ACCELERATION TECHNIQUES

by

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CAS on Intermediate Accelerator Physics Course
Trieste 3-14 October 2005

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Content

Electric field from electro-magnetic waves,

Pill-box type cavities,

Fundamental parameters of accelerating cavities,

Equivalent circuit and input impedance of a cavity,

Criteria for cavity choice,

Different types of single cell cavities,

Different types of multi-cells cavities,

Evolution of RF cavities,

Beam loading.

Acceleration and Energy Gain

ACCELERATION is the main task of an accelerator.

- An accelerator provides KINETIC ENERGY to charged particles and increase their MOMENTUM.
- To do so an ELECTRIC FIELD, **E**, is required, possibly along the momentum **p** :

$$\boxed{\frac{dp}{dt} = eE}$$

The ENERGY GAIN is the work from the electric field force :

$$E^2 = E_0^2 + p^2c^2 \quad \Rightarrow \quad dE = v\,dp$$

$$\frac{dE}{dz} = v\frac{dp}{dz} = \frac{dp}{dt} = eE_z$$

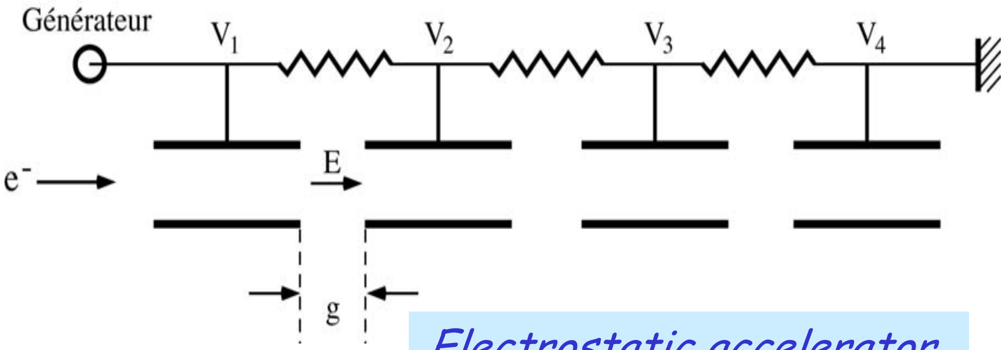
$$dE = dW = eE_z dz \quad \Rightarrow \quad W = e\int E_z dz$$

Methods of acceleration

1_ Electrostatic field

Energy gain : $W=n.e(V_2-V_1)$

limitation : $V_{generator} = \Sigma\ V_i$



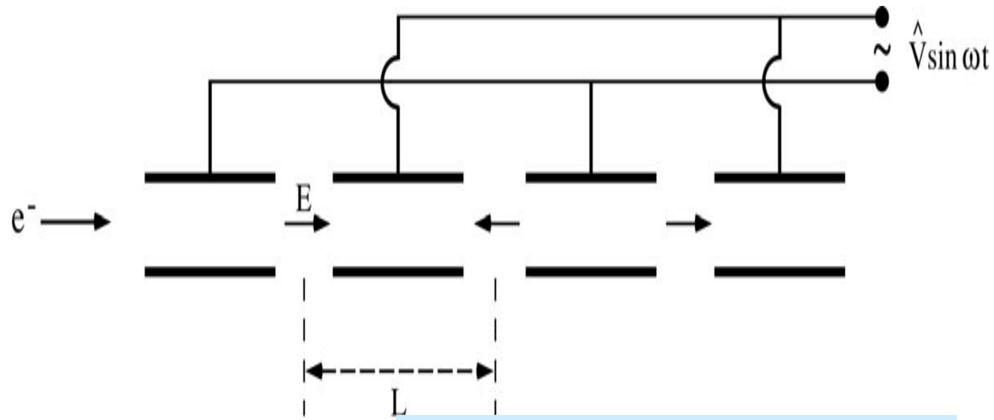
Electrostatic accelerator

2_ Radio-frequency field

Synchronism ➡ $L=vT/2$

v=particle velocity

T= RF period



Wideroe type accelerator

Methods of acceleration (2)

3_ Acceleration by induction

MAXWELL EQUATIONS :

The electric field is derived from a scalar potential V and from a vector potential A
It is the time variation of the magnetic field H that creates the electric field E

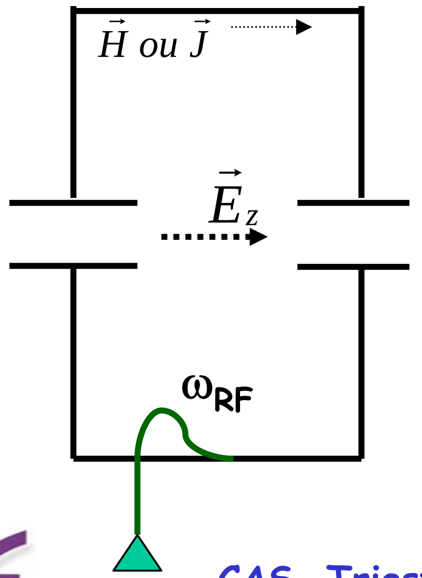
$$E = - \, gradV - \frac{\partial A}{\partial t}$$

$$B = \mu H = rot \; A$$

Towards Radio-Frequency Cavities

- When particles get relatively high velocities the drift spaces get longer and the accelerator lose efficiency. A first solution consists of using a higher RF frequency.

- However the power lost by radiation, due to circulating currents on the electrodes, is proportional to the RF frequency. A second solution hence consists of enclosing the system inside a cavity which resonant frequency is adjusted to the RF generator frequency.

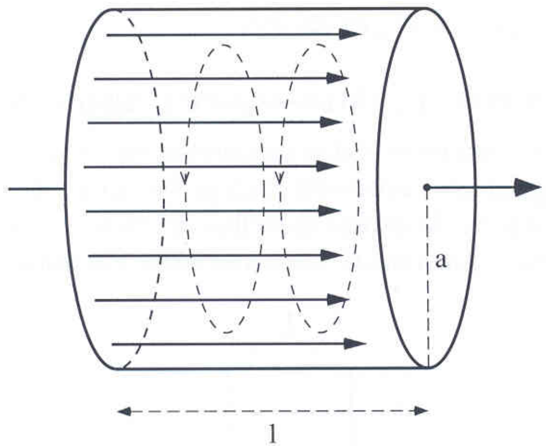


- Each such cavity can be independently fed from a RF power source.

- The electro-magnetic power is stored in the resonant volume.

- Note however that power is dissipated in the cavity walls (joule losses) unless there are made of SC materials.

