Interactions of particles with Matter

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Caveat:

- The talk is intentionally qualitative with minimal math, and no in-depth discussion*
- … it is aimed at illustrating some general and well known concepts about particle atomic and nuclear interactions…
- … in particular the nuclear physics part is kept at a (sub?)minimal level and the maximum energy considered is limited at few hundreds MeV

It will likely be disappointing for many and maybe still obscure for non-experts, I apologize in advance

** Extra material with more details is available in another file on Indico*

Credits, in particular but not only: A.Mairani, V.Patera, P.Sala, F.Salvat, PDG…

Overview:

Photon interactions
- Compton
- Photoelectric
- Coherent scattering

Charged particle atomic interactions:
- (average) stopping power
- Landau fluctuations
- Multiple scattering
- Bremsstrahlung and Pair production
  → radiation length

Nuclear interactions
- Elastic/Non-elastic
- hN nuclear interactions
- hA nuclear interactions
- AA nuclear interactions
- Photonuclear interactions

?

Neutronics:
- Reaction types
- Evaluated data files
- Examples of evaluated cross sections
- Caveats

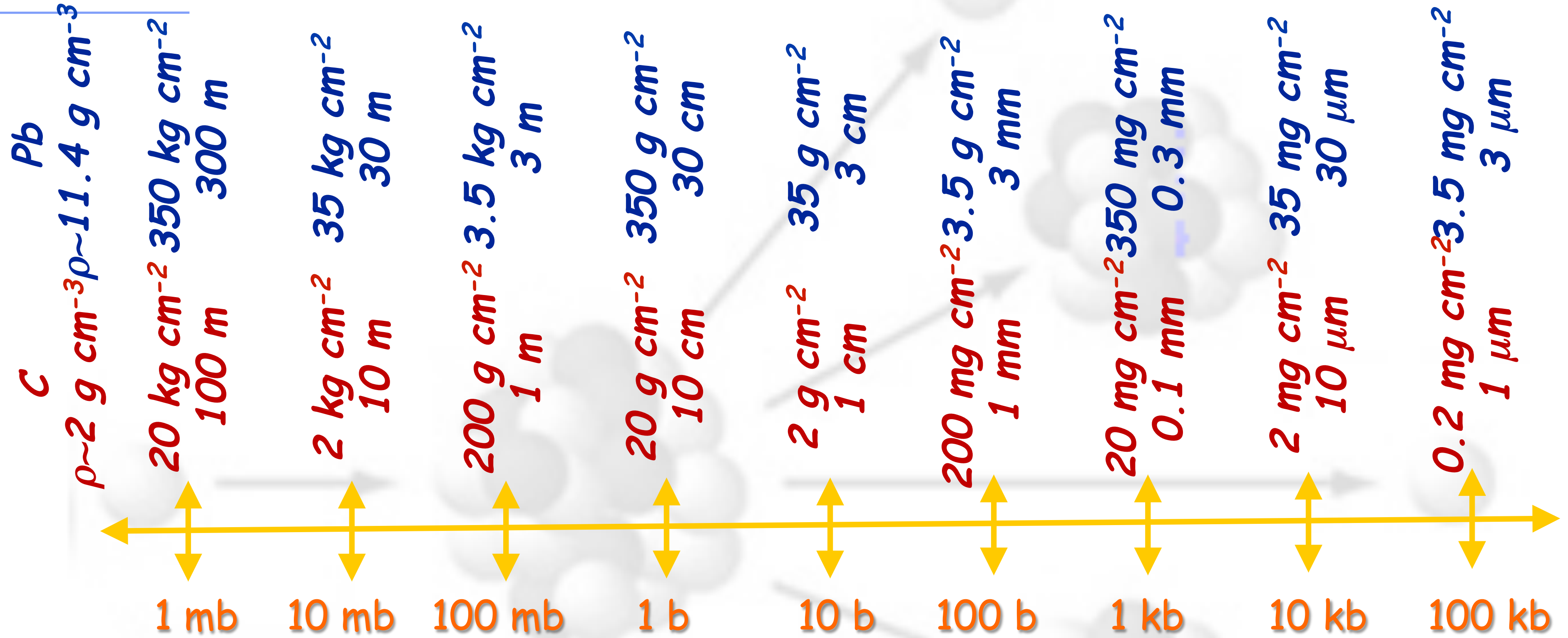
What matters for what:
- Electron machines
- Shielding
- Hadron Therapy
- Isotope production

Miscellaneous
- Deuteron stripping
- ElectroMagnetic Dissociation

High energy showers
- ElectroMagnetic showers
- EM component of hadronic showers
- Spatial development of hadronic showers

Cross section metrics:

Barn (b) = 10^{-24} cm^2

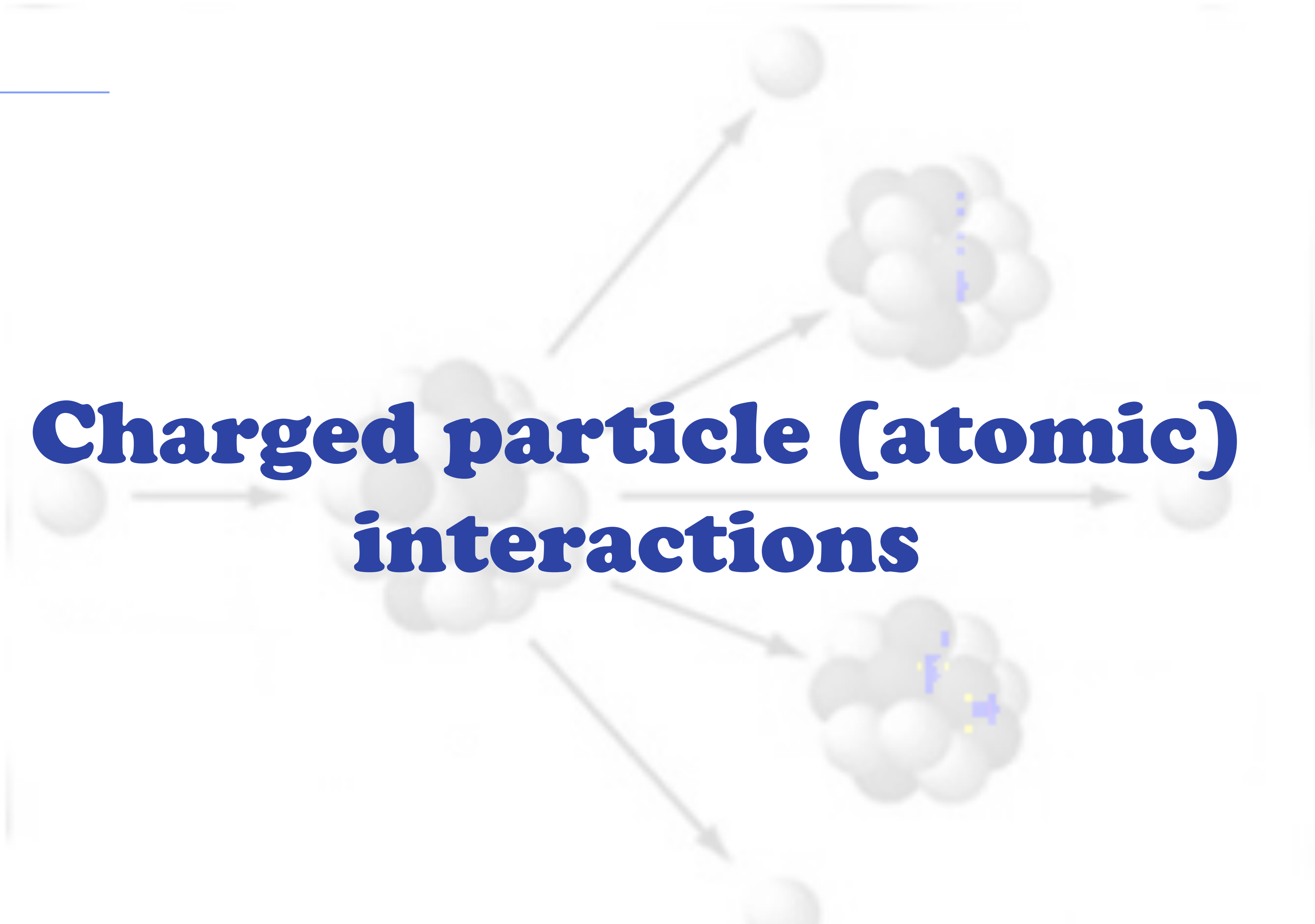


$$\left[\frac{atoms}{volume}\right] = \frac{\rho N_{Av}}{P_A} \qquad \left[\frac{atoms}{mass}\right] = \frac{N_{Av}}{P_A} \qquad \left[\frac{electrons}{mass}\right] = \frac{Z N_{Av}}{P_A}$$

$$\Sigma = \lambda^{-1} = \frac{\rho N_{Av}}{P_A}\sigma \qquad \lambda^{-1}[\mathrm{cm}^{-1}] = \frac{\rho[\mathrm{g/cm^3}] N_{Av}[\mathrm{cm^2 b^{-1} mole^{-1}}]}{P_A[\mathrm{g/mole}]}\sigma[\mathrm{b}]$$

$$\lambda^{-1}[(\mathrm{g/cm^2})^{-1}] = \frac{N_{Av}[\mathrm{cm^2 b^{-1} mole^{-1}}]}{P_A[\mathrm{g/mole}]}\sigma[\mathrm{b}] \quad N_{Av} = 0.6[\mathrm{cm^2 b^{-1} mole^{-1}}] \quad P_A \approx A[\mathrm{g/mole}]$$

Charged particle (atomic) interactions



Charged particles dE/dx

All energy transfers to the target medium are in the end mediated by Coulomb interactions of charged particles following atomic or nuclear interactions

Two main problems:

- Compute the average energy loss, for a given particle, in a given material, at a given energy
- Compute the distribution of actual energy losses around the average value (energy loss fluctuations) *not discussed today*

The problem is to compute the moments of the energy loss distribution

It is a central problem both in dosimetry and in general in radiation physics

Coulomb collisions among charged particles

Rutherford cross section ***(m_{proj}≡m << M_{targ}≡M):***

$$\frac{d\sigma_{Ruth}}{d\Omega} = \frac{z^2 Z^2 r_e^2 m_e^2 c^2}{4\beta^2 p^2 \sin^4\frac{\theta}{2}}$$

using the 4-momentum transfer $q = 2p\sin\frac{\theta}{2}$ $q^2 = 2p^2(1-\cos\theta)$ the cross section becomes:

$$\frac{d\sigma}{dq^2} = \frac{4\pi z^2 Z^2 r_e^2 m_e^2 c^2}{\beta^2 q^4}$$

In this form the cross section is no longer dependent on the ***(m << M)*** assumption and it works in ***every frame!***

Finally, using, ***$T=q^2/2M$*** (T=target recoil energy):

$$\frac{d\sigma}{dT} = \frac{2\pi z^2 Z^2 r_e^2 m_e c^2}{\beta^2 T^2}\left(\frac{m_e}{M}\right)$$

The dependence on the recoil energy is essentially given by the ***$1/T^2$*** term.
It is therefore clear from such formulae, that low energy transfers are much more likely than large ones.

Coulomb collisions: considerations

For a given projectile/energy combination:

- The cross section per atom is given by
  - ✓ **Z times** the cross section on one **electron** ($\div$ Z x 1^2)
  - ✓ **1 time** the cross section on the **atomic nucleus** ($\div$1 x Z^2)
- The q^2 (4-momentum transfer) dependence is the same for light/heavy target/projectile
  - ✓ Energy losses due to interactions on atomic electrons are $\mathbf{M_{nucleus}/m_e}$ **times larger** than those on atomic nuclei ($T=q^2/2M$) for the same q^2
  - ✓ Angular deflections are the same for the same q^2

⇩

- Energy losses are dominated by interactions with electrons (so called electronic stopping power) by a factor $M_{nucleus}/(Z\, m_e) = m_{amu}/m_e\ A/Z$ and are computed as the sum of:
  - Close collisions (collisions energetic enough to be ~ on free electrons)
  - Distant collisions (lower energy transfer, interaction involving the whole atom)
- Angular deflections are mostly due to interactions on atomic nuclei by a factor Z

Close collisions: secondary electrons

The cross section for producing an electron of energy T_e for an incident particle of kinetic energy $T_0 = (\gamma - 1)Mc^2$ (note now $M \equiv m_{proj}$) and charge z is given for spin 0 and spin ½ particles by:

$$\frac{d\sigma}{dT_e} = \frac{2\pi r_e^2}{\beta^2}\frac{m_e c^2}{T_e^2}\left[1-\beta^2\frac{T_e}{T_{max}} + \boxed{\frac{1}{2}\left(\frac{T_e}{T_0+Mc^2}\right)^2}\right]$$

For spin ½ only

The maximum energy transfer, T_{max}, to an electron is dictated by kinematics and given by:

$$T_{\max} = \frac{2m_e c^2\beta^2\gamma^2}{1+2\gamma\frac{m_e}{M}+\left(\frac{m_e}{M}\right)^2}$$

Common approximation

$$T_{\max} \approx 2m_e c^2\frac{p^2}{M^2} \xrightarrow[\beta<<1]{} 4\frac{m_e}{M}T_0 \approx \frac{1}{500}(T_0/A)$$

Similar expressions hold for e^-e^- (Møller, $T_{max}=T_0/2$) and e^+e^- (Bhabha, $T_{max}=T_0$) scattering. In all cases the dependence on the energy of the secondary electron is mostly due to the $1/T^2$ term.

For a 200 MeV/n ion T_{max}~ 400 keV → $R_{CSDA\ H2O}$~2 mm, for 20 MeV e^- → $R_{CSDA\ H2O}$~45 mm
T_{max} determines the extent of the buildup region and of the electronic (dis)equilibrium!

Unrestricted dE/dx for heavy particle:

The (unrestricted) electronic (ionization) stopping power for charged particles heavier than the electron can be obtained summing up distant and close collisions for spin 0 or spin 1/2 particles as:

$$n_e = \frac{\rho Z N_{Av}}{P_A} \quad (n.\ elec.\ per\ unit\ volume)$$

Effective charge

$\sim \ln \beta^4\gamma^4$ relativistic rise

For spin $\frac{1}{2}$ only

$$\left(\frac{dE}{dx}\right)_{hv} = \frac{2\pi n_e r_e^2 m_e c^2 z_{eff}^2}{\beta^2}\left[\ln\left(\frac{2m_e c^2\beta^2 T_{max}}{I^2(1-\beta^2)}\right) - 2\beta^2 + \boxed{\frac{1}{4}\frac{T_{max}^2}{\left(T_0+Mc^2\right)^2}} + 2zL_1(\beta) + 2z^2L_2(\beta) - 2\frac{C}{Z} - \delta\right]$$

Density correction

Mean excitation energy

Barkas (z^3) term

Bloch (z^4) term

Shell corrections

while for electrons ($T_{max} = T_0/2$) and positrons ($T_{max} = T_0$) is given by:

$$\left(\frac{dE}{dx}\right)_{el} = \frac{2\pi r_e^2 n_e m_e c^2}{\beta^2}\left[\ln\frac{T_0^2(\gamma+1)}{2I^2} + \left(1-\beta^2\right) - \frac{2\gamma-1}{\gamma^2}\ln 2 + \frac{1}{8}\left(\frac{\gamma-1}{\gamma}\right)^2 - \delta\right]$$

$$\left(\frac{dE}{dx}\right)_{po} = \frac{2\pi r_e^2 n_e m_e c^2}{\beta^2}\left[\ln\frac{2T_0^2(\gamma+1)}{I^2} - \frac{\beta^2}{12}\left\{23 + \frac{14}{\gamma+1} + \frac{10}{(\gamma+1)^2} + \frac{4}{(\gamma+1)^3}\right\} - \delta\right]$$

dE/dx and range examples:

Continuous Slowing Down Approximation (CSDA) range, ***R_{CSDA}***, or simply ***R*** = total amount of matter traversed by a particle of energy ***E_0*** whenever the energy losses are the average ones

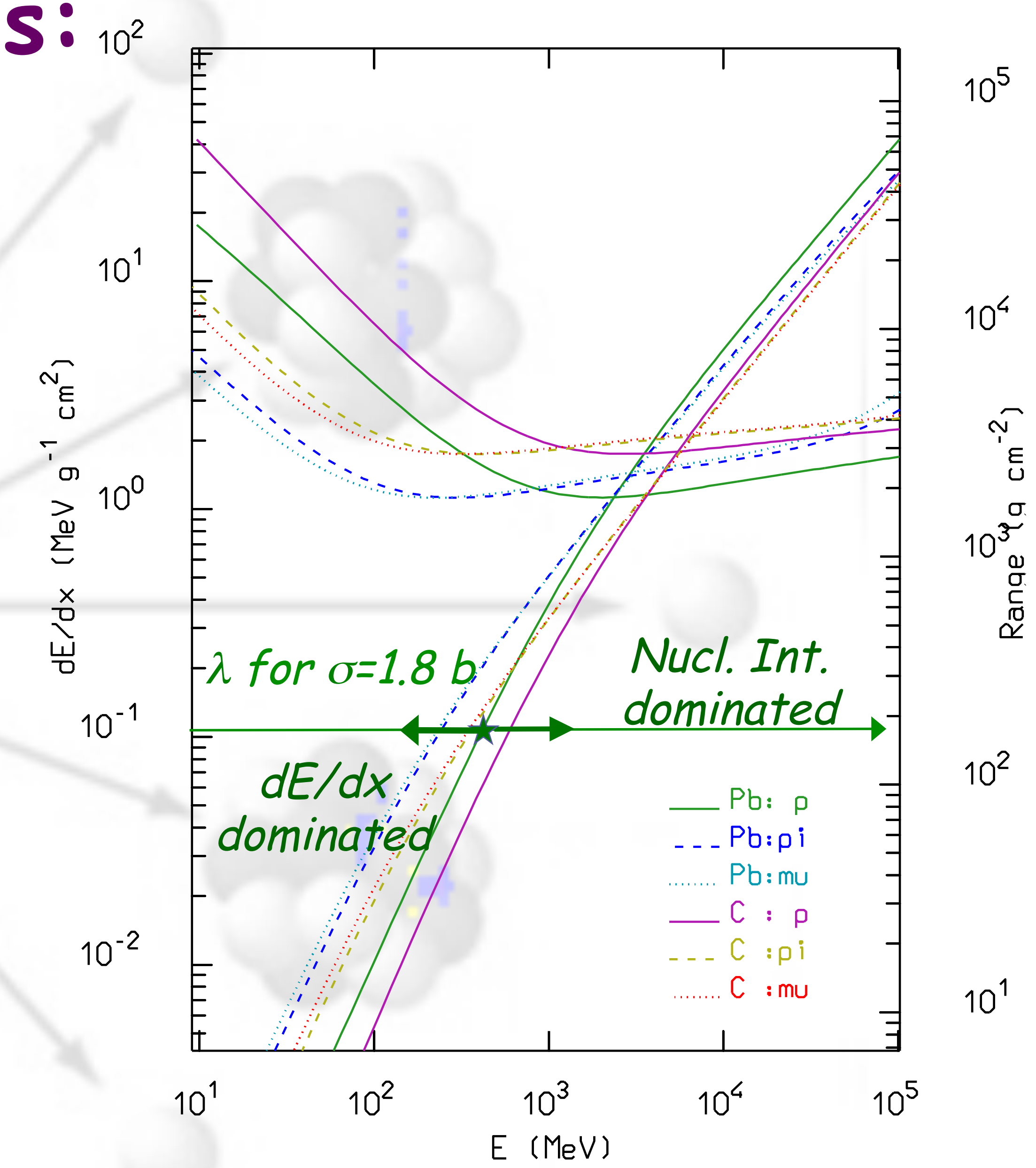
$$R_{csda} = \int_{E_0}^{0} \left(\frac{\mathrm{d}E}{\mathrm{d}x} \right)_{mean}^{-1} \mathrm{d}E$$

It is a useful concept for (heavy) charged particles up to the energy where nuclear reactions dominate.

Since ***dE/dx*** is approx. a function only of the particle velocity, β, and of its charge squared, ***z^2***, the following scaling property holds:

$$R_b(M_b, z_b, p_b) = \left[\frac{M_b / M_a}{z_b^2 / z_a^2} \right] R_a\left(M_a, z_a, p_a = p_b \frac{M_a}{M_b} \right)$$

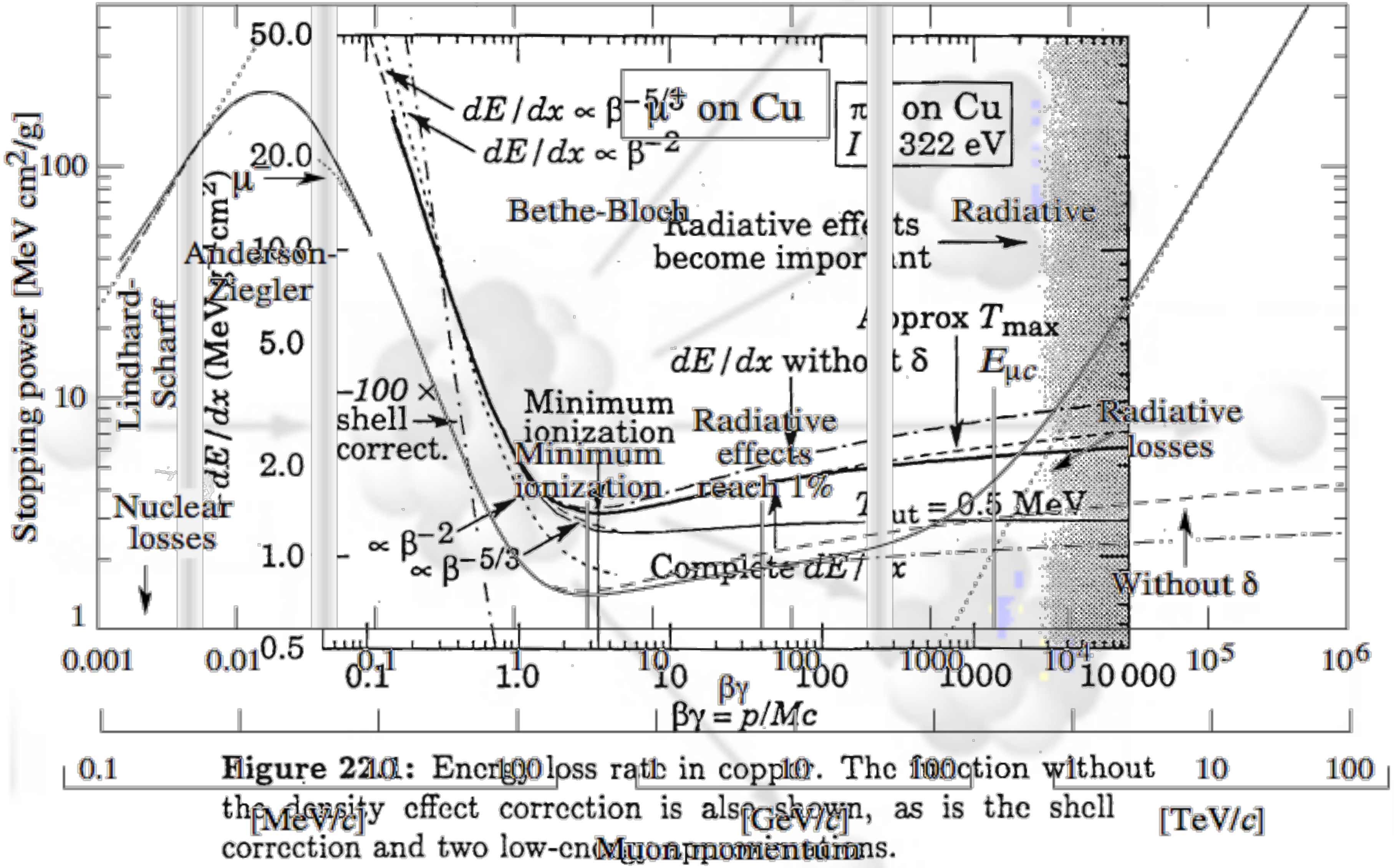
Eg the range of an α particle of momentum ***p_α*** is approximately equal to the range of a proton of momentum ***$p_\alpha/4$***, same momentum per nucleon, eg of energy ***$T_p = T_\alpha/4$*** in the non relativistic regime, and again ***$T_p \sim T_\alpha/4$*** in the relativistic case



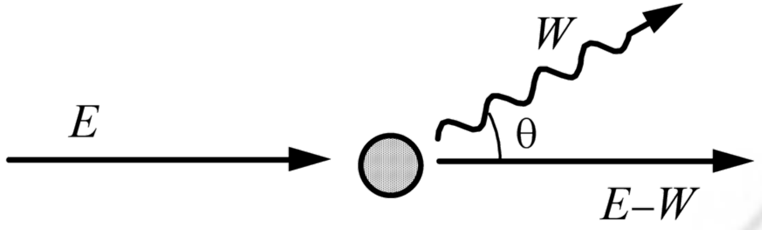
dE/dx: considerations:

