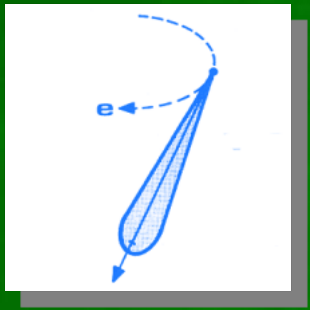

ELECTRON DYNAMICS with SYNCHROTRON RADIATION

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CERN Accelerator School: Introduction to Accelerator Physics
September 5, 2014, Prague, Czech Republic

Useful books and references

H. Wiedemann, *Synchrotron Radiation*
Springer-Verlag Berlin Heidelberg 2003

H. Wiedemann, *Particle Accelerator Physics I and II*
Springer Study Edition, 2003

A. Hofmann, *The Physics of Synchrotron Radiation*
Cambridge University Press 2004

A. W. Chao, M. Tigner, *Handbook of Accelerator Physics and Engineering*, World Scientific 1999

CERN Accelerator School Proceedings

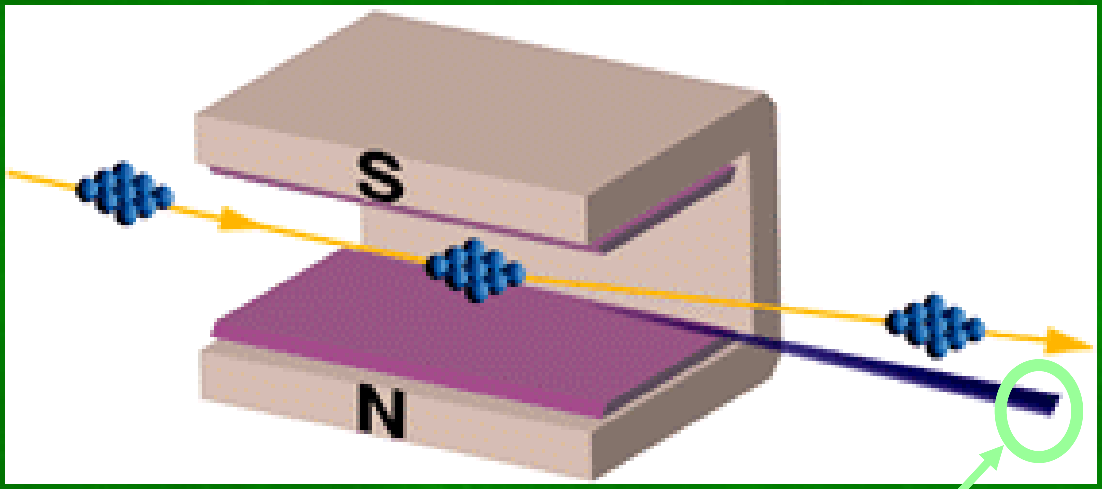
Synchrotron Radiation and Free Electron Lasers

Grenoble, France, 22 - 27 April 1996
(A. Hofmann's lectures on synchrotron radiation)
CERN Yellow Report 98-04

Brunnen, Switzerland, 2 – 9 July 2003
CERN Yellow Report 2005-012

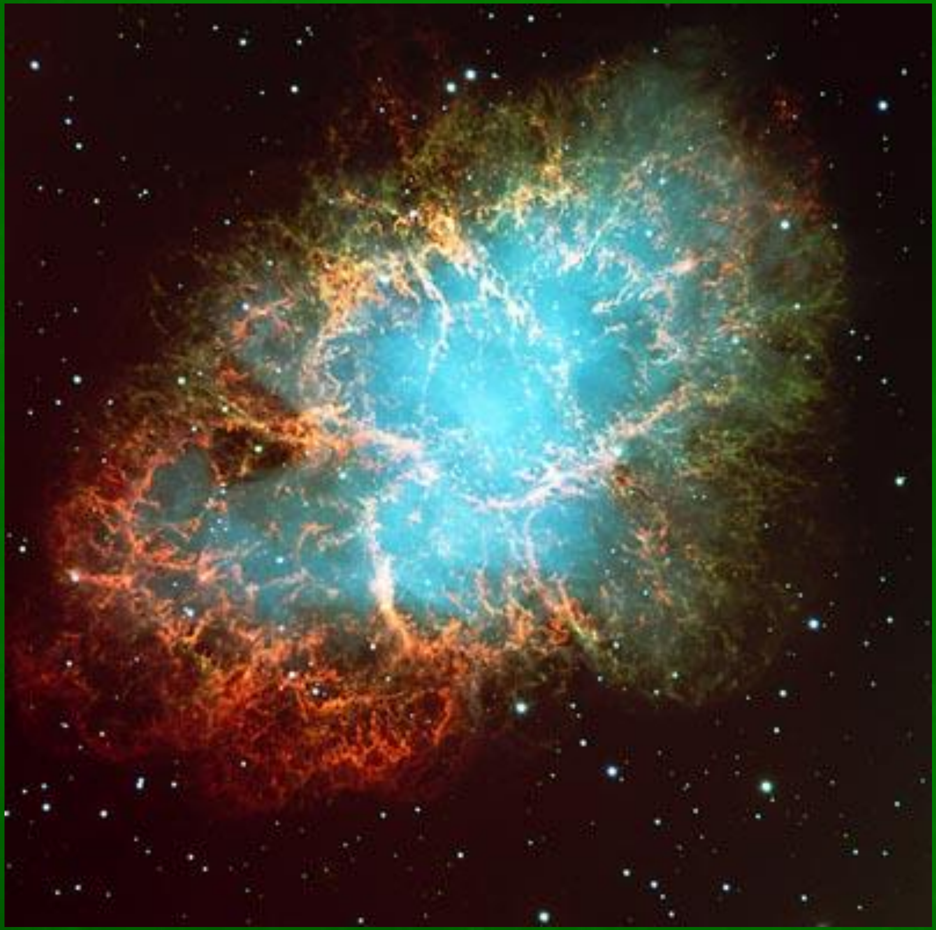
http://cas.web.cern.ch/cas/Proceedings.html

Curved orbit of electrons in magnet field



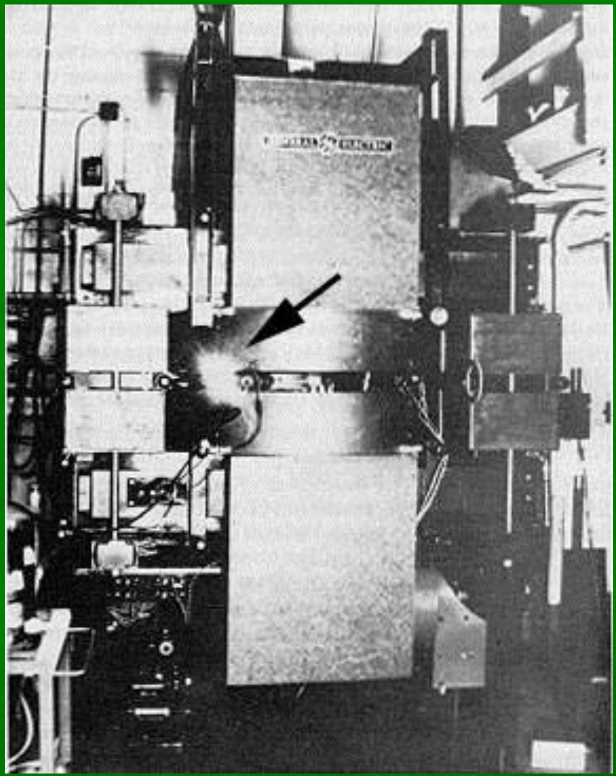
Accelerated charge → Electromagnetic radiation