The Pill-Box Cavity

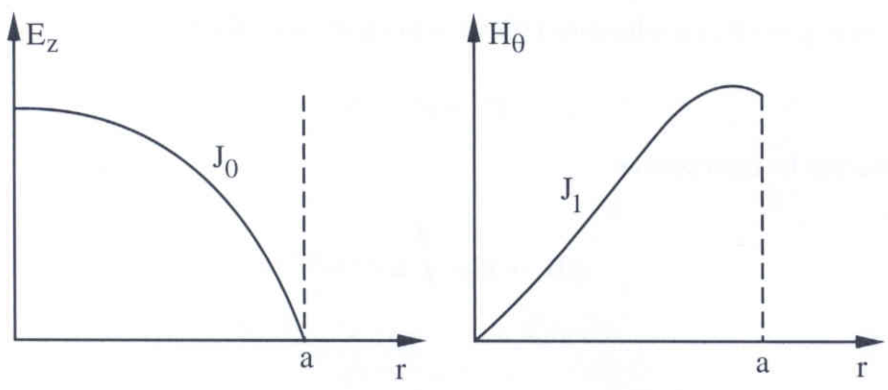


$\longrightarrow$ E_z $\cdots\cdots\rightarrow$ H_θ

From Maxwell equations one gets the wave equations :

$$\nabla^2 A - \varepsilon_0 \mu_0 \frac{\partial^2 A}{\partial t^2} = 0 \qquad (A = E \text{ ou } H)$$

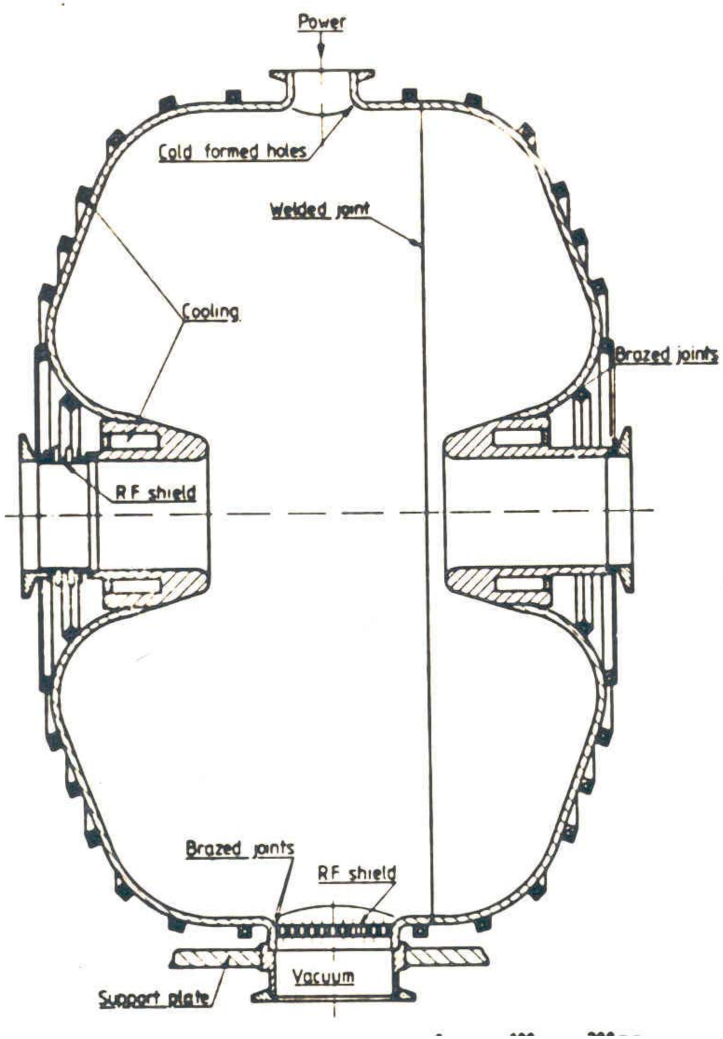
The solutions for E and H are oscillating modes (SW) ,at different frequencies, of types TM or TE. For l<2a the most simple mode is TM_{010}, It has the lowest frequency, and only two field components:



$$\left.\begin{aligned} E_z &= J_0(kr) \\ H_\theta &= -\frac{j}{Z_0} J_1(kr) \end{aligned}\right\} e^{j\omega t}$$

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} \quad \lambda = 2{,}62\,a \quad Z_0 = 377\Omega$$

The Pill-Box Cavity (2)



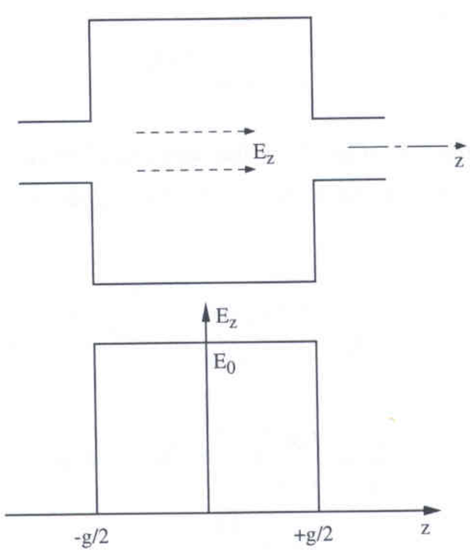
The geometry of a pill-box can be sophisticated in order to improve its characteristics:

-Shaping a nose allows to concentrate the electric field along the axis, where really needed.

-Shaping the body, to avoid sharp corners, allows to spread the magnetic field and reduce the wall losses as well as multipactor.

A good cavity is a cavity which efficiently transform the input RF power into an accelerating electric field.

Transit Time Factor



Consider an oscillating field at frequency ω with constant amplitude in the gap:

$$E_z = E_0 \cos\omega t = \frac{V}{g}\cos\omega t$$

Consider a particle crossing the middle of the gap at time t=0 :

$$z = vt$$

Total energy gain is:

$$\Delta W = \frac{eV}{g}\int_{-g/2}^{g/2}\cos\omega\frac{z}{v}dz$$

$$\Delta W = eV\frac{\sin\theta/2}{\theta/2} = eVT$$

$$\theta = \frac{\omega g}{v} \quad \textit{transit angle}$$

$$T \quad \textit{transit time factor}$$

(0 < T < 1)

Shunt Impedance and Q Factor

The shunt impedance R is defined as the parameter which relates the accelerating voltage V in the gap to the power dissipated in the cavity walls (Joule losses).

$$P_d = \frac{V^2}{R}$$

$$Q = \frac{\omega W_s}{P_d}$$

$$\frac{R}{Q} = \frac{V^2}{\omega W_s}$$

The Q factor is the parameter which compares the stored energy, W_s, inside the cavity to the energy dissipated in the walls during an RF period ($2\pi/\omega$). A high Q is a measure of a good RF efficiency

Filling Time of a SW Cavity

From the definition of the Q factor one can see that the energy is dissipated at a rate which is directly proportional to the stored energy:

$$P_d = -\frac{dW_s}{dt} = \frac{\omega}{Q} W_s$$

leading to an exponential decay of the stored energy:

$$W_s = W_{s0}\, e^{-\frac{t}{\tau}}$$ avec $$\boxed{\tau = \frac{Q}{\omega}}$$ (filling time)

Since the stored energy is proportional to the square of the electric field, the latter decay with a time constant 2τ .

