Field Solvers

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TECHNISCHE UNIVERSITÄT DARMSTADT

CERN Accelerator School 2018

Thessaloniki, Greece, 11-23 November 2018

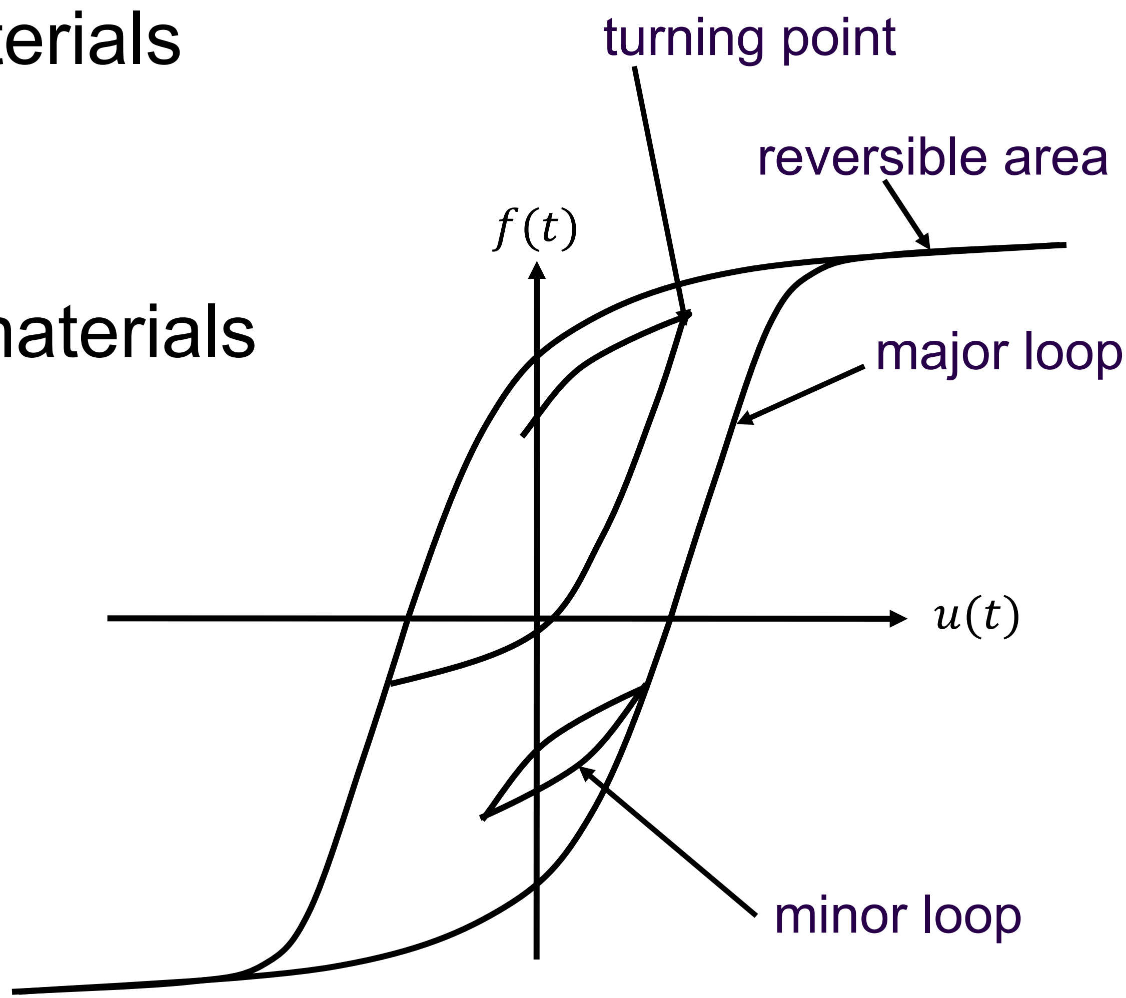
Lecture 4 : Modelling of Hysteresis

Overview

- Hysteresis
- Preisach model
  - elementary hysteresis operator
  - combining hysteresis operators
  - Preisach model
- Properties
  - complete saturation
  - memory formation
  - nonlocal memory
  - wiping-out property
  - congruency property
- Representation
  - first-order transition curves
  - representation theorem
- Numerical Implementation
- Example

Hysteresis

- Ferromagnetic materials
  - remanence
  - hysteresis losses
- Superconductive materials
  - flux pinning
- Friction



Hysteresis and Field Simulation

- as a post-processing step

  e.g. by Steinmetz formula

$$p_{\text{hyst}} = \sigma_{\text{hyst}} k_{\text{hyst}} \frac{f}{50\ \text{Hz}} \left(\frac{|\vec{B}|}{1\ \text{T}} \right)^2$$

- within a time-harmonic field simulation

  complex permeability

$$\left. \begin{aligned} B(t) &= \hat{B} \cos(\omega t) \\ H(t) &= \hat{H} \cos(\omega t - \varphi) \end{aligned} \right\} \Rightarrow \quad \underline{\mu} = \frac{\hat{B}}{\hat{H}} e^{j\varphi}$$

- within a transient field simulation

  by a hysteresis model

Hysteresis Models

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- Preisach model
- Jiles-Atherton model
- Stone-Wolfarth model
- ...

Preisach model

I.D. Mayergoyz, „Mathematical Models of Hysteresis“, Springer-Verlag, New York, 1991, pp. 1-44.

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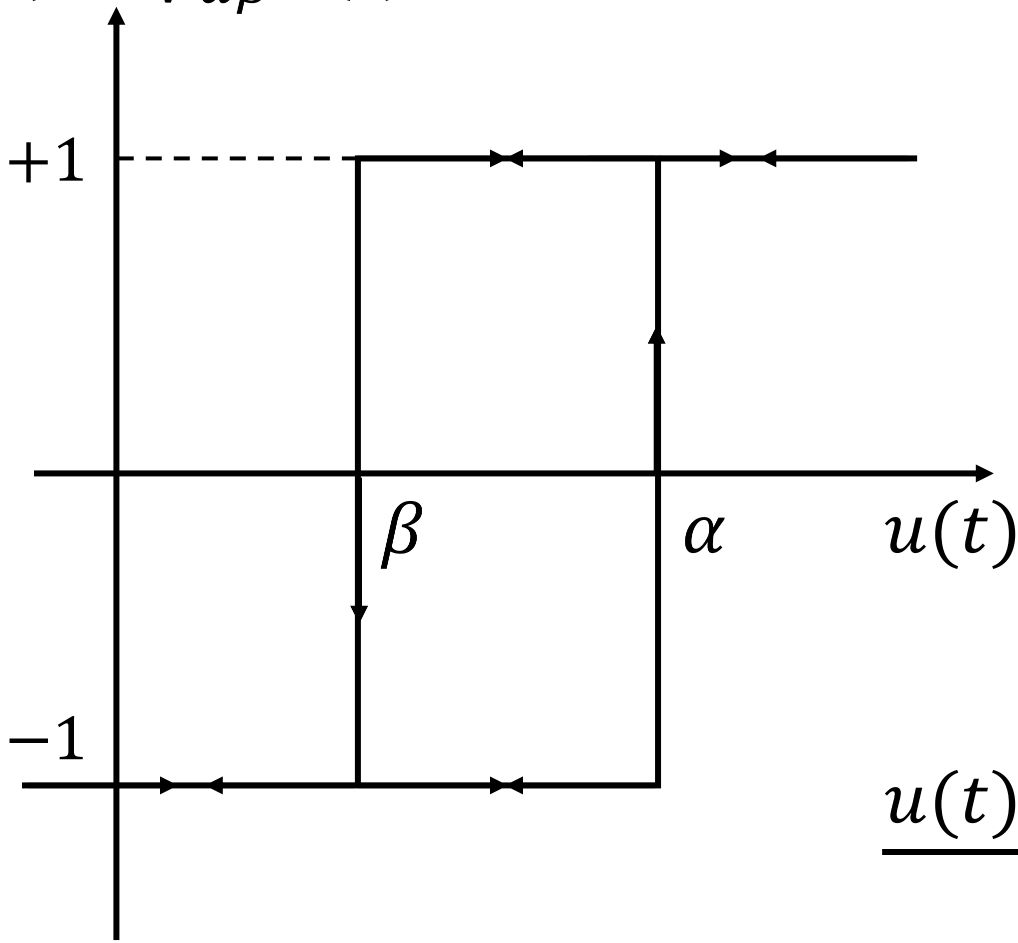
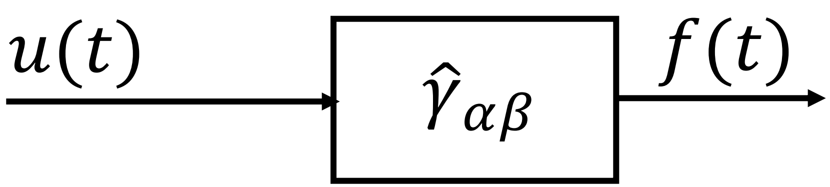
Elementary Hysteresis Operator

$$f(t) = \hat{\gamma}_{\alpha\beta} u(t)$$





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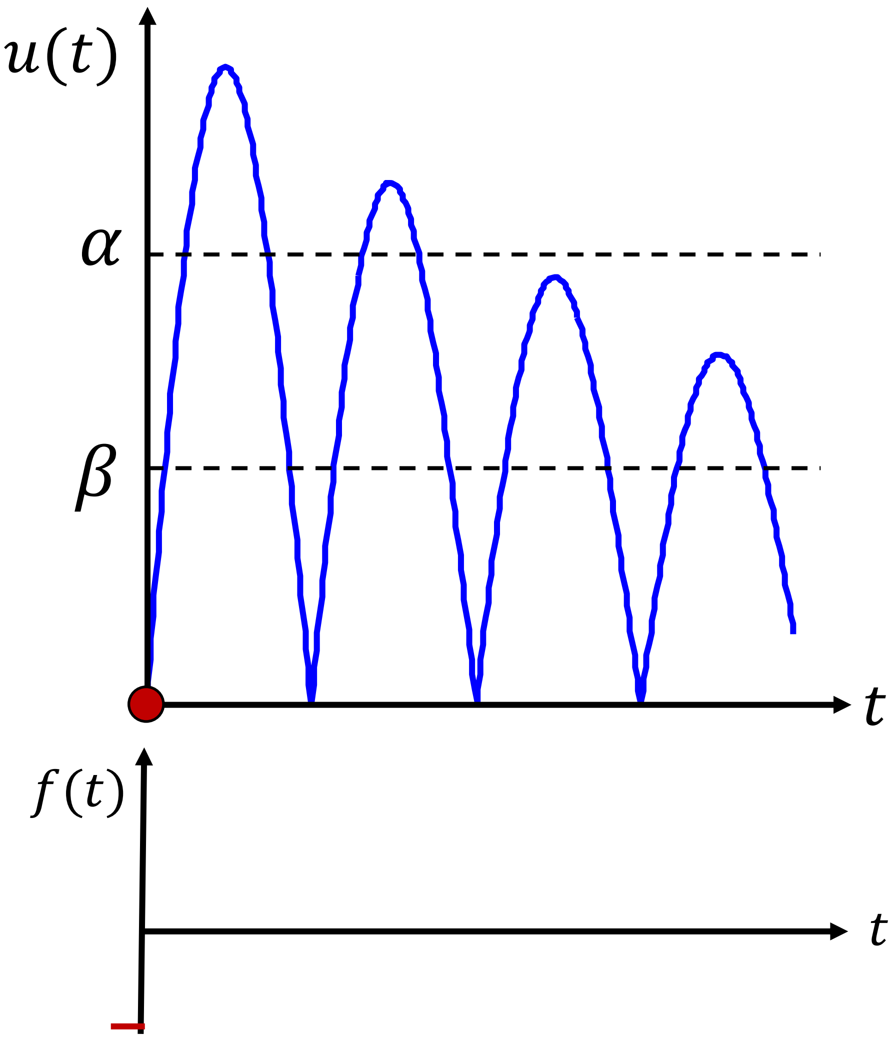

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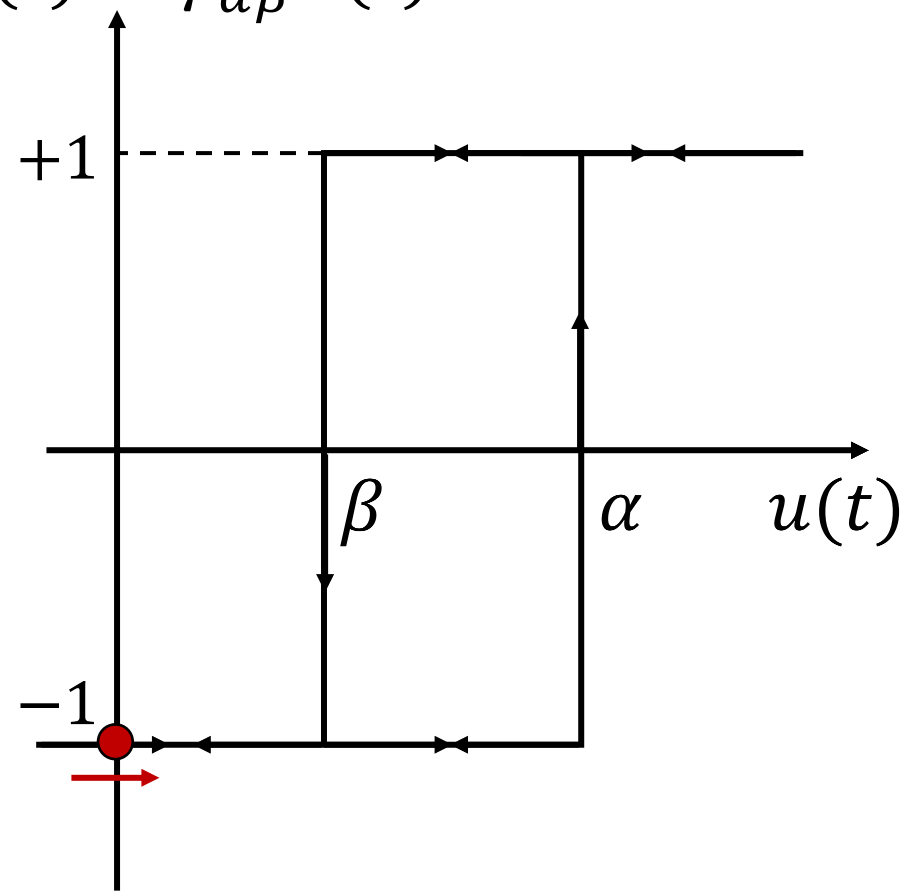

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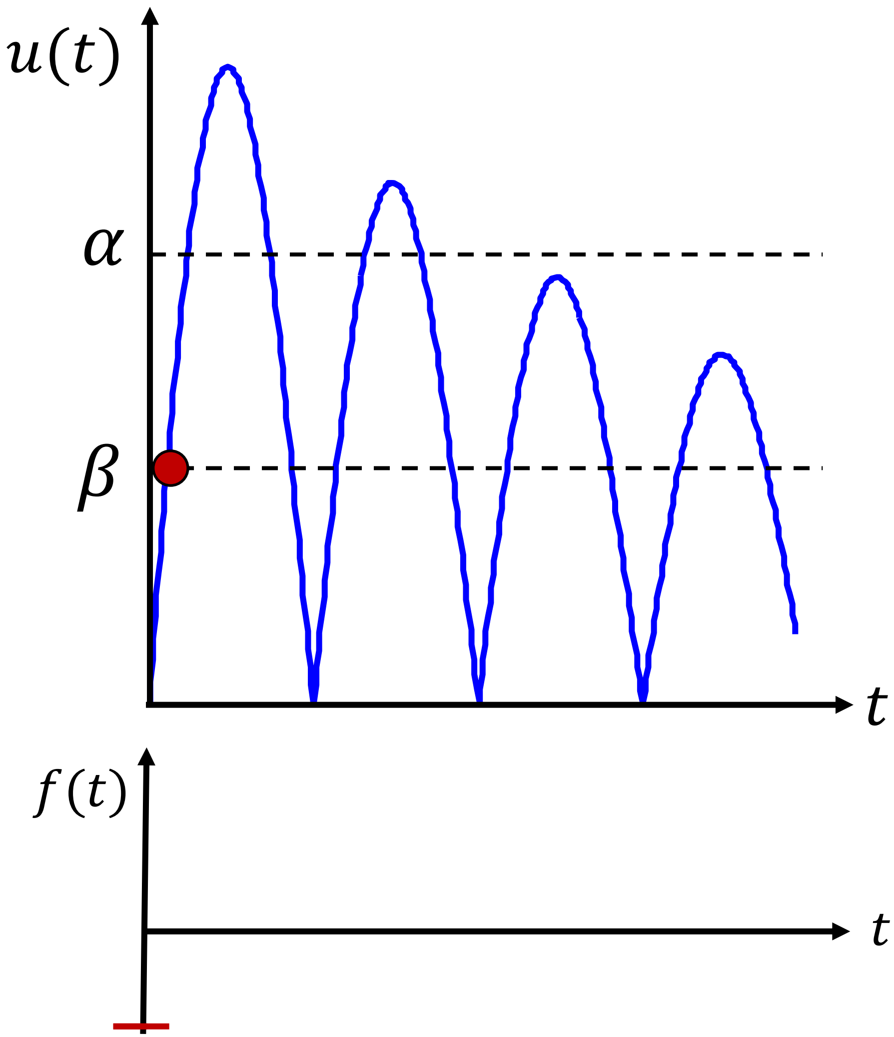

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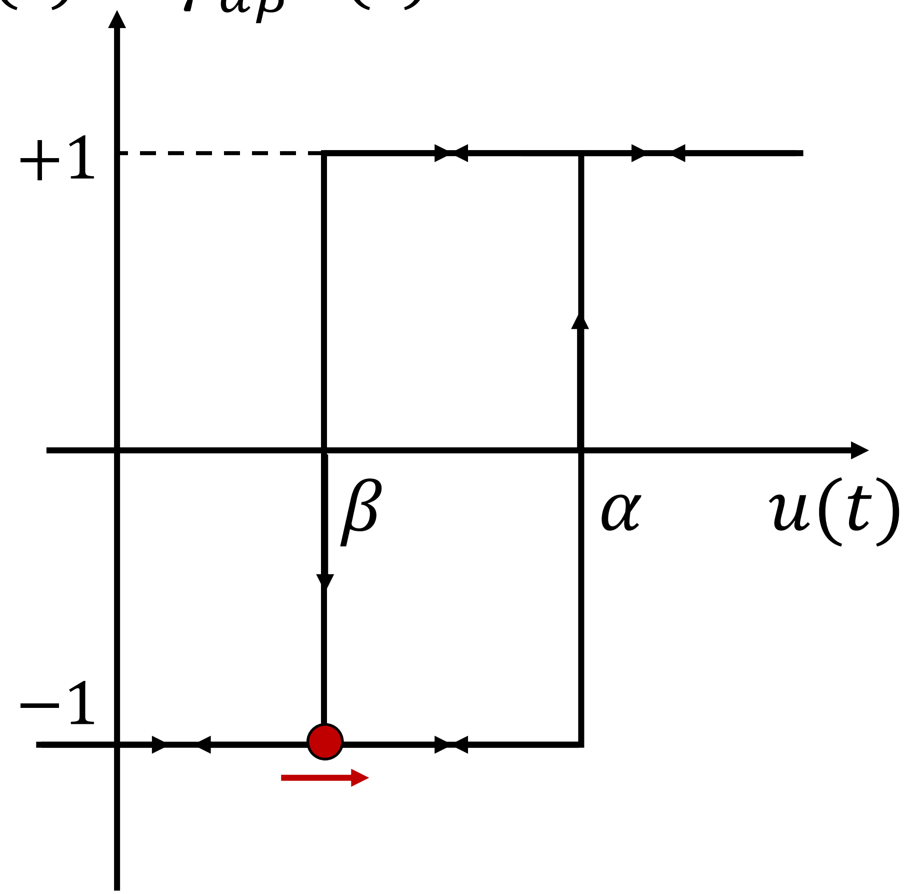

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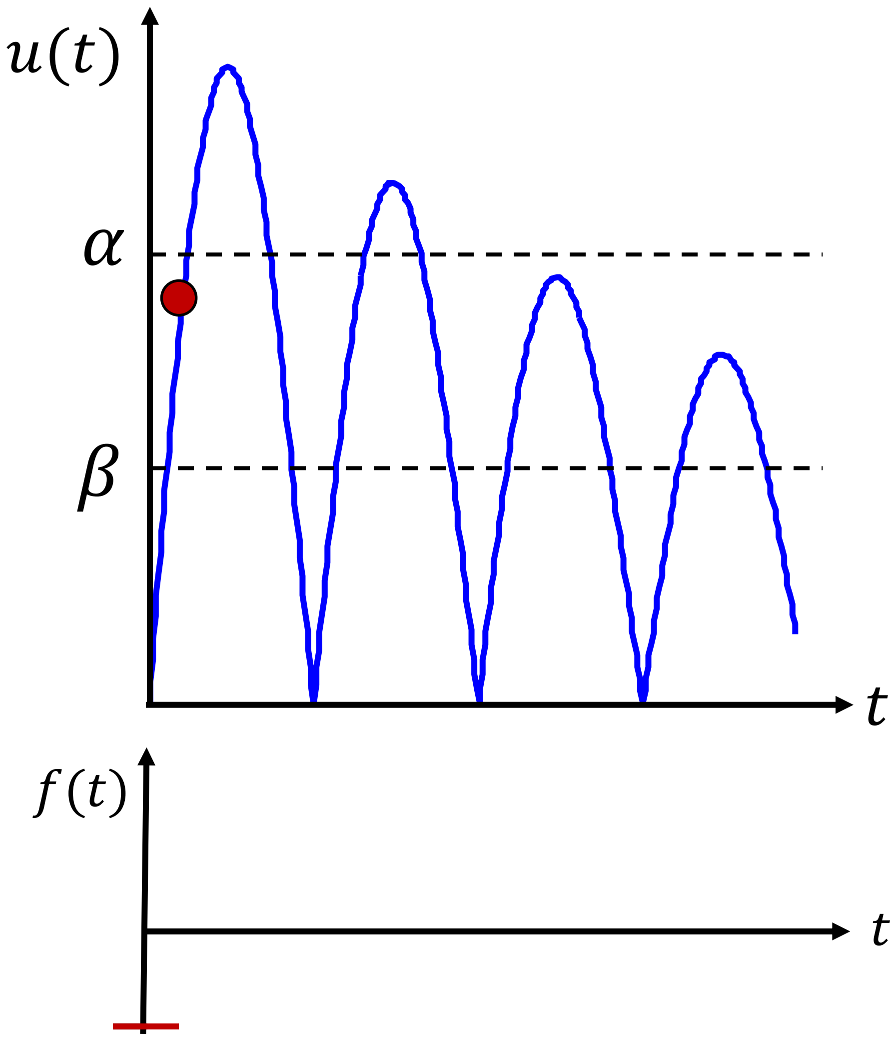

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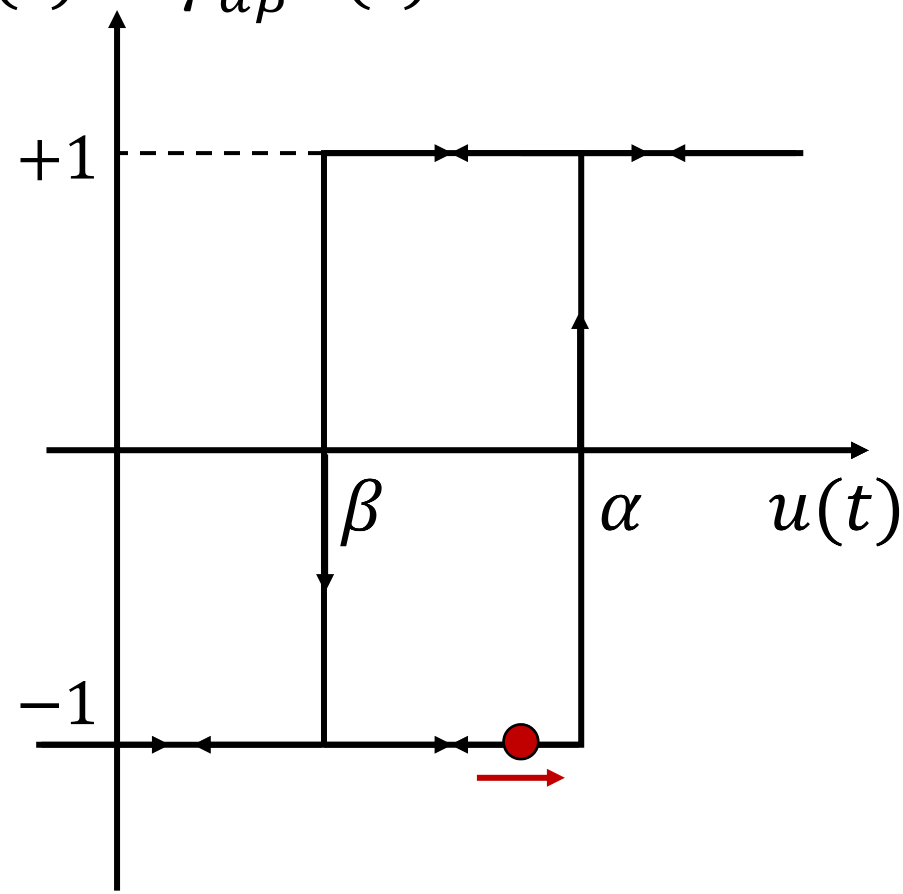

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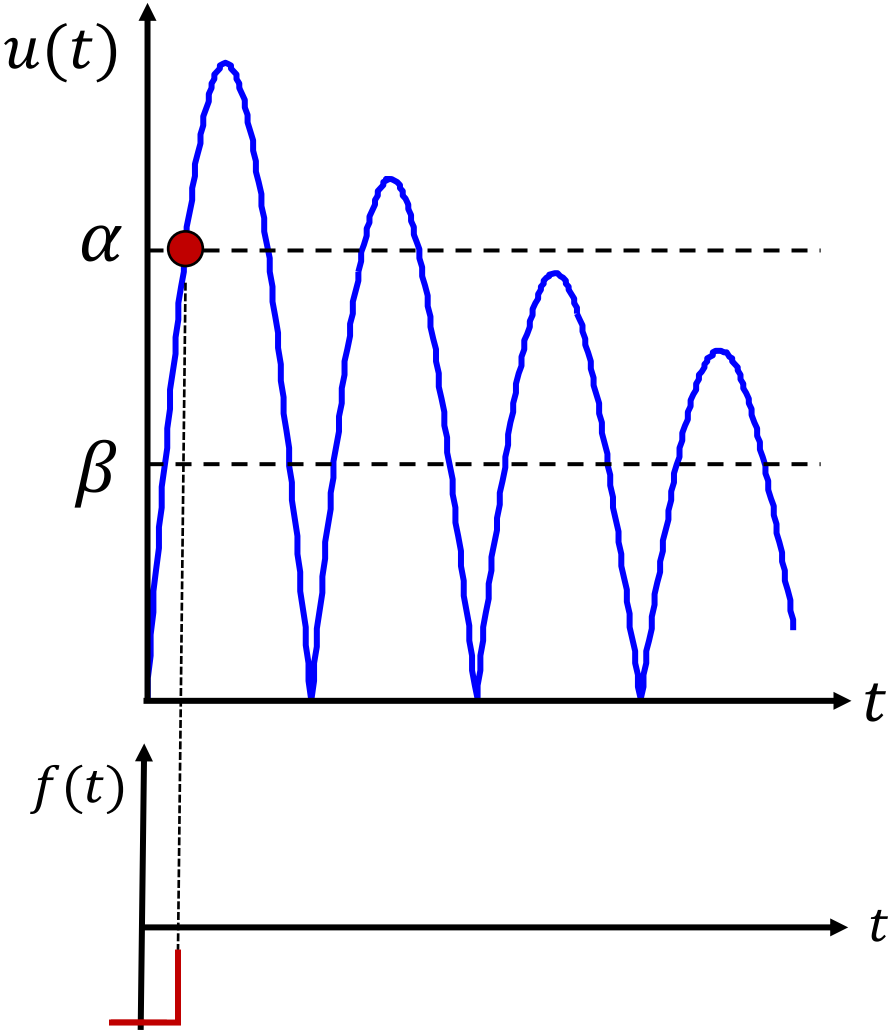

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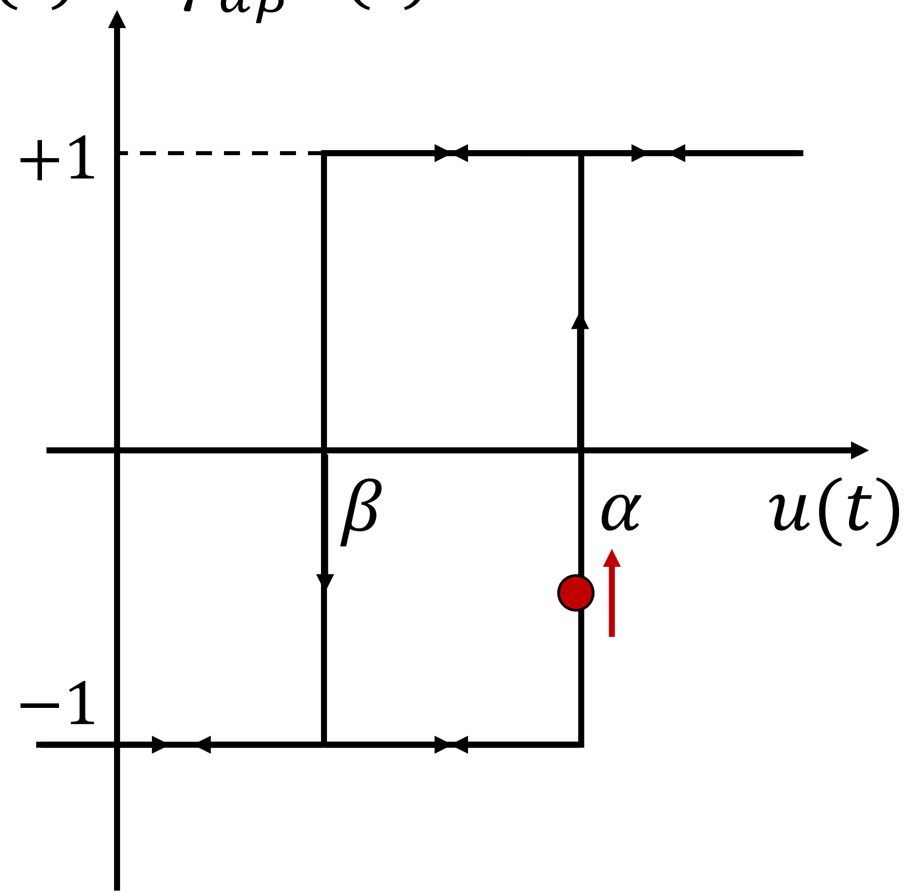

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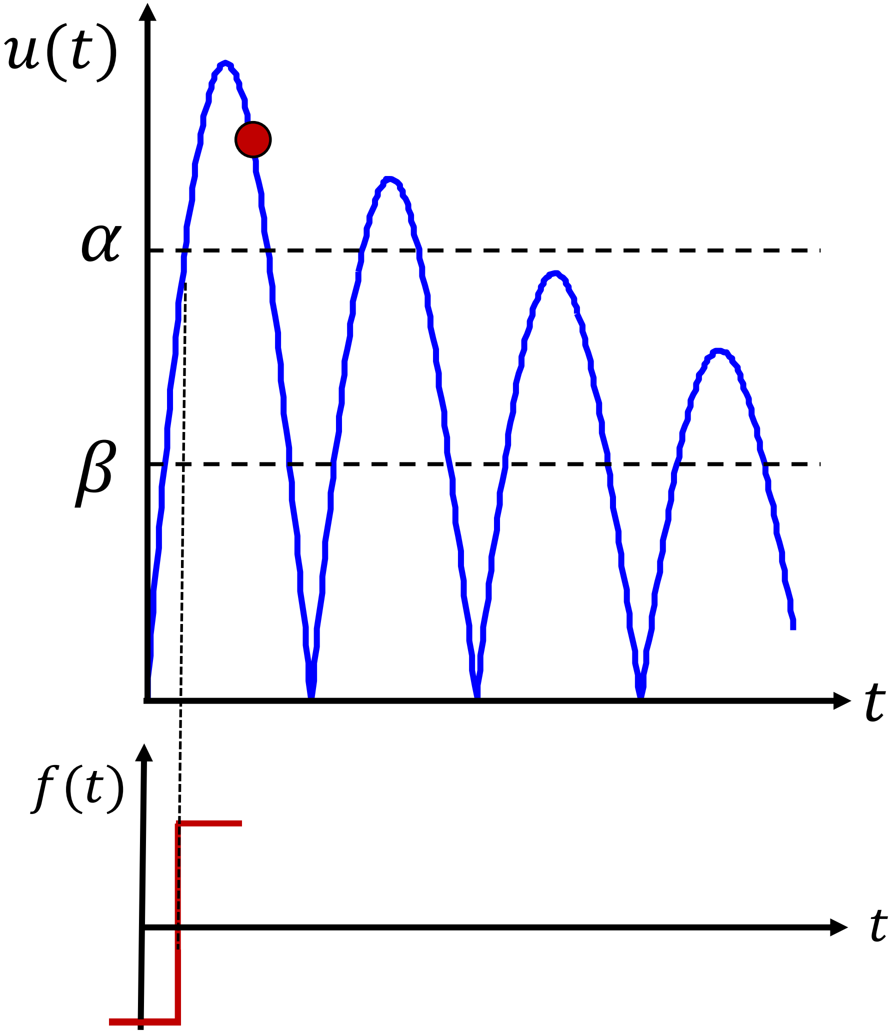