Crab Nebula
6000 light years away



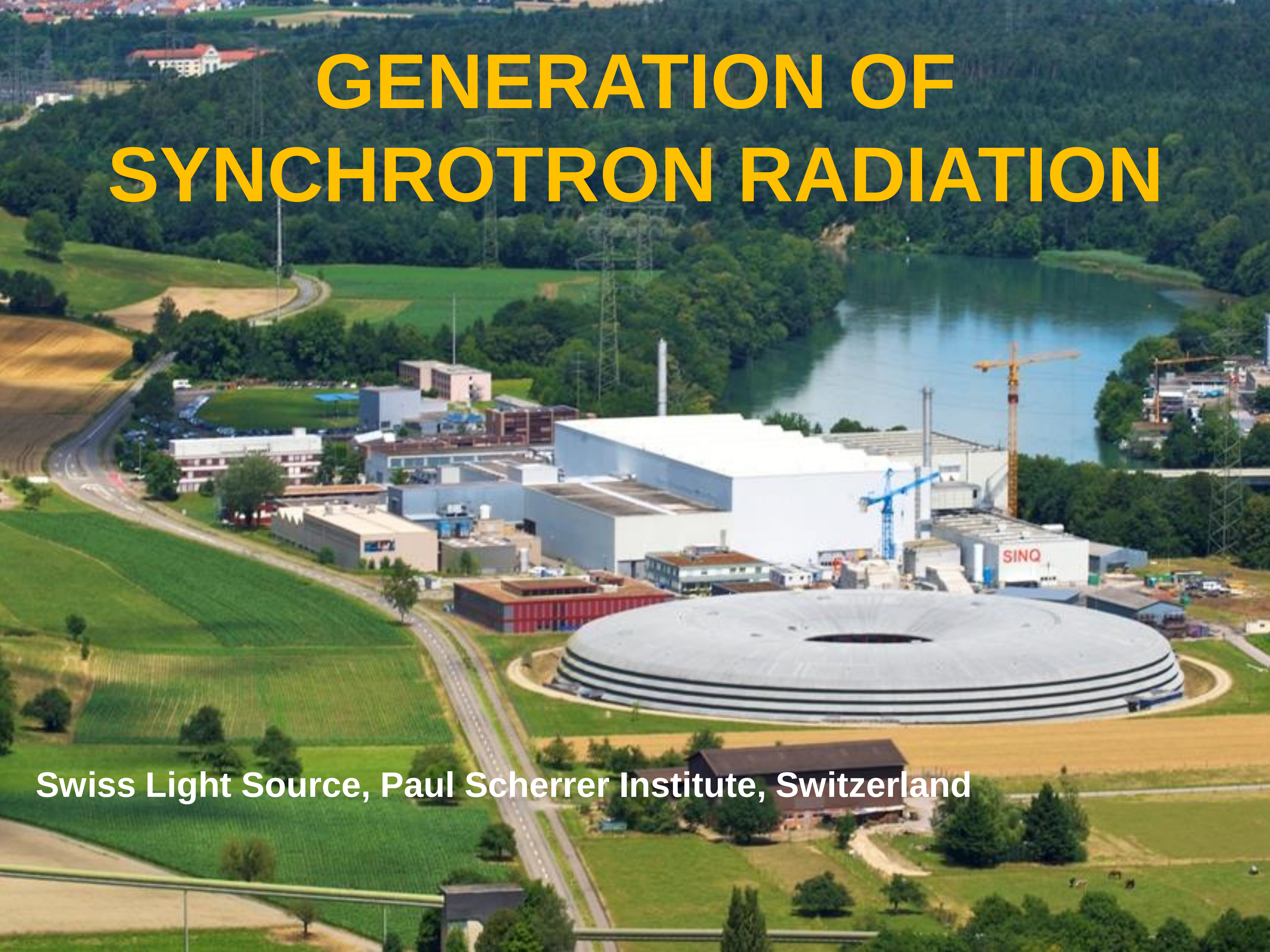
First light observed 1054 AD

GE Synchrotron
New York State



First light observed 1947

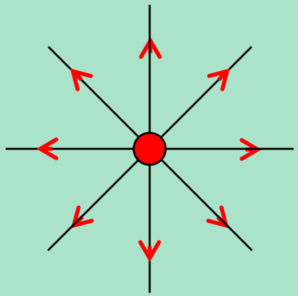
GENERATION OF SYNCHROTRON RADIATION



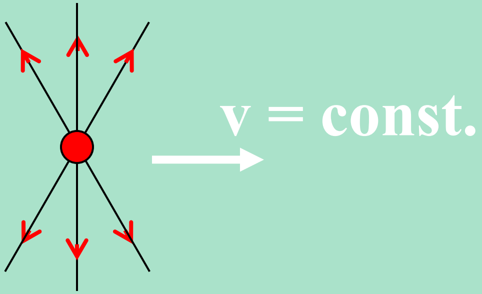
Swiss Light Source, Paul Scherrer Institute, Switzerland

Why do they radiate?

Charge at rest: Coulomb field, no radiation

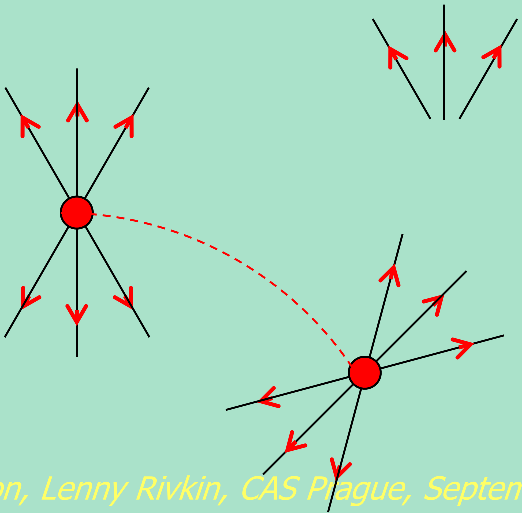


Uniformly moving charge
does not radiate

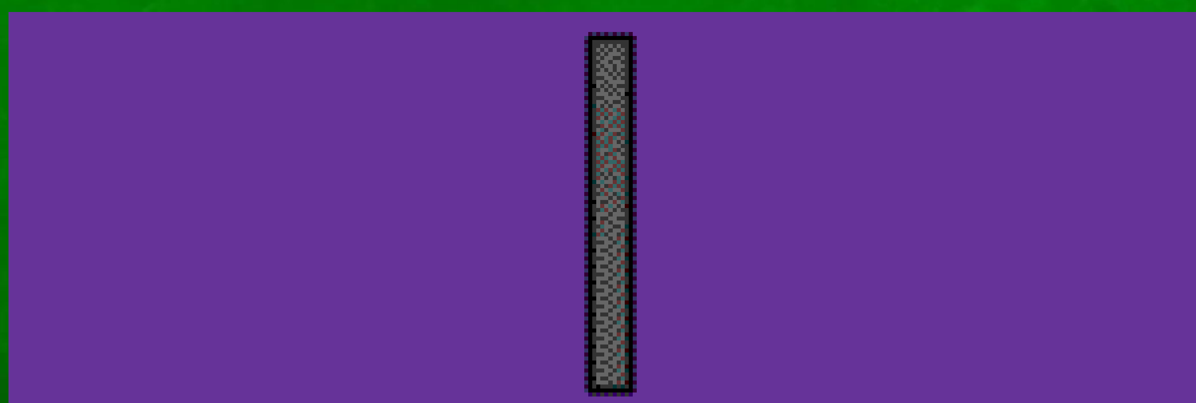


But! Cerenkov!

