Linear Imperfections

- multipole expansion of magnetic fields
- equations of motion with imperfections: smooth approximation
- sources for linear field errors: feed down
- perturbation treatment: driven oscillators and resonances
- transfer matrices with coupling: element and one-turn
- what we have left out (coupling)
- orbit correction for the un-coupled case

Multipole Expansion of Magnetic Fields

Taylor expansion of the magnetic field:

$$B_y + iB_x = \sum_{n=0}^{\infty} \frac{1}{n!} \cdot (b_n - ia_n) \cdot (x + iy)^n \qquad \text{with:} \qquad b_n = \frac{\partial^n B_y}{\partial x^n}$$

multipole	order	B_x	B_y
dipole	0	0	B_0
quadrupole	1	$b_1 \cdot y$	$b_1 \cdot x$
sextupole	2	$b_2 \cdot x \cdot y$	$\frac{1}{2} \cdot b_2 \cdot (x^2 + y^2)$
octupole	3	$\frac{1}{6} \cdot b_3 \cdot (3yx^2 - y^3)$	$\frac{1}{6} \cdot b_3 \cdot (x^3 - 3xy^2)$

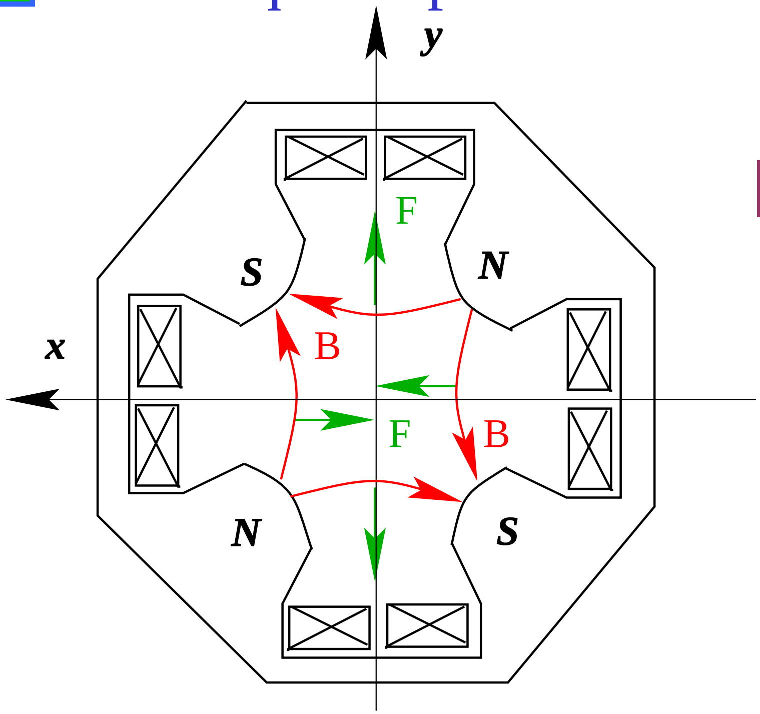
skew multipoles a_n:

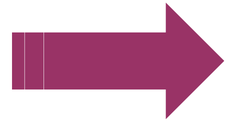
rotation of the magnetic field
by half of the coil symmetry:

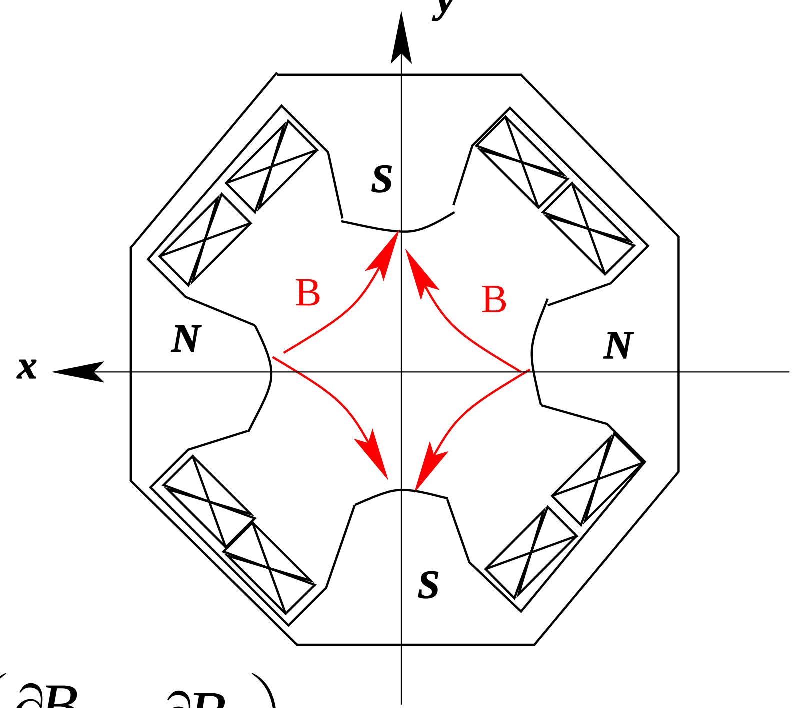
90° for dipole magnets
45° for quadrupole magnets
30° for sextupole magnets

