Superconducting Magnets

Stephan Russenschuck
CERN – AT/MEL/EM

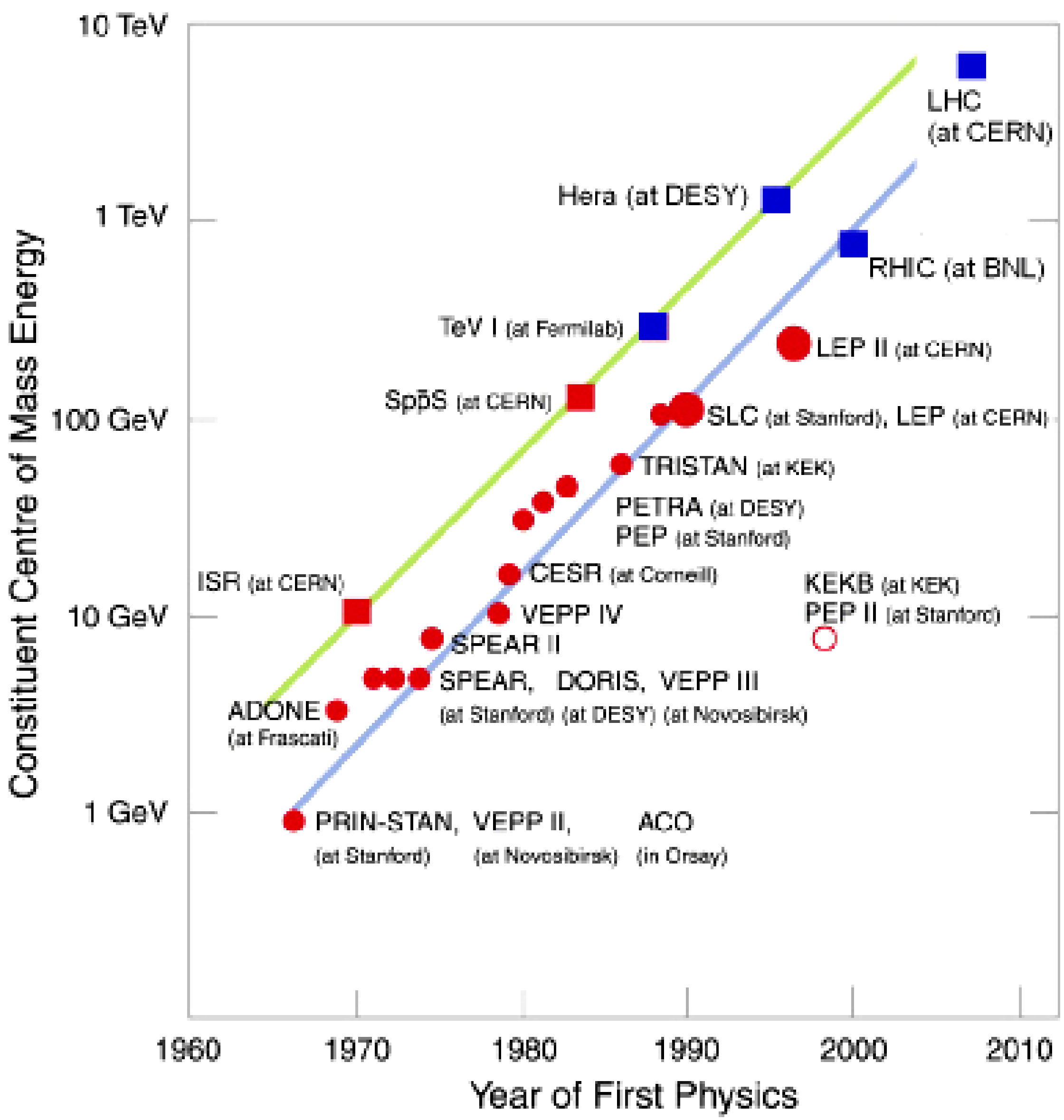
CAS – Trieste 2005

Overview

- Cryogenics
- Vacuum technology
- Material science
- Survey and alignment
- Cold electrical engineering, power supplies, current leads, bus-bars
- Mechanical Engineering
- Electromagnetic design
  - Harmonic fields
  - Complex analysis methods
  - Field of line currents
  - Numerical methods
  - Time transient effects (Persistent currents, quench)
  - Optimization

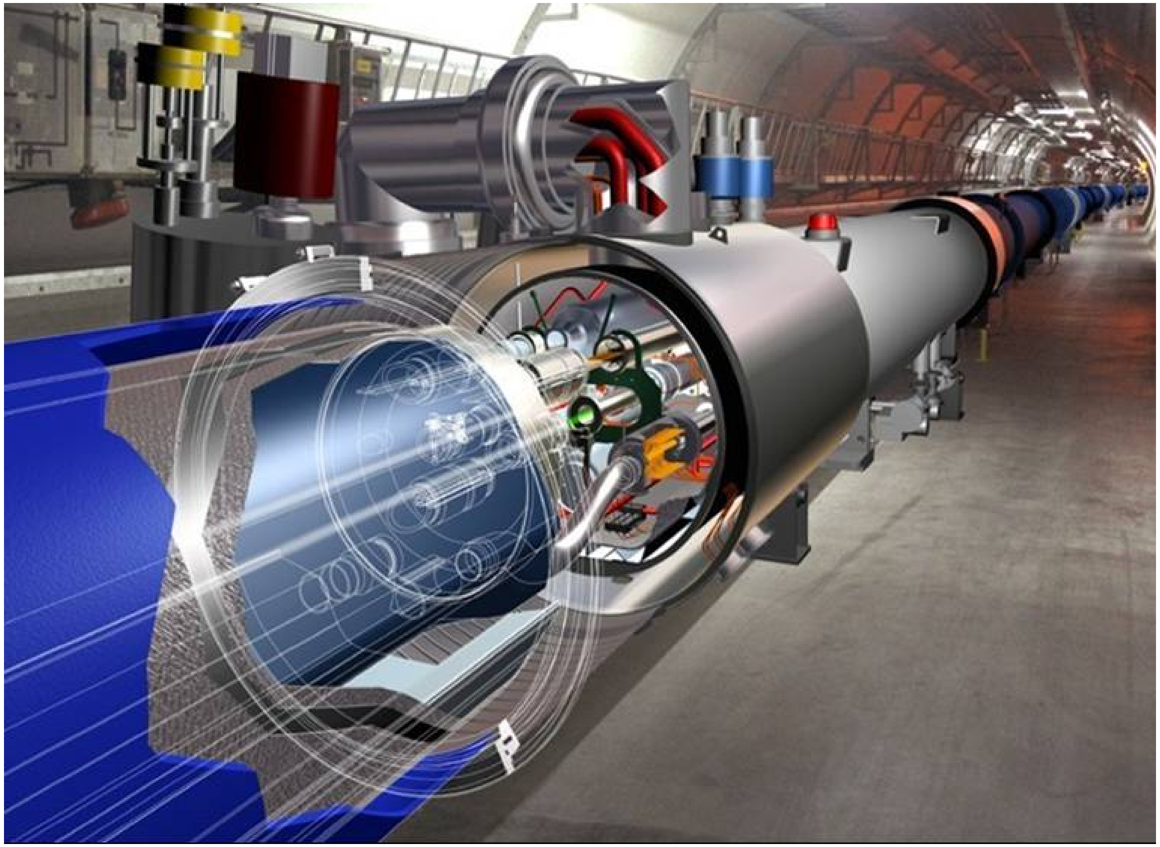
S. Russenschuck: Electromagnetic Design and Mathematical Optimization Methods in Magnet Technology, E-book, www.cern.ch/russ, ISBN:92-9083-242-8

Livingston Plot

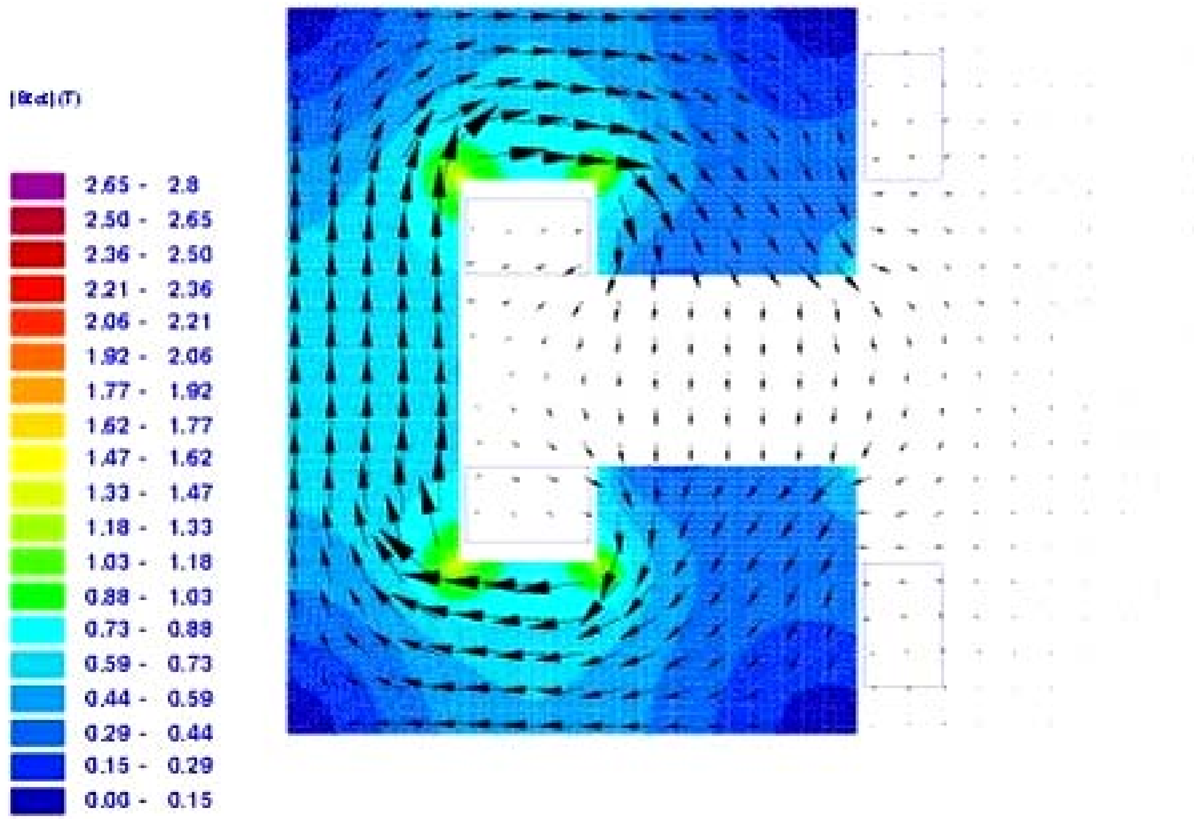


LHC in the LEP Tunnel

$$\{p\}_{\mathrm{GeV/c}} \approx 0.3\{Q\}_e\{R\}_\mathrm{m}\{B_0\}_\mathrm{T}$$

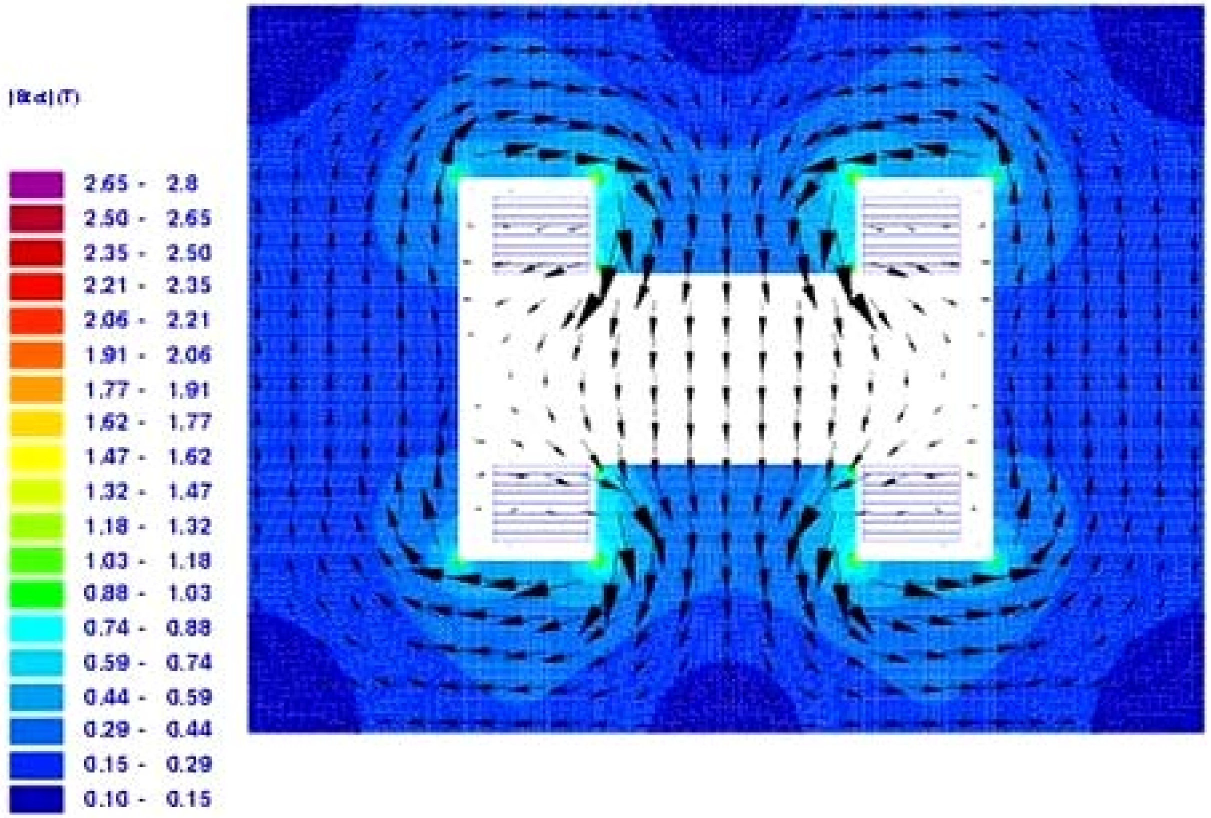


LEP Dipole



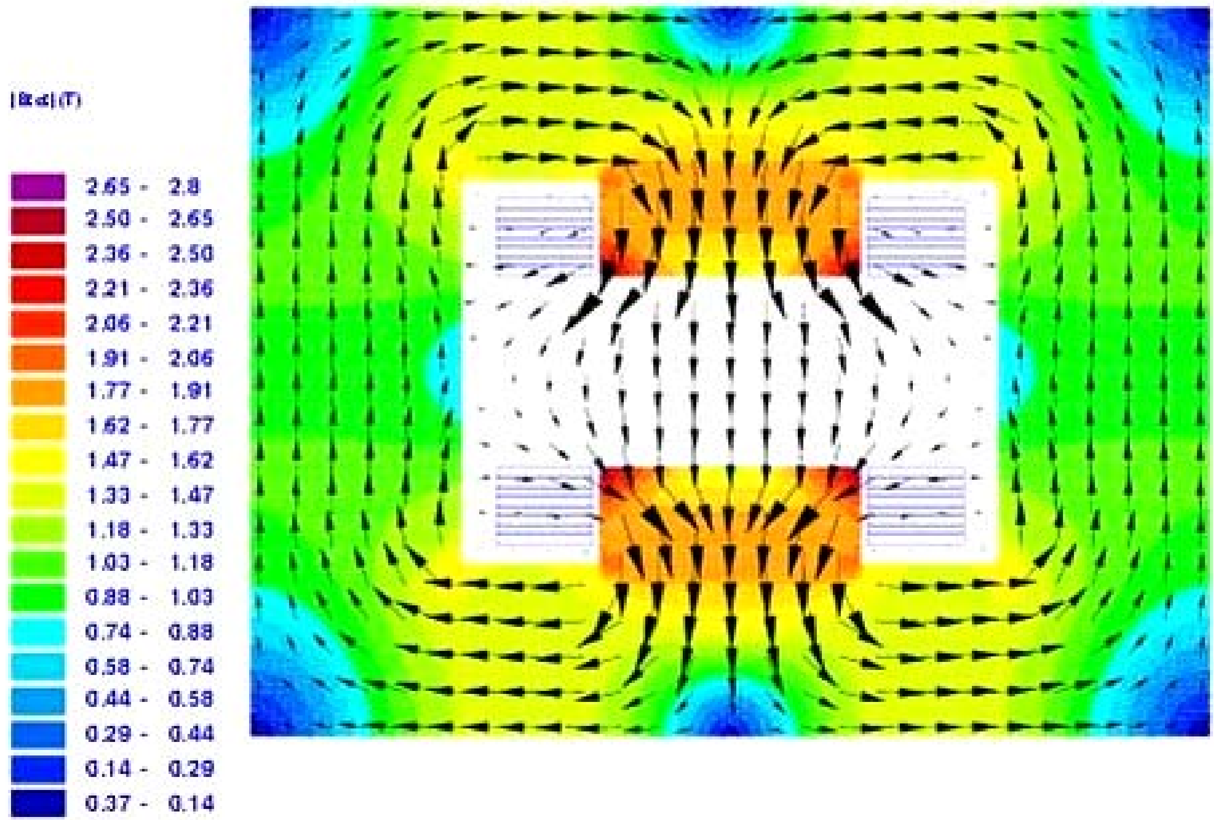
$N \cdot I = 4480$ A $\quad B_1 = 0{,}13$ T $\quad B_S = 0{,}042$ T $\quad$ Fill.fac. 0.27

H Magnet



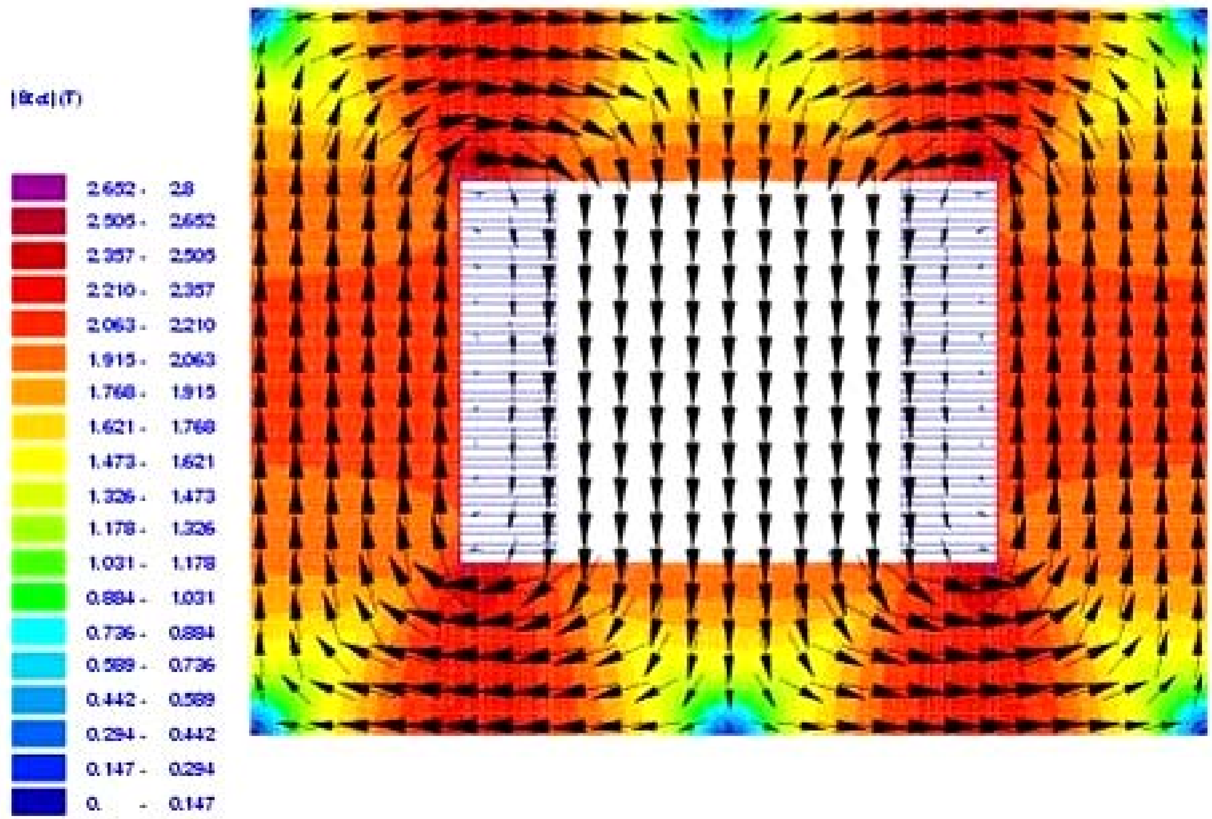
$N \cdot I = 24000\ \mathrm{A}$ $\quad B_1 = 0{,}3\ \mathrm{T}$ $\quad B_s = 0{,}065\ \mathrm{T}$ $\quad$ Fill.fac. 0.98

Super-Ferric H Magnet



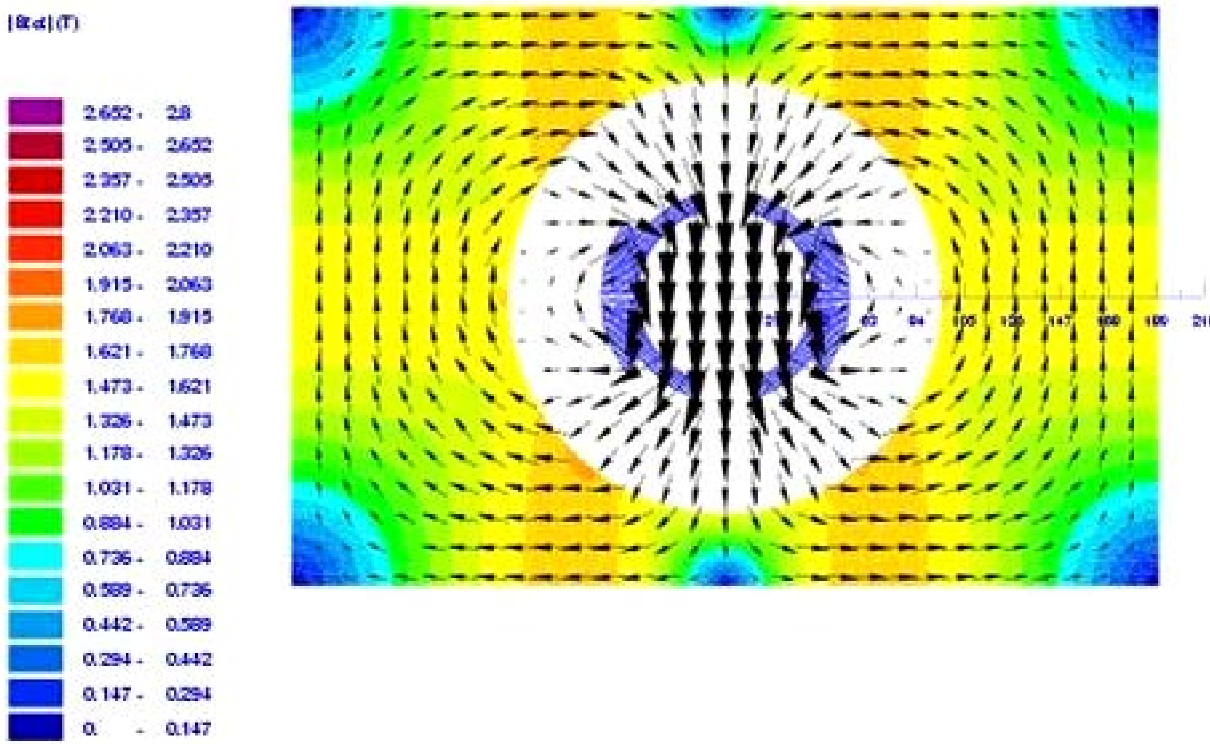
$N \cdot I = 96000\ \mathbf{A} \qquad B_1 = 1.18\ \mathbf{T} \qquad B_S = 0.26\ \mathbf{T}$

