BEAM INSTRUMENTATION

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Linear Imperfections

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Introduction

Orbit and dispersion

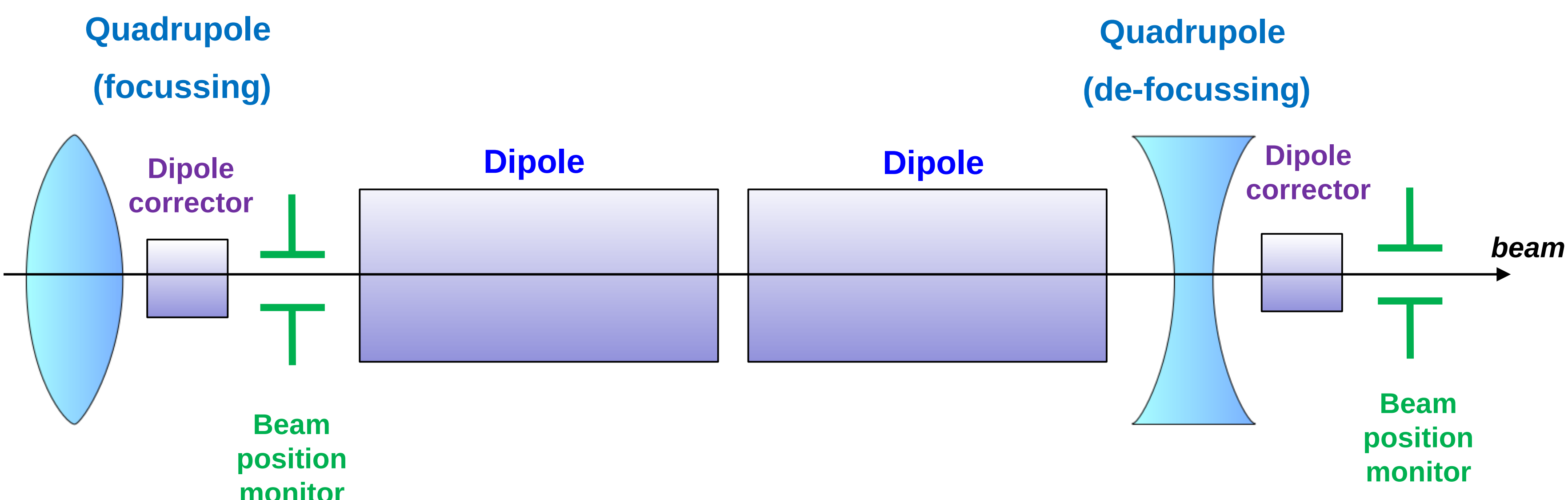
Tune and coupling

Chromaticity

Linear optics

Accelerator lattice cell

- An accelerator is usually build using a number of basic ‘cells’.
- The cell layouts of an accelerator come in many subtle variants.
- A simple FODO cell usually contains:
  - **Dipole magnets** to bend the beams,
  - **Quadrupole magnets** to focus the beams,
  - **Beam position monitors** (BPM) to measure the beam position,
  - **Small dipole corrector magnets** for beam steering.
  - **Sextupole magnets** to control off-energy focussing.



Schematic of a ½ cell

Dipole magnet

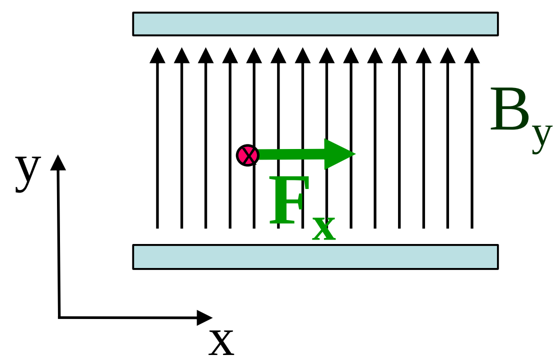
- The dipole has two magnetic poles and generates a **homogeneous field** providing a constant force on all beam particles – used to **deflect** the beam.
  - A dipole corrector is just a small version of such a magnet, dedicated to steer the beam as we will see later.

Lorentz force:

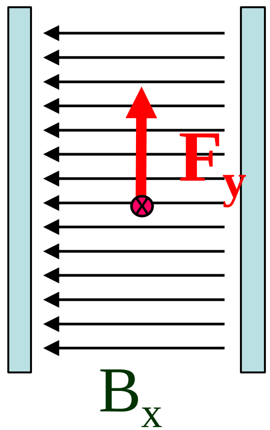
$$\vec{F} = q\,\vec{v} \times \vec{B}$$

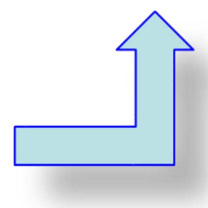
orthogonal to the speed and magnetic field directions

Horizontal deflection



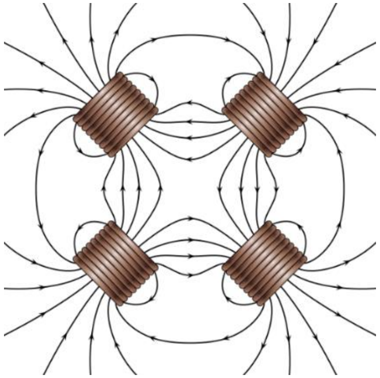
Vertical deflection

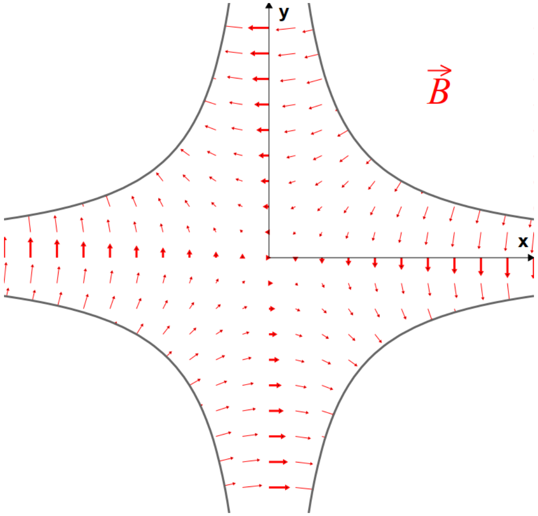


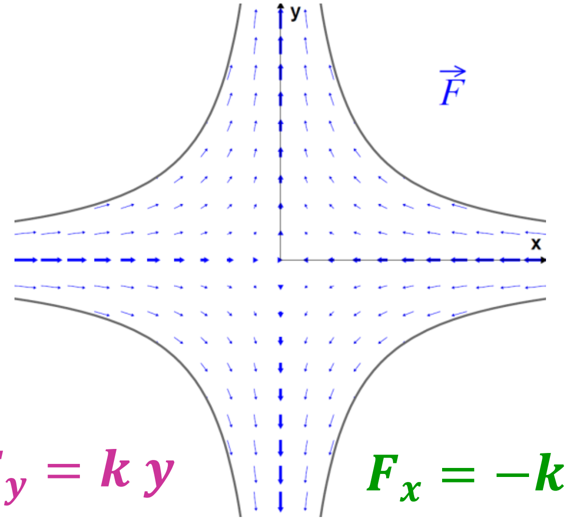


Quadrupole magnet

- A quadrupole has 4 magnetic poles.
- A quadrupole provides a field (force) that **increases linearly** with the distance to the quadrupole center – provides **focussing** of the beam.
  - Similar to an optical lens, except that a quadrupole is focussing in one plane, defocussing in the other plane.





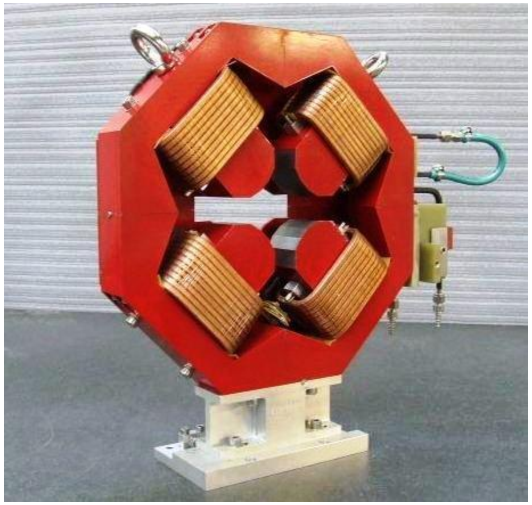


$$F_y = k\,y$$

$$F_x = -k\,x$$

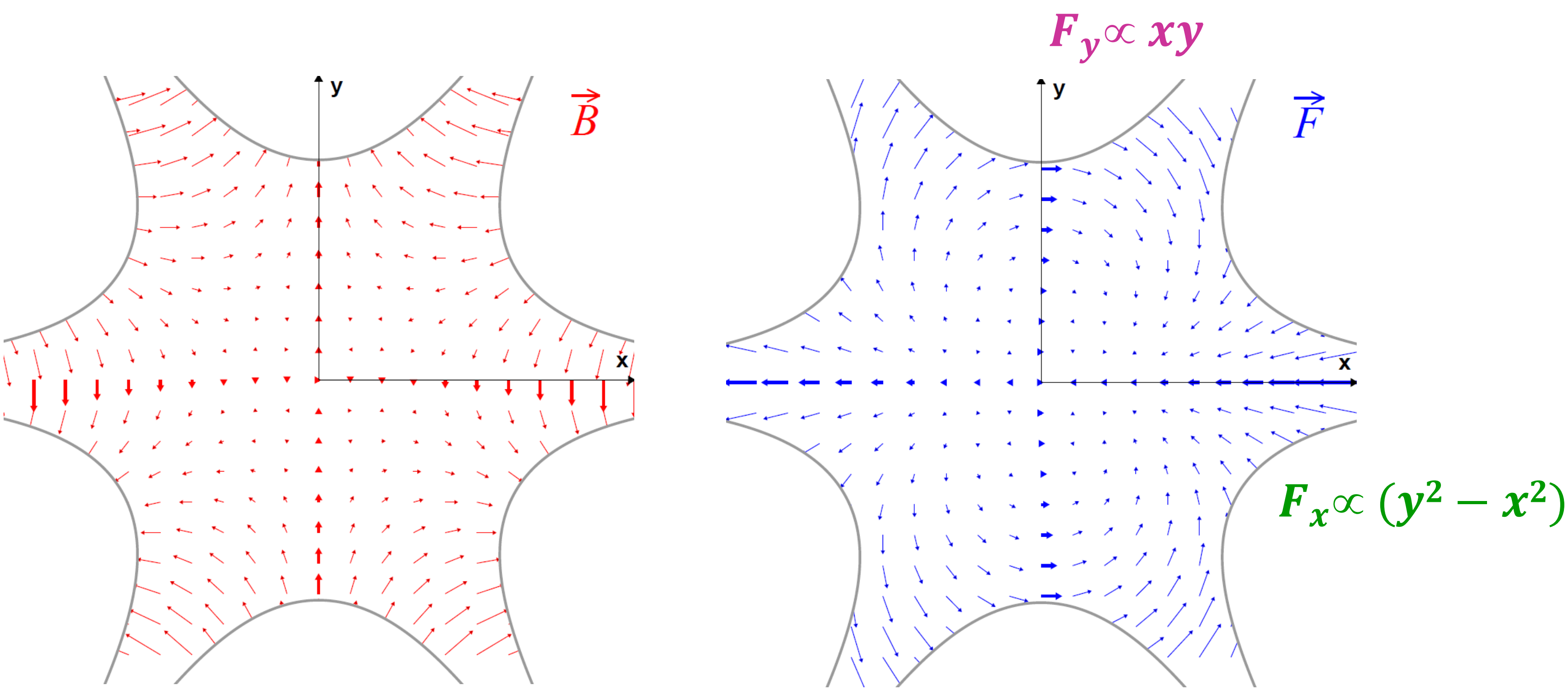
Force pushes the particle away from the center → **defocussing**

Force pushes the particle towards the center → **focussing**



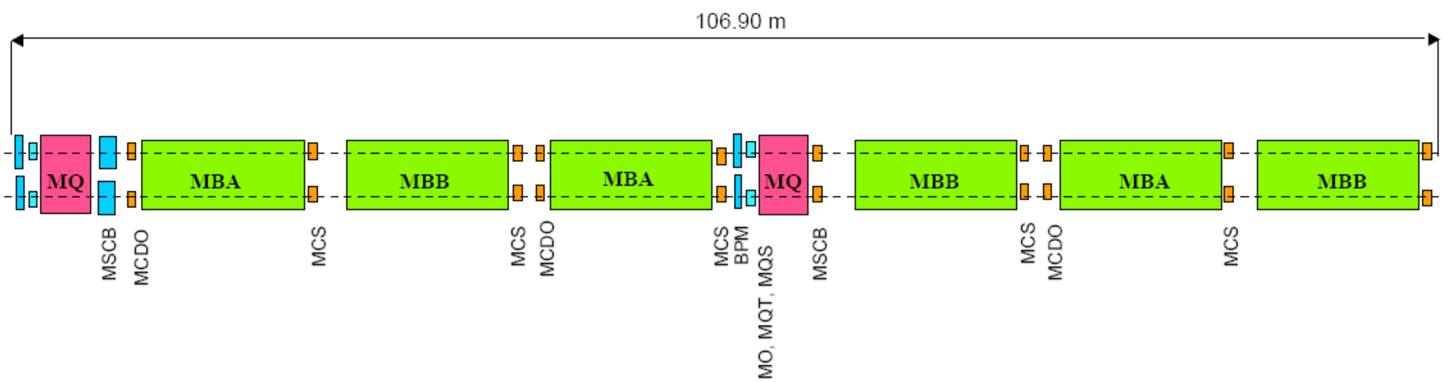
Sextupole

- With sextupoles we are entering the regime of **non-linear dynamics**.
- But sextupoles are also used to correct linear optics errors, for example the chromaticity (tune change with momentum).
  - They may generate linear optics errors through misalignments.
  - A sextupole is ≅ a quadrupole with gradient that increases with distance to centre.



A realistic lattice - LHC

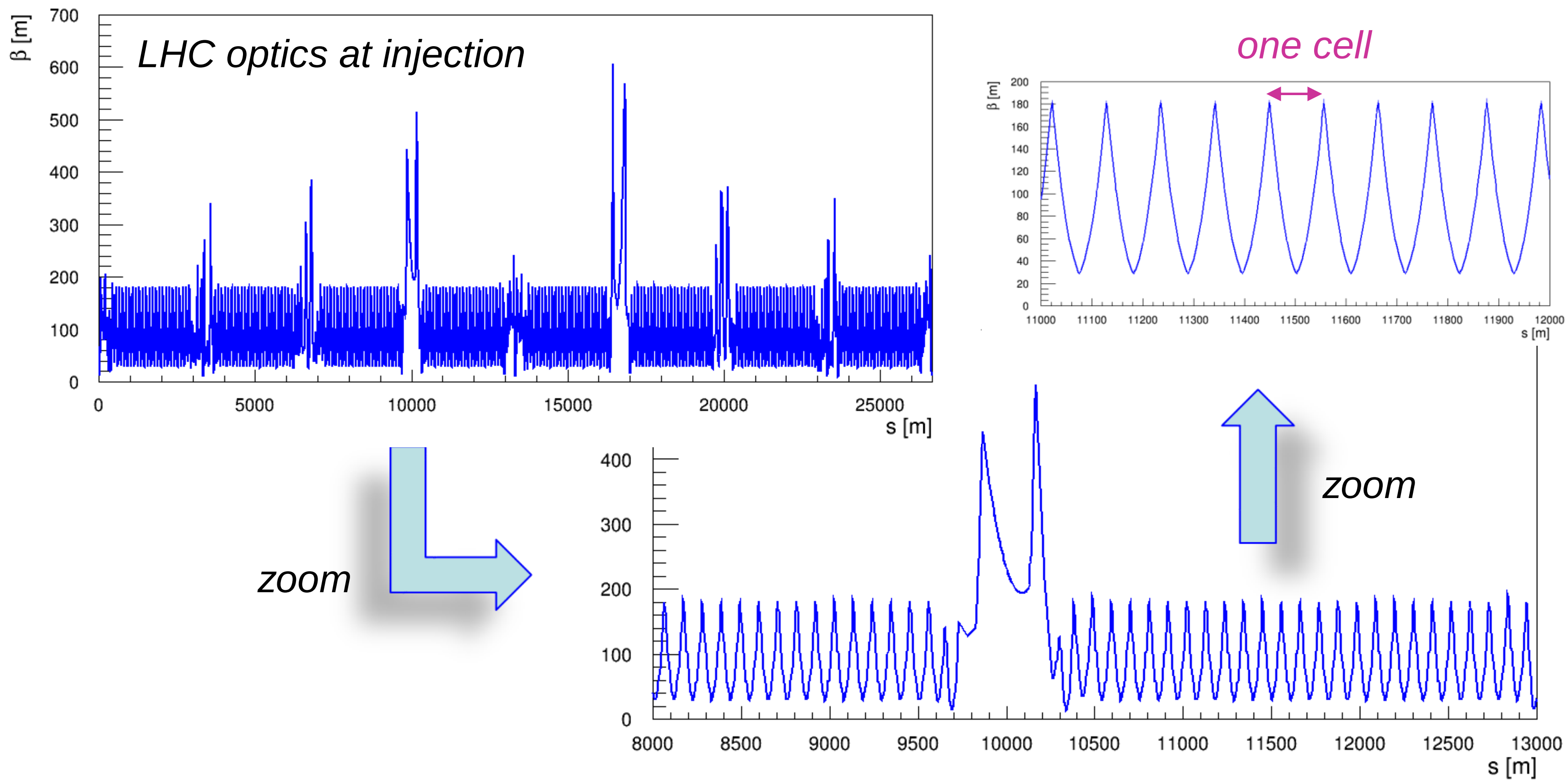
- The LHC arc section are equipped with 107 m long F0D0 cells. Besides our 3 main elements the LHC cell is equipped with other correction (trim) magnets.
  - In modern light sources multipoles are often combined inside the same magnet.



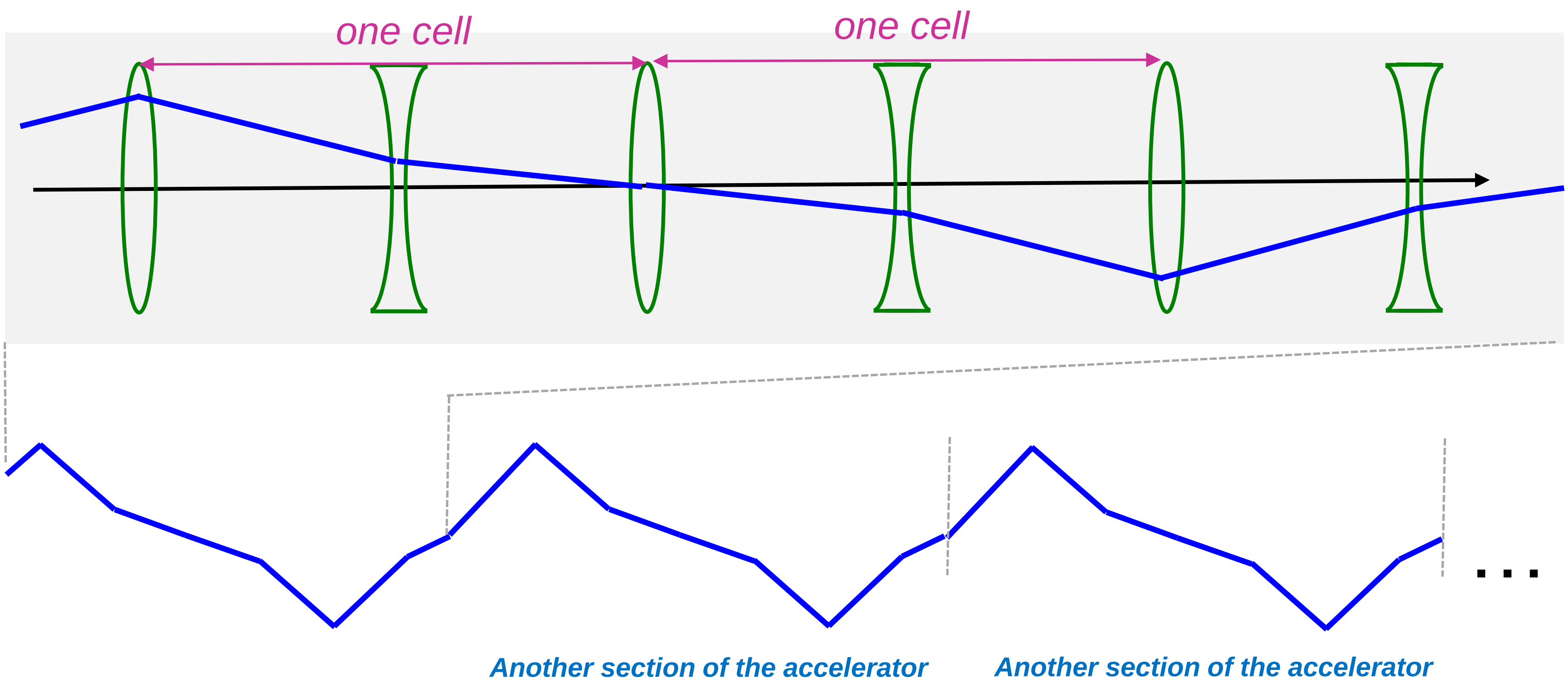
- **MB: main dipole**
- **MQ: main quadrupole**
- **MQT**: trim **quadrupole**
- **MQS**: **skew** trim **quadrupole**
- MO: lattice octupole (Landau damping)
- **MSCB: sextupole** + **orbit corrector dipole**
- MCS: Spool piece **sextupole**
- MCDO: Spool piece 8 / 10 pole
- **BPM: Beam position monitor**

Recap on beam optics

- There are a few quantities related to a beam optics in a circular accelerator that we will need for the lecture:
  - The **betatron function** (**β**) that defines the beam envelope,
    - **Beam size / envelope is proportional to $\sqrt{\beta}$**
  - The **betatron phase advance** (**μ**) that defines the phase of an oscillation.



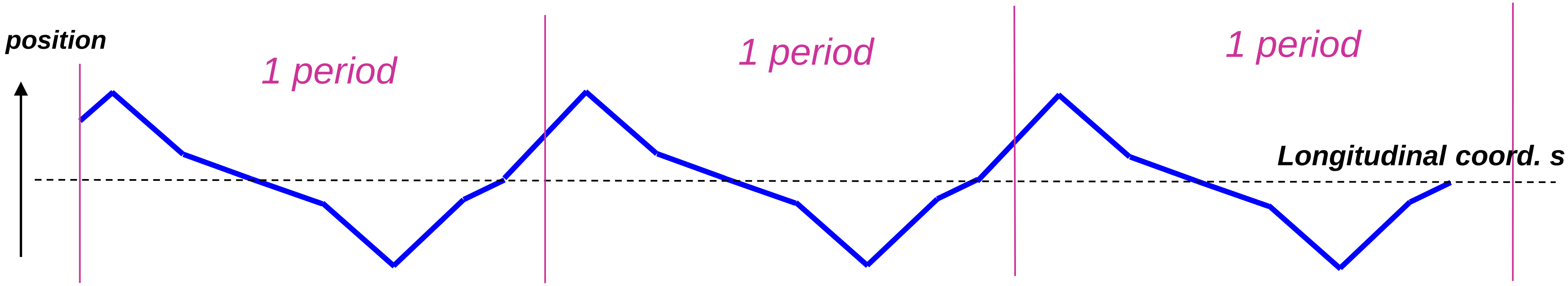
- Consider a particle moving in a section of the accelerator lattice. The focussing elements make it bounce back and forth.



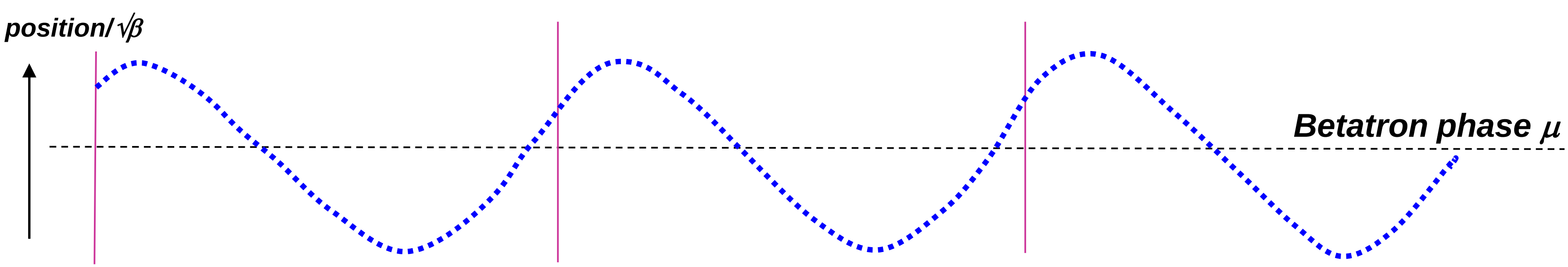
- This periodic oscillation is called a betatron oscillation.

Recap on beam optics for pedestrians

- The number of oscillation periods for one turn of the machine is called the machine **tune** (**Q**) or **betatron tune**.
  - In this **example** Q is around 2.75 – 2 periods and ¾ of a period.



- It is possible to change the **coordinates** (from the longitudinal position in meters to the betatron phase advance in degrees) and transform this **'rocky' oscillation** into a **sinusoidal oscillation**.
  - Convenient (and simpler) way to analyse the beam motion.



From model to reality - fields

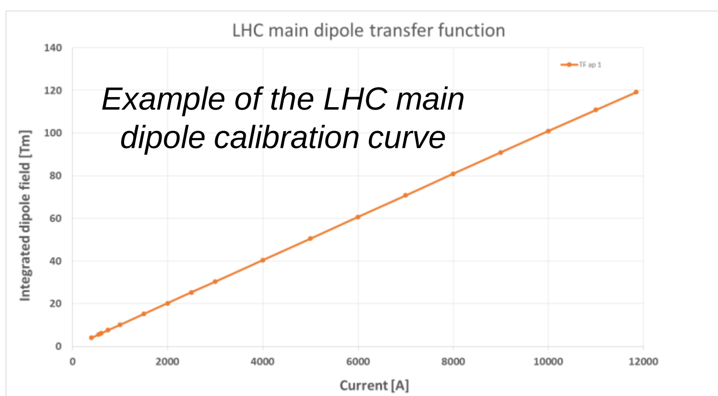
- The **machine model** defined by the accelerator designer must be converted into **electromagnetic fields** and eventually into **currents** for the power converters that feed the magnet circuits (for example).
- **Field imperfections** are introduced when the model is transferred into the real machine. For magnetic fields for example the errors are due to:
  - Beam momentum, magnet measurements and power converter regulation.

Magnet strength → **Magnetic field (gradient)** → **Requested current** → **Actual magnet current**

Beam momentum

Magnet calibration curve (transfer function)

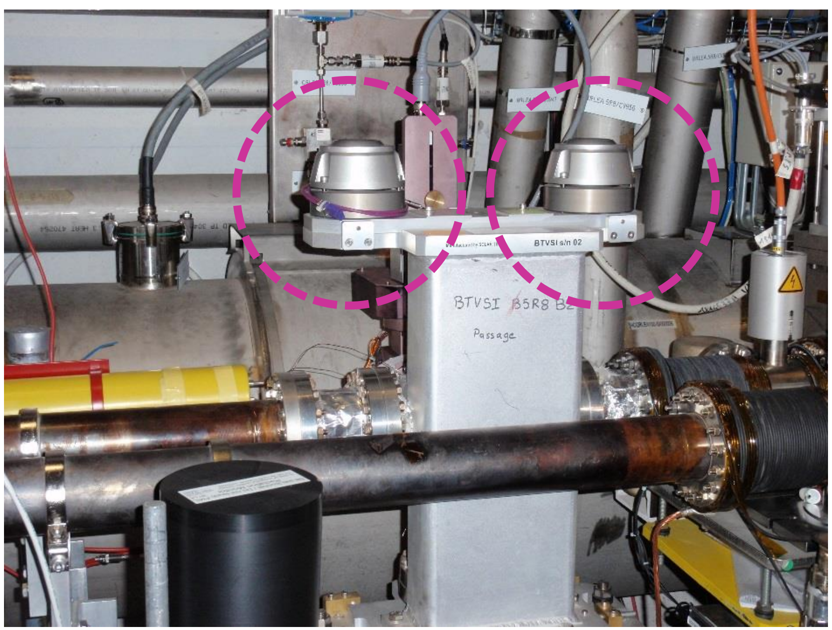
Power converter

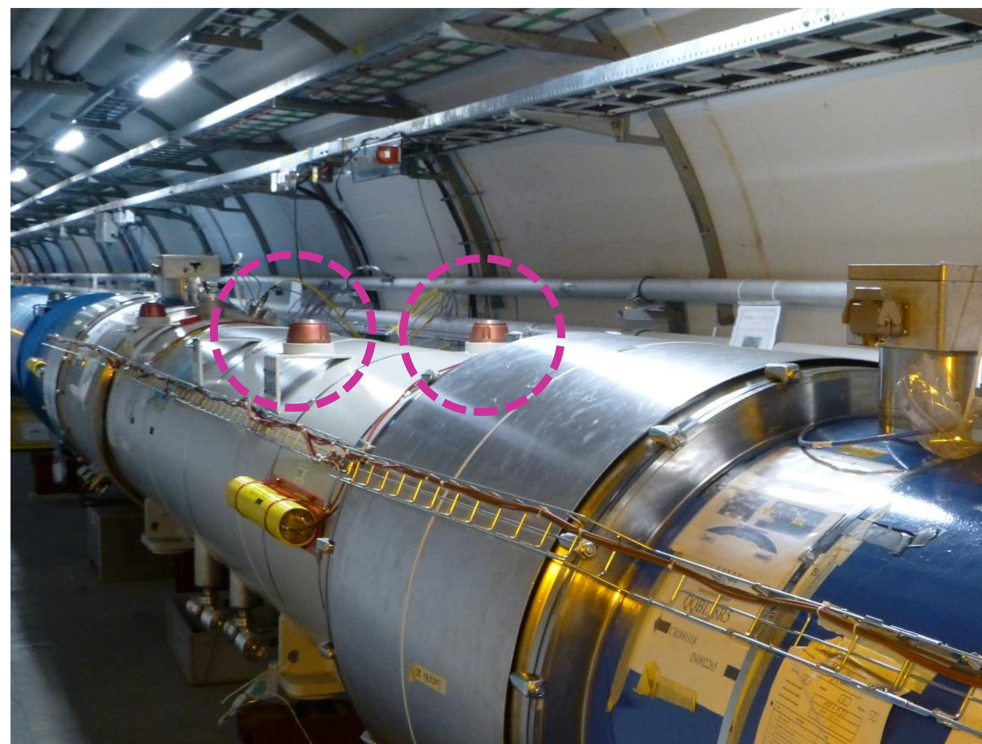


From the lab to the tunnel

From model to reality - alignment

- To ensure that the accelerator elements are in the correct position the alignment must be precise – to the sub-micrometre level for linear colliders !
- The alignment process for a magnet implies:
  – Precise measurements of the magnetic axis in the laboratory with reference to the element alignment markers used by survey teams.
  – Precise in-situ alignment (position and angle) of the element in the tunnel.
- **Alignment errors** are another common source of **imperfections**.