Skew Multipoles: Example Skew Quadrupole

normal quadrupole: → clockwise rotation by 45° → skew quadrupole







$$b_1 = \frac{\partial B_y}{\partial x}$$

$$a_1 = \frac{1}{2}\left(\frac{\partial B_y}{\partial y} - \frac{\partial B_x}{\partial x}\right)$$

$$B_y = b_1 \cdot x \Rightarrow F_x = -q \cdot v \cdot b_1 \, x$$

$$B_x = b_1 \cdot y \Rightarrow F_y = +q \cdot v \cdot b_1 \cdot y$$

$$B_y = +a_1 \cdot y \Rightarrow F_x = -q \cdot v \cdot a_1 \cdot y$$

$$B_x = -a_1 \cdot x \Rightarrow F_y = -q \cdot v \cdot a_1 \cdot x$$

Equation of Motion I

Smooth approximation for Hills equation: $w = x, y$

$$\frac{d^2}{ds^2} w(s) + K(s) \cdot w(s) = 0 \xrightarrow{K(s) = \text{const}} \frac{d^2}{ds^2} w(s) + \omega_0{}^2 \cdot w(s) = 0$$

(constant β-function and phase advance along the storage ring)

$$\longrightarrow \quad w(s) = A \cdot \sin(\omega_0 \cdot s + \phi_0) \qquad \omega_0 = 2\pi \cdot Q_0 / L$$

(Q is the number of oscillations during one revolution)

perturbation of Hills equation:

$$\frac{d^2}{ds^2} w(s) + \omega_0{}^2 \cdot w(s) = F(x(s), y(s), s) / (v \cdot p)$$

in the following the force term will be the Lorenz force of a charged particle in a magnetic field: $F = q \cdot \vec{v} \times \vec{B}$

Equation of Motion II

- perturbed equations of motion:

B_y →

$$\frac{d^2}{ds^2}x + \omega_x^{\ 2}\cdot x = -\sum_{n=0}^{\infty}\frac{1}{n!}\cdot\frac{q}{p}\cdot \mathrm{Re}\left[\left(b_n(s) - i a_n(s)\right)\cdot(x+iy)^n\right]$$

B_x →

$$\frac{d^2}{ds^2}y + \omega_y^{\ 2}\cdot y = +\sum_{n=0}^{\infty}\frac{1}{n!}\cdot\frac{q}{p}\cdot \mathrm{Im}\left[\left(b_n(s) - i a_n(s)\right)\cdot(x+iy)^n\right]$$

- normalized multipole gradients:

$$k_n = 0.3\cdot\frac{b_n[T/m^n]}{p[GeV/c]}$$

with: $[k_n] = 1/m^{n+1}$

$$\kappa_n = 0.3\cdot\frac{a_n[T/m^n]}{p[GeV/c]}$$

$[\kappa_n] = 1/m^{n+1}$

Sources for Linear Field Errors

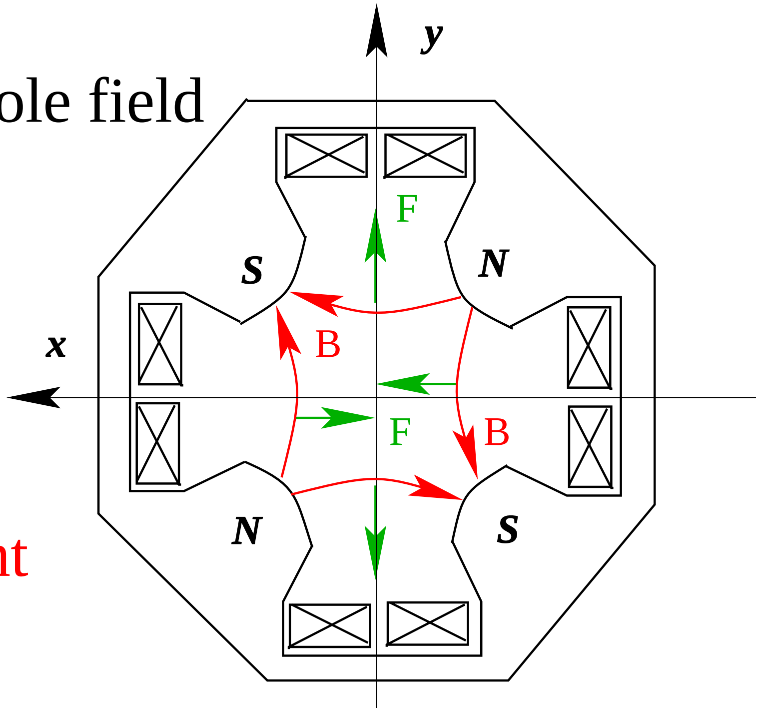
- sources for linear imperfections:
  - -magnetic field errors: b_0, b_1, a_0, a_1
  - -powering errors for dipole and quadrupole magnets
  - -energy errors in the particles ➔ change in normalized strength
  - -roll errors for dipole and quadrupole magnets
  - -feed-down errors from quadrupole and sextupole magnets

➔ example: feed down from a quadrupole field

$$x = \tilde{x} + \Delta x$$

$$B_y = b_1 \cdot (\tilde{x} + \Delta x)$$

$$B_x = b_1 \cdot y$$



➔ dipole + quadrupole field component

Sources for Linear Field Errors

- sources for feed down and roll errors:

