

# Beam-beam effects

(an introduction)

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[\(/afs/ictp/home/w/wfherr/public/CAS/doc/beambeam.pdf\)](/afs/ictp/home/w/wfherr/public/CAS/doc/beambeam.pdf)

[http://cern.ch/lhc-beam-beam/talks/Trieste\\_beambeam.pdf\)](http://cern.ch/lhc-beam-beam/talks/Trieste_beambeam.pdf)

## BEAMS: moving charges

- Beam is a collection of charges
- Represent electromagnetic potential for other charges
- Forces on itself (**space charge**) and opposing beam (**beam-beam effects**)
- Main limit for present and future colliders
- Important for high density beams, i.e. high intensity and/or small beams:  
for **high luminosity** !



## Beam-beam effects

Remember:

$$\Rightarrow \mathcal{L} = \frac{N_1 N_2 f B}{4\pi \sigma_x \sigma_y} = \frac{N_1 N_2 f B}{4\pi \cdot \sigma_x \sigma_y}$$

- Overview: which effects are important for present and future machines (LEP, PEP, Tevatron, RHIC, LHC, ...)
- Qualitative and physical picture of the effects
- Mathematical derivations in:  
Proceedings, Zeuthen 2003



## Beam-beam effects

- A beam acts on particles like an electromagnetic lens, but:
    - Does not represent simple form, i.e. well defined multipoles
    - Very non-linear form of the forces, depending on distribution
    - Can change distribution as result of interaction (time dependent forces ..)
  - Results in many different effects and problems
-

## Fields and Forces (I)

- Start with a point charge  $q$  and integrate over the particle distribution  $\rho(\vec{x})$ .
- In rest frame only electrostatic field:  $\vec{E}'$ , but  $\vec{B}' \equiv 0$
- Transform into moving frame and calculate Lorentz force

$$E_{\parallel} = E'_{\parallel}, \quad E_{\perp} = \gamma \cdot E'_{\perp} \quad \text{with :} \quad \vec{B} = \vec{\beta} \times \vec{E}/c$$

$$\vec{F} = q(\vec{E} + \vec{\beta} \times \vec{B})$$



## Fields and Forces (II)


- Derive potential  $U(x, y, z)$  from Poisson equation:

$$\Delta U(x, y, z) = -\frac{1}{\epsilon_0} \rho(x, y, z)$$

- The fields become:

$$\vec{E} = -\nabla U(x, y, z)$$

- Example Gaussian distribution:

$$\rho(x, y, z) = \frac{Ne}{\sigma_x \sigma_y \sigma_z \sqrt{2\pi}^3} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}\right)$$


## Simple example: Gaussian

- For 2D case the potential becomes  
(see proceedings):

$$U(x, y, \sigma_x, \sigma_y) = \frac{ne}{4\pi\epsilon_0} \int_0^\infty \frac{\exp\left(-\frac{x^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q}\right)}{\sqrt{(2\sigma_x^2+q)(2\sigma_y^2+q)}} dq$$

- Can derive  $\vec{E}$  and  $\vec{B}$  fields and therefore forces
  - For arbitrary distribution (non-Gaussian):  
difficult (or impossible, numerical solution  
required)
-

## Simple example: Gaussian

- Round beams:  $\sigma_x = \sigma_y = \sigma$
  - Only components  $E_r$  and  $B_\Phi$  are non-zero
- Force has only radial component, i.e. depends only on distance  $r$  from bunch centre  
(where:  $r^2 = x^2 + y^2$ ) (see proceedings)

$$F_r(r) = -\frac{ne^2(1 + \beta^2)}{2\pi\epsilon_0 \cdot r} \left[ 1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right]$$



## Beam-beam kick:

- Kick ( $\Delta r'$ ): angle by which the particle is deflected during the passage
- Derived from force by integration over the collision (assume:  $m_1=m_2$  and  $\beta_1=\beta_2$ ):

$$F_r(r, s, t) = -\frac{Ne^2(1 + \beta^2)}{\sqrt{(2\pi)^3\epsilon_0 r \sigma_s}} \left[ 1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right] \cdot \left[ \exp\left(-\frac{(s + vt)^2}{2\sigma_s^2}\right) \right]$$

with Newton's law :

$$\Delta r' = \frac{1}{mc\beta\gamma} \int_{-\infty}^{\infty} F_r(r, s, t) dt$$



## Beam-beam kick:

→ Using the classical particle radius:

$$r_0 = e^2 / 4\pi\epsilon_0 mc^2$$

we have (radial kick and in Cartesian coordinates):

$$\Delta r' = -\frac{2Nr_0}{\gamma} \cdot \frac{r}{r^2} \cdot \left[ 1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right]$$

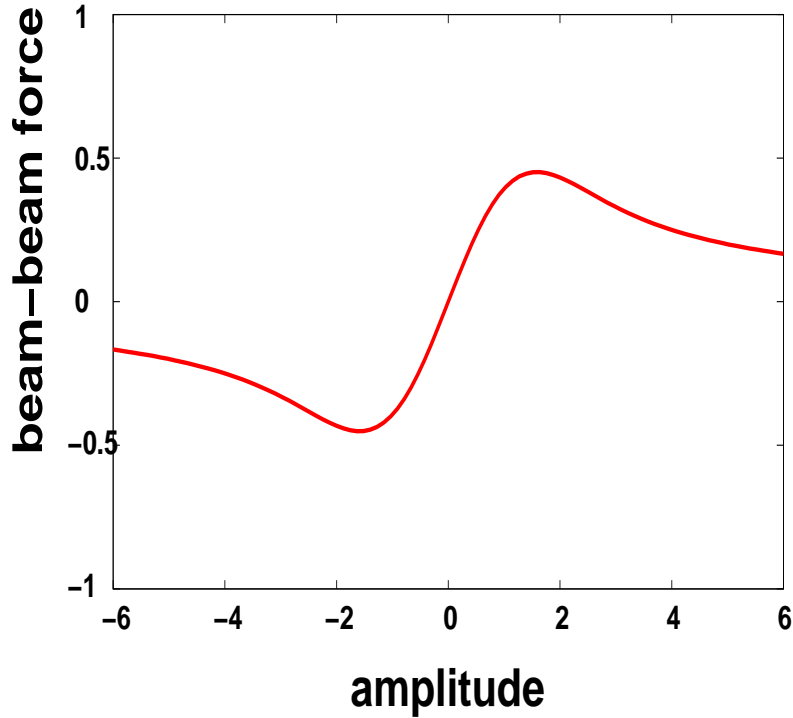
$$\Delta x' = -\frac{2Nr_0}{\gamma} \cdot \frac{x}{r^2} \cdot \left[ 1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right]$$

$$\Delta y' = -\frac{2Nr_0}{\gamma} \cdot \frac{y}{r^2} \cdot \left[ 1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right]$$



# Beam-beam force

beam-beam force, round beams



Force varies strongly with amplitude

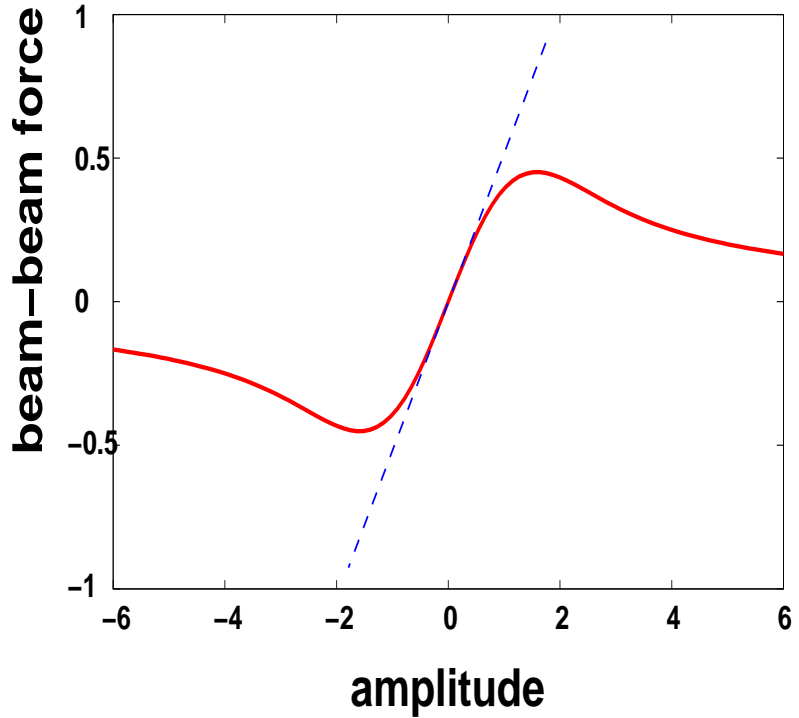
Exponential function:

Contains many high order multipoles



# Beam-beam force

beam-beam force, round beams




- For small amplitudes: linear force
- ▶ amplitude independent tune change (like quadrupole)
- For large amplitudes: very non-linear
- ▶ amplitude dependent tune change (like non-linear fields)



## Can we measure the beam-beam strength ?

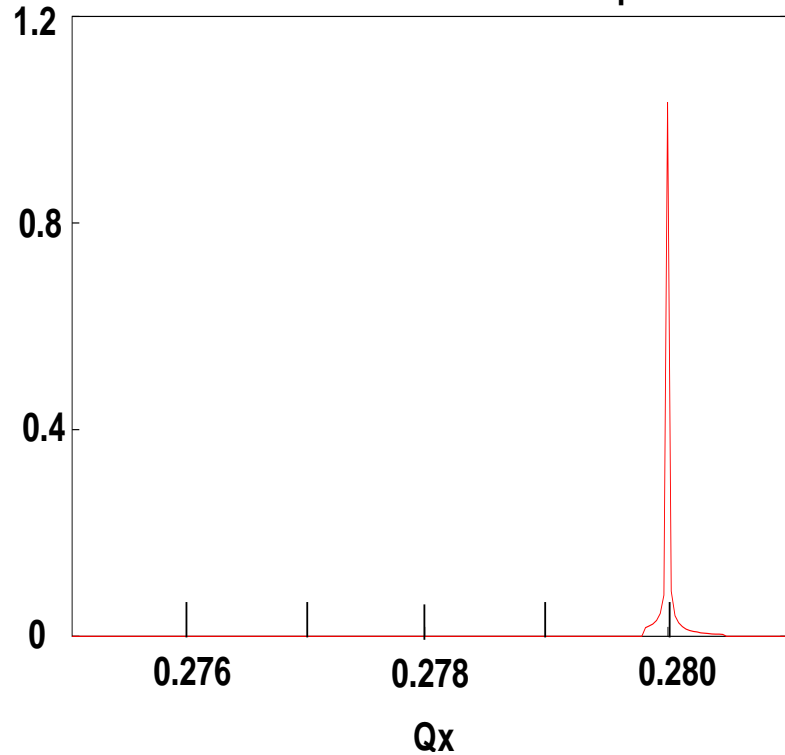
- Try the slope of force at zero amplitude  $\rightarrow$  proportional to (linear) tune shift  $\Delta Q_{bb}$  from beam-beam interaction
- This defines: beam-beam parameter  $\xi$
- For head-on interactions we get:

$$\xi_{x,y} = \frac{N \cdot r_o \cdot \beta_{x,y}}{2\pi \gamma \sigma_{x,y} (\sigma_x + \sigma_y)}$$