Window Frame Magnet



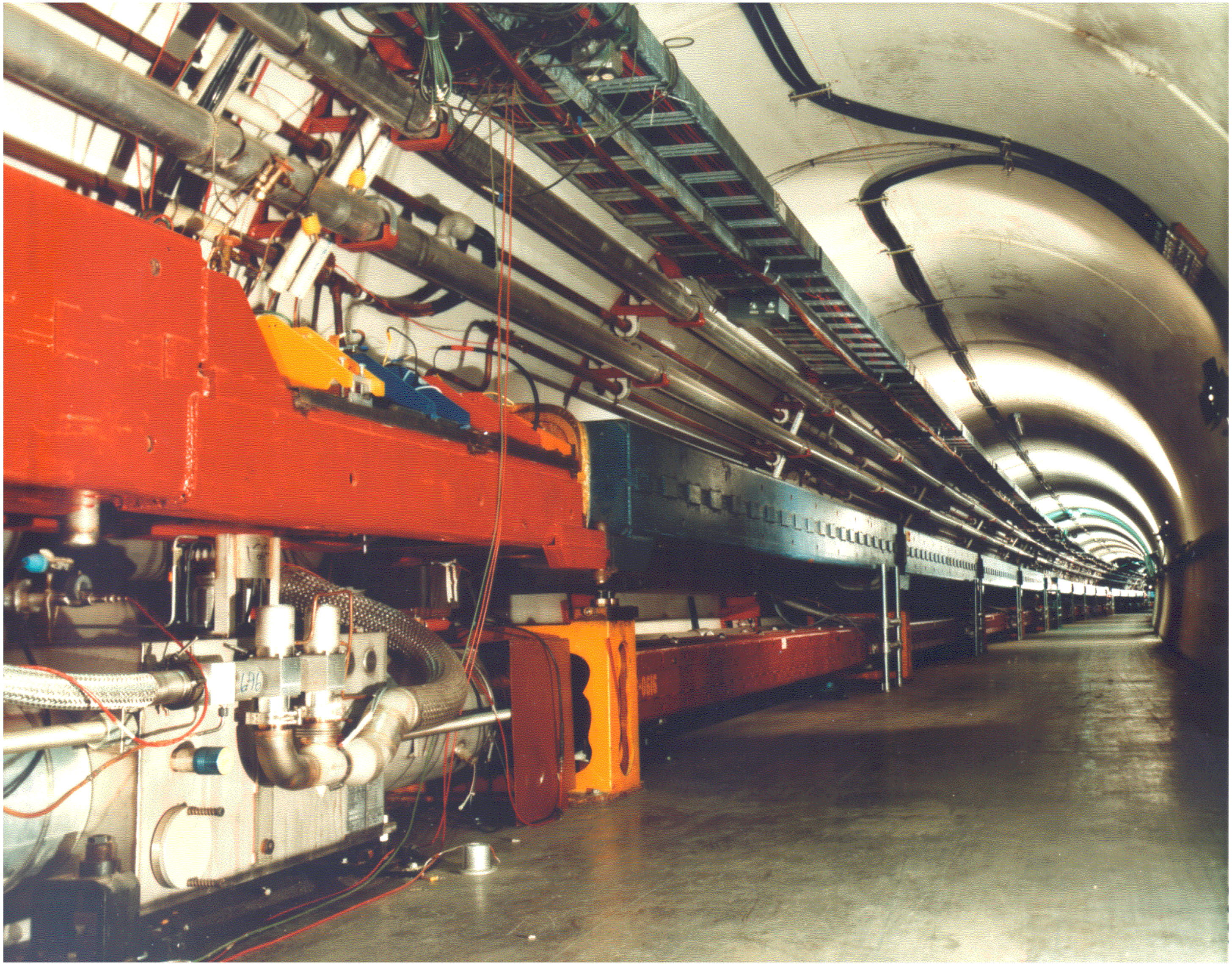
$N \cdot I = 360000$ A $\qquad B_1 = 2.08$ T $\qquad B_s = 1.04$ T

Tevatron Dipole



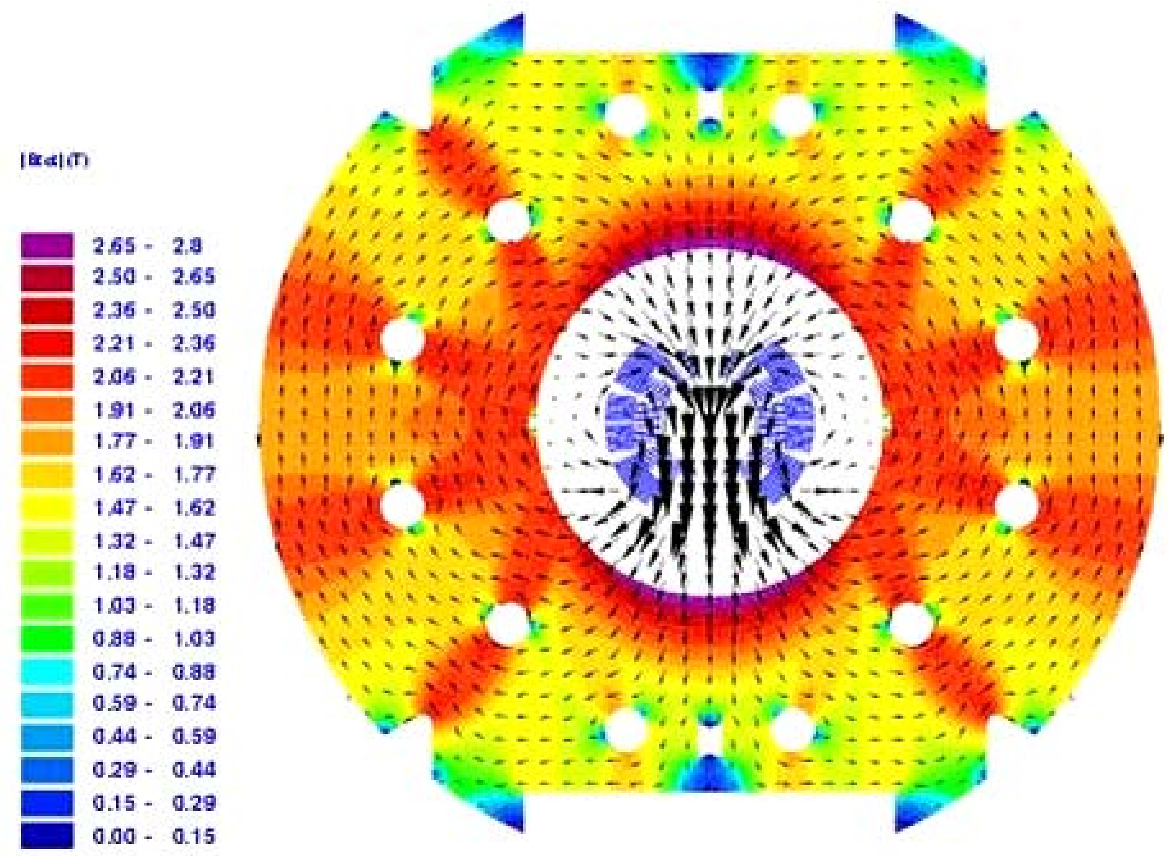
$N \cdot I = 471000 \text{ A} \qquad B_1 = 4.16 \text{ T} \qquad B_S = 3.39 \text{ T}$

The Fermilab Tevatron



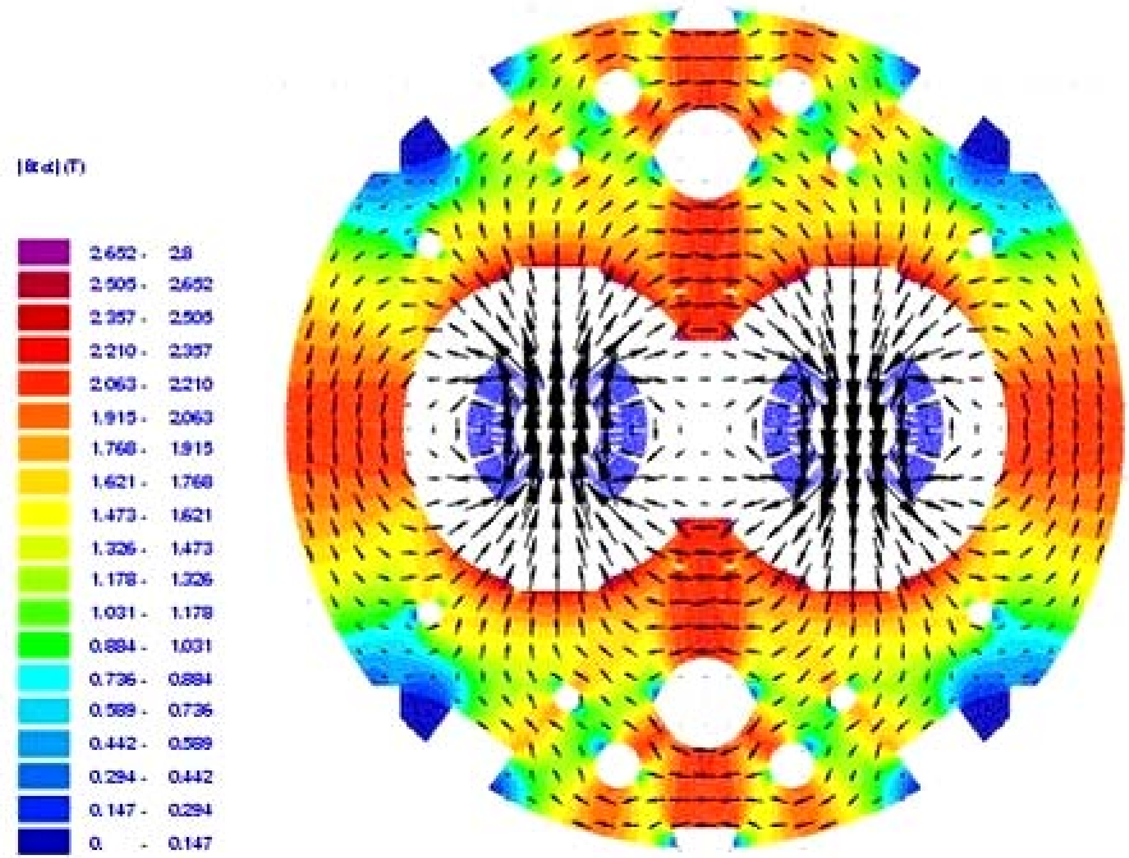
HERA at Desy

LHC Coil Test Facility



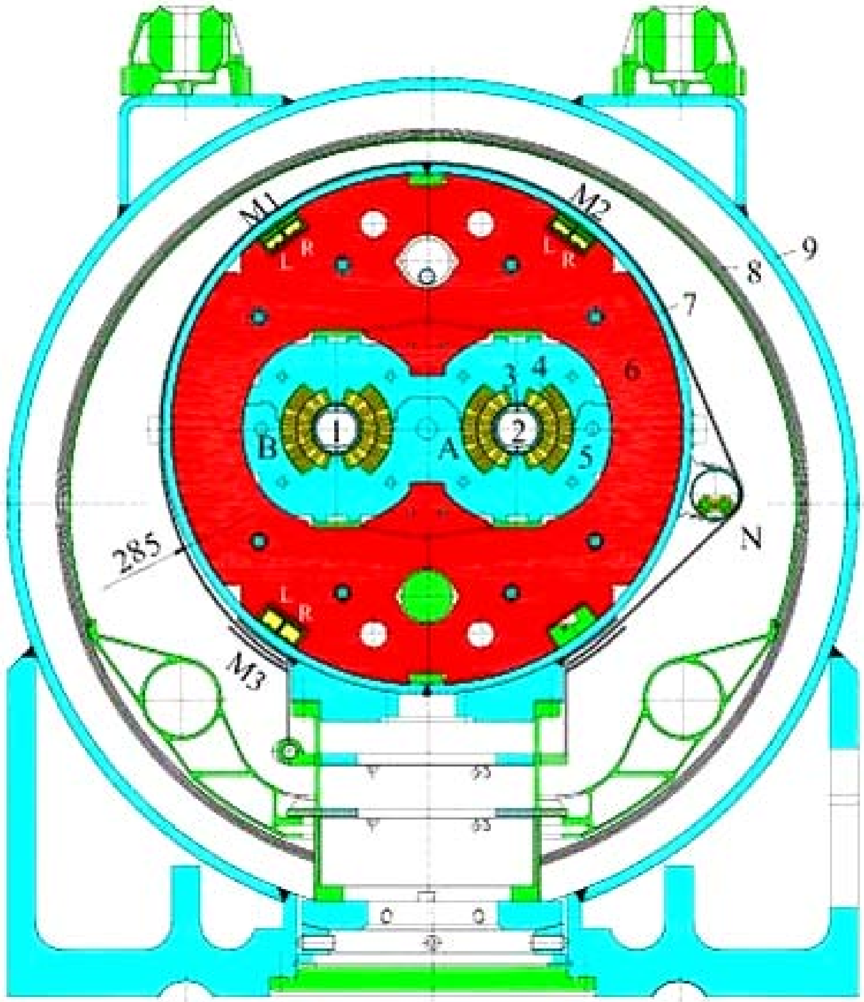
$N \cdot I = 960000$ A $\quad B_1 = 8.33$ T $\quad B_S = 7.77$ T

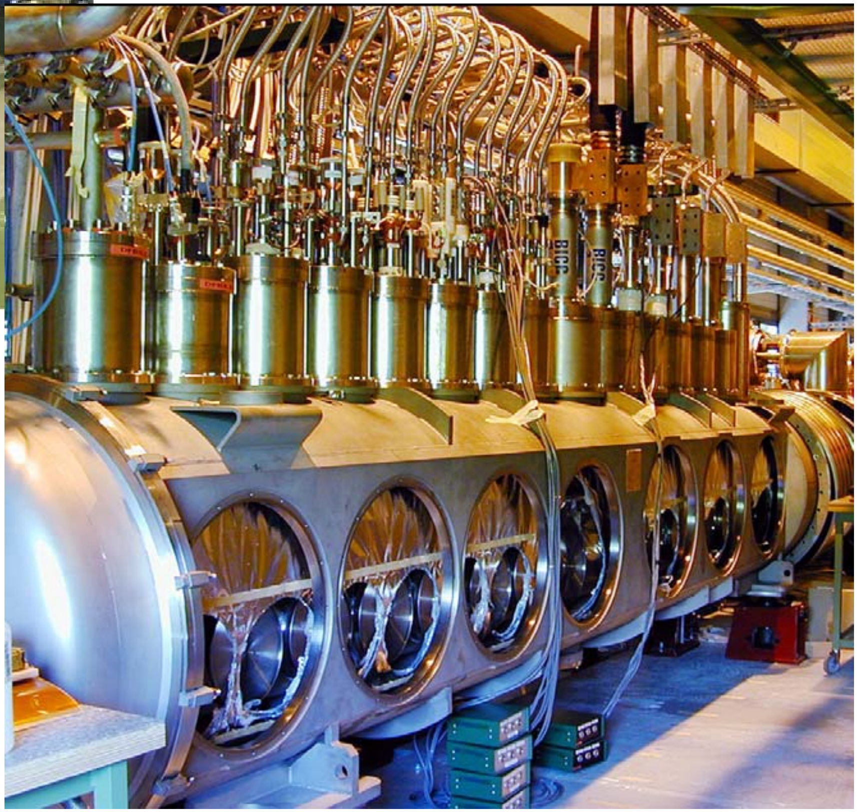
LHC Two-in-one Dipole



$N \cdot I = 2x944000$ A $\qquad B_1 = 8.32$ T $\qquad B_S = 7.44$ T

Cross-section of Cryodipole





Conventional magnets

- Important ohmic losses require water cooling
- Field is defined by the iron pole shape (max 1.5 T)
- Easy electrical and beam-vacuum interconnections
- Voltage drop over one coil of the MBW magnets = 22 V

Superconducting magnets

- Field is defined by the coil layout
- Maximum field limited to 10 T (NbTi), 12 T (Nb_3Sn)
- Enormous electromagnetic forces (400 tons/m in MB for LHC)
- Quench protection system required
- Cryogenic installation (1.8 K)
- Electrical interconnections in cryo-lines
- Voltage drop on LHC magnet string (154 MB) 155 V

Comparison (EM-Design)

➔ Conventional magnets

- Ideal pole shape known from potential theory
- One-dimensional (analytical) field computation for main field
- Commercial FEM software can be used as a black box (hysteresis modeling)

➔ Superconducting magnets

- Decoupling of coil and yoke optimization
- Accuracy of the field solution
- Modeling of the coils
- Filament magnetization
- Quench simulations

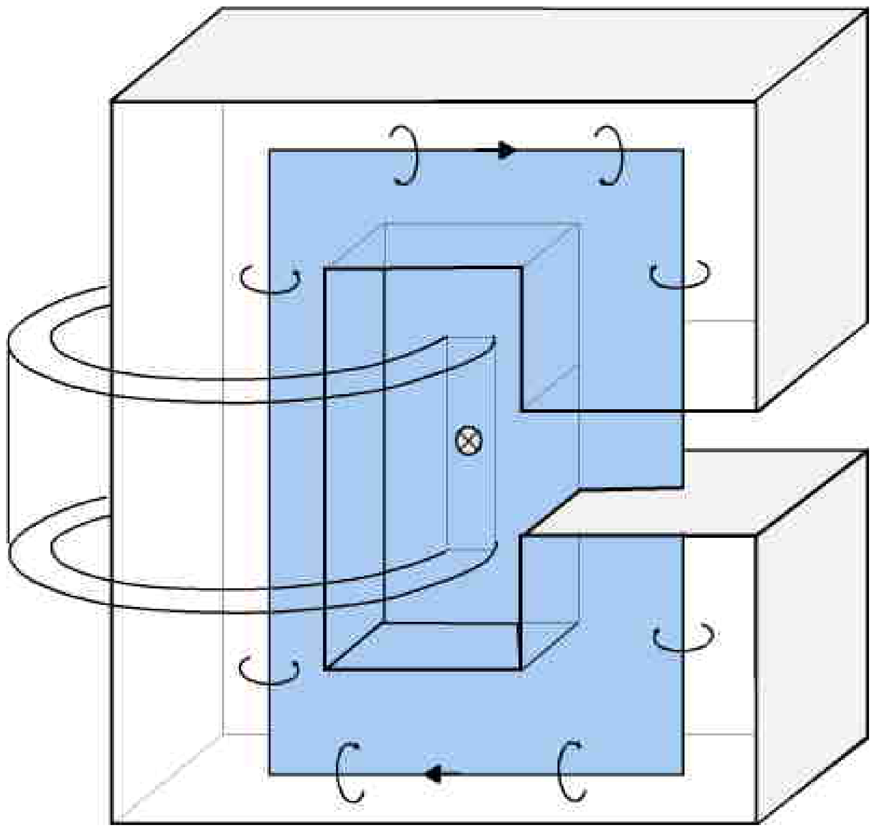
SI-unit	Relation	SI-unit
1A	$V_m(s) = \int_s \mathbf{H} \cdot \mathrm{d}s$	$1\mathrm{A} \cdot \mathrm{m}^{-1}$
1V	$U(s) = \int_s \mathbf{E} \cdot \mathrm{d}s$	$1\mathrm{V} \cdot \mathrm{m}^{-1}$
$1\mathrm{V} \cdot \mathrm{s}$	$\Phi(a) = \int_a \mathbf{B} \cdot \mathrm{d}a$	$1\mathrm{V} \cdot \mathrm{s} \cdot \mathrm{m}^{-2}$
$1\mathrm{A} \cdot \mathrm{s}$	$\Psi(a) = \int_a \mathbf{D} \cdot \mathrm{d}a$	$1\mathrm{A} \cdot \mathrm{s} \cdot \mathrm{m}^{-2}$
1A	$I(a) = \int_a \mathbf{J} \cdot \mathrm{d}a$	$1\mathrm{A} \cdot \mathrm{m}^{-2}$
$1\mathrm{A} \cdot \mathrm{s}$	$Q(V) = \int_V q \cdot \mathrm{d}V$	$1\mathrm{A} \cdot \mathrm{s} \cdot \mathrm{m}^{-3}$

Global physical quantities

Local vector-fields

$$\mathbf{B} \in \mathcal{V}(\Omega) := C^\infty(\Omega \in E_3, V_3) \qquad \mathbf{B} : \Omega \to V_3 : P \mapsto \mathbf{B}(P)$$

$$\int_a \mathbf{J} \cdot \mathrm{d}a = \int_{\partial a} \mathbf{H} \cdot \mathrm{d}s$$



$$\Phi = \int_a \mathbf{B} \cdot \mathrm{d}a$$

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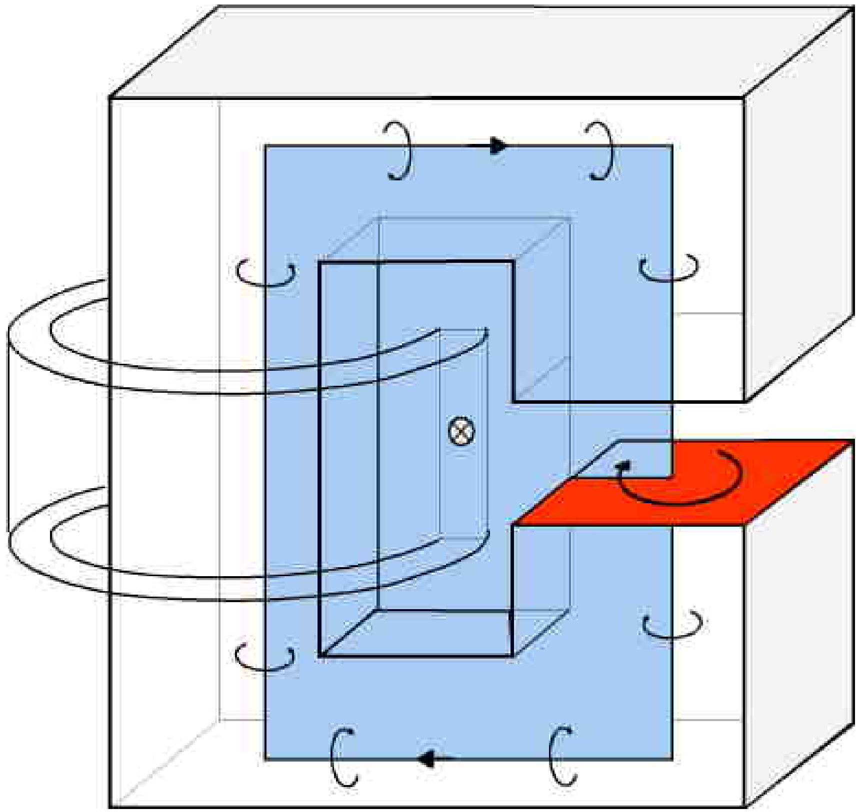
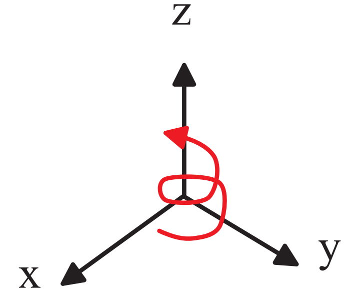
$$\int_a \mathbf{J} \cdot \mathrm{d}a = \int_{\partial a} \mathbf{H} \cdot \mathrm{d}s$$