  -magnet positioning in the tunnel

  transverse position → +/- 0.1 m

  roll error → +/- 0.5 mrad

  -tunnel movements:

  slow drifts

  civilization

  moon

  seasons

  civil engineering

  -closed orbit errors → beam offset inside magnetic elements

  -energy error: → dispersion orbit

Coupling I

- distributed coupling:

$$\frac{d^2}{ds^2}x(s)+\omega_x{}^2\cdot x(s)=-\kappa_1\cdot y(s)$$

$$\frac{d^2}{ds^2}y(s)+\omega_y{}^2\cdot y(s)=-\kappa_1\cdot x(s)$$

- solution by decomposition into 'Eigenmodes':

$$q_1(s)=a\cdot x+b\cdot y \qquad q_2(s)=c\cdot x+d\cdot y$$

with: $a\cdot c+b\cdot d=0$

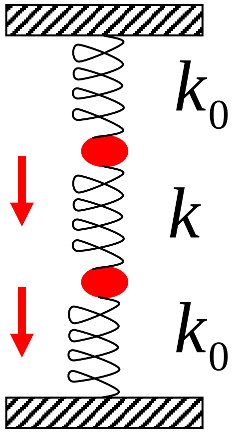
➔ $\frac{d^2}{dt^2}q_1(s)+\omega_1^2\cdot q_1(s)=0 \qquad \frac{d^2}{dt^2}q_2(s)+\omega_2^2\cdot q_2(s)=0$

Coupling II: Identical Coupled Oscillators

- fundamental modes for identical coupled oscillators:

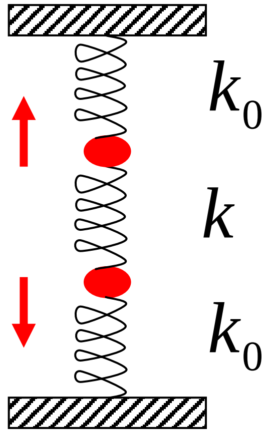
ω mode:

$$q_1(t) = x + y$$



π mode:

$$q_2(t) = x - y$$



$$\frac{d^2}{dt^2} q_1(t) + \omega_\omega^2 \cdot q_1(t) = 0$$

$$\omega_\omega = \sqrt{k_0}$$

$$\frac{d^2}{dt^2} q_2(t) + \omega_\pi^2 \cdot q_2(t) = 0$$

$$\omega_\pi = \sqrt{k_0 + 2k}$$

- weak coupling ($k \ll k_0$): ➔ degenerate mode frequencies

➔ description of motion in unperturbed ‘x’ and ‘y’ coordinates

Coupling IV: Orthogonal Coupled Oscillators

- different oscillation frequencies:

$$\frac{d^2}{ds^2}x+\omega_x{}^2\cdot x=-\kappa_1\cdot y \qquad \frac{d^2}{ds^2}y+\omega_y{}^2\cdot y=-\kappa_1\cdot x$$

- solution by decomposition into 'Eigenmodes':

$$q_1(s)=a\cdot x(s)+b\cdot y(s) \qquad q_2(s)=c\cdot x(s)+d\cdot y(s)$$

yields: $\frac{d^2}{ds^2}q_1(s)+\omega_1^2\cdot q_1(s)=0 \qquad \frac{d^2}{ds^2}q_2(s)+\omega_2^2\cdot q_2(s)=0$

with:

$$\omega_{1,2}^2=\tfrac{1}{2}\cdot\left(\omega_x^2+\omega_y^2\right)\pm\Omega$$

$$\Omega=\sqrt{\kappa_1^2+\left(\frac{\omega_x^2-\omega_y^2}{2}\right)^2}$$

Coupled Oscillators Case Study: Case 1

very different unperturbed frequencies: $\left(\frac{\omega_x^2-\omega_y^2}{2\kappa_1}\right)^2 >> 1$

$$\omega_{1,2}^2 = \tfrac{1}{2}\cdot\left(\omega_x^2+\omega_y^2\right) \pm \tfrac{1}{2}\cdot\left(\omega_x^2-\omega_y^2\right)\cdot\sqrt{1+\left(\frac{2\kappa_1}{\left(\omega_x^2-\omega_y^2\right)}\right)^2}$$

expansion of the square root: $\sqrt{1+\varepsilon} \approx 1+\tfrac{1}{2}\varepsilon$

➔ $\omega_1 = \omega_x + \frac{\kappa_1^2}{\omega_x^2-\omega_y^2} \approx \omega_x$ $\quad$ $\omega_2 = \omega_y - \frac{\kappa_1^2}{\omega_x^2-\omega_y^2} \approx \omega_y$

➔ 'nearly' uncoupled oscillators $a \approx 1; b \approx 0; c \approx 0; d \approx 1$

Coupled Oscillators Case Study: Case 2

almost equal frequencies: $\omega_x = \omega_0 + \frac{1}{2}\Delta \qquad \omega_y = \omega_0 - \frac{1}{2}\Delta$

➔ $\omega_0 = \frac{1}{2}(\omega_x + \omega_y) \qquad \omega_x^2 + \omega_y^2 \approx 2\omega_0^2 \qquad \omega_x^2 - \omega_y^2 \approx 2\omega_0\Delta$

$$(a \approx 1; b \approx 1; c \approx 1; d \approx -1)$$

➔ keep only linear terms in Δ:

$$\omega_{1,2}^2 = \omega_0^2 \cdot \left[1 \pm \sqrt{\frac{\kappa_1^2}{\omega_0^4} + \frac{\Delta^2}{\omega_0^2}}\right]$$

$$\omega_{1,2} = \omega_0 \cdot \sqrt{1 \pm \sqrt{\frac{\kappa_1^2}{\omega_0^4} + \frac{\Delta^2}{\omega_0^2}}}$$

expansion of the square root for small coupling and Δ:

➔ $$\omega_{1,2} = \omega_0 \pm \tilde{\Omega}$$

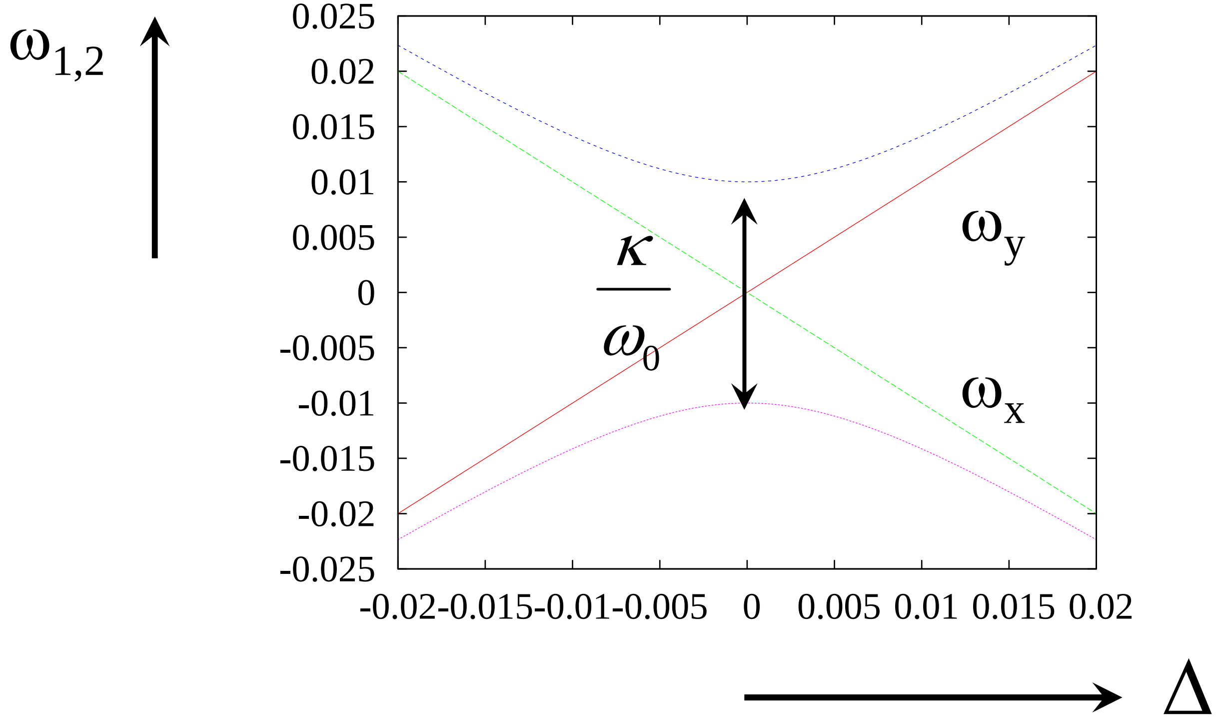
with: $$\tilde{\Omega} = \frac{1}{2} \cdot \sqrt{\frac{\kappa_1^2}{\omega_0^2} + \Delta^2}$$

Coupled Oscillators Case Study: Case 2

- measurement of coupling strength:

$$\omega_{1,2} = \omega_0 \pm \frac{1}{2} \cdot \sqrt{\frac{\kappa_1^2}{\omega_0^2} + \Delta^2}$$

measure the differnce in the Eigenmode frequencies while bringing the unperturbed tunes together:



➔ the minimum separation yields the coupling strength!!

Coupled Oscillators Case Study: Case 2

- initial oscillation only in horizontal plane:

$$x(0) = A; \quad x'(0) = 0; \quad y(0) = 0; \quad y'(0) = 0$$

➔ $q_1 = A \cdot \cos(\omega_1 \cdot s)$ and $q_2 = A \cdot \cos(\omega_2 \cdot s)$

with $\omega_{1,2} = \frac{1}{2} \cdot \left(\omega_x + \omega_y\right) \pm \tilde{\Omega}$ and $q_1(t) = x + y$, $q_2(t) = x - y$

sum rules for sin and cos functions:

➔

$$x(s) = A \cdot \cos\left(\tilde{\Omega} \cdot s\right) \cdot \cos\left(\tfrac{1}{2}\left[\omega_1 + \omega_2\right] \cdot s\right)$$

$$y(s) = -A \cdot \sin\left(\tilde{\Omega} \cdot s\right) \cdot \sin\left(\tfrac{1}{2}\left[\omega_1 + \omega_2\right] \cdot s\right)$$

