Longitudinal instabilities

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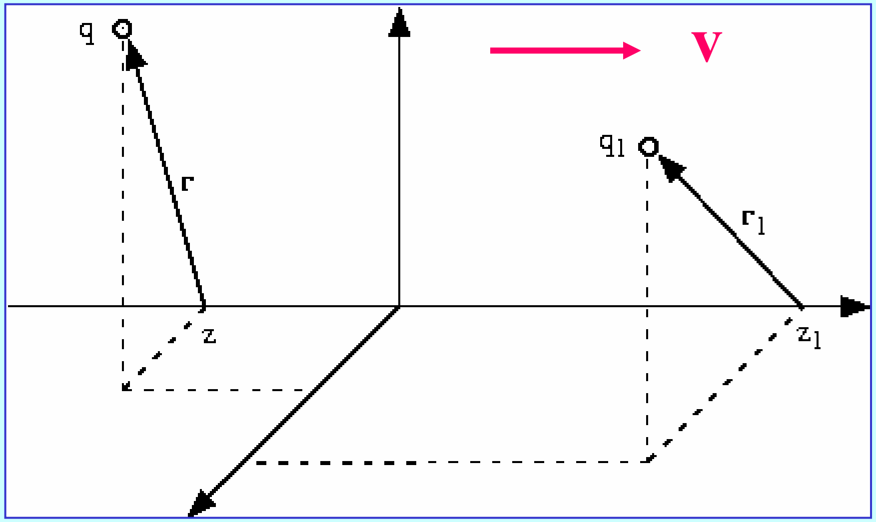
Contents:

- Wake Fields and Coupling Impedance
- Short range and long range wake fields
- Potential well distortion
- Microwave instability
- Robinson Instability
- Coupled bunch Instability

Longitudinal Wake Fields and Impedance



q_1 (z_1, r_1): trailing point charge

q (z,r) : test point charge

The test charge q can **gains** or **loses energy** because of the electromagnetic fields generated behind q_1

$\Delta z = z_1 - z \Longrightarrow$

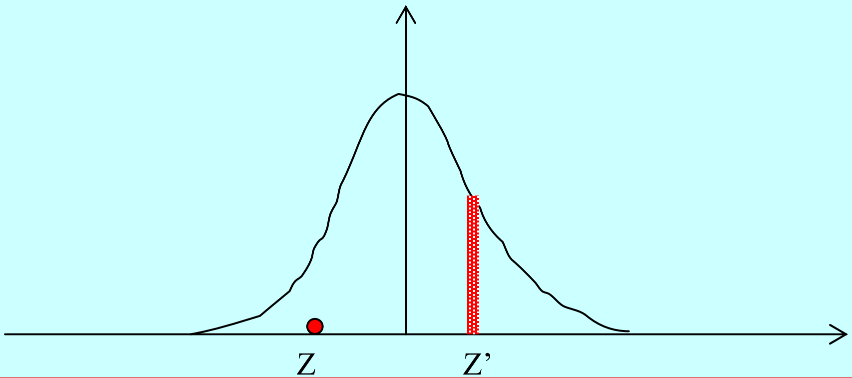
$$U_{\|}(\mathbf{r},\mathbf{r}_1;\Delta z) = -\int_{trajectory} F_{\|}(z,\mathbf{r},z_1,\mathbf{r}_1;t)\,dz$$

with $t = (z_1 + \Delta z)/c$

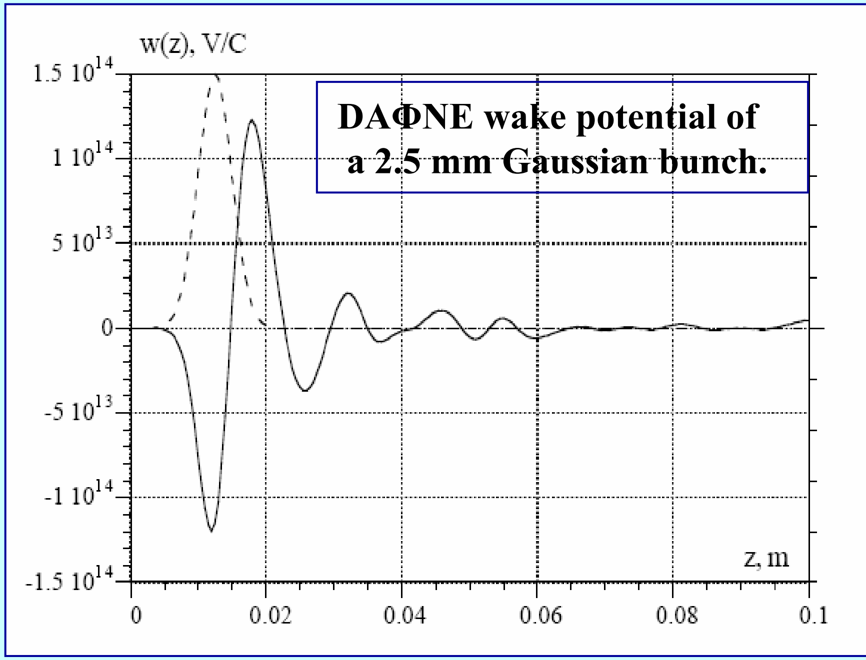
The energy variation of the test charge q, normalized to q and q_1 is called longitudinal wake function (green's function)

$$w_{\parallel}(\mathbf{r},\mathbf{r}_1;\Delta z)=\frac{U_{\parallel}(\mathbf{r},\mathbf{r}_1;\Delta z)}{qq_1}$$

The energy variation of a test charge inside a bunch, due to the distribution ρ(z) , is called longitudinal bunch wake potential

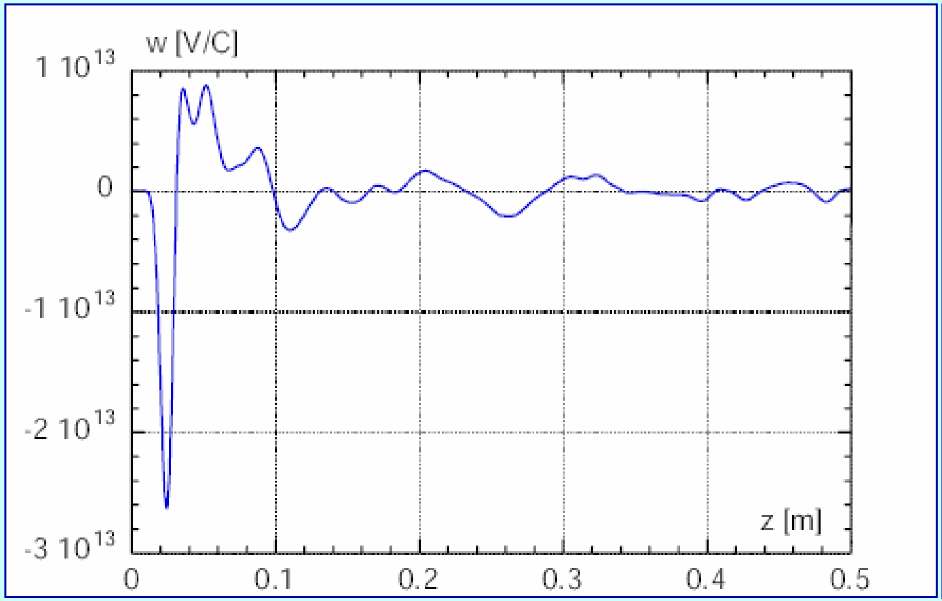


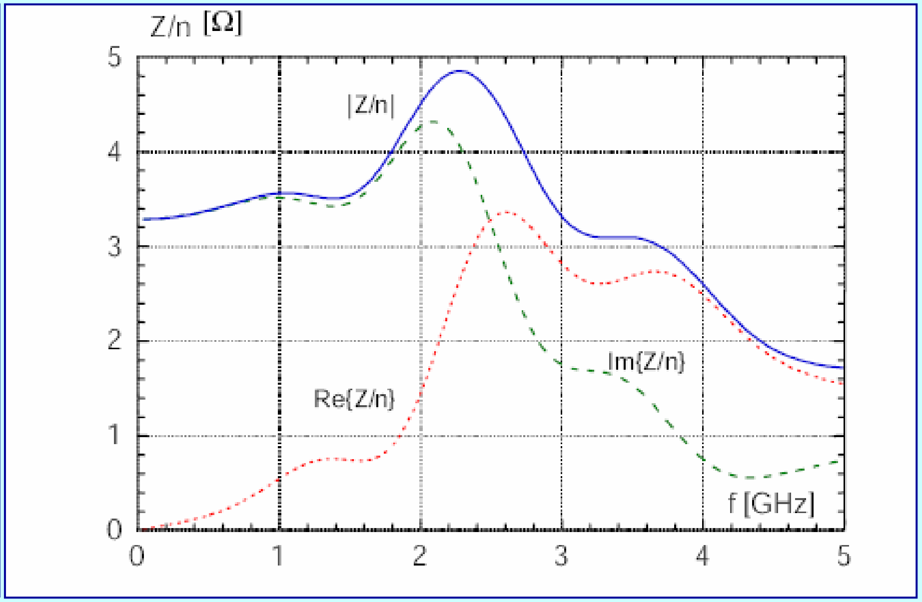
$$W_{\parallel}(z)=e^2N_p\int_{-\infty}^{\infty}\rho(z')\,w_{\parallel}(z'-z)\,dz'$$



The longitudinal coupling impedance is the Fourier transform of the wake function

$$Z_{\parallel}(\mathbf{r},\mathbf{r}_1;\omega)=\frac{1}{c}\int_{-\infty}^{\infty} w_{\parallel}(\mathbf{r},\mathbf{r}_1;z)\exp\left[-i\omega\frac{z}{c}\right]dz$$