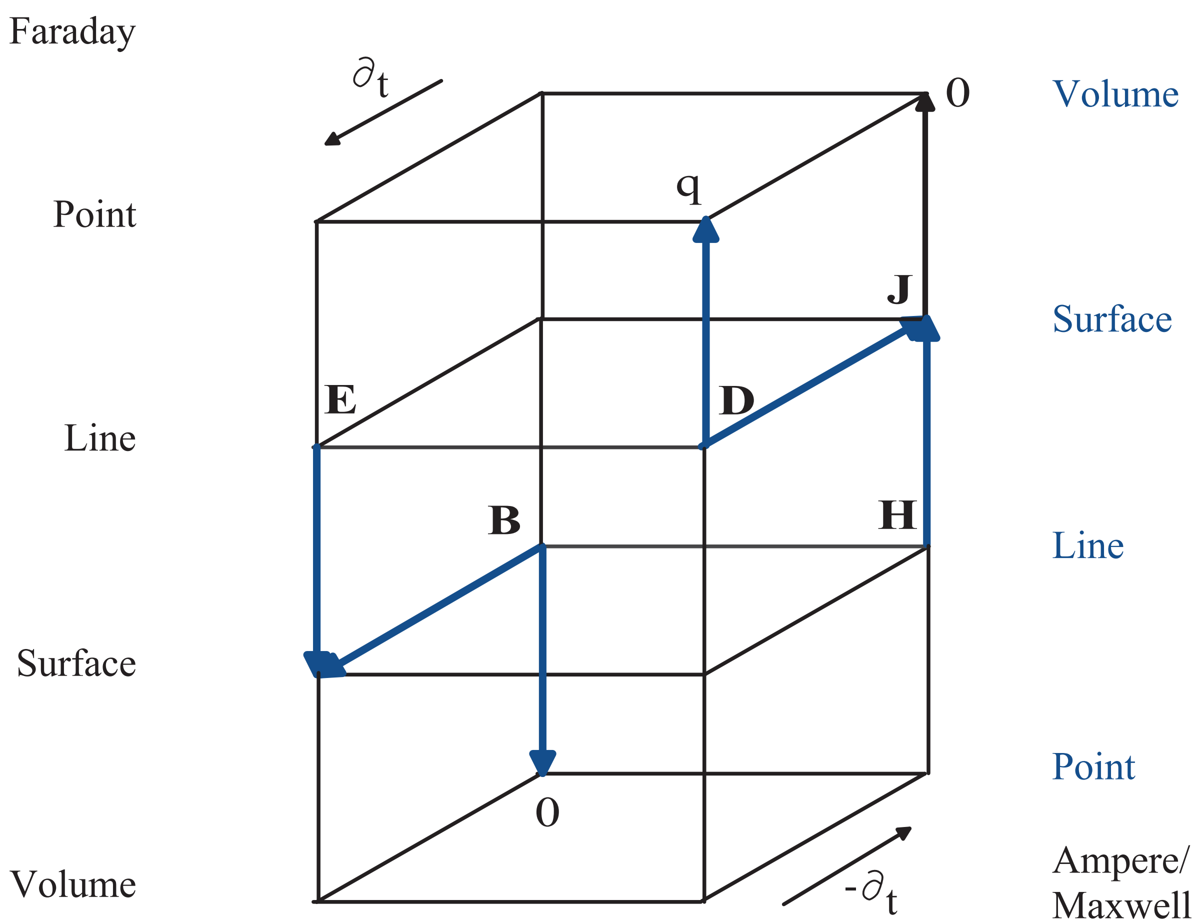




Embedding into oriented ambient space (Origin, coordinates)

$$\Phi = \int_a \mathbf{B} \cdot \mathrm{d}a$$

Outer oriented space elements



Inner oriented space elements

The Grad, Curl and Div Operators

$$\text{grad}\,\phi = \frac{\partial \phi}{\partial x}\mathbf{e}_x + \frac{\partial \phi}{\partial y}\mathbf{e}_y + \frac{\partial \phi}{\partial z}\mathbf{e}_z$$

$$\text{div}\,\mathbf{g} = \frac{\partial g_x}{\partial x} + \frac{\partial g_y}{\partial y} + \frac{\partial g_z}{\partial z}$$

$$\begin{aligned}\text{curl}\,\mathbf{g} = & \left(\frac{\partial g_z}{\partial y} - \frac{\partial g_y}{\partial z}\right)\mathbf{e}_x + \\ & \left(\frac{\partial g_x}{\partial z} - \frac{\partial g_z}{\partial x}\right)\mathbf{e}_y + \\ & \left(\frac{\partial g_y}{\partial x} - \frac{\partial g_x}{\partial y}\right)\mathbf{e}_z\end{aligned}$$

$$\mathbf{v} \cdot \operatorname{grad} \phi \quad := \quad \lim_{\Delta t \to 0} \frac{\phi(\mathbf{r}(t + \Delta t)) - \phi(\mathbf{r}(t))}{\Delta t}$$

$$\mathbf{n} \cdot \operatorname{curl} \mathbf{g} \quad := \quad \lim_{a \to 0} \frac{\int_{\partial a} \mathbf{g} \cdot \mathrm{d}\mathbf{s}}{a}$$

$$\operatorname{div} \mathbf{g} \quad := \quad \lim_{V \to 0} \frac{\int_{\partial V} \mathbf{g} \cdot \mathrm{d}\mathbf{a}}{V}$$

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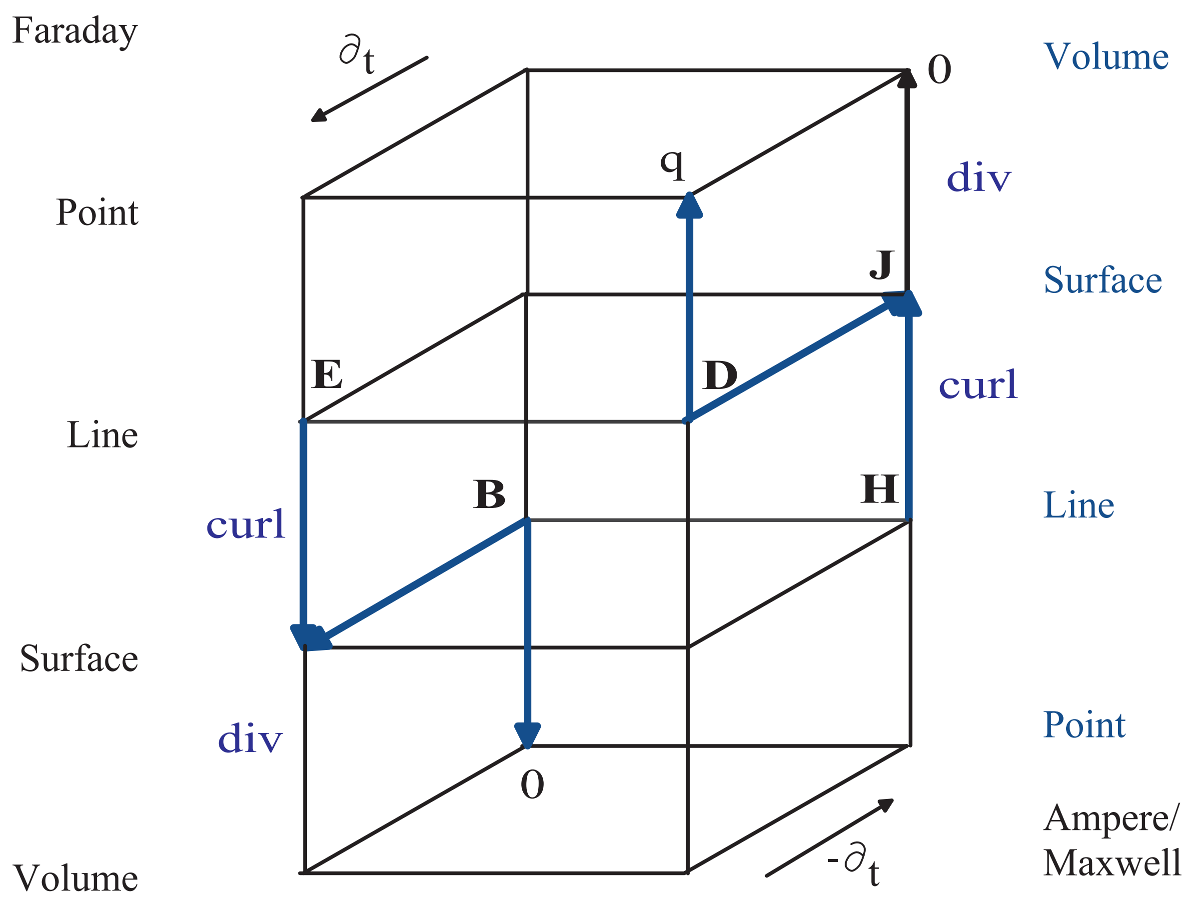
1 Poincaré Lemma

$$\mathbf{a} \xrightarrow{\text{curl}} \mathbf{b} \xrightarrow{\text{div}} 0$$

$$f \xrightarrow{\text{grad}} \mathbf{e} \xrightarrow{\text{curl}} 0$$

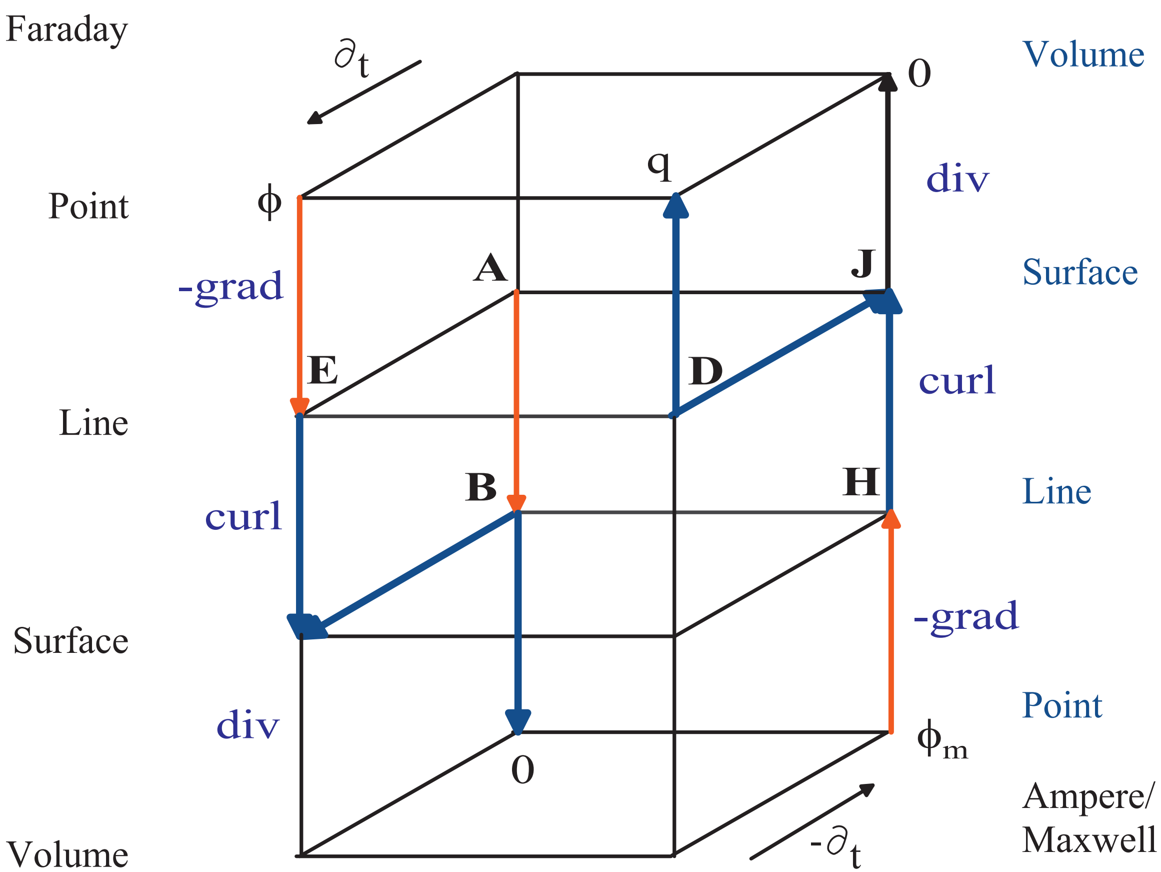


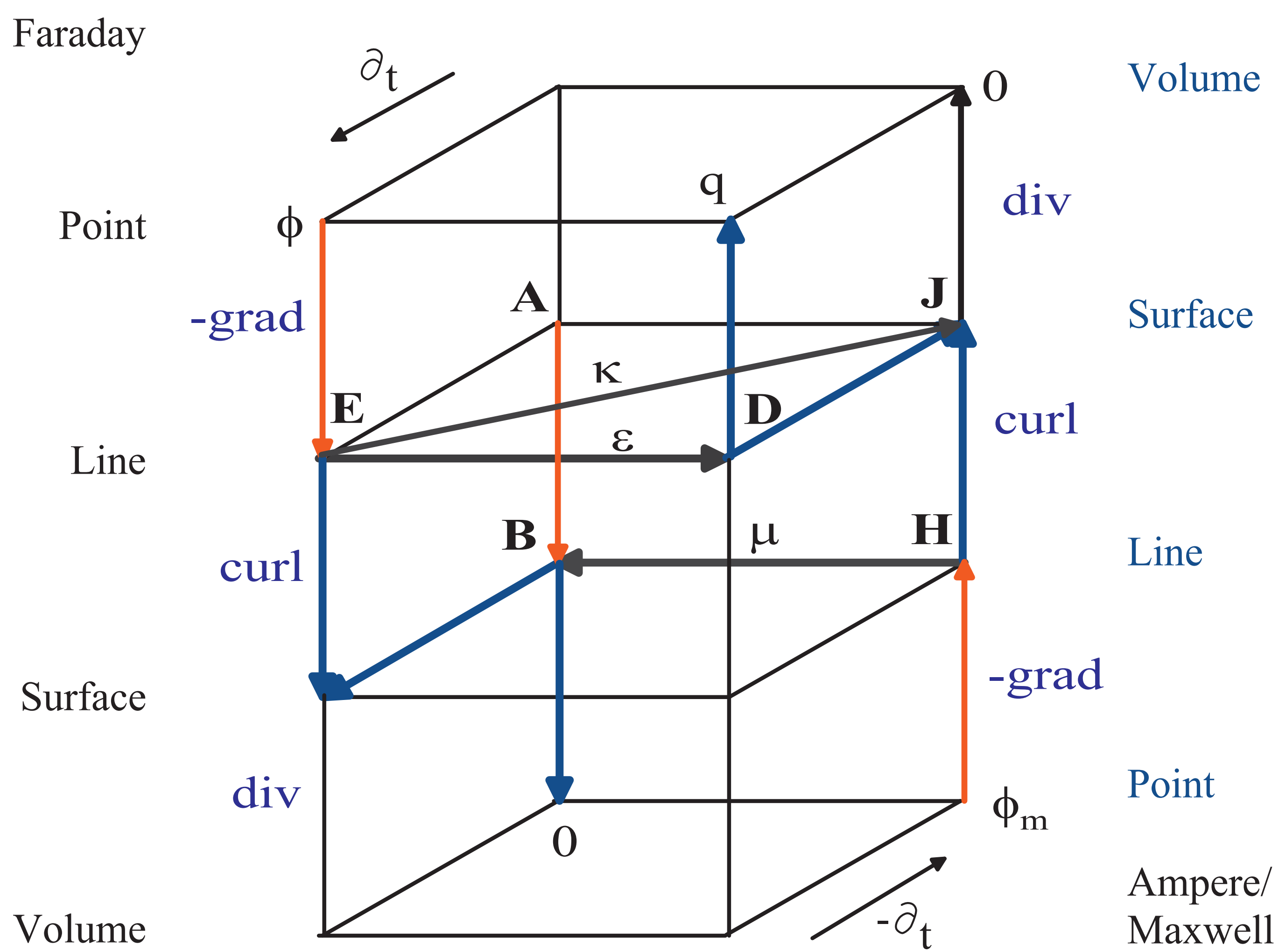
Maxwell's House

Warning: Only for trivial domains, no holes (2D), no bubbles (3D)

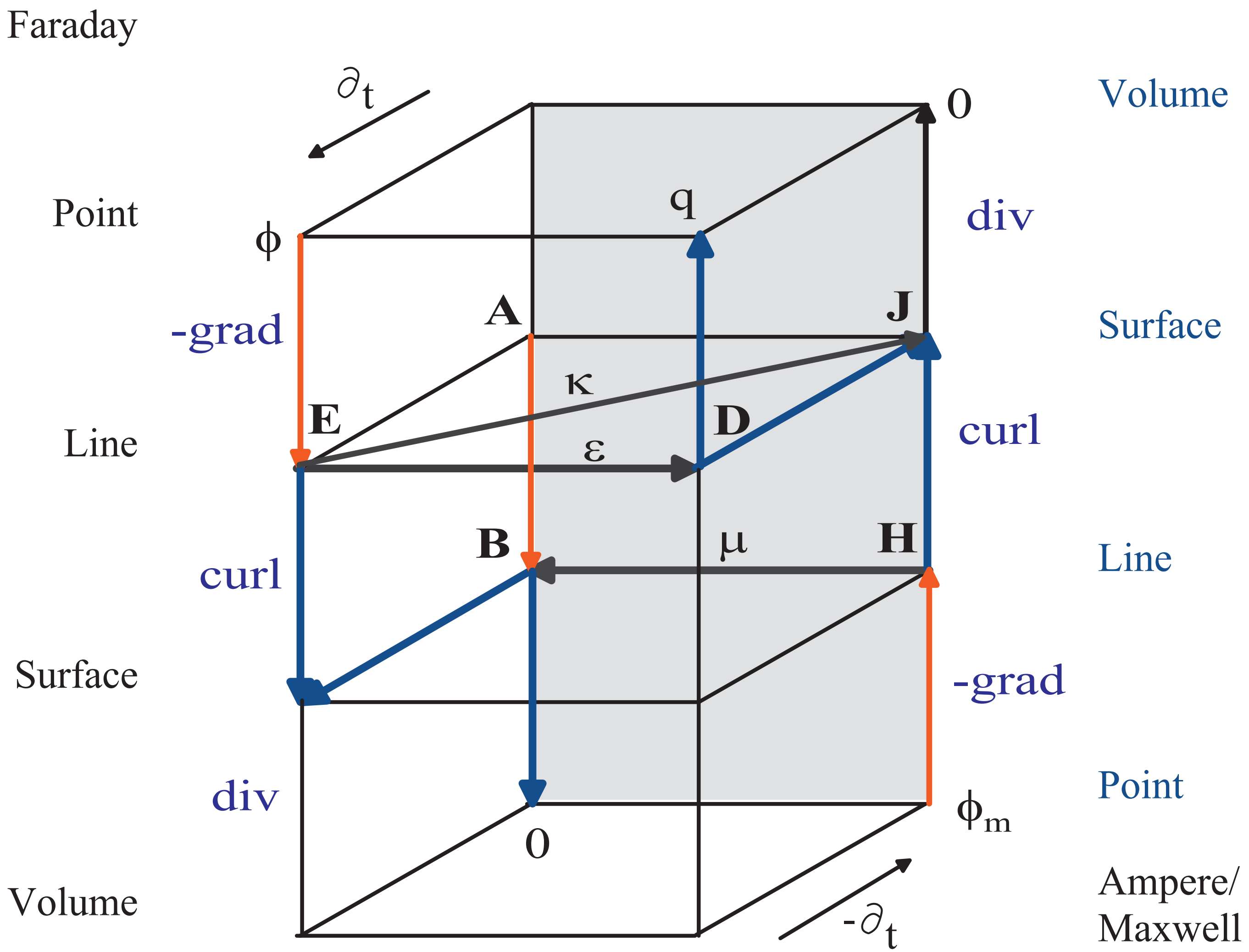
$$\mathbf{a} \xrightarrow{\text{curl}} \underset{}{\overset{H^2}{\mathbf{b}}} \xrightarrow{\text{div}} 0$$

$$f \xrightarrow{\text{grad}} \underset{H^1}{\mathbf{e}} \xrightarrow{\text{curl}} 0$$

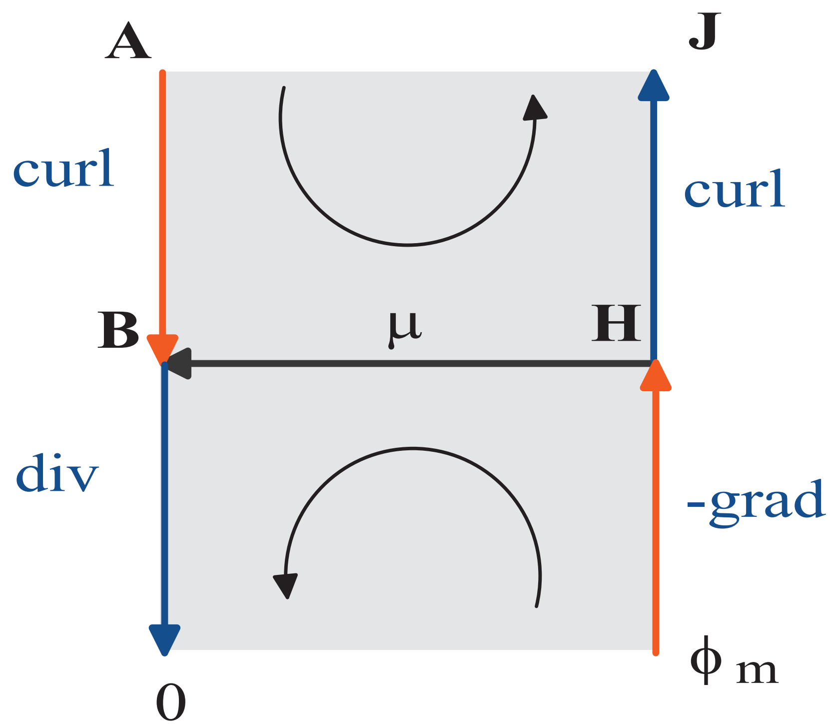



Faraday
∂t
0
Volume
Point
ϕ
q
div
-grad
A
J
Surface
κ
E
D
ε
curl
Line
B
μ
H
curl
Line
-grad
Surface
div
Point
ϕm
0
Ampere/
Maxwell
-∂t
Volume