Restoring the model

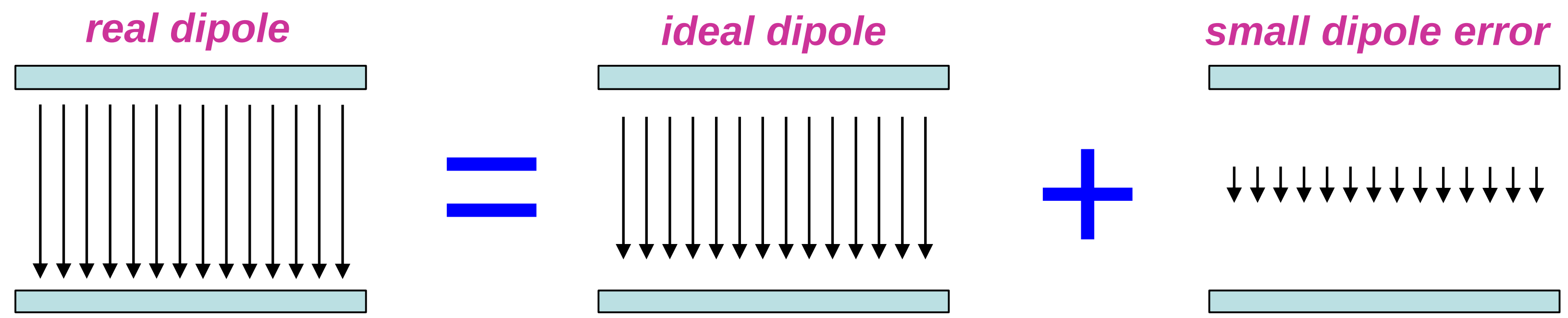
- As a consequence of **imperfections**, the **actual accelerator** may differ from the **model** to a point where the accelerator may not function well / at all.
  - Beam does not circulate due to misalignments,
  - Incorrect optics due to field errors and alignment errors,
  - Missing beam aperture due to alignment errors,
  - ….
- Over the years many tools were developed to measure and correct accelerator parameters in control rooms and to restore design models or update the actual machine model.
  - In many cases the tools are applied iteratively when an accelerator is bootstrapped and commissioned.
- This presentation provides an overview of **linear imperfections**, how to measure and correct them.

Imperfection – undesired deflection

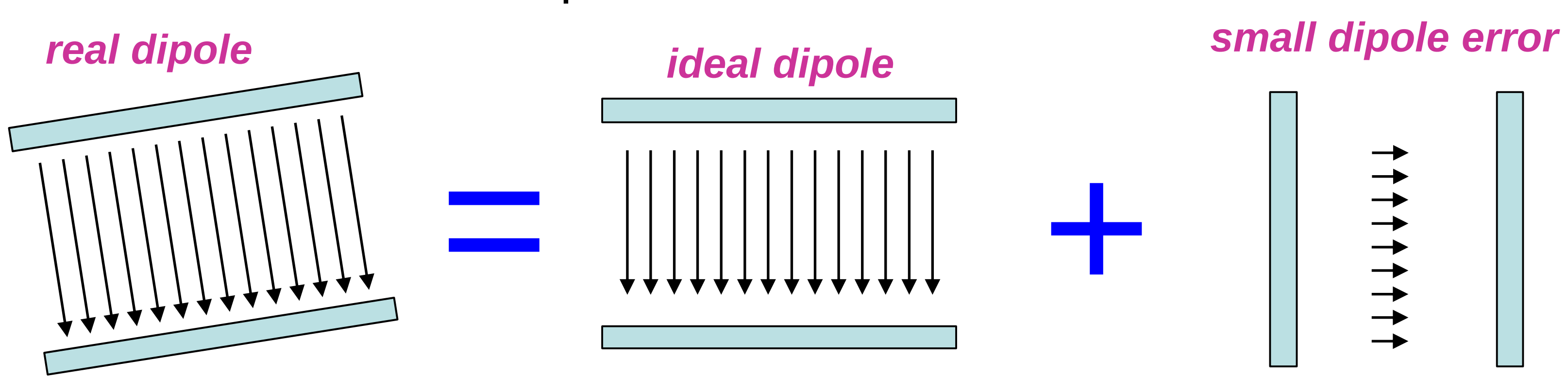
- The presence of an **unintended deflection** along the path of the beam is a first category of imperfections.
- This case is also in general the first one that is encountered when beam is first injected…
- The **dipole orbit corrector** is added to the cell to **compensate** the effect of **unintended deflections**.
  - With the orbit corrector we can generate a deflection of opposite sign and amplitude that compensates locally the imperfection.

Unintended deflection

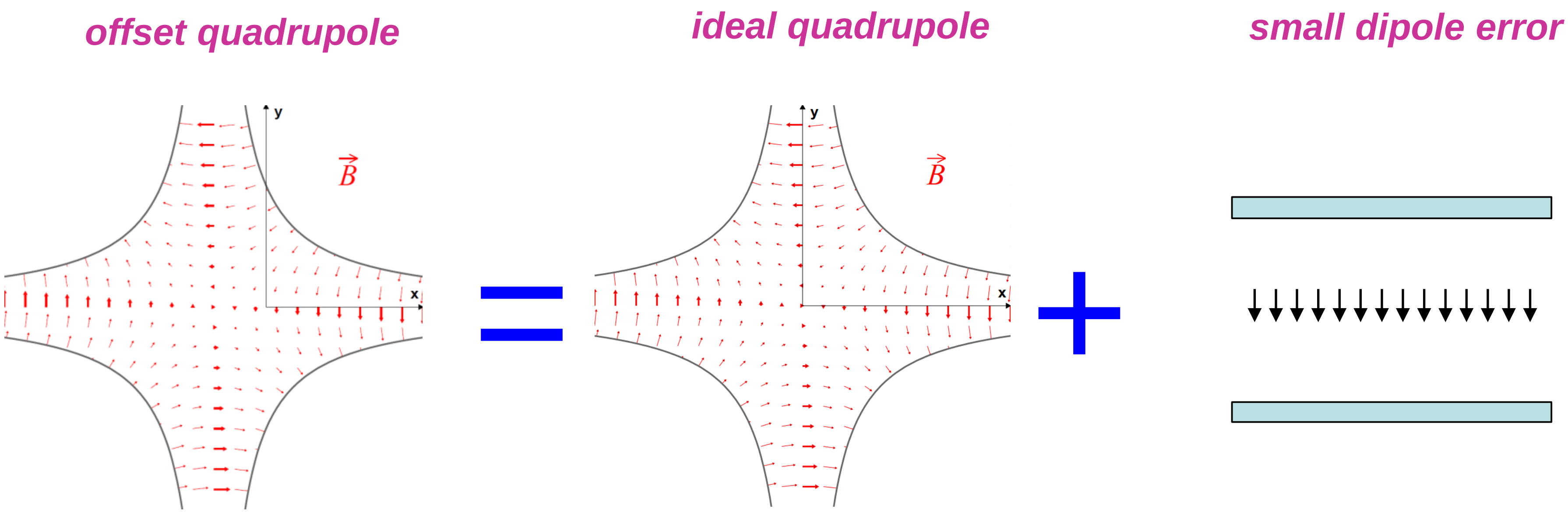
- The first source is a **field error** (deflection error) of a <u>dipole magnet</u>.
- This can be due to an **error** in the **magnet current** or in the **calibration table** (measurement accuracy etc).
  - The imperfect dipole can be expressed as a perfect one + a small error.



- A small **rotation** (**misalignment**) of a <u>dipole magnet</u> has the same effect, but in the other plane.



- The second source is a **misalignment** of a quadupole magnet.
  - The misaligned quadrupole can be represented as a perfectly aligned quadrupole plus a small deflection.



Non-zero magnetic field on the beam axis !

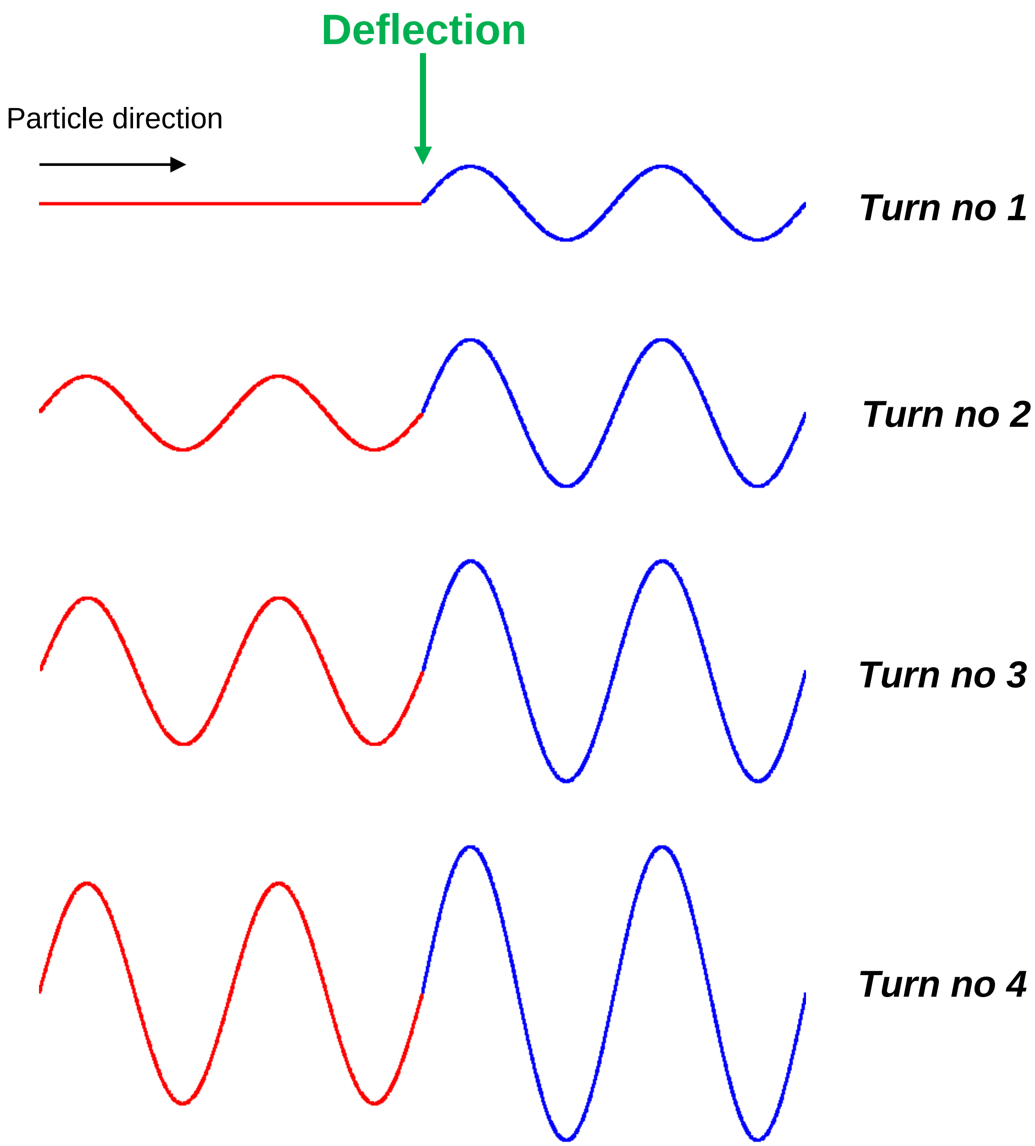
No magnetic field on the beam axis

Linear imperfections table

- Summary table of linear imperfections

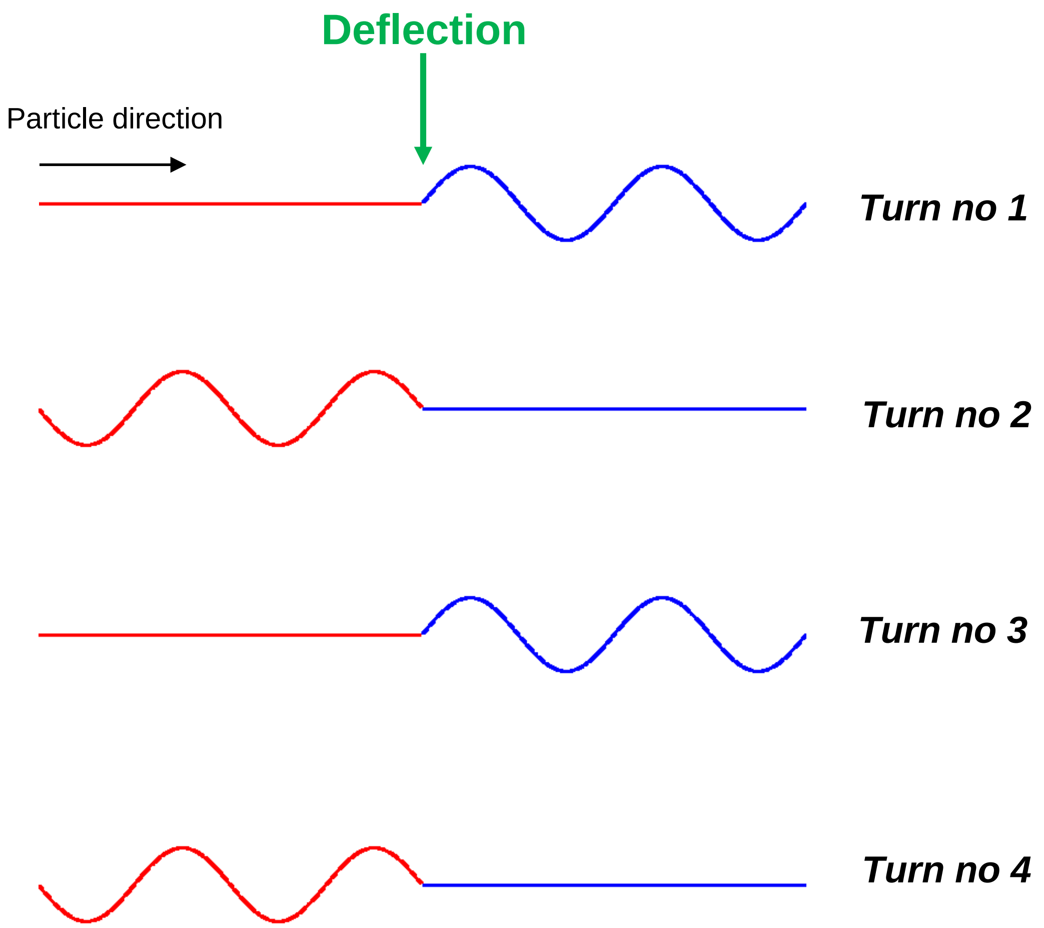
Field type	Imperfection	Error type	Impact
Dipole	**Field error**	**Dipole**	**Orbit / trajectory**
Dipole	**Tilt**	**Dipole**	**Orbit / trajectory**
Quadrupole	Field error	Quadrupole	Tune / optics
Quadrupole	**Offset**	**Dipole**	**Orbit / trajectory**
Quadrupole	Tilt	Skew quadrupole	Coupling
Sextupole	Field error	Sextupole	Chromaticity
Sextupole	Offset horizontal	Quadrupole	Tune / optics
Sextupole	Offset vertical	Skew quadrupole	Coupling

Effect of a deflection



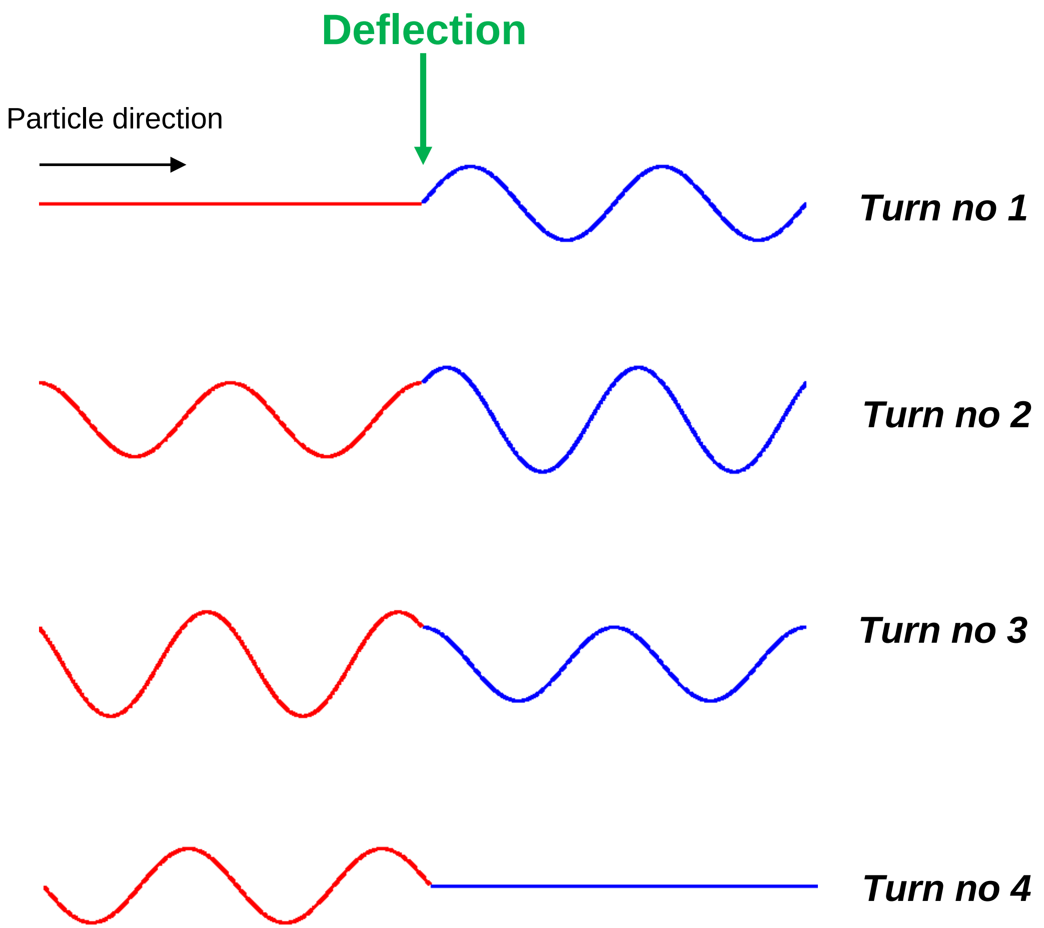
- We set the machine tune to an integer value:
  - $Q = n \in N$
- When the tune is an integer number, **the deflections add up on every turn** !
  - The amplitudes diverge, the particles do not stay within the accelerator vacuum chamber.
- We just encountered our first **resonance** – the integer resonance that occurs when $Q = n \in N$

Effect of a deflection



- We set the machine tune to a half integer value:
  - $Q = n+0.5,\ n \in N$
- For half integer tune values, **the deflections compensate on every other turn** !
  - The amplitudes are stable.
- This looks like a much better working point for Q!

Effect of a deflection

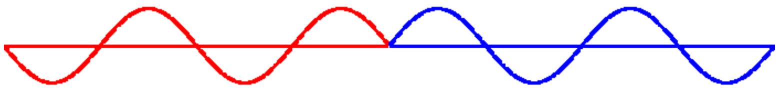


- We set the machine tune to a quarter integer value:
  - Q = n+0.25, n ∈N
- For quarter tune values, **the deflections compensate every four turns** !
  - The amplitudes are stable.
- Also a reasonable working point for Q!

Many turns reveal something

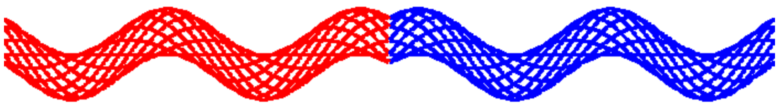
- Let's plot the 50 first turns on top of each other and change Q.
  - All plots are on the same scale

Q = n + 0.5

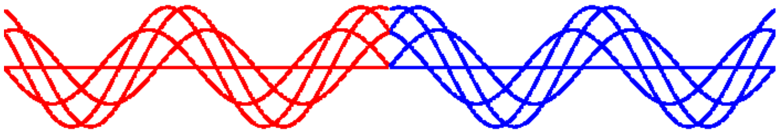


Q = n + 0.4

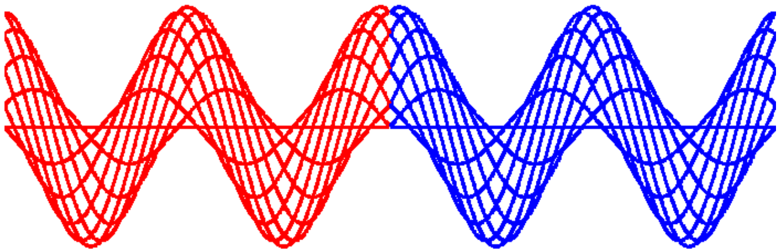
Q = n + 0.3



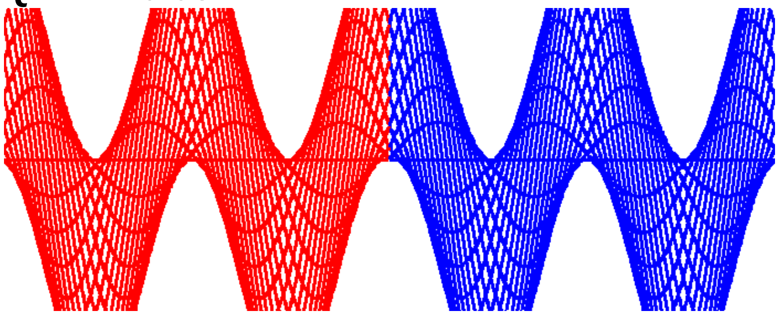
Q = n + 0.2



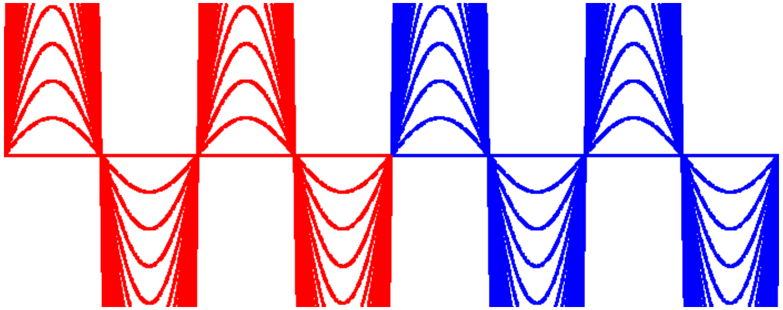
Q = n + 0.1



Q = n + 0.05



Q = n



- The particles **oscillate around a stable mean value** ($Q \neq n$)!
- The amplitude diverges as we approach Q = n → integer resonance

The closed orbit

- The stable mean value around which the particles oscillate is called the **closed orbit**.
  - Every particle in the beam oscillates around the closed orbit.
  - As we have seen the closed orbit 'does not exist' when the tune is an integer value.
- The general expression of the **closed orbit x(s)** in the presence of a **deflection θ** is:

amplitude modulated by the envelope β

oscillating term

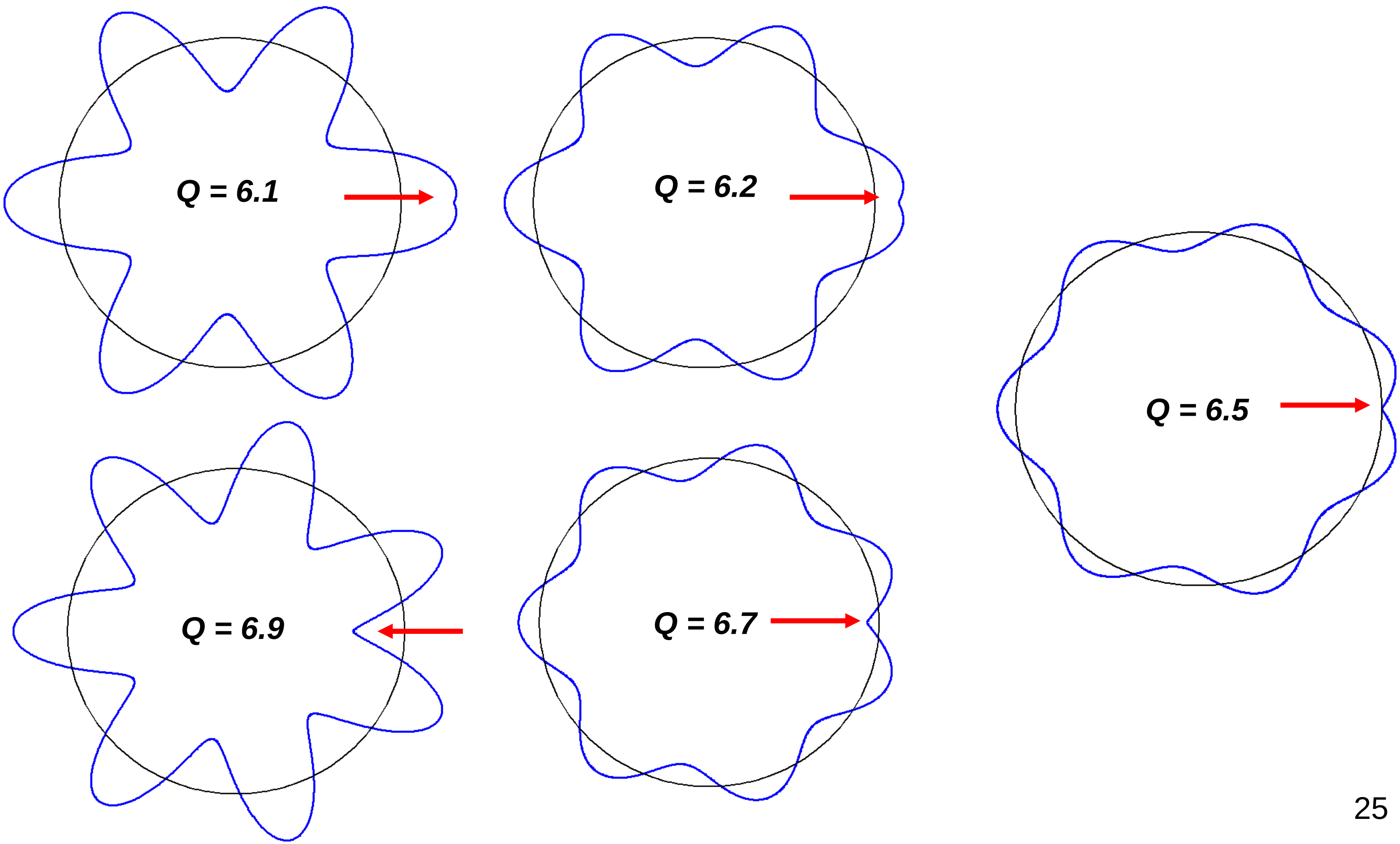
kink at the location of the deflection

$$x(s) = \frac{\sqrt{\beta(s)\beta_\theta}\cos(|\mu(s)-\mu_\theta|-\pi Q)}{2\sin(\pi Q)}\theta$$

divergence for Q = n

Closed orbit example

- Example of the horizontal closed orbit for a machine with tune Q = 6 + q.
- The **kink at the location of the deflection** (**→**) can be used to localize the deflection (if it is not known) → can be used for orbit correction.



- The **position response** Δu_i of the beam at position i due to a deflection $\Delta\theta_j$ at position j is given in linear approximation by:

$$\Delta u_i = R_{ij}\,\Delta\theta_j \qquad \text{where:} \qquad R_{ij} = \frac{\sqrt{\beta_i\,\beta_j}\,\cos(|\mu_i - \mu_j| - \pi Q)}{2\sin(\pi Q)}$$

Closed orbit

$$R_{ij} = \sqrt{\beta_i\,\beta_j}\,\sin(\mu_i - \mu_j) \qquad \mu_i > \mu_j$$

$$R_{ij} = 0 \quad \mu_i < \mu_j$$

Trajectory/Linac

$$\beta = \text{betatron function} \qquad \mu = \text{phase advance} \qquad Q = \text{tune}$$

- To a first approximation we can limit the discussion to deflections generated by misaligned quadrupoles (gradient k, length l_Q) and by steering elements (orbit correctors).
  - For quadrupoles with alignment error δ, the kick is $\Delta\theta = -k l_Q \delta$

- The relation between the positions measured at ***N* BPMs** (beam position monitors) and the deflections due to ***M* steering elements** (in general ($N \geq M$) can be cast into a matrix format (**R** = response matrix):

$$\Delta\vec{u} = \mathbf{R}\,\Delta\vec{\theta}$$

$$\Delta\vec{u} = \begin{pmatrix} \Delta u_1 \\ \Delta u_2 \\ \dots \\ \Delta u_N \end{pmatrix} \quad \mathbf{R} = \begin{pmatrix} R_{11} & R_{12} & \dots & R_{1M} \\ R_{21} & R_{22} & \dots & R_{2M} \\ \dots & \dots & \dots & \dots \\ R_{N1} & \dots & \dots & R_{NM} \end{pmatrix} \quad \Delta\vec{\theta} = \begin{pmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \dots \\ \Delta\theta_M \end{pmatrix}$$

- To steer the beam deterministically this equation must be inverted, something like :

$$\Delta\vec{\theta}_c = \mathbf{R}^{-1}\Delta\vec{u}_m$$

$\mathbf{R}^{-1}$ is in general a 'pseudo' inverse, a real inverse only exists if $N=M$.

- Response matrix **R** obviously contains a lot of information on the machine optics – later we will see that they are tools to take advantage of that fact to determine and correct the lattice functions.

- In general the algorithms for beam steering aim to minimize the least square error:

$$\left\| \Delta\vec{\theta}_c - \mathbf{R}^{-1}\Delta\vec{u}_m \right\|^2 = \min$$

  – We assume here that we know the response matrix ***R*** well enough to apply a correction.

- This equation is rather generic, problems of optics and dispersion correction can be cast in a similar form, and the algorithms used for steering are also used or adapted to those problems.

The early days of orbit correction

- The problem of correcting the orbit deterministically came up a long time ago in the first machines.
- B. Autin and Y. Marti of CERN published a note in 1973 describing an algorithm that is still in use today in many machines:
  - **MICADO***
  - One of the first deterministic correction algorithms !