- ***dE/dx*** in a given material depends only on the particle velocity, β, and charge, ***z***
- thus particles with the **same velocity** and **charge** have roughly the **same energy loss**.
- if one measures distances in units of ***ρdx, g/cm²***, the **energy loss** is **weakly dependent on the material**, as it goes like ***Z/A*** plus the logarithmic dependence on ***I***
- Obviously ***dE/dx*** depends on the projectile charge squared
- In practice, due to shell corrections, ***dE/dx*** **never behaves like 1/E_k** at low energies
- The energy loss, when plotted as a function of ***βγ=p/Mc***, has a broad minimum at ***βγ~3-3.5***.
- This minimum is almost constant up to very high energies, if the ***restricted energy loss*** (that is the energy loss due to energy transfers smaller than some suitable threshold) is considered
- In practice, most relativistic particles have energy losses in active detectors close to the minimum and are called ***minimum ionizing particles***, or ***mip***'s

Energy loss: examples (from PDG)



Bremsstrahlung:



The classical rate of energy loss for a charged particle experiencing an acceleration ***a*** is given by:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{2e^2}{3c^3}a^2$$

In reality things are much more complex and atomic screening plays a major role in determining the actual bremsstrahlung rate,

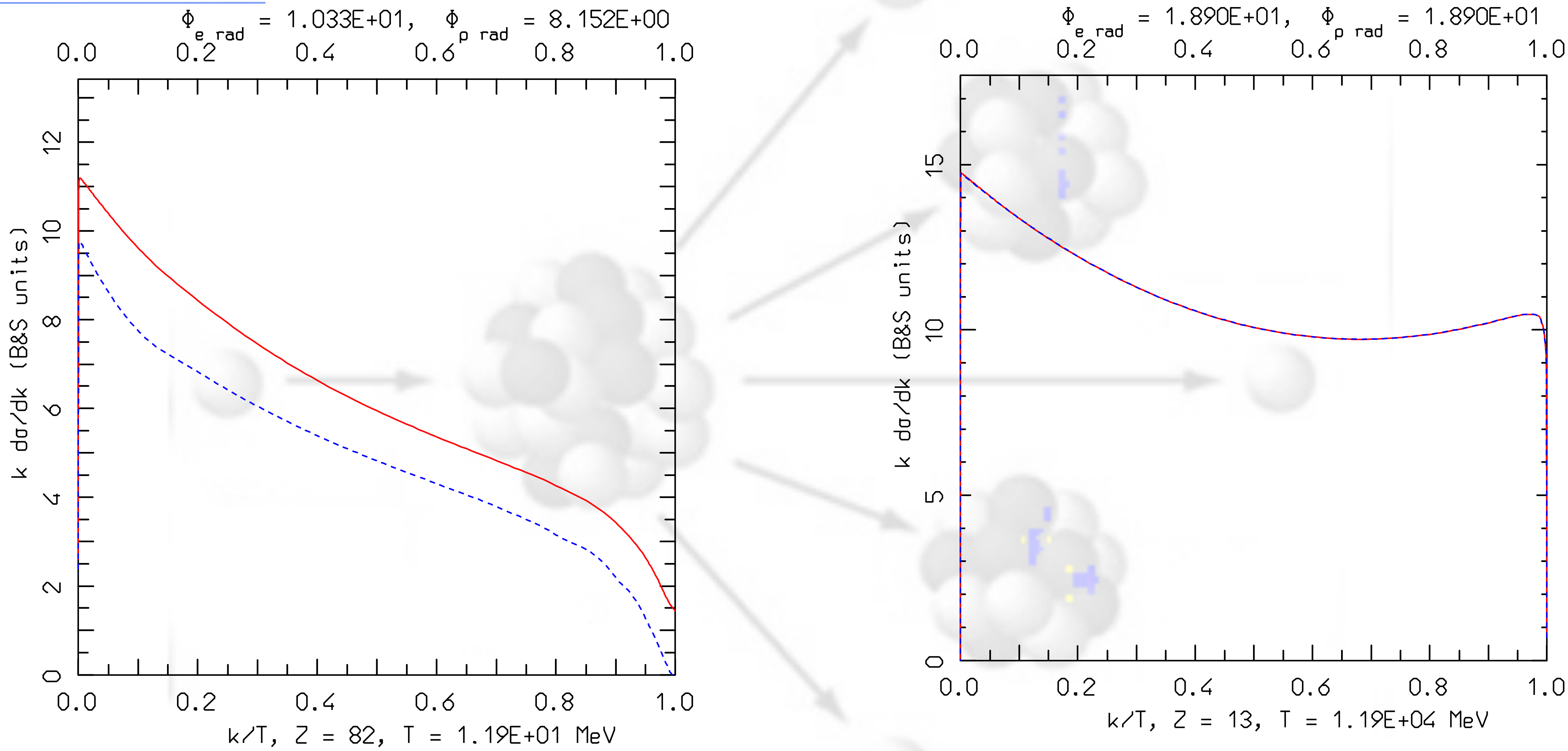
The cross section differential in ***v* = *w*/*E*$_0$** (***w*** photon energy, ***E*$_0$**, incident particle energy)::

$$\frac{\mathrm{d}\sigma_{brem}}{\mathrm{d}v} = \frac{4\alpha r_e^2 z^4 Z^2}{v}\left(\frac{m_e}{m_p}\right)^2 \left\{ \left[\frac{4}{3} - \frac{4}{3}v + v^2\right] \left[\left(\frac{\Phi_1}{4} - \frac{1}{3}\log Z - f_c\right) + \frac{1}{Z}\left(\frac{\Psi_1}{4} - \frac{2}{3}\log Z\right)\right] + \right.$$

$$\left. + \frac{2}{3}(1-v)\left[\frac{\Phi_1 - \Phi_2}{4} + \frac{1}{Z}\frac{\Psi_1 - \Psi_2}{4}\right]\right\}$$

Here ***f*$_c$** is an higher order correction (the so called ***Coulomb correction***), ***Φ*$_1$, *Φ*$_2$** are the (elastic) screening functions for ***nuclear*** bremsstrahlung, and ***Ψ*$_1$, *Ψ*$_2$** are the (inelastic) screening functions for (incoherent) bremsstrahlung on ***atomic electrons***.

Bremsstrahlung spectra: examples



$1/Z^2$ v $d\sigma_{brem}/dv$ for **e- (red)/e+ (blue)** at 12 MeV on Pb (left) and 12 GeV on Al (right)

Pair production:

The matrix elements of the bremsstrahlung are related to those of pair production by the substitution ***k* → - *k*** and ***p* → - *p***, where ***p*** is the four momentum or either the incident particle in the bremsstrahlung emission or the four momentum of one of the pair of particles in the pair production.

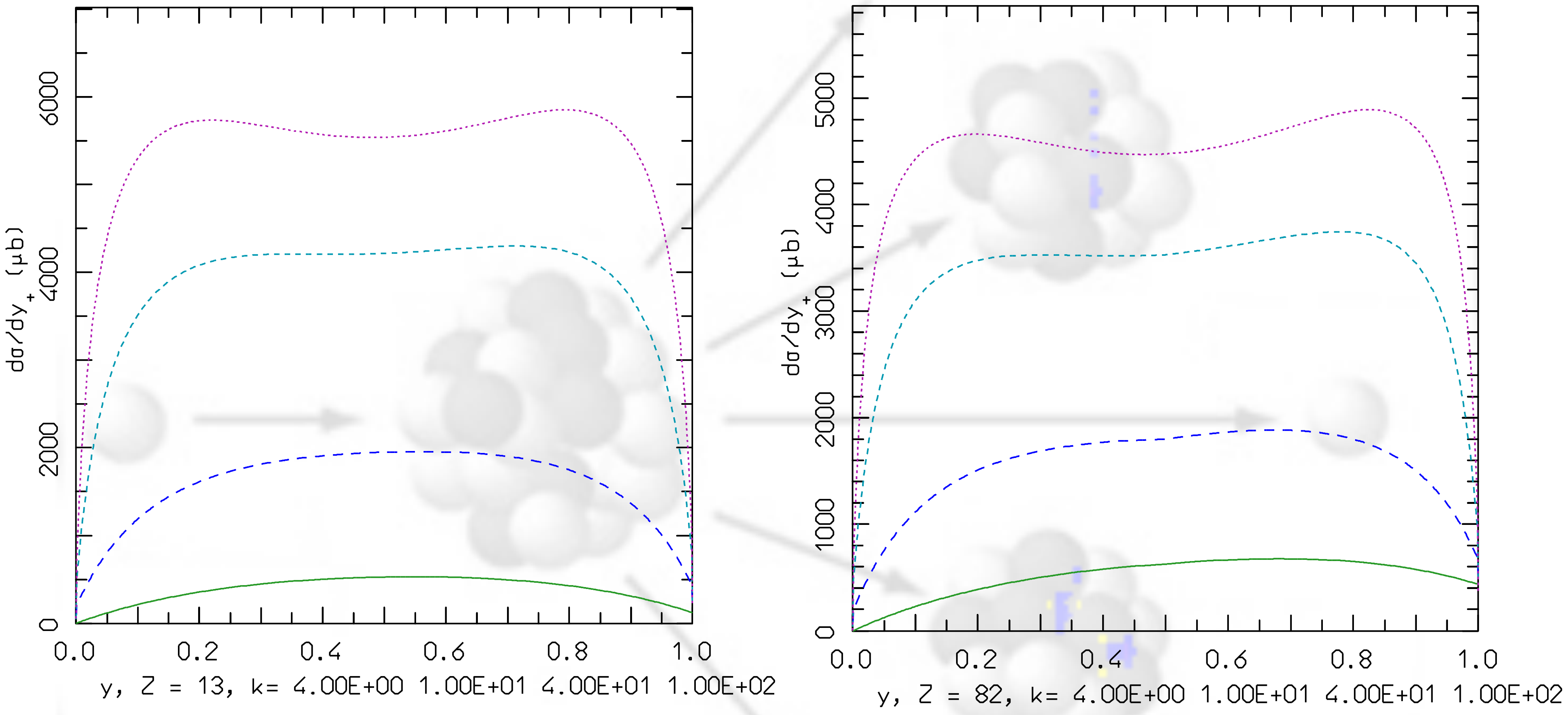
In another way, they Feynman diagram of pair production is the same as the bremsstrahlung one, rotated of 90°, where one of the electron lines, now going back in time, becomes the (outgoing) positron line

The integration of the pair production cross section over all possible angles brings to the usual cross section differential in $u = E_+/k$, as, for instance, reported in the review of Tsai:

$$\frac{d\sigma_{pair}}{du} = 4\alpha r_e^2 z^4 Z^2 \left(\frac{m_e}{m_p}\right)^2 \left\{ \left[\frac{4}{3}u^2 - \frac{4}{3}u + 1\right] \left[\left(\frac{\Phi_1}{4} - \frac{1}{3}\log Z - f_c\right) + \frac{1}{Z}\left(\frac{\Psi_1}{4} - \frac{2}{3}\log Z\right)\right] + \frac{2}{3}u(1-u)\left[\frac{\Phi_1 - \Phi_2}{4} + \frac{1}{Z}\frac{\Psi_1 - \Psi_2}{4}\right]\right\}$$

where again f_c is the ***Coulomb correction***, and Φ_1, Φ_2 are the same (elastic) screening functions as for nuclear bremsstrahlung, while Ψ_1, Ψ_2 the same (inelastic) screening functions as for (incoherent) bremsstrahlung on atomic electrons.

Pair production: examples



$1/Z^2 d\sigma_{pair}/du_+$ for different incoming photon energies (in units of ***$m_e c^2$***)

Radiation length X_0

Integrating the bremsstrahlung cross section gives the result

$$\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{brem} \equiv E_0 X_0^{-1} = \frac{\rho N_A}{P_A} E_0 \int_0^1 \mathrm{d}v\, v \frac{\mathrm{d}\sigma_{brem}}{\mathrm{d}v} =$$

$$= 4\alpha r_e^2 z^4 Z^2 \left(\frac{m_e}{m_p}\right)^2 \frac{\rho N_A}{P_A} E_0 \left[L_{rad}^{fs} + \frac{L'^{fs}_{rad}}{Z} - f_c + \frac{1}{18}\left(1 + \frac{1}{Z}\right)\right]$$

$$\left(\begin{aligned} L_{rad}^{fs} &= \log\left[\frac{m_p}{m_e}\frac{184.15}{Z^{1/3}}\right] \\ L'^{fs}_{rad} &= \log\left[\frac{m_p}{m_e}\frac{1194}{Z^{2/3}}\right] \end{aligned} \right)$$

where the ***radiation length X₀*** has been introduced:

$$X_0 = 1 \Big/ 4\alpha r_e^2 z^4 Z^2 \left(\frac{m_e}{m_p}\right)^2 \frac{\rho N_A}{P_A} \left[L_{rad}^{fs} + \frac{L'^{fs}_{rad}}{Z} - f_c + \frac{1}{18}\left(1 + \frac{1}{Z}\right)\right]$$

X₀ is the length over which the initial energy is reduced to ***1/e***

In a fully analogue manner:

$$\lambda_{pair}^{-1} = \frac{\rho N_A}{P_A} \int_0^1 \mathrm{d}u \frac{\mathrm{d}\sigma_{pair}}{\mathrm{d}u} = 4\alpha r_e^2 z^4 Z^2 \left(\frac{m_e}{m_p}\right)^2 \frac{\rho N_A}{P_A} \left\{ \frac{7}{9}\left[L_{rad}^{fs} + \frac{L'^{fs}_{rad}}{Z} - f_c \right] + \frac{1}{54}\left(1 + \frac{1}{Z}\right)\right\} \rightarrow \lambda_{pair} \approx \frac{9}{7} X_0$$

- Bremsstrahlung (*cm²/g*) scales as (same as pair production):

$$X_0^{-1} \propto \frac{Z^2}{A}$$

Coulomb collisions: Molière cross section

The Molière cross section for particle-Nucleus Coulomb scattering accounting for screening:

$$\frac{d\sigma_{Mol}}{d\Omega} = \frac{d\sigma_{Ruth}}{d\Omega} \times K_{scr}(\theta,\beta) = \frac{d\sigma_{Ruth}}{d\Omega} \times \frac{(1-\cos\theta)^2}{(1-\cos\theta + {}^1\!/_2\,\chi_a^2)^2} = \frac{z^2 Z^2 r_e^2 m_e^2 c^4}{\beta^4 E^2 (1-\cos\theta + {}^1\!/_2\,\chi_a^2)^2} \approx \frac{4 z^2 Z^2 r_e^2 m_e^2 c^4}{\beta^4 E^2 (\theta^2 + \chi_a^2)^2}$$

$$\sigma_{Mol} = \frac{r_e^2 z^2 Z(Z+\xi_e) m_e^2 c^4}{\beta^4 E^2} \frac{4\pi}{\chi_a^2} \frac{1}{1 + {}^1\!/_4\,\chi_a^2}$$

	T(MeV)	χ_a (mrad)	σ_{Mol} (kb)	R (g/cm^2)	Σ^{-1} (g/cm^2)
Al	0.1	38	900	0.0187	4.98×10^{-5}
	1.0	8.4	346	0.555	1.30×10^{-4}
	10.0	1.14	308	5.86	1.46×10^{-4}
Pb	0.1	317	517	0.0311	6.66×10^{-4}
	1.0	35	784	0.784	4.39×10^{-4}
	10.0	4.5	793	6.13	4.34×10^{-4}

MCS: Molière distribution

The Molière distribution is an approximate result which holds for a specific single scattering cross section (the Molière one), for path-lengths not too short, and angles not too large. It is expressed as an universal function, which depends only on one parameter B.

Probability of scattering through an angle θ after travelling a total step length t:

$$F_{Mol}(\theta,t)\mathrm{d}\Omega = 2\pi\chi\mathrm{d}\chi\left[2\,\mathrm{e}^{-\chi^2} + \frac{1}{B}f_1(\chi) + \frac{1}{B^2}f_2(\chi) + \ldots\right]\left[\frac{\sin\theta}{\theta}\right]^{\frac{1}{2}}$$

Bethe correction, not present in the original theory

Gaussian term

$$f_n(\chi) = \frac{1}{n!}\int_0^\infty u\,\mathrm{d}u\,J_0(\chi u)\,\mathrm{e}^{-u^2/4}\left(\frac{u^2}{4}\ln\frac{u^2}{4}\right)^n$$

Single scatt. tail
For $\chi \gg \chi_c$ $f_1 \propto 1/\chi^4$

where the scaled variable is given by: $\chi = \frac{\theta}{\chi_c\sqrt{B}}$ $\qquad \chi_c = \frac{\chi_{cc}t^{1/2}}{\beta^2 E}$

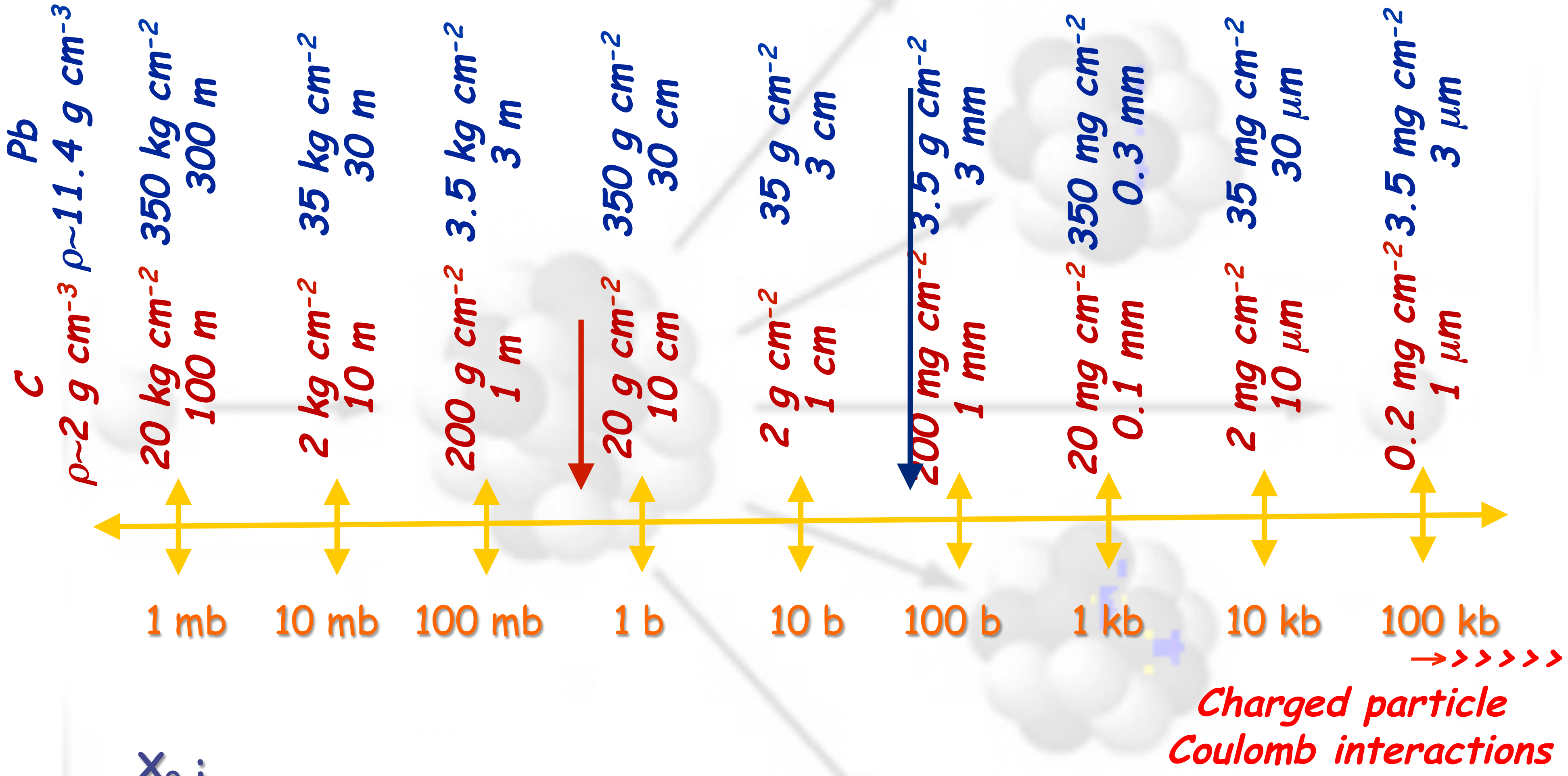
and ***B*** is solution of the transcendental equation (χ_{cc} and b_c are material dependent constants):

$$B - \ln B = b \equiv \ln\Omega_0 \qquad \Omega_0 = \frac{b_c t}{\beta^2}$$

A Gaussian approximation for the MCS distr. (like $\theta_{rms} = \frac{19.2}{\beta p}\sqrt{\frac{t}{X_0}}\left[1 + 0.038\ln\left(\frac{t}{X_0}\right)\right]$ *) can often be found*

However it is not precise and, most important, it ignores the tails!

Cross section metrics: X_0



X_0:
C 42.7 g cm^{-2} (21.3 cm), Cu 12.86 g cm^{-2} (1.44 cm), Pb 6.37 g cm^{-2} (0.56 cm)

Energy loss e⁺/e⁻: examples, Water and Lead

Critical energy ≡ the energy at which collision and radiative energy losses are equal

Given that:

- Bremsstrahlung scales as (same as pair production):