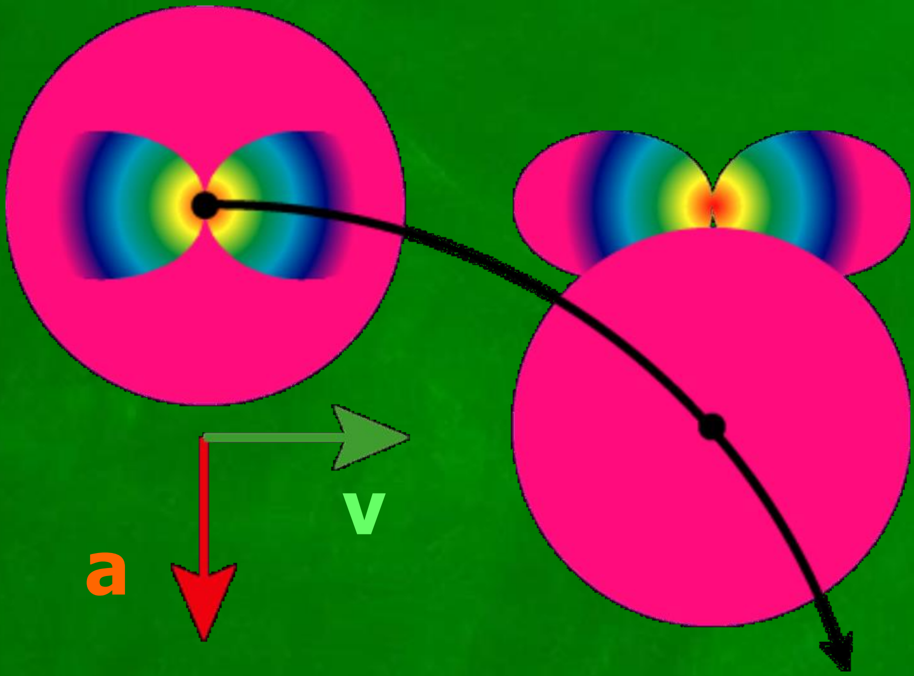
Accelerated charge



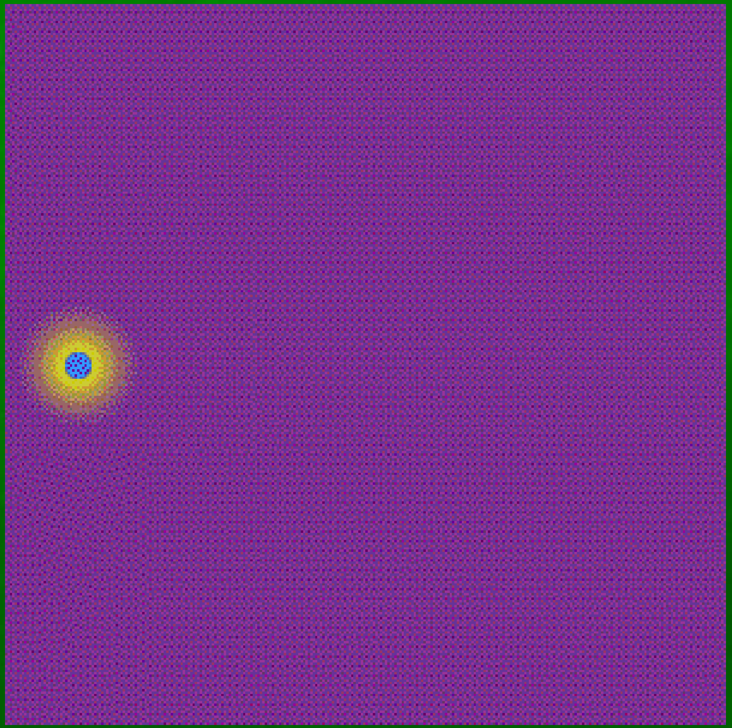
Bremsstrahlung
or
“braking” radiation



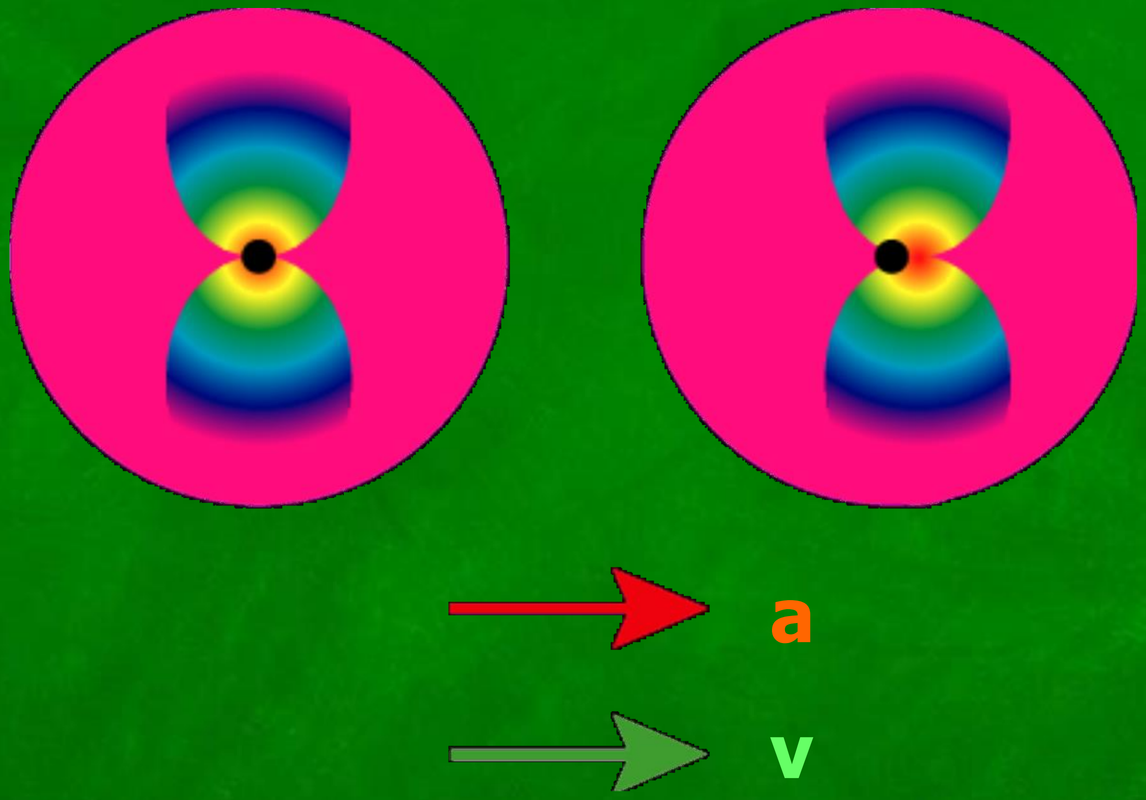
Transverse acceleration



Radiation field quickly separates itself from the Coulomb field



Longitudinal acceleration



Radiation field cannot separate itself from the Coulomb field

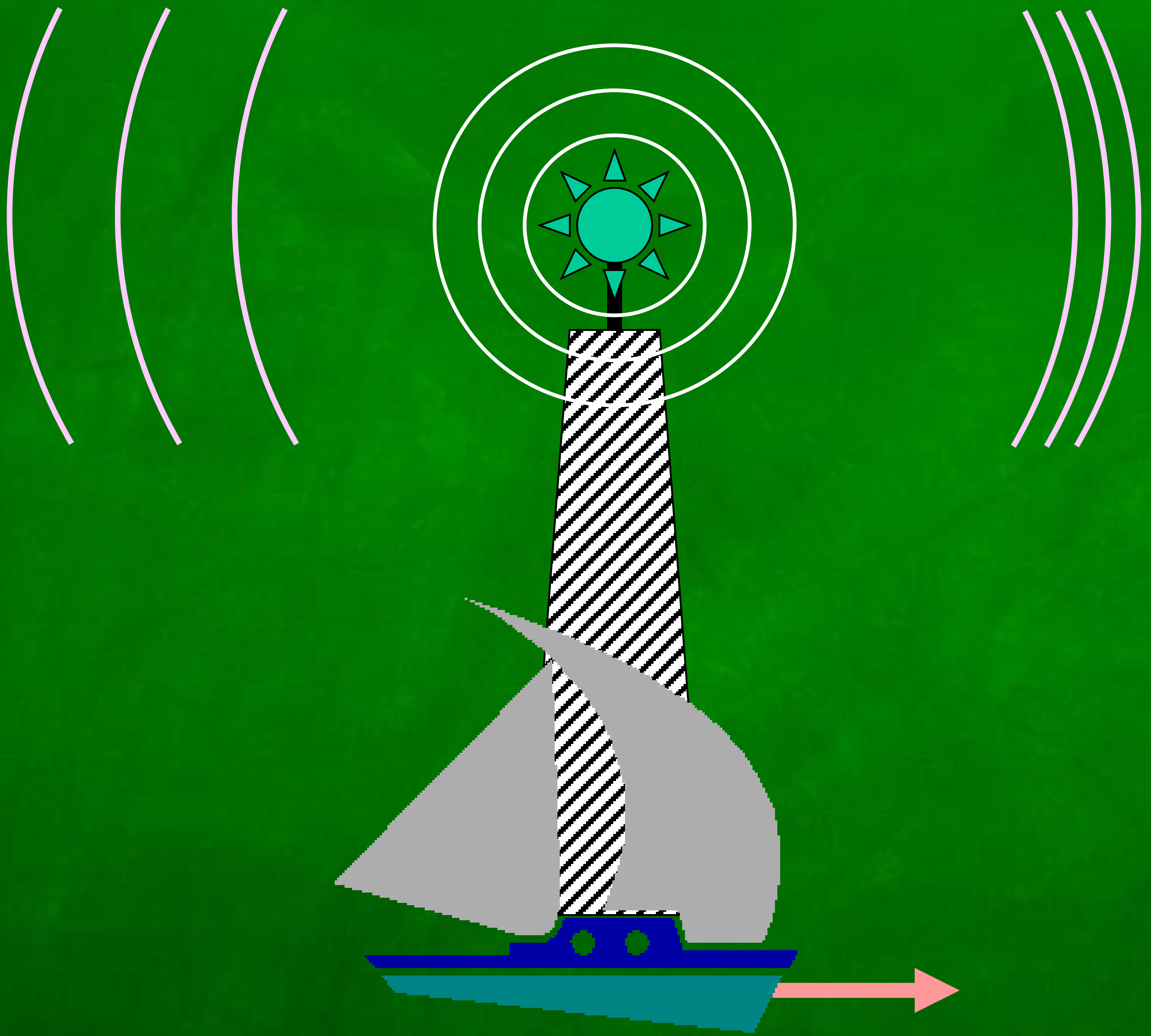
Fields of a moving charge

$$\vec{\mathbf{E}}(t) = \frac{q}{4\pi\varepsilon_0}\left[\frac{\vec{\mathbf{n}} - \vec{\boldsymbol{\beta}}}{\left(1 - \vec{\mathbf{n}} \cdot \vec{\boldsymbol{\beta}}\right)^3 \gamma^2} \cdot \boxed{\frac{\mathbf{1}}{\mathbf{r}^2}}\right]_{ret} +$$

$$\frac{q}{4\pi\varepsilon_0 c}\left[\frac{\vec{\mathbf{n}} \times \left[\left(\vec{\mathbf{n}} - \vec{\boldsymbol{\beta}}\right) \times \boxed{\dot{\vec{\boldsymbol{\beta}}}}\right]}{\left(1 - \vec{\mathbf{n}} \cdot \vec{\boldsymbol{\beta}}\right)^3 \gamma^2} \cdot \boxed{\frac{\mathbf{1}}{\mathbf{r}}}\right]_{ret}$$

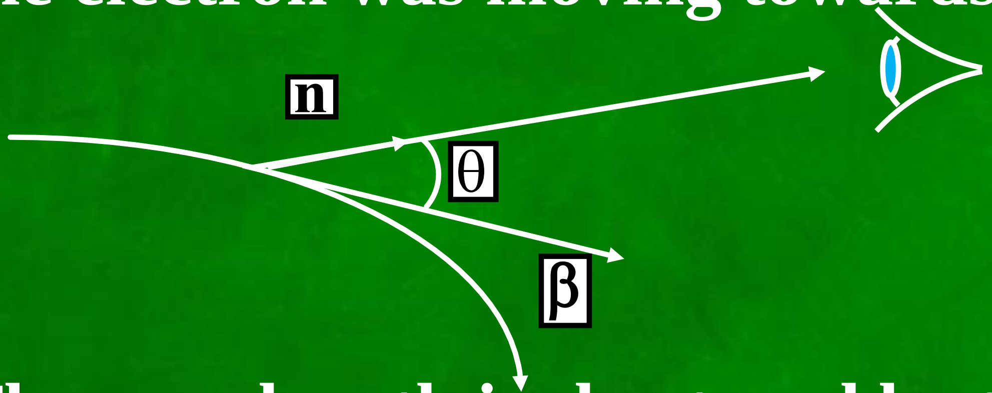
$$\vec{\mathbf{B}}(t) = \frac{1}{\mathbf{c}}\left[\vec{\mathbf{n}} \times \vec{\mathbf{E}}\right]$$

Moving Source of Waves



Time compression

Electron with velocity β emits a wave with period T_{emit} while the observer sees a different period T_{obs} because the electron was moving towards the observer



$$T_{obs} = (1 - \mathbf{n} \cdot \boldsymbol{\beta}) T_{emit}$$

The wavelength is shortened by the same factor

$$\lambda_{obs} = (1 - \beta \cos\theta) \lambda_{emit}$$

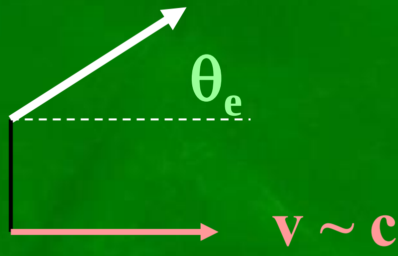
in ultra-relativistic case, looking along a tangent to the trajectory