➔ modulation of the amplitudes

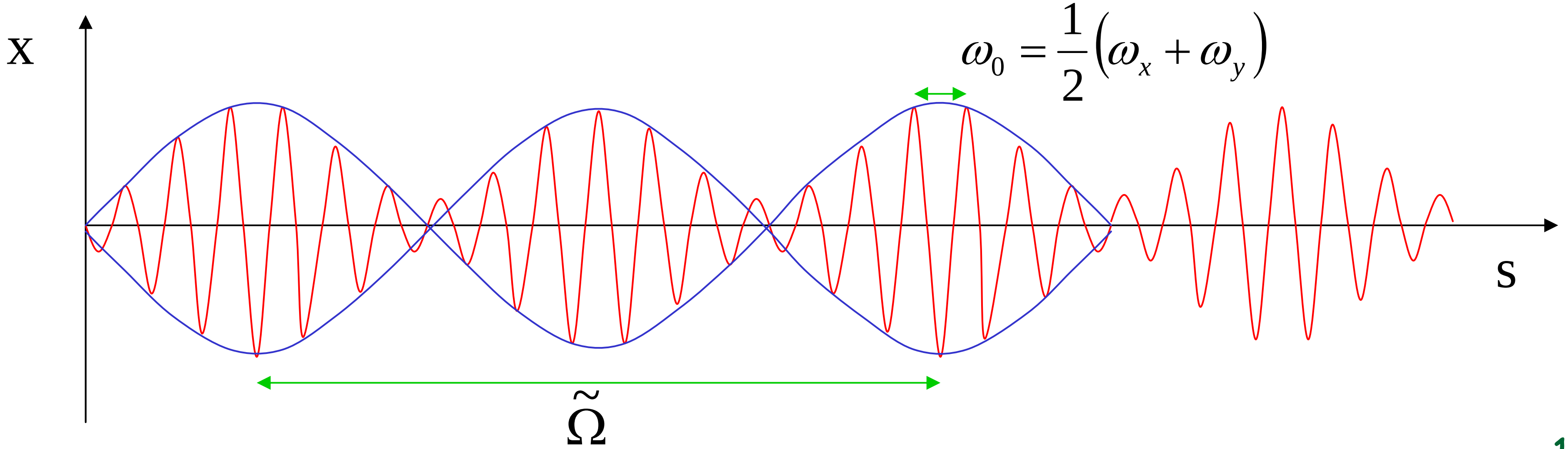
Beating of the Transverse Motion: Case I

- two almost identical harmonic oscillators with weak coupling:

  π-mode and ω=mode frequencies are approximately identical!

  → frequencies can not be distinguished and energy can be exchanged between the two oscillators

- modulation of the oscillation amplitude:



Driven Oscillators

- Perturbation treatment:

substitute the solutions of the homogeneous equation of motion:

$$w(s) = A \cdot \sin(\omega_0 \cdot s + \phi_0)$$

into the right-hand side of the perturbed Hills equation and express the 's' dependence of the multipole terms by their Fourier series (the perturbations must be periodic with one revolution!)

- equation of motion ➔ driven un-damped oscillators:

$$\frac{d^2}{ds^2} w(s) + \omega_w Q^{-1} \frac{d}{ds} w(s) + \omega_w{}^2 w(s) = \sum_{k.l.m} W_{klm} e^{(k \cdot \omega_x \cdot s + l \cdot \omega_y + \frac{2\pi}{L} \cdot m \cdot s + \phi_{klm})}$$

➔ large number of driving frequencies!

Driven Oscillators

- single resonance approximation: $\omega = k\omega_x + l\omega_y + m\frac{2\pi}{L}$

  consider only one perturbation frequency (choose $\omega \approx \omega_0$):

$$\frac{d^2}{ds^2}w(s) + \omega_0 \cdot Q^{-1} \cdot \frac{d}{ds}w(s) + \omega_0{}^2 \cdot w(s) = W(s)\cdot\cos(\omega\cdot s + \phi_0)$$

- general solution: $w(s) = w_{tr}(s) + w_{st}(s)$

- without damping the transient solution is just the HO solution

$$w_{tr}(s) = a\cdot\sin(\omega_0 \cdot s + \phi_0)$$

Driven Oscillators

stationary solution: $$w_{st}(s) = \frac{W(\omega)}{\omega_0^2} \cdot \cos[\omega \cdot s - \alpha(\omega)]$$

where 'ω' is the driving angular frequency!
and W(ω) can become large for certain frequencies!

$$W(\omega) = W_n \cdot \frac{1}{\sqrt{1 - \left(\frac{\omega_n}{\omega_0}\right)^2 + \left(\frac{\omega_n}{Q\omega_0}\right)^2}}$$

resonance condition:

$$\omega_n = \omega_0$$

➔ justification for single resonance approximation:

➔ all perturbation terms with: $\omega_n \neq \omega_0$ de-phase with the transient

➔ no net energy transfer from perturbation to oscillation (averaging)!

Resonances and Perturbation Treatment

example single dipole perturbation:

$$\frac{F(s)}{v \cdot p} = k_0 \cdot \delta_L(s - s_0)$$

Fourier series of periodic δ-function

$$\frac{d^2}{ds^2} w(s) + \omega_0{}^2 \cdot w(s) = l k_0 \cdot \frac{1}{L} \cdot \sum_{n=-\infty}^{\infty} \cos(n \cdot 2\pi \cdot s / L)$$

resonance condition:

$$\omega_0 = n \cdot 2\pi / L \xrightarrow{\omega_0 = 2\pi \cdot Q_0 / L} Q_0 = n$$

avoid integer tunes!

$$\Delta CO(s) = \frac{-\sqrt{\beta(s)}}{2\sin(\pi Q)} \cdot \oint \Delta k_0(t) \cdot \sqrt{\beta(t)} \cdot \cos(|\phi(t) - \phi(s)| - \pi Q) dt$$