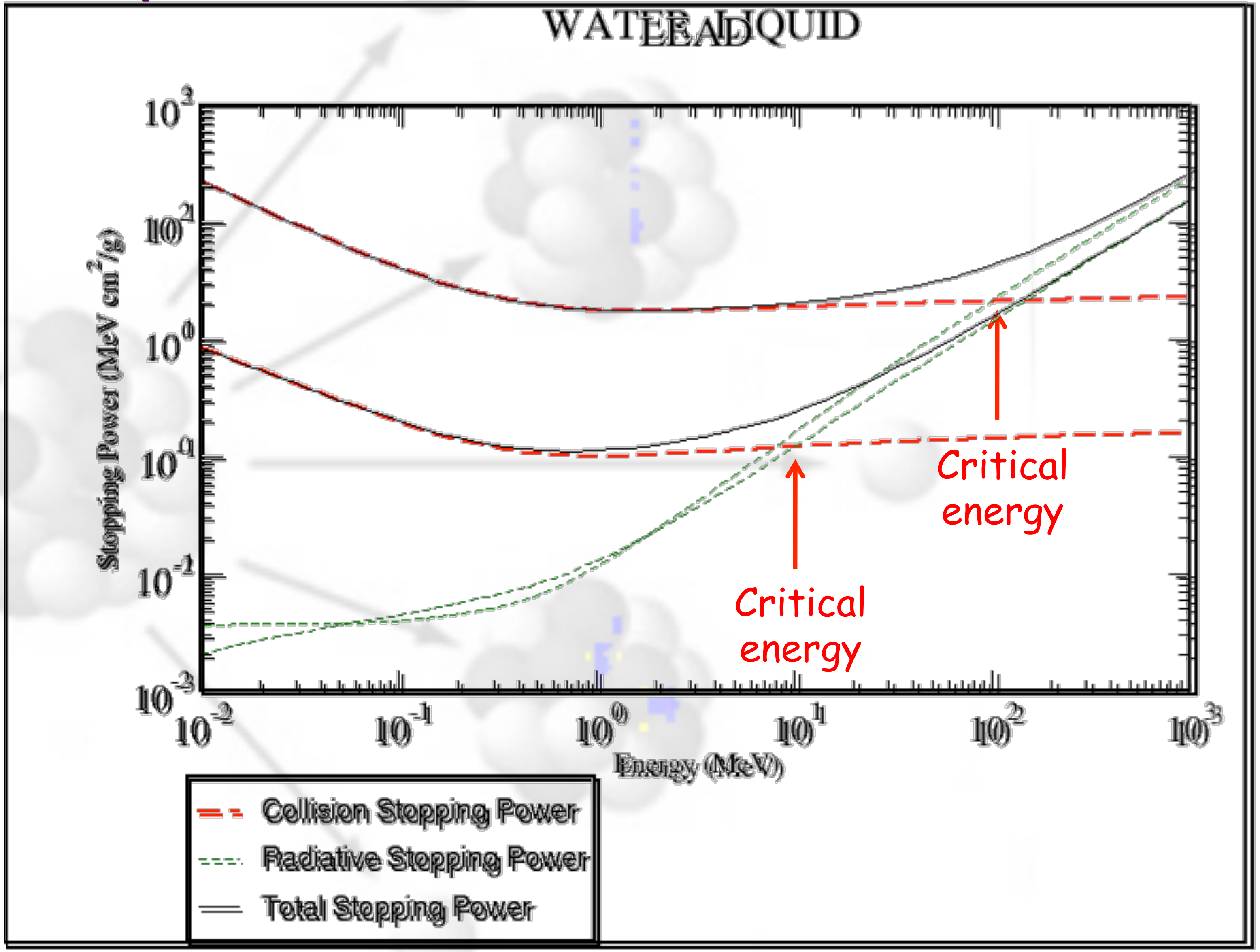
$$X_0^{-1} \propto \frac{Z^2}{A}$$

- Electronic stopping power (dE/dx) scales (roughly) as:

$$\frac{dE}{dx} \propto \frac{Z}{A}$$

→ The critical energy approximately scales as:

$$E_c \propto \frac{1}{Z}$$

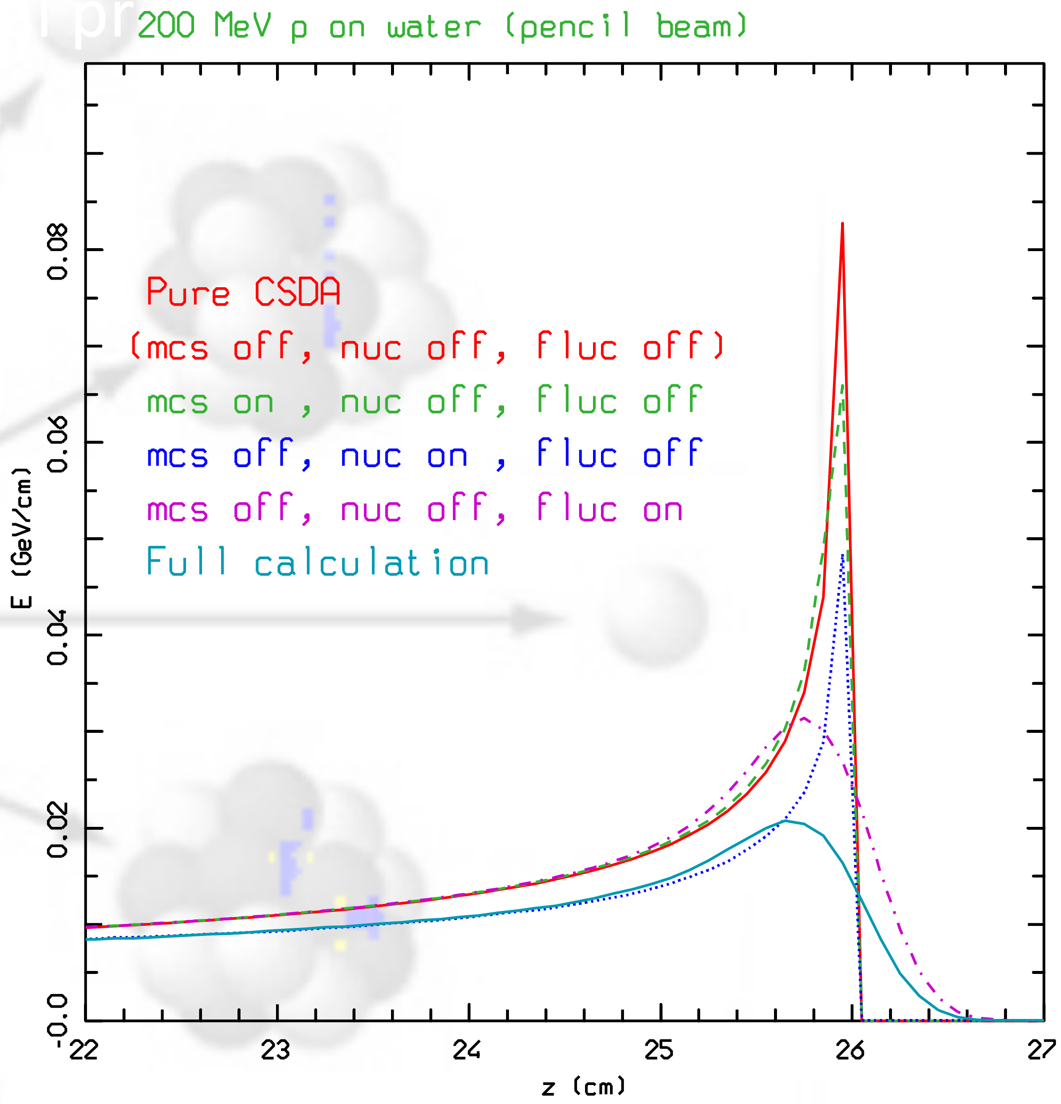


Bragg peaks: ideal proton case

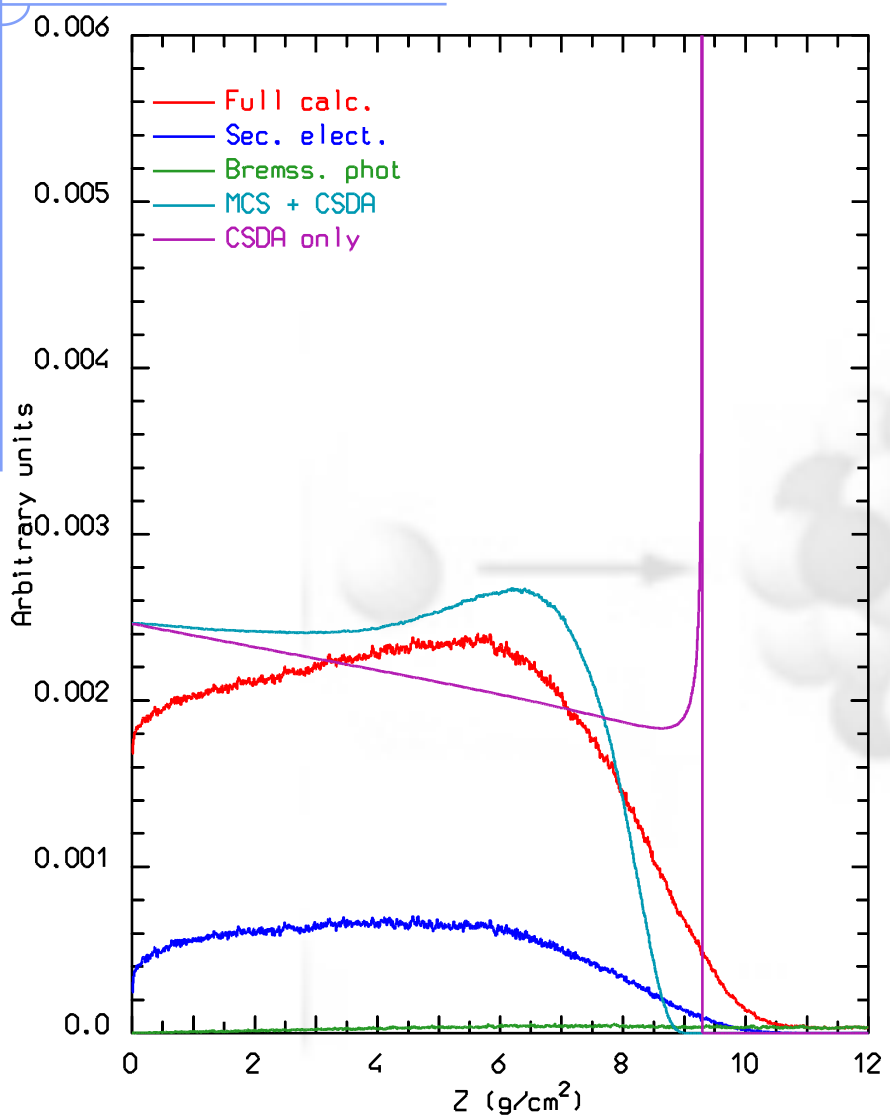
The Bragg peak is a pronounced peak on the *Bragg curve* *(laterally integrated depth-dose curve)* which plots the energy loss of ionizing radiation during its travel through matter. For protons, alpha particles and heavy ions, the peak occurs immediately before the particles come to rest. This is called Bragg peak, for William Henry Bragg who discovered it in 1903.

Curves (200 MeV p on Water):

- **Red**: pure CSDA, no nucl. int.
- **Green**: MCS + CSDA
- **Blue**: CSDA + nuclear int.
- **Purple**: no MCS, no nucl. int, Landau fluct. on.
- **Light Blue**: full calculation



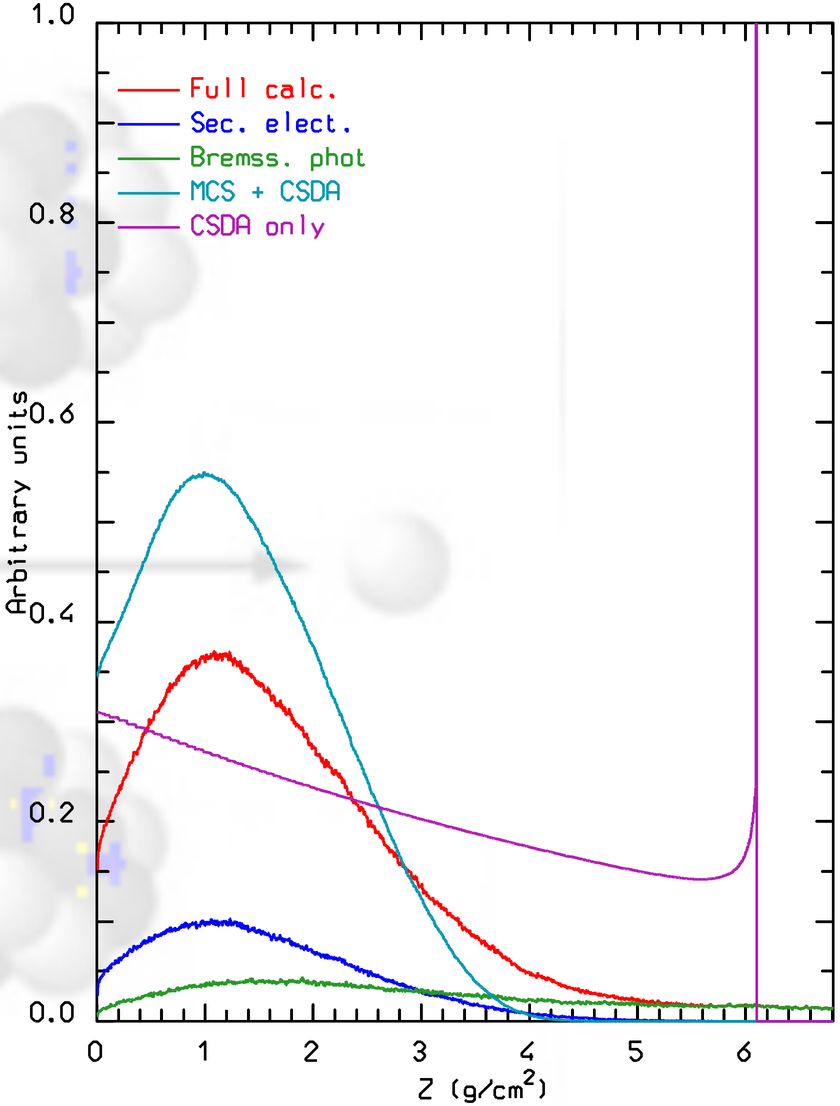
Electron energy losses: complete examples, H_2O and Pb



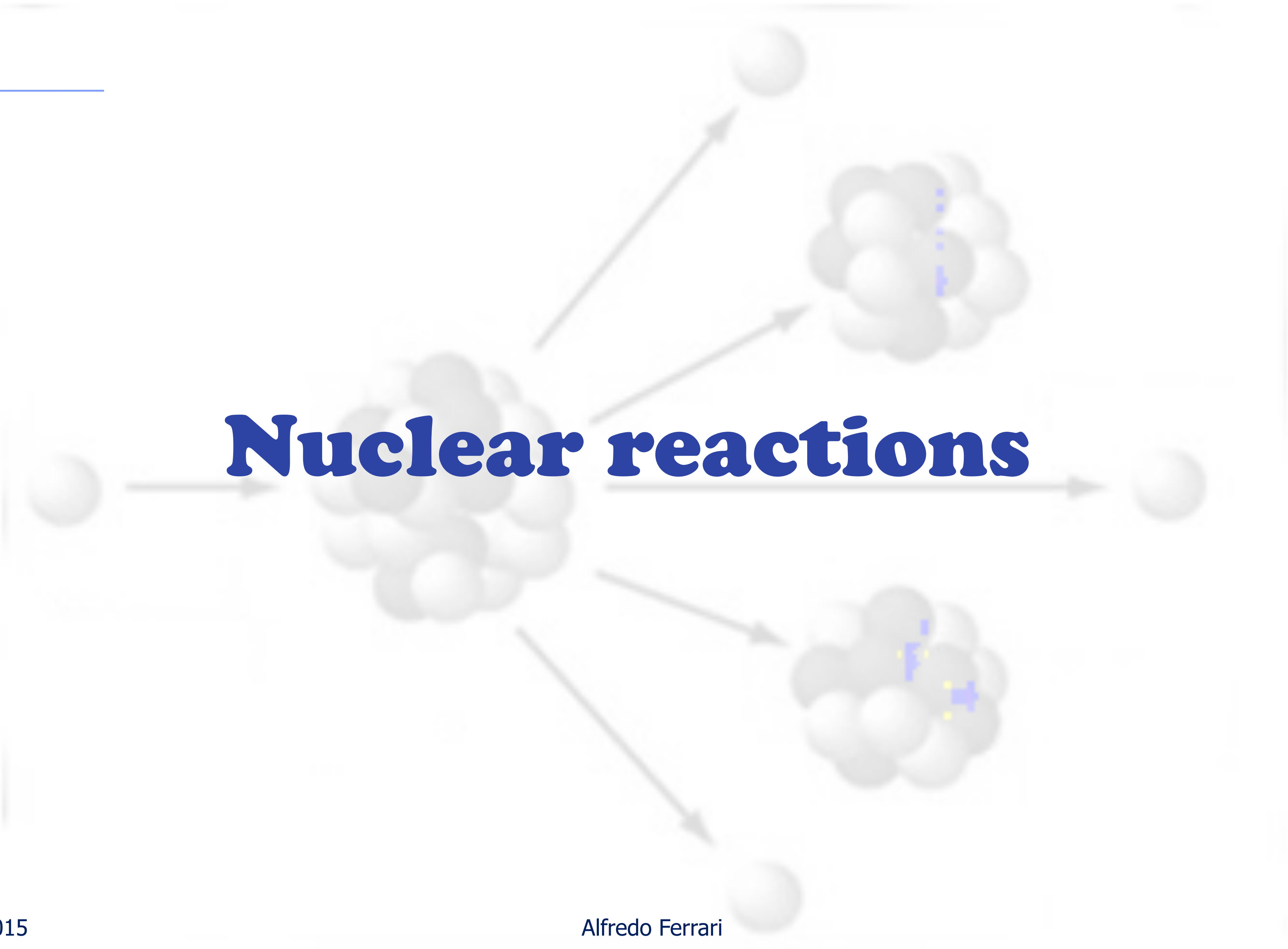
Right: depth-dose curve for 20 MeV e$^-$ on Water
Left: depth dose curve for 10 MeV e$^-$ on Lead

- **Purple:** pure CSDA (bremss. included)
- **Cyan:** MCS + CSDA
- **Red:** full calculation
- **Blue:** secondary electrons (E> 10 keV) contribution
- **Green:** Bremss. photon contribution

Note that the effect of Multiple Coulomb Scattering (MCS) is much more important than for the 200 MeV p example



Nuclear reactions

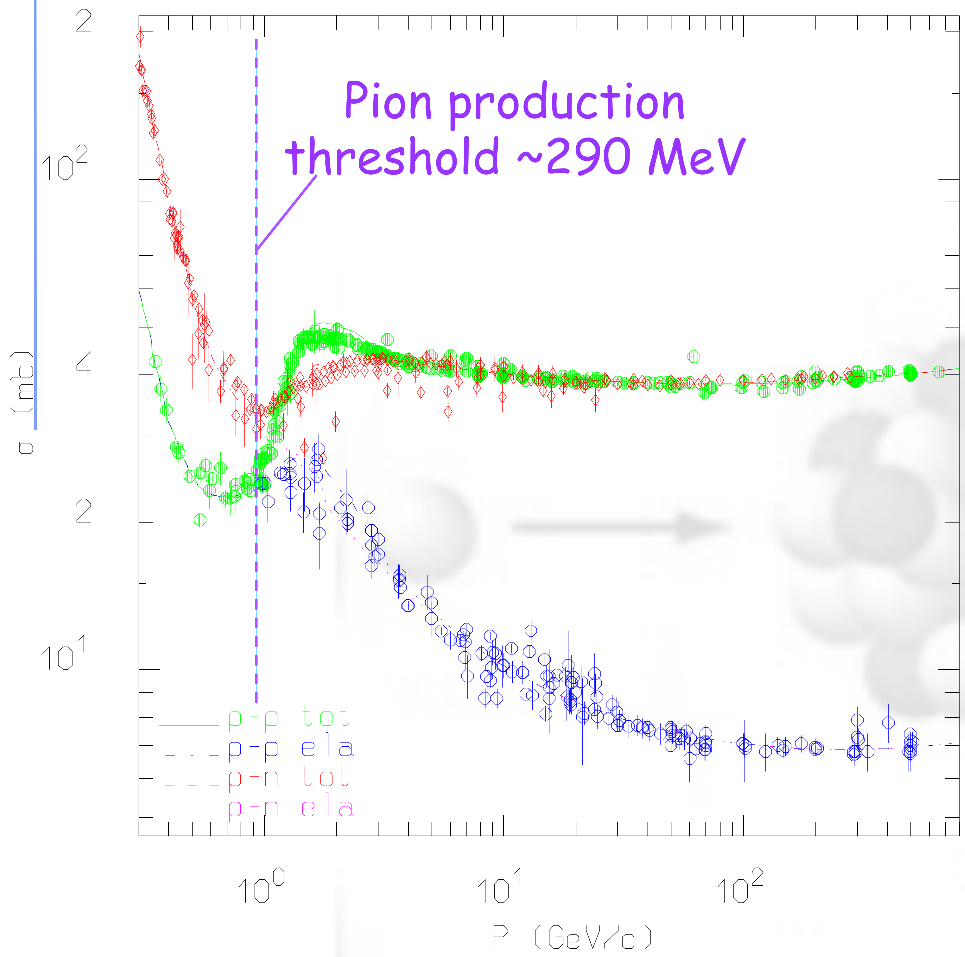


Nuclear interactions: generalities

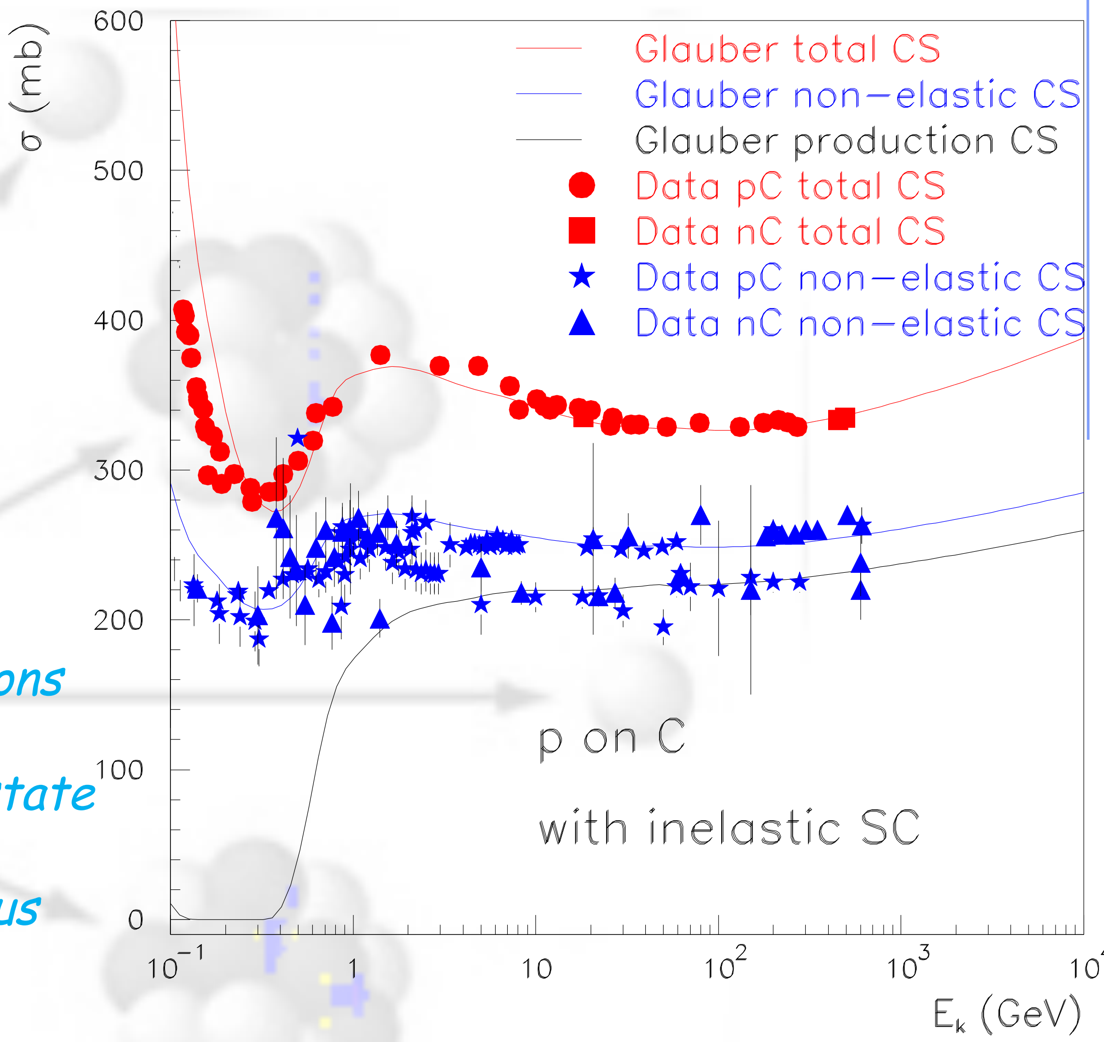
- In order to understand ***Nucleus-Nucleus*** (**AA**) and ***Hadron-Nucleus*** (**hA**) nuclear reactions one has to understand first ***Hadron-Nucleon*** (**hN**) reactions, since nuclei are made up by protons and neutrons
- In general there are two kind of nuclear reactions (for both hN and hA/AA) ***elastic*** and ***non-elastic***:

  - **Elastic** interactions are those that **do not change the internal structure** of the projectile/target and **do not produce new particles**.
    - They transfer part of the projectile energy to the target (lab system)
    - Or equivalently they deflect in opposite directions target and projectile in the centre-of-mass system (CMS) with no change in energy.
    - There is no threshold for elastic interactions
  - **Non-elastic** reactions are those where **new particles are produced** and/or the **internal structure** of the projectile/target **is changed** (e.g. exciting a nucleus).
    - Any specific non-elastic reactions has usually an energy threshold below which the reaction cannot occur (the exception being neutron capture)

From hN to hA cross sections:

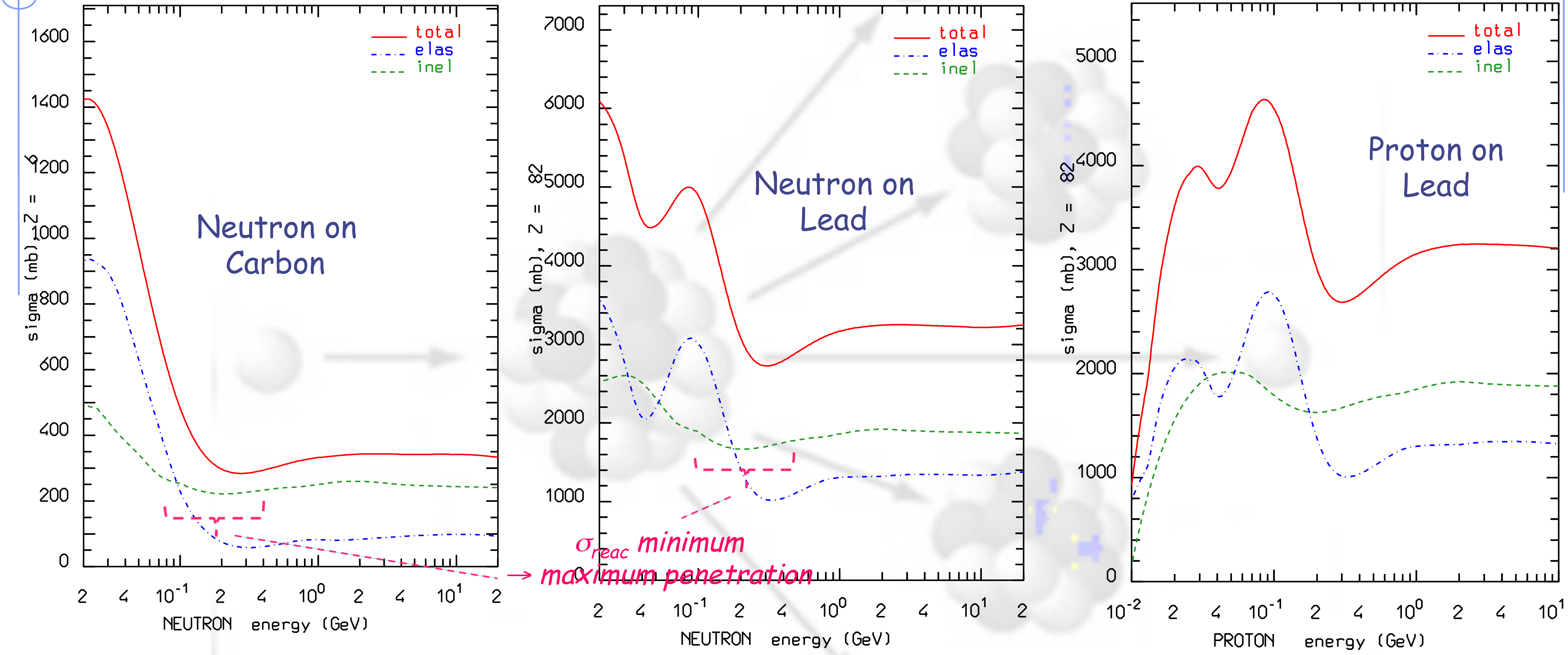


NN cross sections
+
Nucleus ground state
+
Glauber calculus



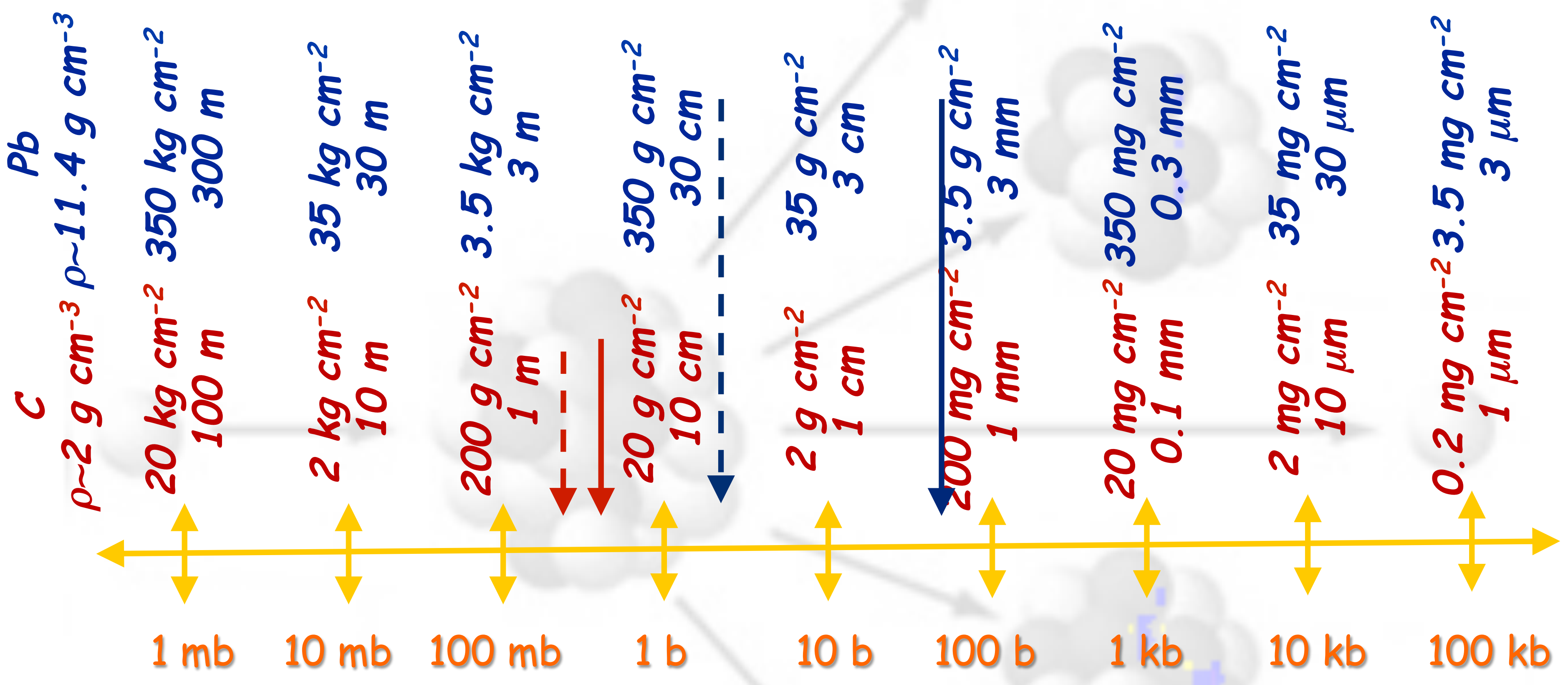
Proton (Neutron) Carbon cross sections computed in self-consistent Glauber approach accounting for inelastic screening starting from NN ones (left)

nA, pA Cross sections:



Hadronic interactions are mostly surface effects ⇒ hadron nucleus cross section scale with the target atomic mass $A^{2/3}$ ($R_{nuc} \div A^{1/3}$)

Cross section metrics: λ_{int}



λ_{int}:

C 85.8 *g* cm^{-2} (42.9 cm), *Cu* 137.3 *g* cm^{-2} (15.3 cm), Pb 199.6 *g* cm^{-2} (17.5 cm)

Simplified scheme of Nuclear interactions

Target nucleus description (**density, Fermi motion**, etc)

⇩

Glauber-Gribov cascade with formation zone

⇩

Generalized IntraNuclear cascade

High energies (above few GeV)

⇩

Preequilibrium stage with current exciton configuration and excitation energy
(not discussed in this lecture)

⇩

Evaporation/Fragmentation/Fission

⇩

γ deexcitation

t (s)

10^{-23}

10^{-22}

10^{-20}

10^{-16}

… INC, a bit like snooker…

The … s
rep … ion can
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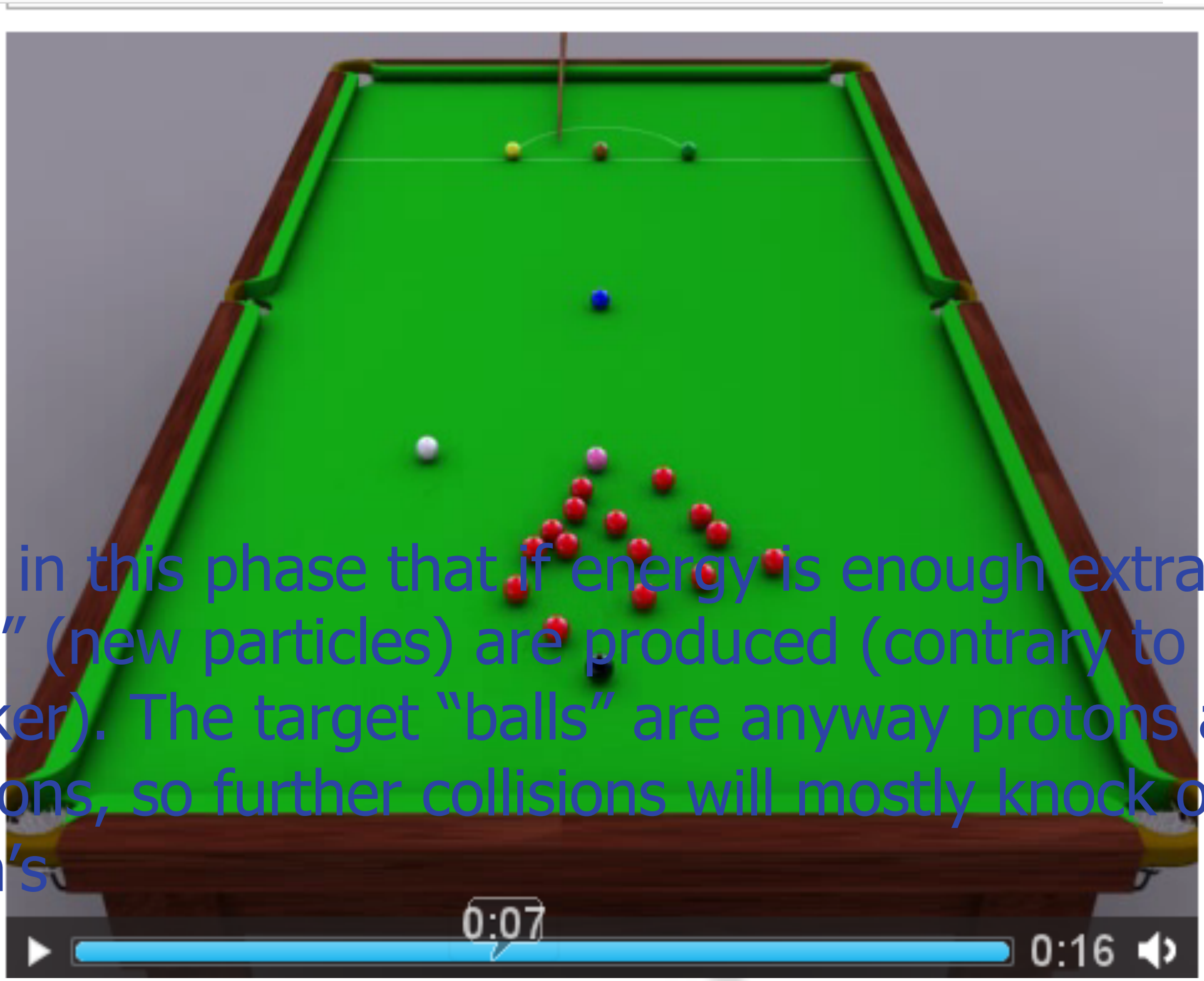


…it is in this phase that if energy is enough extra "balls" (new particles) are produced (contrary to snooker). The target "balls" are anyway protons and neutrons, so further collisions will mostly knock out p's and n's

Evaporation:

After many collisions and possibly particle emissions, the residual nucleus is left in a highly excited "equi *eus*). De-excitation can be described by **statisti** **evaporation** of "droplets", actually **low energy**, i **s (p, n, d, t, 3He, alphas…)** from a "boiling" soup nperature"

Since only neutrons h come, **neutron emission is strongly favoured**.

When the excitation e paration energy (energy needed to break the nuclear b **mpt photons** are emitted during **gamma deexcitation**

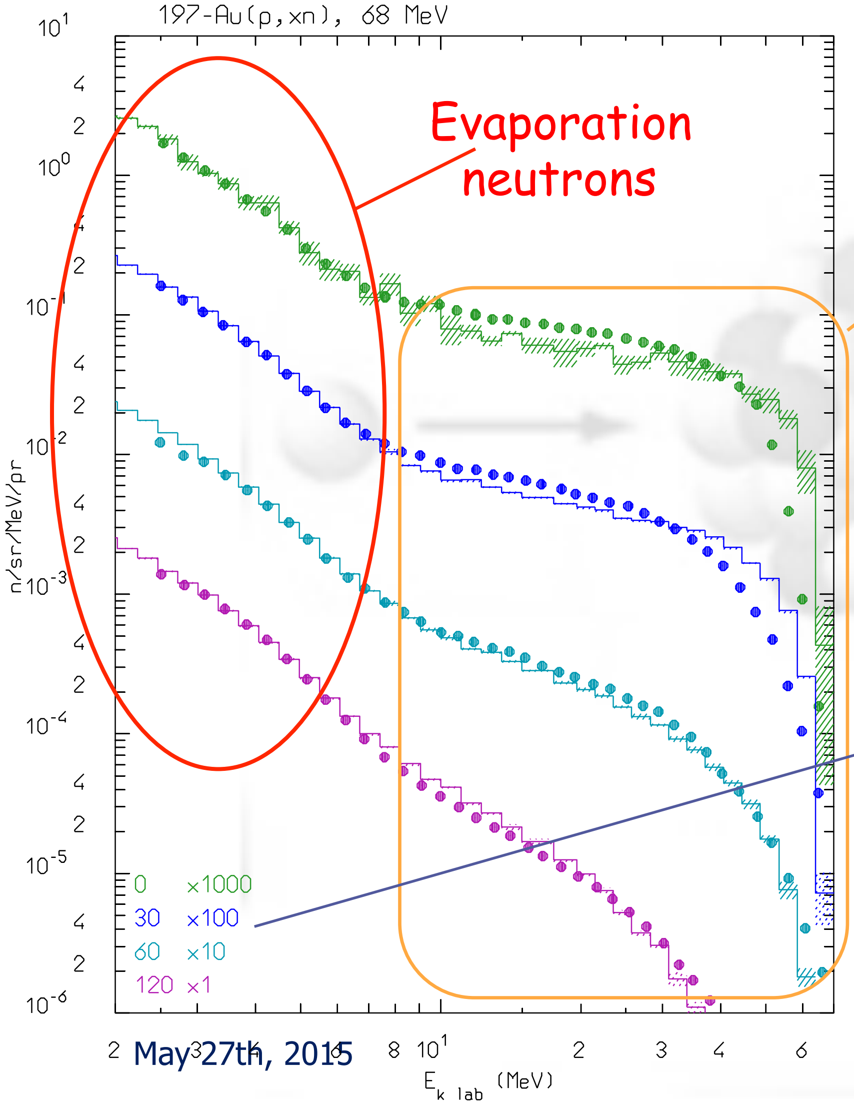
The process is termin s spent

→ the leftover nucle 'cold", with **typical recoil energies ~ MeV**.

For heavy nuclei the initial excitation energy can be large enough to allow breaking into two major chunks (**fission**).

Thick target examples: neutrons

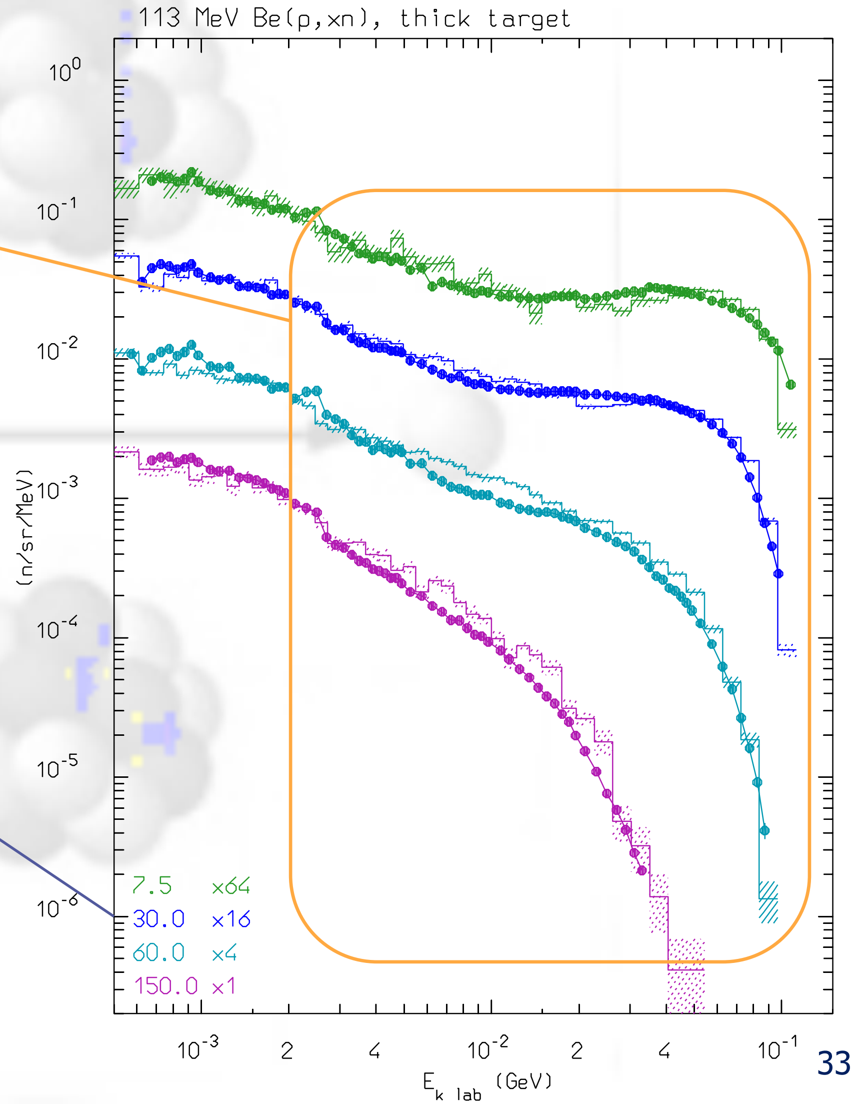
^{197}Au(p,xn) @ 68 MeV, stopping target
Data: JAERI-C-96-008, 217 (1996)



Cascade, preequilibrium neutrons

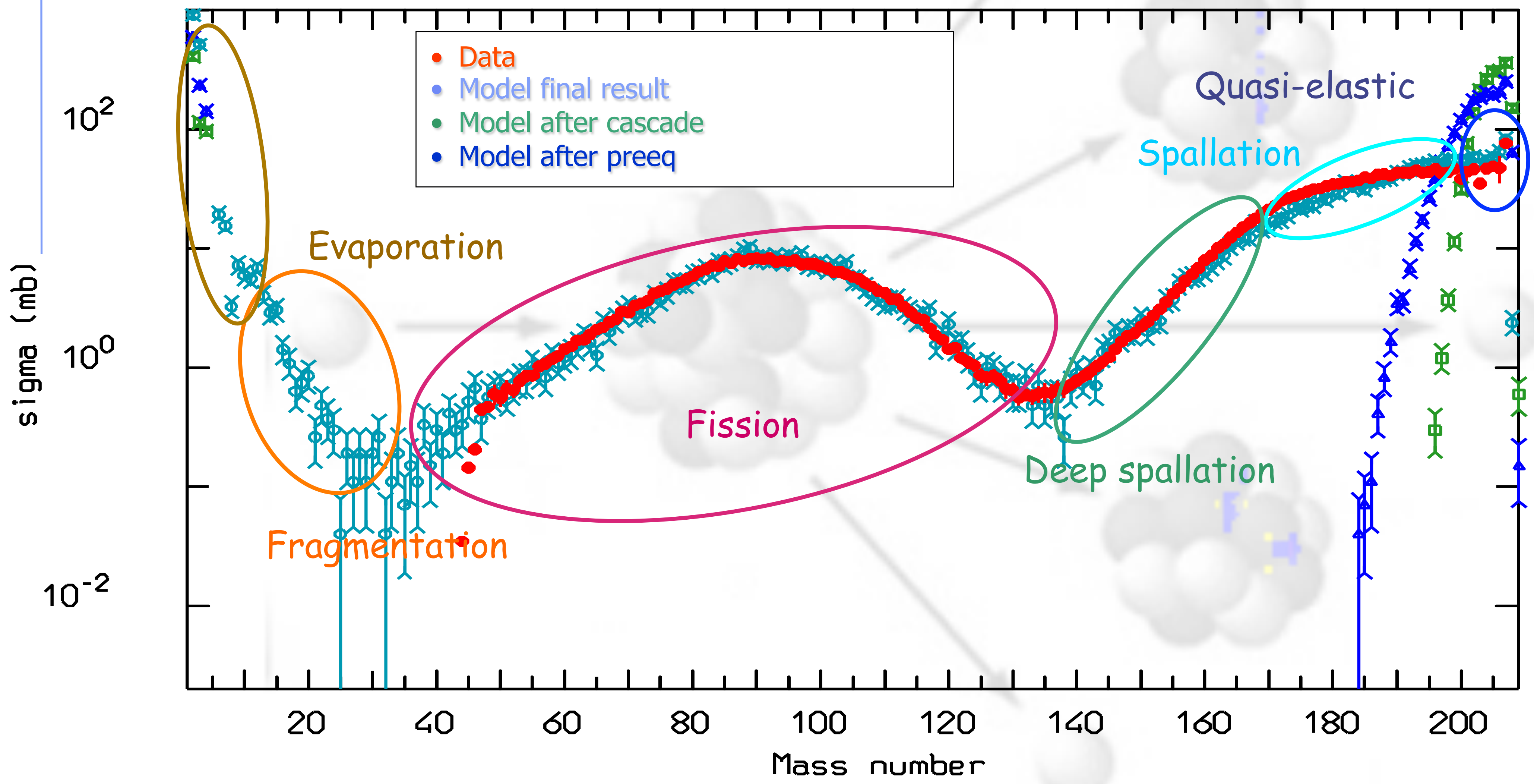
Double differential neutron yield, energy spectra at various angles

^{9}Be(p,xn) @ 113 MeV, stopping target
Data: NSE110, 299 (1992)



Example of fission/evaporation

1 A GeV ^{208}Pb + p reactions Nucl. Phys. A 686 (2001) 481-524

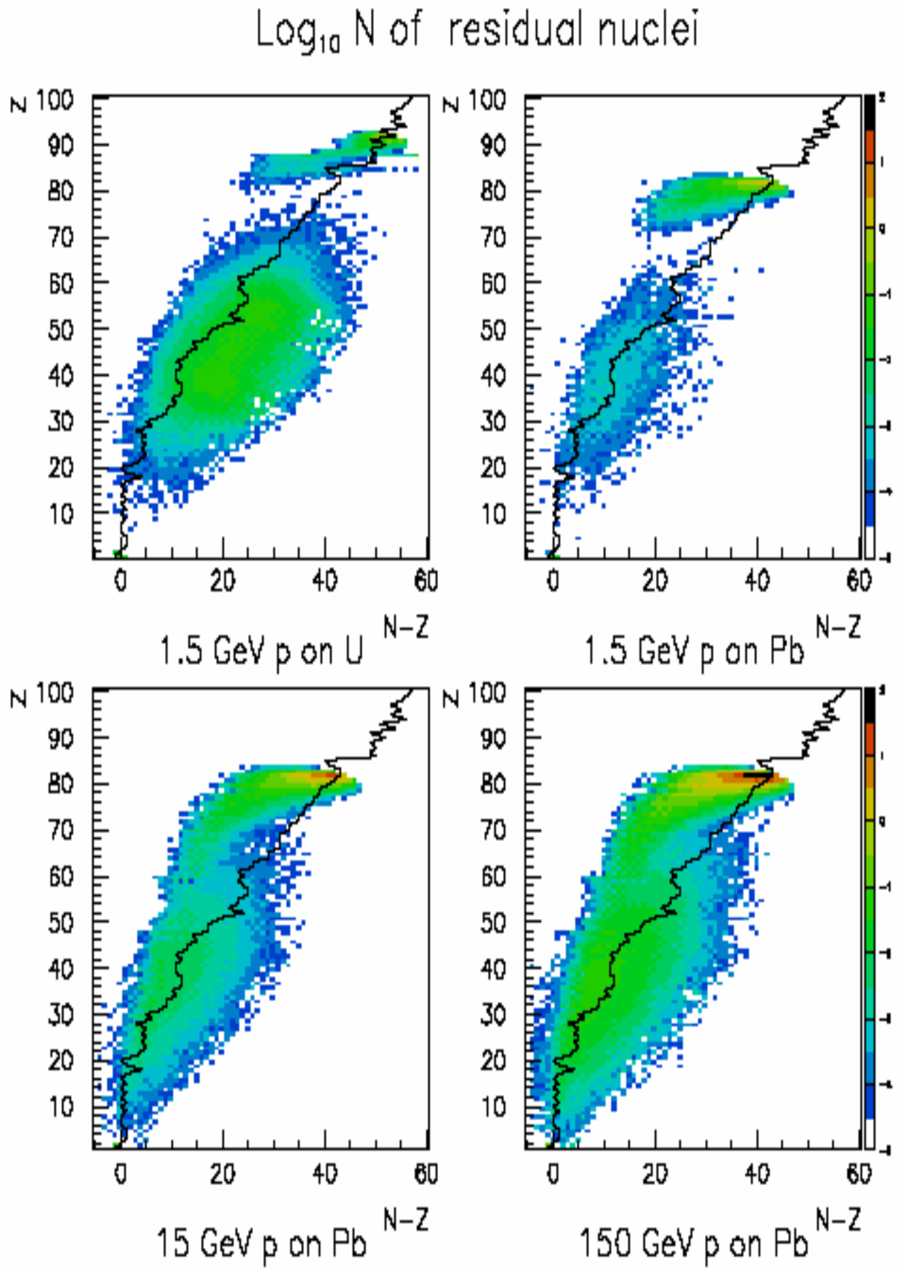


The production of specific residuals is the result of the very last step of the nuclear reaction, thus it is influenced by all the previous stages