Maxwell's Facade



$$\text{curl}\,\frac{1}{\mu}\,\text{curl}\,\mathbf{A} = \mathbf{J} \qquad \frac{1}{\mu}\,\text{curl curl}\,\mathbf{A} = 0 \qquad \nabla^2\mathbf{A} - \text{grad div}\,\mathbf{A} = 0 \qquad \nabla^2 A_z = 0$$



$$\text{div}\,\mu\,\text{grad}\,\phi_{\text{m}} = 0 \qquad \mu_0\,\text{div grad}\,\phi_{\text{m}} = 0 \qquad \nabla^2\phi_{\text{m}} = 0$$

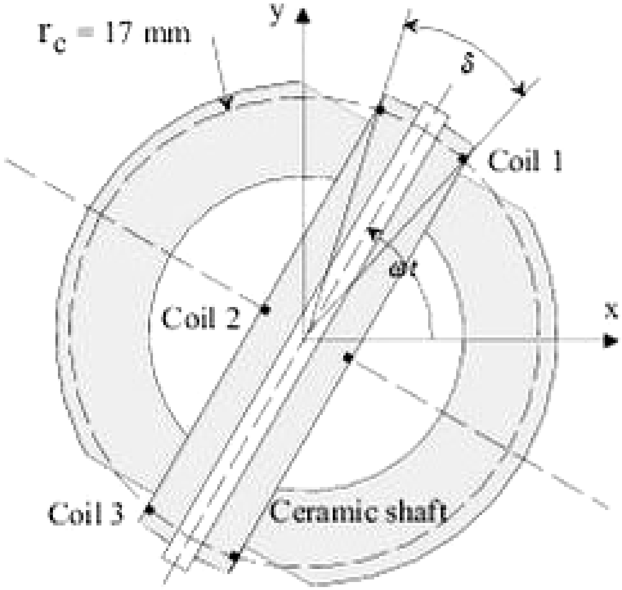
$$A_z(r,\varphi) = \sum_{n=1}^{\infty} (E_n r^n + F_n r^{-n})(G_n \sin n\varphi + H_n \cos n\varphi)$$

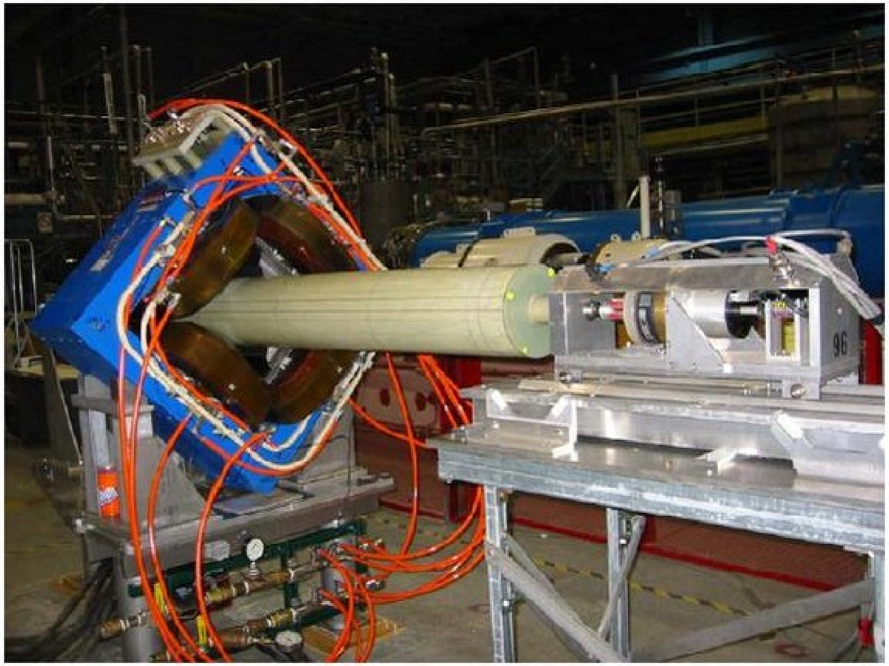
$$B_r(r,\varphi) = \frac{1}{r}\frac{\partial A_z}{\partial \varphi} = \sum_{n=1}^{\infty} n r^{n-1} (\mathcal{C}_n \sin n\varphi + \mathcal{D}_n \cos n\varphi)$$

$$n r^{n-1} \mathcal{C}_n = B_n \qquad n r^{n-1} \mathcal{D}_n = A_n$$

$$B_\varphi(r,\varphi) = -\frac{\partial A_z}{\partial r} = -\sum_{n=1}^{\infty} n r^{n-1} (\mathcal{D}_n \sin n\varphi - \mathcal{C}_n \cos n\varphi)$$

Rotating Coil Measurements





$$\Psi(t) = NL\int_{\varphi-\delta/2}^{\varphi+\delta/2} B_r(r_c,\varphi)r_c d\varphi = \sum_{n=1}^{\infty}\frac{2NLr_c}{n}\sin(\frac{n\delta}{2})[B_n(r_c)\sin(n\omega t + n\Theta) + A_n(r_c)\cos(n\omega t + n\Theta)]$$

$$V(t) = -\frac{d\Psi}{dt} = \sum_{n=1}^{\infty} 2NLr_c\omega\sin(\frac{n\delta}{2})[-B_n(r_c)\cos(n\omega t + n\Theta) + A_n(r_c)\sin(n\omega t + n\Theta)]$$

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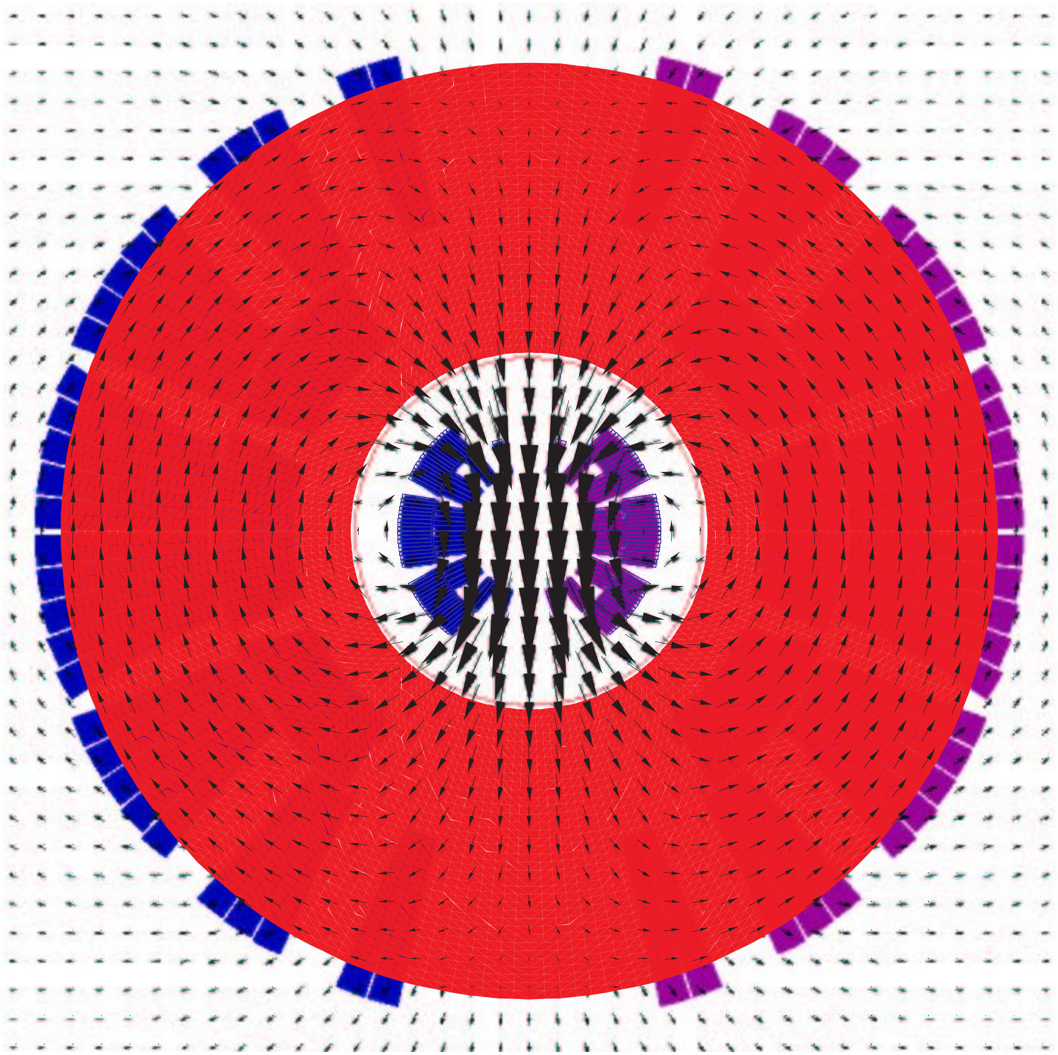
Imaging Method



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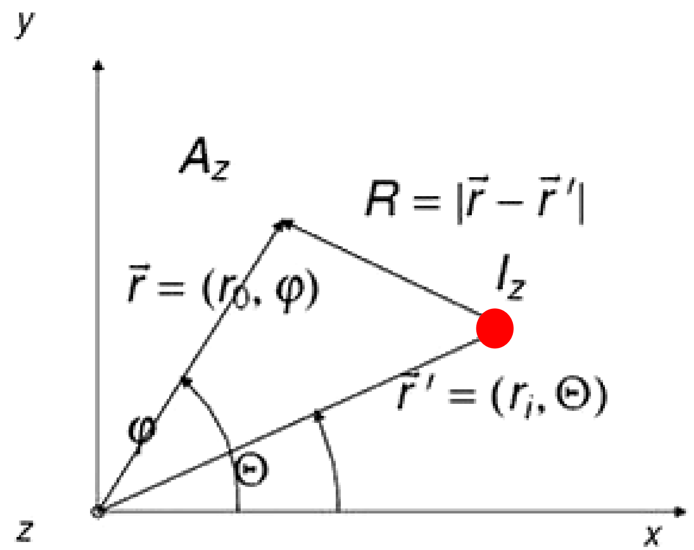
The Field of a Line Current



$$A_z = -\frac{\mu_0 I}{2\pi} ln(\frac{R}{R_{ref}})$$

$$B_r = -\frac{\mu_0 I}{2\pi}\sum_{n=1}^{\infty}(\frac{r_0^{n-1}}{r_i^n})(\sin n\varphi \; \cos n\Theta - \cos n\varphi \; \sin n\Theta)$$

$$B_n(r_0) = -\frac{\mu_0 I}{2\pi}\frac{r_0^{n-1}}{r_i^n} \cos n\Theta \qquad A_n(r_0) = \frac{\mu_0 I}{2\pi}\frac{r_0^{n-1}}{r_i^n} \sin n\Theta$$

Field Components

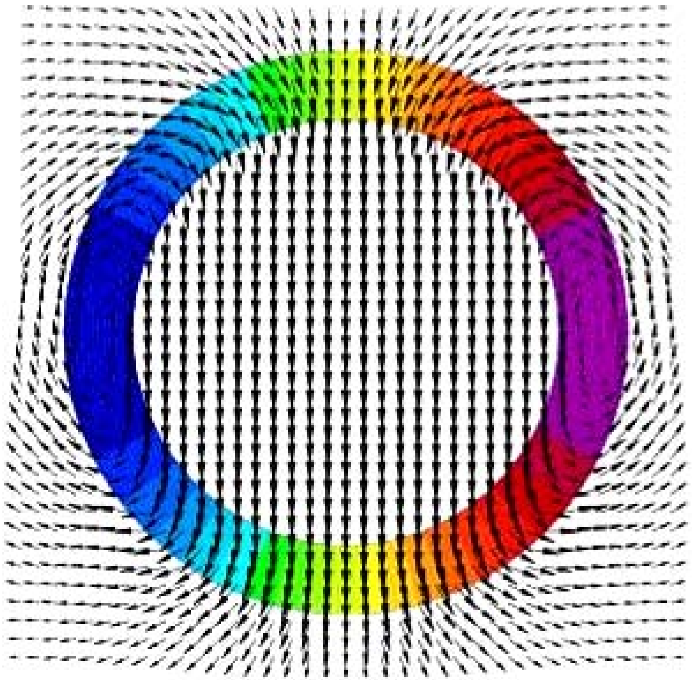
$$B_n(r_0) = -\sum_{i=1}^{n_s} \frac{\mu_0 I_i}{2\pi} \frac{r_0^{n-1}}{r_i^n} \left(1 + \frac{\mu_r - 1}{\mu_r + 1}\left(\frac{r_i}{R_{Yoke}}\right)^{2n}\right) \cos n\Theta_i$$

Influence of the iron yoke (non-saturated)

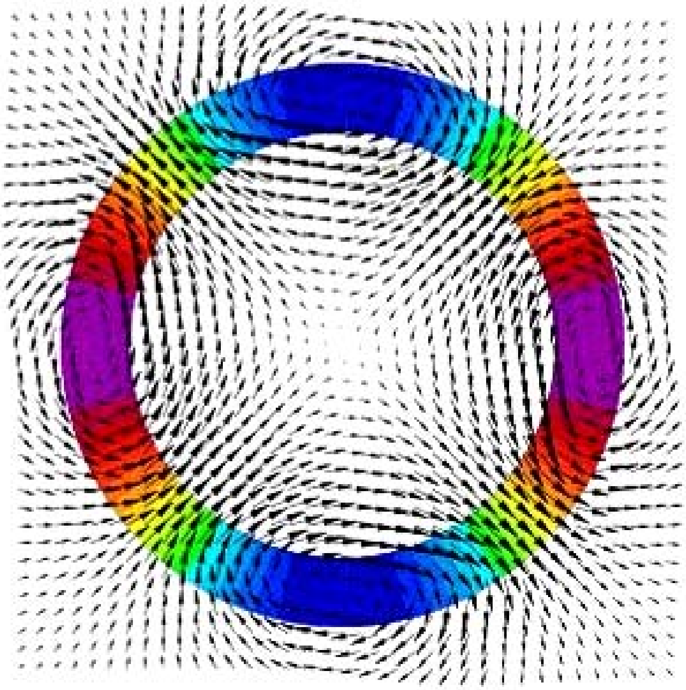
$r_i = 43.5$ mm, $R_{Yoke} = 89$ mm: B_1 - 19% , B_5 - 0.07%

$$B_n(r_0) = -\sum_{i=1}^{n_s} \int_{r_i}^{r_o} \int_0^{2\pi} \frac{\mu_0 J_0 \cos m\Theta}{2\pi} \frac{r_0^{n-1}}{r_i^n} \left(1 + \frac{\mu_r - 1}{\mu_r + 1} (\frac{r_i}{R_{Yoke}})^{2n}\right) \cos n\Theta_i r d\Theta dr$$

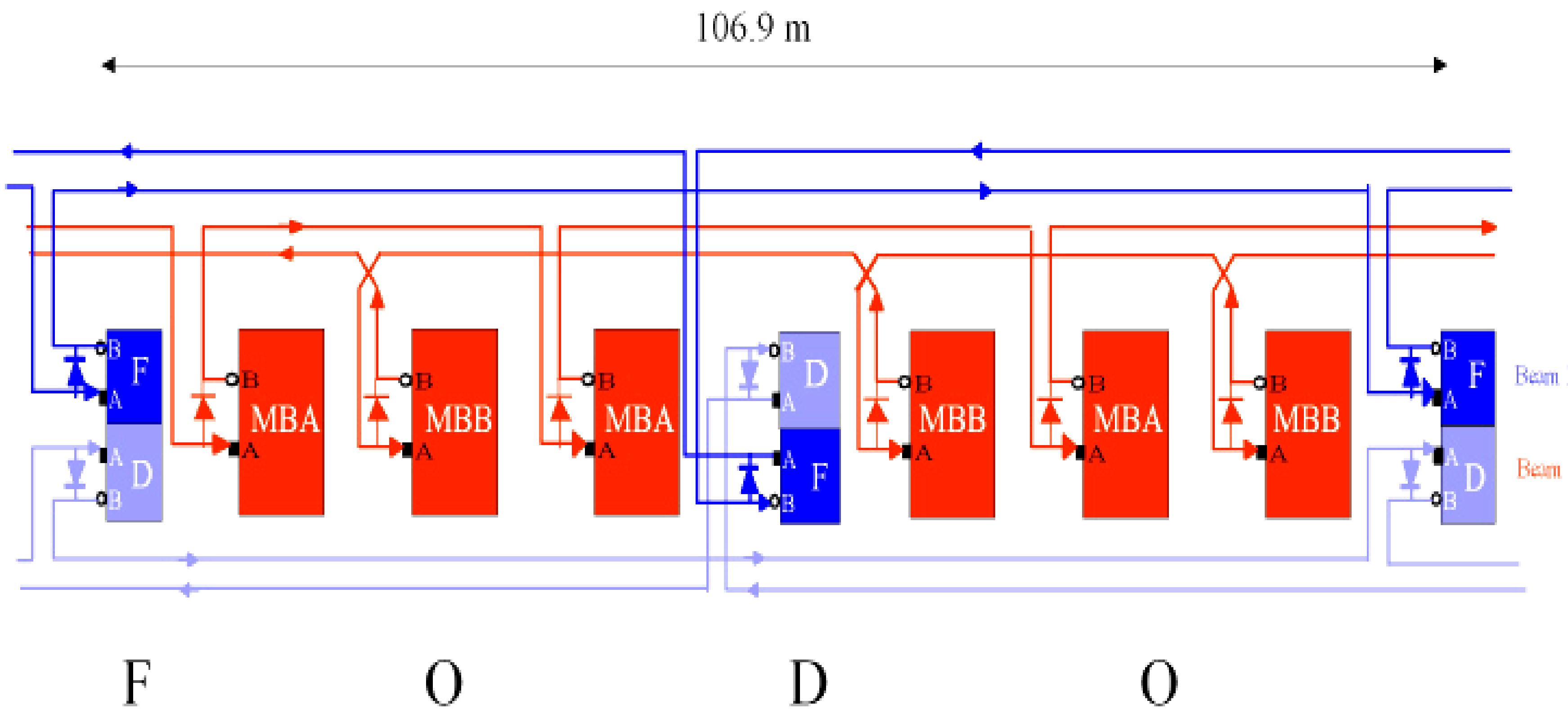
Dipole



Quadrupole



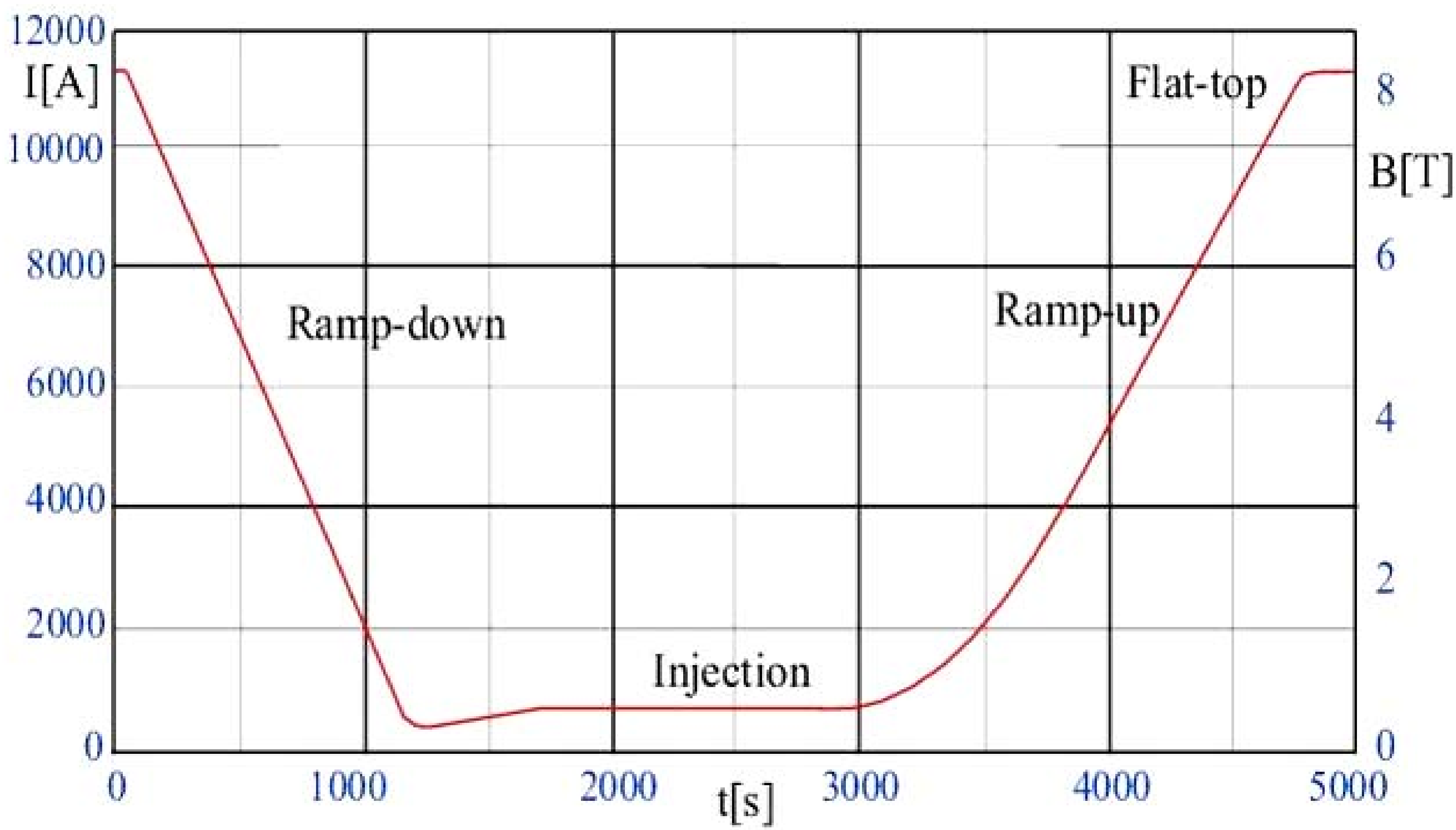
The FODO Cell



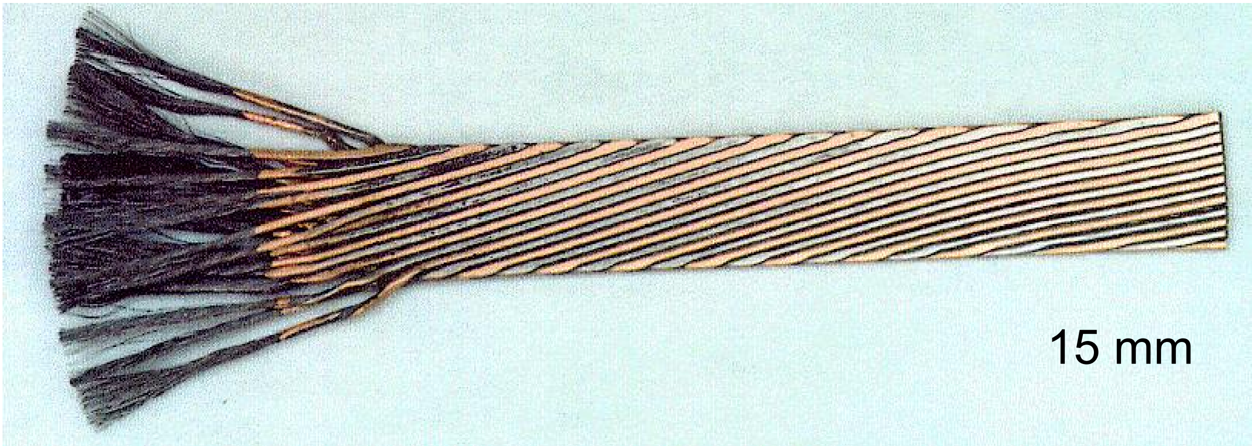
23 Arc cells plus 8 DS cells (154 dipole magnets)