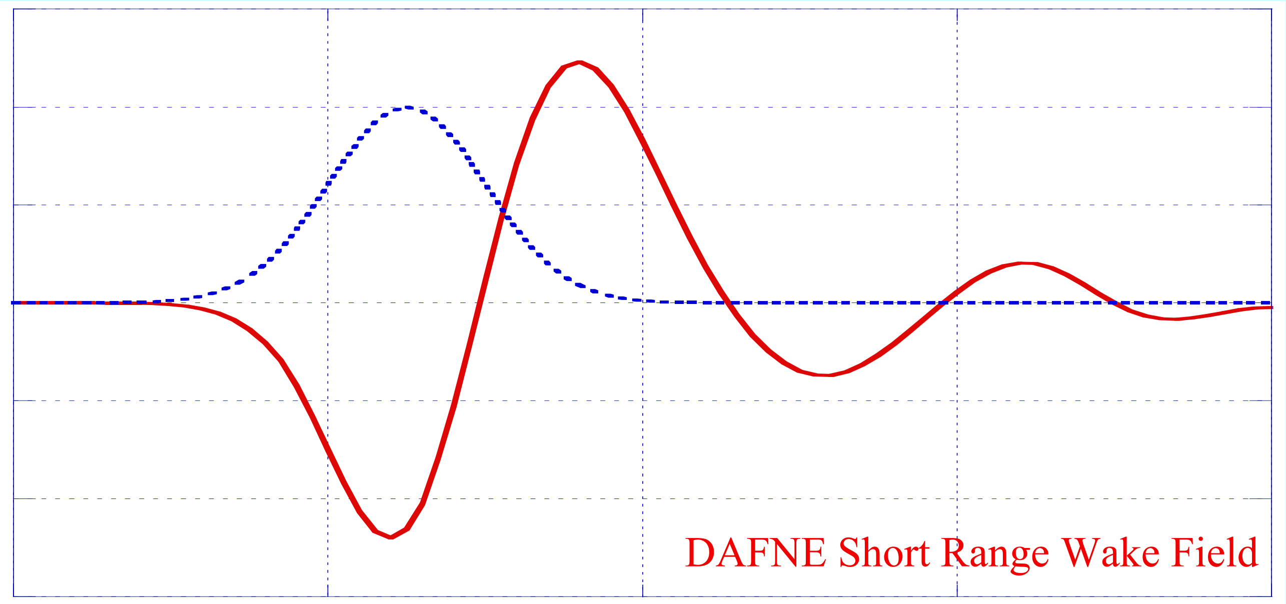




DAΦNE accumulator wake potential of a 2.5 mm Gaussian bunch.

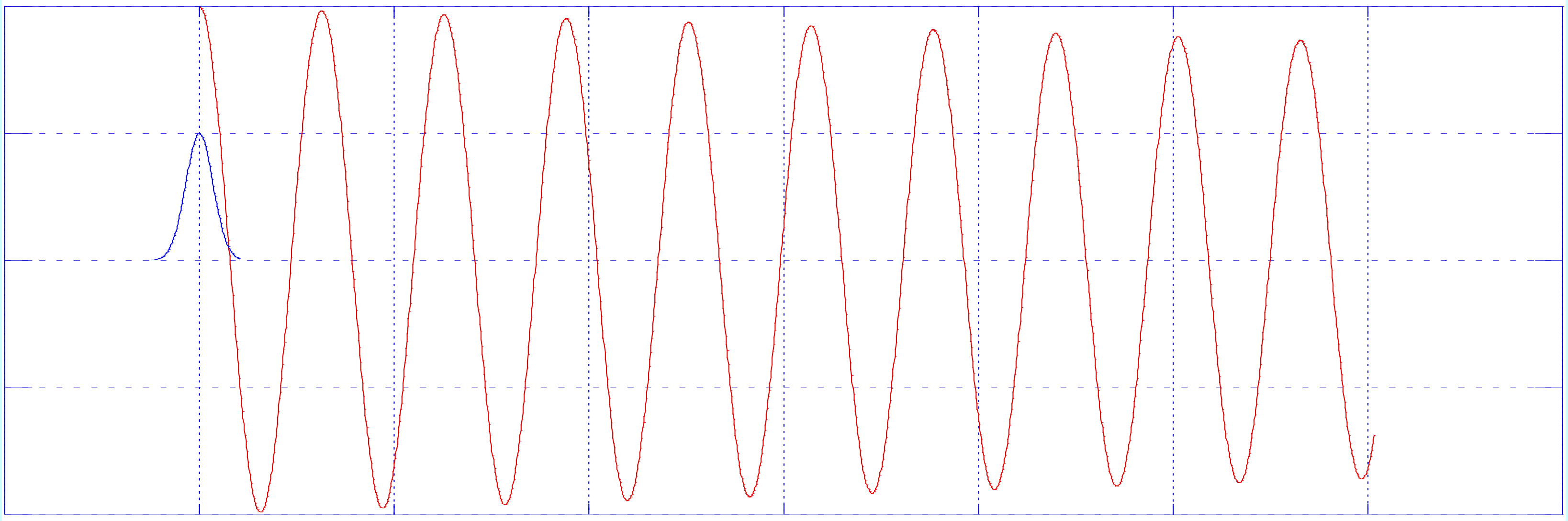
$$\frac{Z_{\parallel}(\omega)}{n}=\frac{Z_{\parallel}(\omega)}{\omega/\omega_o}$$

Short range wakefield acts over the bunch length



- Vanishes after a distance of few bunch lengths
- Low frequency resolution of Fourier transform and of coupling impedance
- Smoother and broader impedance → broad band impedance mode (e.g. Broad Band Resonator Model)

Long range wakefields acts on many bunches/multi-turn



- Fields oscillating over long distances
- produced by high Q resonant modes
- Determined by only 3 parameters: Q, ω_r and R_s
- High peak impedance

In both cases we have

$$Z_{\|}(\omega) = \frac{R_s}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

$$Z_{\|}(\omega) \approx i\frac{\omega}{\omega_r}\frac{R_s}{Q}$$

$$\frac{Z_{\|}(\omega)}{n} \approx i\frac{\omega_o}{\omega_r}\frac{R_s}{Q}$$

$$w_{\|} = \frac{\omega_r R_s}{Q}\exp\left(-\frac{\Gamma\Delta z}{c}\right)\left[\cos\left(\frac{\omega_n \Delta z}{c}\right) - \frac{\omega_r}{2Q\omega_n}\sin\left(\frac{\omega_n \Delta z}{c}\right)\right]H(\Delta z)$$

where $\Gamma = \omega_r / 2Q$, $\omega_n^2 = \omega_r^2 - \Gamma^2$, and $H(\Delta z)$ is the step function.

Notice that for the short range wake field, the Broand Band Resonator with Q ~ 1, is only a useful approximation,

Effects on beam dynamics

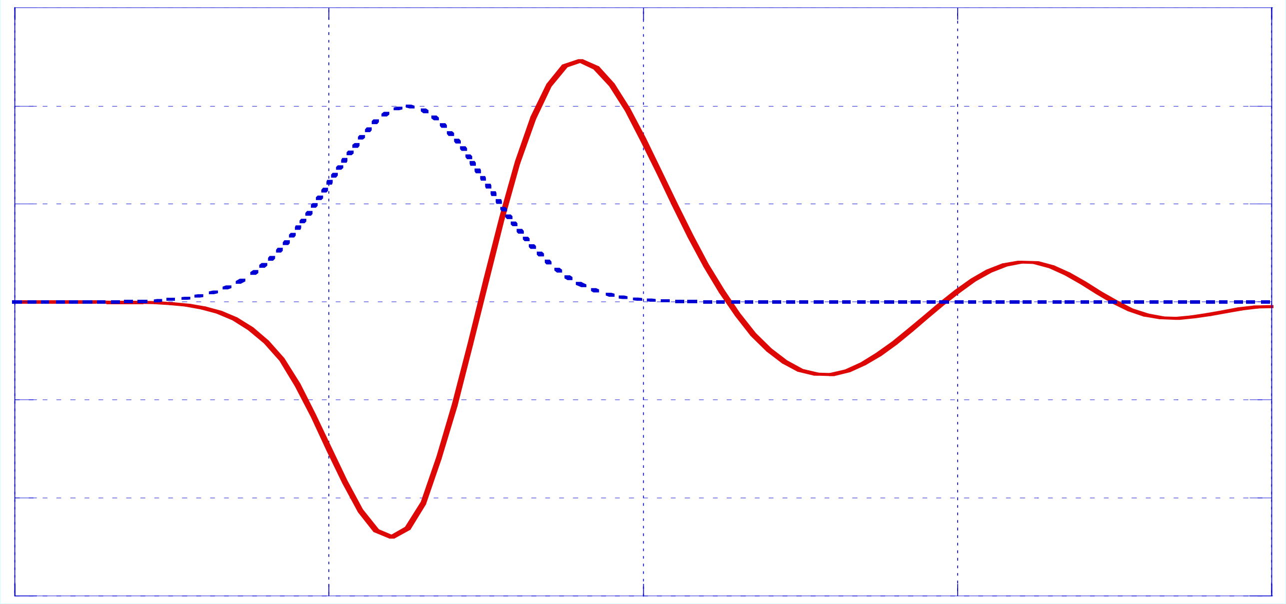
Short range wakefileds:

- Potential well distortion
- Longitudinal emittance growth, microwave instability

Long range wakefileds:

- Robinson instabilities (RF fundamental mode)
- Coupled bunch instability

Short range wakefileds



Potential well distortion

The motion of a particle in the bunch is confined by the potential due to the RF voltage and to the wake fields

$$\Psi(z)=\frac{\alpha_c}{L_0}\int_0^z\left[eV_{RF}(z')-U_0\right]dz' - \frac{\alpha_c e^2 N_p}{L_0}\int_0^z dz'\int_{-\infty}^{\infty}\rho(z'')w_{\parallel}(z''-z')dz''$$

