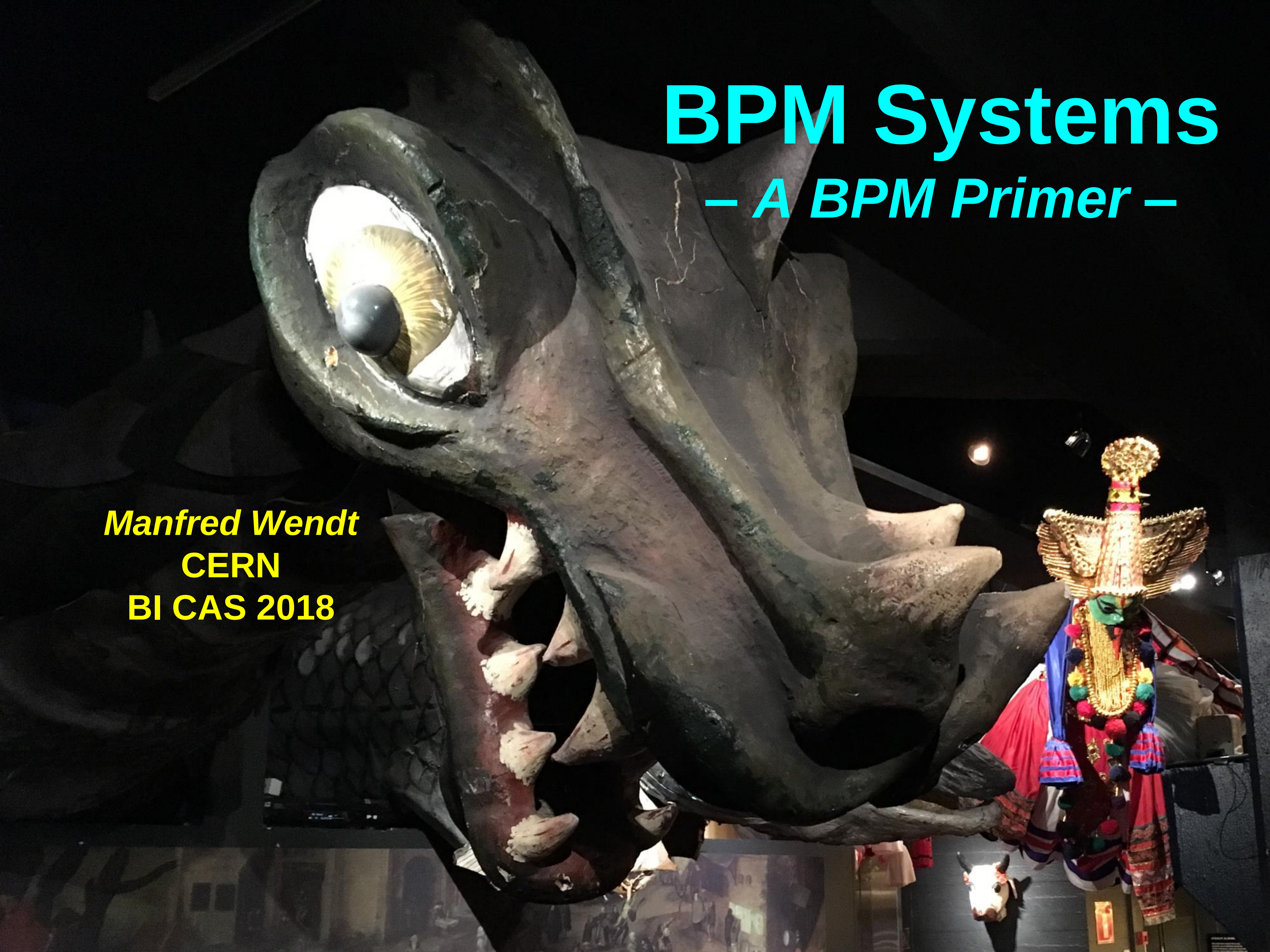
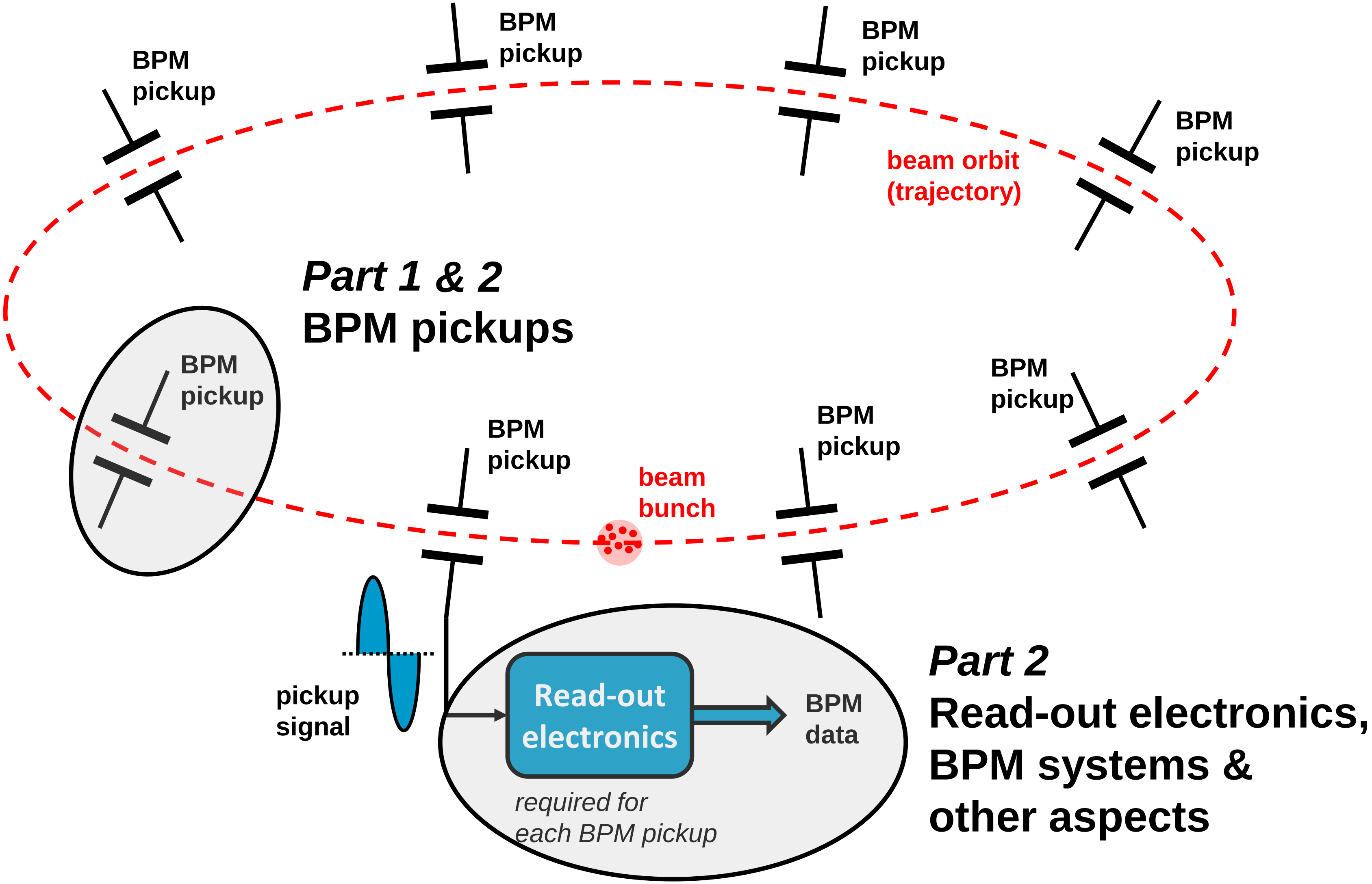


BPM Systems

– A BPM Primer –

Manfred Wendt
CERN
BI CAS 2018

A Beam Position Monitoring System



Part 1 & 2
BPM pickups

Part 2
Read-out electronics,
BPM systems &
other aspects

Learning Objectives

- Introduction into BPM systems
  - From an engineering perspective
    - How to design and build a BPM system!?
  - Focus on the principle of operation
    - Often in graphical format
  - Practical formulas as necessary, but no lengthy derivations
  - Stick to the fundamentals
    - Expert and exotic stuff: please study the related papers
- Very personal selection of BPM topics
  - Focus on popular BPM components and subsystems
    - BPM pickups, such as buttons, stripline, cavity BPM, etc.
    - BPM electronics, e.g. RF signal conditioning, digital signal processing
  - Will not cover BPM applications and beam measurements
    - Covered at the CAS introduction and advanced courses
- Give the design basics for the key elements of a BPM system
  - So lets start…

BPM Systems

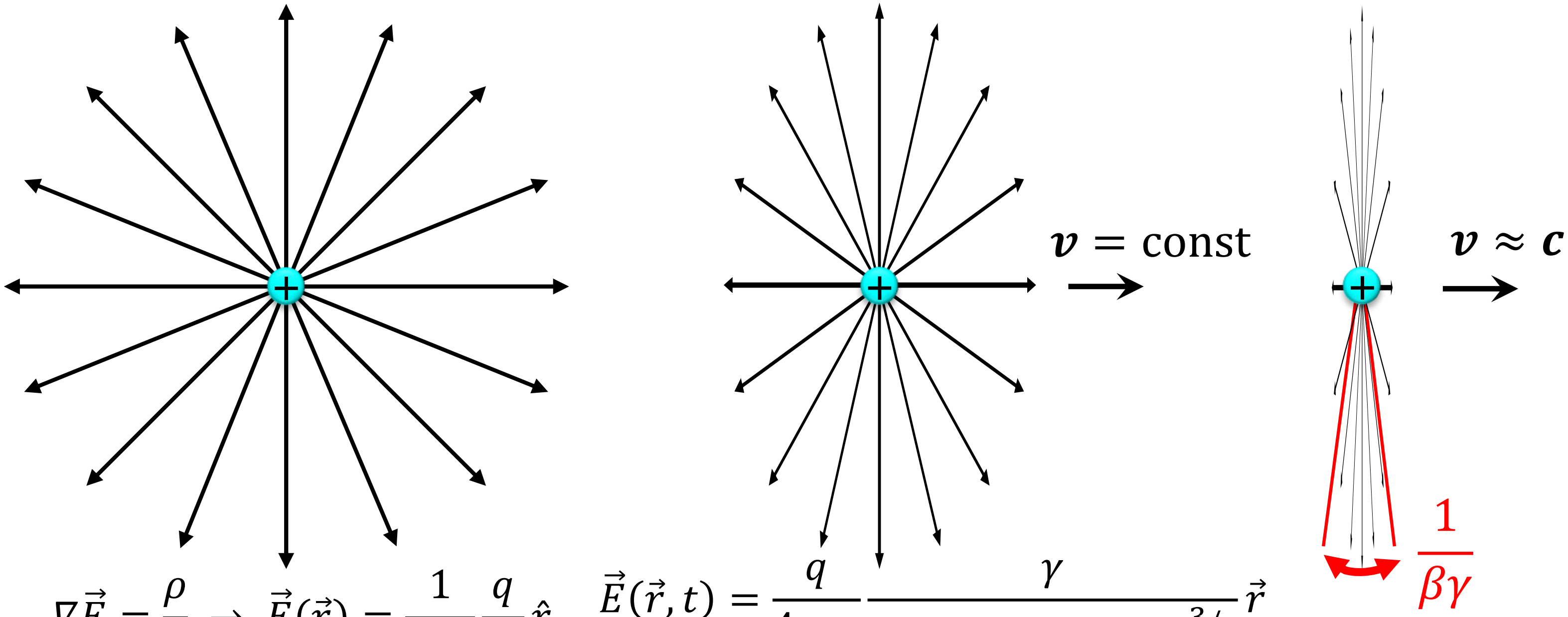
- **are based on Beam Position Monitors (BPM), which are beam detectors located along the accelerator**
  - **BPM: Beam Position Monitor**
    - **Beam pickup with signal processing (read-out) electronics**
      - Often colleagues just refer to the beam pickup as BPM
  - **BPMs are typically located near each quadrupole magnet**
    - **Use 4 or more BPMs per betatron oscillation period**
- **deliver beam orbit (trajectory) information**
  - **Non-invasive monitoring based on the EM-field of the passing beam**
  - **Synchronized BPMs deliver beam timing information**
    - **Beam orbit measurement**
      - turn-by-turn, batch-by-batch, bunch-by-bunch, or averaged over many turns
    - **Beam phase or time-of-flight (TOF) information in linacs**
- **are a powerful beam diagnostics tool**
  - **Machine commissioning, characterization of the beam optics, measurement of beam parameters, trouble-shooting,…**

BPM Systems Part 1

- Principle of operation
  - Wall currents and the electrostatic BPM pickup
- Bunch Beam Signals
- The image charge (current) model for BPMs
  - Position characteristic in a circular beam pipe
  - Numerical analysis and correction of non-linearities
- BPM pickups
  - Split-plane BPM
  - Button BPM
  - Stripline BPM
  - Cavity BPM
- Summary of part 1

E-field of a moving point charge

Point charge at rest | **Moving point charge** | **Relativistic moving point charge**



$$\nabla\vec{E} = \frac{\rho}{\varepsilon} \rightarrow \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}\hat{r}$$

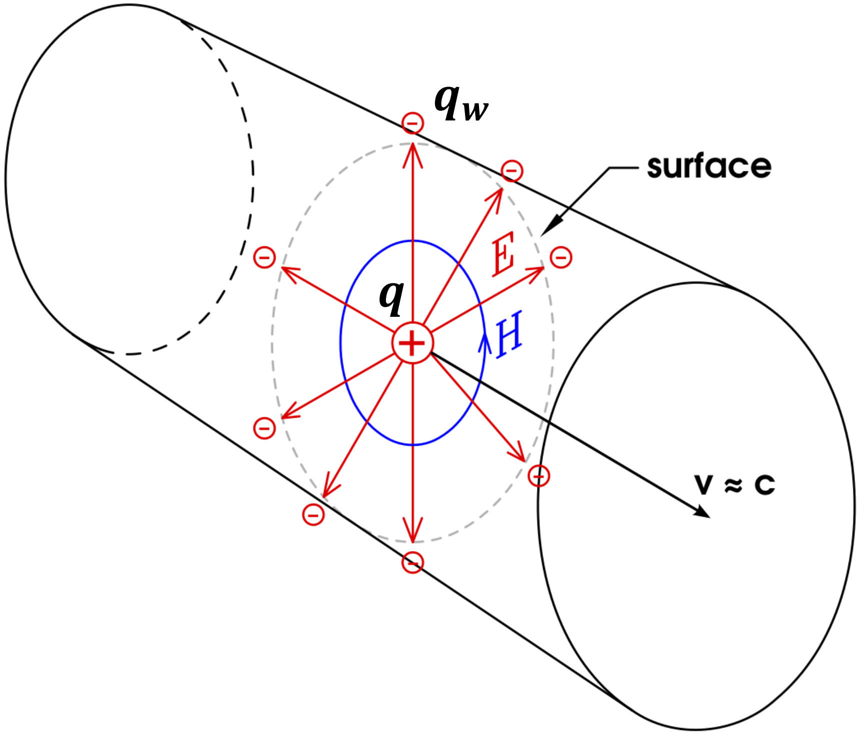
$$\vec{E}(\vec{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{\gamma}{r^3\left(1+\vec{v}_r^2\,\gamma^2/c^2\right)^{3/2}}\vec{r}$$

- **A relativistic moving charge has a “pancake”-like EM-field**

$$\boldsymbol{\beta} = \frac{\boldsymbol{v}}{\boldsymbol{c}} \approx \mathbf{1} \qquad E_z^{'} = \gamma E_z$$

  - **Only Transverse Electric Magnetic (TEM) field components**
    - **almost no longitudinal field components**

Wall current



- **Single proton in a perfect conducting cylindrical beam pipe of radius r**
  - Travelling with: $\beta = \frac{v}{c} \approx 1$
  - Image charges q_w along the azimuth of the beam pipe wall:

$$q = -q_w$$

- **Wall current density:**

$$j_w(t) = -\frac{i_b(t)}{2\pi r}$$

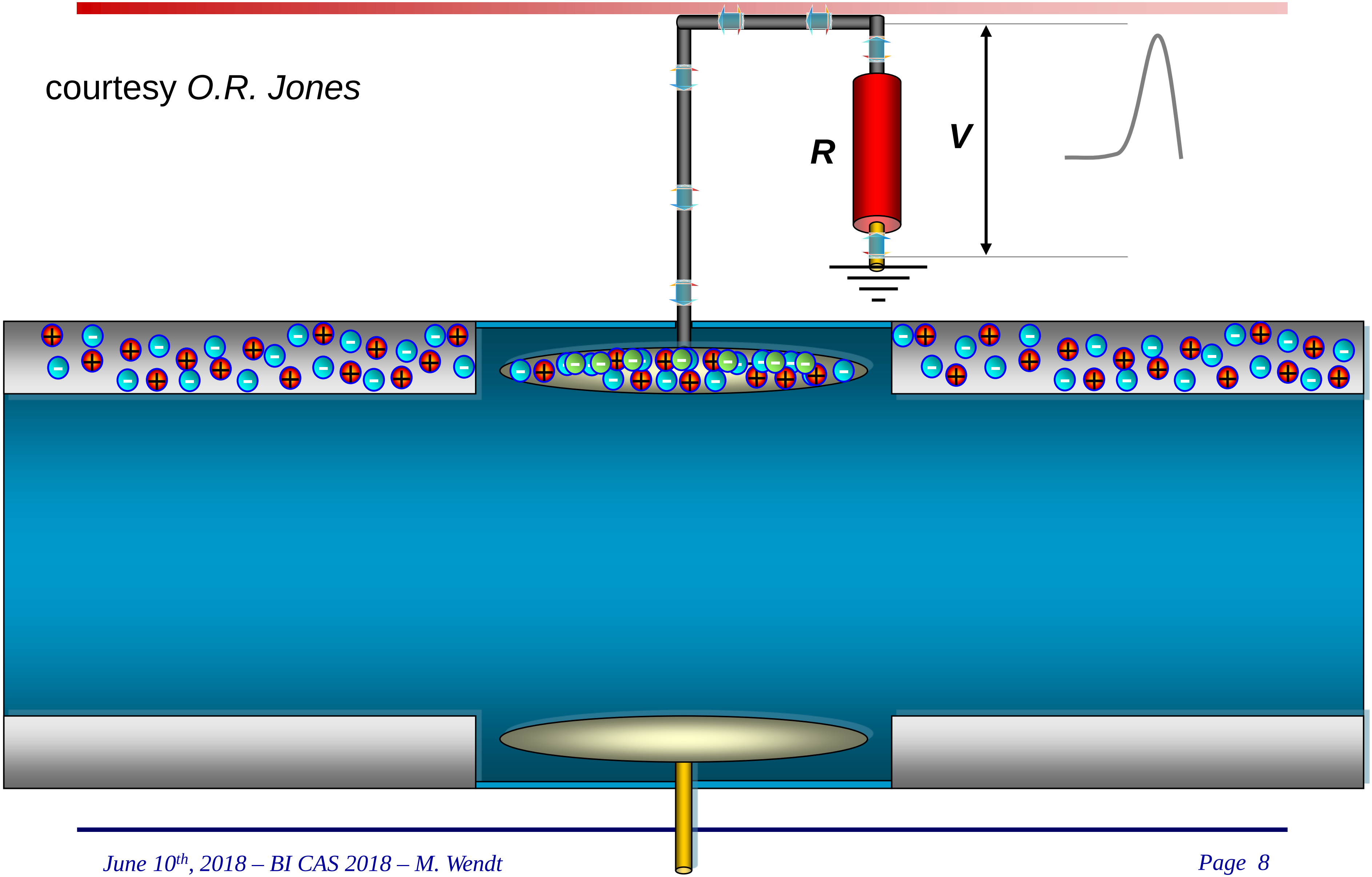
- **Beam current:**

$$i_b(t) = \frac{dq}{dt}$$

$$q = Ne$$

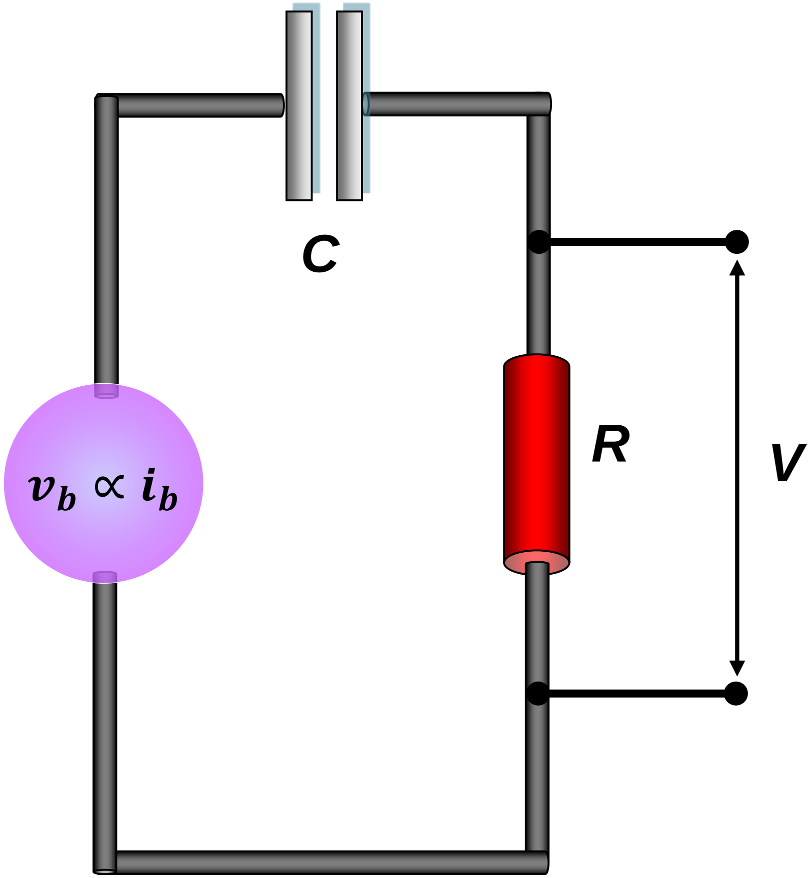
Electrostatic Beam Monitor

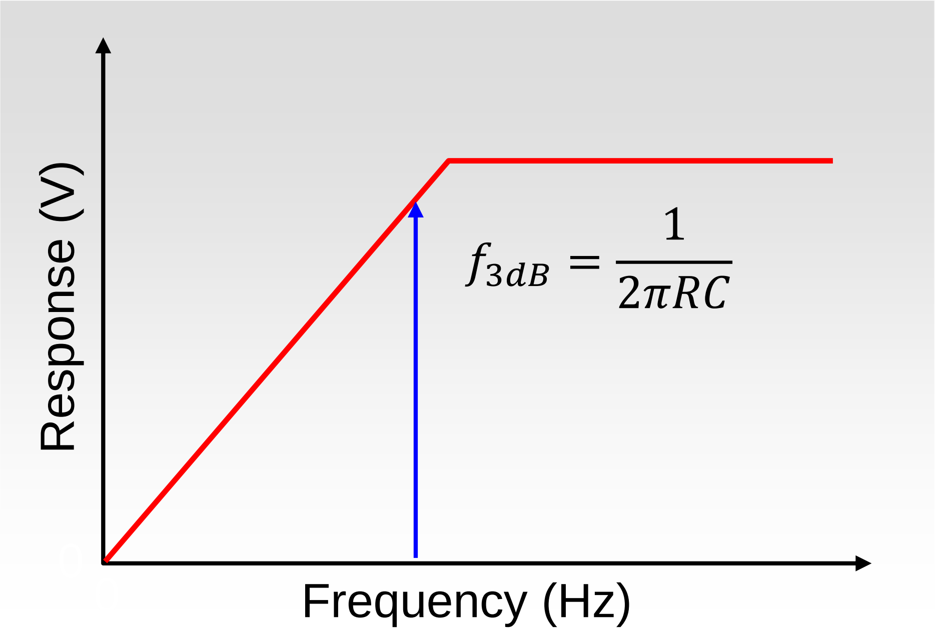
courtesy *O.R. Jones*

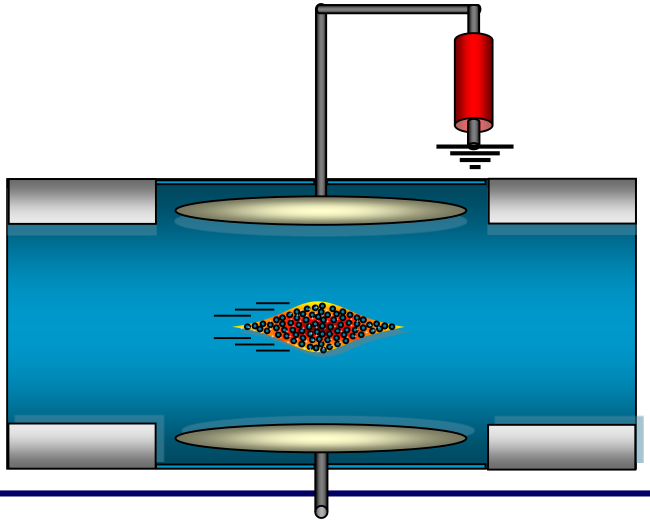


Beam response – High-pass

courtesy *O.R. Jones*

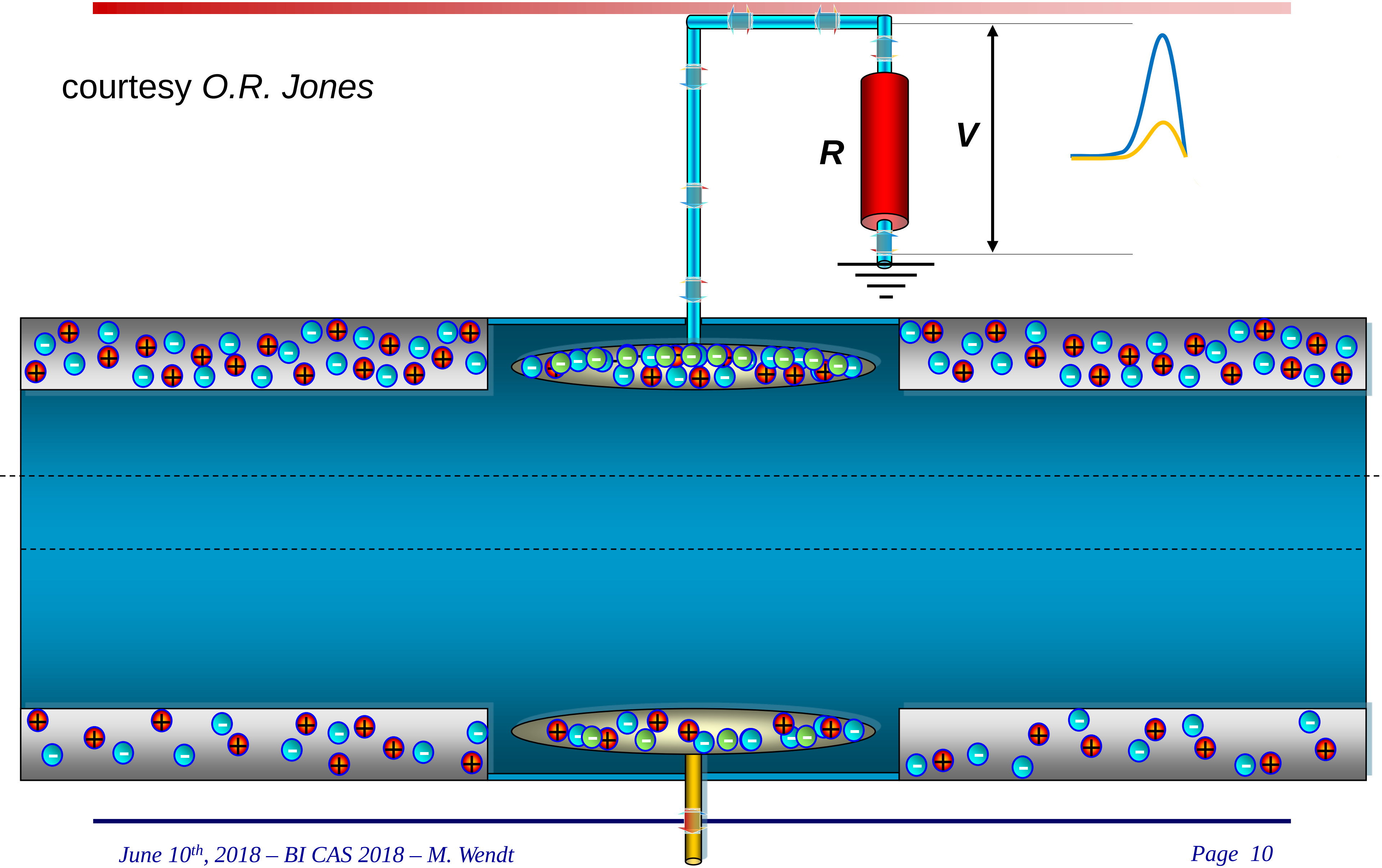






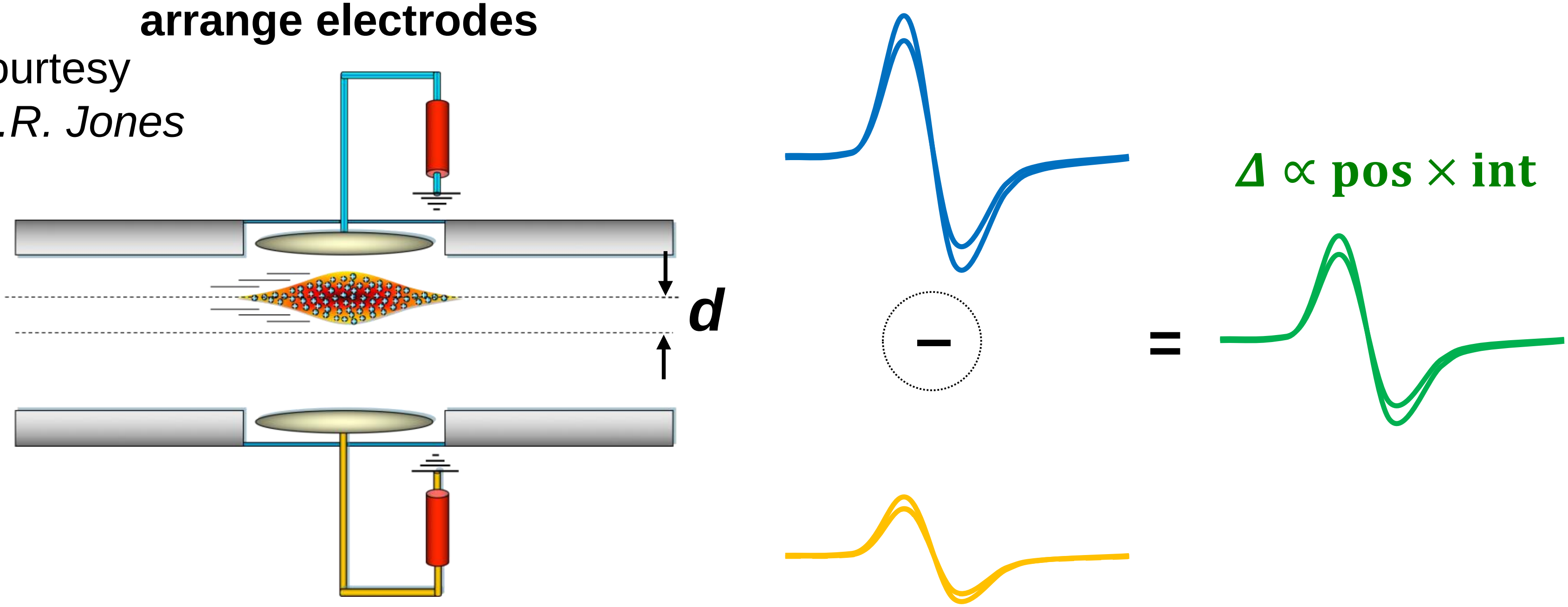
Electrostatic BPM

courtesy *O.R. Jones*



Beam Position Monitor Principle

- **The BPM principle is based on symmetry**
  - **The beam displacement *d* is detected by a pair of symmetrical arrange electrodes**



courtesy
O.R. Jones

  - **BUT: Hoops, wait a minute, not so fast…**
    What happens if the beam / bunch intensity changes?!
    - **The Δ-signal still contains beam intensity information!**
    - **Need to “normalize” the Δ-signal**

EM Pickup for Beam Position Measurements

- **The BPM pickup detects the beam positions by means of:**
  - **identifying asymmetries of the signal amplitudes from two symmetrically arranged electrodes A & B:**

$$\text{norm. beam pos.} \propto \frac{A-B}{A+B}$$

    - ➢ **broadband pickups, e.g. buttons, striplines**
  - **Detecting dipole-like eigenmodes of a beam excited, passive resonator:**
    - ➢ **narrowband pickups, e.g. cavity BPM**
- **BPM electrode transfer impedance:**

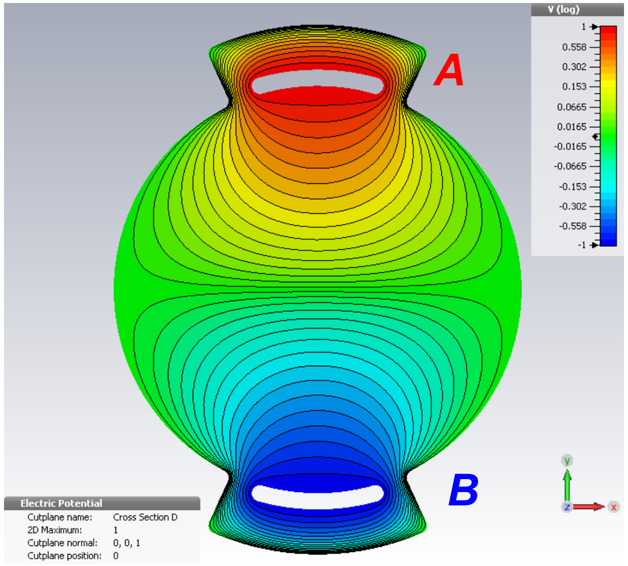
$$V_{elec}(x,y,\omega) = s(x,y,\omega)Z(\omega)I_b(\omega)$$

beam position

applies only for resonant pickups

frequency depending transfer impedance of the BPM electrode

beam intensity



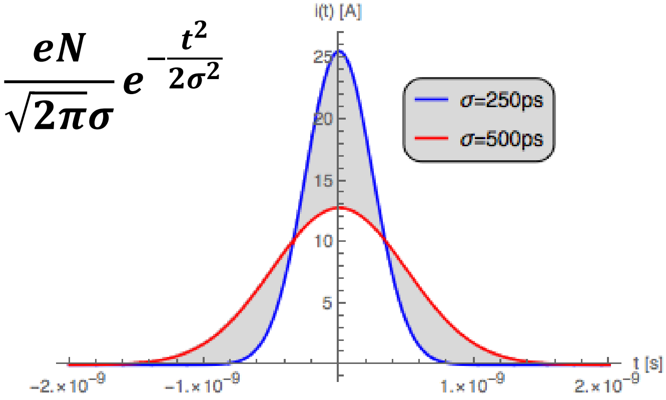
Electric Potential
Cutplane name: Cross Section D
2D Maximum: 1
Cutplane normal: 0, 0, 1
Cutplane position: 0

  - **The beam displacement or sensitivity function $s(x,y)$ is frequency independent for broadband pickups (@ $v \approx c$)**

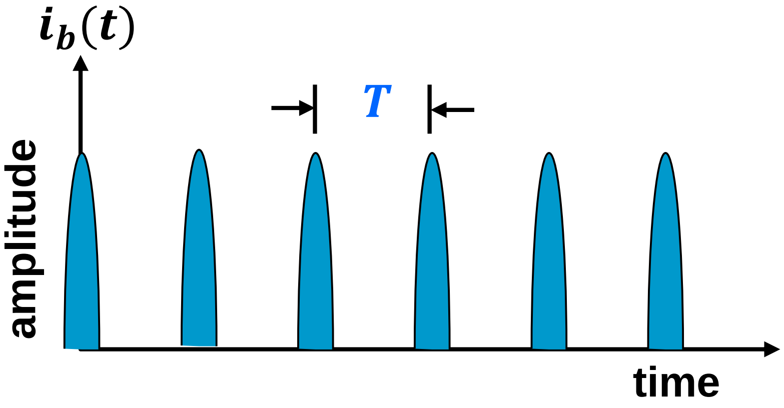
Beam Current Signals

- **Single bunch with Gaussian distribution of the particles (long.)**
  - **Number of particles: N**
  - **Bunch length (time): σ**

$$i_b(t) = \frac{eN}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}}$$



- **Bunches spaced by an equidistant time T**
  - ***Fourier* series expansion:** $i_b(t) = \langle I_{DC} \rangle + 2\langle I_{DC} \rangle \sum_{m=1}^{\infty} A_m \cos(m\omega t)$
  - **Average beam current:** $\langle I_{DC} \rangle = \frac{eN}{T}$
  - **Frequency harmonics spaced by:** $\omega = 2\pi f$
  - **Time distance:** $T = \frac{2\pi}{\omega}$
    - ➢ **Note: Understand the integration time of the BPM electronics**
  - **Harmonic amplitude factor:** $A_m = \cdots$



Beam Signals: Harmonic Amplitude Factors

Bunch shape	Harmonic amplitude factor A_m	Comments
δ-function (point charge)	1	For all harmonics
Gaussian	$\mathbf{exp}\left[\frac{(\boldsymbol{m\omega\sigma})^2}{\mathbf{2}}\right]$	$\boldsymbol{\sigma} = \textbf{RMS bunch length}$
parabolic	$3\left(\frac{\sin\alpha}{\alpha^3} - \frac{\cos\alpha}{\alpha^2}\right)$	$\alpha = {}^{m\pi W}/_{T}$
$(\cos)^2$	$\frac{\sin(\alpha-2)\frac{\pi}{2}}{(\alpha-2)\pi} + \frac{\sin\frac{\alpha\pi}{2}}{\frac{\alpha\pi}{2}} + \frac{\cos(\alpha+2)\frac{\pi}{2}}{(\alpha+2)\pi}$	$\alpha = {}^{2mW}/_{T}$
triangular	$\frac{2(1-\cos\alpha)}{\alpha^2}$	$\alpha = {}^{m\pi W}/_{T}$
square	$\frac{\sin\alpha}{\alpha}$	$\alpha = {}^{m\pi W}/_{T}$

Normaization: $A_m \rightarrow 1$ for $\omega \rightarrow 0$ T: bunch period W: bunch lenght at base

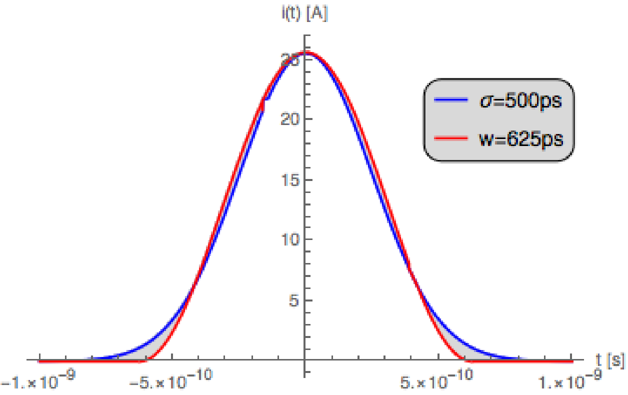
Beam Signal Frequency Response

- **Use *Fourier* transformation**
  - **instead of *Fourier* series expansion with infinite sums**
- **Examples: *Gaussian* and raised cosine (cos^2) pulse**
  - **Time domain** **Frequency domain**

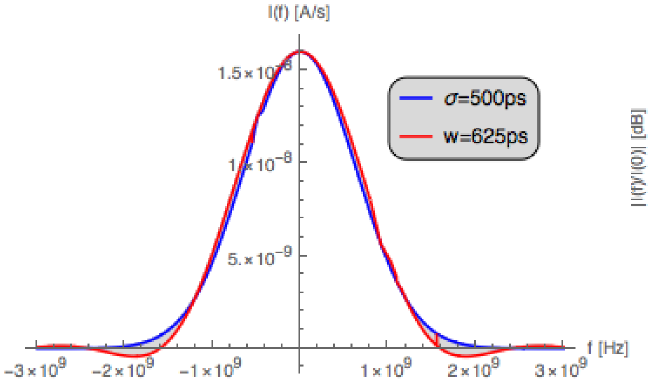
$$i_{Gauss}(t) = \frac{eN}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}} \qquad I_{Gauss}(f) = eN\, e^{-2(\pi f\sigma)^2}$$

$$i_{RaisedCos}(t) = \begin{cases} \frac{eN}{2\,w}\left(1 + \cos\frac{\pi t}{w}\right), & -w < t < w \\ 0, & \text{elsewhere} \end{cases} \qquad I_{RaisedCos}(f) = \frac{eN\sin(2\pi f w)}{2\pi f w(1 - 4f^2w^2)}$$

Time domain (N=10^{11})

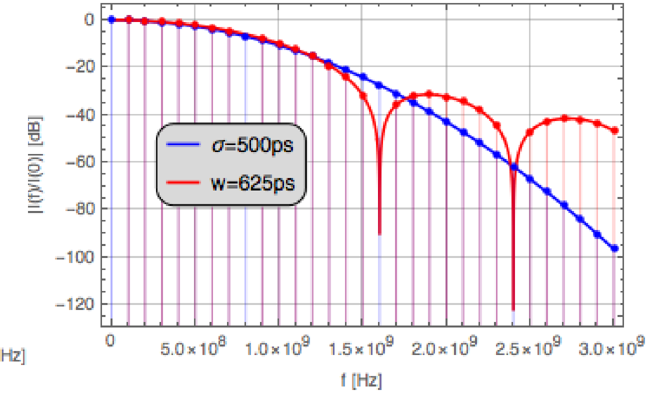


Frequency domain



Frequency domain

log$_{10}$[magn.(f))/log$_{10}$[magn.(0)]



