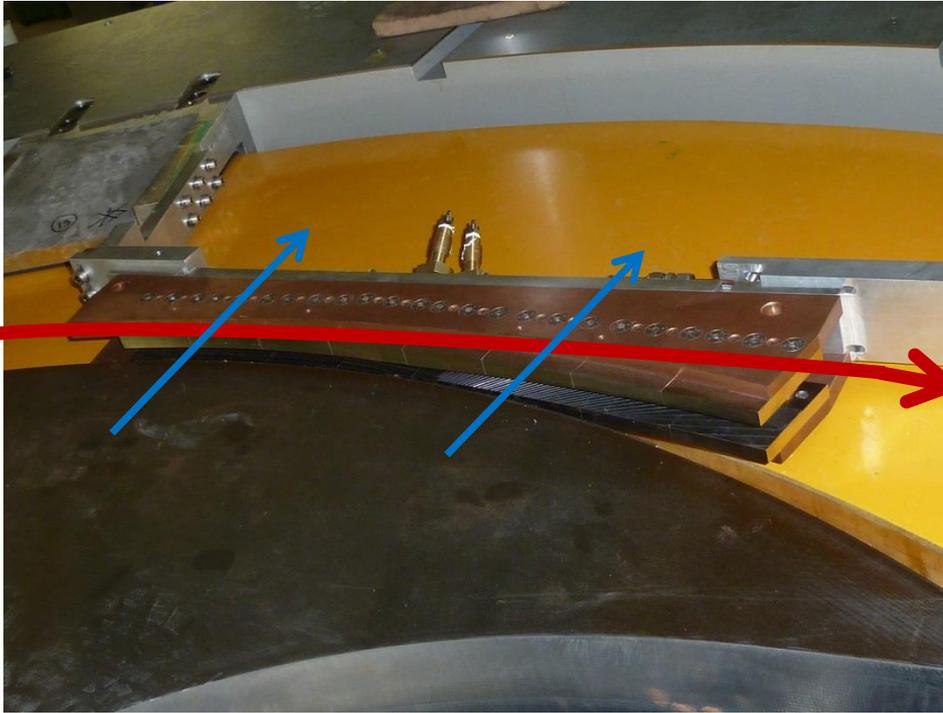
Therefore particles can be accelerated very close to the radius of the pole

Only a small kick is needed to extract the beam => orbit is extracted in  $\frac{1}{4}$  of a turn

# C235 Extraction Scheme

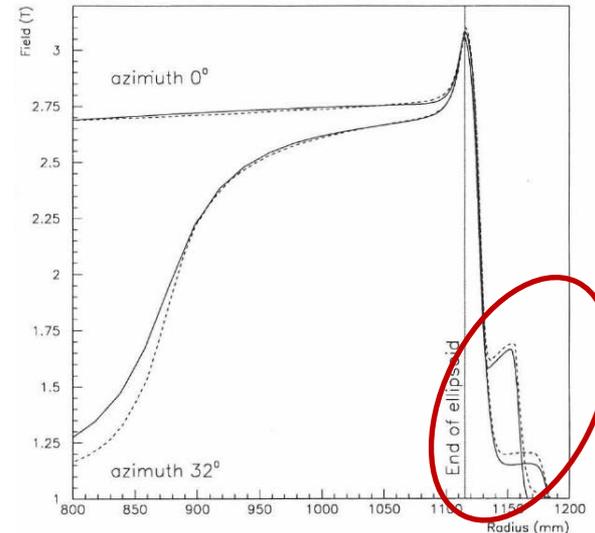


# C235 Gradient Corrector

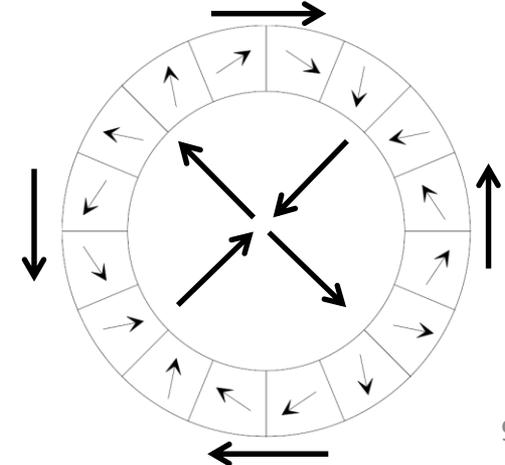
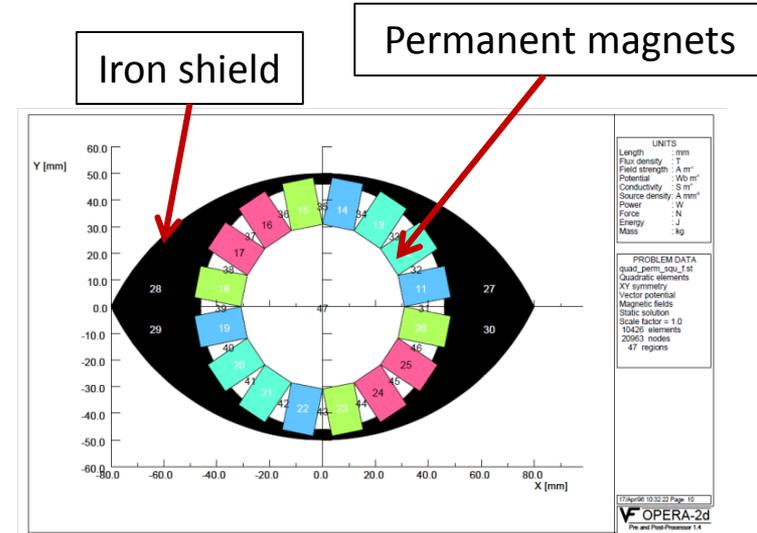