In the low current regime, with gaussian energy distribution, energy spread $\sigma_{\varepsilon 0}$, the longitudinal distribution is described by an integral equation known as the Haissinski equation

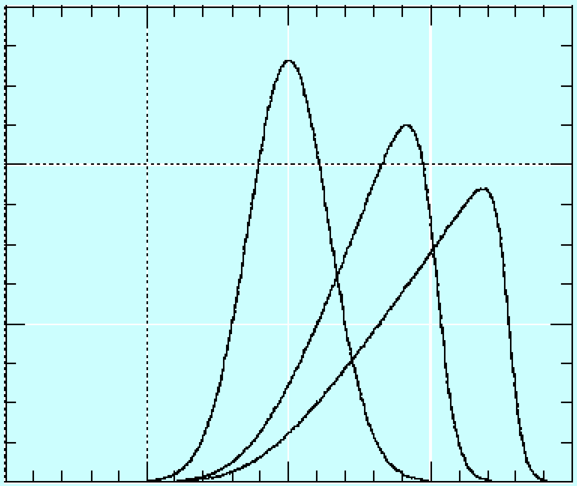
$$\rho_0(z)=\bar{\rho}\exp\left[-\frac{1}{E_0\alpha_c^2\sigma_{\varepsilon 0}^2}\Psi(z)\right]$$

Particular solution of Haissinski equation

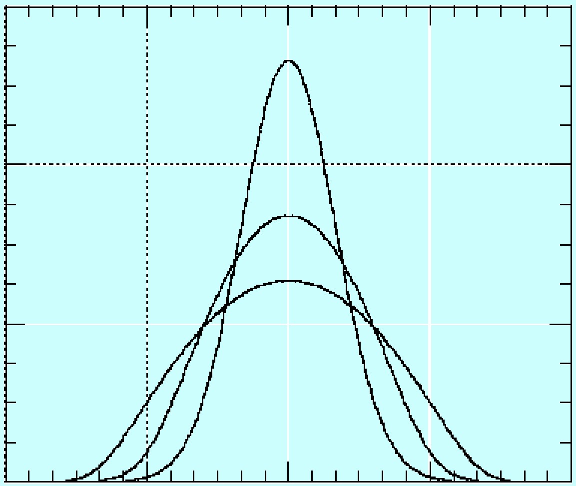
No wake field contribution: a linear expansion of V_{RF} around z=0 gives

$$\rho_0(z) = \bar{\rho}\exp\left[-\frac{z^2}{2\sigma_{z0}^2}\right] \qquad \bar{\rho} = \frac{1}{\sqrt{2\pi}\sigma_{z0}} \qquad \sigma_{z0} = \frac{\alpha_c c\sigma_{\varepsilon 0}}{\omega_{s0}}$$