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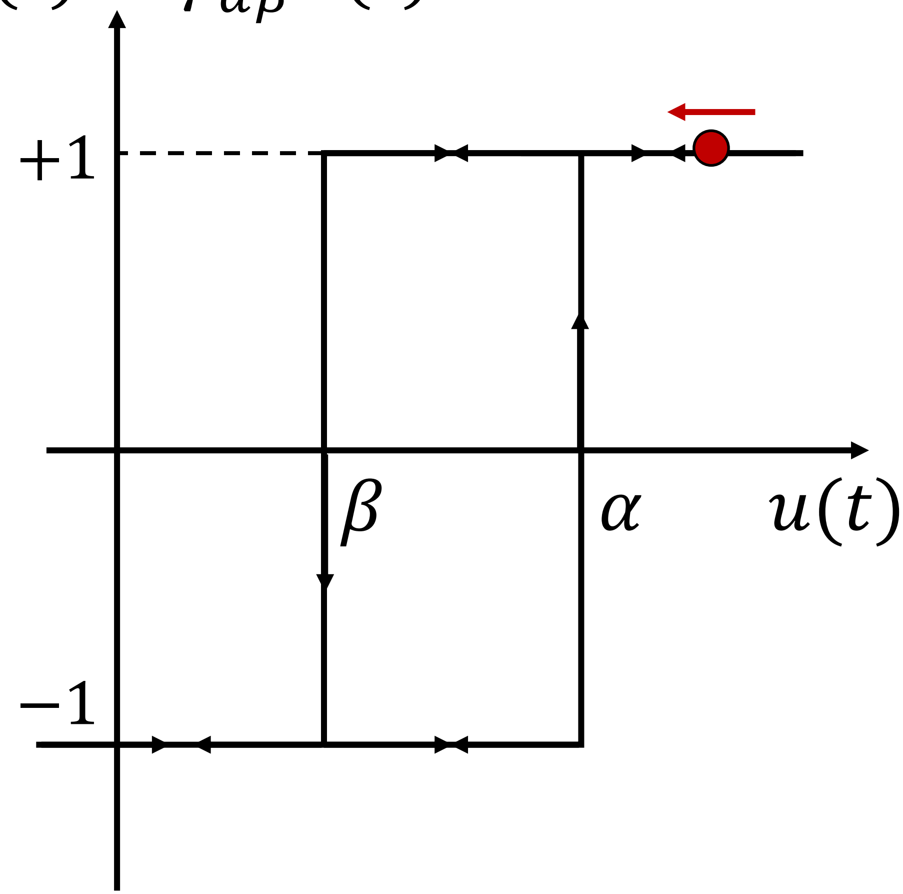

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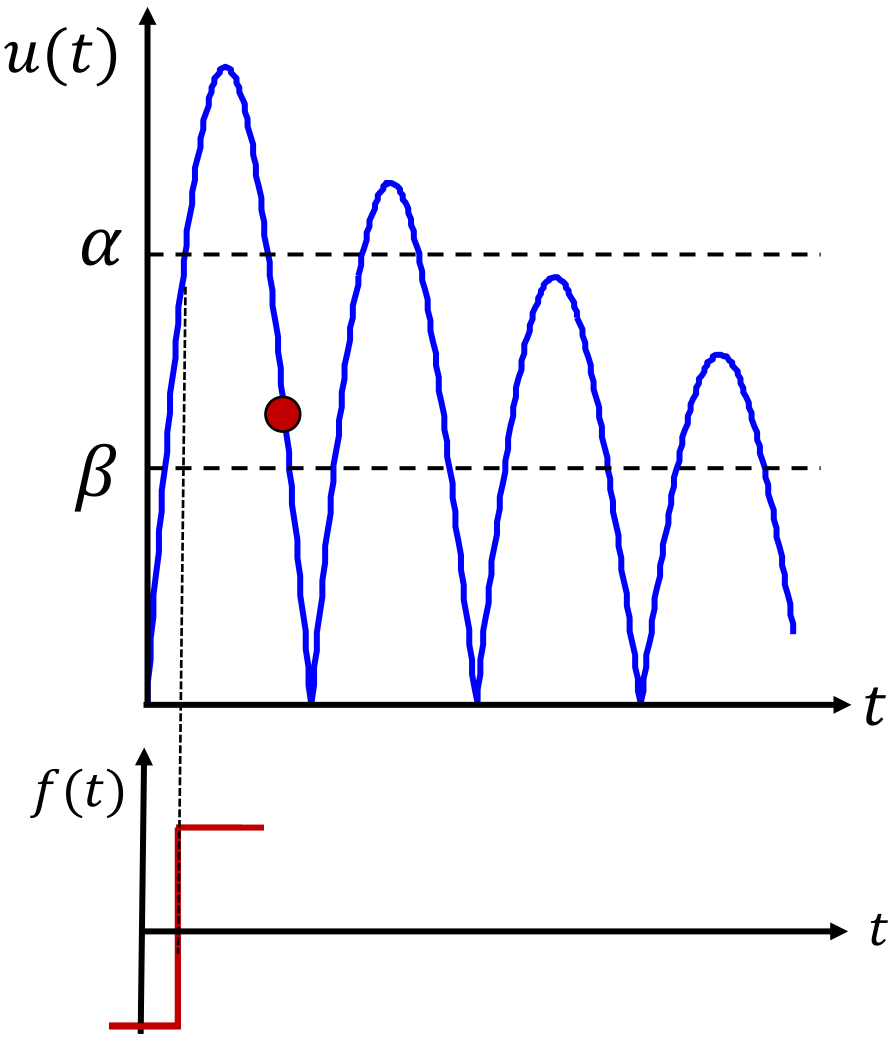

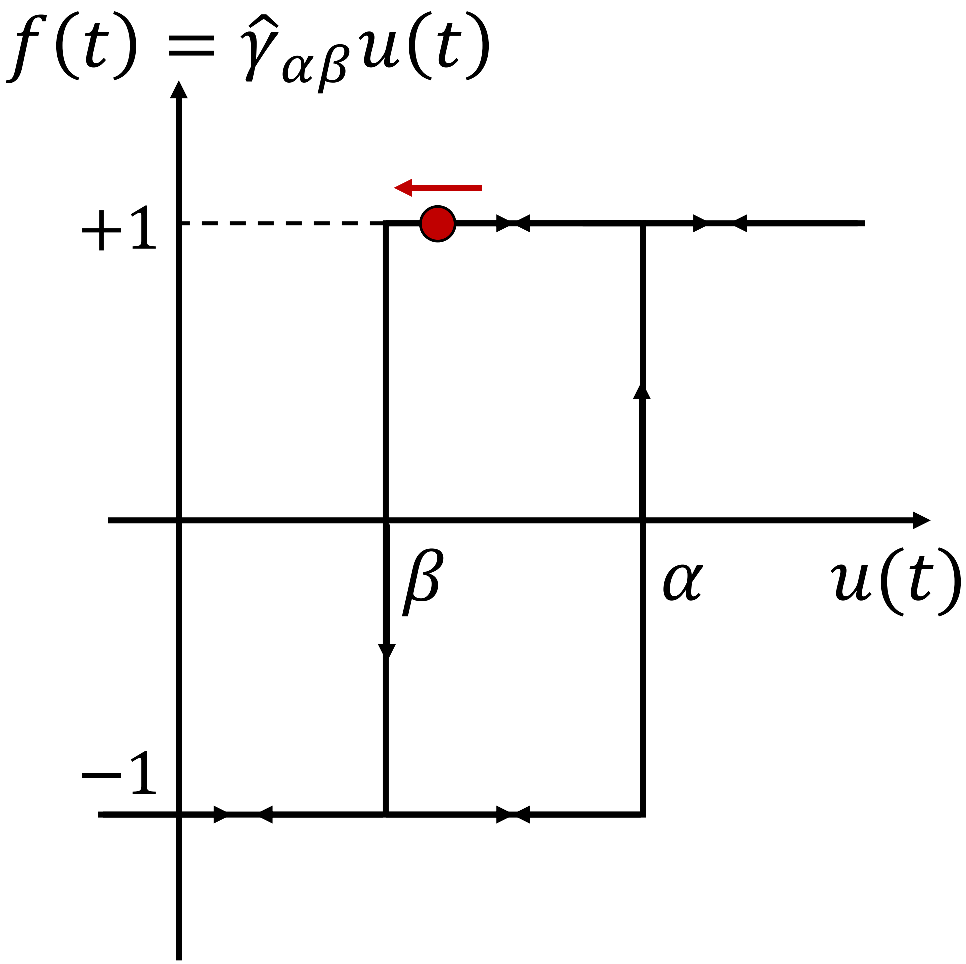

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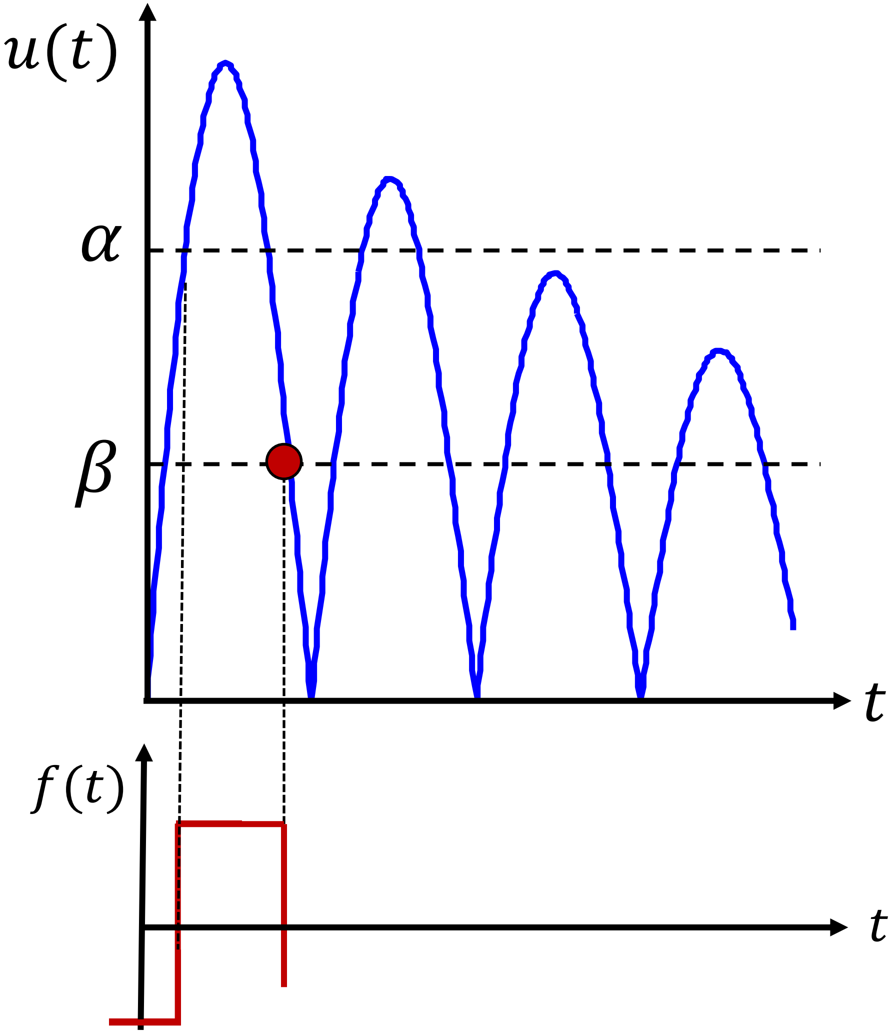

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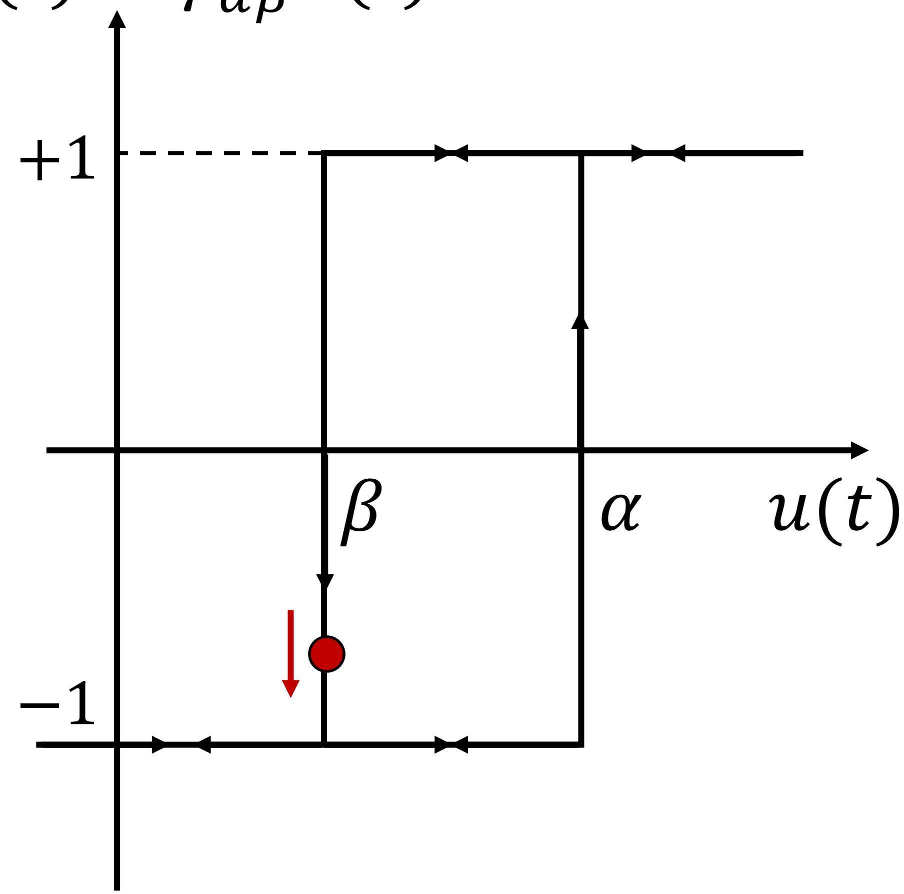

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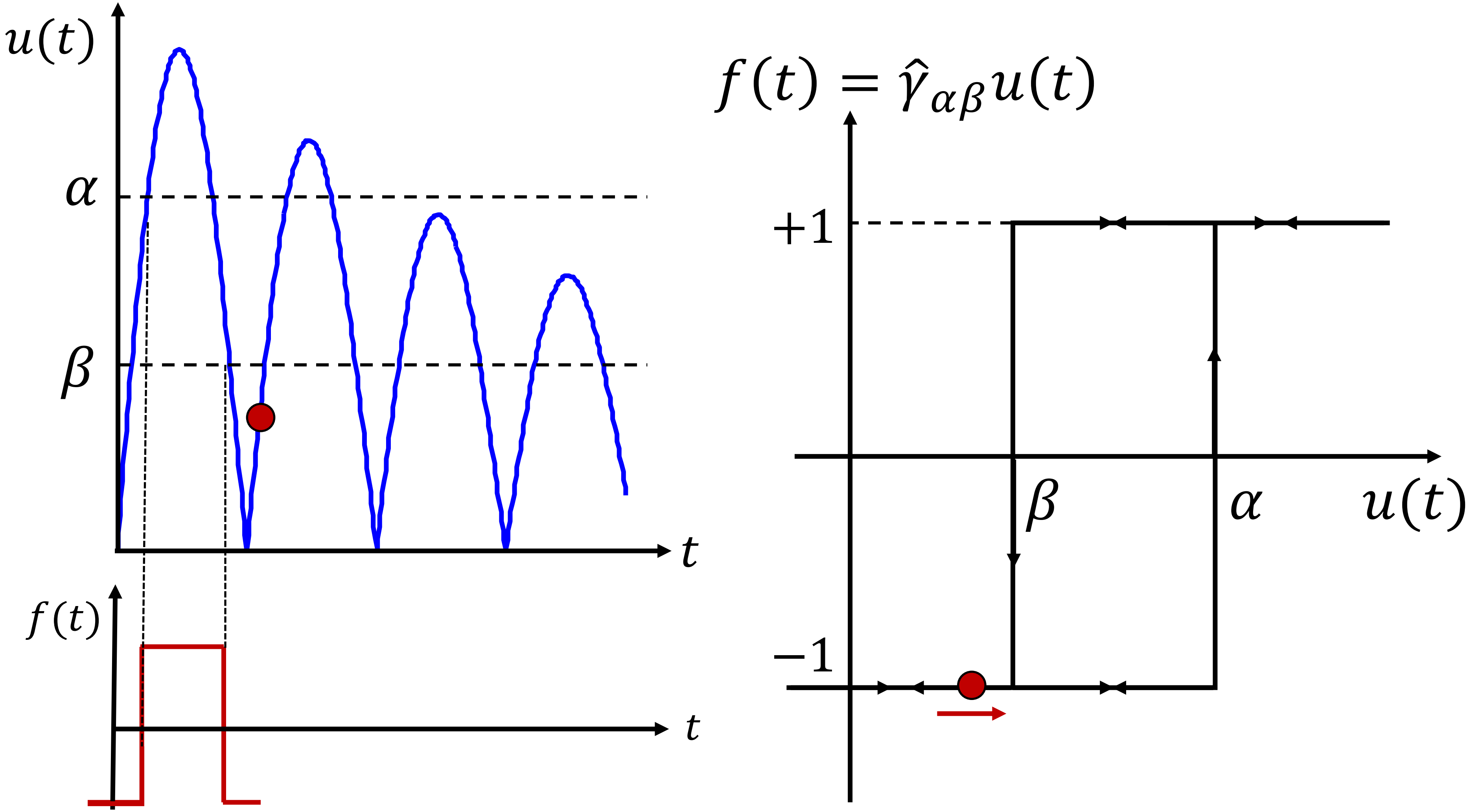

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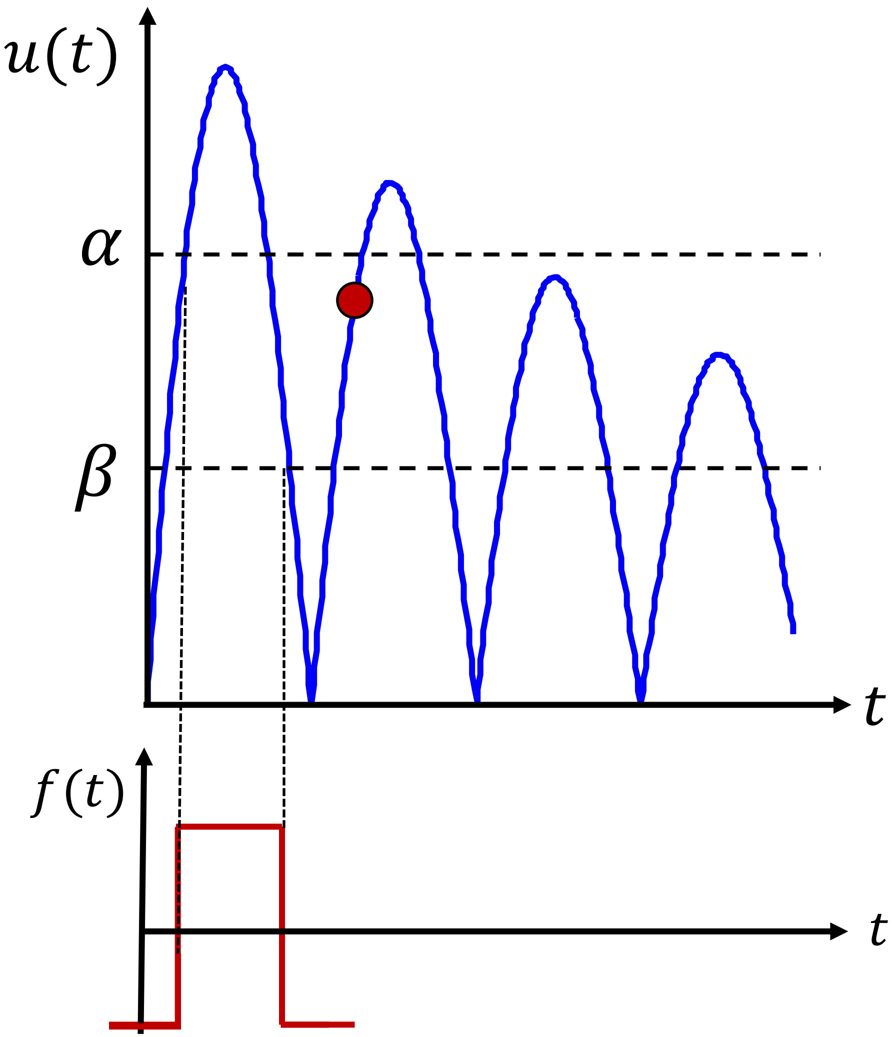

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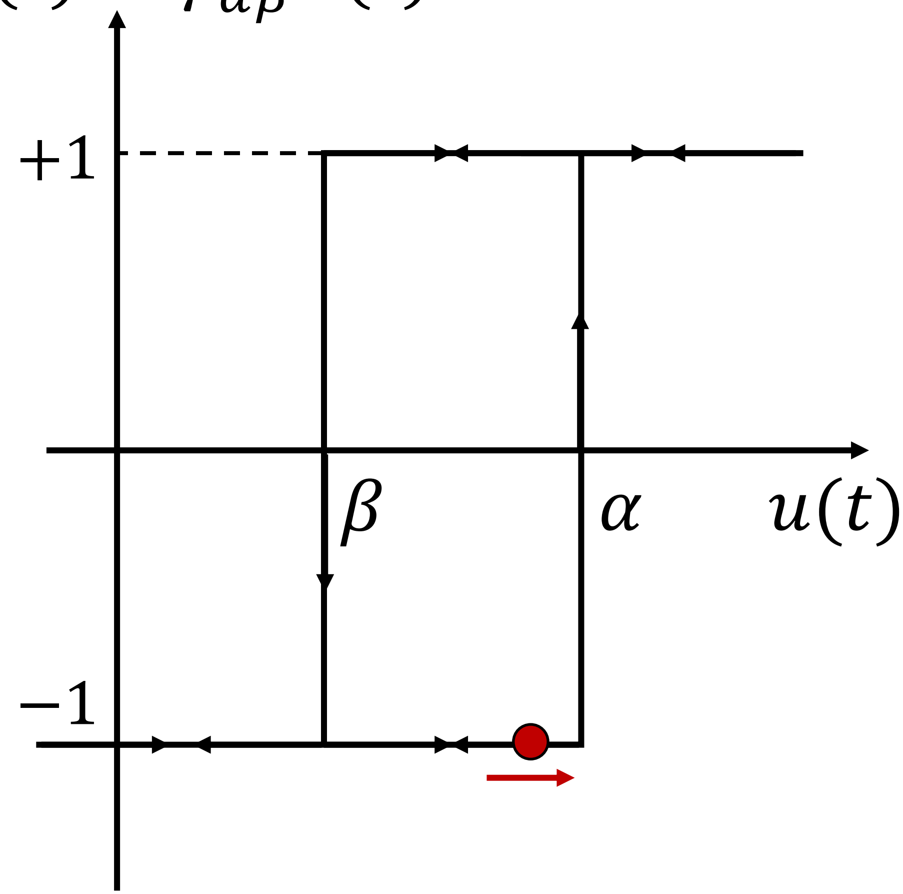

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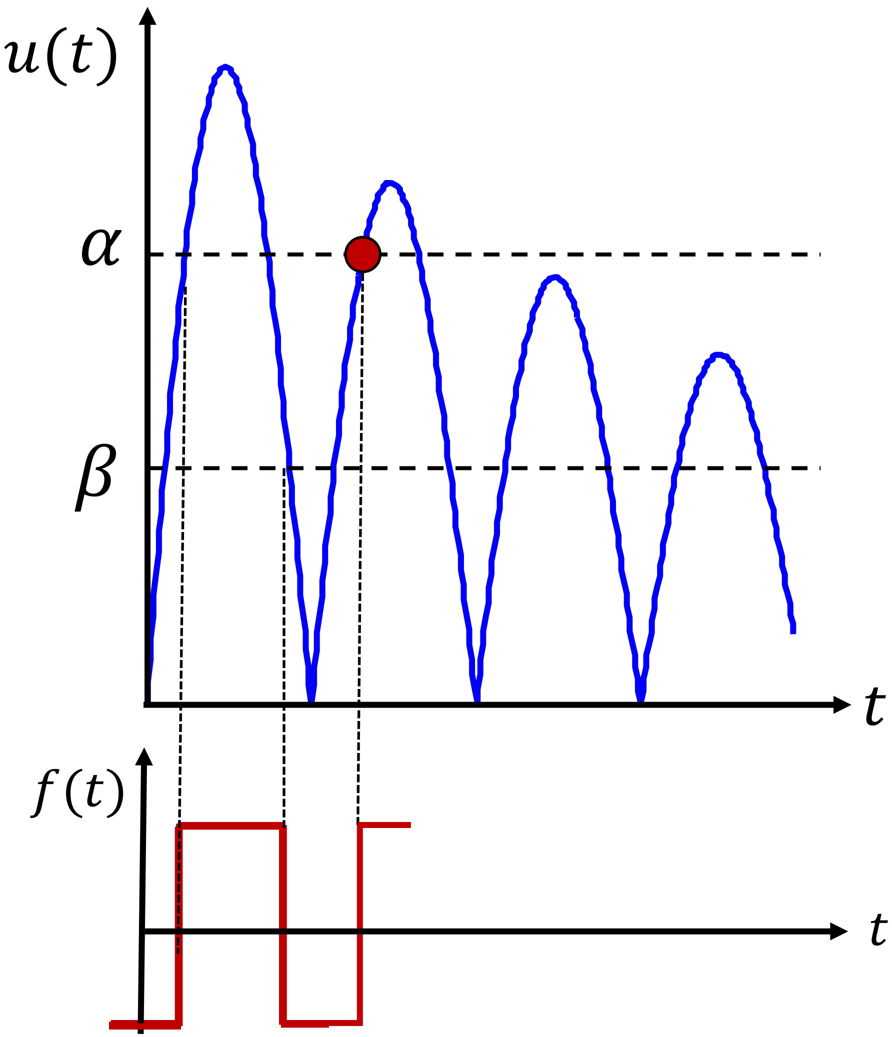

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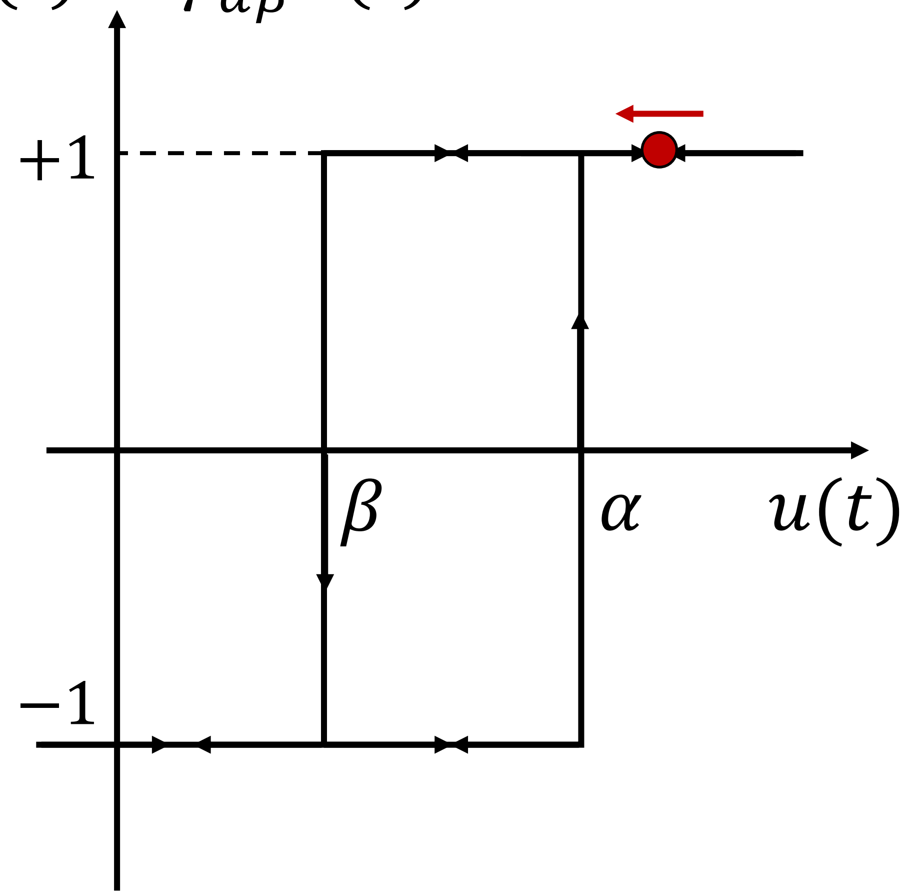

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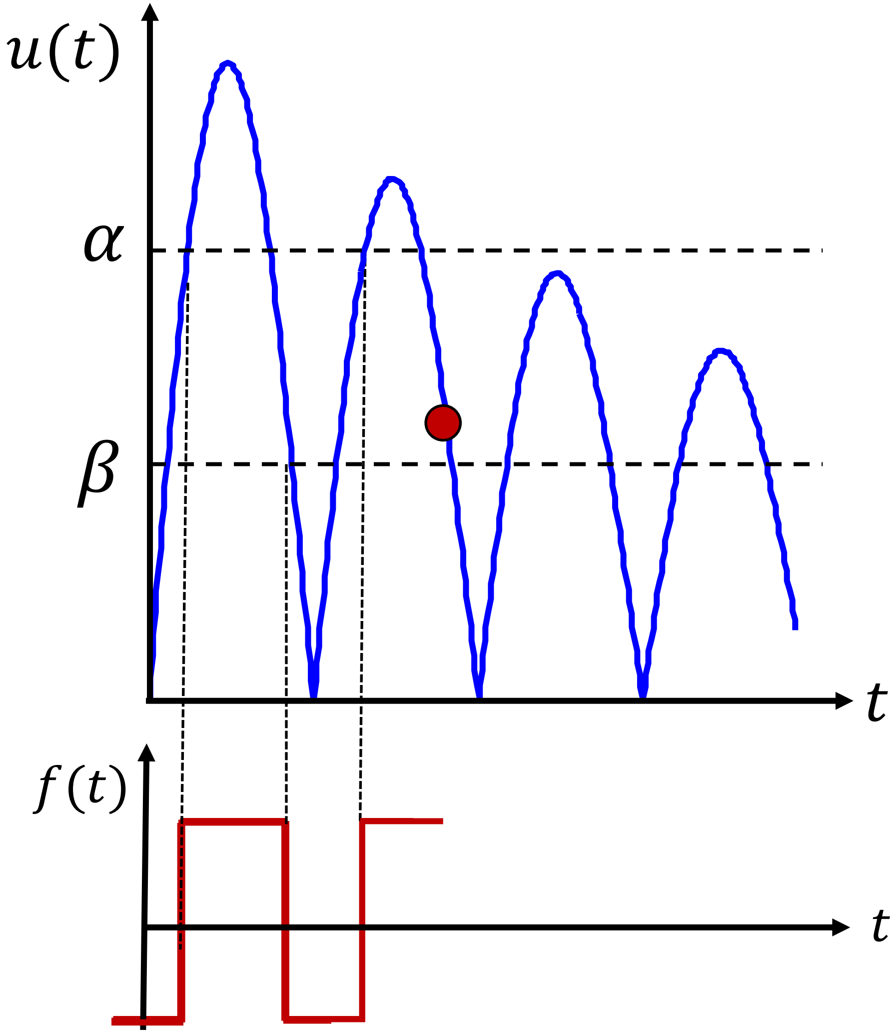

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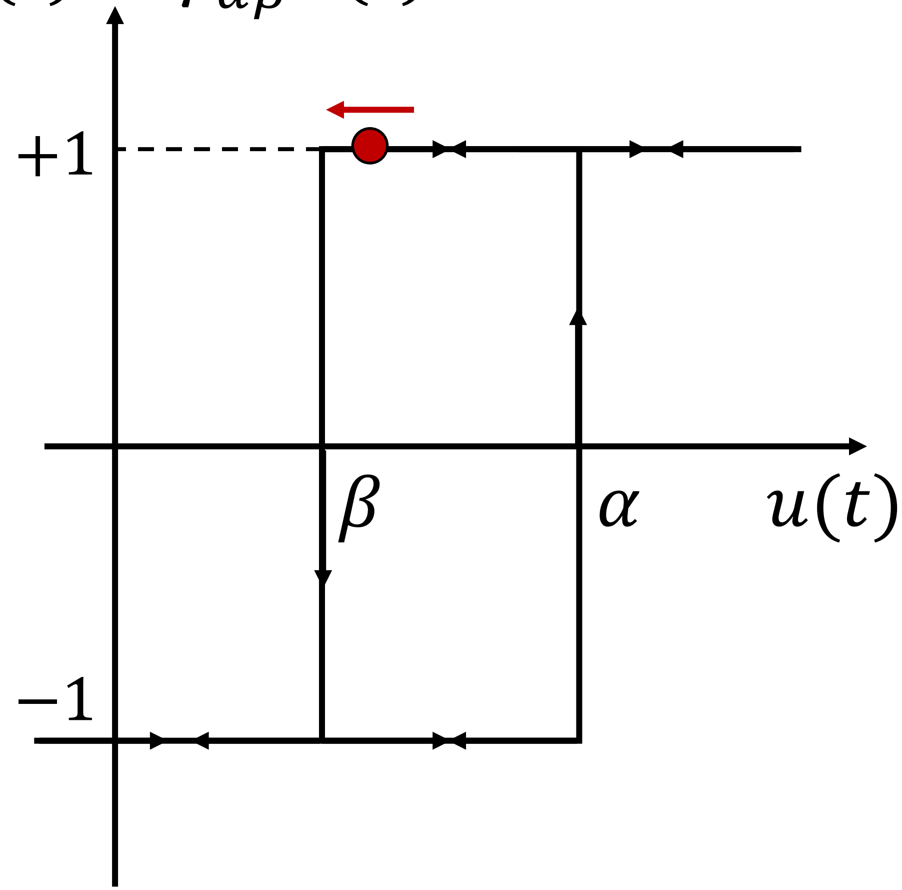

