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Sources of emittance growth

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Summary:

- Introduction
- Emittance growth in single-passage systems
  - Scattering through thin foils
- Emittance growth in multi-passage systems
  - Injection process
  - Scattering processes
  - Others
- Emittance manipulation
  - Longitudinal
  - Transverse

Acknowledgements:

D. Brandt and D. Möhl

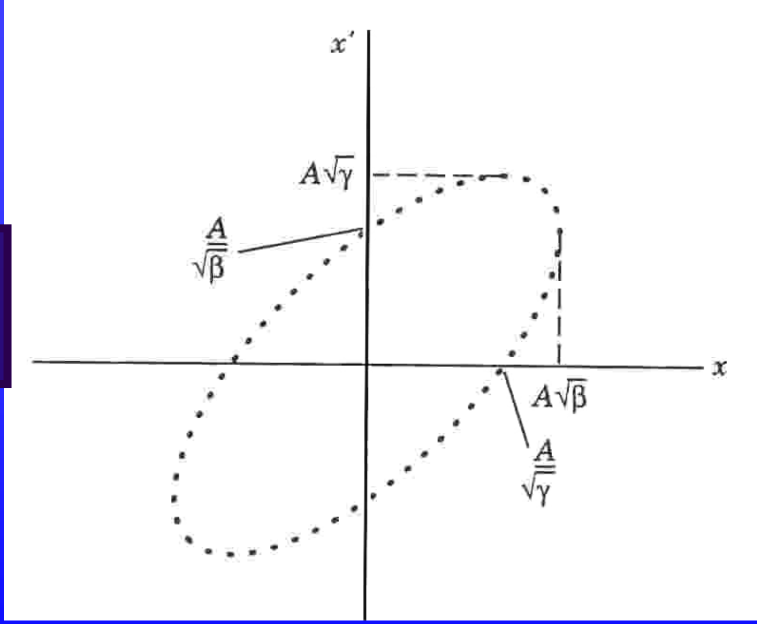
Introduction - I

- The starting point is the well-known Hill's equation

$$x(s)'' + K(s)\, x(s) = 0$$

- Such an equation has an invariant (the so-called Courant-Snyder invariant)

$$A = \gamma\, x^2 + 2\alpha\, x\, x' + \beta\, x'^2$$



Parenthetically: in a bending-free region the following dispersion invariant exists

$$A = \gamma\, D^2 + 2\alpha\, D\, D' + \beta\, D'^2$$

Introduction - II

In the case of a beam, i.e. an ensemble of particles:

- Emittance: value of the Courant-Snyder invariant corresponding to a given fraction of particles.
- Example: rms emittance for Gaussian beams.

Why emittance can grow?

- Hill equation is linear -> in the presence of nonlinear effects emittance is no more conserved.

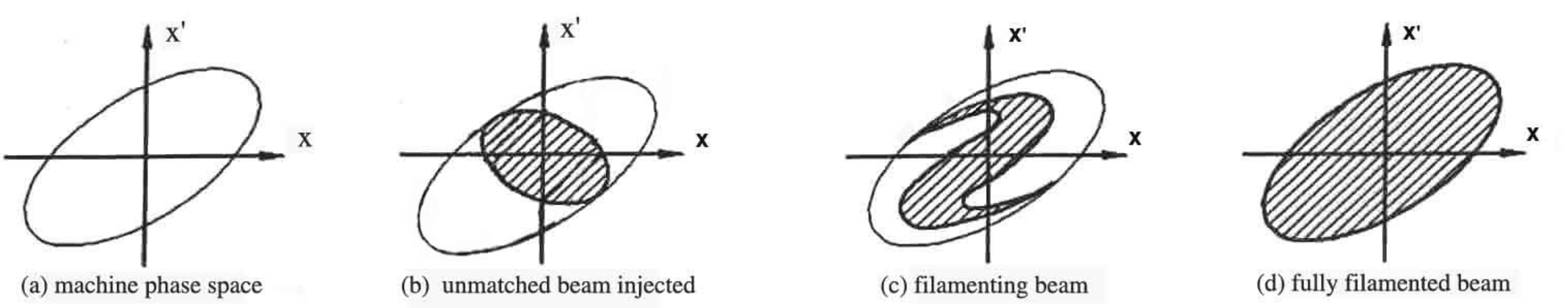
Introduction - III

Why emittance growth is an issue?

- Machine performance is reduced, e.g. in the case of a collider the luminosity (i.e. the rate of collisions per unit time) is reduced.
- It can lead to beam losses.

Introduction - IV

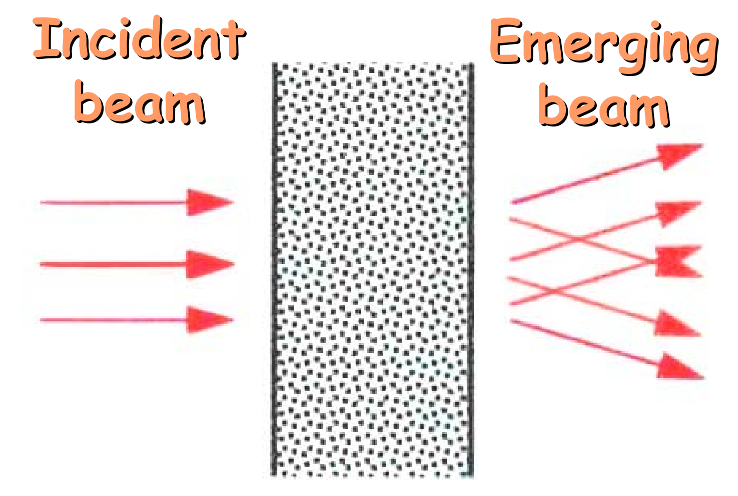
- Filamentation is one of the key concepts for computing emittance growth
  - Due to the presence of nonlinear imperfections, the rotation frequency in phase space is amplitude-dependent.
  - After a certain time the initial beam distribution is smeared out to fill a phase space ellipse.

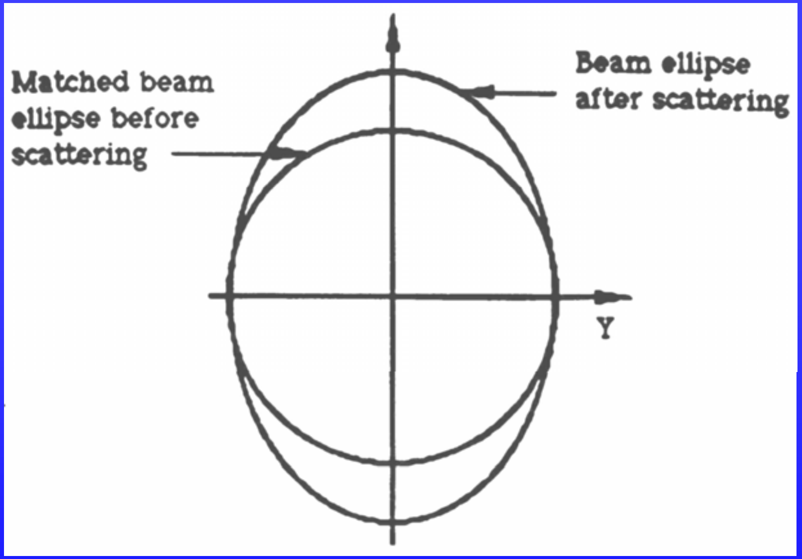


(a) machine phase space (b) unmatched beam injected (c) filamenting beam (d) fully filamented beam