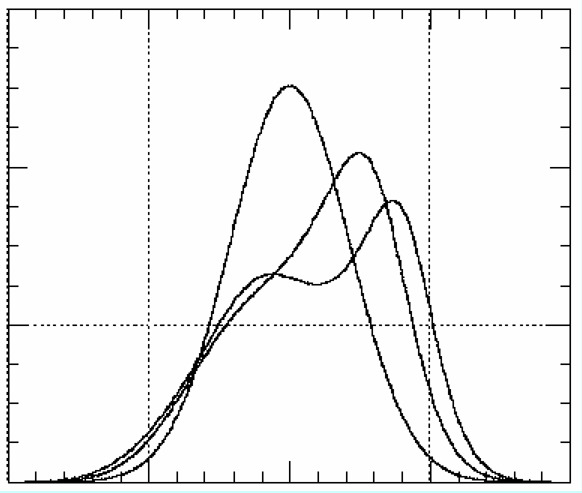
Pure resistive impedance



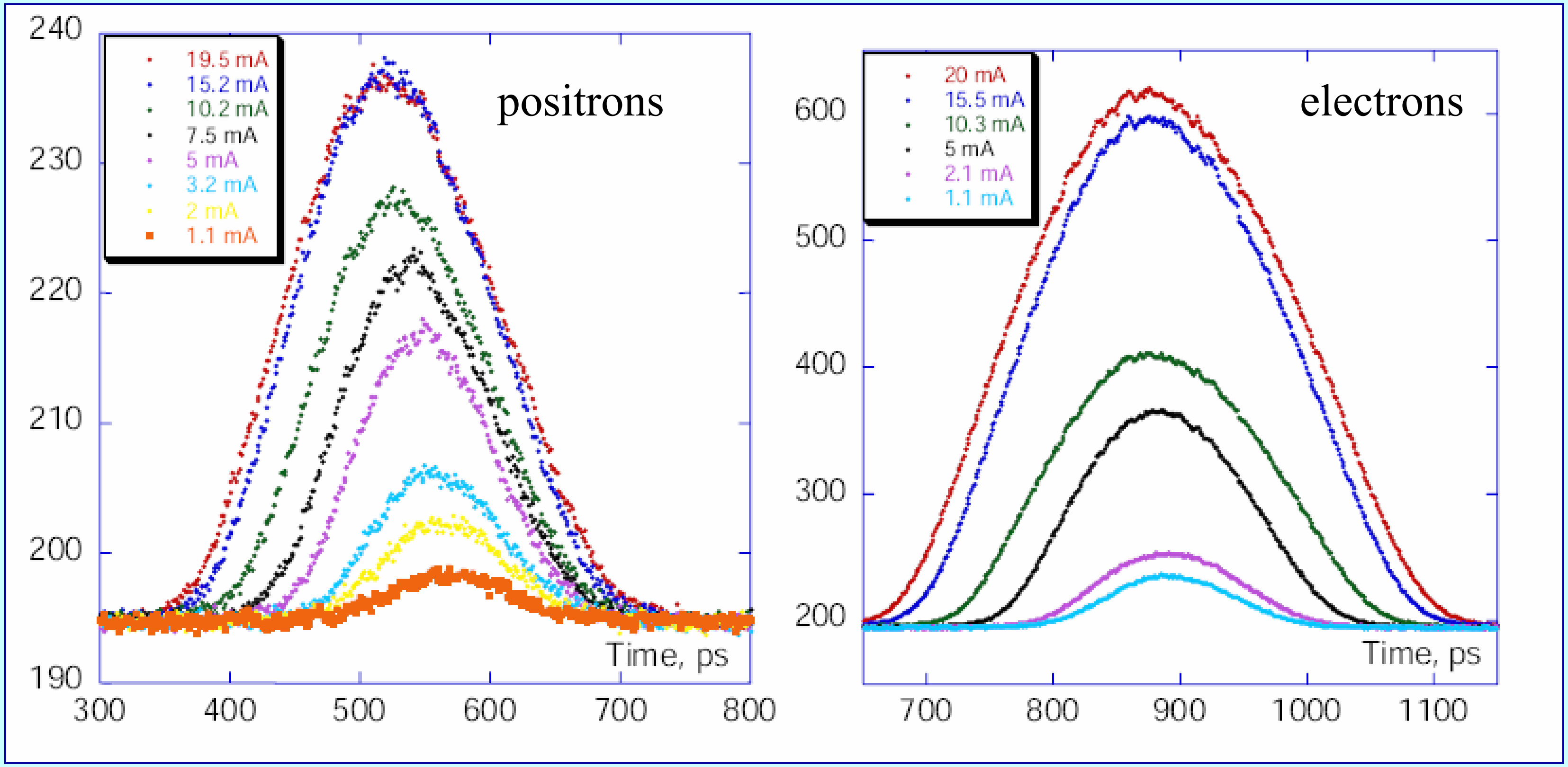
Pure inductive impedance



Broad band resonator

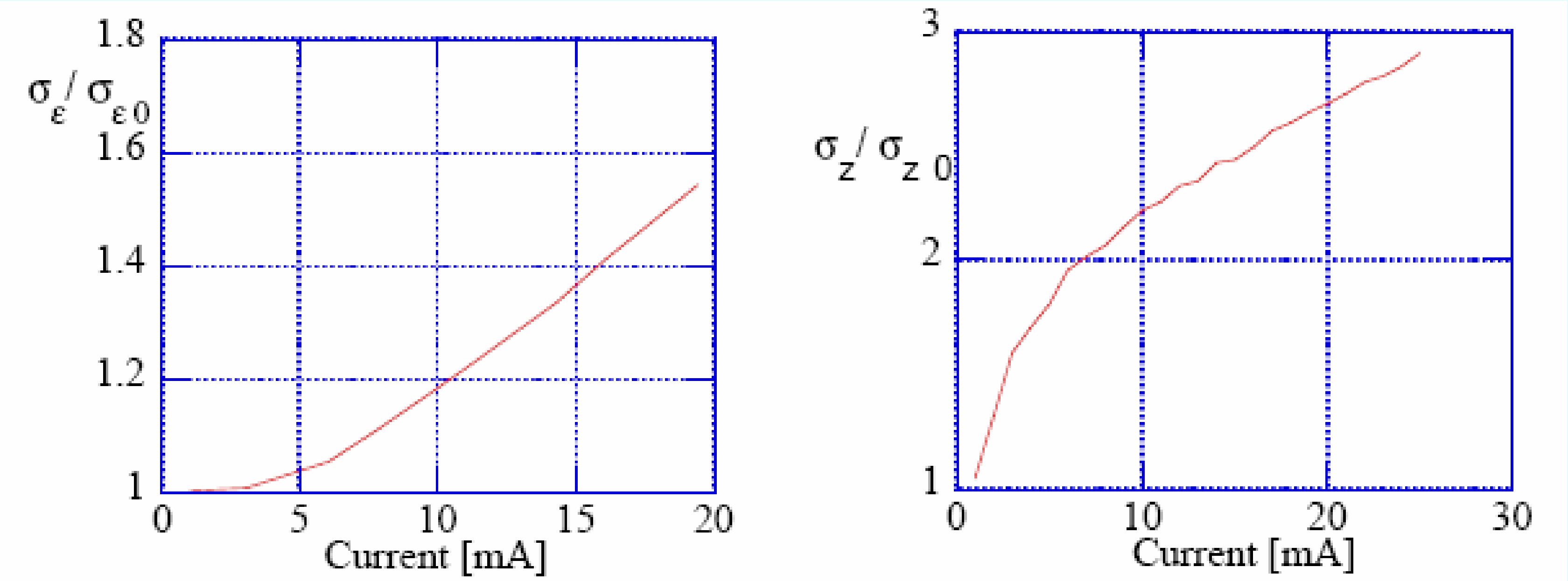


Typical measured bunch distributions in the Dafne Rings. The head is to the left



Longitudinal emittance growth, microwave instability

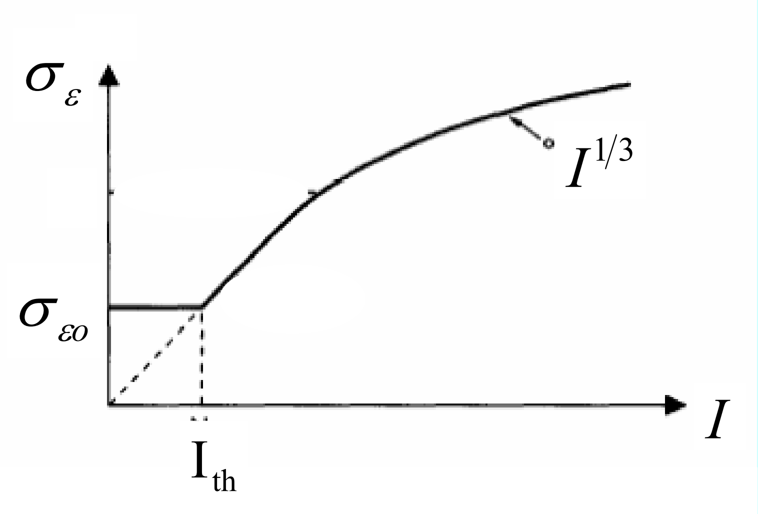
• Observe energy spread and bunch length as a function of the current.



• σ_ε is almost constant up to a threshold current after which it starts to increase with the current according to a given power law (in most cases 1/3 power).

• σ_z starts to increase from the very beginning (potential well distortion), and, after the same threshold current, it grows with the same power law.

Longitudinal emittance growth & microwave instability



Threshold current:

$$\frac{\hat{I}\left|Z_{\parallel}/n\right|}{2\pi\alpha_c\left(E_0/e\right)\sigma_\varepsilon^2} \leq 1$$

$$\hat{I} = \frac{ceN_p}{\sqrt{2\pi}\sigma_z}$$

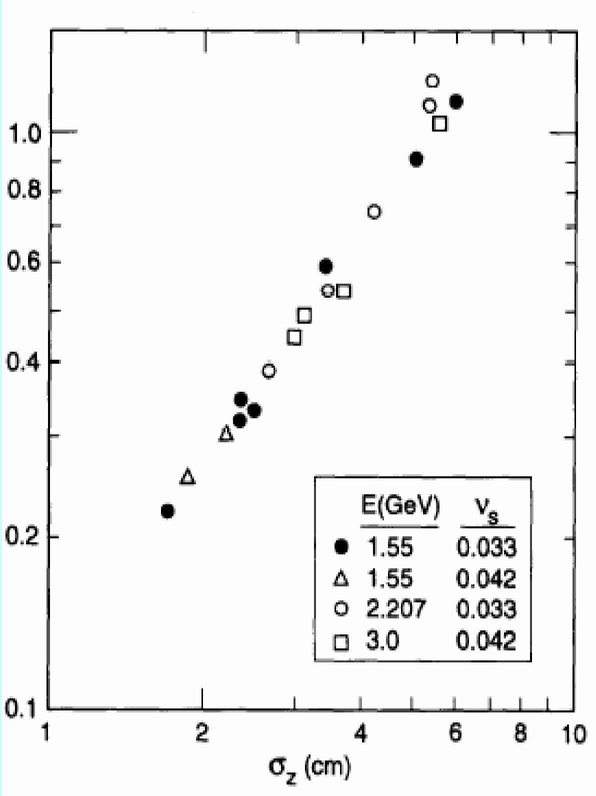
$$n = \frac{\omega}{\omega_o}$$

Above threshold: Boussard criterion

$$\sigma_z = \left(\frac{R^3\left|Z/n\right|\xi}{\sqrt{2\pi}}\right)^{1/3} \qquad \xi = \frac{I\alpha_c}{\upsilon_s^2 E_0/e}$$

Longitudinal emittance growth & microwave instability

Chao – Gareyte scaling law:



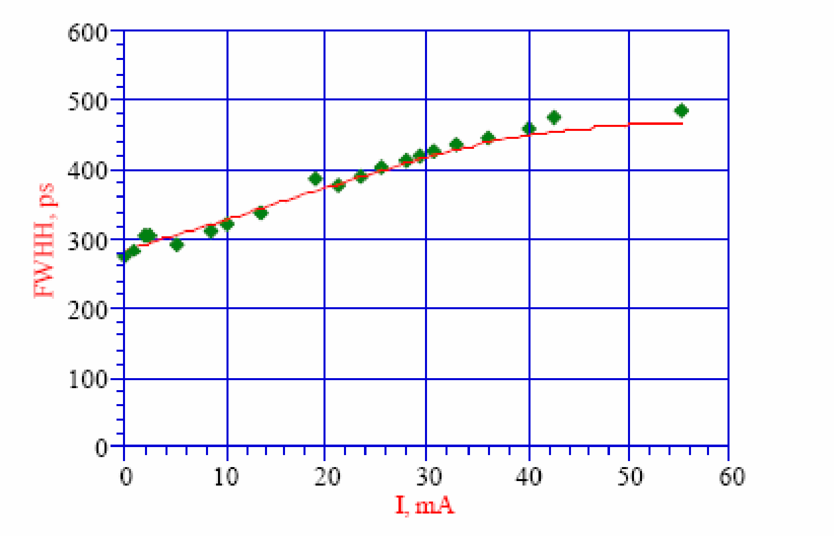
Assume a power-law behavior of $Z_{||}(\omega)$

$$\left|\frac{Z}{n}\right| \propto Z_0 \omega^{a-1} \quad \text{then} \quad \sigma_z \propto \left(\xi Z_0 R^3\right)^{1/(2+a)}$$

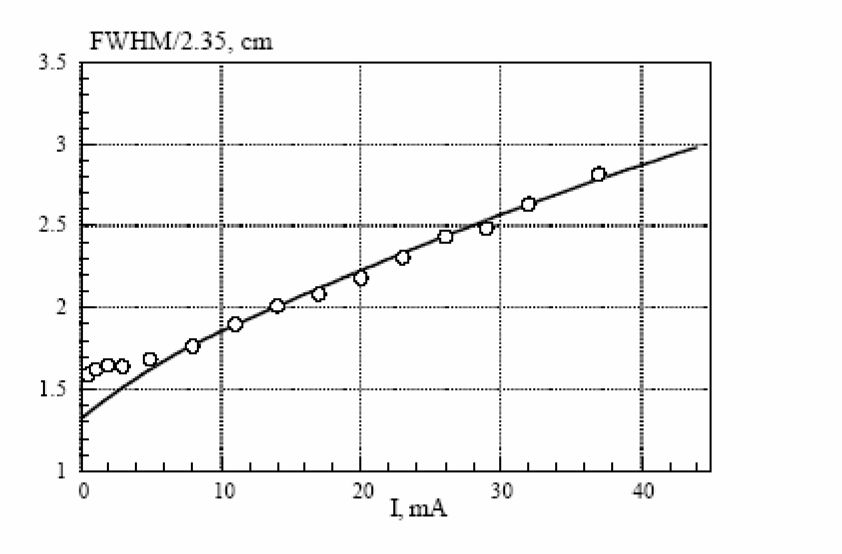
For SPEAR a = 0.68

From: A. W. Chao, J. Gareyte, Particle Accelerators, Vol. 25, pp. 229-234, 1990

Bunch lengthening in DAFNE



DAFNE Accumulator.
Dots: measurement results
Solid line : numerical simulation.

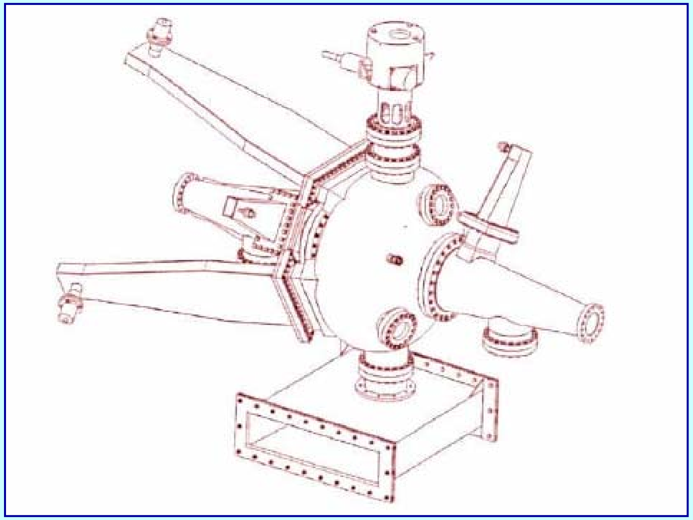


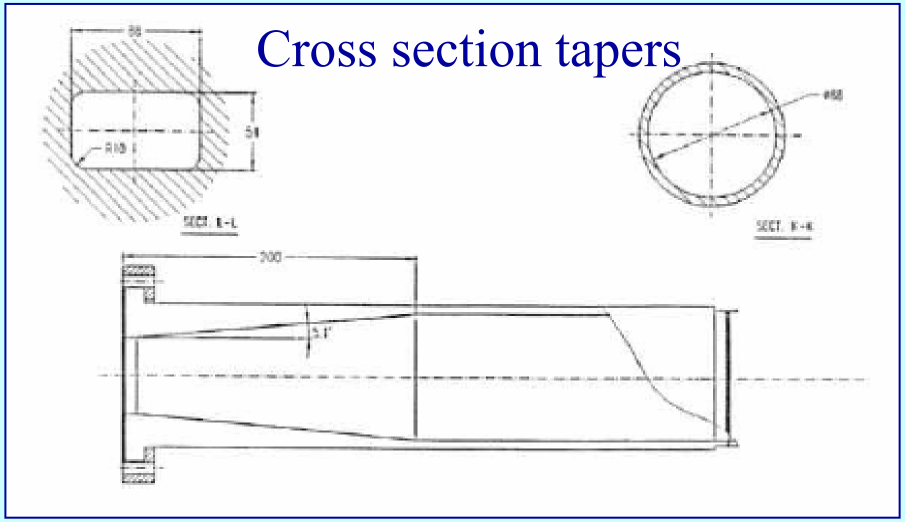
DAFNE main rings
Circles - measurement results.
Solid line - numerical calculations

