



Fundamental processes: Atomic Physics

CERN Accelerator School: Ion Sources
Senec, Slovakia 2012

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Outline and intro

- Electrons in an atom
- Electron configurations
- Periodic table of elements
- Ionization energies
- Negative ions – electron affinity
- Atomic processes in ion sources
- Ways to ionize atoms:
 - Hot surface
 - Particle impact
 - Photons
- Interest of atomic physics:
 - study of the atom as an isolated system of electrons and an atomic nucleus
 - Processes: atom ionization and excitation by photons or collisions with particles
- Atomic physics for ion sources:
 - Energy required for ionization
 - Efficiency of ionization



Physical quantities and units

- Kinetic energy of charged particles is measured in *electron volts* (eV).
- 1 eV: energy acquired by singly charged particle moving through potential of 1 Volt.

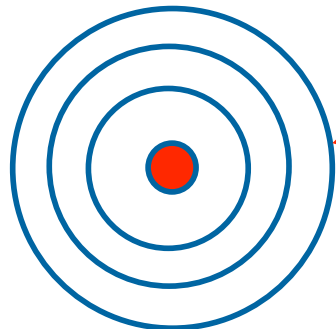


- $1 \text{ eV} = e * (1 \text{ Volt}) = 1.6022 * 10^{-19} \text{ J}$
- Mass of electron: $m_e = 9.109 * 10^{-31} \text{ kg}$
- Mass of proton: $m_p = 1.672 * 10^{-27} \text{ kg}$
- Atomic mass unit = 1/12 carbon-12 mass: $1 \text{ u} = 1.6606 * 10^{-27} \text{ kg}$
- Elementary charge of particle is $e = 1.6022 * 10^{-19} \text{ C}$ (or A*s)
- Electron with 1 eV kinetic energy is moving with a velocity of about 594 km/s
- 1eV = thermal energy at 11 600 K

Electrons in an atom

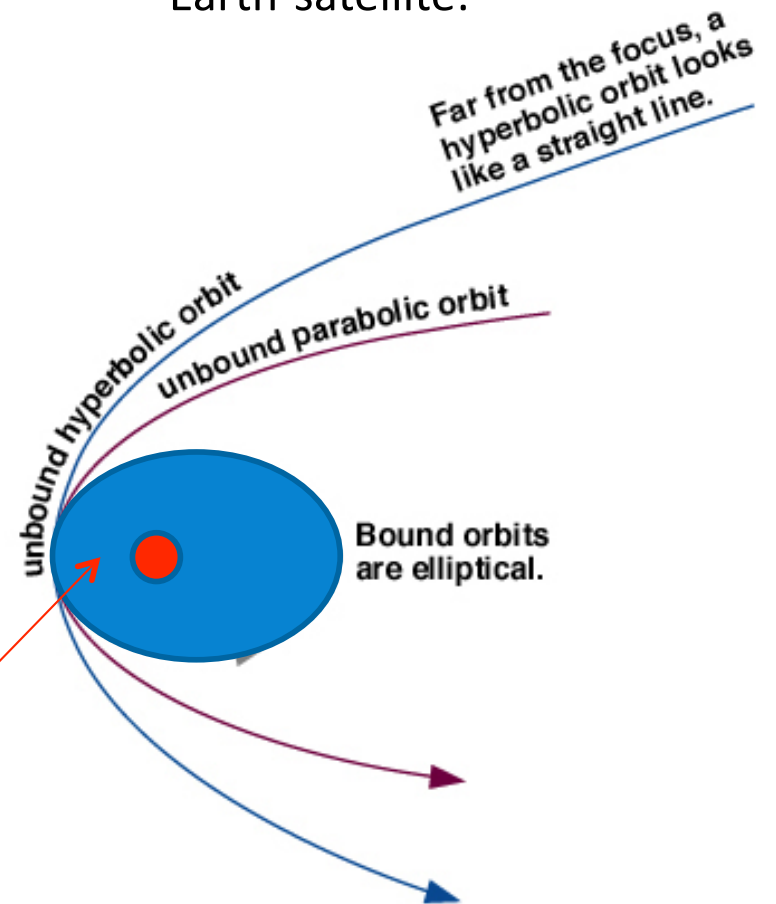
- Analogy:
 - Satellite orbiting the Earth contains gravitational potential energy
 - Satellite can orbit the Earth at any height. Or, it can contain any amount of gravitation energy—its gravitational potential energy is continuous
- Similarly, electron orbiting nucleus possesses electric potential energy. But it can only stay in a finite number of discrete energy levels (or orbits)
- => **energy levels of atoms are 'quantized'**

Nucleus-electrons



Allowed states

Earth-satellite:



Far from the focus, a hyperbolic orbit looks like a straight line.

Bound orbits are elliptical.

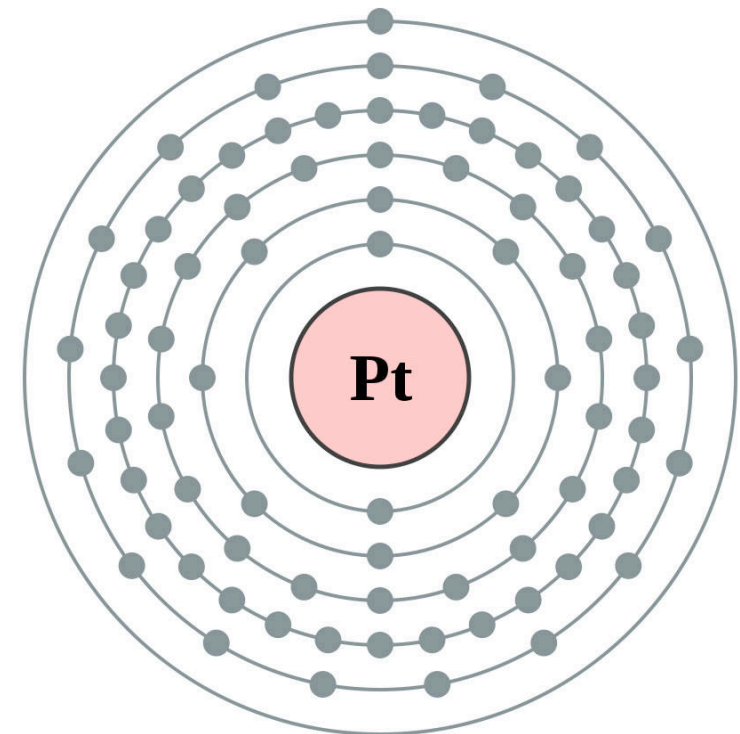
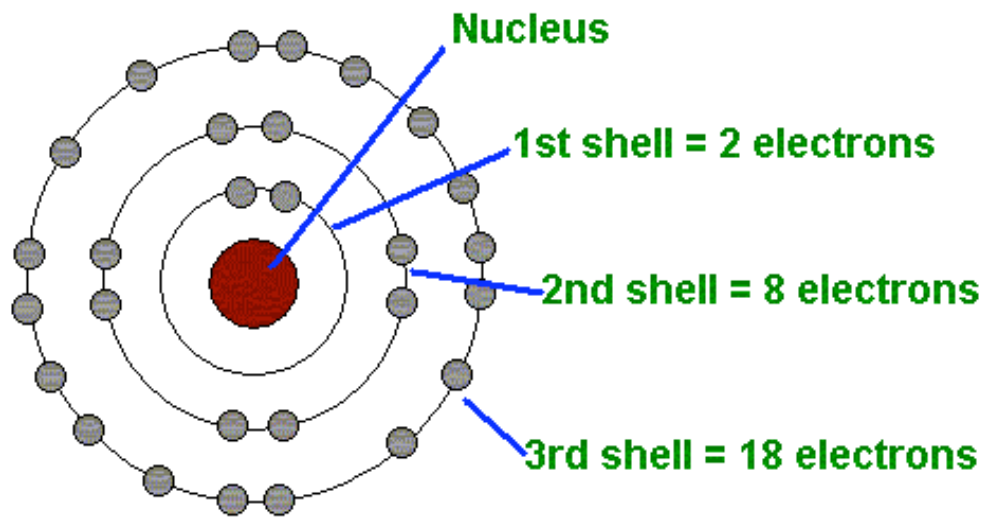


Electrons in an atom

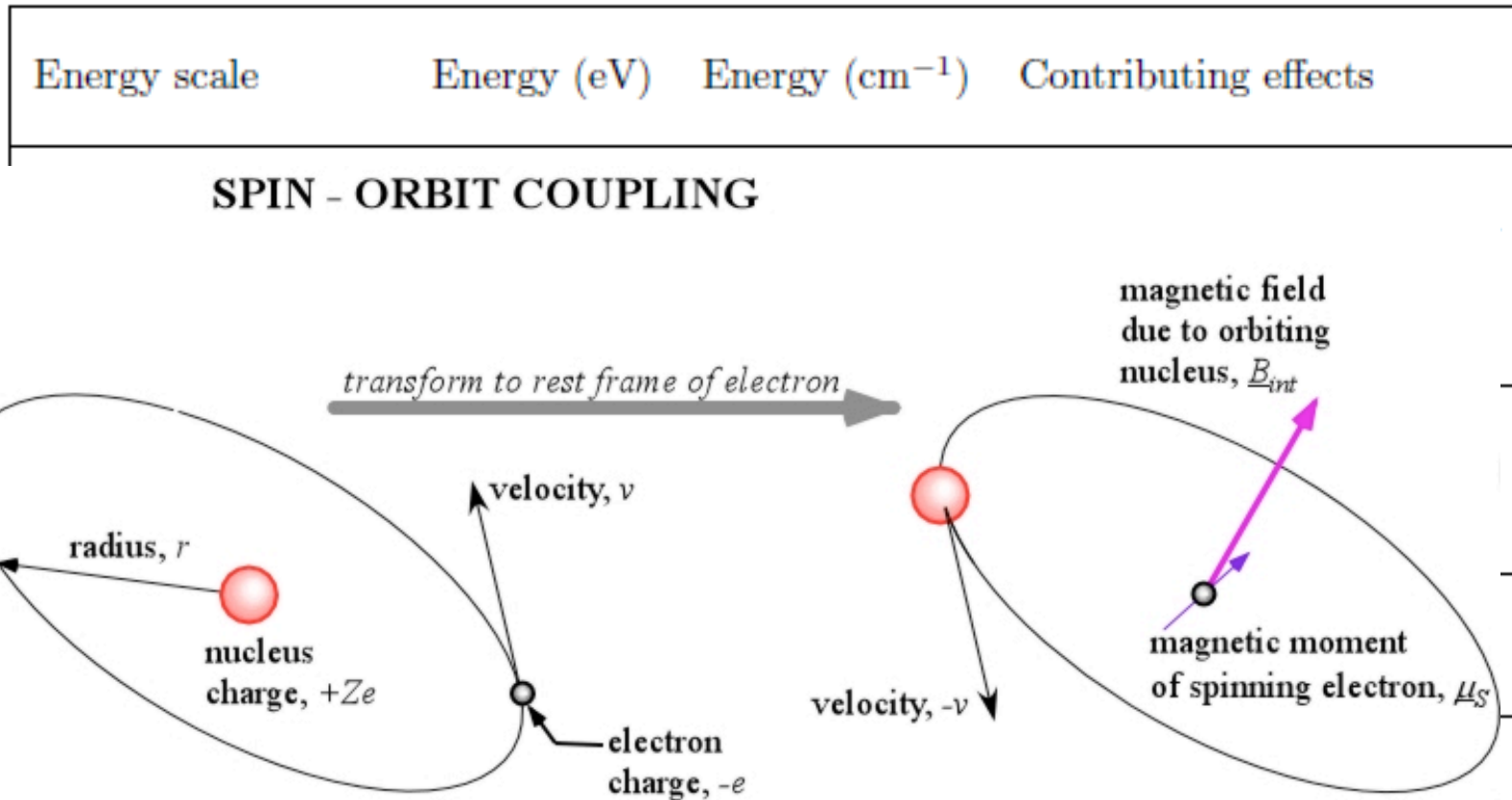
- Electrons orbit the atomic nucleus on orbits of fixed energy
- Energy of each electron level/orbit/shell is determined mostly by attraction of the nucleus and to a smaller degree by the repulsion of other electrons
- **Factors influencing electron energy: nucleus, el-el interaction, spin-orbit**
- Quantum mechanics is behind the existence of shells and the number of electrons on each shell

78: Platinum

2,8,18,32,17,1



Atomic shell structure



Spin-orbit interaction: interaction between electron's spin and orbital angular momentum (i.e. magnetic field generated by the electron's orbit around nucleus)

Hyperfine structure: nuclear spin experiences magnetic field due to current loop of electron and dipolar interaction of the electronic and nuclear spins

Electron quantum numbers

- n Principal Quantum Number (QN)
 - Specifies shell (radial dependence)
- l Azimuthal QN
 - s, p, d, f, ... correspond to $l = 0, 1, 2, 3, \dots, n-1$
 - Gives orbital angular momentum $L^2 = \hbar^2 l(l+1)$
- m_l Magnetic QN
 - Projection of azimuthal QN along an axis ($-l < m_l < l$)
 - Projected angular momentum is $L_z = m_l \hbar$
- m_s Spin projection QN
 - Electron spin = $(-1/2, 1/2)$
- j total orbital angular momentum
 - Spin-orbit coupling: $j = 1/2, 3/2 \dots n-1/2$, total: $J^2 = \hbar^2 j(j+1)$
- m_j orbital angular momentum
 - $m_j = -j, -j+1 \dots j$ and satisfies $m_j = m_l + m_s$

Quantum Numbers: describe the conserved quantities of the system: e.g. energy and angular momentum, spin

With spin-orbit interaction l, m, s no longer commute with Hamiltonian => change over time. Need new QN's



Electron quantum numbers

- In an electronic configuration, electrons can't have the same quantum number
 - Pauli exclusion principle
 - Applying this can be a bit complicated at times
 - Some states may be indistinguishable
 - Have degeneracy rather than new configuration
 - Example: Configuration: $2p^2$ ($n = 2$ with 2p electrons)
 - Important since this represents the electronic configuration of many of the most abundant ions
 - Total Spin: $S = 0$ (Singlet) or 1 (Triplet)
 - Orbital Angular Momentum: $L = 0, 1, 2$
 - Thus $J (= L + S)$ could range from 0 to 3
- } Capital letters used for many-electron systems

When including interaction with nucleus of spin I:

Total angular momentum F; F quantum number $F = |I - J|, \dots, I + J$



Electron configuration

Quantum number	symbol	Value
principal	n	any integer > 0
orbital	l	integer up to $(n - 1)$
magnetic	m_l	integer from $-l$ to $+l$
spin	m_s	$\pm 1/2$

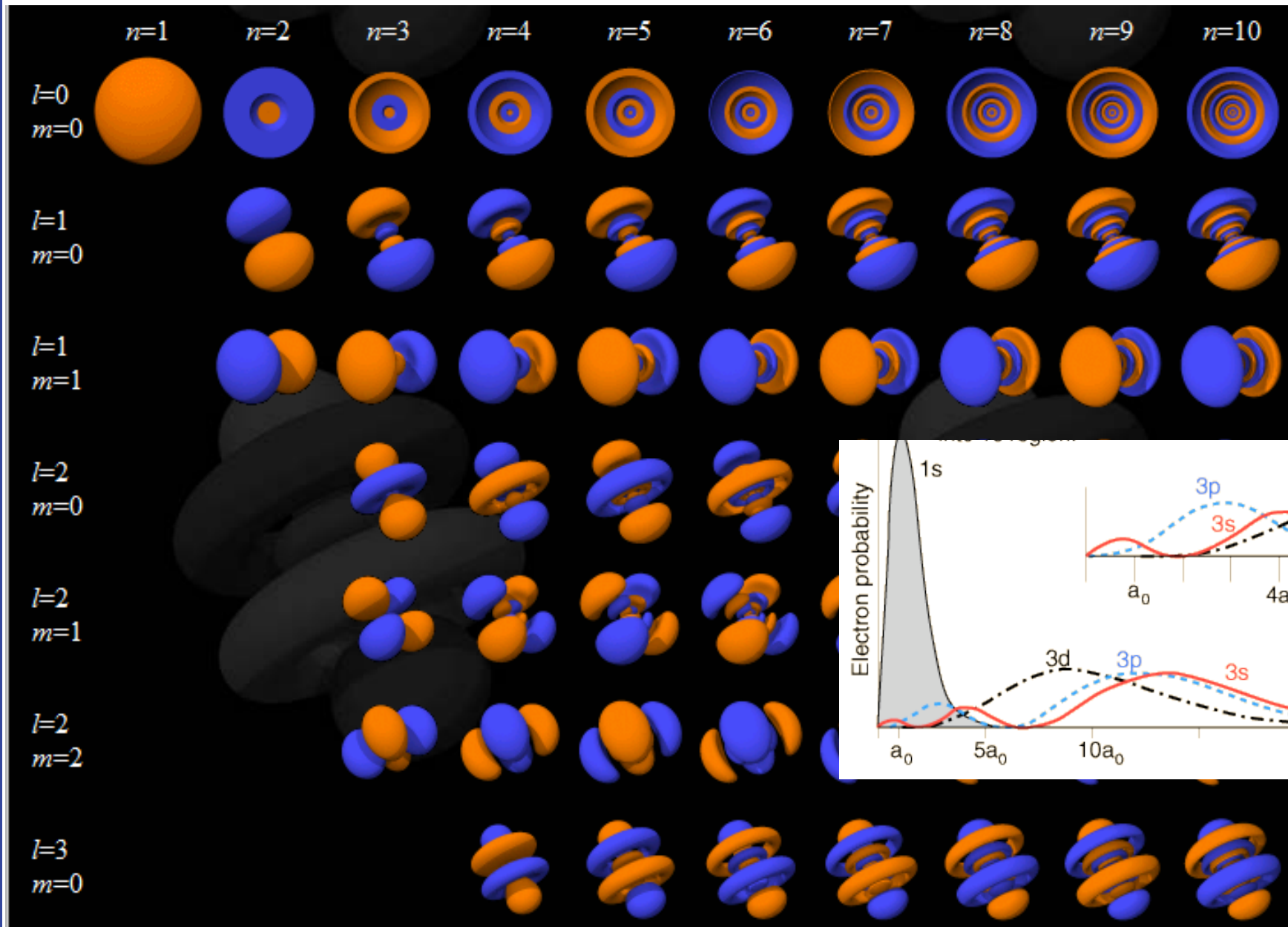
$n = 1, 2, 3, \dots$	$l = 0, 1, 2, \dots, n - 1$	$m = 0, \pm 1, \pm 2, \dots, \pm l$	Orbital
1	0	0	1s
2	0	0	2s
2	1	0	2p _z
2	1	+	2p _x
2	1	-	2p _y

Orbital names: s, p, d, f, g, h ...:

sharp, principal, diffuse, fundamental, and then alphabetic



Electron orbitals



Electron terms

- Coming from valence electrons (those in outermost orbit)

Term symbols contain 3 pieces of information:

“Spin multiplicity”

$$= 2S+1$$

(Where S is the total spin quantum number for the atom)

Gives L , the total orbital angular momentum quantum number

for the atom:

S: $L = 0$

P: $L = 1$

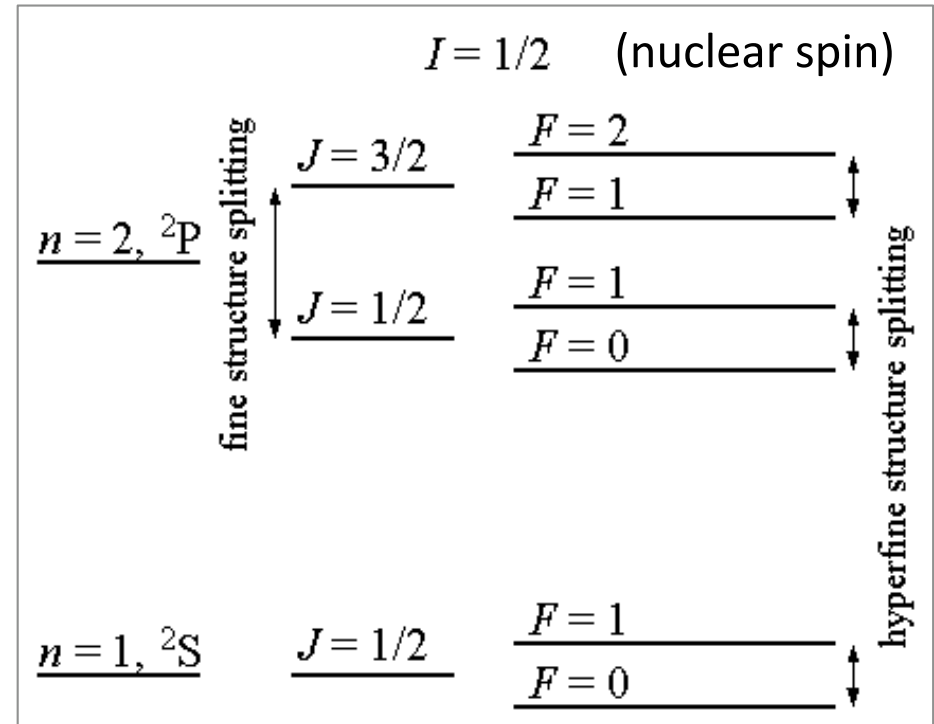
D: $L = 2$

F: $L = 3$, etc.