$$\lambda_{obs} = \frac{1}{2\gamma^2} \lambda_{emit}$$

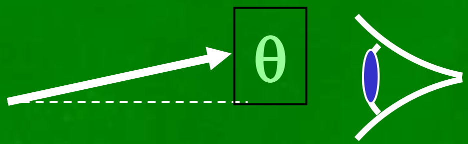
since

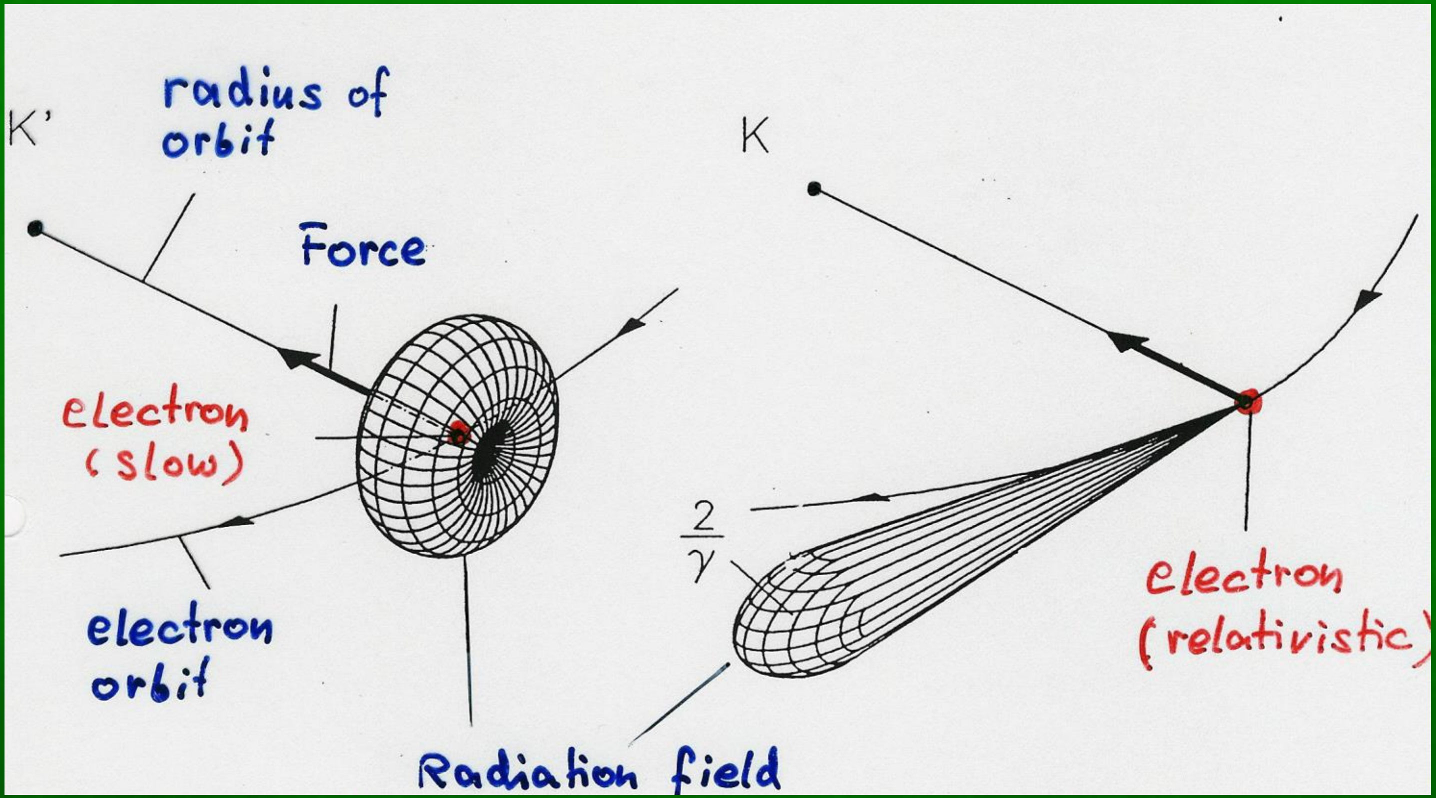
$$1 - \beta = \frac{1 - \beta^2}{1 + \beta} \cong \frac{1}{2\gamma^2}$$

Radiation is emitted into a narrow cone



$$\theta = \frac{1}{\gamma} \cdot \theta_e$$



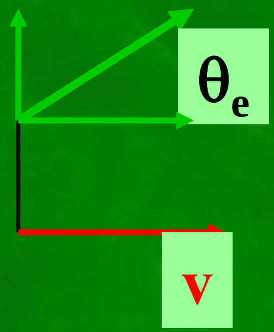


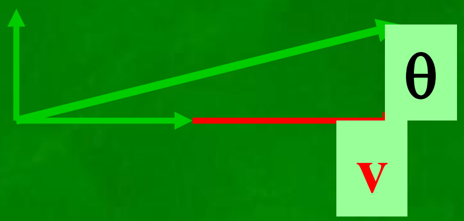
v << c

v ≈ c

Sound waves (non-relativistic)

Angular collimation





$$\theta = \frac{v_{s\perp}}{v_{s\parallel} + \mathbf{v}} = \frac{v_{s\perp}}{v_{s\parallel}} \cdot \frac{1}{1 + \frac{\mathbf{v}}{v_s}} \approx \theta_e \cdot \frac{1}{1 + \frac{\mathbf{v}}{v_s}}$$

Doppler effect (moving source of sound)

$$\lambda_{heard} = \lambda_{emitted} \left(1 - \frac{\mathbf{v}}{v_s} \right)$$

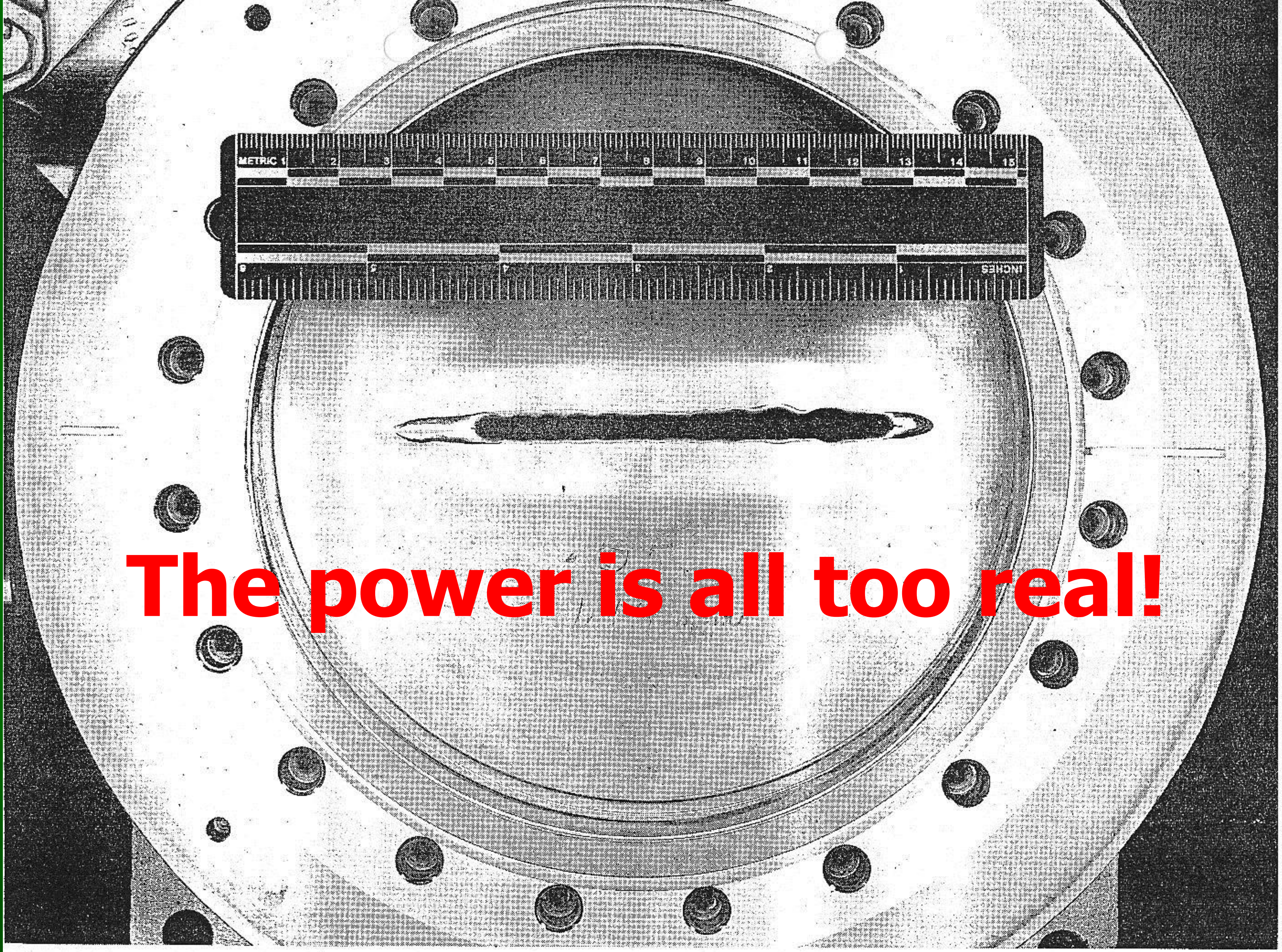
Synchrotron radiation power

Power emitted is proportional to:

$$\boldsymbol{P \propto E^2 B^2}$$

$$P_\gamma = \frac{c C_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{\left(m_e c^2\right)^3} = 8.858 \cdot 10^{-5} \left[\frac{\mathrm{m}}{\mathrm{GeV}^3}\right]$$

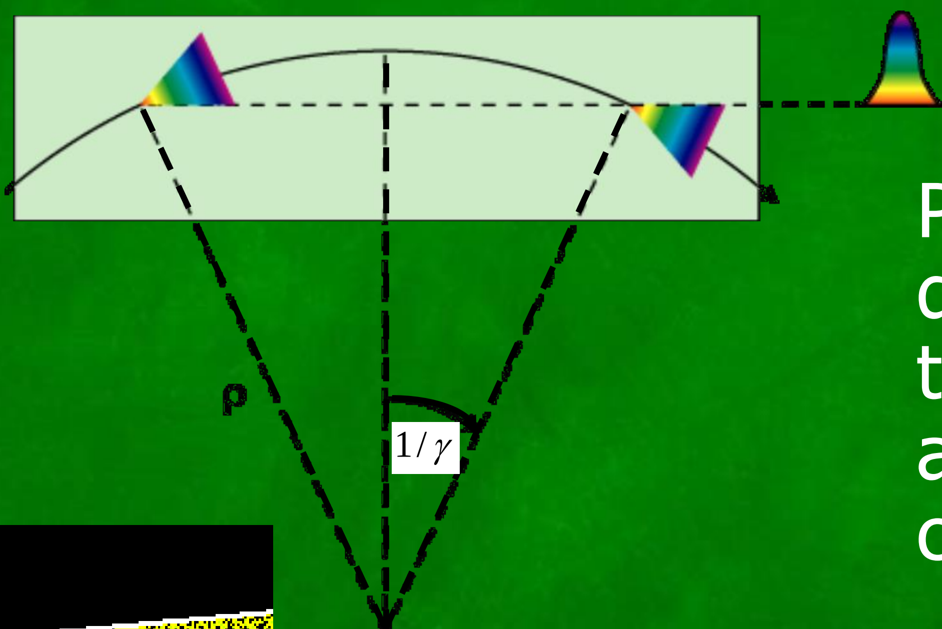


ig. 12. Damaged X-ray ring front end gate valve. The power incident on the valve was approximately 1 kW for a duration estimated to 2–10 min and drilled a hole through the valve plate.