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN ISR-MA/73-17

CLOSED ORBIT CORRECTION OF A.G. MACHINES

USING A SMALL NUMBER OF MAGNETS

by

B. Autin & Y. Marti

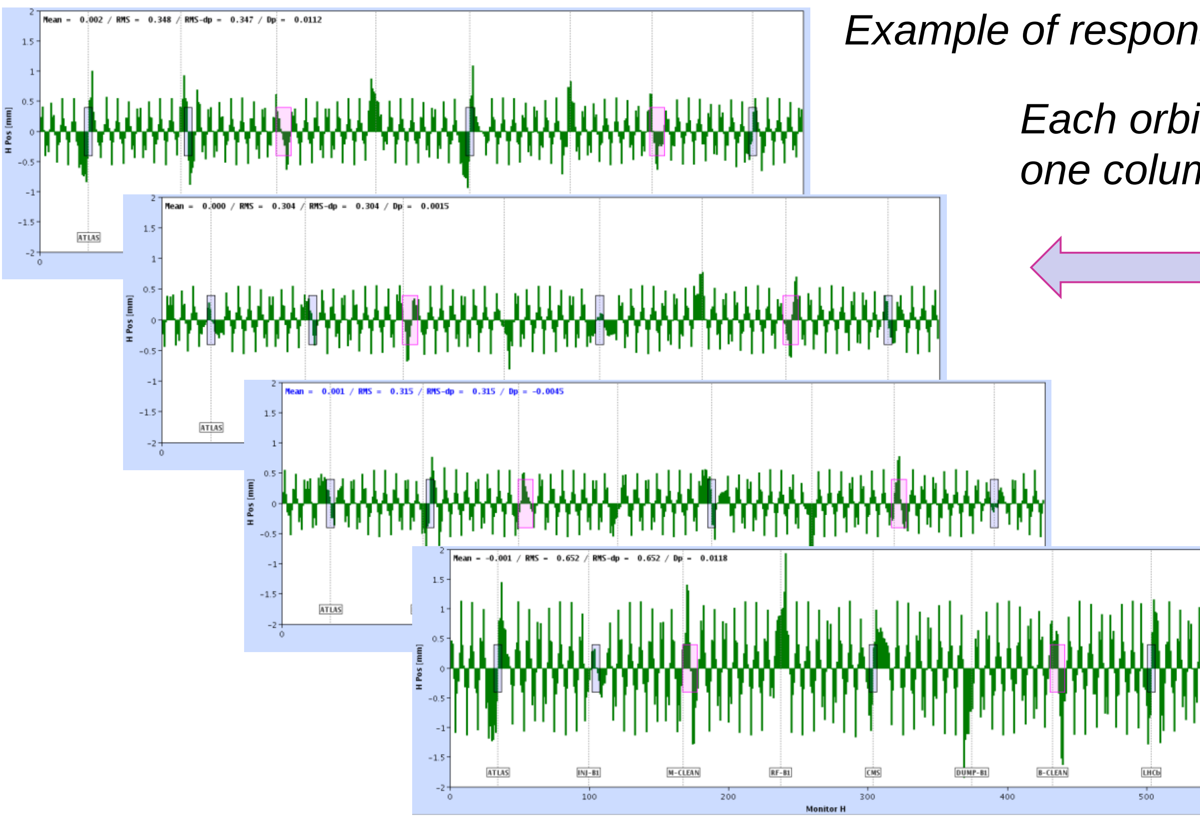
```
CALL MICADO (A, B, NDIM, M, N, AP, XA, NA, NB, NC, EPS, ITER, DP, X, NX, R, RHO).
```

* MInimisation des CArrés des Distortions d'Orbite.

(Minimization of the quadratic orbit distortions)

MICADO principle

- The intuitive principle of MICADO is rather simple.
- Preparation:
  - The machine model must be used to build the $\boldsymbol{R}$ matrix,
  - Each column of $\boldsymbol{R}$ correspond to the response of all N BPMs to one of the correctors.



Example of responses for LHC

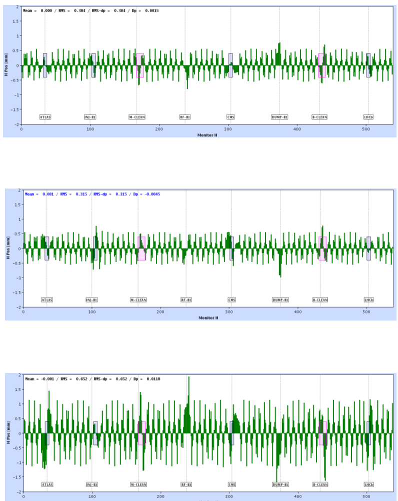
Each orbit response corresponds to one column of $\boldsymbol{R}$

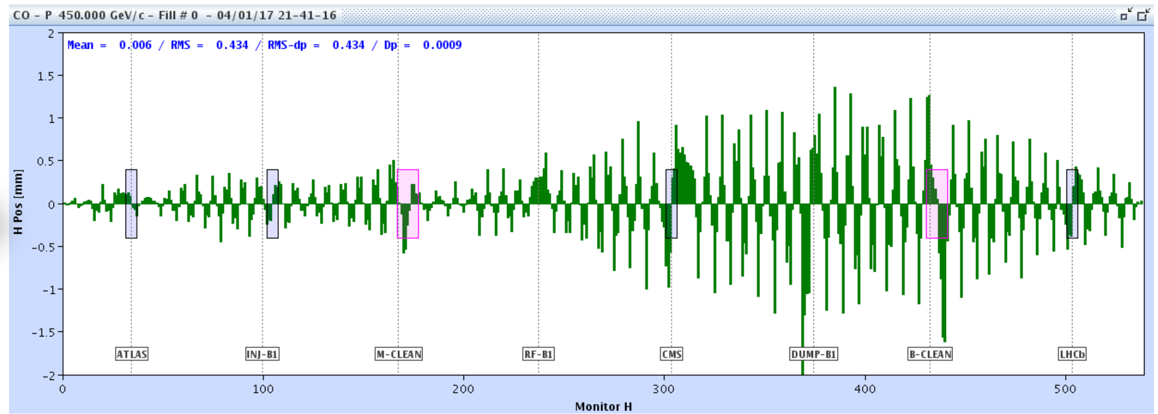
$$\mathbf{R} = \begin{pmatrix} R_{11} & R_{12} & \dots & R_{1M} \\ R_{21} & R_{22} & \dots & R_{2M} \\ \dots & \dots & \dots & \dots \\ R_{N1} & \dots & \dots & R_{NM} \end{pmatrix}$$

. . .

MICADO principle II

- MICADO compares the response of every corrector with the raw orbit.





…

- MICADO picks out the corrector that hast the **best match** with the orbit, and that will give the largest reduction of $\left\|\Delta\vec{\theta}_c - \mathbf{R}^{-1}\Delta\vec{u}_m\right\|^2$
- The procedure can be **iterated** using the remaining correctors until the orbit is good enough (stop after K steps using K correctors) or as good as it can be by using all available correctors.

Singular Value Decomposition

- **Singular Value Decomposition** (**SVD**) is a generic operation applicable to any matrix **R** that is decomposed into 3 matrices **Z**, **W** and **V**:

$$R = ZWV^T$$

P2 P3

- **W** is a diagonal 'eigenvalue' matrix while **V** is a square matrix that is also ortho-normal:

$$W = \begin{pmatrix} w_1 & 0 & \dots & 0 \\ 0 & w_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & w_M \end{pmatrix} \quad V = \begin{pmatrix} v_{11} & v_{12} & \dots & v_{1M} \\ v_{21} & v_{22} & \dots & v_{2M} \\ \dots & \dots & \dots & \dots \\ v_{M1} & v_{M2} & \dots & v_{MM} \end{pmatrix} \quad Z = \begin{pmatrix} z_{11} & z_{12} & \dots & z_{1M} \\ z_{21} & z_{22} & \dots & z_{2M} \\ \dots & \dots & \dots & \dots \\ z_{N1} & z_{N2} & \dots & z_{NM} \end{pmatrix}$$

$$VV^T = V^T V = 1 \qquad Z^T Z = 1$$

The columns (vectors) of **V** are related to the columns (vectors) of **Z** by the eigenvalues:

$$\vec{v}_j = \begin{pmatrix} v_{1j} \\ v_{2j} \\ \dots \\ v_{Mj} \end{pmatrix} \qquad \mathbf{R}\vec{v}_j = w_j \vec{z}_j = w_j \begin{pmatrix} z_{1j} \\ z_{2j} \\ \dots \\ z_{Nj} \end{pmatrix}$$

$$VV^T = V^T V = 1$$

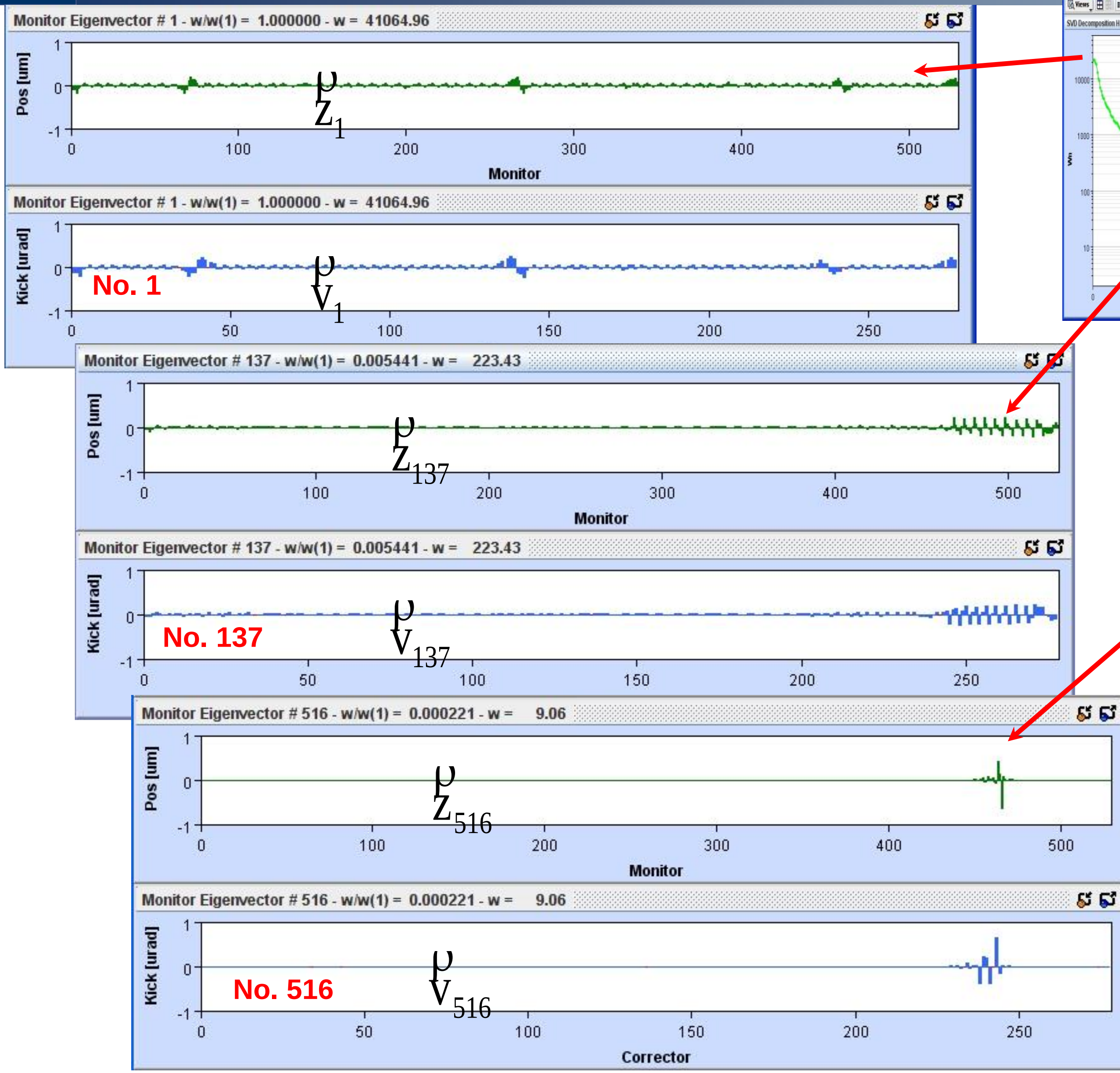
$$\downarrow$$

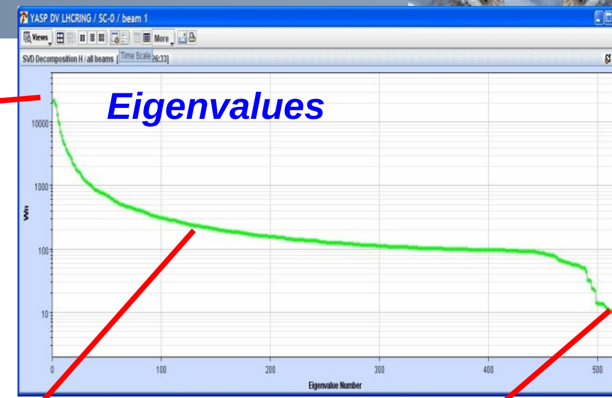
$$\vec{v}_i \cdot \vec{v}_j = \delta_{ij}$$

The vectors form an ortho-normal basis of the 'corrector space'

$\cdot$ = scalar product

Eigenvectors examples for LHC





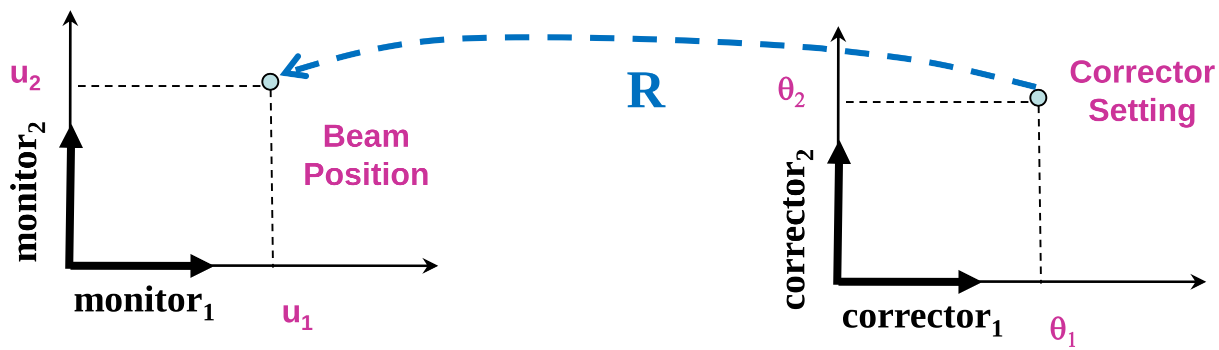
As the eigenvalues decrease, the associated eigenvectors correspond to increasingly local 'structures'

$$\vec{z}_i = \frac{1}{w_i}\mathbf{R}\vec{v}_i$$

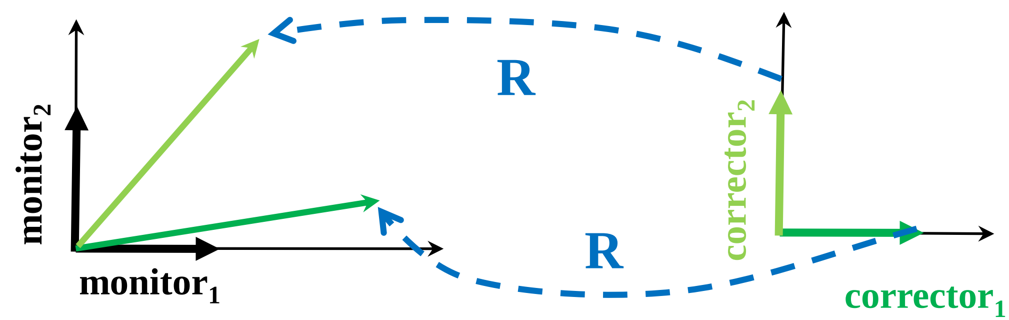
Meaning of eigenvectors

What are those eigenvectors and what is the useful ‘trick’ behind an SVD decomposition?

- The response matrix **R** maps points in ‘corrector space’ to point in ‘beam position’ space.
  - The natural basis vectors of those spaces are **physical monitors and correctors**.



- The problem is that **R** maps orthogonal vectors (=correctors) into non-orthogonal responses in monitor space.

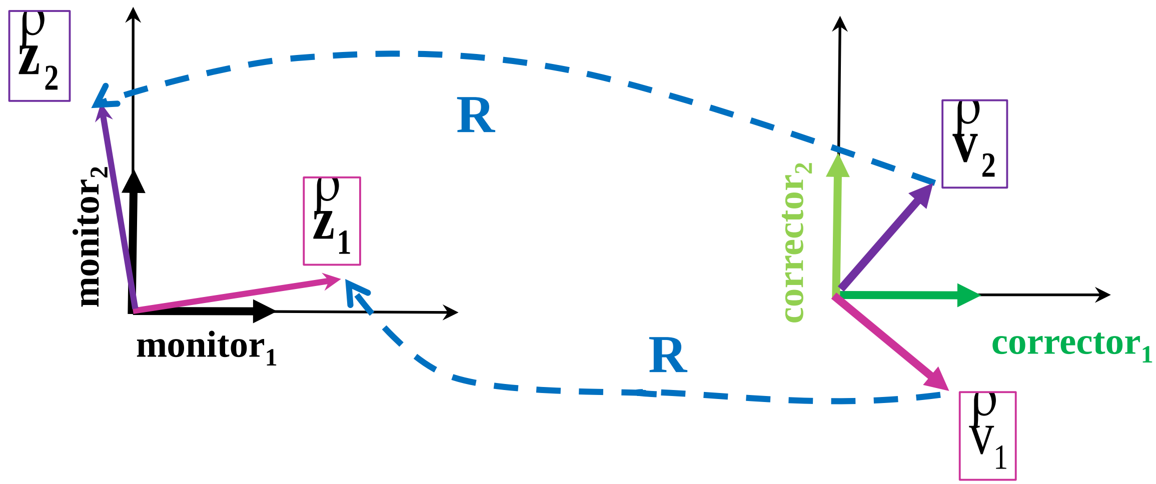


Meaning of eigenvectors II

- The SVD decomposition identifies a **new orthonormal basis of the orbit corrector space** such that **their responses are also orthogonal** !

M vectors for a **N×N** dimensional space

M vectors for a **M×M** dimensional space



$$\vec{\mathbf{z}}_\mathrm{i} \cdot \vec{\mathbf{z}}_\mathrm{j} = \delta_{ij}$$

$$\vec{\mathbf{v}}_\mathrm{i} \cdot \vec{\mathbf{v}}_\mathrm{j} = \delta_{ij}$$

- Every corrector setting can be decomposed into the **v** vectors.
- Every orbit can be decomposed into the **z** vectors plus a residual un-correctable remainder:

$$\vec{\mathbf{u}}_\mathrm{m} = \sum_{i=1}^{M} c_i \vec{\mathbf{z}}_\mathrm{j} + \vec{\mathbf{u}}_\mathrm{residual}$$

- SVD can be used to solve the determine a correction using k out of M eigenvalues.
  - The eigenvalues w_j are typically sorted in descending order: $w_{j+1} \leq w_j$.

$$\Delta\vec{\theta}_c = -\left(VW^{-1}Z^T\right)\Delta\vec{u}_m = -\tilde{R}^{-1}\Delta\vec{u}_m$$

$$W^{-1} = \begin{pmatrix} 1/w_1 & 0 & \dots & \dots & \dots & 0 \\ 0 & \dots & 0 & \dots & \dots & \dots \\ \dots & 0 & 1/w_k & \dots & \dots & \dots \\ \dots & & 0 & 0 & \dots & \dots \\ \dots & & & & \dots & 0 \\ 0 & \dots & \dots & \dots & 0 & 0 \end{pmatrix}$$

- This operation corresponds intuitively to decompose the measured orbit into the orbit eigenvectors z_i – unique ! – and then to correct the effect of the k largest eigenvectors (since z_i is associated to v_i).

$$Z^T\Delta\vec{u}_m \qquad \vec{z}_j \cdot \vec{u}_m = c_i \qquad \forall i \leq k$$

$$\left(W^{-1}Z^T\right)\Delta\vec{u}_m \quad \Longleftrightarrow \quad c_i / w_i$$

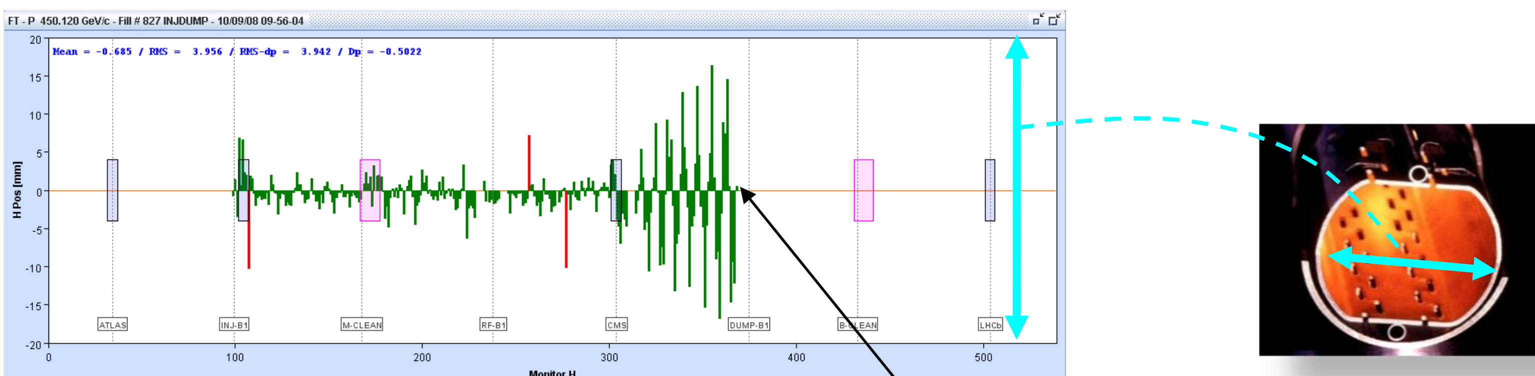
$$-\left(VW^{-1}Z^T\right)\Delta\vec{u}_m \qquad -\sum_i^k (c_i / w_i)\vec{v}_i$$

SVD - MICADO

- MICADO picks out individual correctors.
  - With a perfect match of model and machine it will help localize local sources.
  - Well suited in case of clean measurements to identify a single / dominant source.
  - MICADO can be in trouble if the R matrix presents singularities, associated to poor BPM or corrector layout (poor phase conditions…).
- SVD will always use all correctors.
  - With few eigenvalues for the correction, even a local perturbation will be corrected with many elements.
    - Can be a pro if the strength of correctors is limited.
  - The number of eigenvalues controls the locality /quality of the correction. With more eigenvalues local structures will be corrected better.
  - By limiting the number of eigenvalues it is possible to avoid correcting on noise, in particular with eigenvectors that drive large strength and provide little position change.
    - See also later the **MIA technique**.
  - Since the SVD correction can be cast into a simple matrix operation, it is well suited (and always used) for real-time **orbit feedbacks**.

- The first problem encountered during machine commissioning is to bring the beam to the end of the linac, respectively circulate it in the storage ring.
  - Level of difficulty depends on the alignment errors and the length of the machine.
- For small accelerators it is usually not a too serious issue as the number of undesired deflections encountered over the length of the accelerator is not too large, but for many km long machines this is far for guaranteed.
  - Trajectory excursion build up randomly along the path of the beam s.
  - For random errors the trajectory amplitudes scale roughly ~ $\sqrt{s}$.

LHC during first turn steering

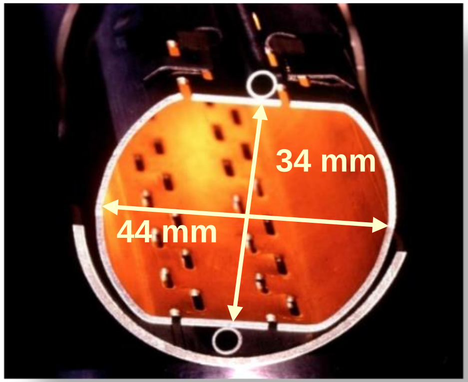


Beam stopped on a collimator

- For a typical **rms alignment error σ_a** it is possible to estimate the resulting rms orbit error σ_{orb} for a machine (length L) build with a homogenous lattice consisting of N_c FODO cells:

$$\sigma_{orb} \approx \frac{|k| l_Q \beta_{eff}}{4\sin(\pi Q)} \sqrt{N_c}\, \sigma_a \equiv \kappa \sigma_a \quad \propto \sqrt{L}$$

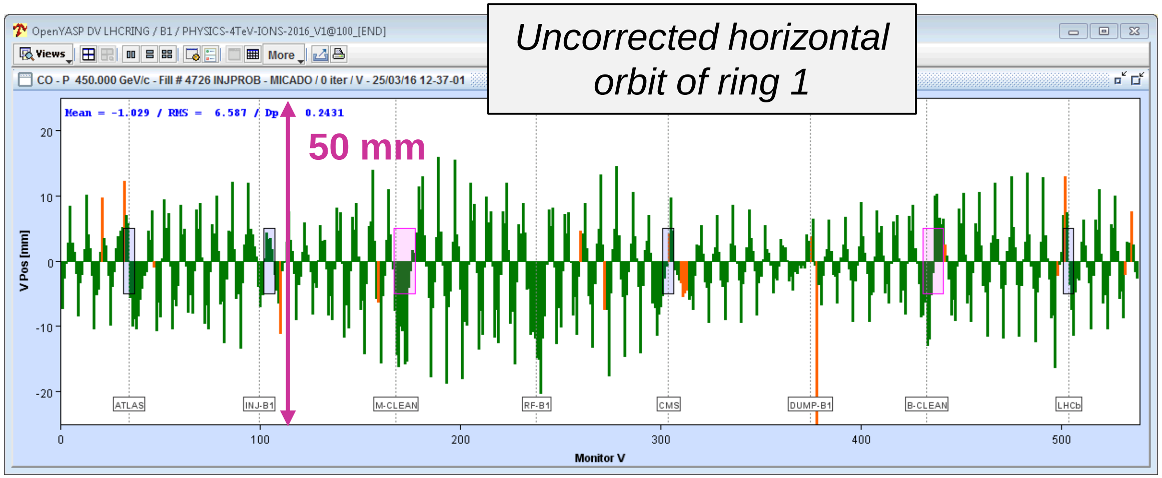
LHC vacuum chamber



- **Example**: for the **LHC injection optics** **$\kappa \cong$ 20-30**. For σ_a ~ 0.3 mm, the expected orbit rms σ_{orb} is 6-9 mm. Excursions of $\pm 2\sigma_{orb}$ already exceed the mechanical aperture of the vacuum chamber.
- The situation is likely become worse at FCC-hh !
- **Example**: for the 100 km long FCC-ee with N_c ~ 1500, **$\kappa \cong$ 45**. For σ_a ~ 0.1 mm, the expected orbit rms σ_{orb} is ~5 mm. This does not look too bad, but the FCC-ee lattice is so non-linear (strong focussing) that the beam does not make it around, although the vacuum chamber height is ~ twice as large than LHC.

LHC orbit correction example

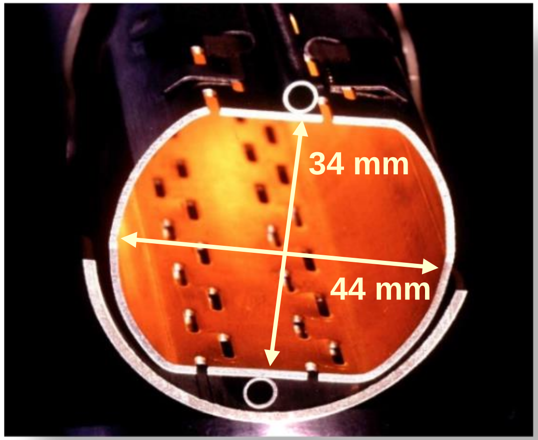
- The raw orbit at the LHC can have large errors (in this example the correctons were unfolded !), but proper correction bring the deviations down by more than a factor 20.

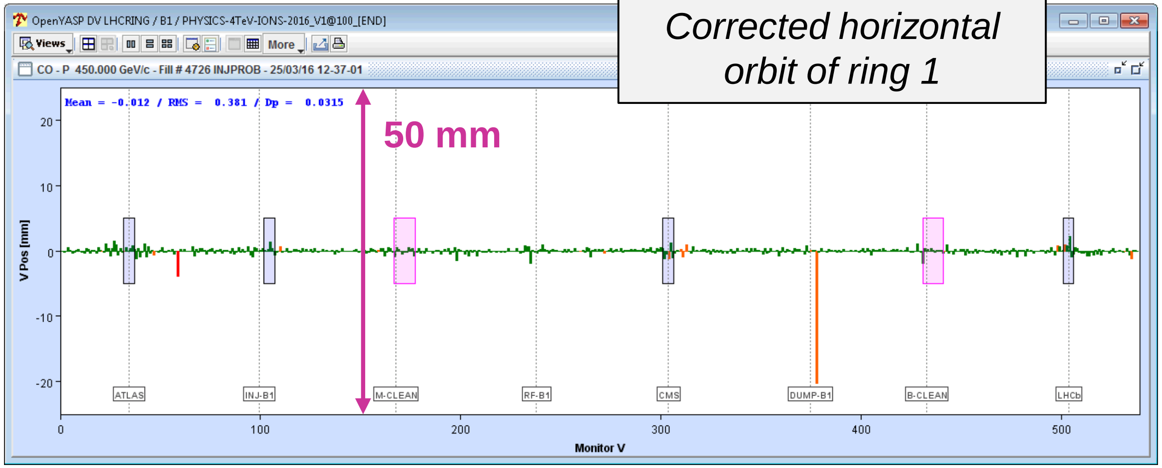


Uncorrected horizontal orbit of ring 1

MICADO & SVD

LHC vacuum chamber

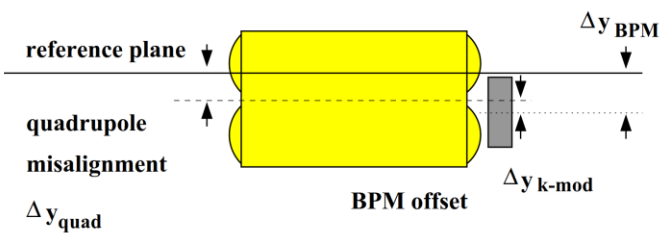




Corrected horizontal orbit of ring 1

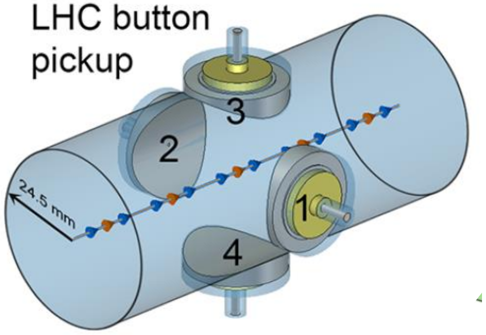
BPM errors

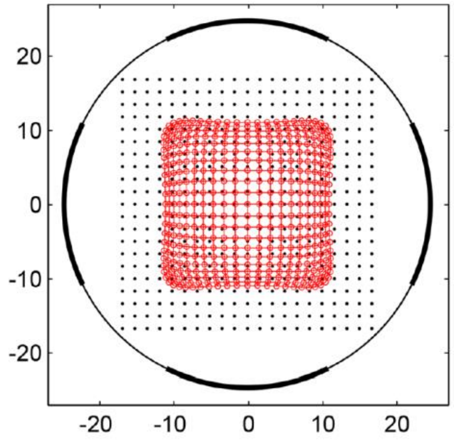
- The **quality of the BPM measurements** used for measurements and corrections are affected by:
  - **Offsets**,
  - **Scale errors and non-linearities,**
  - **Intensity and beam pattern effects.**



- Non-linearities and beam/intensity systematics must be simulated or obtained from laboratory measurements.
  - Non-linearities are due to electrode geometries and to the electronics.
  - Improper correction of such effect can bias beam based measurements.

