

LATTICES FOR LIGHT SOURCES

Amor NADJI Synchrotron SOLEIL

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Ultra relativistic electrons can be deviated by the constant magnetic field of bending magnets in which their trajectory is an arc of circle



They emit photons in a direction tangent to their trajectory **This is synchrotron radiation**

Such conditions are met in electron Storage Rings

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A Storage Ring : A typical Cell

The arrangement of magnets along the desired beam path is called the magnet **LATTICE**





Standard Magnetic Elements

The Bending Magnets deviate the beam in the Horizontal plane (2π in total)



around an off centered orbit

from the center. It can focus the beam in one plane with the reverse effect in the other plane



A parameter of prime importance in experiments with synchrotron radiation sources is the spectral brilliance (brightness) defined as :

$$B = \frac{dN_{ph}}{dA \, d\Omega \, dt \, d\lambda / \lambda}$$

 $\frac{photons \ per \ sec \ ond}{mm^2 mrad^2 0.1\% b.w}$

High intensity, small e- beam cross section, good focusing of photon beam and high monochromaticity are looked for in order to maximize the photon flux.

✤ Apart from *diffraction effects*, we have :

$$dA d\Omega \approx \varepsilon_x \varepsilon_z$$

 ε_x and ε_z are the transverse e- beam emittances, which are the areas occupied by the beam respectively in horizontal (x,x') and vertical (z,z') phase planes.

HIGH PHOTON BEAM BRILLIANCE ⇒ **LOW** ELECTRON BEAM EMITTANCE



- Additionally undulator magnets must be used to fully take advantage of a low beam emittance.
- The radiation from undulators :

$$\lambda_n = \frac{\lambda_o}{2n\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$

The natural spectral width of the radiation :

$$\frac{\Delta\lambda_n}{\lambda_n} = \frac{1}{nN}$$

The finite divergence of the beam can enlarge this width :

$$\frac{\Delta\lambda_n}{\lambda_n} = \left\{ \left(\frac{1}{nN}\right)^2 + \frac{\gamma^2}{\left(1 + \frac{K^2}{2}\right)} \left(\sigma_x^{\prime 2} + \sigma_z^{\prime 2}\right) \right\}^{1/2}$$

A small beam emittance is desirable when the spectral purity is of importance





The successive generations of storage ring based sources

<u>1st generation</u>

exploitation of the light from the bending magnets of e+/e- colliders originally built for elementary particle physics

2nd generation

exploitation of the light from the bending magnets and a few insertion devices at a later stage of dedicated storage rings ! lower e beam emittances and ! better optimisation of light extraction gain of about 1 000

<u>3rd generation</u>

brilliance is the figure of merit rings designed for very low emittances priority to undulators large number of straight sections gain of about 10 000 moderate gain of about 100 on bending magnet light



Due to the random location (random amplitude has the same effect) of the emission, at the exit they are well separated : **the e⁻ beam is heated up by radiation** introducing a sort of noise or dilution. An e- beam size and divergence are created (**plus E spread**)

Concept of e- beam emittance in [m*rad] units

 $\mathcal{E}_{x[m^*rad]} = \frac{1}{\pi} \oint dx dx'$ associated with a given E

Note that the phase space dilution is less for a short bending magnet

Generation of emittance by radiation

Two e⁻ with same E and same trajectory at the entrance of the bending magnet (they cannot be distinguished : no e- beam size)

They both radiate ΔE and exit with $E - \Delta E$

1 emits ΔE at the origin, smaller ρ 2 emits ΔE but at the end They follow different trajectories

$$dx(s) = \eta(s) \frac{\Delta E}{E}$$
 $dx'(s) = \eta'(s) \frac{\Delta E}{E}$

 $\eta(s)$ and $\eta'(s)$ are respectively the dispersion function and its derivative



e- beam horizontal phase space



The concept of emittance

Accelerator physicists used to write

$$\alpha_X : \beta_X : \gamma_X \qquad \gamma_X = \frac{1 + \alpha_X^2}{\beta_X} \qquad \alpha_x = -\frac{1}{2} \frac{d\beta_x}{ds}$$

ellipse (Twiss) parameters which vary along the storage ring structure

The quantity :