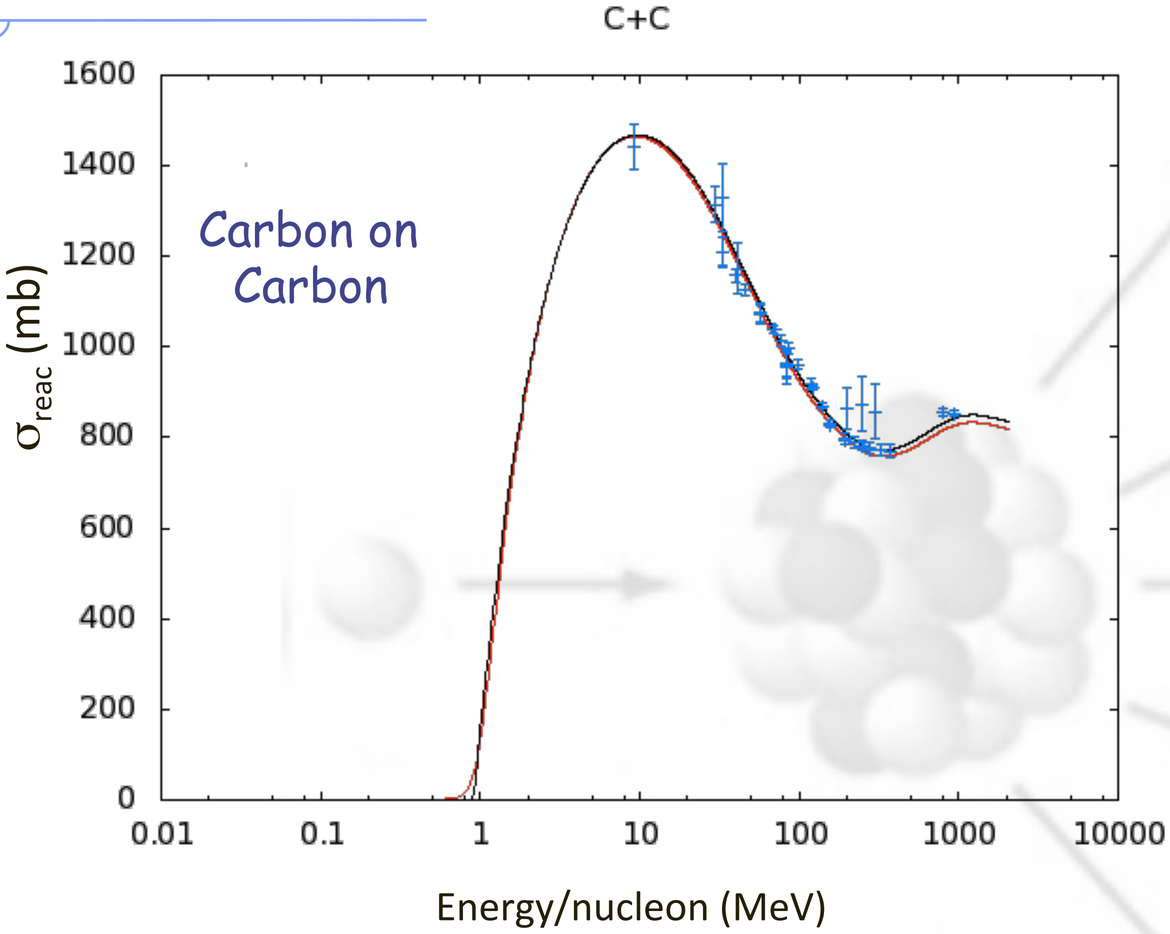
Residual Nuclei

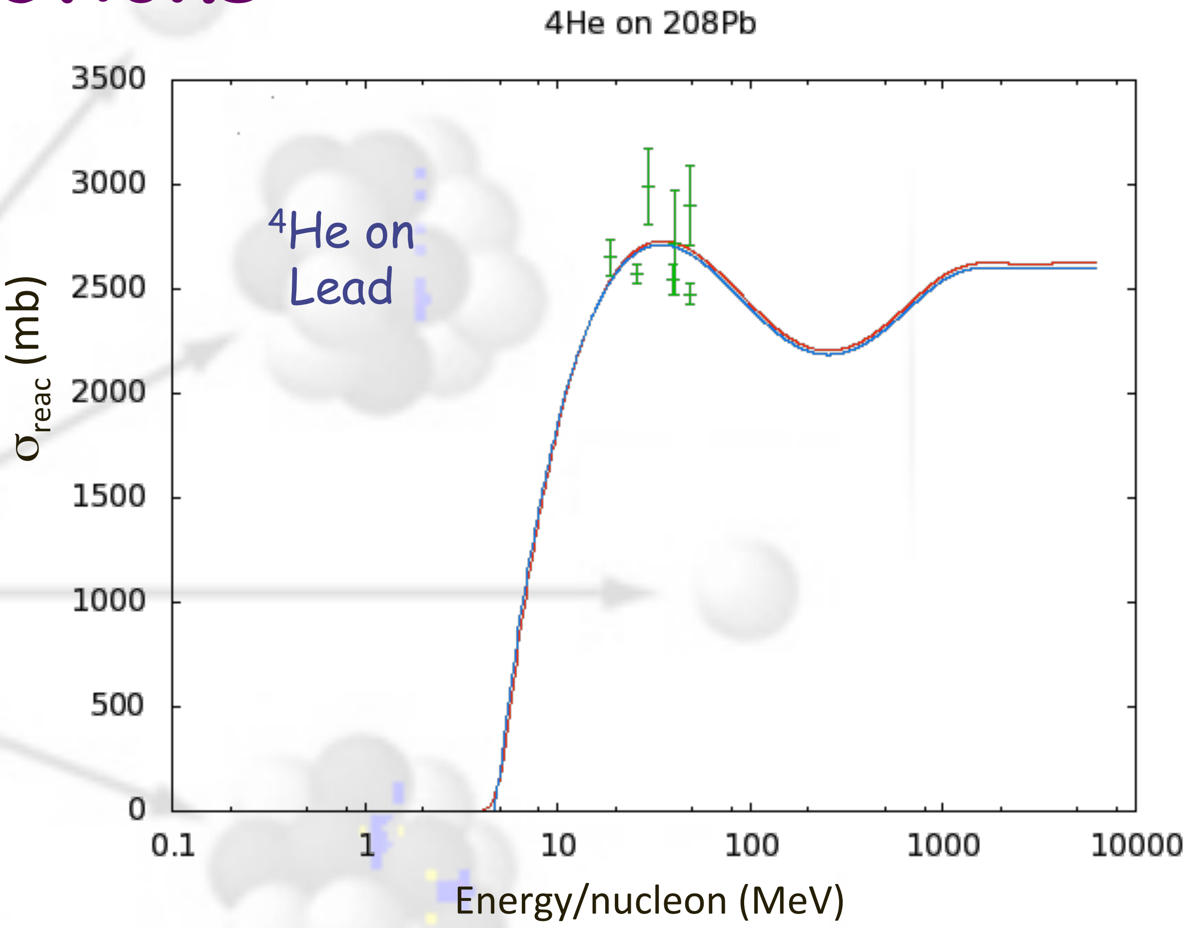
Right: color scale of isotope production as a function of **neutron excess** (x-axis) and **atomic number** (y-axis) for various proton energy/target combo's. The **black line** is the **stability line**

- Particle beams tend to produce proton rich isotopes because of the preference for evaporating neutrons rather than charged particles
- Isotopes produced by fission are typically neutron rich (at least for fission on actinides)
- → there is an obvious complementarity between the two techniques



Ion-Ion reaction cross sections:





$$\sigma_{reac} \approx \pi \left(R_A + R_B\right)^2 \left(1 - \frac{V_{Coul}}{E_k}\right)$$

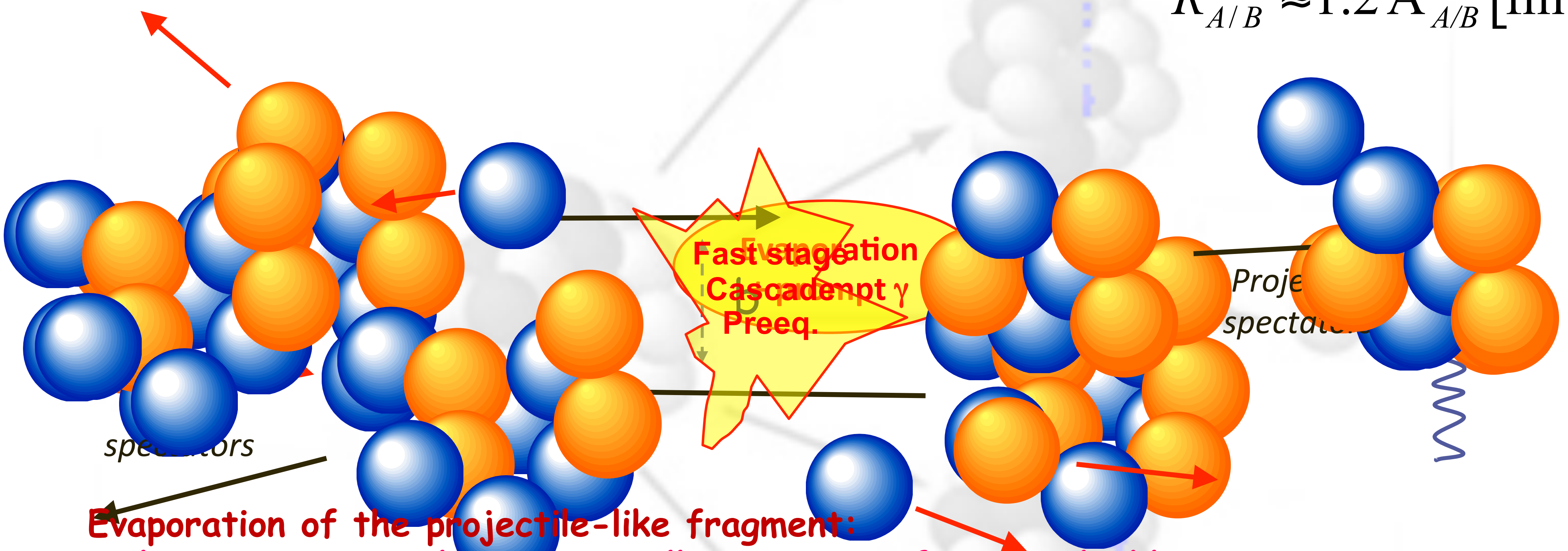
$$V_{Coul} \approx 1.4 \frac{Z_A Z_B}{A_A^{1/3} + A_B^{1/3}} [\mathrm{MeV}]$$

$$R_{A/B} \approx 1.2\, A_{A/B}^{1/3} [\mathrm{fm}]$$

"Heavy" ion nuclear interactions:

$$\sigma_{reac} \approx \pi \left(R_A + R_B\right)^2$$

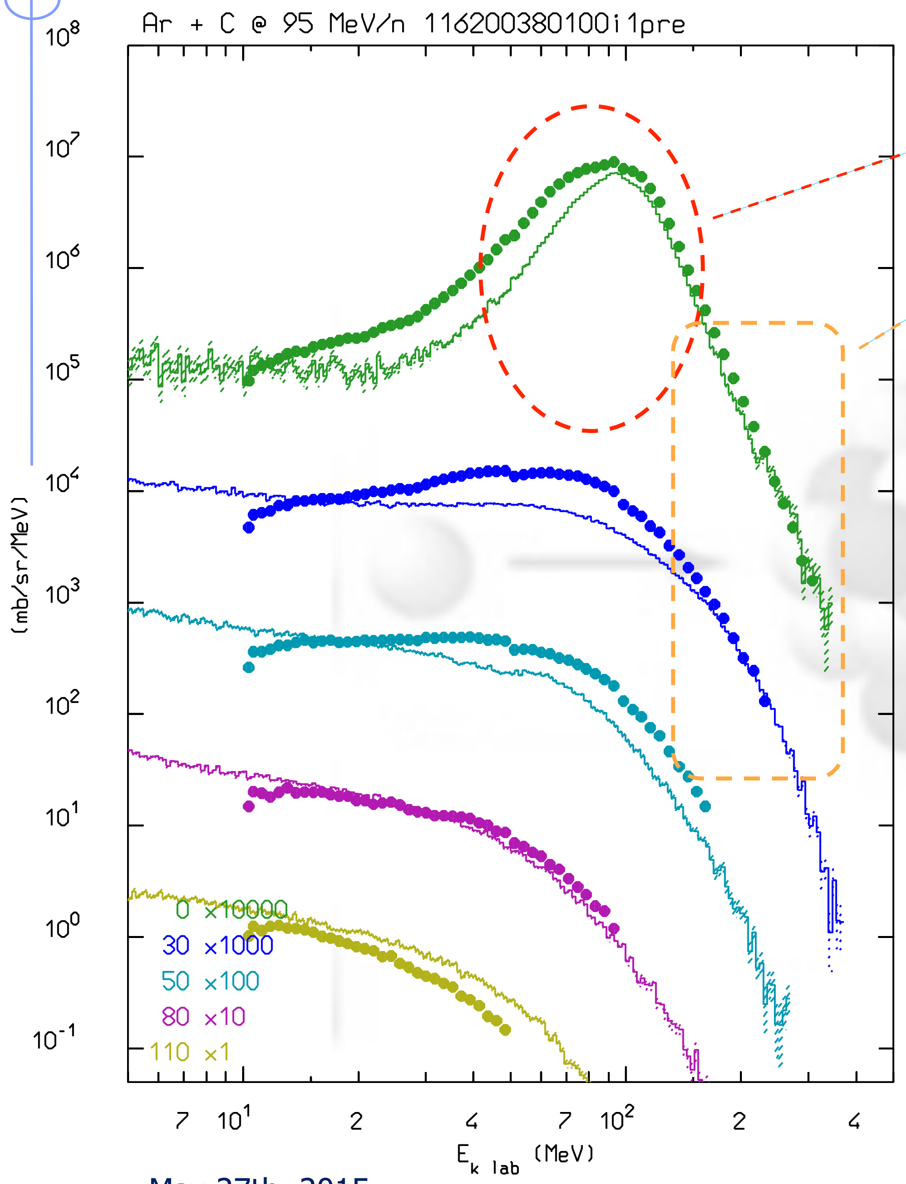
$$R_{A/B} \approx 1.2\,\mathrm{A}_{A/B}^{1/3}\,[\mathrm{fm}]$$



Evaporation of the projectile-like fragment:

- *low energy particles isotropically in its rest frame → highly energetic ones, strongly forward peaked, in the lab frame (including the final residual)!*
- *Contrary to hA collisions there could be a projectile-like residual flying forward*

Examples of (thin target) neutron emission spectra:



Projectile evaporation forward peak

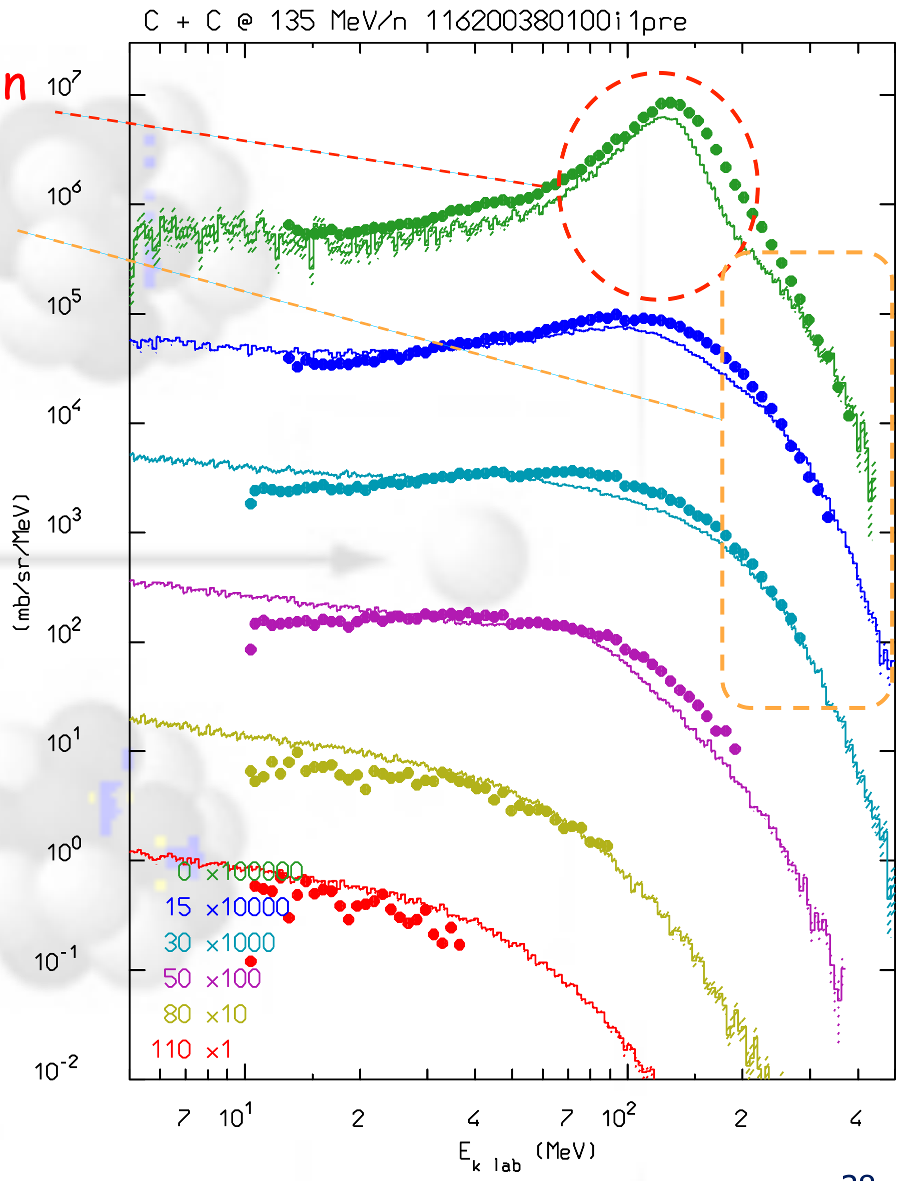
High energy tails ($\gg E_{beam}/n$)

C on C @ 135 MeV/n (0,15,30,50,80,110 deg)

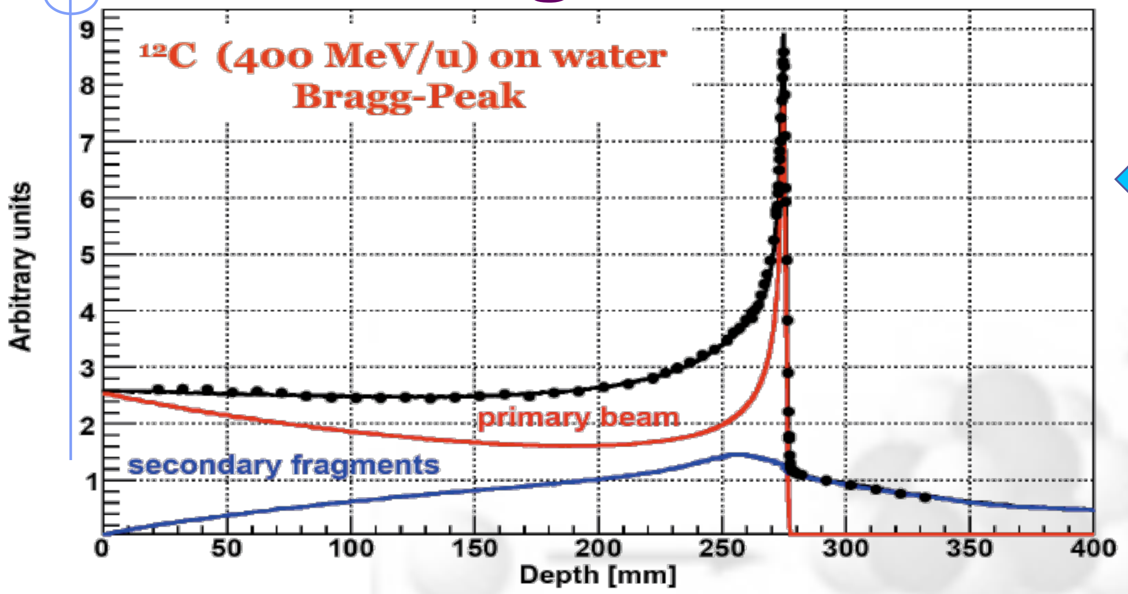
Ar on C @ 95 MeV/n (0,30,50,80,110 deg

Neutron double differential spectra at various angles

Symbols: exp. data

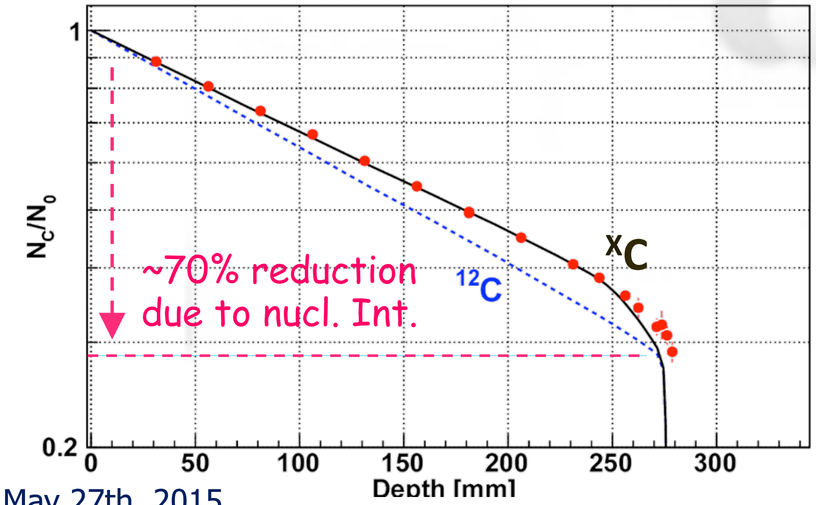


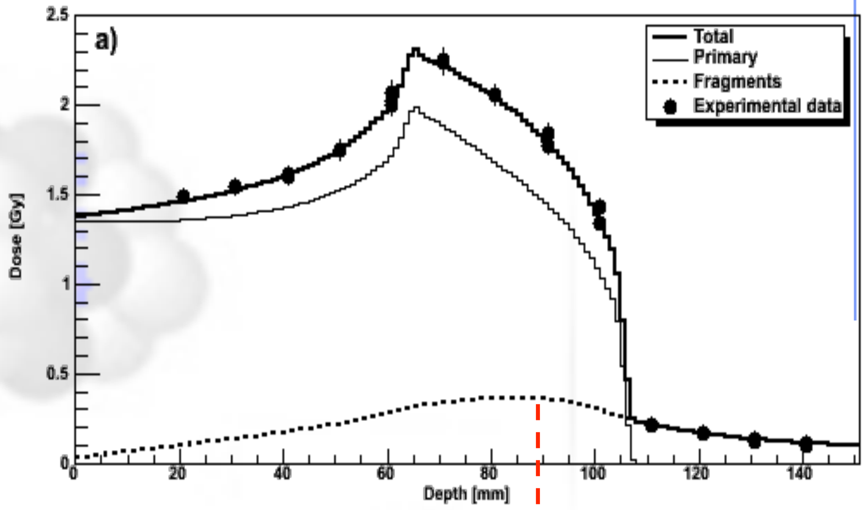
Monoenergetic and SOBP for ^{12}C:



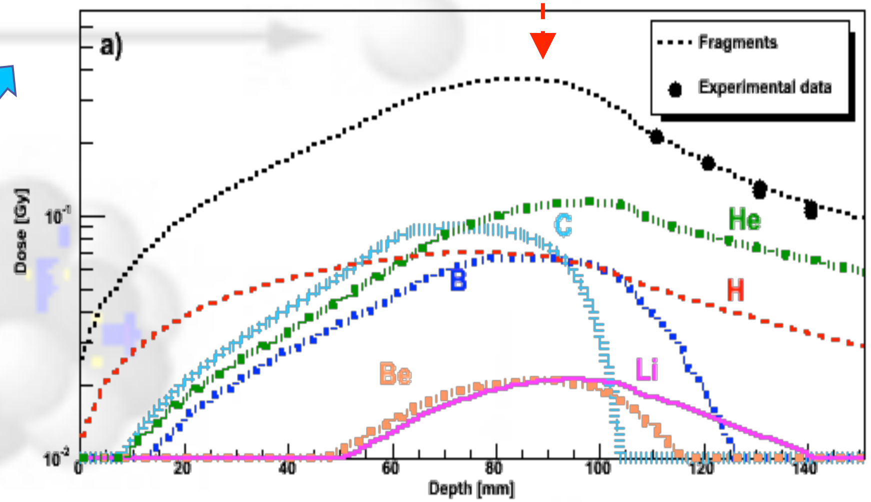
Exp. Data (points) from Haettner et al, Rad. Prot. Dos. 2006
Simulation: A. Mairani PhD Thesis, 2007, Nuovo Cimento C, 31, 2008