Kleeven/Zaremba CAS-2015: Cyclotrons - magnetic design and beam dynamics

- A passif channel, magnetized by the cyclotron magnetic field
- Placed between the main coils, against (almost touching) the hill sector.
- A descending 'slider' of gradually decreasing magnetic field that guides the beam gently through the fringe field

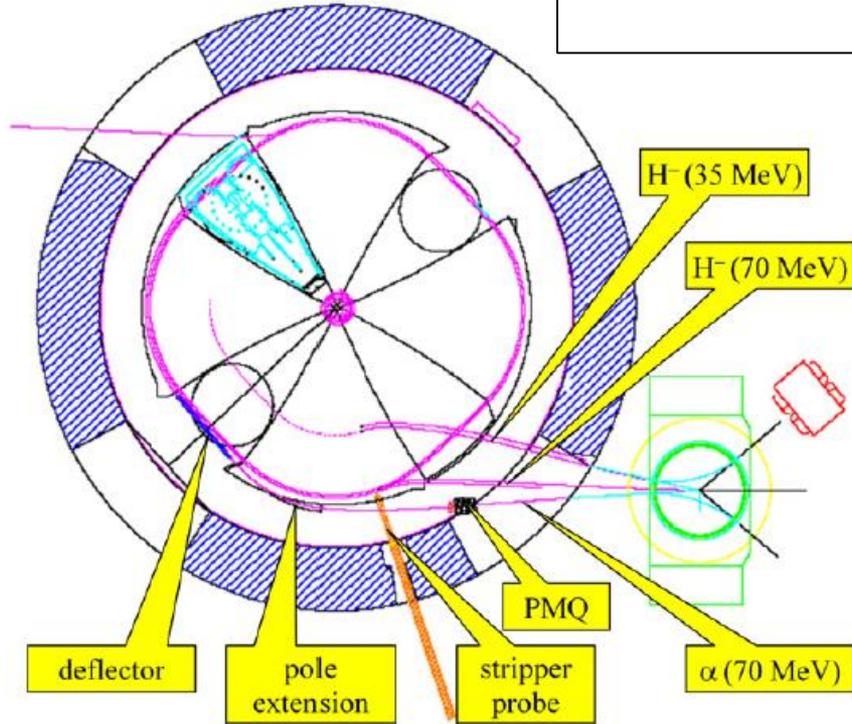


# C235 Permanent Magnet Doublet Placed in the return yoke



# Two extraction systems in one cyclotron

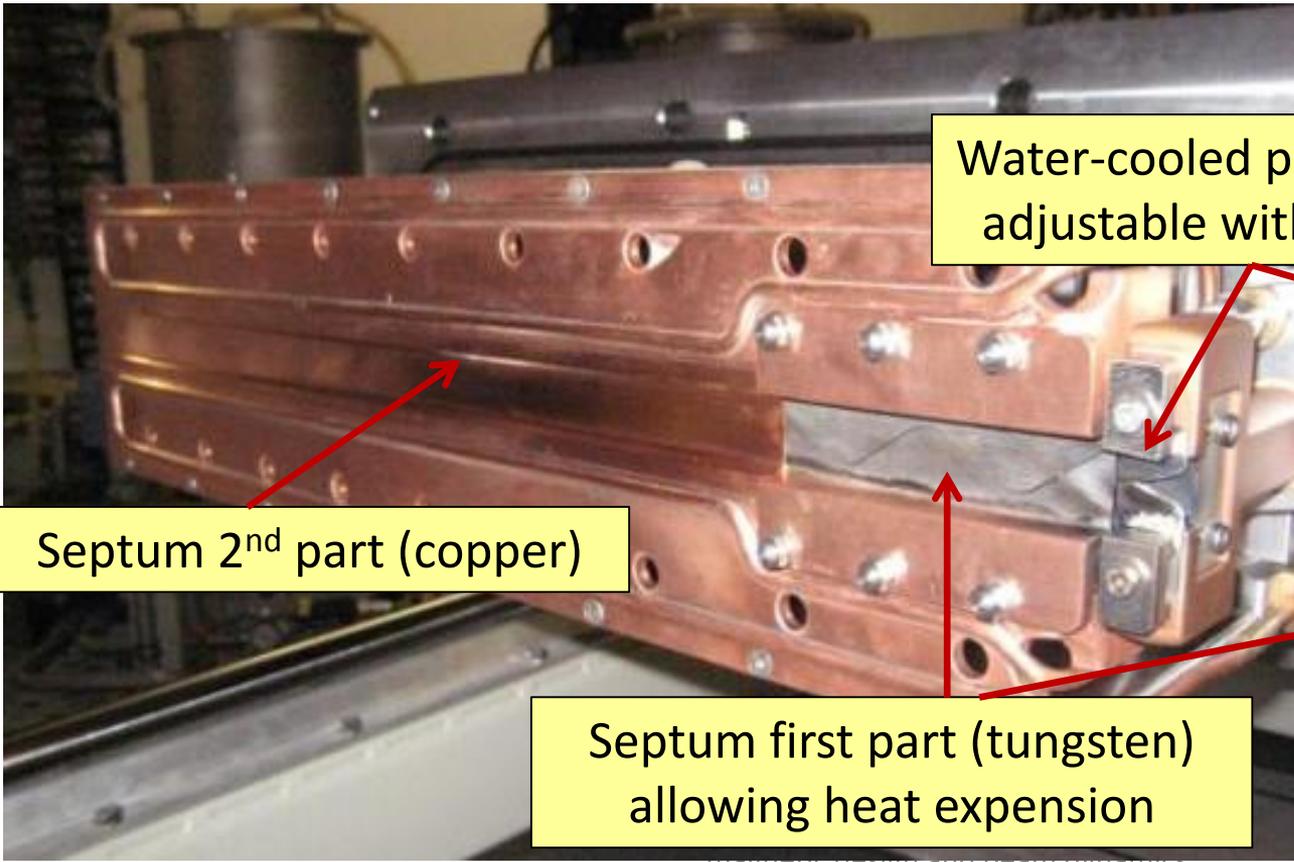
C70 multiple particle cyclotron for Arronax in Nantes  
 $H^-$ ,  $D^-$ ,  $\alpha$ ,  $H_2^+$