NOTICE
Numerical simulations performed before measurements : good impedance model of the machine

Design strategy: proper design of vacuum chamber

- Single bunch: low broad band impedance Z/n





Reduce parasitic loss, taper discontinuities

Impedance budget

Element	Im Z_L/n [Ω]
Tapers	0.156
Transverse feedback kickers (low frequency)	0.128
Scrapers	0.062
Bellows	0.024
Resistive wall (at roll - off frequency)	0.013
BPMs	0.01
Vacuum pump screens	0.02
Injection port	0.0031
Antechamber slots	0.0005
Synchrotron radiation	$<$ 0.015
Space charge	-0.0021
Other inductive elements	0.1
Total	**0.53 Ω**

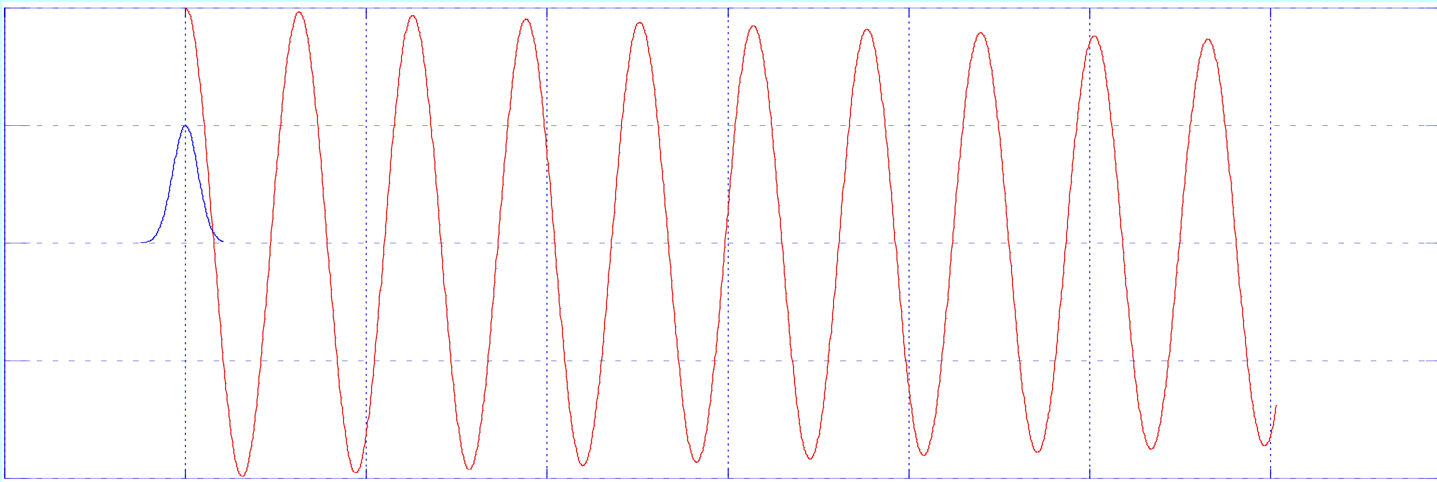
Element	k_l, V / pC at $\sigma_z = 3$ cm
RF cavity	0.129
Third harmonic cavity	0.157
Longitudinal feedback kicker	0.120
Transverse kickers	0.064
Injection kickers	0.047
IR taper system	0.0026
Scrapers	0.00007
Injection port	0.00004
Total	**0.52**

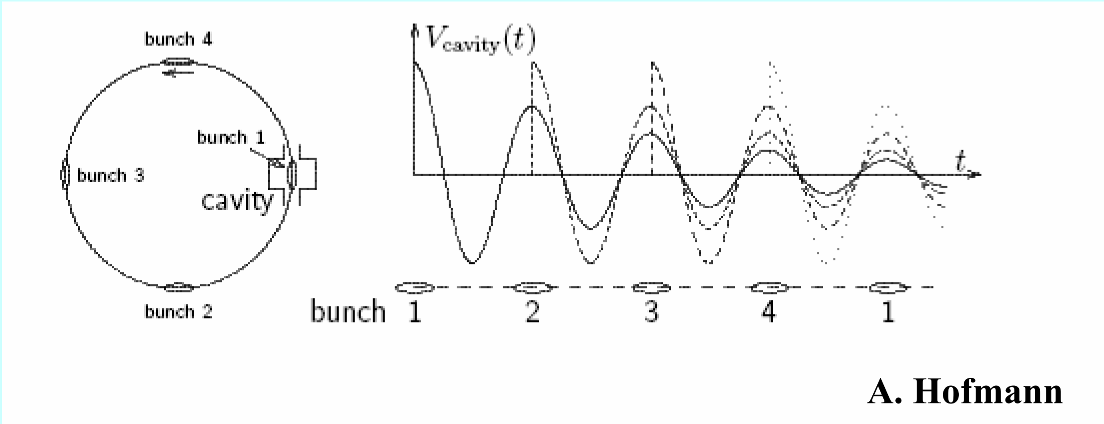
Longitudinal Microwave instability is is fast but not destructive

Cures ?

- Landau damping

Long range wakefields





Interaction with RF fundamental mode: Robinson Instabilities

Single particle equation of motion (neglecting quantum fluctuation)

$$\dot{z} = -c\alpha_c \varepsilon$$

$$\dot{\varepsilon} = \frac{eV_{RF} - U_0}{T_0 E_0} - \frac{D}{T_0}\varepsilon \qquad D = \frac{2U_0}{E_0}$$ is the damping coefficient

Combined they give a second order differential equation

$$\ddot{z} + \frac{D}{T_0}\dot{z} + \omega_{s0}^2 z = 0 \qquad \text{with} \qquad \omega_{s0}^2 = \frac{c^2\alpha_c 2\pi h e\hat{V}\sin(\phi_s)}{L_0^2 E_0}$$

$$\cos(\phi_s) = \frac{U_0}{e\hat{V}} \qquad \left(0 \le \phi_s \le \frac{\pi}{2}\right)$$ synchronous phase

Robinson instability …

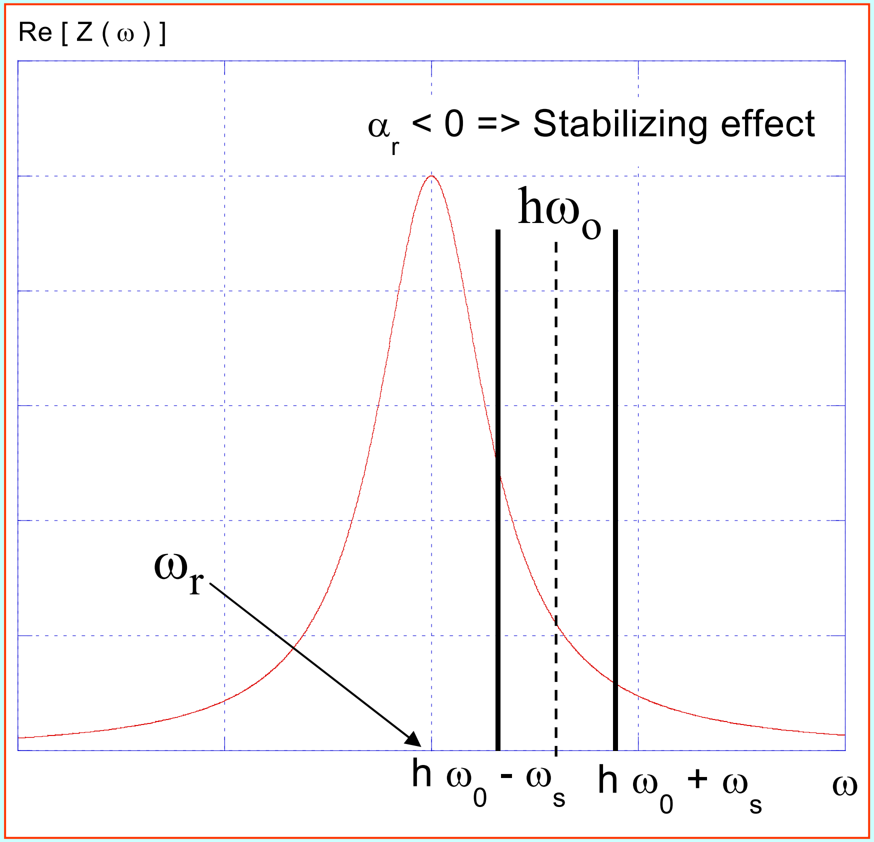
By including also the beam loading effect we have (see A. Hofmann lecture)

$$\ddot{z}+\left(\frac{D}{T_0}-\alpha_r\right)\dot{z}+\omega_s^2 z=0$$

$$\alpha_r=\frac{eN_p\alpha_c h\omega_0}{\omega_s\left(E_0/e\right)T_0^2}\mathrm{Re}[\Delta Z]$$