LHC button BPM





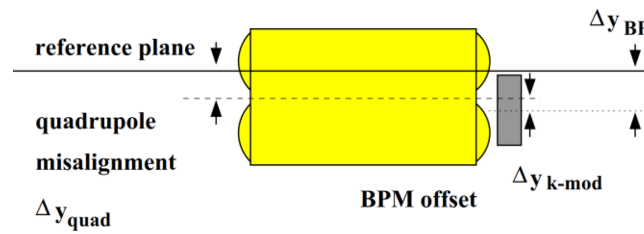
Black points: real beam position,

Red points: distorted positions based on a position reconstruction with a linear assumption:

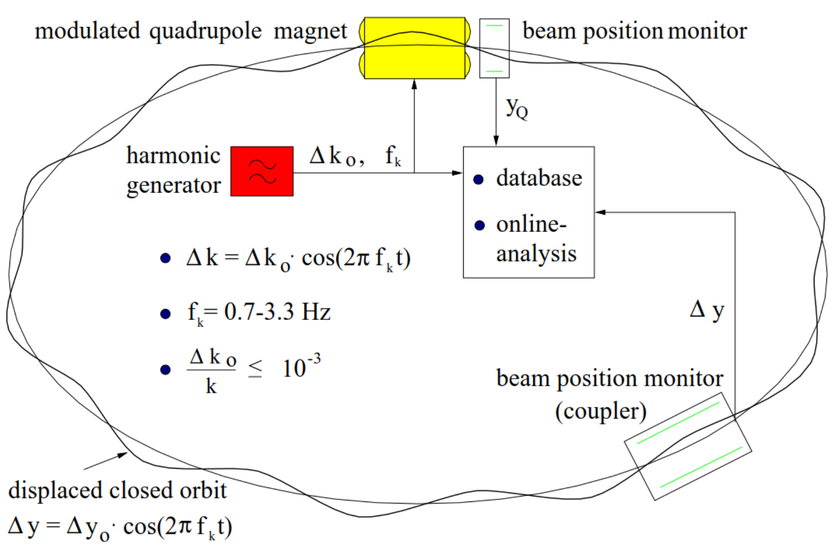
$$x = K_x \frac{S_1 - S_2}{S_1 + S_2} \qquad y = K_y \frac{S_3 - S_4}{S_3 + S_4}$$

BPM-NL

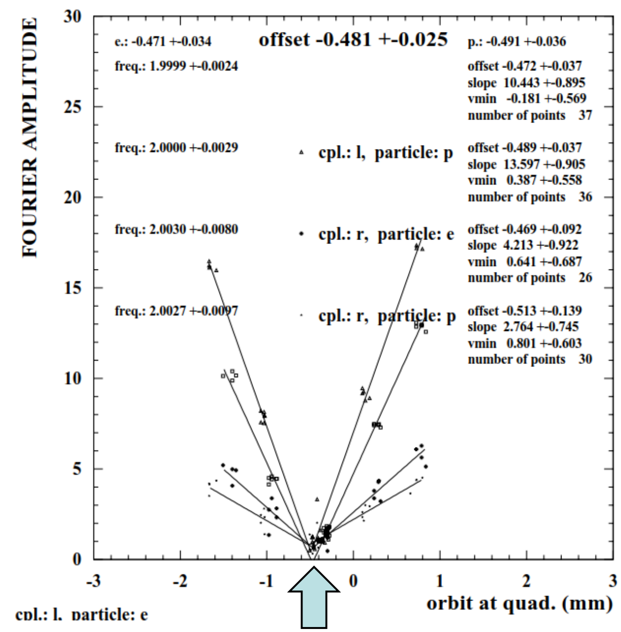
- BPM **offsets**, an important nuisance for orbit corrections, can be measured by a technique called **k-modulation**:
  - The gradient of a selected quadrupole is modulated slightly at a frequency f_{mod},
  - The beam position is varied inside the quadrupole using orbit bumps.
- The beam orbit is modulated at f_{mod}, the modulation amplitude vanishes when the beam is centred in the quadrupole.



P1



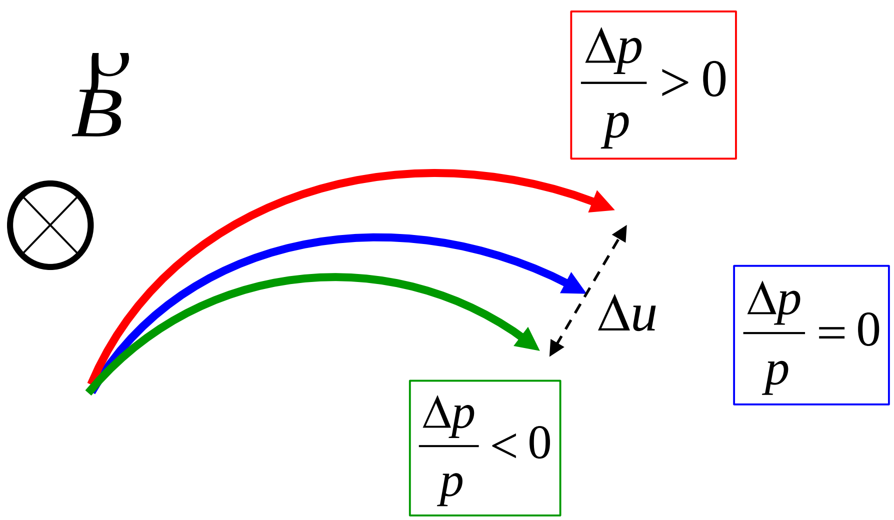
K-modulation at LEP



Oscillation amplitude as a function of the orbit bump amplitude

Dispersion

- The bending of charged particles by magnetic fields depends on the momentum (Lorentz force).
- As a consequence for a beam that is subject to a dipole deflection, the trajectories of the particles will be sorted by energy: **dispersion** !
  - In a circular accelerator there is always some dispersion.



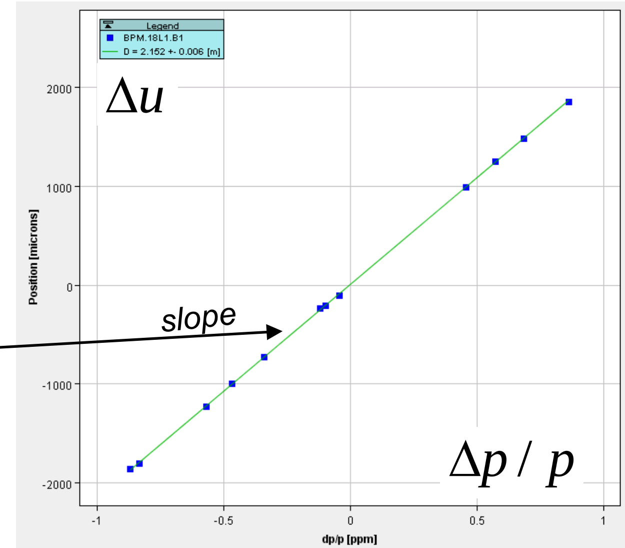
The dispersion measures the position difference per unit of energy deviation

$$D_u(s) = \frac{\Delta u(s)}{\Delta p / p}$$

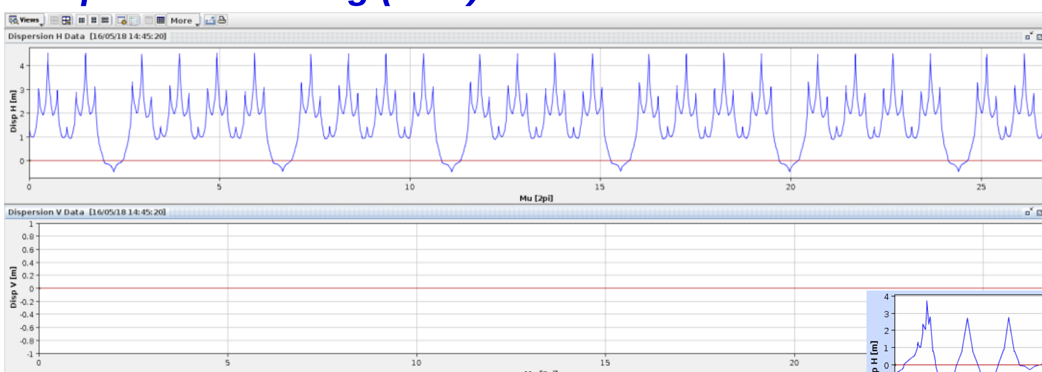
Dispersion

- Closely associated to the orbit is the **dispersion**, which is the derivative of the orbit wrt energy (u=x,y) :

$$D_u(s) = \frac{\Delta u(s)}{\Delta p / p}$$

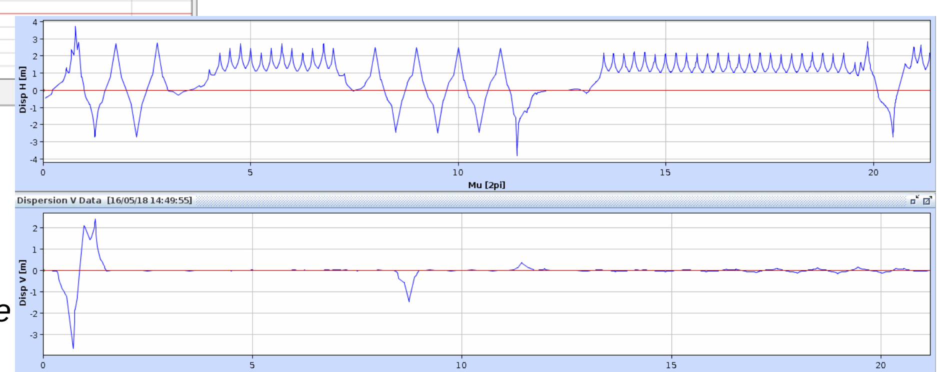


Dispersion in a ring (SPS)



Model dispersion is purely horizontal due to the flat ring

Dispersion in a transfer line



Model dispersion also present in the vertical plane due to vertical bending

Dispersion errors

- In a storage ring there is always non-zero horizontal dispersion (bending!). For flat machines the vertical dispersion is usually ‘0’ by design.
  - For hadron machines the dispersion is in general not critical. Controlling and correcting it follows similar lines to optics correction.
  - At **e^+e^- machines the vertical dispersion can lead to significant vertical emittance ε_y growth**, and to important luminosity performance loss.

$$\varepsilon_y \propto \left(\frac{\Delta p}{p}\right)^2 \left(\gamma D_y^2 - 2\alpha D_y D'_y + D'^2_y\right)$$

- **Dispersion errors** may be driven by:
  - **Optics errors** → handled together with the general optics correction,
  - **Coupling** → mainly from the horizontal to the vertical plane,
  - **Steering and alignment errors** → mainly an issue at e^+e^- machines where the emittances may be spoiled – storage rings and linacs !
- Optics and coupling correction will be discussed later, for the moment we focus on the third point, namely steering driven dispersion.

Dispersion free steering

- A technique combining regular steering and dispersion correction was used at SLC and LEP – **dispersion free steering (DFS)** – will be outlined here.

P3

- The principle of DFS relies on the extension of the orbit response matrix to dispersion including the **dispersion response** $\mathbf{S}$ and a **weight factor** α between orbit and dispersion:

$$\Delta\vec{u} = \mathbf{R}\,\Delta\vec{\theta} \quad \Longrightarrow \quad \begin{pmatrix} (1-\alpha)\,\Delta\vec{u} \\ \alpha\,\Delta\vec{D}_u \end{pmatrix} = \begin{pmatrix} (1-\alpha)\mathbf{R} \\ \alpha\,\mathbf{S} \end{pmatrix} \Delta\vec{\theta}$$

  – The dispersion response may be estimated analytically or from a simulation program like MAD etc.

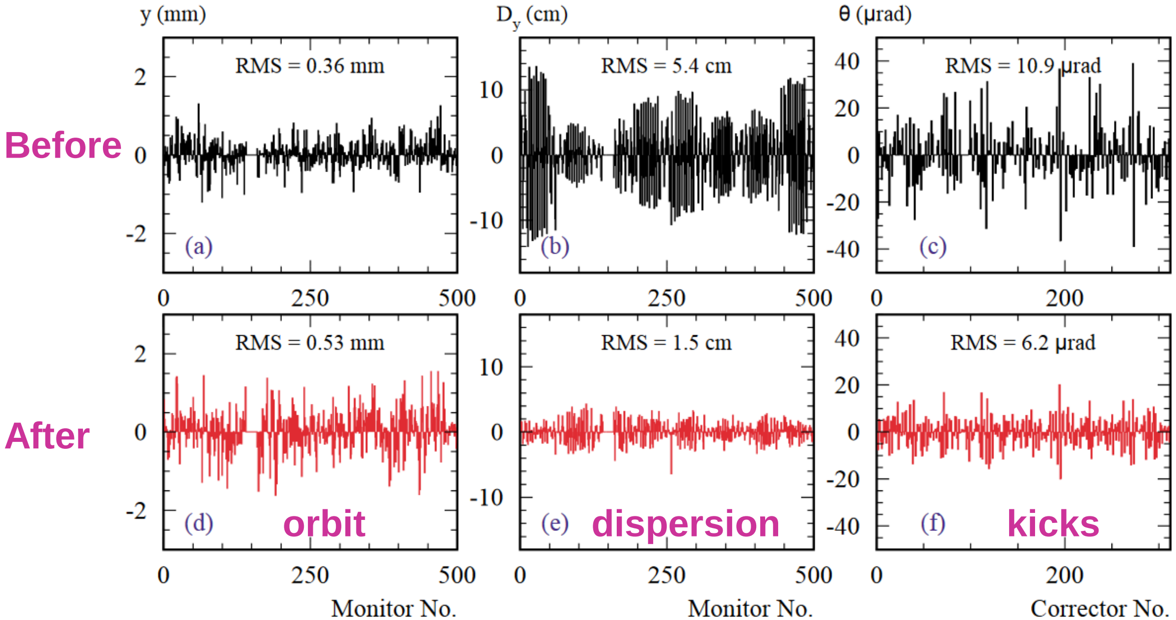
DISP-RES

- The combined orbit and dispersion correction system can be solved in exactly the same way than the 'normal' steering.

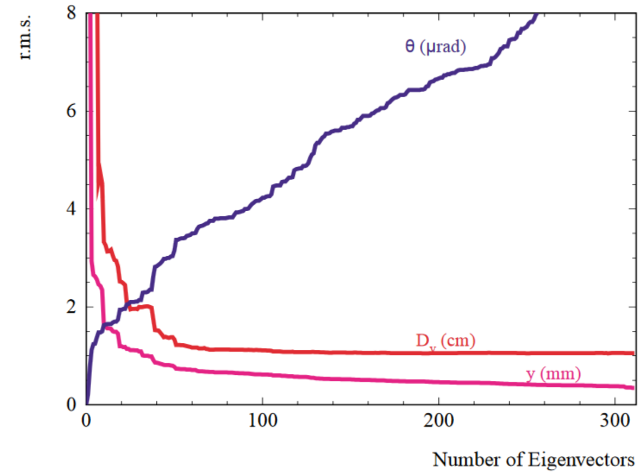
  – This does however not apply to all sources of dispersion (due to coupling between planes for example).

Dispersion free steering II

DFS at LEP



DFS convergence at LEP



P3

Observe that the orbit rms is <u>degraded</u> while the dispersion rms is <u>improved</u> !

- DFS provides controlled correction of the orbit and the dispersion with the dispersion is dominated by deflection (errors).

Introduction

Orbit and dispersion

Tune and coupling

Chromaticity

Linear optics

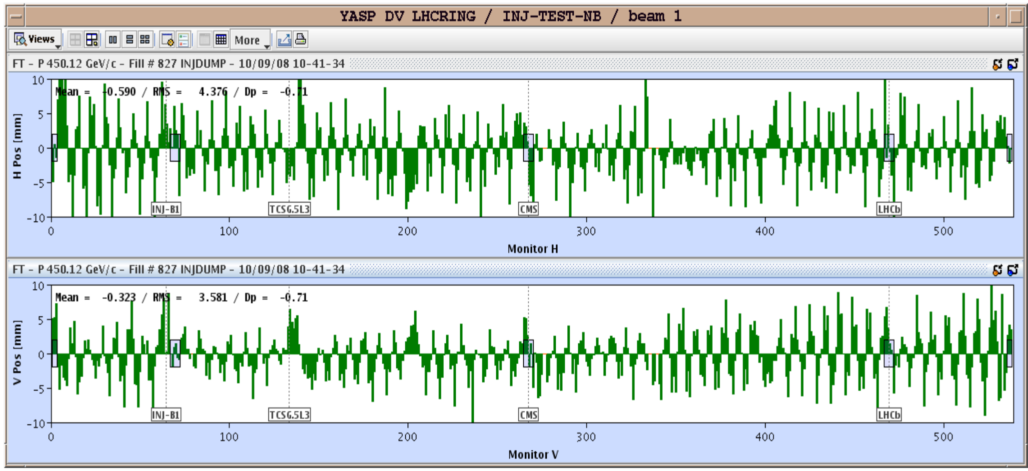
Tune measurement

- The **tune** (number of betatron oscillations per turn):

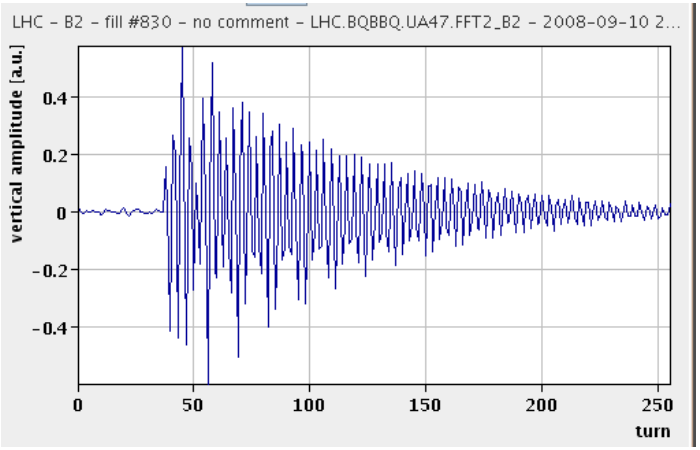
$$Q = N + q$$

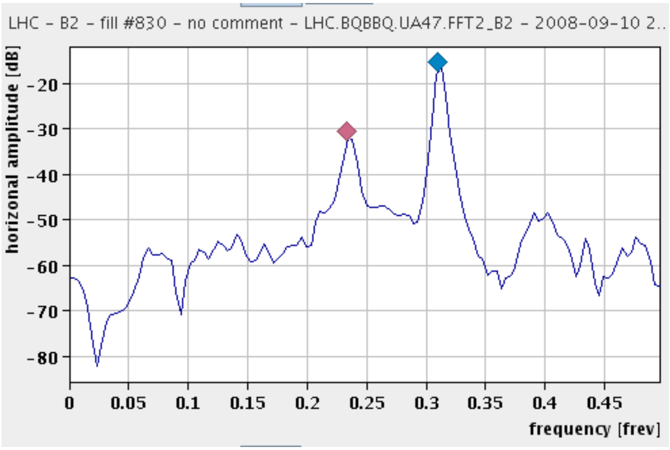
- The **integer tune N** is obtained from a poor closed orbit or a kick on the orbit.
  - In large machine N may be wrong before correction !

Uncorrected orbit at the LHC



- The **fractional tune q** is obtained from the turn-by-turn data (TbT) of a single position monitor is the beam is given a kick or is oscillating naturally.
  - Fast Fourier Transform (FFT) of oscillation data provides the resonant frequency q.

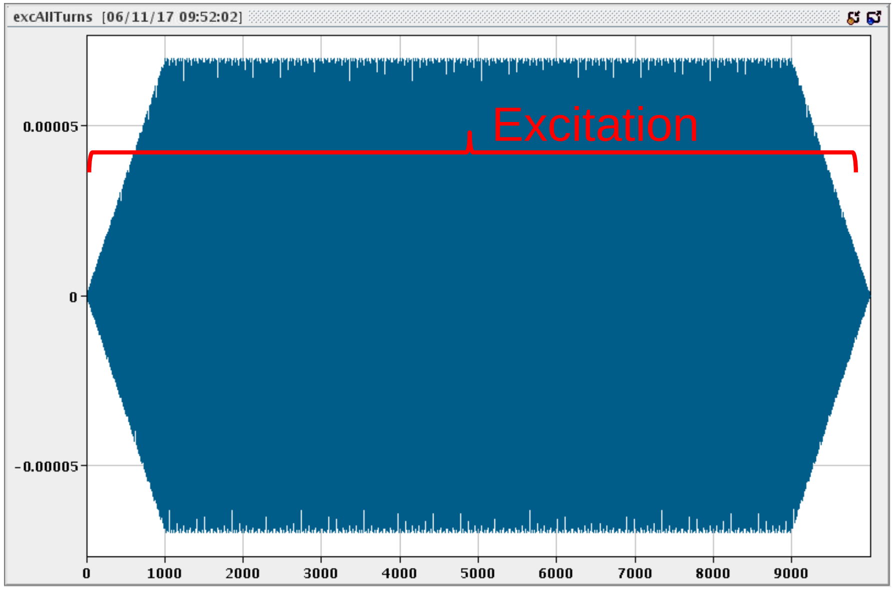




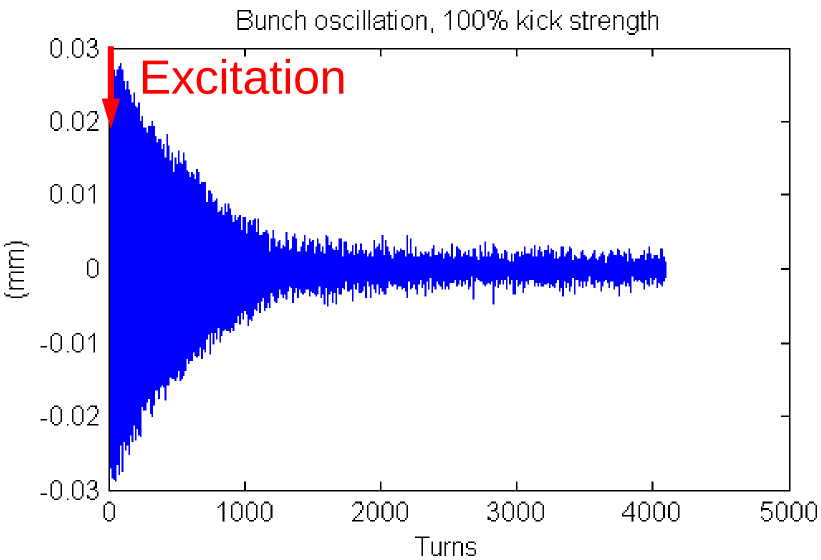
Two common beam excitation methods:

- The single kick method followed by a **free** oscillation, damped by decoherence (tune spread due to non-linearities),
- The AC dipole **forced** excitation at a fixed frequency, usually close to the tune.
  - Emittance growth free excitation for hadrons (if distance to tunes sufficient large),
  - Long excitation duration (accuracy thanks to many turns).
  - As this is a forced oscillation it does not provide the tune – but can be used for coupling and optics measurements.

Forced AC dipole oscillation (LHC)



Single kick and free oscillation (LHC)

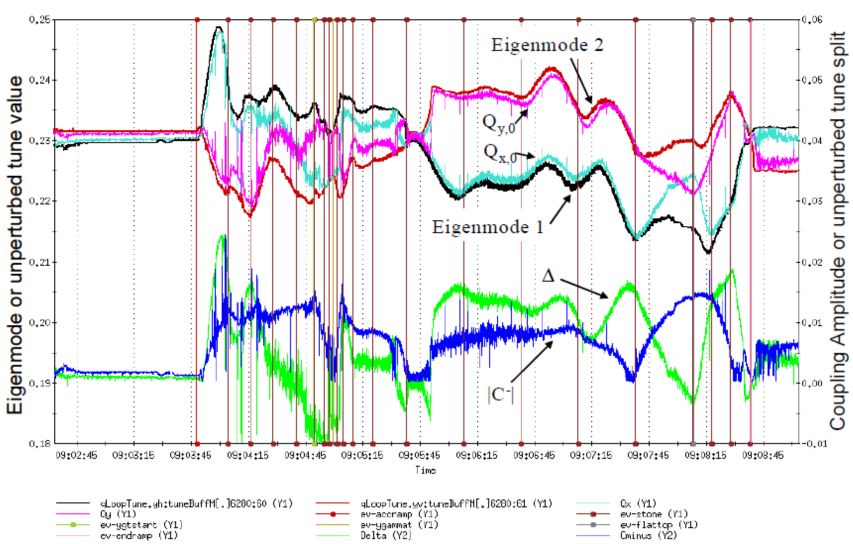


Excitations for tune measurements (II)

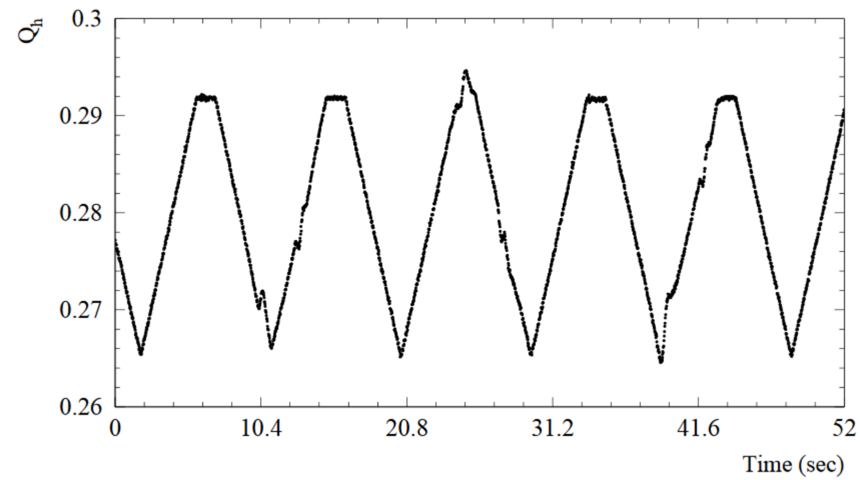
- Phased locking an excitation-measurement system provides a means to track tunes continuously.
  - An exciter shakes the beam on the tune, it remains locked on the tune using the phase of the beam response wrt excitation.
  - 'Ideal' for e^+e^- rings where damping will erase the effect of the excitation.
  - Problematic for hadrons as this technique tends to produce emittance blowup.
    - Used at RHIC, but not at LHC (too strong transverse feedback).

Q-FB-RHIC

Phased locked tune and coupling (RHIC ramp)

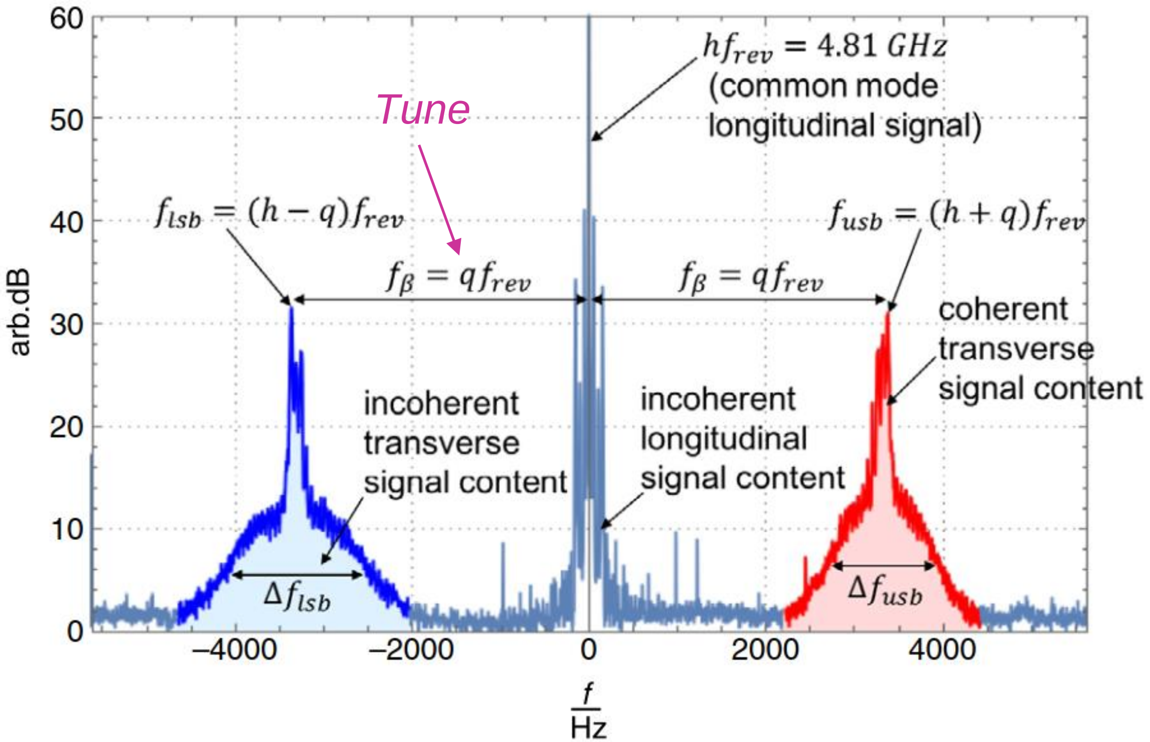


Phased locked tune (and chromaticity) measurement at LEP



- A direct and non-invasive technique to measure the tunes is based the Shottky spectrum of the beam.
  - Relies on noise in the beam, no excitation is required – ideal for hadrons.
    - But small signals: detection electrodes must be close to the beam, or the beam must have large charge Z (Shottky signal amplitude ~ Z^2).
  - Can be problematic for bunched beams due to large coherent signals,
  - This devices is also able to provide chromaticity and emittance.