- Stripping extraction for negative particles
- ESD for  $\alpha$ -particle
- Two opposite exit ports
- Simultaneous dual beam capability for  $H^-$  and  $D^-$
- Variable energy for  $H^-$  and  $D^-$
- External switching magnet to direct different energies and particle into the beam lines

# The C70 electrostatic deflector (ESD)

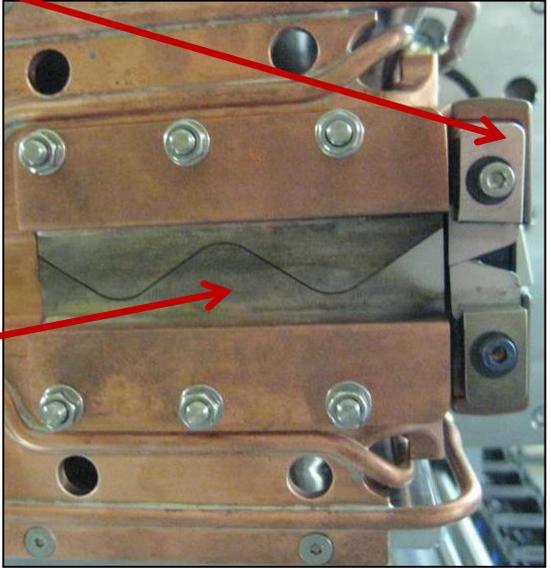
High intensity isotope production cyclotron



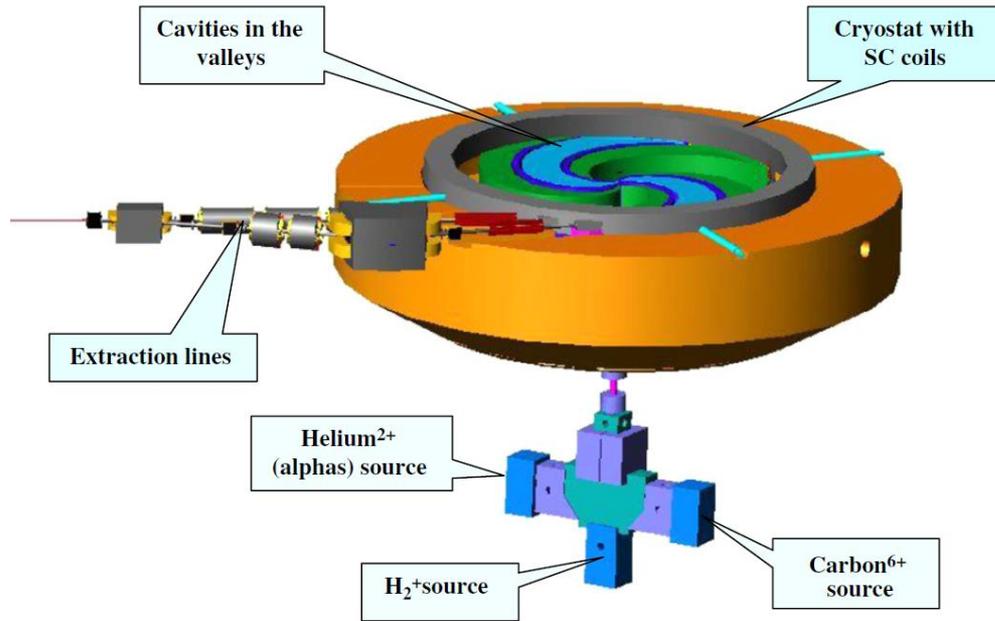
Water-cooled pre-septum, adjustable with V-shape

Septum 2<sup>nd</sup> part (copper)

Septum first part (tungsten) allowing heat expansion



# The IBA C400 cyclotron



particles	$^{12}\text{C}^{6+}$ ; $\text{H}_2^+$ ; $^4\text{He}^{2+}$
Final energy	
ions	400 MeV/A
protons	265 MeV
Bending limit	K=1600
Weight	700 t
Diameter	6.6 m
Hill field	4.5 Tesla
Valley field	2.45 Tesla
Number of cavities	2
RF frequency	75 MHz; h=4
Vdee	80-160 kV
Number of turns	2000
SC coil	NbTi; Helium cooled
Ischronism of $\text{H}_2^+$	Coil in 2 parts