Typical frequency of synchrotron light

Due to extreme collimation of light observer sees only a small portion of electron trajectory **(a few mm)**

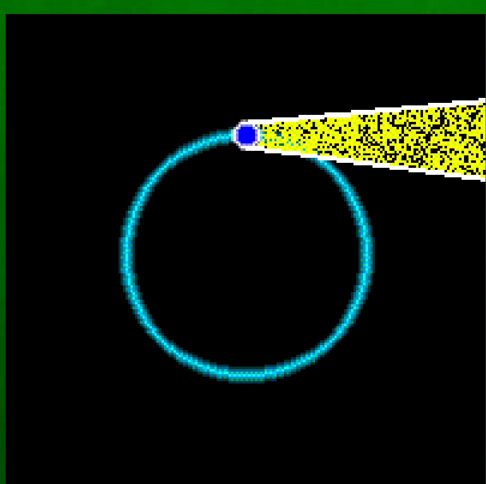
$$l \sim \frac{2\rho}{\gamma}$$



Pulse length: difference in times it takes an electron and a photon to cover this distance

$$\Delta t \sim \frac{l}{\beta c} - \frac{l}{c} = \frac{l}{\beta c}(1-\beta)$$

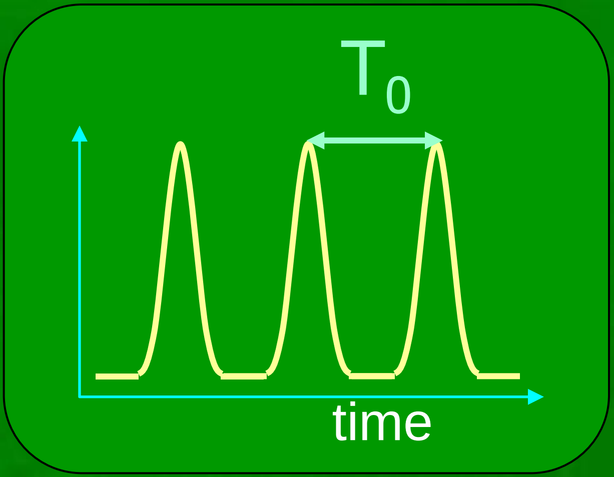
$$\Delta t \sim \frac{2\rho}{\gamma c} \cdot \frac{1}{2\gamma^2}$$



$$\omega \sim \frac{1}{\Delta t} \sim \gamma^3 \omega_0$$

Spectrum of synchrotron radiation

- Synchrotron light comes in a series of flashes every T_0 (revolution period)



- the spectrum consists of harmonics of

$$\omega_0 = \frac{1}{T_0}$$

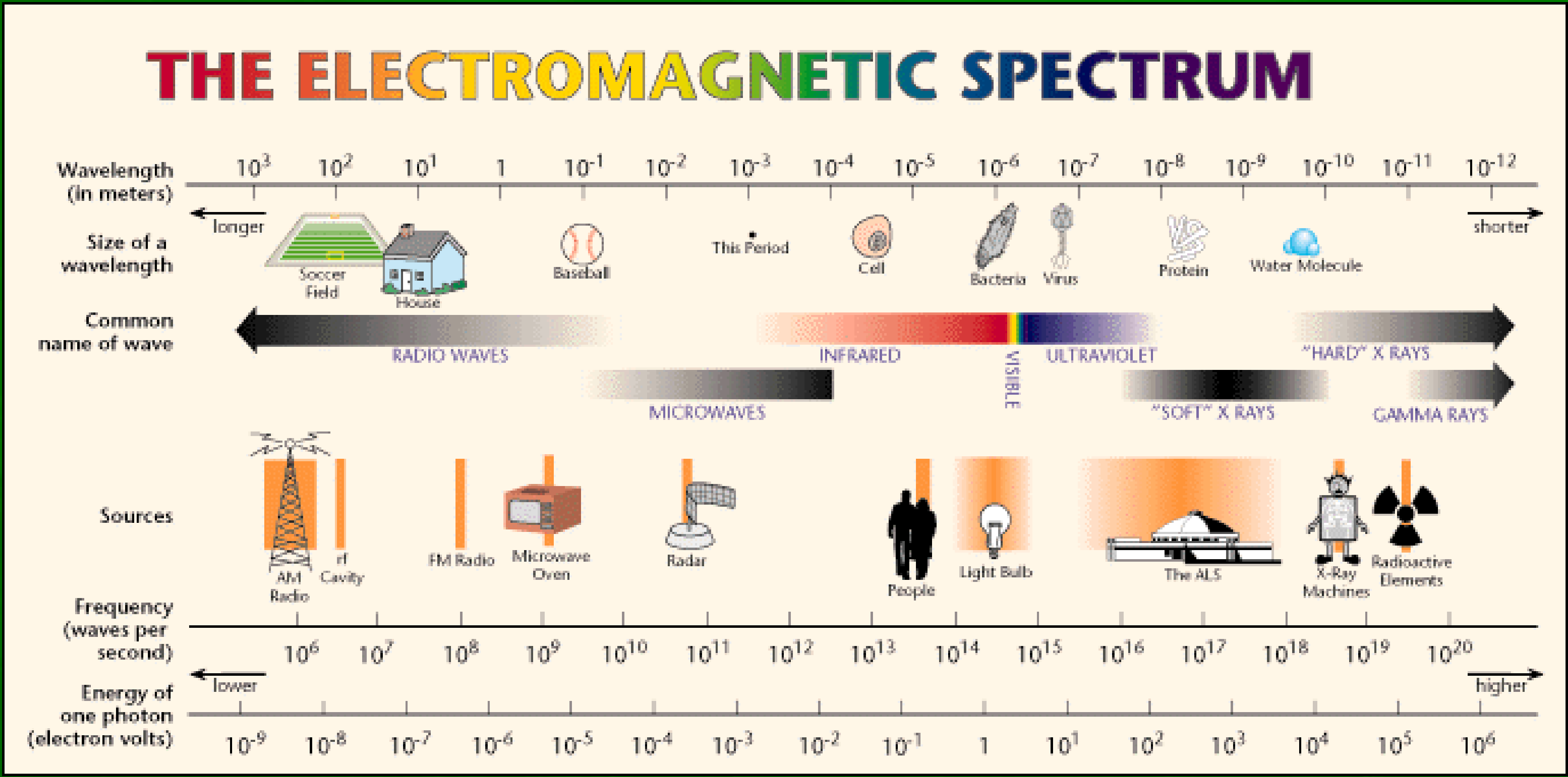
- flashes are extremely short: harmonics reach up to very high frequencies

$$\omega_{typ} \cong \gamma^3 \omega_0$$

$$\omega_0 \sim 1\ \mathrm{MHz}$$
$$\gamma \sim 4000$$
$$\omega_{typ} \sim 10^{16}\ \mathrm{Hz!}$$

- At high frequencies the individual harmonics overlap

continuous spectrum !



Wavelength continuously tunable !

Synchrotron radiation power

Power emitted is proportional to:

$$\boldsymbol{P \propto E^2 B^2}$$

$$P_\gamma = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

$$P_\gamma = \frac{2}{3} \alpha \hbar c^2 \cdot \frac{\gamma^4}{\rho^2}$$

$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5} \left[\frac{\mathrm{m}}{\mathrm{GeV}^3} \right]$$

$$\alpha = \frac{1}{137}$$

$$\hbar c = 197 \ \mathrm{Mev} \cdot \mathrm{fm}$$

Energy loss per turn:

$$U_0 = C_\gamma \cdot \frac{E^4}{\rho}$$

$$U_0 = \frac{4\pi}{3} \alpha \hbar c \frac{\gamma^4}{\rho}$$

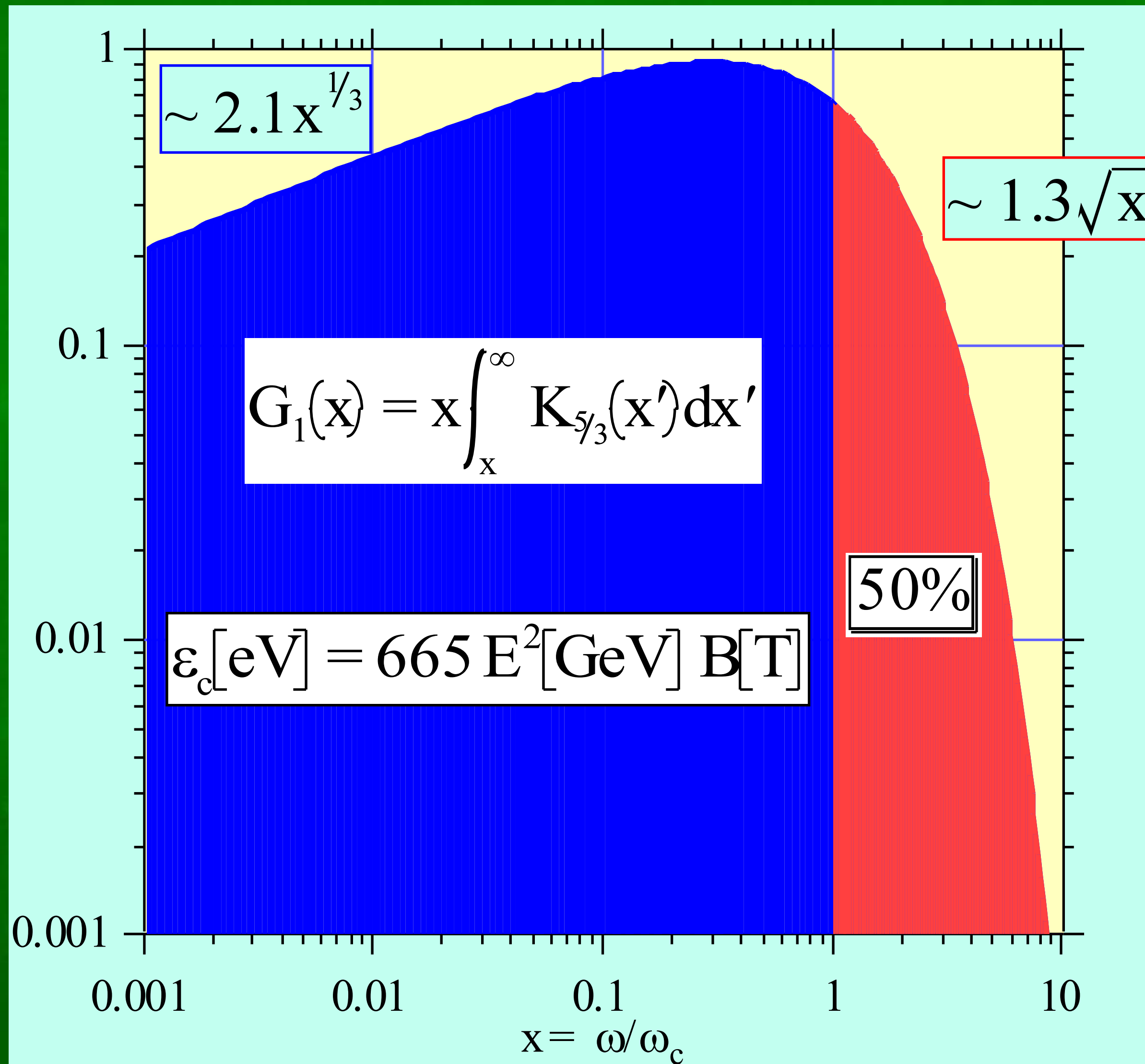
$$\frac{dP}{d\omega} = \frac{P_{tot}}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)$$