- BUT: does not describe non-linear part of beam-beam force (so far: only an additional quadrupole !)
- 

# Tune measurement: linear optics

Tune distribution for linear optics



Linear force: 

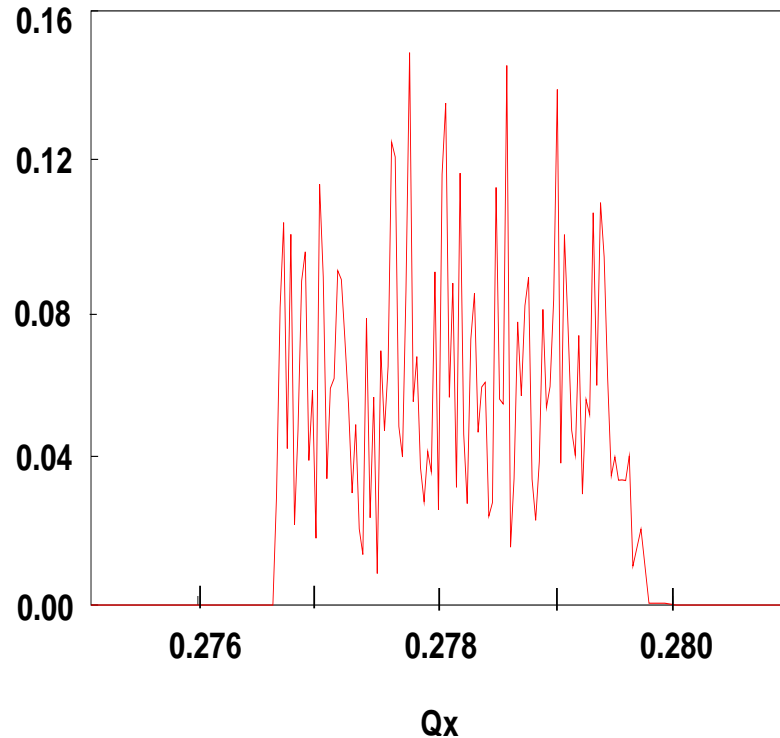
all particles have same tune

 Only one frequency (tune) visible



# Tune: linear optics with beam-beam

Tune distribution for optics with beam-beam

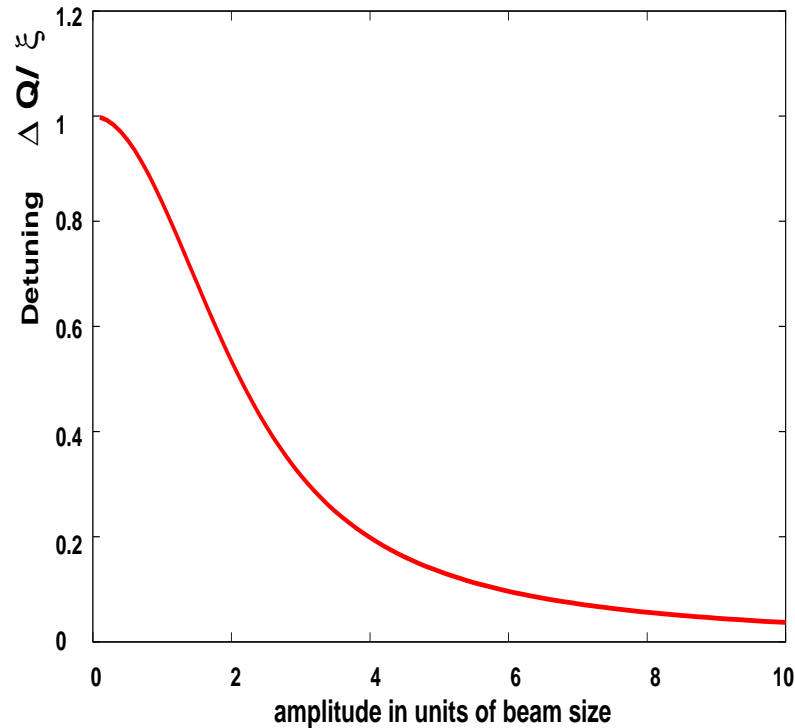


- Non-linear force: →
- particles with different amplitudes have different frequencies (tunes)
- ▶ We get frequency (tune) spectra
- ▶ Width of the spectra: about  $\xi$



# Amplitude detuning

Detuning with amplitude – round beams



■ Non-linear force:  tune depends on amplitude

■ Largest effect for **small** amplitudes

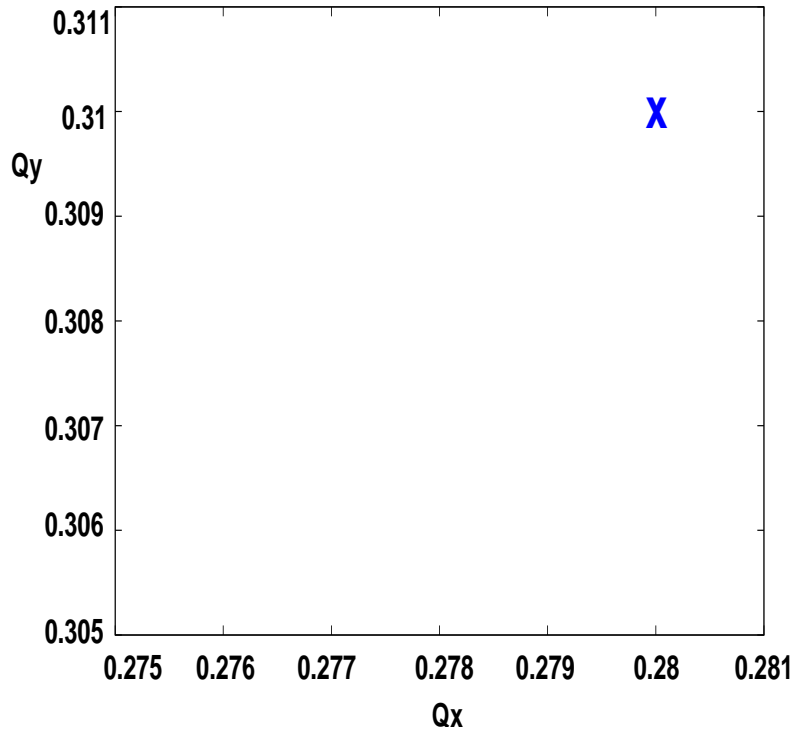
▶ Calculation in the **proceedings**





# Working point - two dimensions

working point - two dimensions



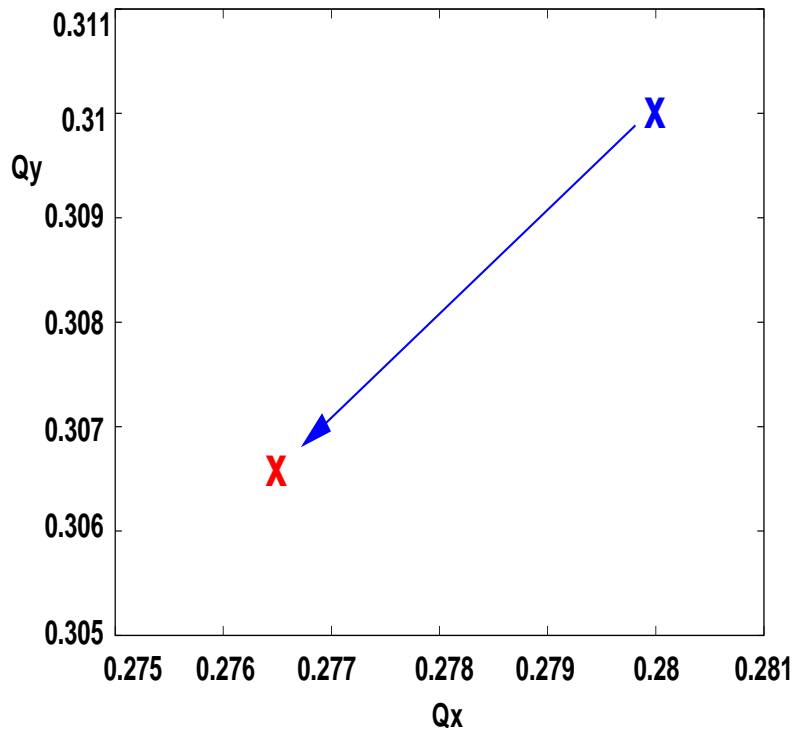
■ Start with standard working point

■ Beam-beam will change working point



# Linear tune shift - two dimensions

Tune shift for head-on collision

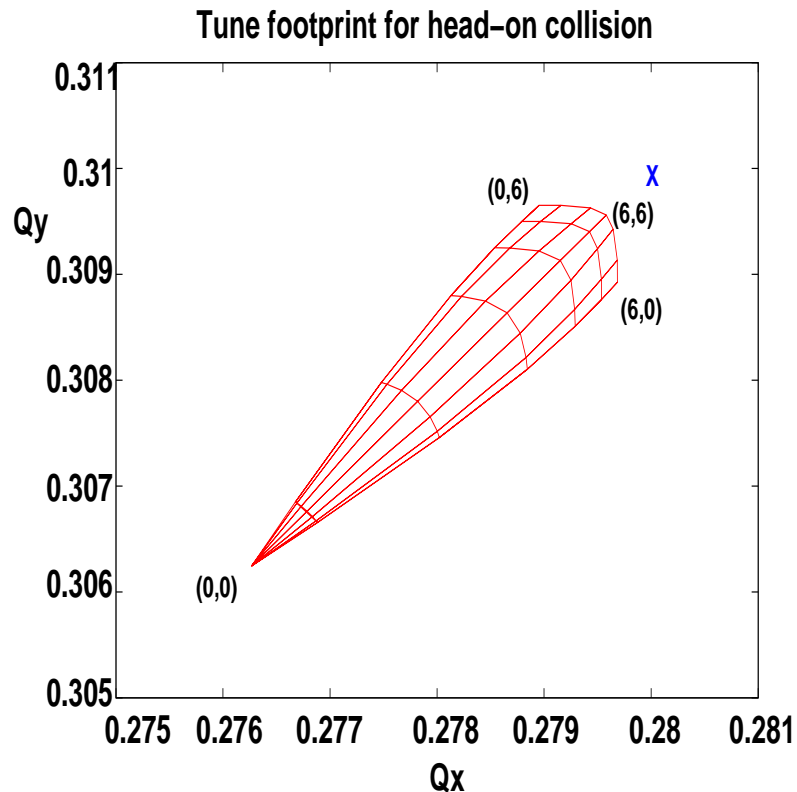


■ Start with standard working point

■ Tunes shifted in both planes



## Non-linear tune shift - two dimensions



- Tunes depend on x **and** y amplitudes
- No single tune in the beam
- Compute and plot for every amplitude (pair) the tunes in both planes
- In 2 dimensions:  
plotted as **footprint**