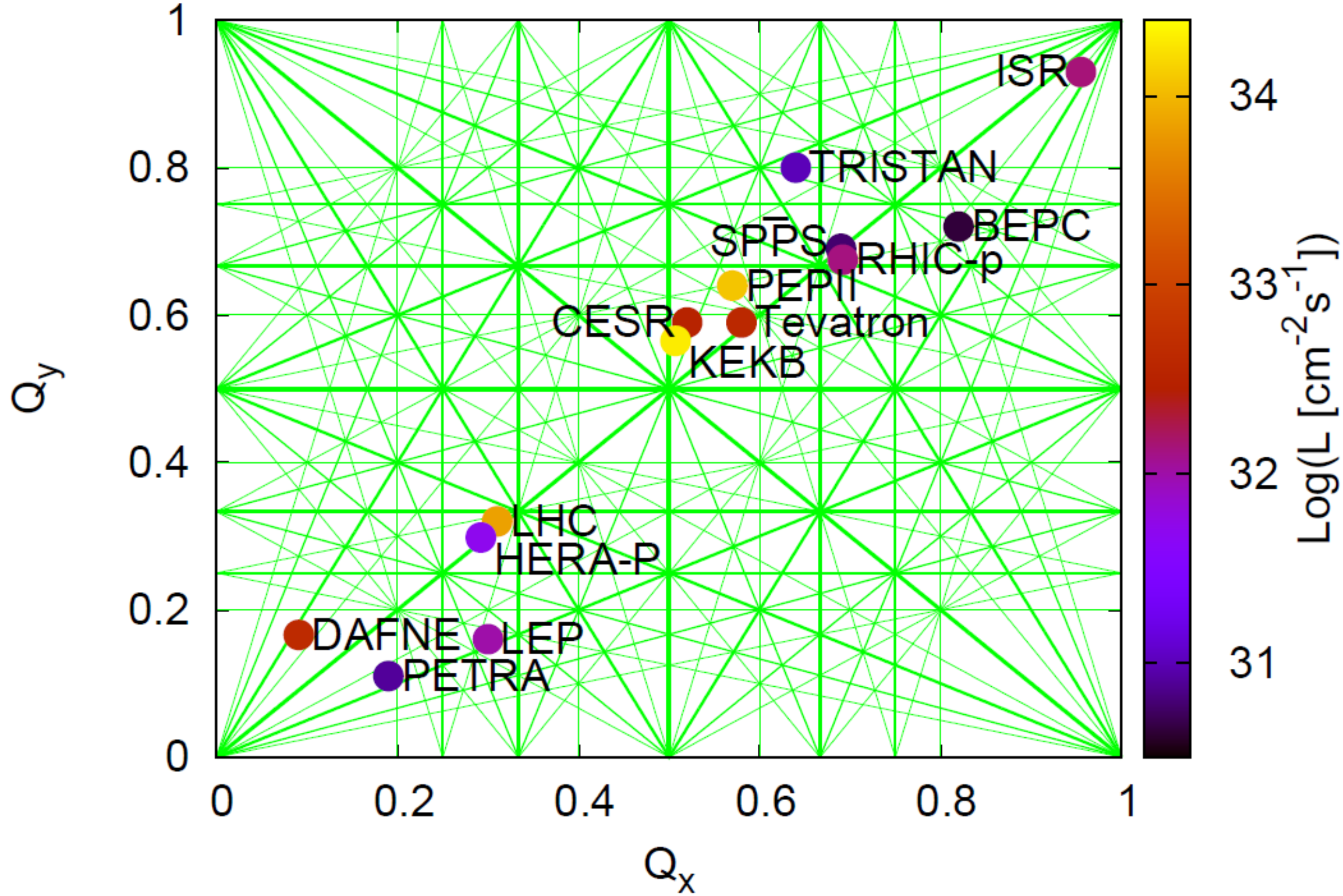
Shottky signals at the LHC



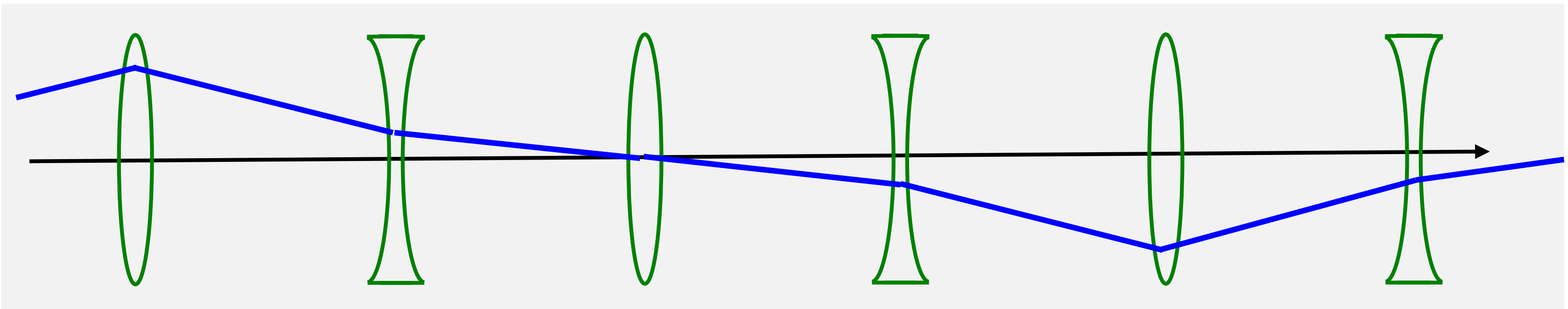
Q-SHOT-LHC

Tune working points

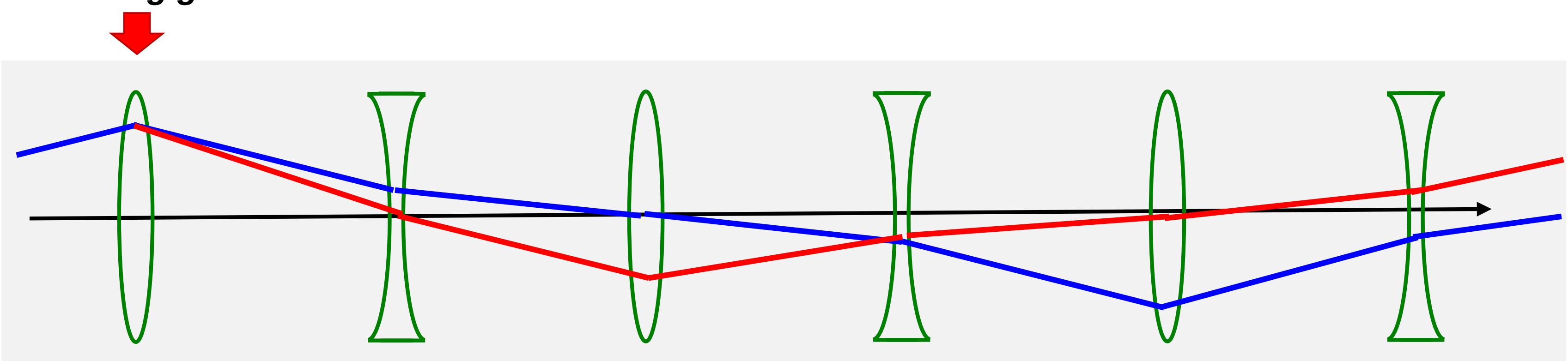
- Collider tune working points.
  - Hadron machines work usually very close to the diagonal (tune space).
    - De-coupling of the planes is important.
  - The tune value may depend on the number of IPs. For example for e^+e^- the tune increment from one IP to the next is often chosen to be close to 0.5.



- What is the impact of a quadrupole gradient error?
  - Let us consider a particle oscillating in the lattice.



Too strong gradient / lens



*The oscillation period is affected → **change of tune**, here Q increases !*

Tune change by quadrupole

- The tune change ΔQ_u (u = x,y) induced by a small quadrupole strength change Δk is given to first order by:

$$\Delta Q_u = \frac{1}{4\pi}\beta_u L_Q \Delta k$$

L_Q is the quadrupole length,

β_u is the betatron function inside the quarupole

- In general **the vertical and horizontal betatron functions differ** at the quadrupole, therefore the **tune changes are different** for H and V.
- By combining two (groups of) quadrupoles with different β_x and β_y, it is possible to construct combinations that affect Q only in one plane.
  - Provides a means for orthogonal corrections of the two planes.
  - In larger machine there are often distributed trim quadrupoles grouped in two (or more) families to spread out the correction and reduce optics errors.

Quadrupole 1

$$\Delta Q_{x,1} = \frac{1}{4\pi}\beta_{x,1} L_{Q,1} \Delta k_1$$

$$\Delta Q_{y,1} = -\frac{1}{4\pi}\beta_{y,1} L_{Q,1} \Delta k_1$$

Quadrupole 2

$$\Delta Q_{x,2} = \frac{1}{4\pi}\beta_{x,2} L_{Q,2} \Delta k_2$$

$$\Delta Q_{y,2} = -\frac{1}{4\pi}\beta_{y,2} L_{Q,2} \Delta k_2$$

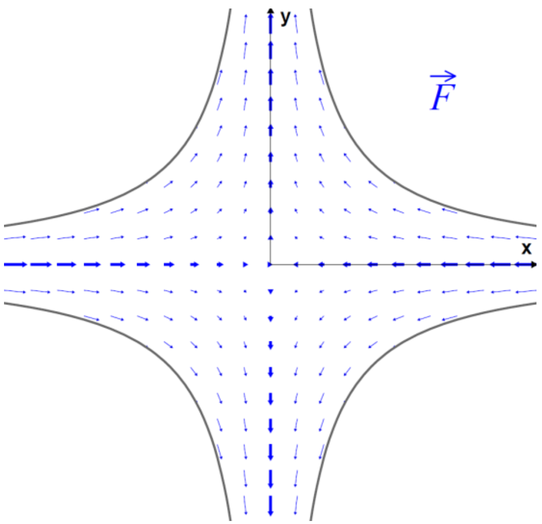
$$\Rightarrow \begin{pmatrix} \Delta Q_x \\ \Delta Q_y \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} \Delta k_1 \\ \Delta k_2 \end{pmatrix}$$

$m_{ij} = f(\beta_{x,1}, \beta_{y,1}, \beta_{x,2}, \beta_{y,2})$

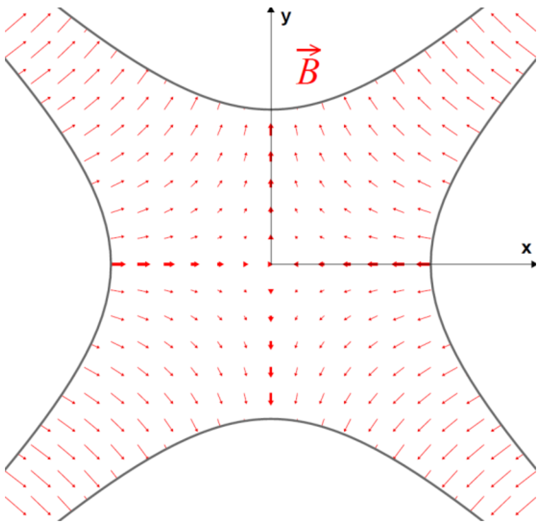
Tilted quadrupole

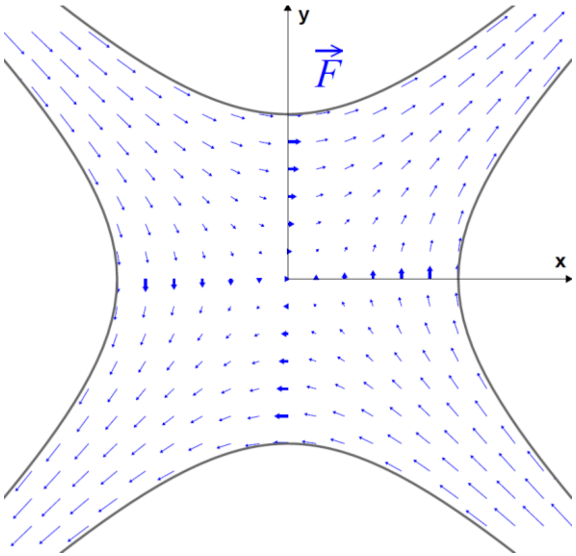
- If a quadrupole is rotated by 45º ('skew quadrupole') one obtains an element where the force (deflection) in x depends on y and vice-versa: the horizontal and vertical planes are **<u>coupled</u>**.

normal quadrupole



skew quadrupole





$$F_x = -k\,x$$

$$F_y = k\,y$$

No mixing of planes

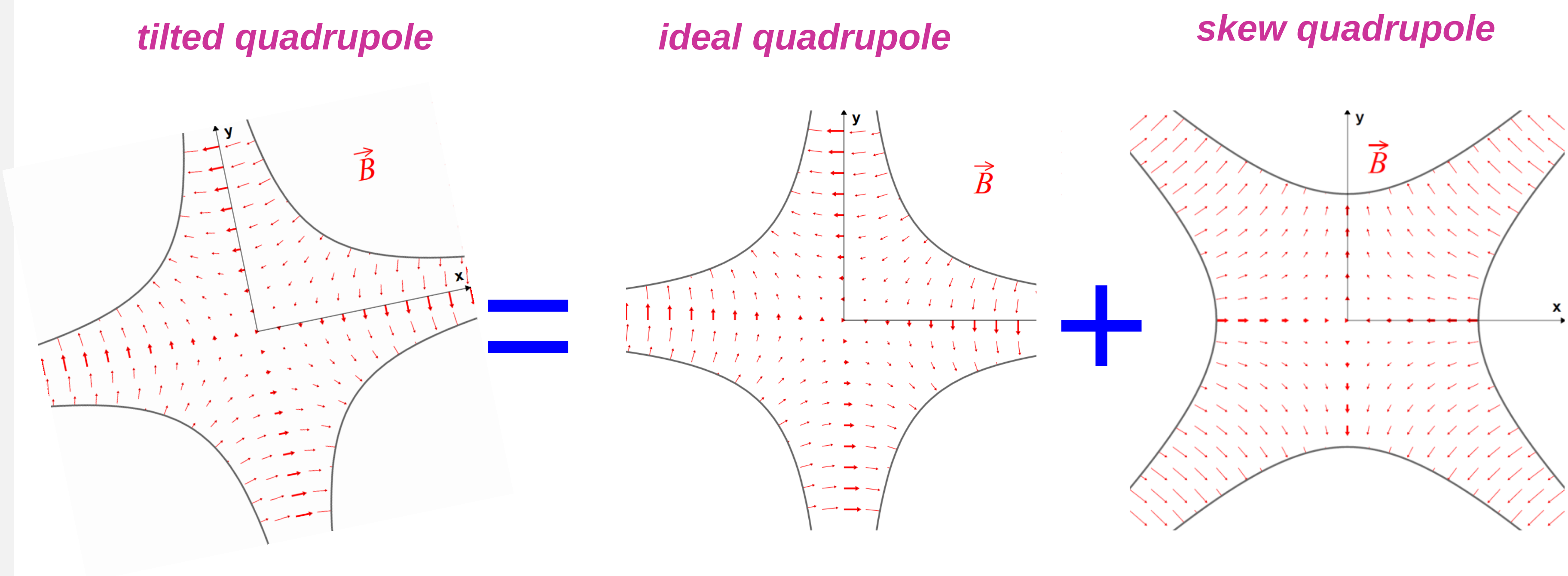
Full mixing of planes

$$F_x = k\,y$$

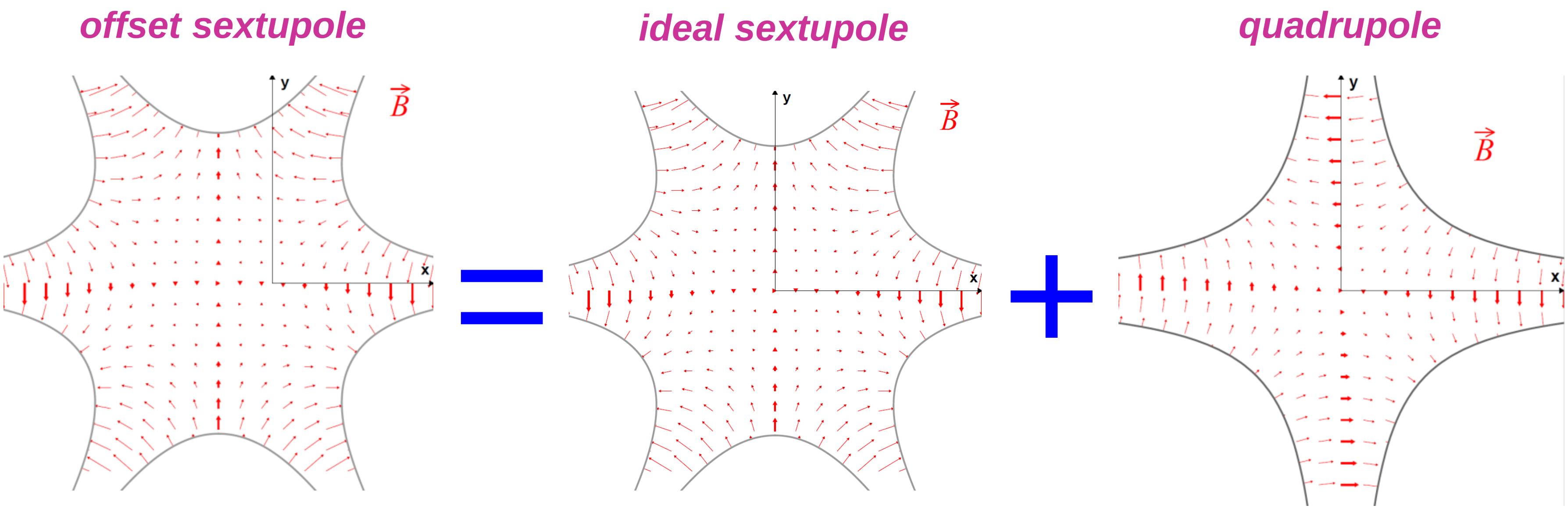
$$F_y = -k\,x$$

Coupling (I)

- Small quadrupole tilts lead to **coupling of the x and y planes**.
- The coupling can be corrected by installing dedicated **skew quadrupoles** to compensate for alignment or skew quadrupolar field errors.



- Causes of undesired coupling in machines:
  - Element misalignments (for examples tilt angles of quadrupoles),
  - Skew quadrupolar field errors,
  - Solenoids (experiments, coolers etc),
  - Vertical orbit offsets in sextupoles (or misaligned sextupoles)
    - Vertical offsets → skew quadrupole → coupling
    - Horizontal offset → normal quadrupole → tune, beta-beating



Linear imperfections table

- Summary table of linear imperfections

Field type	Imperfection	Error type	Impact
Dipole	**Field error**	**Dipole**	**Orbit / trajectory**
Dipole	**Tilt**	**Dipole**	**Orbit / trajectory**
Quadrupole	**Field error**	**Quadrupole**	**Tune / optics**
Quadrupole	**Offset**	**Dipole**	**Orbit / trajectory**
Quadrupole	**Tilt**	**Skew quadrupole**	**Coupling**
Sextupole	Field error	Sextupole	Chromaticity
Sextupole	**Offset horizontal**	**Quadrupole**	**Tune / optics**
Sextupole	**Offset vertical**	**Skew quadrupole**	**Coupling**

Linear imperfections table

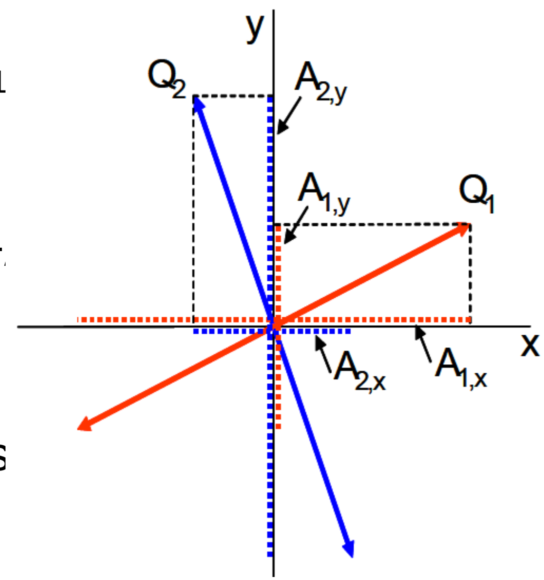
- Summary table of linear imperfections.
- Alignment **offsets** generate field errors of **lower order**:
  - Quadrupole → dipole,
  - Sextupoles → quadrupole,
  - Etc..

'Feed-down'

Field type	Imperfection	Error type	Impact
Dipole	Field error	Dipole	Orbit / trajectory
Dipole	Tilt	Dipole	Orbit / trajectory
Quadrupole	Field error	Quadrupole	Tune / optics
Quadrupole	**Offset**	**Dipole**	**Orbit / trajectory**
Quadrupole	Tilt	Skew quadrupole	Coupling
Sextupole	Field error	Sextupole	Chromaticity
Sextupole	**Offset horizontal**	**Quadrupole**	**Tune / optics**
Sextupole	**Offset vertical**	**Skew quadrupole**	**Coupling**

Coupling measurement

- In the presence of **coupling** the beam **eigen-modes** Q_1 and Q_2 no longer coincide with Q_x and Q_y.
- Large coupling can be an issue for the **tune working point** (hadron collider) or for the **vertical emittance** (e+ machines).

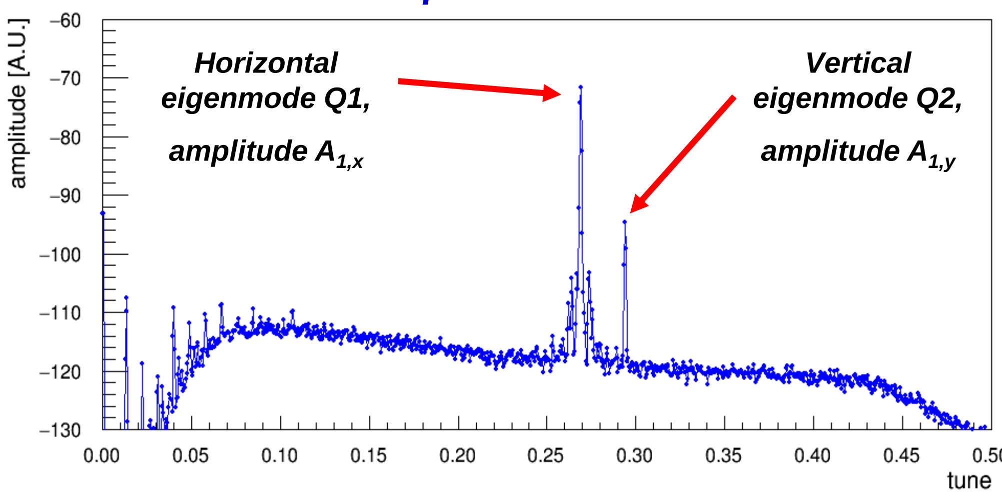


- A first technique to characterise the coupling coefficient consists in measuring the crossed tune peak amplitudes
  - Vertical tune in horizontal spectrum and vice-versa.
  - Simple measurement, but no phase information.
  - Only the local coupling is obtained, which can differ from the global coupling.

$$\left\| C^{-} \right\| = \frac{2\sqrt{r_1 r_2}\,|Q_1 - Q_2|}{1 + r_1 r_2}$$

$$r_1 = \frac{A_{1,y}}{A_{1,x}} \qquad r_2 = \frac{A_{2,x}}{A_{2,y}}$$

Horizontal tune spectrum at the LHC



Coupling measurement II

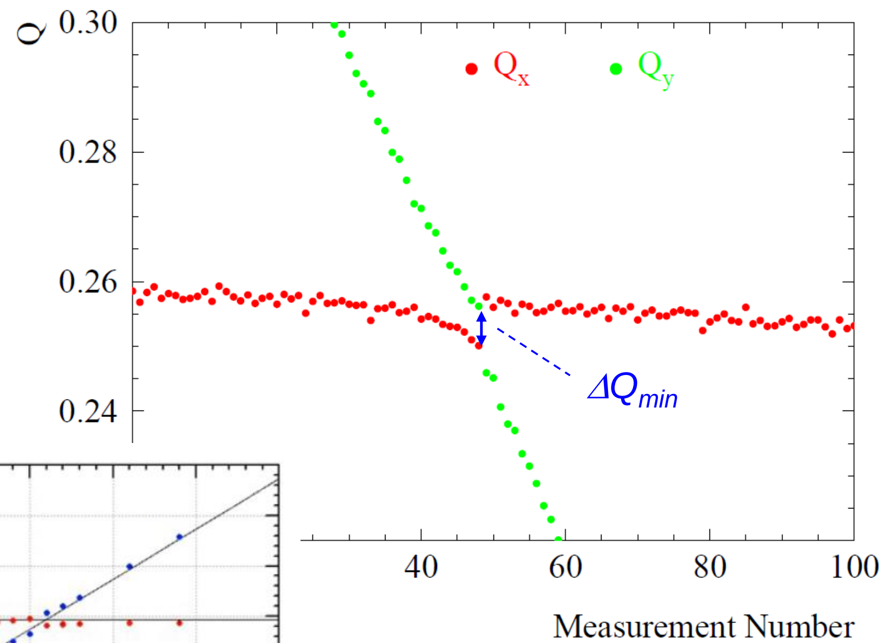
- The global machine coupling can be also determined directly using the **closest tune approach**.
  - Requires to move the tunes close to / across each other.
  - Provides global coupling information from a single location.

LEP-PERF

- The closest distance of approach ΔQ_{min} corresponds to the coupling parameter C^{-}:

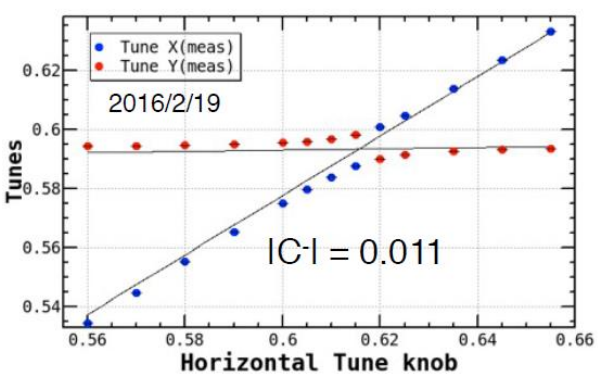
$$\Delta Q_{\min} = \left\| C^{-} \right\|$$

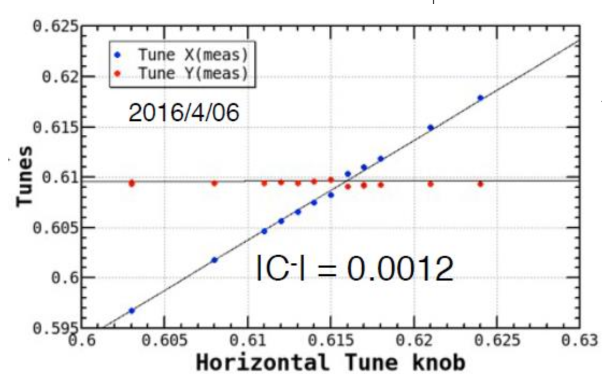
Closest tune measurement at LEP



Q-COUP-KEKB

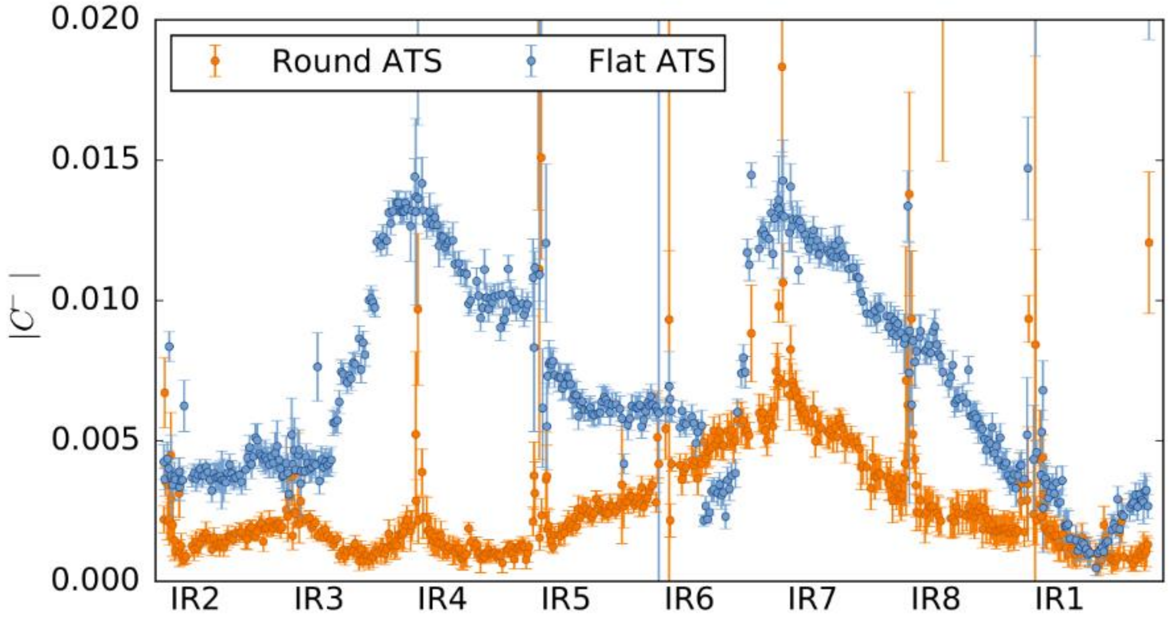
Closest tune measurements at superKEKB





- The coupling can also be determined along the machine using multi-turn beam position data (with e.g. AC dipole excitation) using pairs or groups of BPMs to reconstruct the coupling locally.
  - Provides detailed local coupling information, including phase.
  - Global coupling is obtained by integration of the local coupling.
  - Provides excellent deterministic corrections, but kicking the beam is required.

Local coupling measurement at LHC



Q-COUP-LHC

O-NL-LHC

Correction of coupling

The coupling correction scheme depends on machine design.

- Ideally experimental solenoids should be compensated by **local anti-solenoids** to correct the coupling source locally.
  - At high energy hadron colliders like LHC, the solenoids of the 4 experiments contribute very little to he machine coupling (due to the high momentum).
- The global machine coupling is usually corrected with **distributed skew quadrupoles**, either using 2 orthogonal knobs (similar to a tune correction) or with more refined local corrections.
  - For measurements that do not provide phase information (only global coupling), the two orthogonal knobs must be scanned by trial and error to determine a correction.
- As an alternative, **orbit bumps in sextupoles** may also be used for coupling corrections, but this can lead to problems with dispersion etc. Requires a careful combined correction of both parameters.

Introduction

Orbit and dispersion

Tune and coupling

Chromaticity

Linear optics

Chromaticity measurement

- The **linear chromaticity** defines the tune dependence on momentum:

$$\Delta Q = Q' \frac{dp}{p}$$

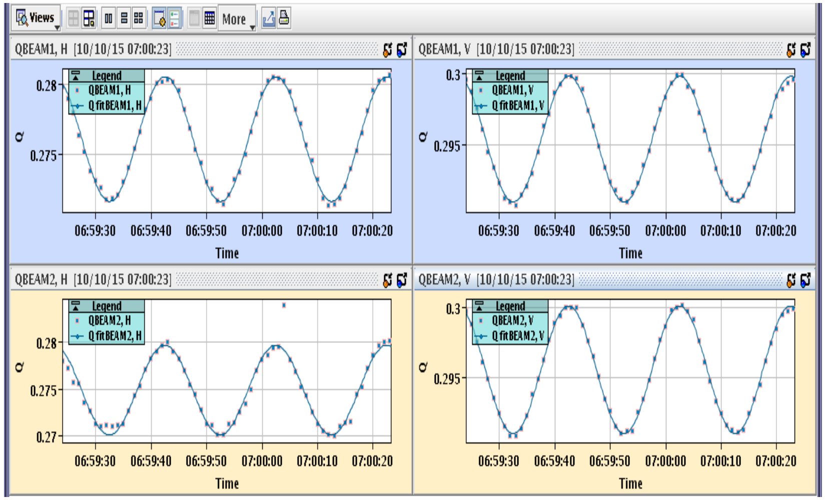
- For a lattice Q' is usually negative unless Q' is corrected with sextupoles. The typical operation range of Q' is ~ **+1 to +20 above transition**, **slightly negative below transition energy** – due to collective effects.