2P_{3/2}

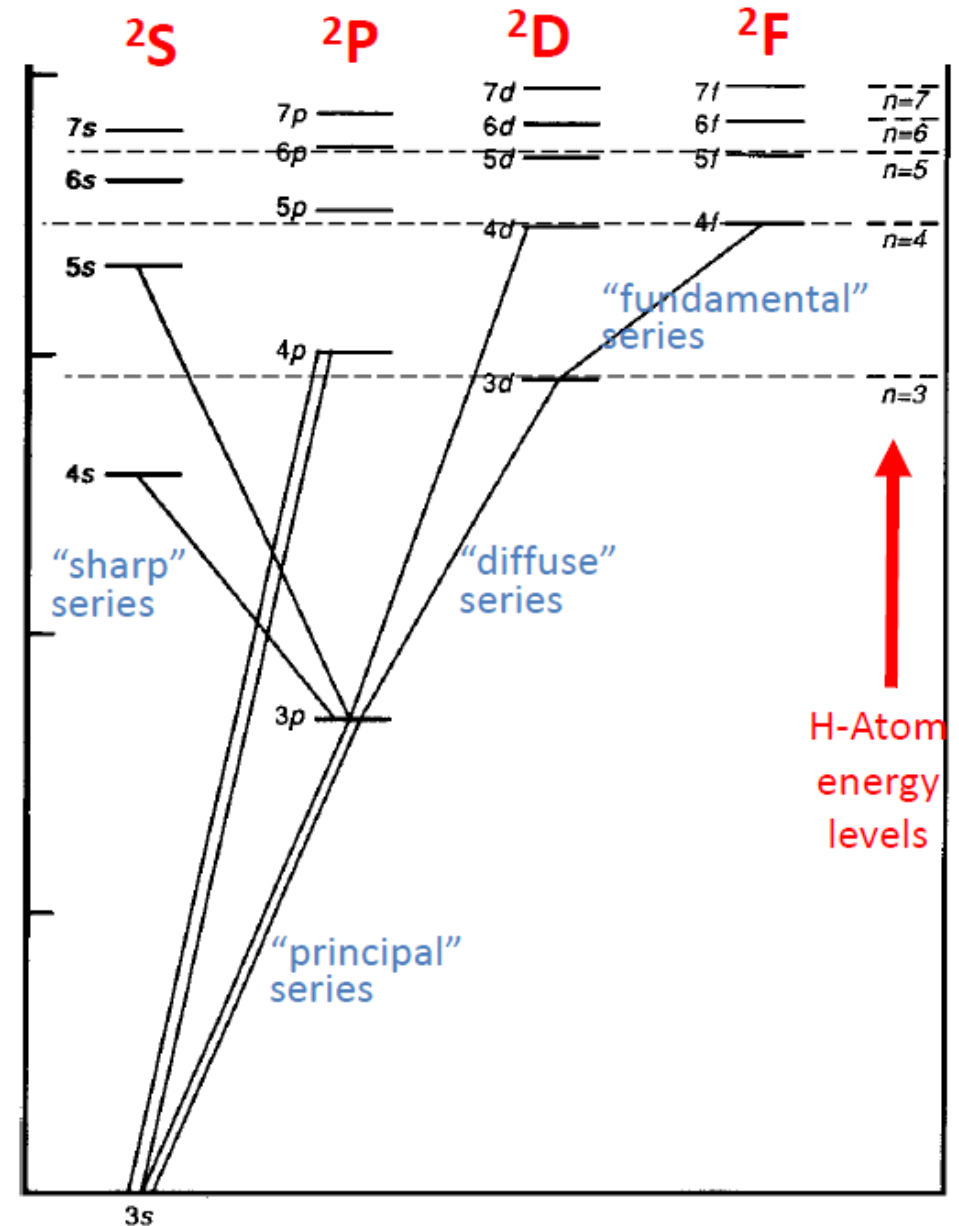
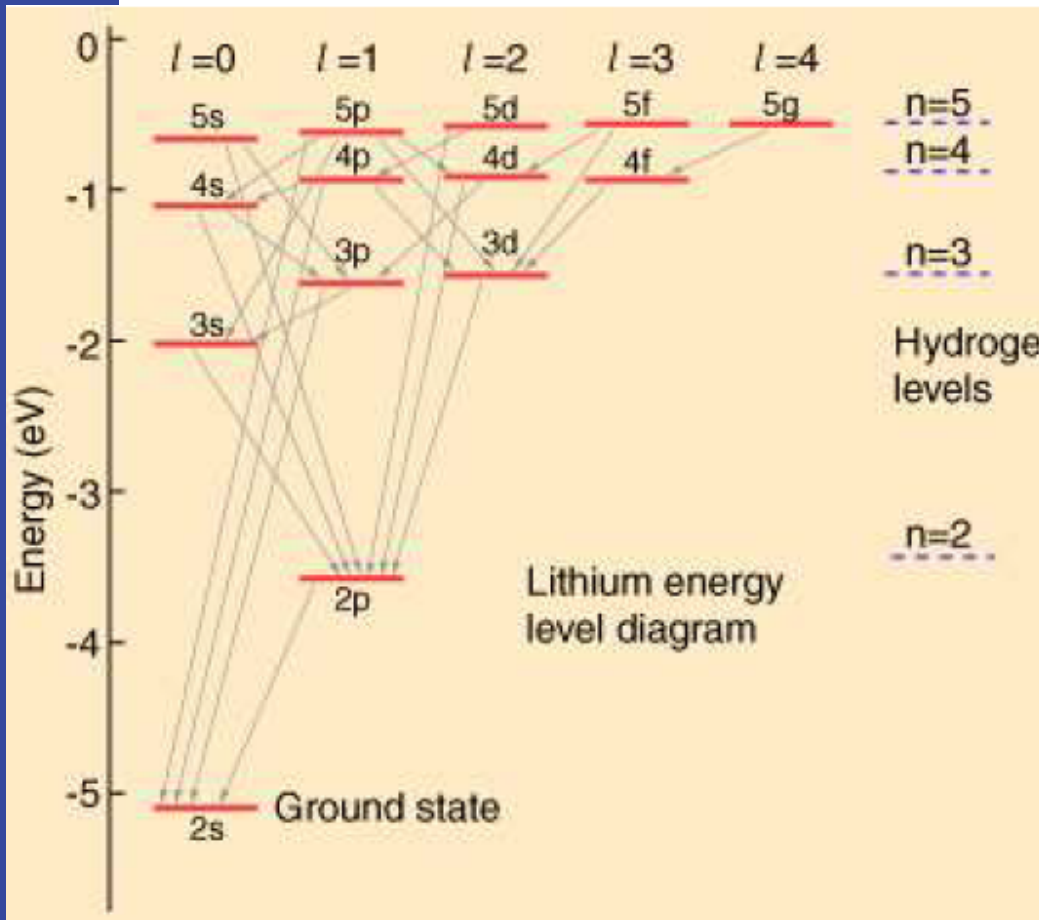
J , the total angular momentum quantum number for the atom. *i.e.*, how L and S are coupled

Hydrogen terms



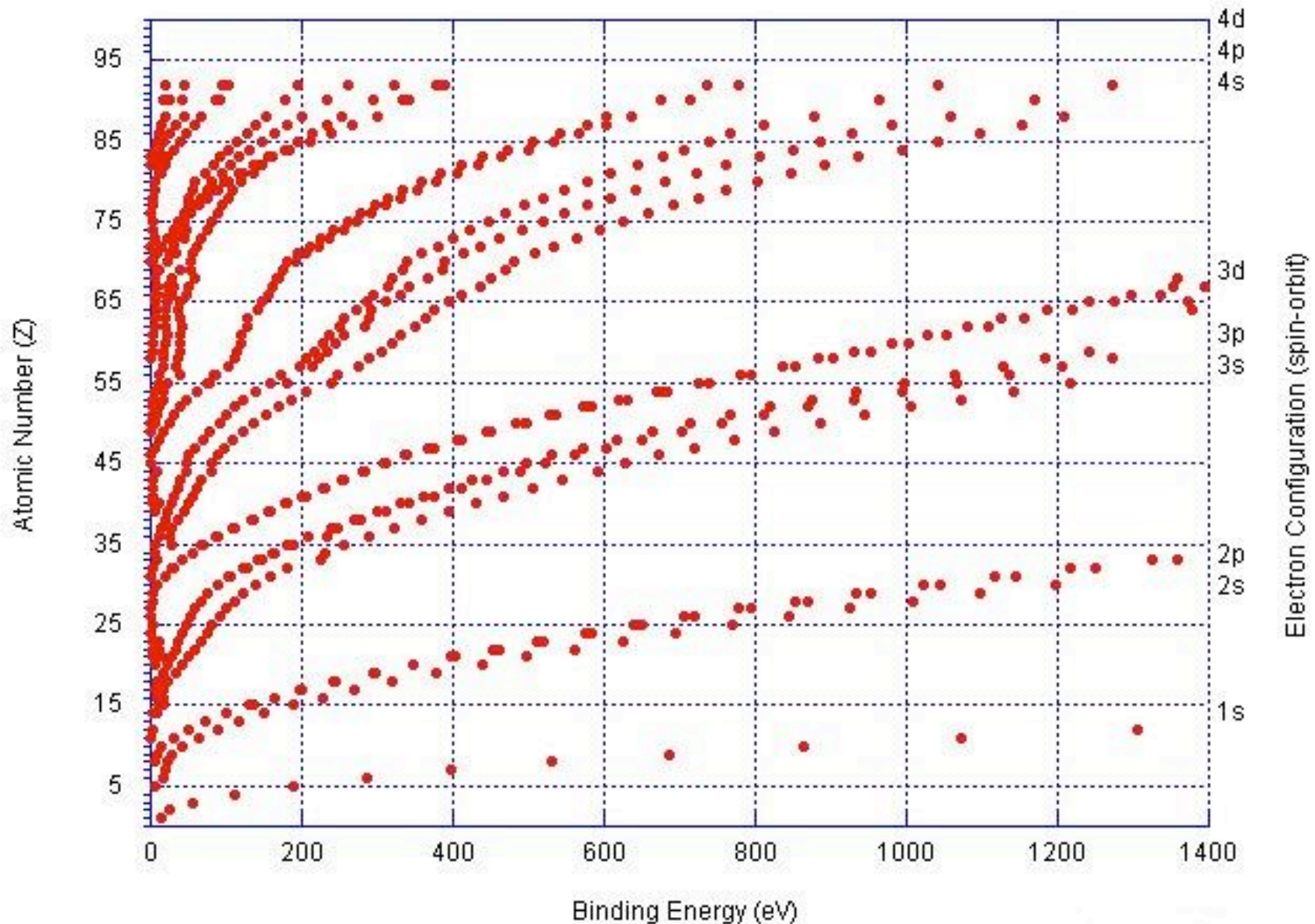
Electron configuration

● Examples:







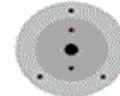
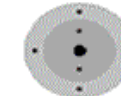
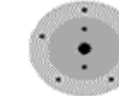
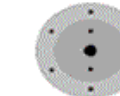
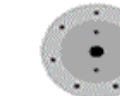
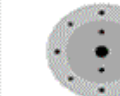
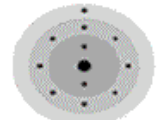
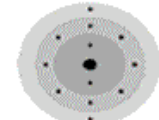
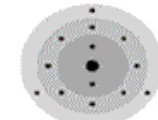
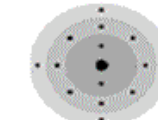
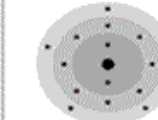
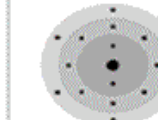
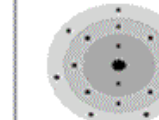
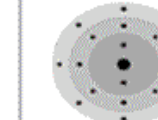
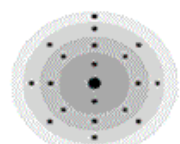
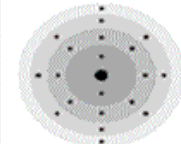
Electron binding energies

Binding Energy vs Atomic # vs Electron Configuration



Periodic table of elements

Periodic table

group 1							group 8
HYDROGEN 1  1.01							HELIUM 2  4.00
LITHIUM 3  6.94	BERYLLIUM 4  9.01	BORON 5  10.81	CARBON 6  12.01	NITROGEN 7  14.01	OXYGEN 8  16.00	FLUORINE 9  19.00	NEON 10  20.18
SODIUM 11  22.99	MAGNESIUM 12  24.31	ALUMINUM 13  26.98	SILICON 14  28.09	PHOSPHORUS 15  30.97	SULFUR 16  32.07	CHLORINE 17  35.45	ARGON 18  39.95
POTASSIUM 19  39.10	CALCIUM 20  40.08						

Chemical properties dictated by valence electrons



Periodic table of elements

- 1 (1A) - alkali metals ($1s^1-7s^1$)
- 2 (2A) - alkaline earth metals ($1s^2-7s^2$)
- 17 (7A) - halogens ($2p^5-6p^5$)
- 18 (8A) - noble gases ($1s^2, 2p^6-6p^6$)
- 14 (4A) - the Carbon family

s-block												Non-Metals					s-block		
1 New Designation												13 IIIA					18		
IA Original Designation												14 IVA					VIIIA		
												15 VA							
												16 VIA							
												17 VIIA							
												18							
1	1 H 1.0094																2 He 4.00260		
s-block												p-block							
2	3 Li 6.941	4 Be 9.0122	d-block										5 B 10.81	6 C 12.011	7 N 14.007	8 O 15.999	9 F 18.998	10 Ne 20.179	
		Transition Metals																	
3	11 Na 22.990	12 Mg 24.305	3 IIB	4 IVB	5 VB	6 VIB	7 VIIB	8	9	10 VIII	11 IB	12 IIB	13 Al 26.982	14 Si 28.086	15 P 30.974	16 S 32.06	17 Cl 35.453	18 Ar 39.948	
4	19 K 39.098	20 Ca 40.08	21 Sc 44.956	22 Ti 47.88	23 V 50.942	24 Cr 51.996	25 Mn 54.938	26 Fe 55.847	27 Co 58.933	28 Ni 58.69	29 Cu 63.546	30 Zn 65.39	31 Ga 69.72	32 Ge 72.59	33 As 74.922	34 Se 78.96	35 Br 79.904	36 Kr 83.80	
5	37 Rb 85.468	38 Sr 87.62	39 Y 88.906	40 Zr 91.224	41 Nb 92.906	42 Mo 95.94	43 Tc (98)	44 Ru 101.07	45 Rh 102.91	46 Pd 106.42	47 Ag 107.87	48 Cd 112.41	49 In 114.82	50 Sn 118.71	51 Sb 121.75	52 Te 127.60	53 I 126.91	54 Xe 131.29	
6	55 Cs 132.91	56 Ba 137.33	57 to 71	72 Hf 178.49	73 Ta 180.95	74 W 183.85	75 Re 186.21	76 Os 190.2	77 Ir 192.22	78 Pt 195.08	79 Au 196.97	80 Hg 200.59	81 Tl 204.38	82 Pb 207.2	83 Bi 208.98	84 Po (209)	85 At (210)	86 Rn (222)	
7	87 Fr (223)	88 Ra 226.03	89 to 103	104 Unq (261)	105 Unp (262)	106 Unh (263)	107 Uns (262)	108 Uno (265)	109 Une (266)	110 Uun (267)	(Mass Numbers in Parentheses are from the most stable of common isotopes.)							Phases	
		Metals															Solid Liquid Gas		
Rare Earth Elements		d-block										f-block							
Lanthanide Series		57 La 138.91	58 Ce 140.12	59 Pr 140.91	60 Nd 144.24	61 Pm (145)	62 Sm 150.36	63 Eu 151.96	64 Gd 157.25	65 Tb 158.93	66 Dy 162.50	67 Ho 164.93	68 Er 167.26	69 Tm 168.93	70 Yb 173.04	71 Lu 174.97			
Actinide Series		89 Ac 227.03	90 Th 232.04	91 Pa 231.04	92 U 238.03	93 Np 237.05	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (252)	100 Fm (257)	101 Md (258)	102 No (259)	103 Lr (260)			

Ionization energy

- **Ionization energy (IE): minimum energy required to remove an electron from gaseous atom or ion (not solid or liquid)**
- First ionization energy: minimum energy needed to remove an electron from the highest occupied sub-shell (outermost electron) of gaseous atom
- Second ionization energy: minimum energy needed to remove the second electron from the highest occupied sub-shell of gaseous atom
- Third, fourth, ... ionization energy – analogous
- “Total” ionization energy: minimum energy required to remove all electrons from gaseous atom
- Naming: known also as ionization potential
- Units: eV or kJ/mol in chemistry
- It governs chemical properties of atoms

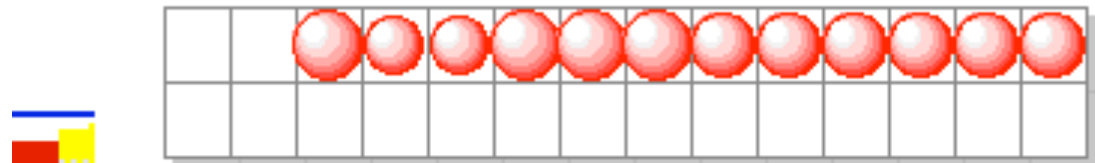
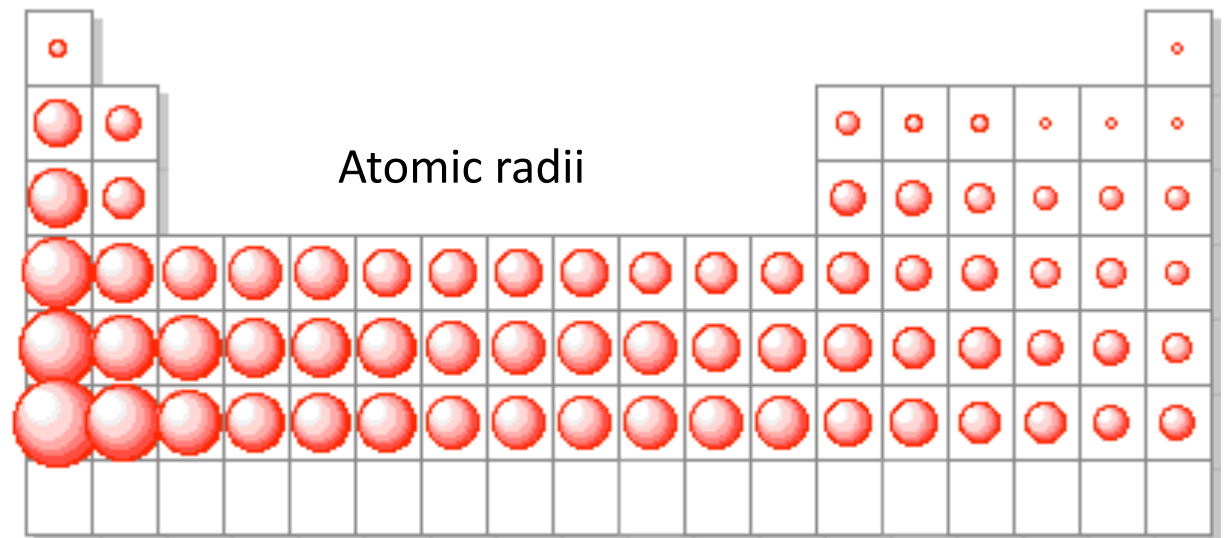
Ionization energy and shell structure

- IE shows how easy it is to pull electron completely from atomic nucleus
- IE is influenced by (in order of importance):
 - Nuclear charge – nucleus-electron attraction increases with nuclear charge
 - Number of shells – in presence of levels closer to nucleus outermost electrons are further from nucleus and are not so strongly attracted
 - Shielding – electrons on orbits closer to nucleus shield/protect outermost electron from attraction of nucleus

● Atomic radii and IE are connected

● What has lower 1st IE:

- Mg or Ne
- K or Ca
- K or Rb
- P or Ar
- Etc ...