## LEP - LHC

	LEP	LHC
Beam sizes	160 - 200 $\mu\text{m}$ · 2 - 4 $\mu\text{m}$	16.6 $\mu\text{m}$ · 16.6 $\mu\text{m}$
Intensity N	4.0 · 10 <sup>11</sup> /bunch	1.15 · 10 <sup>11</sup> /bunch
Energy	100 GeV	7000 GeV
$\beta_x^*$ · $\beta_y^*$	1.25 m · 0.05 m	0.55 m · 0.55 m
Crossing angle	0.0	285 $\mu\text{rad}$
Beam-beam parameter( $\xi$ )	<b>0.0700</b>	<b>0.0034</b>



## Weak-strong and strong-strong

- Both beams are very strong (**strong-strong**):
  - Both beam are affected and change due to beam-beam interaction
  - Examples: LHC, LEP, RHIC, ...
- One beam much stronger (**weak-strong**):
  - Only the weak beam is affected and changed due to beam-beam interaction
  - Examples: SPS collider, Tevatron, ...



## Incoherent effects

(single particle effects)

- Single particle dynamics: treat as a particle through a static electromagnetic lens
- Basically non-linear dynamics
- All single particle effects observed:
  - Unstable and/or irregular motion
  - beam blow up or bad lifetime



## Observations hadrons

- Non-linear motion can become chaotic
  - reduction of "dynamic aperture"
  - particle loss and bad lifetime
- Strong effects in the presence of noise or ripple
- Very bad: unequal beam sizes (studied at SPS, HERA)
- Evaluation is done by simulation



## Observations leptons

Remember:

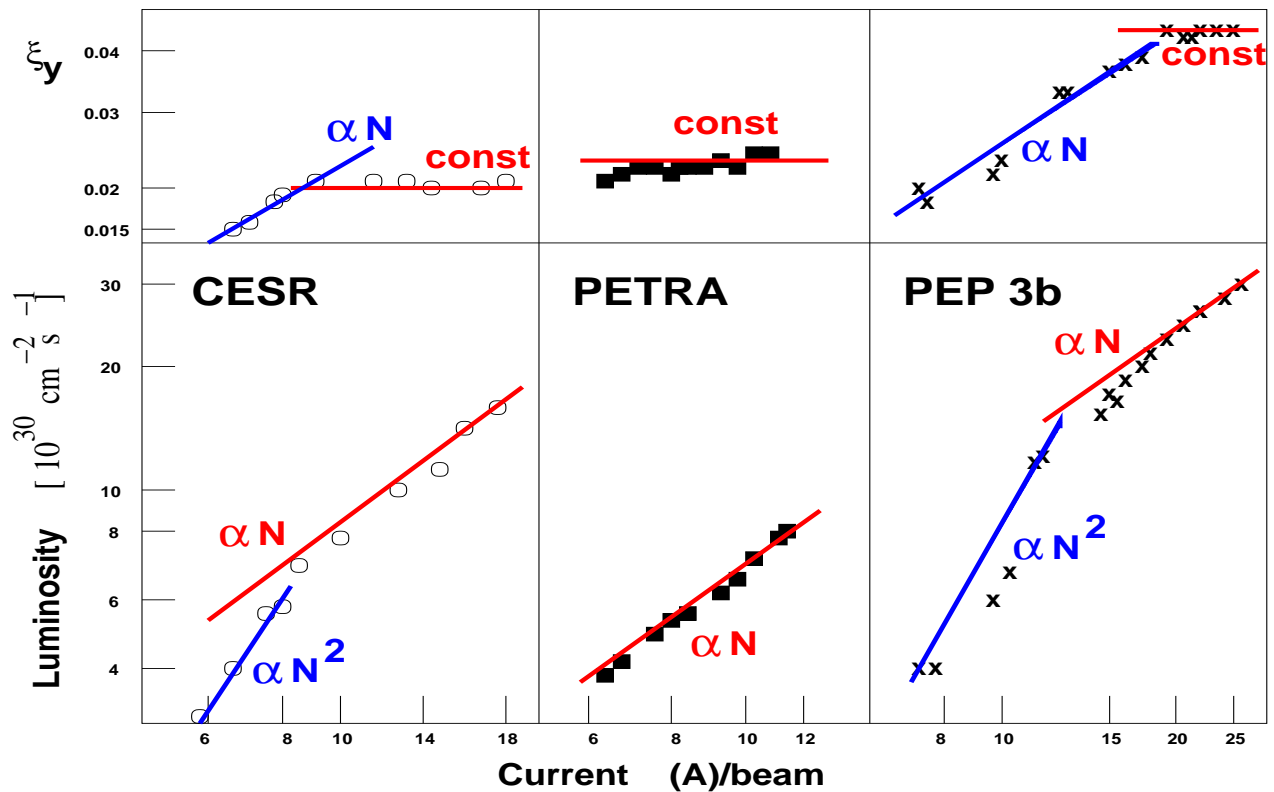
$$\Rightarrow \mathcal{L} = \frac{N_1 N_2 f B}{4\pi\sigma_x\sigma_y}$$

- Luminosity should increase  $\propto N_1 N_2$
- for:  $N_1 = N_2 = N$  →  $\propto N^2$
- Beam-beam parameter should increase  $\propto N$
- But:

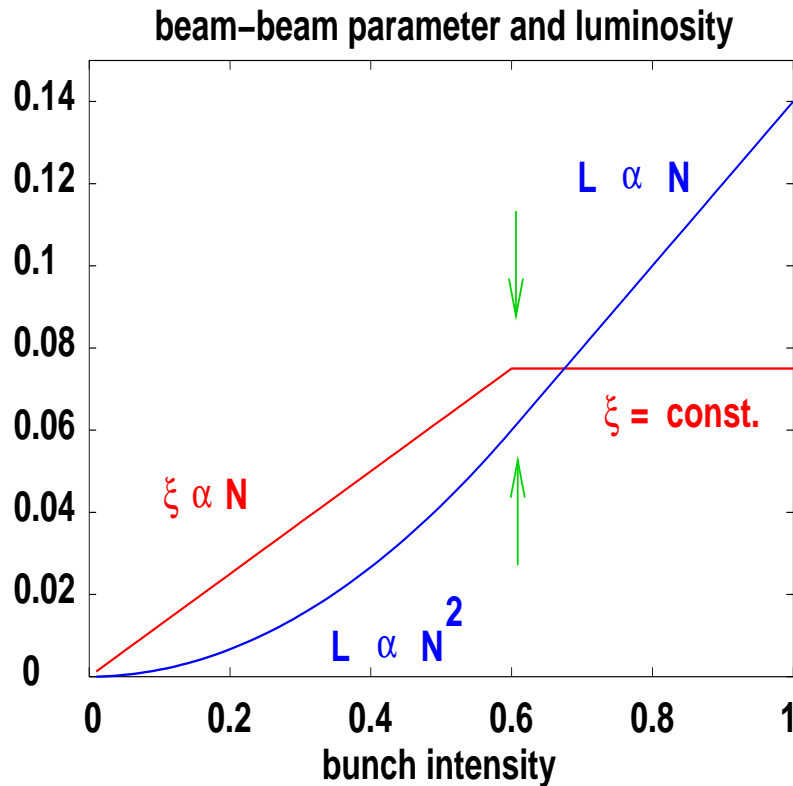




# Examples: beam-beam limit



# Beam-beam limit (schematic)



■ Beam-beam parameter increases linearly with intensity

■ Saturation above some intensity

▶ Luminosity increases linearly

▶ So-called **beam-beam limit**

## What is happening ?

we have 
$$\xi_y = \frac{Nr_0\beta_y}{2\pi\gamma\sigma_y(\sigma_x + \sigma_y)} \quad (\sigma_x \gg \sigma_y) \quad \approx \quad \frac{r_0\beta_y}{2\pi\gamma(\sigma_x)} \cdot \frac{N}{\sigma_y}$$

and 
$$\mathcal{L} = \frac{N^2 f B}{4\pi\sigma_x\sigma_y} = \frac{N f B}{4\pi\sigma_x} \cdot \frac{N}{\sigma_y}$$

- Above beam-beam limit:  $\sigma_y$  increases when  $N$  increases to keep  $\xi$  constant  $\rightarrow$  **equilibrium emittance !**
  - Therefore:  $\mathcal{L} \propto N$  and  $\xi \approx$  constant
  - $\xi_{limit}$  is NOT a universal constant !
  - Difficult to predict
-

## The next problem

Remember:

$$\Rightarrow \mathcal{L} = \frac{N_1 N_2 f \cdot B}{4\pi\sigma_x\sigma_y}$$

- How to collide many bunches (for high  $\mathcal{L}$ ) ??
- Must avoid unwanted collisions !!
- Separation of the beams:
  - Pretzel scheme (SPS, LEP, Tevatron)
  - Bunch trains (LEP, PEP)
  - Crossing angle (LHC)