More on Image Charges and Image Currents

- **Relativistic beams v ≈ c:**
  - **Electrostatic problem of a line charge in a conductive circular cylinder**
- **Solution based on the image charge method:**
  - **Image current density**

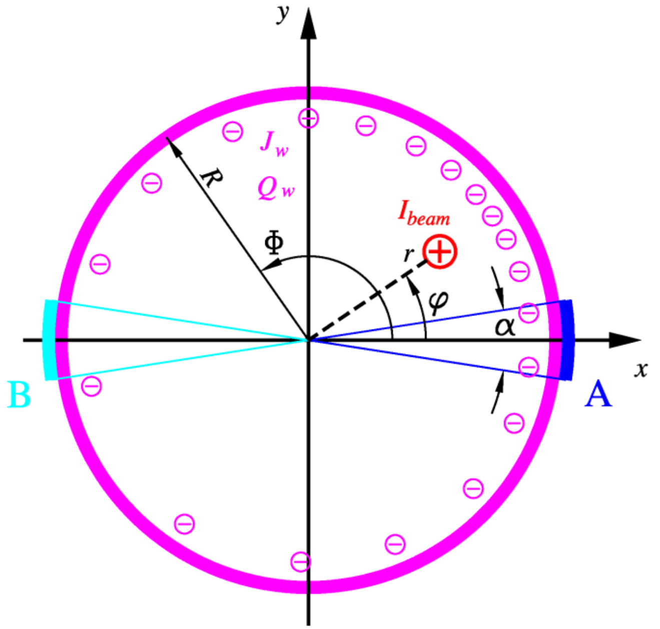
$$J_w(R,\Phi_w) = -\frac{I_{beam}}{2\pi R}\left[1 + 2\sum_{n=1}^{\infty}\left(\frac{r}{R}\right)^n \cos n(\Phi_w - \varphi)\right] = -\frac{I_{beam}}{2\pi R}\frac{R^2 - r^2}{R^2 + r^2 - 2r\cos(\Phi_w - \varphi)}$$

  - **Image current integrated on BPM electrode A**

$$I_A = R\int_{-\alpha/2}^{+\alpha/2} J_w(R,\Phi_w)\,d\Phi_w = -\frac{I_{beam}}{2\pi} s_A\left({}^{r}/_{R}, \varphi, \alpha\right)$$

$$s_A\left({}^{r}/_{R}, \varphi, \alpha\right) = \alpha + 4\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{r}{R}\right)^n \cos(n\varphi)\sin\left(\frac{n\alpha}{2}\right)$$

  - **Similar solution for electrode B**
    or for vertically arranged electrodes



Normalized Beam Position Characteristic

- **Pair of symmetric horizontal electrodes:**

  $A = I_A$ (right electrode) $B = I_B$ (left electrode)

  - **Horizontal and vertical beam position:**

    $x = r\cos\varphi \qquad y = r\sin\varphi$

- **Normalized beam position characteristic *Δ/Σ* (horizontal):**
  - Approximation and **closed form solution**

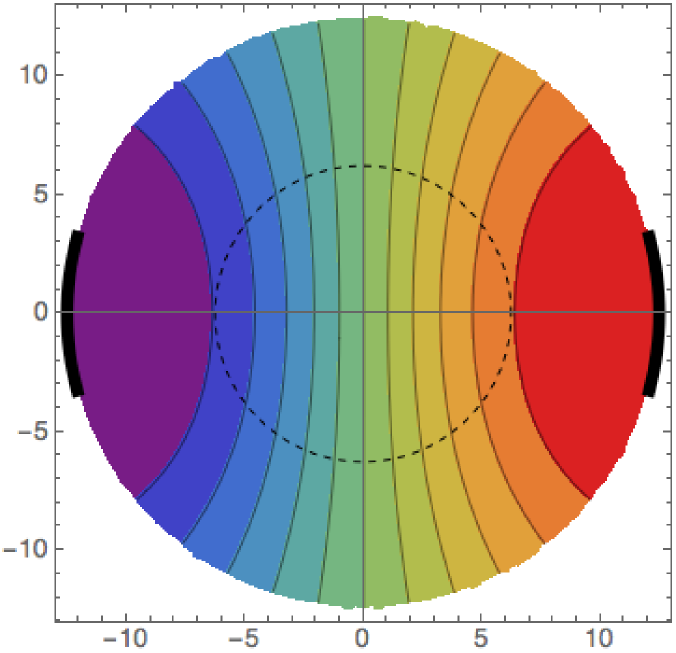
$$\boxed{\begin{aligned}\textbf{hor. position} = \frac{\boldsymbol{\Delta}}{\boldsymbol{\Sigma}} = \frac{\boldsymbol{A-B}}{\boldsymbol{A+B}} &= \frac{4\sin\frac{\alpha}{2}}{\alpha}\frac{x}{R} + \text{higher}-\text{order terms} \\ &= \frac{\boldsymbol{f(x,y,R,\alpha) - f(-x,y,R,\alpha)}}{\boldsymbol{f(x,y,R,\alpha) + f(-x,y,R,\alpha)}}\end{aligned}}$$

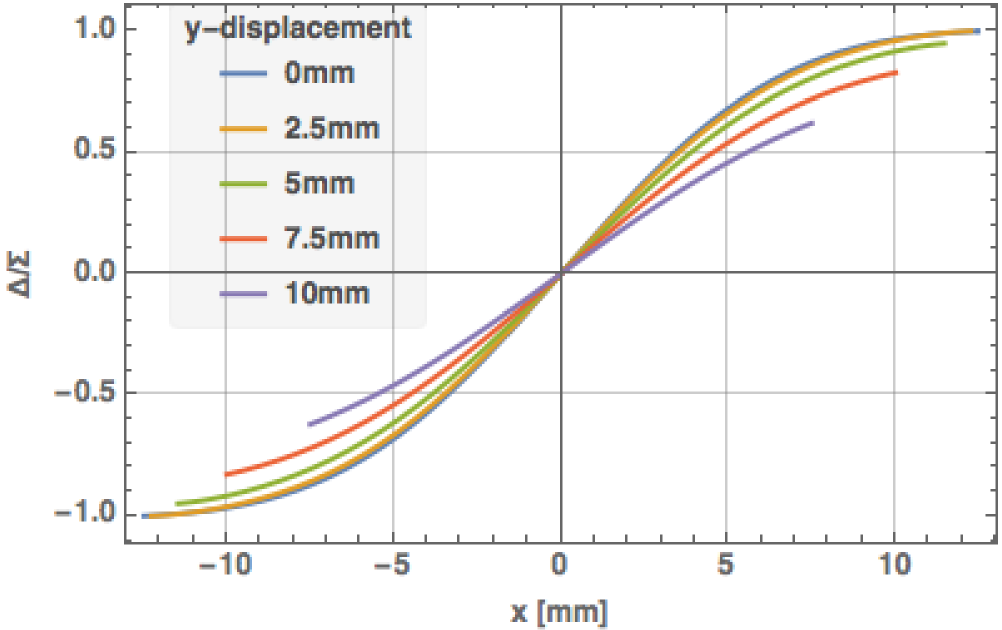
$$\boldsymbol{f(x,y,R,\alpha)} = \pi\int_{-\alpha/2}^{+\alpha/2} J_w(R,\Phi_w)\,d\Phi_w = \textbf{tan}^{\boldsymbol{-1}}\frac{\boldsymbol{\left[(R+x)^2+y^2\right]}\,\textbf{tan}\boldsymbol{\left(\frac{\alpha}{4}\right) - 2Ry}}{\boldsymbol{x^2+y^2-R^2}} +$$

$$\textbf{tan}^{\boldsymbol{-1}}\frac{\boldsymbol{\left[(R+x)^2+y^2\right]}\,\textbf{tan}\boldsymbol{\left(\frac{\alpha}{4}\right) + 2Ry}}{\boldsymbol{x^2+y^2-R^2}}$$

Example

- **Position characteristic for R = 12.5 mm, α = 30^0:**





- **Approximations**
  - **Point-like electrodes:**

$$\text{hor. position} = \frac{A-B}{A+B} = \frac{1}{1+\frac{x^2+y^2}{R^2}}\frac{2}{R}x, \qquad \alpha \to 0$$

  - **Small beam positions:**

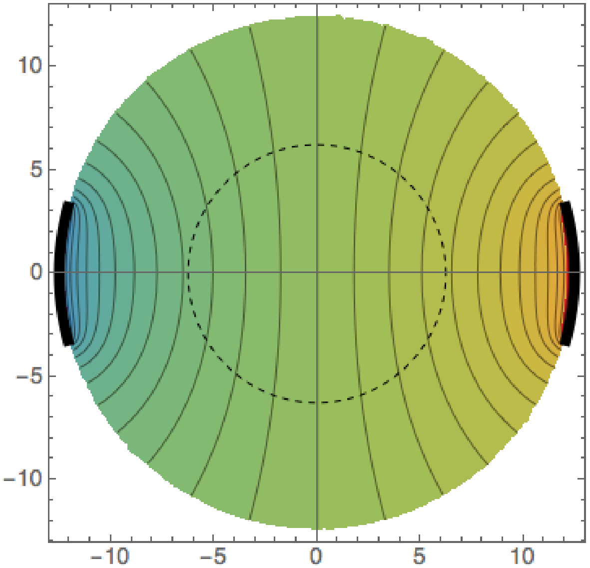
$$\text{hor. position} = \frac{A-B}{A+B} \approx \frac{2}{R}x, \qquad (x^2+y^2) \ll R^2$$

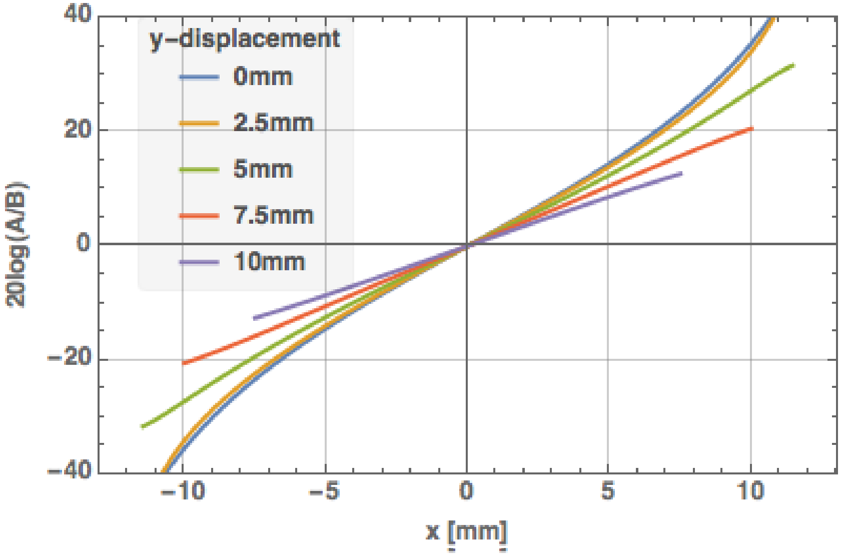
Logarithmic Ratio Normalization

- **Results in a more linear position characteristic**

$$\textbf{hor. position} = \mathbf{20log_{10}}\left(\frac{\boldsymbol{A}}{\boldsymbol{B}}\right) = 20\log_{10}\left(\frac{\sin\frac{\alpha}{2}}{\alpha}\frac{x}{R} + \text{higher}-\text{order terms}\right)$$
$$= \mathbf{20log_{10}}\left[\frac{\boldsymbol{f(x,y,R,\alpha)}}{\boldsymbol{f(-x,y,R,\alpha)}}\right]$$

  - **The example for *R* = 12.5 mm, *α* = 30^0 gives a sensitivity of 2.75 dB/mm near the origin**

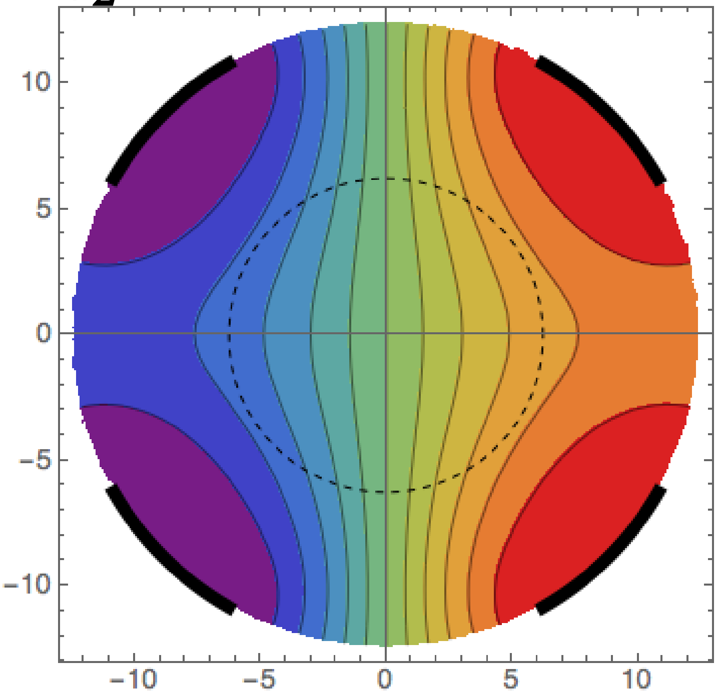


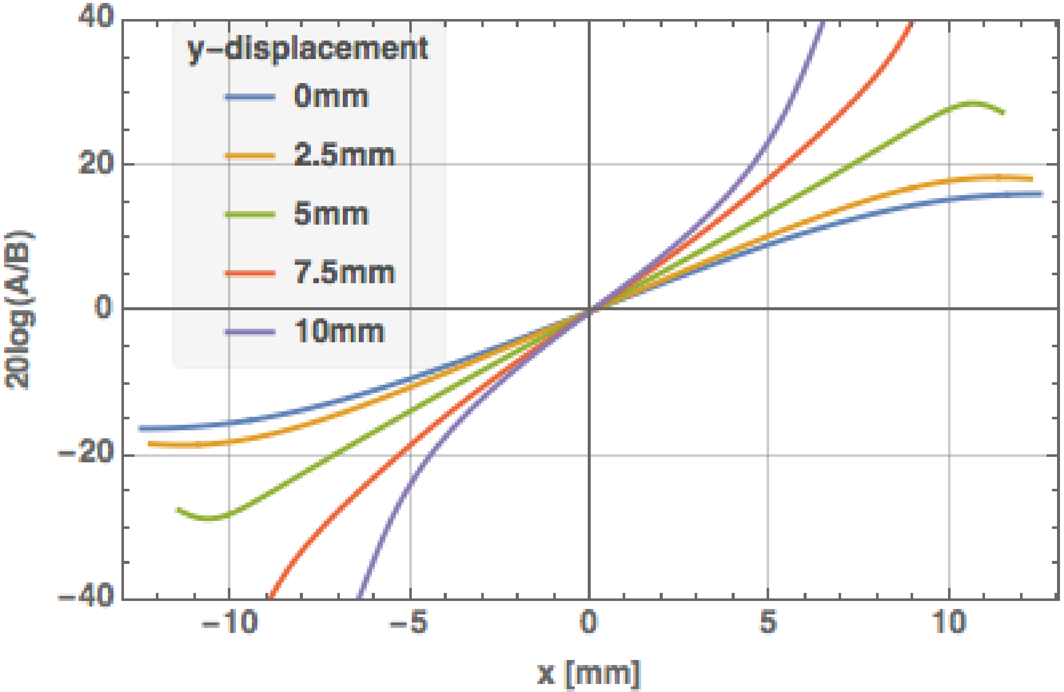


BPM with 45^0 Rotated Electrodes

- **Popular in synchrotron light and damping rings, and in the LHC!**
  - **Prevents that sync light, or collision debris, hits the electrodes**
    - **More non-linear position behavior**
    - **Reduced position sensitivity (this example: 1.94 dB/mm)**
  - **Same mathematical procedure, but with coordinate rotation of 45^0**

$\Delta / \Sigma = const.$





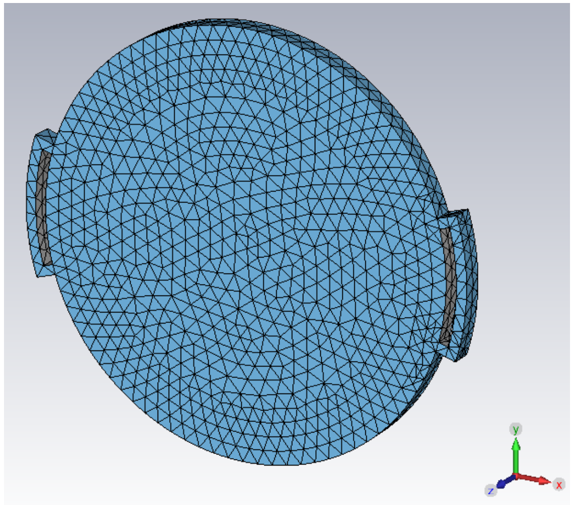
Numerical Analysis (1)

- **For most button or stripline type BPMs: 2D E-static analysis**
  - **Relativistic beam** $^1/_{\gamma^2} \ll \left(^{\sigma_\ell}/_R\right)^2$ **long bunches** $\sigma_\ell \gg R$
  - **Solve the 2D E-static potential problem**
    - **BUT: without varying** ρ **for many positions** $\vec{r} \in (x, y)$
  - **Apply Green's reciprocity theorem and solve the Laplace equation in 2D:** $\boldsymbol{\nabla_\perp^2 \Phi_{elec}(r) = 0 \rightarrow \Phi_{elec}(x, y)}$

line charge density

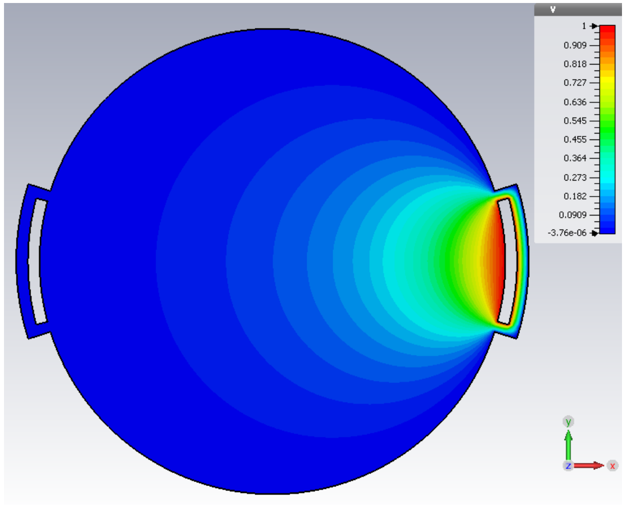
$$\nabla_\perp^2 \Phi_{elec}(r) = \frac{\rho}{\phi}\delta(\vec{r} - \vec{r}_0)$$

potential



2D "slice", prepared with tetrahedral mesh for the numerical analysis

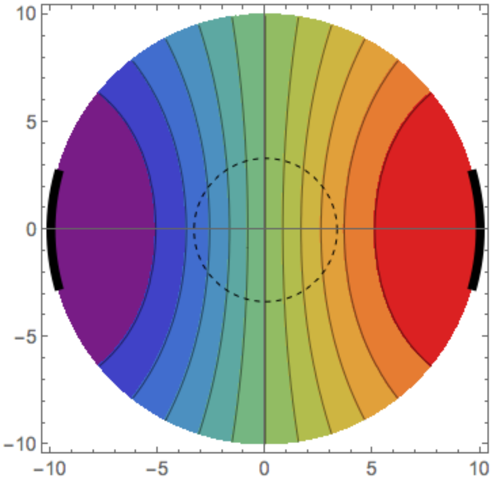
Result of the quasi-2D numerical analysis: equipotentials of the right electrodes



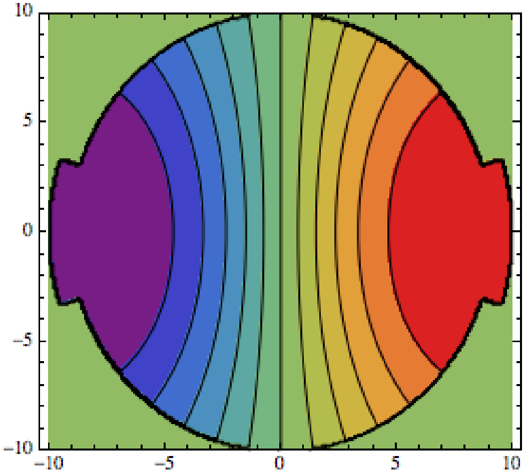
Numerical Analysis (2)

- **For a horizontal BPM**
  - **Mirror the potential field** $\Phi_{elec}(x,y) = \Phi_A(x,y) = \Phi_B(-x,y)$
  - **And then combine the fields** $\boldsymbol{\Phi_{hor}(x,y) = \frac{\Phi_A - \Phi_B}{\Phi_A + \Phi_B} = \frac{\Delta}{\Sigma}}$
    - ➢ **Can be performed for any symmetric arrangement of the electrodes**
- **Comparison:**