If the cavity is fed from an RF power source, the stored energy increases as follows:

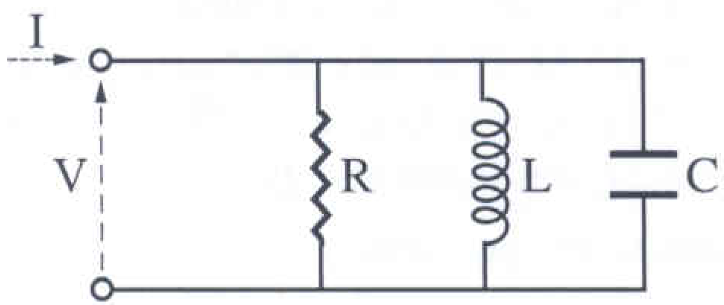
$$\boxed{W_s = W_{s0}\left(1 - e^{-\frac{t}{2\tau}}\right)^2}$$

Equivalent Circuit of a Cavity

RF cavity: *on the average, the stored energy in the magnetic field equal the stored energy in the electric field, $W_{se}=W_{sm}$*

$$\frac{\varepsilon_0}{2}\int_V |E|^2 dV = \frac{\mu_0}{2}\int_V |H|^2 dV$$

RLC circuit: *the previous statement is true for this circuit, where the electric energy is stored in C and the magnetic energy is stored in L:*



$$W_{se} = \frac{1}{4} VV^* C \qquad \omega_0 = (LC)^{-\frac{1}{2}}$$

$$W_{sm} = \frac{1}{4} L\, I_l\, I_L^* \quad avec \quad V = \omega_0 L I_l$$

$$W_s = W_{se} + W_{sm} = \frac{1}{2} CVV^*$$

Leading to:

$$P_d = \frac{1}{2}\frac{VV^*}{R} \qquad Q = \omega_0 RC = \frac{R}{\omega_0 L}$$

Input Impedance of a Cavity

The circuit impedance as seen from the input is:

$$Z_e = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C \right)^{-1} \quad avec \quad \omega = \omega_0 + \Delta\omega$$

Within the approximation $\Delta\omega \ll \omega_0$ the impedance becomes:

$$Z_e \approx \frac{\omega_0^2 RL}{\omega_0^2 L + j2R\Delta\omega} = \frac{R}{1 + j2Q\dfrac{\Delta\omega}{\omega_0}}$$

When $\Delta\omega$ satisfies the relation $Q=\omega_0/2\Delta\omega$ one has $Z_e= 0{,}707\ |Z_e|_{max}$, with $|Z_e|_{max}= R$. The quantity $2\Delta\omega/\omega_0$ is called the bandwidth (BW) :

$$\boxed{Q = \frac{1}{BW}}$$

Loaded Q

If R represents the losses of the equivalent resonant circuit of the cavity, then the Q factor is generally called Q_0.

Introducing additional losses, for instance through a coupling loop connected to an external load, corresponding to a parallel resistor R_L , then the total Q factor becomes Q_l (loaded Q):

$$Q_l = \frac{R_t}{\omega_0 L} \qquad avec \qquad R_t = \frac{R R_L}{R + R_L}$$

Defining an external Q as, $Q_e = R_L / \omega_0 L$, one gets:

$$\boxed{\frac{1}{Q_l} = \frac{1}{Q_0} + \frac{1}{Q_e}}$$

Some Criteria for Cavity Choice

frequency

low	**high**
-better long. capture (p^+ synch.)	-higher efficiency :$R/l \propto \omega^{1/2}$
-high beam transv. acceptance	-use of klystrons (high P_{RF})
-low v/c (transit time)	-reduced size

Q

small	large
-large BW; variable ω_{RF} (protons)	-reduced losses
-short filling time	-better use of P_{RF}

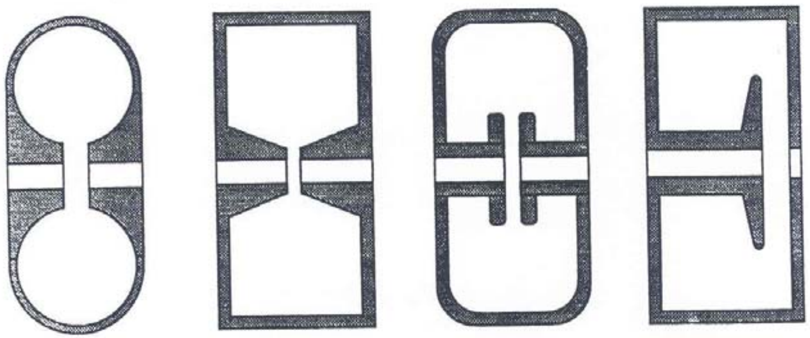
Main Cavities Families

- Single gap cavity
- Multi-cells cavities
  - Multiple RF sources
  - Common RF source
    - Standing wave (SW) cavities
    - Traveling wave (TW) cavities