Y. Jongen et.al., NIM  
A624 (2010) 47-53

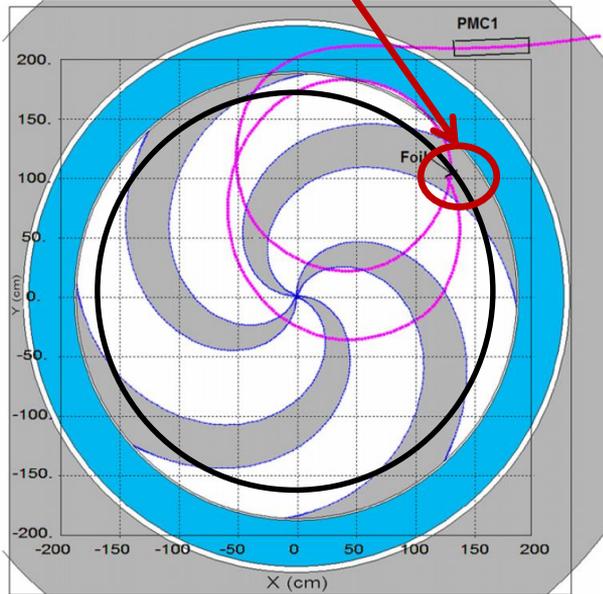
- Full detailed design study was done in collaboration with JINR
- Possibly/hopefully to be industrialized by the French company Normandy Hadrontherapy in which IBA is minority shareholder

# Extraction from the C400

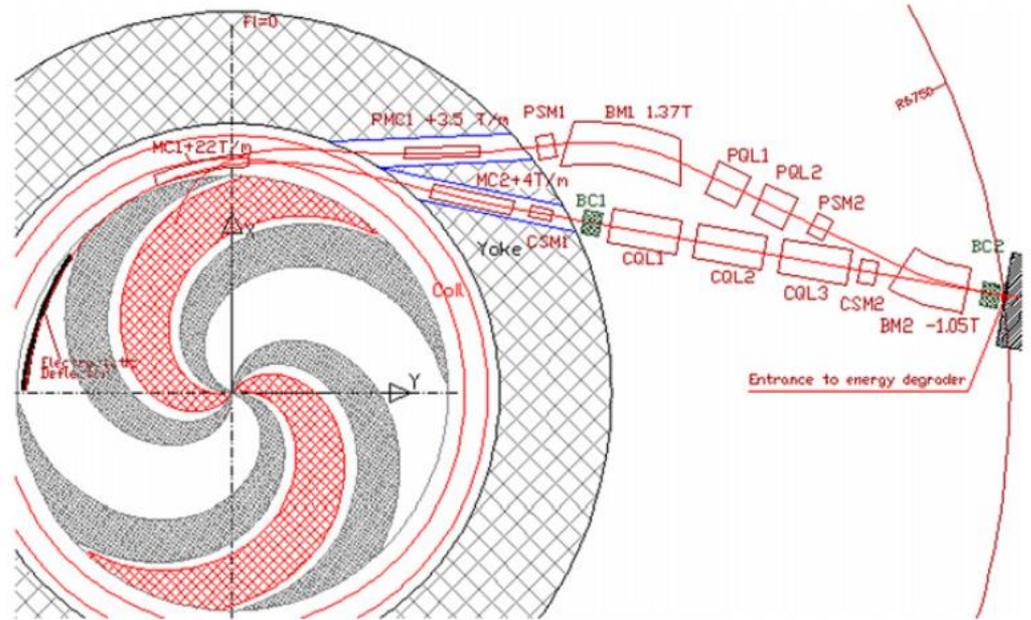
Protons => stripping of  $H_2^+$

$^{12}C^{6+}$  => Electrostatic deflector

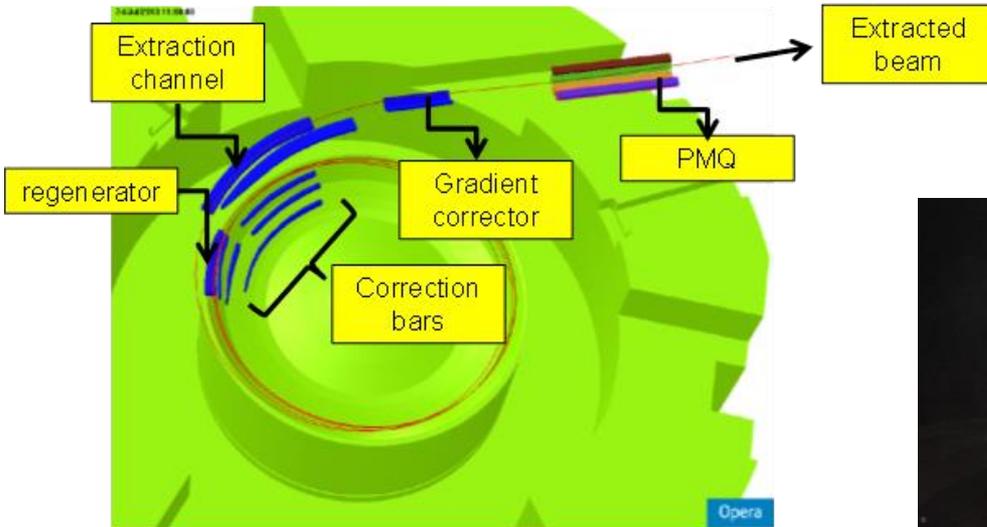
$H_2^+$  two-turn extraction after stripping