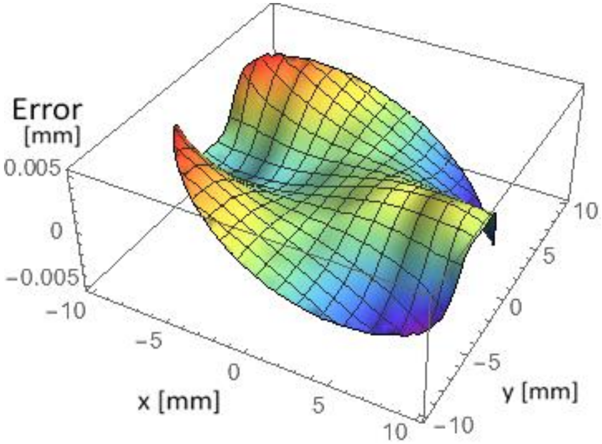
analytical analysis



numerical analysis

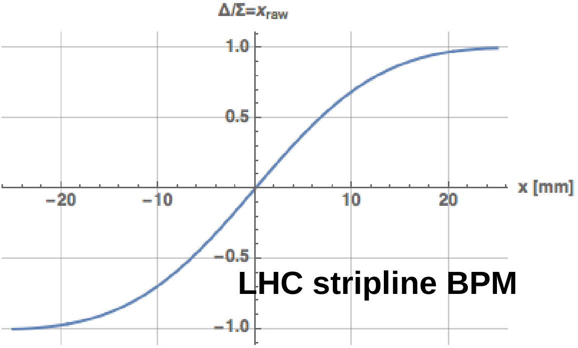


analytical - numerical



1D Non-linear Correction

- The 2D electrostatic analysis enables
  - optimization of the characteristic impendence of a stripline
  - Coverage factor, centered beam sensitivity $s(x = 0, y = 0)$
  - Correction of the non-linear position behavior in 1D or 2D
    - ➢ By look-up tables or a polynomial fit function



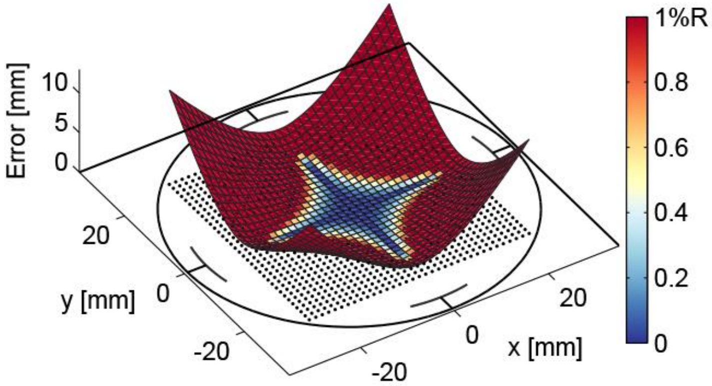
LHC stripline BPM

- Find a 1D polynomial fit function for horizontal beam displacements $x \neq 0, y = 0$
  - Relationship between raw (measured) and true beam position: $x_{raw} = f(x)$
  - Fit the inverted function $x = f^{-1} x_{raw}$ by a polynomial of power p
    - ➢ Larger p, better fit
      - A too high p may lead to an unstable fit
    - ➢ In most cases a x-y symmetry is given, and the same correction polynomial can be applied to the vertical axes
    - ➢ In practice, the quadradic correction area has to be limited, e.g. $\mathbb{R} = 40\%$

$$\boldsymbol{x_{bpm}^{1D} = \sum_{i=0}^{p} c_i x_{raw}^i = U_p(x_{raw}) \approx x}$$

2D Non-linear Correction

- **1D non-linear correction example:**
  - Remaining errors for the LHC stripline BPM applying correction polynomials $U_5(x), U_5(y)$ at an area $\mathbb{R} = 68\%$



- **Find a 2D polynomial fit function for $x, y \neq 0$**

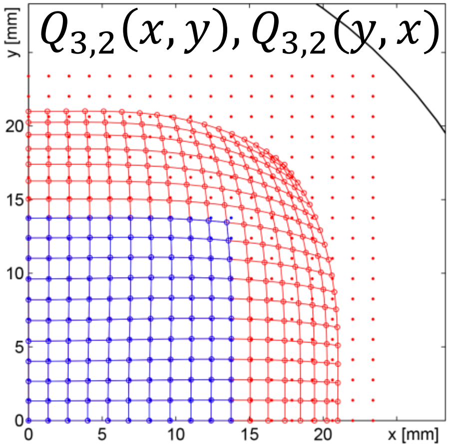
$$\begin{cases} x = f(x_{raw}, y_{raw}) \\ y = g(x_{raw}, y_{raw}) \end{cases}$$

  - Raw and true beam position are given by: with: $f = g$ and $y = f(y_{raw}, x_{raw})$
    - ➢ Notice the swap!
  - Fit a 2D surface polynomial for $f(x, y)$ of power p and q for x and y

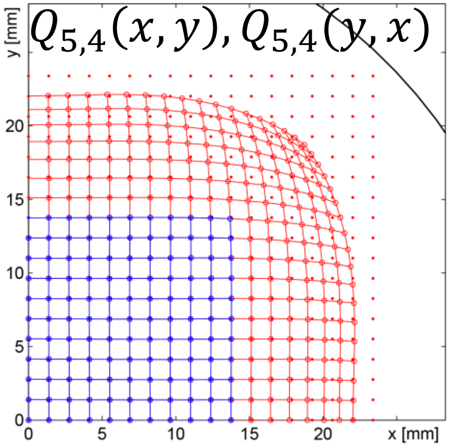
$$\begin{cases} x_{bpm}^{2D} = \sum_{i,j=0}^{p,q} \left(c_{ij} x_{raw}^{i} y_{raw}^{j} \right) = Q_{p,q}(x_{raw}, y_{raw}) \approx x \\ y_{bpm}^{2D} = \sum_{i,j=0}^{p,q} \left(c_{ij} y_{raw}^{i} x_{raw}^{j} \right) = Q_{p,q}(y_{raw}, x_{raw}) \approx y \end{cases}$$

Examples for 2D corrections

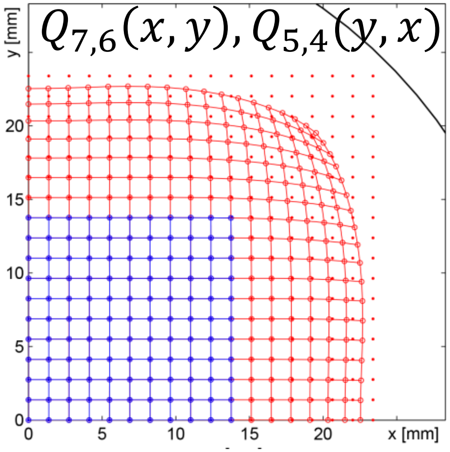
- **LHC stripline BPM**
  - **Pin-cushion maps ($\mathbb{R} = 40\%$)**
    - For symmetry reasons some cross-terms are 0, or very small (negligible)



$Q_{3,2}(x,y), Q_{3,2}(y,x)$

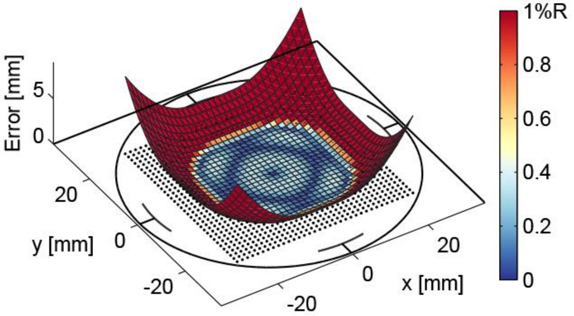


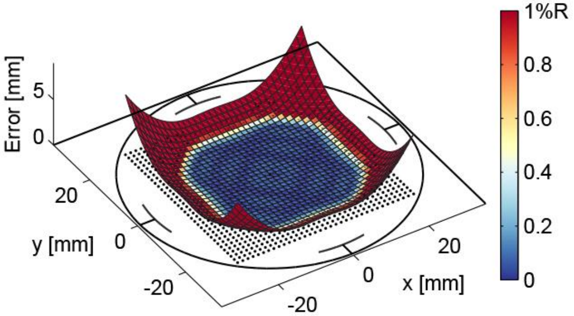
$Q_{5,4}(x,y), Q_{5,4}(y,x)$

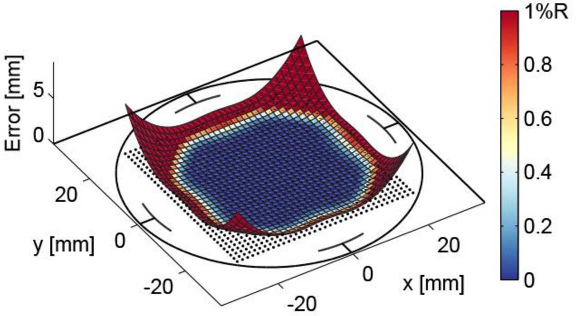


$Q_{7,6}(x,y), Q_{5,4}(y,x)$

  - **Remaining errors:**





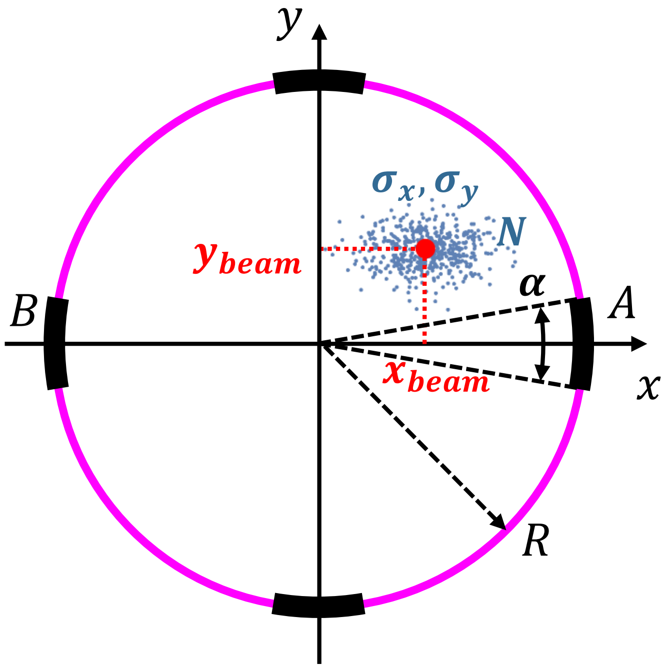


Higher Order Moments

- Particle<->BPM EM-fields are linear and time-invariant
  - Superposition principles apply
  - Many particles -> BPM detects (approximately) the center-of-charge
    - Point charge $q = eN$
- But: Non-linear effects allow the detection of higher moments:
  - Electrode A signal to N particles:

$$I_A = -\frac{I_{beam}}{2\pi}\left[\alpha + 4\sum_{i=1}^{N}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{r_i}{R}\right)^n \cos(n\varphi_i)\sin\left(\frac{n\alpha}{2}\right)\right]$$

  - After some math gym follows:



$$I_A = -\frac{I_{beam}}{\pi}\left[\overset{\text{monopole moment} \propto \text{intensity (common mode)}}{\frac{\alpha}{2}} + \underbrace{\frac{2}{R}\sin\left(\frac{\alpha}{2}\right)x_{beam}}_{\text{dipole moment} \propto \text{position}/R} + \underbrace{\frac{1}{R^2}\sin(\alpha)\left(\sigma_x^2-\sigma_y^2+x_{beam}^2-y_{beam}^2\right)}_{\text{quadrupolar moment} \propto (\Delta\text{size}+\Delta\text{pos})/R^2} + \cdots\right]$$

Effects of the Beam Size

Ideal 2-dim model:

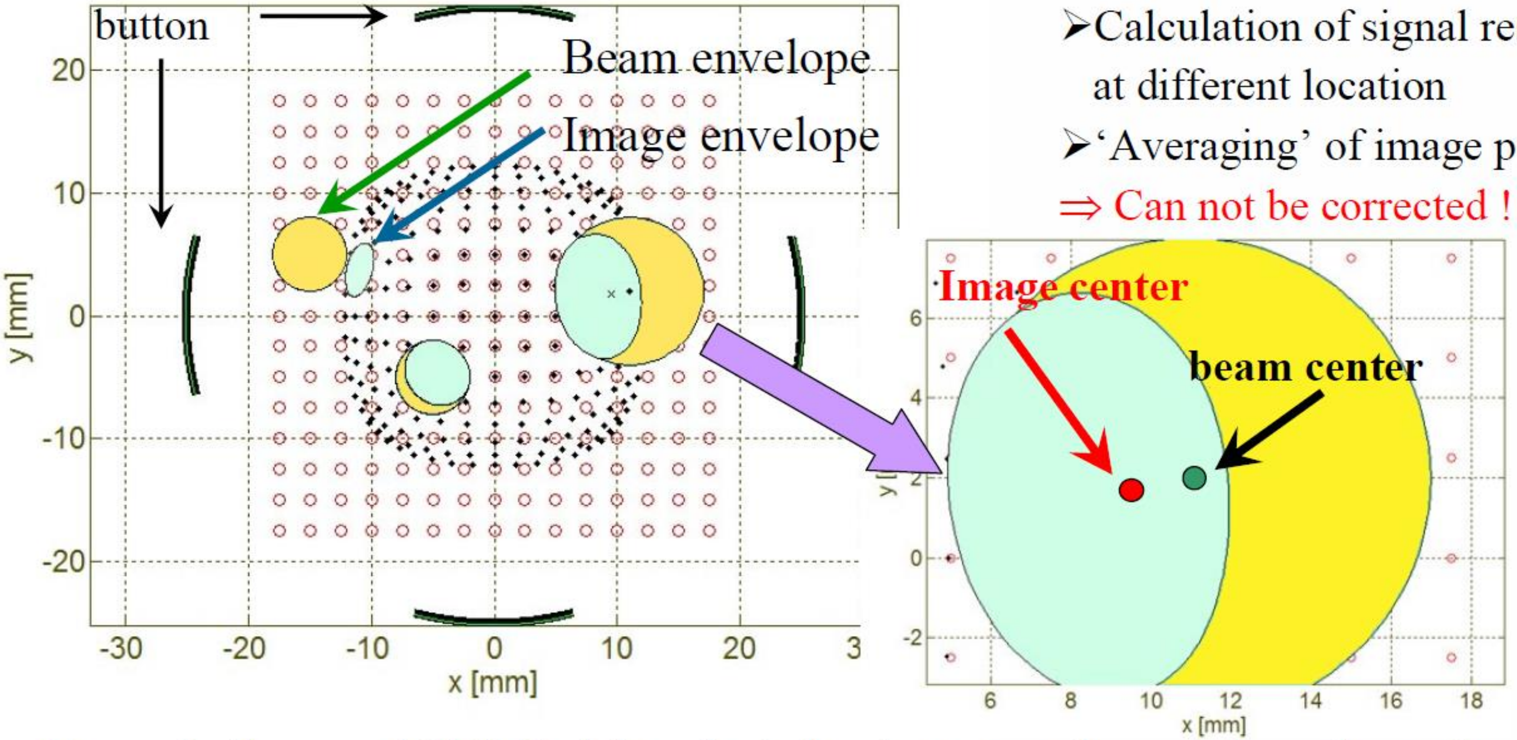
courtesy *M. Bozzolan*

Due to the non-linearity, the beam size enters in the position reading.

Finite beam size:

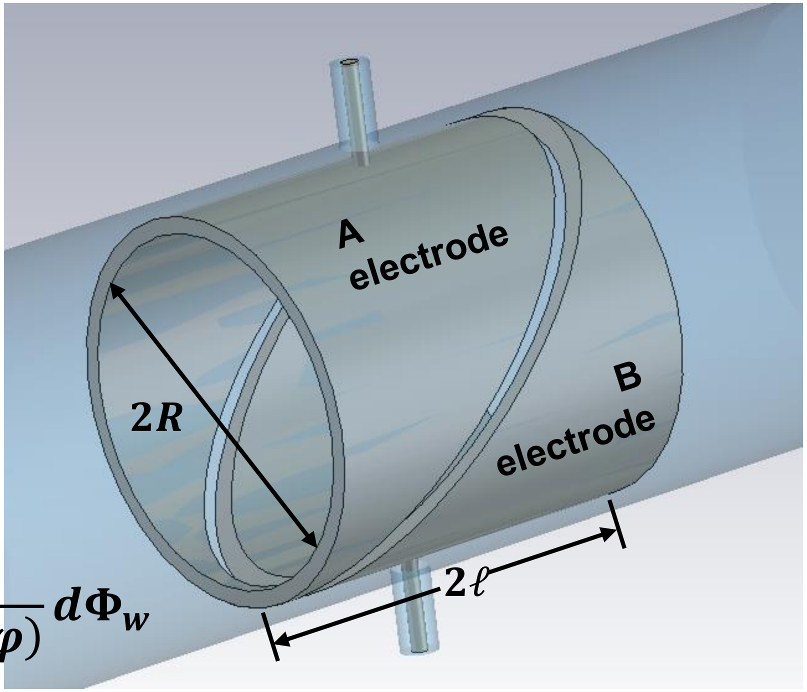
- Calculation of signal response at different location
- 'Averaging' of image position

⇒ Can not be corrected !



Remark: For most LINACs: Linearity is less important, because beam has to be centered → correction as feed-forward for next macro-pulse.

BPM with Linear Position Response



- **Split-plane tube electrode**
  - radius: R
  - length: $\ell(\Phi_w) = \ell(1 + \cos\Phi_w)$
- **A beam of**
  - charge: q_b, position: (r, φ)

induces an image charge on the electrode:

$$Q_{elec} = -q_b \ell \int_0^{2\pi} \frac{(1+\cos\Phi_w)(R^2+r^2)}{R^2+r^2-2Rr\cos(\Phi_w-\varphi)} d\Phi_w$$

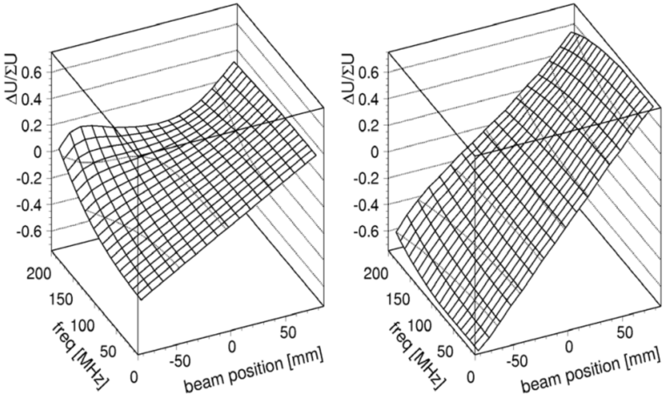
  - with a linear position response

$$Q_{elec} = -q_b \ell \left(1 + \frac{r\cos\varphi}{R}\right) = -q_b \ell \left(1 + \frac{x}{R}\right)$$

$$\Longrightarrow \boxed{\text{hor. position} = \frac{\Delta}{\Sigma} = \frac{A-B}{A+B} = \frac{x}{R}}$$

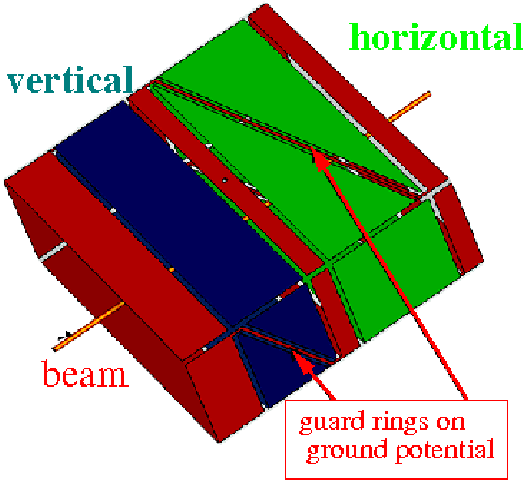
“Shoe-box” BPM

- **Split-plane (“shoe-box”) BPM with ground-guards**
  - **Improved sensitivity and reduced cross-talk**
  - **Improved linearity**

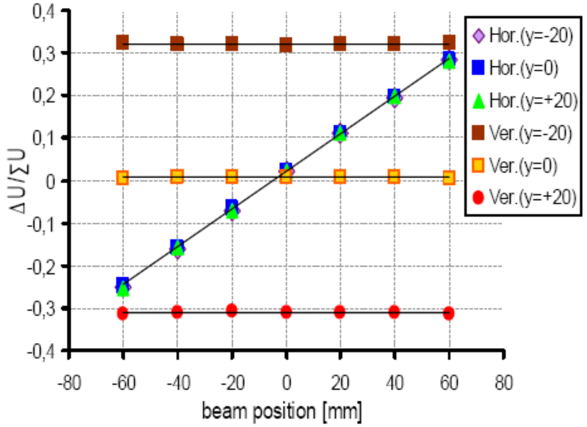


without with

ground guards



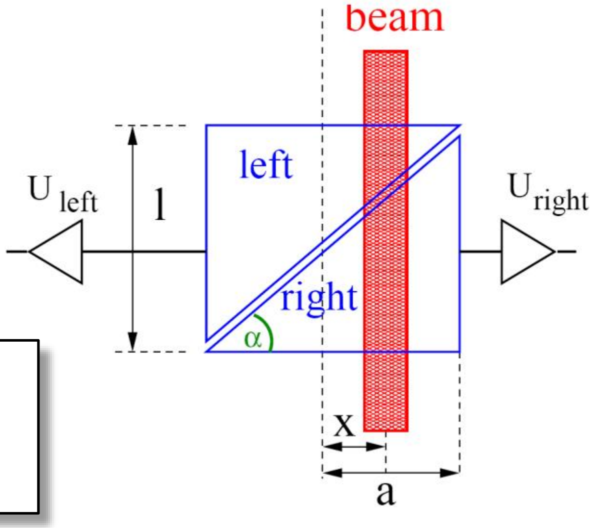
courtesy *P. Kowina*



$$V_{left} = (a + x)\tan\alpha$$

$$V_{right} = (a - x)\tan\alpha$$

$$\boxed{\boldsymbol{x = a\frac{V_{right} - V_{left}}{V_{right} + V_{left}} = \frac{\Delta}{\Sigma}}}$$