$$S(x) = \frac{9\sqrt{3}}{8\pi} x \int_x^\infty K_{5/3}(x')dx'$$

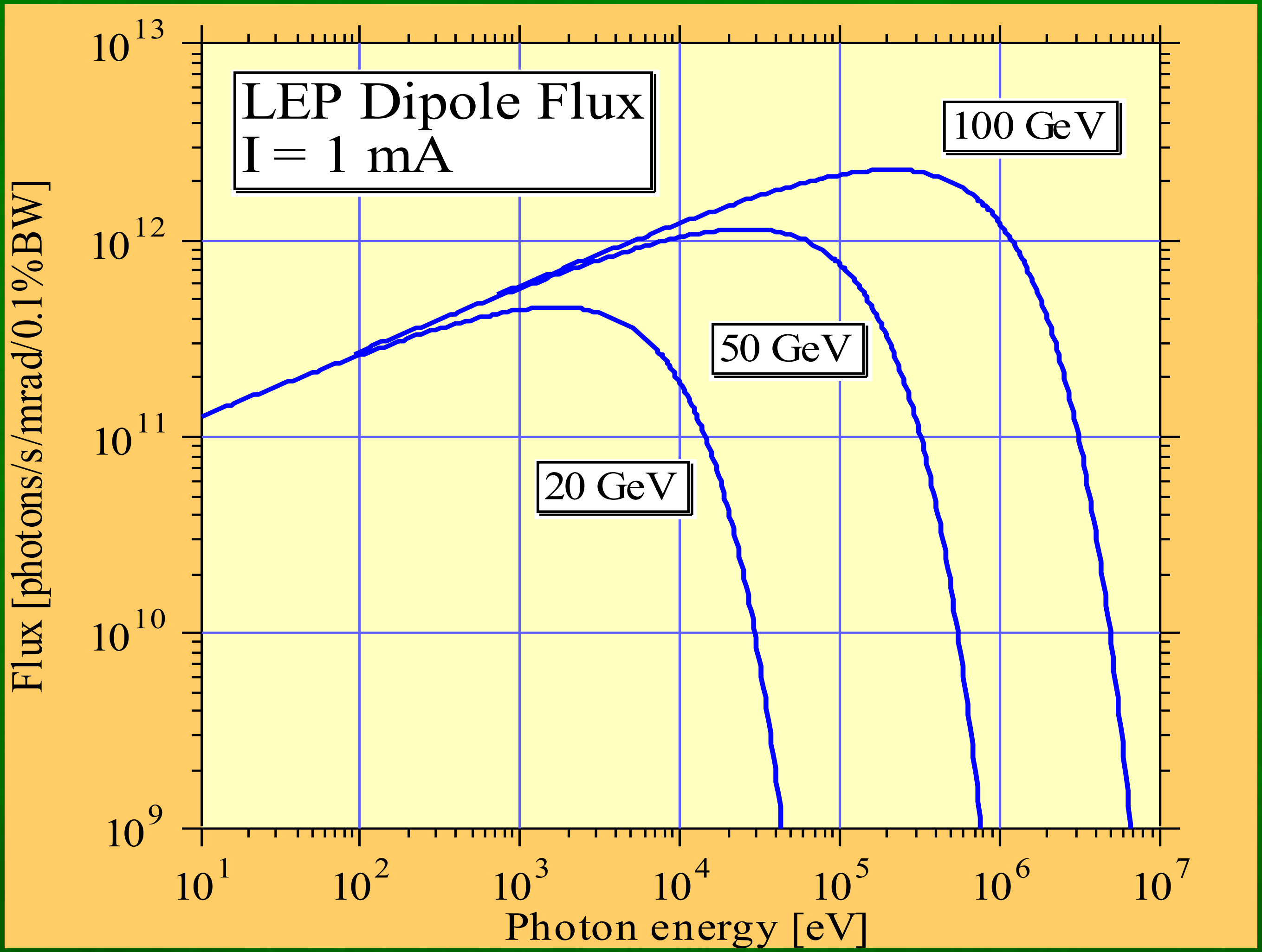
$$\int_0^\infty S(x')dx' = 1$$

$$P_{tot} = \frac{2}{3} \hbar c^2 \alpha \frac{\gamma^4}{\rho^2}$$

$$\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho}$$



Synchrotron radiation flux for different electron energies



Angular divergence of radiation

The rms opening angle R'

- **at the critical frequency:**

$$\omega = \omega_c \qquad R' \approx \frac{0.54}{\gamma}$$

- **well below**

$$\omega \ll \omega_c \qquad R' \approx \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3} \approx 0.4\left(\frac{\lambda}{\rho}\right)^{1/3}$$

independent of γ !

- **well above**

$$\omega \gg \omega_c \qquad R' \approx \frac{0.6}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/2}$$

Radiation effects in electron storage rings

Average radiated power restored by RF

$$U_0 \cong 10^{-3} \;\text{ of }\; E_0$$

- Electron loses energy each turn
- RF cavities provide voltage to accelerate electrons back to the nominal energy

$$V_{RF} > U_0$$

Radiation damping

- Average rate of energy loss produces DAMPING of electron oscillations in all three degrees of freedom (if properly arranged!)

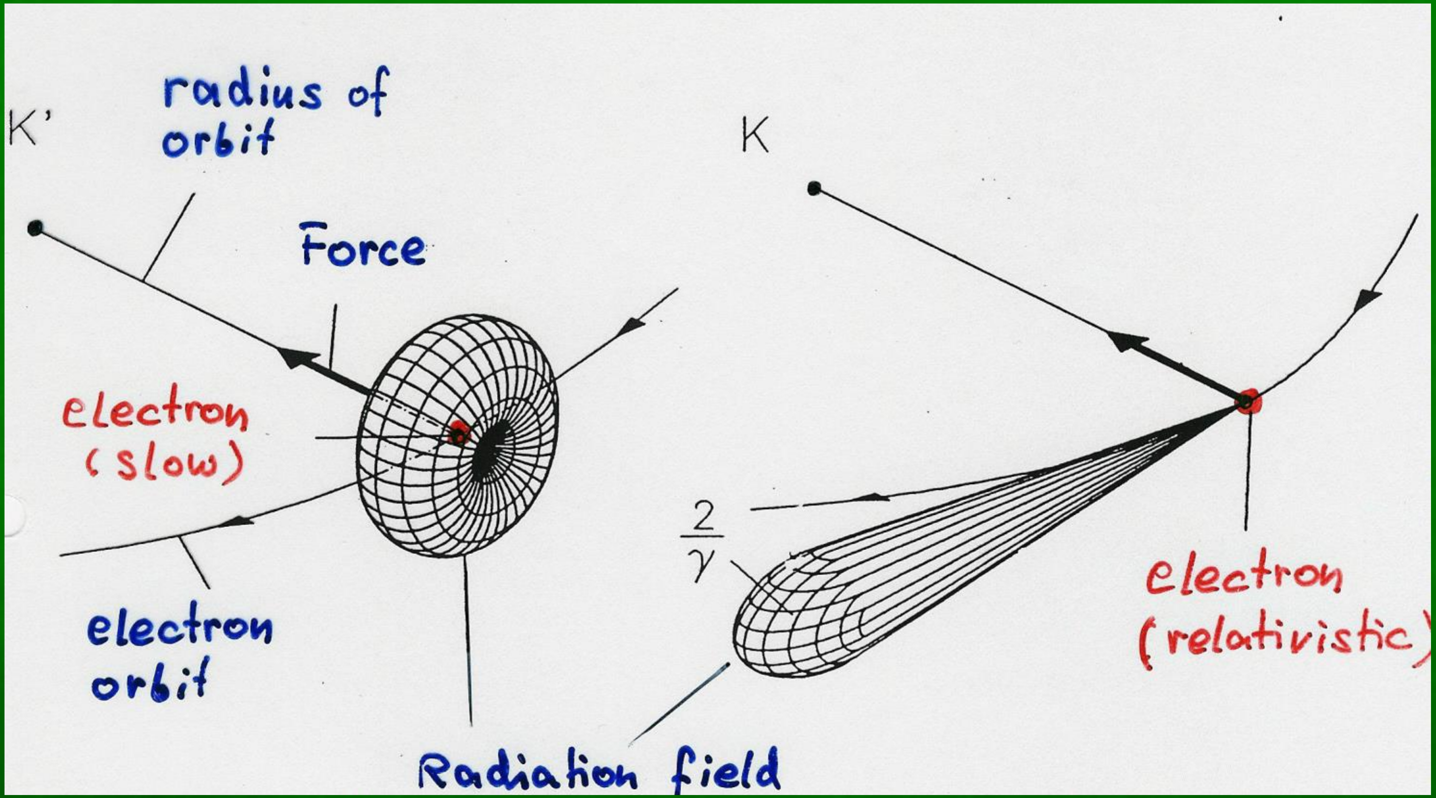
Quantum fluctuations

- Statistical fluctuations in energy loss (from quantised emission of radiation) produce RANDOM EXCITATION of these oscillations

Equilibrium distributions

- The balance between the damping and the excitation of the electron oscillations determines the equilibrium distribution of particles in the beam

