An introduction to linear imperfections



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Contents

- ★ CERN complex
- ★ Some CERN lattices
- ★ Imperfections
- ★ Correction techniques
- $\star~\beta$ -beating in the CERN synchrotrons
- ★ Dynamic imperfections

CERN accelerator complex



The Proton Synchrotron Booster (PSB)



The LHC Interaction Region (IR)



The first linear imperfection is...



...gravity!. The LHC vacuum chamber has a 22 mm radius. Everything takes 67 ms to fall. Why do protons not fall?

Dipole magnetic field



Lorentz force:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Dipole errors

- An error in the strength of a main dipole causes a perturbation on the horizontal closed orbit.
- A tilt error in a main dipole causes a perturbation on the vertical closed orbit.



Orbit perturbation formula

From distributed angular kicks θ_i the closed orbit results in:

$$CO(s) = \frac{\sqrt{\beta(s)}}{2\sin\pi Q} \sum_{i} \sqrt{\beta_i} \theta_i \cos(\pi Q - |\phi(s) - \phi_i|)$$

Attention to the denominator $sin(\pi Q)$ that makes closed orbit to diverge at the integer resonance $Q \in \mathbb{N}$.

Another source of orbit errors is offset quadrupoles.

Quadrupole field and force on the beam



Note that $F_x = -kx$ and $F_y = ky$ making horizontal dynamics totally decoupled from vertical.

Offset quadrupole - Feed-down



An offset quadrupole is seen as a centered quadrupole plus a dipole. This is called feed-down.

Quadrupole strength error



Quadrupole strength error - Formulae

Tune change (single source):

$$\Delta Q_x \approx \frac{1}{4\pi} \beta_x \Delta k_i, \quad \Delta Q_y \approx -\frac{1}{4\pi} \beta_y \Delta k_i$$

 β -beating from many sources:

$$\frac{\Delta\beta(s)}{\beta} \approx \sum_{i} \frac{\Delta k_i \beta_i}{2\sin(2\pi Q)} \cos(2\pi Q - 2|\phi(s) - \phi_i|)$$

Attention to the denominator $\sin(2\pi Q)$ that makes β -beating diverge at the integer and half integer resonances, $2Q \in \mathbb{N}$.

Quadrupole strength error - Tune change



Quadrupole errors can push tunes into resonances (dangerous) $nQ_x + mQ_y = N$

Luminosity imbalance CMS/ATLAS



Colliders in the resonance world





Collision debris can quench IR triplets

 ★ Luminosity limited to ≈ 1.7 × 10³⁴ cm⁻² s⁻¹ (LHC cannot beat KEKB on paper)

★ HL-LHC with large quadrupoles $\rightarrow 5 \times 10^{34}$

Can LHC operate at $Q_x \approx Q_y \approx 0.5$?



- Half integer resonance représents a challenge for optics control
- ★ First exploration at injection in 2011 \rightarrow Need further demonstrations

Interlude: Farey sequences (1802)

The Farey sequence of order n is the sequence of completely reduced fractions between 0 and 1 which, when in lowest terms, have denominators less than or equal to $N \rightarrow$ **Resonances of order** N **or lower** (in one plane)

Farey diagram of order 5 (F_5)



Some properties of Farey sequences

- ★ The distance between neighbors in a Farey sequence (aka two consecutive resonances) a/b and c/d is equal to 1/(bd)
- ★ The next leading resonances in between two consecutive resonances a/b and c/d is (a+c)/(b+d).
- ★ The number of 1D resonances of order N or lower tends asymptotically to $3N^2/\pi^2$

Optics measurements

Standard approach is to record BPM data of betatron oscillations after a single kick:

$$x(N,s) = \sqrt{2\beta_x(s)J_x}\cos(2\pi Q_x N + \phi_x(s)) + CO(s)$$

Beta function, tune, phase advance, closed orbit

 β and ϕ are related by:

$$\phi_{0\to 1} = \phi(s_1) - \phi(s_0) = \int_{s_0}^{s_1} \frac{\mathrm{d}s}{\beta(s)}$$

so β and ϕ carry the same information, ϕ being a calibration independent observable.

Turn-by-turn BPM data



 \star

















Forced oscillations with AC dipole

- ★ An AC dipole forces betatron oscillations
- If addiabatically ramped up & down causes no emittance blow up (contrary to kick)
- Can be used as many times as needed with the same beam



Measuring β -beating versus β^*



Tilted quadrupole



A tilted quadrupole is seen as a normal quadrupole plus another quadrupole tilted by 45° (this is called a skew quadrupole).

Skew quadrupole \rightarrow x-y Coupling



Note that $F_x = ky$ and $F_y = kx$ making horizontal and vertical dynamics to couple.

Skew quadrupole — x-y Coupling



Coupling makes it impossible to approach tunes below $\Delta Q_{min} = |C^-|$, where C^- is a complex number characterizing the difference resonance $Q_x - Q_y = N$.

Coupling



Coupling can push tunes into resonances.

Sextupole field and force



Ooops, We are entering the non-linear regime, however...

Offset sextupole



A sextupole horizontally (vertically) displaced is seen as a centered sextupole plus an offset quadrupole (skew quadrupole). Offset sextupoles are also sources of quadrupole and skew quadrupole errors.

Longitudinal misalignments



Longitudinal misalignments can be seen as perturbations at both ends of the magnet with opposite signs. Tolerances are generally larger for longitudinal misalignments.