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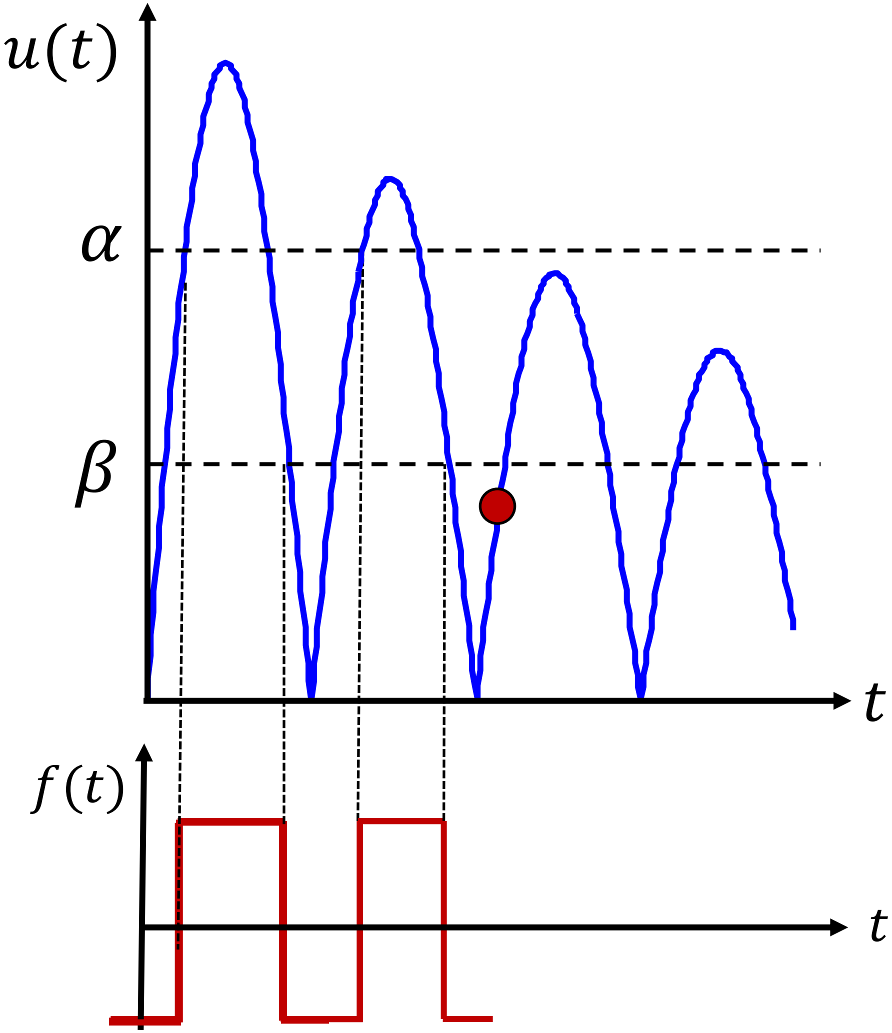

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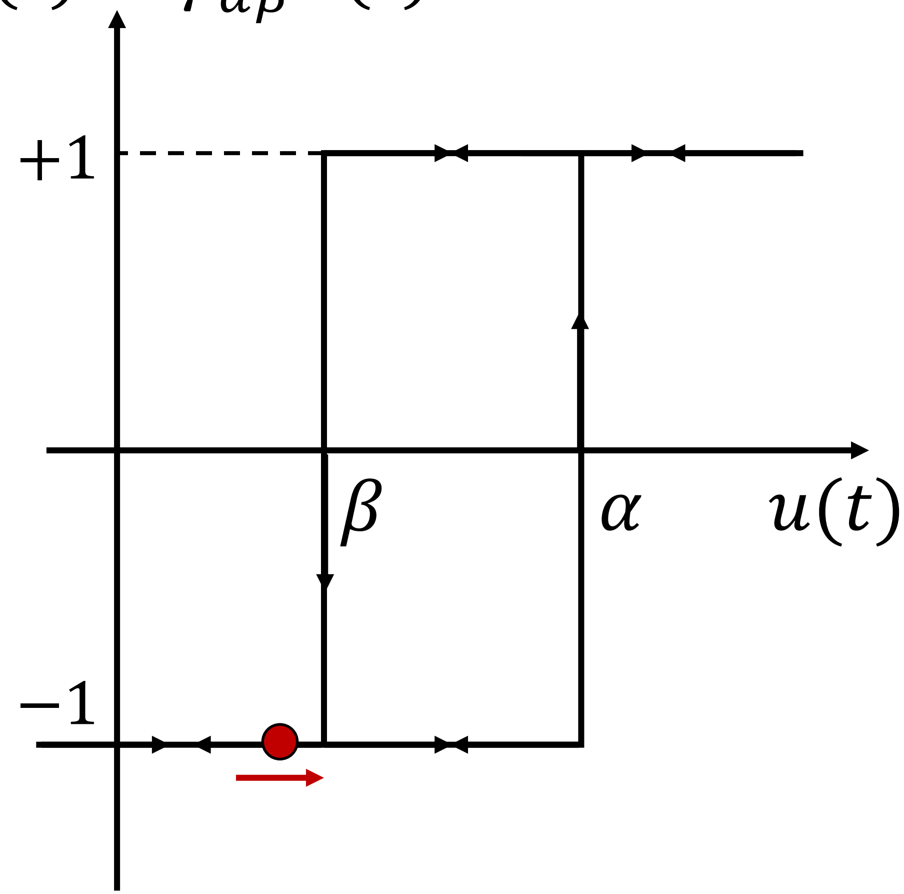

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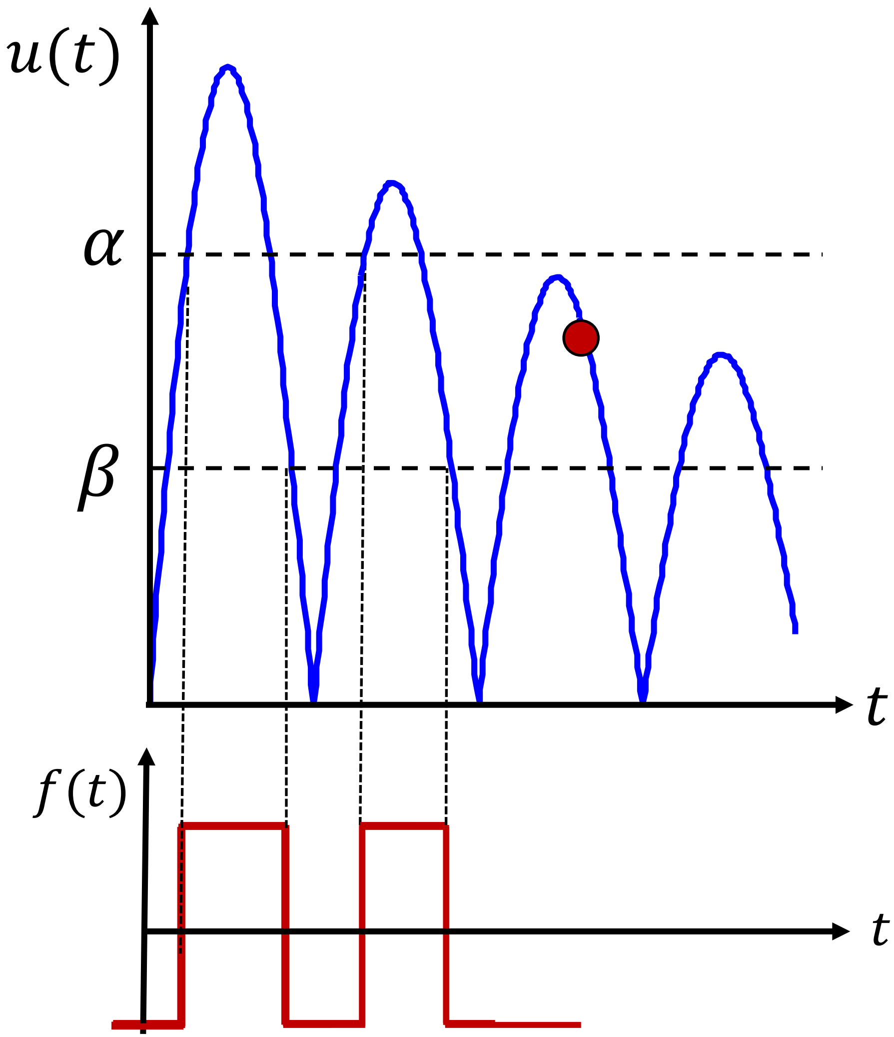

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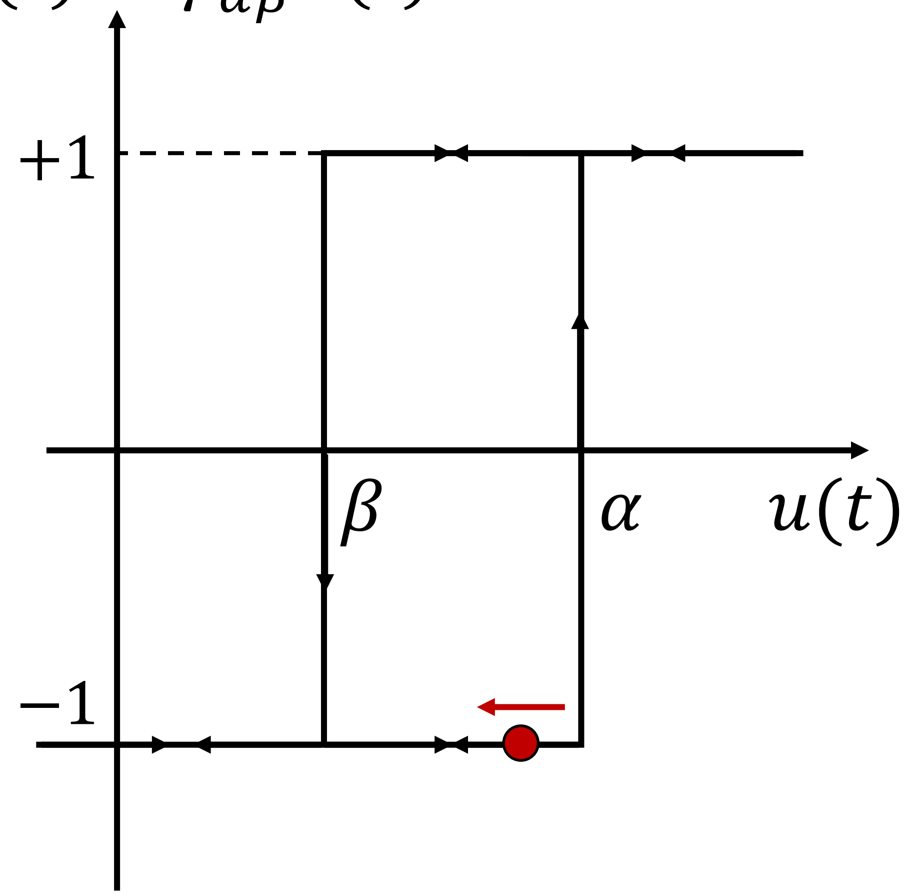

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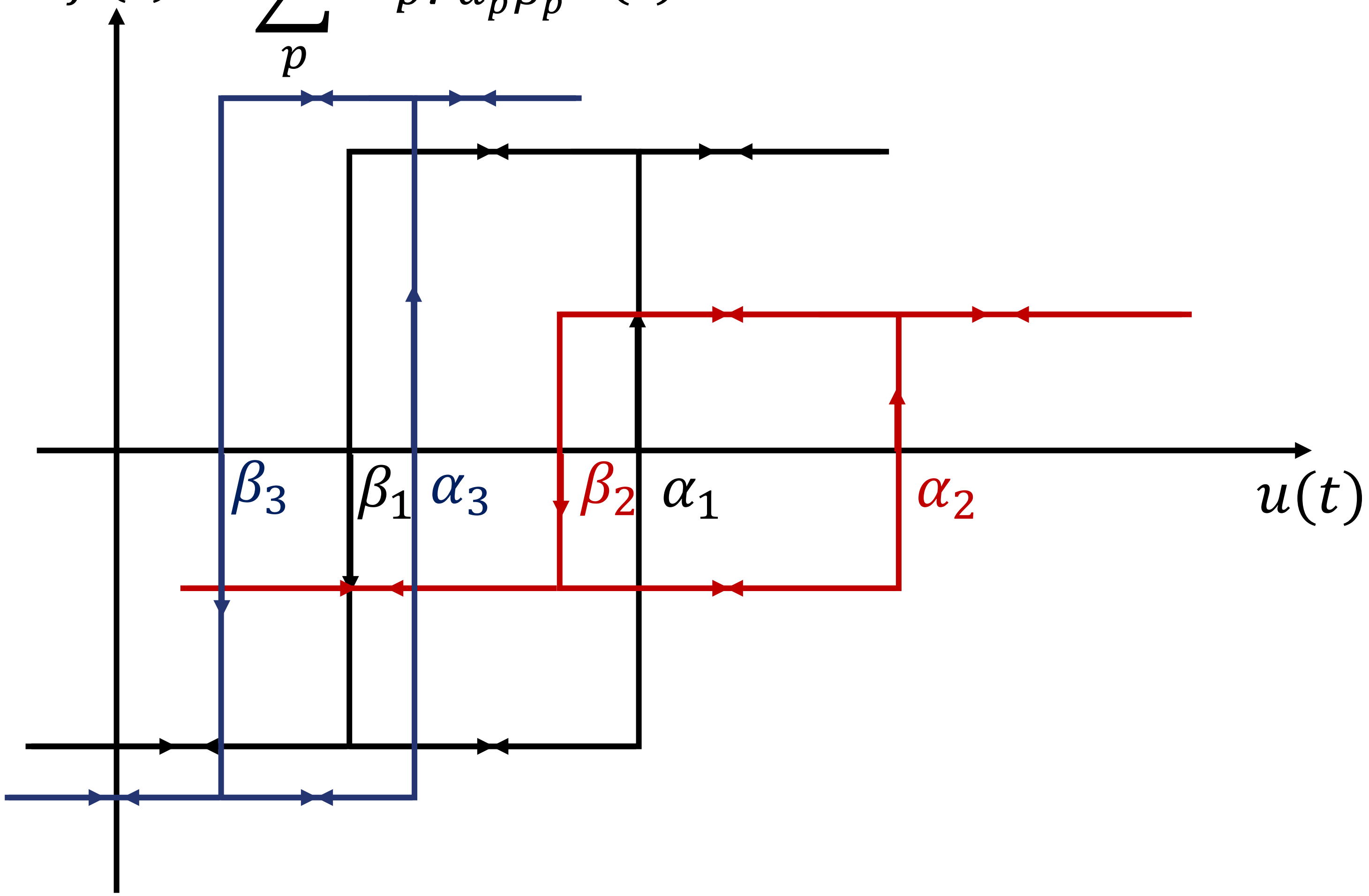
Combining Hysteresis Operators

$$f(t) = \sum_p m_p \hat{\gamma}_{\alpha_p \beta_p} u(t)$$



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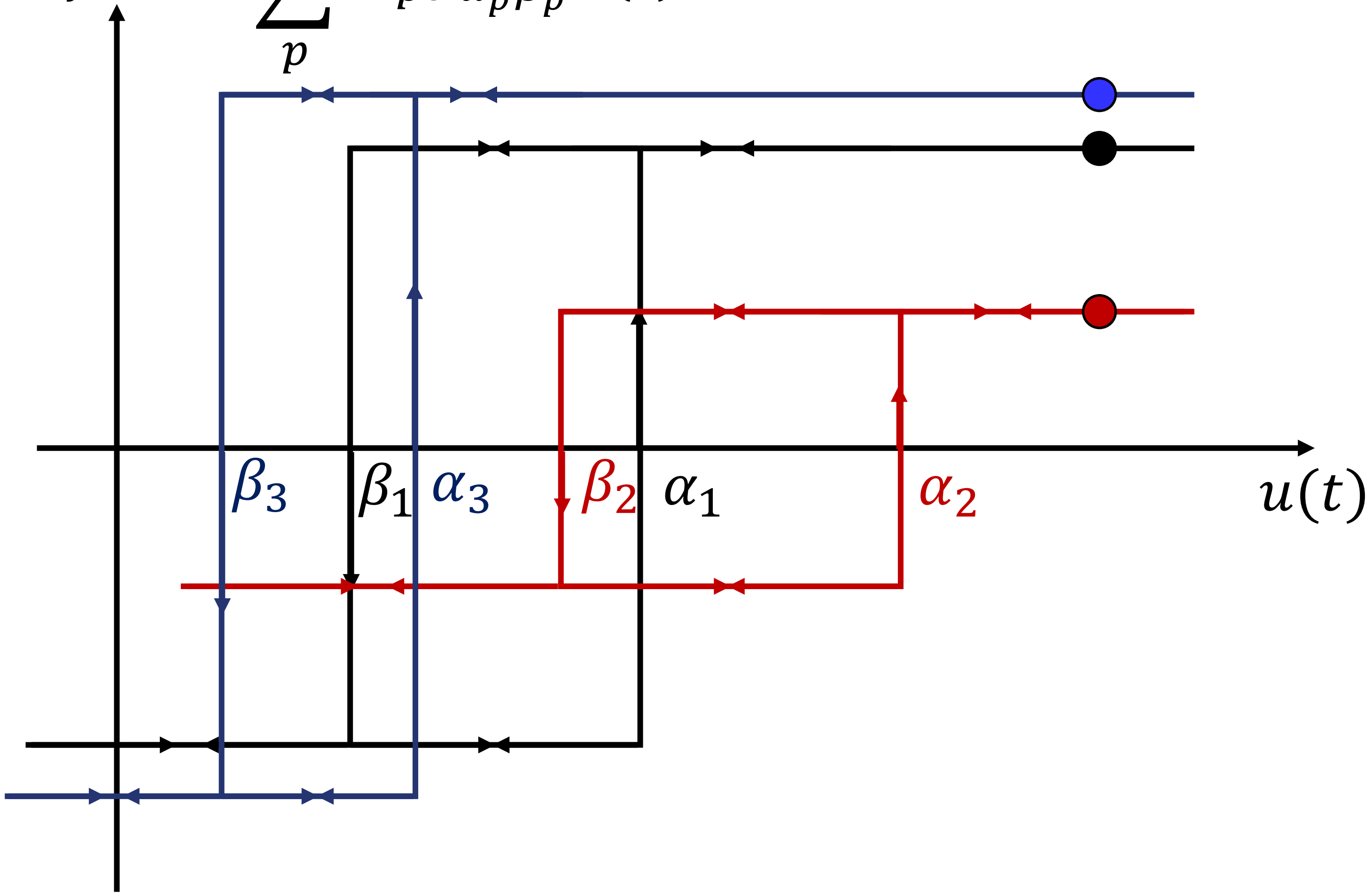

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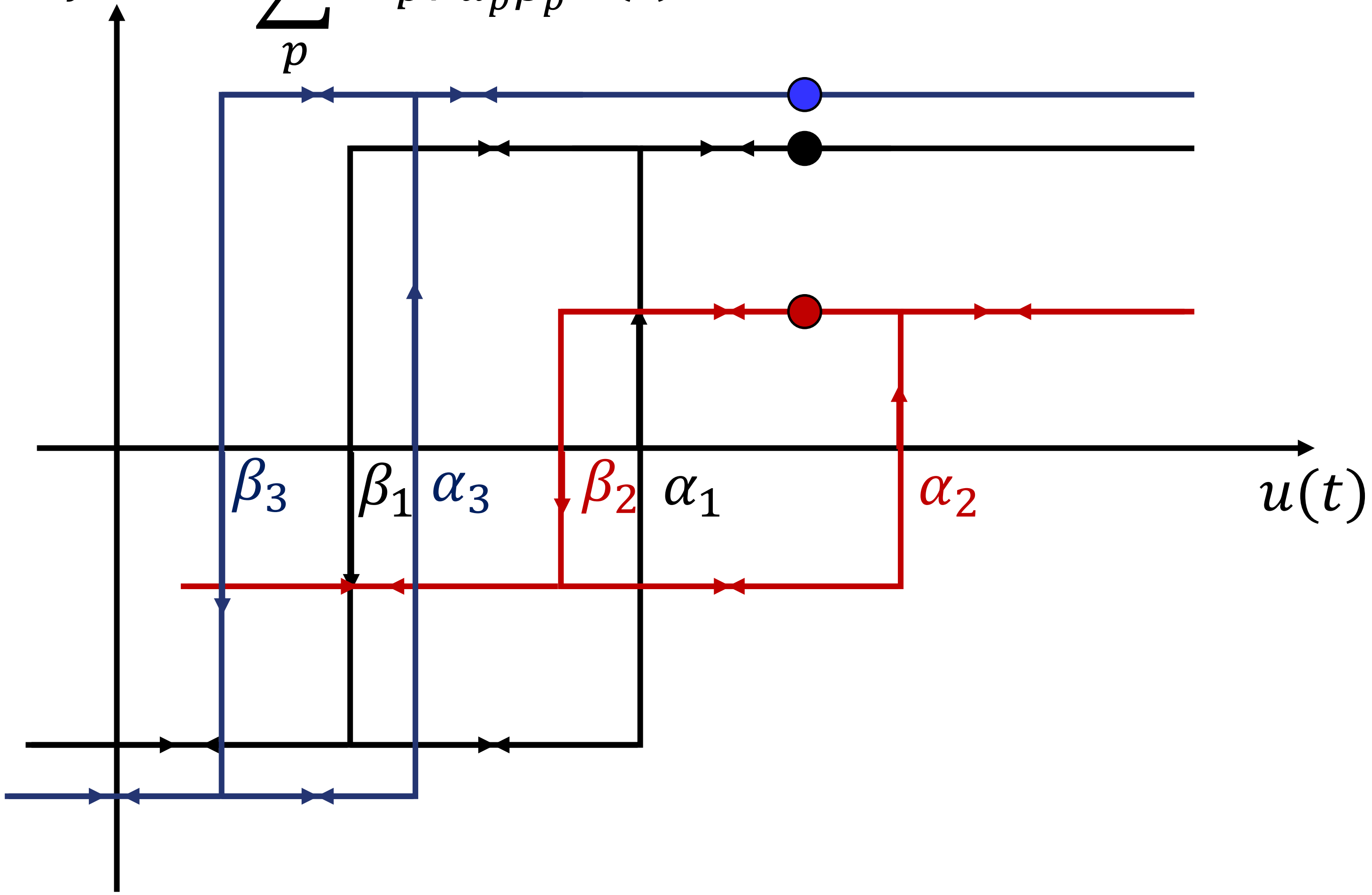

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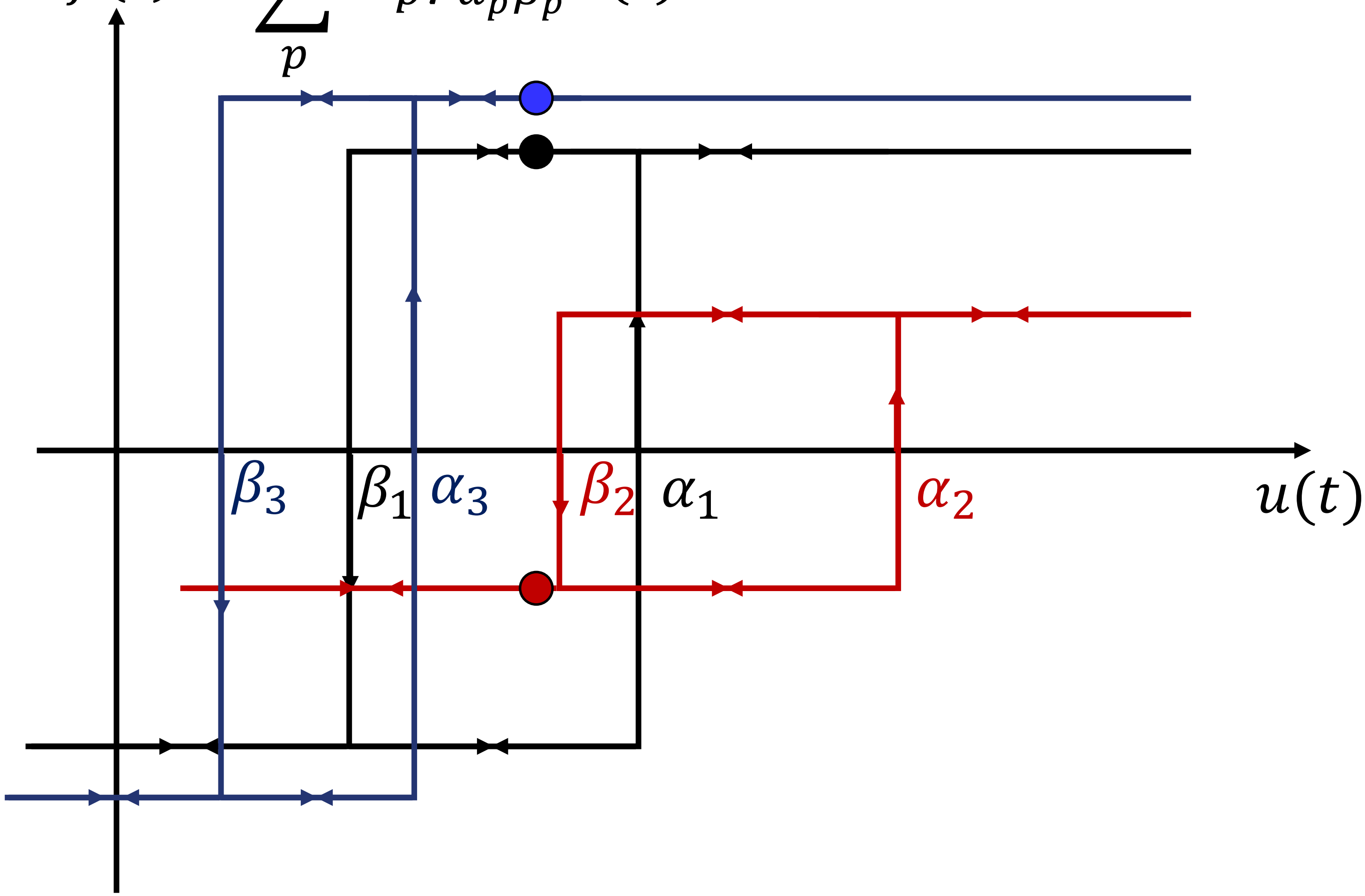

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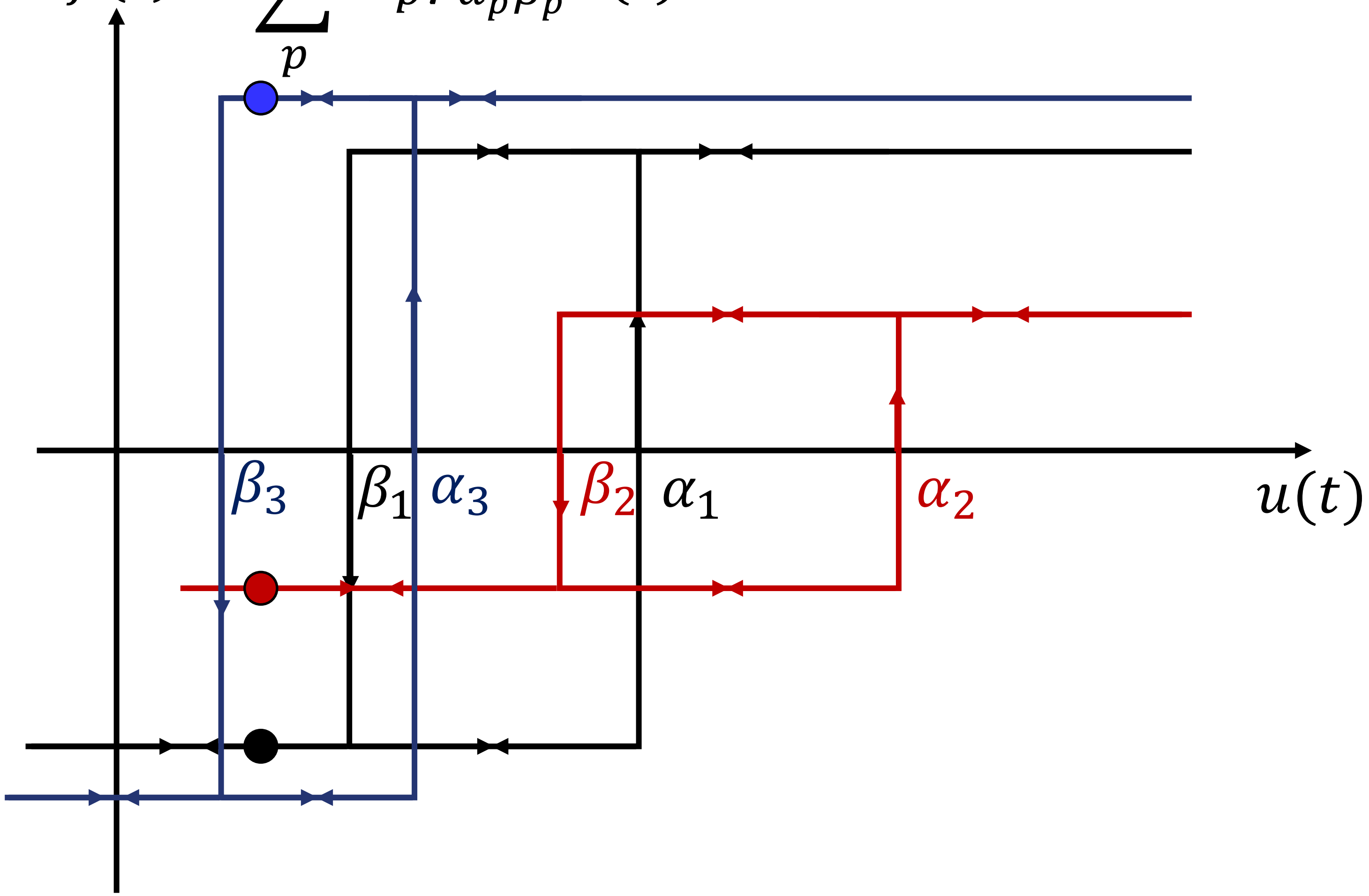

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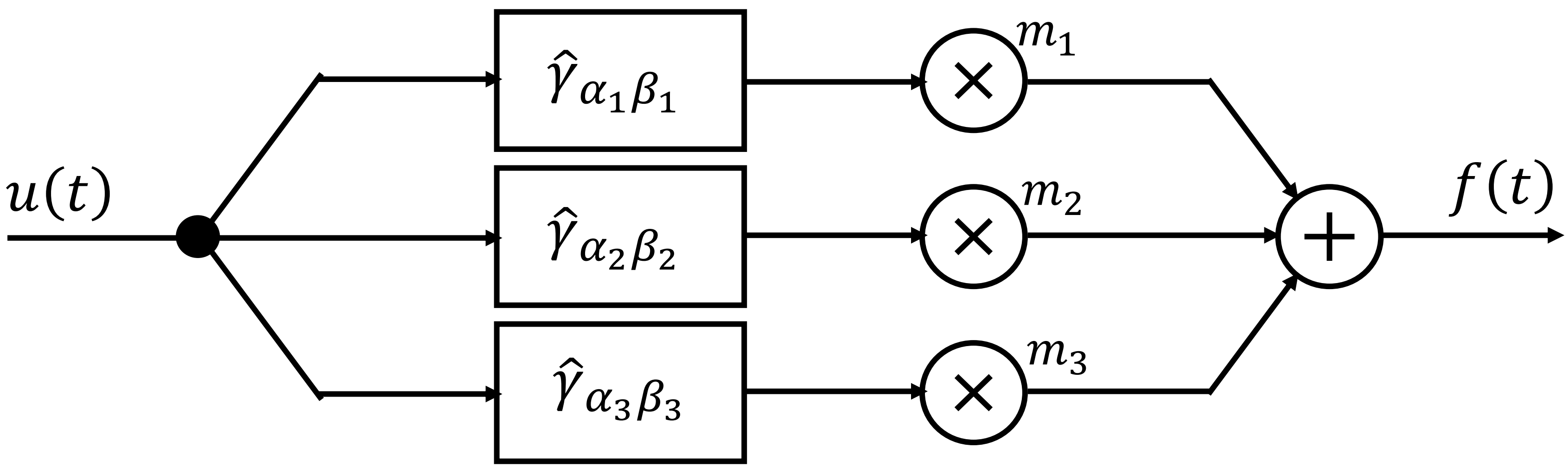


Combining Hysteresis Operators

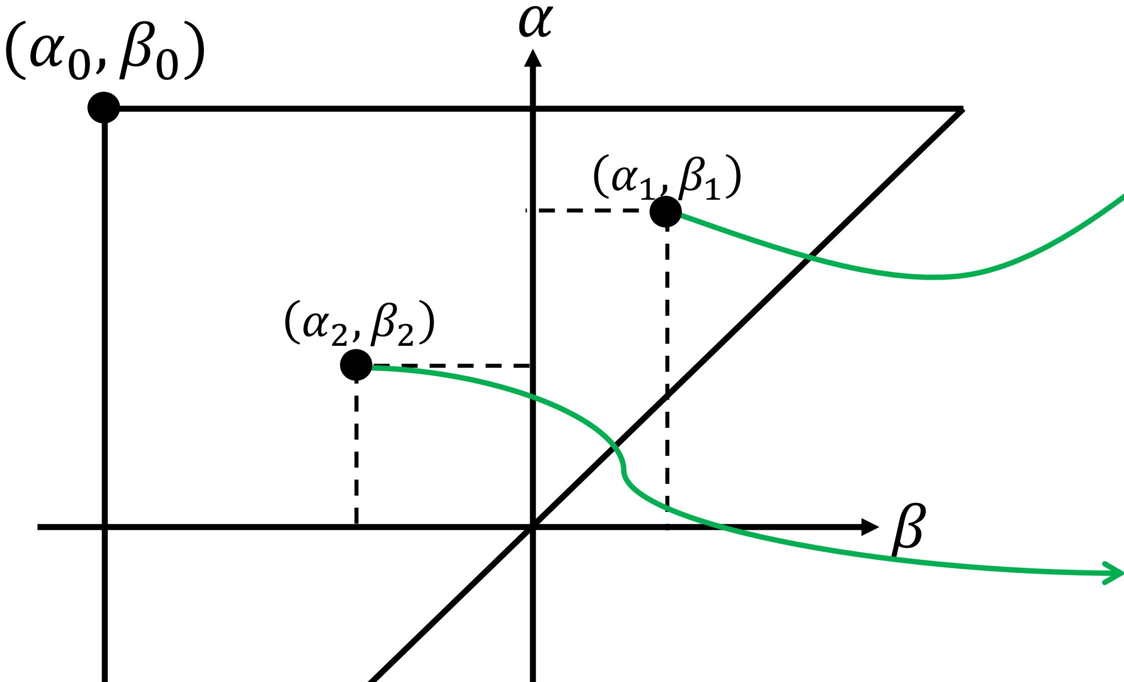
$$f(t) = \sum_p m_p \hat{\gamma}_{\alpha_p \beta_p} u(t)$$