Combining both beams into one beam line



# The IBA S2C2 extraction system



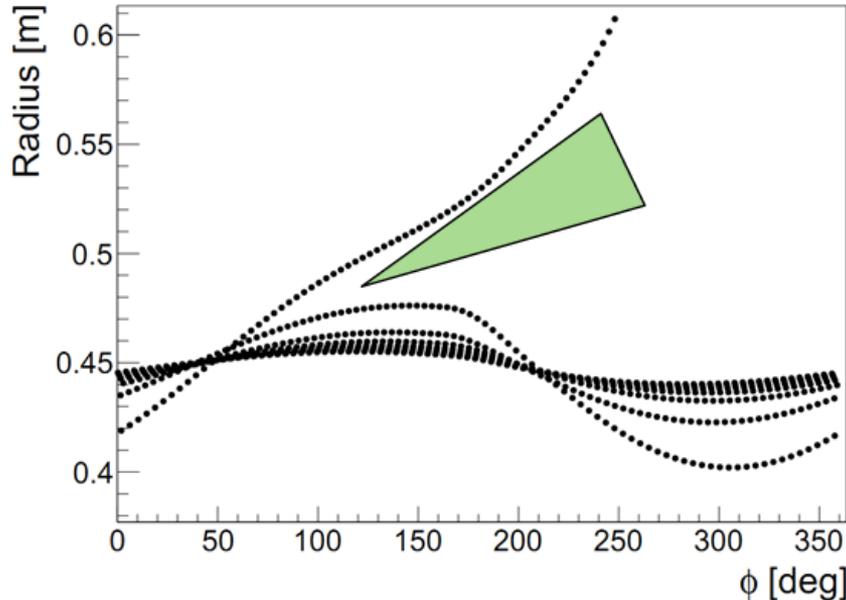
- Horizontal focusing by gradient corrector and permanent magnet quadrupole (PMQ) in strongly

- Fully passive system => only soft iron
- Use resonant extraction based on  $2Q_h$
- Strong local field bump produced by r frequency and locks it to unity.
- Unstable orbit is pushed towards the c

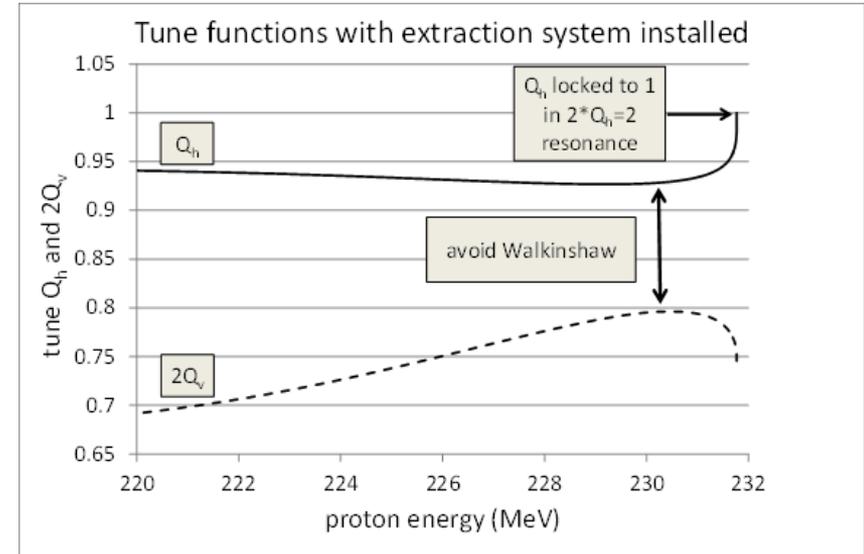


# Regenerative extraction based on $2\nu_r=2$ resonance

- A strong regenerator bump increases  $\nu_r$  and locks it to 1
- A steady shift of the beam towards the extraction channel builds up



- Avoid Walkinshaw resonance ( $\nu_r=2\nu_z$ )



# Part V: A little bit about magnetic design

# Main design choices are made before any calculation starts

- i. What is the application=> particle-type(s) and energy
- ii. Choice of cyclotron type: isochronous or synchrocyclotron
- iii. Coil technology => super-conducting or normal-conducting
- iv. Extraction => stripping, ESD, regenerative
- v. Maximum rigidity => max  $\langle B \rangle$ -field => pole radius
- vi. For example for an isochronous cyclotron
  - a. Number of sectors, pole gap, pole angle, valley depth, pole spiral etc
  - b. Number of dees, dee-voltage, harmonic mode, RF-frequency
  - c. Injection: internal or external ion source

# Back of the envelope => a few examples

Maximum energy determines magnetic rigidity  $B\rho = \sqrt{T^2 + 2 T E_0} / (300 Z)$

Relation between field and coil Amp-turns  $\oint \vec{H} \cdot d\vec{l} = \frac{1}{\mu_0} \cdot (B_{hill} \cdot gap) + \frac{1}{\mu_0 \mu_{iron}} \cdot (B_{iron} \cdot L_{iron}) = nI$

Estimation of hill and valley field

Average field vs hill/valley field

$$\langle B \rangle = \alpha B_{hill} + (1 - \alpha) B_{val}$$

Total produced magnetic flux

$$\Phi = 2\pi R_{pole}^2 \langle B \rangle \cong B_{ret} A_{ret}$$

Relativistic mass increase

$$B = B_0 \gamma_{rel}$$

Estimation of the flutter.....