$V \approx 2\,E\,/\,I\,t$

E = 1.15 TJ (320 kWh), I = 11800 A, Ramp rate 10 A/s, 155 V

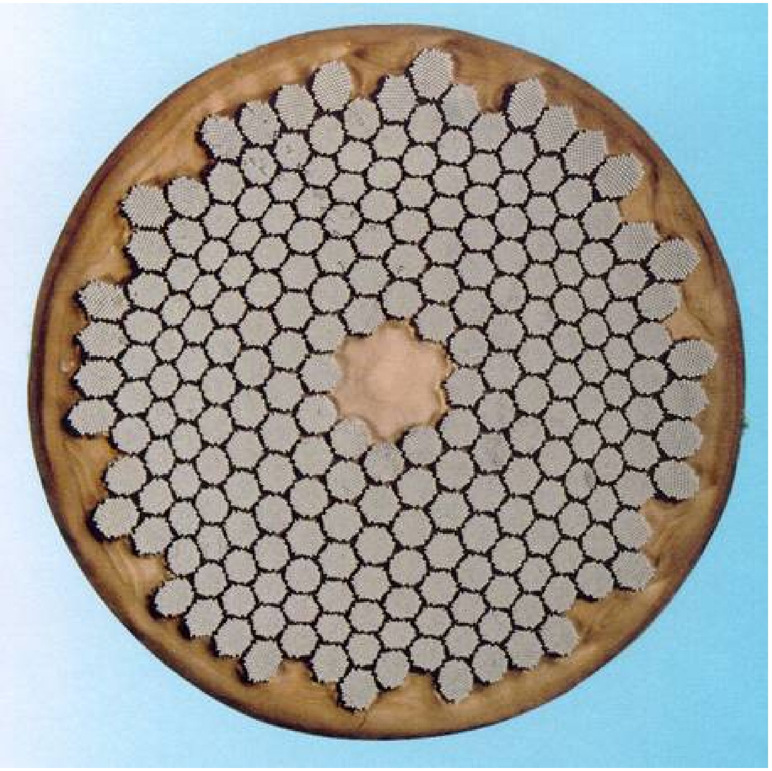


Rutherford Cable, Strand, NbTi Filament



15 mm

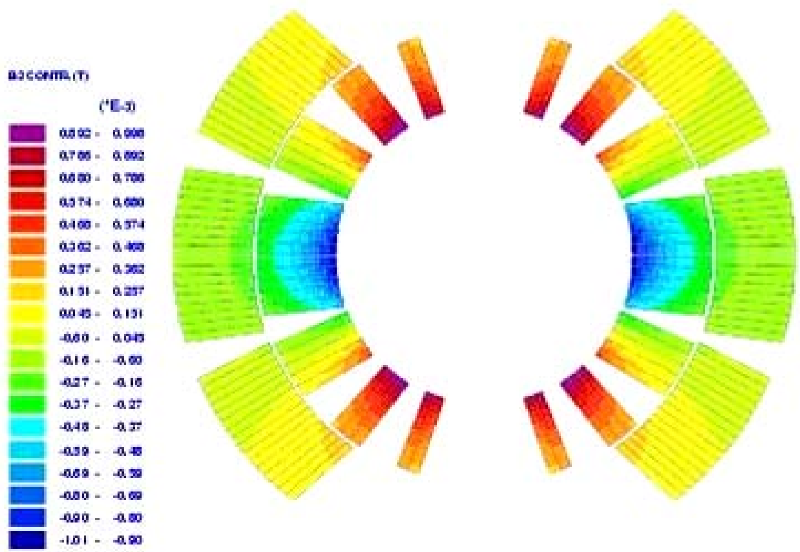
1 mm

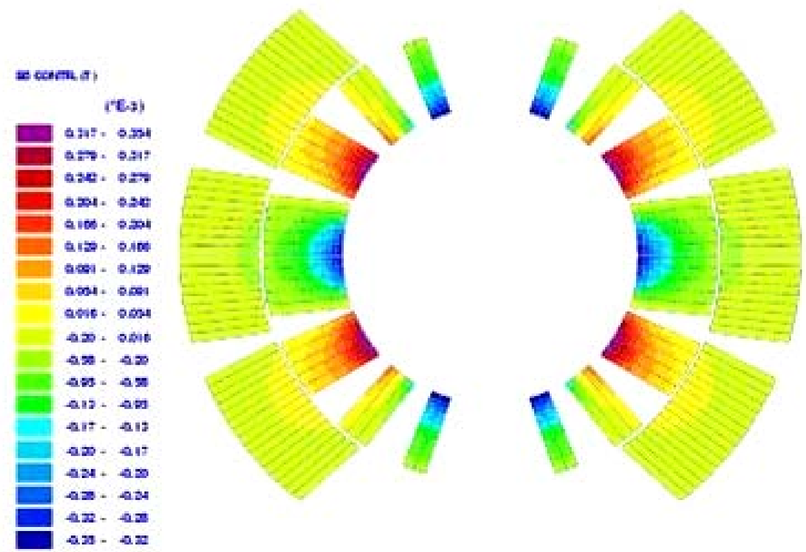


6 µm

Coil Block Approximation

$$B_n(r_0) = -\frac{\mu_0 I_i}{2\pi} \frac{r_0^{n-1}}{r_i^n} \left(1 + \frac{\mu_r - 1}{\mu_r + 1} \left(\frac{r_i}{R_{\mathrm{Yoke}}}\right)^{2n}\right) \cos n\Theta_i$$





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Sensitivity to Manufacturing Errors

$$B_n(r_0) = -\frac{\mu_0 I_i}{2\pi}\frac{r_0^{n-1}}{r_i^n}\left(1 + \frac{\mu_r - 1}{\mu_r + 1}\left(\frac{r_i}{R_{\mathrm{Yoke}}}\right)^{2n}\right)\cos n\Theta_i$$

$$\frac{\partial B_n(r_0)}{\partial \Theta_i} = -\frac{\mu_0 I_i}{2\pi}\frac{n r_0^{n-1}}{r_i^n}\left(1+\left(\frac{r_i}{R_{\mathrm{Yoke}}}\right)^{2n}\right)\sin n\Theta_i$$

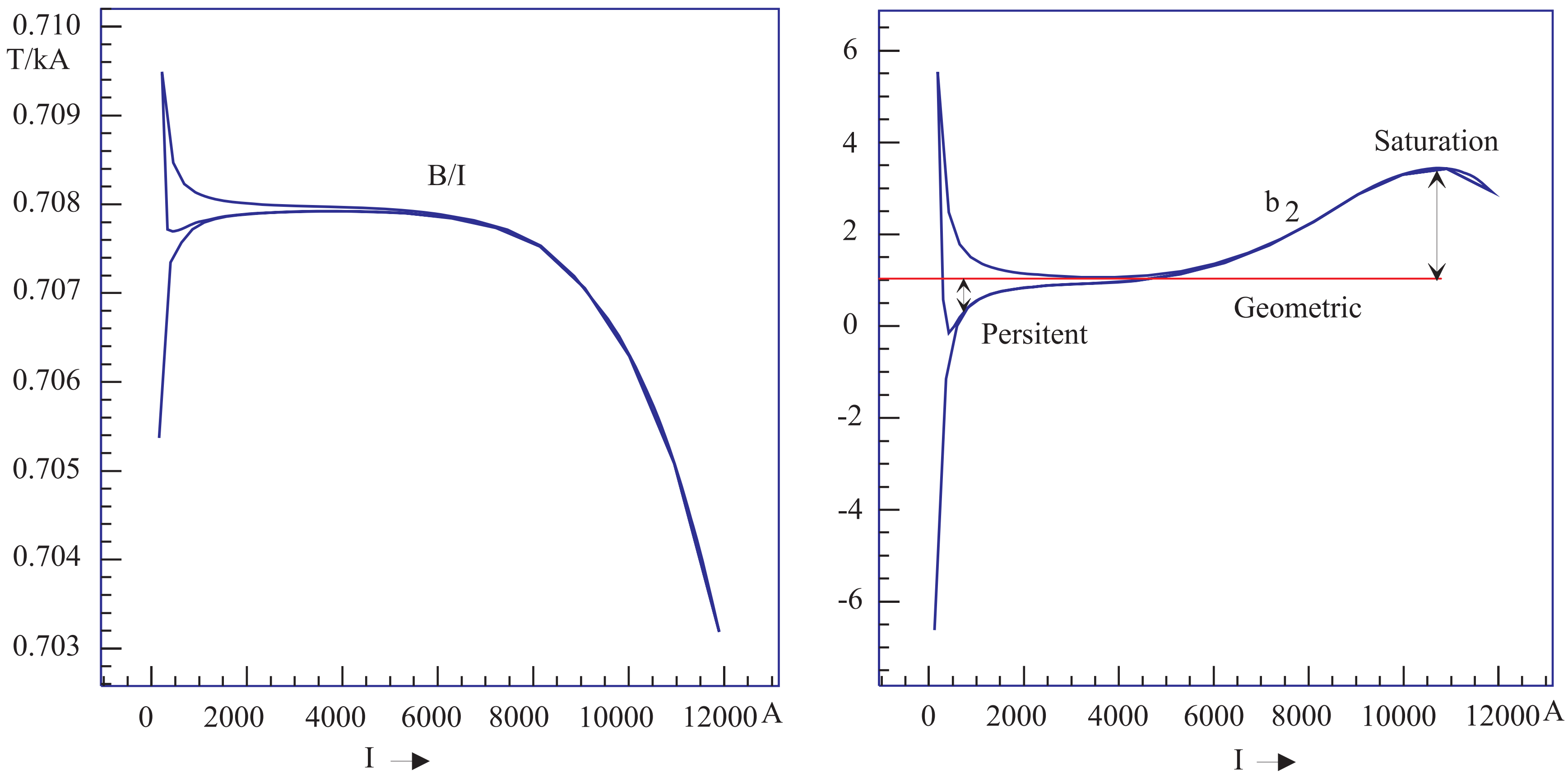
$$\frac{\partial B_n(r_0)}{\partial r_i} = \frac{\mu_0 I_i}{2\pi}\frac{n r_0^{n-1}}{r_i^{n+1}}\left(1-\left(\frac{r_i}{R_{\mathrm{Yoke}}}\right)^{2n}\right)\cos n\Theta_i$$

Increase of the azimuthal coil size by 0.1 mm produces (in units of 10^{-4}):

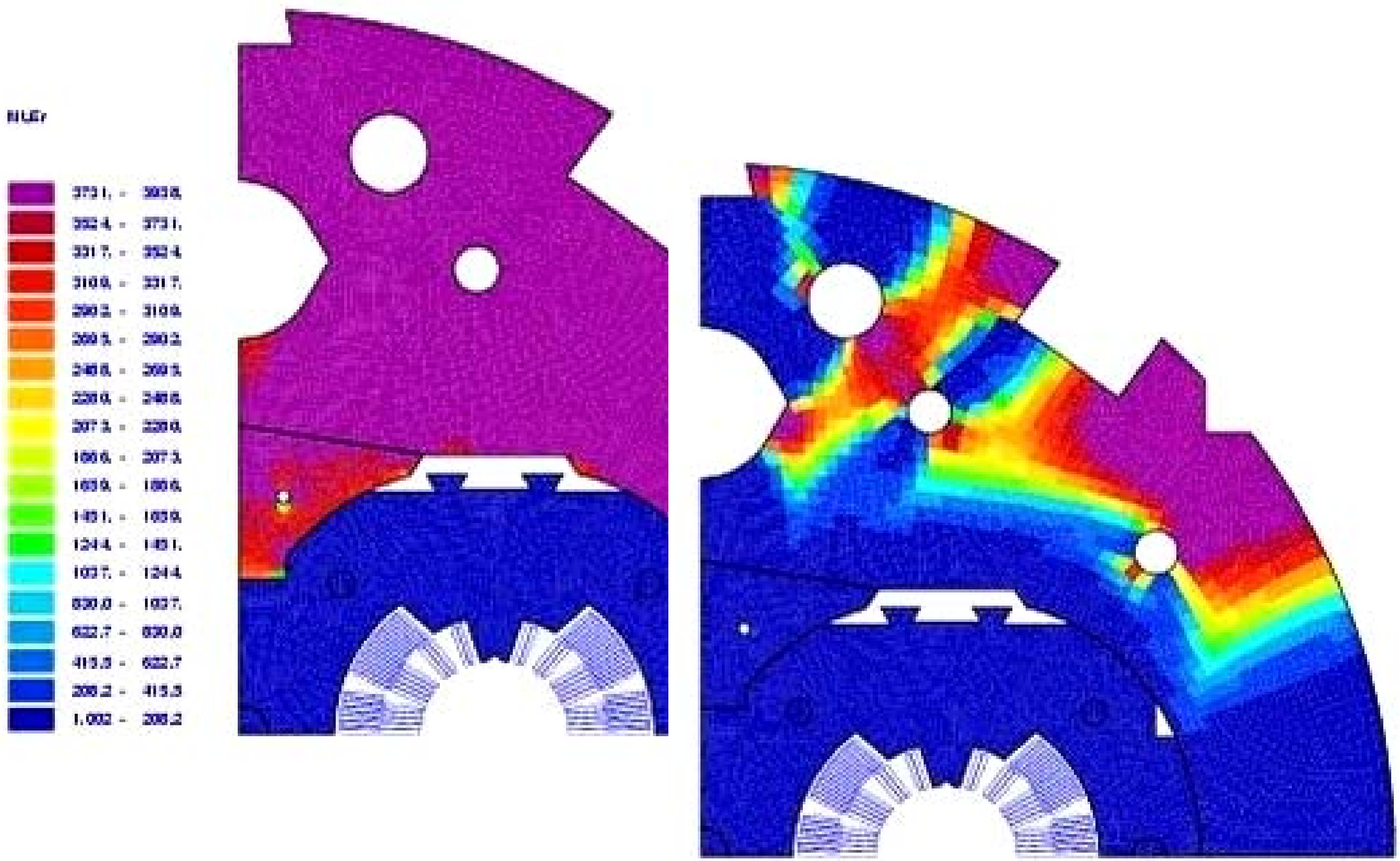
$b_1 = -14.\qquad b_3 = 1.2\qquad b_5 = 0.03$

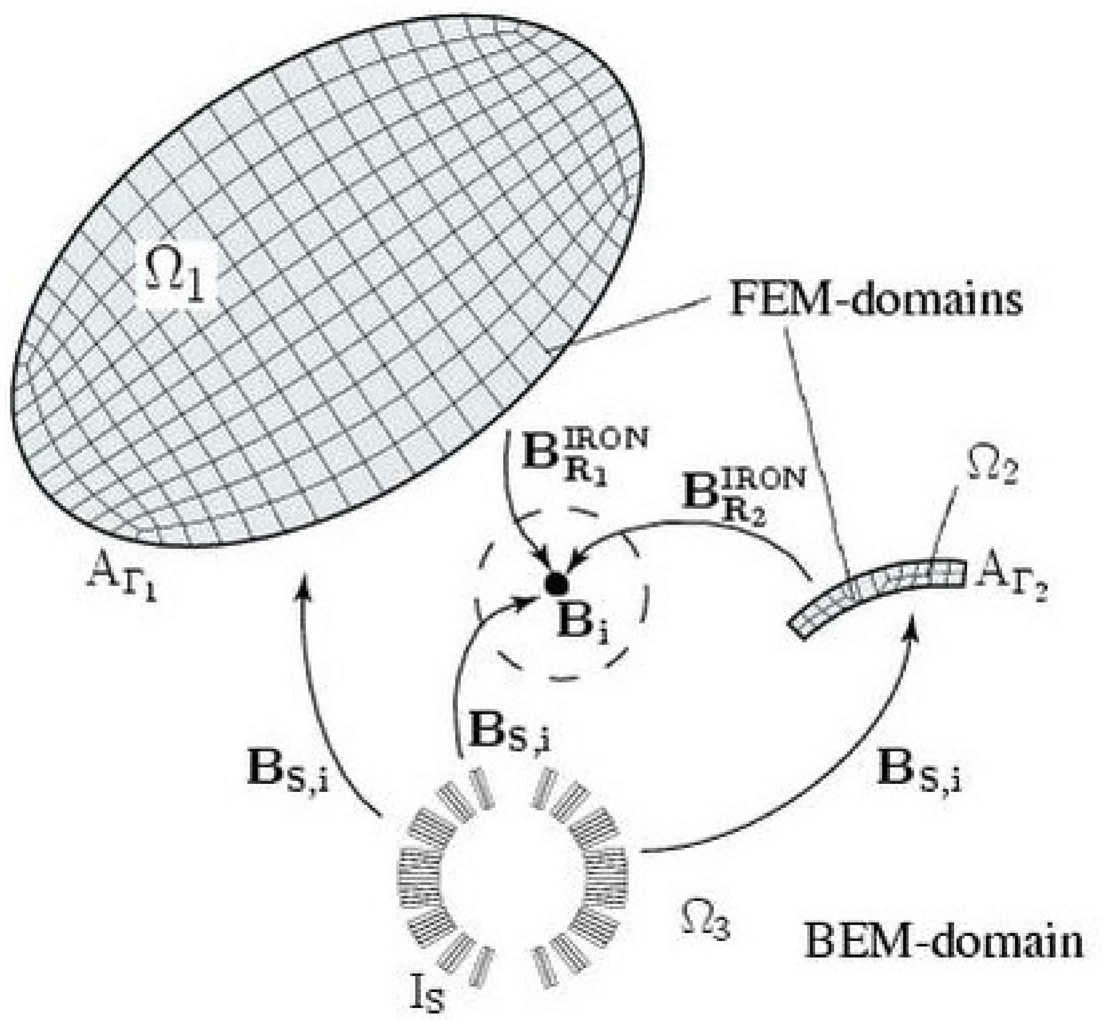
Specified tolerances on coils: $\pm\, 0.025$ mm

Coil Winding and Curing

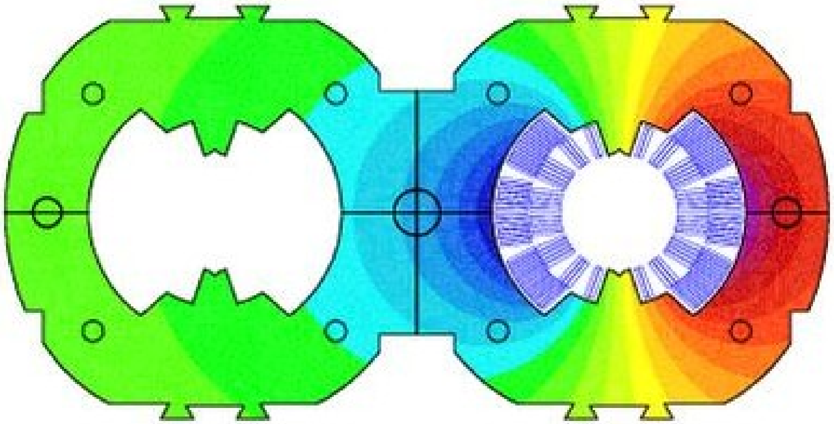

0.710
T/kA
0.709
0.708
0.707
0.706
0.705
0.704
0.703
B/I
0
2000
4000
6000
8000
10000
12000
A
I
6
4
2
0
-2
-4
-6
Saturation
b 2
Geometric
Persitent
0
2000
4000
6000
8000
10000
12000
A
I

Saturation Effects in the Dipole Iron Yoke




Ω1
FEM-domains
$B_{R_1}^{IRON}$
$B_{R_2}^{IRON}$
Ω2
A_{Γ_1}
A_{Γ_2}
B_i
$B_{S,i}$
$B_{S,i}$
$B_{S,i}$
Ω3
BEM-domain
I_S