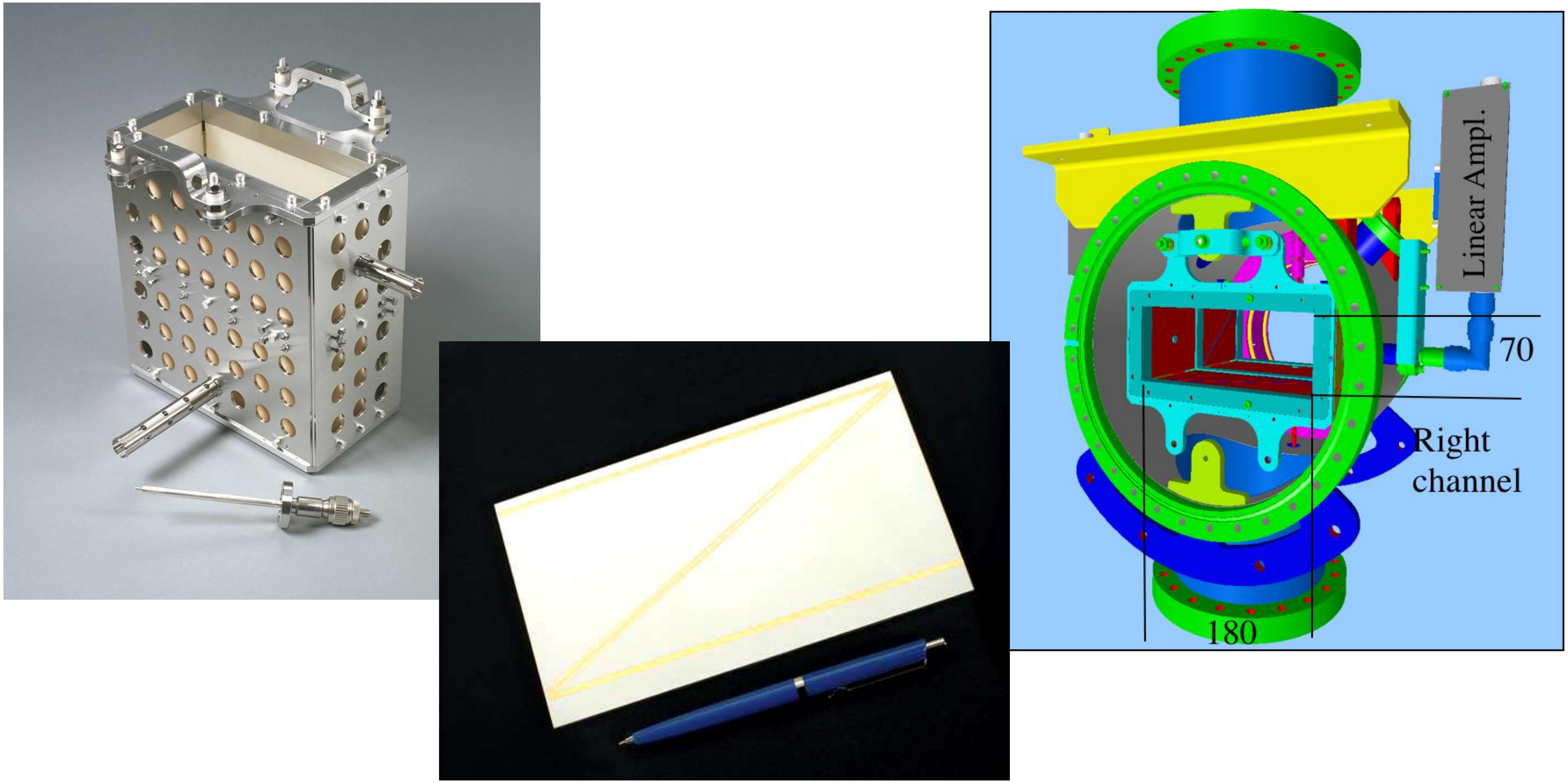


Spit-Plane BPM

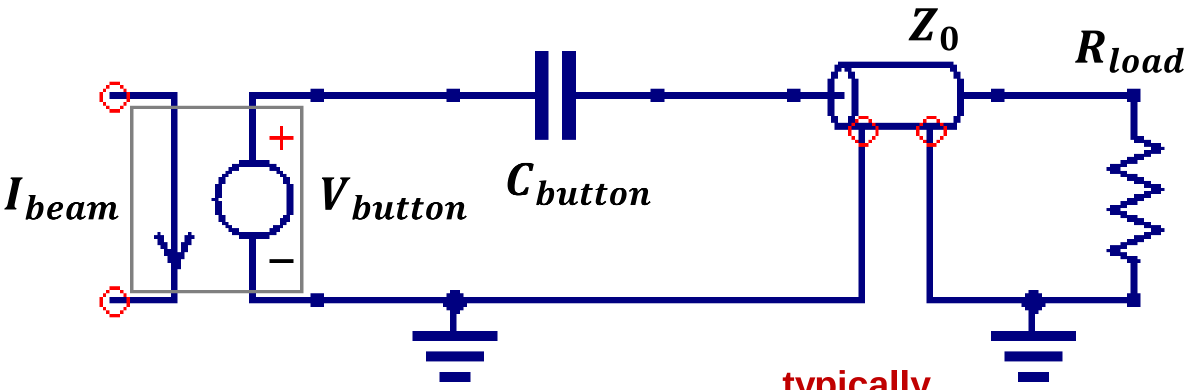
- + **Pros**
  - – Linear position response
    - ➢ No correction for non-linearities required, simple analog read-out electronics without post-processing
  - – high transfer impedance at low frequencies
    - ➢ Good match for long bunches, low-β beam
    - ➢ High signal levels, allows operation at low frequencies with high load impedance
- - **Cons**
  - – Complicated mechanics, requires sufficient real-estate
  - – High capacitive source impedance, reflective
  - – Eigenmodes at low frequencies, high beam coupling impedance
    - ➢ Limited to low frequency operation, typically <300 MHz

“Shoe-box” BPM Example

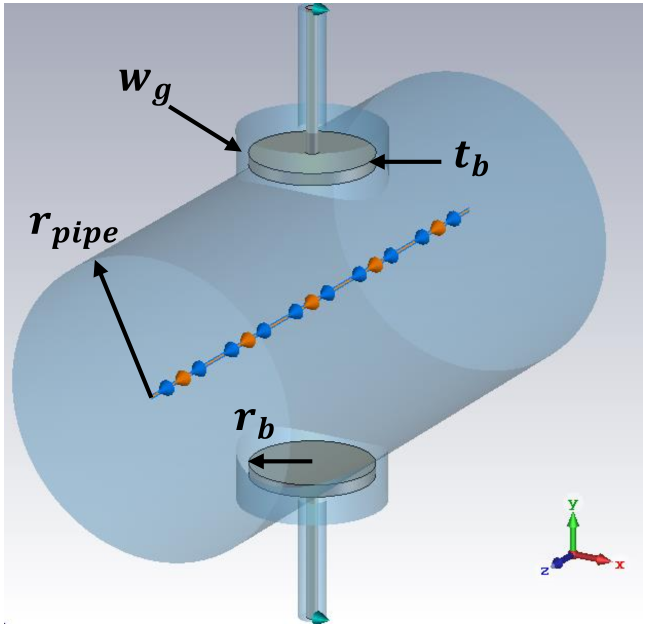
Technical realization at the HIT synchrotron of 46 m length for 7 MeV/u→ 440 MeV/u
BPM clearance: 180x70 mm^2, standard beam pipe diameter: 200 mm.



Button BPM Equivalent Circuit



typically
50 Ω



$$Z_{button}(\omega) = \frac{V_{button}(\omega)}{s(x,y)I_{beam}(\omega)} = \phi R_{load} \frac{\omega_1}{\omega_2} \frac{{j\omega}/{\omega_1}}{1 + {j\omega}/{\omega_1}}$$

$$\omega_1 = \frac{1}{R_{load}C_{button}} \qquad \omega_2 = \frac{c}{2r_{button}} \qquad \phi = \frac{r_{button}}{4r_{pipe}} = s(x=0, y=0)$$

coverage factor

time constant

1/electrode transit time

button capacitance, very simplified
typically 2…15 pF

$$C_{button} \approx \frac{2\pi\varepsilon_0}{\ln\left(\frac{r_{button}+w_{gap}}{r_{button}}\right)} t_{button} + \cdots$$

Increase	Effect
d_{button} (larger area & C_{button})	Higher signal level Lower resolution
t_{button} (larger volume)	Lower wake impedance Lower resolution, higher weight
w_{gap} (more trapped modes)	Higher resolution Higher wake impedance

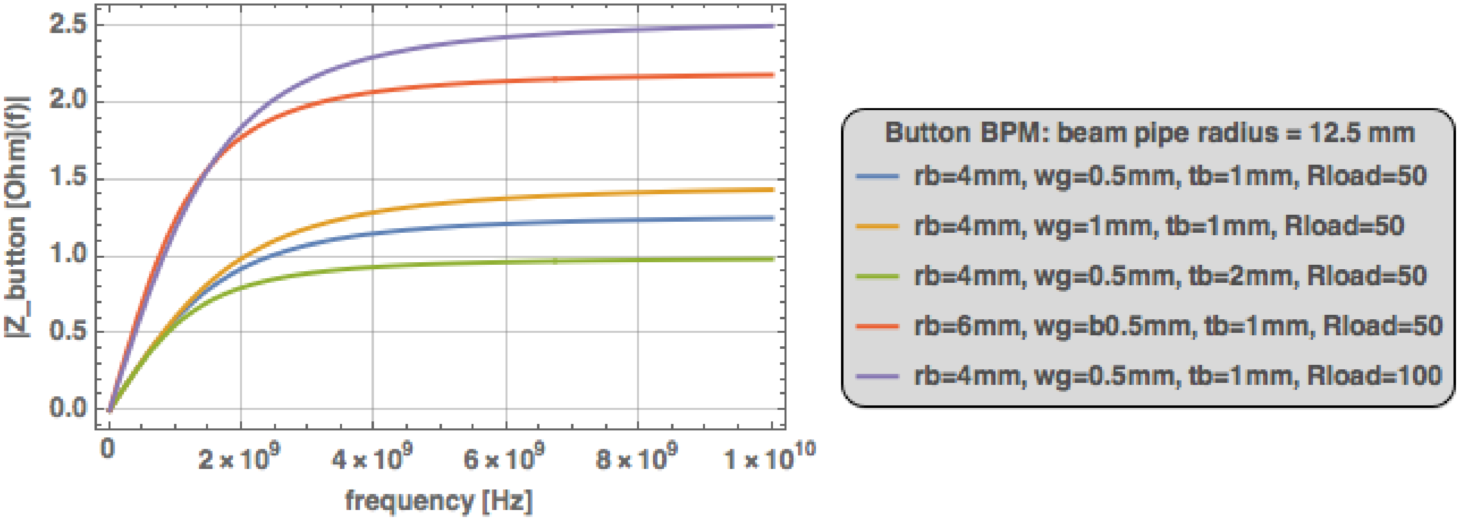
Button BPM

\+ **Pros**

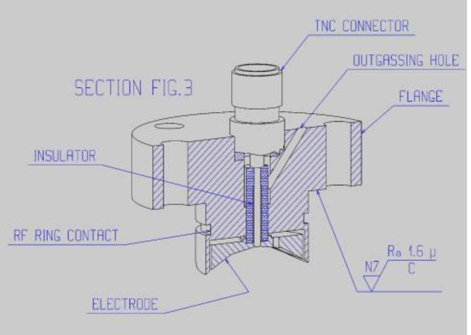
- Robust simple construction, cost effective, minimum real-estate
  - RF UHV feedthrough and button is a single element

\- **Cons**

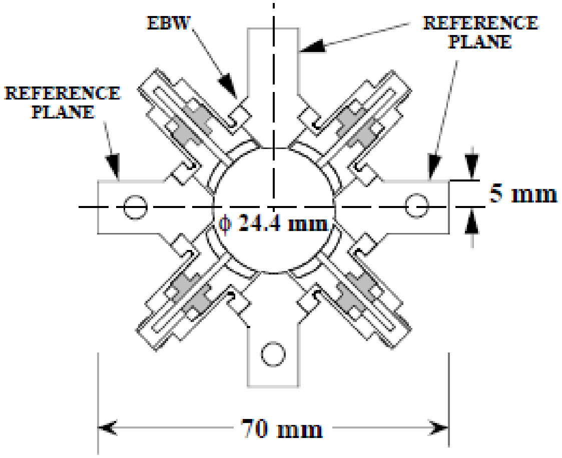
- High-pass characteristic with high cut-off frequency
  - Typically >= 500 MHz,
  - bad match to operate with long bunches, or at low frequencies
- Capacitive source impedance, reflective



Examples of Button BPMs



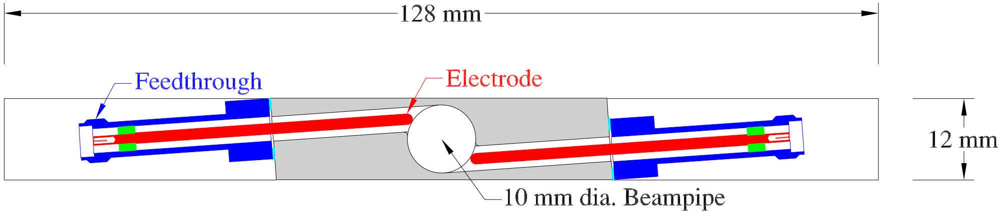
LHC Button BPM (*CERN*)



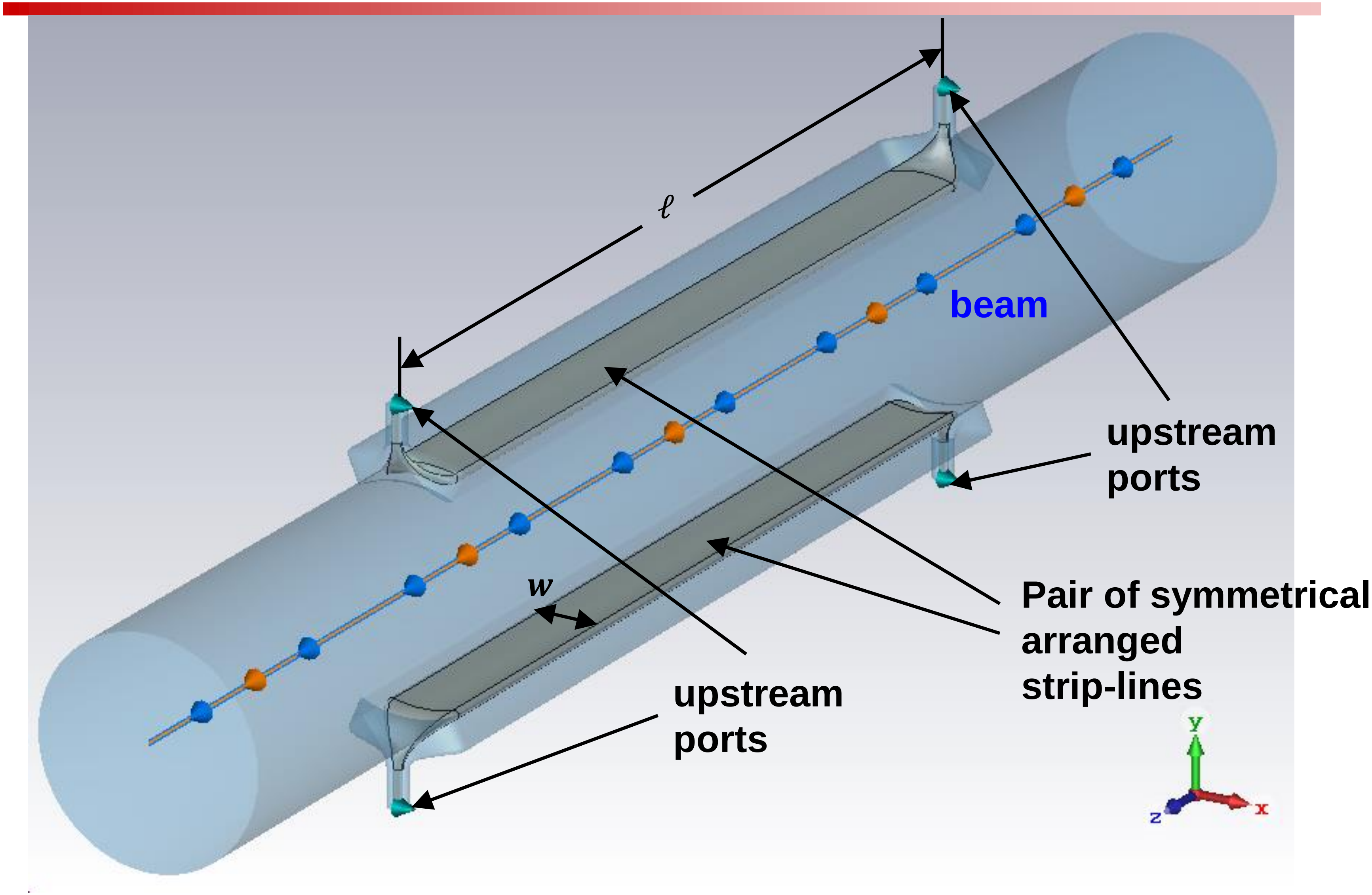
ATF Button BPM (*KEK*)

NSLS-II Button BPM (*BNL*)