 $\gamma_{x} \mathbf{x}^{2} + 2\alpha_{x} \mathbf{x} \mathbf{x}' + \beta_{x} \mathbf{x}'^{2}$

is an invariant when following a particle trajectory and disregarding radiation

In an electron storage ring, the distributions in X and X' are gaussian

$$\sigma_{Xbet}^{2}(s) = \beta_{X}(s)\varepsilon_{X}$$

$$\sigma_{X'bet}^{2}(s) = \gamma_{X}(s)\varepsilon_{X}$$

ε_x

give the rms horizontal size and divergence in terms of the emittance



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ELLE Transport of an e-beam by means of magnetic elements



Given an emittance,

$$\varepsilon_{x[m^*rad]} = \frac{1}{\pi} \oint dx dx'$$

it can be transported through a channel with bending magnets and focusing quadrupole



 $\epsilon_{\rm X}$ is invariant when radiation is neglected

The transverse motion (X and Z planes) of individual particles around the beam center of mass is called betatron (bet) oscillation

If the outer phase space rms limit contour is an ellipse, it will remain an ellipse with different parameters but same inner area



Fortunately there is a cooling effect

The heating in the horizontal X plane produced by radiation in every bending magnet would lead to a continuous blow up of the emittance, size and divergence however, there is the RF cavity that restores the energy lost per turn and cools down the transverse horizontal oscillations ΔF



The two adverse effects (heating+cooling) lead to an equilibrium emittance ϵ_x and gaussian distributions in X and X'



- The two counteracting effects (heating + cooling) lead to an equilibrium emittance
- The natural horizontal emittance for an isomagnetic ring, i.e. all bending magnets having same bending radius is :

$$\varepsilon_x = \frac{C_q \gamma^2 \langle H \rangle_{dipole}}{J_x \rho}$$

 J_x is the horizontal damping partition number. $J_x \sim 1$ (zero field gradient in bending magnet) $J_x \sim 2$ (vertical focusing in bending magnet) : (potentially) emittance reduction of a factor two

 C_q = 3.83 x 10⁻¹³ m and γ is the Lorentz factor (ratio of the particle energy to its rest energy). ρ is the bending radius.

- H is the so called lattice invariant or dispersion's emittance or H-function
- (...) an average taken only in the part of the circumference where photons are emitted, that is in the bending magnets (and Insertion Devices).

$$H(s) = \gamma_x(s)\eta^2(s) + 2\alpha_x(s)\eta(s)\eta'(s) + \beta_x(s)\eta'^2(s)$$

*****In practical units, ε_x is given by :

$$\varepsilon_{x}[nm.rad] = 1470 E[GeV]^{2} \frac{\langle H \rangle_{dipole}}{\rho J_{x}}$$

 ε_x is completely determined by the energy, bending field and lattice functions.

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♦ β₀, α₀, γ₀, η₀, η'₀, are the values of the lattice functions <u>at the beginning</u> of the **BM** ♦ We assume a lattice where η₀=η'₀ =0 (achromatic condition).

✤ We get after integration ov	ver one BM :	$\langle H \rangle = \beta_0 B + \alpha_0 \rho A + \gamma_0 \rho^2 C$
For small bending angles: (sufficiently accurate for most storage rings design application)	$B \approx \frac{1}{3} \Theta^2 \left(1 - \frac{1}{5} \Theta^2 \right),$ $A \approx -\frac{1}{4} \Theta^3 \left(1 - \frac{5}{18} \Theta^2 \right)$ $C \approx \frac{1}{20} \Theta^4 \left(1 - \frac{5}{14} \Theta^2 \right)$), $\begin{array}{c} \theta = \frac{l_b}{\rho} \\ l_b \end{array}$ the length of the BM . Θ the full BM deflexion angle

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✤ We get for the natural horizontal e- beam emittance :

$$\varepsilon_{x} \cong \frac{C_{q} \gamma^{2} \Theta^{3}}{J_{x}} \left[\frac{1}{3} \frac{\beta_{0}}{l_{b}} - \frac{1}{4} \alpha_{0} + \frac{1}{20} \gamma_{0} l_{b} \right]$$

♦ There is clearly a cubic dependence of the beam emittance on the deflexion angle It is a general lattice property, there is no assumption on the lattice type.