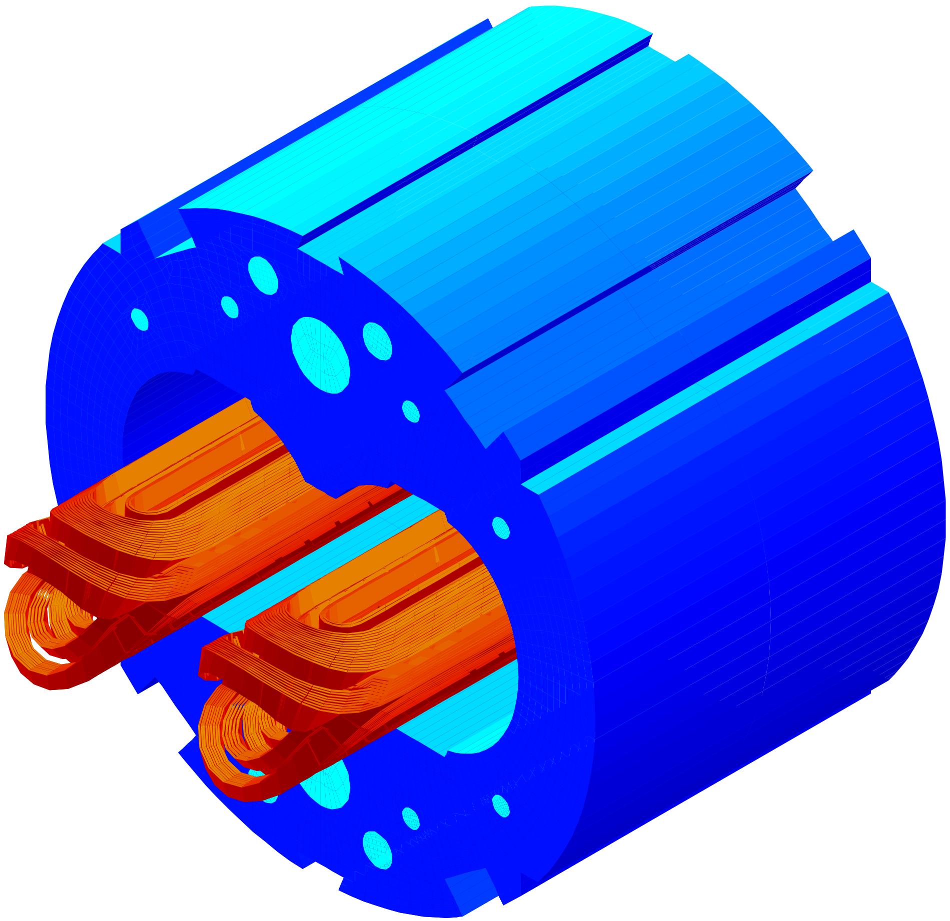
Open Boundary Problem



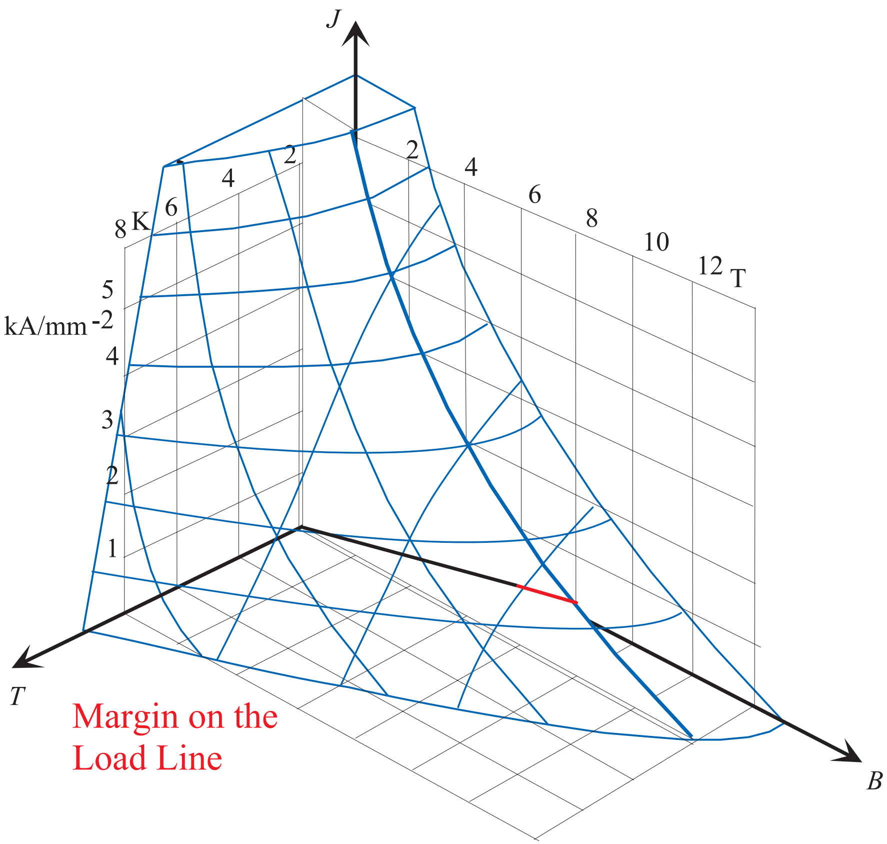
Collared Coil Field Problem

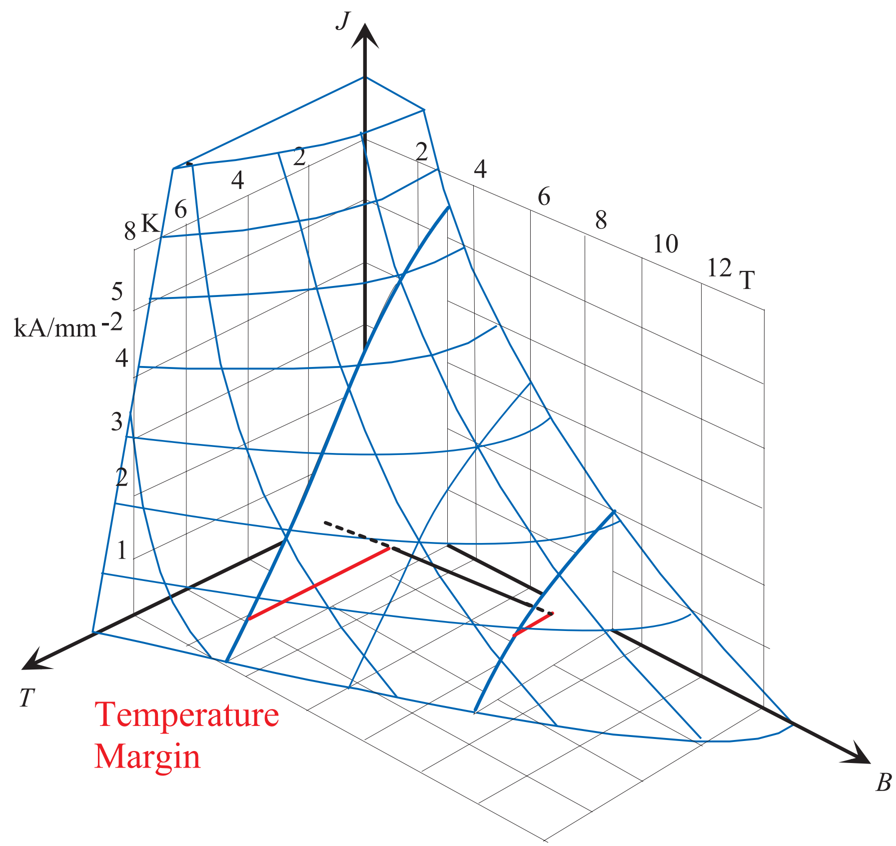
Collared Coil Measurements in Industry

Dipole Model Extremities

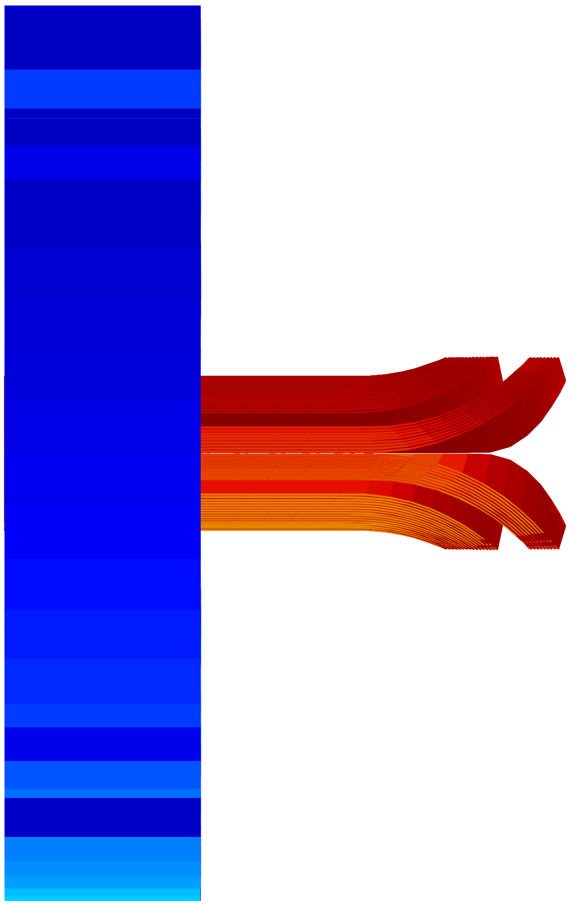


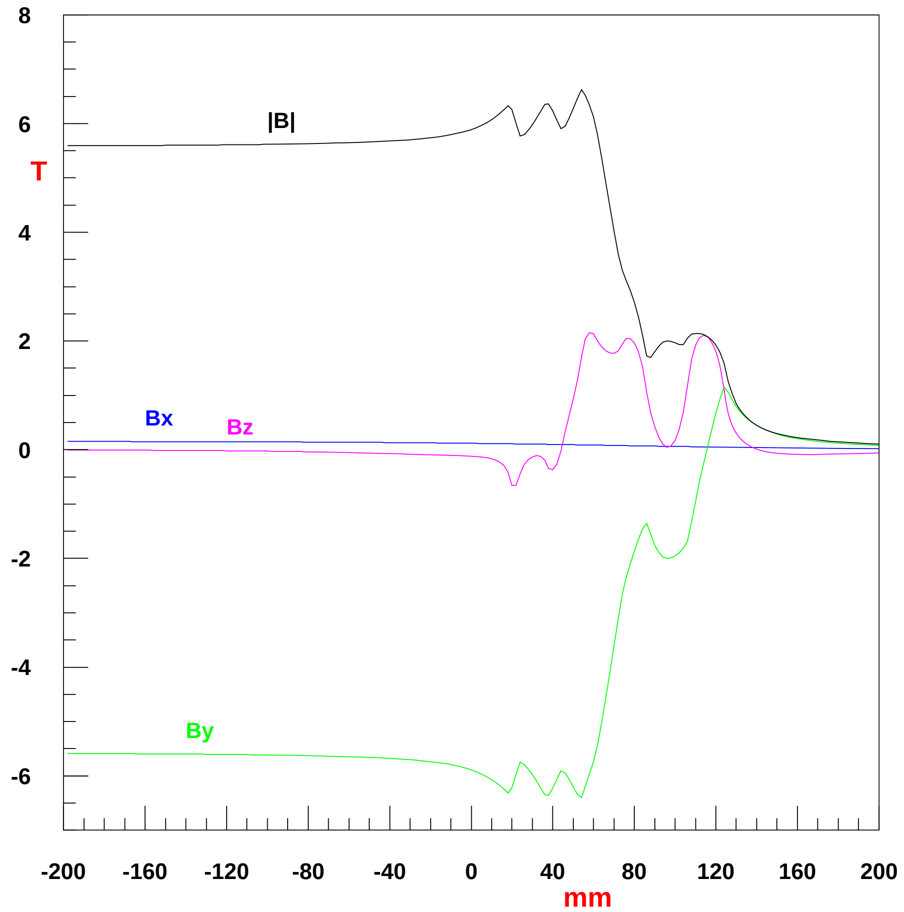
Critical Surface of NbTi



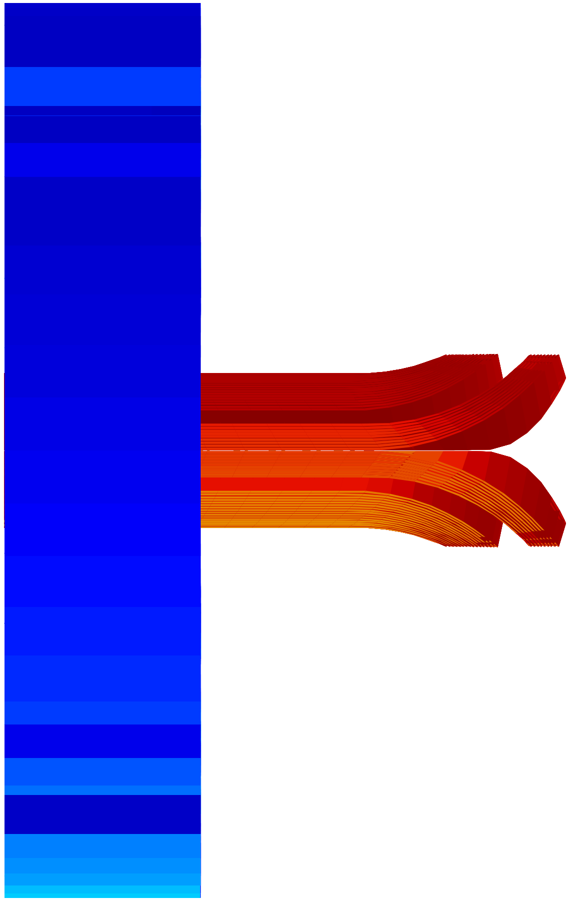


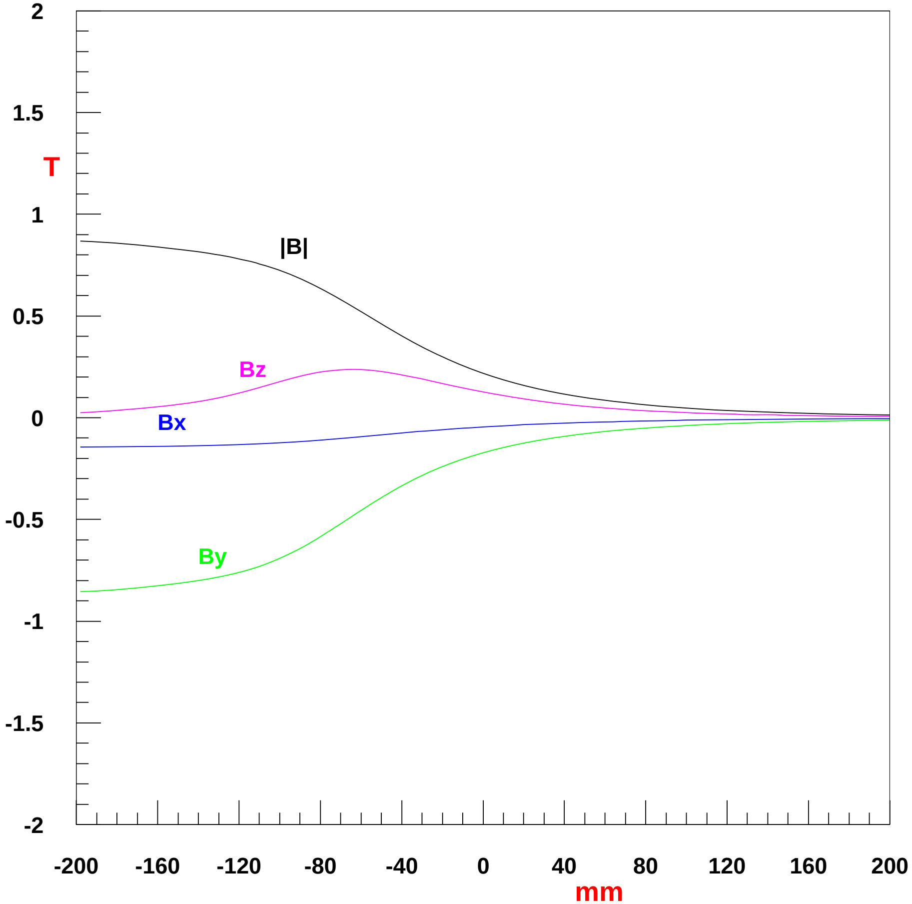
Source Field



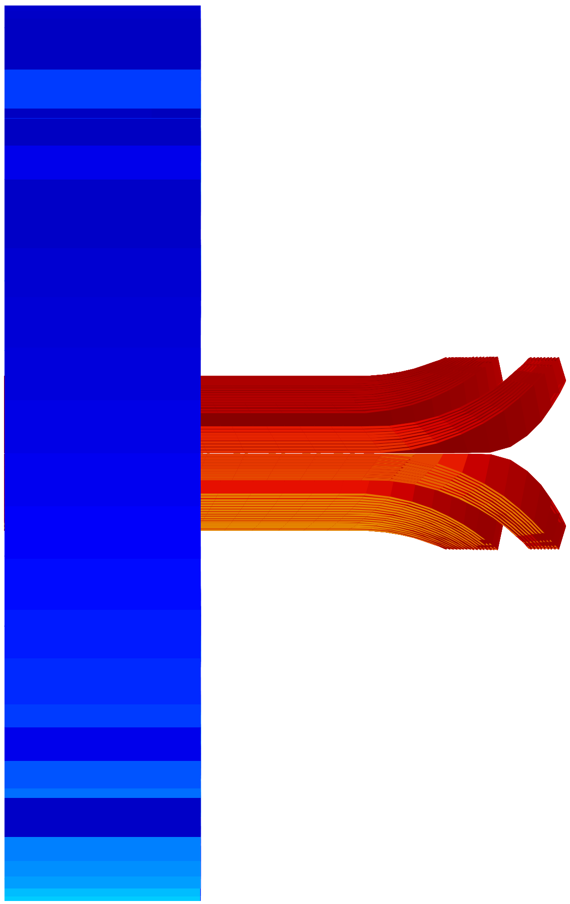


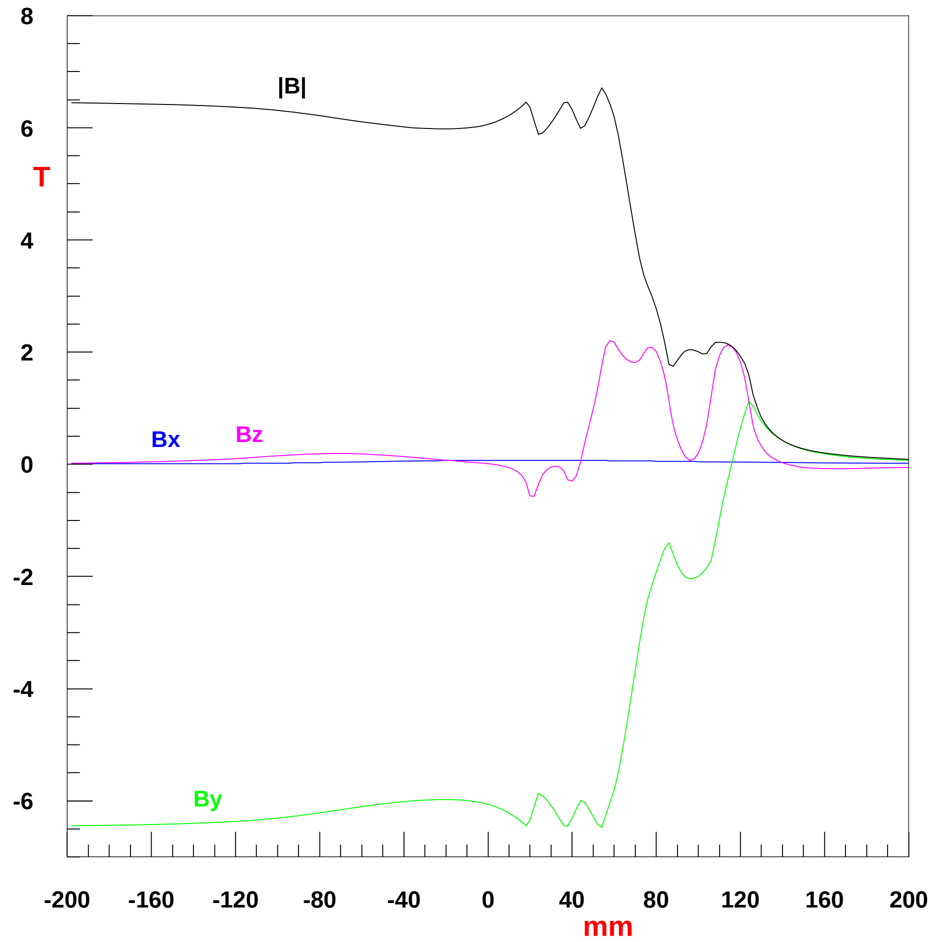
Reduced Field

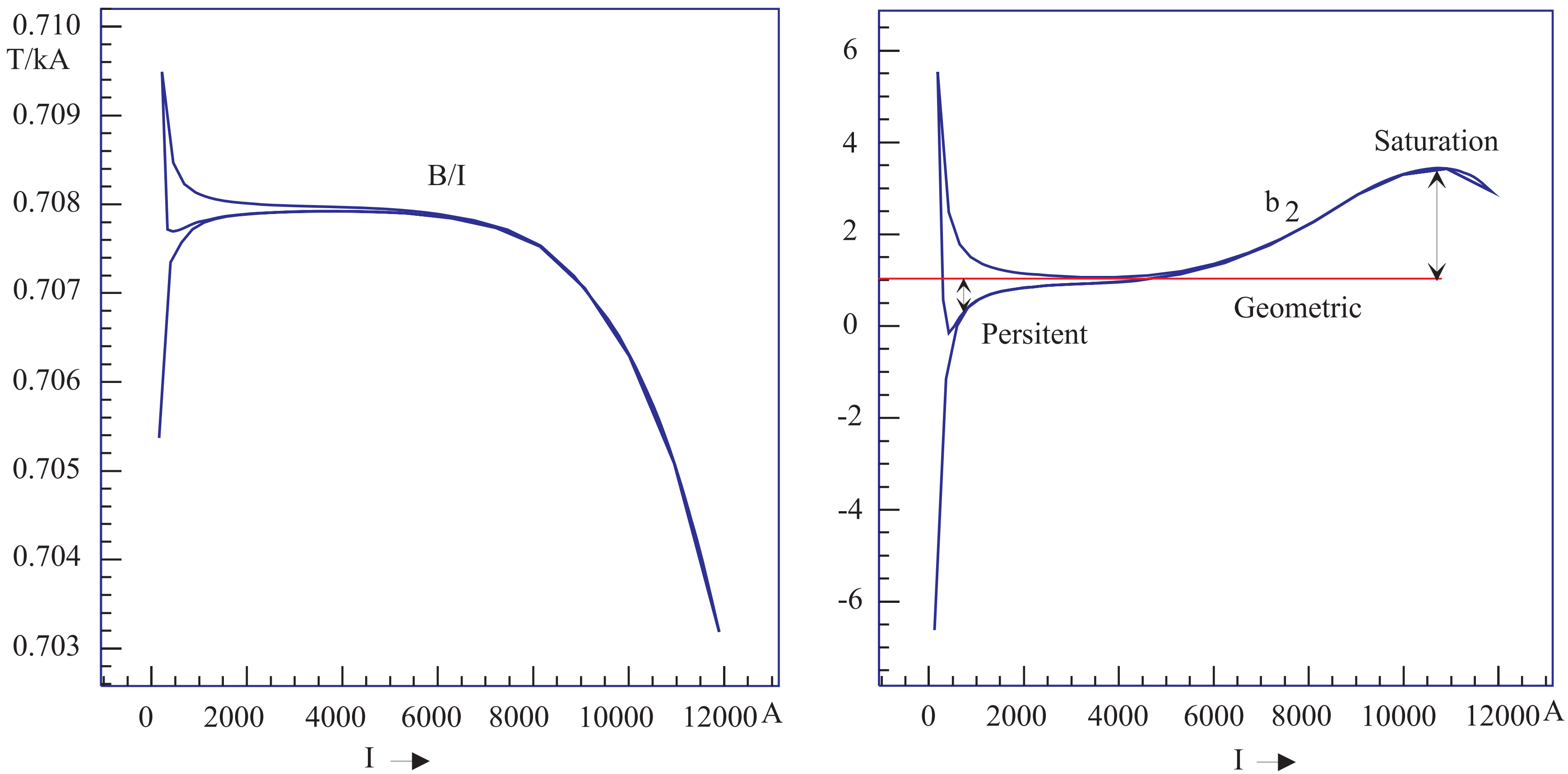


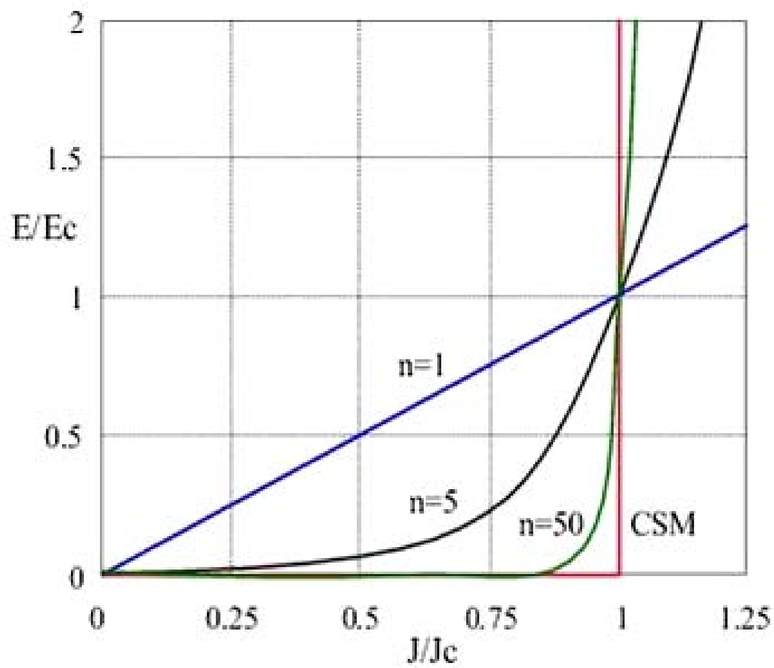


Total Field





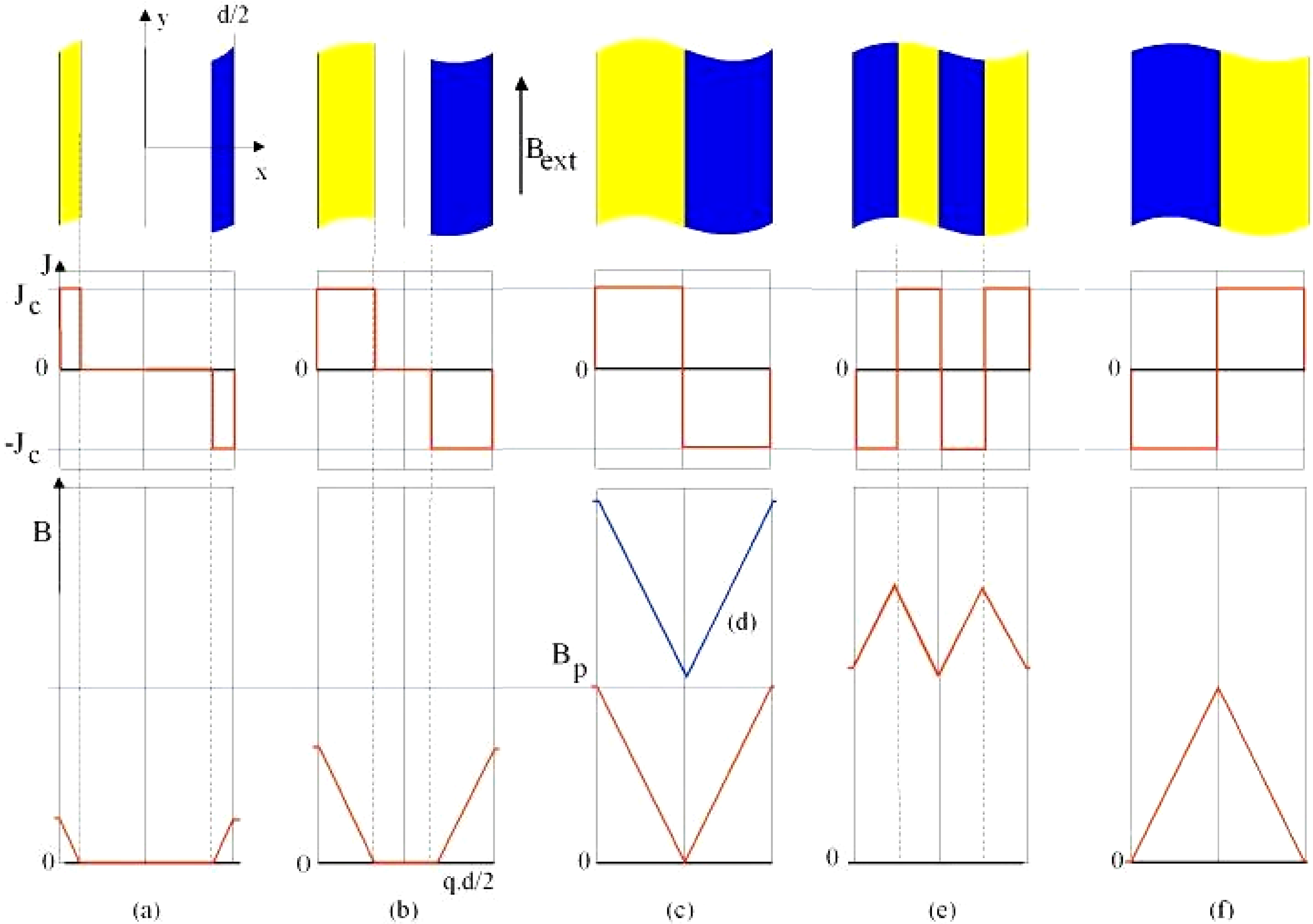

0.710
T/kA
0.709