TTFII Button BPM (*DESY*)
located inside an undulator magnet

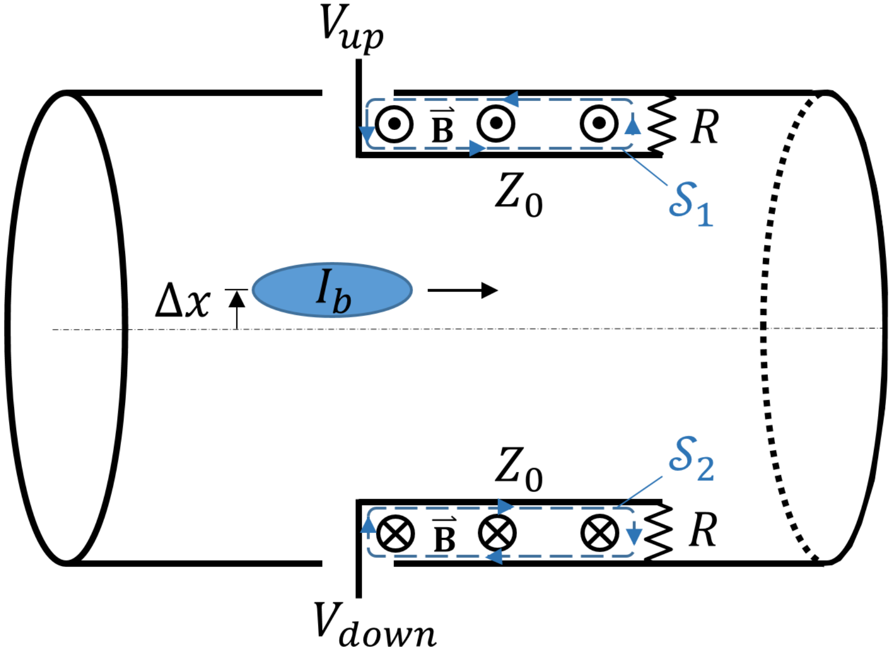


Stripline or Directional Coupler BPM

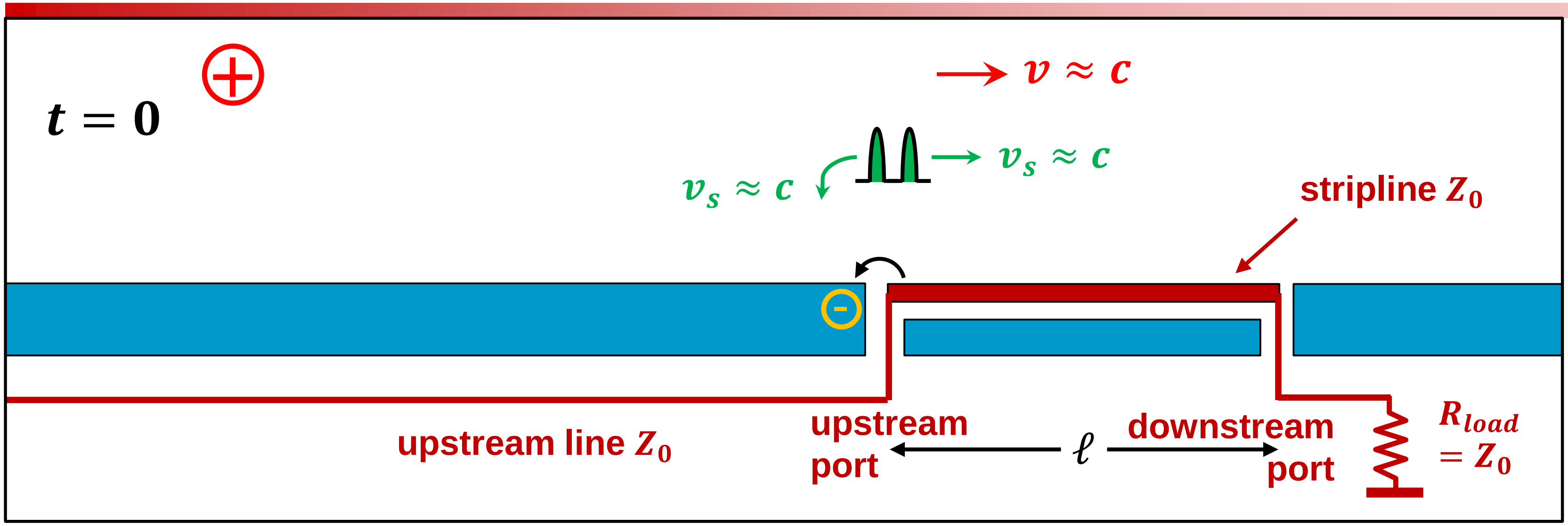


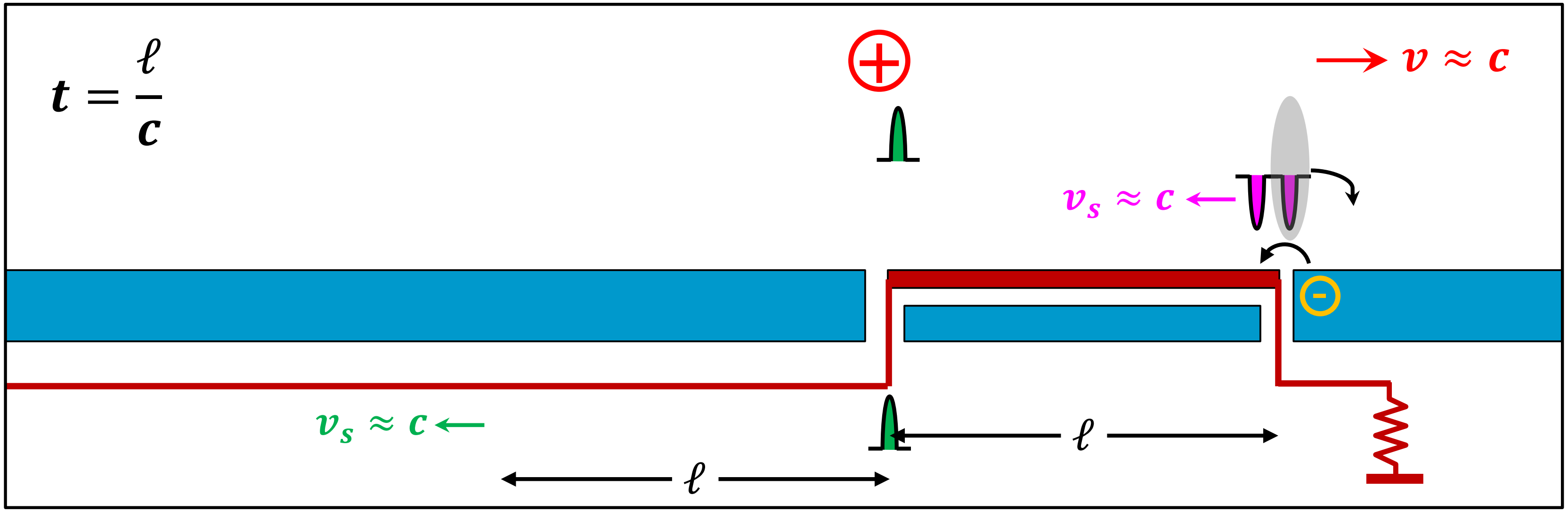
Stripline BPM

- **Strip transmission-line of characteristic impedance $Z_0 = 50\ \Omega$**
- **No dielectric materials ($\varepsilon_r = 1$, vacuum)**
  - **TEM fields of the signal, except near the transition at the ports**
- **Coupling to the TEM field of the beam**
  - **Electric and magnetic field coupling**
- **Only the upstream port delivers a beam signal!**
  - **Directional coupler**
- **Both ports are terminated**
  - **Sometimes the downstream port is shorted to ground**
    - **➢ No directivity**

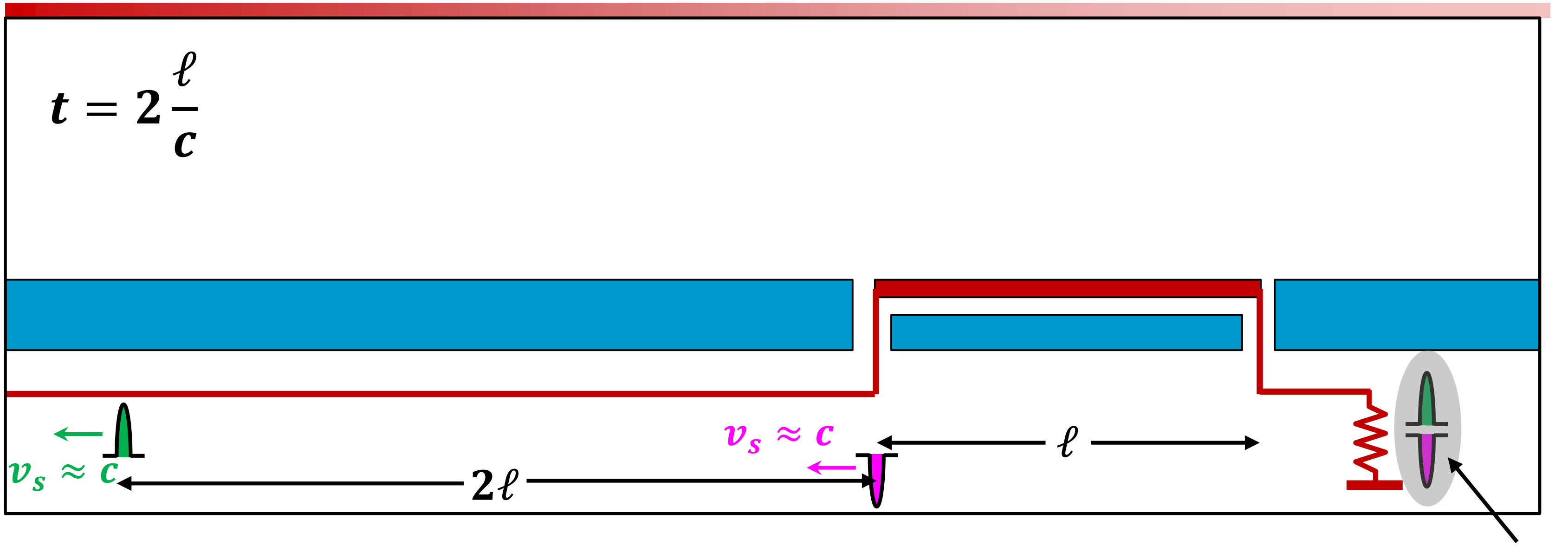


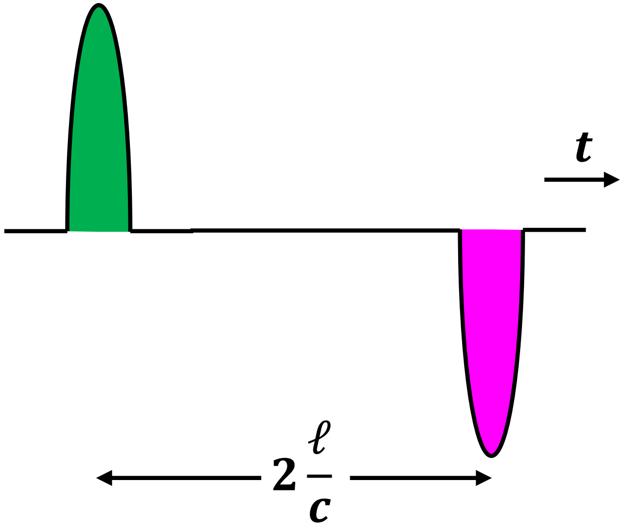
Stripline BPM Principle (1)





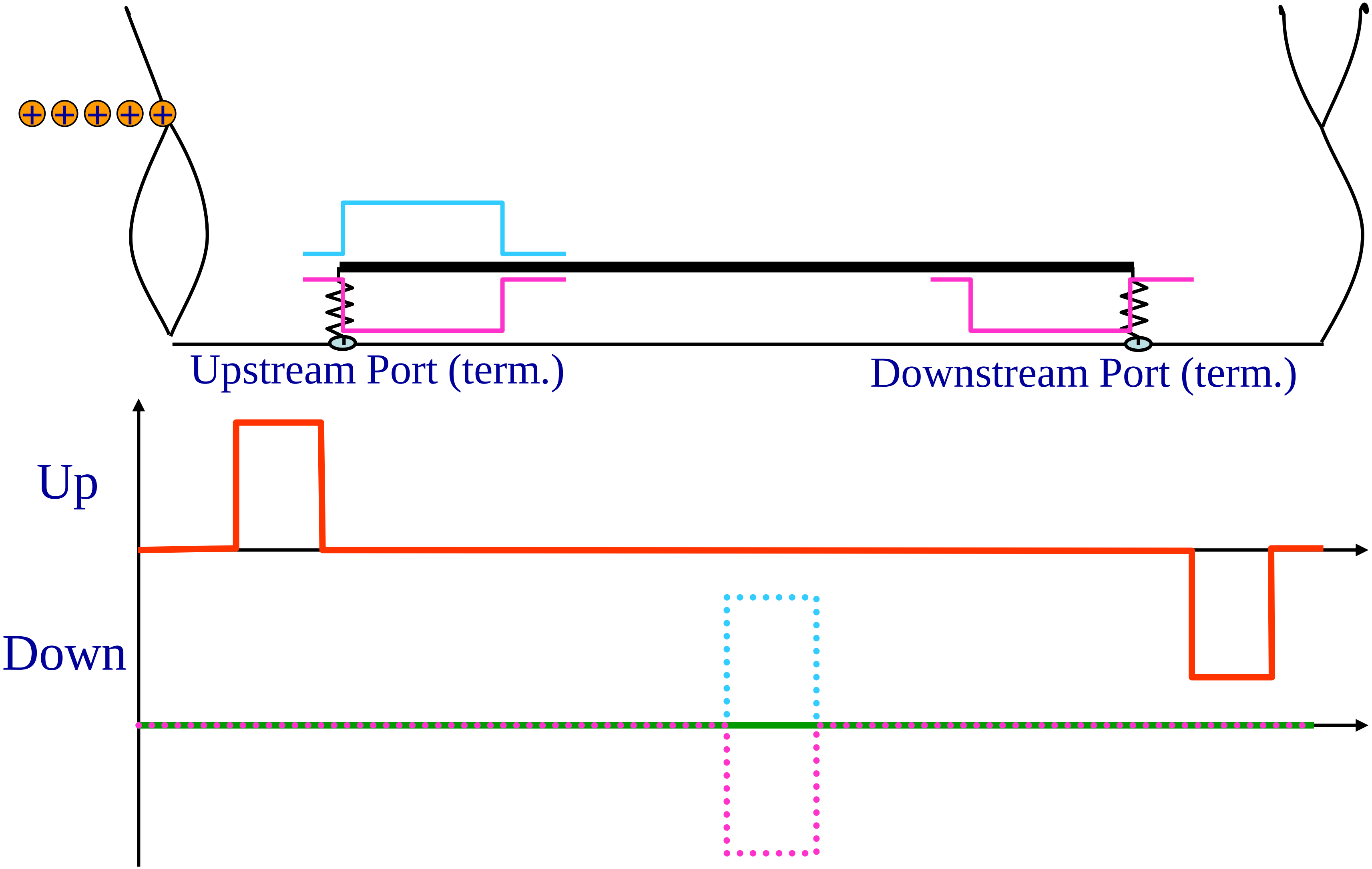
Stripline BPM Principle (2)



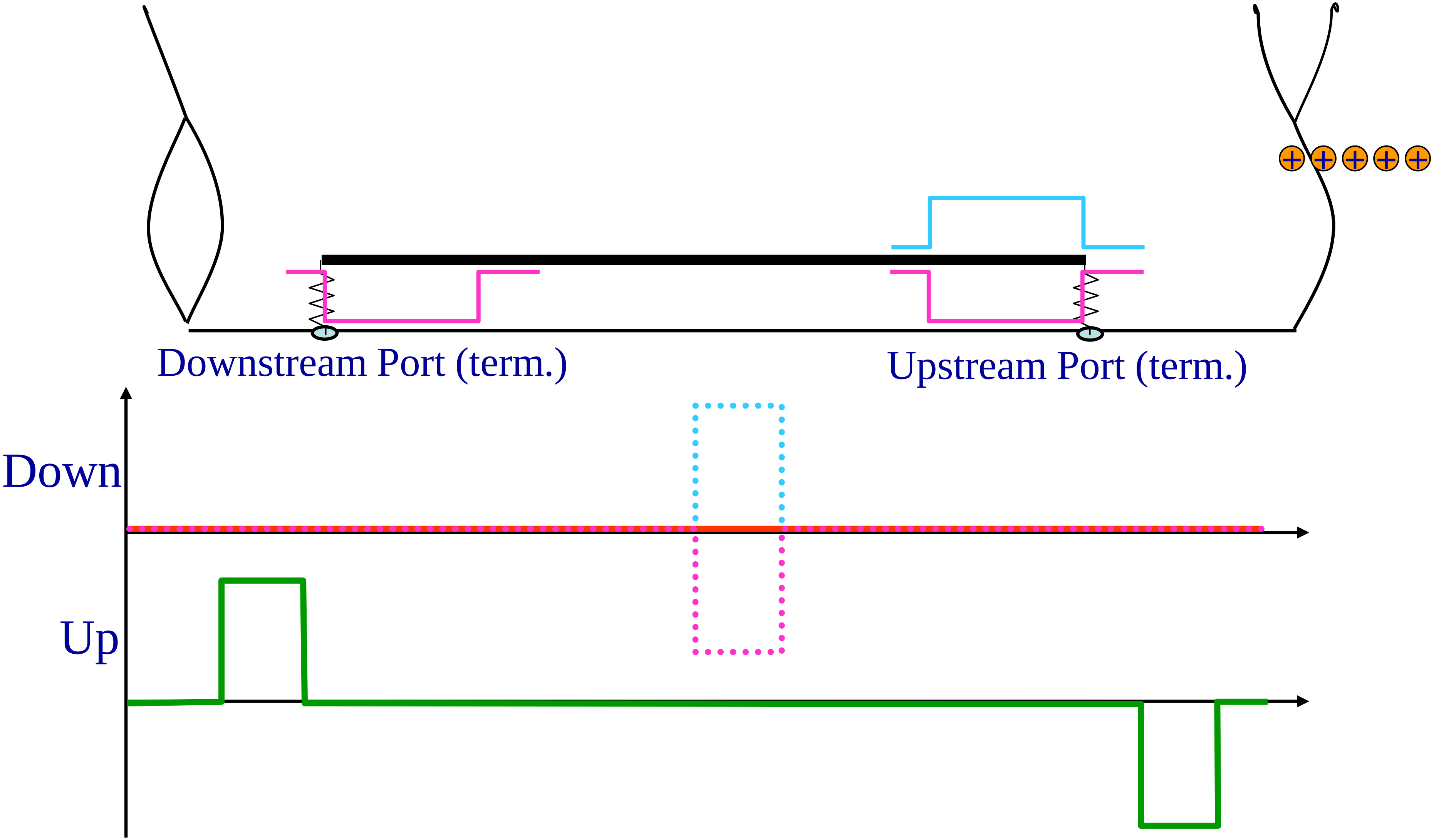


- **Stripline BPM response:**
  - **δ-doublet pulse, space by $t = 2\,{}^{\ell}/_{c}$**

Operational Principle: Beam from Left



Operational Principle: Beam from Right



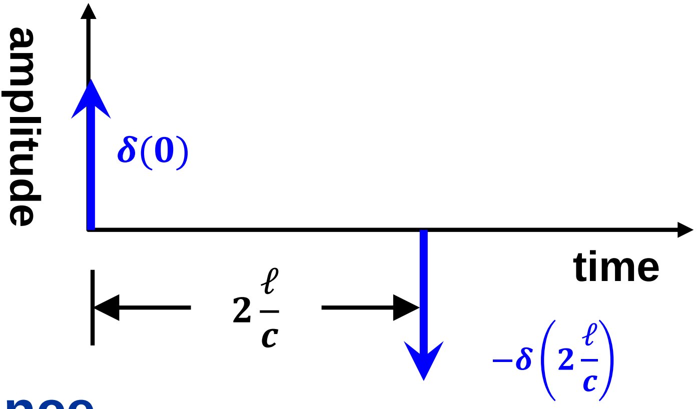
Transfer Impedance

- **Time domain impulse response: δ-doublet pulse**

characteristic impedance
typically 50 Ω

$$z(t) = \phi \frac{Z_0}{2}\left[\delta(t) - \delta\left(t - 2\frac{\ell}{c}\right)\right]$$

coverage
factor



- **Frequency domain transfer impedance**

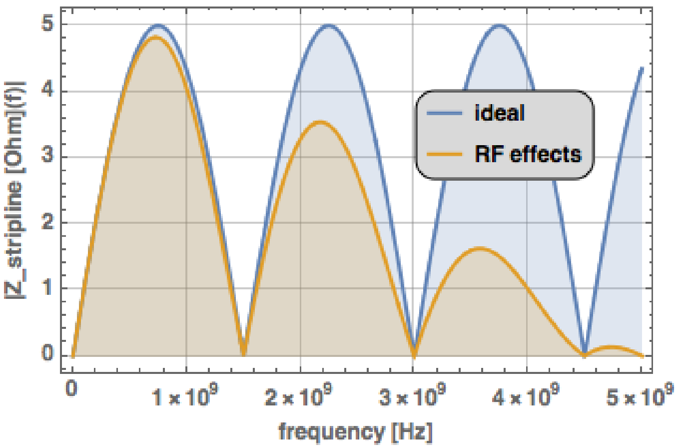
$$Z(\omega) = j\phi Z_0 e^{j\frac{\pi}{2}} e^{-j\omega\frac{\ell}{c}} \sin\left(\omega\frac{\ell}{c}\right)$$

  - **Lobes at:** $f_c = (2n-1)\frac{c}{4\ell}$
  - **3dB lobe bandwidth:**

$$f_{lo} = \frac{f_c}{2} \qquad f_{BW} = f_{hi} - f_{lo} = f_c$$

$$f_{hi} = 3\, f_{lo}$$

$Z_0 = 50\Omega, \ell = 100\ mm, \phi = 0.1$



Stripline BPM

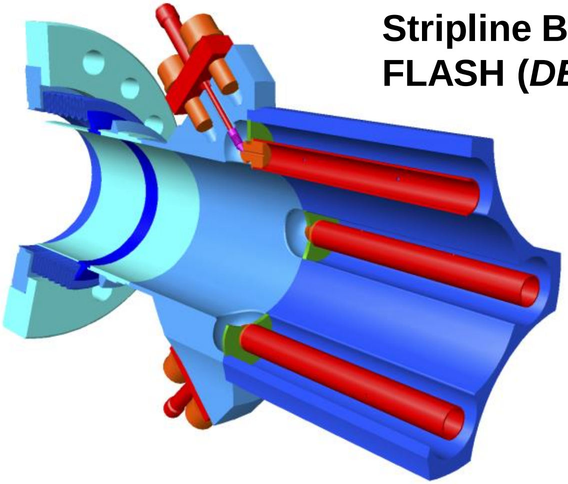
\+ **Pros**

- **Well defined, and high transfer impedance function**
  - **For striplines with *l* >> *w,* TEM operation**
  - **Can be “tuned” to match a dominant beam frequency**
    - Optimizing the length of the stripline
- **Matched characteristic impedance with 50 Ω source**
  - **Only for terminated ports**
    - Shorted downstream port: 0 Ω source, no directivity
  - **Reduced reflection effects between generator (stripline BPM) and read-out electronics!**
- **Directivity**
  - **Separate beam position measurement of counter-rotating beams**
    - e.g. near the IP in a ring collider