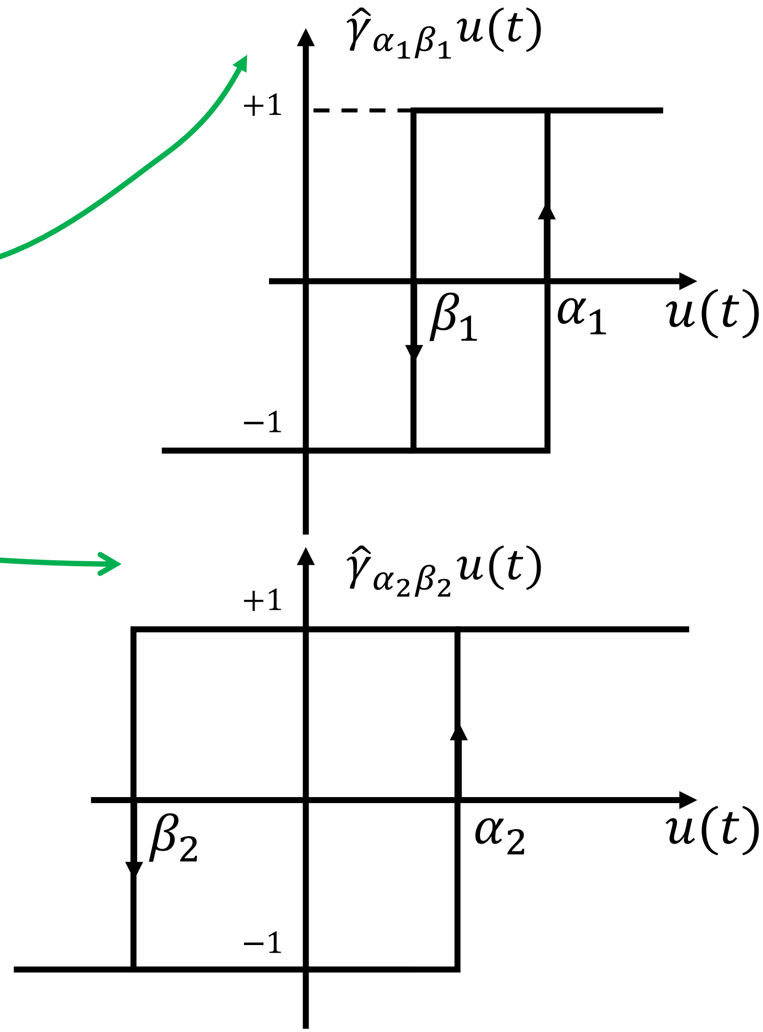
continuous analogue

$$f(t) = \hat{\Gamma} u(t) = \iint_{\alpha \geq \beta} \mu(\alpha, \beta)\ \hat{\gamma}_{\alpha\beta}\, u(t)\, d\alpha\, d\beta$$

Preisach model

Preisach Model



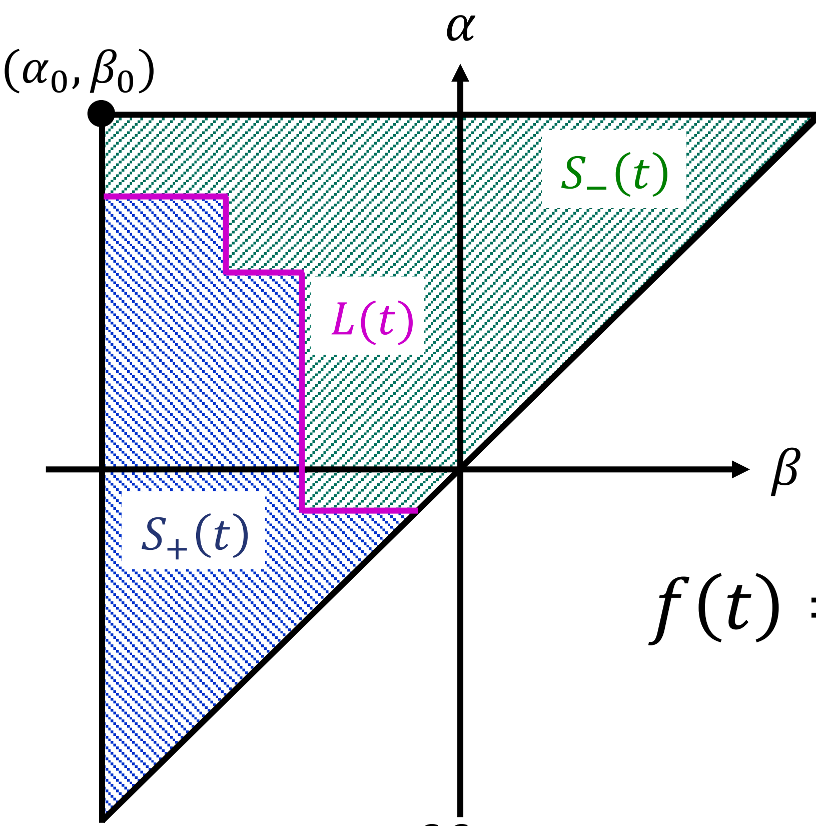


$\mu(\alpha, \beta)$: finite function with support within T

limiting triangle

➡ closed major loops

Preisach Model



$$f(t) = \iint_T \mu(\alpha, \beta)\ \underbrace{\hat{\gamma}_{\alpha\beta} u(t)}_{\pm 1}\, d\alpha\, d\beta$$

$$f(t) = \iint_{S_+(t)} \mu(\alpha, \beta)\, d\alpha\, d\beta - \iint_{S_-(t)} \mu(\alpha, \beta)\, d\alpha\, d\beta$$

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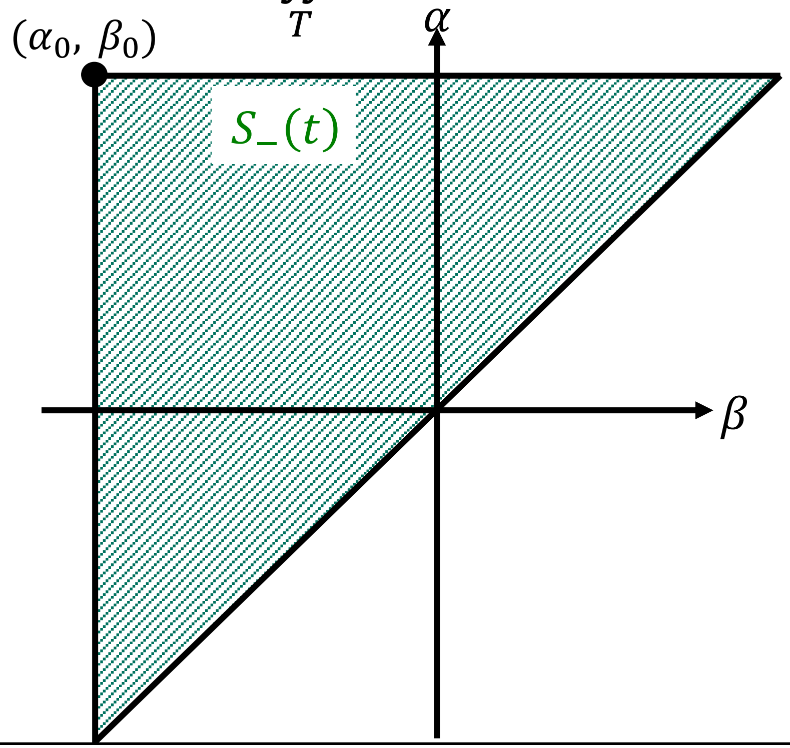
Complete Saturation

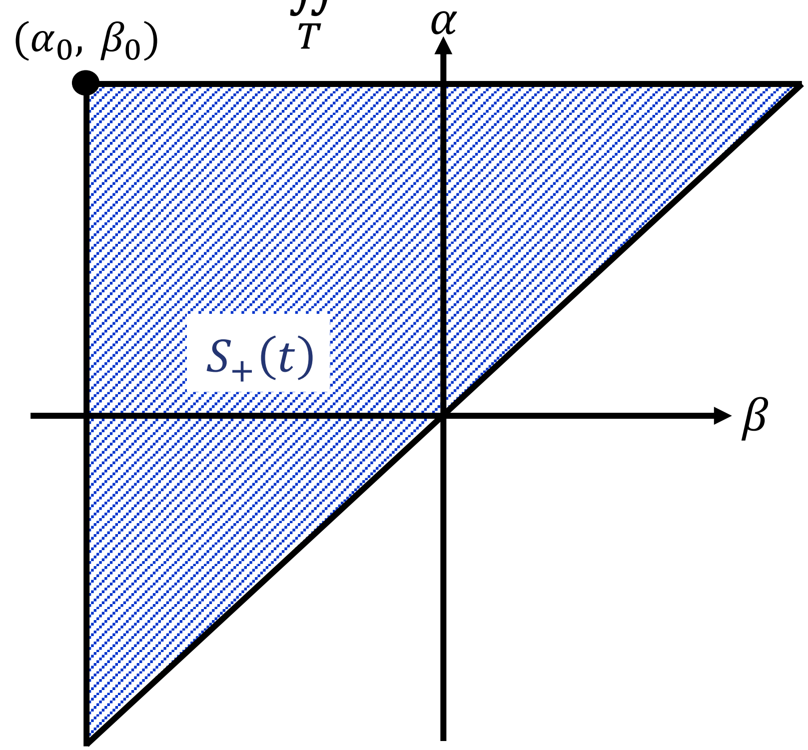
negative saturation
⟺ all hysteresis operators „down"

$$f_{\mathrm{neg}} = -\iint_T \mu(\alpha,\beta)\, d\alpha\, d\beta$$



positive saturation
⟺ all hysteresis operators „up"

$$f_{\mathrm{pos}} = -f_{\mathrm{neg}} = \iint_T \mu(\alpha,\beta)\, d\alpha\, d\beta$$



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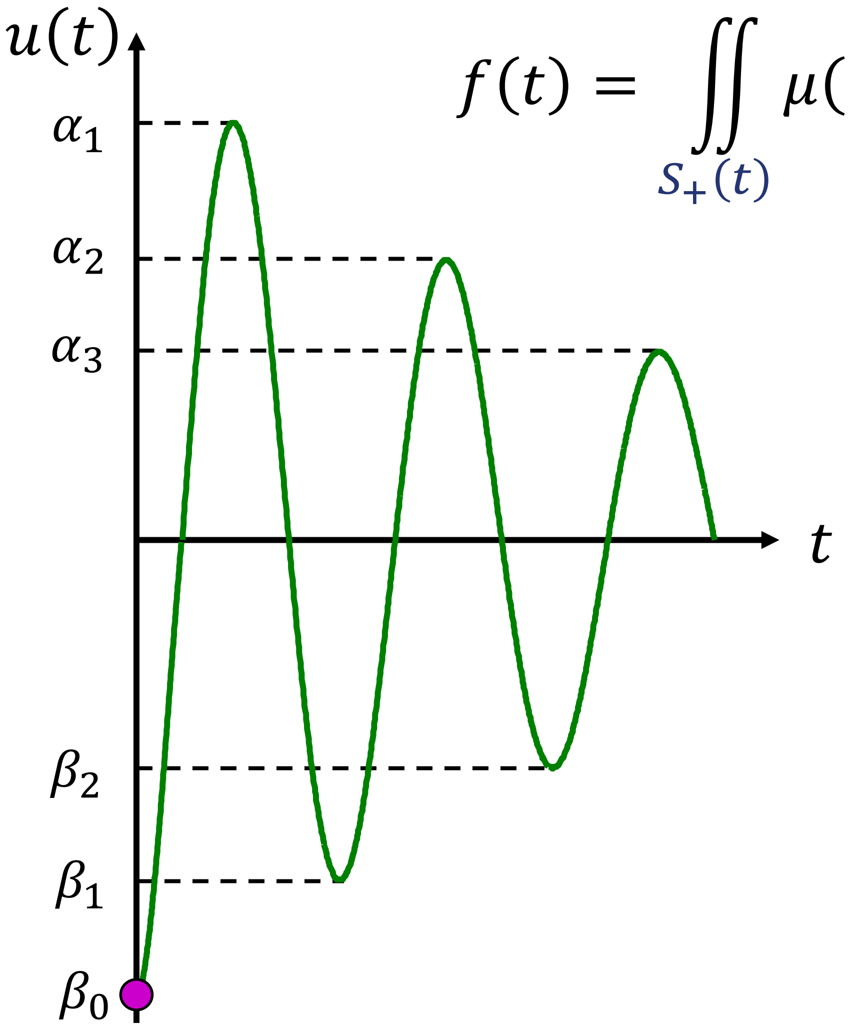

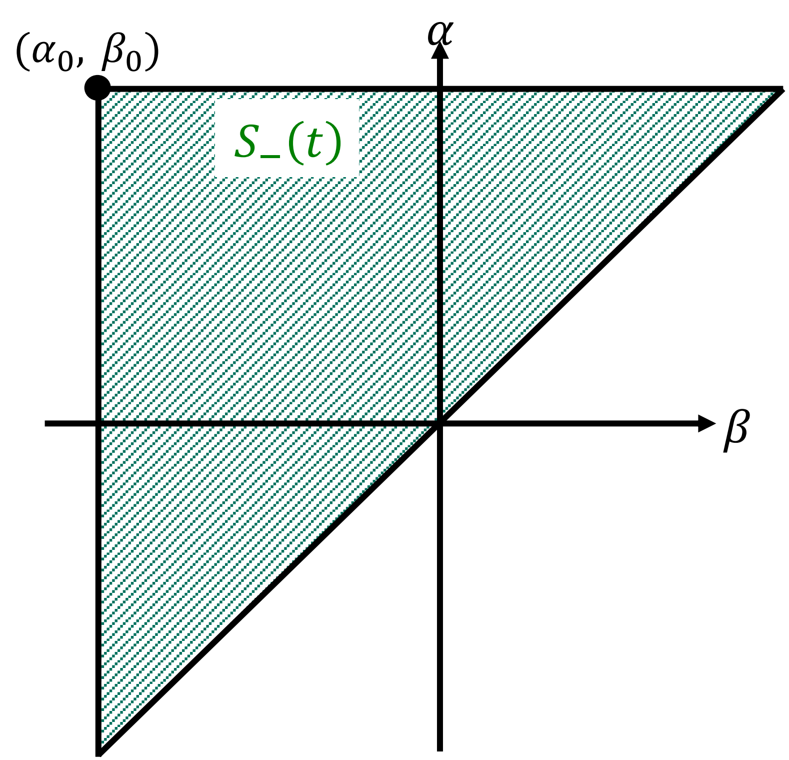

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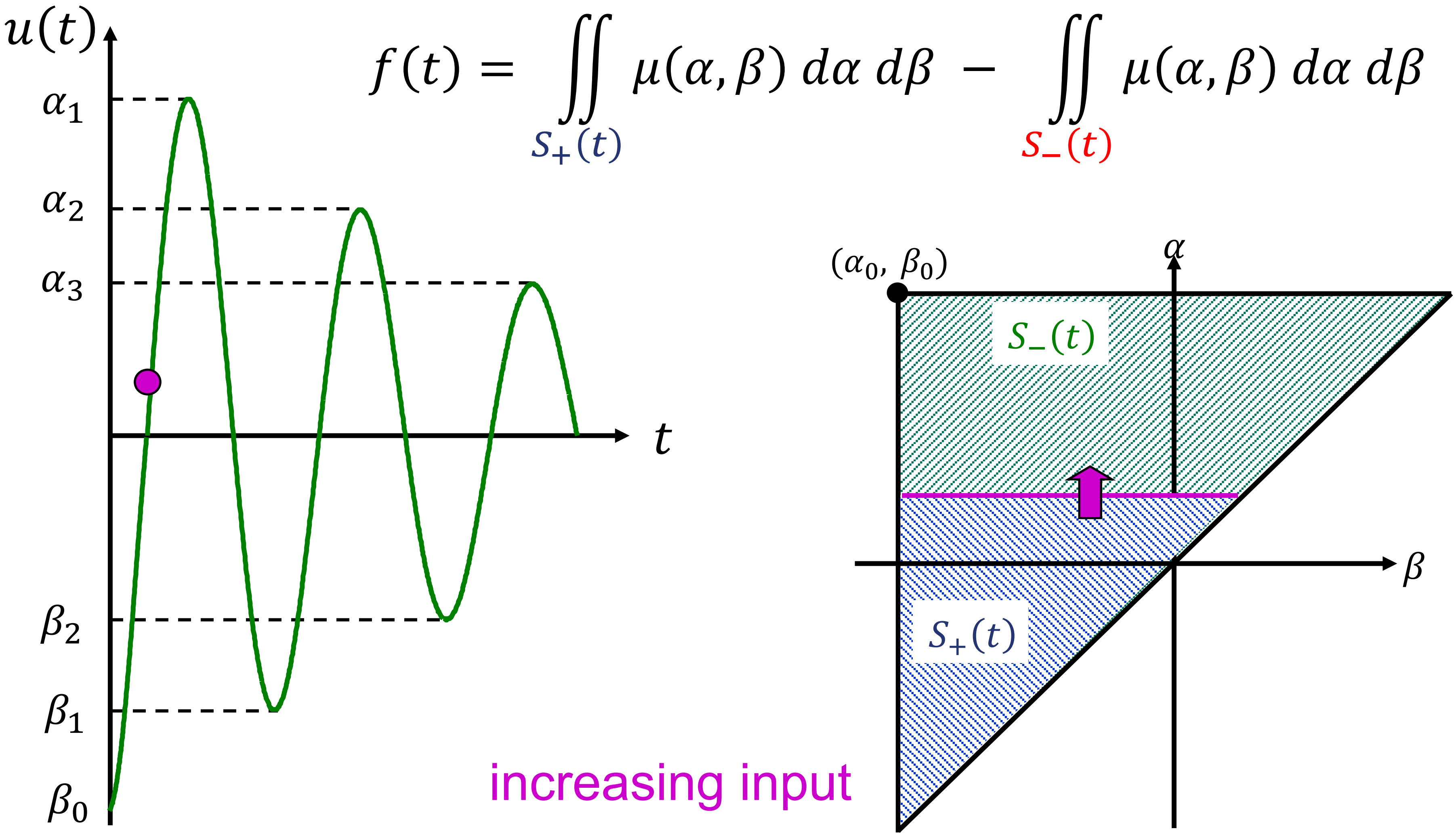

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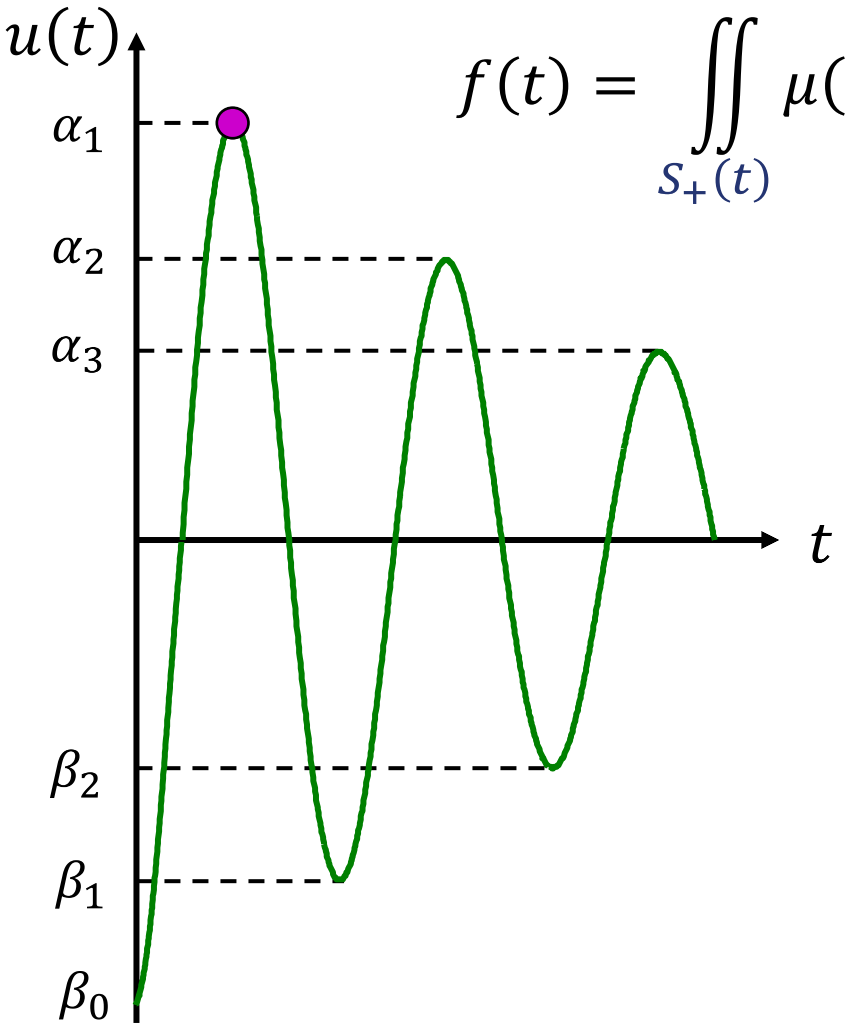

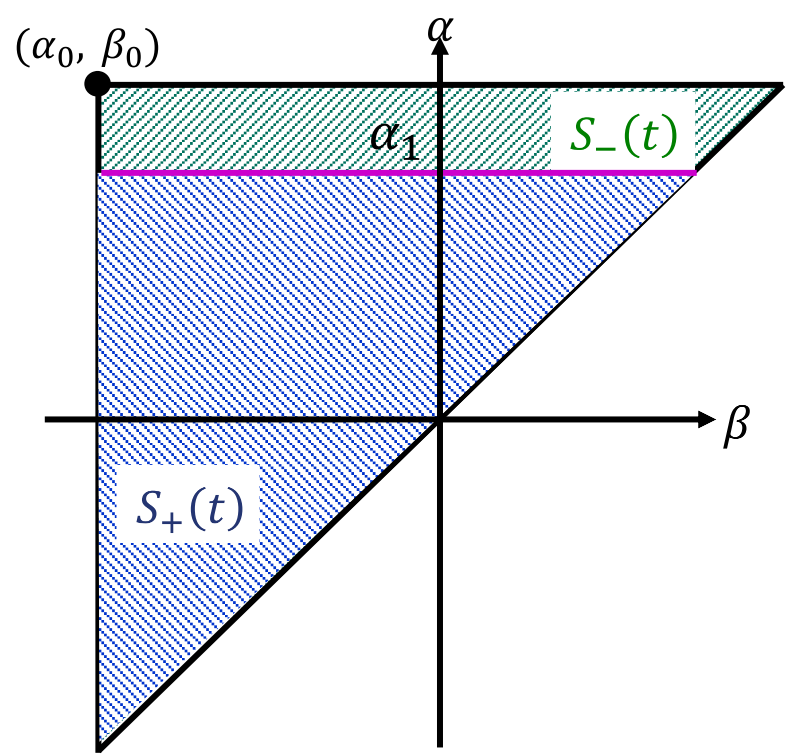

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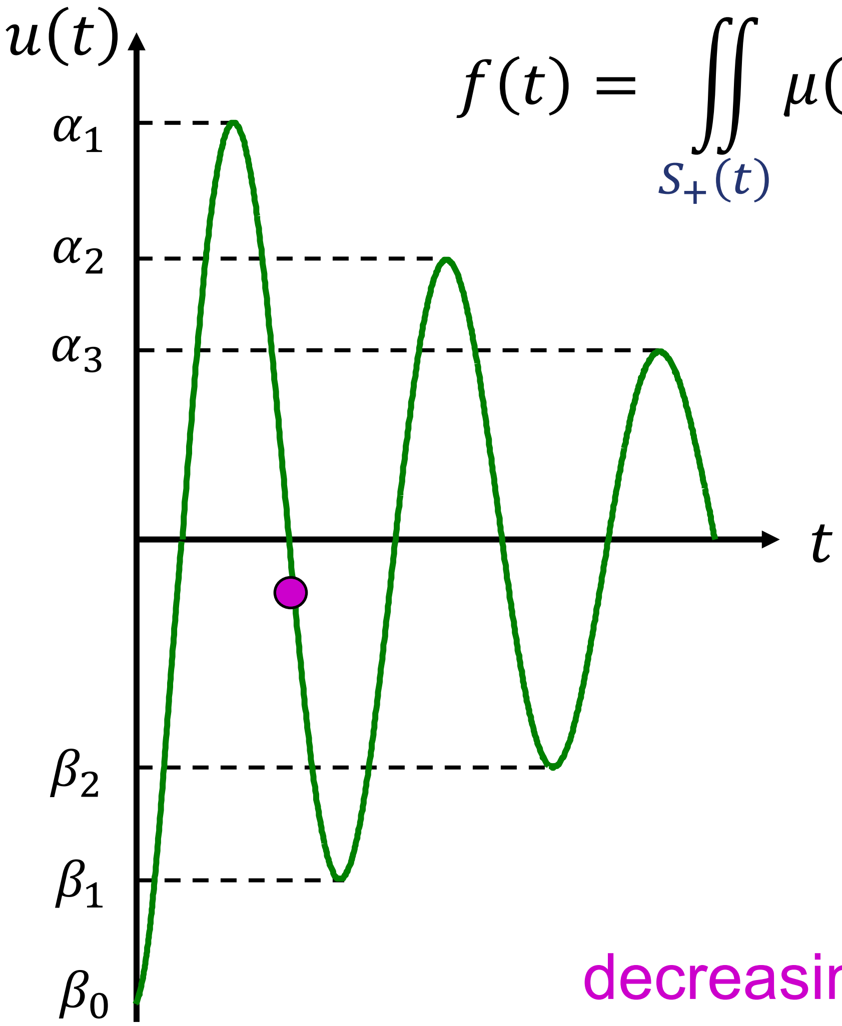

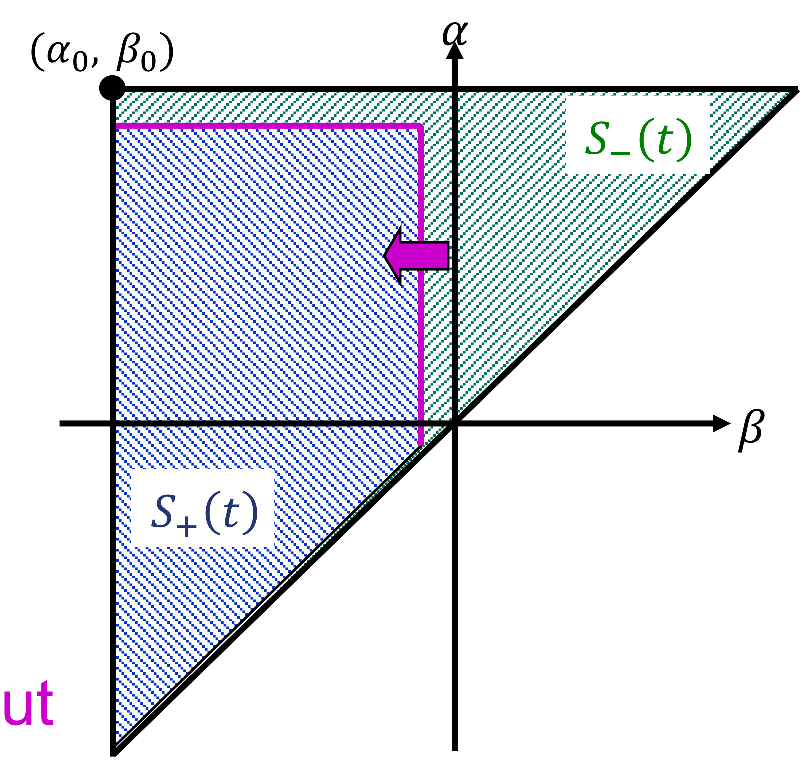

decreasing input

Memory Formation

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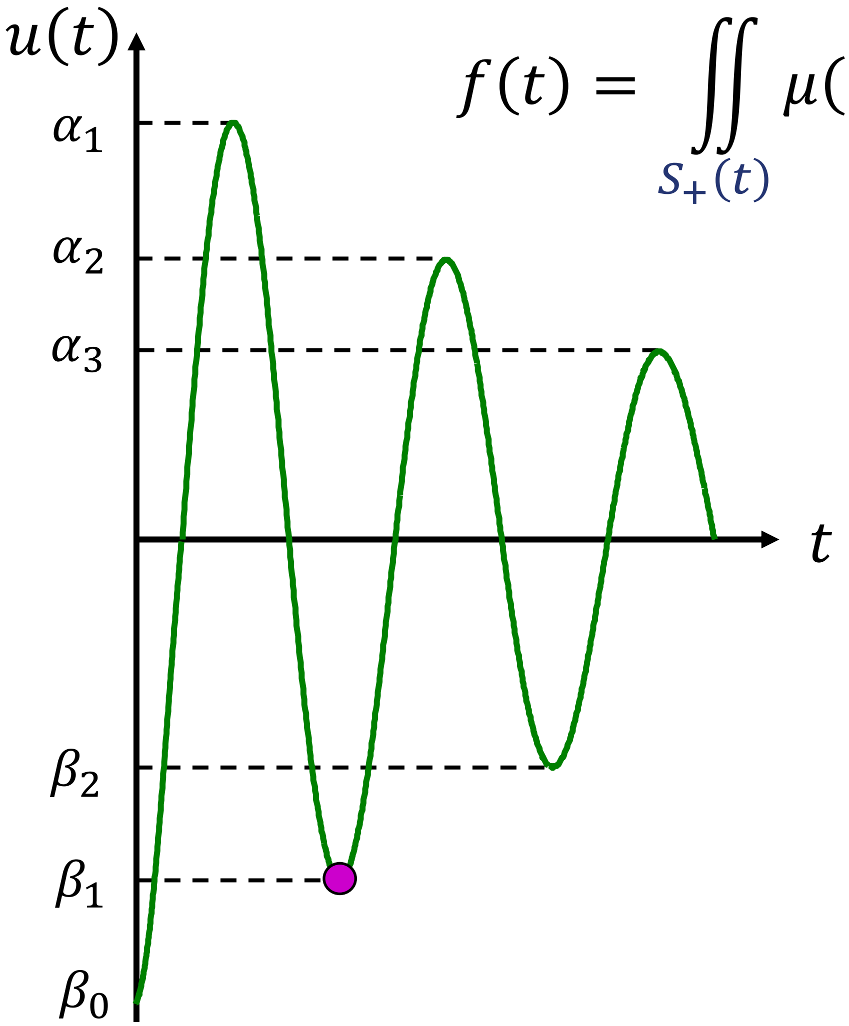

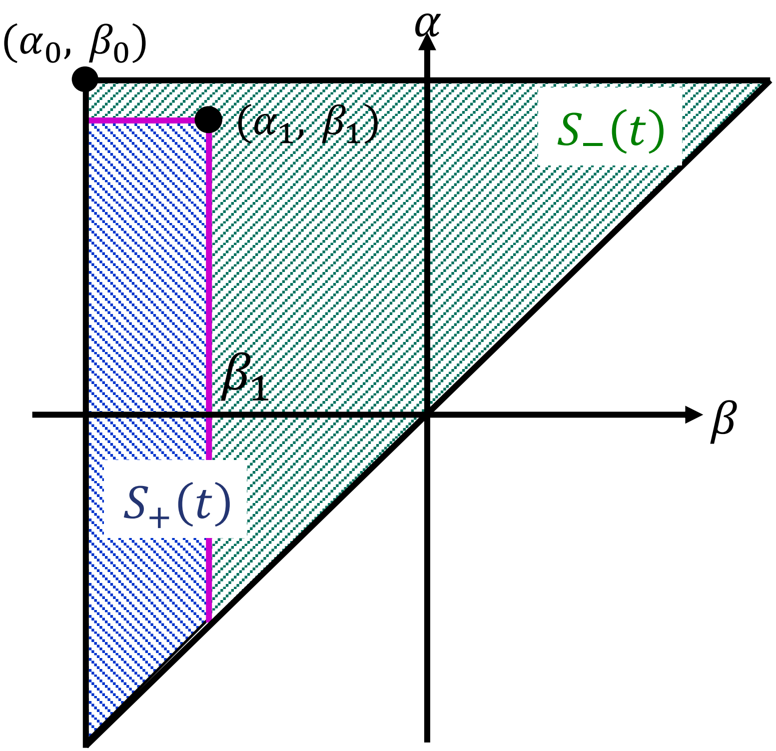

Memory Formation

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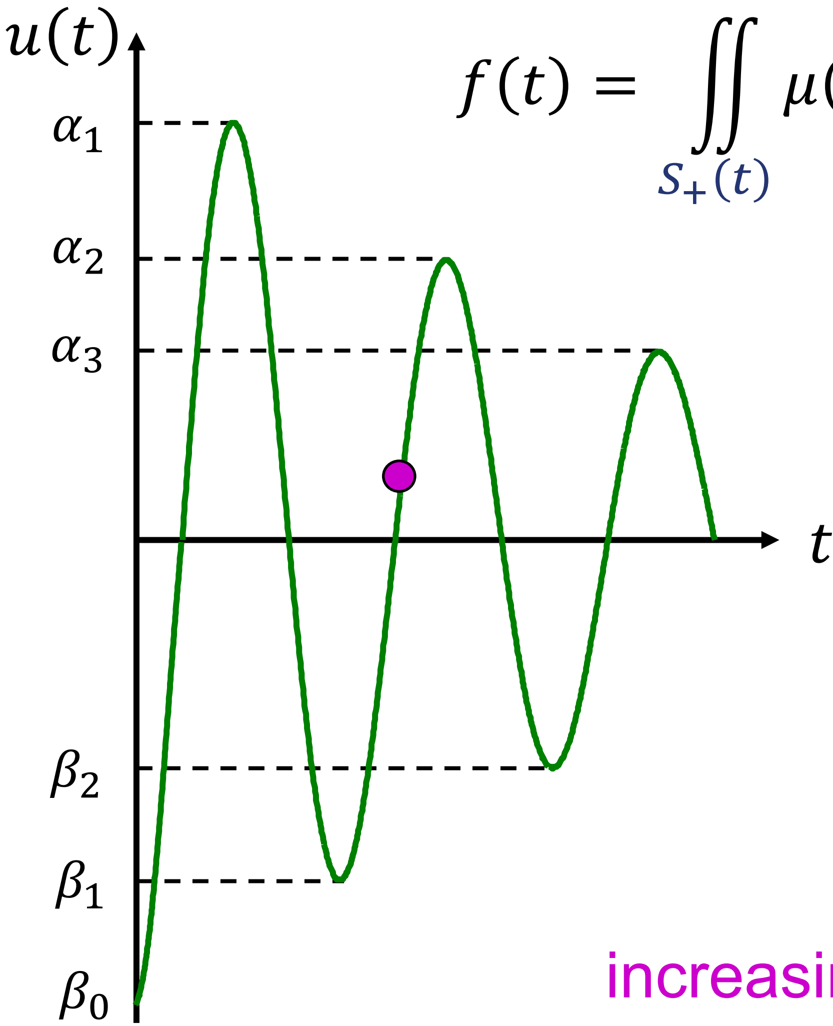