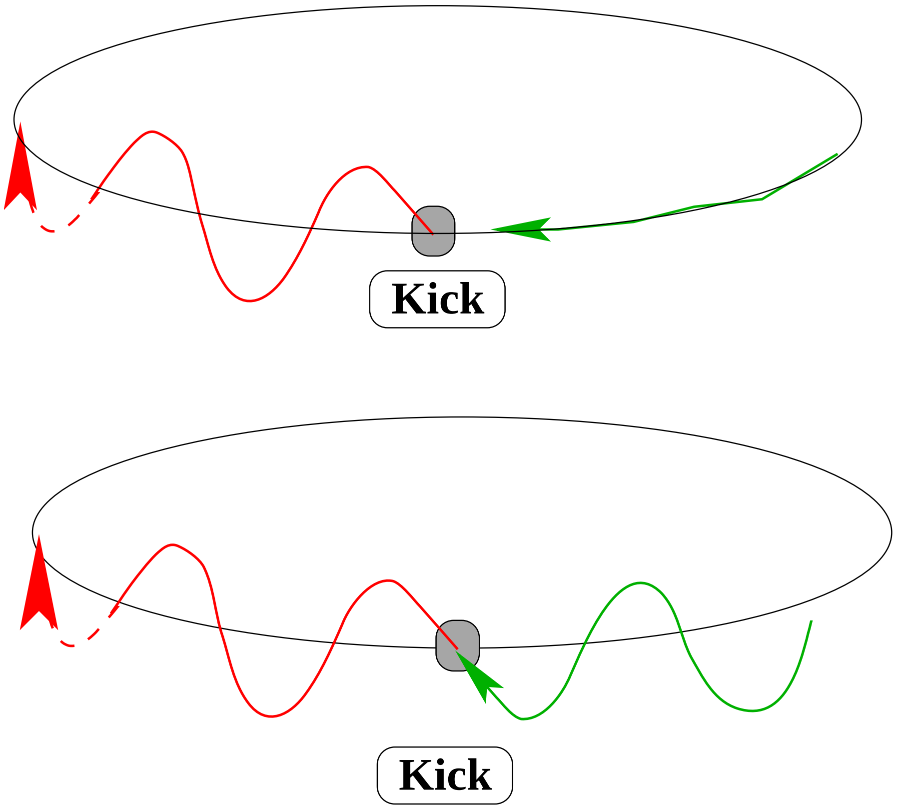
(see general CAS school for more details)

Resonances and Perturbation Treatment

integer resonance for dipole perturbations:

assume:

Q = integer



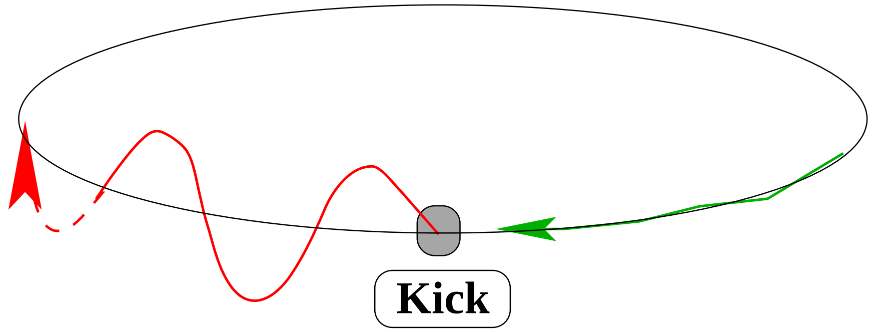
➔ dipole perturbations add up on consecutive turns! ➔ Instability

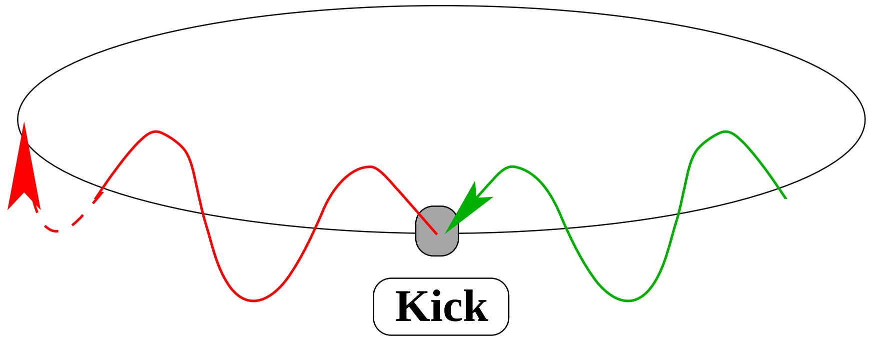
Resonances and Perturbation Treatment

integer resonance for dipole perturbations:

assume:

Q = integer/2





➔dipole perturbations compensate on consecutive turns!

➔ stability

Resonances and Perturbation Treatment

example single quadrupole perturbation:

with: $$\frac{F(s)}{v\cdot p}=k_1\cdot x \qquad w_0(s)=A\cdot\cos(\omega_{0,x}\cdot s+\phi_0)$$

$$\longrightarrow \frac{d^2}{ds^2}w(s)+\omega_{x,0}{}^2\cdot w(s)=A\cdot\frac{lk_1}{2L}\sum_{n=-\infty}^{\infty}\cos([2\pi\cdot n/L\pm\omega_{0,x}]\cdot s\pm\phi_0)$$

→ resonance condition: $2\cdot\omega_0=n\cdot 2\pi/L \xrightarrow{\omega_0=2\pi\cdot Q_0/L} Q_0=\frac{n}{2}$

→ avoid half integer tunes!

$$\frac{\Delta\beta(s)}{\beta_0(s)}=\frac{-1}{2\sin(2\pi Q)}\cdot\oint\Delta k_1(t)\cdot\beta(t)\cdot\cos(2\,|\phi(t)-\phi(s)|-2\pi Q)dt$$