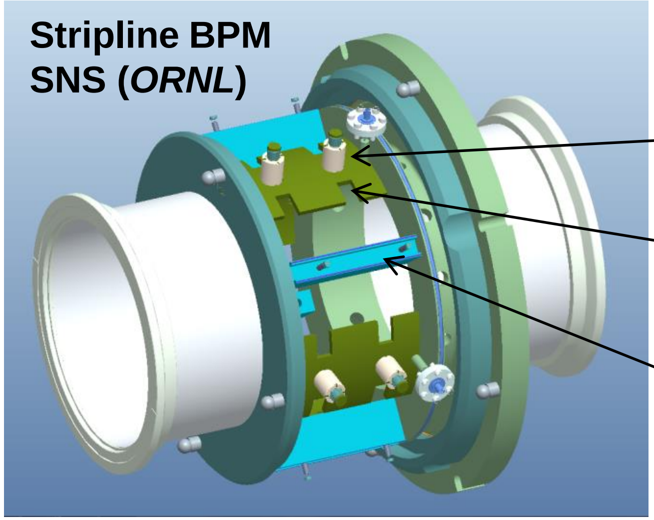
\- **Cons**

- **Complicated, fragile mechanics, high costs, requires sufficient real-estate**
  - **More RF feedthroughts required, impedance control of the stripline, strip-electrode may require a support structure**

Examples of Stripline BPMs



Stripline BPM FLASH (*DESY*)

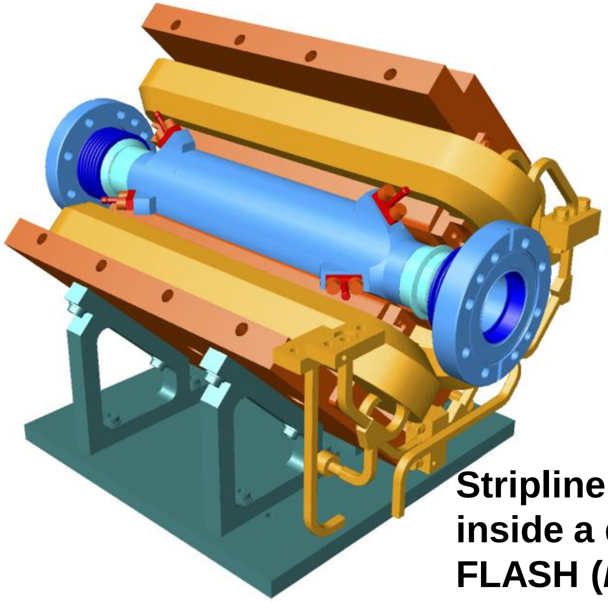


Stripline BPM SNS (*ORNL*)

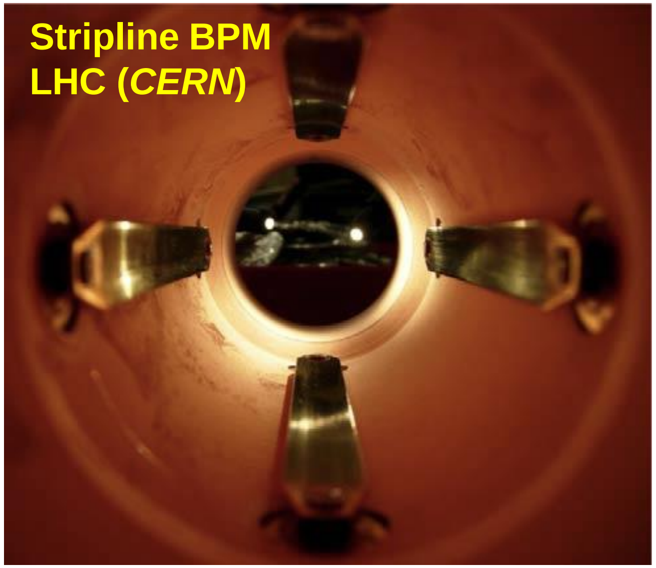
Ceramic posts hold the electrode

Impedance Match at the post

Inner-shielding bar reduces electrode to electrode coupling



Stripline BPM inside a quad FLASH (*DESY*)

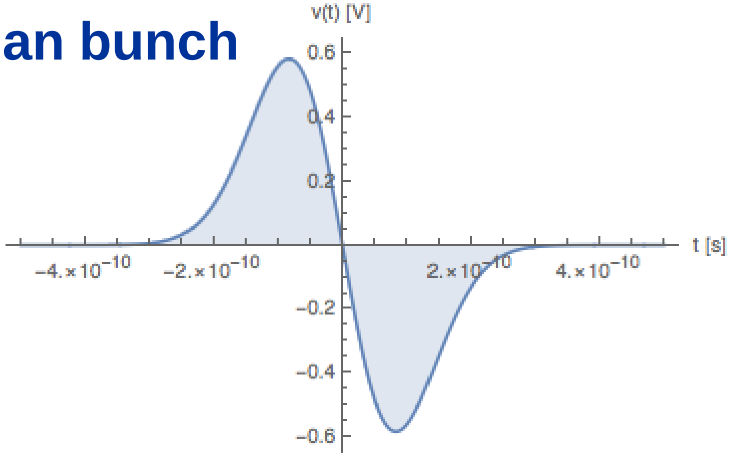


Stripline BPM LHC (*CERN*)

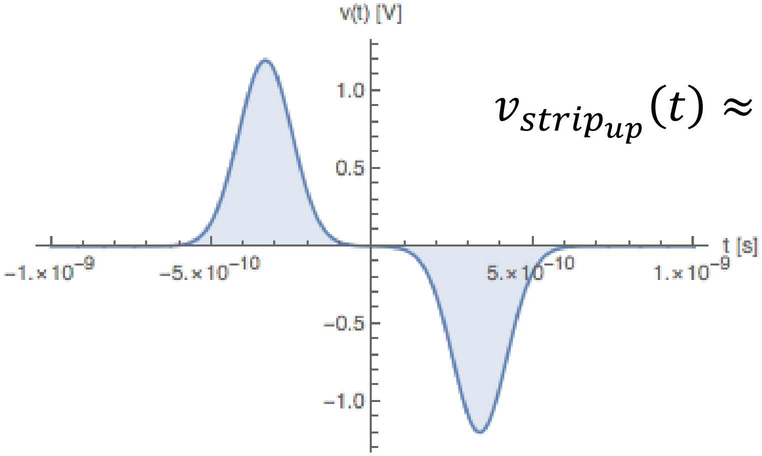
Bunch Signals from broadband BPMs

- **Button BPM output signal to a Gaussian bunch**

$$v_{button}(t) \approx \frac{A_{button}R_{load}}{\pi r_{pipe}\beta c}\frac{di_{beam}(t)}{dt}$$
$$= \frac{r_{button}^2 R_{load}}{2r_{pipe}\beta c}\frac{eN}{\sqrt{2\pi}\sigma^3}te^{-\frac{t^2}{2\sigma^2}}$$



- **Stripline BPM output signal to a Gaussian bunch**
  - **at the upstream port**

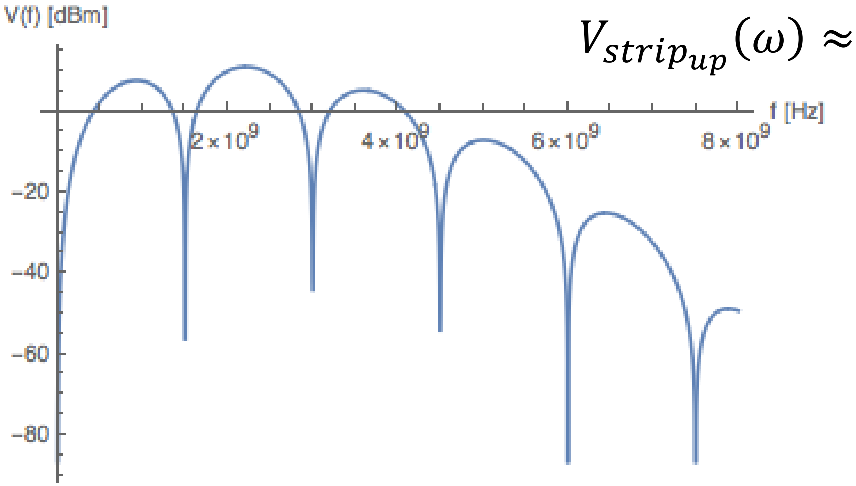


$$v_{strip_{up}}(t) \approx \frac{\phi Z_0}{2}\frac{eN}{\sqrt{2\pi}\sigma}\left\{\exp\left[-\frac{(t+\tau)^2}{2\sigma^2}\right] - \exp\left[-\frac{(t-\tau)^2}{2\sigma^2}\right]\right\}$$

$$\text{with: } \tau = \frac{\ell}{2c}\left(\frac{1}{\beta}+\frac{1}{\beta_{strip}}\right)$$
$$Z_0 = R_{load} = 50\ \Omega$$

Limits of the Analytical Analysis

- **Stripline BPM response in the frequency domain**



$$V_{strip_{up}}(\omega) \approx \sqrt{2}\phi Z_0 \frac{eN}{T} \mathrm{A}(\omega) \sin\left[\frac{\omega\ell}{2c}\left(\frac{1}{\beta}+\frac{1}{\beta_{strip}}\right)\right]$$

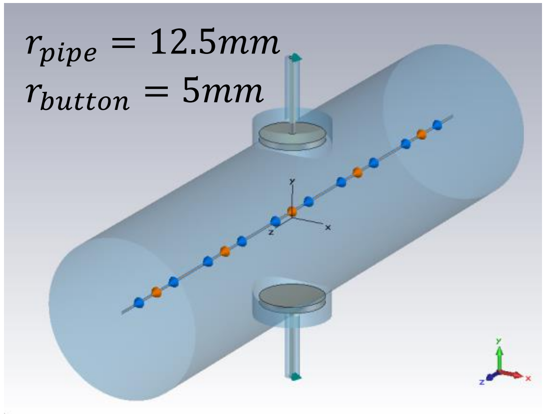
$$\text{with: } A(\omega) = \exp\left(-\frac{\omega^2\sigma^2}{2}\right)$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

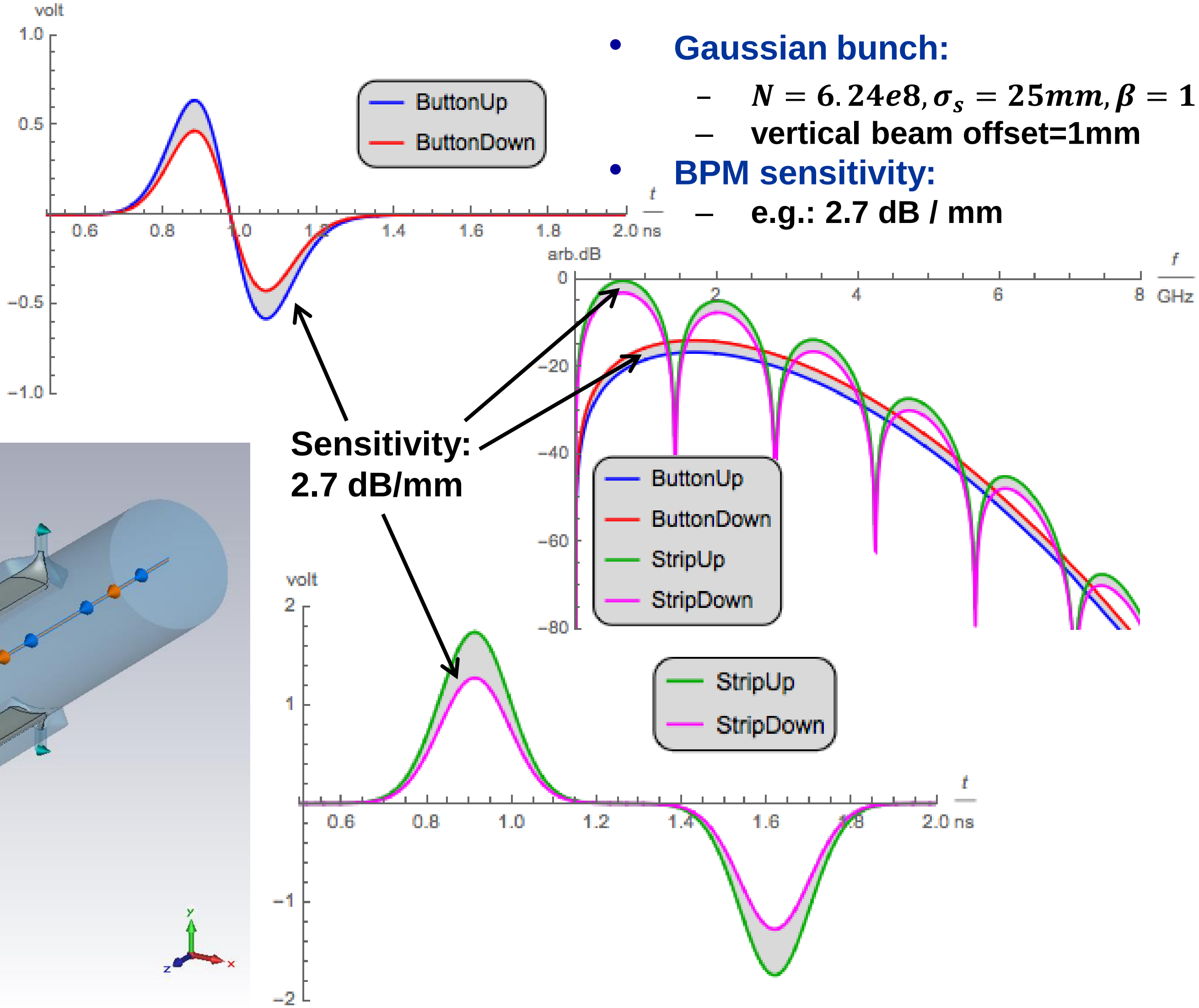
- **No closed form expression which includes important details**
  - **high frequency effects a the feedthrough transition**
  - **attenuation and dispersion of long coaxial cables**
  - **effects of low-β beams**
- **Numerical analysis of the EM problem**
  - **With a beam field as stimulus signal**

Numerically generated BPM Signals

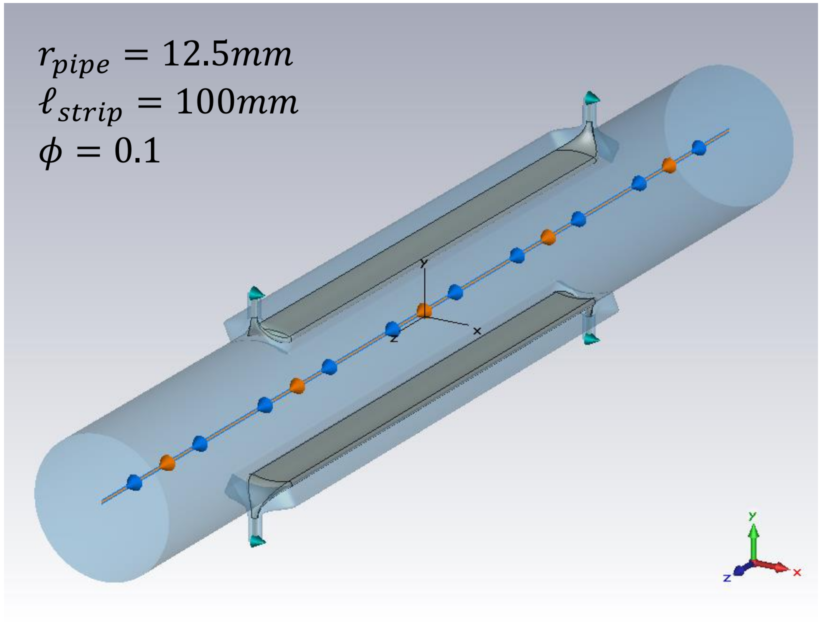
- **Gaussian bunch:**
  - $N = 6.24e8, \sigma_s = 25mm, \beta = 1$
  - **vertical beam offset=1mm**
- **BPM sensitivity:**
  - **e.g.: 2.7 dB / mm**



$r_{pipe} = 12.5mm$
$r_{button} = 5mm$



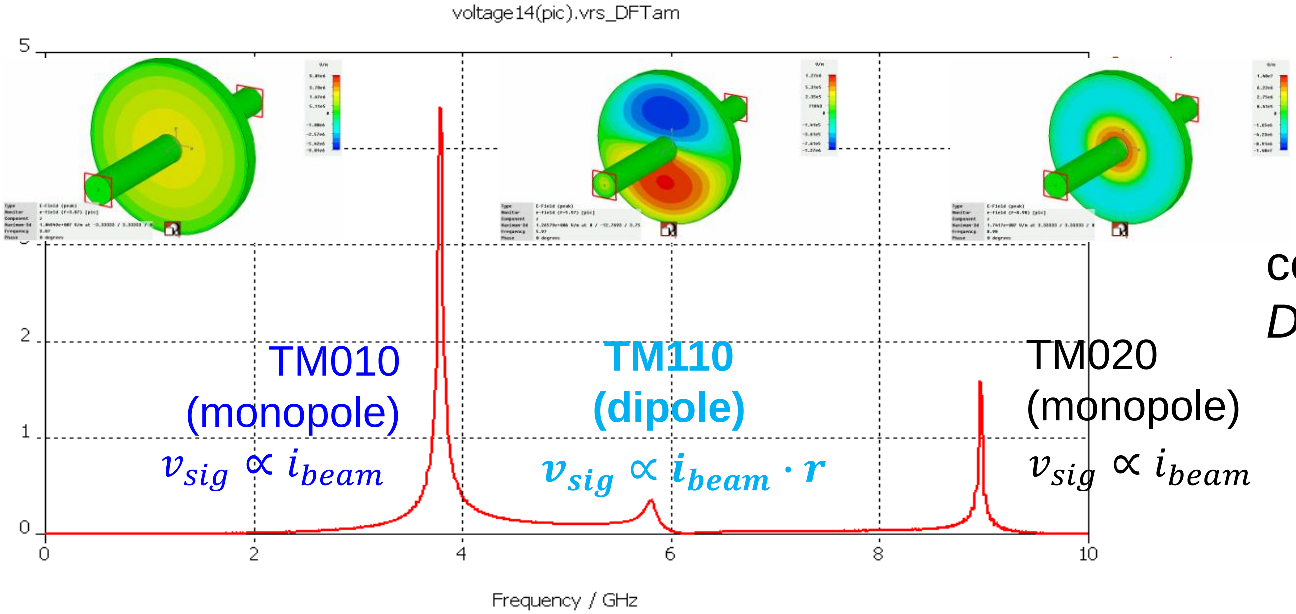
Sensitivity:
2.7 dB/mm



$r_{pipe} = 12.5mm$
$\ell_{strip} = 100mm$
$\phi = 0.1$

Resonant Cavity BPM

- **Based on a beam-excited, passive resonator**
  - **Often a cylindrical “pillbox” cavity is used**
  - **Operating on the TM110 dipole-eigenmode offers a higher resolution potential than comparable broadband BPMs (button, stripline).**
    - **No common-mode Σ signal, only a difference Δ signal**
    - **High transfer impedance, typically in the kΩ/mm range**



TM010
(monopole)
$v_{sig} \propto i_{beam}$

TM110
(dipole)
$v_{sig} \propto i_{beam} \cdot r$

TM020
(monopole)
$v_{sig} \propto i_{beam}$

courtesy
D. Lipka

Towards a Cavity BPM…

- **Eigenmodes in a brick-style resonator**
  - **1st step towards a cavity BPM**
  - **Unfortunately you need to go through the math of the modal expansion of the vector potential Ψ…😊**

Laplace equation:

$$\boldsymbol{\Delta\Psi + k_0^2 \varepsilon_r \mu_r \Psi = 0}$$

$$k_0^2 = \omega^2 \varepsilon_0 \mu_0$$

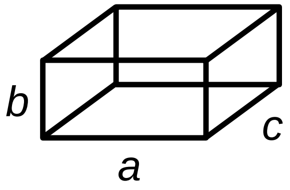
$$k_0 = {}^{2\pi}/_{\lambda_0}$$

k_0: free space wave number

λ_0: free space wave length

Product ansatz (*Cartesian* coordinates):

$$\boldsymbol{\Psi = X(x)Y(y)Z(z)}$$



General solution (field components):

$$\Psi = \begin{Bmatrix} A\cos(k_x x) + B\sin(k_x x) \\ \acute{A}e^{-jk_x x} + \acute{B}e^{-jk_x x} \end{Bmatrix} \begin{Bmatrix} C\cos(k_y y) + D\sin(k_y y) \\ \acute{C}e^{-jk_y y} + \acute{D}e^{-jk_y y} \end{Bmatrix} \begin{Bmatrix} E\cos(k_z z) + F\sin(k_z z) \\ \acute{E}e^{-jk_z z} + \acute{F}e^{-jk_z z} \end{Bmatrix}$$

standing waves

travelling waves

separation condition:

$$\boldsymbol{k_x^2 + k_y^2 + k_z^2 = k_0^2 \varepsilon_r \mu_r}$$

$$k_x = \frac{m\pi}{a} \qquad k_y = \frac{n\pi}{b} \qquad k_z = \frac{p\pi}{c}$$

Eigen frequencies:

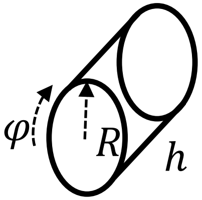
$$\boxed{\boldsymbol{f_{mnp} = \frac{c_0}{2\pi\varepsilon_r\mu_r}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2}}}$$