Scattering through thin foil - I

- Typical situation:
  - Vacuum window between the transfer line and a target (in case of fixed target physics)
  - Vacuum window to separate standard vacuum in transfer line from high vacuum in circular machine





The beam receives an angular kick

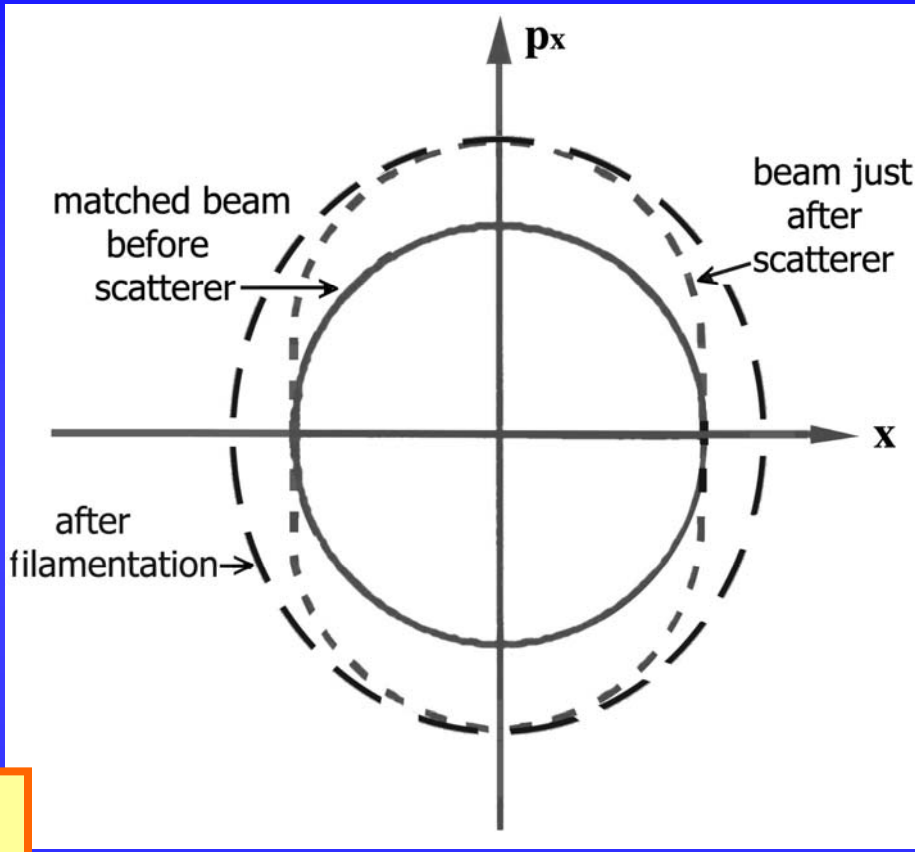
Scattering through thin foil - II

- Multiple Coulomb scattering due to beam-matter interaction is described by means of the rms scattering angle:

$$\boxed{\theta_{rms} = \frac{14\ MeV\ /c}{p\beta_p} q_p \sqrt{\frac{L}{L_{rad}}}\left(1+\varepsilon_{corr}\right)}$$

- Downstream of the foil the transformed coordinates are given by

$$\boxed{\begin{aligned} x_i &\rightarrow x_i = A_{io}\sin(\psi_i) \\ p_{xi} &\rightarrow p_{xi} + \Delta p = A_{io}\cos(\psi_i) + \beta\,\theta \end{aligned}}$$



$$\boxed{p_x = \alpha\, x + \beta\, x'}$$

Normalised coordinate

α = 0 at the location of the foil

Scattering through thin foil - III

- By assuming that:
  - Scattering angle and betatronic phase are uncorrelated
  - Averaging of the phase, i.e. assuming filamentation in the transfer line or the subsequent machine

$$\left\langle A_i^{\,2} \right\rangle = \left\langle x_i^{\,2} + p_{xi}^{\,2} \right\rangle = \left\langle A_{i0}^{\,2} \right\rangle + \left\langle \beta^2 \theta_i^{\,2} \right\rangle$$

  - Therefore the final result is

$$2\sigma_x^{\,2} = 2\sigma_{x0}^{\,2} + \beta^2 \theta_{rms}^{\,2}$$

  - Taking into account the statistical definition of emittance

$$\Delta\varepsilon_{rms} = \frac{\pi}{2}(\theta_{rms})^2 \beta$$

Scattering through thin foil - IV

- Few remarks
  - A special case with $\alpha = 0$ at the location of the thin foil is discussed -> it can be generalised.
  - The correct way of treating this problem is (see next slides):
    - Compute all three second-order moments of the beam distribution downstream of the foil
    - Evaluate the new optical parameters and emittance using the statistical definition
  - The emittance growth depends on the beta-function!

THE SMALLER THE VALUE OF THE BETA-FUNCTION AT THE LOCATION OF THE FOIL THE SMALLER THE EMITTANCE GROWTH

Scattering through thin foil - V

- Correct computation (always for α=0):

$$\begin{aligned} x &= A_o\sqrt{\beta_o}\cos(\psi_o) = A_1\sqrt{\beta_1}\cos(\psi_1) \\ x' &= -A_o\sqrt{1/\beta_o}\sin(\psi_o) + \theta = -A_1\sqrt{1/\beta_1}\sin(\psi_1) \end{aligned}$$

- By squaring and averaging over the beam distribution

$$\begin{aligned} <x^2> &= \frac{<A_0^2>\beta_0}{2} = \frac{<A_1^2>\beta_1}{2} \\ <x'^2> &= \frac{<A_0^2>}{2\beta_0} + \theta_{rms}^2 = \frac{<A_1^2>}{2\beta_1} \end{aligned}$$

- Using the relation

$$\varepsilon_{rms} = \pi\frac{\langle A^2\rangle}{2}$$

Scattering through thin foil - VI

- The solution of the system is

$$\frac{\langle A_1^{2}\rangle^{2} - \langle A_0^{2}\rangle^{2}}{\langle A_0^{2}\rangle} = 2\beta_0\,\theta_{rms}^{2}$$

$$\frac{\beta_1}{\beta_0} = \frac{\langle A_0^{2}\rangle}{\langle A_1^{2}\rangle}$$

- This can be solved exactly, or by assuming that the relative emittance growth is small, then