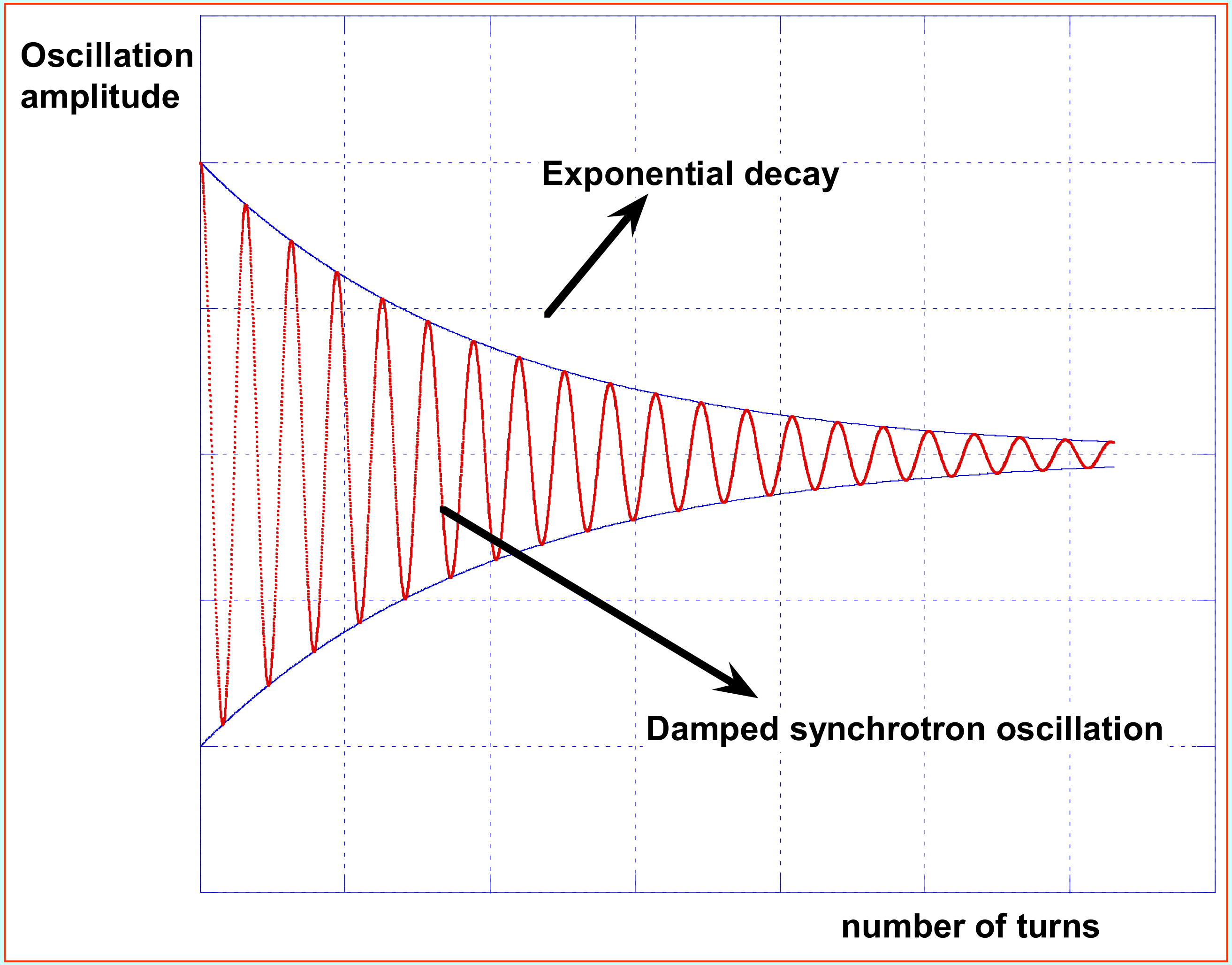
$$\mathrm{Re}[\Delta Z]=\mathrm{Re}[Z(n\omega_0+\omega_s)-Z(n\omega_0-\omega_s)]$$

$$z=A_0\exp\left[-\frac{D}{T_0}+\alpha_r\right]\cos[\omega_s t+\theta_0]$$



Robinson instability …….

Example of stability

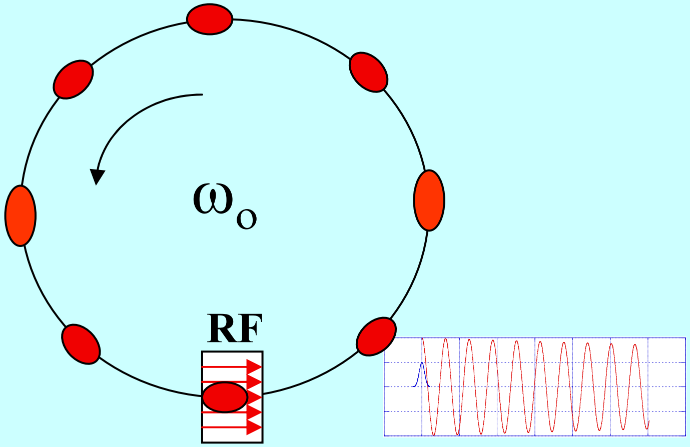


Interaction with HOMs : Coupled bunch instability
(Macroparticle model)

The equations of motion are the same of the single particle. The difference is in the voltage induced by other bunches in the HOMs

$$\dot{z}_n = -c\alpha_c \varepsilon_n$$

$$\dot{\varepsilon}_n = \frac{eV_{RF}(z_n) - U_0}{T_0 E_0} - \frac{eV_w^n}{T_0 E_0} - \frac{D}{T_0}\varepsilon_n$$



V_w^n is the voltage seen by the n^th bunch and induced by the long range wake fields.

$$\ddot{z}_n + \frac{D}{T_0}\dot{z}_n = -\frac{c\alpha_c}{T_0 E_0}\left[eV_{RF}(z_n) - U_0 - eV_w^n\right]$$

By linearizing the RF voltage and the HOM induced wake fields with respect to z_n we obtain three terms

1) A term independent on z that modifies synchronous phase

$$e\hat{V}\cos(\phi_{sn}) = U_0 + e\sum_{h=0}^{N_b-1}\sum_{q=-\infty}^{\infty} Q_h w_{\parallel}\left[\left(q - \frac{h}{N_b} + \frac{n}{N_b}\right)L_0\right]$$

2) A term dependent on z_n (t) that modifies synchronous frequency

$$\omega_{sn}^2 = \frac{c^2\alpha_c e}{L_0 E_0}\left[\frac{2\pi h\hat{V}\sin(\phi_{sn})}{L_0} + \sum_{h=0}^{N_b-1}\sum_{q=-\infty}^{\infty} Q_h \left.\frac{dw_{\parallel}}{dz}\right|_{\left(q-\frac{h}{N_b}+\frac{n}{N_b}\right)L_0}\right]$$

3) Other terms dependent on z_h at previuos passages that are seen as 'external coupling forces'

$$\ddot{z}_n + \frac{D}{T_0}\dot{z}_n + \omega_{sn}^2 z_n = \frac{c\alpha_c e}{T_0 E_0}$$

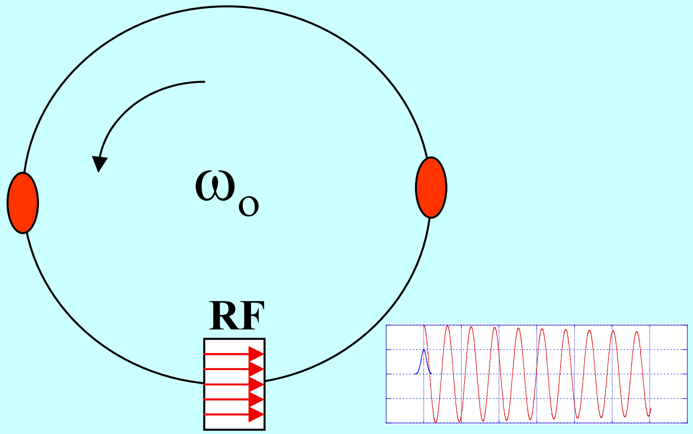
$$\sum_{h=0}^{N_b-1}\sum_{q=-\infty}^{\infty} Q_h \left.\frac{dw_{\parallel}}{dz}\right|_{\left(q-\frac{h}{N_b}+\frac{n}{N_b}\right)L_0} z_h\left(t - qT_0 + \frac{h}{N_b}T_0 - \frac{n}{N_b}T_0\right)$$

To solve the equation system we seek a solution of the kind $z_n\,(t) = a_n\,\exp[i\Omega t]$ and obtain

$$\lambda\left(\Omega^{(\mu)}\right) = -i\frac{c^2\alpha_c e^2 N_p N_b}{L_0^2 E_0}\sum_{q=-\infty}^{\infty}\left[\Omega^{(\mu)} + (qN_b + \mu)\omega_0\right] Z_{\parallel}\left[\Omega^{(\mu)} + (qN_b + \mu)\omega_0\right]$$

$$\lambda(\Omega) = \Omega^2 - i\frac{D}{T_0}\Omega - \omega_{sn}^2 \qquad \mu = 0,\ldots,N_b - 1$$