→ Should use many short BMs to get low emittance.

If the ring consists of N identical **BMs** :
$$\Theta = \frac{2\pi}{N}$$
 \longrightarrow $\varepsilon_x \propto \frac{1}{N^3}$

Need for a magnet focusing (quadrupoles) providing small waist for the optical functions

Minimum Emittance (with Achromatic Arc Condition)

✤ In order to get the minimum possible emittance we have to vary the initial conditions β₀ and α₀ until the minimum is found.

$$\frac{\partial \varepsilon_x}{\partial \alpha_0} = \frac{C_q \gamma^2 \Theta^3}{J_x} \frac{\partial}{\partial \alpha_0} \left(\frac{1}{3} \frac{\beta_0}{l_b} - \frac{1}{4} \alpha_0 + \frac{1}{20} \frac{1 + \alpha_0^2}{\beta_0} l_b \right) = \frac{C_q \gamma^2 \Theta^3}{J_x} \left(-\frac{1}{4} + \frac{l_b}{10} \frac{\alpha_0}{\beta_0} \right) = 0.$$

and $\frac{\partial z_x}{\partial \beta_0} = \frac{\partial q}{J_x} \left[\frac{1}{3} - \frac{\boldsymbol{\iota}_b}{20} \frac{1+\alpha_0}{\beta_0^2} \right] = 0.$

• We can calculate the unknown initial conditions β_0 and α_0 :

 $\beta_{0,\min} = \sqrt{\frac{12}{5}} l_b$ $\alpha_{0,\min} = \sqrt{15}$

The minimum possible emittance is determined only by the **BM** length, l_b

✤ The minimum equilibrium beam emittance in an isomagnetic ring with an Achromatic Arc Condition, $\eta_0 = \eta'_0 = 0$, at the entrance of the **BM** is:

$$\varepsilon_{x,\min} = \frac{C_q \gamma^2 \Theta^3}{4\sqrt{15} J_x}$$

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By breaking the achromatic condition (non-zero dispersion in straight sections) we can obtain the configuration in which the emittance becomes the smallest.

$$\varepsilon_{x,\min} = \frac{C_q \gamma^2 \Theta^3}{12 \sqrt{15} J_x}$$

✤ It is smaller by a factor 3 than in the achromatic arc configuration.

The optimum values of betatron functions at the entrance of the BM are:

$$\beta_{0,\min} = \frac{8}{\sqrt{15}} l_b \quad \alpha_{0,\min} = \sqrt{15} \quad \eta_{0,\min} = \frac{1}{6} \rho \Theta^2 \quad \eta_{0,\min} = -\frac{1}{2} \Theta$$

It is interesting to note that in this case, the dispersion and the betatron function are parabola with the symmetry axis at the middle of the bending magnet :

$$\eta(x) = \frac{\rho\Theta}{2} \left(x - \frac{1}{2}\right)^2 + \frac{\rho\Theta^2}{24} \qquad \beta(x) = \frac{15\beta_0}{4} \left(x - \frac{1}{2}\right)^2 + \frac{\beta_0}{16} \qquad x = \frac{\theta}{\Theta} \qquad \text{Minimum at center}$$
of the **BM**.