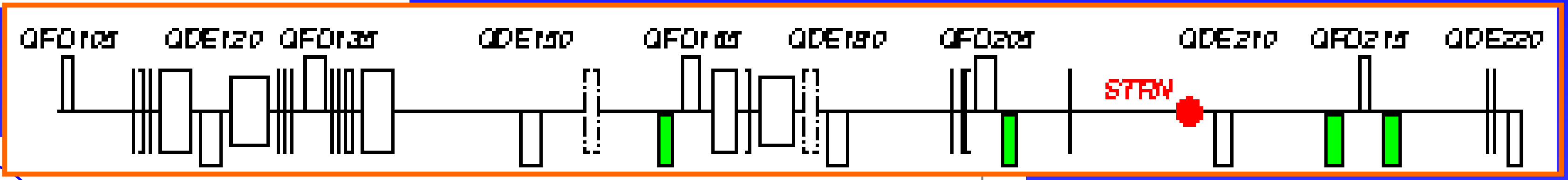
$$\Delta\varepsilon_{rms} = \frac{\pi}{4}\theta_{rms}^{2}\beta_0$$

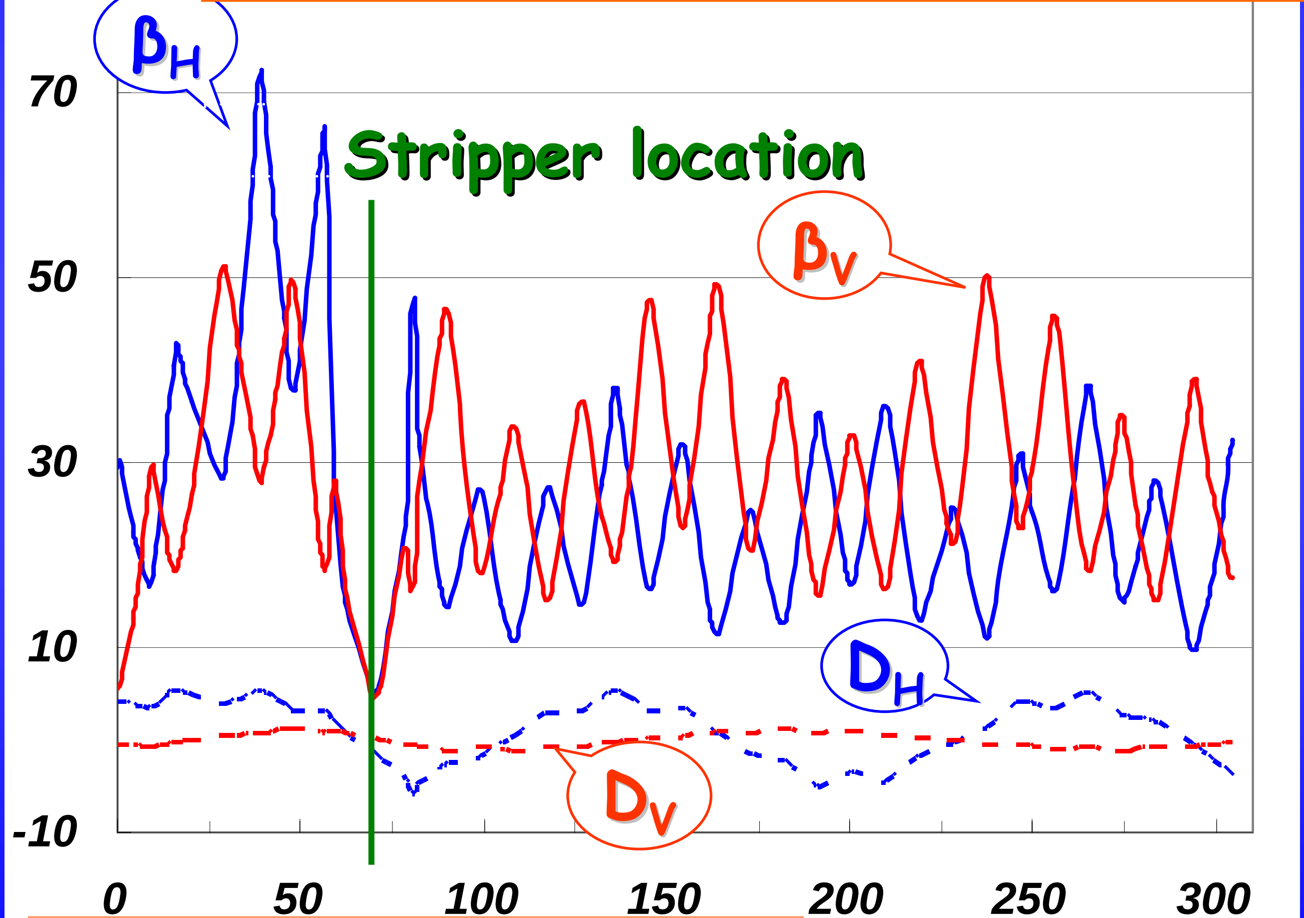
$$\beta_1 = \beta_0\left[1 - \frac{\pi}{4}\frac{\theta_{rms}^{2}}{\varepsilon_{rms}}\right]$$

NB: the emittance growth is now only half of the previous estimate!

Downstream of the foil the transfer line should be matched using the new Twiss parameters α1, β1

Example: ion stripping for LHC lead beam between PS and SPS





- Pb^{54+} -> Pb^{82+}
- A low-beta insertion is designed (beta reduced by a factor of 5)
- Stripping foil, 0.8 mm thick Al, is located in the low-beta insertion

Courtesy M. Martini - CERN

Exercises…

- Compute the Twiss parameters and emittance growth for a THICK foil
- Compute the Twiss parameters and emittance growth for a THICK foil in a quadrupolar field

Injection process - I

- Two sources of errors:
  - Steering errors.
  - Optics errors (Twiss parameters and dispersion).
- In case the incoming beam has an energy error, then the next effect will be a combination of the two.
- In all cases filamentation, i.e. nonlinear imperfection in the ring, is the source of emittance growth.

Injection process - II

- Steering errors:
  - Injection conditions, i.e. position and angle, do not match position and angle of the closed orbit.
- Consequences:
  - The beam performs betatron oscillations around the closed orbit. The emittance grows due to the filamentation
- Solution:
  - Change the injection conditions, either by steering in the transfer line or using the septum and the kicker.
  - In practice, slow drifts of settings may require regular tuning. In this case a damper (see lecture on feedback systems) is the best solution.