Example with 2 bunches



$$\ddot{z}_1 + \frac{D}{T_0}\dot{z}_1 + \omega_s^2 z_1 = \frac{c\alpha_c eQ}{T_0 E_0}\sum_{q=-\infty}^{\infty}\left\{ \left.\frac{dw_{\parallel}}{dz}\right|_{qL_0} z_1\left(t - qT_0\right) + \left.\frac{dw_{\parallel}}{dz}\right|_{\left(q-\frac{1}{2}\right)L_0} z_2\left(t - qT_0 - \frac{1}{2}T_0\right)\right\}$$

$$\ddot{z}_2 + \frac{D}{T_0}\dot{z}_2 + \omega_s^2 z_2 = \frac{c\alpha_c eQ}{T_0 E_0}\sum_{q=-\infty}^{\infty}\left\{ \left.\frac{dw_{\parallel}}{dz}\right|_{qL_0} z_2\left(t - qT_0\right) + \left.\frac{dw_{\parallel}}{dz}\right|_{\left(q-\frac{1}{2}\right)L_0} z_1\left(t - qT_0 - \frac{1}{2}T_0\right)\right\}$$

Seek for a solution of the kind

$z_1(t) = a_1 \exp[i\Omega t]$ and $z_2(t) = a_2 \exp[i\Omega t]$ and obtain

$$\left(\Omega^2 - i\frac{D}{T_0}\Omega - \omega_s^2\right)a_1 = -i\frac{c^2\alpha_c eQ}{L_0^2 E_0}\sum_{q=-\infty}^{\infty}(\Omega + q\omega_0)Z_{\|}(\Omega + q\omega_0)\left(a_1 + a_2 e^{i\pi q}\right)$$

$$\left(\Omega^2 - i\frac{D}{T_0}\Omega - \omega_s^2\right)a_2 = -i\frac{c^2\alpha_c eQ}{L_0^2 E_0}\sum_{q=-\infty}^{\infty}(\Omega + q\omega_0)Z_{\|}(\Omega + q\omega_0)\left(a_1 e^{i\pi q} + a_2\right)$$

- Homogeneus system of two equations.
- Non trivial solution the matrix determinant most be zero.
- Consider a single narrow band HOM:

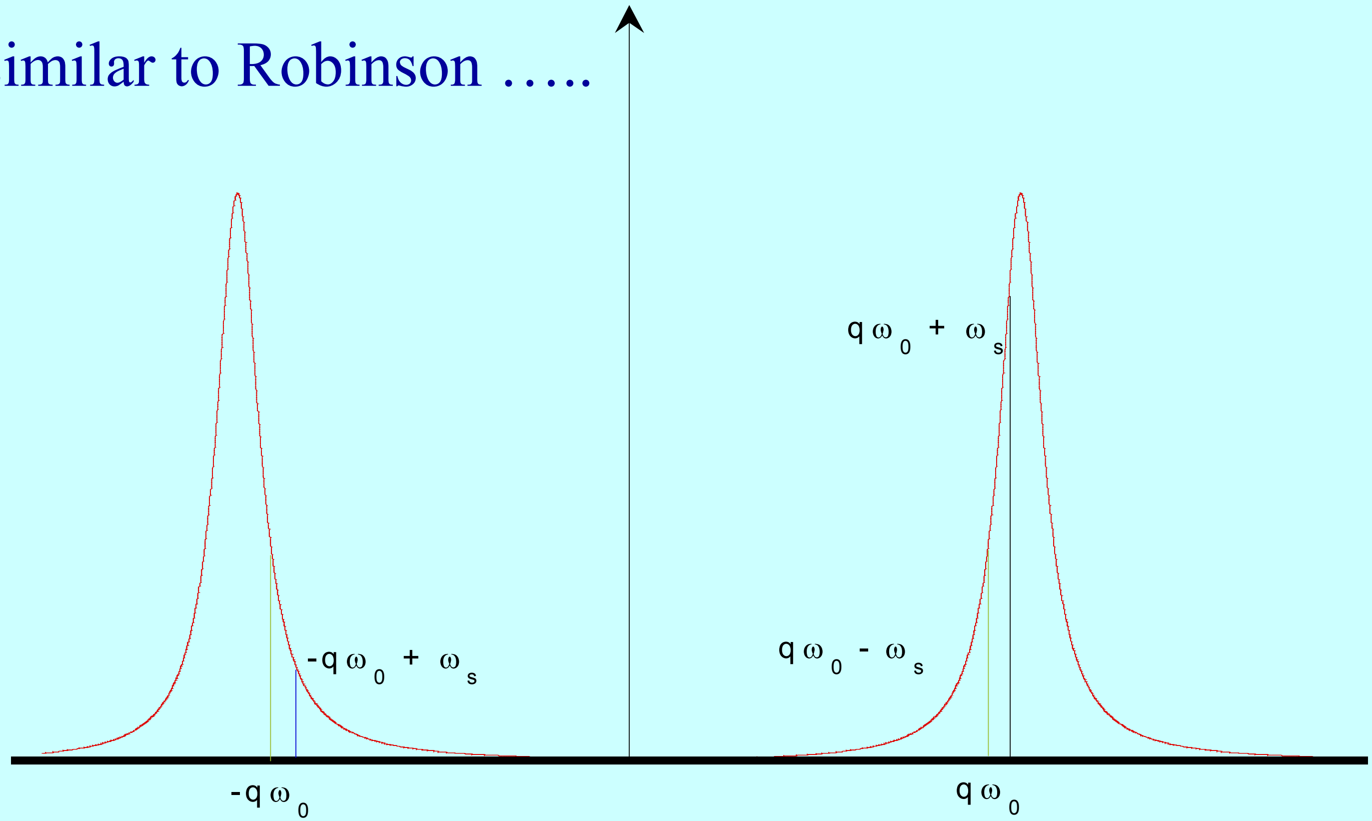
$$\left(\Omega^2 - i\frac{D}{T_0}\Omega - \omega_s^2\right) =$$

$$-i\frac{2c^2\alpha_c e^2 Q}{L_0^2 E_0}\left[(q\omega_0 + \Omega)Z_{\|}(\Omega + q\omega_0) - (q\omega_0 - \Omega)Z_{\|}(\Omega - q\omega_0)\right]$$

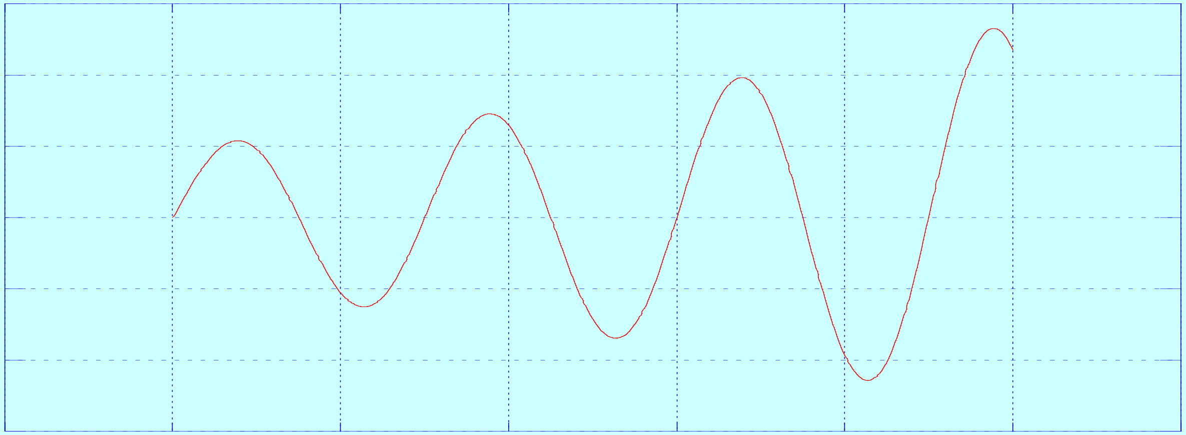
That can be further simplified ($\Omega \approx \omega_s$)

$$\Omega = \omega_s + i\frac{D}{2T_0} - i\frac{c^2\alpha_c eQ}{L_0^2 E_0 \omega_s}\left[(q\omega_0+\omega_s)Z_{\|}(\omega_s+q\omega_0) - (q\omega_0-\omega_s)Z_{\|}(\omega_s-q\omega_0)\right]$$

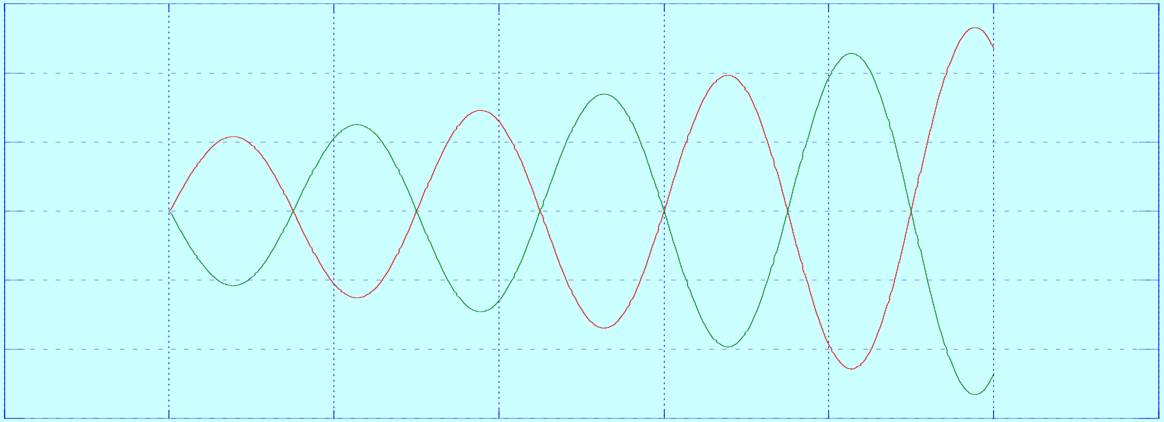
Looks similar to Robinson .....