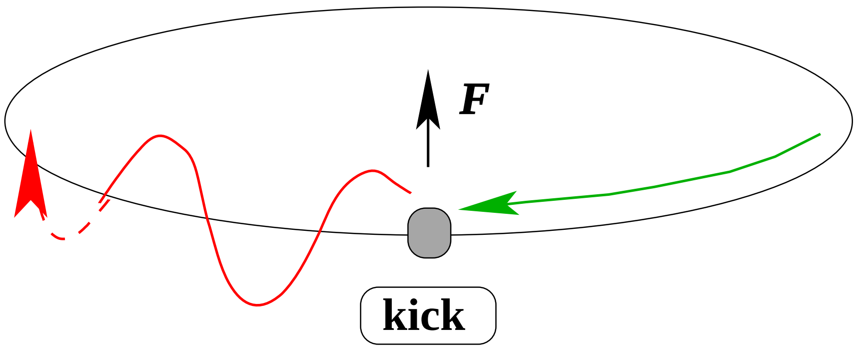
→ (see general CAS school for more details)

Resonances and Perturbation Treatment

- half integer resonance for quadrupole perturbations:

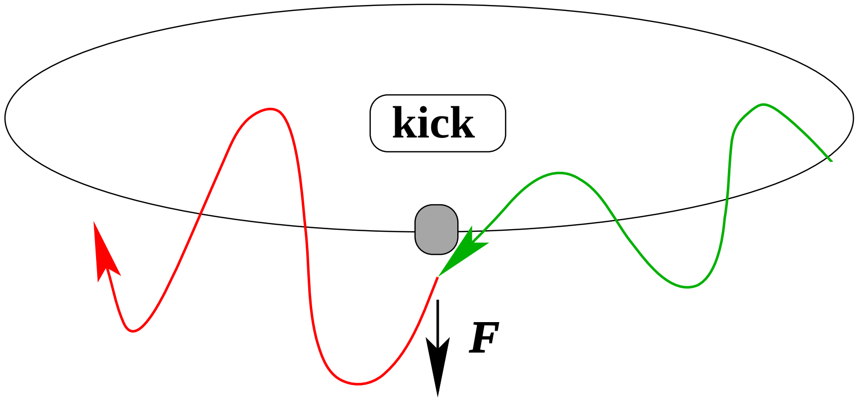
assume:

Q = integer + 0.5



feed down error:

$$B_x = b_1 \cdot y \Rightarrow F_y = +q \cdot v \cdot b_1 \cdot y$$



➔quadrupole perturbations add up on consecutive turns!

➔ Instability

Resonances and Perturbation Treatment

example single skew quadrupole perturbation:

with: $$\frac{F_x(s)}{v \cdot p} = \kappa_1 \cdot y \qquad y_0(s) = A \cdot \cos(\omega_{0,y} \cdot s + \phi_0)$$

$$\longrightarrow \quad \frac{d^2}{ds^2} x(s) + \omega_{x,0}{}^2 \cdot x(s) = A \cdot \frac{l\kappa_1}{2L} \sum_{n=-\infty}^{\infty} \cos([2\pi \cdot n / L \pm \omega_{0,y}] \cdot s \pm \phi_0)$$

resonance condition:

$$\longrightarrow \quad \omega_{x,0} \pm \omega_{y,0} = n \cdot 2\pi / L \xrightarrow{\omega = 2\pi \cdot Q / L} Q_x \pm Q_y = n$$

→ avoid sum and difference resonances!

→ difference resonance ➔ stable with energy exchange
sum resonance ➔ instability as for externally driven dipole

Resonances and Perturbation Treatment: Case 1

coupling with: $Q_x >> Q_y \quad or \quad Q_x << Q_y$

➔ drive and response oscillation de-phase quickly
no energy transfer between motion in ‘x’ and ‘y’ plane

➔ small amplitude of ‘stationary’ solution: $W(\omega) = W_0 \cdot \frac{1}{\sqrt{[1-(\frac{\omega}{\omega_0})^2]^2 + (\frac{\omega}{Q\omega_0})^2}}$

➔ no damping of oscillation in ‘x’ plane due to coupling

➔ coupling is weak ➔ tune measurement in one plane will
show both tunes in both planes but
with unequal amplitudes

➔ tune measurement is possible for both planes

Resonances and Perturbation Treatment: Case 2

coupling with: $Q_x \approx Q_y$

➔ drive and response oscillation remain in phase and energy can be exchanged between motion in 'x' and 'y' plane:

➔ large amplitude of 'stationary' solution: $W(\omega) = W_0 \cdot \frac{1}{\sqrt{[1-(\frac{\omega}{\omega_0})^2]^2 + (\frac{\omega}{Q\omega_0})^2}}$

➔ damping of oscillation in 'x' plane and growth of oscillation amplitude in 'y' plane

➔ 'x' and 'y' motion exchange role of driving force!

➔ each plane oscillates on average with: $\frac{1}{2}\left(Q_x + Q_y\right)$

➔ Impossible to separate tune in 'x' and 'y' plane!