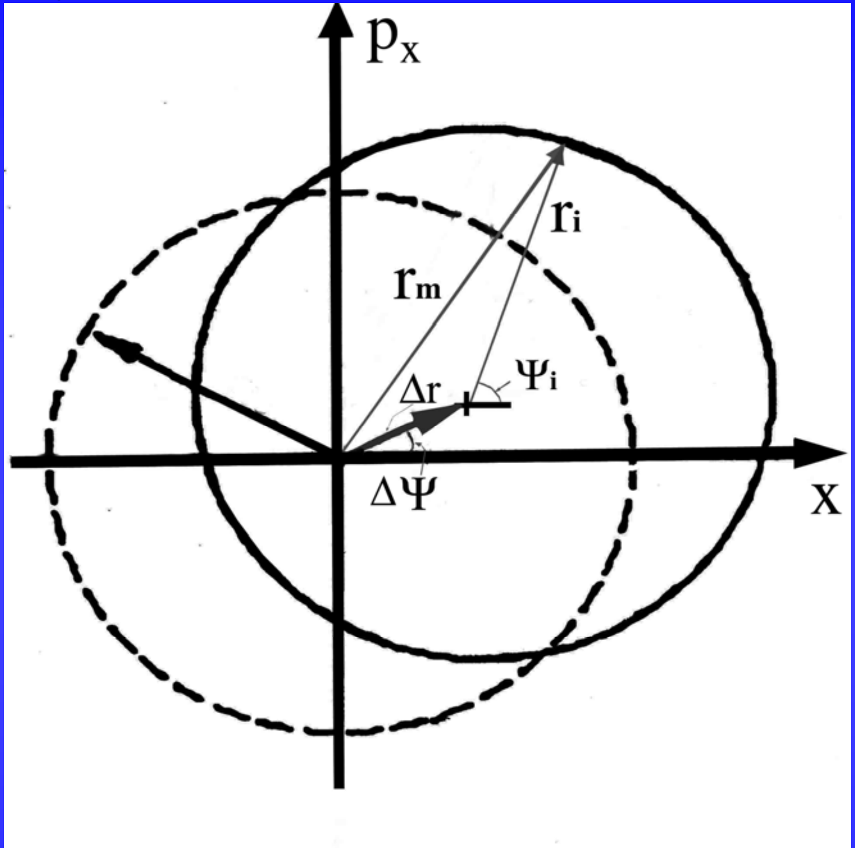
Injection process - III

- Analysis in normalised phase space (i stands for injection m for machine):

$$x_m = r_i \cos\psi_i + \Delta r \cos\psi$$
$$p_m = r_i \sin\psi_i + \Delta r \cos\psi$$

- Squaring and averaging gives

$$< r_m^2 > = < x_m^2 + p_m^2 >$$
$$< r_m^2 > = < r_i^2 > + \Delta r^2$$



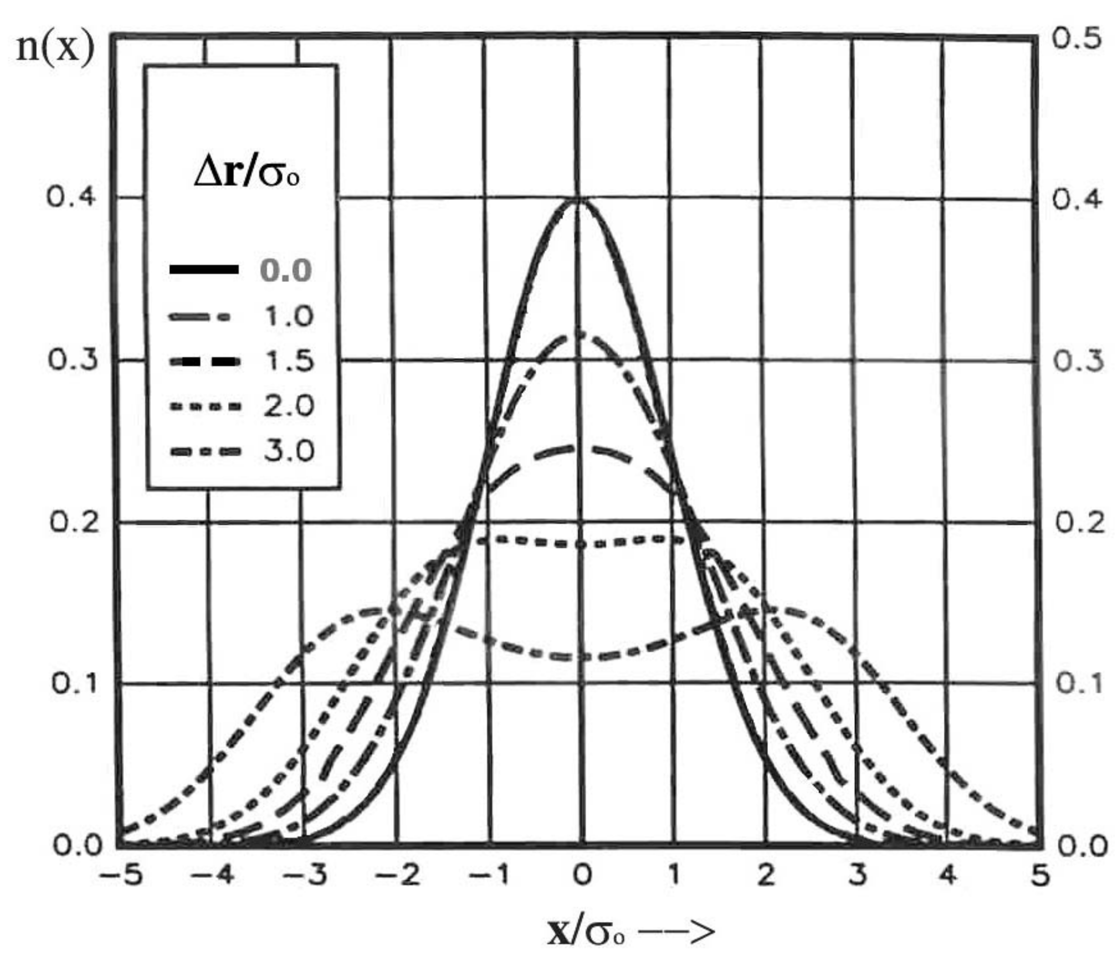
- After filamentation

$$< x^2 > = \tfrac{1}{2} < r_m^2 > = \tfrac{1}{2} < r_i^2 > + \tfrac{1}{2}\Delta r \;\Rightarrow\; \sigma^2 = \sigma_i^2 + \tfrac{1}{2}\Delta r^2$$

Injection process - IV

Example of beam distribution generated by steering errors and filamentation.

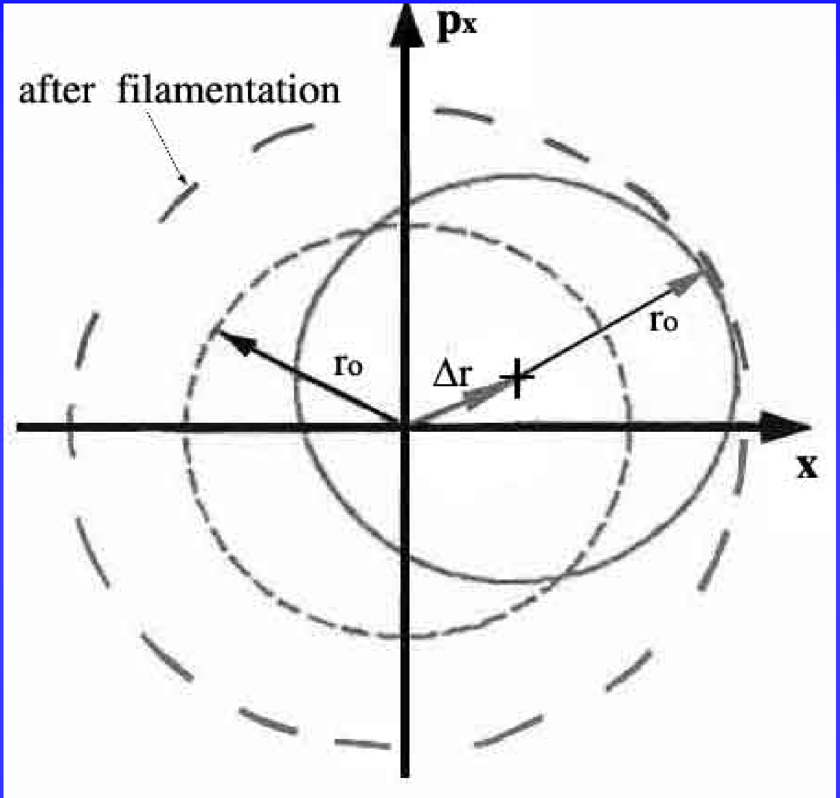
The beam core is displaced -> large effect on emittance growth