0.708
0.707
0.706
0.705
0.704
0.703
B/I
0
2000
4000
6000
8000
10000
12000
A
I
6
4
2
0
-2
-4
-6
Saturation
b 2
Geometric
Persitent
0
2000
4000
6000
8000
10000
12000
A
I



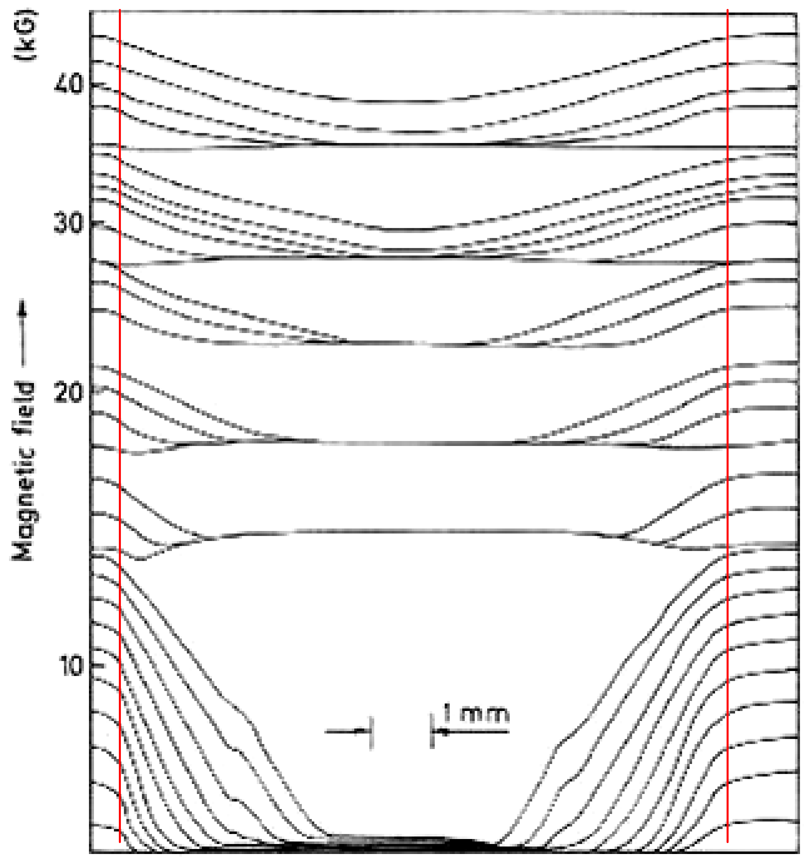
$$\vec{E} = E_c \left(\frac{|\vec{J}|}{J_c} \right)^{n-1} \frac{\vec{J}}{J_c}$$

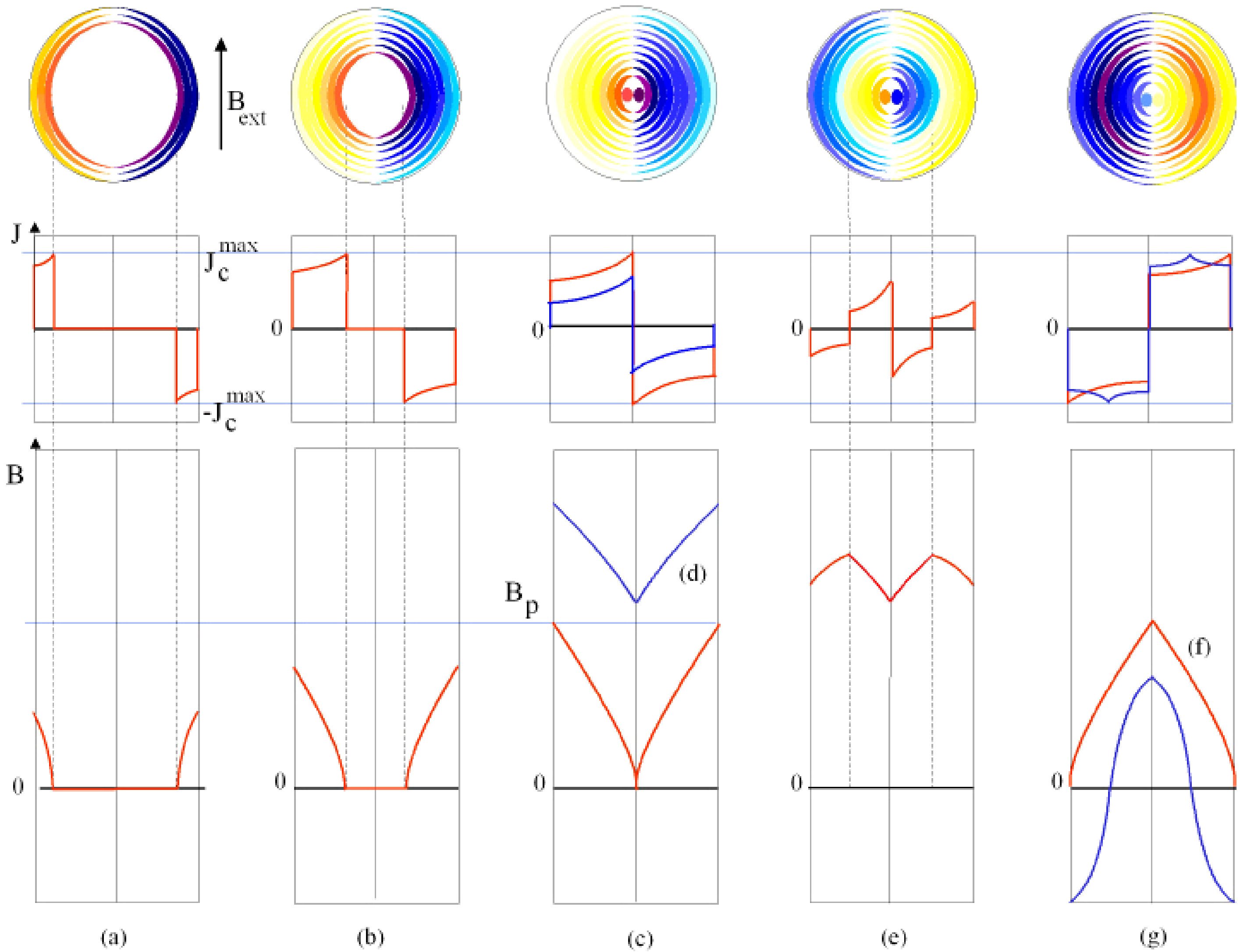
$$E_c = 1 \mu V/cm \qquad J_c(5T, 4.2K) = 3 \cdot 10^9 A/m^2 \qquad n(1.8K, 4T) = 50$$

Bean's Critical State Model (CSM)



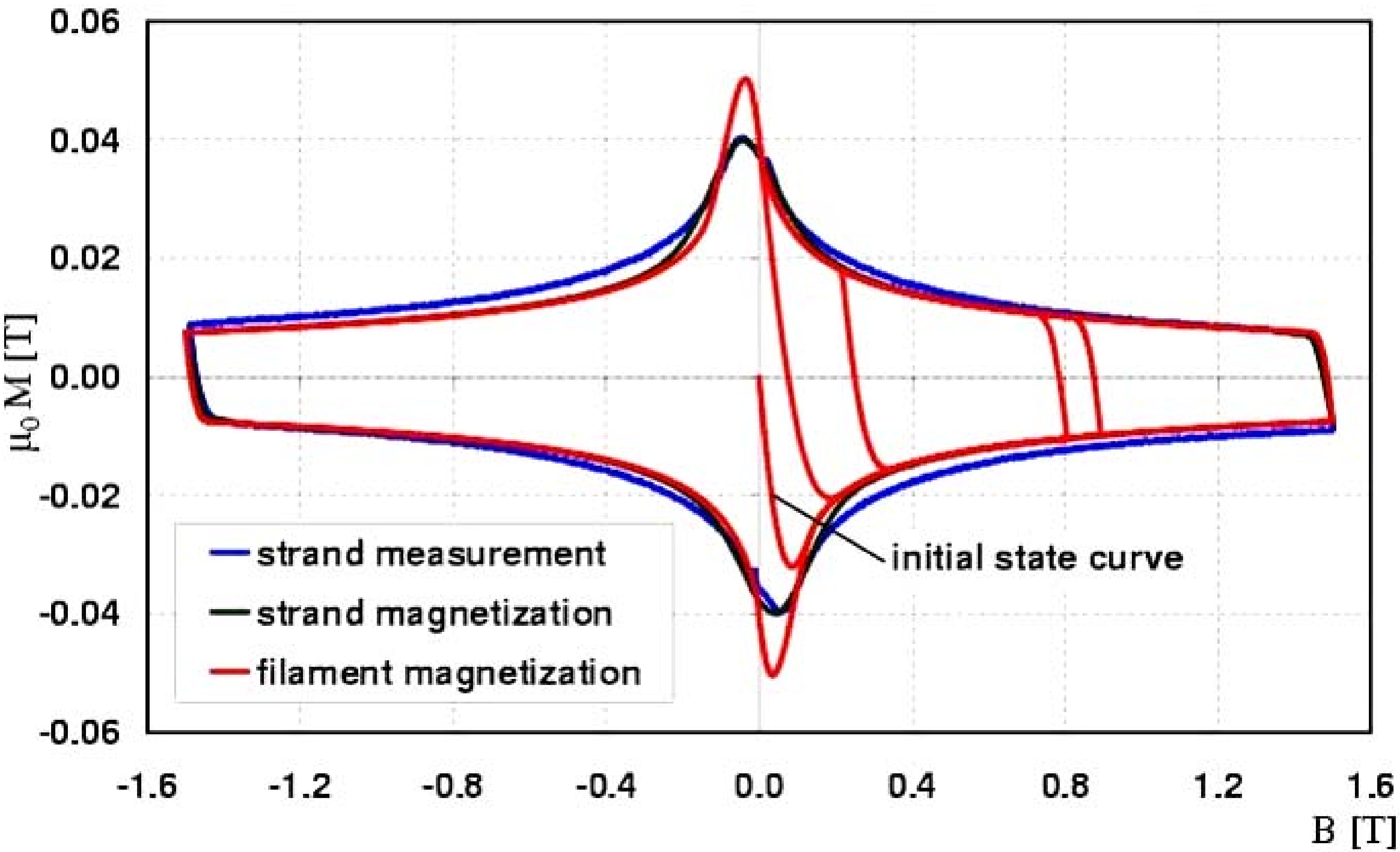
Stephan Russenschuck, CERN-AT-MEL


(kG)
40
30
20
10
Magnetic field
1 mm

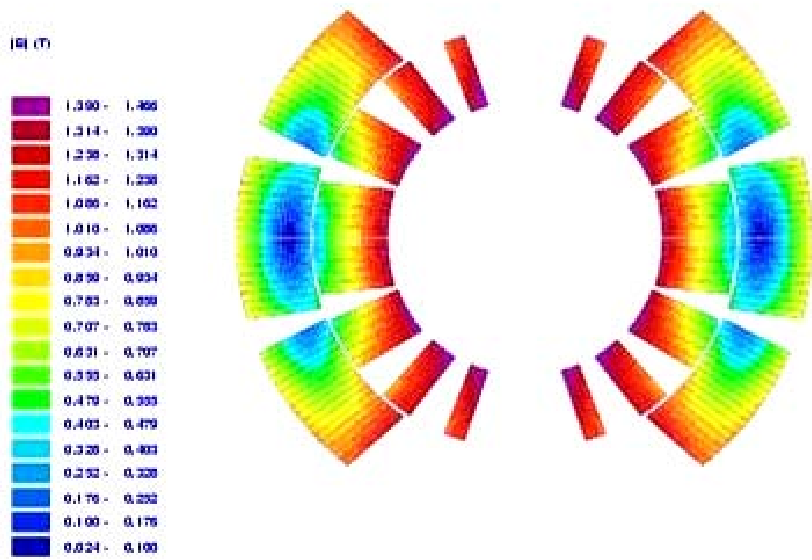


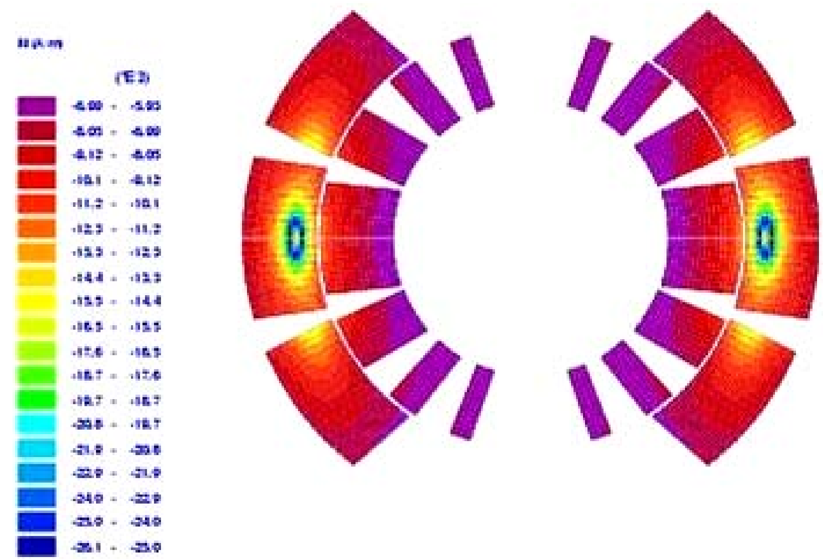
B
ext
J
max
J
c
-J
c
0
B
B
p
(d)
(f)
(a)
(b)
(c)
(e)
(g)