Single Gap Cavities

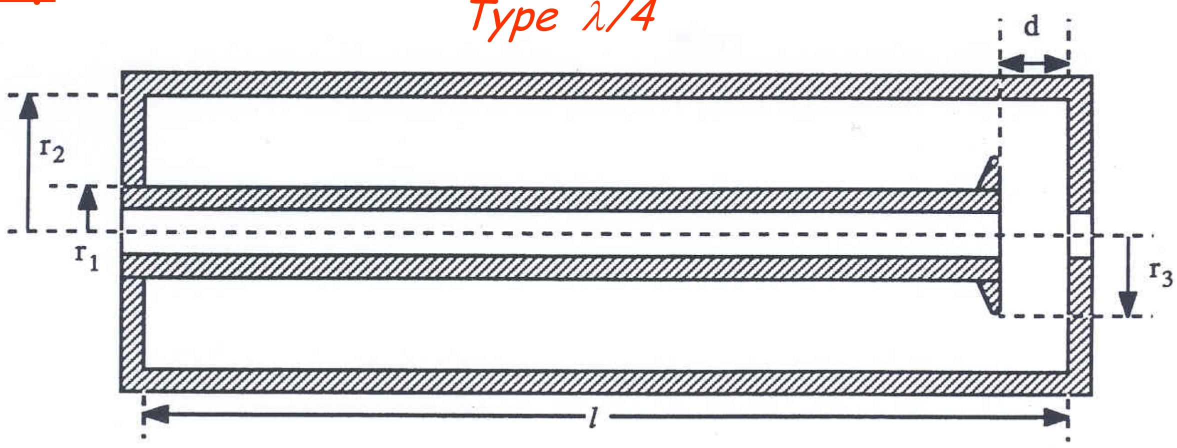
"Pill-Box" variants

with nose *with disk*

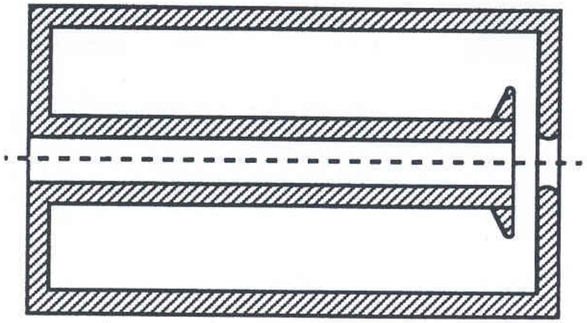


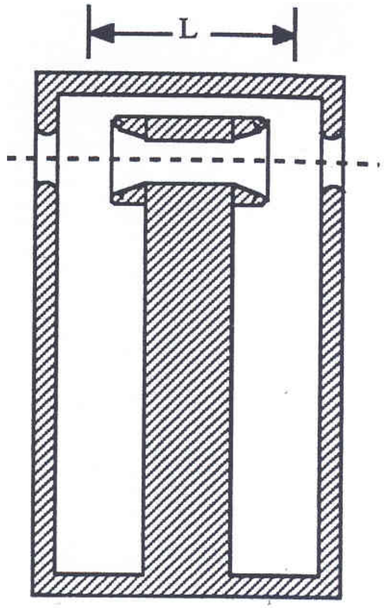
Coaxial cavity

Type $\lambda/4$

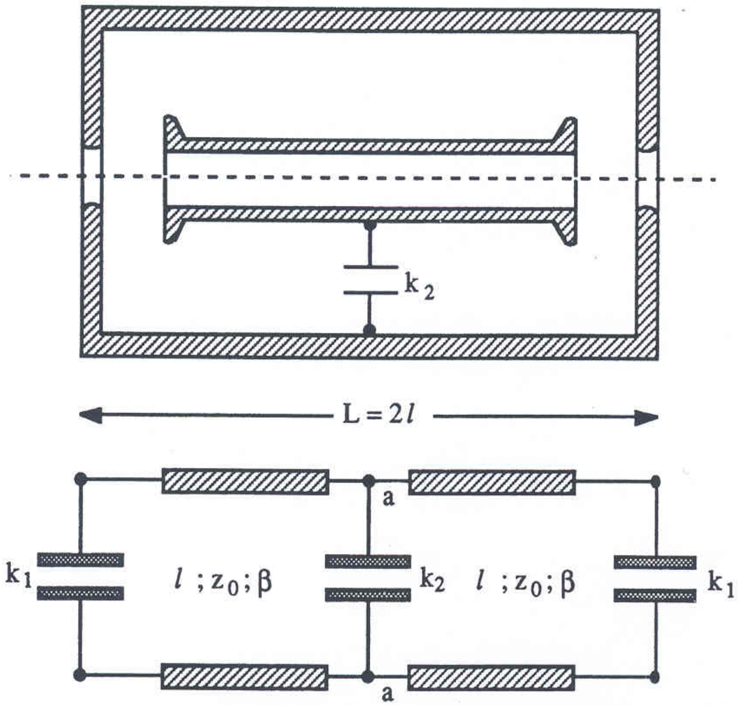


Coaxial Cavities Variants





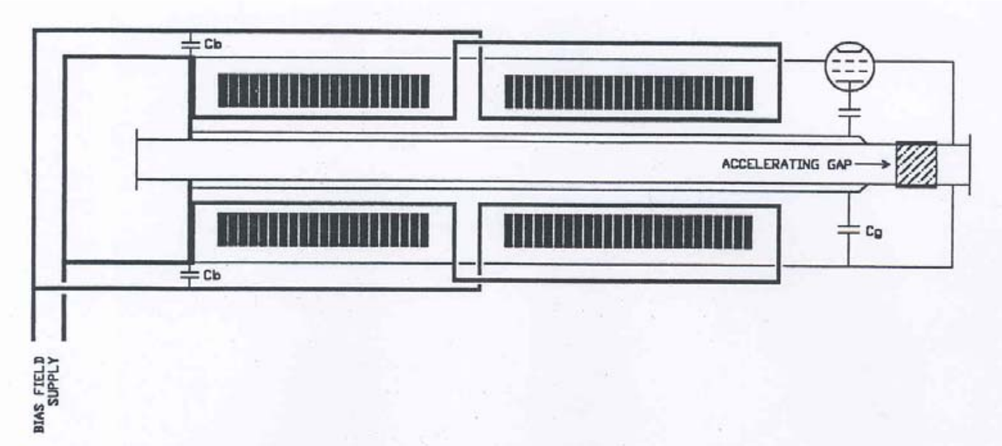
Type λ/2

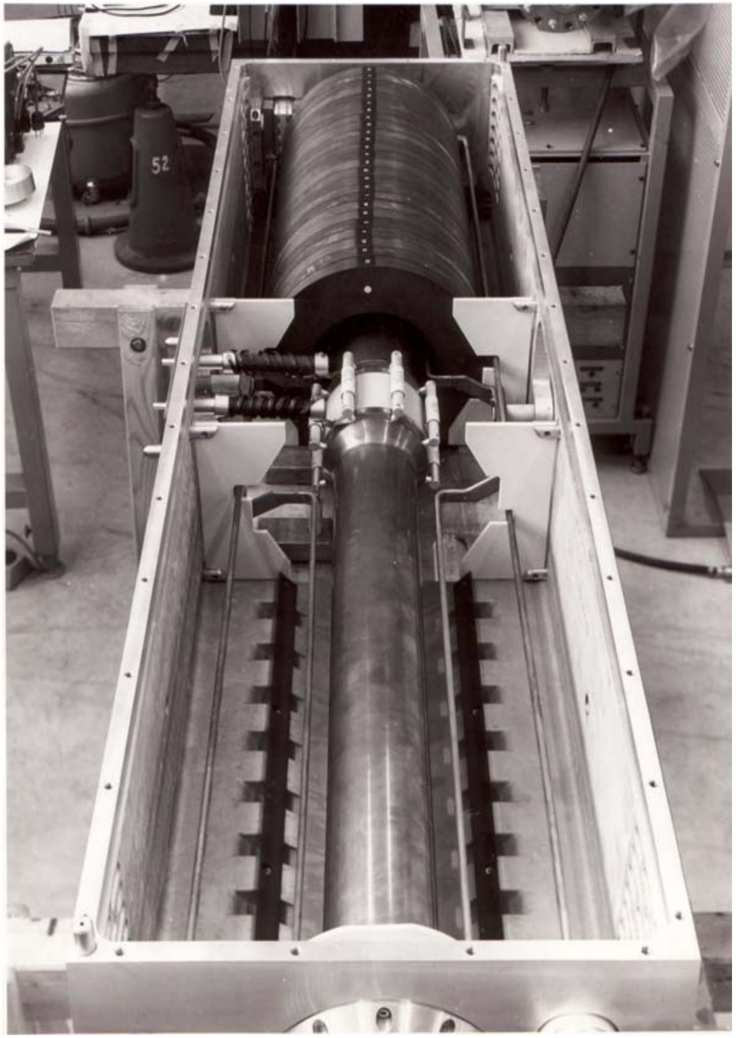


Ferrite Loaded Cavities

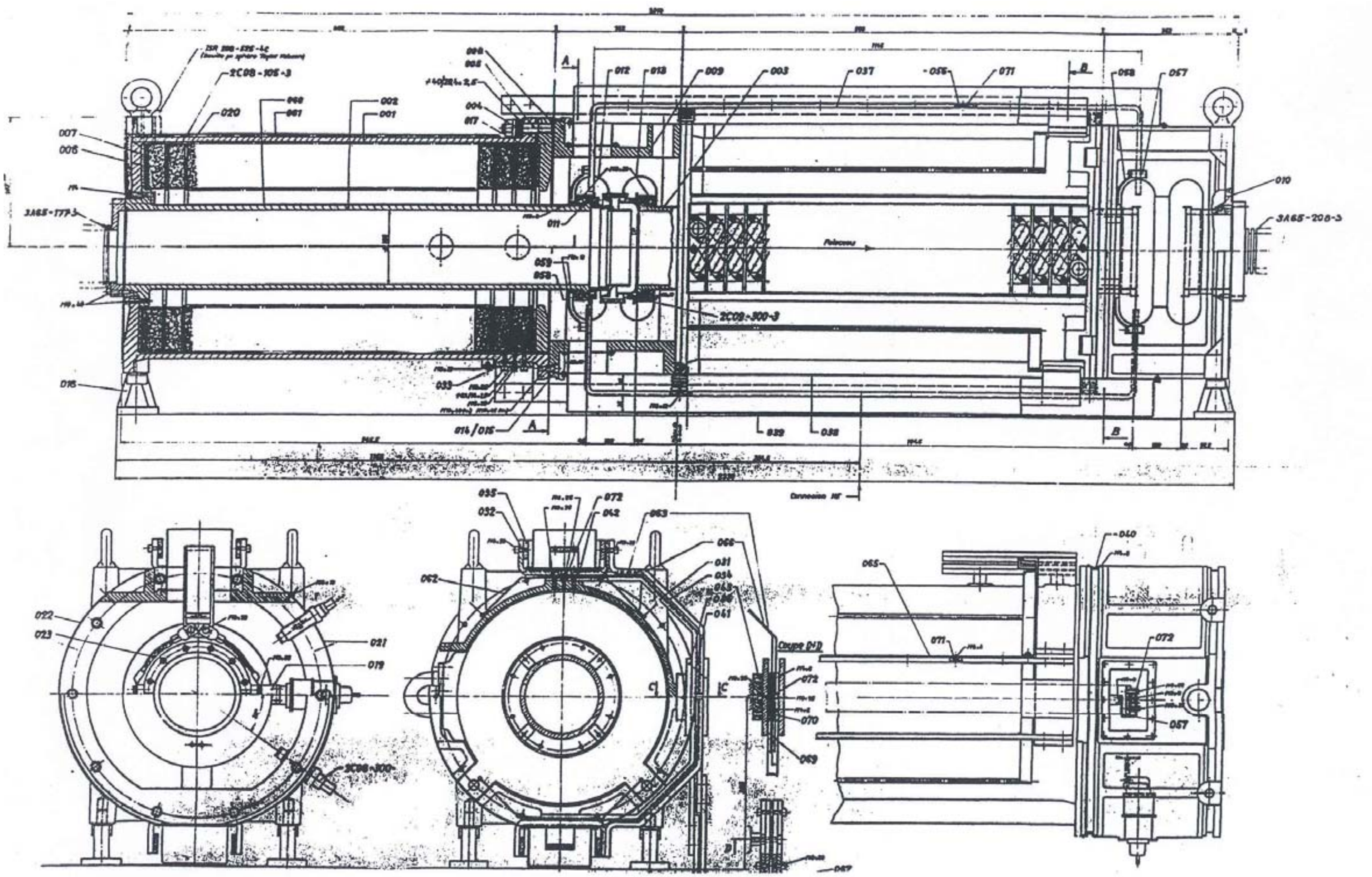
- ferrite toroids distributed around the beam tube permit to reach low frequency within reasonable size.

- polarizing theferrites one can change the cavity resonant frequency to satisfy the synchronism condition while ramping in energy in protons (ions) synchrotrons.