Radiation is emitted into a narrow cone of only a few mrads opening angle



Synchrotron radiation power

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$$\boxed{P_\gamma = \frac{2}{3}\alpha\hbar c^2 \cdot \frac{\gamma^4}{\rho^2}}$$

$$C_\gamma = \frac{4\pi}{3}\frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5}\left[\frac{\mathrm{m}}{\mathrm{GeV}^3}\right]$$

$$\boxed{\alpha = \frac{1}{137}}$$

$$\boxed{\hbar c = 197\ \mathrm{Mev} \cdot \mathrm{fm}}$$

Energy loss per turn:

$$\boxed{U_0 = C_\gamma \cdot \frac{E^4}{\rho}}$$

$$\boxed{U_0 = \frac{4\pi}{3}\alpha\hbar c\frac{\gamma^4}{\rho}}$$

RADIATION DAMPING

TRANSVERSE OSCILLATIONS

Average energy loss and gain per turn

- Every turn electron radiates small amount of energy

$$E_1 = E_0 - U_0 = E_0\left(1 - \frac{U_0}{E_0}\right)$$

- only the amplitude of the momentum changes

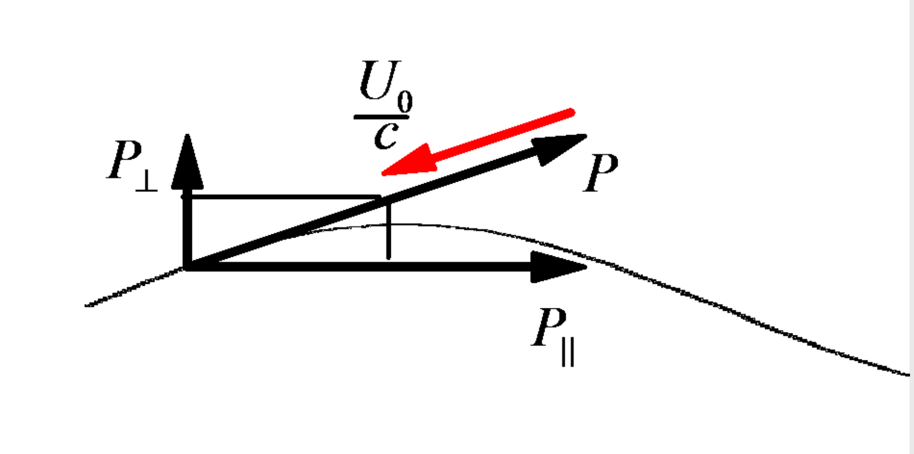
$$P_1 = P_0 - \frac{U_0}{c} = P_0\left(1 - \frac{U_0}{E_0}\right)$$

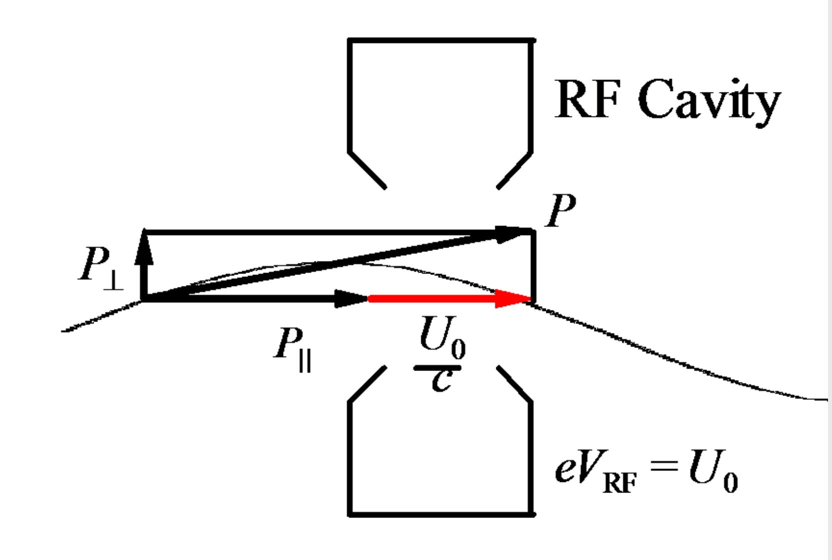
- Only the longitudinal component of the momentum is increased in the RF cavity

- Energy of betatron oscillation

$$\boxed{E_\beta \propto A^2}$$

$$A_1^2 = A_0^2\left(1 - \frac{U_0}{E_0}\right) \quad \text{or} \quad A_1 \cong A_0\left(1 - \frac{U_0}{2E_0}\right)$$





Damping of vertical oscillations

- But this is just the exponential decay law!

$$\frac{\Delta A}{A} = -\frac{U_0}{2E}$$

$$A = A_0 \cdot e^{-t/\tau}$$

- The oscillations are exponentially **damped** with the **damping time (milliseconds!)**

$$\tau = \frac{2E\,T_0}{U_0}$$

the time it would take particle to 'lose all of its energy'

- In terms of radiation power

$$\tau = \frac{2E}{P_\gamma}$$ and since $$P_\gamma \propto E^4$$

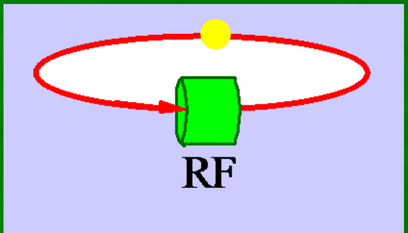
$$\tau \propto \frac{1}{E^3}$$

Adiabatic damping in linear accelerators

In a **linear accelerator:**

$$x' = \frac{p_\perp}{p} \ \text{ decreases } \propto \frac{1}{E}$$

In a **storage ring** beam passes many times through same RF cavity

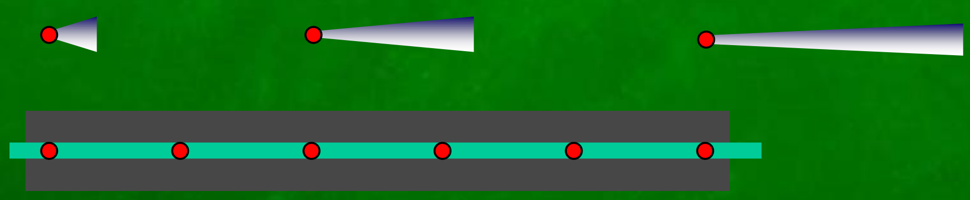


- **Clean loss of energy every turn (no change in x')**
- **Every turn is re-accelerated by RF (x' is reduced)**
- **Particle energy on average remains constant**

Emittance damping in linacs:

$\varepsilon \qquad \frac{\varepsilon}{2} \qquad \frac{\varepsilon}{4}$

$\Omega \qquad \frac{\Omega}{2} \qquad \frac{\Omega}{4}$



$\gamma \qquad 2\gamma \qquad 4\gamma$

$$\varepsilon \propto \frac{1}{\gamma}$$

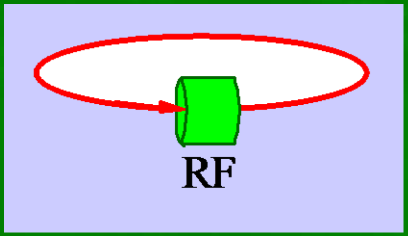
or

$$\gamma\varepsilon = \text{const.}$$

RADIATION DAMPING

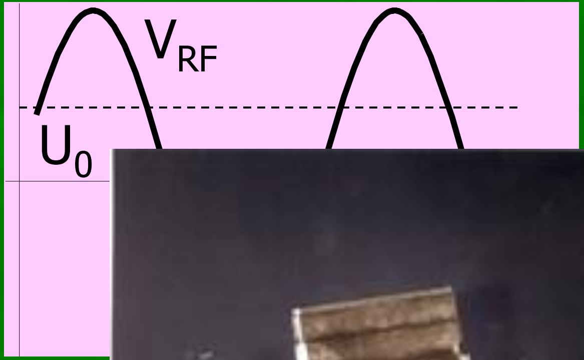
LONGITUDINAL OSCILLATIONS

Longitudinal motion: compensating radiation loss U_0



- RF cavity provides accelerating field with frequency
  - h – harmonic number

$$f_{RF} = h \cdot f_0$$

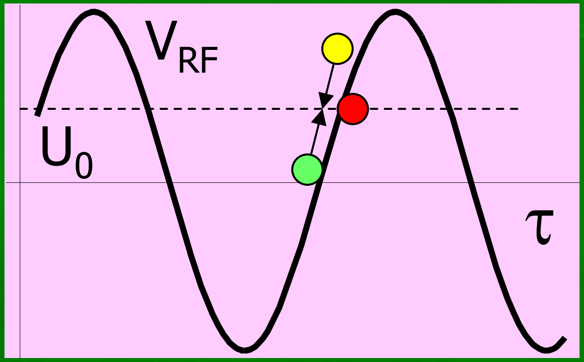


- The energy gain:

$$U_{RF} = eV_{RF}(\tau)$$

- Synchronous particle:
  - has design energy
  - gains from the RF on the average as as it loses per turn U_0

Longitudinal motion: phase stability

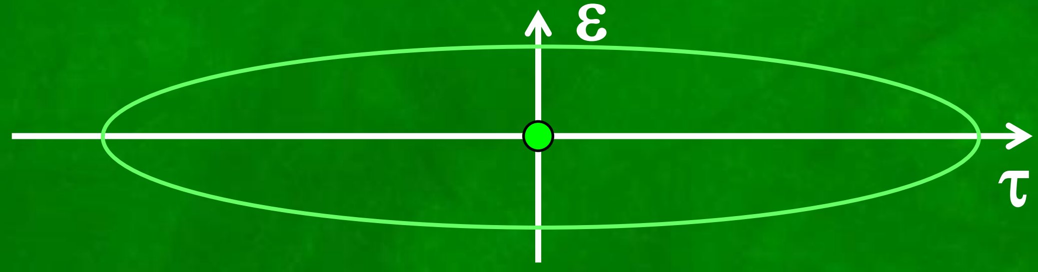


- Particle ahead of synchronous one
  - gets too much energy from the RF
  - goes on a longer orbit (not enough B)
    >> takes longer to go around
  - comes back to the RF cavity closer to synchronous part.

- Particle behind the synchronous one
  - gets too little energy from the RF
  - goes on a shorter orbit (too much B)
  - catches-up with the synchronous particle

Longitudinal motion: energy-time oscillations

energy deviation from the design energy, or the energy of the synchronous particle

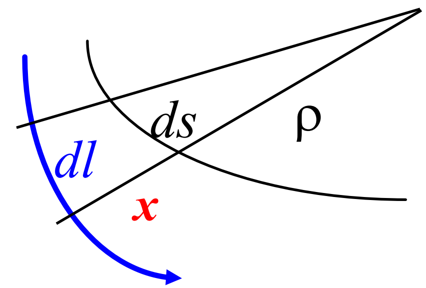


longitudinal coordinate measured from the position of the synchronous electron

Orbit Length

Length element depends on x

$$dl = \left(1 + \frac{x}{\rho}\right) ds$$



Horizontal displacement has two parts:

$$x = x_\beta + x_\varepsilon$$

- To first order x_β does not change L
- x_ε – has the same sign around the ring

Length of the off-energy orbit $$L_\varepsilon = \oint dl = \oint \left(1 + \frac{x_\varepsilon}{\rho}\right) ds = L_0 + \Delta L$$

$$\Delta L = \delta \cdot \oint \frac{D(s)}{\rho(s)} ds \quad \text{where} \quad \delta = \frac{\Delta p}{p} = \frac{\Delta E}{E}$$

$$\frac{\Delta L}{L} = \alpha \cdot \delta$$

Something funny happens on the way around the ring...