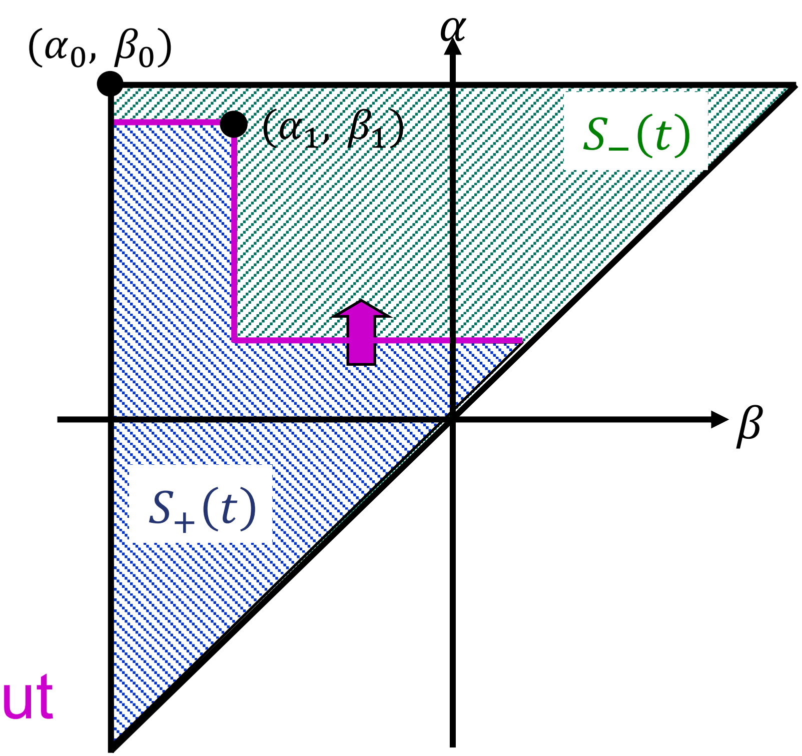

increasing input

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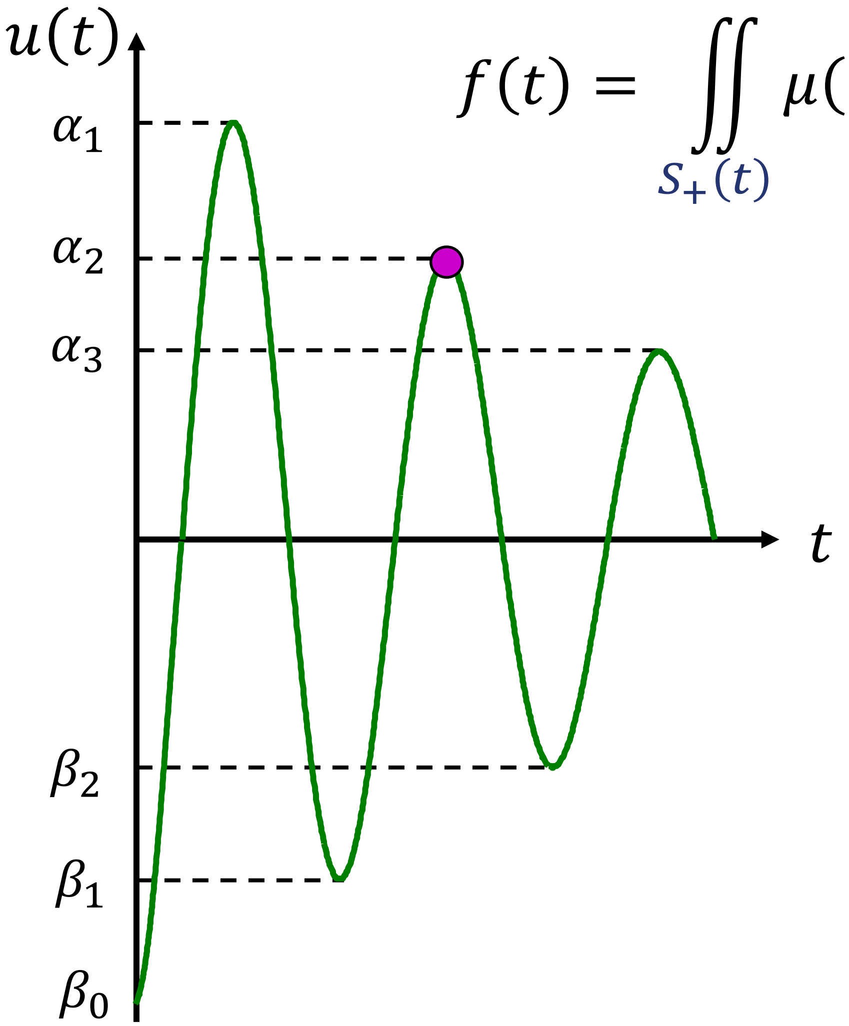

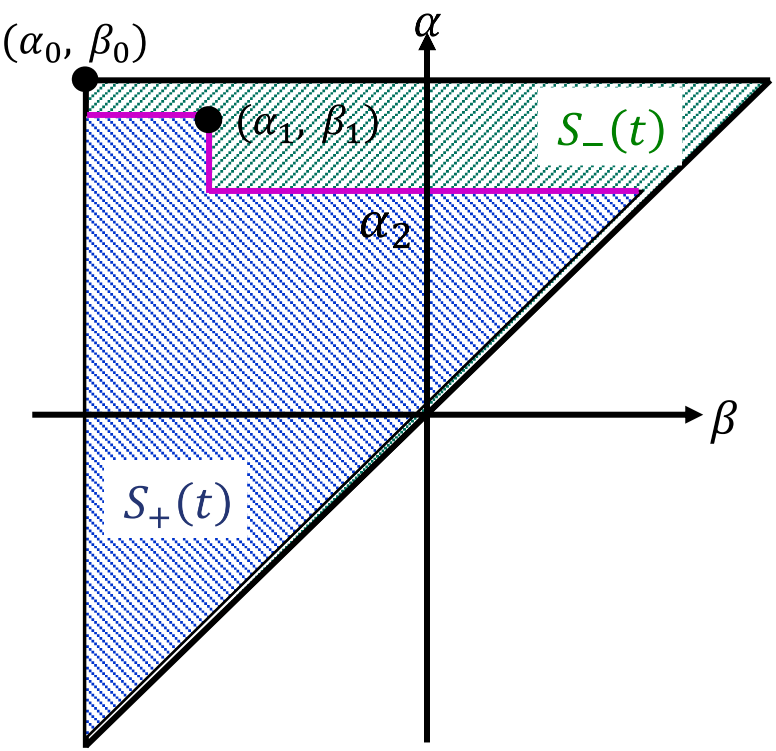

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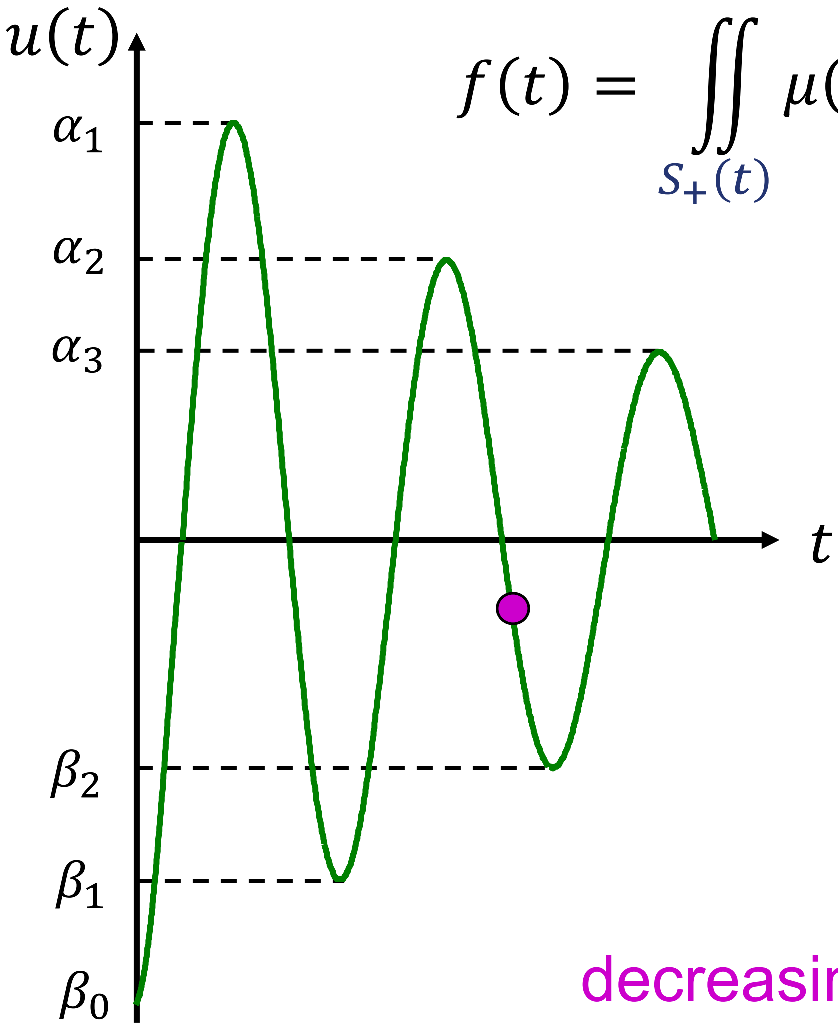

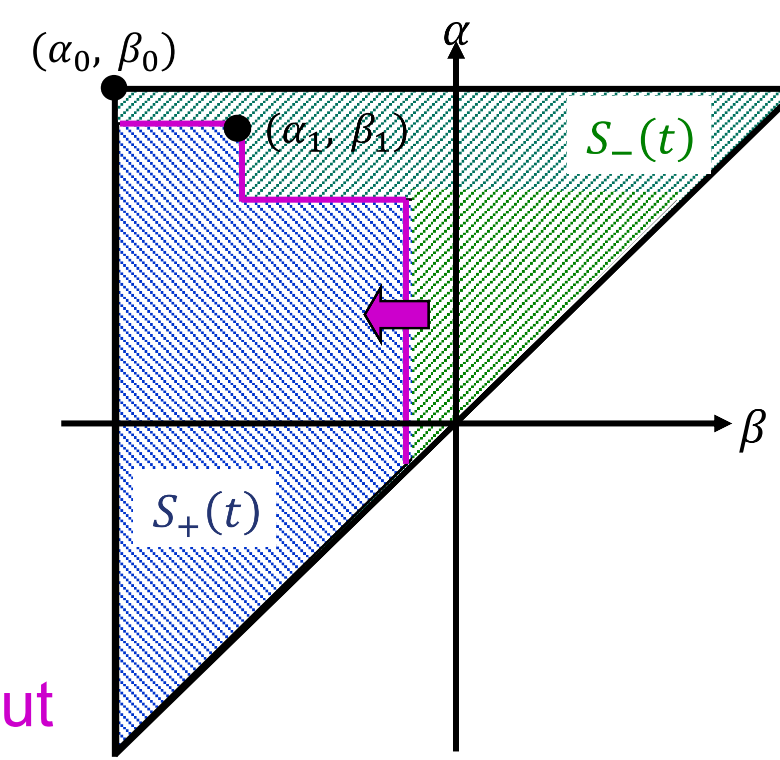

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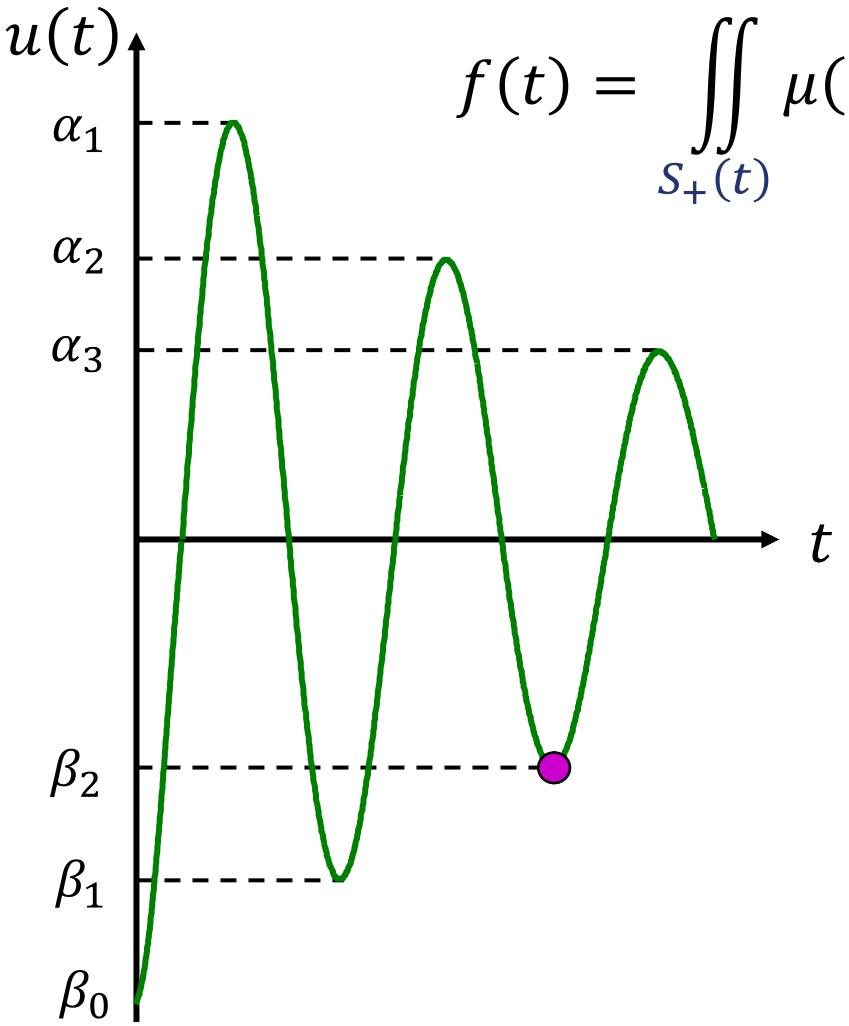
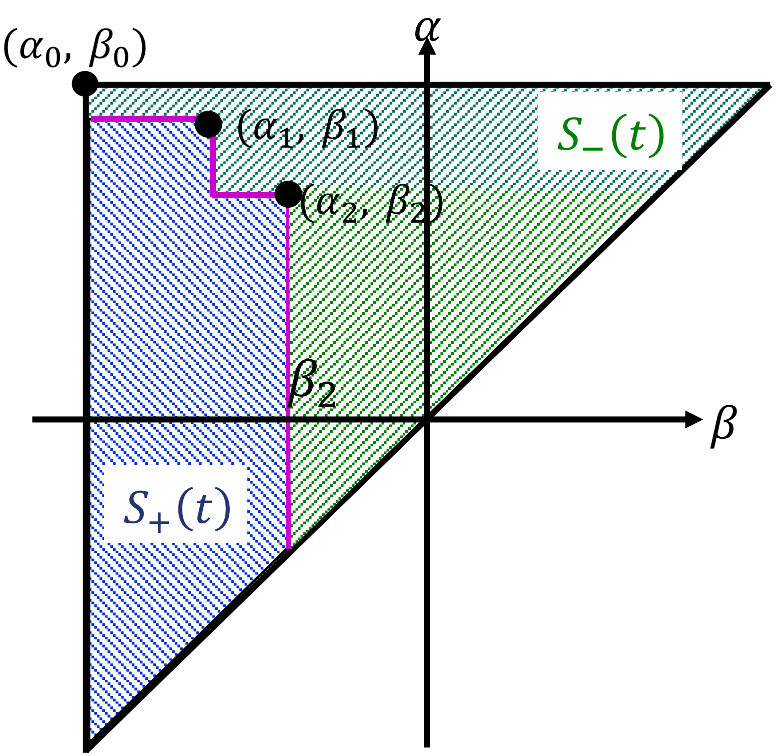
Memory Formation

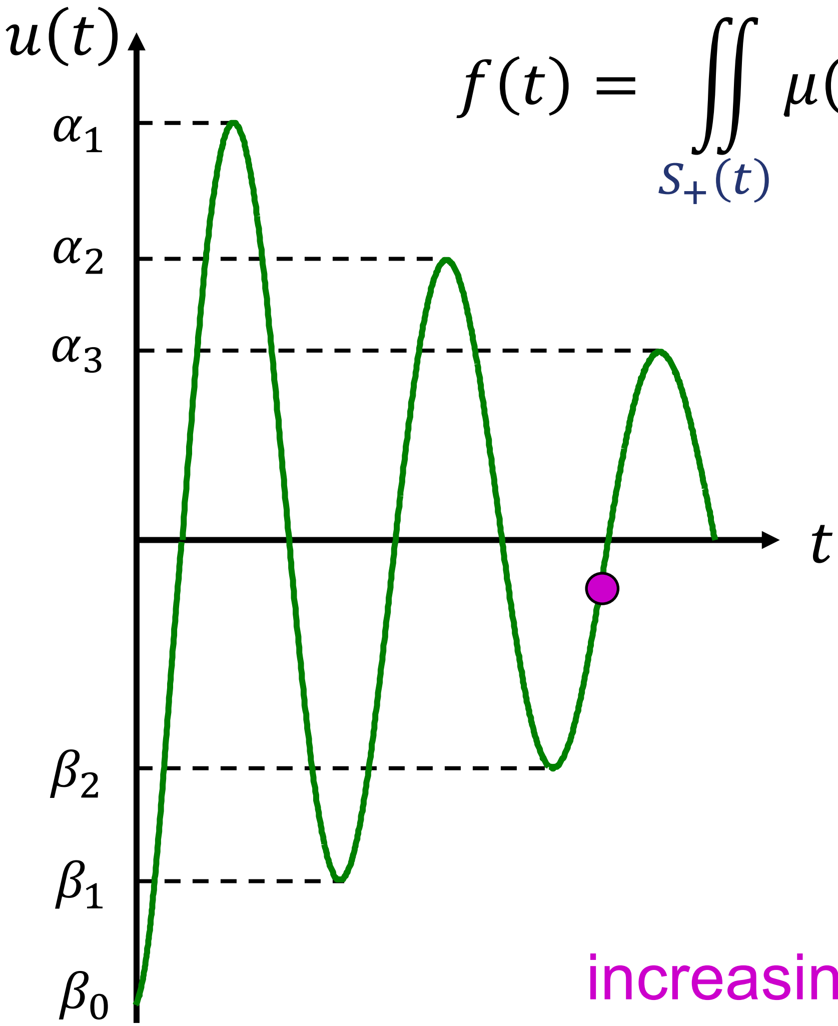
$$f(t) = \iint_{S_+(t)} \mu(\alpha,\beta)\, d\alpha\, d\beta - \iint_{S_-(t)} \mu(\alpha,\beta)\, d\alpha\, d\beta$$

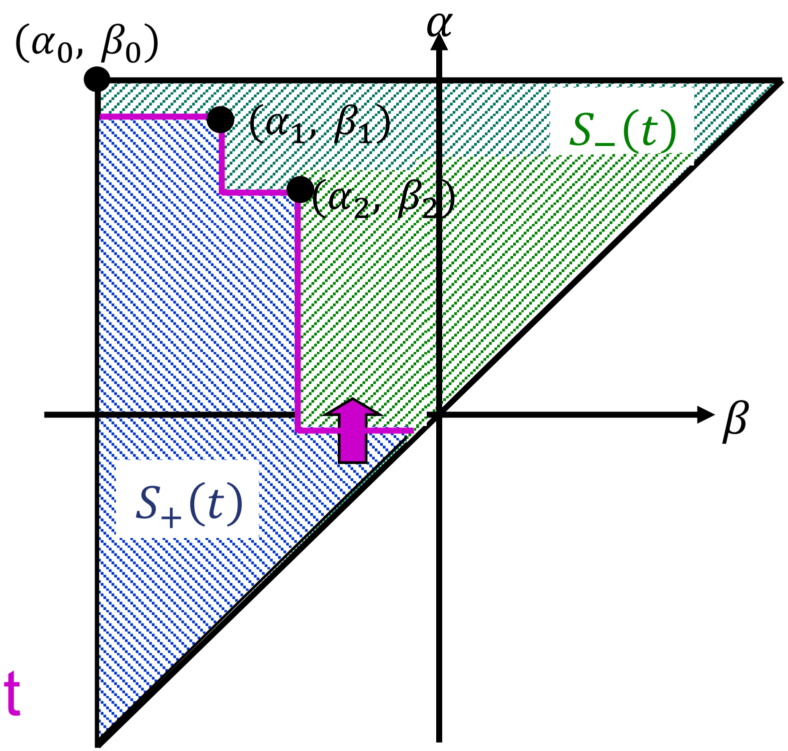




Memory Formation

$$f(t) = \iint_{S_+(t)} \mu(\alpha,\beta)\, d\alpha\, d\beta - \iint_{S_-(t)} \mu(\alpha,\beta)\, d\alpha\, d\beta$$



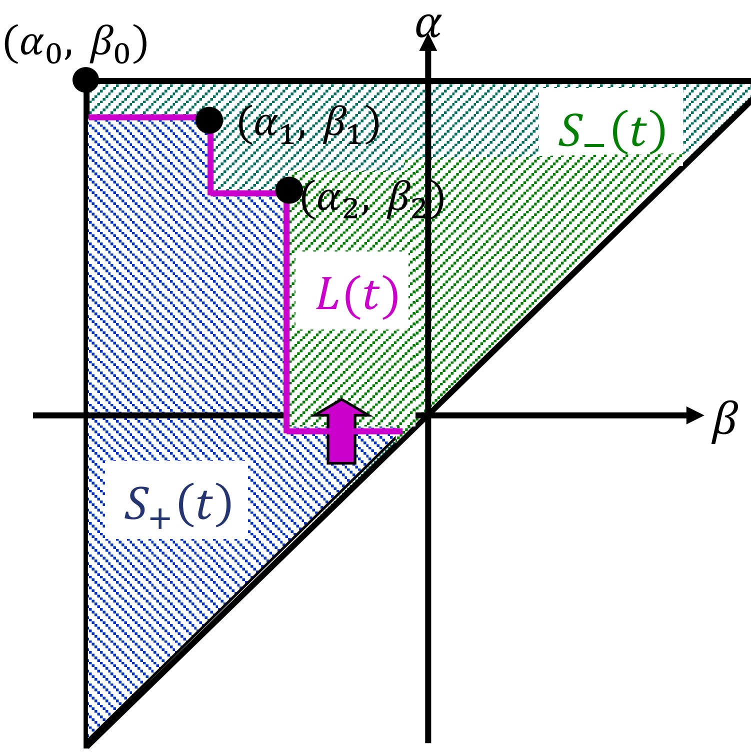


increasing input

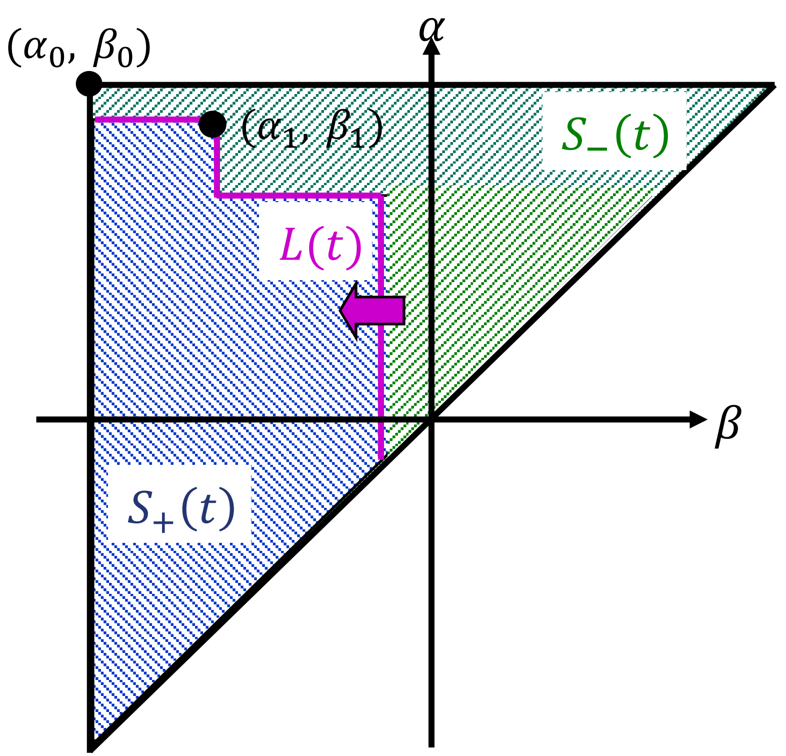
Memory Formation

state of the hysteretic material
determined by staircase interface L(t)

increasing input



decreasing input



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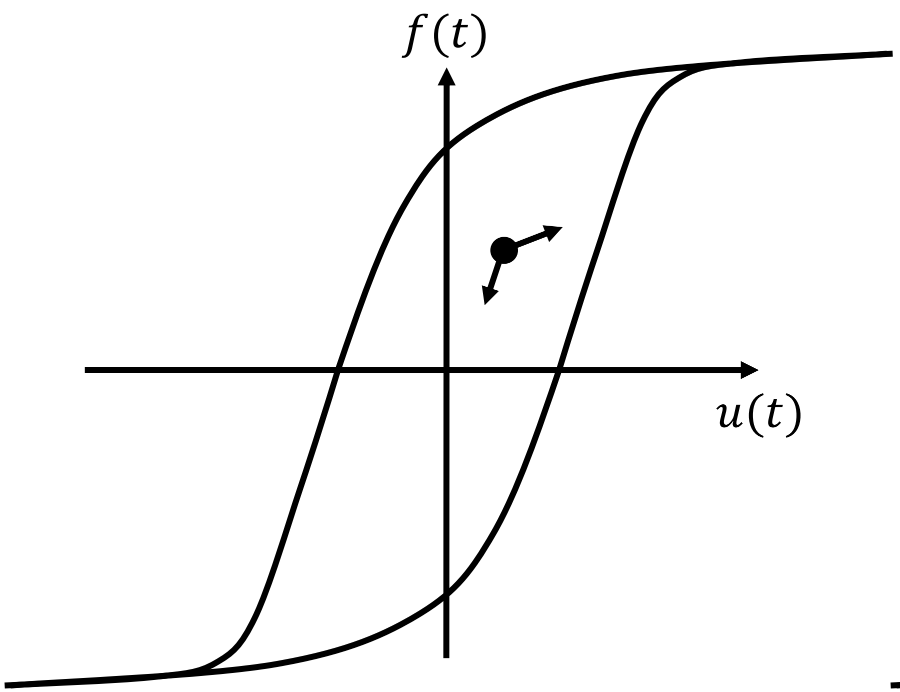
(Non)local Memory

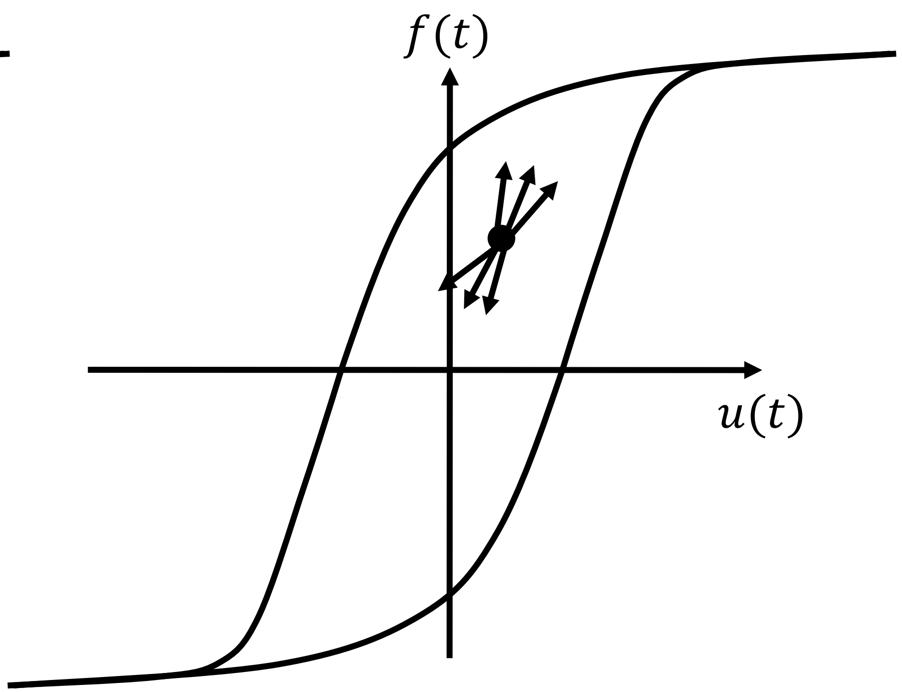
local memory

slope only determined by position



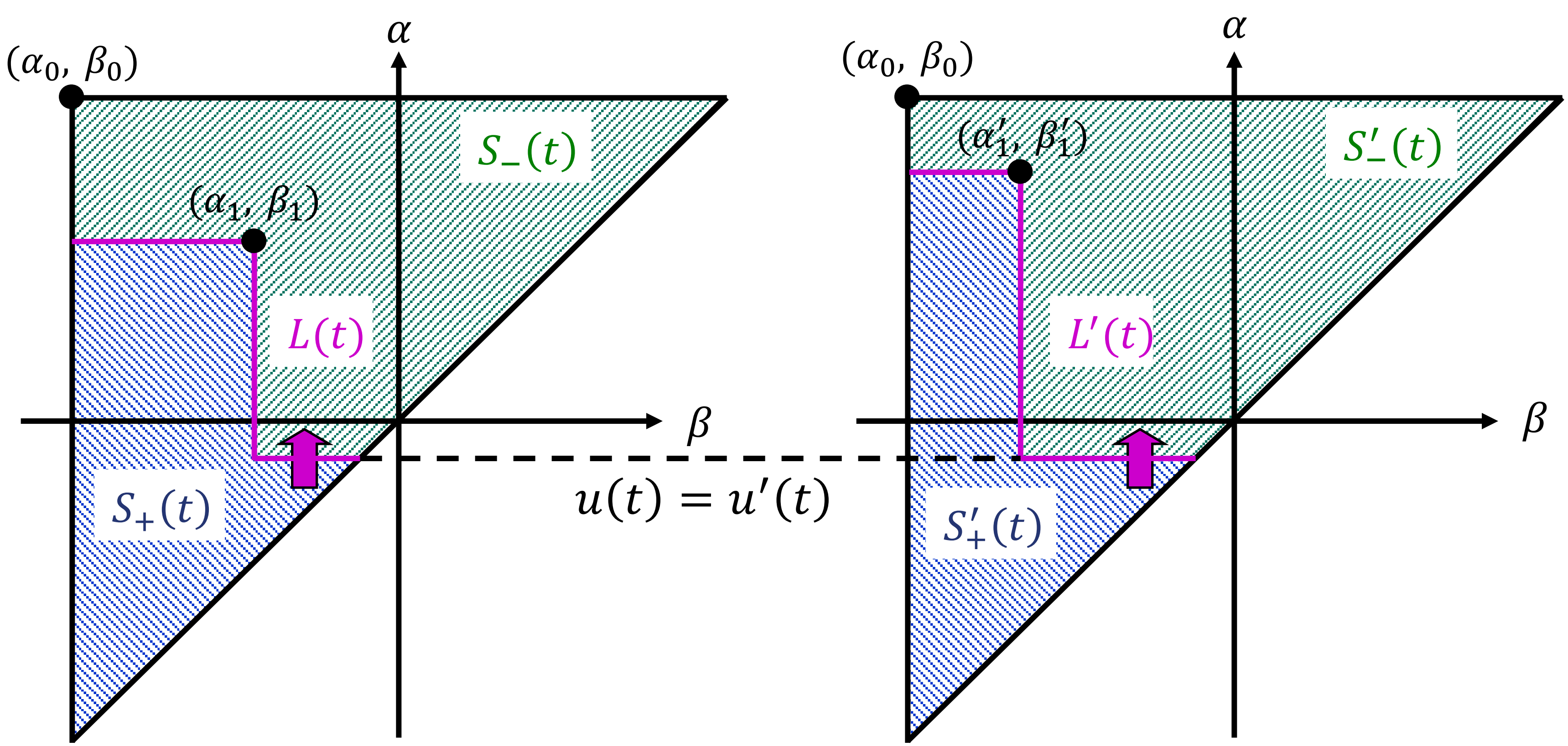
nonlocal memory

slope determined by position and history



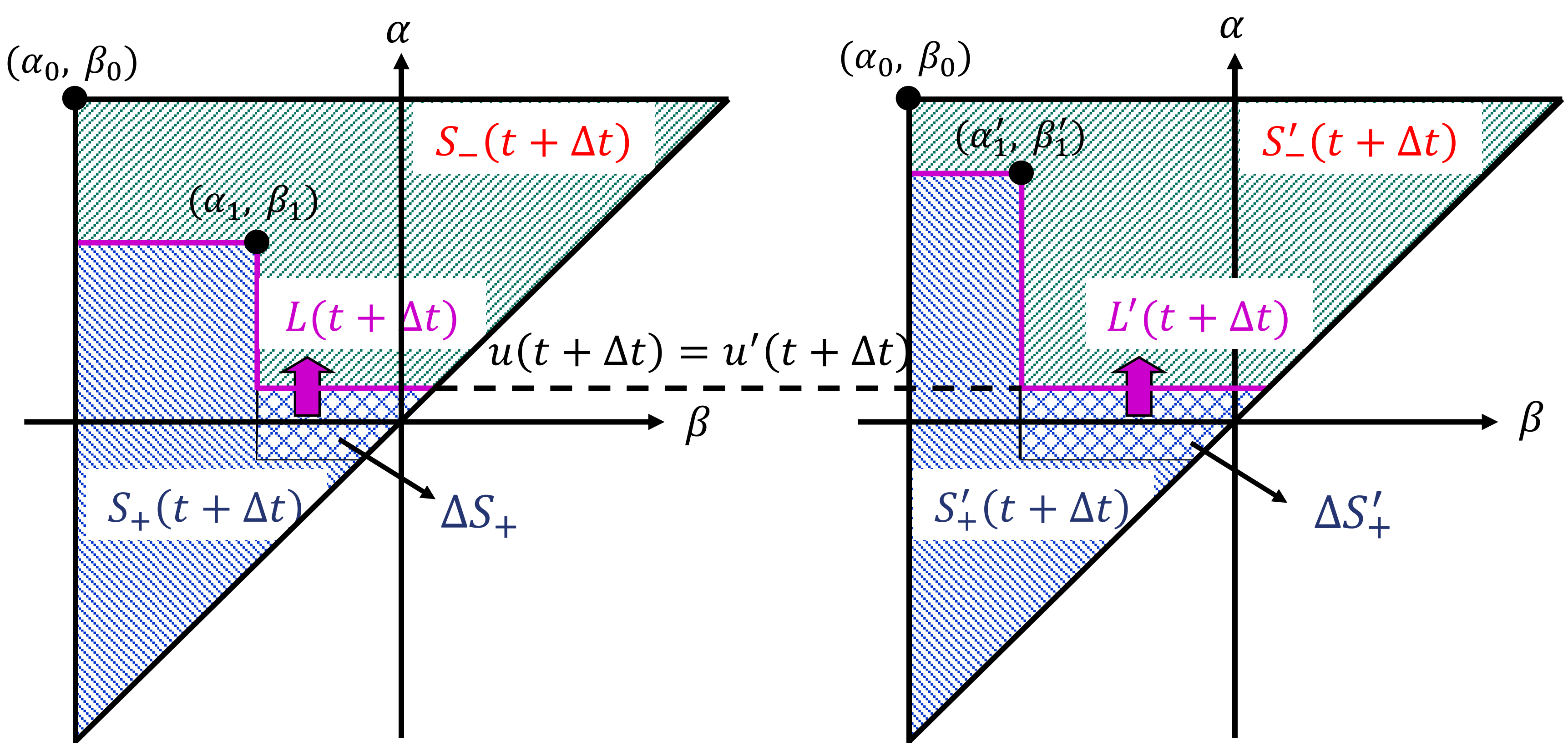
Nonlocal Memory

suppose $f(t) = f'(t) = \iint\limits_{S_+(t)} \mu - \iint\limits_{S_-(t)} \mu = \iint\limits_{S'_+(t)} \mu - \iint\limits_{S'_-(t)} \mu$