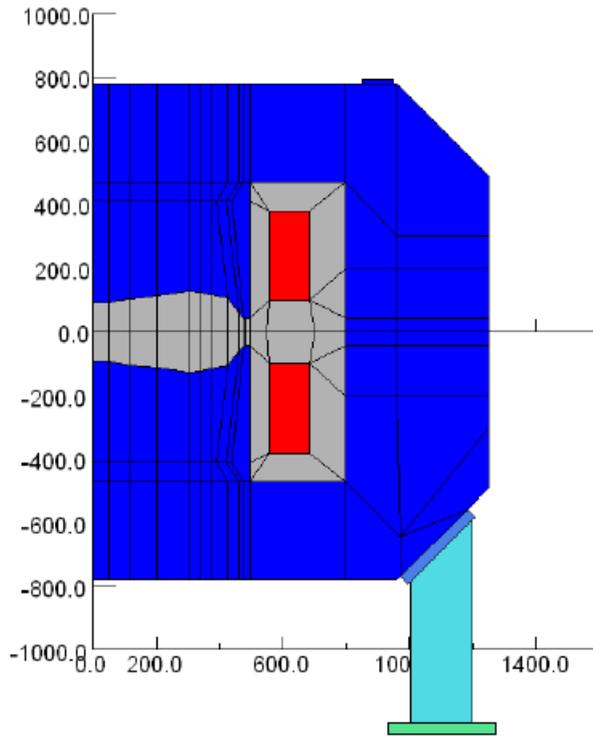
Estimation of betatron frequencies..... => required spiral angle

Etc.

# Tools for magnetic modeling in OPERA

- OPERA2D =>
  - Perfect for a synchro-cyclotron
  - use stacking factors for modeling of AVF cyclotron ( $\langle B \rangle$ , return yoke)
- OPERA3D => **modeler interface**
  - Easy to use and easy to include fine geometrical details
  - 3D FE-mesh automatically generated;
    - Tetrahedral mesh => less regular => magnetic fields may be more noisy
- OPERA3D => **pre-processor interface**
  - More difficult to use and to include geometrical details
  - 3D FE-mesh fully created by the user and more regular
    - Hexahedral mesh => less noisy magnetic fields => more precise prediction of magnetic forces

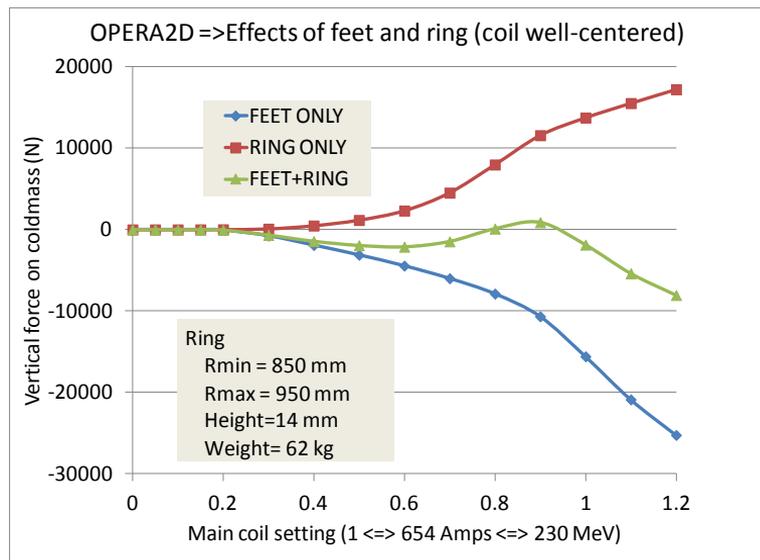
# OPERA2D - example



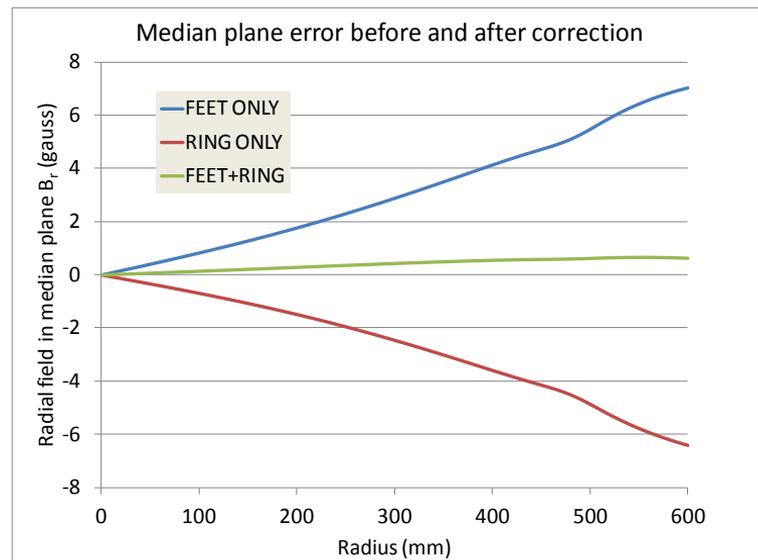
- Initial design of a synchro-cyclotron can very well be done in OPERA2D => rotational symmetry
- Fast optimization of dimensions
  - Pole profile => magnetic field maps => tune functions
  - Yoke dimensions => stray-fields
  - Coil dimensions => Maximum field on the coils
- Yoke-penetrations + feet => include by stacking factors
- Extraction-elements => assume fully saturated iron
- Study of special features
  - Vertical asymmetry
    - Median plane errors
    - Forces on the cold-mass
  - Compensation of vertical asymmetry