- The chromaticity is measured by changing / modulating the energy offset dp/p through the RF frequency while recording the tune change ΔQ.

$$\frac{dp}{p} = \left(\frac{1}{\gamma^2} - \alpha\right)^{-1} \frac{df_{RF}}{f_{RF}} \cong \frac{-1}{\alpha} \frac{df_{RF}}{f_{RF}}$$

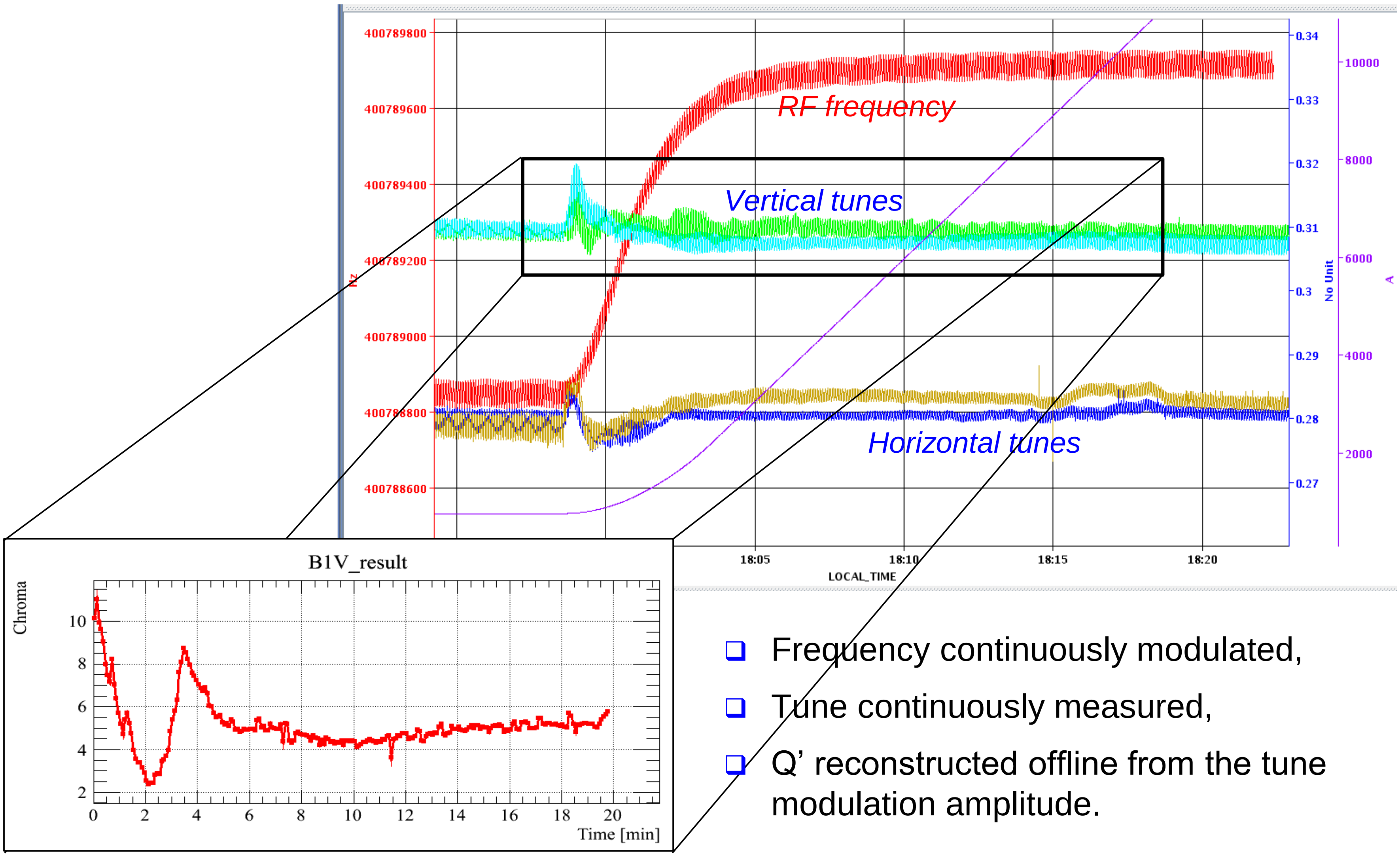
At high energy machines, $\gamma >> 1$

Q' measurement by RF frequency modulation at LHC



Chromaticity measurement II

Q' measurement along a ramp by RF frequency modulation at LHC

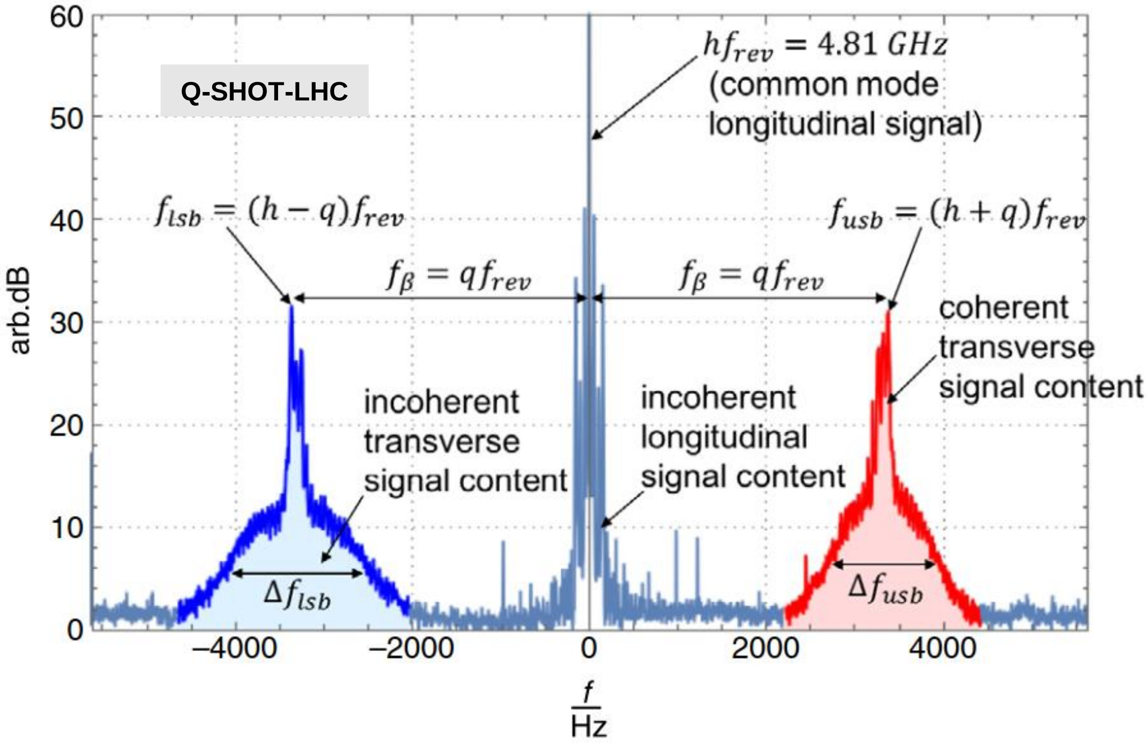


- Frequency continuously modulated,
- Tune continuously measured,
- Q' reconstructed offline from the tune modulation amplitude.

- A Shottky monitor can be used to determine Q' for hadron beams without the need for a radial modulation.
  - Completely non-invasive measurement.
- Q' is related to the difference in width of the upper and lower Shottky side-bands:

$$Q' = \eta \left(n \frac{\Delta f_{\mathrm{lsb}} - \Delta f_{\mathrm{usb}}}{\Delta f_{\mathrm{lsb}} + \Delta f_{\mathrm{usb}}} + q \right)$$

Shottky signal at the LHC

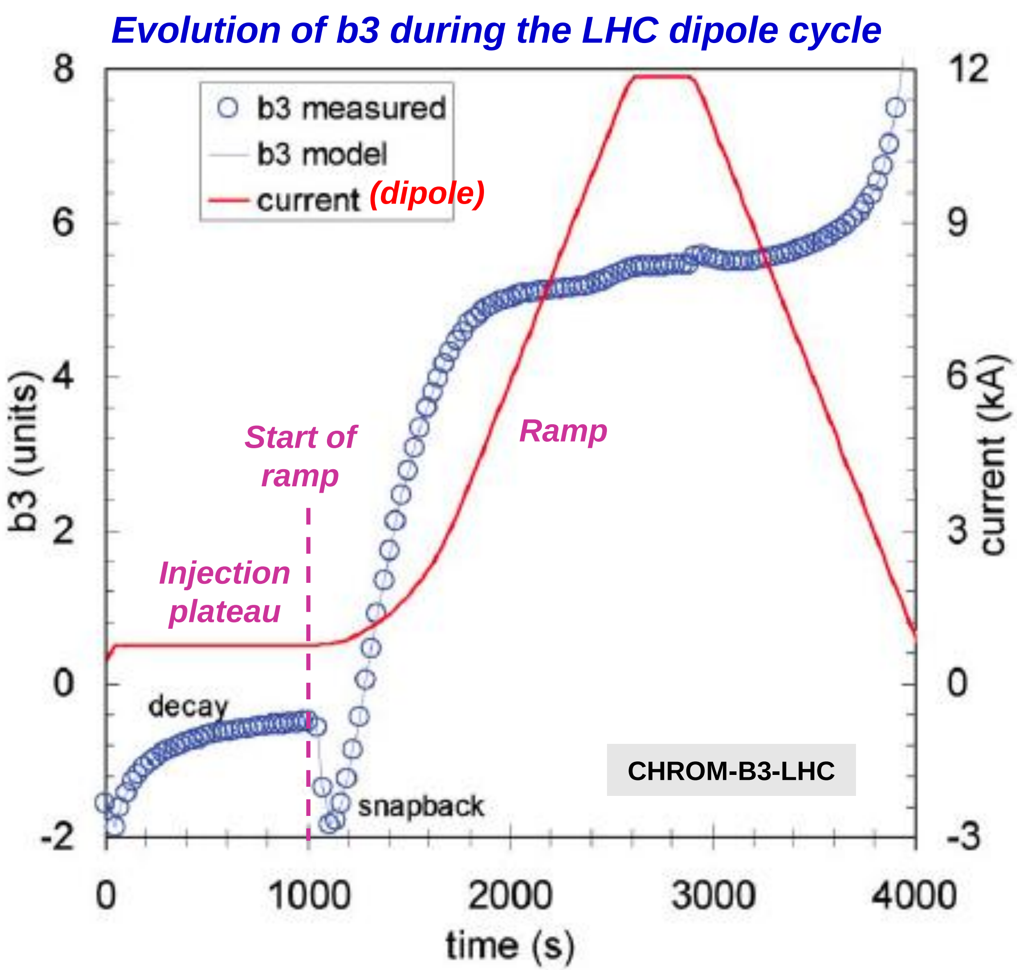


At LHC for example, the Shottky provides reasonable Q' data at injection for protons, and at ~ all energies for ions (due to Z^2 sensitivity).

- In **superconducting machines** like Tevatron, LHC or FCC-hh the **sextupolar field errors** (b_3) play an important role and may generate huge chromaticity errors.
- The **field errors** are usually generated in the **dipoles** and due to the important integrated length of the dipoles, the b_3 errors can add up to hundred’s of units of Q’.
- Furthermore superconducting machine are affected by **decay** and **snapback** phenomena:
  - **Decay**: while the machine ‘sits’ at injection for filling, persistent currents (~ eddy currents) flow between the cable strands and may induced large field errors, in particular b_3 components. This leads to a drift of the chromaticity over time.
  - **Snapback**: At the start of the energy ramp, the persistent currents ‘snap back’ to their initial value before decay over a very narrow energy range, leading to large dynamic swings of Q’.

b3 errors at LHC

- The b3 errors for the LHC dipole magnets were measured on test benches and on the series production magnets.
  - Higher order field errors were also measured (up to b7).

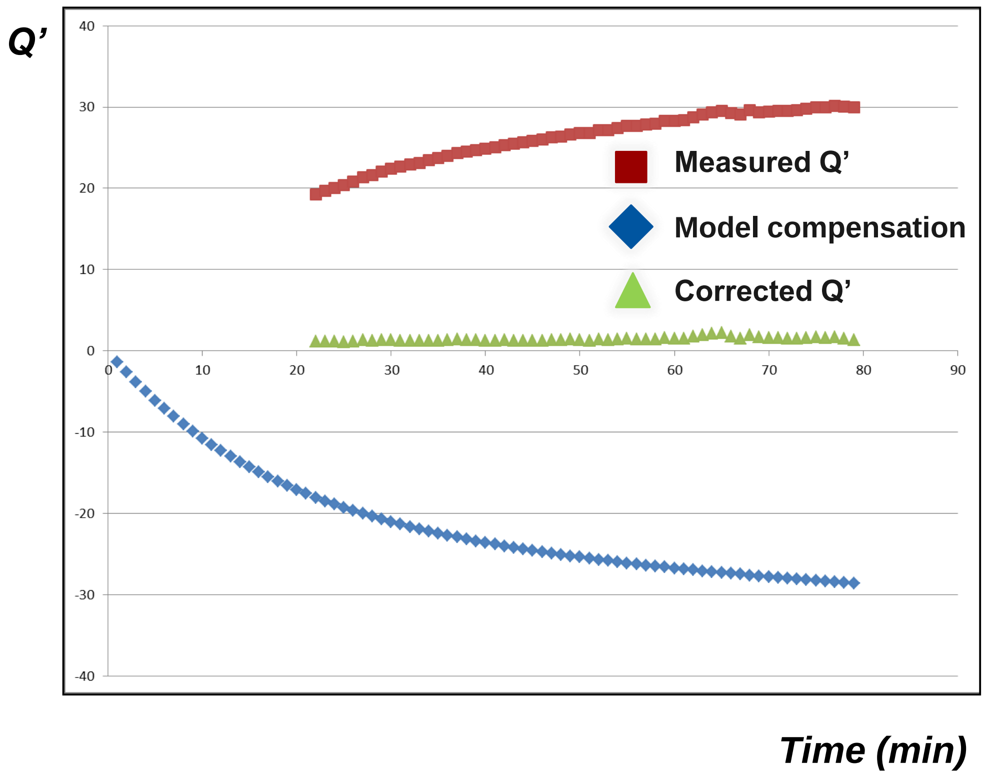


- During injection b_3 decays by ~1 unit ≅ 38 units of Q'.
  - **1 unit = relative field (error) of 10^{-4} at a radius of 17 mm**.
- Over the full cycle the swing of b3 is ~7 units ≅ 270 units of Q'.
- Typically one would like to control Q' to a few units (±1-2).

b3 errors at LHC II

- To control the LHC chromaticity at injection it is not possible to modulate the frequency for continuous measurements, a model must therefore be available to stabilize Q’.

Evolution of the H chromaticity at injection for LHC

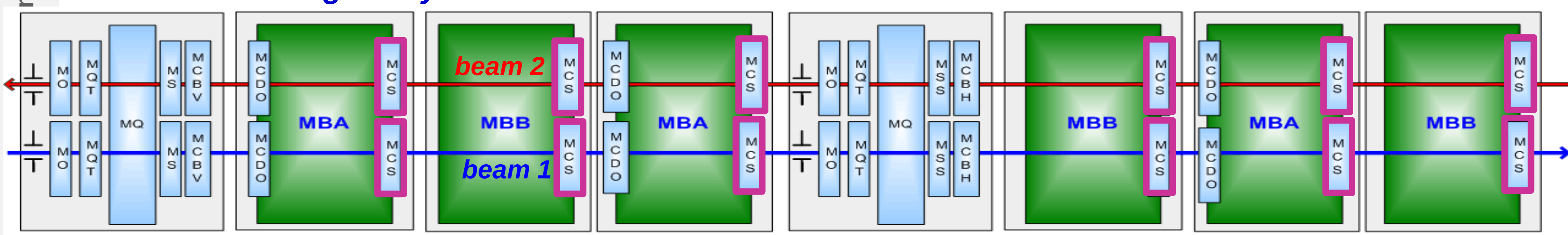


- Continuous Q’ measurements over many hours are used to establish models of the decay.
  - Parameters depend on the flat top energy etc
- A decay model is used to stabilize Q’ during injection.
  - In general correct within ~ ±2.
- For the snapback a model is used to correct Q’ during this short phase.
  - Lasts ~ 30 s at LHC.

Chromaticity correction

- The chromaticity is usually corrected with the **lattice sextupoles**. The corrections are usually distributed over all / many sextupoles in the form of orthogonal knobs for the two planes.
- For systematic field errors as they appear in large super-conducting machines, a correction with the lattice sextupoles may lead to poor dynamic aperture due to strong non-linear fields.
- A better compensation of field errors is obtained using **dedicated b3 correctors** (~small sextupoles) that are installed next to the dipoles and distributed over the entire machine – **local corrections**.

LHC arc cell magnet layout



quadruple *dipole* *dipole* *dipole*

b3 corrector

MCS: Sextupole corrector (b3)
MCDO: Assembly of spool correctors consists of an octupole insert **MCO** (b4) and a decapole magnet **MCD** (b5)
MQT: Trim quarupole corrector
MS: arc sextupole corrector
MQS: skew quad lattice corrector
MCBH: Horizontal dipole corrector
MCBV: Vertical dipole corrector
MO: Lattice octupole

Introduction

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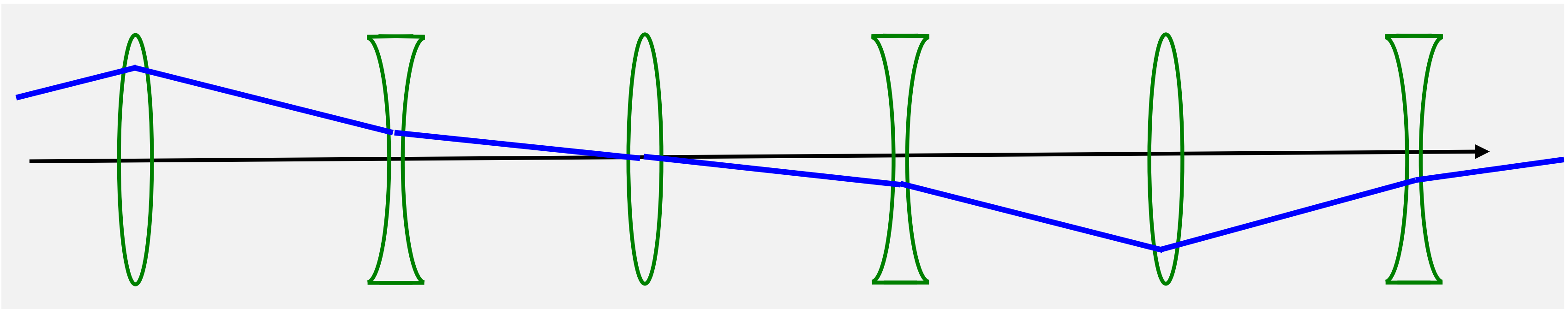
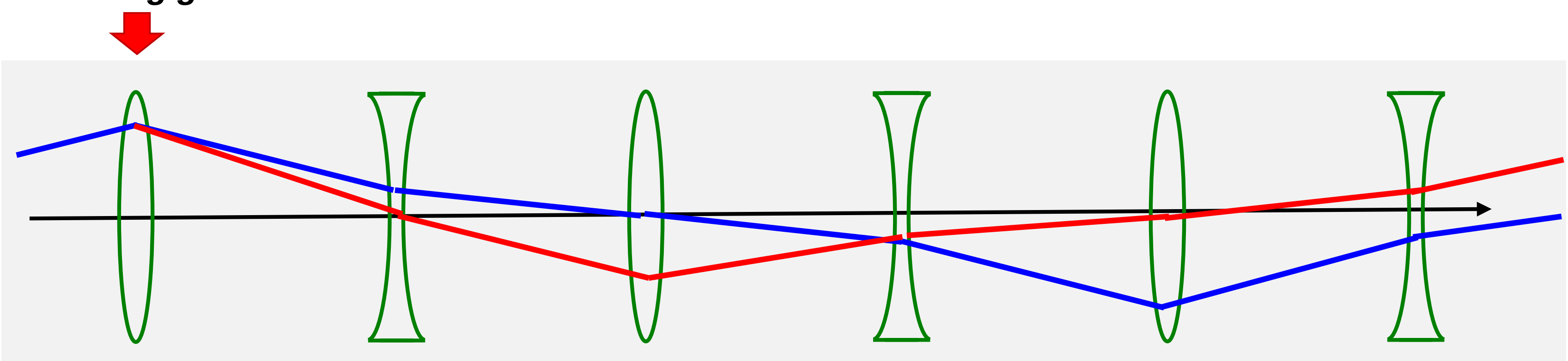
Linear optics

- What is the impact of a quadrupole gradient error?



Too strong gradient / lens



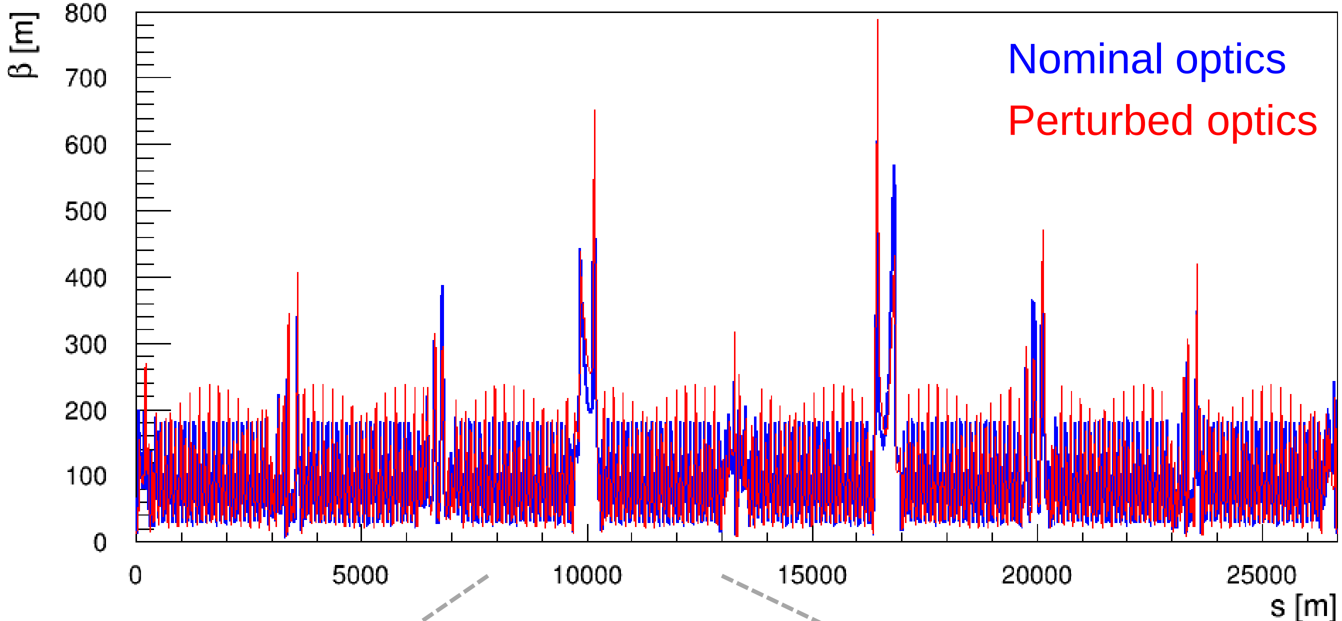
The oscillation period is affected, leading to a ***change of tune,***

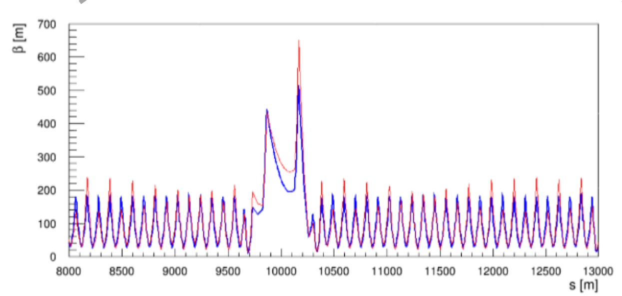
The focussing error also ***affects the beam envelope (β function)****.*

Optics perturbation

- The focussing error affects not only the tune, but also the beam optics and envelope (size) over the entire ring !

Example for LHC: one quadrupole gradient is incorrect

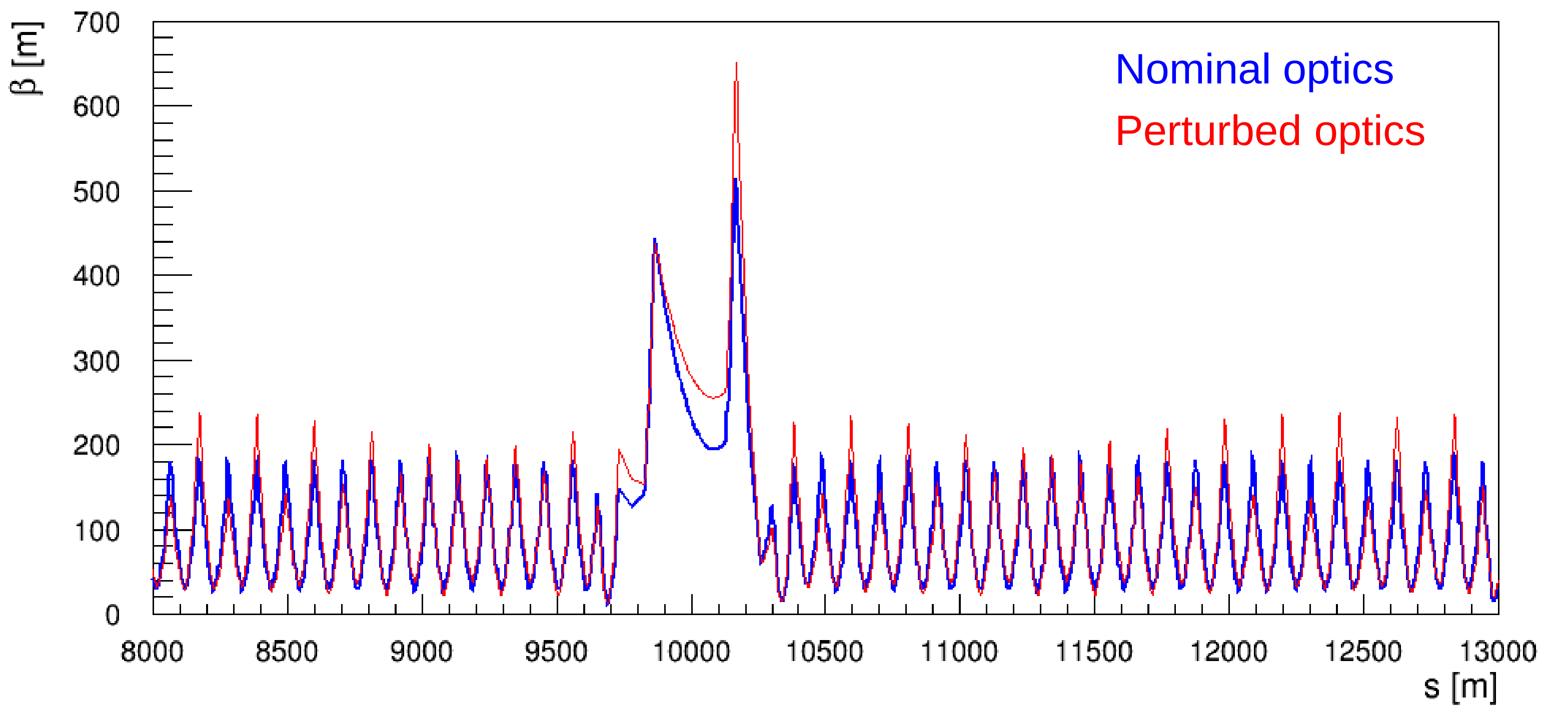




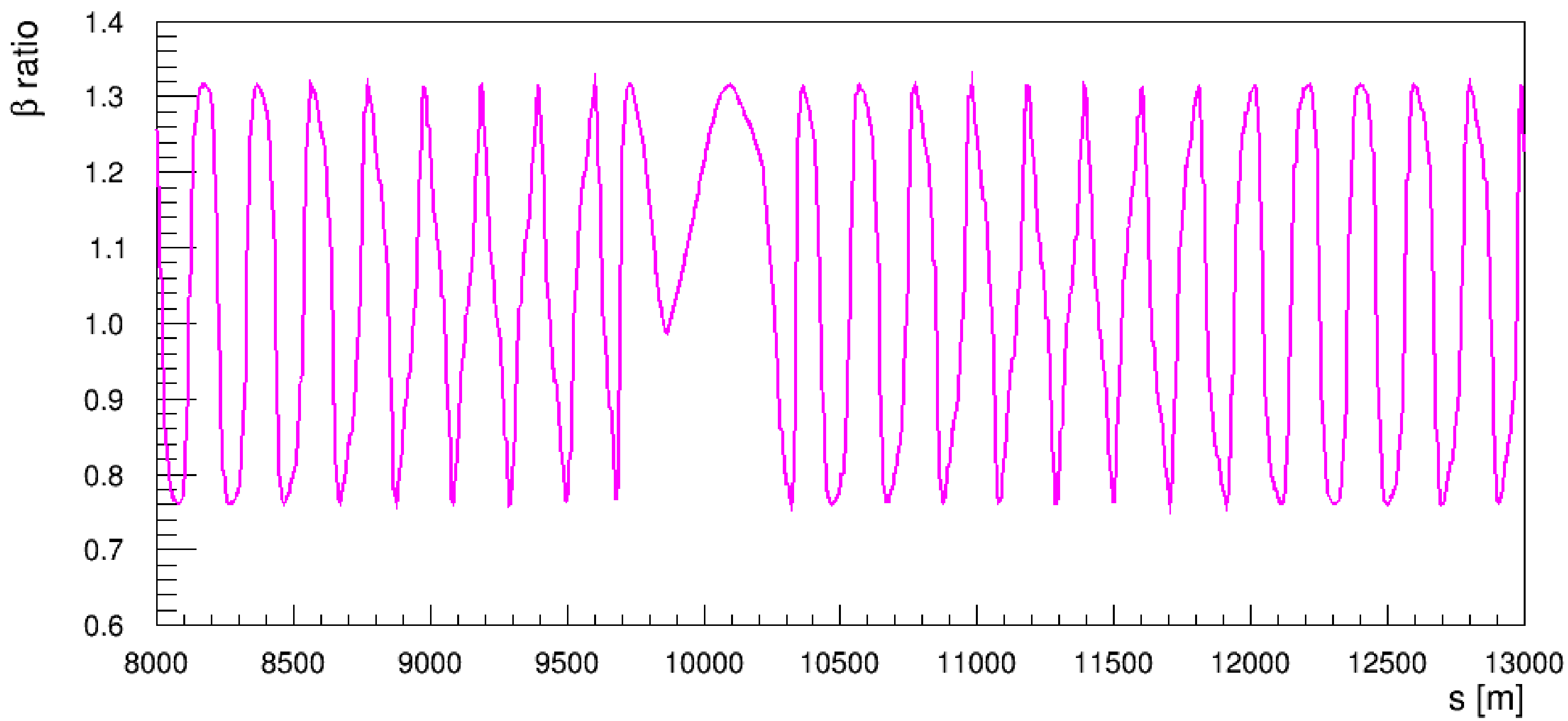
Zoom into a subsection

Optics perturbation

- The beam optics perturbation exhibits an oscillating pattern.