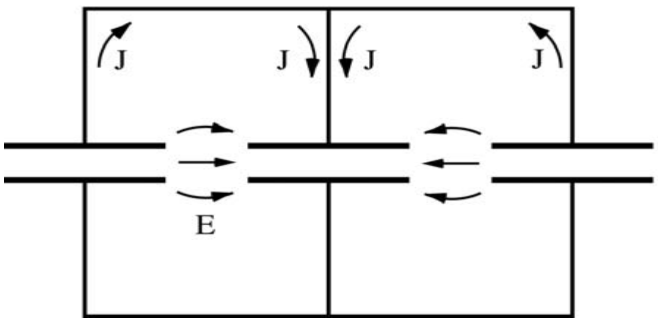
Exemple of Low Frequency Ferrite Loaded Cavity



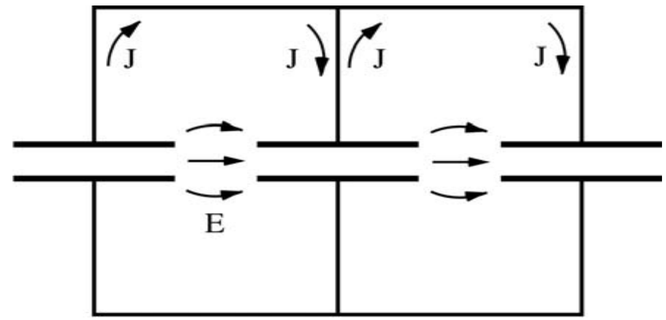
CERN PS accelerating cavity, ferrite loaded
Operating frequency range 2.5 - 10 MHz

Standing Wave Multi-cells Cavities

Relativistic particles



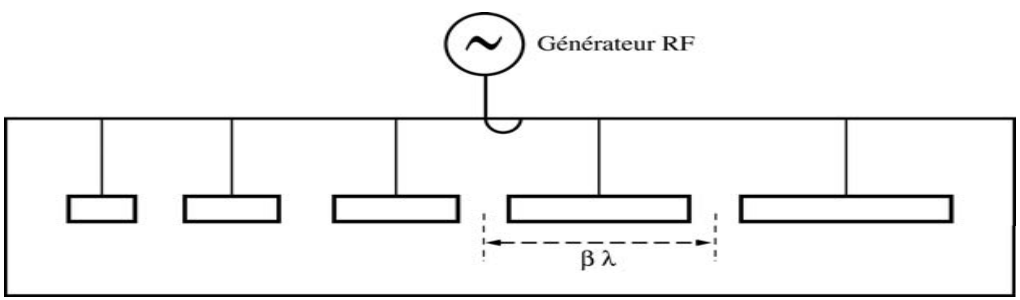
Mode π L= vT/2



Mode 2π L= vT = $\beta\lambda$

In « WIDEROE » the radiated power $\propto \omega$ CV

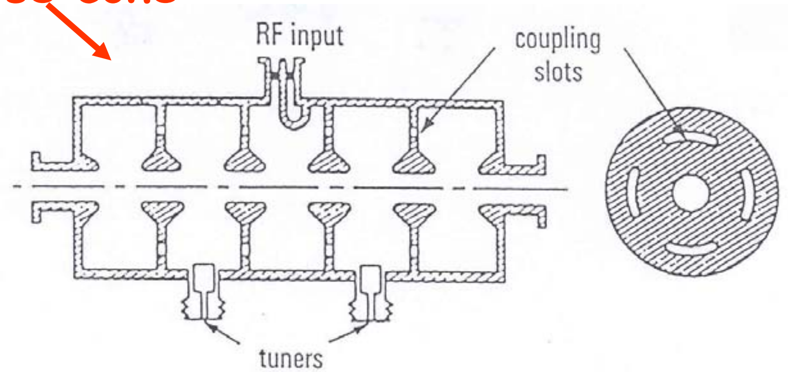
In order to limit the radiated power the gap is enclosed inside a resonant cavity at the operating frequency. A zero circulating current in a wall makes this wall useless (Maxwell).