Beam Sizes

The local effective emittance relevant for the brilliance has to include the projection of energy spread σ_e to the horizontal dimension via the dispersion η .

$$\varepsilon_{x,eff} = \sqrt{\varepsilon_{x0}^2 + \varepsilon_{x0} H(s)\sigma_e^2}$$

$$\sigma_x(s) = \sqrt{\varepsilon_x \beta_x(s) + (\sigma_e \eta_x(s))^2}$$
 Horizontal beam size

Where
$$\sigma_e = 6.64 \times 10^{-4} \sqrt{\frac{B(T)E(GeV)}{J_s}}$$

the statistical process caused by the emission of synchrotron radiation affects also the energy-time phase space : a gaussian distribution of the individual particle energies . J_s is the longitudinal damping partition number.

❖ Ideally, a flat lattice (i.e. with no vertical BMs) has no vertical emittance (*H* = 0.). In reality, however, spurious vertical dispersion and linear coupling create a finite vertical emittance, ε_z. Vertical beam size (negligible vertical dispersion) : $\sigma_z(s) = \sqrt{\varepsilon_z \beta_z(s)}$

f $\kappa^2 = \frac{\varepsilon_x}{\varepsilon_z}$, the natural horizontal emittance ε_{x0} is divided into horizontal and vertical emittances

 κ^2 is generally of the order of a few %. It can be controlled by means of skew quadrupoles.

$$\varepsilon_x = \frac{\varepsilon_{x0}}{1+\kappa^2}$$
 $\varepsilon_z = \frac{\kappa^2 \varepsilon_{x0}}{1+\kappa^2}$



LOW EMITTANCE (HIGH BRILLIANCE) LATTICES

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Design Criteria For (Modern) Synchrotron Light Sources

- * Emittance is the design criteria
- Compactness
- Extensive use of Insertion Devices such as **Undulators** and Wigglers (highest ratio of available straight sections to the circumference).
- Straights of variable length, based on both machine and user needs
- Very long straight sections
- Tunability
- Stability
- Upgrade potential

The Double Bend Achromat Lattice (DBA)

R. Chasman and K. Green proposed (~1975) a cell with 2 BMs separated by 1
 Focusing Quadrupole. It is the basic <u>Chasman-Green</u> lattice.

> Two quadrupoles (FQ1 and DQ1) in straight section to match the β -functions in the **BM** to reach the minimum emittance.

➤The strength of the CFQ is adjusted so that the dispersion generated by the first dipole is cancelled by passing through the second dipole (achromatic condition).

the structure is rather limited in tunability

- > no simultaneous control of β-functions in straight section
- restricted area for the sextupoles and diagnostics





Example of ELETTRA (Achromatic Condition):

A triplet of quadrupoles for the control of dispersion in the achromat section, two quadrupole triplets for the adjustment of beam sizes in straight sections.



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Example of ESRF (Achromatic Condition):

- Four quadrupoles are used between the two BMs.
- Different β -functions in straight sections: to provide parallel beam in one insertion and small beam sizes in the other.



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Example of ESRF (non-Achromatic Condition):

The DBA lattice of ESRF starting with dispersion free straight sections was later tuned into a dispersive mode in order to reduce the emittance further.



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Expanded Chasman-Green Lattices (4)

Super-ACO : ABSOLUTE MINIMUM EMITTANCE OPTICS



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Super-ACO : Zoom on optical functions in the dipole.





Example of SOLEIL (non-Achromatic Condition): SOLEIL plans for a dispersive lattice right from the beginning

C = 354m

24 straight sections

(variable length) 4 x 12 m 12 x 7 m 8 x 3.6m

Different β -functions in straight sections

Finite dispersion In straight sections



(increase the number of straight sections by drifting apart the two focusing quadrupoles in the achromat)



Triple Bend Achromat

★ Large flexibility since the phase advance in the achromat can be varied between π and 2π by changing the position of the achromat quadrupoles and/or the length of the central bending magnet / outer ones.



to keep the emittance small, the dispersion in the inner BM must be reduced ⇒ large sextupole strengths

- More suitable for small rings, where the deflexion angle per BM, and correspondly the dispersion is larger.

-More compact than a DBA lattice of the same emittance, however provides fewer straight sections.