Left: ^{12}C @ 400 MeV/n in water Depth-dose distribution (top) and beam attenuation (bottom)

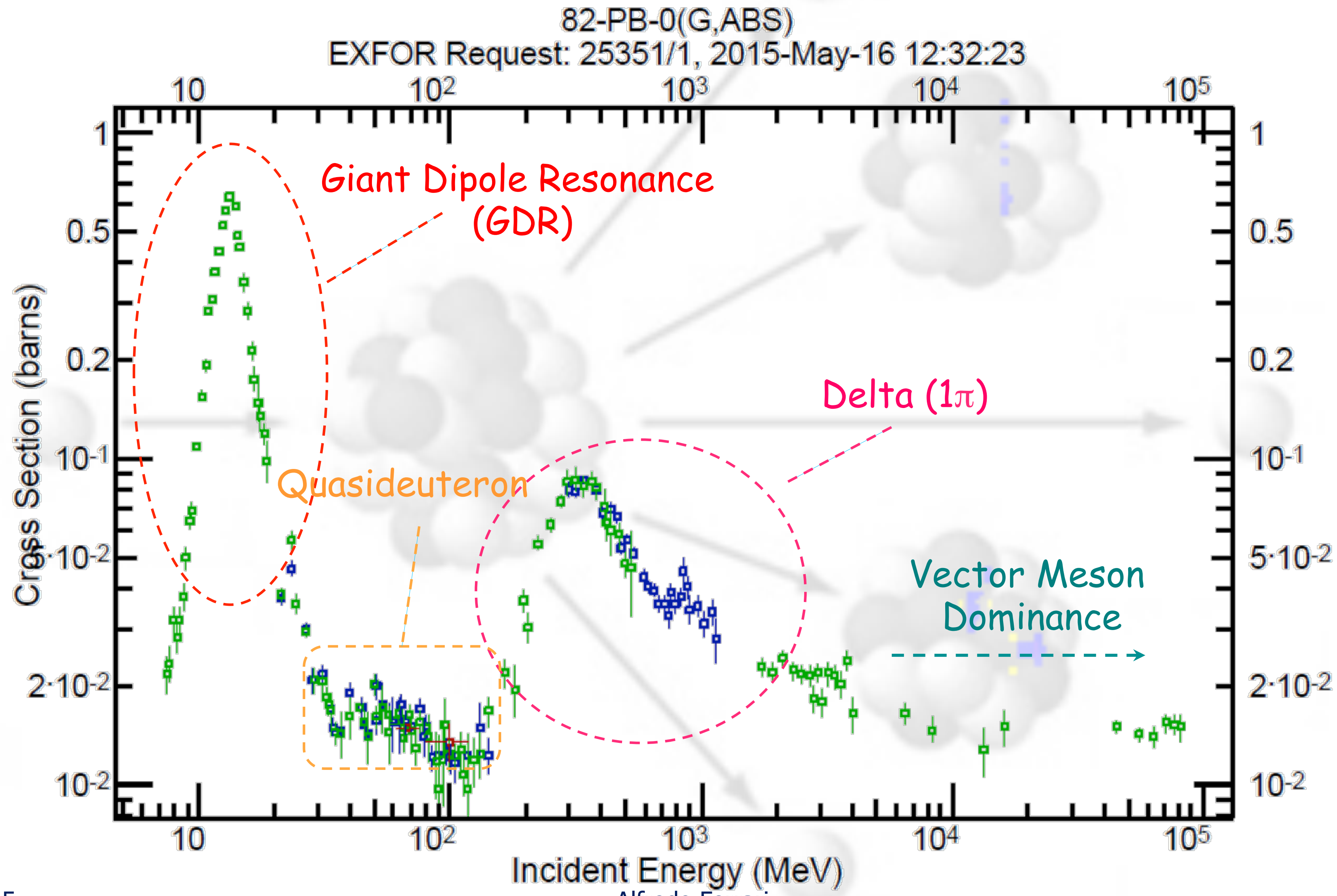


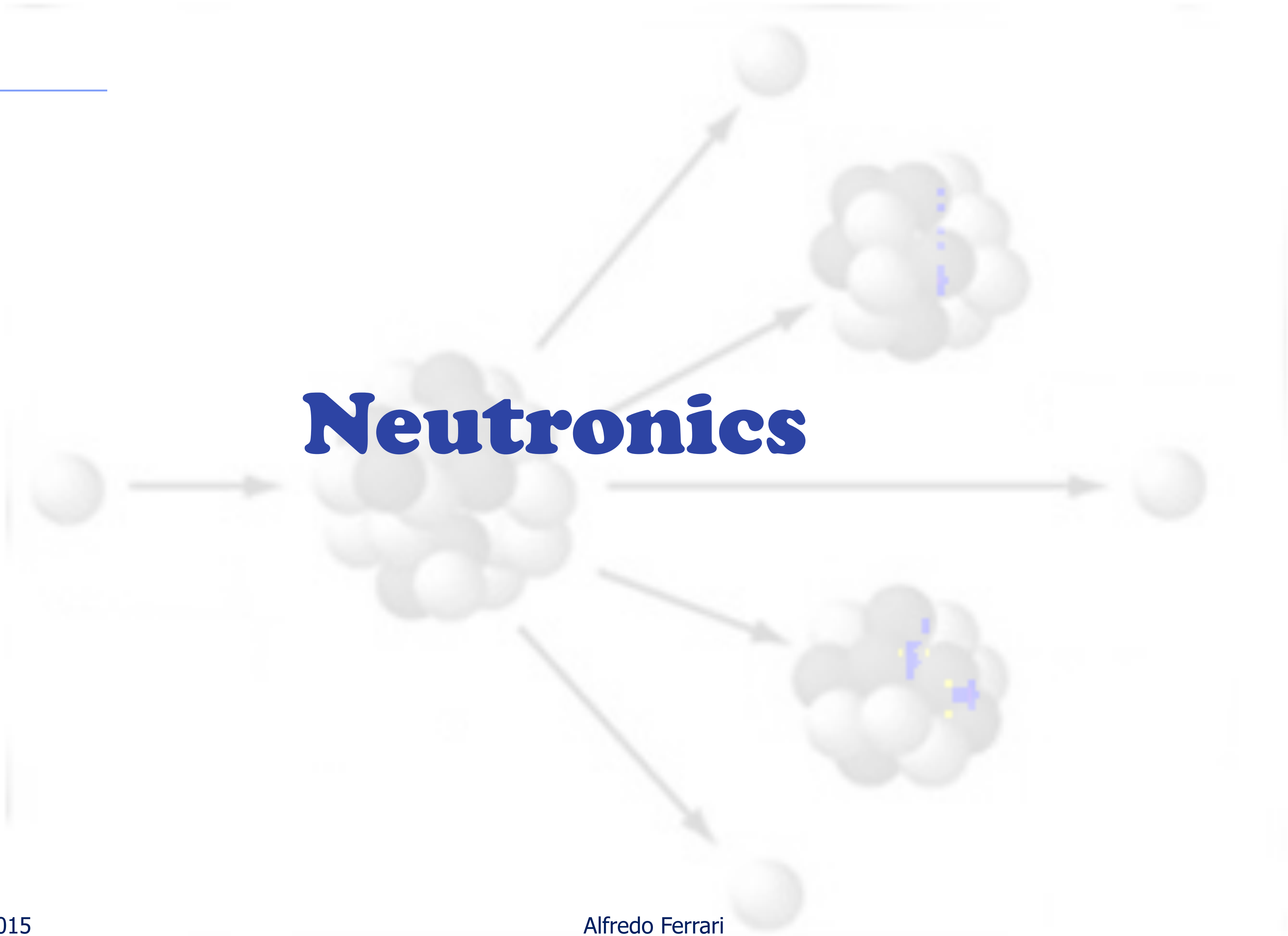


Right: Spread Out Bragg Peak, global depth-dose distribution (top) and detail of ion fragment contributions (bottom)



Photonuclear reactions: Pb(γ,x)





Neutronics

Low energy neutron interactions:

*Neutrons, being the only **"stable"** ($T_{1/2}$~10 min) **neutral** particle are the dominating component at low energies. They undergo elastic and non-elastic nuclear interactions until in most cases they are thermalized and captured by a nucleus ($n + {}^{A}_{Z}X \rightarrow {}^{A+1}_{Z}X + \gamma$'s). The slowing down is mostly accomplished via elastic interactions since non-elastic ones (apart capture) have thresholds*

At energies below 10-20 MeV, the specific nuclear structure of individual isotopes starts to play a major role, and cross sections are no longer a smooth function of A (mass number), rather…

→ Evaluated nuclear data files (ENDF, JEFF, JENDL…)

- typically provide neutron σ (cross sections) and secondary particles (sometimes only neutrons) inclusive distributions for E < 20 MeV for all channels. Recent evaluations include data up to 150/200 MeV for a few isotopes
- σ are stored as continuum + resonance parameters

"Low" energy neutrons:

- **Thermal neutrons**: Maxwellian distribution (most probable energy @ 293 K ~ **0.025 eV**)
- **Epithermal neutrons**, resonance neutrons, slow neutrons: **0.4 eV – 0.1 MeV**
- **Fast neutrons**: **> 0.1 MeV**

- **Non-elastic: (n,2n)** (>~8 MeV), (n,3n), (n,p), (n,α), (n,d), (n,np)… E_{th} ~ several MeV
- **Inelastic: (n,n')** (γ's emitted with the n), E_{th}~MeV's (even-even nuclei), ~keV's (heavy odd-odd nuclei)
- **Elastic:** no thresh., energy transfer (θ^*= cms scattering angle)

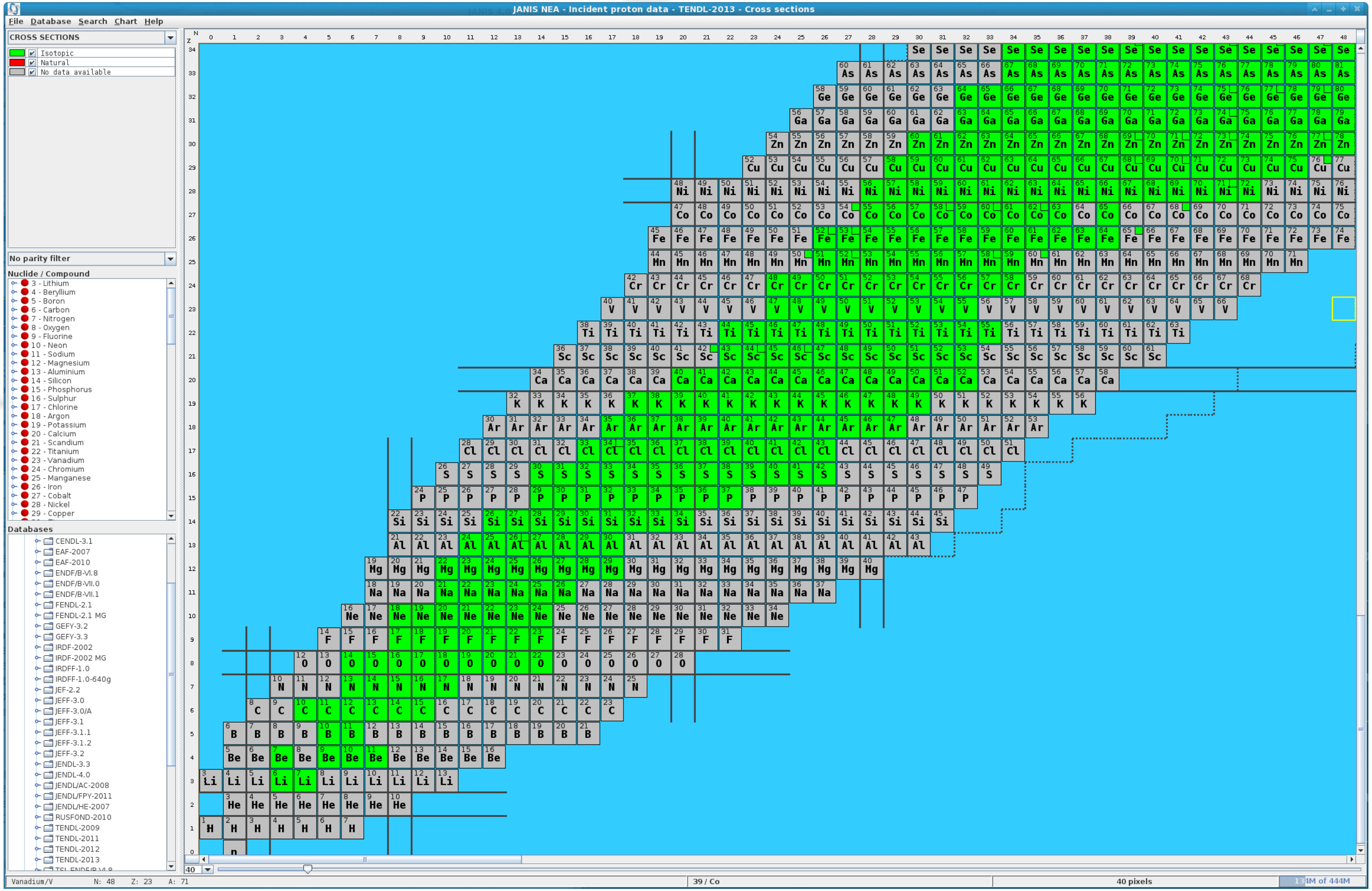
$$T_{rec} \approx 2E_{kin\ n}\left(1-\cos\theta^*\right)\frac{m_n M_{A,Z}}{(m_n + M_{A,Z})^2}$$

  - Proton target: $T_{rec\ max}=E_{kin\ n}$, $\langle T_{rec}\rangle=1/2\ E_{kin\ n}$
  - Lead target: $T_{rec\ max}=0.019\ E_{kin\ n}$, $\langle T_{rec}\rangle=0.009\ E_{kin\ n}$ (for isot. scatt., E_n <0.5 MeV)
- **Capture:** no thresh., important in the thermal (and resonance) regions, mostly $n + {}^{A}_{Z}X \rightarrow {}^{A+1}_{Z}X + \gamma$'s, notable exceptions:
  - $^3He(n,p)^3H$, Q = 764 keV
  - $^{14}N(n,p)^{14}C$, Q = 626 keV
  - $^{10}B(n,\alpha)^7Li$, Q = 2790 keV

Neutron data: examples (eg from http://www.oecd-nea.org/janis/):

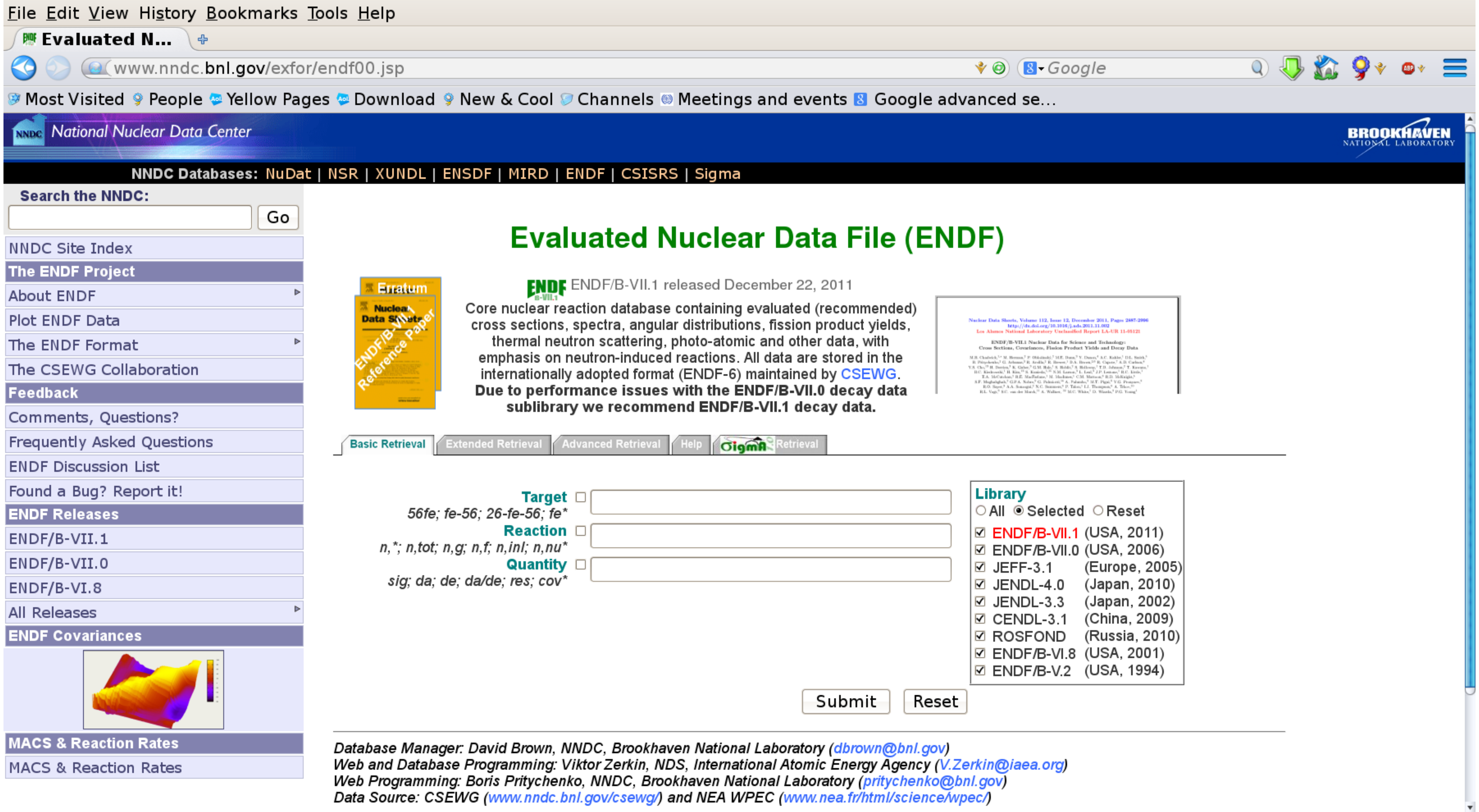
ENDF/B-7.1: US
JENDL-4.0: Japan
JEFF-3.1.2: Europe
...
TENDL-2013: Model (TALYS)

Some of them include data for incident charged particles as well, and/or evaluations up to 150/200 MeV for some isotopes

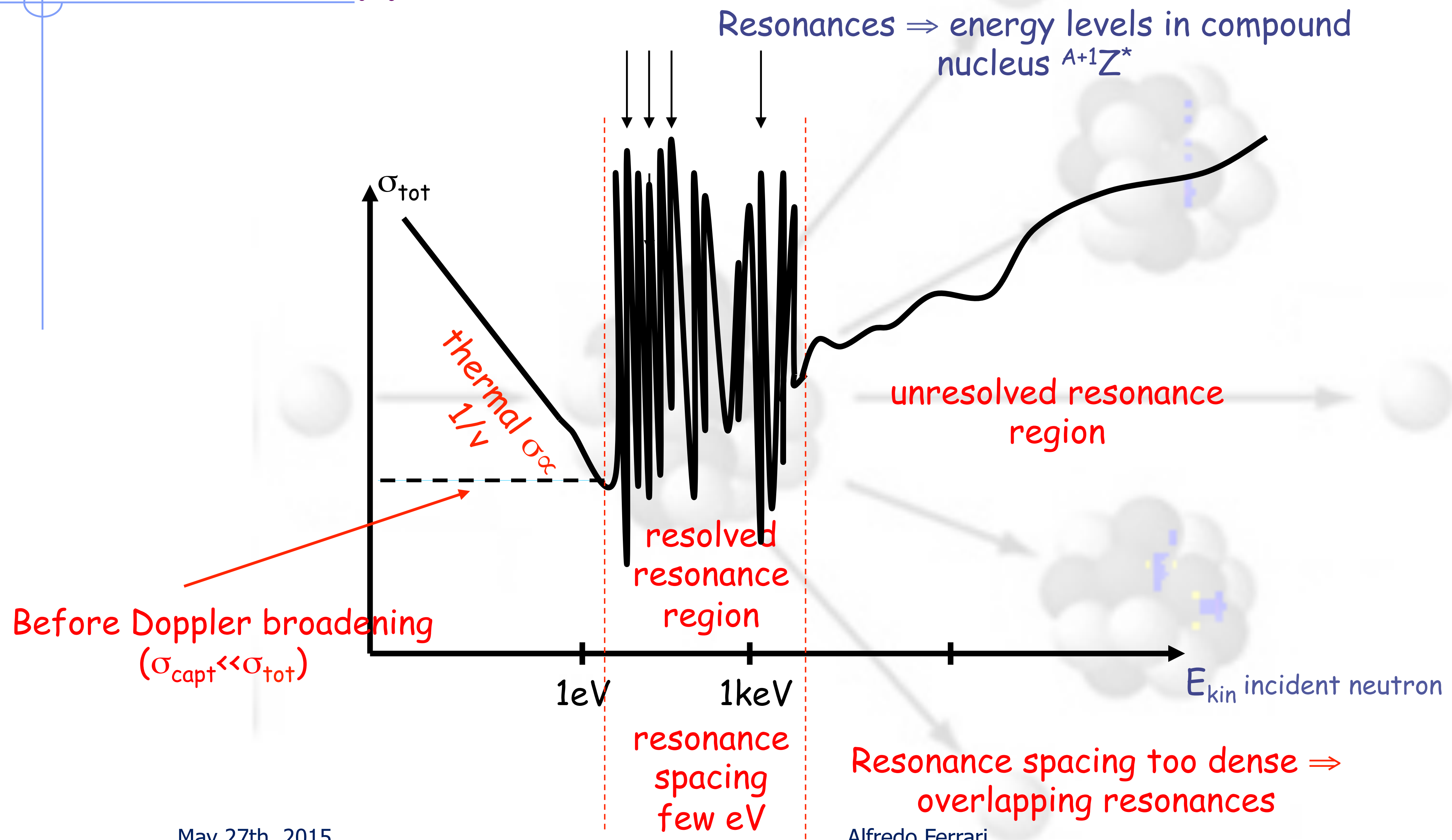


May 27th, 2015

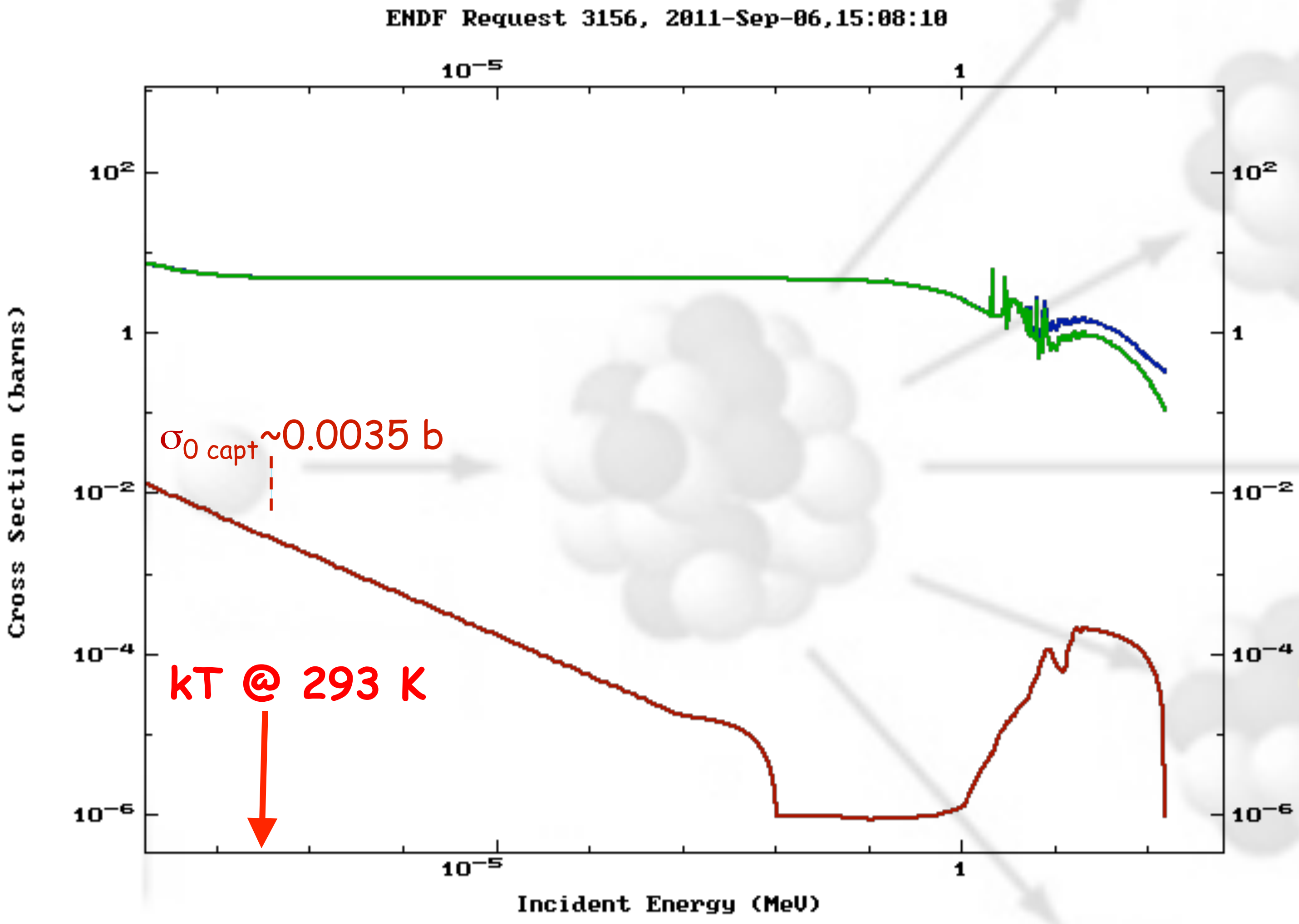
… or from http://www.nndc.bnl.gov :