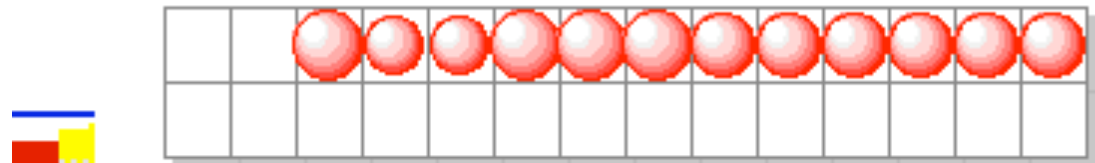
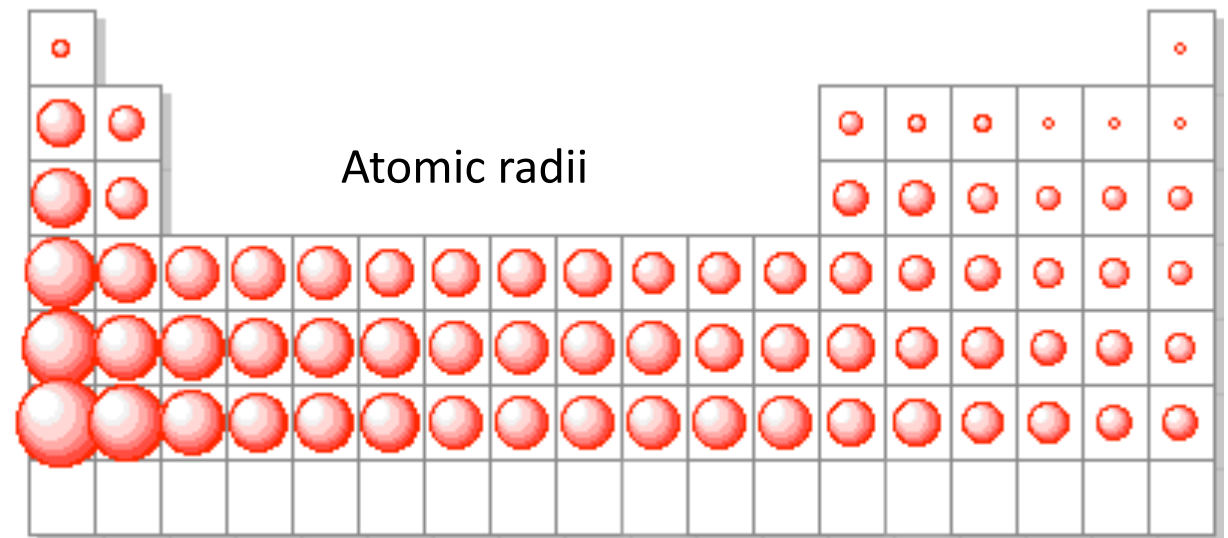


Ionization energy and shell structure

- Atomic size increases down a Periodic Table group while ionization energy decreases, as negative valence electron is further from positive nucleus
- 1st ionization energy generally increases across periods of Periodic Table due to increase in nuclear charge: additional electrons go into same shell, are about the same distance from nucleus, but positive nuclear charge increases
- 2nd ionization energy is larger than 1st, as electron is removed from positive ion
- As number of electrons in outermost shells increases across a period so too does the first ionization energy, therefore a high first ionization energy indicates shell or sub-shell that is almost full

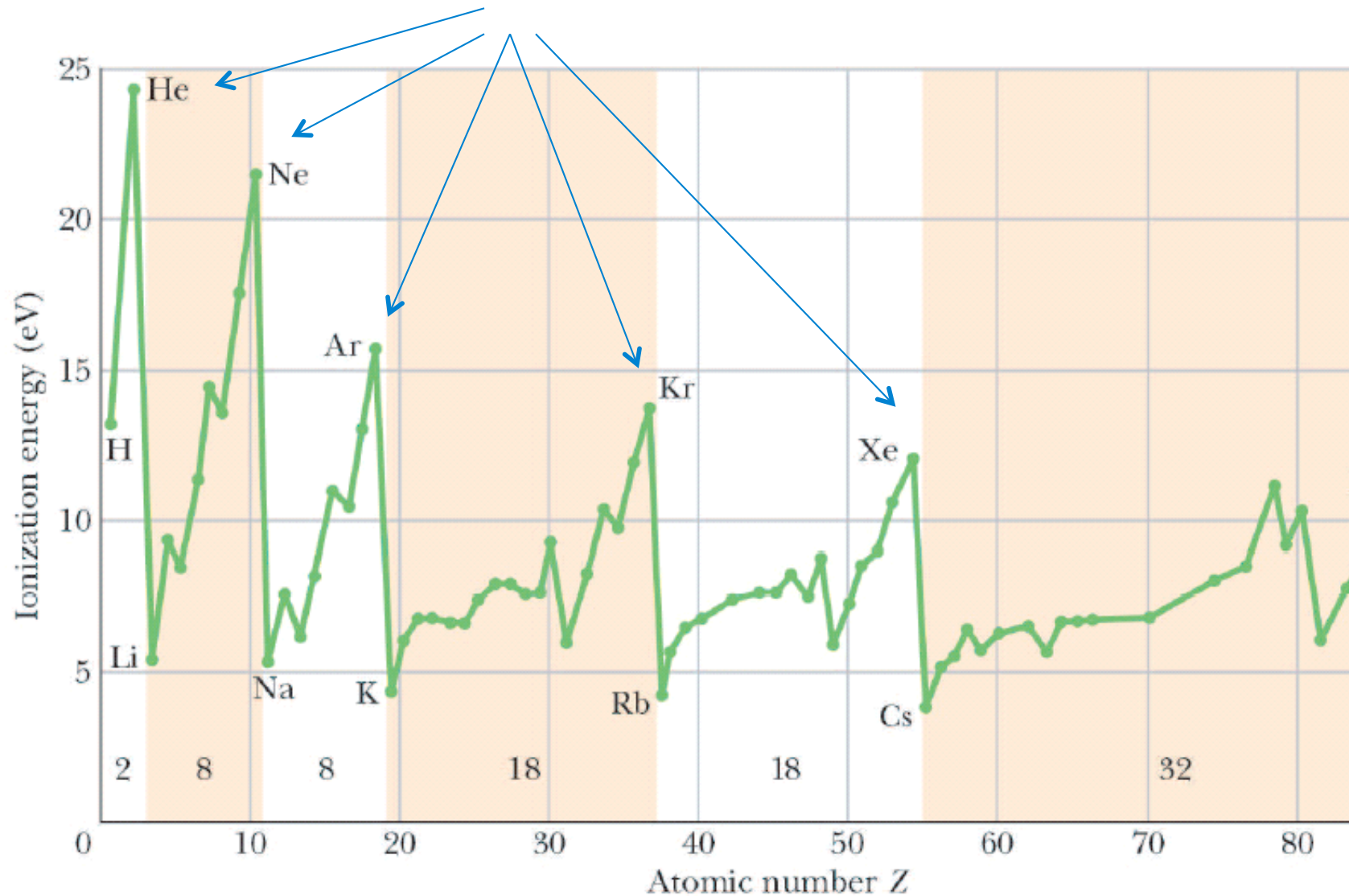
● What has lower 1st IE:

- Mg or Ne
- K or Ca
- K or Rb
- P or Ar
- Etc ...



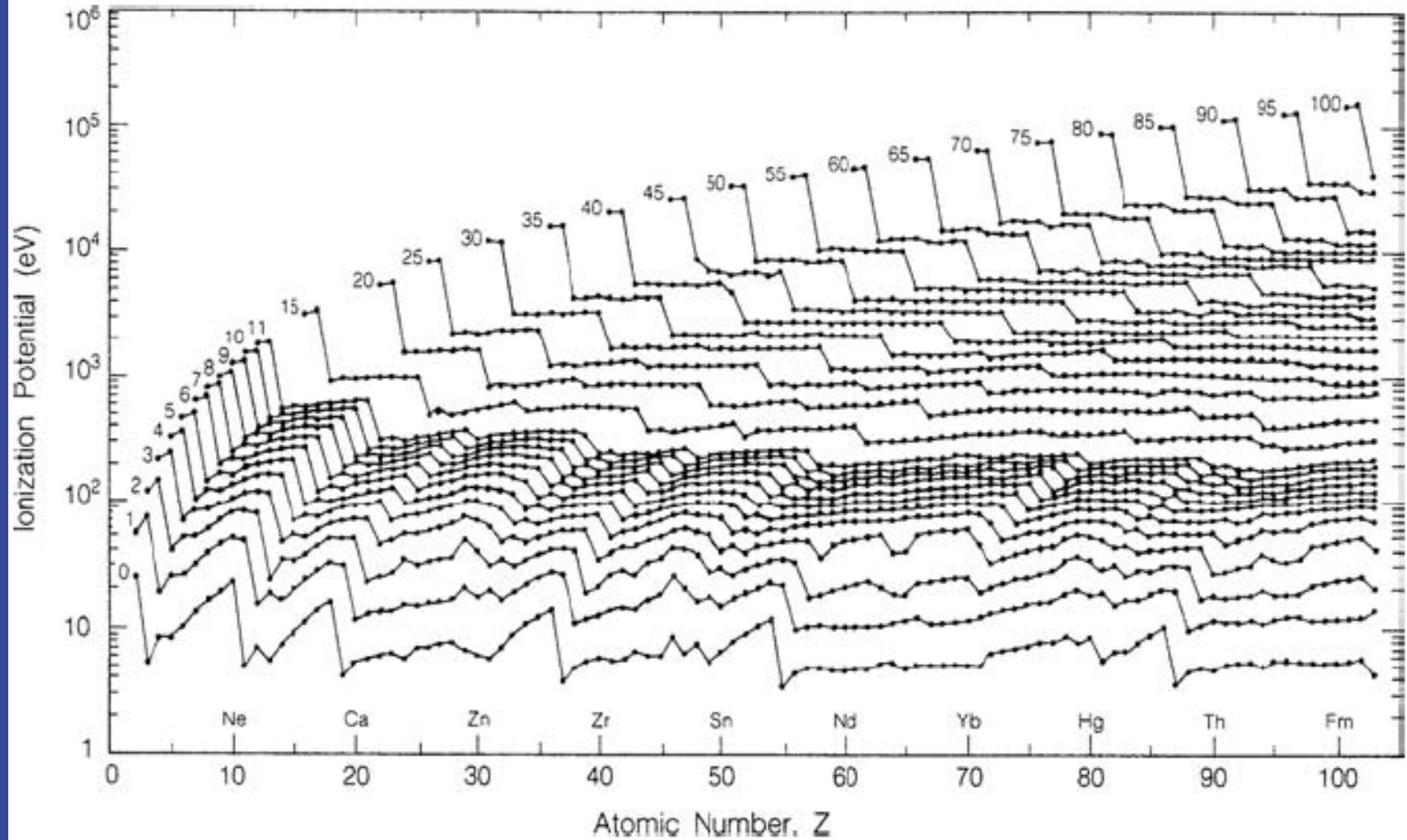
First ionization energy

● Closed-shell atoms



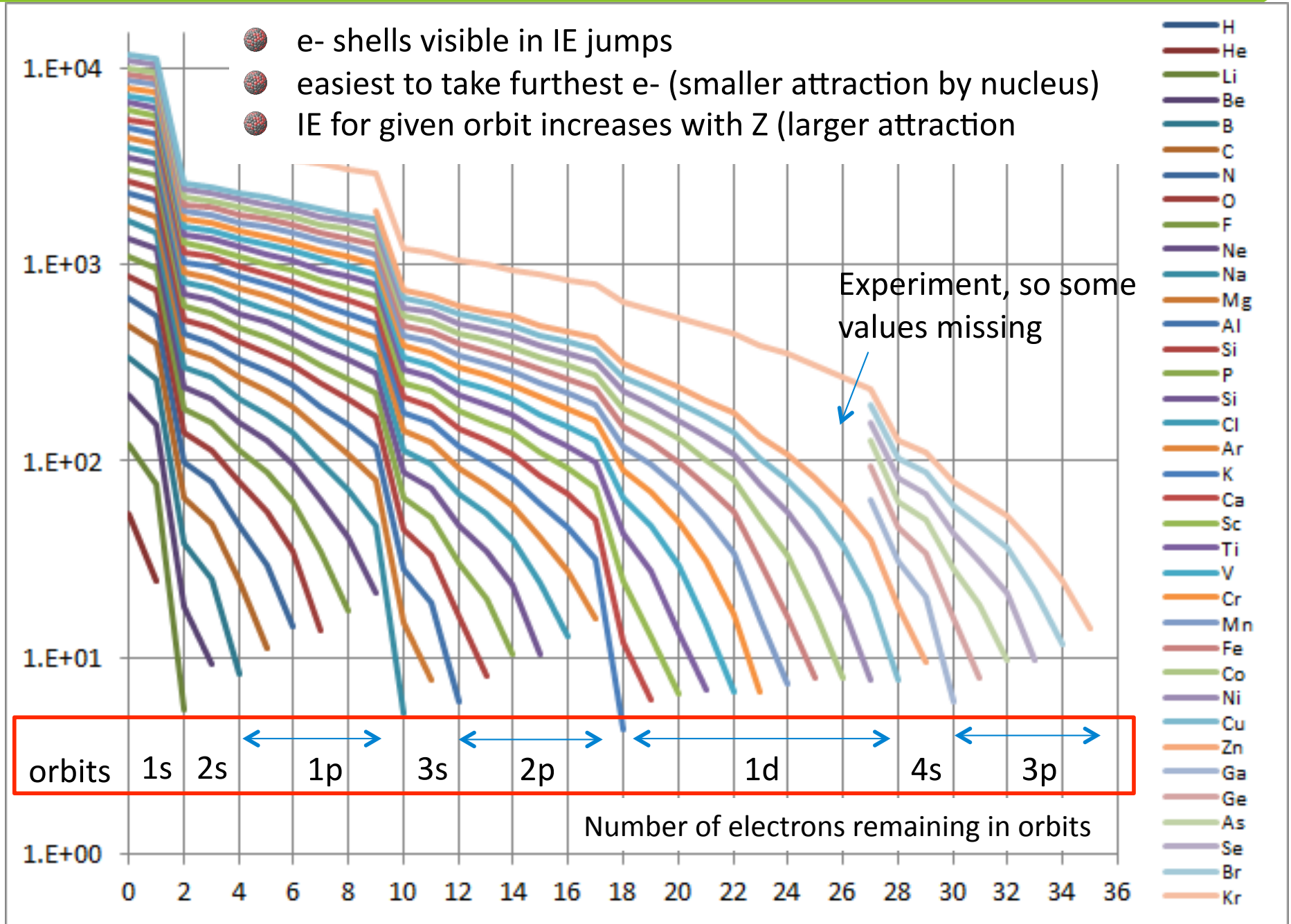
Ionization energy

- Ionization energy for multiply charged ions



Ionization energy

IE: Ionization energy for last electron (eV)



Ionization energy

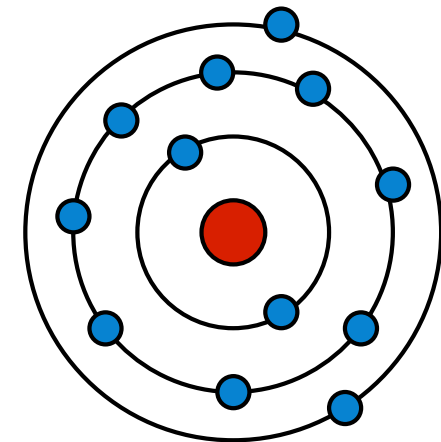
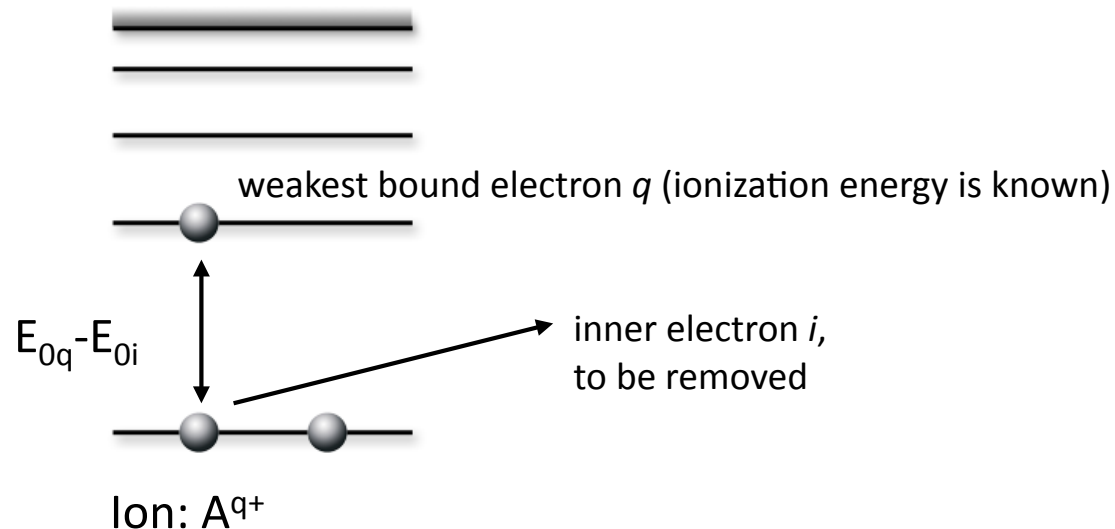
- When ionization energies for more bound electrons are not known, **Carlson-correction** is used:
- Ionization energy $P_{q,i}$ is calculated from ionization energy $W_i(q)$ of ion with charge state q and the atomic binding energies of electrons (measured or calculated)

$$P_i = E_{0i} + W_i(q) - E_{0q}$$

E_{0i} : binding energy of an electron in the i -th shell of an atom

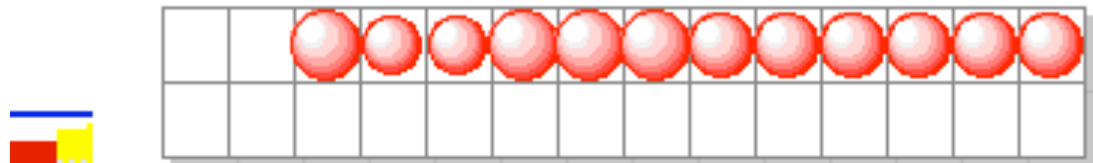
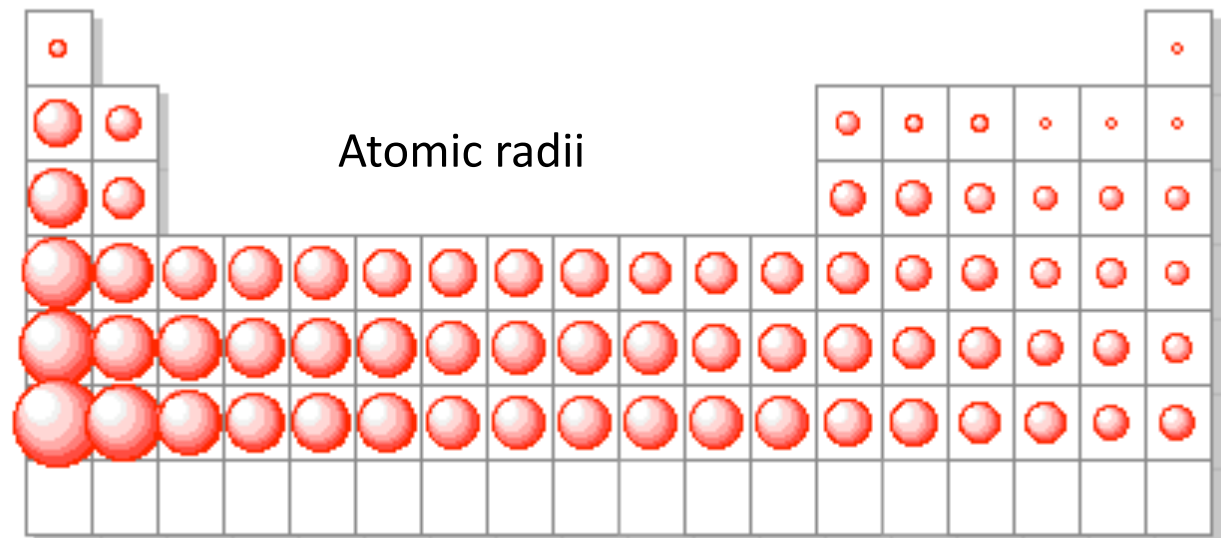
E_{0q} : atomic binding energy of the electron, which is the weakest bound electron in the ion of the charge state q

$W_i(q)$: ionization energy of the ion (describes always the weakest bound electron)