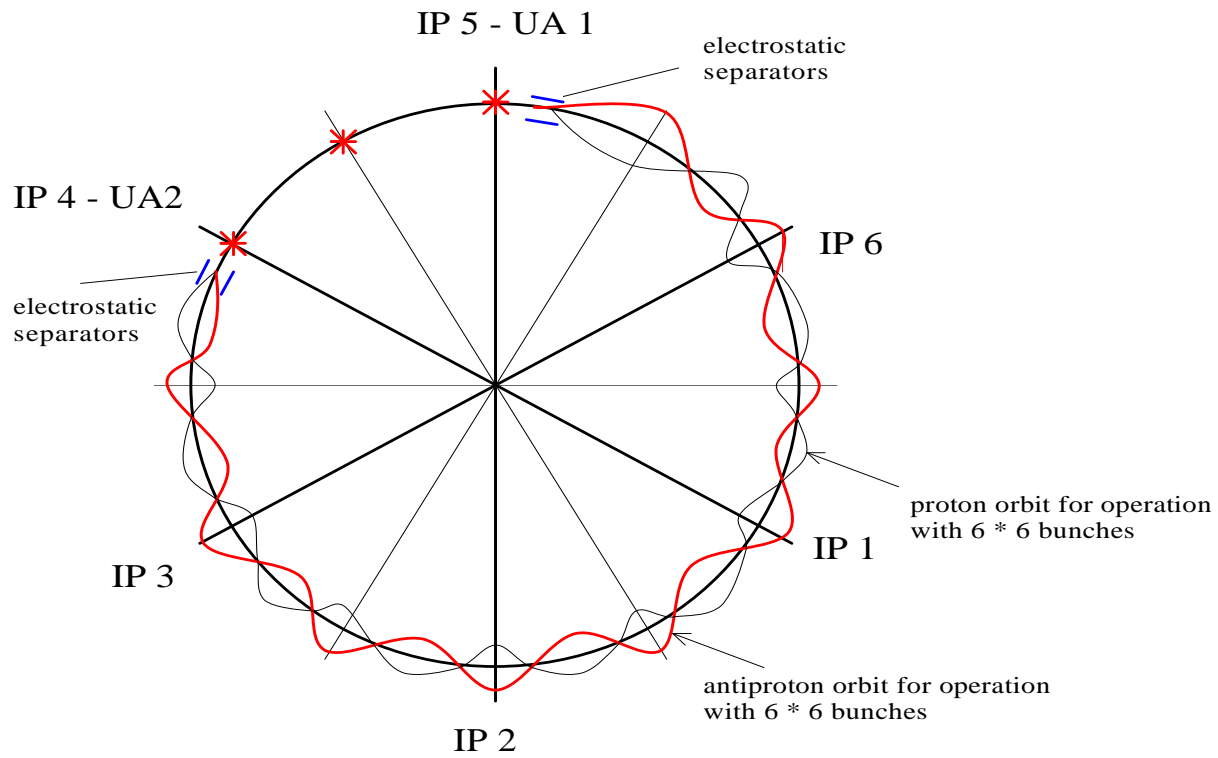


## Separation: SPS

- $\Rightarrow$  Few equidistant bunches  
(6 against 6)
- Beams travel in same beam pipe  
(12 collision points !)
- Two experimental areas
- Need **global** separation
- Horizontal pretzel around most of the circumference



# Separation: SPS

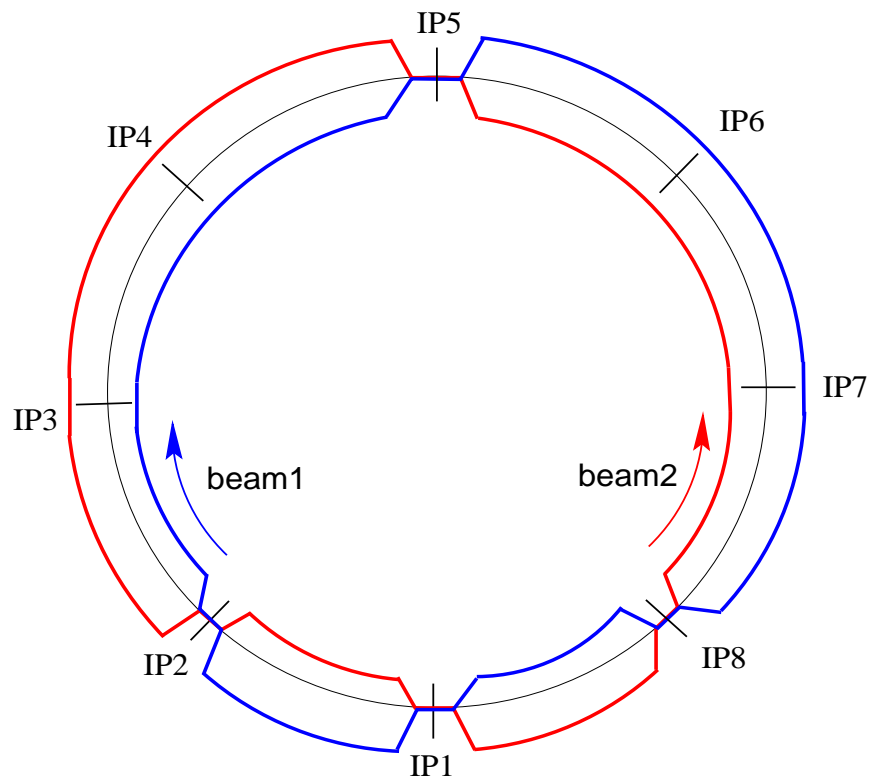


## Separation: LHC

- $\Rightarrow$  Many equidistant bunches
- Two beams in separate beam pipes except:
  - Four experimental areas
  - Need **local** separation
- Two horizontal and two vertical crossing angles

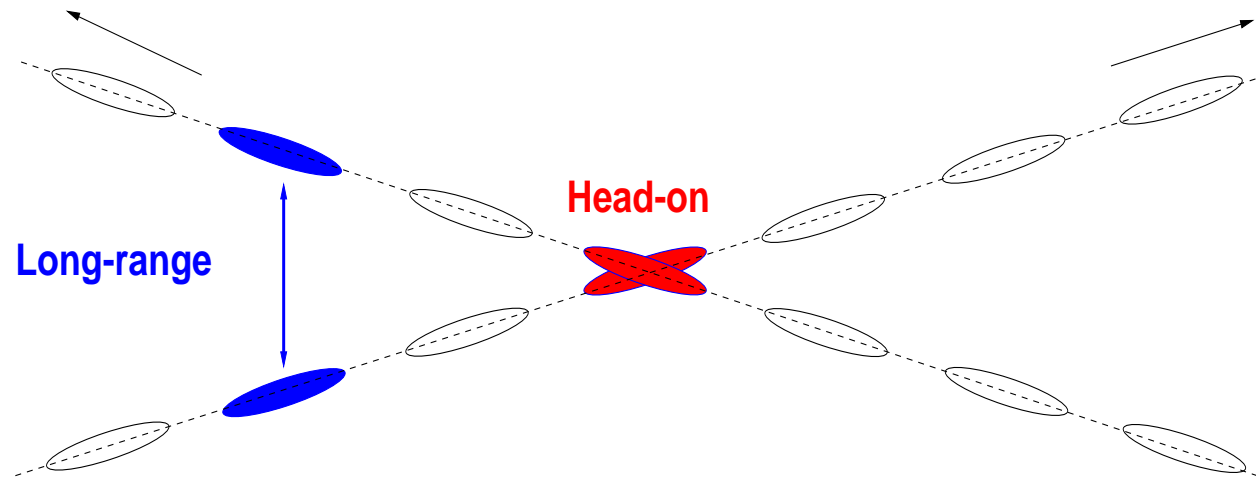


# Layout of LHC





## Crossing angles (example LHC)

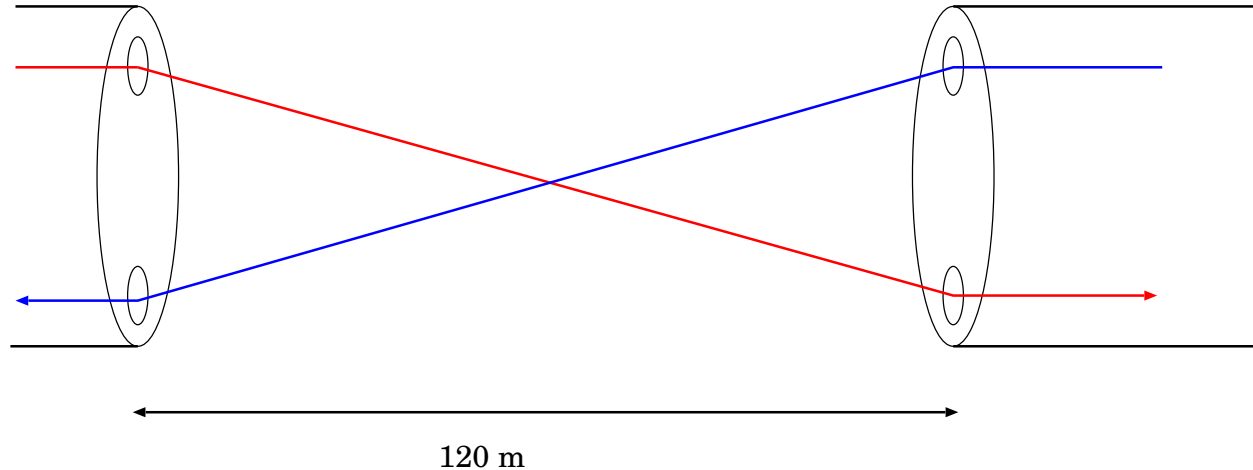


- Still some parasitic interactions
  - Particles experience distant (weak) forces
- We get so-called **long range interactions**



## Example: LHC

- Two beams, 2808 bunches each, every 25 ns
- In common chamber around experiments



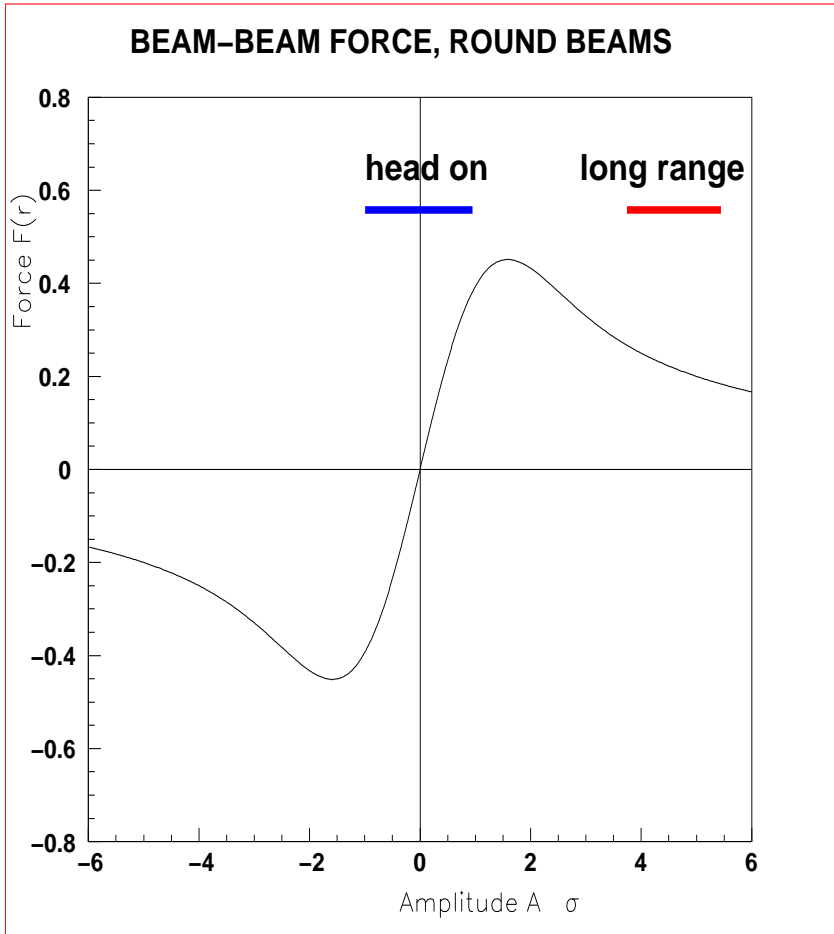
- Around each IP: 30 long range interactions
- Separation typically 6 - 12  $\sigma$