Injection process - V

- Dispersion mismatch: analysis is similar to that for steering errors.
- In this case

$$\Delta r^2 = \left[\Delta D^2 + \frac{1}{2}\left(\beta \Delta D' + \alpha \Delta D\right)^2\right]\left(\frac{\sigma_p}{p}\right)^2$$

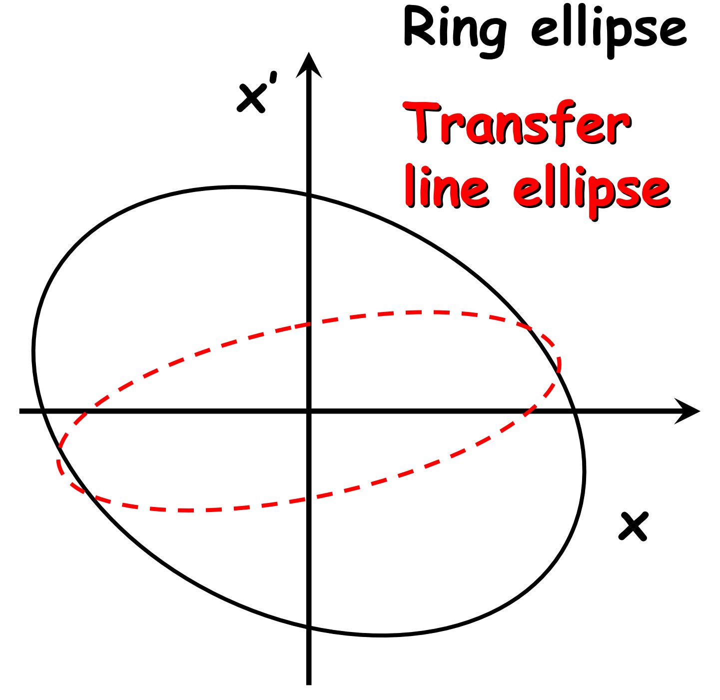


- The final result is

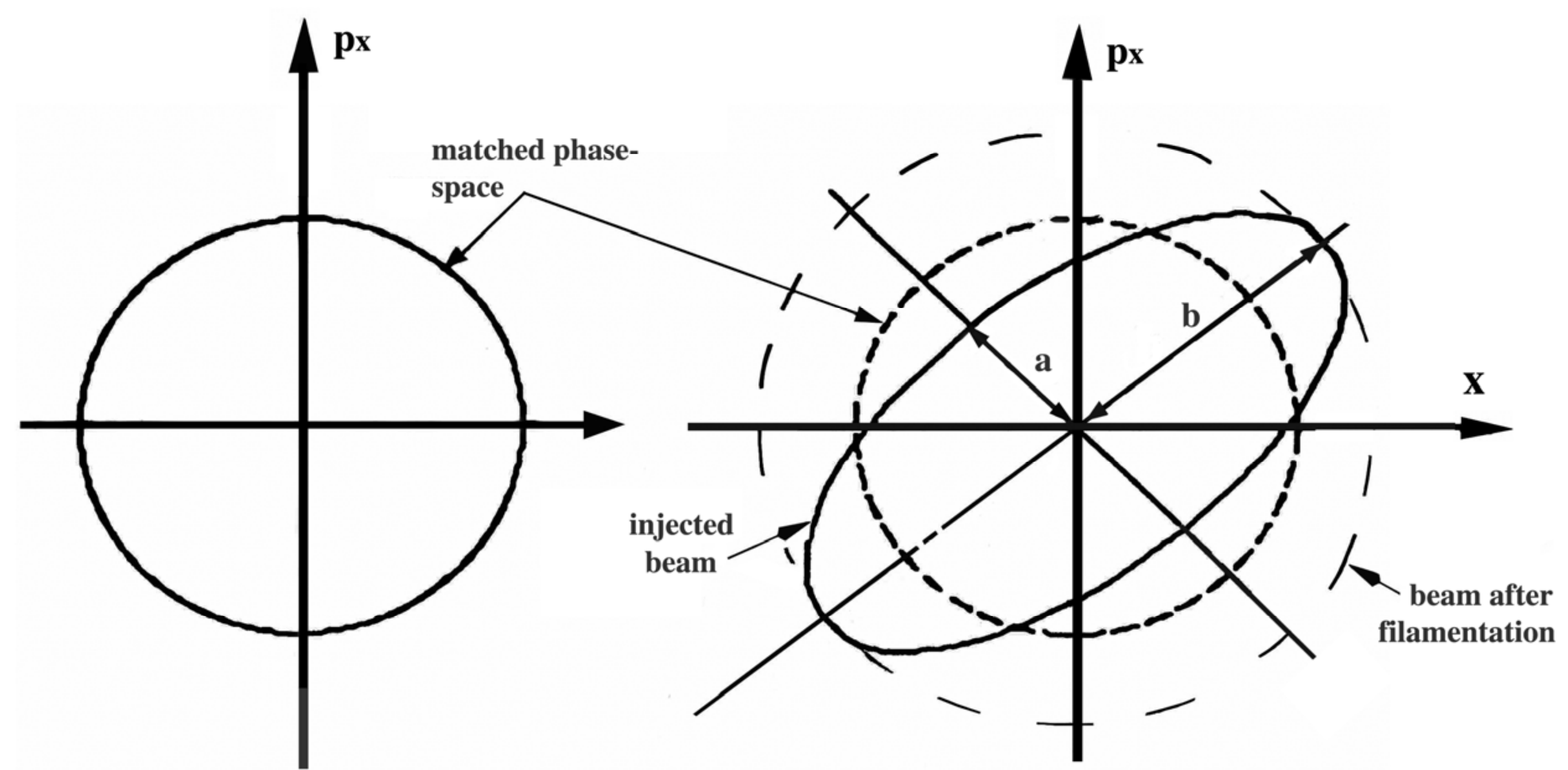
$$\varepsilon_{rms}^{after\ fil.} = \varepsilon_{rms}\left\{1 + \frac{1}{2}\left[\Delta D^2 + \left(\beta\,\Delta D' + \alpha\,\Delta D\right)^2\right]\left(\frac{\sigma_p}{p}\right)^2 / \sigma_0^{\,2}\right\}$$

Injection process - VI

- Optics errors:
  - Optical parameters (Twiss and dispersion) of the transfer line at the injection point are different from those of the ring
- Consequences:
  - The beam performs quadrupolar oscillations (size changes on a turn-by-turn basis). The emittance grows due to the filamentation
- Solution:
  - Tune transfer line to match optics of the ring



Injection process - VII



In normalised phase space (that of the ring) the injected beam will fill an ellipse due to the mismatch of the optics…