# Compensation of median plane error due to cyclotron feet calculated with OPERA2D

Force on cold mass  
compensated by iron ring on  
top of yoke

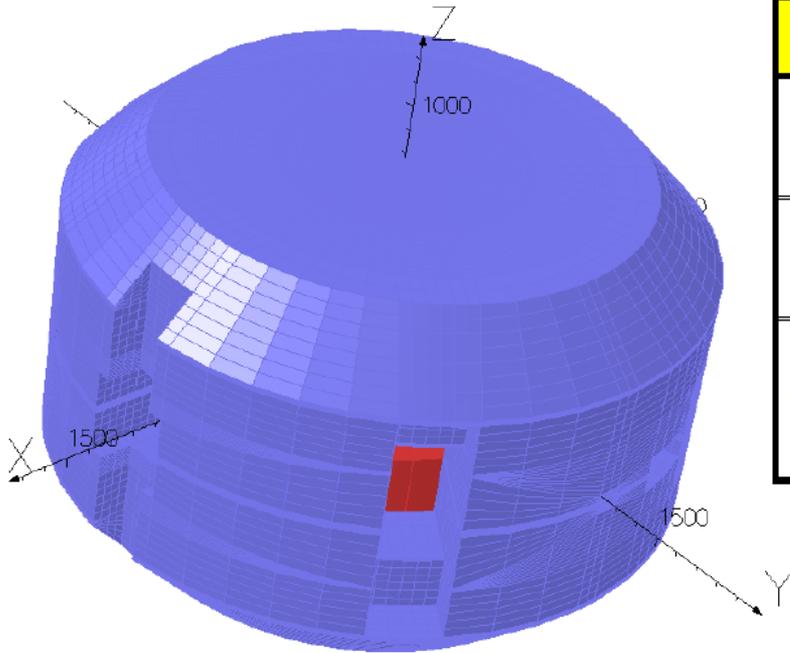


Median plane magnetic field  
error also compensated



# Coil forces in the S2C2 calculated with the pre-processor

A pre-processor model with the typical hexahedral mesh



Differential forces on the cold-mass due to translations or rotation can be calculated with better precision in the pre-processor

FORCES AND TORQUES ACTING ON THE MAIN COIL SYSTEM DUE TO COIL DISPLACEMENTS AND ROTATIONS							
		FORCES			TORQUES		
		dFx ton/mm	dFy ton/mm	dFz ton/mm	dTx Nm/mm	dTy Nm/mm	dTz Nm/mm
coil shift	x-direction	1.99	-0.05	0.00	0	-9	8
	y-direction	-0.05	2.00	0.00	10	2	41
	z-direction	0.00	0.00	0.56	-80	-201	0
coil rotation		dFx ton/deg	dFy ton/deg	dFz ton/deg	dTx Nm/deg	dTy Nm/deg	dTz Nm/deg
	around x-axis	-0.02	0.00	-0.12	91559	-4609	-80
	around y-axis	-0.05	-0.01	-0.30	-4484	91305	79

- All forces vary linear with displacement or rotation
- All coil movements are unstable => forces want to increase their cause

# OPERA3D model design approach

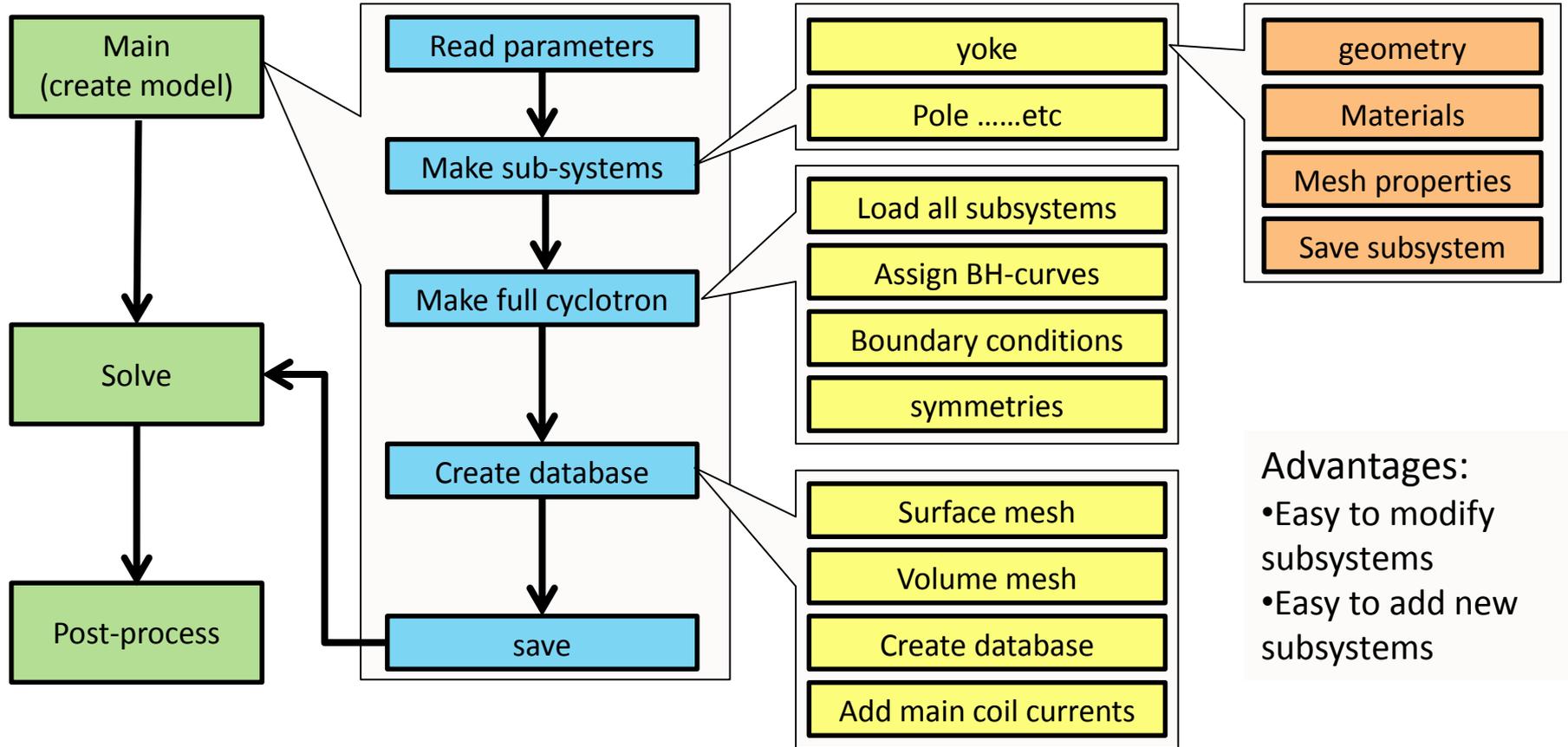
- Full parameterization of 3D models
- Automatic generation of models using macro-structures
- One common macro-structure for all different types of IBA cyclotrons:
  - S2C2, C230, C30-family, C3, .....
- Verify iron BH-curves with in-house permeability meter