## What is special about them ?

- Break symmetry between planes, also odd resonances
- Mostly affect particles at **large** amplitudes
- Tune shift has **opposite** sign in plane of separation
- Cause effects on closed orbit
- PACMAN effects



# Opposite tuneshift ???



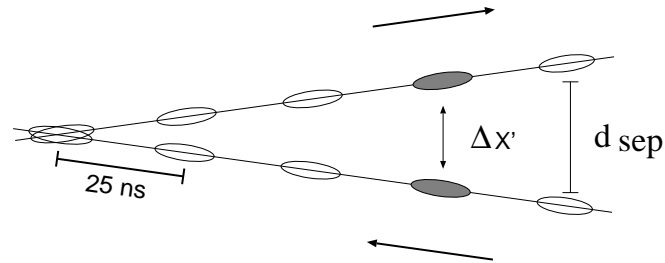
Local slope of force has opposite sign for large separation

▶ Opposite sign for focusing

▶ Used for partial compensation



# Long range interactions (LHC)

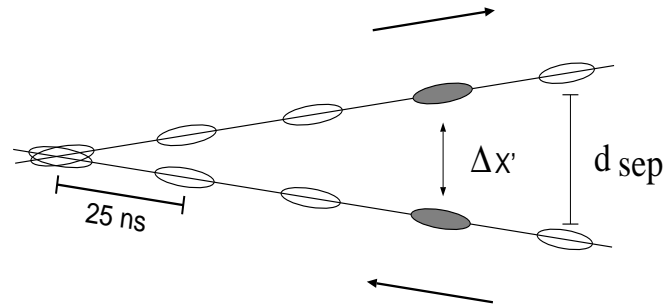


→ For horizontal separation  $d$ :

$$\Delta x'(x + d, y, r) = -\frac{2Nr_0}{\gamma} \cdot \frac{(x + d)}{r^2} \left[ 1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right]$$

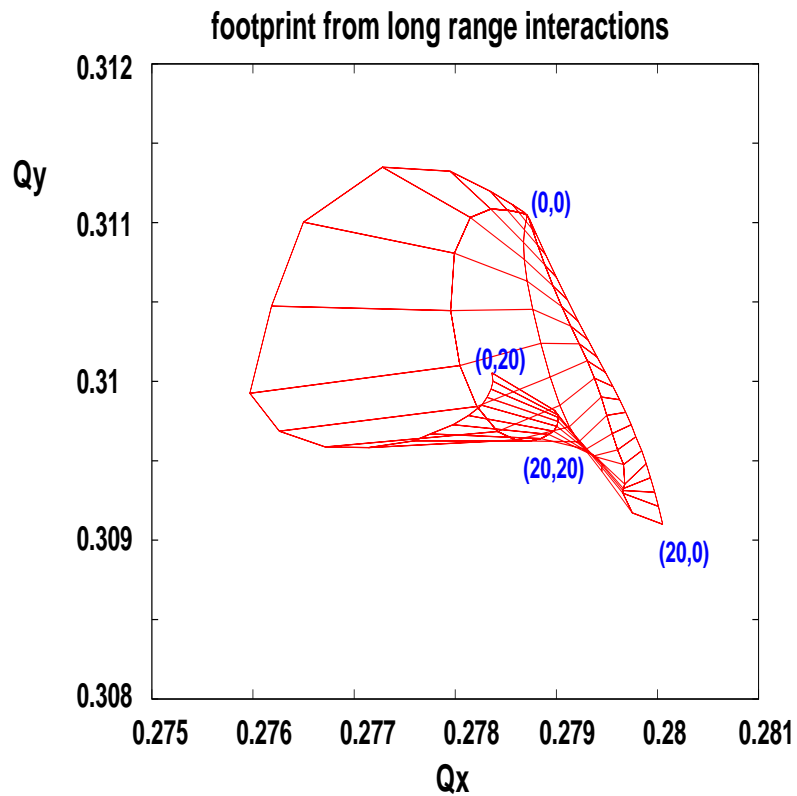
$$(with: r^2 = (x + d)^2 + y^2)$$

# Long range interactions (LHC)



- Number of long range interactions depends on spacing and length of common part
- In LHC 15 collisions on each side, 120 in total !
- Effects depend on separation:  $\Delta Q \propto -\frac{N}{d^2}$  (for large enough  $d$  !)    footprints ??

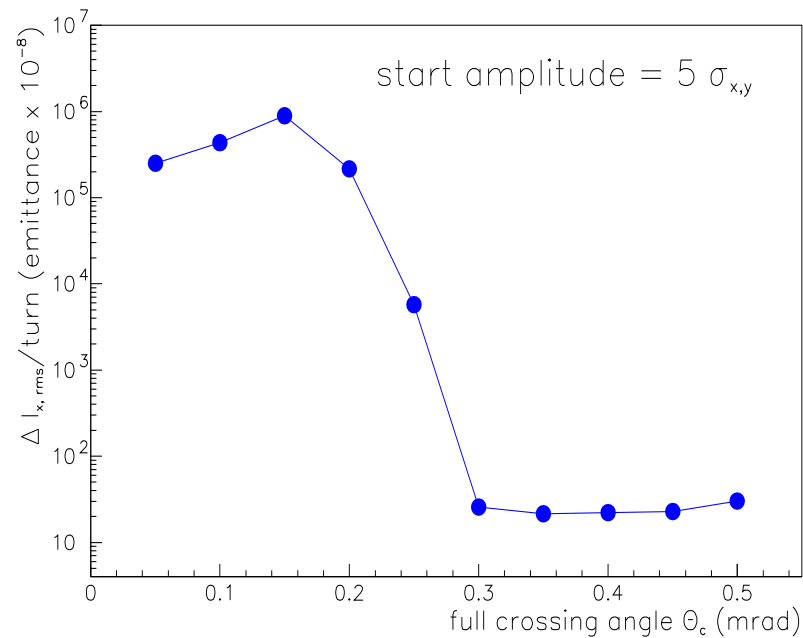
# Footprints



- ▶ Large fo largest amplitudes where non-linearities are strong
- ▶ Size proportional to  $\frac{1}{d^2}$
- ▶ Must expect problems at small separation
- ▶ Footprint very asymmetric

## Particle losses

- Small crossing angle  $\iff$  small separation
- Small separation: particles become unstable and get lost



- Minimum crossing angle for LHC:  $285 \mu\text{rad}$

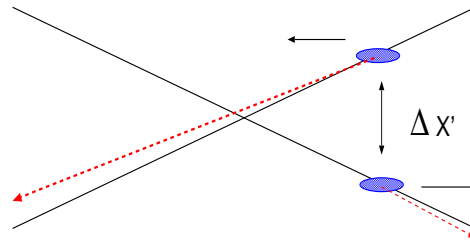


## Closed orbit effects

$$\Delta x'(x + d, y, r) = -\frac{2Nr_0}{\gamma} \cdot \frac{(x + d)}{r^2} \left[ 1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right]$$

For well separated beams ( $d \gg \sigma$ ) the force (kick) has an amplitude independent contribution:  $\rightarrow$  orbit kick

$$\Delta x' = \underbrace{\frac{\text{const.}}{d}} \cdot \left[ 1 - \frac{x}{d} + O\left(\frac{x^2}{d^2}\right) + \dots \right]$$

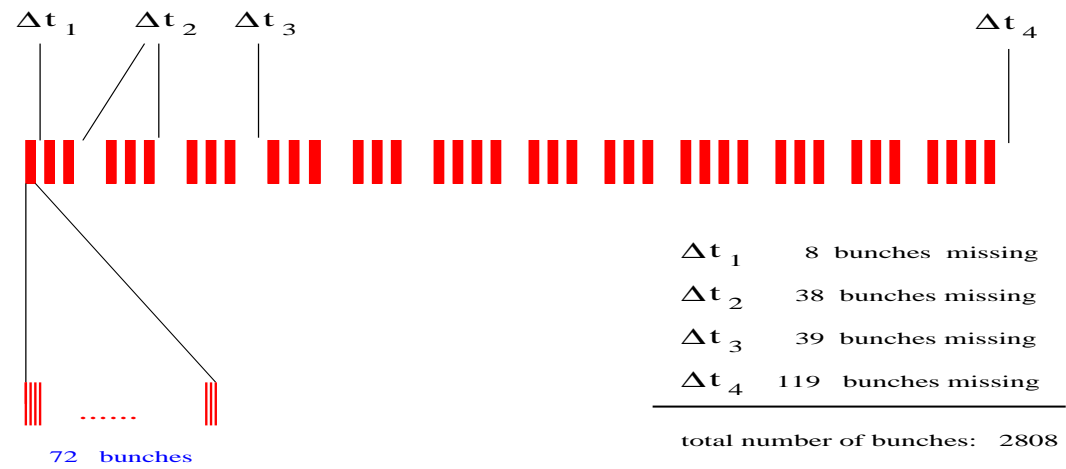


## Closed orbit effects

- Beam-beam kick from long range interactions changes the orbit
  - Has been observed in LEP with bunch trains
  - Self-consistent calculation necessary
  - Effects can add up and become important
- Orbit can be corrected, **but:**



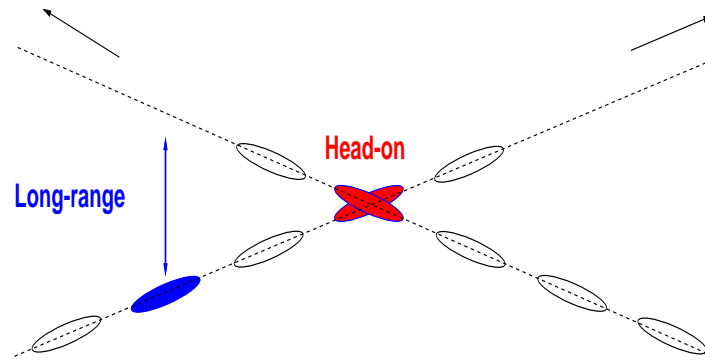
# PACMAN bunches



- LHC bunch filling not continuous: holes for injection, extraction, dump ..
- 2808 of 3564 possible bunches → 1756 "holes"
- "Holes" meet "holes" at the interaction point
- But not always ...