Injection process - VIII

Given the initial condition of the beam end of the transfer line

$$x_l = A_i \sqrt{b/a}\ \sin\psi_i, \qquad p_l = A_i \sqrt{a/b}\ \cos\psi_i$$

By squaring and averaging over the particles' distribution

$$< r_i^2 > = < A_i^2 > \frac{\left(b/a\ +\ a/b\right)}{2}$$

The emittance after filamentation is given by

$$\varepsilon_{rms}^{after\ fil.} = \varepsilon_{rms} \frac{\left(b/a\ +\ a/b\right)}{2}$$

$$\varepsilon_{rms}^{after\ fil.} = \varepsilon_{rms} F$$

$$F = \frac{1}{2}\left(\frac{\beta_l}{\beta_m} + \frac{\beta_m}{\beta_l} + \left(\frac{\alpha_m}{\beta_m} - \frac{\alpha_l}{\beta_l}\right)^2 \beta_m \beta_l\right)$$

Scattering processes - I

- Two main categories considered:
  - Scattering on residual gas -> similar to scattering on a thin foil (the gas replaces the foil...)

$$\Delta\varepsilon_{k\sigma} = \frac{\pi}{2}k^2 q_p^2 \left(\frac{14\ MeV/c}{p\beta_p}\right)^2 \bar{\beta}\,\frac{\beta_p c\, t}{L_{rad}} \quad where \quad \bar{\beta} \quad average\ beta$$

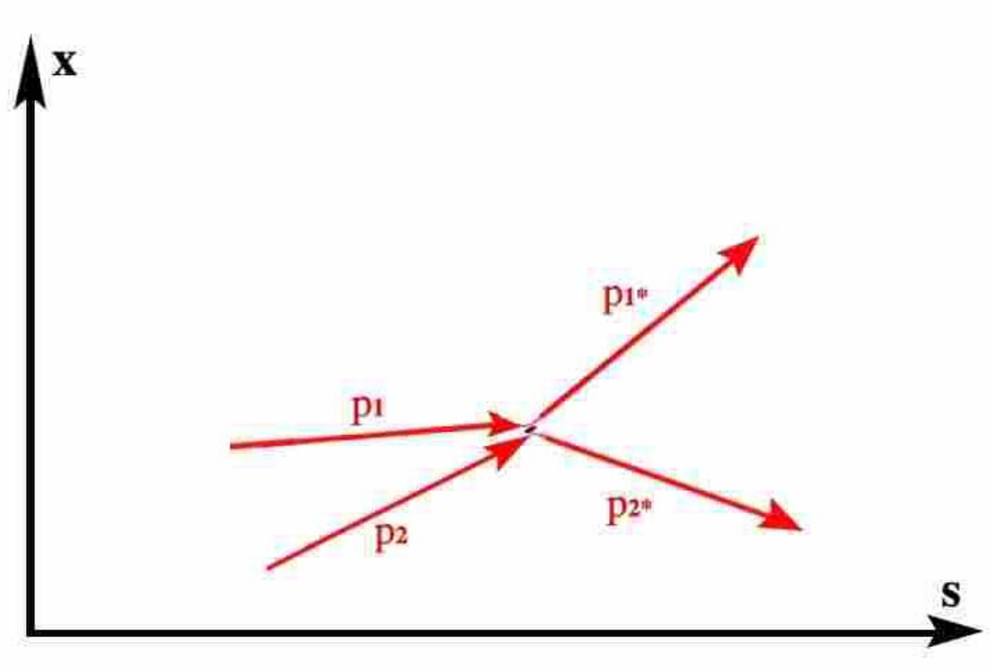
NB: $\beta_p ct$ represents the scatterer length until time t

This is why good vacuum is necessary!

  - Intra-beam scattering, i.e. Coulomb scattering between charged particles in the beam.

Scattering processes - II

- Intra-beam scattering
  - Multiple (small angle) Coulomb scattering between charged particles.
  - Single scattering events lead to Touscheck effect.
  - All three degrees of freedom are affected.



Scattering processes - III

How to compute IBS?

- Transform momenta of colliding particles into their centre of mass system.
- Rutherford cross-section is used to compute change of momenta.
- Transform the new momenta back to the laboratory system.
- Calculate the change of the emittances due to the change of momenta at the given location of the collision.

1. For each scattering event compute average over all possible scattering angles (impact parameters from the size of the nucleus to the beam radius).
2. Take the average over momenta and transverse position of the particles at the given location on the ring circumference (assuming Gaussian distribution in all phase space planes.
3. Compute the average around the circumference, including lattice functions, to determine the change per turn.

Scattering processes - IV

Features of IBS

- For constant lattice functions and below transition energy, the sum of the three emittances is constant.
- Above transition the sum of the emittances always grows.
- In any strong focusing lattice the sum of the emittances always grows.
- Even though the sum grows from the theoretical point of view emittance reduction in one plane predicted by simulations, but were never observed.

Scattering processes - V

Scaling laws of IBS

- Accurate computations can be performed only with numerical tools.
- However, scaling laws can be derived.
- Assuming

$$\frac{1}{\tau_{x,y,l}} = \frac{1}{\tau_0} F_{x,y,l}$$

- Then

$$1/\tau_0 = \frac{N_b r_0^2 \left(\frac{q^2}{A}\right)^2}{(4/\pi^2)\,\gamma\,\varepsilon_x^* \varepsilon_y^* \varepsilon_l^* / E_0} \propto \frac{N_b \left(\frac{q^2}{A}\right)^2}{\gamma\,\varepsilon_x^* \varepsilon_y^* \varepsilon_l^*}$$