Negative ions: electron affinity

- **Electron affinity (EA):** energy given off when neutral atom in gas phase gains extra electron to form *negatively charged ion*
- IE is influenced by the same effects as EI:
 - Nuclear charge
 - Number of shells
 - Shielding
- Atomic radii and IA are connected



Electron affinity

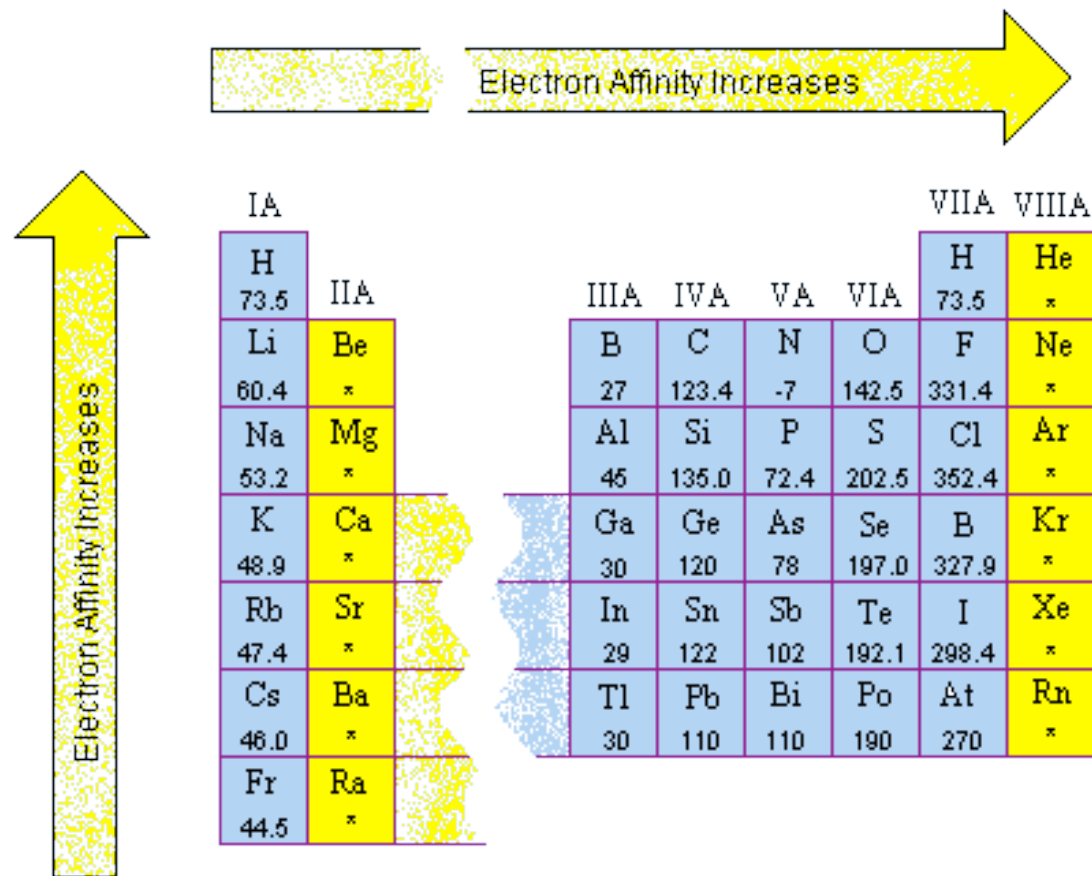
- Electron affinity < 0 – negative ion is not stable

Electron affinities and ionization energies of elements

Group I A	Ionization potential (eV) – Electron affinity (eV)						VIII A
1 H 13.59 0.75	II A	III A	IV A	V A	VI A	VII A	2 He 24.58 0.078
3 Li 3.39 0.62	4 Be 9.32 < 0	5 B 8.30 0.28	6 C 11.26 1.26	7 N 14.54 ≤ 0	8 O 13.61 1.46	9 F 17.42 3.39	10 Ne 21.56 < 0
11 Na 5.14 0.54	12 Mg 7.64 < 0	13 Al 5.98 0.46	14 Si 8.15 1.38	15 P 10.55 0.74	16 S 10.36 2.07	17 Cl 13.01 3.61	18 Ar 15.76 < 0
19 K 4.34 0.50	20 Ca 6.11 ≈ 0	31 Ga 6.00 0.3	32 Ge 7.88 1.2	33 As 9.81 0.80	34 Se 9.75 2.02	35 Br 11.84 3.36	36 Kr 14.00 < 0
37 Rb 4.18 0.48	38 Sr 5.69 < 0	49 In 5.78 0.3	50 Sn 7.34 1.25	51 Sb 8.64 1.05	52 Te 9.01 1.97	53 I 10.45 3.06	54 Xe 12.13 < 0
55 Cs 3.89 0.47	56 Ba 5.21 < 0	81 Tl 6.11 0.3	82 Pb 7.41 1.1	83 Bi 7.29 1.1	84 Po 8.43 1.9	85 At 9.5 2.8	86 Rn 10.74 < 0

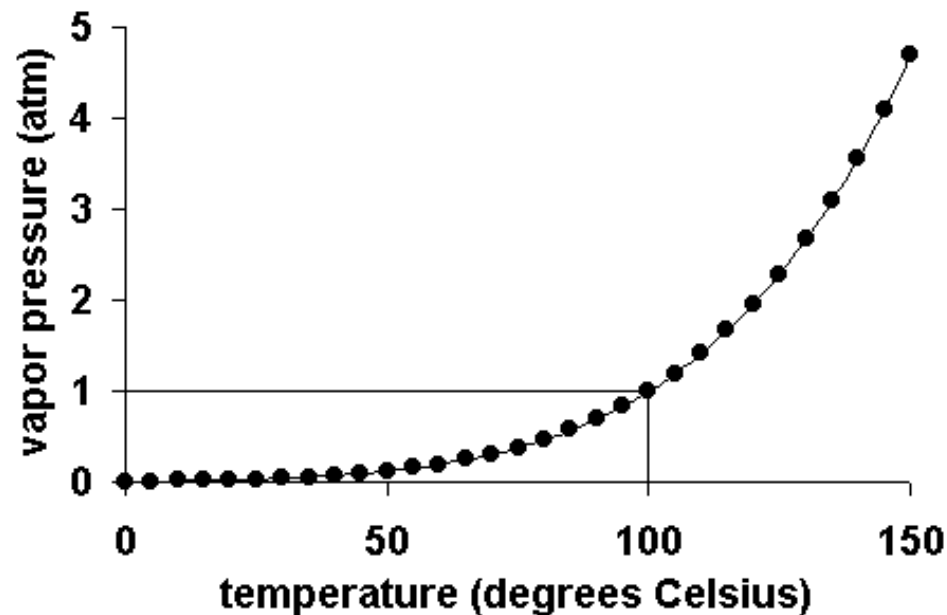
Electron affinity patterns

- Electron affinities get smaller when going down a column of periodic table:
 - electron added to atom is placed in larger orbitals, where it spends less time near nucleus
 - number of electrons on atom increases as we go down a column, so repulsion between electron being added and electrons already present on a neutral atom becomes larger
- Electron affinity data are complicated because repulsion between electron added to atom and electrons already present depends on atom's volume

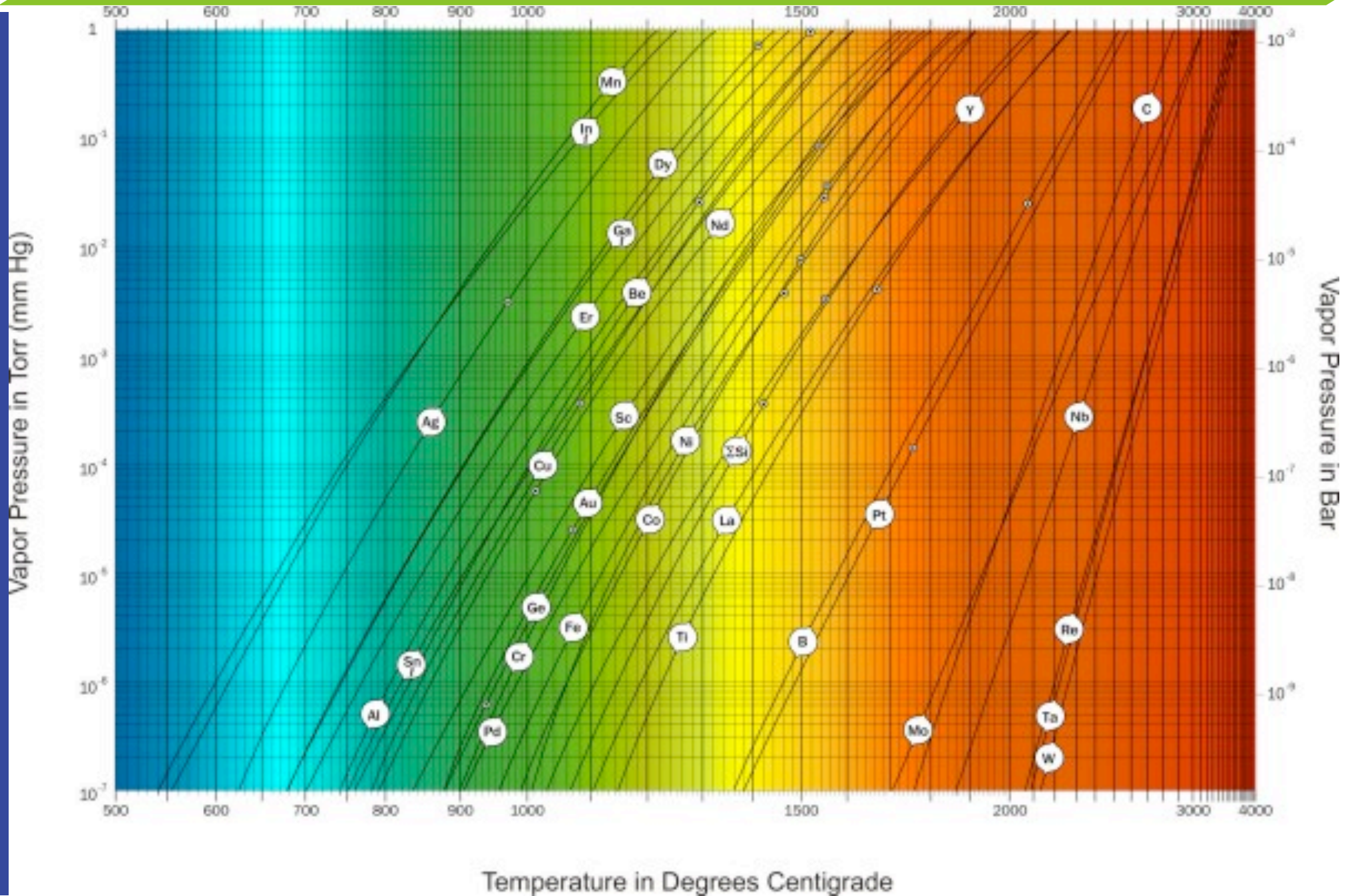


Vapour pressure

- vapor pressures: equilibrium pressure of vapour above its liquid (or solid); pressure of the vapour resulting from evaporation of a liquid (or solid) above a sample of the liquid (or solid) in a closed container
- vapor pressures at ambient temperatures increase with decreasing boiling points
- Important especially in surface ion sources



Vapour pressure



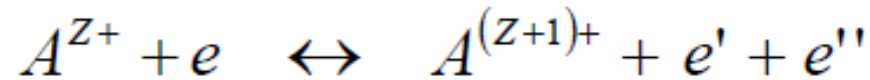
Atomic processes in ion sources

In most ion sources, ions are produced in a plasma.

The basic atomic processes (selection) in plasmas are:

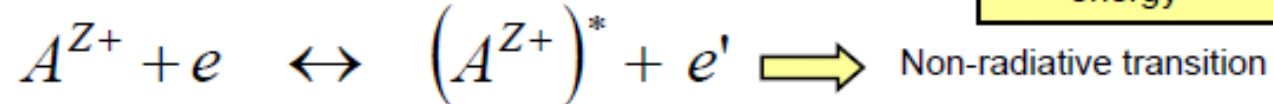
electrons collisions with

Impact Ionization



Three-Body-Recombination (TBR)

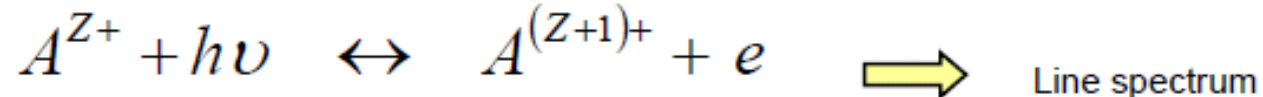
Impact excitation



Impact disexcitation

A^{Z+} : Atom of species A with charge state Z
 e' : electron changed energy

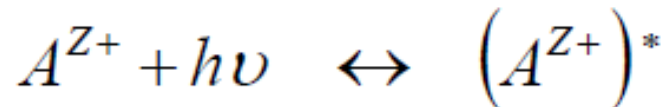
Photo ionization



Radiative Recombination (RR)

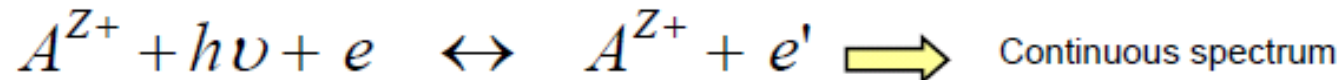
photons collisions with

Excitation



Spontaneous emission

Photo absorption



Bremsstrahlung

The electron changes from one free state to another free state with lower energy.

Atomic processes in ion sources

From these multiple processes arise the dynamic balance quantities:

- Distribution of the abundance of all charge states Z ($=0 \dots Z_{\max}$), **Ionization equilibrium**
- Number of emitted and absorbed photons per time interval, **Radiative equilibrium**

The density of the particle species are determined from so-called **rate equations**:

$$\frac{dn}{dt} = \text{souces} - \text{sinks}$$

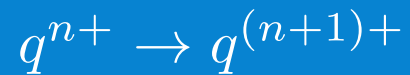
Example: impact ionization:
$$\frac{dn_{z+1}}{dt} = n_e \cdot n_z \cdot v_e \sigma_{z \rightarrow z+1} - n_e \cdot n_{z+1} \cdot \beta_{z+1, TBR}$$

$\beta_{z+1, TBR}$: rate coefficient for Three-Body-Recombination (TBR)

The **rate coefficients** often not be calculated with sufficient precision; experimental data are only available to a limited extent. Therefore one tries to obtain data from thermo dynamical equilibrium.

With decreasing electron density the TBR drops, so that the impact ionization is not in equilibrium with the TBR anymore. The RR rate also decreases but not as strong. With decreasing n_e also the photo ionization becomes unlikely. As result the **impact ionization** and the **RR-process** dominate. The photons leave the plasma without being re-absorbed.

Atomic processes in ion sources



- **Ionization**

- single-ionization
- double-ionization

The production of higher charge states is a successive process

The ionization has energy threshold

→ higher charge states need higher projectile energies (electron energies)

- **Charge exchange**
(for low charge states)

- **Recombination**

- radiative recombination
The cross section is larger for lower electron temperatures
- dielectronic recombination
(resonant process)

- **Charge exchange**
(for high charge states)

depending on the neutral particle density residual gas)

cross section are larger for higher charge states



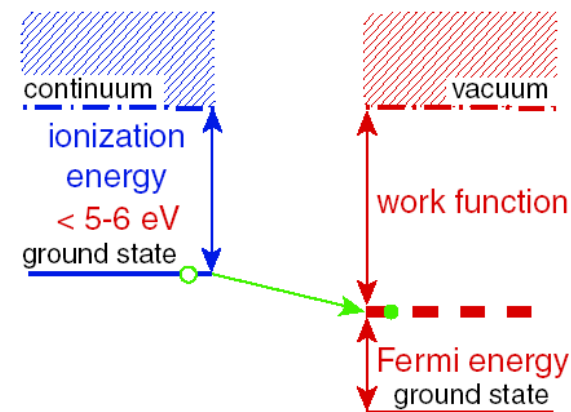
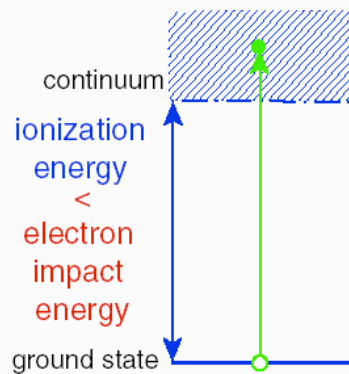
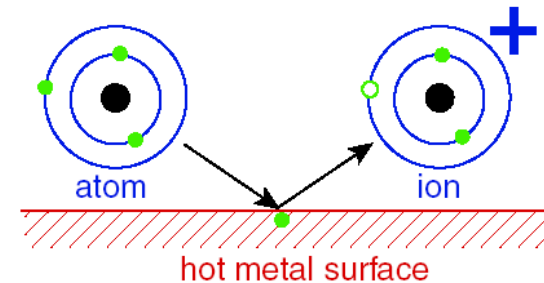
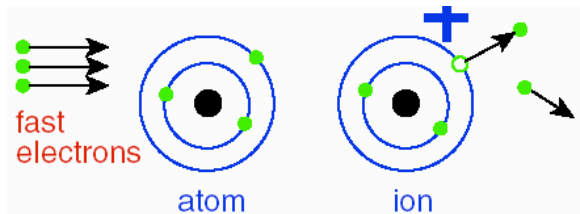
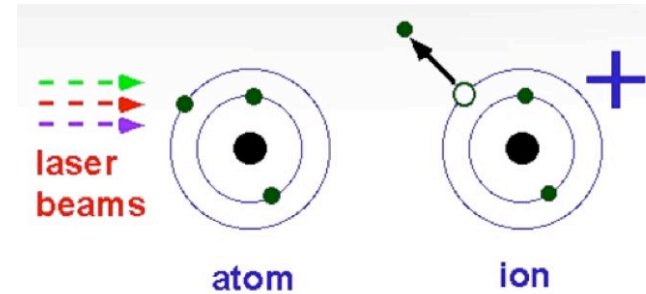
Ways to ionize atoms

Positive ions:

- Electron impact
- Photons
- Hot surfaces

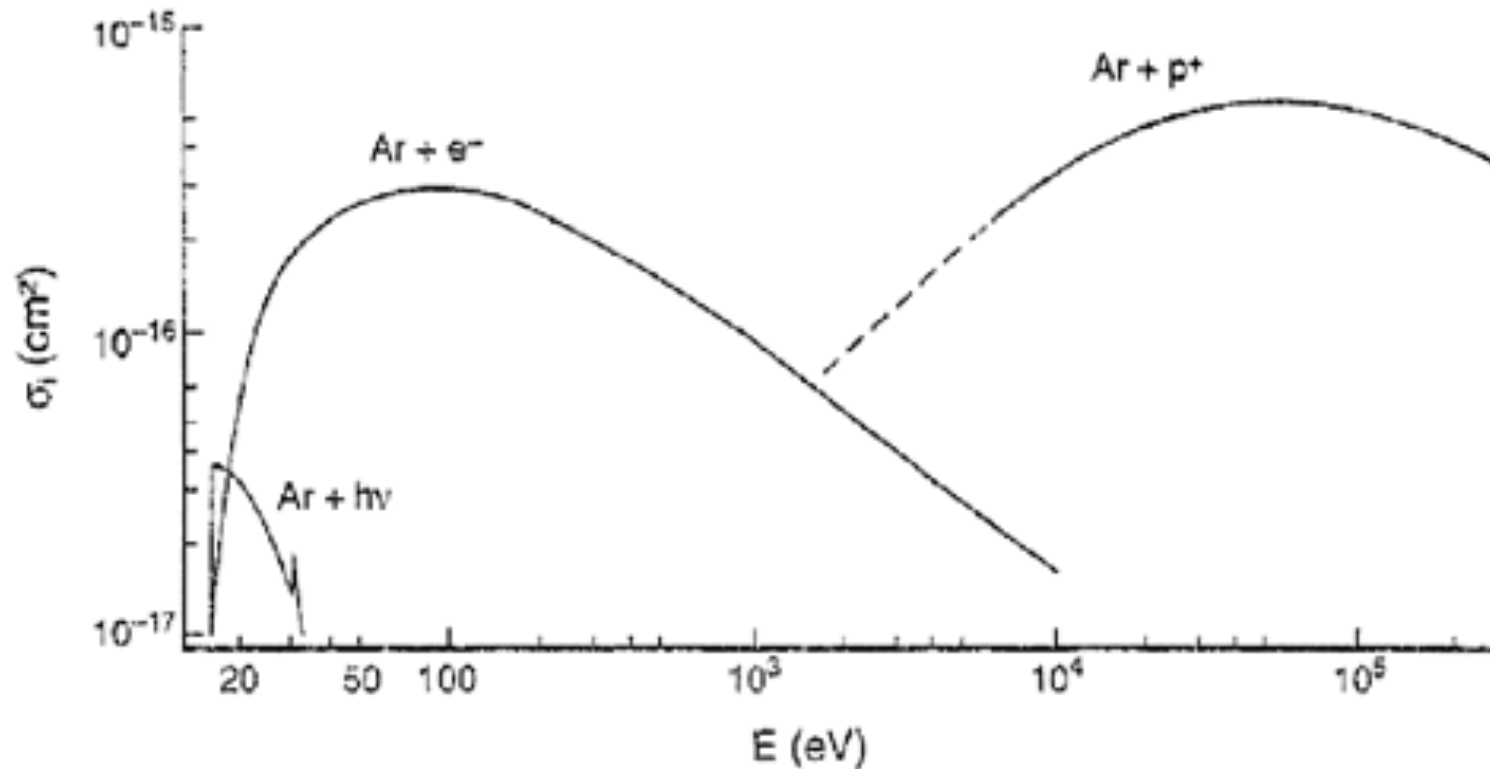
Negative ions:

- Electron attachment
- Charge exchange of a positive ion on a hot surface or in metal vapour
- (Molecule dissociation)
- (Molecule excitation)



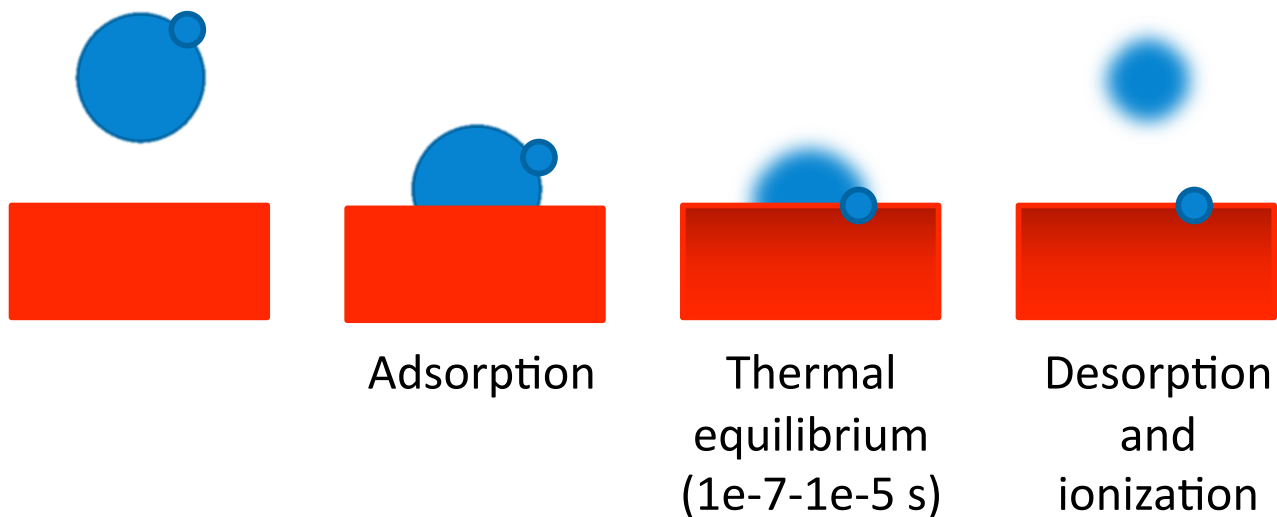
Ways to ionize atoms

- Ionization cross sections for Ar vs energy of ionizing collisions with:
 - Photons
 - Electrons
 - protons



Surface ionization

- **Surface ionization: ionization by contact with a (metal) surface**
- Requirements:
 - Atom sticks (is adsorbed) to the surface long enough to reach thermodynamic equilibrium => atom valence electron is “broadened” and can move between atom and surface
 - Surface is hot enough to desorb particles: some are neutral atoms, or positive/negative ions
- Material work function (W) = minimal energy required for an electron to escape the material surface
- Work function for elements follows a trend similar to ionization energy



Surface ionization

- Degree of positive surface ionization: **Saha-Langmuir equation**

Constant describing atom properties

Material **work function**

Atom **ionization energy**

Boltzman's constant

Material **temperature**

$$P_i = \frac{\text{ions}}{\text{atoms}} = G \exp\left(\frac{W - E_i}{kT}\right)$$

- Not important: Charge state before adsorbing on the surface (due to equilibrium)
- Important: material work function and state of particles before desorption (but after adsorption), i.e. atom ionization energy

$$G = \frac{g_i}{g_A} \frac{1-r}{1-r_0} = \frac{2J_i+1}{2J_A+1} \frac{1-r}{1-r_0}$$

$g_i/g_A=1/2$ for group I and 2 for group II

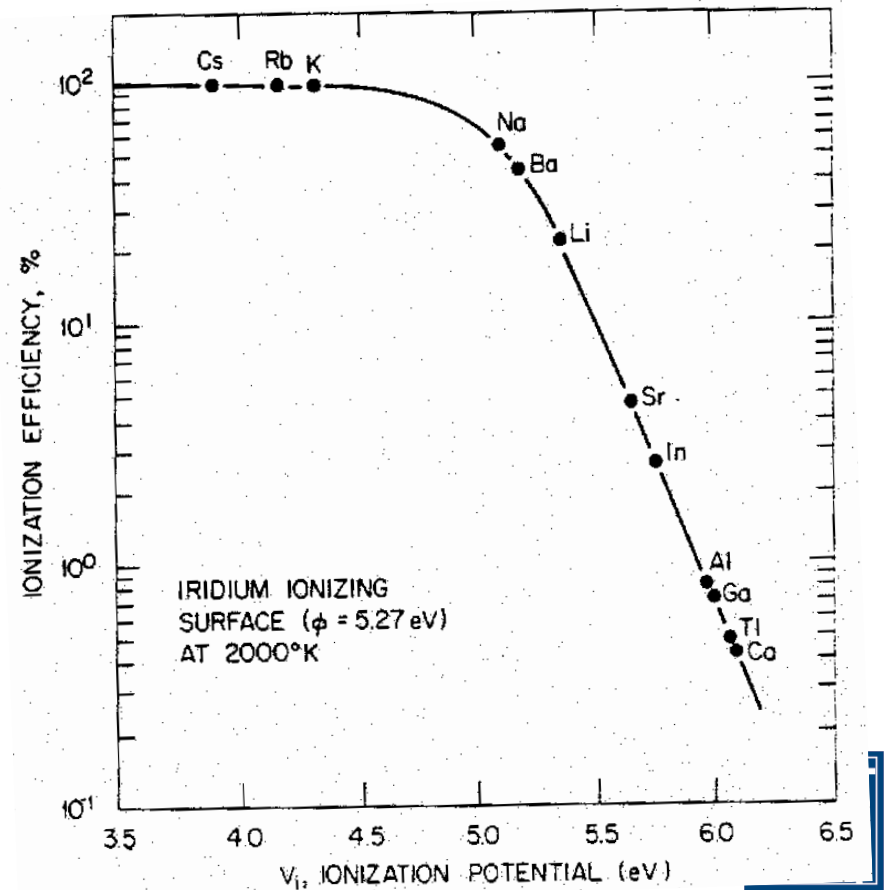
$J = 0, 1, \dots$ = quantum number (electron total angular momentum)
 $r = 0$ to 1 , reflection coefficient



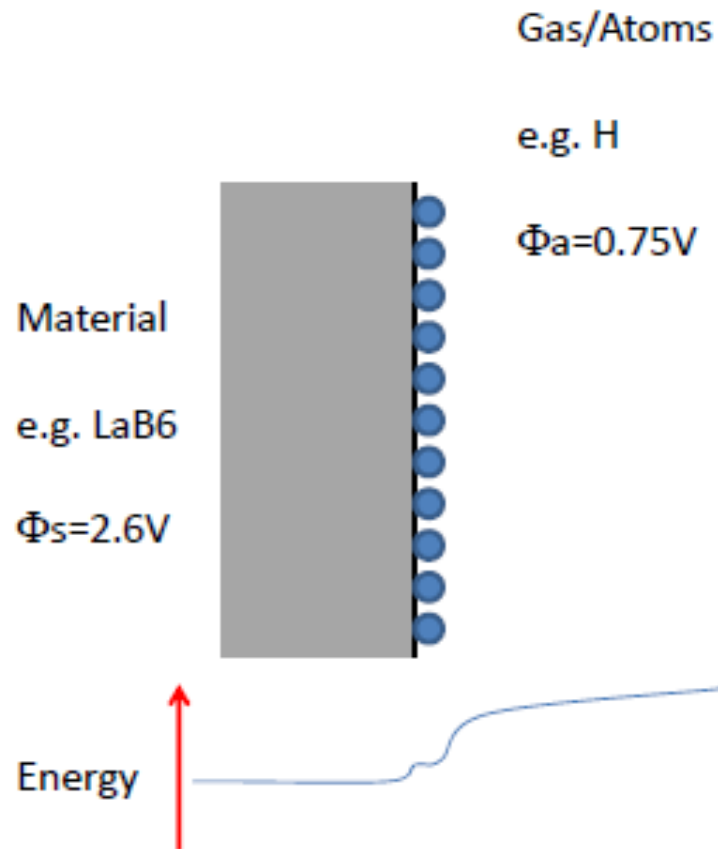
Surface ionization

particle	E_i (eV)	$W-E_i$ (eV)	$P_i(1000\text{ K})$	$P_i(1500\text{ K})$	$P_i(2000\text{ K})$	$P_i(2000\text{ K})$
Cs	3.88	0.64	790	72	20	10
K	4.32	0.20	6.3	2.2	1.6	1.3
Na	5.12	-0.60	$5e-4$	$5e-3$	$1.6e-2$	$3e-2$
Li	5.40	-0.88	$2e-5$	$6e-4$	$3e-3$	$8e-3$

- P_i decreases with increasing T if $W-E_i > 0$
- Surface ionization possible also for $W-E_i < 0$
- T must be sufficiently high to evaporate given element (e.g. Li, Na).
- On the contrary, the diffusion of surface material must be low enough (< 10% of a mono-layer), to keep the ionization conditions as constant as possible
- => trade-off required



Surface negative ionization



- If the base material has a low work-function, and the atoms a high electron affinity-
- The electrons have a finite probability of being present on the electron.
- Use plasma bombardment to liberate the atoms from the surface.
- Use a alkali metal coating deposited on the surfaces inside a plasma discharge source, to produce the ions.

● See 1st lecture

Resonant ionization by photons

RILIS (Resonant ionization laser ion source)

absorption

$$h\nu + E_l \rightarrow E_u$$

emission

spontaneous
(isotropic)

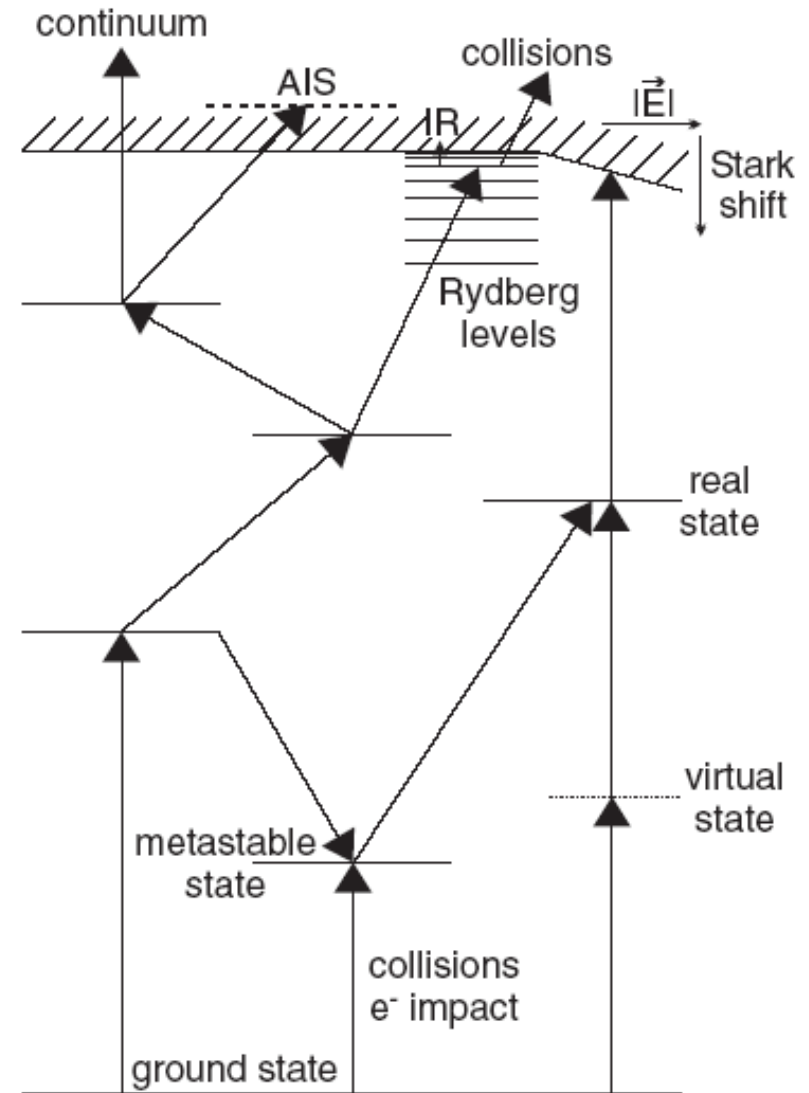
$$E_u \rightarrow E_l + h\nu$$

stimulated
(direction of incoming photon)

$$h\nu + E_u \rightarrow E_l + 2h\nu$$

Orders of magnitudes for the cross sections:

- non-resonant (direct ionization):
 $\sigma = 10^{-19} - 10^{-17} \text{ cm}^2$
- resonant:
 $\sigma = 10^{-10} \text{ cm}^2$
- AIS (auto-ionizing states): $\sigma = 1.6 \times 10^{-14} \text{ cm}^2$
- Rydberg states: $\sigma \sim 10^{-14} \text{ cm}^2$



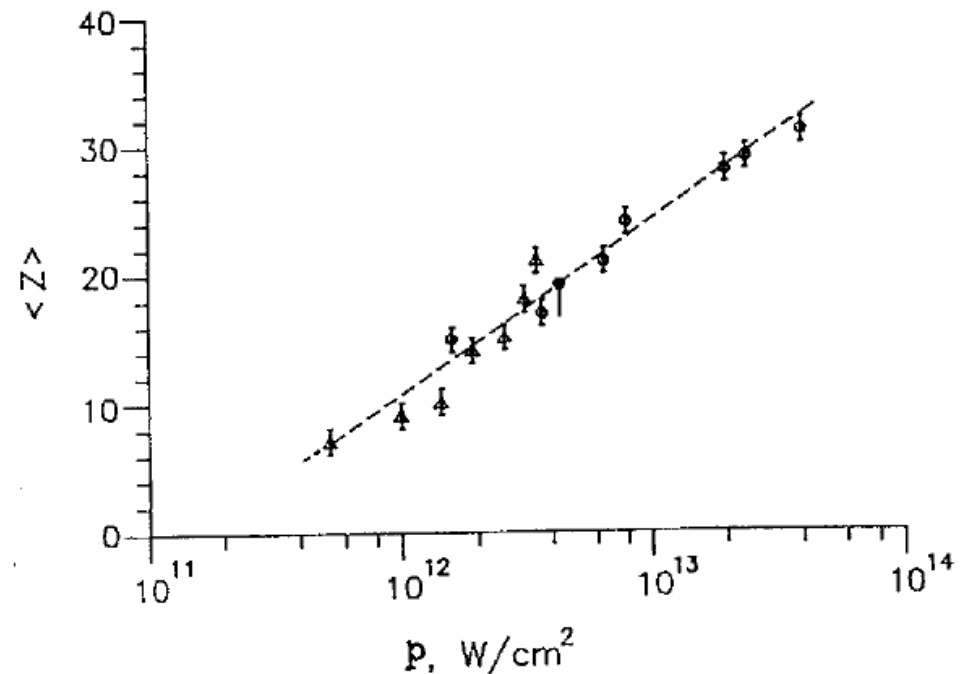
Excitation schemes used for
resonant laser ionization



Non-resonant photo-ionization

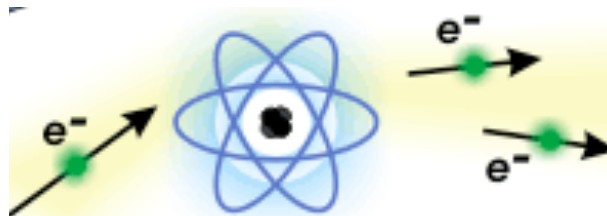
- In contrast to resonant ionization, ions/plasma within this source is generated via energy deposition
- Laser frequency couples to free electron plasma frequency (either in material, or formed plasma)
- The laser beam penetrates material
- Local heating of electrons by inverse Bremsstrahlung + excitation of atoms
- Material is ablated and an expanding plasma-plume develops
- If plasma density is lowered and cut-off frequency drops below laser-frequency, laser light can re-enter plasma.
- => Electrons inside plasma are accelerated up to 100 keV

Charge state vs laser power



Ionization by particle impact

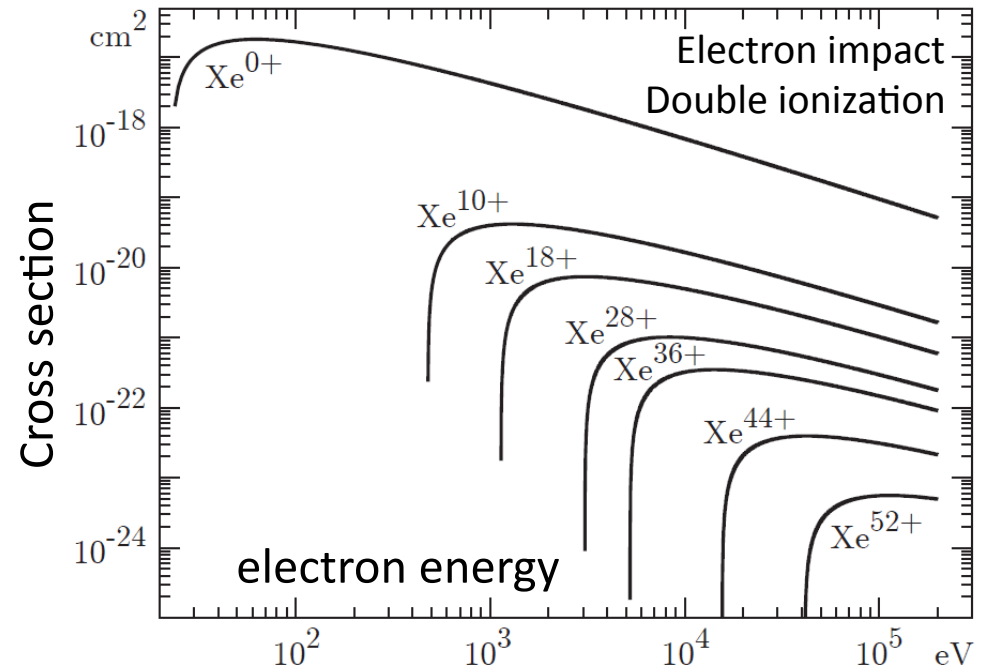
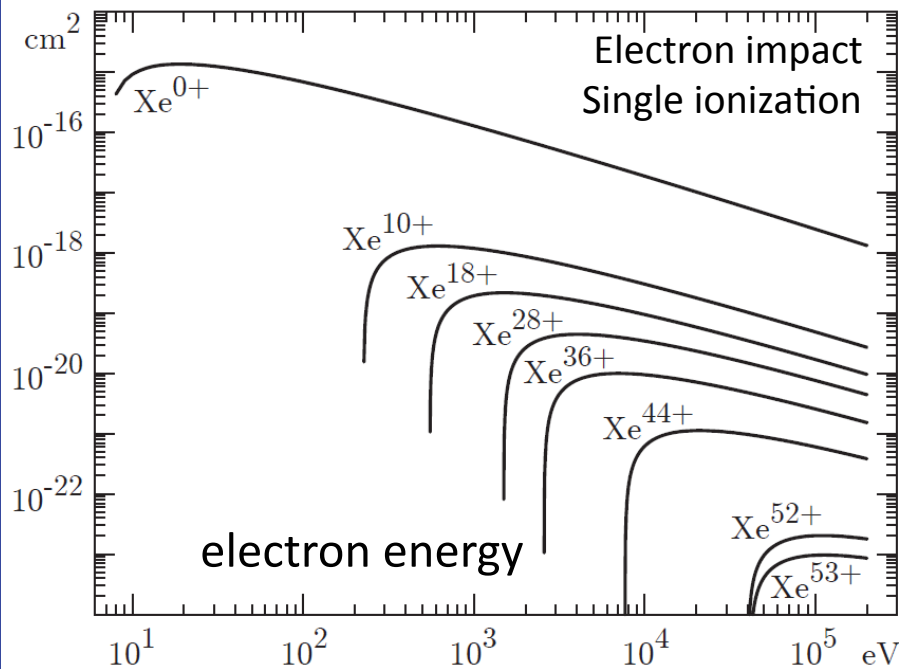
- Impact ionization is by orders of magnitudes higher than cross section for photo ionization
- Cross section depends on mass of colliding particle: energy transfer of a heavy particle is lower, proton needs for an identical ionization probability an ionization energy three orders of magnitudes higher than electron
- Thus, electrons are most common ionizing particles