Typical neutron cross section

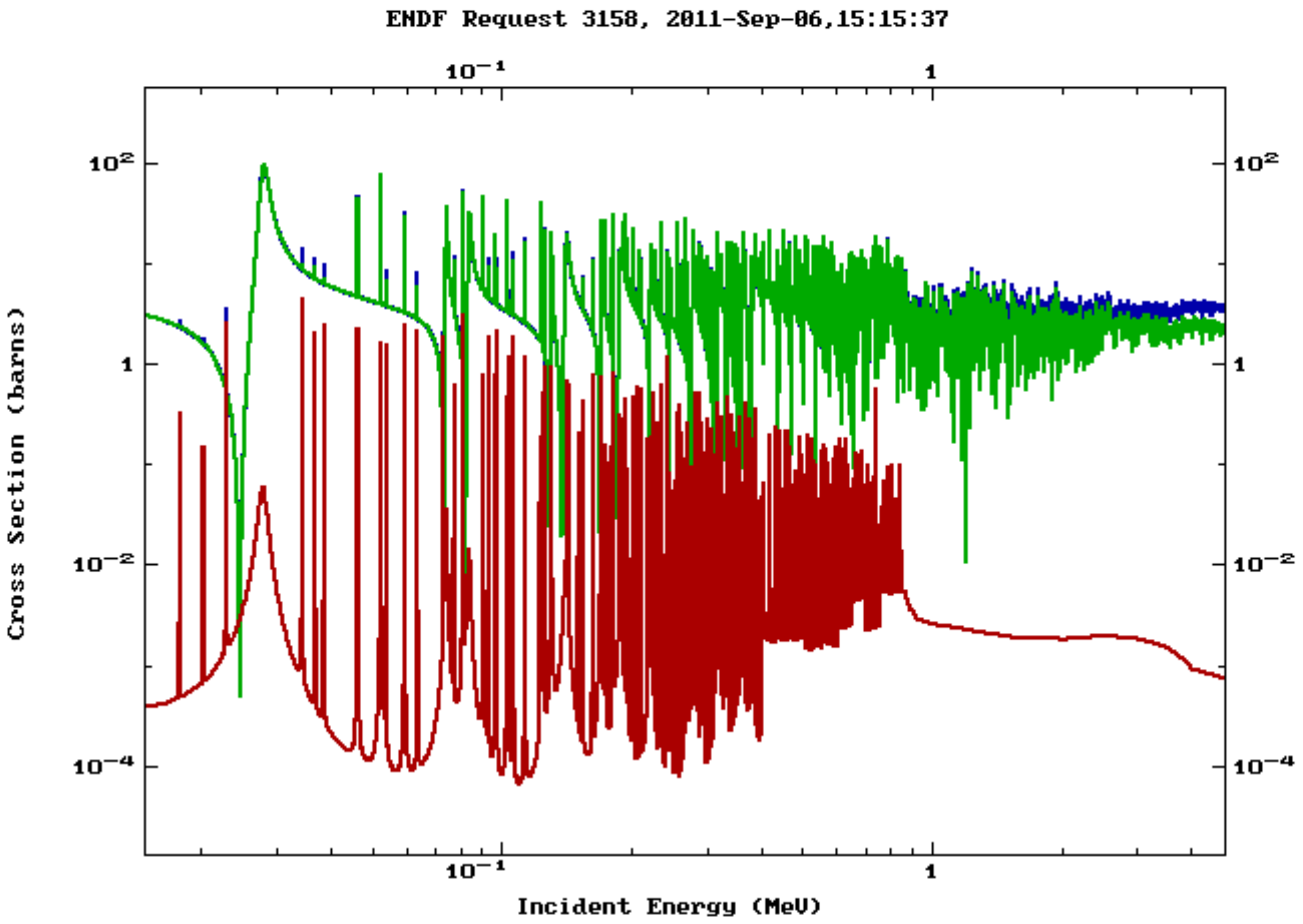


Low Energy Neutron Cross sections: C



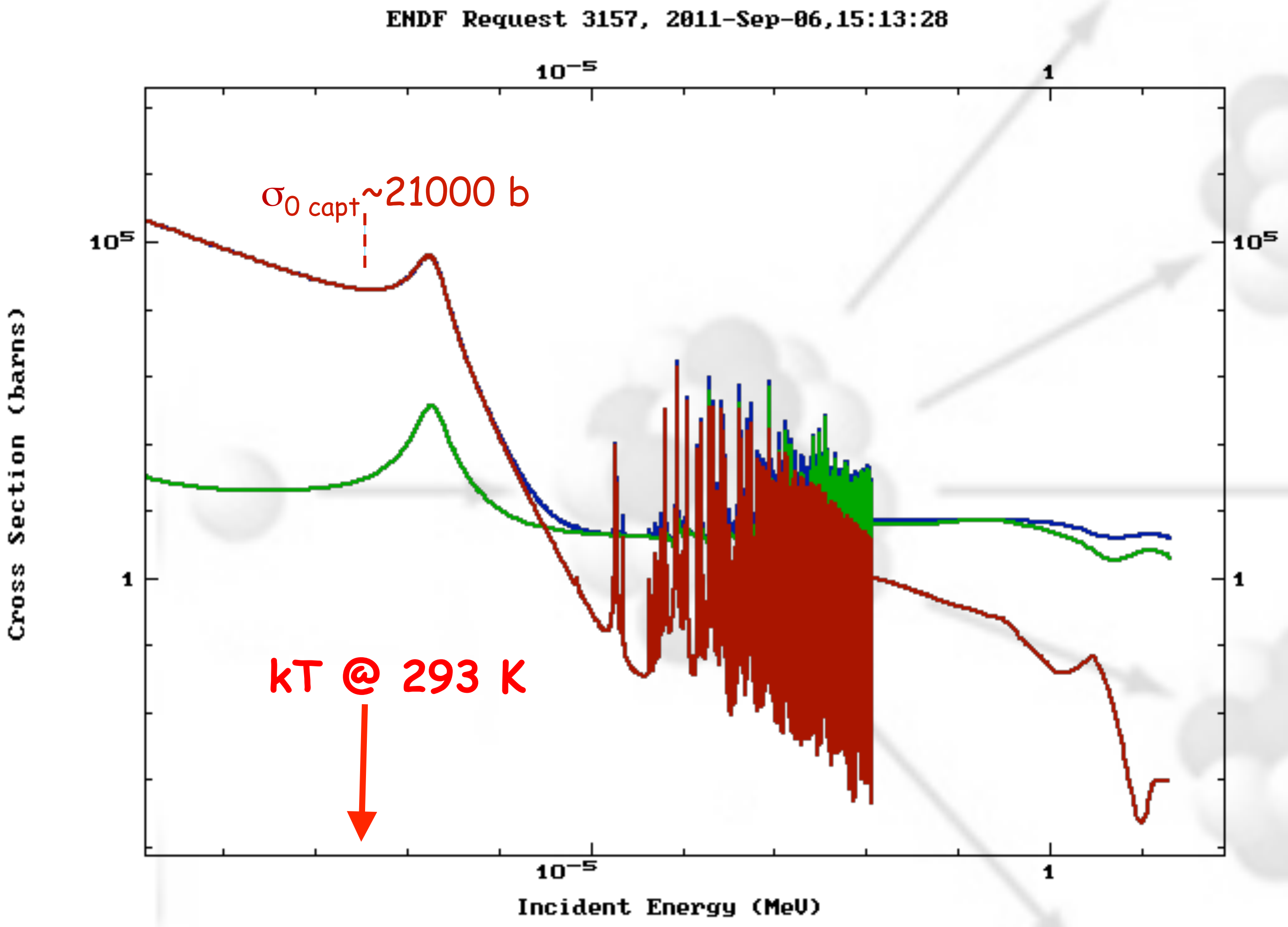
Blue: total cross section
Green: elastic cross section
Red: capture cross section

Low Energy Neutron Cross sections: ^{56}Fe



Blue: total cross section
Green: elastic cross section
Red: capture cross section

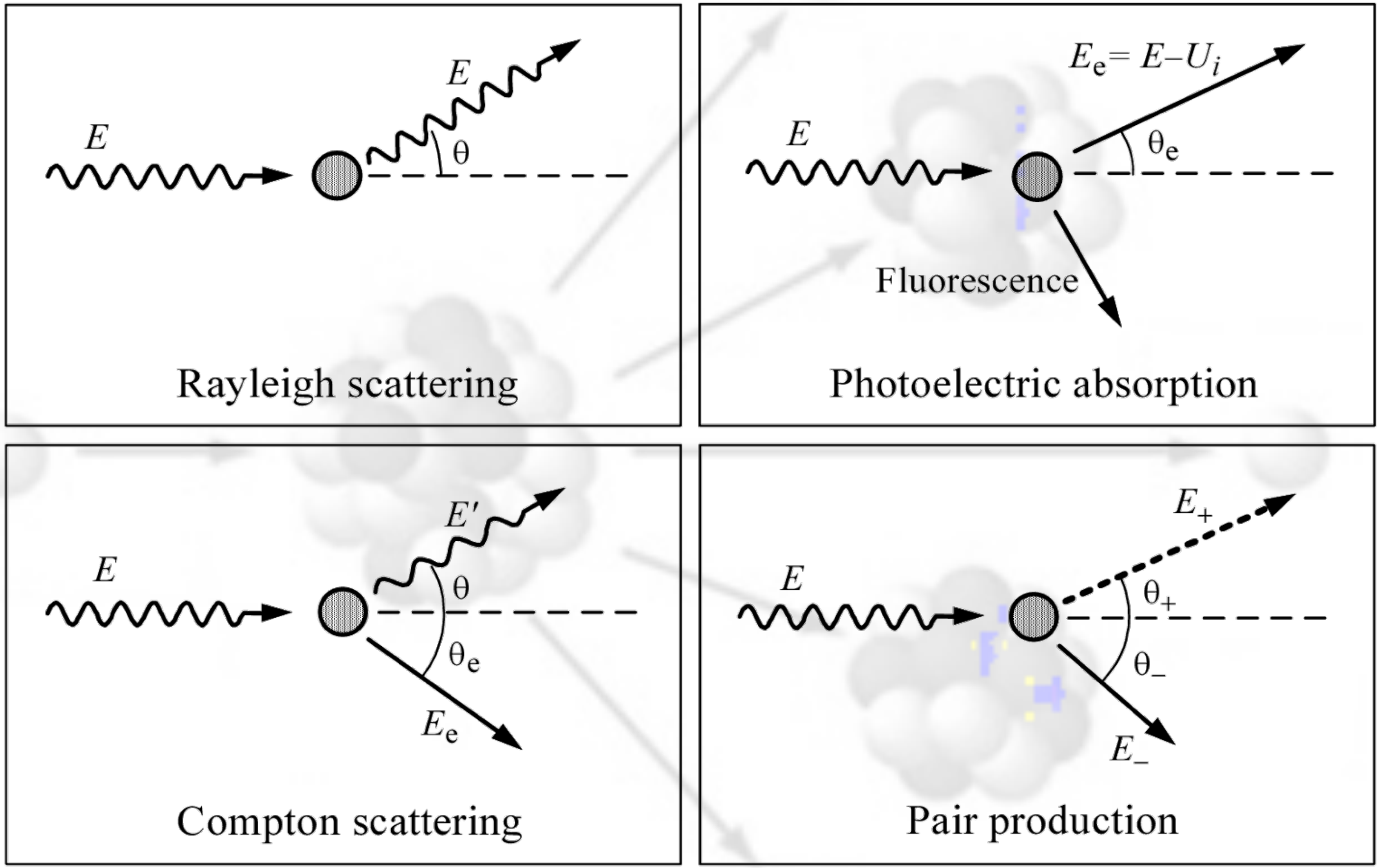
Low Energy Neutron Cross sections: ^{113}Cd



Blue: total cross section
Green: elastic cross section
Red: capture cross section

Photon (atomic) Interactions

Photon interactions:



Photon cross sections: scaling

- The photoelectric macroscopic cross section (cm^2/g) scales as:

$$\Sigma_{pe} \propto \frac{Z^5}{A}$$

- The Compton macroscopic cross section (cm^2/g) scales as:

$$\Sigma_{incoh} \propto \frac{Z}{A}$$

- The coherent (Rayleigh) macroscopic cross section (cm^2/g) scales as:

$$\Sigma_{coh} \propto \frac{Z^2}{A}$$

- The pair production macroscopic cross section (cm^2/g) scales as:

$$\Sigma_{pair} \propto \frac{Z^2}{A}$$

Compton scattering: dynamics

Klein-Nishina cross section (see for example Heitler, "The Quantum Theory of Radiation"):

$$\frac{d\sigma_{KN}}{d\Omega} = \frac{1}{4} r_e^2 \frac{k'^2}{k^2}\left[\frac{k}{k'} + \frac{k'}{k} - 2 + 4\cos^2\Theta\right]$$

Let $\vec{e}$ be the polarization vector of the incident photon, and $\vec{e}\,'$ that of the scattered one:

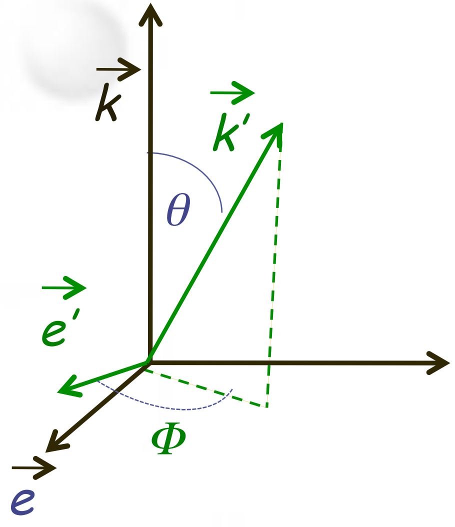
$$\cos\Theta = \vec{e}\cdot\vec{e}\,'$$

Split σ into the two components, $\perp$ and || to $\vec{e}$ respectively (actually with $\vec{e}\,'$ $\perp$ to the plane *(e,k')*, or contained in the plane *(e,k')*):

$$\frac{d\sigma_{\perp}}{d\Omega} = \frac{1}{4} r_e^2 \frac{k'^2}{k^2}\left[\frac{k}{k'} + \frac{k'}{k} - 2\right]$$

$$\frac{d\sigma_{\parallel}}{d\Omega} = \frac{1}{4} r_e^2 \frac{k'^2}{k^2}\left[\frac{k}{k'} + \frac{k'}{k} + 2 - 4\sin^2\vartheta\cos^2\Phi\right]$$

$$\cos^2\Theta = 1 - \sin^2\vartheta\cos^2\Phi$$



The effect of polarization is important for polarized photon beams (eg synchrotron radiation source)!! It breaks the scattering azimuthal symmetry!!!

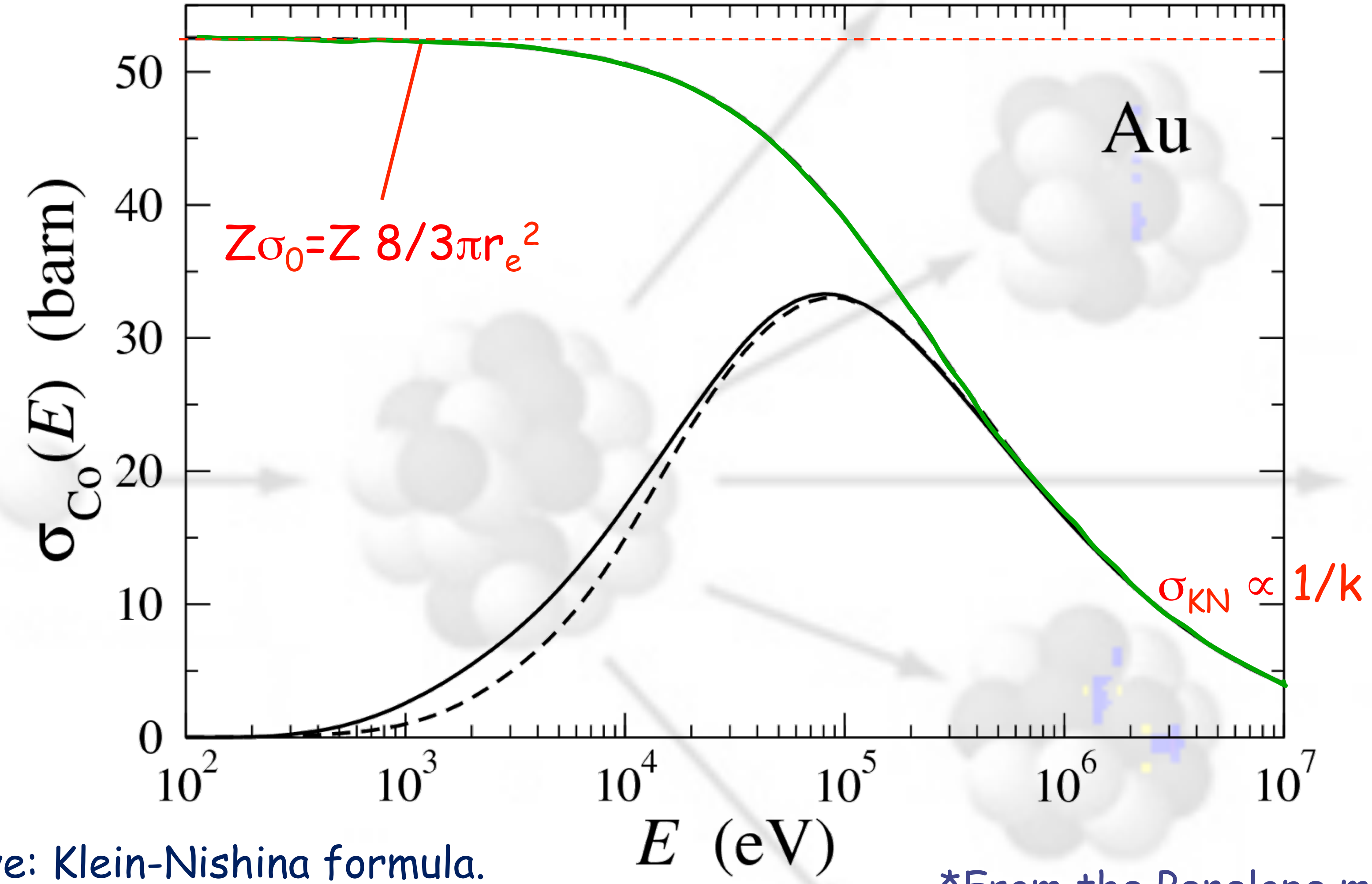
Atomic corrections:

Atomic electrons are bound → atomic corrections are important

- Mild effects on Compton (except at low energies):
  - ✓ Small angles (→ small energy/momentum transfers) suppressed
  - ✓ Compton line broadened by bound electron motion
- Dominant effect on coherent scattering
  - ➢ Atomic effect dominant, only small angle scattering is left
  - ➢ Medium-large angle scattering suppressed because of loss of coherence

Under certain approximations atomic effects can be described via the inelastic and elastic atomic form factors

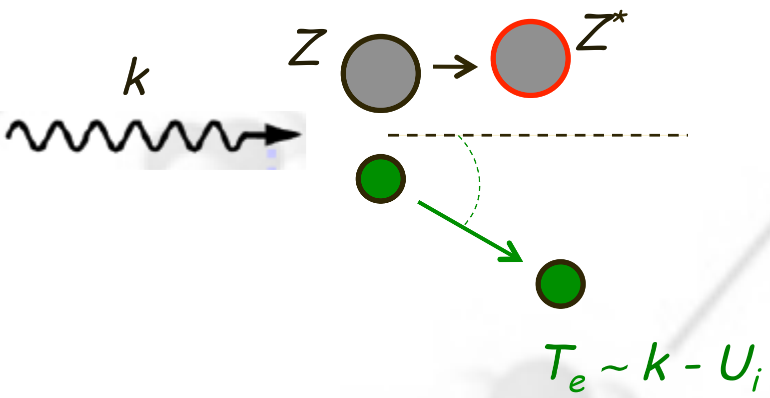
Compton cross section: Gold*



Green curve: Klein-Nishina formula.
Dashed curve: Waller-Hartree approximation.
Black curve: relativistic impulse approximation.

*From the Penelope manual

Photoelectric effect: just a reminder (more in the backup slides)

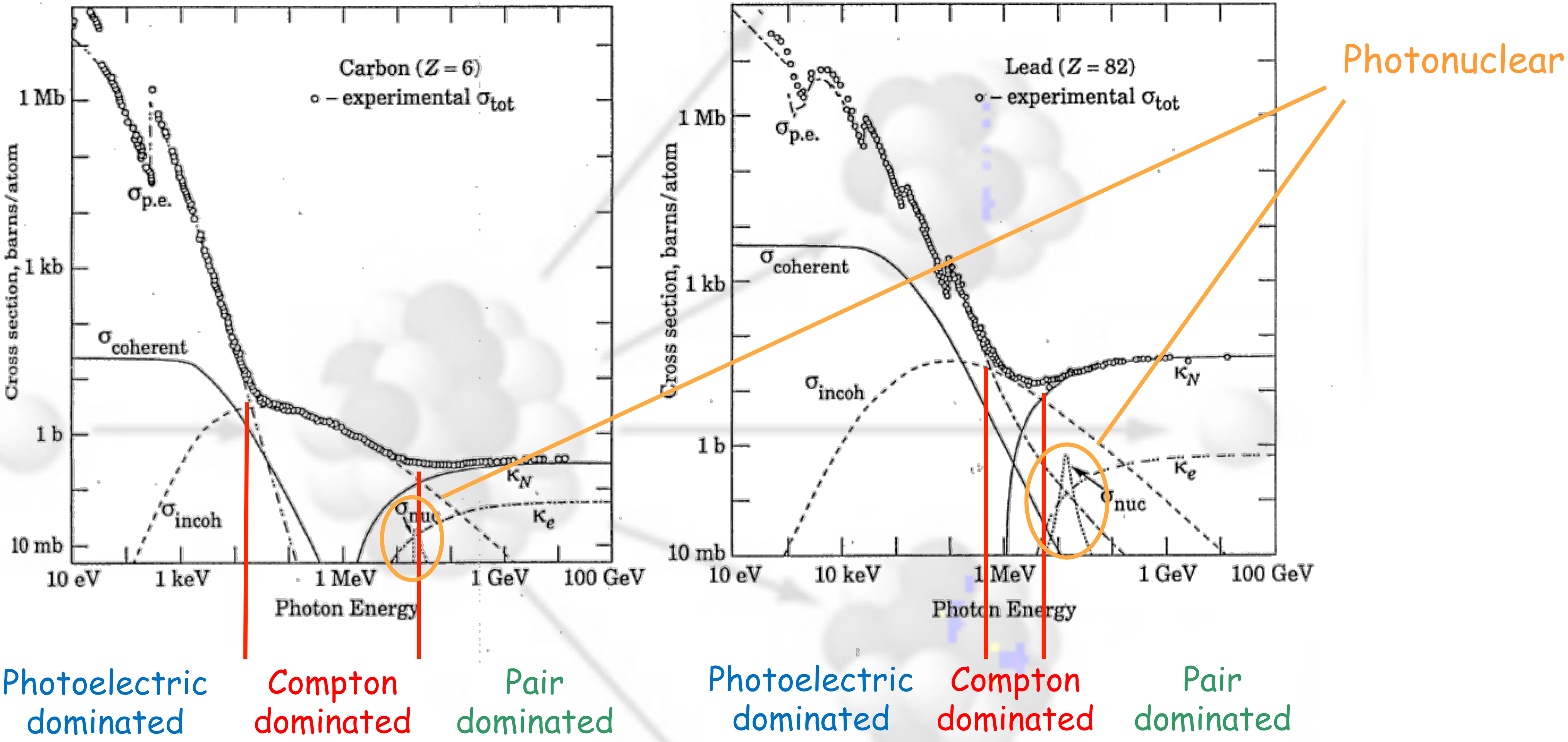


The incident photon is absorbed by an atomic electron which is emitted with kinetic energy roughly equal to the incident photon energy minus the (electron) binding energy. The atom is left in an excited state

The electron must be bound to fulfill energy-momentum conservation
What is required to describe fully p.e. interactions?