Strong dependence on charge

$\varepsilon_{x,y,l}^*$ are normalised emittances

N_b of particles/bunch

r_0 classical proton radius

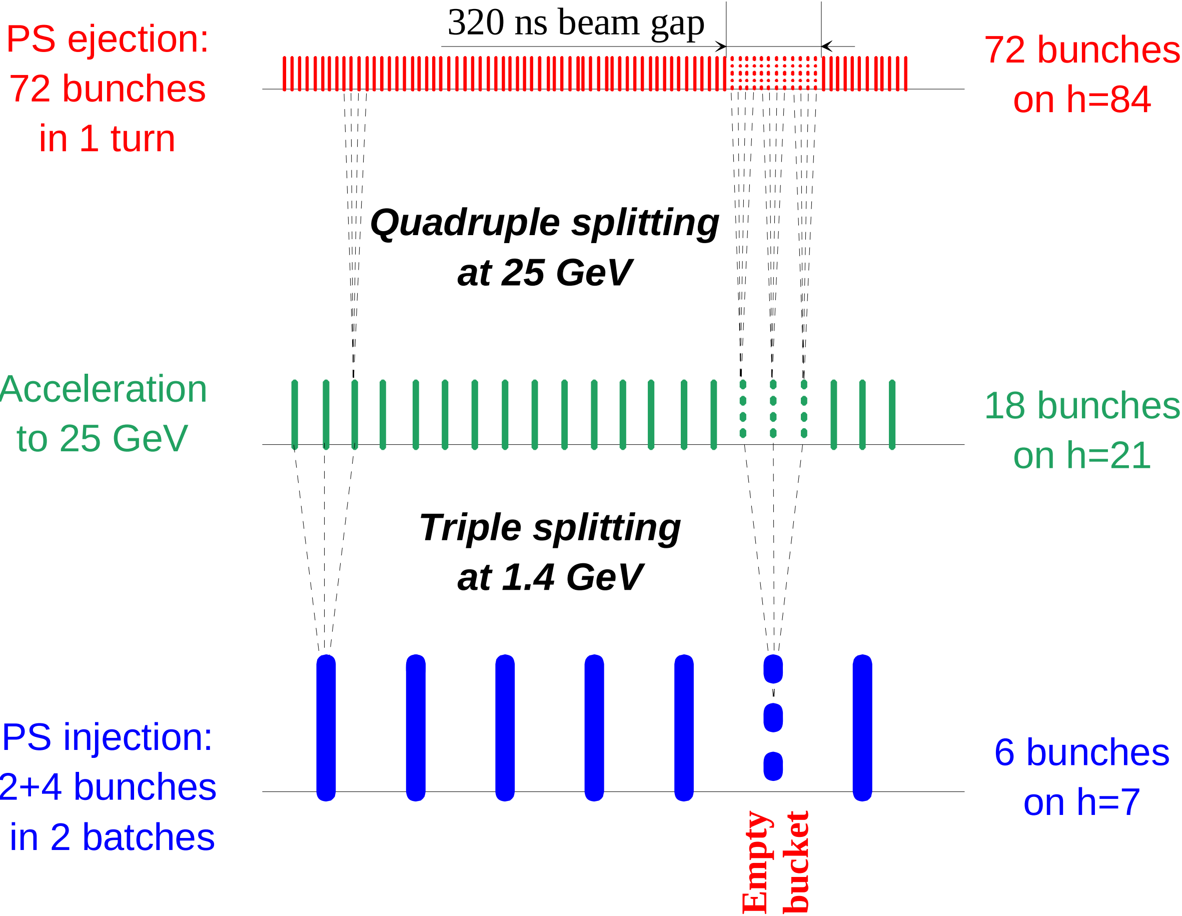
Others

- Diffusive phenomena:
  - Resonance crossings
  - Ripple in the power converters
- Collective effects
  - Space charge (soft part of Coulomb interactions between charged particles in the beam) -> covered by a specific lecture.
  - Beam-beam -> covered by a specific lecture.
  - Instabilities -> covered by a specific lecture.

Emittance manipulation

- Emittance is (hopefully) conserved…
- Sometimes, it is necessary to manipulate the beam so to reduce its emittance.
  - Standard techniques: electron cooling, stochastic cooling.
  - Less standard techniques: longitudinal or transverse emittance splitting.

Longitudinal manipulation: LHC beam in PS machine - I



0.35 eVs (bunch)
1.1 × 10^{11} ppb

40 %
Blow-up
(pessimistic)

1 eVs (bunch)
4.4 × 10^{11} ppb

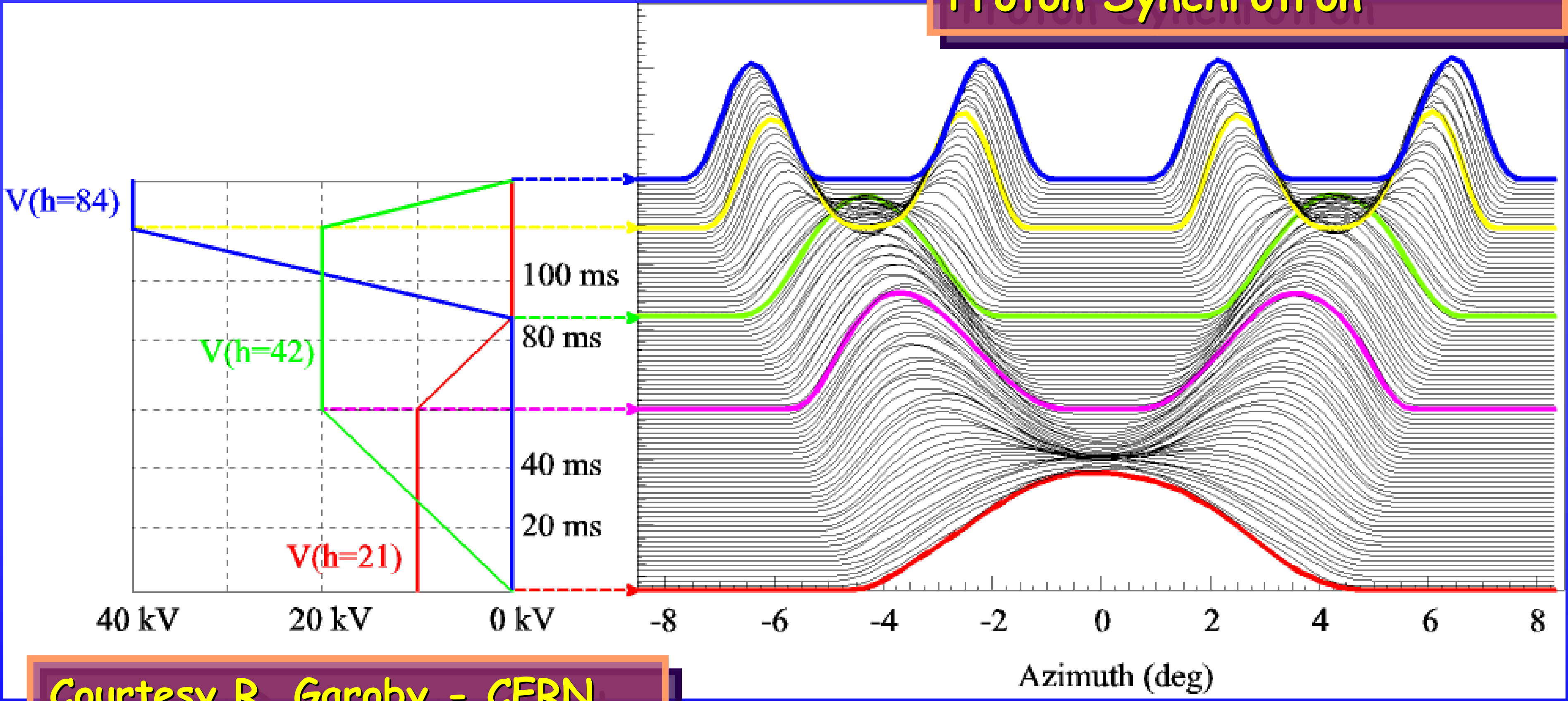
115 %
Blow-up
(voluntary)