- Processes involved:
 - Direct knockout ionization
 - Indirect processes (based on inner-shell excitation and subsequent autoionization); more important for heavier atoms

Ionization by electron impact

- Ionization cross section: electron energy has to be $>$ ionization energy



- For energetic reasons most probable process is ionization releasing only one electron from atomic shell
- To produce highly charged ions, kinetic energy of projectile electrons has to be at least equivalent to n -th ionization potential

Ionization by electron impact

- Approximation of cross section and ionization time for production of bare ions from H-like ions using Mosley's law, for X-ray frequencies emitted in transitions from continuum to K-shell:

$$E_{i \rightarrow k}(Z) = 13.6 \cdot Z^2 [eV] \rightarrow$$

σ – single ionization cross-section cm^2
 j_e – electron current density A/cm^2

$$\sigma_{z-1 \rightarrow z} = 4.5 \cdot 10^{-14} \cdot \frac{\ln e}{e \cdot 13.6^2 Z^4} = \frac{9 \cdot 10^{-17}}{Z^4}$$

with $E = e \cdot E_{i \rightarrow k}(Z)$

$$v_{z-1 \rightarrow z} = \frac{j_e}{e} \sigma_{z-1 \rightarrow z} = \frac{1}{\tau_{z-1 \rightarrow z}}$$

$$\rightarrow j_e \tau_{z-1 \rightarrow z} = \frac{e}{\sigma_{z-1 \rightarrow z}} \approx \frac{eZ^4}{9 \cdot 10^{-17}} \approx \left(\frac{Z}{5}\right)^4$$

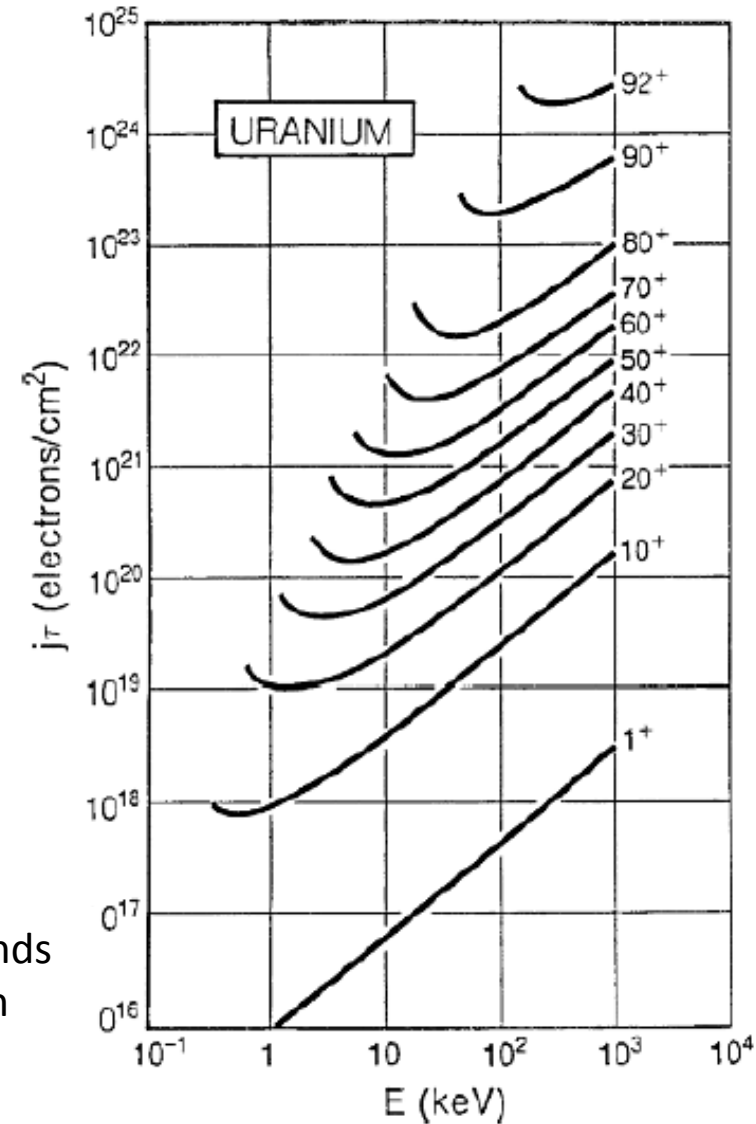
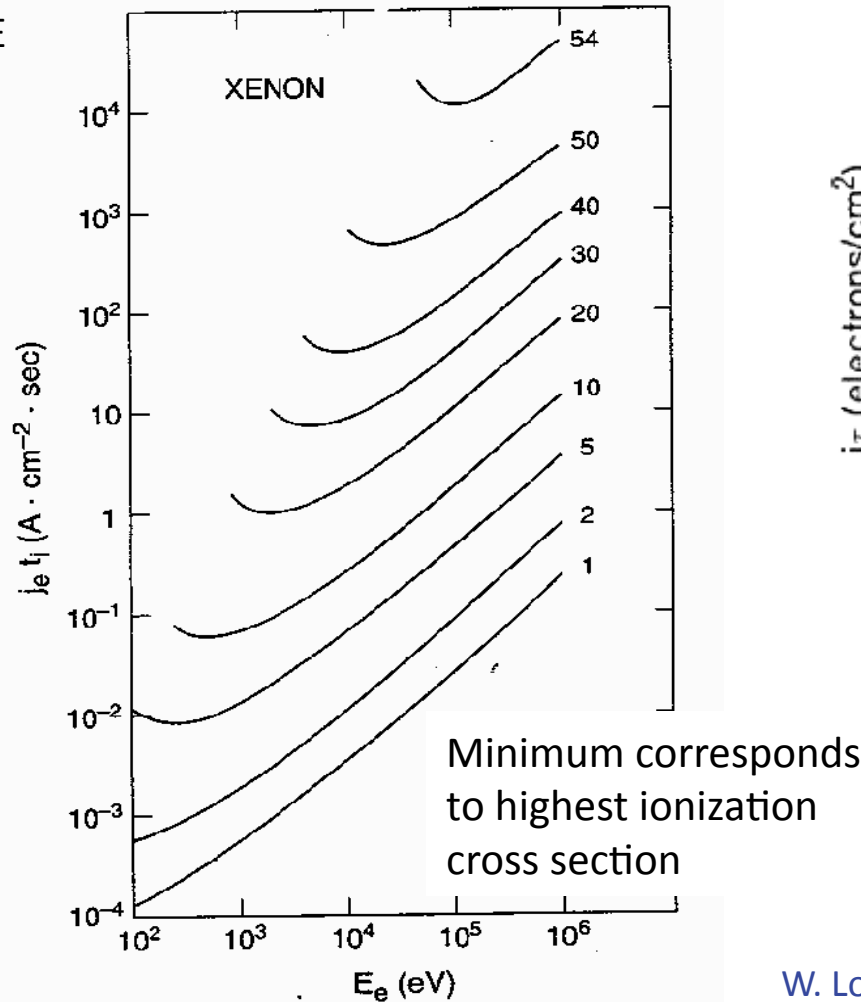
Ionization by electron impact

- Lotz formula: semi-empirical formula for cross-section of multiple ionization

$$\sigma_{q \rightarrow q+1} = 4.5 \cdot 10^{-14} \cdot \sum_{nl} \frac{\ln\left(\frac{E_{kin}}{P_i}\right)}{E_{kin} \cdot P_i} \quad [\text{cm}^2]$$

Higher E

Ionization factor $j_e \cdot \tau$ for different charge states of Xe depending on electron energy



Ionization by electron impact



Lotz formula in detail:

- 3 parameters to represent cross sections for single ionization from ground-state
- Approximates almost all data within 10% and within exp. errors for up to 10keV-electrons
- Empirical formula, but with proper theory basis: follows earlier theoretical work on e-e scattering, uses approximation of starting and final wave functions, gets parameters from fits

Total ionization cross-section σ is the sum over subshells. N is the number of subshells. $a_i q_i$ is the number of electrons in subshell i . P_i is the ionization potential of subshell i . b_i and c_i are empirical parameters.

$$\sigma = \sum_{i=1}^N a_i q_i \frac{\ln(E/P_i)}{E P_i} \{1 - b_i \exp[-c_i(E/P_i - 1)]\};$$

$E < E(\text{max cross section})$:

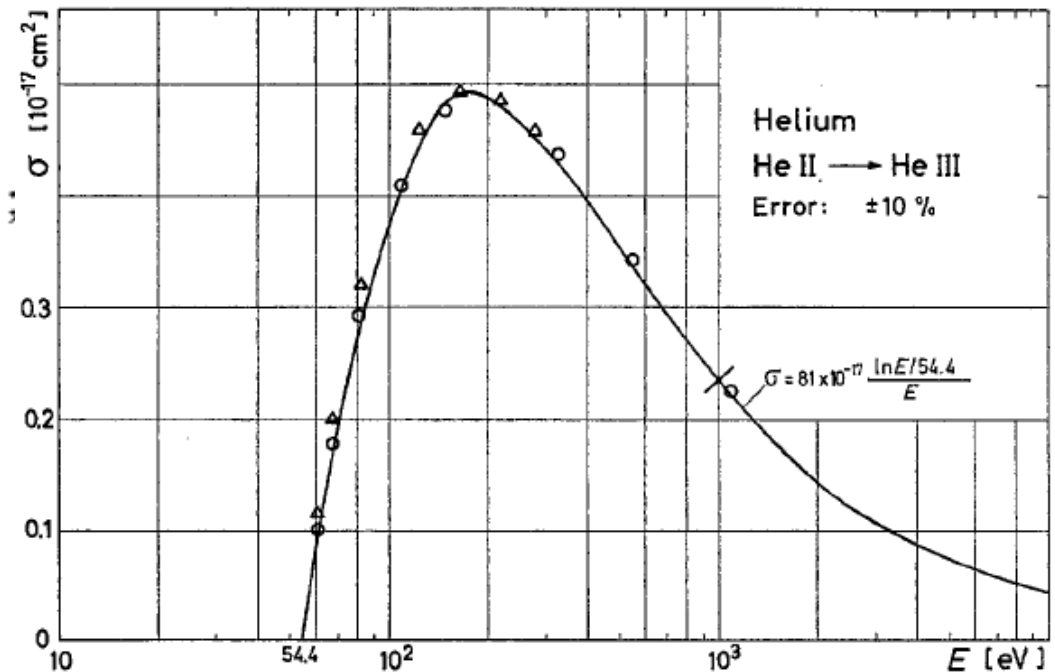
$$\sigma = \sum_{i=1}^N a_i q_i \frac{\ln(E/P_i)}{E P_i} \{1 - b_i \exp[-c_i(E/P_i - 1)]\};$$

E close to P_i

$$\sigma \approx a_1 q_1 \frac{(E/P_1 - 1)}{P_1^2} (1 - b_1) \propto U - 1, \quad U = E/P_1.$$

$$E \gg P_i \quad \sigma_i = a_i q_i \frac{\ln(E/P_i)}{E P_i} \propto \frac{\ln E}{E}.$$

$a_i = 2.6 - 4.5 \text{ e-14 cm}^2$ (empirically)



Optimal electron energy

- Where $\sigma(E)$ has maximum, ionization factor $j_e \cdot \tau$ has minimum there
- Cross section for the last electron, which is removed, determines ionization time

$$\frac{d\sigma_{z \rightarrow z+1}}{dE} = 4.5 \cdot 10^{-14} \cdot \sum_{i=1}^N \frac{d}{dE} \left(\frac{\ln \left(\frac{E_{kin}}{P_i} \right)}{E_{kin} \cdot P_i} \right) = 0$$

$$\sum_{i=1}^N \frac{1}{P_i E^2} \left(1 - \ln \left(\frac{E}{P_i} \right) \right) = 0 \Rightarrow E_{\max} = \exp \left(\frac{\sum_{i=1}^N \frac{1 + \ln P_i}{P_i}}{\sum_{i=1}^N \frac{1}{P_i}} \right) = e \cdot \exp \left(\frac{\sum_{i=1}^N \frac{\ln P_i}{P_i}}{\sum_{i=1}^N \frac{1}{P_i}} \right)$$

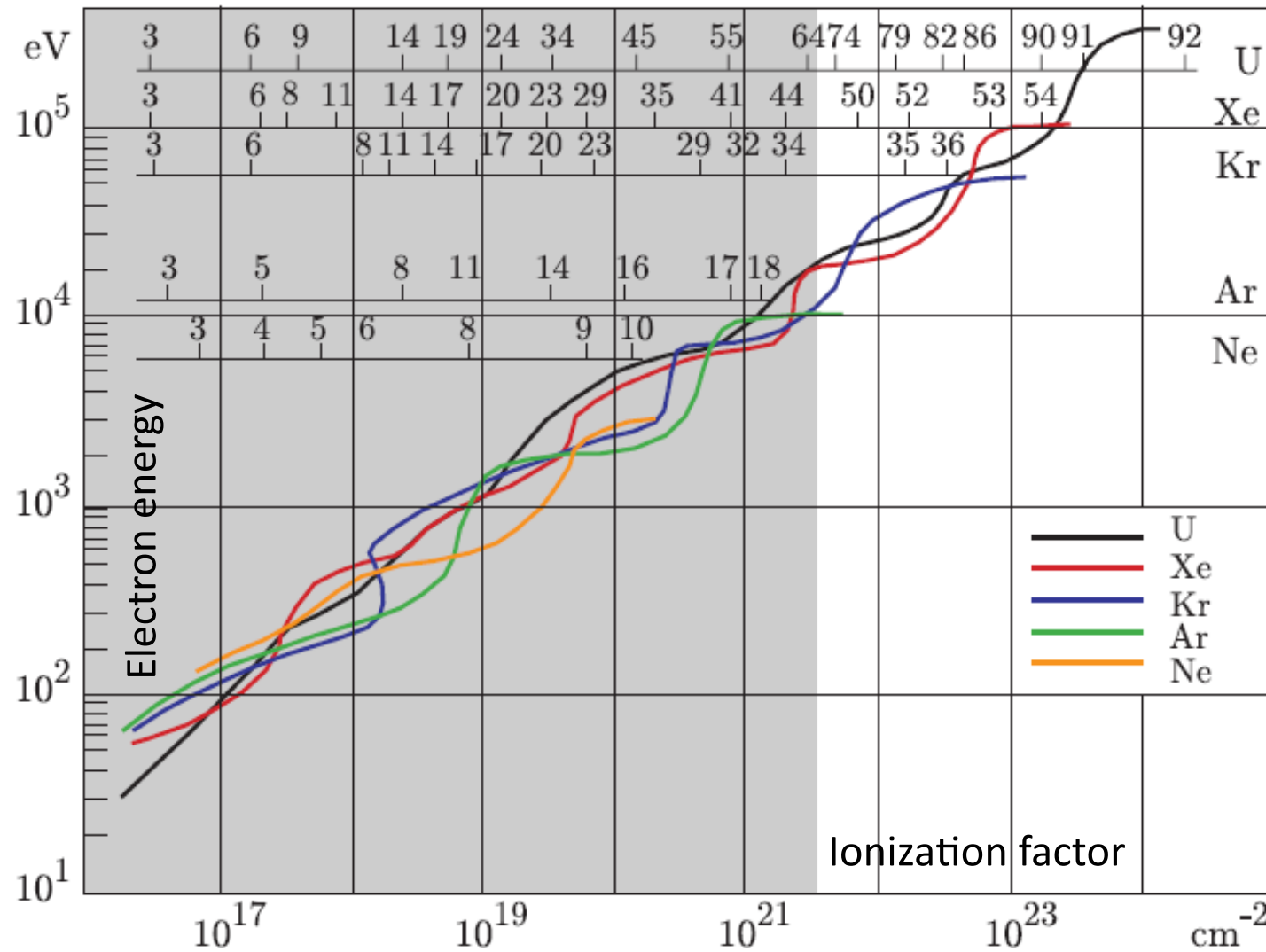
- Optimal energy of last electron, which is removed:

$$E_{\max} \approx e \cdot \exp \left(\frac{\ln P_z}{\frac{P_z}{1/P_z}} \right) = e \cdot P_z$$

- => Optimal energy is nearly e-times ionization energy of last electron removed from ion with charge state z



Ion. factor and optimal electron energy



Summary

- Atomic physics governs many aspects of ion sources
- Electron shell structure determines energy required to excite electrons and even to eject them from atoms (ionization)
- These energies dictate chemical properties of elements
- There are different paths to atom ionization (surface, particle, photon impact)
- Their details depend on electron structure of ionized atoms and involve many atomic physics processes

Literature

- B. Wolf, *Handbook of ion sources*, CRC Press 1995
- G. Brown, *The physics and technology of ion sources*, 2004, Wiley VCH Verlag GmbH, ISBN 3-527-40410-4
- F.J. Currell, *The Physics of Multiply and Highly Charged Ions Volume 1: Sources, Applications and Fundamental Processes*, 2004, Kluwer
- O. Kester, *Lectures on Ion Sources*:
<http://acc.physik.uni-frankfurt.de/vorlesung/vorlesung.html>
- Ch. Foot, *Atomic Physics*, Oxford University Press 2005



Additional slides



Ionization by particle impact

Approximate Ionization Energies, Ionization Cross Sections, and Required $j\tau$ Values for Bare Ions

Ion	E_i (eV)	σ (cm ²)	$j\tau$ (Cb/cm ²)
C ⁶⁺	490	7.7×10^{-20}	2.1
N ⁷⁺	666	4.2×10^{-20}	3.8
O ⁸⁺	870	2.4×10^{-20}	6.5
Ne ¹⁰⁺	1360	1×10^{-20}	16
Ar ¹⁸⁺	4,400	9.5×10^{-22}	170
Kr ³⁶⁺	17,600	6×10^{-23}	2,700
Xe ⁵⁴⁺	39,700	1.2×10^{-23}	13,600
Pb ⁸²⁺	91,400	2.2×10^{-24}	72,300
U ⁹²⁺	115,000	1.4×10^{-24}	115,000

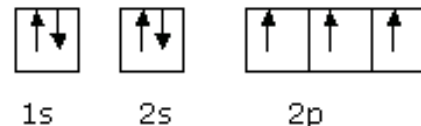
Examples: Argon can be ionized by 10 keV electrons, ions of the heavy elements by up to 100 keV electrons and Uranium by 150 keV electrons. The resulting values for the ionization energy, cross section and ionization factor are summarized in the table on the left side.