## Effect of holes



- A bunch can meet a hole (at beginning and end of bunch train)
- Results in left-right asymmetry
- Example LHC: between 120 (max) and 40 (min) long range collisions for different bunches

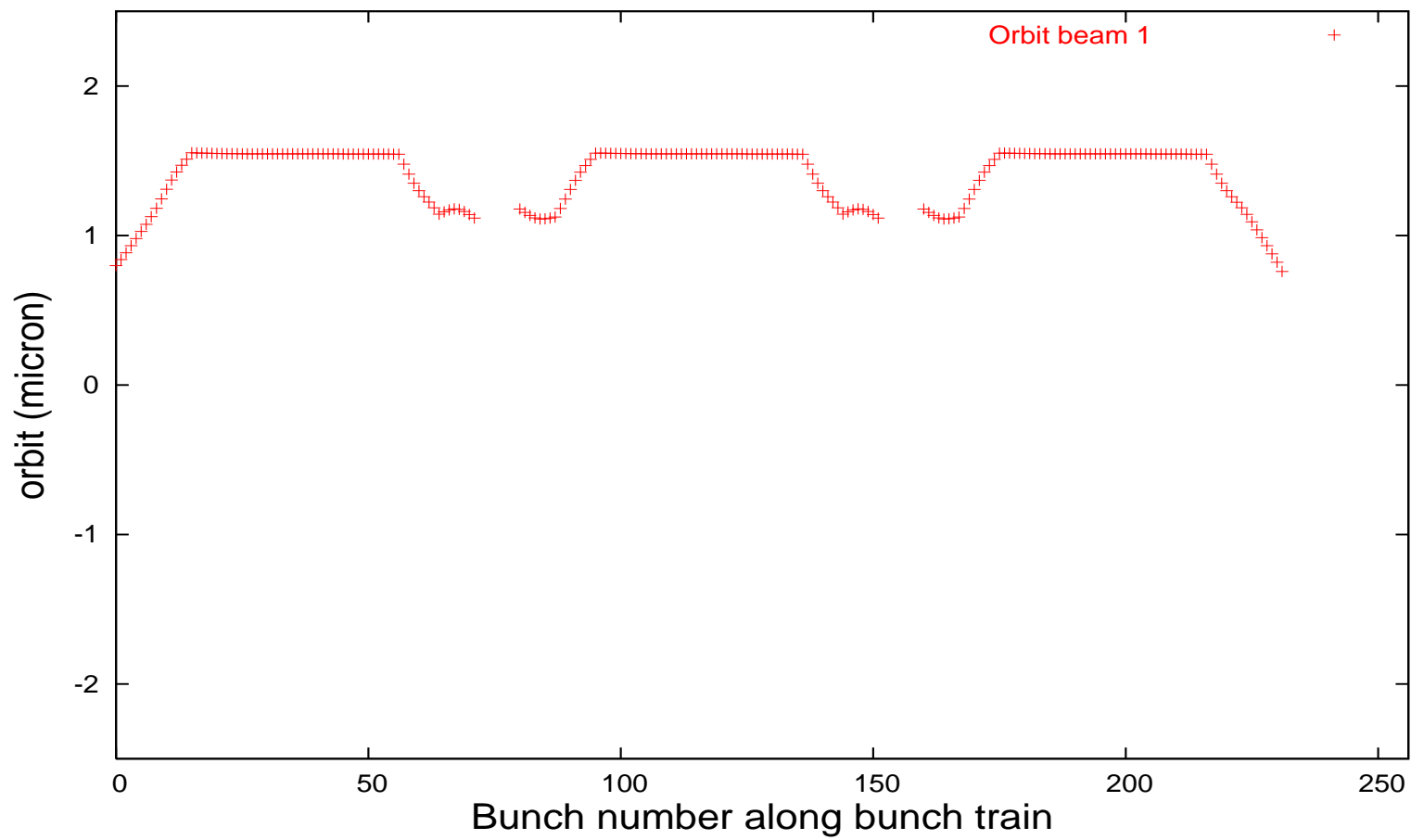
## PACMAN bunches

- When a bunch meets a "hole":
  - Miss some long range interactions, PACMAN bunches
  - They see fewer unwanted interactions in total
  - Different integrated beam-beam effect
- Example: orbit and tune effects



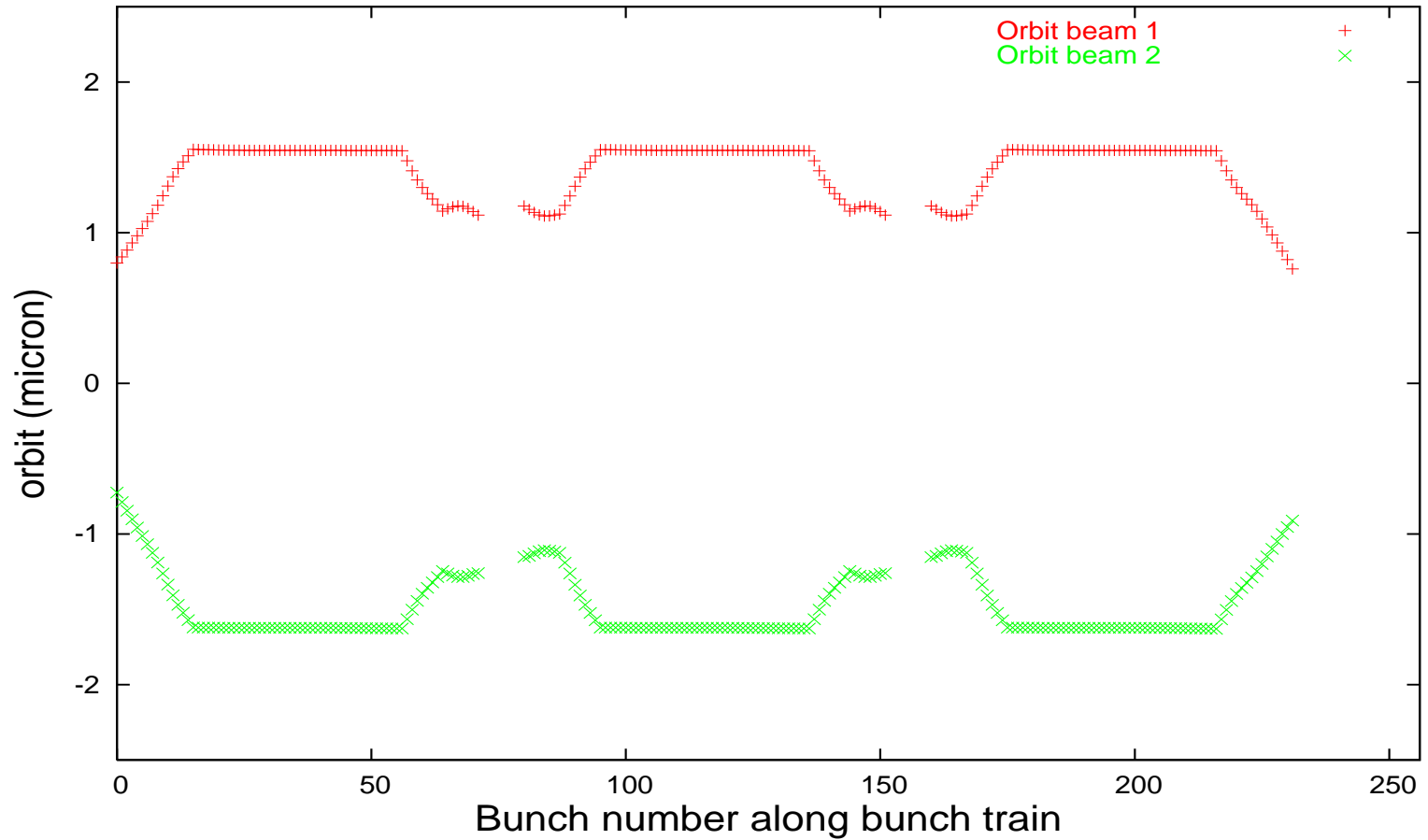
# Orbit along batches: beam 1

## Orbit along bunches



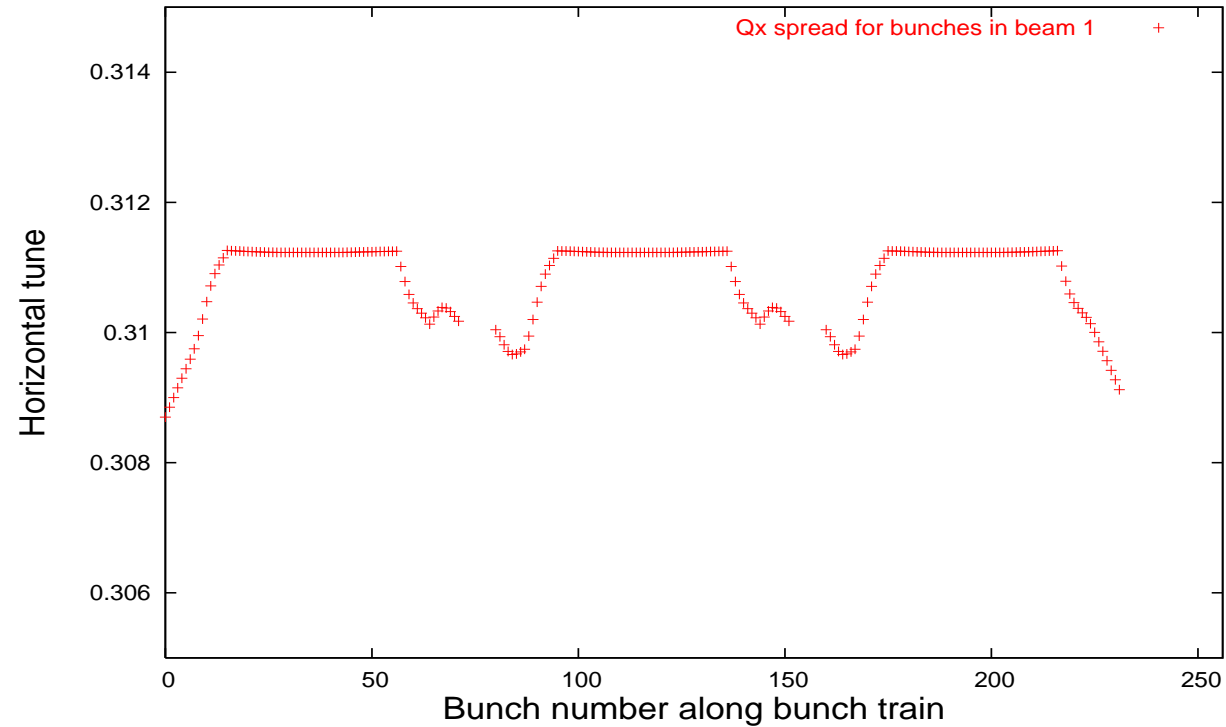
# Orbit along batches: beam 1 and beam 2