Correction

- ★ Local corrections
 - Ideal correction: Error source identification and repair.
 - Effective local error correction.
 - MICADO (ISR-MA/73-17): Best few correctors (no guarantee of locality).
- ★ Global corrections
 - Pre-designed knobs for varying particular observables in the least invasive way (like tunes, coupling, β^* , etc.)
 - MICADO: Best N correctors
 - Response matrix approach

Local correction: segment-by-segment



ments and find corrections.

Pre-designed knobs - Tunes

- ★ In most machines it is OK to use all focusing quads to change Q_x and all defocusing quads for Q_y : PSB, PS, SPS
- In the LHC dedicated tune correctors (MQT) are properly placed to minimize impact on other quantities:



Pre-designed knobs - Coupling

- ★ The full control of the difference resonance (C⁻) needs two independent families of skew quadrupoles.
- ★ PSB, PS and SPS can survive only with one family since $int(Q_x) = int(Q_y)$, making errors in phase with correctors.
- ★ In LHC there are two families to vary the real and imaginary parts of C^- independently.

Best corrector concepts



Elephant = 80 kg. Available corrector weigths: 78 kg, 1 kg, 30 kg, 50 kg

Which is the best corrector? Which is the best second corrector? (using the 1st) Which are the two best correctors?

Best N-corrector challenge

- LHC has about 500 orbit correctors per plane and per beam.
- Imagine you want to find the best 20 correctors
- How many combinations of these 500 correctors taking 20 at a time exist?
- * ...
- ★ (MICADO finds a good approximation to this problem)

Response matrix approach

- **\star** Available correctors: \vec{c}
- ***** Available observables: \vec{a}
- * Assume for small changes of correctors linear approximation is good: $R\Delta \vec{c} = \Delta \vec{a}$
- \star Use, e.g., MADX to compute R
- ★ Invert or pseudo-invert R to compute an effective global correction based on measured $\Delta \vec{a}$:

$$\Delta \vec{c} = R^{-1} \Delta \vec{a}$$

\star This works for orbit, $\Delta\beta/\beta$, coupling, etc.

Pseudo-inverse via SVD

$$R = U \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \\ 0 & 0 & 0 \end{pmatrix} V'$$

Imagine $\sigma_3 \ll \sigma_2 \leq \sigma_1$, then just neglect σ_3 :

$$R^{-1} = V \begin{pmatrix} \frac{1}{\sigma_1} & 0 & 0\\ 0 & \frac{1}{\sigma_2} & 0\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{pmatrix} U'$$

PSB β-beating



Peak β -beating of \approx 5%

PS β-beating



Peak β -beating of $\approx 4\%$

SPS β-beating (Q20)



LHC β -beating, before correction



Peak β -beating of \approx 100% !!!

LHC β -beating, after correction



LHC optics makes history

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Record low β beating in the LHC

R. Tomás,* T. Bach, R. Calaga, A. Langner, Y. I. Levinsen, E. H. Maclean, T. H. B. Persson, P. K. Skowronski, M. Strzelczyk, and G. Vanbavinckhove *CERN, CH 1211 Geneva 23, Switzerland*

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Lepton Collider	Circum. [km]	Peak $\Delta eta / eta$ [%]	Hadron Collider	Circum. [km]	Peak $\Delta eta / eta$ [%]
PEP II	2.2	30	HERA-p	6.3	20
LEP	27	20	Tevatron	6.3	20
KEKB	3	20	RHIC	3.8	20
CESR	0.8	7	LHC	27	7

also, CMS and ATLAS luminosities in 2012 got equal

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Dynamic linear imperfections

- Ground motion and vibrations in quadrupoles produce sinusoidal dipolar fields
- Electrical noise can cause currents in quadrupoles and dipoles to oscillate in time
- Electromagnetic pollution can act directly on the beam.
- ★ Slow variations ($f << Q_{x,y} \times f_{rev}$) just cause a time varying orbit and optics
- ★ Fast variations ($f \approx Q_{x,y} \times f_{rev}$) can cause resonances and emittance growth

An oscillating dipolar field

- ★ Let $Q_{dip} = f_{dip}/f_{rev}$ be the tune of the dipolar field oscillation.
- This causes the appearance of new resonances
- ★ Linear resonances: $Q_x \pm Q_{dip} = N$
- ★ Non-linear resonances of sextupolar order:

$$Q_x \pm 2Q_{dip} = N$$

$$2Q_x \pm Q_{dip} = N$$

★ Note that $mQ_{dip} = N$ is not a problem

Oscillating dipolar field, $Q_x \neq Q_{dip}$



Orbit oscillates with Q_{dip} but there is no emittance growth far from resonances.

Oscillating dipolar field, $Q_x = Q_{dip}$



An oscillating quadrupolar field

- ★ Let $Q_{quad} = f_{quad}/f_{rev}$ be the tune of the quadrupolar field oscillation.
- This causes the appearance of new resonances
- ★ Linear resonances: $2Q_x \pm Q_{quad} = N$

Oscillating quadrupolar field, $2Q_x \neq Q_{quad}$



Tune is modulated with Q_{quad} , displaying sidebands at $Q_x \pm Q_{quad}$ but there is no emittance growth far from resonances.

Oscillating quadrupolar field, $2Q_x = Q_{quad}$



Exponential growth, clear signatures depending on the oscillating field type.

Questions?