Electron configurations

- Hund's rule of Maximum Multiplicity

N ($Z = 7$)

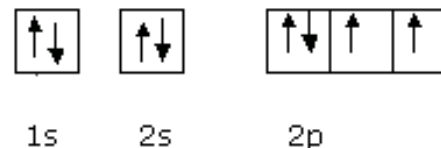
$1s^2 2s^2 2p^3$



The eighth electron in oxygen must enter one of the three half-filled 2p orbitals and pair up with (have opposing spin to) the electron already present

O ($Z = 8$)

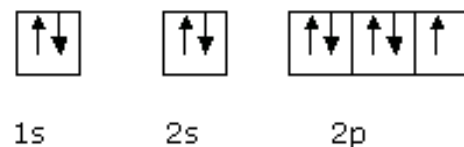
$1s^2 2s^2 2p^4$



Fluorine's ninth electron enters either one of the two remaining half-filled 2p orbitals

F ($Z = 9$)

$1s^2 2s^2 2p^5$



Neon's tenth electron enters the one unfilled orbital in the 2p subshell

Ne ($Z = 10$)

$1s^2 2s^2 2p^6$



PERIODIC TABLE Atomic Properties of the Elements

NIST
National Institute of
Standards and Technology
U.S. Department of Commerce

Group	1 IA	2 IIA	Frequently used fundamental physical constants										13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA								
Period	1 H Hydrogen 1.00794 1s 13,5984	2 He Helium 4.002602 1s ² 24,8674	<p>For the most accurate values of these and other constants, visit physics.nist.gov/constants</p> <p>1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ¹³³Cs</p> <p>speed of light in vacuum c 299 792 458 m s⁻¹ (exact) Planck constant h 6,52561 x 10⁻³⁴ J s ($\hbar = h/2\pi$) elementary charge e 1,6022 x 10⁻¹⁹ C electron mass m_e 9,1094 x 10⁻³¹ kg $m_e c^2$ 0,5110 MeV proton mass m_p 1,6726 x 10⁻²⁷ kg fine-structure constant α 1/137,036 Rydberg constant R_∞ 10 973 732 m⁻¹ $R_\infty c$ 3,289 842 x 10¹⁵ Hz $R_\infty hc$ 13,6057 eV Boltzmann constant k 1,3807 x 10⁻²³ J K⁻¹</p>										5 B Boron 10,811 1s ² 2s ² 2p 8,2960	6 C Carbon 12,0107 1s ² 2s ² 2p ² 11,2603	7 N Nitrogen 14,0067 1s ² 2s ² 2p ³ 14,5341	8 O Oxygen 15,9994 1s ² 2s ² 2p ⁴ 13,6181	9 F Fluorine 18,9984032 1s ² 2s ² 2p ⁵ 17,4228	10 Ne Neon 20,1797 1s ² 2s ² 2p ⁶ 21,5645	11 Na Sodium 22,98976928 [Ne]3s 6,1391	12 Mg Magnesium 24,3050 [Ne]3s ² 7,6462	13 Al Aluminum 26,9815386 [Ne]3s ² 3p 6,9858	14 Si Silicon 28,0855 [Ne]3s ² 3p ² 8,1517	15 P Phosphorus 30,973762 [Ne]3s ² 3p ³ 10,4867	16 S Sulfur 32,065 [Ne]3s ² 3p ⁴ 10,3600	17 Cl Chlorine 35,453 [Ne]3s ² 3p ⁵ 12,9676	18 Ar Argon 39,948 [Ne]3s ² 3p ⁶ 15,7596
	19 K Potassium 39,0983 [Ar]4s 4,3407	20 Ca Calcium 40,078 [Ar]4s ² 6,1132	21 Sc Scandium 44,955912 [Ar]3d ¹ 4s ² 6,5615	22 Ti Titanium 47,867 [Ar]3d ² 4s ² 6,7462	23 V Vanadium 50,9415 [Ar]3d ³ 4s ² 6,7465	24 Cr Chromium 51,9961 [Ar]3d ⁵ 4s 6,7685	25 Mn Manganese 54,938045 [Ar]3d ⁵ 4s ² 7,4340	26 Fe Iron 55,845 [Ar]3d ⁶ 4s ² 7,8024	27 Co Cobalt 58,933195 [Ar]3d ⁷ 4s ² 7,8399	28 Ni Nickel 58,9334 [Ar]3d ⁸ 4s ² 7,6399	29 Cu Copper 63,546 [Ar]3d ¹⁰ 4s 7,7254	30 Zn Zinc 65,38 [Ar]3d ¹⁰ 4s ² 8,3942	31 Ga Gallium 69,723 [Ar]3d ¹⁰ 4s ² 4p 6,8993	32 Ge Germanium 72,64 [Ar]3d ¹⁰ 4s ² 4p ² 7,8994	33 As Arsenic 74,92160 [Ar]3d ¹⁰ 4s ² 4p ³ 8,7986	34 Se Selenium 78,96 [Ar]3d ¹⁰ 4s ² 4p ⁴ 9,7524	35 Br Bromine 79,904 [Ar]3d ¹⁰ 4s ² 4p ⁵ 11,8135	36 Kr Krypton 83,798 [Ar]3d ¹⁰ 4s ² 4p ⁶ 13,9936								
	37 Rb Rubidium 85,4678 [Kr]5s 4,1771	38 Sr Strontium 87,62 [Kr]5s ² 5,6949	39 Y Yttrium 88,90585 [Kr]4d ¹ 5s ² 6,2173	40 Zr Zirconium 91,224 [Kr]4d ² 5s ² 6,6339	41 Nb Niobium 92,90638 [Kr]4d ⁴ 5s 6,7589	42 Mo Molybdenum 95,96 [Kr]4d ⁵ 5s 7,0924	43 Tc Technetium (98) [Kr]4d ⁵ 5s ² 7,26	44 Ru Ruthenium 101,07 [Kr]4d ⁷ 5s 7,3605	45 Rh Rhodium 102,90550 [Kr]4d ⁸ 5s 7,4569	46 Pd Palladium 106,42 [Kr]4d ¹⁰ 8,3369	47 Ag Silver 107,8682 [Kr]4d ¹⁰ 5s 7,5762	48 Cd Cadmium 112,411 [Kr]4d ¹⁰ 5s ² 8,9938	49 In Indium 114,818 [Kr]4d ¹⁰ 5s ² 5p 5,7854	50 Sn Tin 118,710 [Kr]4d ¹⁰ 5s ² 5p ² 7,3439	51 Sb Antimony 121,760 [Kr]4d ¹⁰ 5s ² 5p ³ 8,6084	52 Te Tellurium 127,60 [Kr]4d ¹⁰ 5s ² 5p ⁴ 9,0096	53 I Iodine 126,90447 [Kr]4d ¹⁰ 5s ² 5p ⁵ 10,4513	54 Xe Xenon 131,293 [Kr]4d ¹⁰ 5s ² 5p ⁶ 12,1298								
	55 Cs Cesium 132,9054519 [Xe]6s 3,6839	56 Ba Barium 137,327 [Xe]6s ² 5,2117	57 La Lanthanum 138,90547 [Xe]5d ¹ 6s ² 5,5769	58 Ce Cerium 140,116 [Xe]4f ¹ 5d ¹ 6s ² 5,5367	59 Pr Praseodymium 140,90765 [Xe]4f ³ 6s ² 5,473	60 Nd Neodymium 144,242 [Xe]4f ⁴ 6s ² 5,5250	61 Pm Promethium (145) [Xe]4f ⁵ 6s ² 5,562	62 Sm Samarium 150,36 [Xe]4f ⁶ 6s ² 5,6437	63 Eu Europium 151,964 [Xe]4f ⁷ 6s ² 5,6704	64 Gd Gadolinium 157,25 [Xe]4f ⁷ 5d ¹ 6s ² 6,1498	65 Tb Terbium 168,9348 [Xe]4f ⁹ 6s ² 5,6638	66 Dy Dysprosium 162,500 [Xe]4f ¹⁰ 6s ² 5,9369	67 Ho Holmium 164,93032 [Xe]4f ¹¹ 6s ² 6,0215	68 Er Erbium 167,259 [Xe]4f ¹² 6s ² 6,1077	69 Tm Thulium 168,93421 [Xe]4f ¹³ 6s ² 6,1843	70 Yb Ytterbium 173,054 [Xe]4f ¹⁴ 6s ² 6,2542	71 Lu Lutetium 174,967 [Xe]4f ¹⁴ 5d ¹ 6s ² 6,4250									
	87 Fr Francium (223) [Rn]7s 4,0727	88 Ra Radium (226) [Rn]7s ² 5,2784	104 Rf Rutherfordium (261) [Rn]5f ¹⁴ 6d ² 7s ² 6,07	105 Db Dubnium (268)	106 Sg Seaborgium (271)	107 Bh Bohrium (272)	108 Hs Hassium (277)	109 Mt Meitnerium (276)	110 Ds Darmstadtium (281)	111 Rg Roentgenium (280)	112 Cn Copernicium (285)	113 Uut Ununtrium (284)	114 Uuq Ununquadium (289)	115 Uup Ununpentium (288)	116 Uuh Ununhexium (293)	117 Uus Ununseptium (294)	118 Uuo Ununoctium (294)									
	89 Ac Actinium (227) [Rn]5f ⁷ 6s 5,3807	90 Th Thorium 232,03806 [Rn]6s ² 7s ² 6,3067	91 Pa Protactinium 231,03688 [Rn]5f ² 6s ² 5,49	92 U Uranium 238,02891 [Rn]5f ³ 6s ² 7s ² 6,1939	93 Np Neptunium (237) [Rn]5f ⁴ 6s ² 7s ² 6,2657	94 Pu Plutonium (244) [Rn]5f ⁶ 7s ² 6,2260	95 Am Americium (243) [Rn]5f ⁷ 7s ² 5,9738	96 Cm Curium (247) [Rn]5f ⁸ 7s ² 5,9914	97 Bk Berkelium (247) [Rn]5f ⁹ 7s ² 6,1979	98 Cf Californium (251) [Rn]5f ¹⁰ 7s ² 6,2917	99 Es Einsteinium (252) [Rn]5f ¹¹ 7s ² 6,3676	100 Fm Fermium (257) [Rn]5f ¹² 7s ² 6,50	101 Md Mendelevium (258) [Rn]5f ¹³ 7s ² 6,58	102 No Nobelium (259) [Rn]5f ¹⁴ 7s ² 6,65	103 Lr Lawrencium (262) [Rn]5f ¹⁴ 7s ² 7p ¹ 4,97											
			<p>Lanthanides</p> <p>Actinides</p>																							

■ Solids
■ Liquids
■ Gases
■ Artificially Prepared

Atomic Number: 58
Ground-state Level: 1G₄
Symbol: **Ce**
Name: Cerium
Atomic Weight: 140,116
Ground-state Configuration: [Xe]4f¹5d¹6s²
Ionization Energy (eV): 5,5387

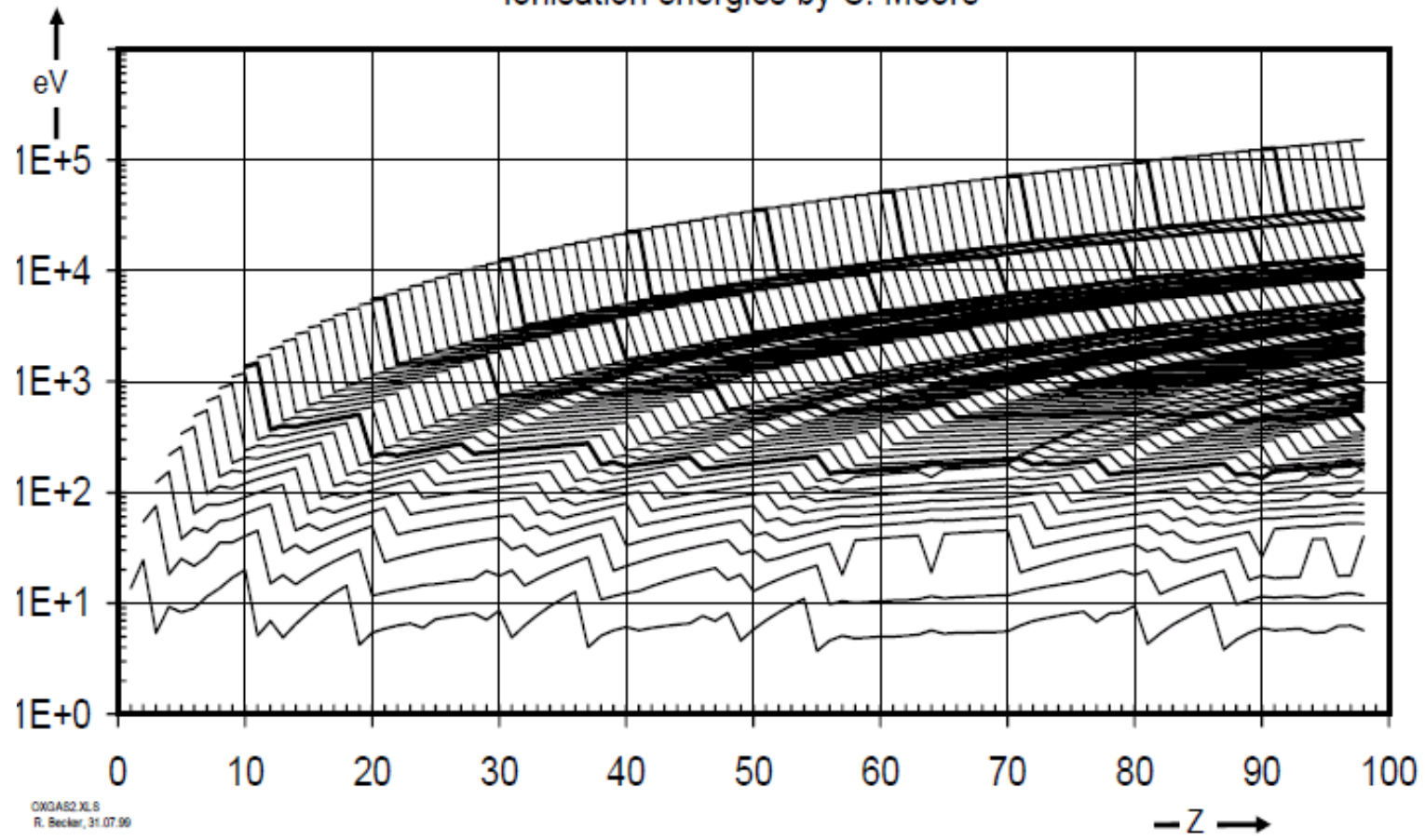
¹Based upon ¹²C. () indicates the mass number of the longest-lived isotope.

For a description of the data, visit physics.nist.gov/data

NIST SP 966 (September 2010)



Ionisation energies by C. Moore



Charge exchange

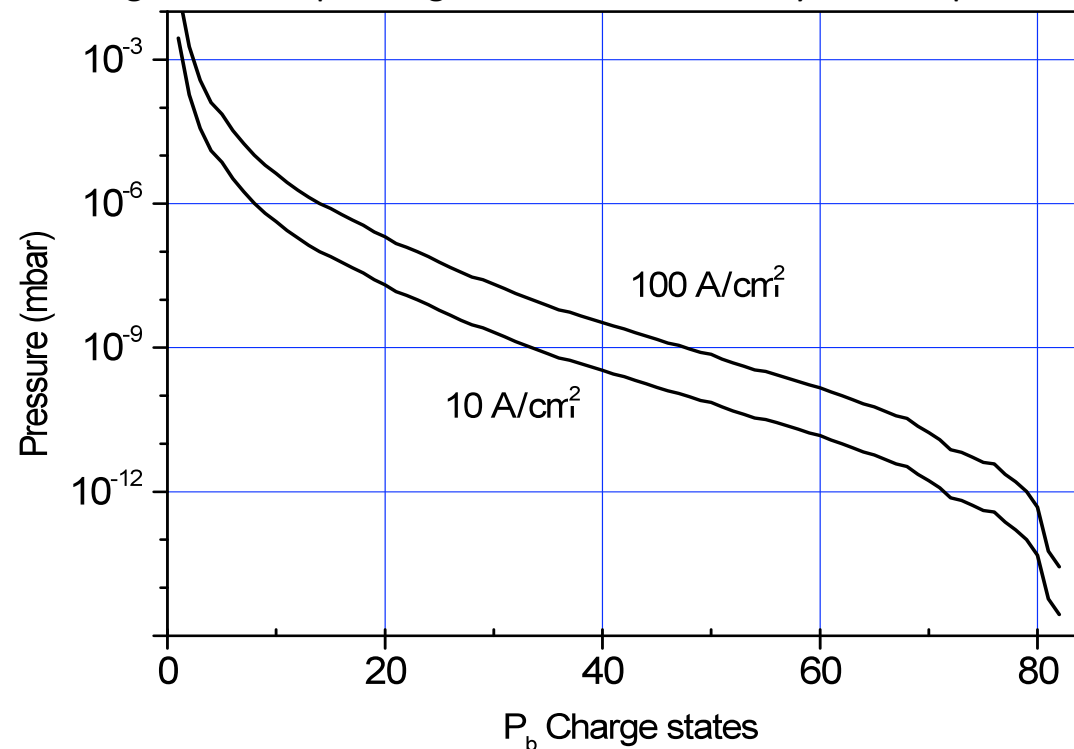
Charge exchange:

For the approximation of the charge exchange cross section, commonly the equation by **Müller and Salzborn** is used:

$$\sigma_{i \rightarrow i-1}^{ex} \approx 1.43 \times 10^{-12} i^{1.17} E_{ion, gas}^{-2.76} \text{ [cm}^2 \text{]}$$

Therein i describes the charge state of the highly charged ion and $E_{ion, gas}$ is the ionization potential of the gas atoms interacting with the ion.

Example: Reachable charge states depending on the current density and the pressure



Radiative recombination

For the approximation of the RR cross sections the semi-classical expression by Kim und Pratt is used. Their formula bases on the first theoretical description of the Radiative Recombination by Kramers (1923):

$$\sigma(E) = \frac{8\pi}{3\sqrt{3}} \alpha (\lambda_e)_r^2 \chi \ln \left(1 + \frac{\chi}{2(n_0)_{\text{eff}}} \right) \quad \text{with} \quad \chi = 2 Z_{\text{eff}}^2 Ry/E$$

with:

- the effective nuclear charge $Z_{\text{eff}} = \frac{1}{\sigma} (Z + q)$ to take into account the shielding of the nuclear potential by the bound electrons