Nonlocal Memory

but $\Delta S'_+ \neq \Delta S_+ \Longrightarrow f(t+\Delta t) \neq f'(t+\Delta t)$



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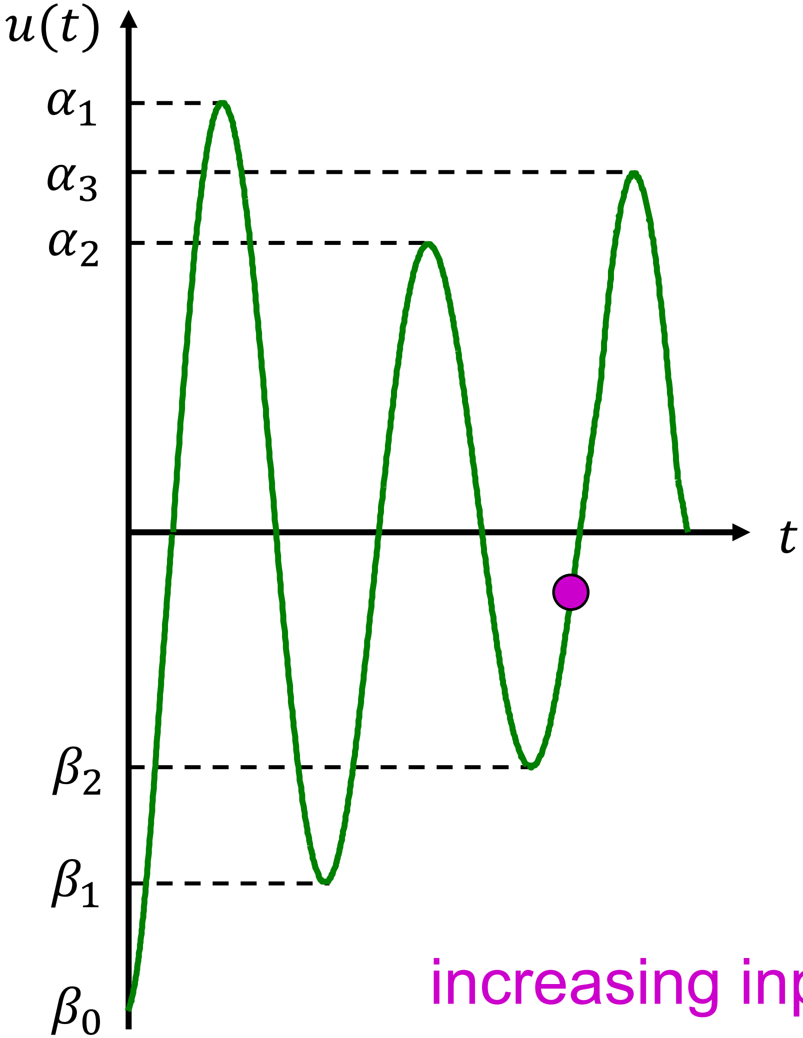
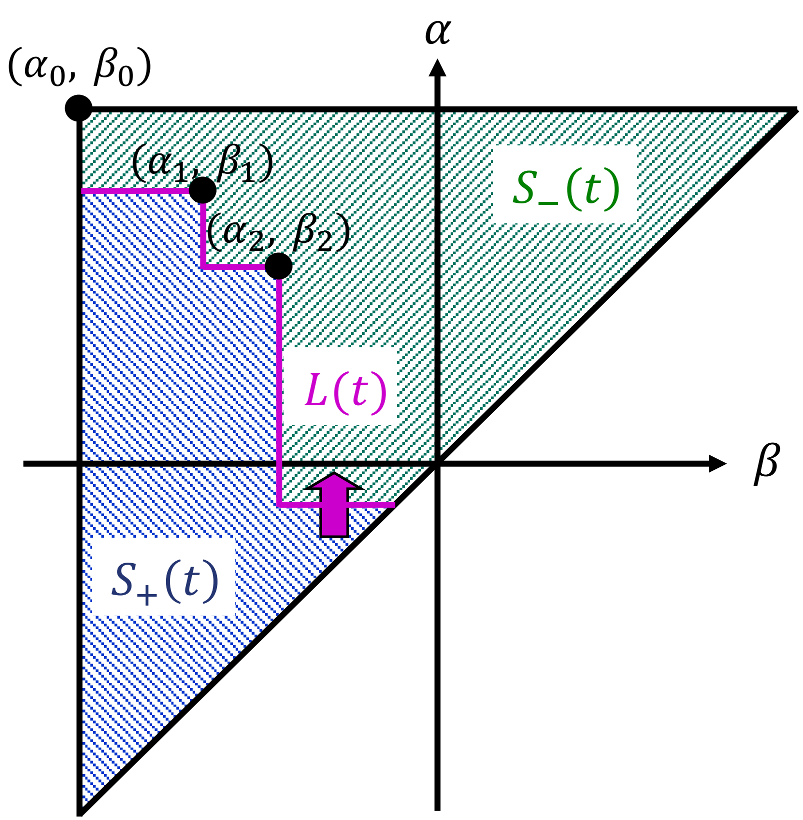
Wiping-Out Property





increasing input

Wiping-Out Property

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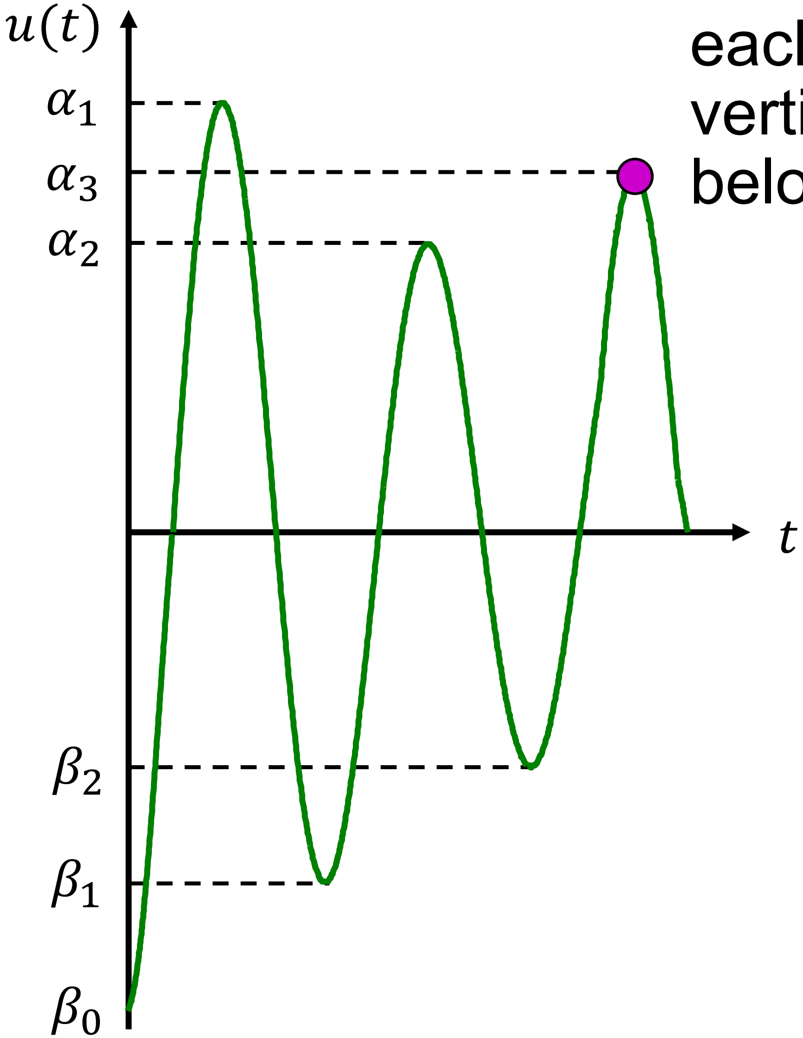
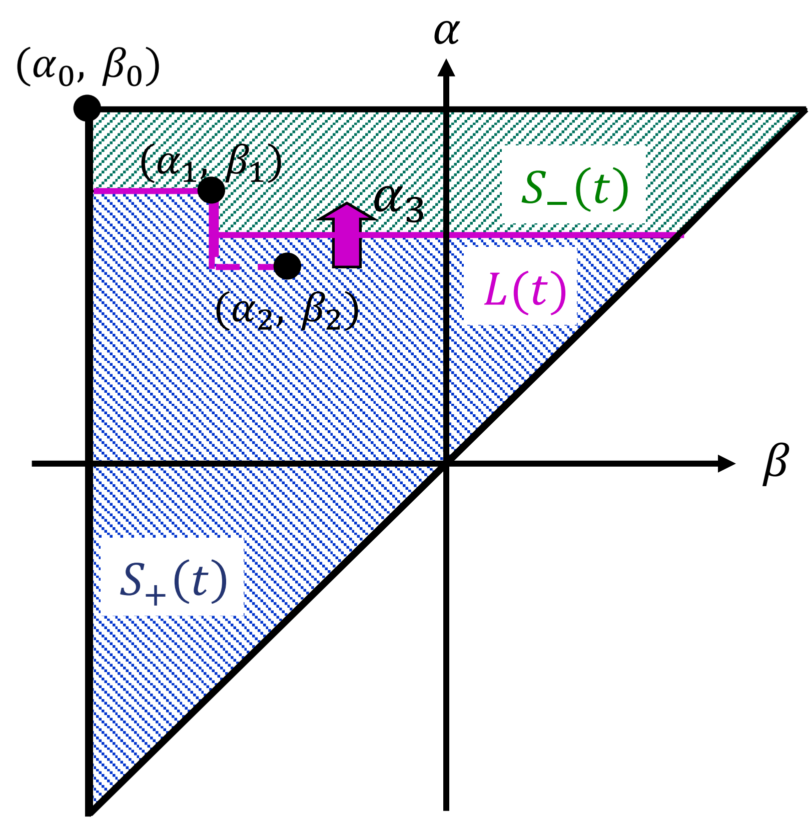


each local input maximum wipes out the vertices of L(t) whose α-coordinates are below this maximum



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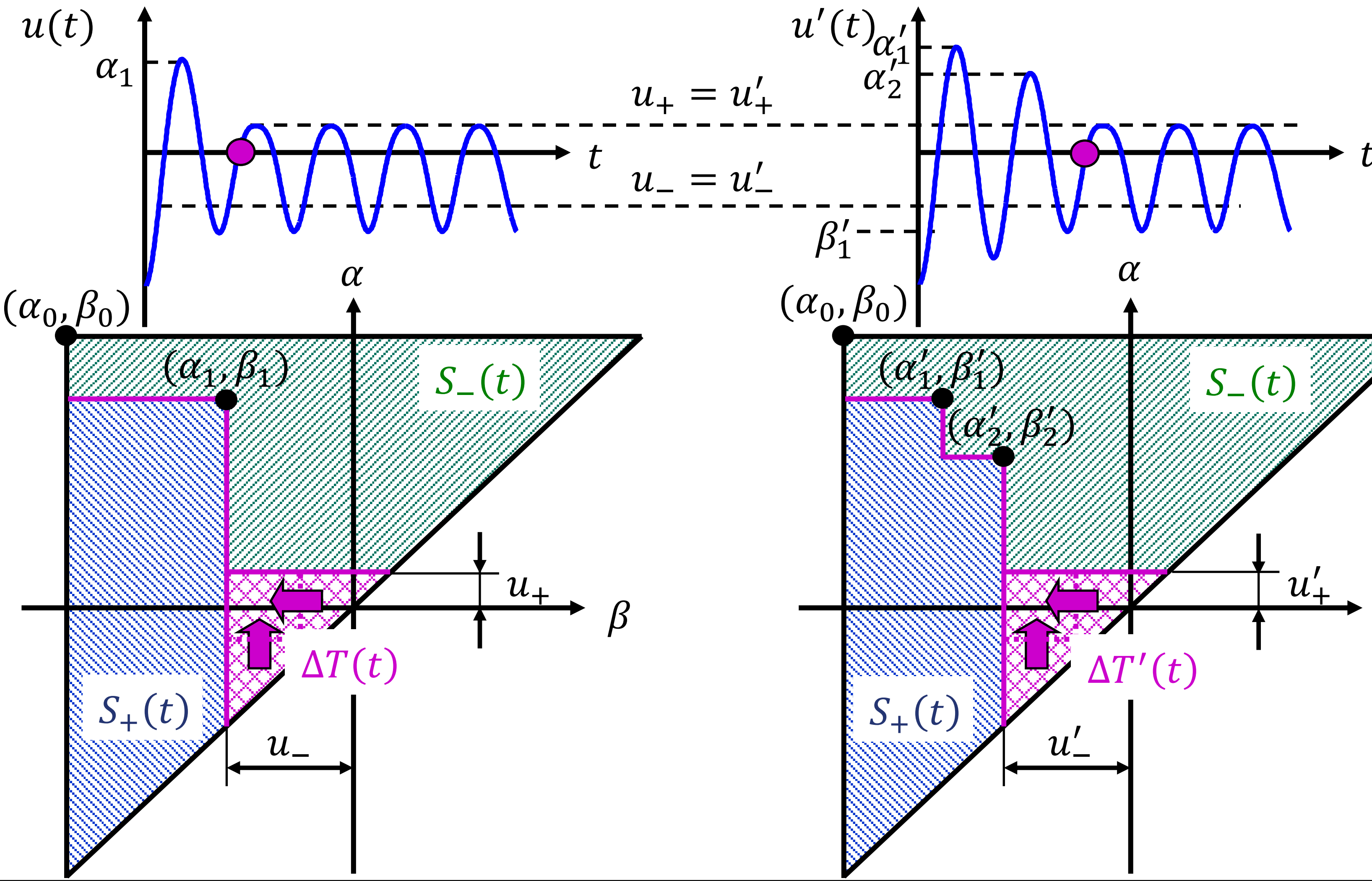
Congruency Property



Congruency Property

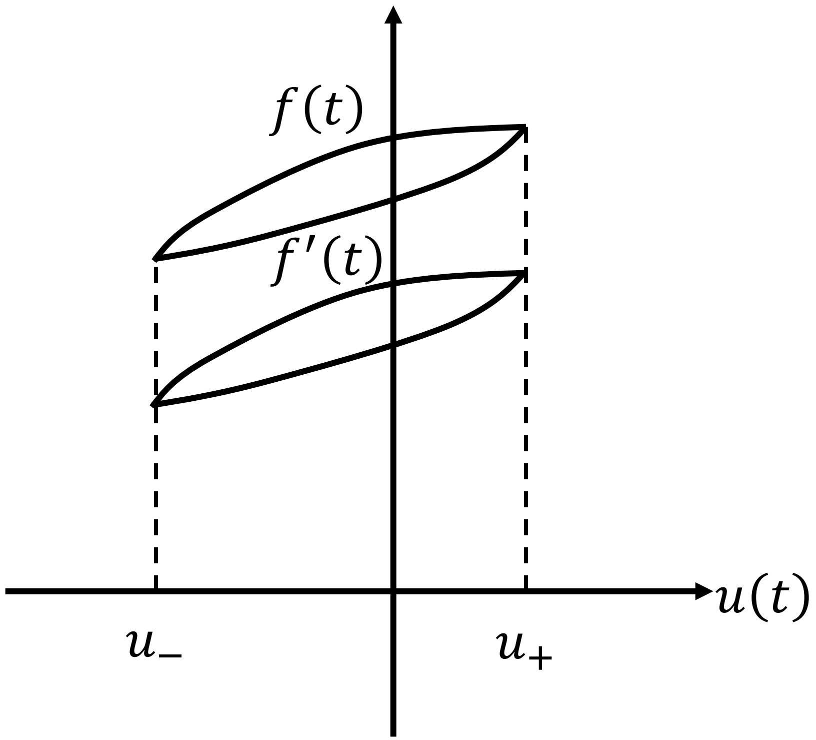
$$\Delta f(t) = 2 \iint\limits_{\Delta T(t)} \mu(\alpha,\beta)\, d\alpha d\beta$$

$$\Delta f'(t) = 2 \iint\limits_{\Delta T'(t)} \mu(\alpha,\beta)\, d\alpha d\beta$$

$$\Delta T'(t) = \Delta T(t)$$

$$\Downarrow$$

$$\Delta f'(t) = \Delta f(t)$$



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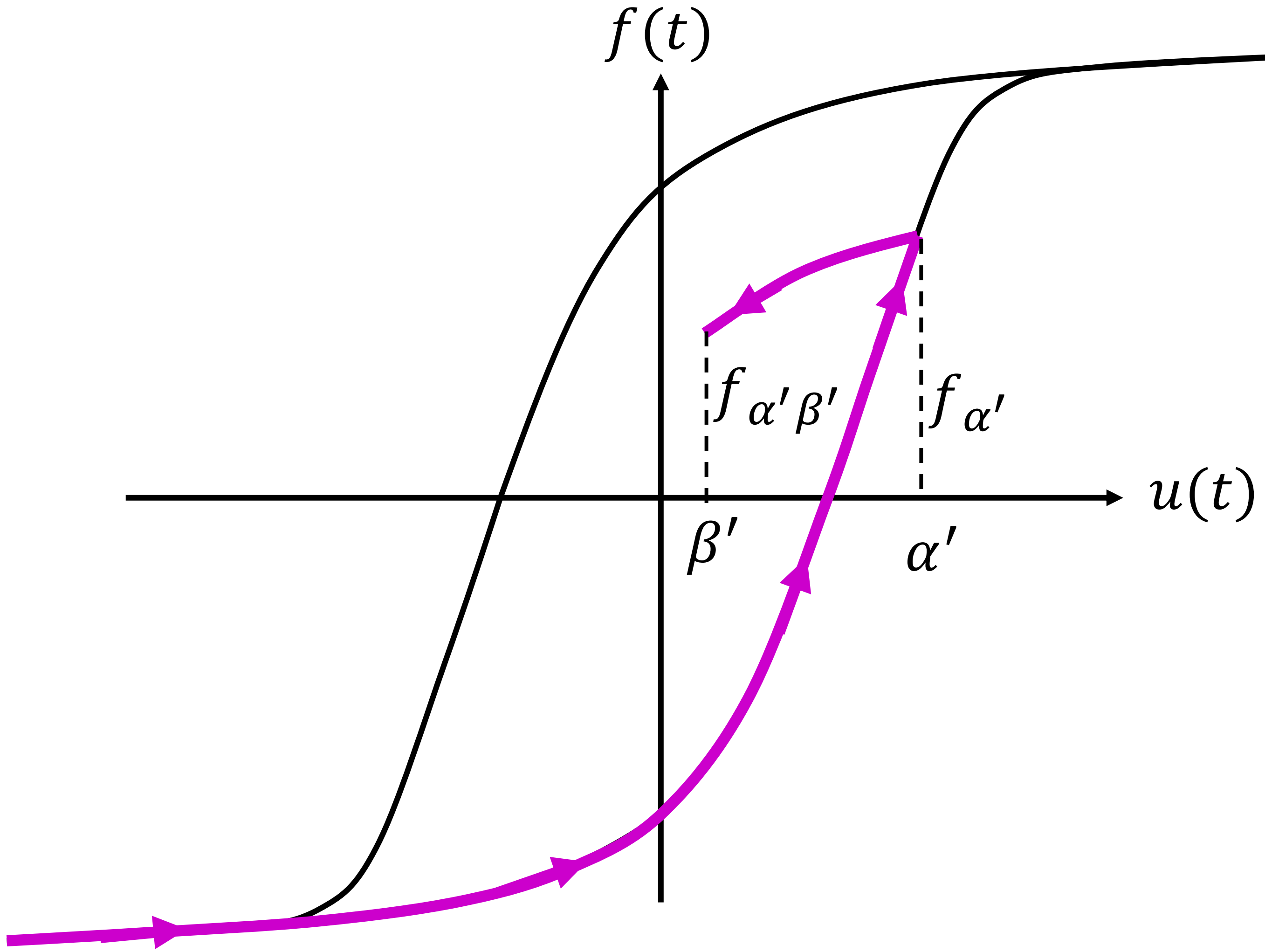
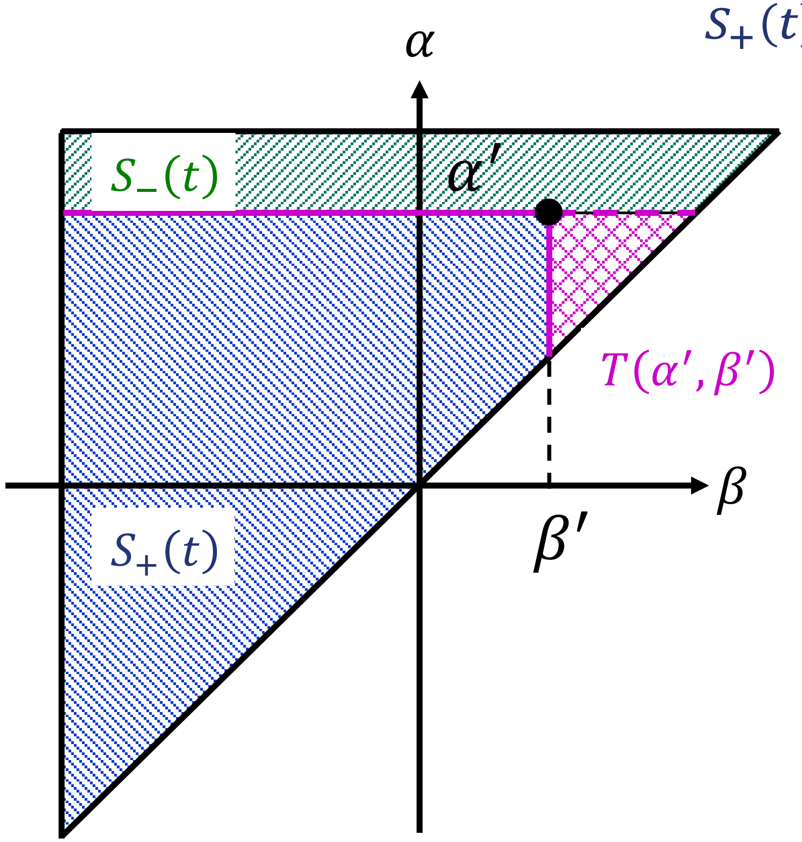
First-Order Transition Curves



Identification

$$f(t) = \iint_{S_+(t)} \mu(\alpha,\beta)\, d\alpha\, d\beta - \iint_{S_-(t)} \mu(\alpha,\beta)\, d\alpha\, d\beta$$



$$F(\alpha',\beta') = \frac{f_{\alpha'} - f_{\alpha'\beta'}}{2}$$

$$F(\alpha',\beta') = \iint_{T(\alpha',\beta')} \mu(\alpha,\beta)\, d\alpha d\beta$$

$$\mu(\alpha',\beta') = -\frac{\partial^2 F(\alpha',\beta')}{\partial\alpha'\partial\beta'}$$

$$\mu(\alpha',\beta') = \frac{1}{2}\frac{\partial^2 f_{\alpha'\beta'}}{\partial\alpha'\partial\beta'}$$

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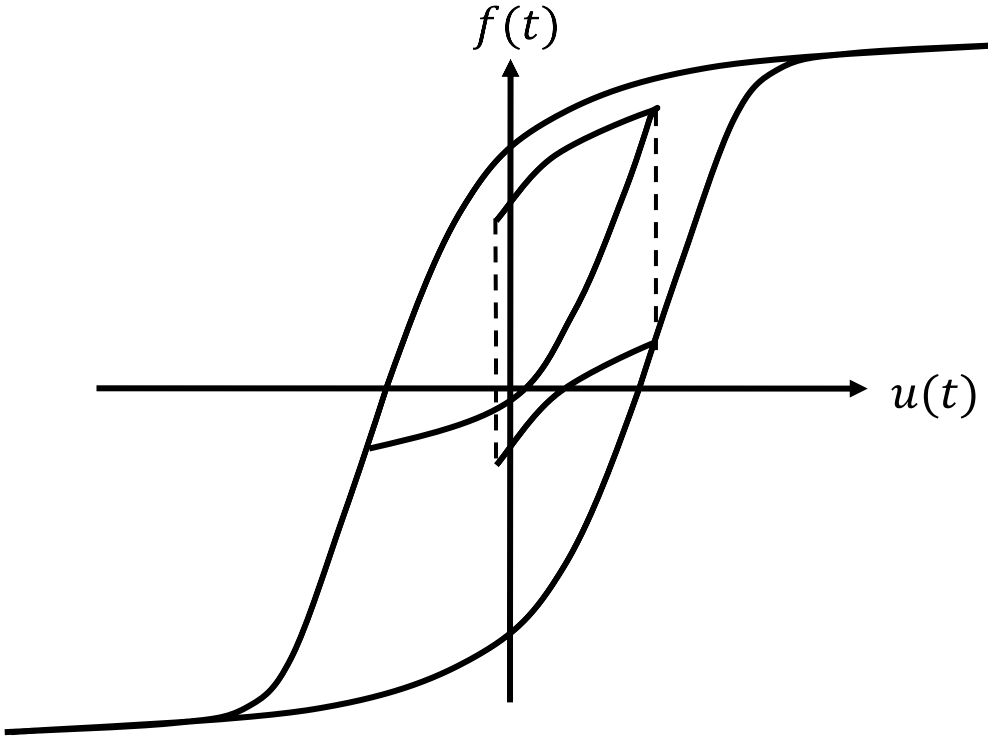
Representation Theorem

The wiping-out property and the congruency property constitute the necessary and sufficient conditions for a hysteresis phenomenon to be represented by the Preisach model based on a set of piecewise monotonic inputs



congruency

$\Downarrow$

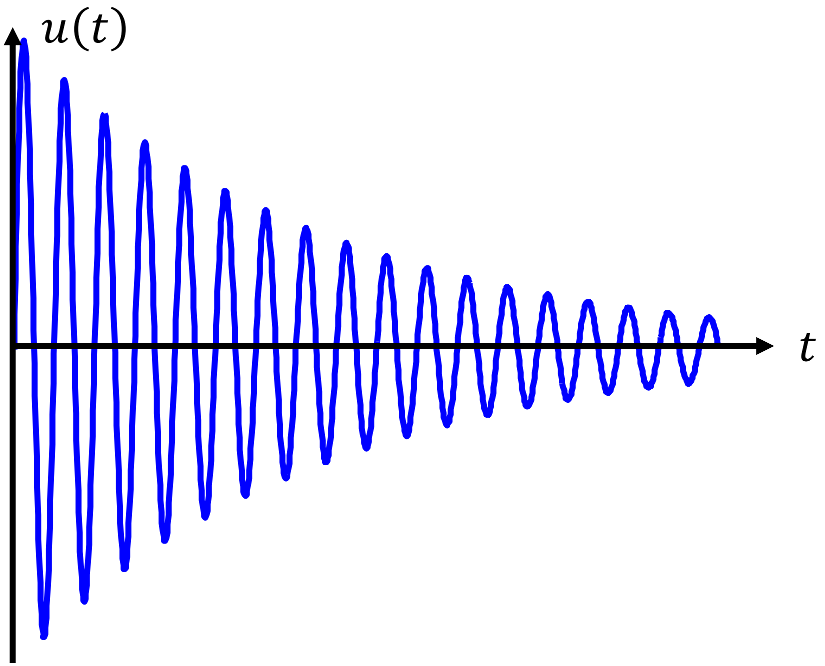
every minor loop is congruent to a first-order transition curve

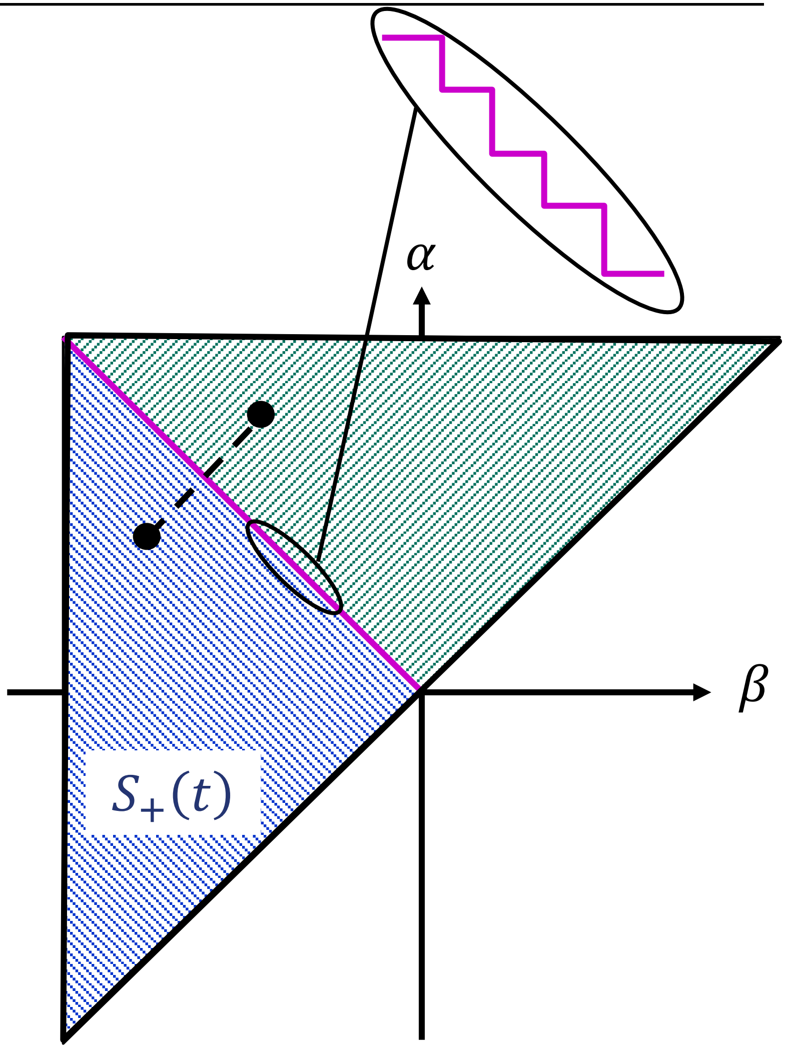
Representation Properties

1. symmetry for physical reasons :

$$\mu(-\beta', -\alpha') = \mu(\alpha', \beta')$$

2. demagnetized state : $f(t) = 0$





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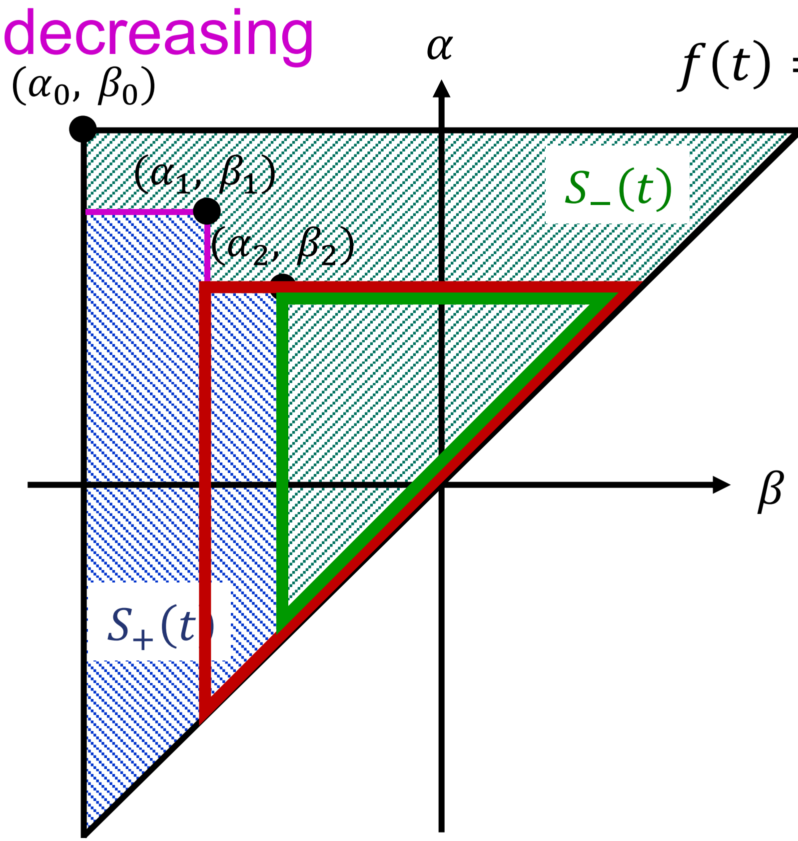
Numerical Implementation

$$f(t) = \iint_{S_+(t)} \mu(\alpha,\beta)\, d\alpha\, d\beta - \iint_{S_-(t)} \mu(\alpha,\beta)\, d\alpha\, d\beta$$



$$f(t) = \underbrace{-\iint_{T} \mu(\alpha,\beta)\, d\alpha\, d\beta}_{F(\alpha_0,\beta_0)} + 2 \underbrace{\iint_{S_+(t)} \mu(\alpha,\beta)\, d\alpha\, d\beta}_{\sum_{k=1}^{n(t)} \iint_{Q_k(t)} \mu(\alpha,\beta)\, d\alpha d\beta}$$

$$\iint_{Q_k(t)} = \iint_{T(\alpha_k,\beta_{k-1})} - \iint_{T(\alpha_k,\beta_{k-1})}$$