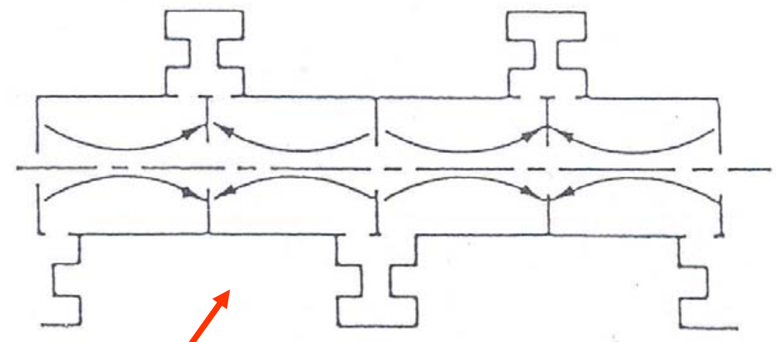


ALVAREZ structure: (proton LINAC)

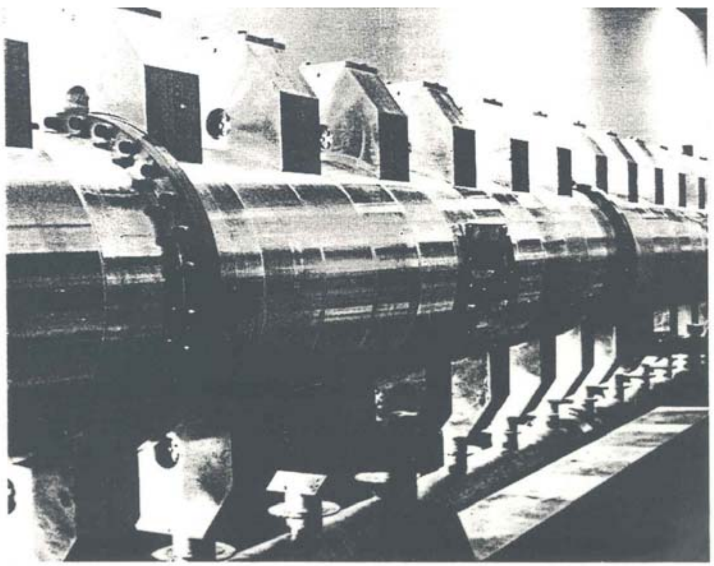
Standing Wave Multi-cells Cavities (2)

nose cone

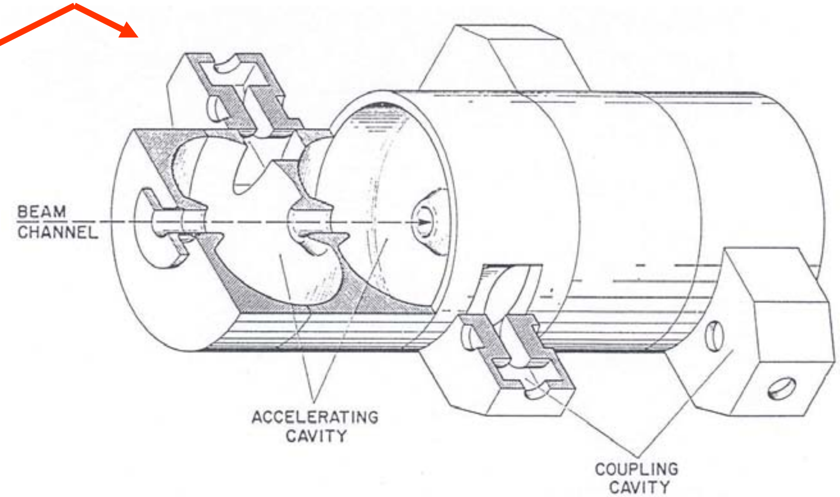




side coupled



Side-coupled waveguide tank

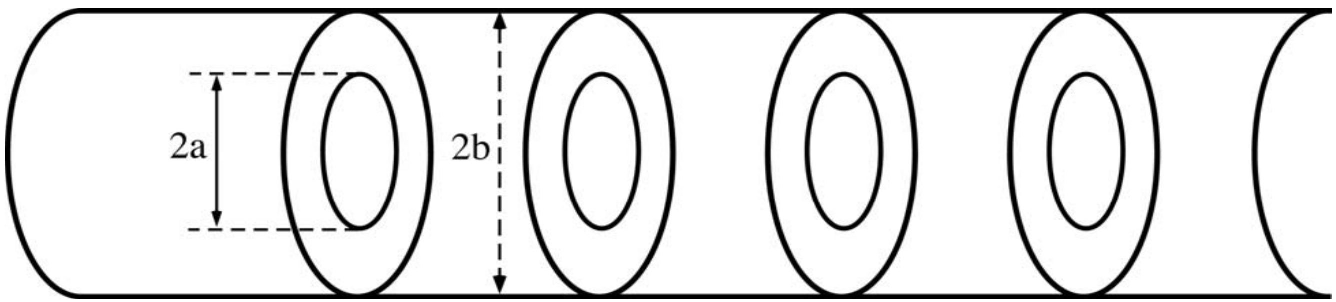


Traveling Wave Multi-cells Cavities

B- Ultra-relativistic particles $v \sim c$, $\beta \sim 1$

- L increases … unless the frequency $\omega = 2 \pi f$ is increased.
  Following the development of klystrons for radars, it became possible after 1945 to get high RF power at high frequencies, $\omega \sim 3000$ MHz

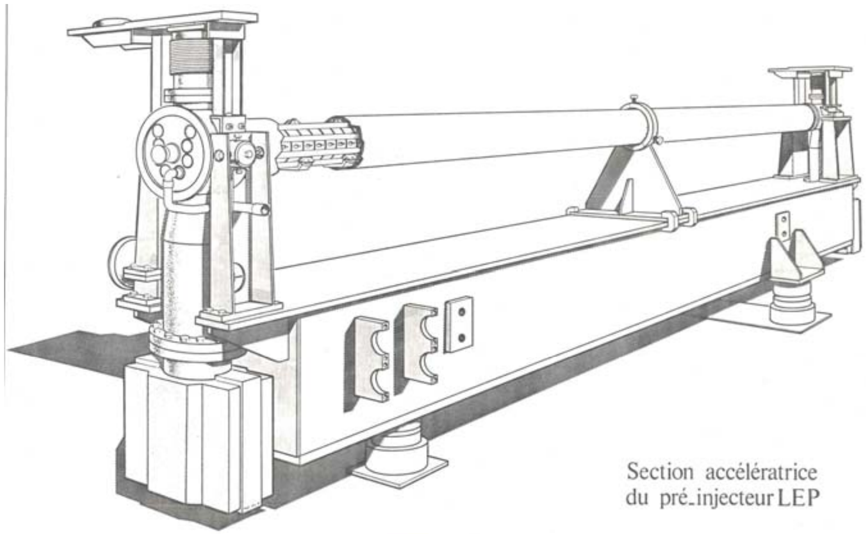
- Next came the idea of suppressing the drift spaces by using a traveling wave. However to benefit from a continuous acceleration the phase velocity of the wave should equal that of the particle (~c).