1.4 eVs (bunch)
13.2 × 10^{11} ppb

Courtesy R. Garoby - CERN

Longitudinal manipulation: LHC beam in PS machine - II

Measurement results obtained at the CERN Proton Synchrotron



Courtesy R. Garoby - CERN

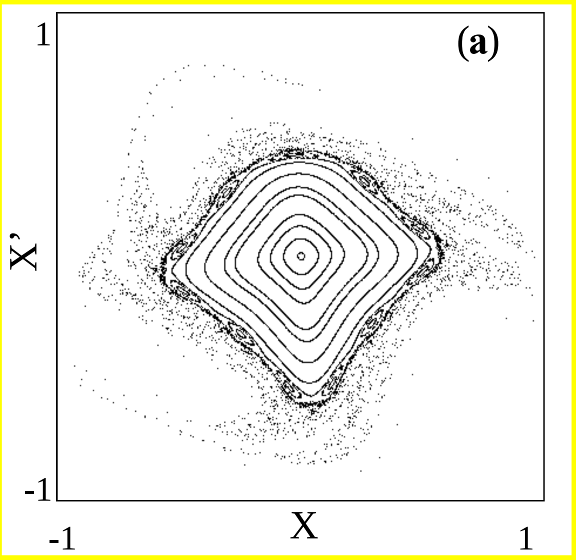
Transverse manipulation: novel multi-turn extraction - I

The main ingredients of the novel extraction:

- The beam splitting is not performed using a mechanical device, thus avoiding losses. Indeed, the beam is separated in the transverse phase space using
  - Nonlinear magnetic elements (sextupoles ad octupoles) to create stable islands.
  - Slow (adiabatic) tune-variation to cross an appropriate resonance.

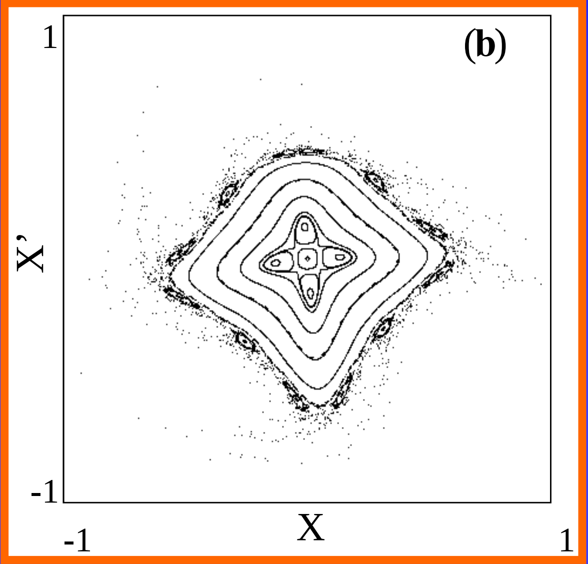
NB: third-order extraction is based on separatrices (see O. Brüning lecture)

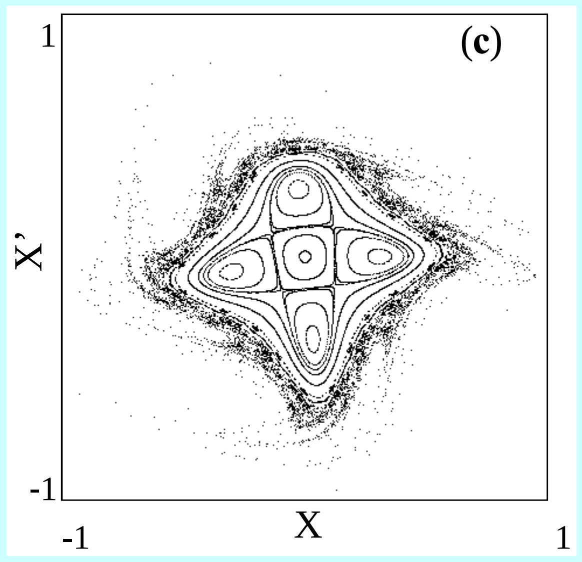
Transverse manipulation: novel multi-turn extraction - II



Left: initial phase space topology. No islands.

Right: intermediate phase space topology. Islands are created near the centre.





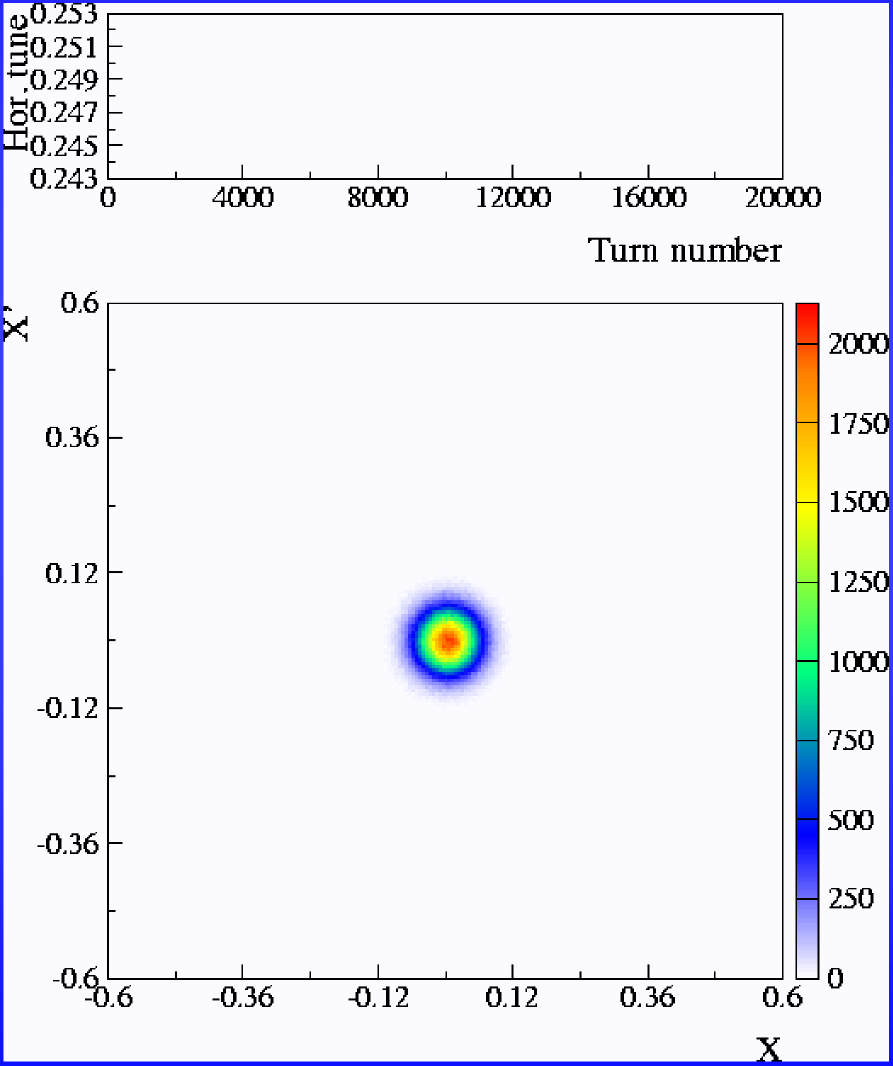
Bottom: final phase space topology. Islands are separated to allow extraction.

Four islands require tune = 0.25

Transverse manipulation: novel multi-turn extraction - III

Tune variation

Phase space portrait



Simulation parameters:

Hénon-like map (i.e. 2D polynomial - degree 3 - mapping) representing a FODO cell with sextupole and octupole

A movie to show the evolution of beam distribution

A series of horizontal beam profiles have been taken during the capture process.

Measurement results obtained at the CERN Proton Synchrotron

Some references

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