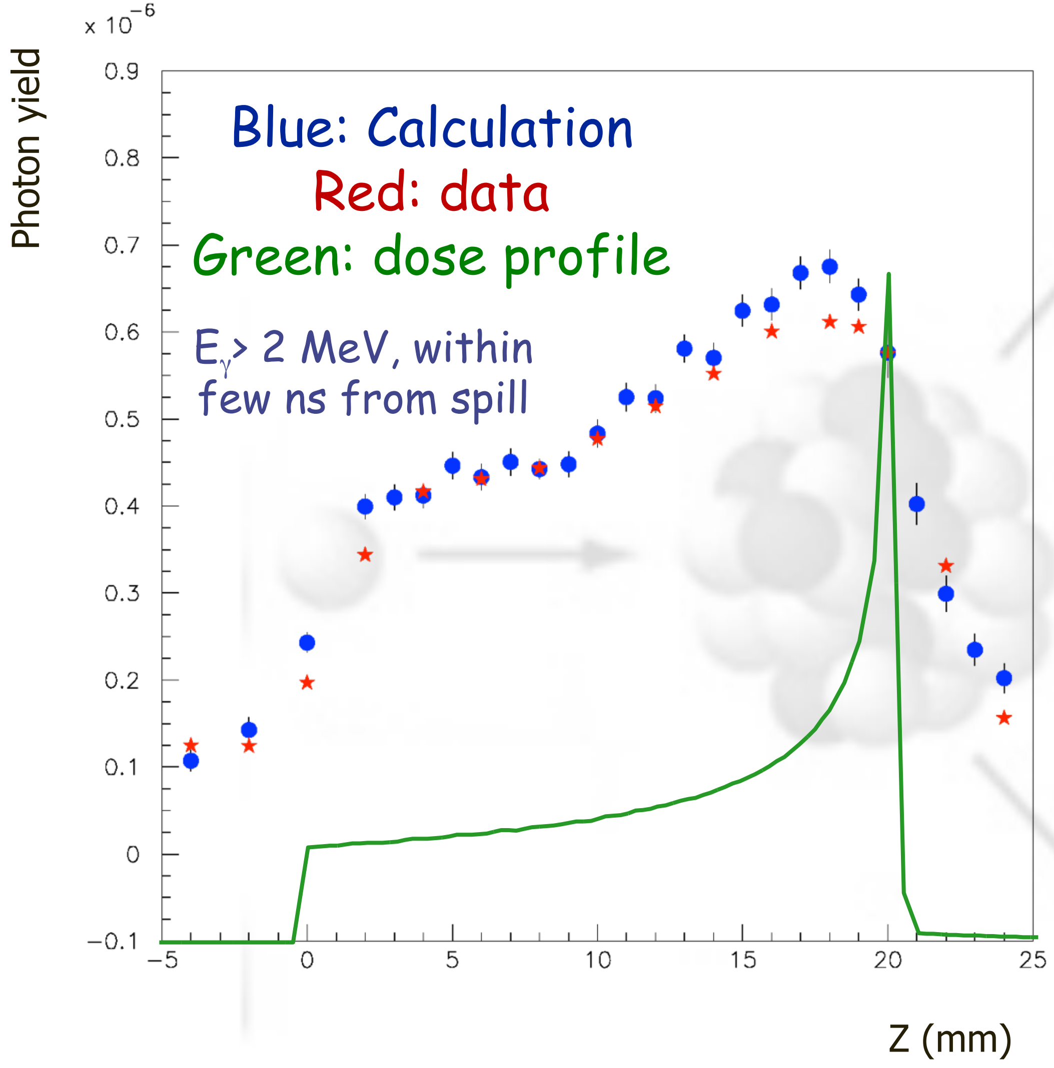
- Cross sections for each atomic shell
- Angular distribution of photoelectrons
- Effect of (possible) photon polarization
- De-excitation of atomic ions left after the interaction
  - ✓ Fluorescence (X-rays) (radiative, between shells)
  - ✓ Auger emission (electrons, between shells)
  - ✓ Coster-Kronig emission (electrons, intra-shell)

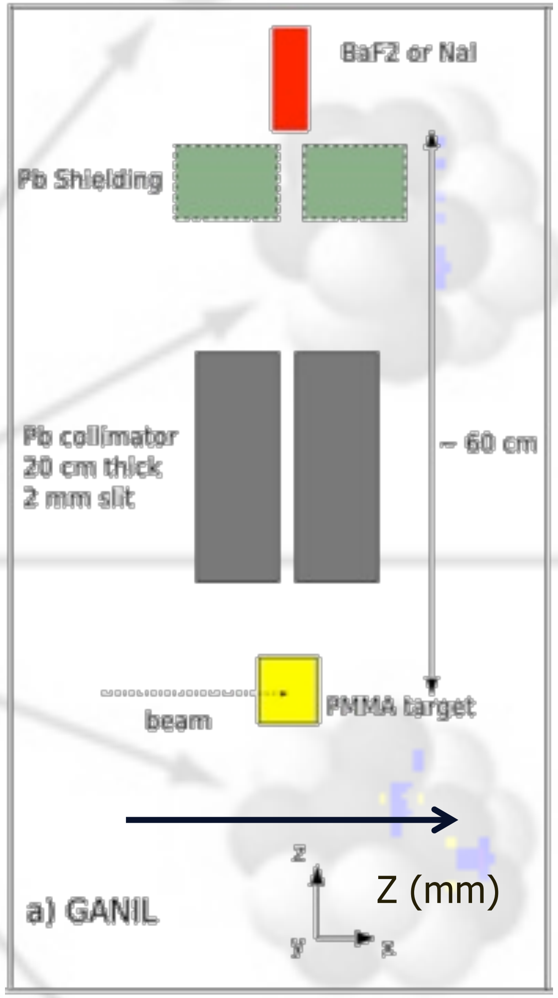
Photon cross sections: summary



$\sigma_{p.e.}$=photoelectric cross section; σ_{incoh}=Compton cross section;
$\sigma_{coherent}$=Rayleigh cross section; σ_{nuc} =photonuclear cross section;
κ_N=pair production cross section, nuclear field;
κ_e=pair production cross section, electron field

How to make good use of (unwanted) nuclear interactions:



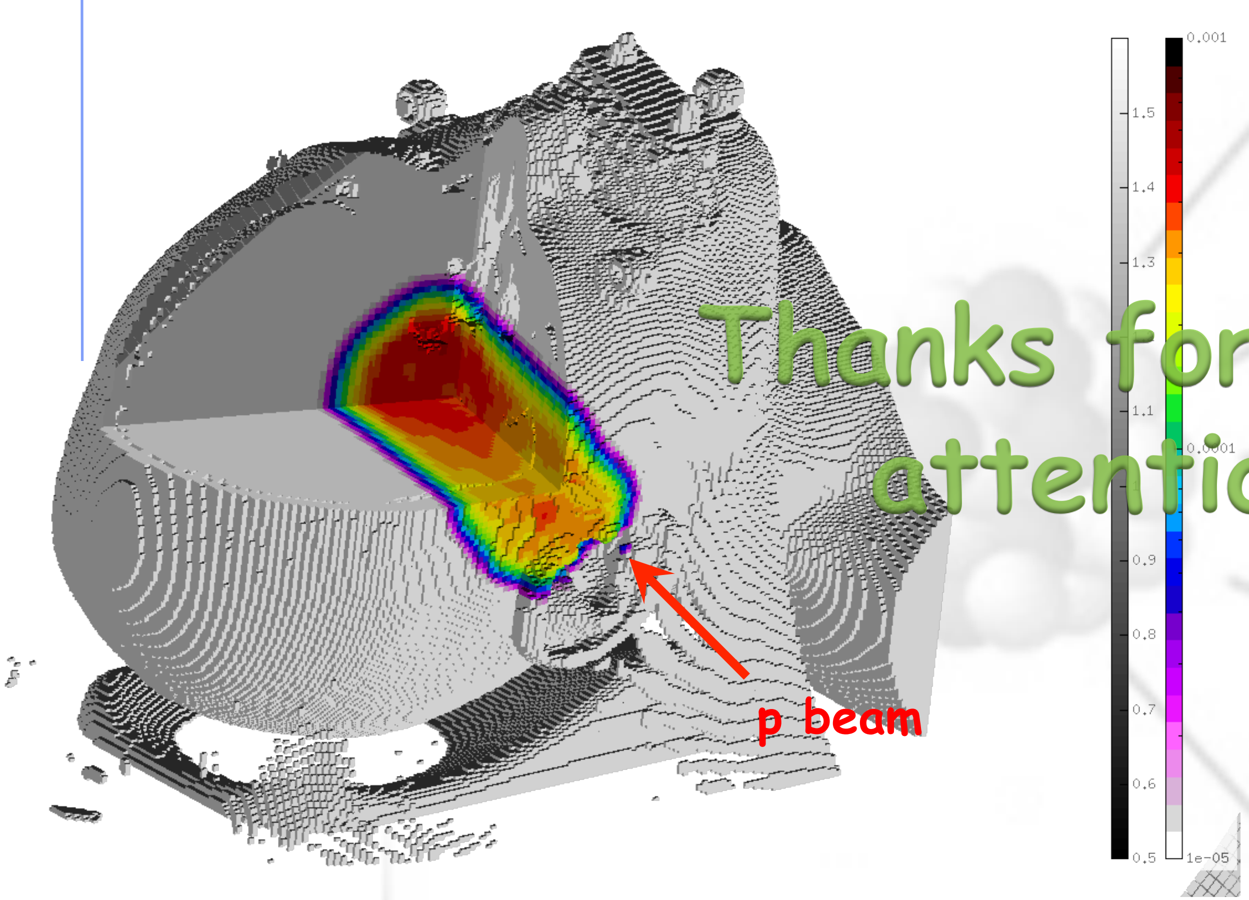


GANIL: 90 deg prompt* photon yields by 95 MeV/n ^{12}C in PMMA

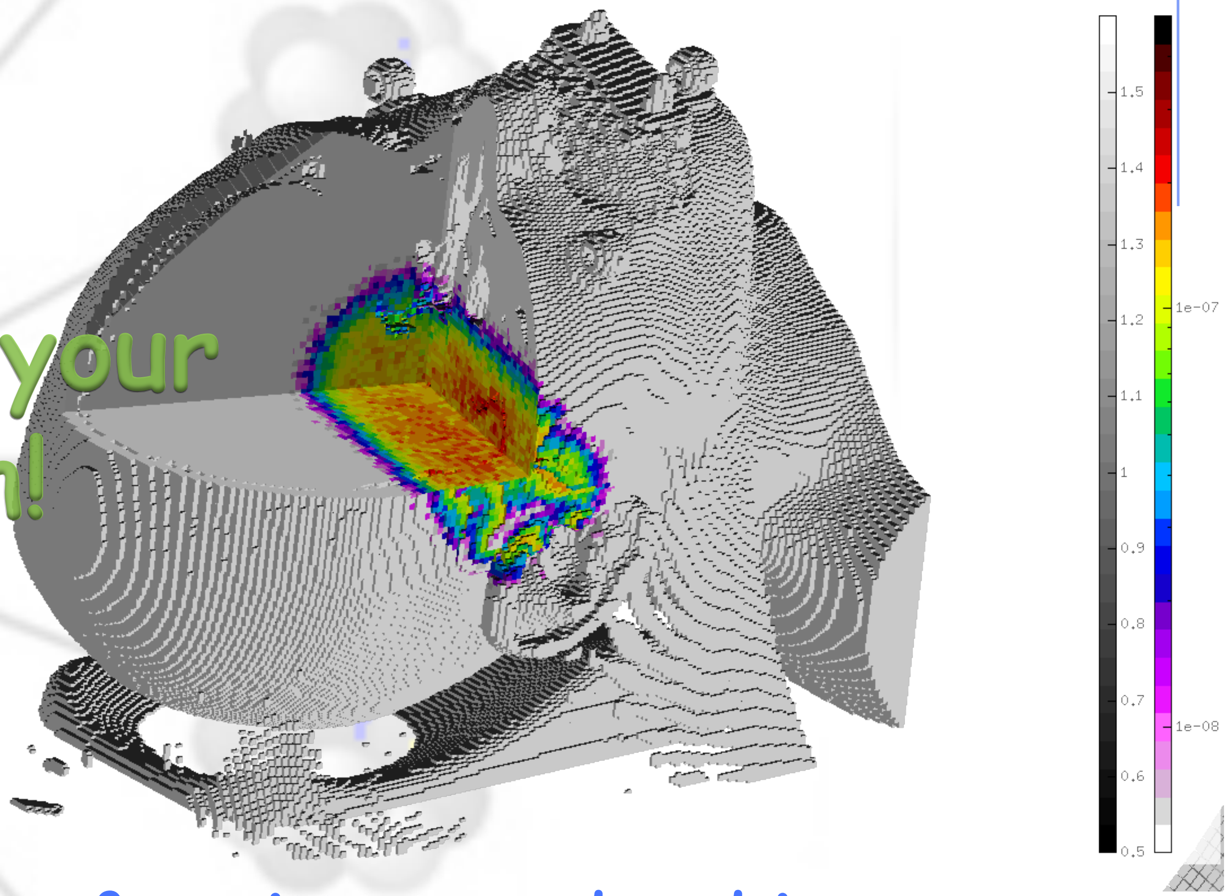
** Prompt photons: nuclear de-excitation photons emitted in nuclear interactions*

[sketch and exp. data taken from F. Le Foulher et al IEEE TNS 57 (2009), E. Testa et al, NIMB 267 (2009) 993. Exp. data have been reevaluated in 2012 with substantial corrections]

Unwanted nuclear physics turned useful: β^+ isotope prod.



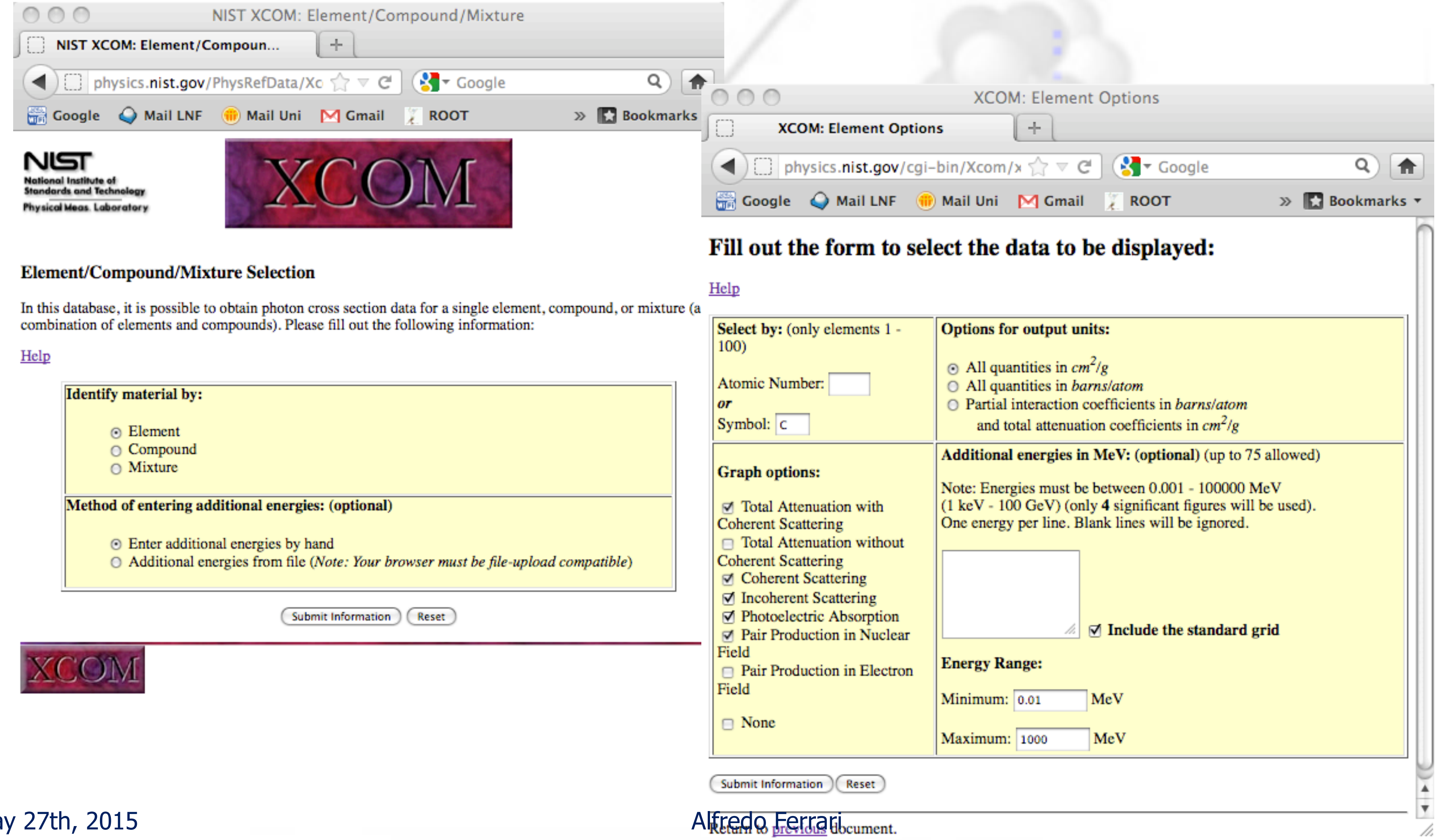
Dose map



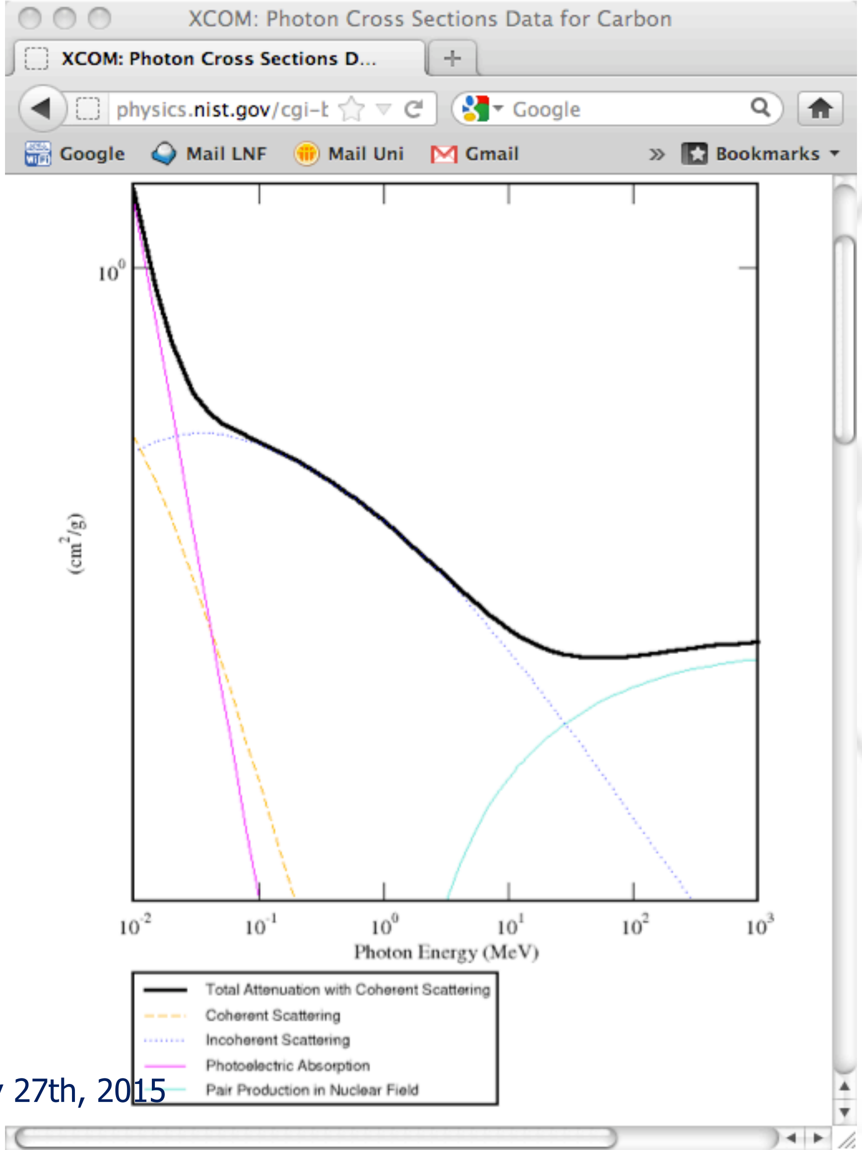
β^+ emitters produced in nuclear interactions map

Thanks for your attention!

NIST Database: http://physics.nist.gov/PhysRefData/Xcom/Text/chap4.html



NIST Database:
http://physics.nist.gov/PhysRefData/Xcom/Text/chap4.html



XCOM: Photon Cross Sections Data for Carbon

Edge	(required) Photon Energy	Scattering: Coherent	Scattering: Incoherent	Photoelectric Absorption	Pair Production: In Nuclear Field	Pair Production: In Electron Field	Total Attenuation: With Coherent Scattering	Total Attenuation: Without Coherent Scattering
	MeV	cm^2/g	cm^2/g	cm^2/g	cm^2/g	cm^2/g	cm^2/g	cm^2/g
	1.000E-02	1.620E-01	1.352E-01	2.076E+00	0.000E+00	0.000E+00	2.373E+00	2.211E+00
	1.500E-02	9.787E-02	1.510E-01	5.585E-01	0.000E+00	0.000E+00	8.074E-01	7.096E-01
	2.000E-02	6.478E-02	1.595E-01	2.177E-01	0.000E+00	0.000E+00	4.420E-01	3.772E-01
	3.000E-02	3.365E-02	1.655E-01	5.706E-02	0.000E+00	0.000E+00	2.562E-01	2.225E-01
	4.000E-02	2.045E-02	1.653E-01	2.193E-02	0.000E+00	0.000E+00	2.076E-01	1.872E-01
	5.000E-02	1.371E-02	1.630E-01	1.042E-02	0.000E+00	0.000E+00	1.871E-01	1.734E-01
	6.000E-02	9.807E-03	1.598E-01	5.671E-03	0.000E+00	0.000E+00	1.753E-01	1.655E-01
	8.000E-02	5.711E-03	1.531E-01	2.169E-03	0.000E+00	0.000E+00	1.610E-01	1.553E-01
	1.000E-01	3.719E-03	1.466E-01	1.031E-03	0.000E+00	0.000E+00	1.514E-01	1.476E-01
	1.500E-01	1.685E-03	1.327E-01	2.706E-04	0.000E+00	0.000E+00	1.347E-01	1.330E-01
	2.000E-01	9.541E-04	1.219E-01	1.063E-04	0.000E+00	0.000E+00	1.229E-01	1.220E-01
	3.000E-01	4.264E-04	1.062E-01	2.980E-05	0.000E+00	0.000E+00	1.066E-01	1.062E-01
	4.000E-01	2.403E-04	9.521E-02	1.272E-05	0.000E+00	0.000E+00	9.547E-02	9.523E-02
	5.000E-01	1.539E-04	8.699E-02	6.839E-06	0.000E+00	0.000E+00	8.715E-02	8.700E-02
	6.000E-01	1.069E-04	8.047E-02	4.253E-06	0.000E+00	0.000E+00	8.058E-02	8.048E-02
	8.000E-01	6.017E-05	7.070E-02	2.144E-06	0.000E+00	0.000E+00	7.076E-02	7.070E-02
	1.000E+00	3.852E-05	6.358E-02	1.333E-06	0.000E+00	0.000E+00	6.362E-02	6.358E-02
	1.022E+00	3.688E-05	6.292E-02	1.229E-06	0.000E+00	0.000E+00	6.296E-02	6.293E-02
	1.250E+00	2.465E-05	5.686E-02	8.348E-07	1.439E-05	0.000E+00	5.690E-02	5.687E-02
	1.500E+00	1.712E-05	5.169E-02	6.062E-07	7.992E-05	0.000E+00	5.179E-02	5.177E-02
	2.000E+00	9.632E-06	4.410E-02	3.826E-07	3.187E-04	0.000E+00	4.443E-02	4.442E-02
	2.044E+00	9.220E-06	4.356E-02	3.702E-07	3.435E-04	0.000E+00	4.391E-02	4.390E-02
	3.000E+00	4.281E-06	3.470E-02	2.147E-07	9.125E-04	1.214E-05	3.563E-02	3.562E-02
	4.000E+00	2.408E-06	2.894E-02	1.478E-07	1.482E-03	4.956E-05	3.047E-02	3.047E-02

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