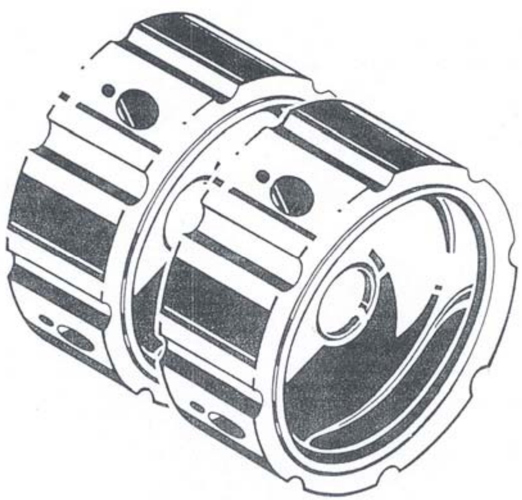
The solution consists of using slow waveguide ⇒ iris loaded waveguide

(Typically electron LINAC)

Traveling Wave Multi-cells Cavities (2)



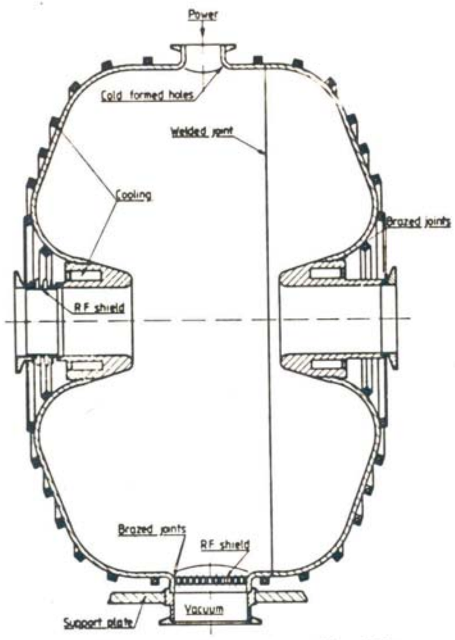
Section accélératrice du pré-injecteur LEP

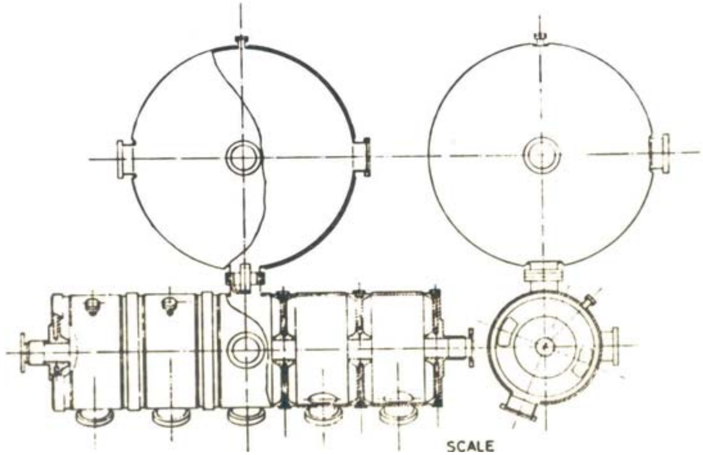


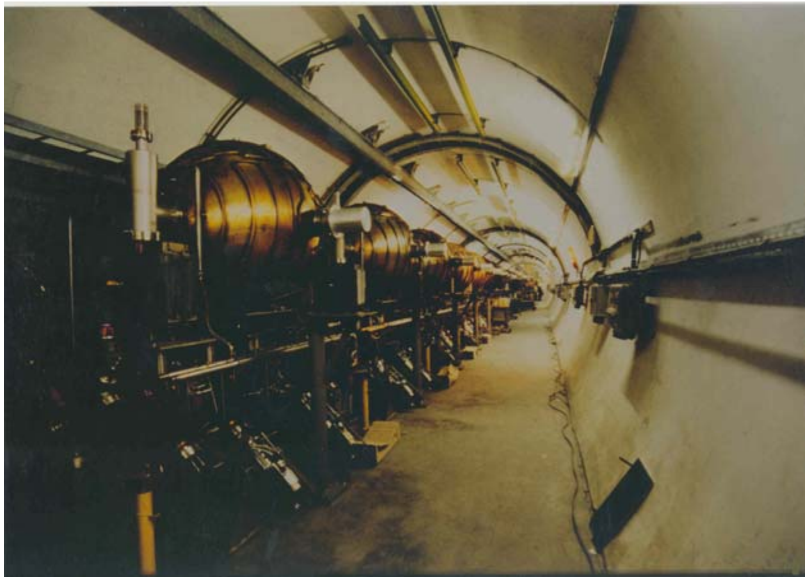
L'onde progressive est extraite par un coupleur en sortie et absorbée dans une charge

Photo d'une section CGR-MeV

A first Evolution of RF Cavities Towards Higher Efficiency







Le champ magnétique dans les parois, allié à la résistivité du matériau, conduit à des pertes de puissance par effet Joule:

$$P_j = V^2/R_s$$

A Second Evolution : Super-conducting Cavities

-the use of Super-conducting material (Nb) at low temperature (2-4 K) considerably reduces the ohmic losses and almost all the RF power from the source is available for the beam (~100% efficiency).

-in contrast with normal conducting cavities ,SC cavities will favor the use of lower frequencies, hence offering bigger opening to the beam that reduces the beam cavity interaction responsible for instability.

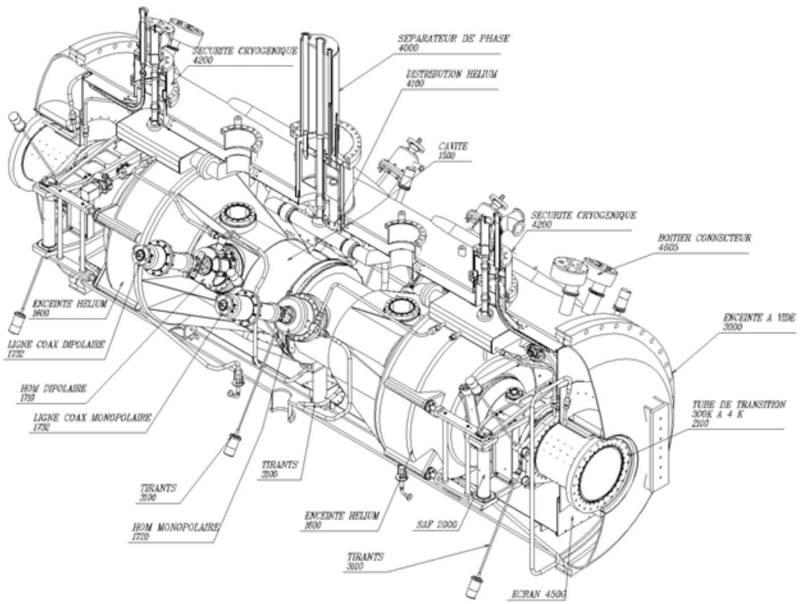
-Q_0 factor as high as 10^9 – 10^{10} are achievable. It leads to much longer filling times which means that it favors CW operation (synchrotron or Linac).