Exact Solution for Transport in Skew Quadrupole

- coupled equation of motion: $x'' + \kappa_1 \cdot y = 0$ and $y'' + \kappa_1 \cdot x = 0$

- can be solved by linear combinations of 'x' and 'y':

$$(x+y)'' + \kappa_1 \cdot (x+y) = 0 \quad \text{and} \quad (x-y)'' - \kappa_1 \cdot (x-y) = 0$$

➔ solution as for focusing and defocusing quadrupole

- transport matrix for 'x-y' and 'x+y' coordinates for $\kappa_1 > 0$:

$$\begin{pmatrix} x-y \\ x'-y' \end{pmatrix}_{end} = \begin{pmatrix} \cos(l\sqrt{\kappa_1}) & \frac{\sin(l\sqrt{\kappa_1})}{\sqrt{\kappa_1}} \\ \sqrt{\kappa_1}\cdot\sin(l\sqrt{\kappa_1}) & \cos(l\sqrt{\kappa_1}) \end{pmatrix} \cdot \begin{pmatrix} x-y \\ x'-y' \end{pmatrix}_{ini}$$

$$\begin{pmatrix} x+y \\ x'+y' \end{pmatrix}_{end} = \begin{pmatrix} \cosh(l\sqrt{\kappa_1}) & \frac{\sinh(l\sqrt{\kappa_1})}{\sqrt{\kappa_1}} \\ \sqrt{\kappa_1}\cdot\sinh(l\sqrt{\kappa_1}) & \cosh(l\sqrt{\kappa_1}) \end{pmatrix} \cdot \begin{pmatrix} x+y \\ x'+y' \end{pmatrix}_{ini}$$

Transport Map with Coupling

- transport map for skew quadrupole: $\vec{z}_{end} = \underline{M}_{sq} \cdot \vec{z}_{ini}$

with: $\vec{z} = \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}$ and $\underline{M}_{sq} = \begin{pmatrix} a & b & c & d \\ -\kappa_1 d & a & -\kappa_1 b & c \\ c & d & a & b \\ -\kappa_1 b & c & -\kappa_1 d & a \end{pmatrix}$

- transport map for linear elements without coupling:

$\vec{z}_{end} = \underline{M}_{l} \cdot \vec{z}_{ini}$ with $\underline{M}_{l} = \begin{pmatrix} m_{11} & m_{12} & 0 & 0 \\ m_{21} & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} \\ 0 & 0 & m_{43} & m_{44} \end{pmatrix}$

One-Turn Map with Coupling

- one-turn map around the whole ring:

$$\vec{z}(s_0 + L) = \underline{T}(s_0) \cdot \vec{z}(s_0)$$

with: $\underline{T} = \prod_i \underline{M}_i$ and starting at s_0 ➔ $\underline{T}$ is a 4x4 symplectic matrix

$$\underline{T} = \begin{pmatrix} \underline{M} & \underline{n} \\ \underline{m} & \underline{N} \end{pmatrix}$$

with: $\underline{M}, \underline{N}, \underline{m}, \underline{n}$ being 2x2 matrices

One-Turn Map with Coupling

rotated coordinate system:

➔ using a linear combination of the horizontal and vertical position vectors the matrix can be put in 'symplectic rotation' form

$$\underline{T} = \begin{pmatrix} \underline{I}\cos(\phi) & \underline{D}^{-1}\sin(\phi) \\ -\underline{D}\sin(\phi) & \underline{I}\cos(\phi) \end{pmatrix} \cdot \begin{pmatrix} \underline{A}_1 & \underline{0} \\ \underline{0} & \underline{A}_2 \end{pmatrix} \cdot \begin{pmatrix} \underline{I}\cos(\phi) & \underline{-D}^{-1}\sin(\phi) \\ \underline{D}\sin(\phi) & \underline{I}\cos(\phi) \end{pmatrix}$$

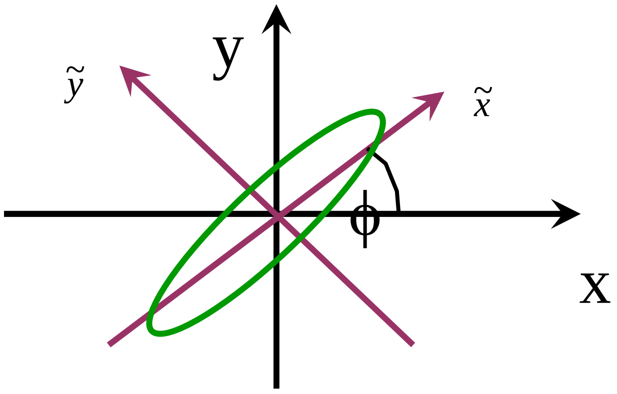
or: $\underline{T} = \underline{R} \cdot \underline{U} \cdot \underline{R}^{-1}$ with: $\underline{I}, \underline{D}, \underline{A}_1, \underline{A}_2, \underline{0}$ being 2x2 matrices

$$\underline{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \underline{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

One-Turn Map with Coupling

rotated coordinate system:

$$\tan(2\phi) = \frac{-\sqrt{\det\left(m + n^{+}\right)}}{\frac{1}{2} Tr(M - N)}$$



with: $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \xrightarrow{+} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

rotated coordinate system:

➔ new Twiss functions and phase advances for the rotated coordinates

$$\underline{A}_i = \underline{I} \cdot \cos(\mu_i) + \underline{J}_i \cdot \sin(\mu_i) \qquad \underline{J}_i = \begin{pmatrix} \alpha_i & \beta_i \\ -\gamma_i & -\alpha_i \end{pmatrix}$$

$$\cos(\mu_1) - \cos(\mu_2) = \left[\tfrac{1}{2} Tr\left(M - N\right)\right]^2 + \det(m + n^{+})$$

Summary One-Turn Map with Coupling

- coupling changes the Twiss functions and tune values in the horizontal and vertical planes

➔ a global coupling correction is required for a restoration of the uncoupled tune values (can not be done by QF and QD adjustments)

- coupling changes the orientation of the beam ellipse along the ring

➔ a local coupling correction is required for a restoration of the uncoupled oscillation planes

(mixing of horizontal and vertical kicker elements and correction dipoles)