Numerical Implementation

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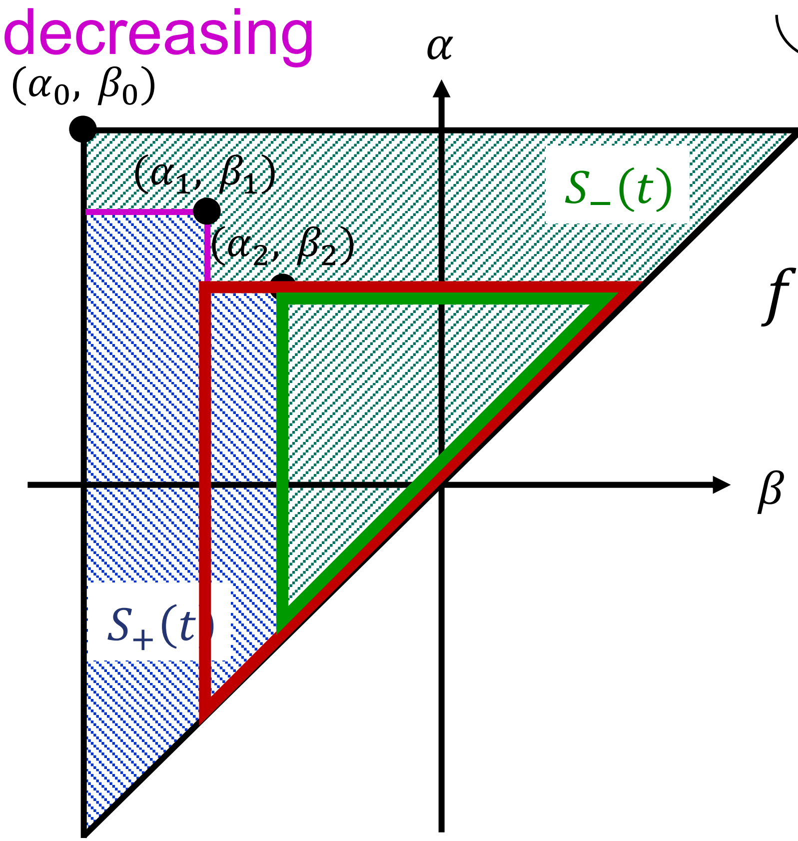
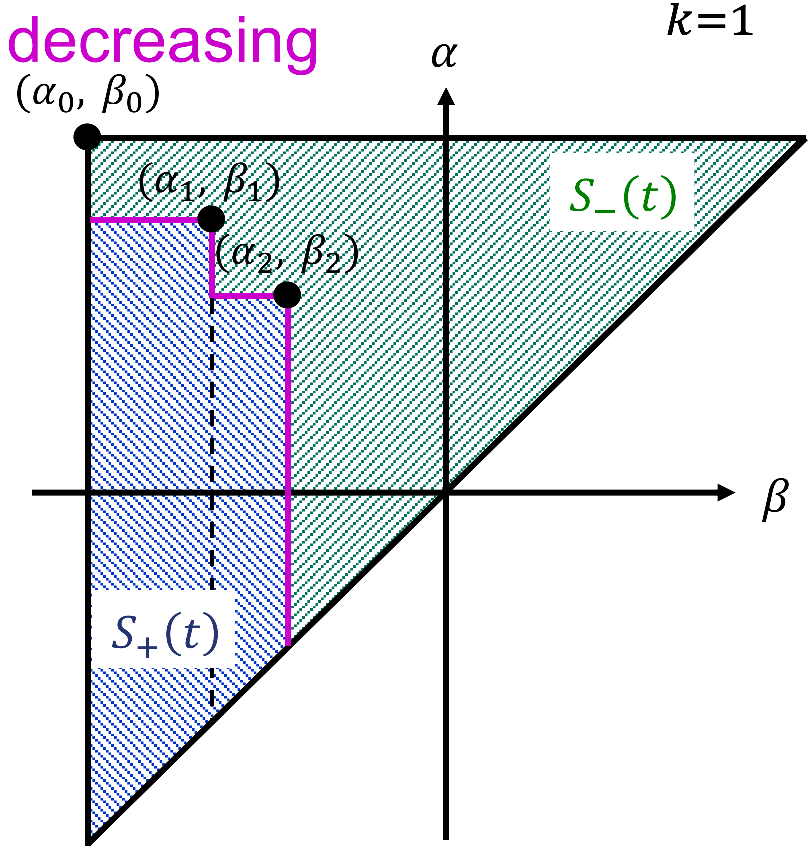
$$\iint\limits_{Q_k(t)} \mu(\alpha,\beta)\, d\alpha d\beta = \underbrace{\iint\limits_{T(\alpha_k,\beta_{k-1})} \mu(\alpha,\beta)\, d\alpha d\beta}_{F(\alpha_k,\beta_{k-1})} - \underbrace{\iint\limits_{T(\alpha_k,\beta_{k-1})} \mu(\alpha,\beta)\, d\alpha d\beta}_{F(\alpha_k,\beta_k)}$$



$$f(t) = -F(\alpha_0,\beta_0) + 2\sum_{k=1}^{n(t)-1} \left[F(\alpha_k,\beta_{k-1}) - F(\alpha_k,\beta_k)\right] + 2\left[F(\alpha_n,\beta_{n-1}) - F\big(\alpha_n,u(t)\big)\right]$$

Numerical Implementation

$$f(t) = -f^+ + 2\sum_{k=1}^{n-1}\left(f_{\alpha_k\beta_k} - f_{\alpha_k\beta_{k-1}}\right) + f_{\alpha_n}u(t) - f_{\alpha_n\beta_{n-1}}$$



set of first-order transition curves only for particular α and β

⇩

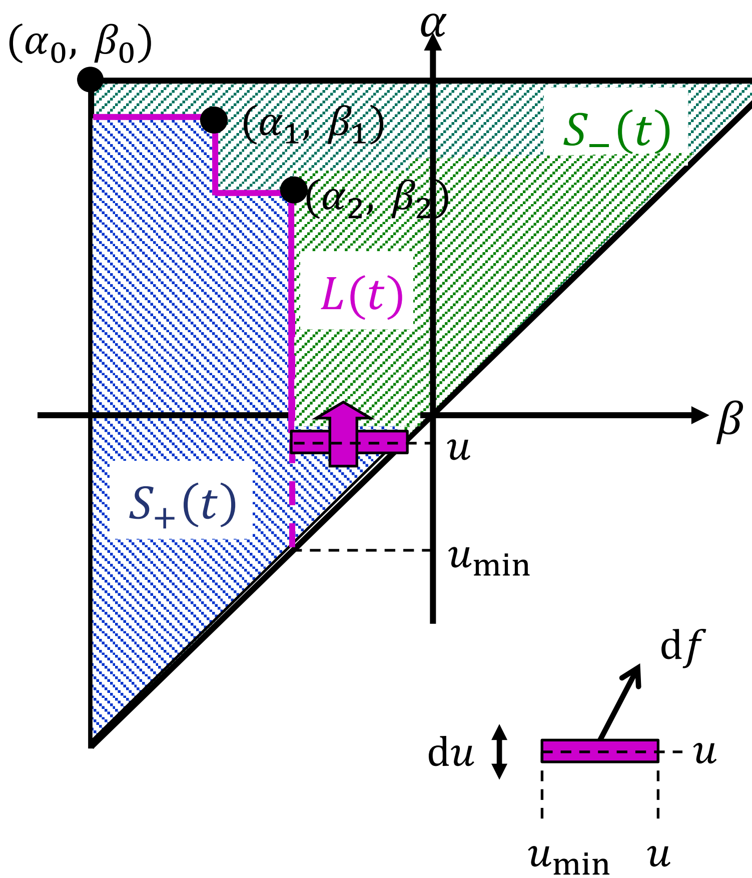
represent $f_{\alpha\beta}$ by bilinear interpolation

$$f_{\alpha\beta} = c_0 + c_1\alpha + c_2\beta + c_3\alpha\beta$$

store α_k and β_k as representation of the material state

Slope (increasing input)

increasing input



slope (increasing branch)

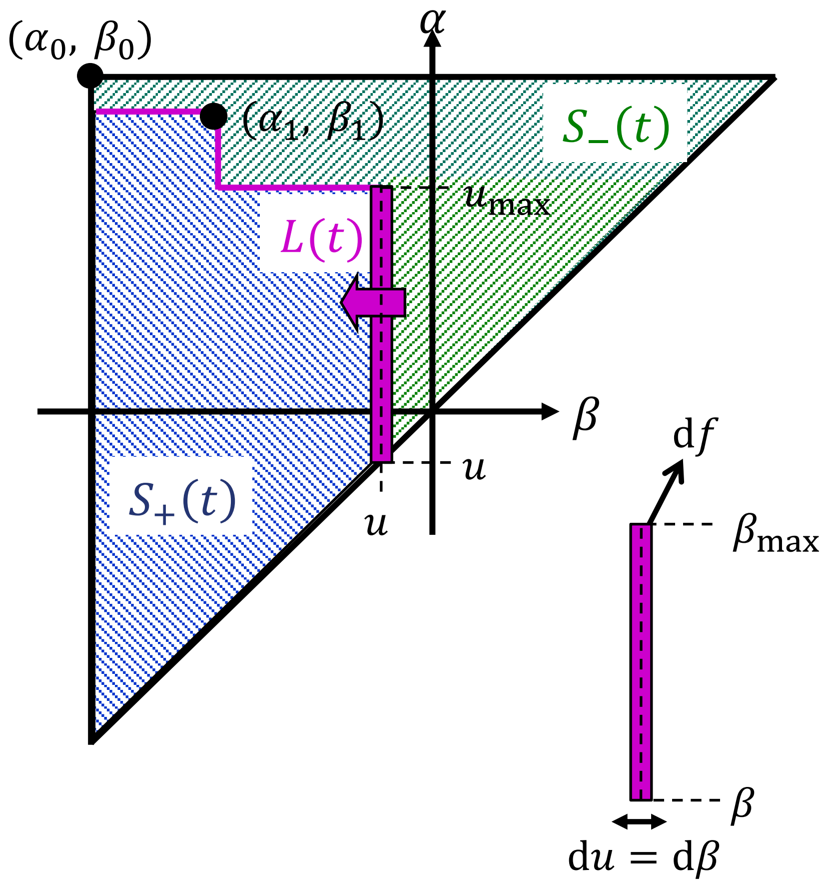
$$\mu_{\mathrm{incr}}^{\partial}(t) = \frac{df(t)}{du}$$

$$\mu_{\mathrm{incr}}^{\partial}(t) = \frac{d}{du}\left(2\int_{u_{\min}}^{u} \mathrm{d}\beta \int_{u}^{u+\mathrm{d}u} \mu(\alpha,\beta)\,\mathrm{d}\alpha\right)$$

$$\mu_{\mathrm{incr}}^{\partial}(t) = 2\int_{u_{\min}}^{u} \mu(u,\beta)\,\mathrm{d}\beta$$

Slope (decreasing input)

decreasing input



slope (decreasing branch)

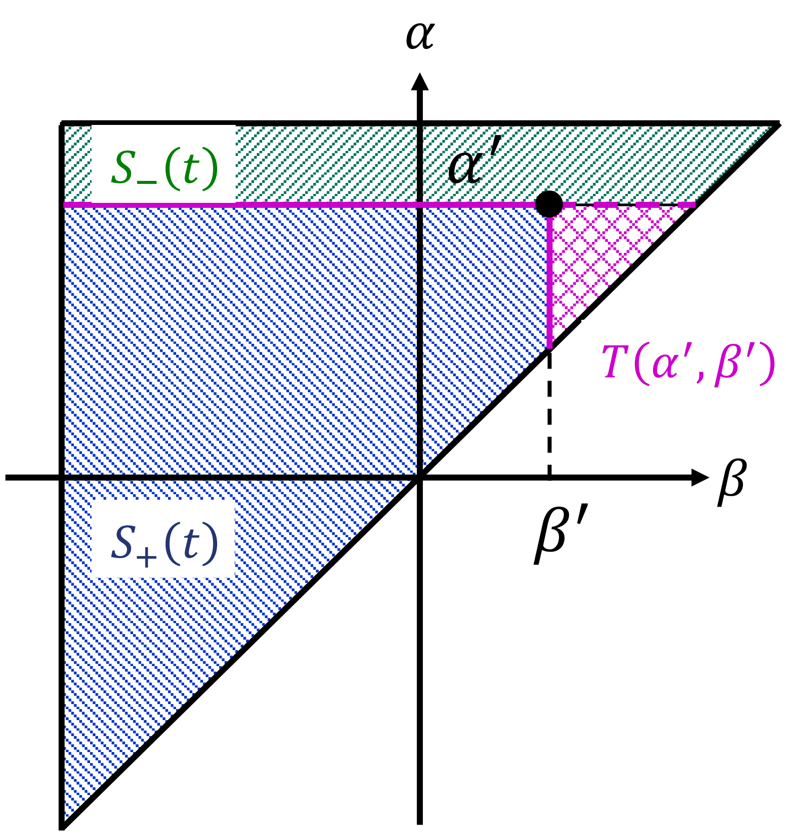
$$\mu_{\mathrm{decr}}^{\partial}(t) = \frac{df(t)}{du}$$

$$\mu_{\mathrm{decr}}^{\partial}(t) = \frac{d}{du}\left(2\int_{u}^{u_{\max}} \mathrm{d}\alpha \int_{u}^{u+\mathrm{d}u} \mu(\alpha,\beta)\mathrm{d}\beta\right)$$

$$\mu_{\mathrm{decr}}^{\partial}(t) = 2\int_{u}^{u_{\max}} \mu(\alpha,u)\,\mathrm{d}\alpha$$

Exercise (1)

(simple) Preisach distribution function : $\mu(\alpha,\beta)=\gamma=\text{ct}$



Everett function :

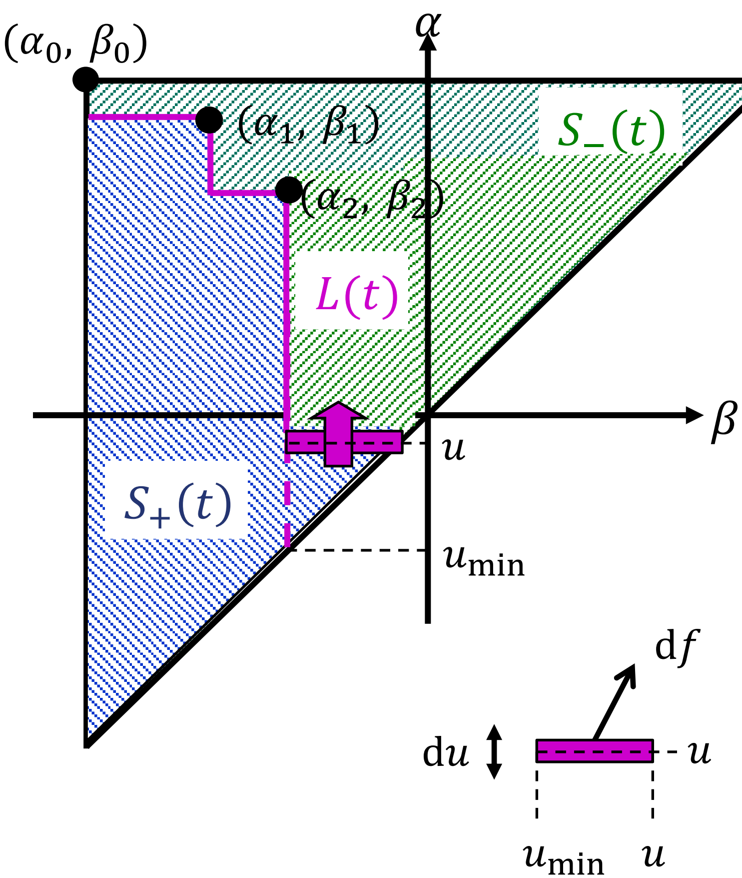
$$F(\alpha',\beta')=\iint_{T(\alpha',\beta')}\mu(\alpha,\beta)\,d\alpha d\beta$$

$$F(\alpha',\beta')=\gamma\frac{(\alpha'-\beta')^2}{2}$$

check that $\mu(\alpha',\beta')=-\frac{\partial^2 F(\alpha',\beta')}{\partial\alpha'\partial\beta'}$

Exercise (2)

increasing input



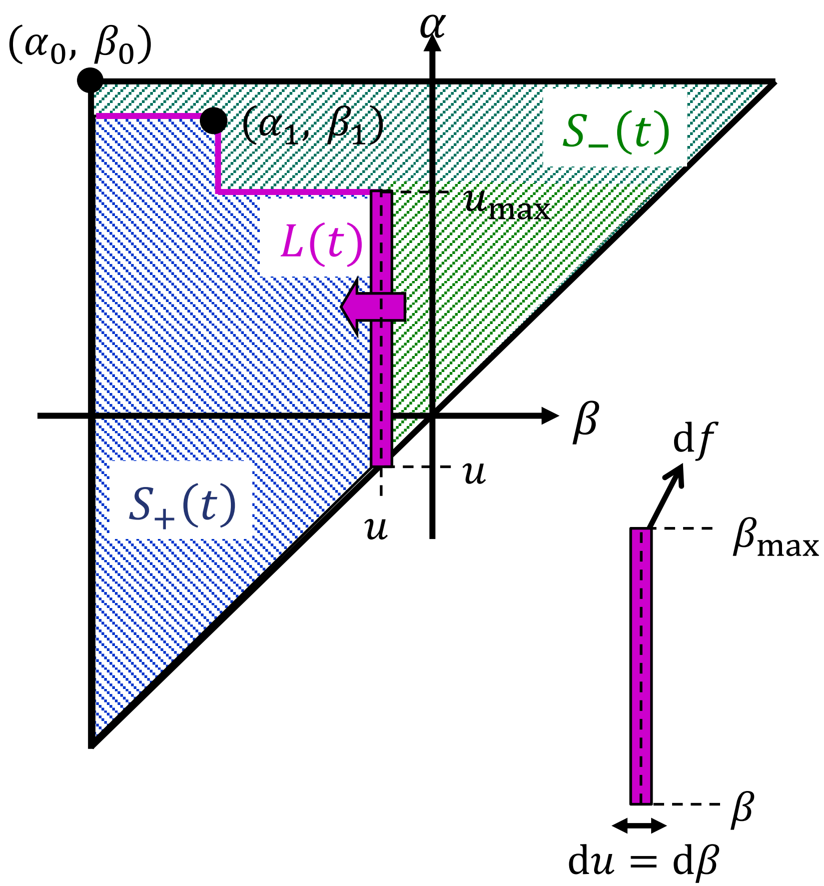
slope (increasing branch)

$$\mu_{\mathrm{incr}}^{\partial}(t) = 2\int_{u_{\min}}^{u} \mu(u,\beta)\,\mathrm{d}\beta$$

$$\mu_{\mathrm{incr}}^{\partial}(t) = 2\gamma(u - u_{\min})$$

Exercise (3)

decreasing input



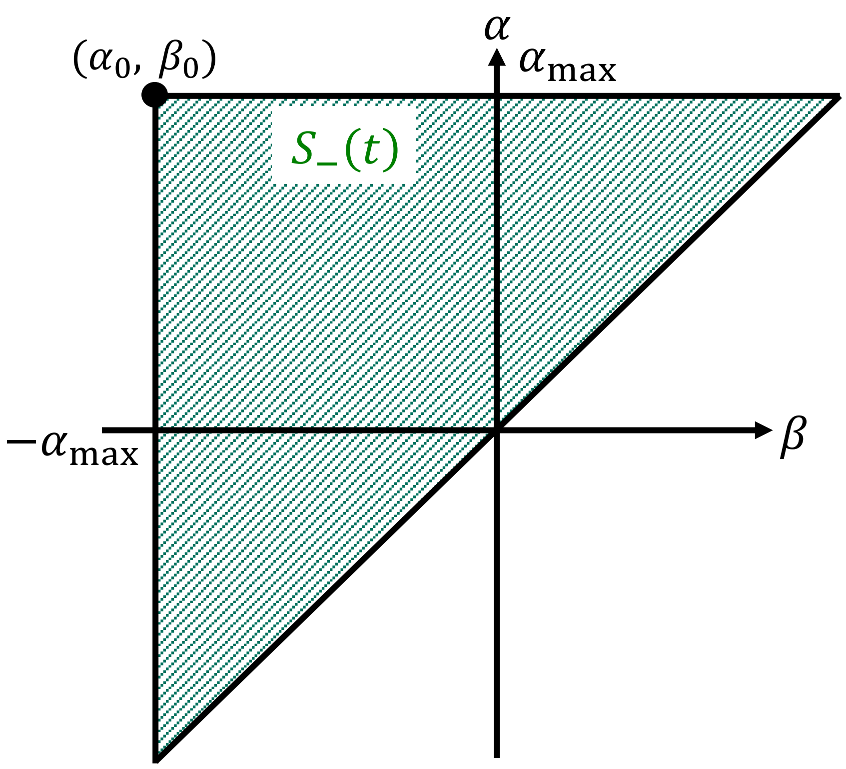
slope (decreasing branch)

$$\mu^{\partial}_{\mathrm{decr}}(t) = 2 \int_{u}^{u_{\max}} \mu(\alpha, u)\, \mathrm{d}\alpha$$

$$\mu^{\partial}_{\mathrm{decr}}(t) = 2\gamma(u_{\max} - u)$$

Exercise (4)

time instant t_{00} (starting point): $u = -\alpha_{\max}$ (negative saturation)



$$u(t_{00}) = -\alpha_{\max}$$

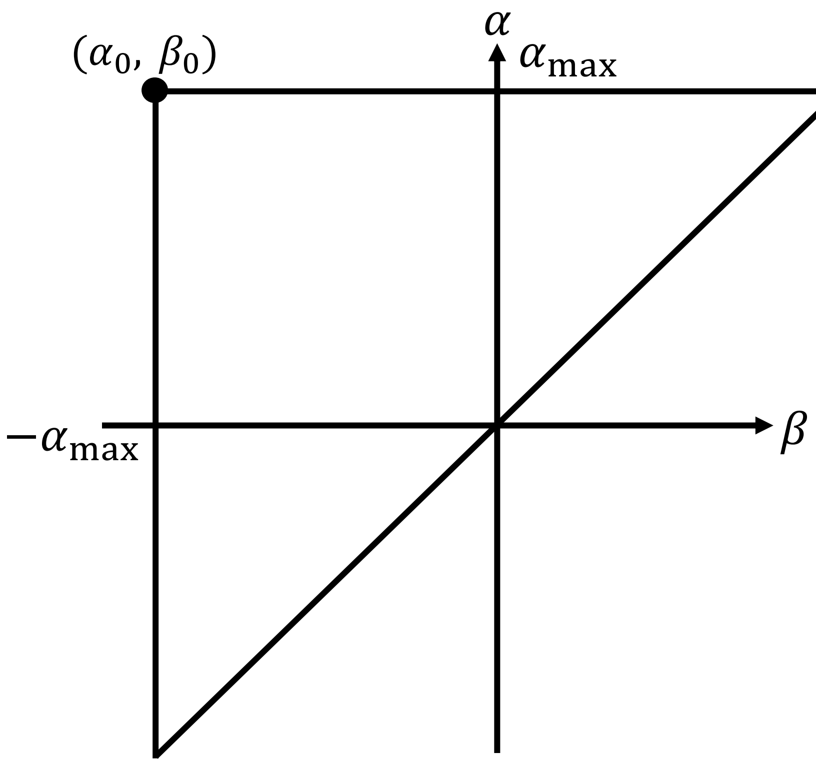
$$f(t_{00}) = -F(\alpha_0, \beta_0) =$$

$$\mu^{\partial}_{\mathrm{incr}}(t_{00}) =$$

$$\mu^{\partial}_{\mathrm{decr}}(t_{00}) =$$

Exercise (5)

time instant t_{10} : increase to $u(t_{10}) = \alpha_{\max}/2$



$u(t_{10}) = \alpha_{\max}/2$

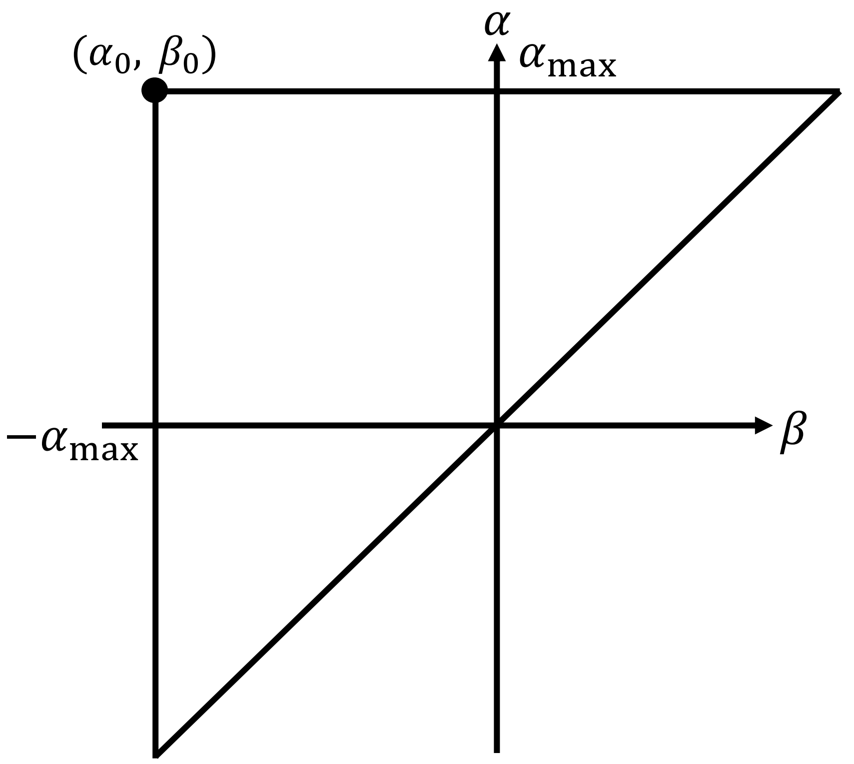
$f(t_{10}) =$

$\mu^{\partial}_{\text{incr}}(t_{10}) =$

$\mu^{\partial}_{\text{decr}}(t_{10}) =$

Exercise (6)

time instant t_{11} : decrease to $u(t_{11}) = -\alpha_{\max}/2$



$u(t_{11}) = -\alpha_{\max}/2$

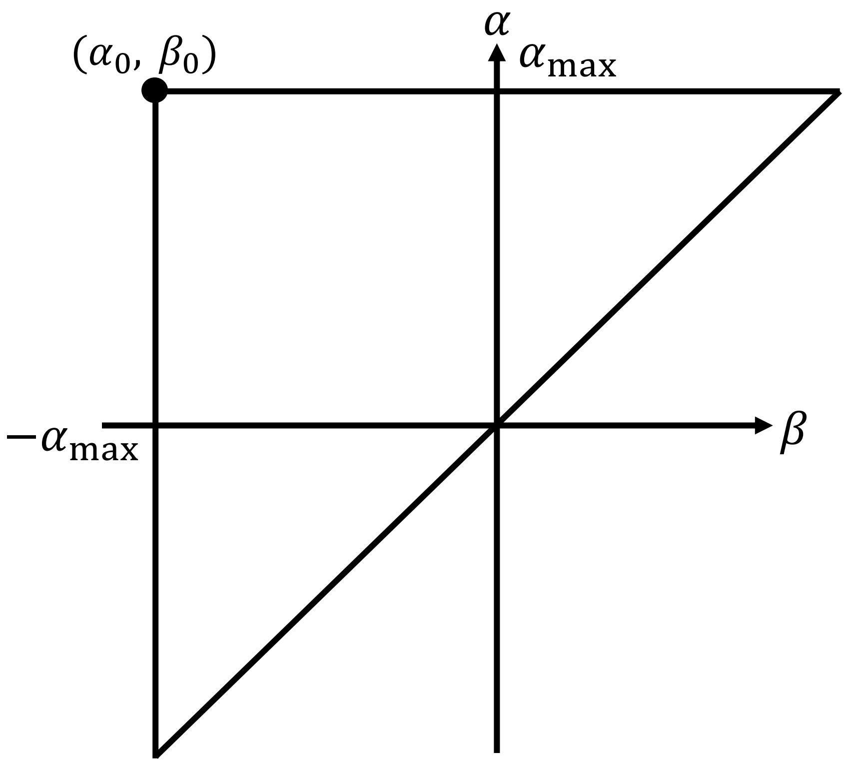
$f(t_{11}) =$

$\mu^{\partial}_{\mathrm{incr}}(t_{11}) =$

$\mu^{\partial}_{\mathrm{decr}}(t_{11}) =$

Exercise (7)

time instant t_{21} : increase to $u(t_{21}) = 0$



$u(t_{21}) = 0$

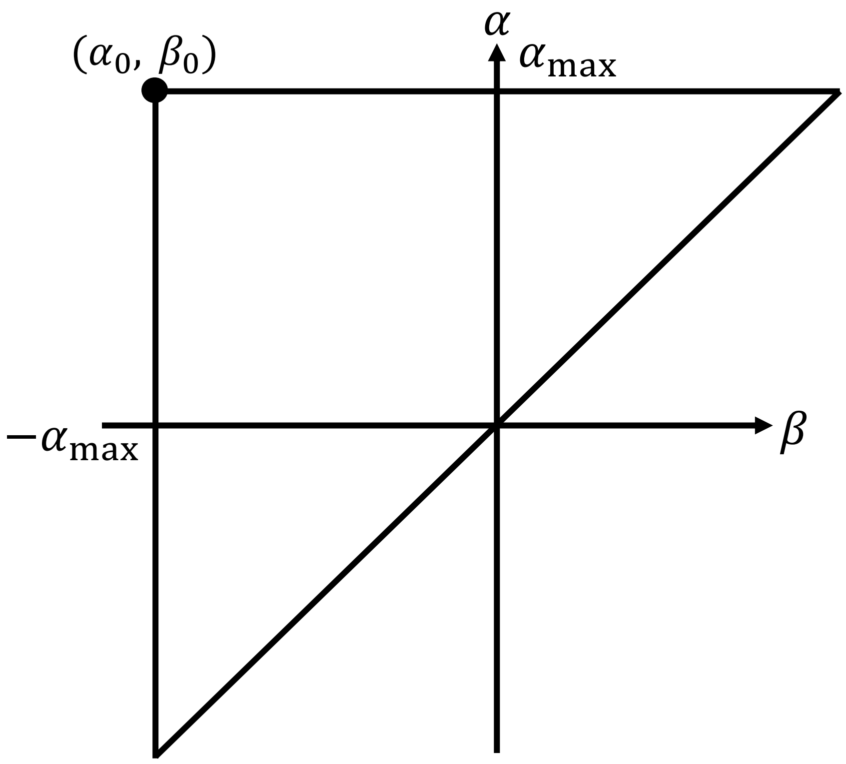
$f(t_{21}) =$

$\mu^{\partial}_{\text{incr}}(t_{21}) =$

$\mu^{\partial}_{\text{decr}}(t_{21}) =$

Exercise (8)

time instant t_{22} : decrease to $u(t_{22}) = -3\alpha_{\max}/4$



$u(t_{22}) = -3\alpha_{\max}/4$

$f(t_{22}) =$

$\mu_{\text{incr}}^{\partial}(t_{22}) =$

$\mu_{\text{decr}}^{\partial}(t_{22}) =$

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Linearisation

operating point $P:(H^{(n)},B^{(n)})$

chord reluctivity $\nu^{(n)}$

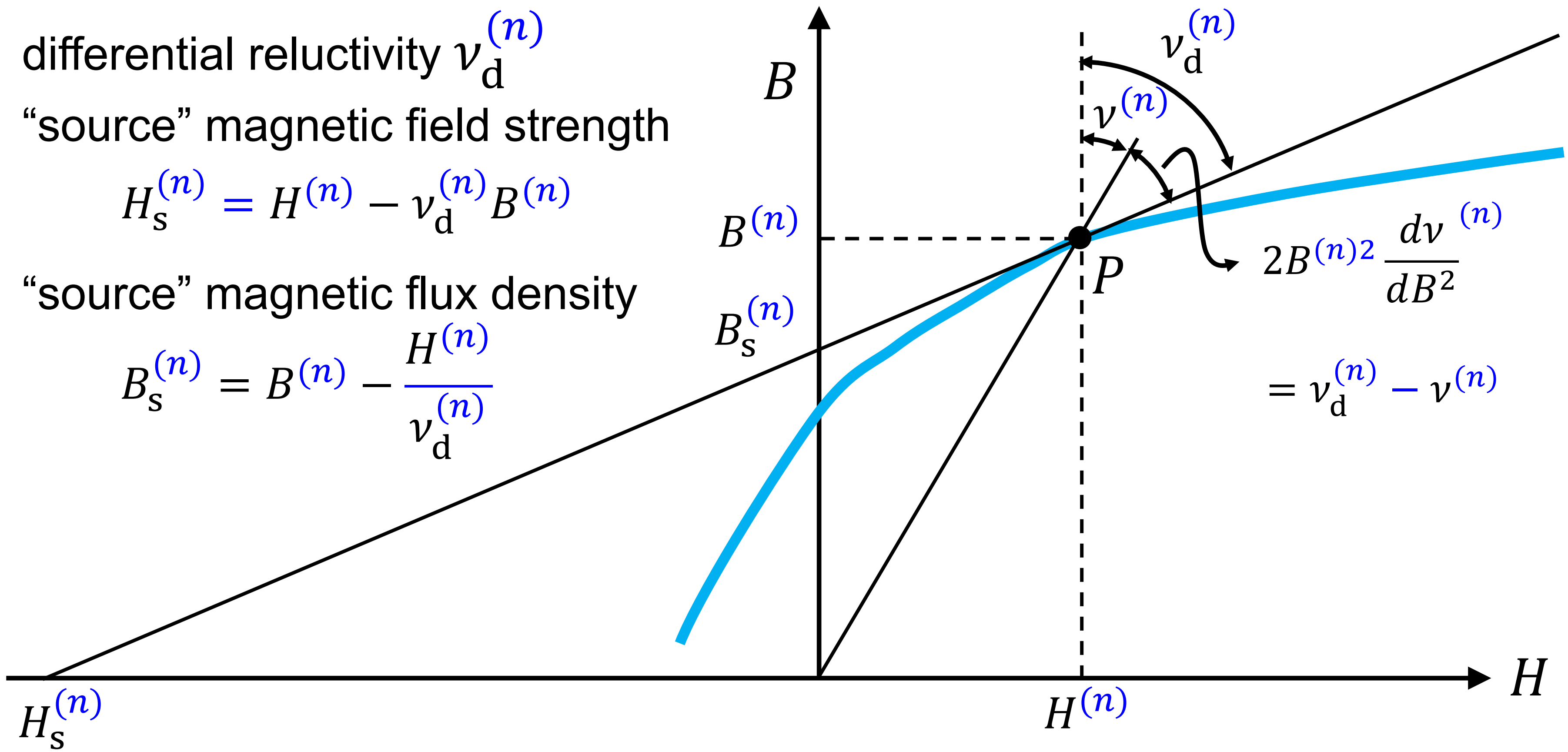
differential reluctivity $\nu_{\mathrm{d}}^{(n)}$