## Orbit along bunches



# Tune along batches

Tune along bunches



- Spread is too large for safe operation



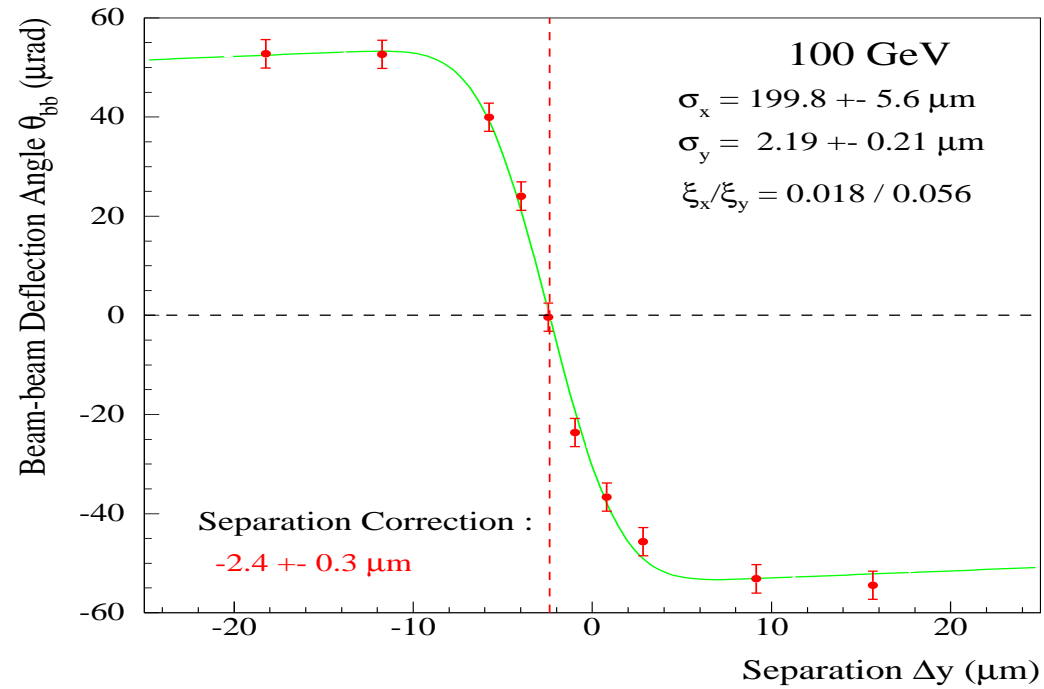


## Beam-beam deflection scan

- The orbit effect can be useful when one has only a few bunches, i.e. not PACMAN effects
- Effect can be used to optimize luminosity
- Scanning two beams against each other
- Two beams get a orbit kick, depending on distance



# Deflection scan (LEP measurement)



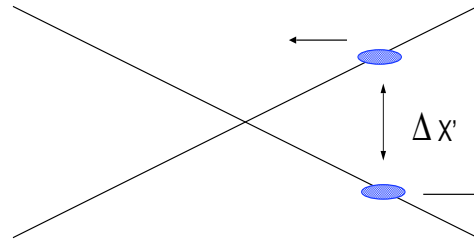
(Courtesy J. Wenniger)

## Deflection scan

- Calculated kick from orbit follows the force function
- Allows to calculate parameters
- Allows to centre the beam
- Standard procedure at LEP



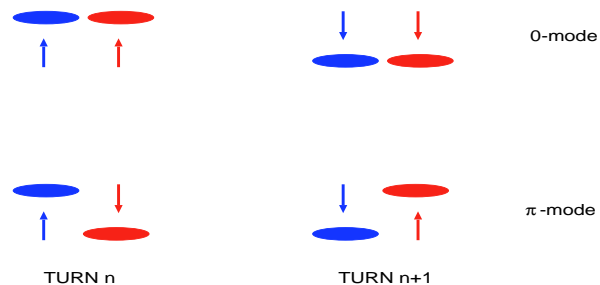
## Coherent beam-beam effect



- Whole bunch sees a kick as an entity (coherent kick)
- The coherent kick of separated beams can excite coherent dipole oscillations
- All bunches couple together because each bunch "sees" many opposing bunches: many coherent modes possible !

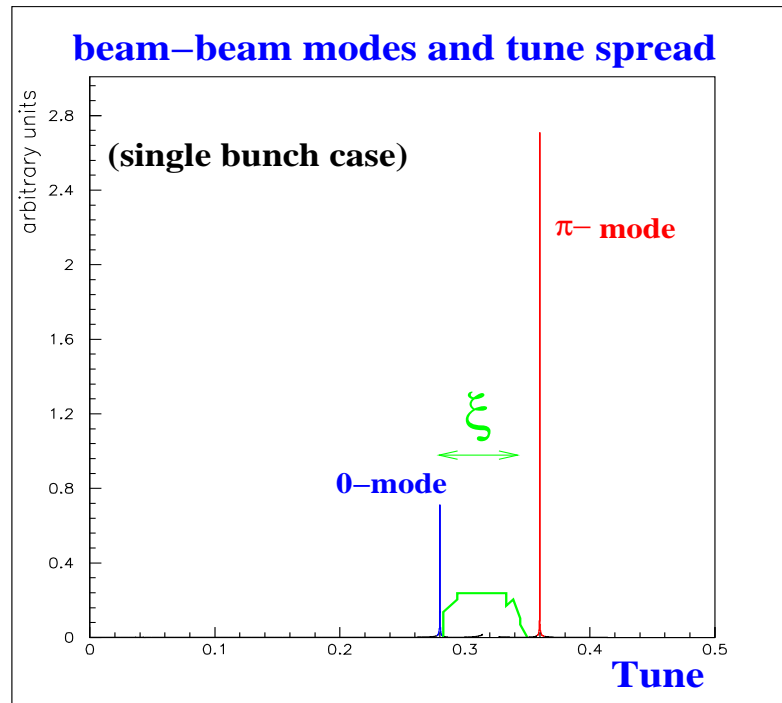
# Coherent beam-beam effect

Simplest case: one bunch per beam:



- Coherent mode: two bunches are "locked" in a coherent oscillation
- 0-mode is stable (Mode with **NO** tune shift)
- $\pi$ -mode can become unstable (Mode with **LARGEST** tune shift)

# Coherent beam-beam frequencies (schematic)



▶ 0-mode is at unperturbed tune

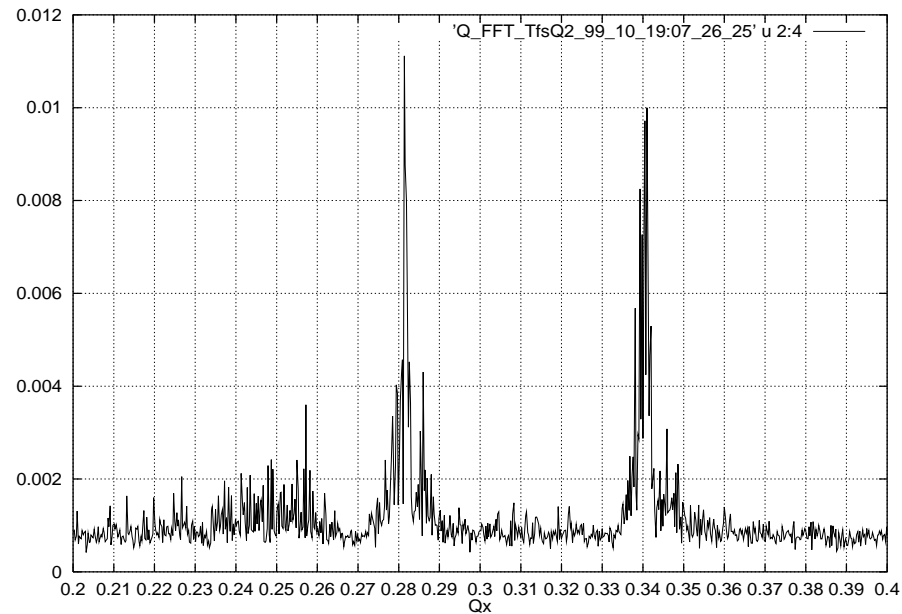
▶ π-mode is shifted by  $1.1 - 1.3 \cdot \xi$

▶ Incoherent spread between  $[0.0, 1.0] \cdot \xi$

➡ Strong-strong case: π-mode shifted outside tune spread

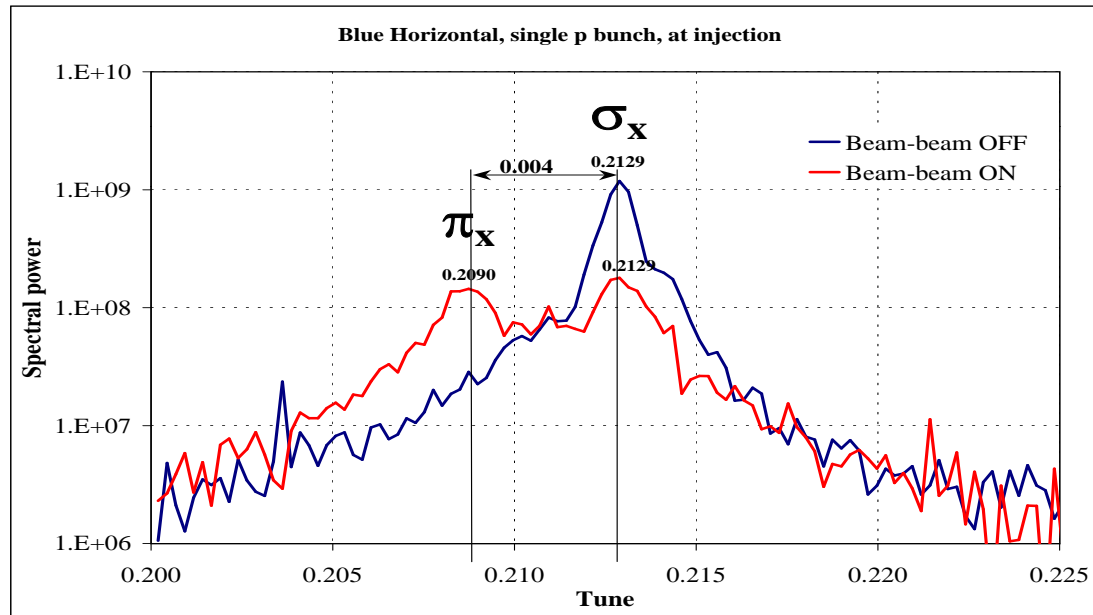
➡ No Landau damping possible

## Measurement: LEP



- Two modes clearly visible
- Can be distinguished by phase relation, i.e. sum and difference signals