Cylindrical “Pillbox” Cavity Resonator

- **Same procedure, but now with cylindrical functions** 😢😢

Product ansatz (cylindrical coordinates):

$$\boldsymbol{\Psi = R(\rho)F(\varphi)Z(z)}$$



General solution (field components):

$$\Psi = \begin{Bmatrix} A\,J_m(k_r\rho) + B\,N_m(k_r\rho) \\ \acute{A}H_m^{(2)}(k_r\rho) + \acute{B}H_m^{(2)}(k_r\rho) \end{Bmatrix} \begin{Bmatrix} C\cos(m\varphi) + D\sin(m\varphi) \\ \acute{C}e^{-jm\varphi} + \acute{D}e^{-jm\varphi} \end{Bmatrix} \begin{Bmatrix} E\cos(k_z z) + F\sin(k_z z) \\ \acute{E}e^{-jk_z z} + \acute{F}e^{-jk_z z} \end{Bmatrix}$$

standing waves

travelling waves

$J_m, N_m, H_m^{(1,2)}$: cylindical functions $(Bessel, Hankel, Neumann)$

$^{(1,2)}$see *Abramowitz* and *Stegun*

separation condition:

$$\boldsymbol{k_r^2 + k_z^2 = k_0^2\varepsilon_r\mu_r}$$

Eigen frequencies:

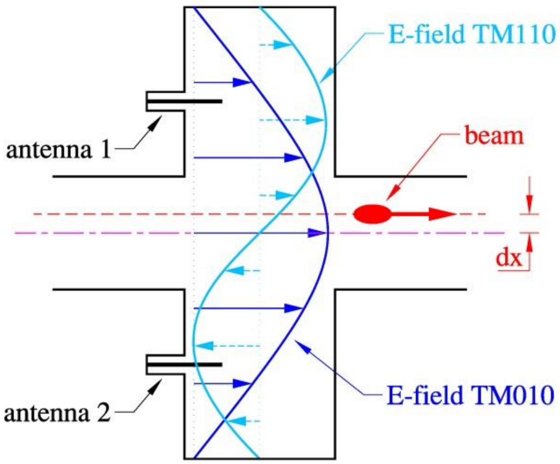
$$\boxed{\boldsymbol{f_{TMmnp} = \frac{c_0}{2\pi\varepsilon_r\mu_r}\sqrt{\left(\frac{j_{mn}}{R}\right)^2 + \left(\frac{p\pi}{h}\right)^2}}}$$

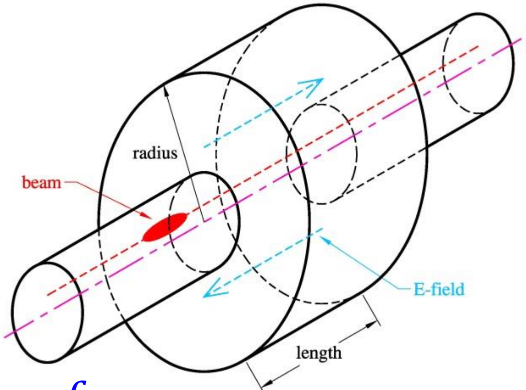
$$\boxed{\boldsymbol{f_{TEmnp} = \frac{c_0}{2\pi\varepsilon_r\mu_r}\sqrt{\left(\frac{j'_{mn}}{R}\right)^2 + \left(\frac{p\pi}{h}\right)^2}}}$$

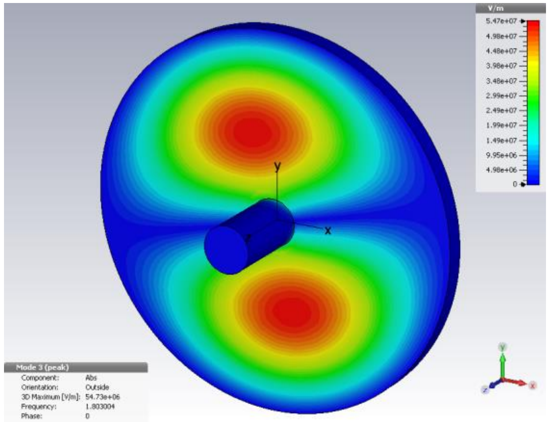
j_{mn} being the n^{th}root of $J_m(x)$

j'_{mn}being the n^{th}root of $J'_m(x)$

Cavity BPM



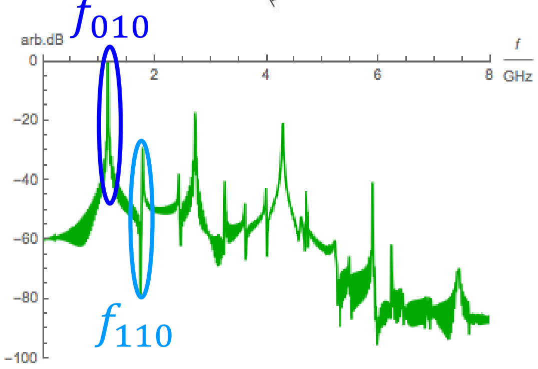


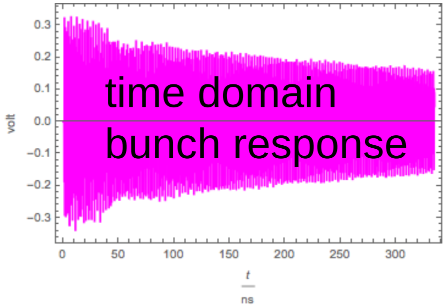


$r_{pipe} = 12.5mm$

$r_{cav} = 100mm$

$\ell_{cav} = 10mm$





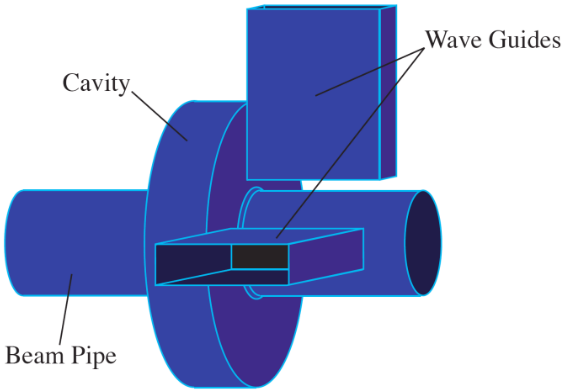
- **Beam couples to:**

$$E_z = C\, J_1\left(\frac{j_{11} r}{R}\right) e^{i\omega t} \cos\varphi$$

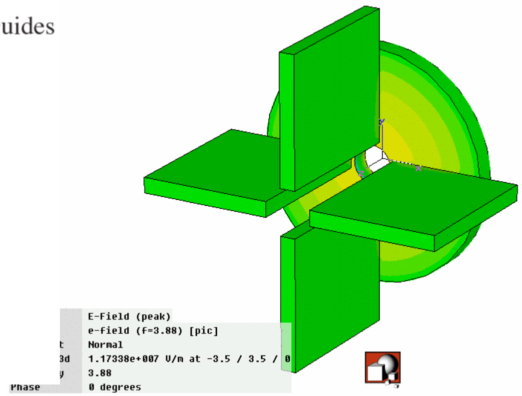
dipole (TM_{110}) and monopole (TM_{010}) & other modes

- **Common mode (TM_{010}) frequency discrimination**
- **Decaying RF signal response**
  - **Position signal: TM_{110}**
    - **Requires normalization and a phase reference**
  - **Intensity signal: TM_{010}**

Common-mode free Cavity BPMs

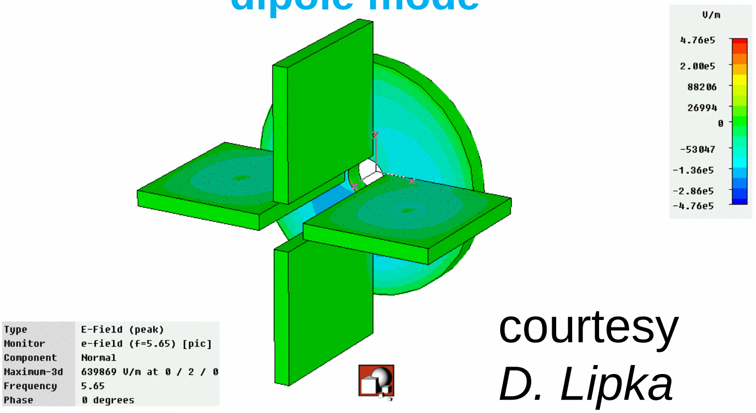


monopole mode



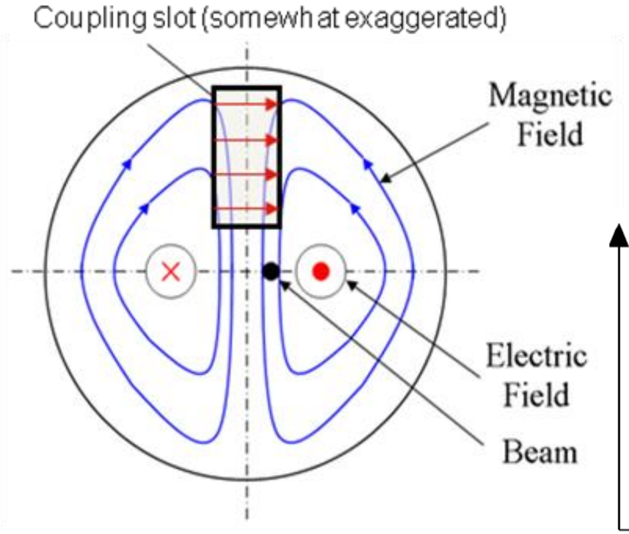
	E-Field (peak)
	e-field (f=3.88) [pic]
t	Normal
3d	1.17338e+007 V/m at -3.5 / 3.5 / 0
y	3.88
Phase	0 degrees

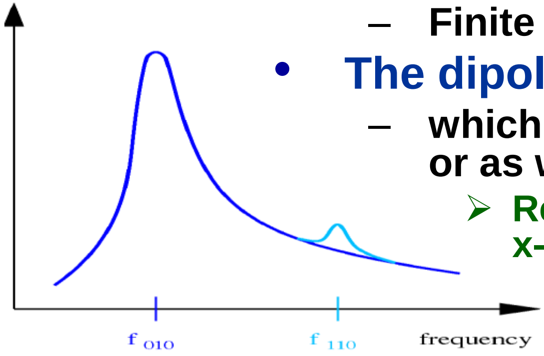
dipole mode



Type	E-Field (peak)
Monitor	e-field (f=5.65) [pic]
Component	Normal
Maximum-3d	639869 V/m at 0 / 2 / 0
Frequency	5.65
Phase	0 degrees

courtesy
D. Lipka





- **Add slot-coupled waveguide TE_{01}-mode high-pass filter**

$$f_{010} < f_{10} = \frac{1}{2a\sqrt{\varepsilon\mu}} < f_{110}$$

between cavity and coaxial output port.
  - **Finite Q of TM_{010} still leaks into TM_{110}!**
- **The dipole mode has two polarizations**
  - **which will orientate along imperfections, or as wanted along the coupling slots**
    - ➢ **Requires tight tolerances to minimize x-y cross talk**

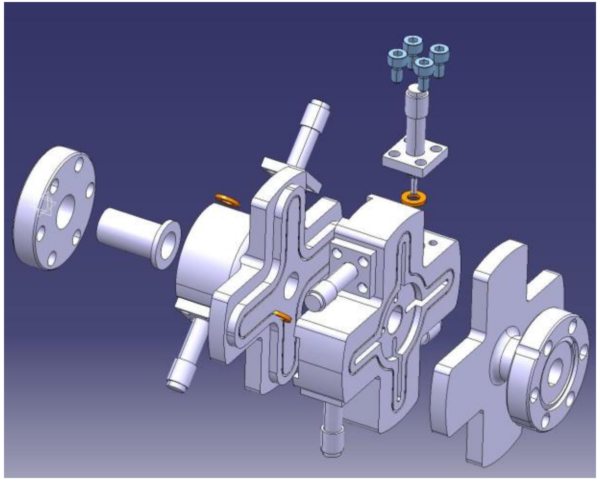
Cavity BPM

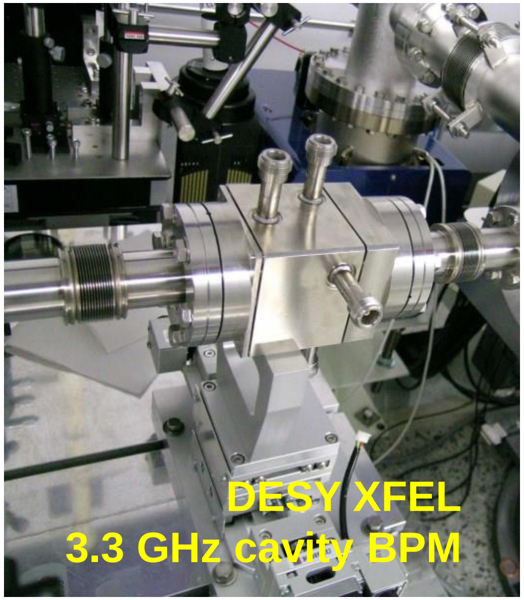
+ **Pros**
  - **No or minimum common mode signal contribution in the Δ-signal**
    - **Frequency discrimination of dipole (TM110) and monopole (TM010) modes**
  - **High resolution potential**
    - **High shunt (transfer) impedance of the TM110 mode**
      - Even for lower Q tuning of the TM110 mode
    - **Sub-μm signal pass resolution potential**

\- **Cons**
  - **High beam coupling impedance**
    - **No free lunch: high impedance may cause beam break-up and/or instabilities**
    - **No or very limited use in ring accelerators**
  - **Requires a reference monopole mode (TM010) resonator**
    - **Beam phase and intensity**
  - **Limited position range**
    - **~half aperture**
  - **Requires advanced RF read-out electronics**
  - **High-Q resonator may not be suitable for single bunch position measurements**

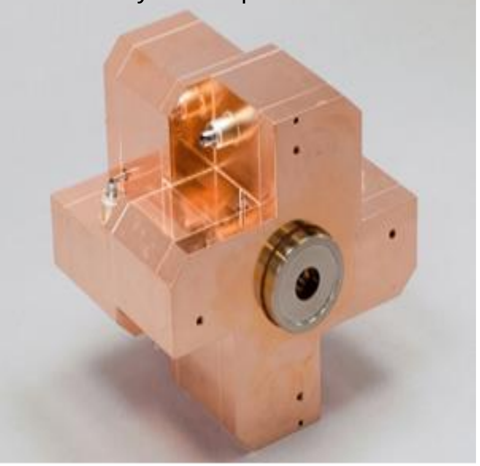
Examples of Cavity BPMs

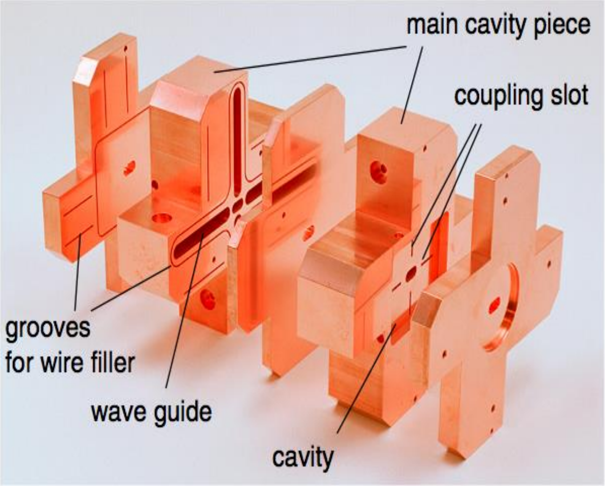

Setup of several 15 GHz cavity BPMs at the CLIC Test Facility linac

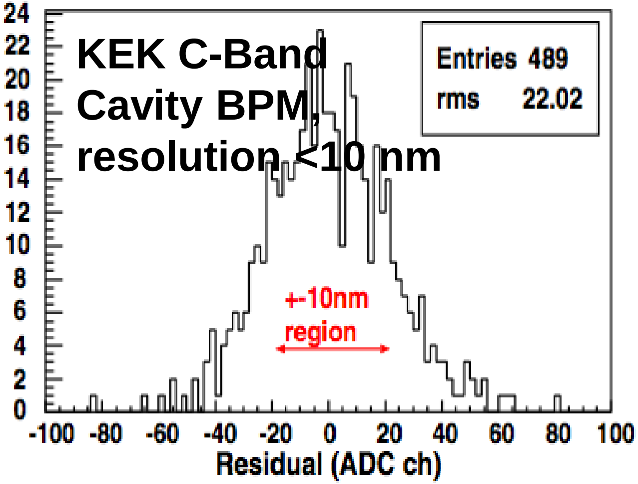



DESY XFEL 3.3 GHz cavity BPM

Courtesy of D. Lipka & Y. Honda

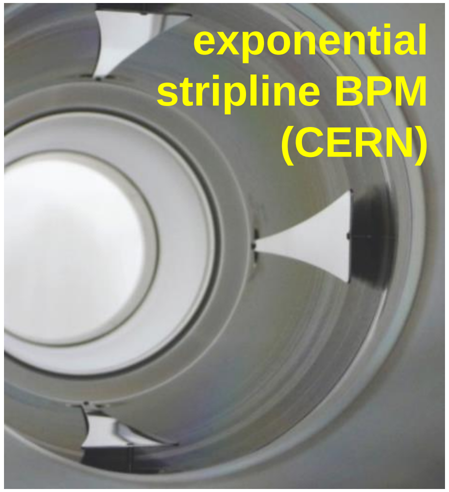


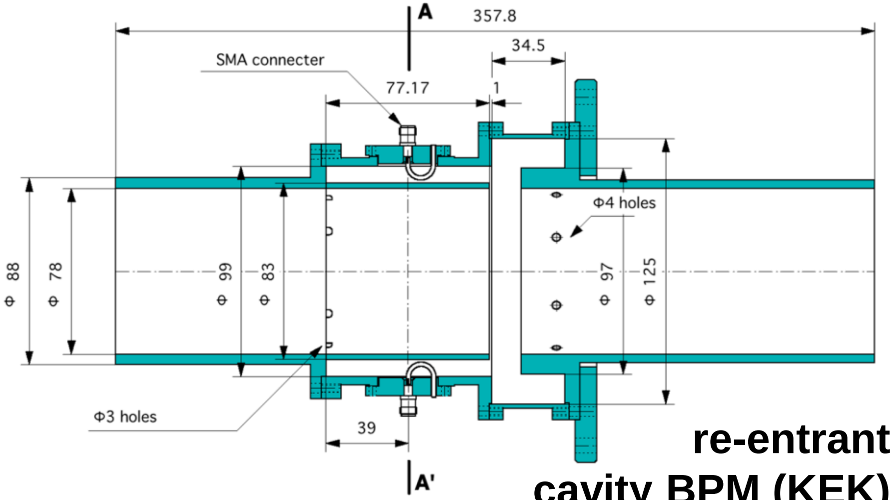



KEK C-Band Cavity BPM, resolution <10 nm

Other Types of BPM Pickups

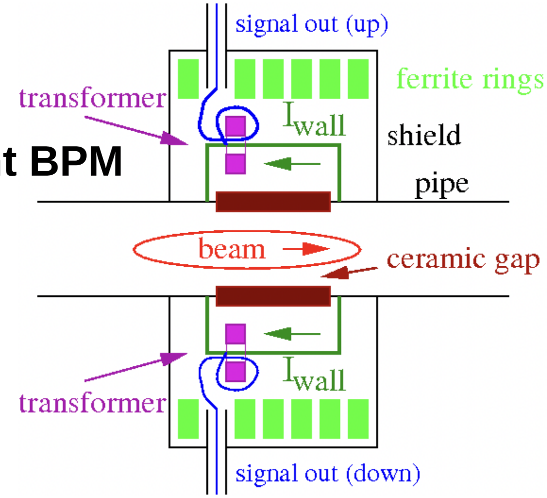
- **Less popular, but sometimes better suited for a specific application**
  - **Stripline BPM with shorted downstream ports**
  - **Exponentially tapered stripline BPM**
  - **Re-entrant cavity BPM**
  - **Resonant stripline of button BPM**
  - **Inductive BPM, …**
    - ➢ **In common: based on symmetry**


exponential stripline BPM (CERN)



re-entrant cavity BPM (KEK)

inductive wall-current BPM (CERN)



Summary Part 1

- **What have we learned?!**
  - **It is hard to stay up on Sunday morning and to follow a BPM session…**
- **What else have we learned,… about BPMs?!**
  - **BPMs are based on symmetry**
    - ➢ **Detect the asymmetric beam position by a pair of symmetrically arranged electrodes or by a symmetric resonant, beam driven cavity**
  - **BPM pickups are non-invasive**
    - ➢ **Detect the beam's center-of-charge by electromagnetic coupling**
  - **Broadband BPM electrode (stripline, button, etc.) signals**
    - ➢ **contain information of the beam position and the beam intensity**
      - Need to "normalize" to extract a position signal
    - ➢ **May have a non-linear position characteristic**
    - ➢ **Have a broad spectral response**
  - **Resonant cavity BPMs**
    - ➢ **Have a frequency selective beam response**
    - ➢ **Have a high resolution potential**
      - No or only little "common mode" signal contribution in the dipole mode