-it also permits to reach high electric field gradients for acceleration (25-30 MV/m) , hence saving on the number of cavities or giving more energy.

-in practice SC cavities are either designed as single cell cavities or multi-cells cavities.

-they are now used for e^- and p^+, for synchrotron and linac : *a technological revolution as a matter of fact.*

Super-conducting Cavities (2)



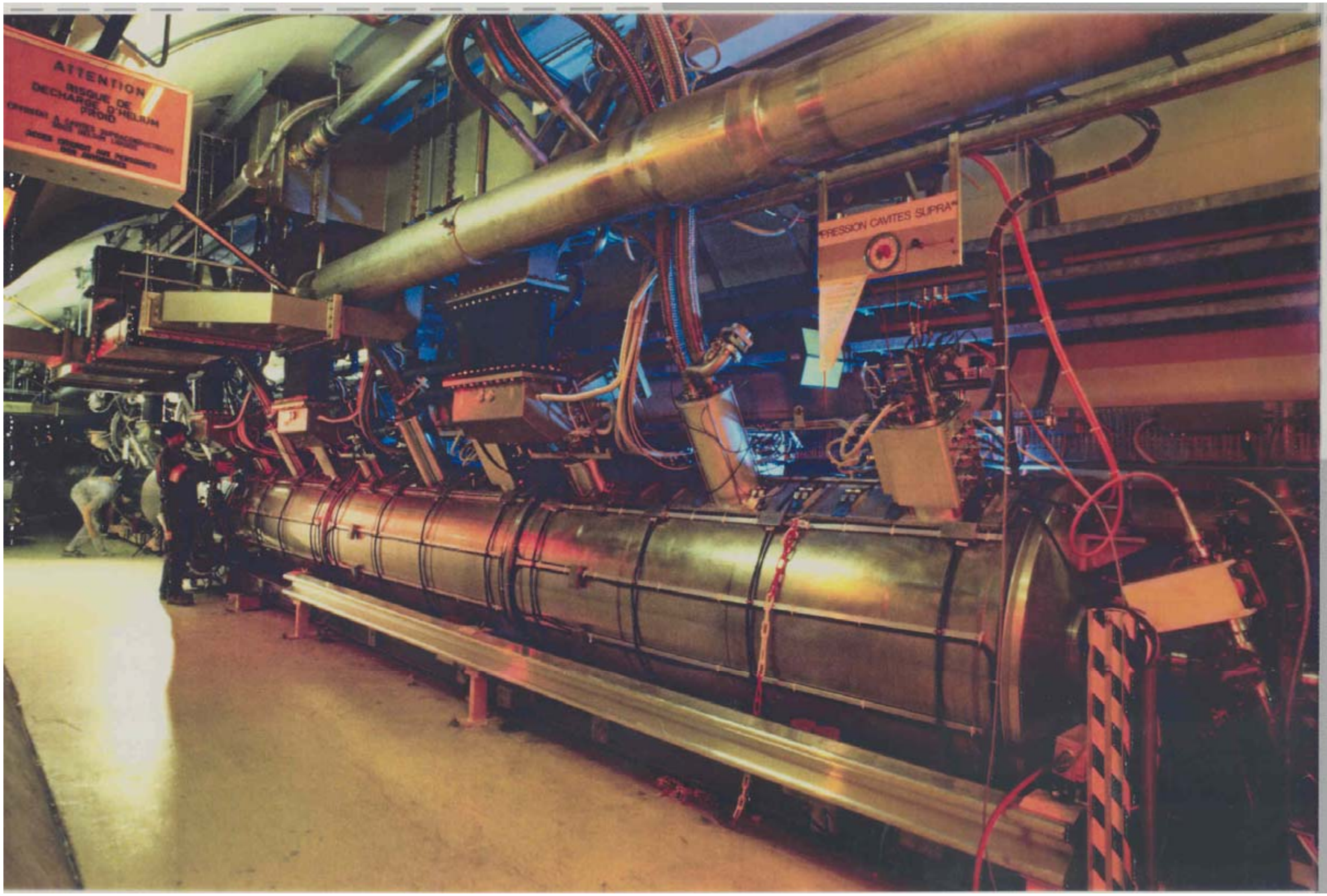
The SC cavity of the synchrotron light source SOLEIL made of two single cells

Super-conducting Cavities (3)

A super-conducting cavity is often a multi-cells assembly.Many such cavities can be put in a single cryostat.

CEBAF SC Cavities

LEP 2: 2x100 GeV with SC cavities



Acceleration of Intense Beams

Obviously the accelerated beam gets its energy from the stored energy in the cavity:

$$P_{RF} = P_{diss.} + P_{beam}$$

The cavity voltage is the vector sum of the voltage due to the generator and the "beam loading":

$$V_t = V_{RF} + V_{beam} = Z_{RF}\, I_g + Z_b\, I_b$$

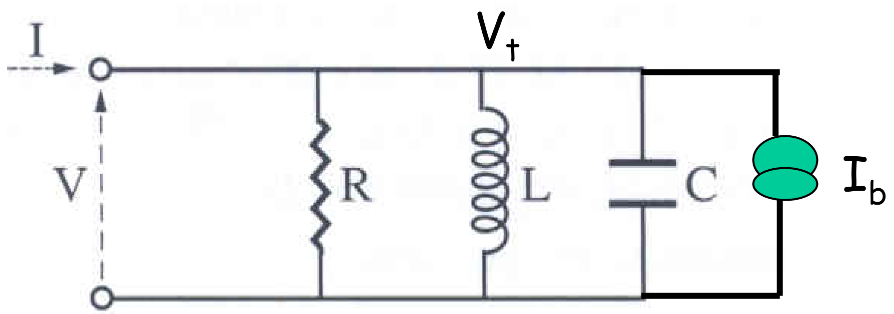
Under proper matching and tuning (cavity on-resonance) the impedance is just the shunt impedance R.

Since the beam loading is just like a power loss one can introduce a corresponding Q factor, Q_b . The loaded Q becomes:

$$\frac{1}{Q_l} = \frac{1}{Q_0} + \frac{1}{Q_e} + \frac{1}{Q_b}$$

Acceleration of Intense Beams (2)

Equivalent circuit with beam



During acceleration a synchronous phase is established between the current and the voltage:

$$P_b = \frac{1}{2} V_t I_b \sin \phi_s$$

The resulting effect is a detuning of the cavity ; a feed back system is used to compensate for that.

Optimum power transfer to the cavity and beam is made by proper matching of the power supply to the cavity through a feeder and a coupling loop.