# Measurement: RHIC

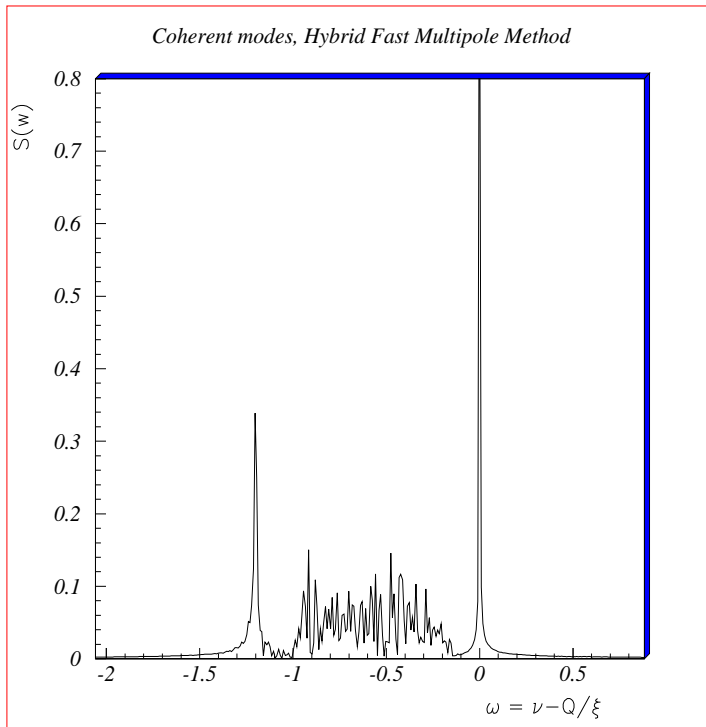


Courtesy W. Fischer (BNL)

→ Compare spectra with and without beams : two modes visible with beams

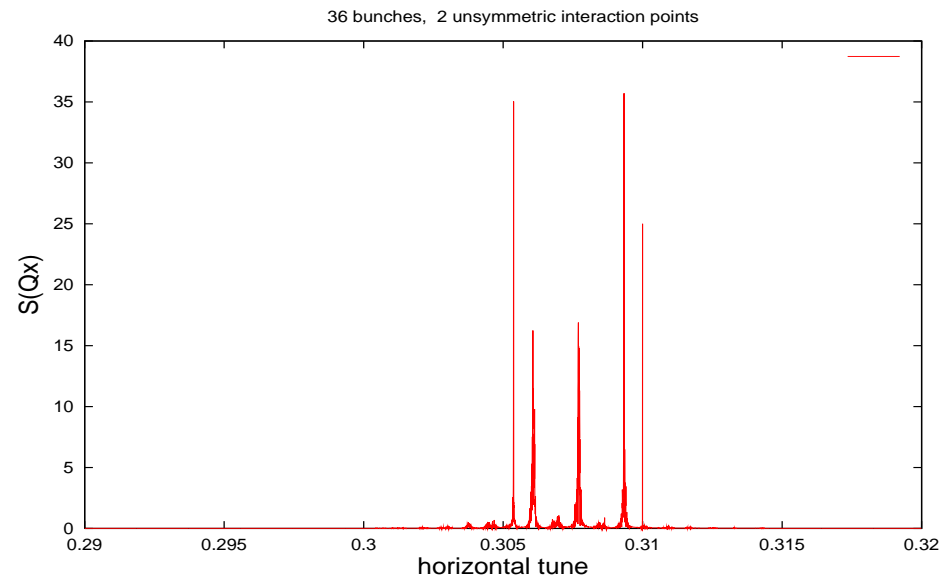


## Simulation of coherent spectra



- ▶ Full simulation of both beams required
- ▶ Use up to  $10^8$  particles in simulations
- ▶ Must take into account changing fields
- ▶ Requires computation of arbitrary fields

## Many bunches and more interaction points



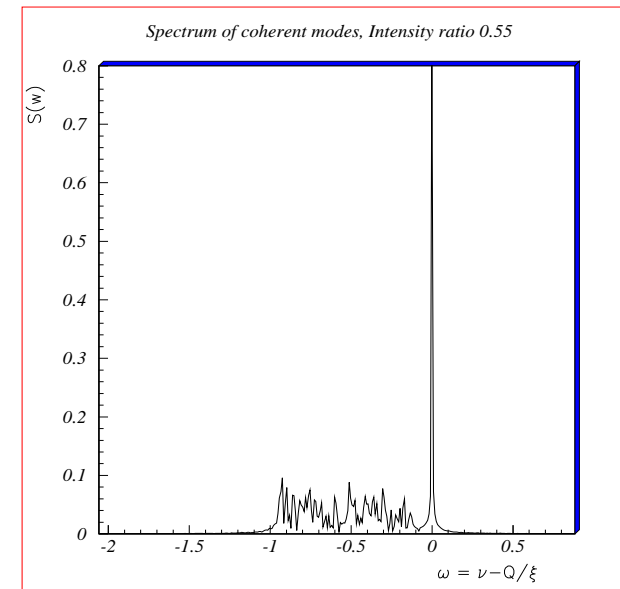
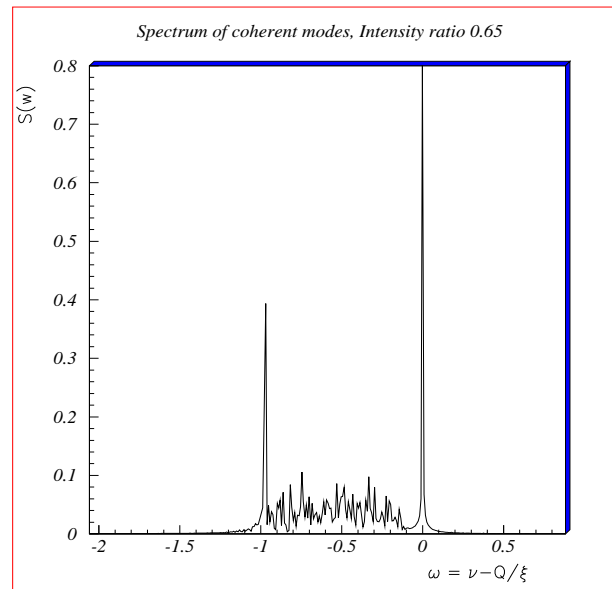
- Bunches couple via the beam-beam interaction
- Additional coherent modes become visible
- Potentially undesirable situation

## What can be done to avoid problems ?

- Coherent motion requires 'organized' motion of many particles
- Therefore high degree of symmetry required
- Possible countermeasure: (symmetry breaking)
  - Different bunch intensity
  - Different tunes in the two beams

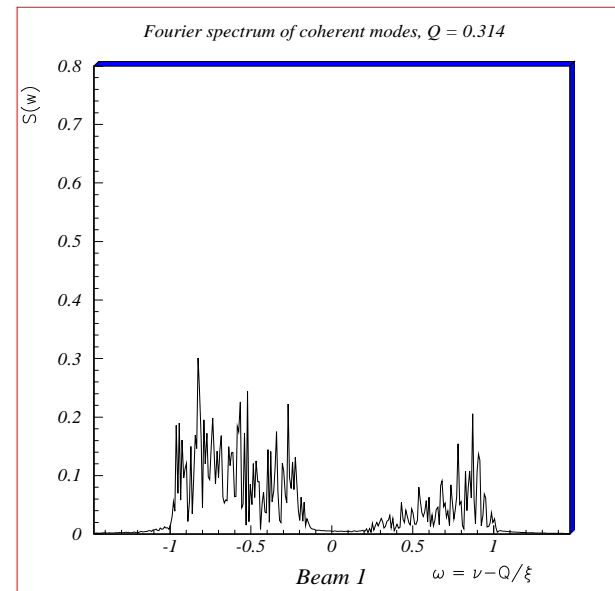
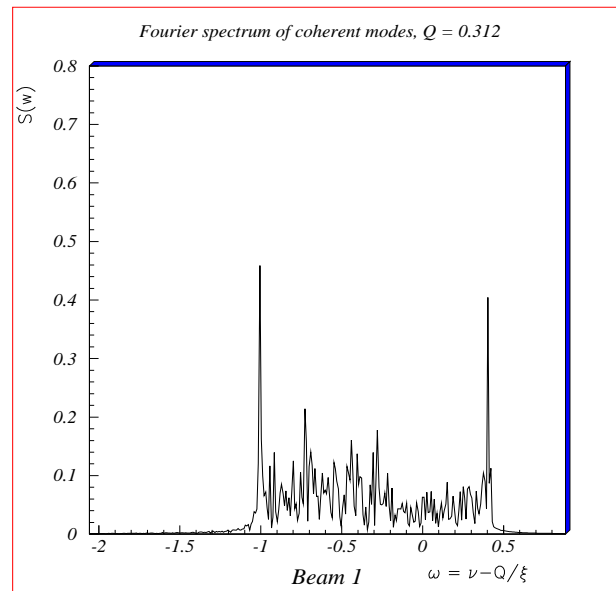


## Beams with different intensity



→ Bunches with **different intensities** cannot maintain coherent motion

## Beams with different tunes



→ Bunches with **different tunes** cannot maintain coherent motion

## Can we suppress beam-beam effects ?

- Find 'lenses' to correct beam-beam effects
  - Head on effects:
    - Linear "electron lens" to shift tunes
    - Non-linear "electron lens" to reduce spread
    - Tests in progress at FNAL
  - Long range effects:
    - At very large distance: force is  $1/r$
    - Same force as a wire !
  - So far: mixed success with **active** compensation
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## Others: Möbius lattice

- Principle:

- Interchange horizontal and vertical plane each turn

- Effects:

- Round beams (even for leptons)

- Some compensation effects for beam-beam interaction

- First test at CESR at Cornell



## Not mentioned:

- Effects in linear colliders
- Asymmetric beams
- Coasting beams
- Beamstrahlung
- Synchrotron coupling
- Monochromatization
- Beam-beam experiments
- ... and many more





# Bibliography



Some bibliography in the hand-out

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