Triple Bend Achromat

Example of SLS :

C = 288m

12 straight sections

(variable length) 3 x 11.7 m 3 x 7 m 6 x 4m

Different β -functions in straight sections

Zero dispersion In straight sections





Example of Factor of Merit

Source	Energy			ΣL_{SS}	8.0	F
	(GeV)	Θ	C(m)		(nm.rad)	
ALS	1.9	0.1745	197	81	5.6	0.48
BESSYII	1.9	0.1963	240	89	6.4	0.68
DIAMOND	3	0.1309	562	218.2	2.74	2.11
ESRF	6	0.09817	844	201.6	4	1.73
ELETTRA	2	0.2618	258	74.78	7	3.05
SLS	2.4	0.2440	288	63	5	6.13
SOLEIL	2.75	0.1963	354	159.6	3.7	10.66

$$F = 10^{5} \times \left(\frac{\sum L_{SS}}{Circumference}\right) / (\mathcal{E}_{n})^{2}$$
$$\mathcal{E}_{n} = \frac{\varepsilon_{x0}}{(Energy)^{2} \times (\Theta)^{3}}$$

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 Looking into the details of all these lattices, one will find that the optical functions do not exactly fit the conditions needed to reach the minimum emittance.
 Example of SOLEIL :

Ideal values :	Design values :
$\alpha_{0,min}$ = $\sqrt{15}$ = 3.873	$\alpha_0 = 1.8$
$\beta_{0,min} = 2.17m$	$\beta_0 = 1.5 m$
$\varepsilon_{x0,min} = 1.8 nm.rad$	$\varepsilon_{x0} = 3.7 \ nm.rad$

★ The ideal value $\alpha_{0,min} = \sqrt{15}$ causes the betatron function to reach a sharp minimum inside the **BM** and then to increase from there on to large values in the quadrupoles, leading to extremely high **chromaticity** (the quadrupoles do not provide the same focussing strength for particle with energy deviation).

$$\xi_{x,z} = dv_{x,z}/(d \Delta p/p)$$

$$v_{x,z}$$
: betatron tunes



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This must not be tolerated for two reasons :

Momentum acceptance : some variation of energy deviation has to be accepted by the storage ring for reasons of **beam lifetime**.

- Head tail instability: collective oscillation of electrons in head and tail of the bunch leading to very fast beam loss. (m)
- We must operate with zero or positive chromaticity

Chromaticity :
$$\xi \sim \int (-K_Q \beta + m_s \beta \eta) ds \ge 0$$

quadrupole strength (introduce negative ξ)

sextupole strength (correct the ξ)

Strong chromaticity correction sextupoles reduce the **dynamic aperture** and this negatively impacts on the beam lifetime.



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Beam Lifetime

2 main limitations :





High density-Touschek scattering of particles - large longitudinal transfers of energy - loss unless large acceptance : RF acceptance, physical aperture, dynamic aperture for large E deviations

 $\tau_T \propto \mathcal{E}_{acc}^{>2}$ (SOLEIL, SLS : ε_{acc} = 4 to 6%)

② Elastic Scattering : scattering with residual gaz of pression p



Transverse deflexion and subsequent lossin regions of low aperture, which are usually the narrow vertical gaps in undulators (g).

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Beam position stability is crucial

• Users specifications : σ_{cm} < 0.10 σ_{Beam} and σ'_{cm} < 0.10 σ'_{Beam}

	σ _{cm} (μm)	σ' _{cm} (µrad)
Horizontal	18	3
Vertical	0.8	0.5

Amplification factors :

$$=\frac{\boldsymbol{\sigma}_{cm}}{\boldsymbol{\sigma}_{girder}}$$

 A_{\cdot}

$$A = \frac{\sqrt{\beta_{obs}}}{2\sqrt{2} \sin \pi \nu} \sqrt{\sum_{i} \beta_{i} (Kl_{i})^{2}}$$

$$A_x = 14 \sigma g_x = 1.28 \mu m$$

 $A_z = 3$ $\sigma g_z = 0.27 \ \mu m$

This corresponds to 1µm peak to peak tolerance in the vertical plane

Sub-micron level !!!