# OPERA3D

## Different types of parameters

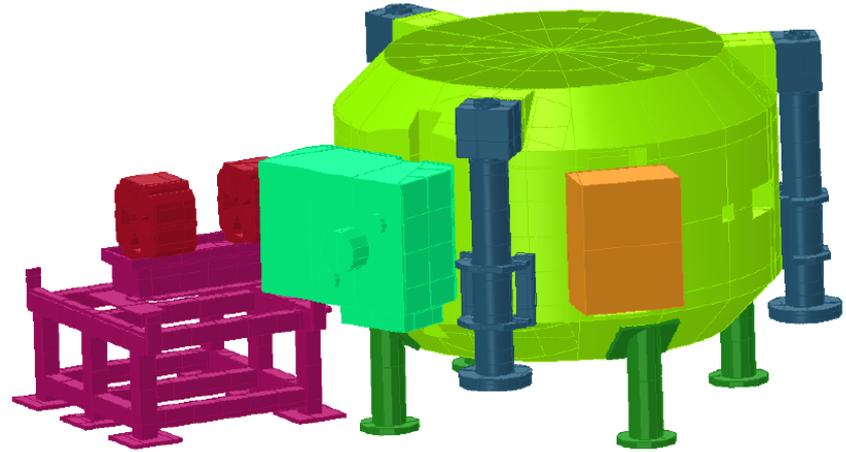
- All dimensional parameters and pole profiles
- Main coil settings
- Material properties (different BH-curves for different subsystems)
- Finite element mesh sizes
- Solver tolerances
- Switches for (de-) selection of separate subsystems
- Switches for filling separate subsystems with air or iron
- .....

# General macro structure for a cyclotron model in OPERA3D



# Elements included in S2C2 OPERA3D model

- i. Yoke+poles+coils
- ii. Yoke penetrations
- iii. Extraction system (regenerator, channels, first harmonic correctors)
- iv. External systems
  - a) Cyclotron feet
  - b) Yoke lifting system
  - c) Shields (cryo-coolers + rotco)
  - d) External quadrupoles



Due to saturation of yoke iron:

- external systems have to be included in the magnetic design studies
- Cryo-coolers and rotco must be shielded

# Major milestones in cyclotron development (1)

## 1. Classical cyclotron (Lawrence)

- Uniform magnetic field => loss of isochronism due to relativistic mass increase => energy limited
- CW but weak focusing => low currents

## 2. Synchro-cyclotron (McMillan-Veksler)

- $B(r)$  decreasing but time varying RF frequency => high energies achievable
- Pulsed operation and weak focusing => very low currents

## 3. The isochronous AVF cyclotron (Thomas focusing)

- Azimuthally varying magnetic fields with hills and valleys
- Allows both isochronism and vertical stability
- CW-operation, high energies and high currents
- Radial sectors => edge-focusing
- Spiral sectors => alternating focusing

# Major milestones in cyclotron development (2)

4. The separate sector cyclotron (Willax)
  - No more valleys=> hills constructed from separate dipole magnets
  - More space for accelerating cavities and injection elements
  - Example PSI-cyclotron at Villingen-Switzerland
  - Very high energy (590 MeV) and very high current (2.5 mA) => 1.5 MWatt
5.  $H^-$  cyclotron (Triumf)
  - Easy extraction of  $H^-$  by stripping
  - Low magnetic field (center 3 kG) because of electromagnetic stripping
  - Triumf is largest cyclotron in the world (17 m pole diameter)
6. Superconducting cyclotron: Fraser/Chalk River/Blosser/MSU
  - High magnetic field (up to 5 Tesla) => high energies at compact design
7. Superconducting synchrocyclotrons (Wu-Blosser-Antaya)
  - Very high average magnetic fields (9 Tesla (Mevion) and almost 6 Tesla (IBA))
  - Very compact => cost reduction => future proton therapy machines?

**THANK YOU FOR  
YOUR ATTENTION**