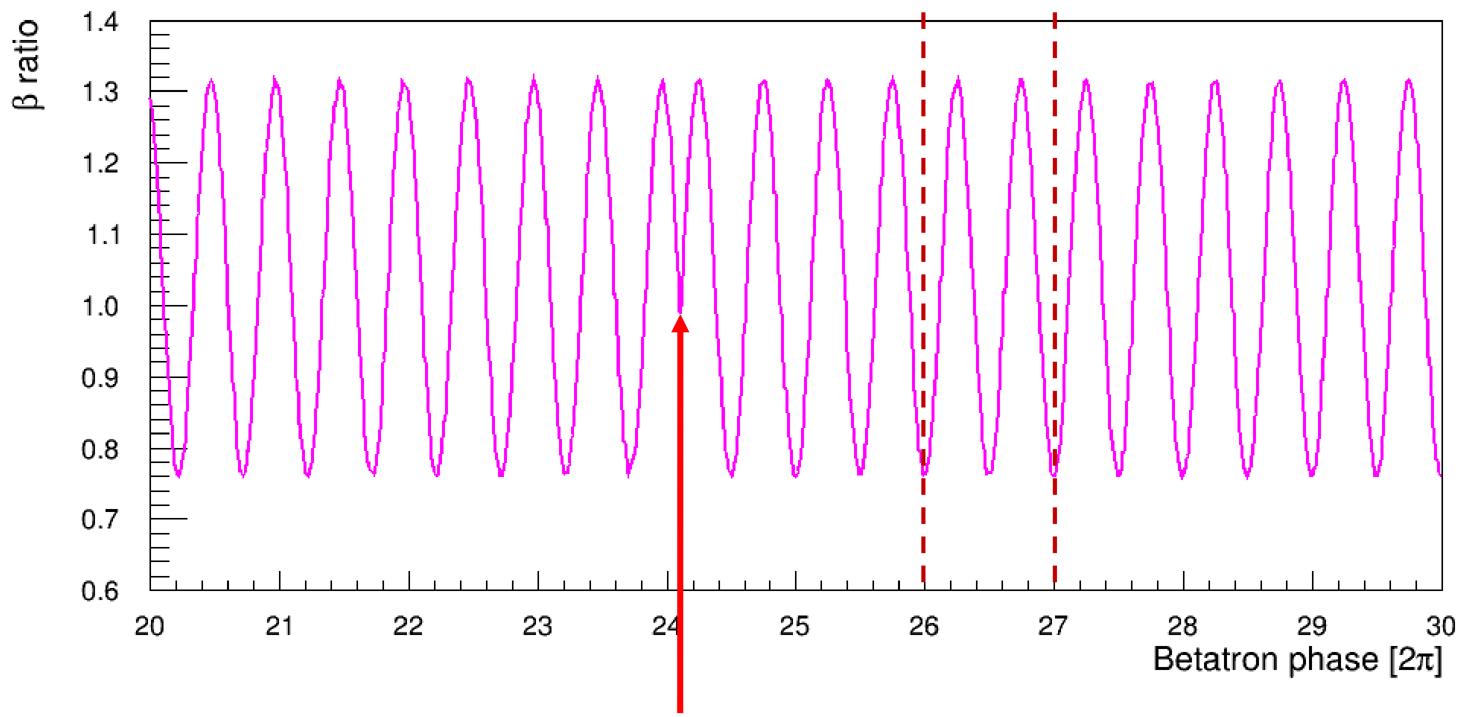


- The error is easier to analyse and diagnose if one considers the ratio of the betatron function perturbed/nominal.
- The ratio reveals an oscillating pattern called the **betatron function beating** ('beta-beating'). The amplitude of the perturbation is the same all over the ring !



Optics perturbation

- The beta-beating pattern comes out even more clearly if we replace the longitudinal coordinate with the betatron phase advance.
- The result is very similar to the case of the closed orbit kick, the error reveals itself by a kink !
  - If you watch closely you will observe that there are **two oscillation periods per 2π (360 deg) phase**. The beta-beating frequency is **twice** the frequency of the orbit !



Optics errors

- The knowledge of the **beam optics** is essential for the good performance of an accelerator. Tools to measure and correct the optics towards a design model are essential at any modern facility.
- The **betatron function error** (beta-beating) at an observation point j due to a number of strength errors Δk_i is given to first order by:

$$\frac{\Delta\beta_j}{\beta_j} \approx \sum_i \frac{\Delta k_i L_i \beta_i}{2\sin(2\pi Q)} \cos(2\pi Q - 2|\mu_j - \mu_i|) = \sum_i B_{ji} \Delta k_i$$

- Contrary to the case of orbit kicks, gradient errors have a **non-linear effect** on the betatron function. A correct treatment must be self-consistent, the equation above is only an approximation.
  - The problem may however be linearized using the matrix elements B_{ji} and solved **iteratively** using the SVD / MICADO algorithms based on measurements of $\Delta\beta_j$. After each correction iteration the matrix elements B_{ji} must be revaluated.

Optics measurements

There are three main technique to measure and reconstruct the machine optics, and they may be used in combination.

- **K-modulation**: the strength of individual quadrupoles is modulated to determine the local optics function.
- **Orbit (trajectory) response**: the orbit or trajectory response matrix is measured with orbit corrector kicks (→ see orbit correction), a fit to the response is used to reconstruct and correct the machine model.
- **Multi-turn beam position data**: the beam is excited and mutli-turn beam position data is recorded to determine the betatron phase advance between beam position monitors. The betatron function is reconstructed from the phase advance information.

- **K-modulation** was already described as a means to determine BPM offsets wrt quadrupole magnetic centres.
- This technique can also be used to determine the average betatron function inside the modulated quadrupole since the tune change ΔQ due to a gradient change Δk is given by:

$$\Delta Q_u = \frac{1}{4\pi}\int_{s_0}^{s_0+l} \Delta k \beta_u(s) \mathrm{d}s$$

(integral over the quadrupole length l)

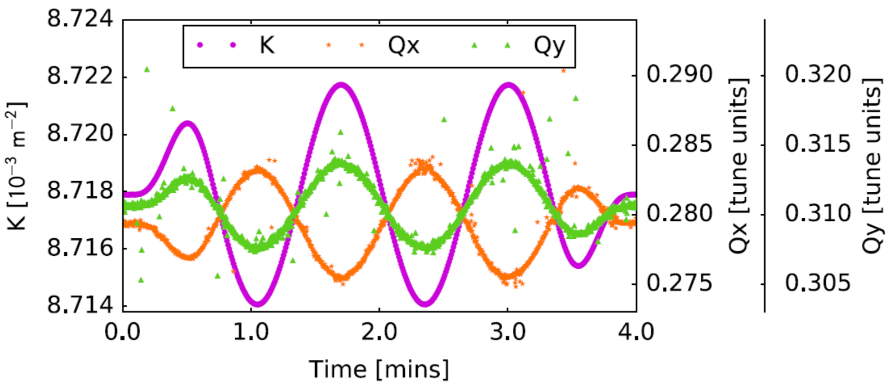
- The betatron function in the quadrupole is then given by:

$$\beta_u = \frac{2}{l\Delta k}\left[\cot(2\pi Q_u) - \frac{\cos(2\pi(Q_u + \Delta Q_u))}{\sin(2\pi Q_u)}\right]$$

O-KMOD-LHC

O-MT-LHC2

K-modulation at LHC



- This technique is powerful and simple but requires quadrupoles to be powered individually which is often the case on synchrotron light sources but applies only to a subset of quadrupoles in large storage rings!
  - Large collider arc quadrupoles are generally powered in series.

Orbit (trajectory) response

- The concept of **orbit / trajectory response** is to exploit the large amount of information that is encoded in the steering response matrix (**ORM**).
- The principle, available in the popular **LOCO code**, is to excite all / many steering elements in a ring / line and record the BPM response. This provides a measurement of the **response matrix** folded with **BPM calibration factors** b_i and **orbit corrector deflection calibration** factors c_j:

$$\Delta \vec{u} = \mathbf{R}_M \, \Delta \vec{\theta} \qquad R_{ij,M} = \frac{b_i c_j \sqrt{\beta_i \beta_j} \cos(|\mu_i - \mu_j| - \pi Q)}{2 \sin(\pi Q)}$$

O-LOCO-BDYN

- All the elements of $\boldsymbol{R_M}$ are observables for a model fit, fit variables include:
  - BPM and orbit corrector calibration factors ($b_{i,}$, c_j),
  - BPM and orbit corrector roll angles as they generate measurement 'coupling',
  - Any selection of quadrupole gradient to fit for β, μ,
  - Skew quadrupoles for coupling.
  - …

- The ORM matrix is linearized in the form of a vector $\vec{r}$:
  - The vector size = the number of elements of $\boldsymbol{R_M}$.

$$r_k = \mathrm{R}_{\mathrm{ij}}^{\mathrm{meas}} - \mathrm{R}_{\mathrm{ij}}^{\mathrm{model}} \quad \forall i, j$$

- The fit parameter vector is composed of (for example): $\vec{c}$
  - BPM calibrations and roll angles,
  - Steering elements calibrations and roll angles,
  - Quadrupole gradients (skew and normal),
  - …

- A response matrix $\mathbf{G}$ ist constructed with the dependence of each r_i on any c_j.
  - Some elements are obtained analytically, others need a simulation tool (like MAD, PTC, ELEGANT…).

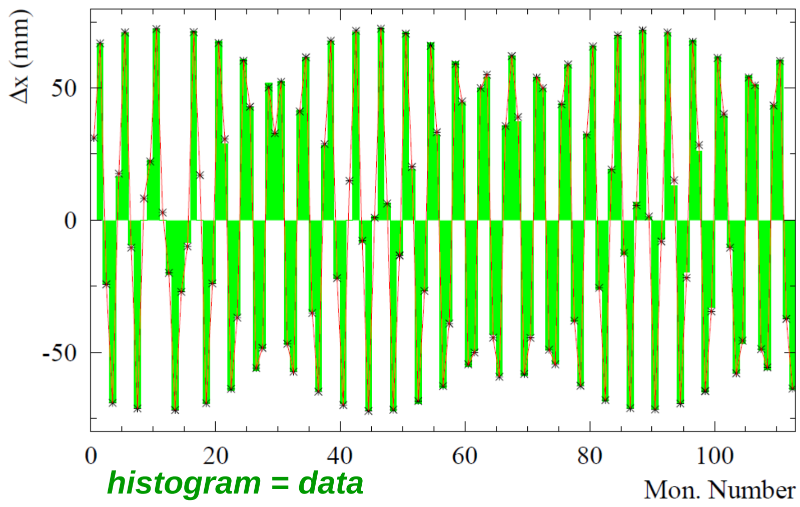
$$\mathrm{G} = \begin{pmatrix} \frac{\partial r_1}{\partial c_1} & \cdots & \frac{\partial r_1}{\partial c_M} \\ \cdots & \cdots & \cdots \\ \frac{\partial r_N}{\partial c_1} & \cdots & \frac{\partial r_N}{\partial c_M} \end{pmatrix}$$

- The response analysis is coupled to an accelerator design tool like MAD, PTC etc in order to determine the sensitivity wrt to the quadrupole gradients etc.
- Once the system is cast in matrix form, it is solved by SVD inversion – structurally equivalent to an orbit correction.
  - The tricks to filter noise by eliminating small eigenvalues as discussed for beam steering can be employed here !
  - A few iteration may be required to converge. At each iteration **G** must be re-evaluated.

$$\left\| \vec{r} - \mathbf{G}^{-1} \Delta \vec{c} \right\|^2 = \min$$

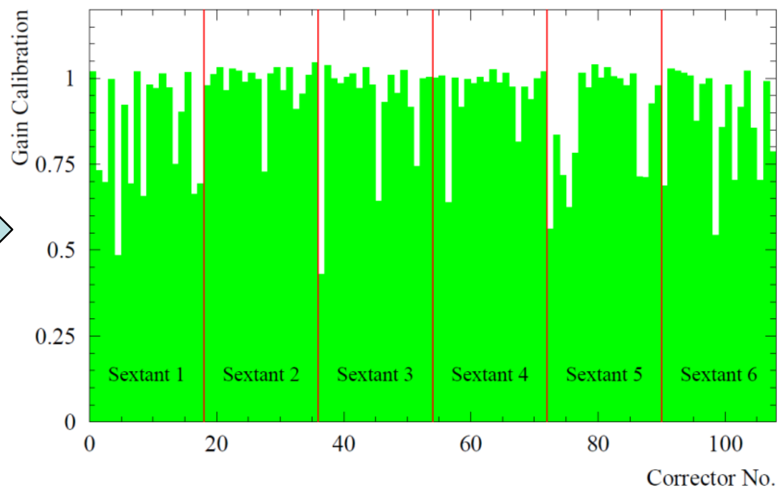
- Note that the size of matrix G grows rapidly !
  - For a machine with 100 BPMs and 100 steering elements, there are 10’000 lines and over 200 columns !

Orbit response example after fit at SPS

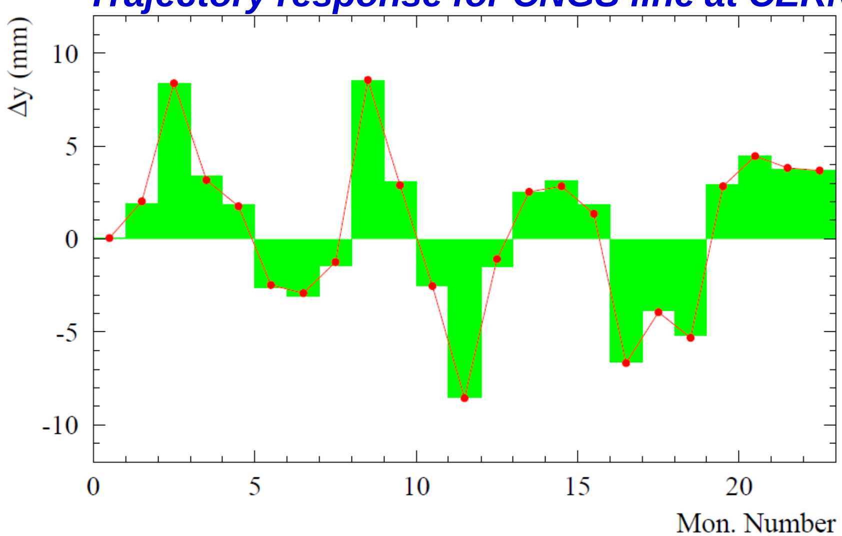


histogram = data

Horizontal orbit corrector calibrations at SPS



Trajectory response for CNGS line at CERN

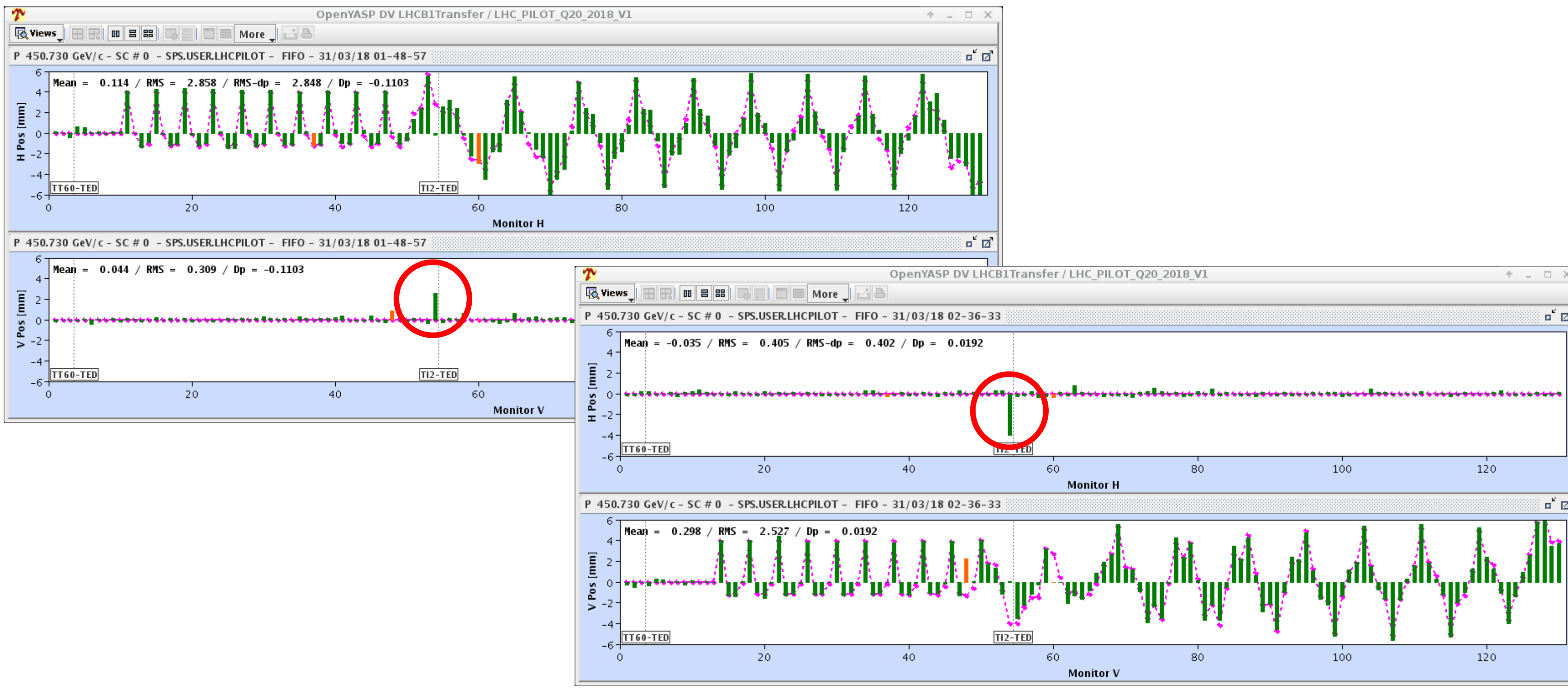


Some correctors are damaged by radiation (from the time when SPS was a lepton injector for LEP !)

O-LOCO-SPS

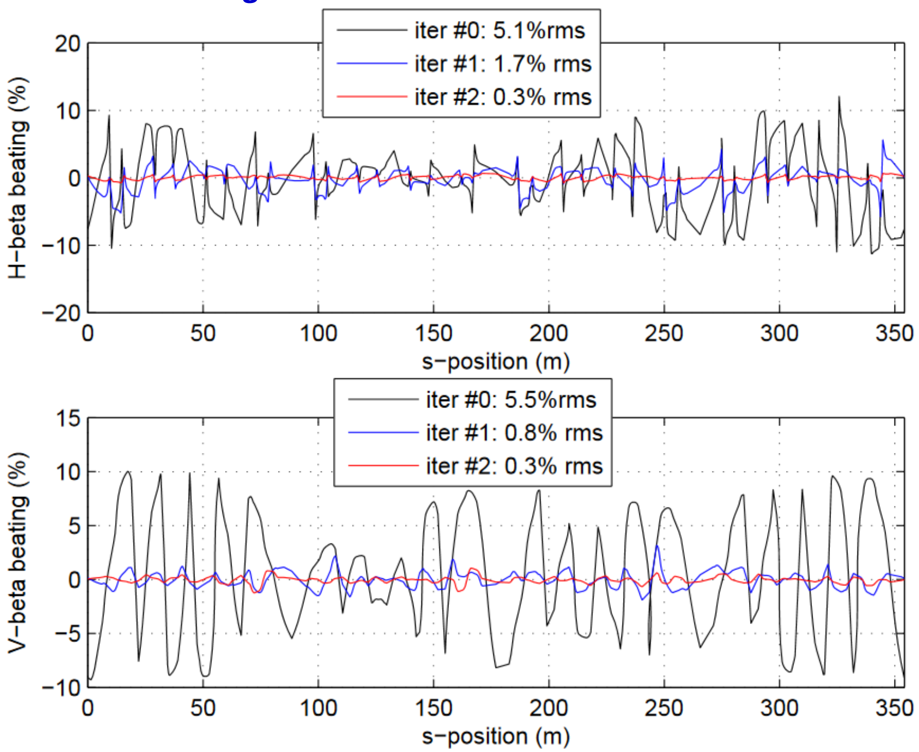
O-LOCO-CNGS

- Example of am inverted BPM plane captured by a response measurement in a transfer line.
  - The BPM clearly appears for both horizontal and vertical plane measurements.
  - Data in **green**, model in **magenta**.



- Orbit response has been used with a lot of success at synchrotron light sources where it is a standard tool.
  - Typically ~ 100 BPMs, steerers and quadrupoles.
- At very large machines like LHC and FCC-hh, the data volumes are immense and multi-turn methods are faster, therefore the response techniques are mainly useful to calibrate BPMs and steerers.

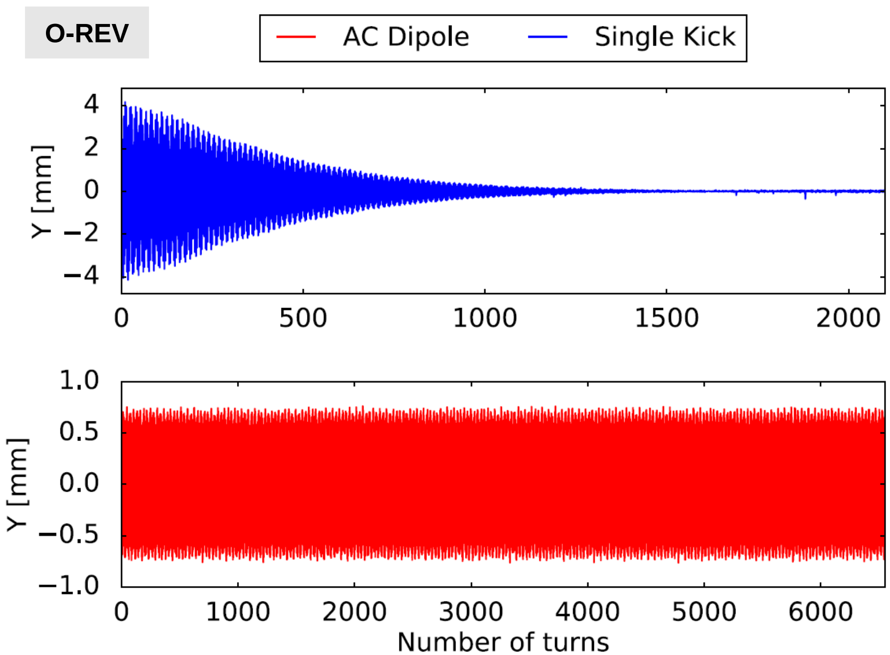
Beta-beating correction with LOCO at SOLEIL



O-LOCO-SOLEIL

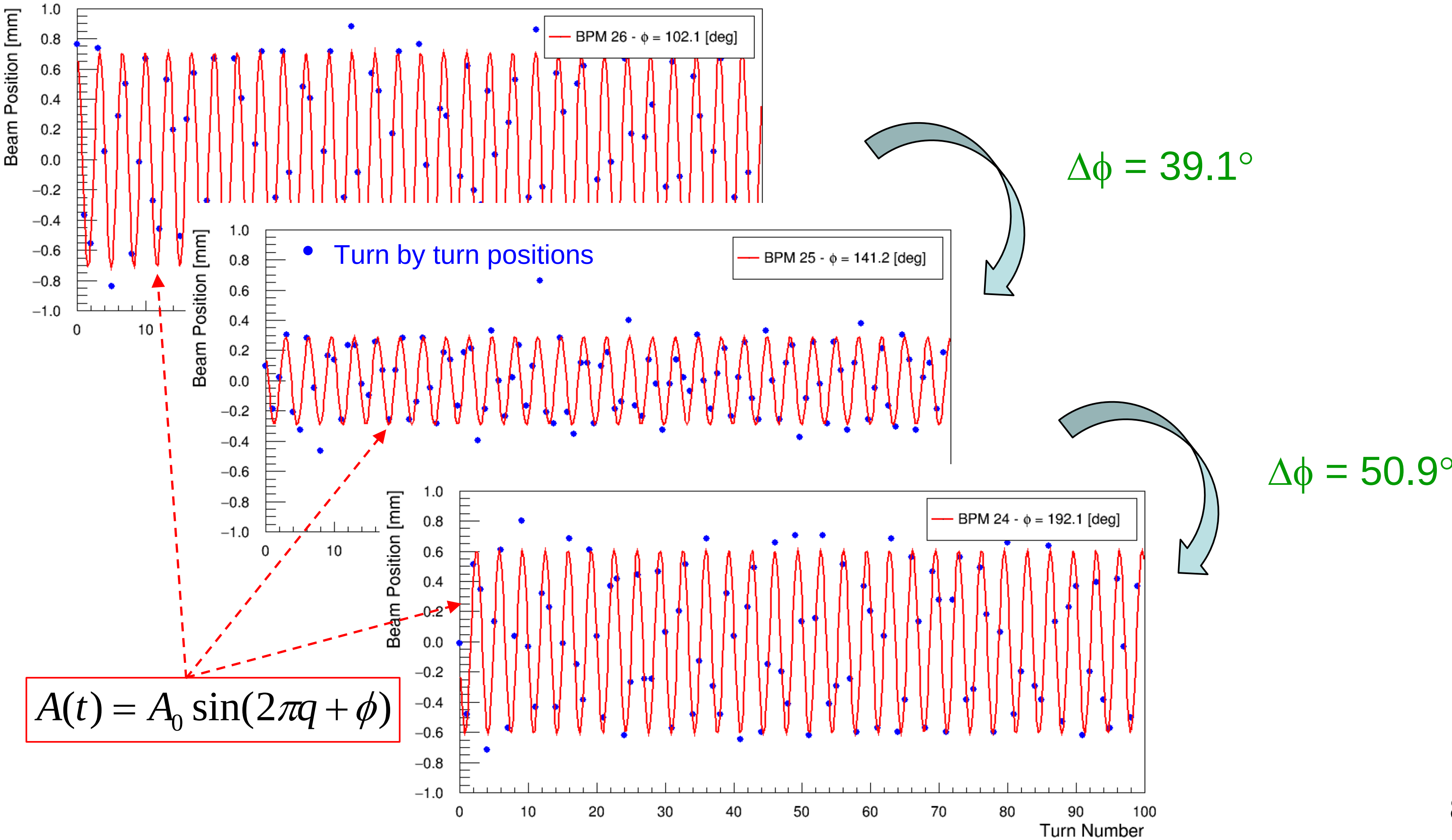
Multi-turn optics measurements

- Multi-turn optics measurements rely on a **beam excitation** for a certain number of turns, typically few x 1000 to obtains sufficient resolution.
- The beam oscillation phase is extracted for each BPM, this phase corresponds to the betatron phase μ at each BPM.
  - The betatron phase advance $\Delta\mu$ between BPMs can therefore be extracted in a straightforward way.
  - The phase measurement does not depend on the BPM calibration (but is sensitive to BPM non-linearities).



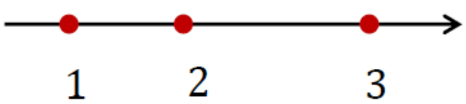
- Exciting the beam with a **single kick** is often limited by the decoherence of the oscillation (or by radiation damping).
  - Limited no of turns → measurement error.
- Excitation by an **AC-dipole** is more favourable since the number of turns can be increased if necessary.

- The betatron phase advance is directly obtained from the phase of the oscillation between adjacent BPMs.
  - The BPMs must be correctly synchronized to the same turn.

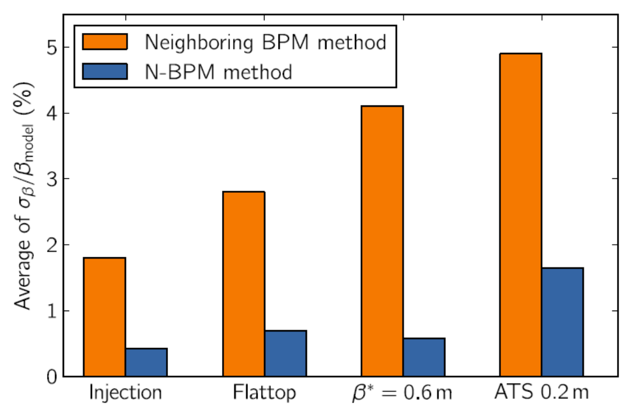


- The betatron function can be reconstructed from the phases measured for 3 BPMs – assuming that they are no sources of errors between the 3 BPMs – but a model is required:

$$\beta_1^{meas} = \beta_1^{model} \frac{\cot\Delta\phi_{12}^{meas} - \cot\Delta\phi_{13}^{meas}}{\cot\Delta\phi_{12}^{model} - \cot\Delta\phi_{13}^{model}}$$



- The raw turn data is often **filtered for noise by SVD** (→ see later) before the phase is extracted, and **multi-BPM interpolation techniques** have been developed to improve the accuracy of this technique.
  - Improved accuracy when the phase advances to neighbouring BPMs are unfavourable.



O-REV

O-MULTI-BPM

O-REV

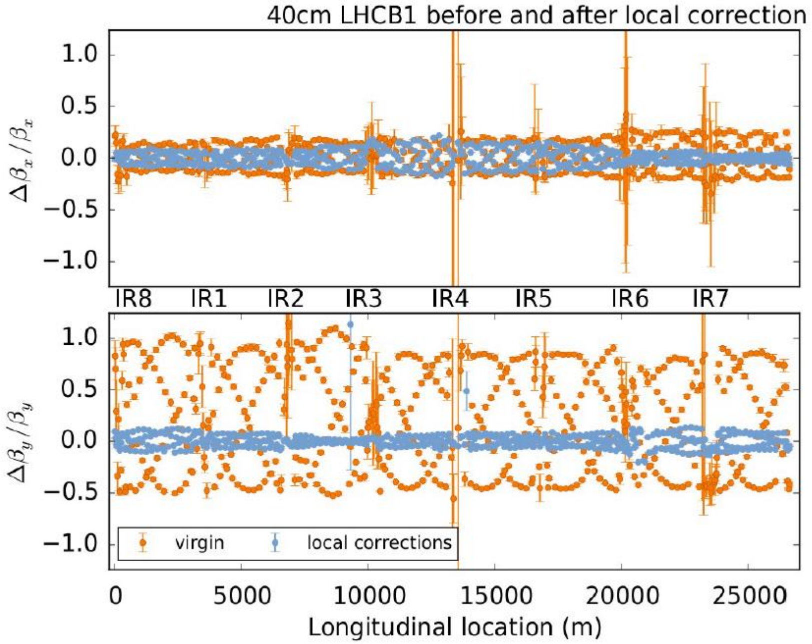
- The advantage of the MT technique is that the optics for both planes is obtained with two measurements – which can be very fast – and for large machines the data size is not as immense as with ORM.
  - In addition this method is not sensitive to BPM calibrations.
- The disadvantage of the MT technique is that a fast kicker is required, and the beam must be excited to **sufficiently large amplitude** compared to the BPM turn-by-turn resolution.
  - Can be an issue when the free aperture for kicking the beam is limited.
  - For ORM the BPM noise is in general not an issue (average over many turns).
- Once the data is prepared an optics modelling tool (MAD, PTC etc) must to used to fit machine errors to data, respectively establish corrections.
  - Alternatively SVD correction can be applied iteratively from the phase or betatron function response matrix.

Multi-turn optics measurements IV

- In a collider with a low-beta section where the peak β is >> than the ring average, the local optics errors may dominate completely the beta-beating.
- In such a configuration it is favourable to first correct the local errors before trying to flatten the beta-beating in the rest of the machine.