"source" magnetic field strength

$$H_{\mathrm{s}}^{(n)}=H^{(n)}-\nu_{\mathrm{d}}^{(n)}B^{(n)}$$

"source" magnetic flux density

$$B_{\mathrm{s}}^{(n)}=B^{(n)}-\frac{H^{(n)}}{\nu_{\mathrm{d}}^{(n)}}$$



Magnetoquasistatics

modified magnetic vector potential

$$\nabla \cdot \vec{B} = 0 \quad \Rightarrow \quad \vec{B} = 0 + \nabla \times \vec{A}^*$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \Rightarrow \quad \vec{E} = -\frac{\partial \vec{A}^*}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \cdot \vec{J} = 0$$

$$\vec{H} = \nu \vec{B} + \vec{H}_c$$

$$\vec{J} = \sigma \vec{E} + \vec{J}_s$$

$$\boxed{\nabla \times \left(\nu \nabla \times \vec{A}^*\right) + \sigma \frac{\partial \vec{A}^*}{\partial t} = \vec{J}_s - \nabla \times \vec{H}_c}$$

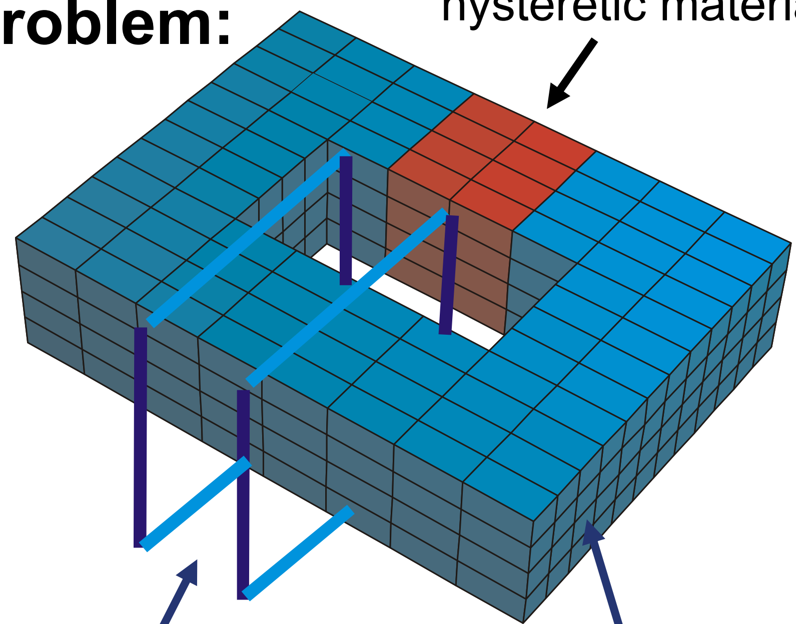
$$\Downarrow$$

$$\tilde{\mathbf{C}}\mathbf{M}_\nu \mathbf{C}\widehat{\mathbf{a}} + \mathbf{M}_\sigma \frac{d\widehat{\mathbf{a}}}{dt} = \widehat{\widehat{\mathbf{j}}}_s - \tilde{\mathbf{C}}\widehat{\mathbf{h}}_c$$

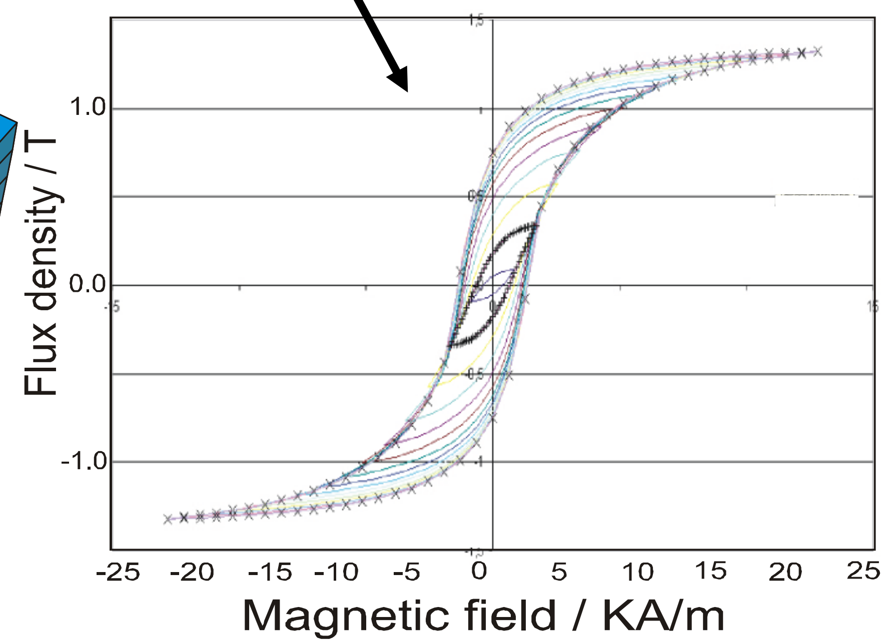
Example

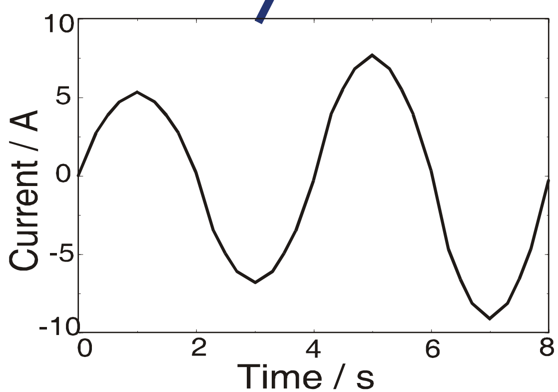
test problem:

hysteretic material:



measured hysteretic first order transient curves



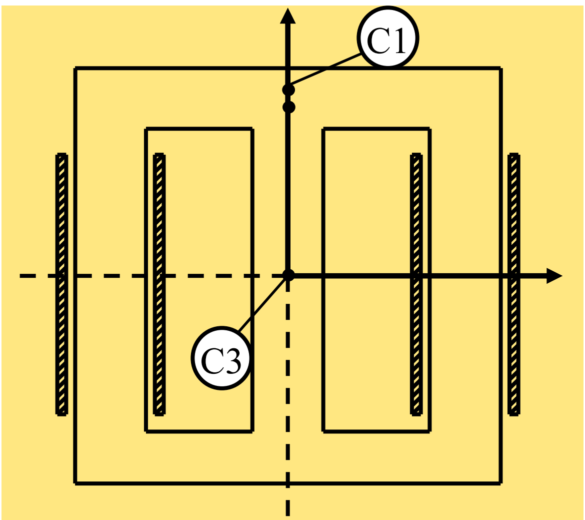


non-hysteretic nonlinear material

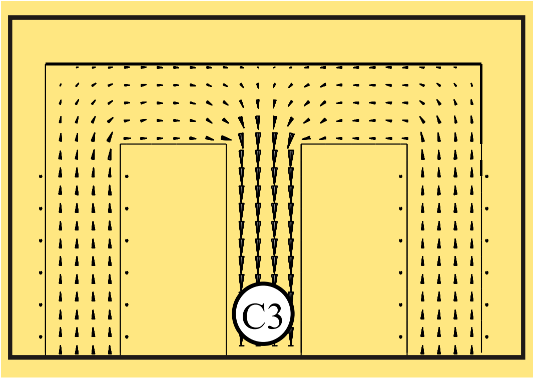
research Dr.-Ing. Jing Yuan

Vector Hysteresis

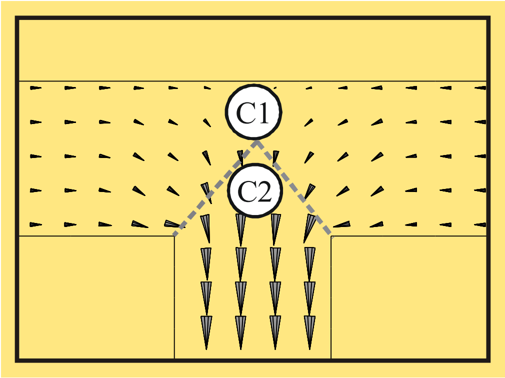
transformator

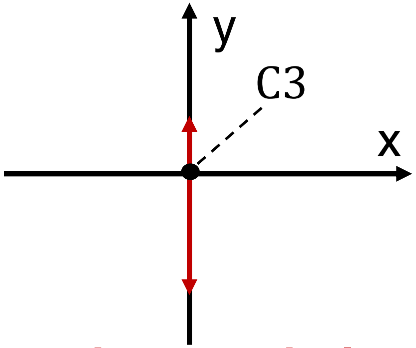


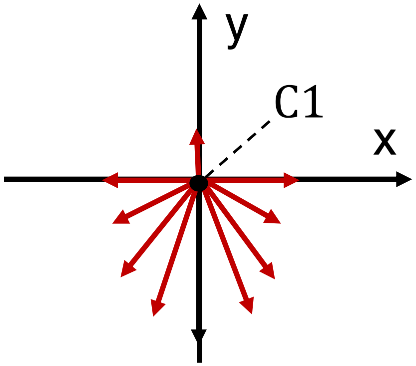
flux density at position C3



flux density at position C1







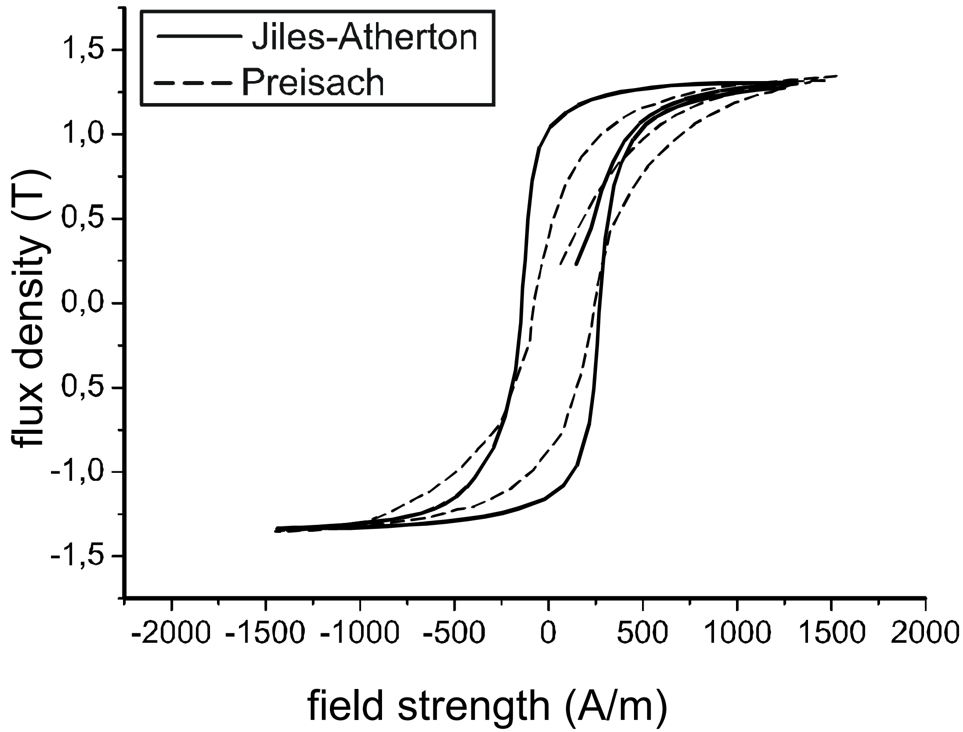
simplified vector-Preisach model =

scalar hysteresis model + direction factor

Scalar Hysteresis Models

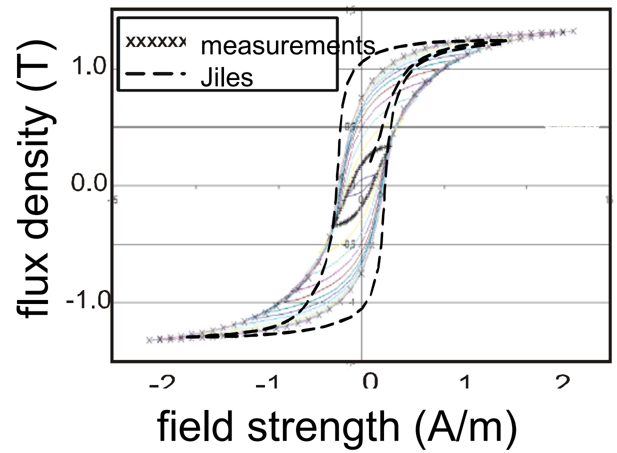
Comparison:

Preisach model with Jiles-Atherton model

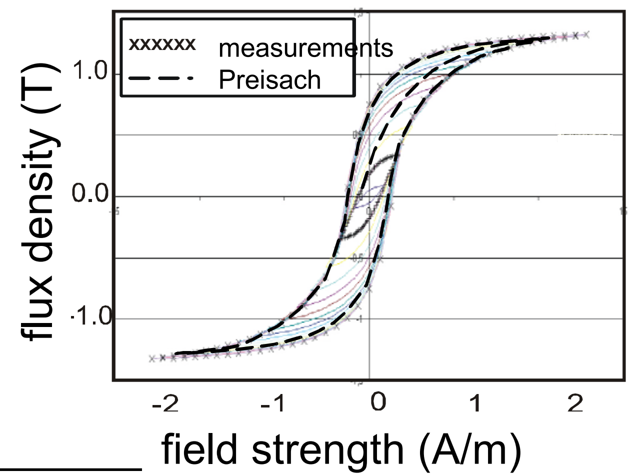


- magnetoquasistatische simulations with BDF1
- 70 time steps

Jiles-Atherton model and measurements

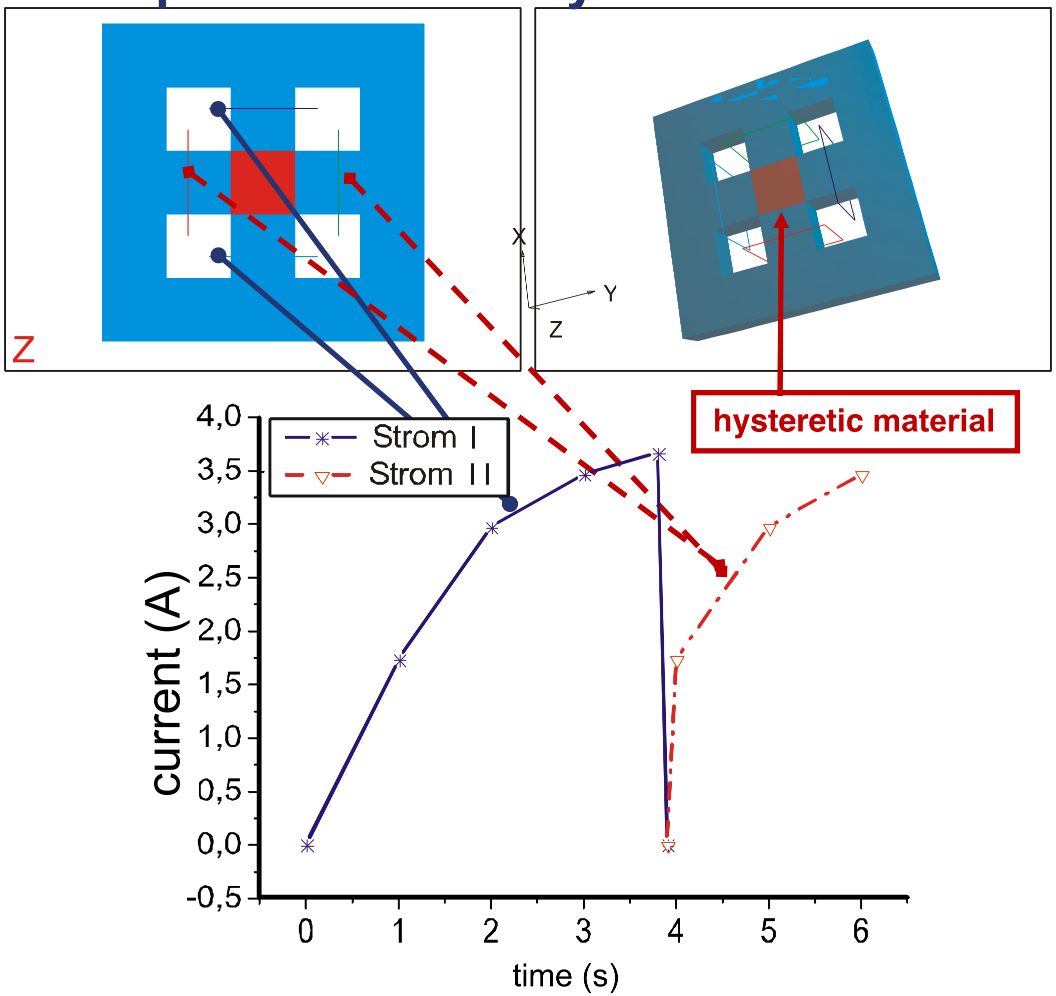


Preisach model and measurements



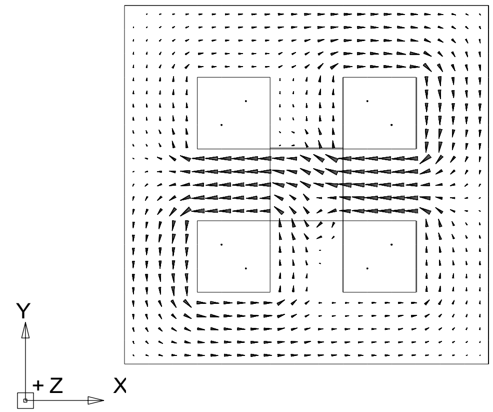
Vector-Hysteresis Model

3D example of a vector-hysteresis model

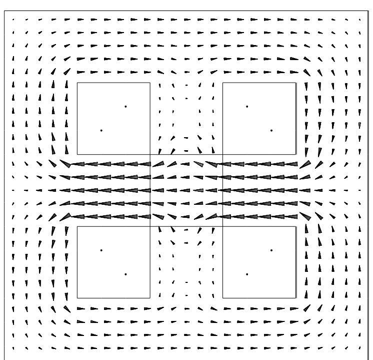


t=4s

with hysteresis

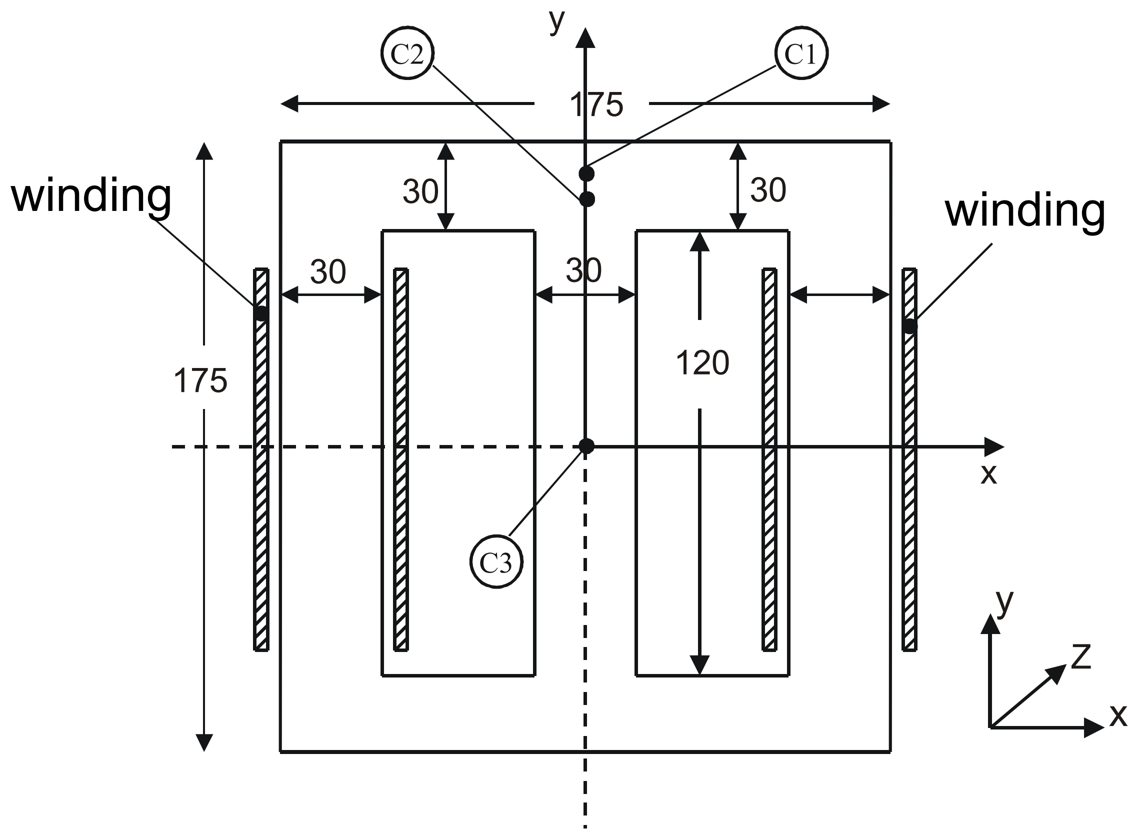


without hysteresis

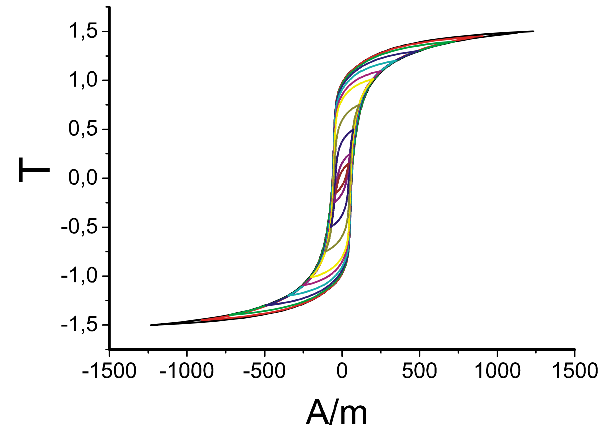


TEAM Benchmark Problem 32

Cross-section of the TEAM Benchmark Problem 32



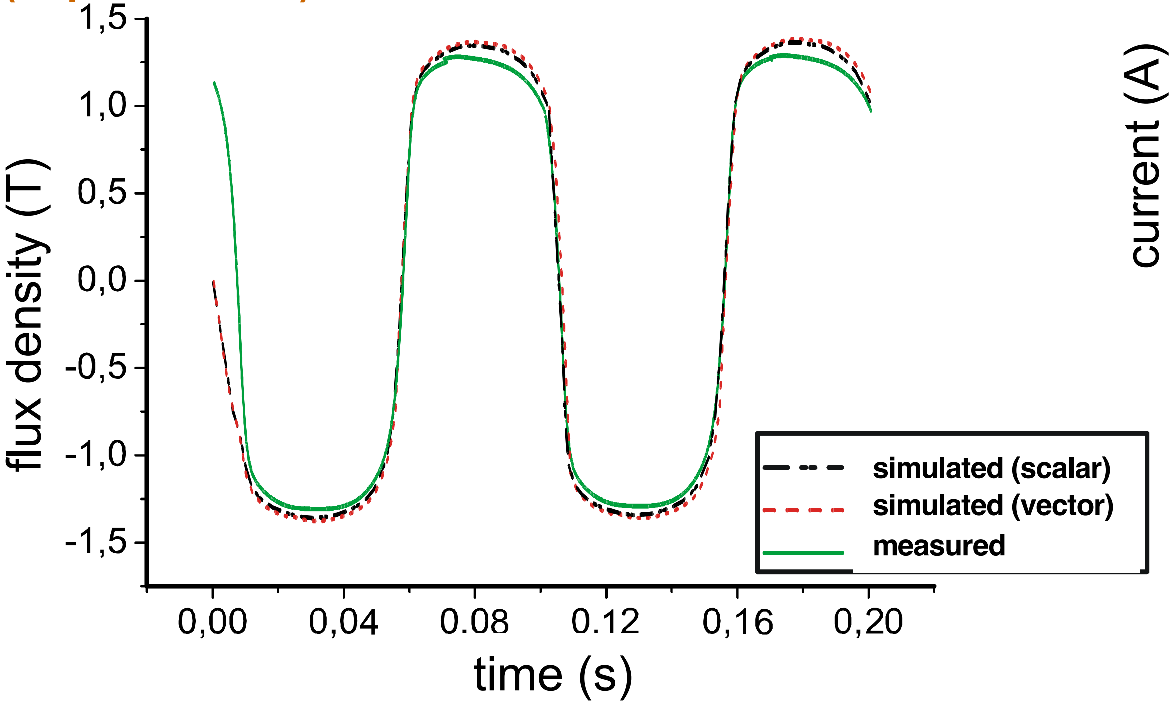
hysteresis curves



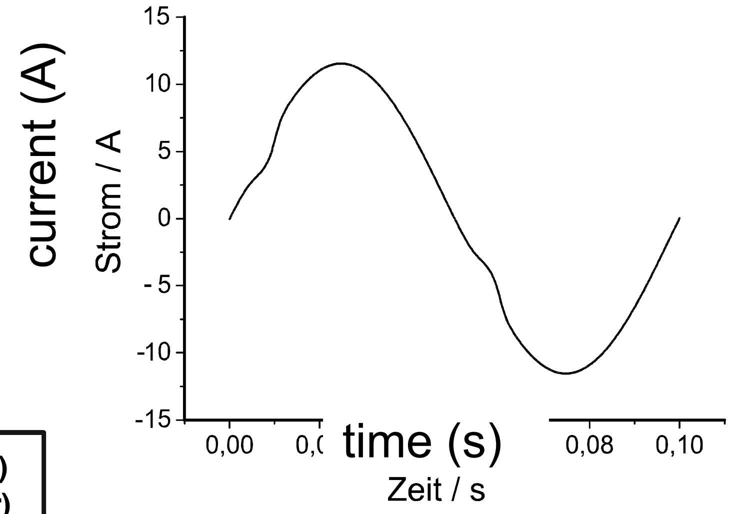
(dimensions in mm, Z=2.5mm)

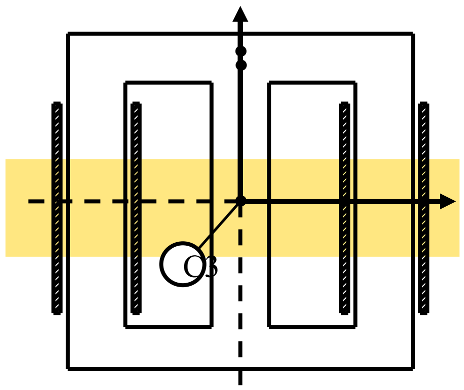
TEAM Benchmark Problem 32

Comparison simulation to measurements
(at position C3)



current excitation:

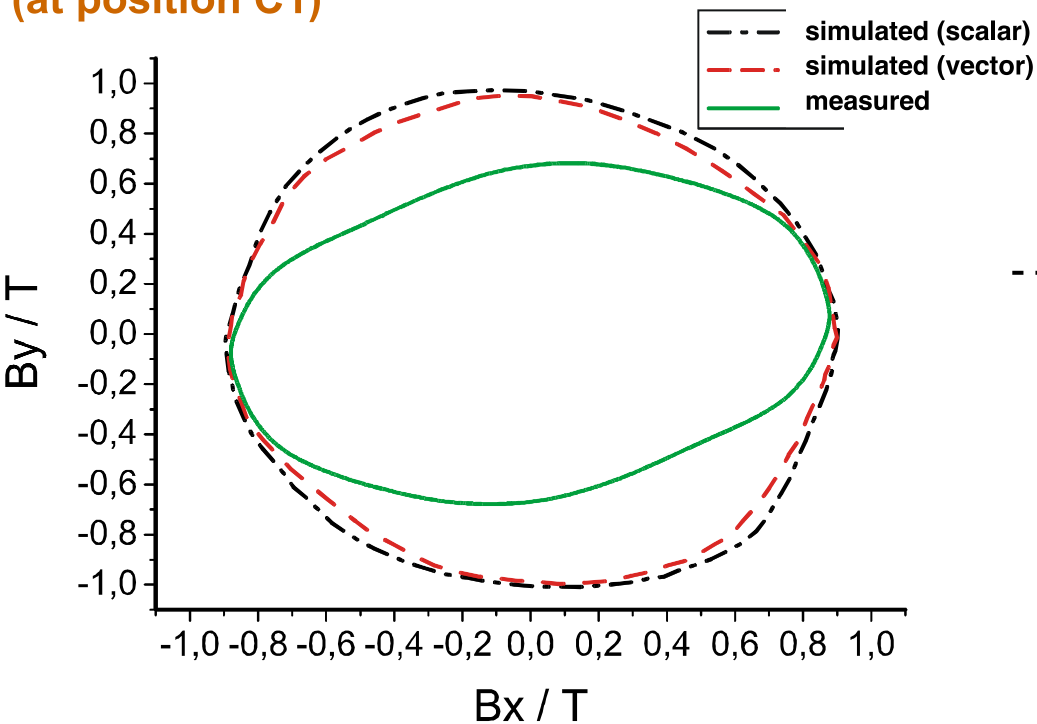


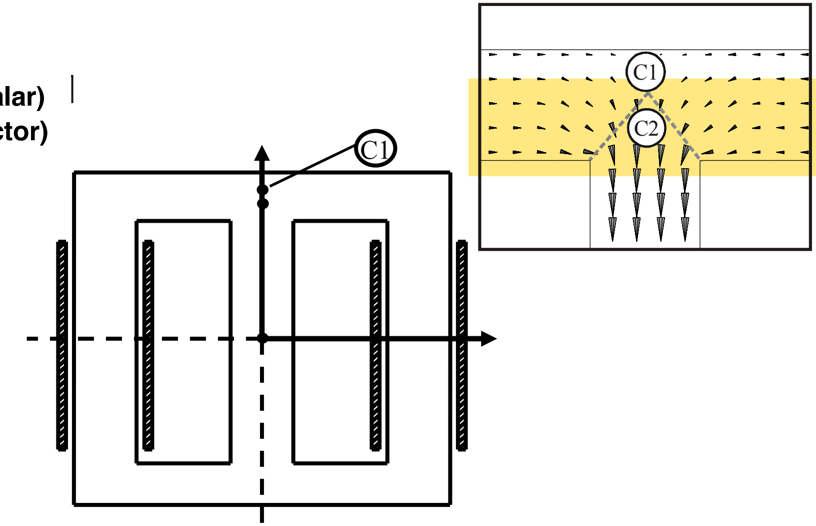


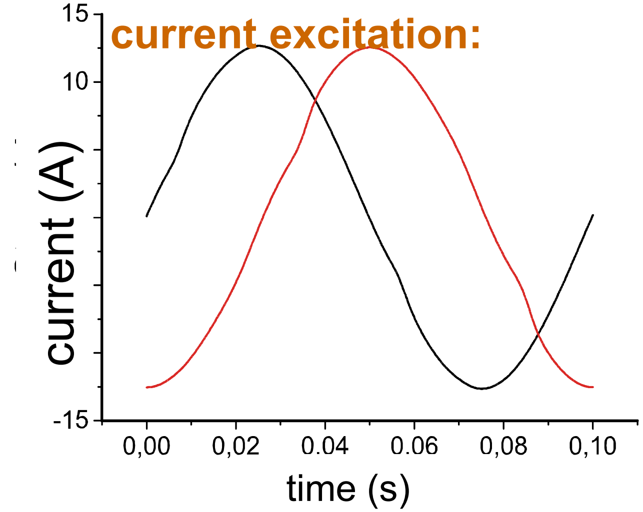
- magnetoquasistatische simulation with BDF1 time integrator
- inverse Preisach model
- Conjugate-Gradient method with SSOR preconditioning
- linearisation by the Newton method

TEAM Benchmark Problem 32

Comparison simulation to measurements
(at position C1)







- magnetoquasistatische simulation with BDF1 time integrator
- inverse Preisach model
- Conjugate-Gradient method with SSOR preconditioning
- linearisation by the Newton method