Brilliance reduction, Emittance growth Time dependent Orbit Oscillations Magnets motion Girders (support) motion Ground Vibration

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Optimisation Method (1)





Tune Diagram (Working point)





Optimization Method (2)







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X-Ray case : To obtain a high degree of coherence, constraints are to be satisfied by the optics of the ring





to produce X-ray diffraction limited beams in both planes $\mathcal{D}a$ reduction of ε_x by a factor $\mathcal{D}0$

Increased storage ring size (Chasman-Green)

 N_{D} = 2 N_{D0} ; ($\theta_{D} = \theta_{D0}/2$) number of cells x by 2; circumference x 2 for a 3 GeV ring, 32 periods, 700 m circumference (~ ESRF) $\eta = \eta_{0}/4$ stronger ξ sextupoles, lower dynamic aperture, ($\alpha = \alpha_{0}/4$); $\epsilon_{c} = \epsilon_{c0}$; $\tau_{x} = 2\tau_{x0}$;

☐ gain a factor $2^3 = 8$ (heating by radiation/16 and damping/2) significant reduction of **H** (V maintained at the limit) and // beam sizes . Intrabeam scattering makes emittance reduction (<4 for 500 mA) much less effective **Increased damping**

All $\eta=0$ ID straight sections (200 metres) filled with damping wigglers thus occupying all the high brilliance sections ; $\Delta E=4 \Delta E_0$

reduction of the damping time $\tau_x = \tau_{x0}/4$

gain another factor

Resulting Touschek lifetime is dramatically shortened (factor 15 V at the diff. limit) • Acceptable for damping rings but not for light source storage rings

Saturation of SR X-Ray light source Brilliance in the low 10²² range slightly below the diffraction limit in the horizontal plane

Emittance Minimisation with Longitudinal Dipole Field Variation (a new step?)

The idea proposed by A. Wrulich is to set the bending field higher where H(s) is lower and vice versa.

✤R. Nagaoka applied the idea to SOLEIL and proposed several laws of dipole field variation (exponential decay, polynomials).

♦ SOLEIL-like :Using an exponentially decaying field with a peak value of 4.4 T, superposed by the transverse gradient of 18 T/m : $ε_x$ =0.9 nm.rad.

A factor **4** lower than that of SOLEIL emittance.

✤ A lattice of 0.8nm.rad is calculated by A. Ropert et al. For and ESRF-type with a maximum field of 1.8T.

Guo and Raubenheimer proposed independently the same idea applied for the NLC damping ring.







SOLEIL PROJECT : ACCELERATORS

LINAC specification :

• (500 mA in 416 bunches): Output LINAC charge 8 nC in

300 ns

• In temporal structure (100 mA in 8 bunches):

• Output LINAC charge 1.5 nC in 3 bunches



BOOSTER:

2 super periods 36 Dipoles : 0.67 T / 2.17 m 44 Qpoles: 10.3 T/m/0.4 mDrifts: 3.17 m Circumference: 157 m **Emittance**: 150 nm Power supplies cycling at 3 Hz

(SLS concept)

TOP UP Injection : injection every 2 min (for a beam lifetime as bas as 4h).

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2.75 GeV **Energy**: Circumference: 354.097 m Emittance H / V: 3.73 nm.rad / 37.3 pm.rad Number of cells / super periods: 16/4Straight sections: 12 m x 4 ; 7 m x 12 ; 3.8 m x 8 18.19 / 10.29 Betatron tunes, v_{y}/v_{z} : Natural Chromat. ξ_{χ}/ξ_{τ} : -52.42 / -22.76 Momentum compaction: 4.49×10⁻⁴ 1.02 10-3 **Energy dispersion : Revolution Frequency :** 0.846 MHz



SOLEIL PROJECT : BEAMLINES



• 10 beamlines

in summer 2006

• 24 beamlines in 2009







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