Notice : even q's corresponds to the two bunches oscillating in phase

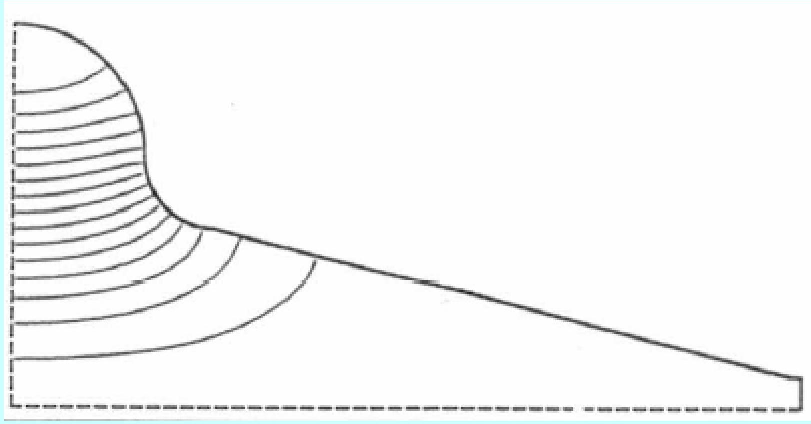


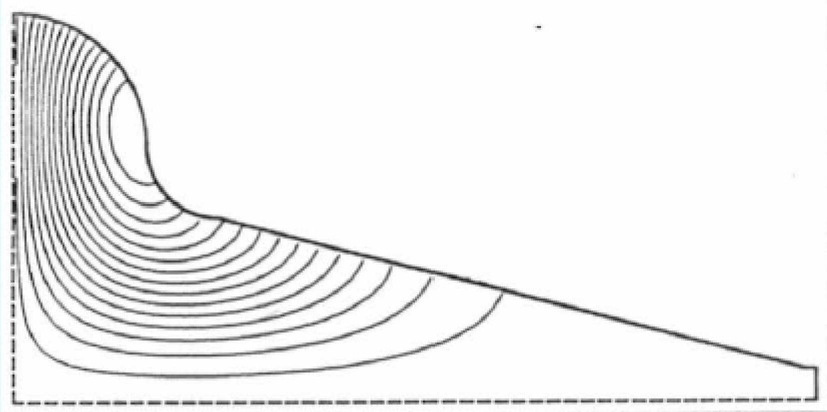
odd q's corresponds to the two bunches oscillating with π phase shift

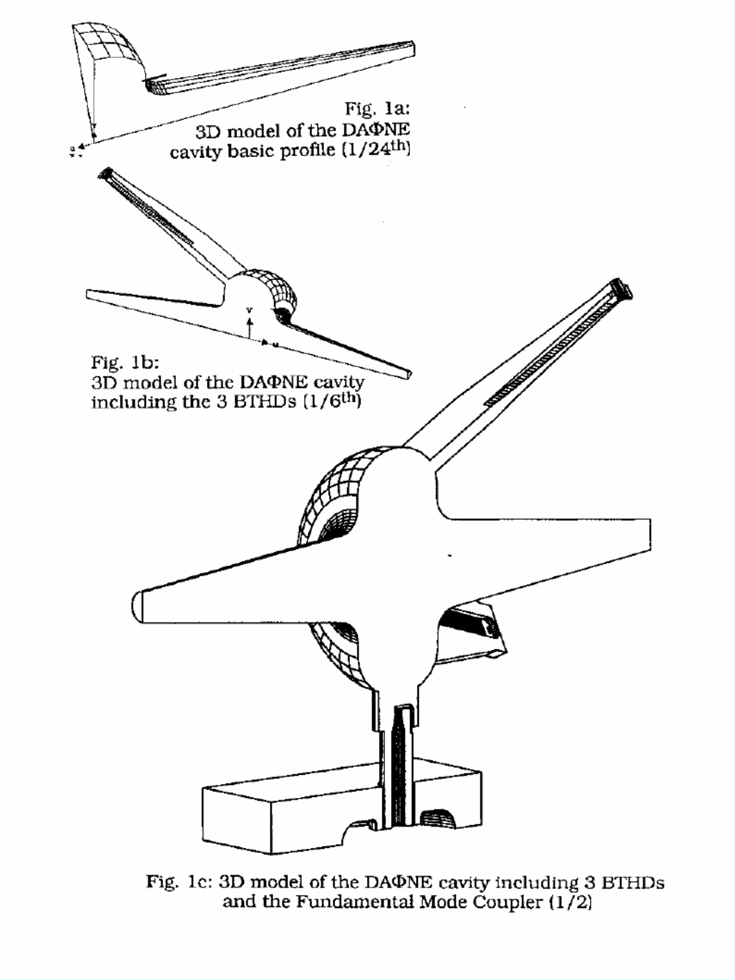


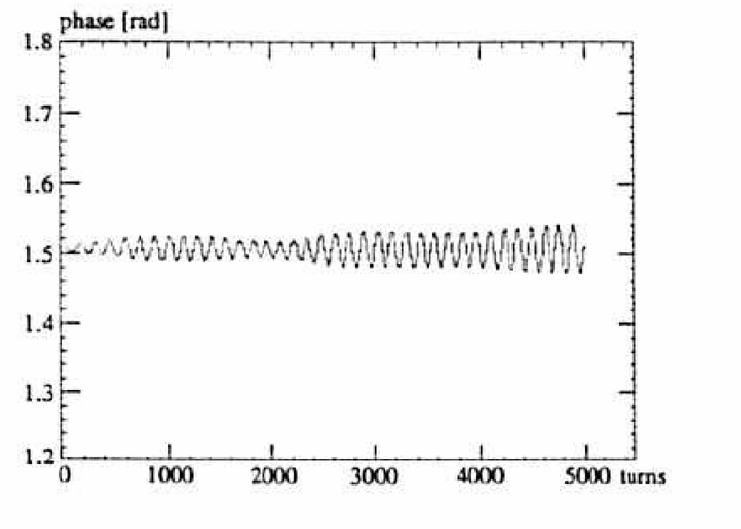
Design strategy: proper design of resonant devices

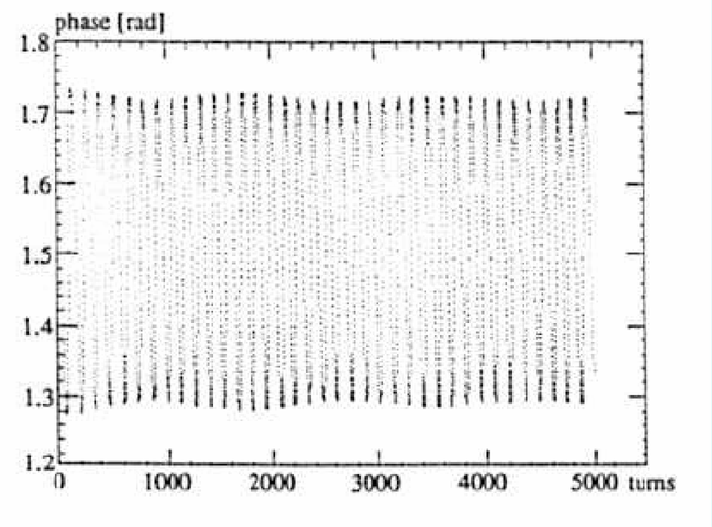
- Reduce HOM's, low Rs / Q and Q

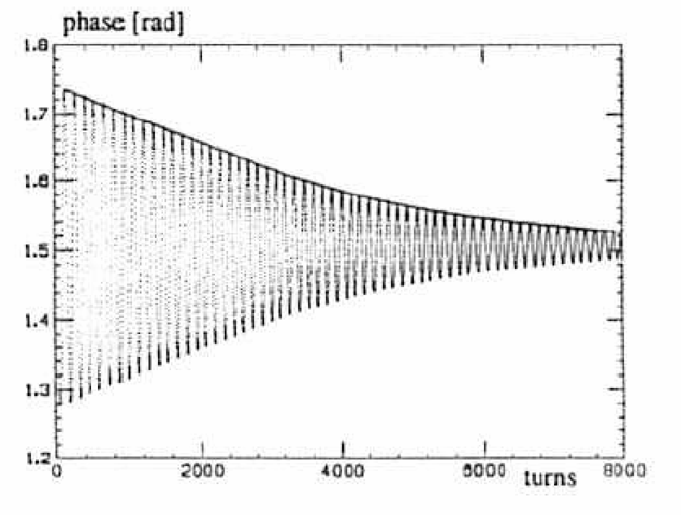












Cures

- Longitudinal feedbacks
- Landau damping

In general, if we suppose a single narrow band HOM and $\Omega^{(\mu)} \approx \omega_s$ then only two (different) oscillation modes are excited, and we obtain

$$\Omega^{(\mu+)} = \omega_s + i\frac{D}{2T_0} - i\frac{c^2\alpha_c e^2 N_p N_b}{2L_0^2 E_0 \omega_s}\left[(q_1 N_b + \mu_+)\omega_0 + \omega_s\right] Z_\parallel\left[(q_1 N_b + \mu_+)\omega_0 + \omega_s\right]$$

$$\Omega^{(\mu-)} = \omega_s + i\frac{D}{2T_0} + i\frac{c^2\alpha_c e^2 N_p N_b}{2L_0^2 E_0 \omega_s}\left[(q_2 N_b - \mu_-)\omega_0 - \omega_s\right] Z_\parallel\left[-(q_2 N_b - \mu_-)\omega_0 + \omega_s\right]$$

(q_1 and $q_2 > 0$)

μ+ (positive synchrotron sideband) is the unstable oscillation mode
μ- (negative synchrotron sideband) is the stable oscillation mode

Conclusions

- The Longitudinal Instability mechanisms are well understood;
- With an accurate model of the machine impedance one can predict the single bunch and multibunch dynamics;
- Single bunch instabilities are not destructive but lead to beam heating (increase of energy spread and bunch length)
- Multibunch instabilities are destructive and require the installation of a fast feedback system on the ring.
- Necessary an accurate design of the vacuum chamber and RF devices