O-MT-LHC2

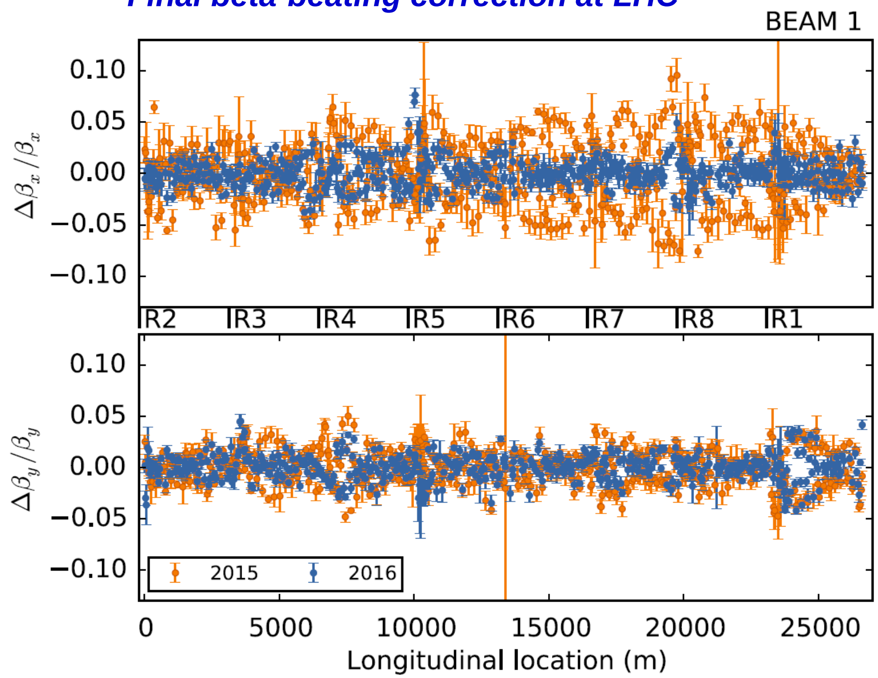
Local beta-beating correction at LHC



Raw β-beating ~ 100%

β-beating with local correction ~10-15%

Final beta-beating correction at LHC



Final β-beating ~ 1-2%

From optics measurements to corrections

- **K-modulation** and **multi-turn** techniques provide measurements of the real optics (phases, β-functions).
  - To restore the design optics (or at least reduce the deviations), corrections must be evaluated in a second step using directly an optics modelling program or using correction techniques based on response matrices with SVD (or even MICADO) type of algorithms.
- The **response matrix** technique (LOCO) can directly provide errors on gradients etc in the fitting procedure if the appropriate variables are used in the procedure.
  - To restore the design model it is sufficient to apply the errors as corrections to the real machine.
- In all cases, due to measurement uncertainties and to the non-linear response of optical functions to gradient changes, iterations may be required to converge to a satisfactory situation.

Introduction

Orbit and dispersion

Tune and coupling

Chromaticity

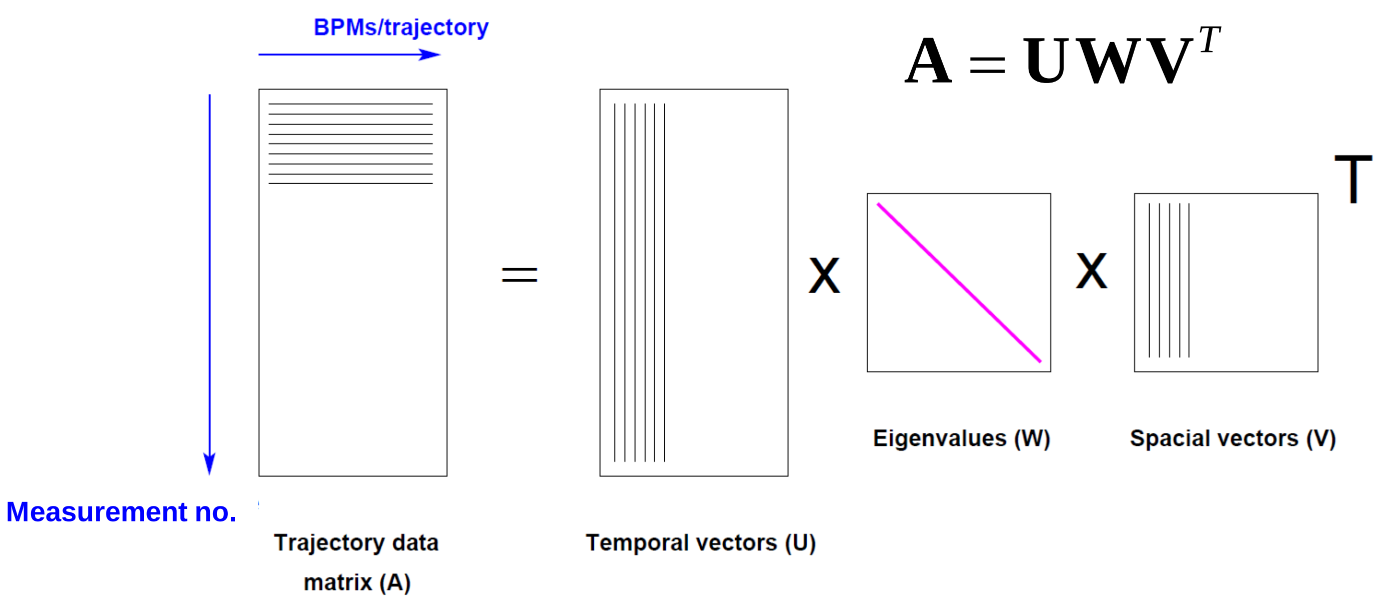
Linear optics

Appendix: model independent analysis

- The properties of the SVD decomposition find an interesting application in **Model Independent Analysis** (MIA) and **noise filtering**.
- The idea behind MIA is to identify 'patterns' in data, in particular in time series, without using a model (at least not initially).
  - SVD is able to find main components in data series.
- Consider for example of a series of position measurements that are repeated at a certain time interval, not necessarily periodic.
  - For each measurement the beam position at N BPMs is recorded,
  - The data is stored in a matrix that is decomposed by SVD.

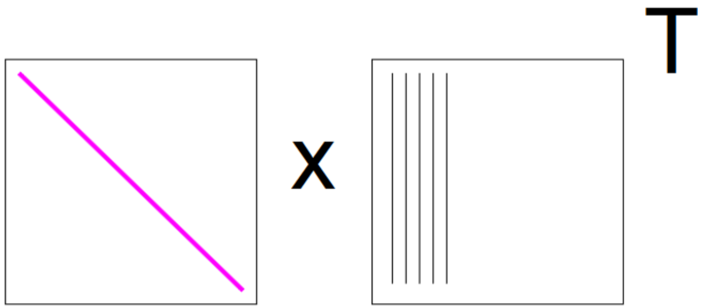
O-MIA-SLAC

$$\mathbf{A} = \mathbf{U}\mathbf{W}\mathbf{V}^T$$



Model Independent Analysis II

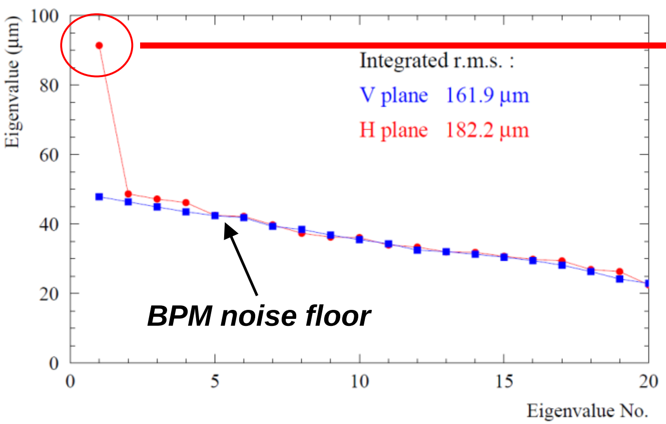
- We are interested in matrices W (eigenvalues) and V (special vectors):
  - The columns of V^T contain orthogonal patterns that are characteristic for the data set ($VV^T = V^TV = 1$).
  - The eigenvalues define the relative 'importance' of each pattern.



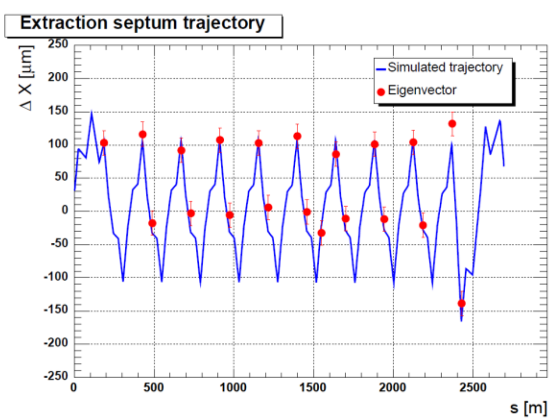
Eigenvalues (W) **Spacial vectors (V)**

- Example application to the SPS-LHC transfer line TI8.

MIA eigenvalues for trajectories



Largest MIA special eigenvector

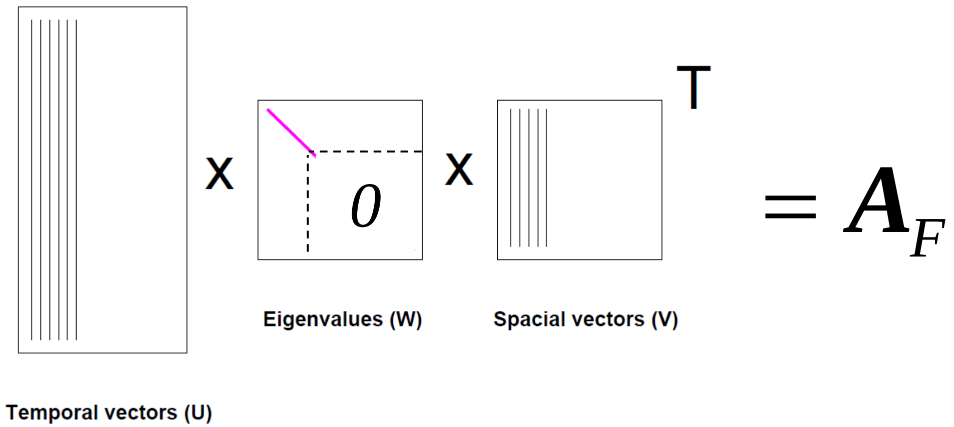


O-MIA-TI8

A analysis of the spacial pattern with a model points to the extraction septum magnet as source (→ ripple)

Model Independent Analysis III

- The example presented on the last slide highlights how a coherent pattern emerges from the data without need of applying a model.
- The components of matrix $\boldsymbol{V}$ are by construction orthogonal.
  - When there are multiple sources of patterns, the eigenvectors do not necessarily correspond to physical elements, but rather to linear combinations that form an orthogonal set.
  - A model is required to identify and disentangle the real physical sources.
- The same technique may also be used to apply **noise reduction** on a data set. For that purpose, after SVD, a filtered version $\boldsymbol{A_F}$ of matrix $\boldsymbol{A}$ is '*reconstructed*' using only the dominant eigenvalues and vectors.
  - This is a common technique to improve the quality of multi-turn data for optics measurements, keeping only the few dominant eigenvalues (~10) that contain the beam oscillation information.



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CERN

Threading

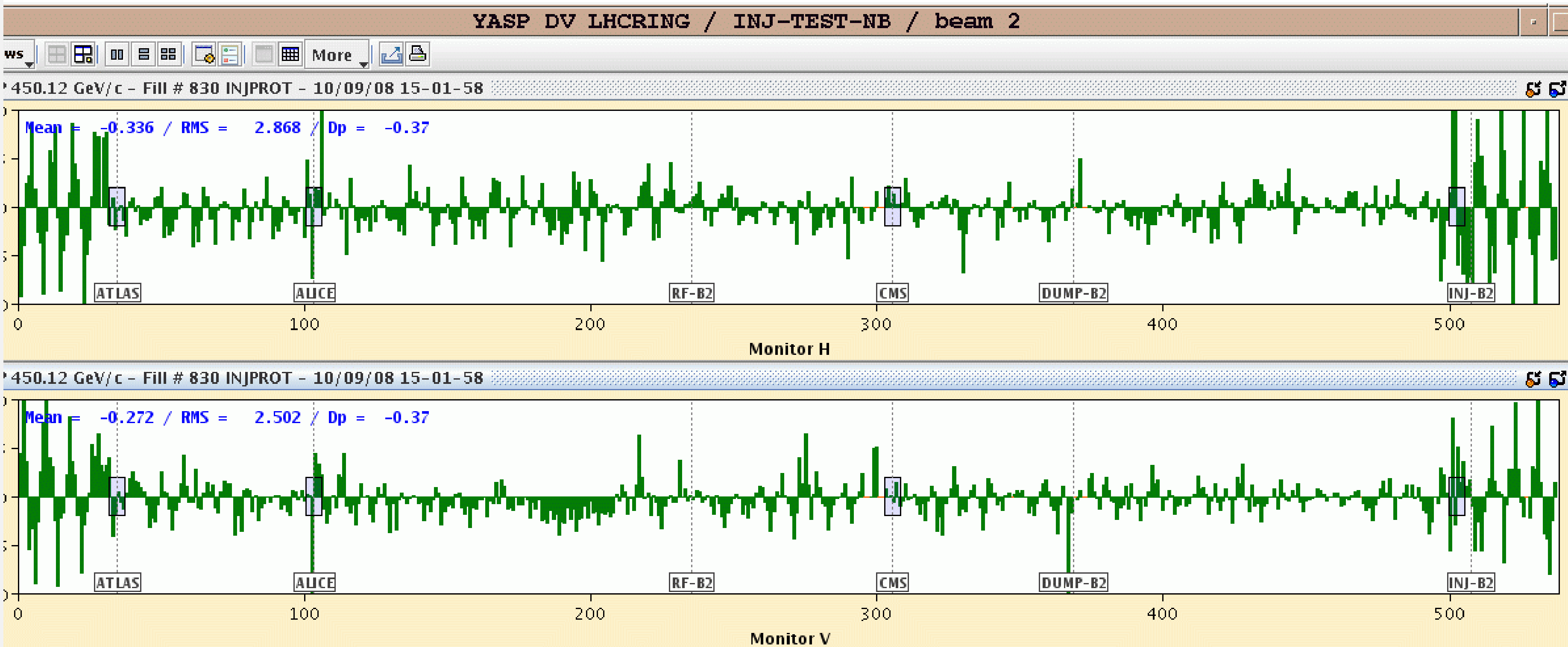
- At synchrotrons of very **large dimension** (10's km) and/or with a **very strong focussing optics**, the beam may not necessary circulate without any corrections when it is injected the first time.
  - Can also be the case for a corrected machine when the tunes are poorly set.
- In such a case it is necessary to **thread the beam around the ring** segment by segment until a few turns are obtained.
  - For superconducting machine one may have to operate at very low intensity / stop the beam at intermediate positions with collimators to avoid quenches.
- Once a few turns are obtained:
  - It may be possible to estimate (and correct) the tunes,
  - The average of the N first turns may be used as estimate for the closed orbit, opening the option to correct directly this estimated closed orbit.
  - To note after a few turns, the exact number depending on the machine, the beam may be debunched and no longer measureable by the BPMs, requiring the RF to be setup for capture for further progress.
    - For example at the LHC the beam debunches over ~30-40 turns.

Very first LHC threading (arc) sector by sector (Sept 2008):

- One beam at the time & one hour per beam, correction with MICADO, 1-3 correctors.
- Collimators used to intercept the beam (1 bunch, 2×10^9 p - 2% of nominal bunch).
- Beam through a sector (1/8 ring), correct trajectory, open collimator and move on.

Beam 2 threading

Beam direction

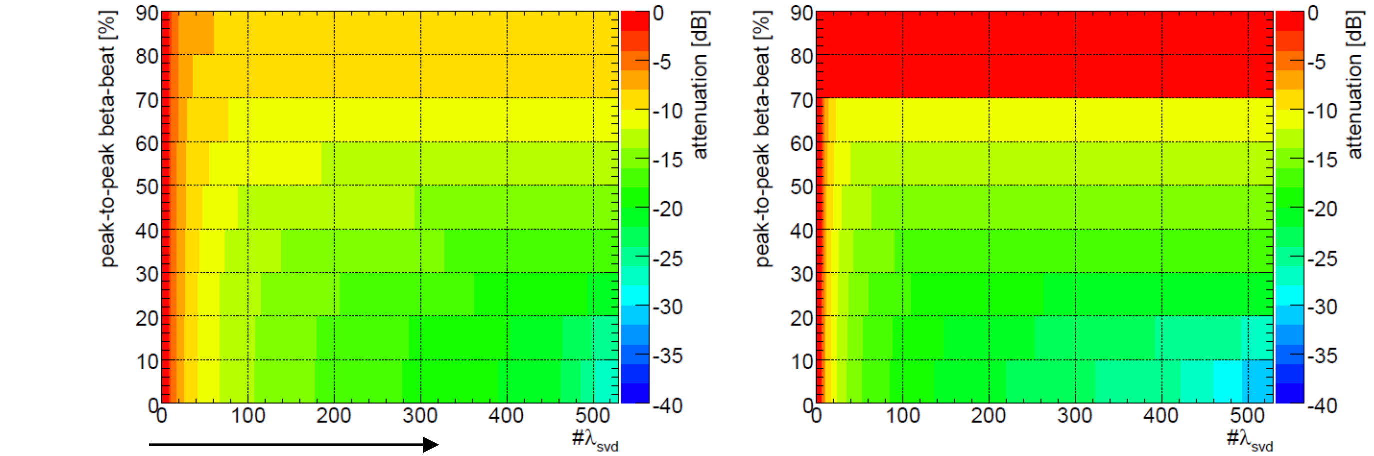


Impact of optics errors

- In particular during the machine commissioning, significant optics errors may be present that could generate divergent orbit corrections.
  - The orbit degrades with a correction instead of improving.
- Typically up to 30-50% beta-beating there are no severe problems, but corrections may require iterations to converge well.
  - Beyond that point corrections can diverge strongly, in particular if low beta sections are present.
  - Errors on the tunes, and in particular **wrong integer tunes**, can generate strongly diverging corrections !

OFB-LHC

LHC orbit correction sensitivity to beta-beating

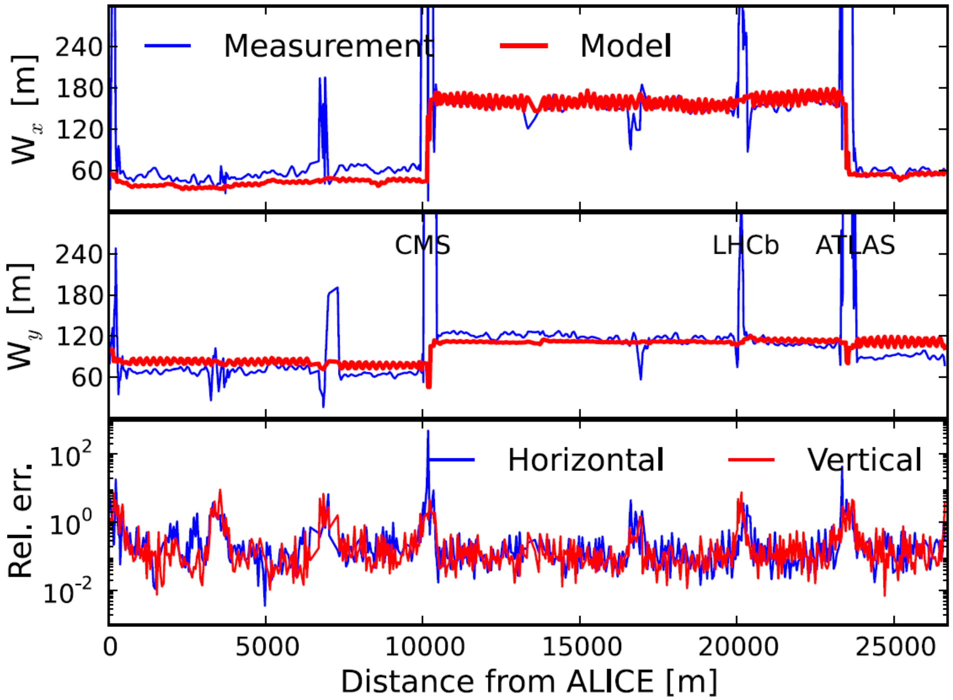


(a) injection optics (b) collision optics

- For lattices with low-beta section and / or very strong focusing measuring and correcting the on-momentum optics is generally not sufficient.
- The optics / ORM must also be measured at different momentum offsets to ensure that the optics correction procedure does not degrade the off-momentum properties of the optics.

Montague W function at LHC beam 2

O-MT-LHC1

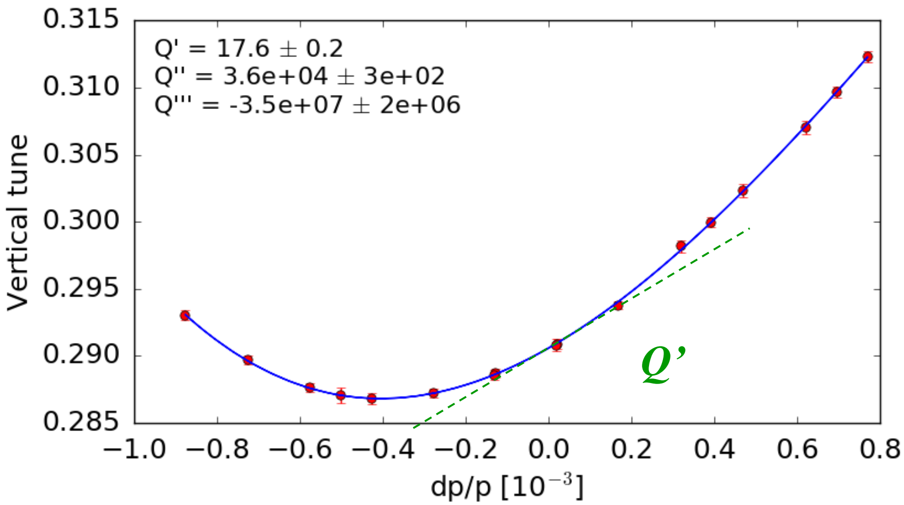


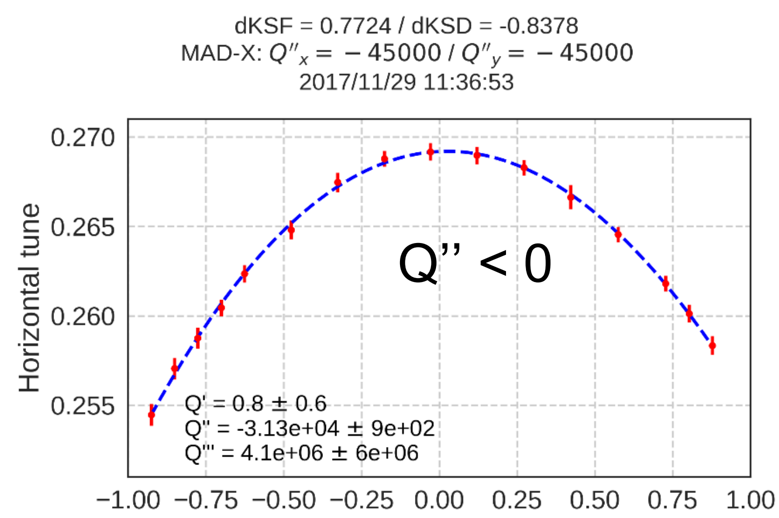
- So far we discussed the linear chromaticity which corresponds to the regime of small *dp/p*:

$$Q' = \lim_{dp/p \to 0} \frac{\Delta Q}{dp/p}$$

- Non-linear chromaticities Q'', Q''' etc (higher order derivatives wrt *dp/p*) are however important for the machine dynamic aperture and for collective effects ('Laudau damping' through tune spread).
  - Measurement of the tune versus dp/p.
  - High order chromaticity gives insight into off-momentum behaviour of optics, feed-down from higher order fields (or field errors) etc.

NL chromaticity measurement at LHC injection



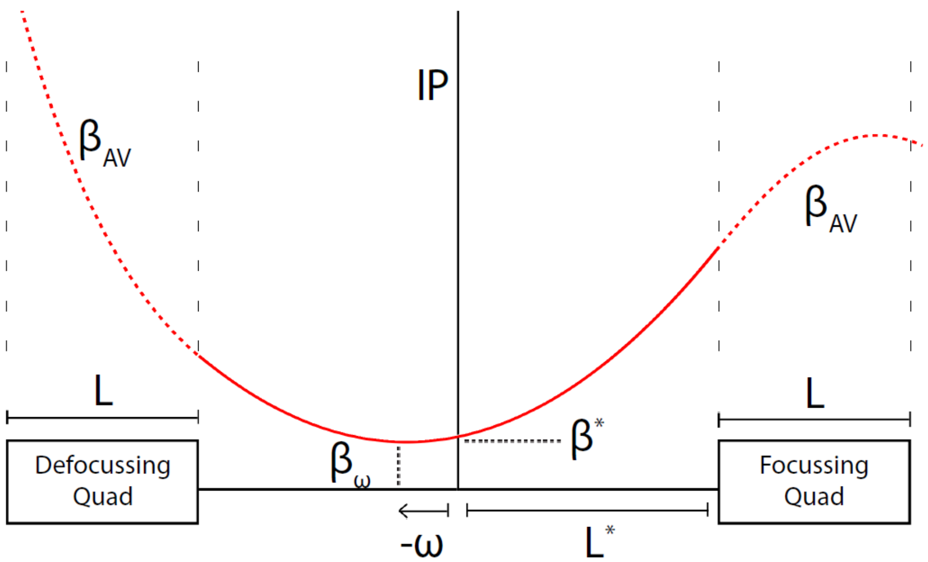


β* from K-modulation

- The knowledge of **the betatron function β* at the interaction points** (IPs) is important for the collider performance.
- The IP is usually surrounded by (low-beta) quadrupoles that focus the beam at the IP, with a drift space between the IP and the first quadrupole.
  - Due to errors, the **betatron function waist β_w** (minimum) may not coincide with the **betatron function β* at the IP**.
  - The β-function in the surrounding quadrupoles can be obtained from k-modulation and interpolated to the IP to obtain β*.
  - For e^+e^- the experimental solenoid can have a large impact due to the lower beam energy → no longer a 'drift' between IP and low-beta quadrupole.

Example of an IP layout, here the LHC

O-BSTAR-LHC



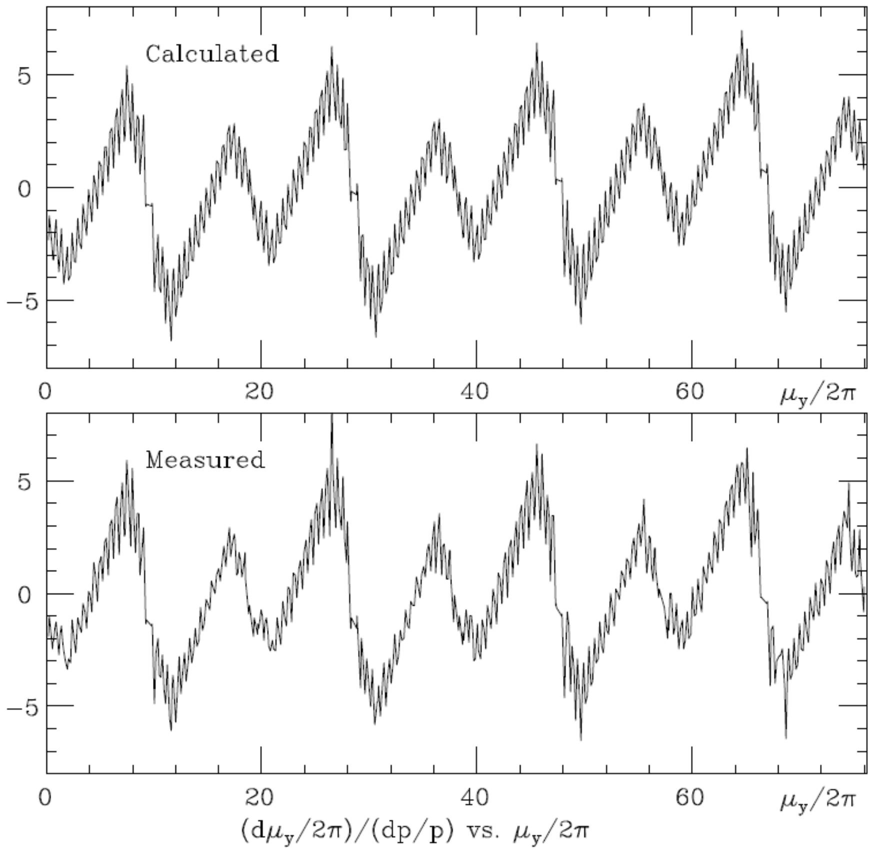
Ideally:

$$\beta_w = \beta^*$$

and

$$\omega = 0$$

Chromatic phase advance at LEP



O-CHROM-LEP

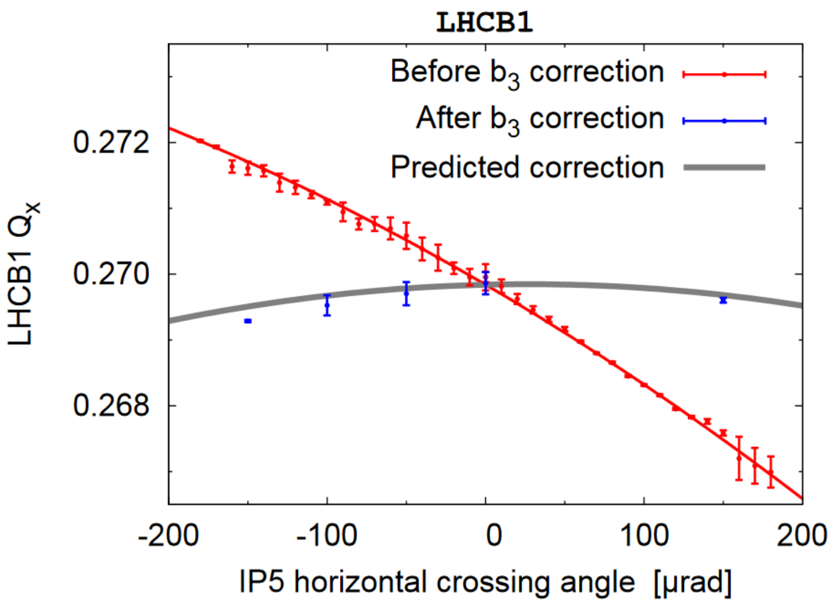
- MT phase advance measurements can also be used to probe the distribution of the sextupoles and the chromaticity correction.
- As an alternative to the Montague functions, it is possible to determine the phase advance for different dp/p settings from which the **chromatic phase advance** change wrt dp/p can be reconstructed between BPMs:

$$\frac{d\mu}{dp/p}$$

Non-linear optics correction

- Higher order field errors may need to be corrected with non-linear (NL) optics corrections.
  - The impact of low-beta quadrupole field errors may be boosted by the very large β-functions at those locations.
- NL optics measurements usually rely on scanning local orbit bumps and recording the tune, or scanning the momentum offset.
- Correction elements include sextupoles, octupoles and local NL correctors (low beta sections).

Local b3 correction for LHC low-beta section



O-NL-LHC