Magnetization in LHC Strands

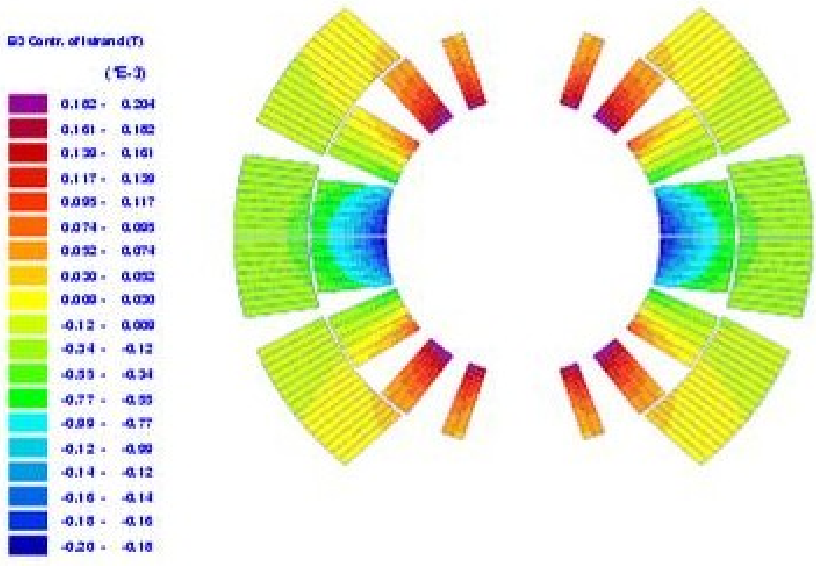


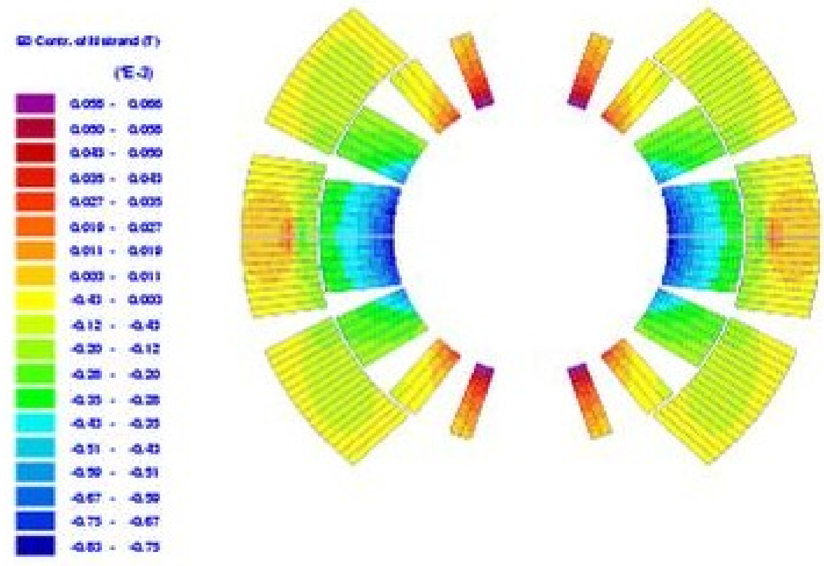
Field and SC Magnetization



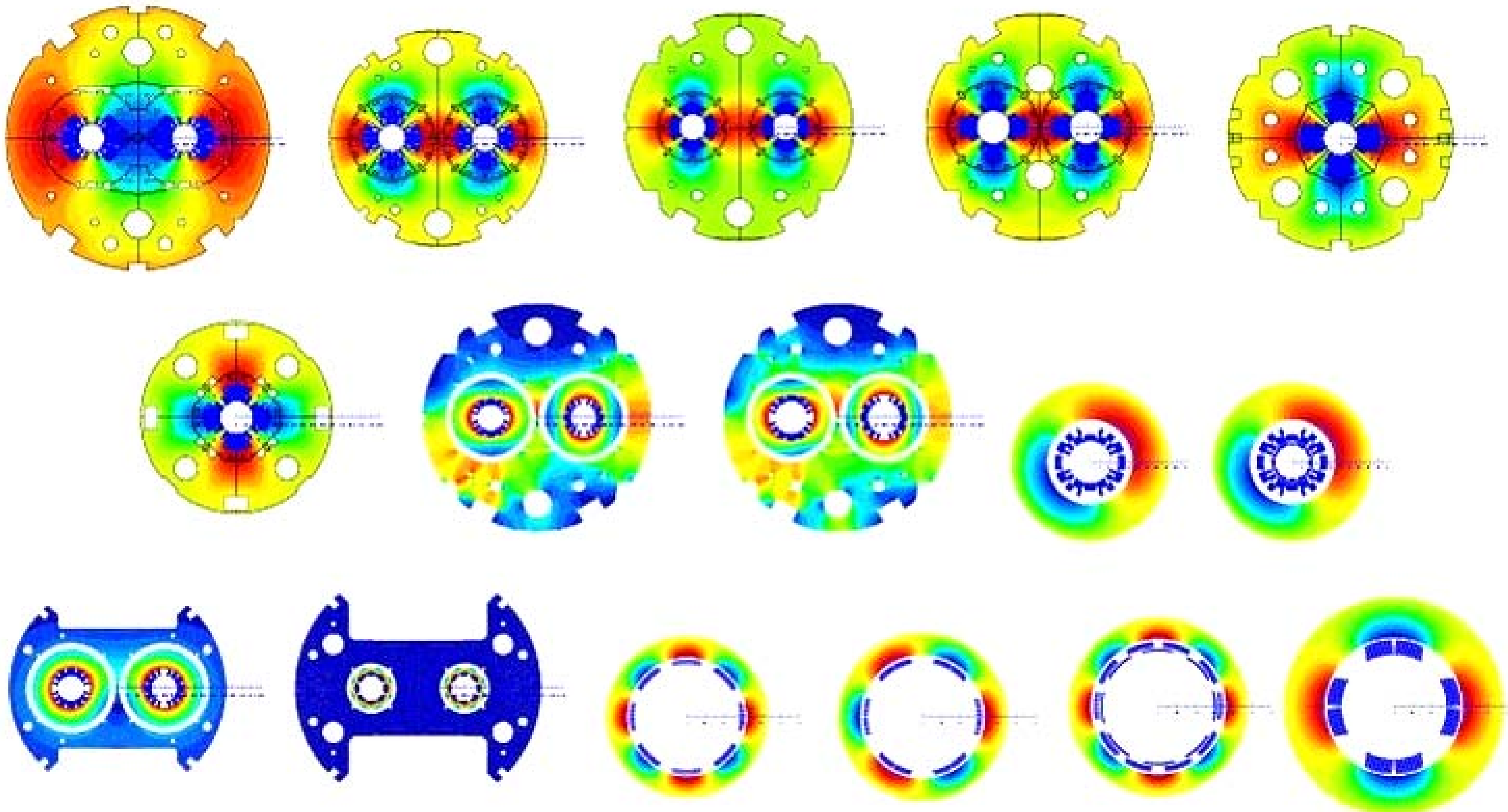


B_3 Contribution from Transport Current and Magnetization

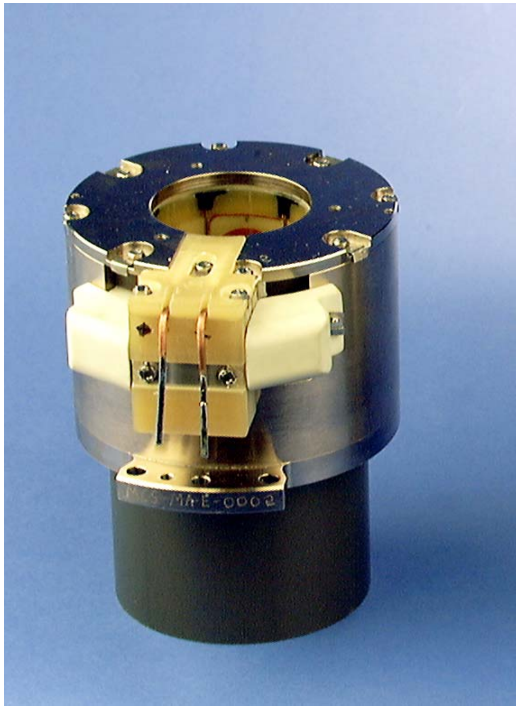




The LHC Magnet Zoo

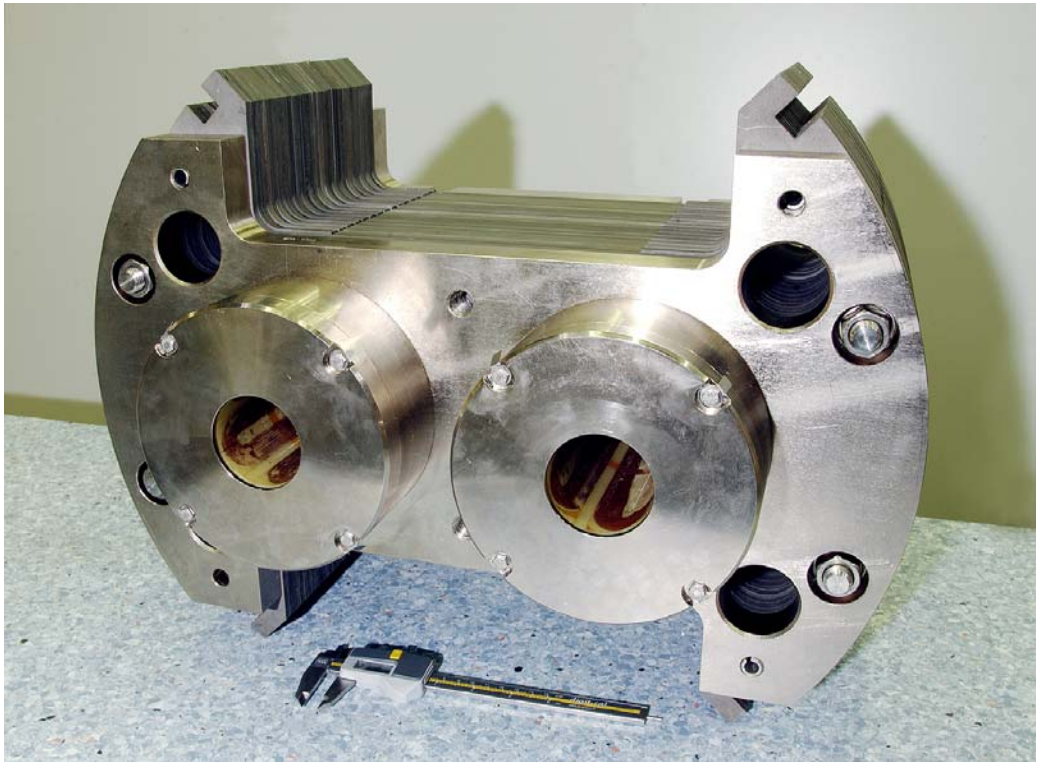


Corrector Magnets



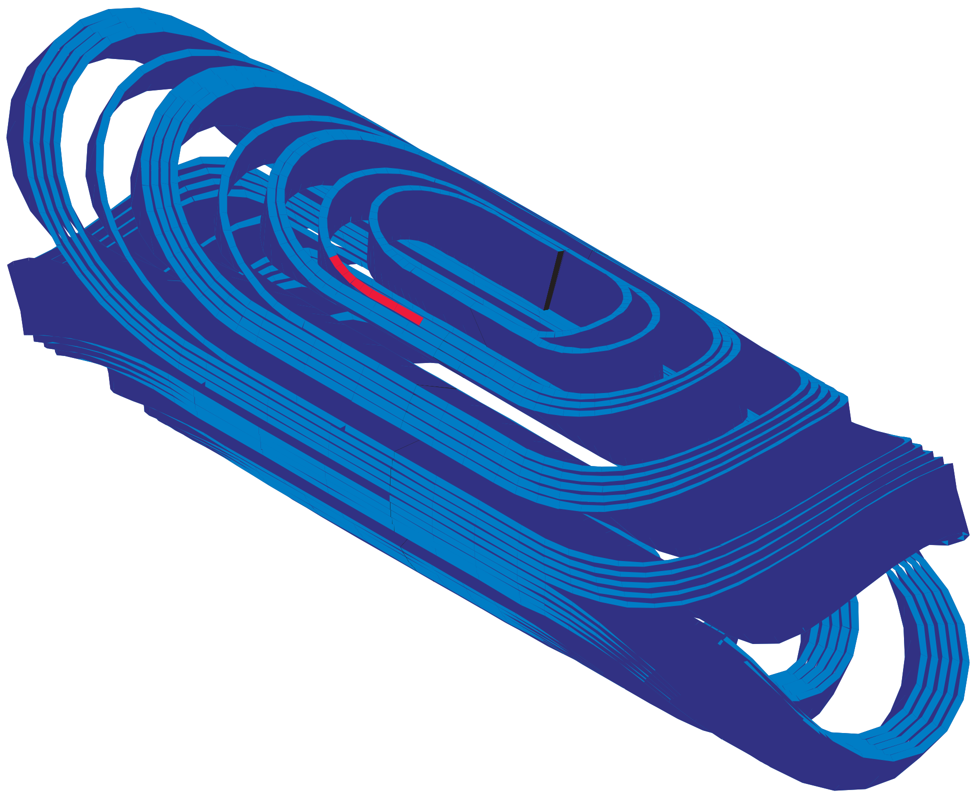
Sextupole-spool pieces

Octupole



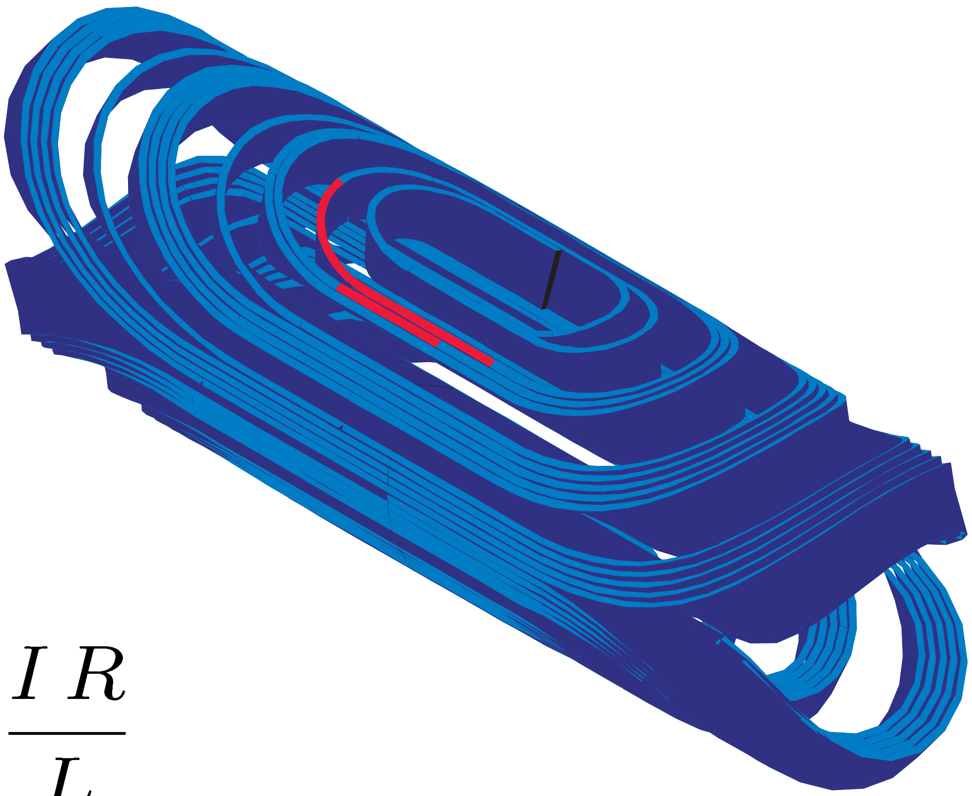
Accelerator Technology Department

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{I(t)^2 \rho_{Cu}}{a_{Cu} a_T C}$$



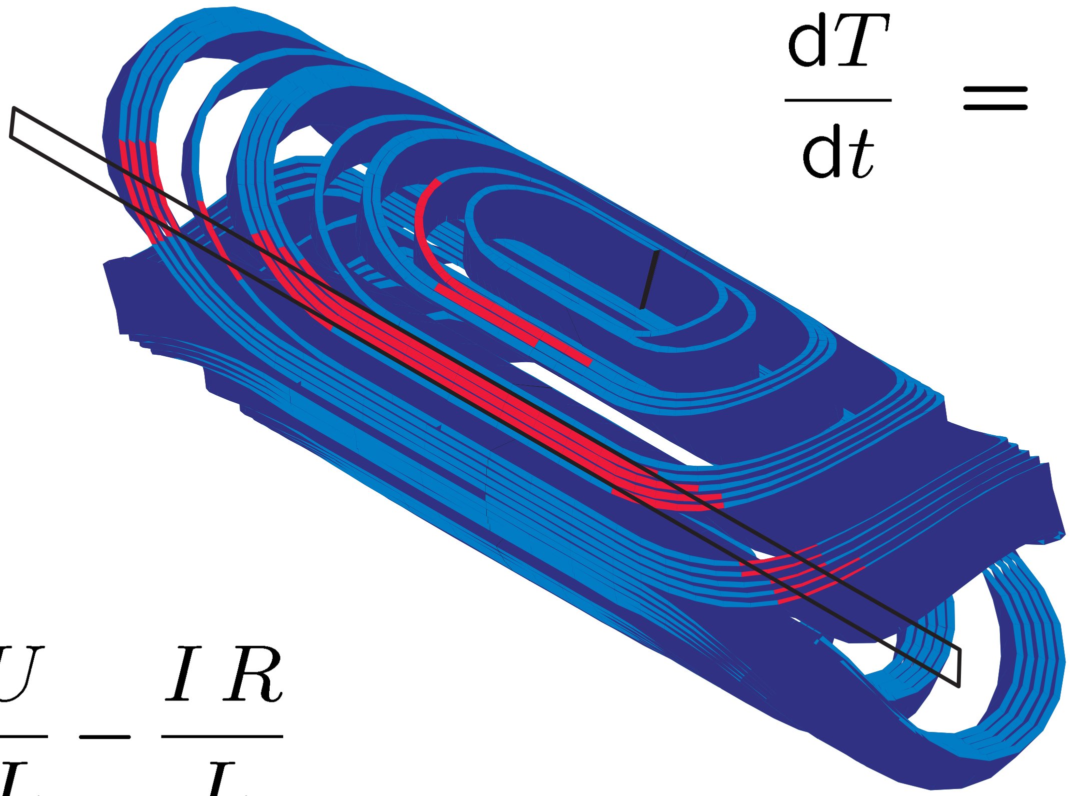
Quench Analysis (Circuit Equation)

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{I(t)^2 \rho_{Cu}}{a_{Cu} a_T C}$$



$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{U}{L} - \frac{I\,R}{L}$$

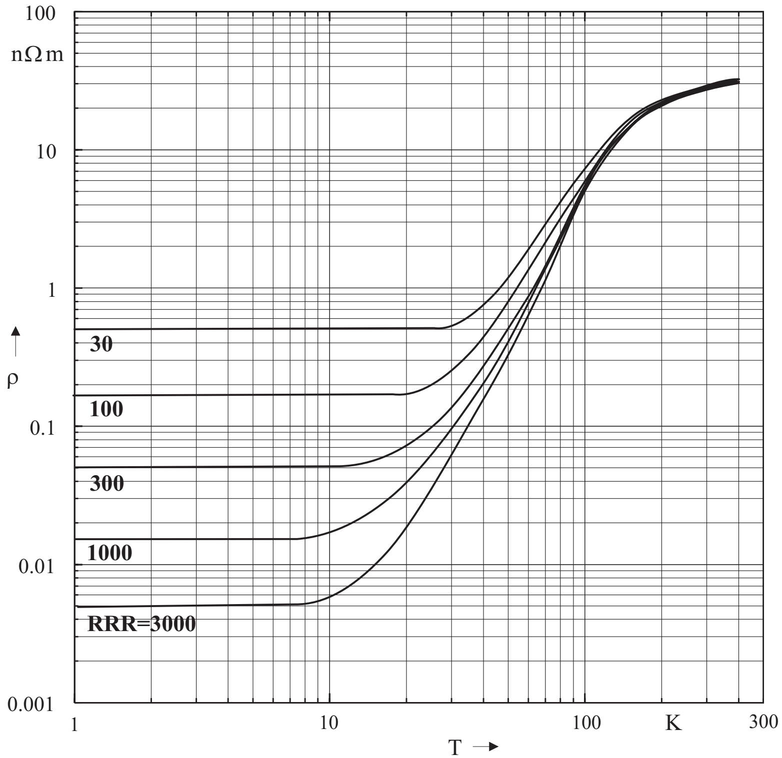
$$R = \sum_{i=1}^{N} l_{\mathrm{mag}} \frac{\rho_{Cu}}{a_{\mathrm{Cu}}} u(t - t_{\mathrm{q}i})$$



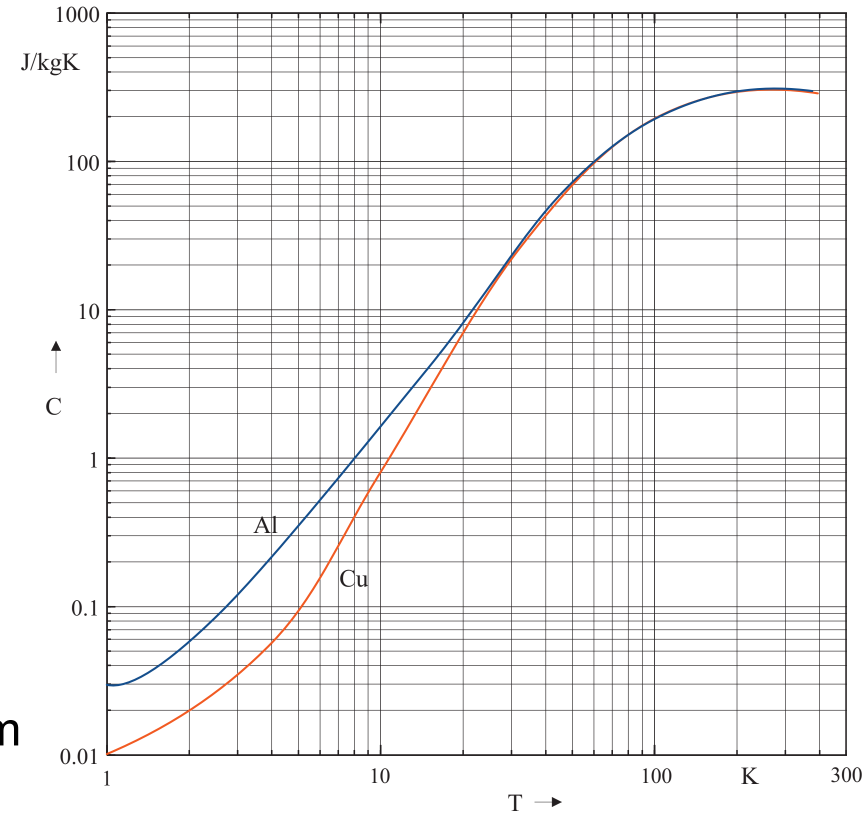
$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{I(t)^2 \rho_{Cu}}{a_{Cu} a_T C}$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{U}{L} - \frac{I\,R}{L}$$

Resistivity as Function of T and RRR

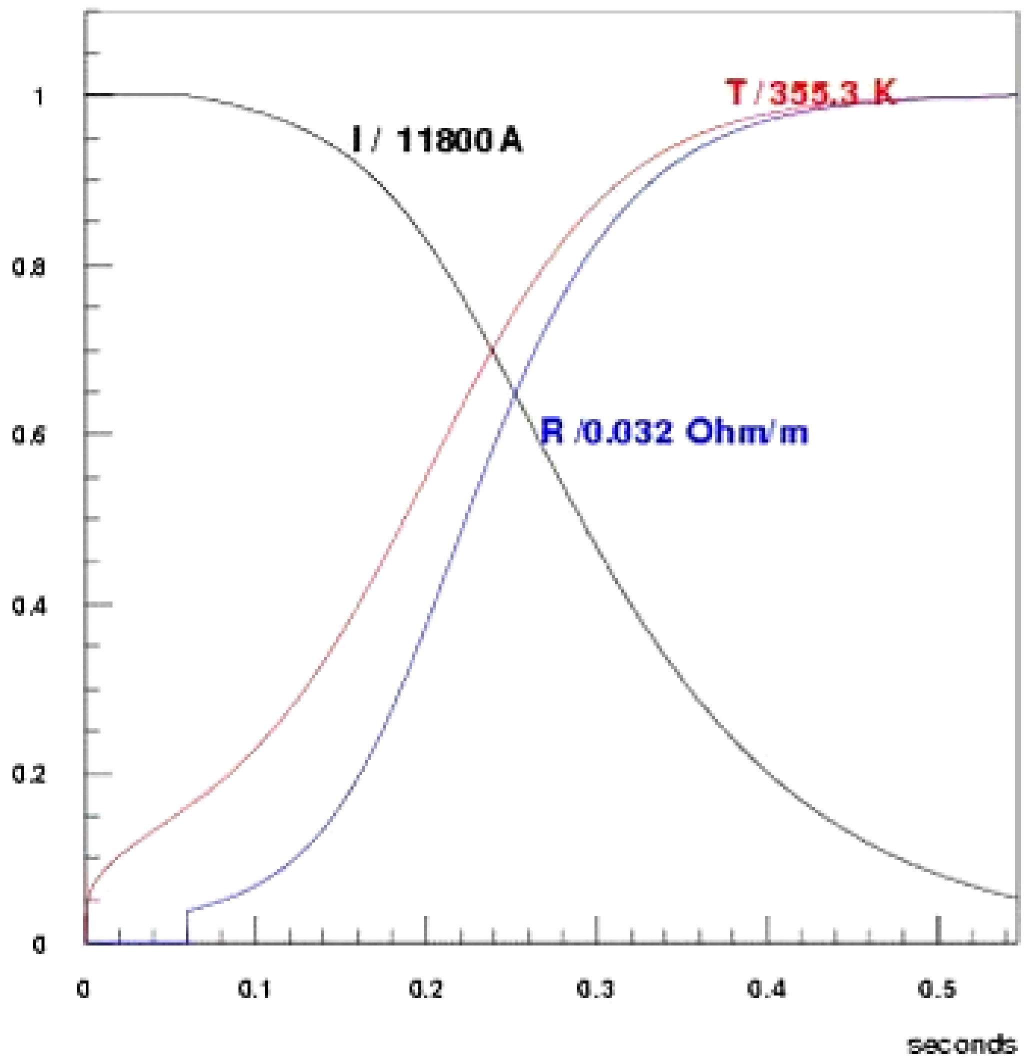


Resistivity of copper

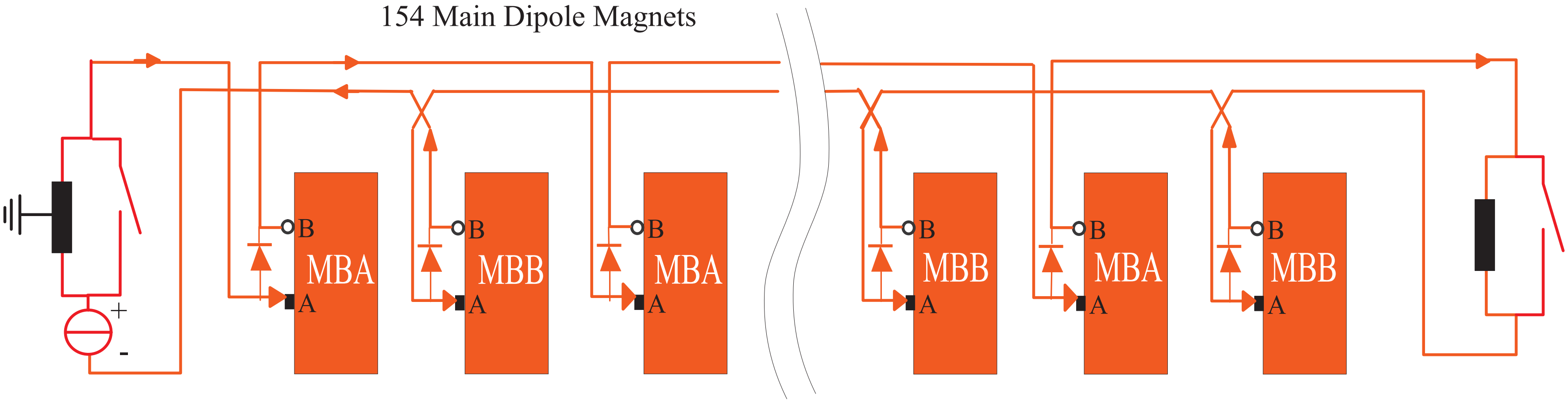


Heat capacity of copper and aluminium

Current, Resistance and Hot-spot Temperature



String of Dipole Magnets



Switches and Dump Resistors