- the effective main quantum number

$$(n_0)_{\text{eff}} = n + (1 - w_0) - 0.3$$

to include the capture into states with different angular momentums

Ry: Rydberg energy 13,6 eV

α : fine structure constant

w_0 ratio between the numbers of occupied and unoccupied states

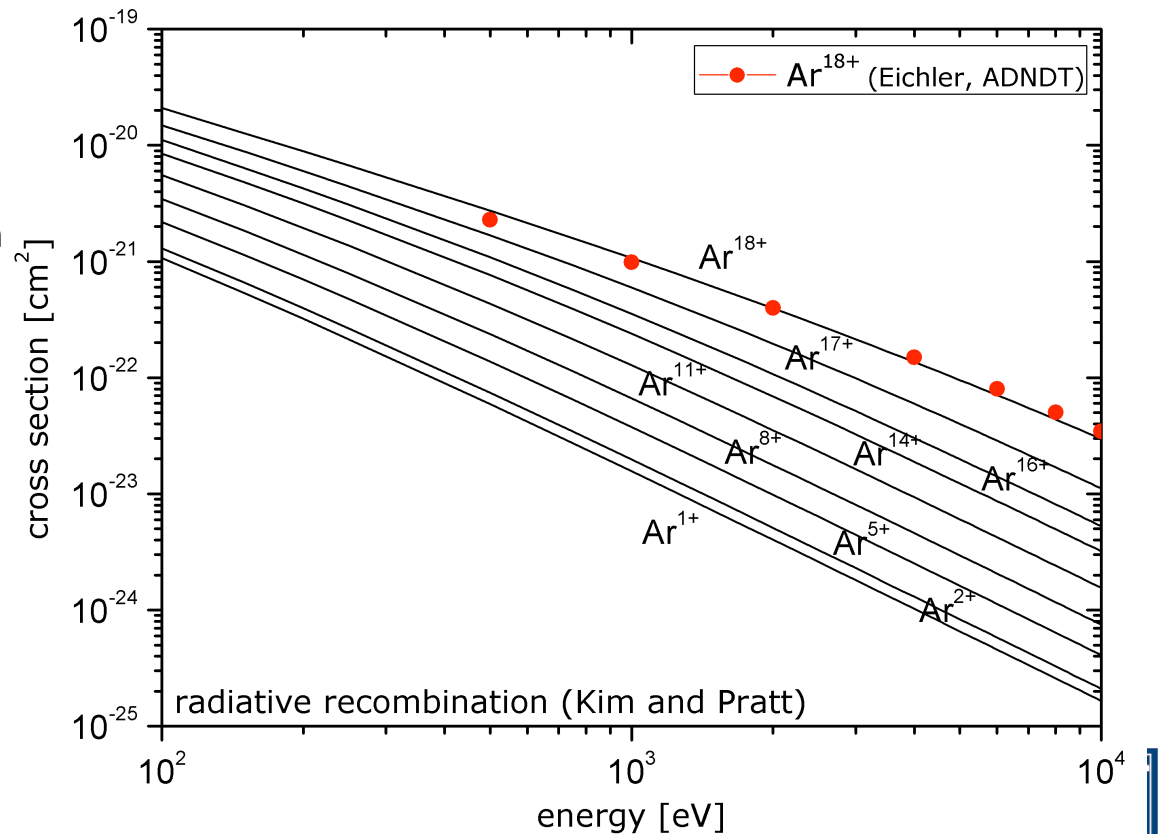
Z: nuclear charge

q: charge state

n: main quantum number

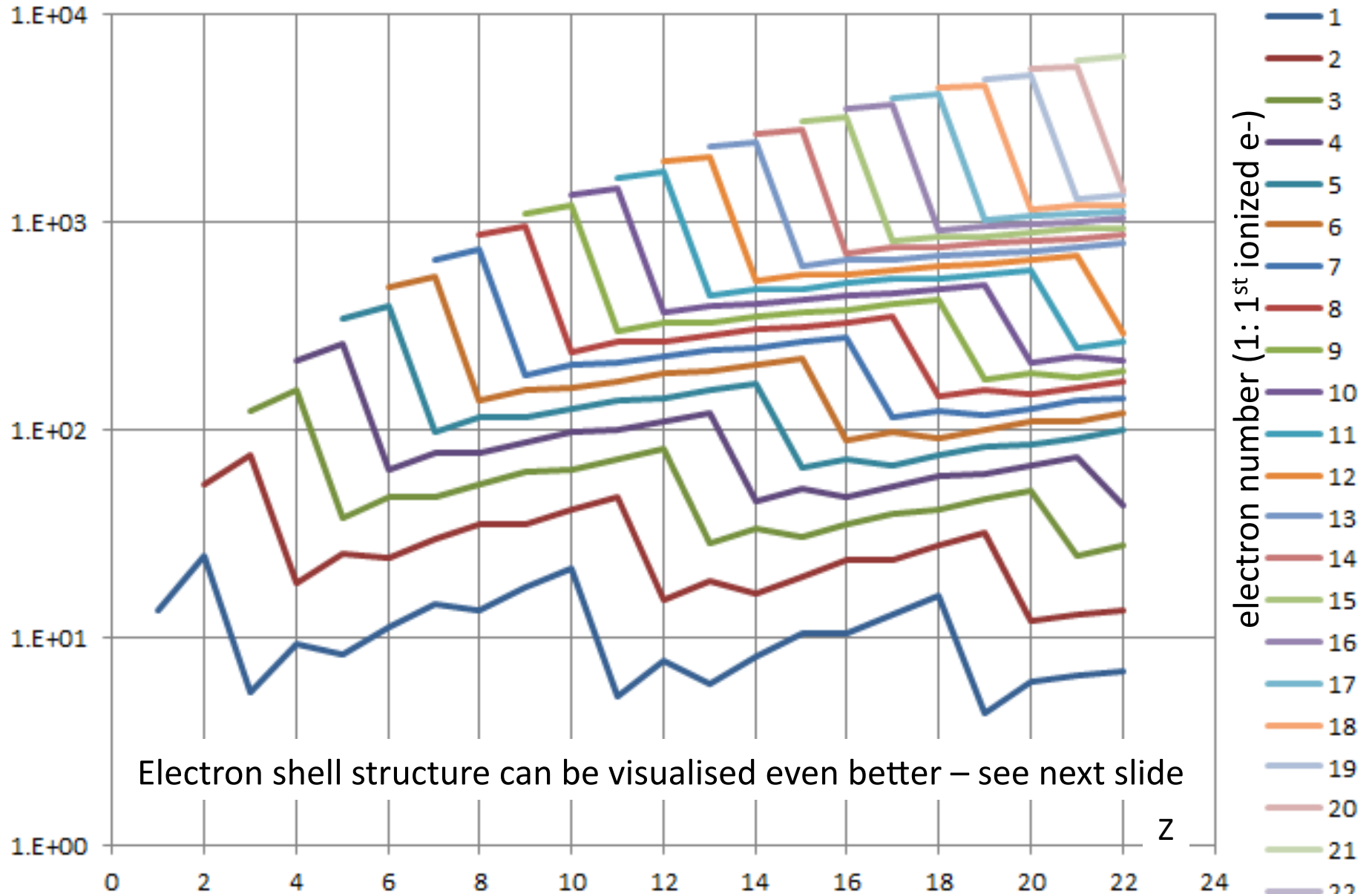
$(\lambda_e)_r$: reduced Compton-wavelength

E: electron energy



Ionization energies

IE: Ionization energy for last electron (eV)



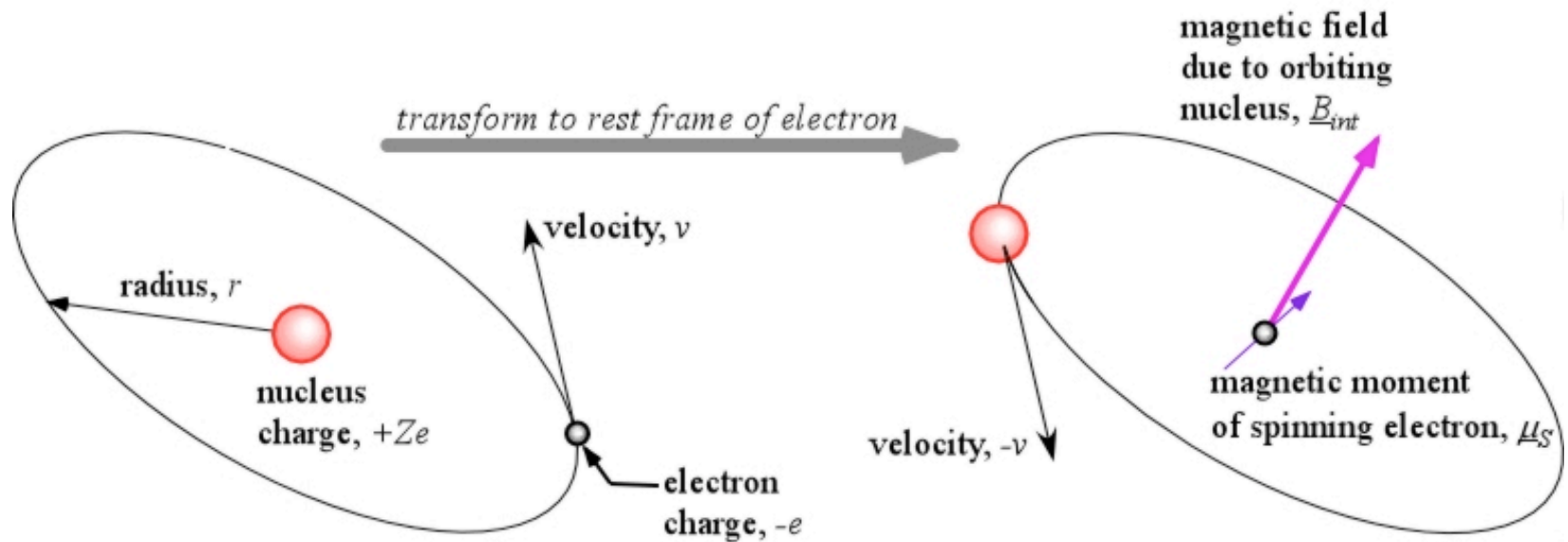
Electron shell structure can be visualised even better – see next slide

Z



Spin-orbit coupling

SPIN - ORBIT COUPLING



Ionization by photons

The reaction is: $A + h\nu = A^+ + e^-$

Atoms of a gas can be ionized by an intensive beam of photons with the adequate energy (**photo ionization**). Therefore the photon energy has to be $h \cdot \nu > e \cdot \varphi_{q,i}$

The energy of a photo electron is: $\frac{1}{2} m v_{\max}^2 = h\nu - e \cdot \varphi_{q,i}$

Cross section σ_p :

- σ_p has a **strong dependence on the photon energy and the nuclear charge Z**: $\sigma_p \propto \frac{Z^{4-5}}{(\hbar \cdot \omega)^{7/2}}$
- For a given atomic shell the cross section σ_p is **the largest close to threshold**, meaning where the photon energy reaches the ionization energy I (resonance/ threshold behavior):

$$\sigma_{\max} : \hbar \cdot \omega \approx I_K, I_L, I_M$$

- For high photon energies $\hbar \cdot \omega \gg I_K$ the **ionization of the s-orbital is most probable** and the K-shell ionization delivers the dominant contribution:

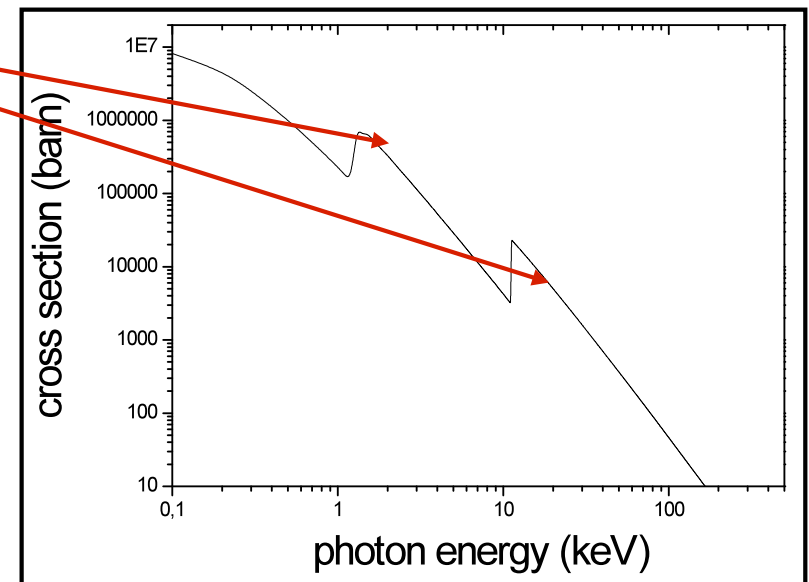
$$\sigma_{p_n} = \frac{1}{n^3} \cdot \sigma_K \quad \rightarrow \quad \sigma_p = \sigma_{pK} \cdot \sum_{n=1}^{\infty} \frac{1}{n^3} = 1.2021 \cdot \sigma_{pK}$$

σ_{pn} : cross section for RR into the K-shell

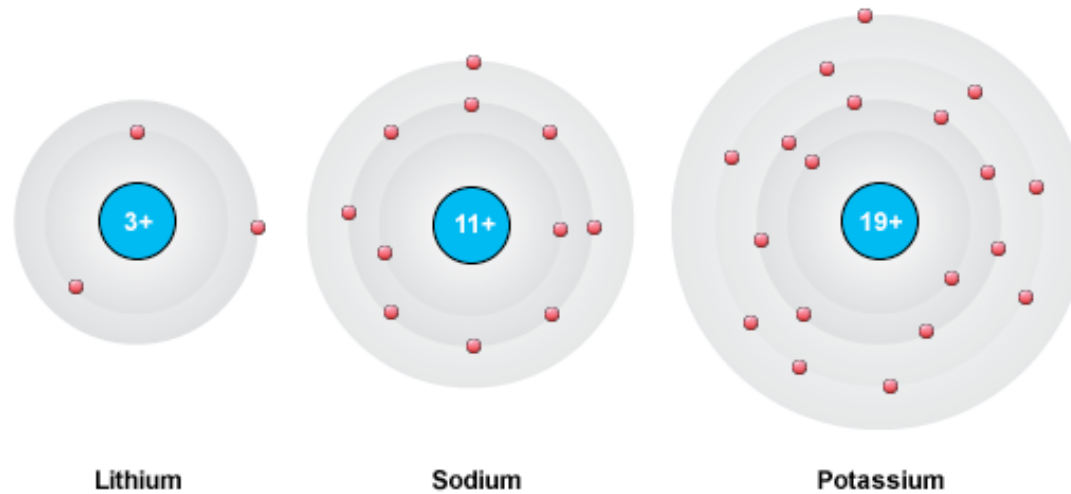
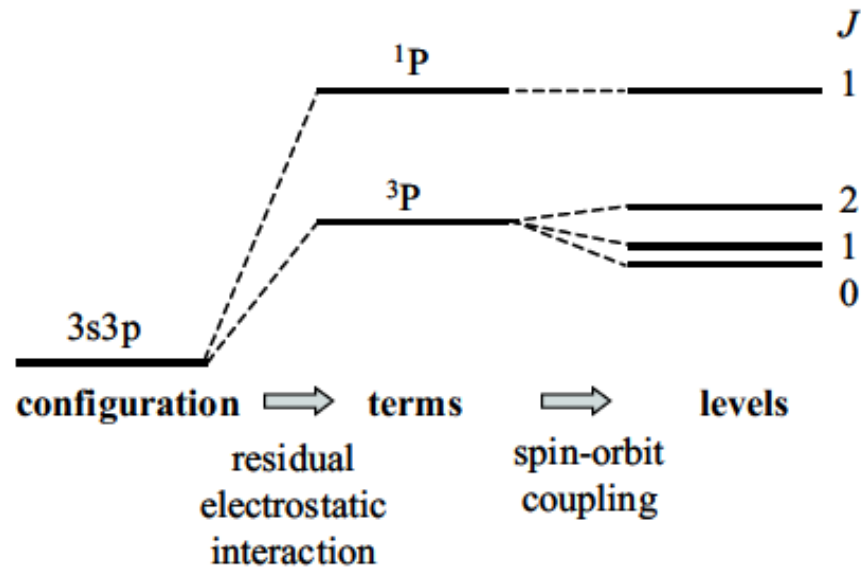
n: main quantum number

- Description of σ_p as **time-inverse effect** to the radiative recombination by the **Milne-formula**:

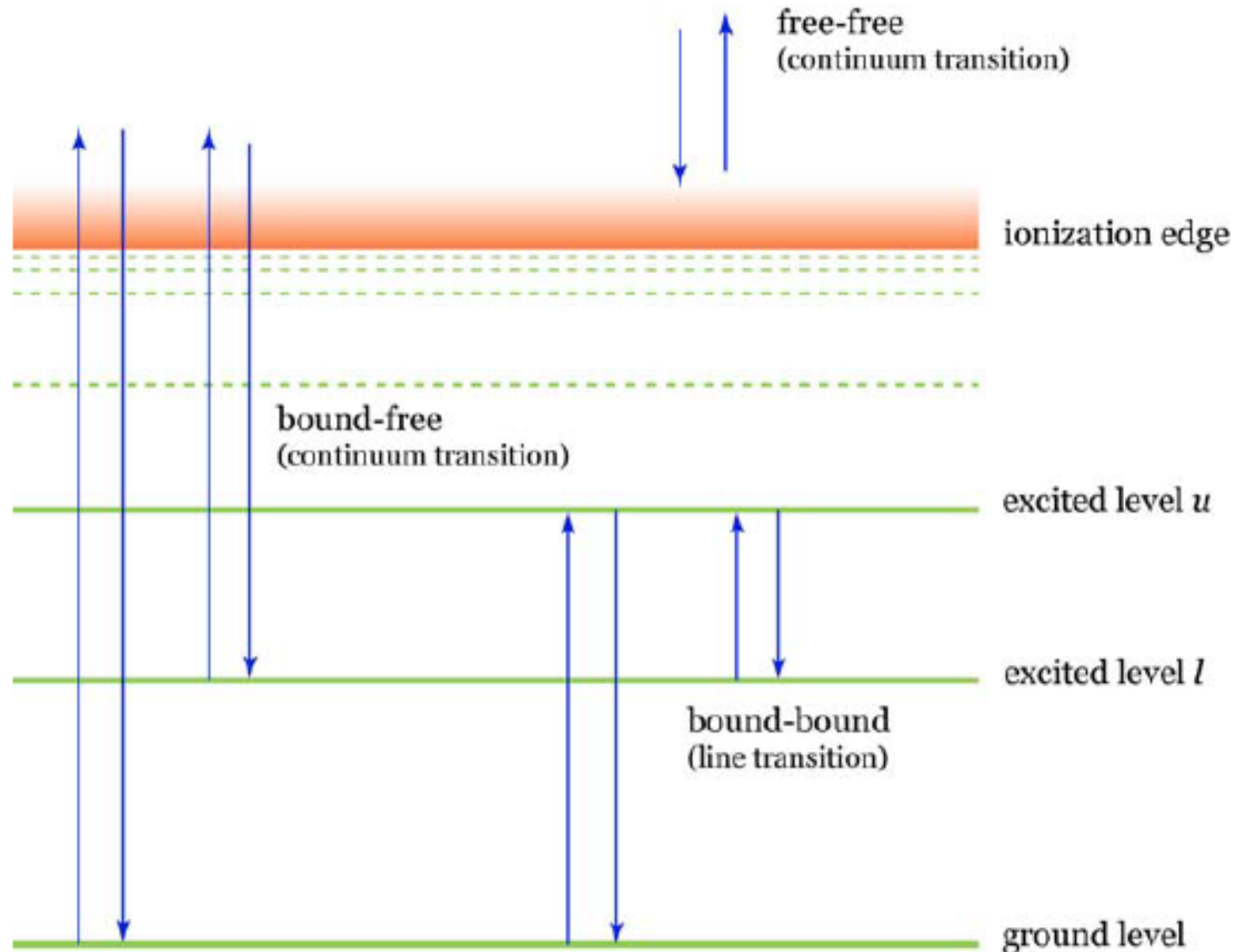
$$g_{q+1} \cdot \sigma_{RR} = \frac{(\hbar\omega)^2}{2m_e c^2 E} g_q \cdot \sigma_p \quad \text{with } g: \text{statistical weights}$$



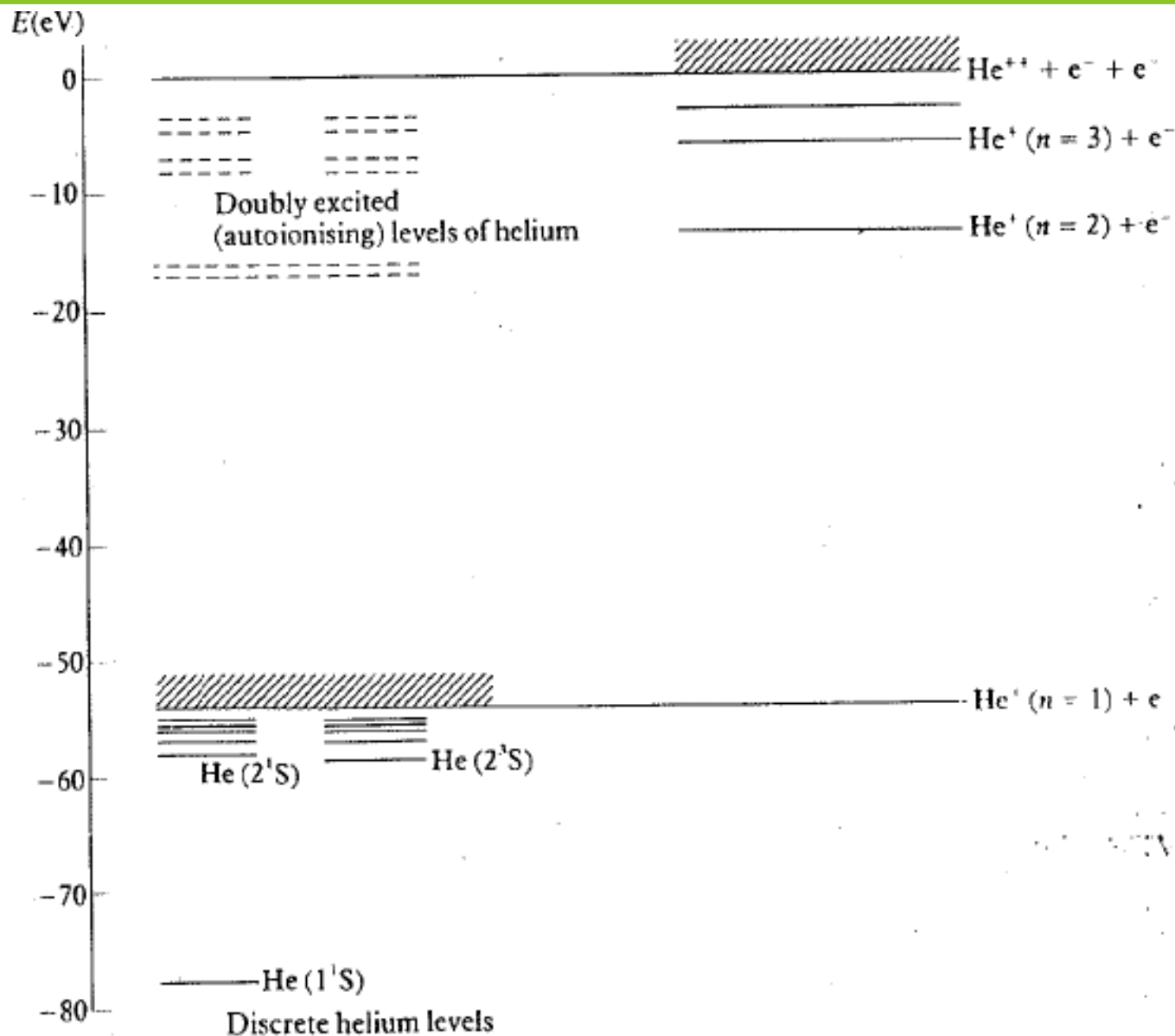
Atomic shell structure



Atomic shell structure



Energy spectrum: He example



6.3 The 'complete' energy level spectrum of helium. The energy scale is relative to the threshold for the ionisation of both electrons and the zero of energy is 54 eV (the ground state energy of He^+), above the zero energy of the scale of Fig. 6.2.

Ionization energy