Revolution time changes with energy

$$\boxed{T_0 = \frac{L_0}{c\beta}}$$

$$\boxed{\frac{\Delta T}{T} = \frac{\Delta L}{L} - \frac{\Delta\beta}{\beta}}$$

- Particle goes faster (not much!) $\boxed{\frac{d\beta}{\beta} = \frac{1}{\gamma^2} \cdot \frac{dp}{p} \qquad \text{(relativity)}}$
- while the orbit length increases (more!) $\boxed{\frac{\Delta L}{L} = \alpha \cdot \frac{dp}{p}}$
- The "slip factor" $\eta \cong \alpha \quad \text{since} \quad \alpha >> \frac{1}{\gamma^2}$

$$\boxed{\frac{\Delta T}{T} = \left(\alpha - \frac{1}{\gamma^2}\right) \cdot \frac{dp}{p} = \eta \cdot \frac{dp}{p}}$$

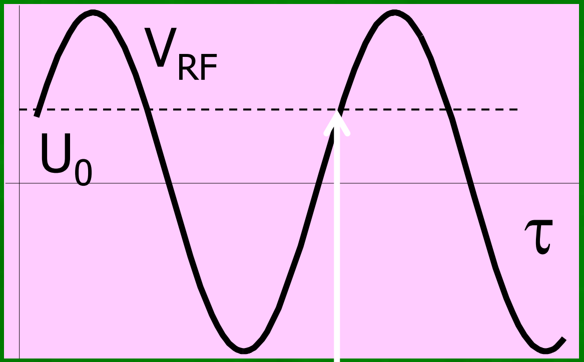
- Ring is above "transition energy" $\boxed{\alpha \equiv \frac{1}{\gamma_{tr}^2}}$

isochronous ring: $\boxed{\eta = 0 \;\; \text{or} \;\; \gamma = \gamma_{tr}}$

Not only accelerators work above transition

Dante Aligieri
Divine Comedy

RF Voltage



$$V(\tau) = \hat{V}\sin(h\omega_0\tau + \psi_s)$$

here the synchronous phase

$$\psi_s = \arcsin\left(\frac{U_0}{e\hat{V}}\right)$$

Momentum compaction factor

$$\alpha \equiv \frac{1}{L}\oint \frac{D(s)}{\rho(s)} ds$$

Like the tunes Q_x, Q_y - α depends on the whole optics

- A quick estimate for separated function guide field:

$$\alpha = \frac{1}{L_0\rho_0} \oint_{\text{mag}} D(s)\, ds = \frac{1}{L_0\rho_0}\langle D\rangle \cdot L_{mag}$$

$\rho = \rho_0$ in dipoles
$\rho = \infty$ elsewhere

- But $L_{mag} = 2\pi\rho_0$

$$\alpha = \frac{\langle D\rangle}{R}$$

- Since dispersion is approximately

$$D \approx \frac{R}{Q^2} \Rightarrow \alpha \approx \frac{1}{Q^2} \text{ typically } < 1\%$$

and the orbit change for ~ 1% energy deviation

$$\frac{\Delta L}{L} = \frac{1}{Q^2} \cdot \delta \approx 10^{-4}$$

Energy balance

Energy gain from the RF system: $U_{RF} = eV_{RF}(\tau) = U_0 + e\dot{V}_{RF} \cdot \tau$

- synchronous particle ($\tau = 0$) will get exactly the energy loss per turn
- we consider only linear oscillations

$$\dot{V}_{RF} = \left.\frac{dV_{RF}}{d\tau}\right|_{\tau = 0}$$

- Each turn electron gets energy from RF and loses energy to radiation within one revolution time T_0

$$\Delta\varepsilon = \left(U_0 + e\dot{V}_{RF} \cdot \tau\right) - \left(U_0 + U' \cdot \varepsilon\right)$$

$$\frac{d\varepsilon}{dt} = \frac{1}{T_0}\left(e\dot{V}_{RF} \cdot \tau - U' \cdot \varepsilon\right)$$

- An electron with an energy deviation will arrive after one turn at a different time with respect to the synchronous particle

$$\frac{d\tau}{dt} = -\alpha \frac{\varepsilon}{E_0}$$

Synchrotron oscillations: damped harmonic oscillator

Combining the two equations $$\boxed{\frac{d^2\varepsilon}{dt^2} + 2\alpha_\varepsilon \frac{d\varepsilon}{dt} + \Omega^2 \varepsilon = 0}$$

- where the oscillation frequency $\Omega^2 \equiv \frac{\alpha e \dot{V}_{RF}}{T_0 E_0}$
- the damping is slow: $\alpha_\varepsilon \equiv \frac{U'}{2T_0}$ typically $\alpha_\varepsilon << \Omega$
- the solution is then:

$$\varepsilon(t) = \hat{\varepsilon}_0 e^{-\alpha_\varepsilon t} \cos(\Omega t + \theta_\varepsilon)$$

- similarly, we can get for the time delay:

$$\tau(t) = \hat{\tau}_0 e^{-\alpha_\varepsilon t} \cos(\Omega t + \theta_\tau)$$

Synchrotron (time - energy) oscillations

The ratio of amplitudes at any instant

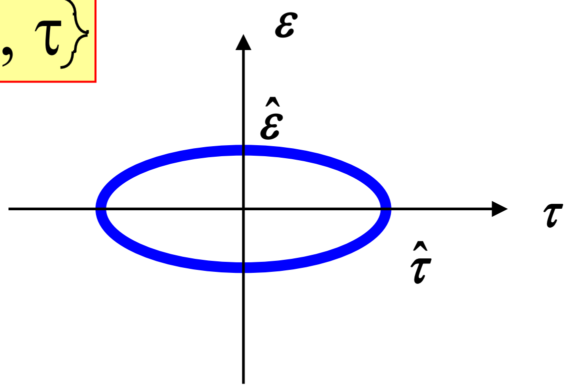
$$\hat{\tau} = \frac{\alpha}{\Omega E_0}\hat{\varepsilon}$$

Oscillations are 90 degrees out of phase

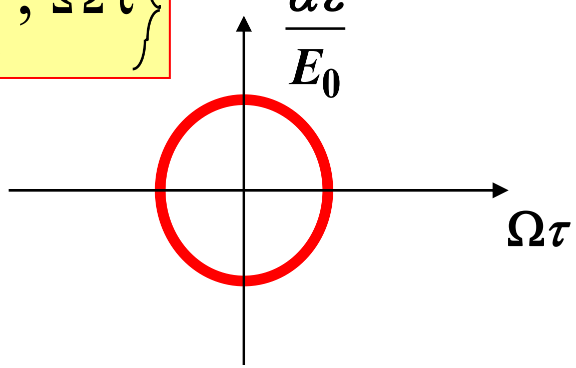
$$\theta_\varepsilon = \theta_\tau + \frac{\pi}{2}$$

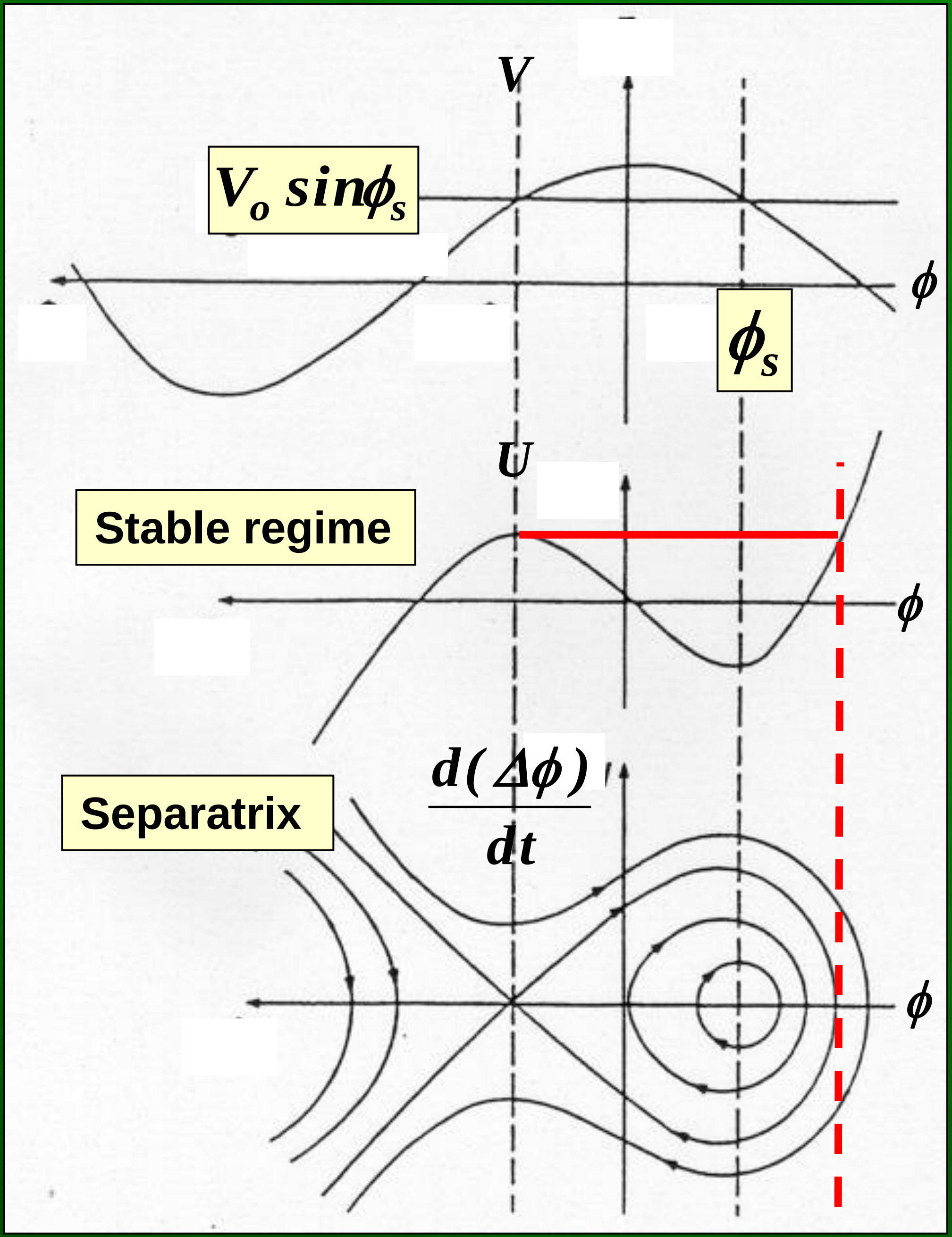
The motion can be viewed in the phase space of conjugate variables

$$\{\varepsilon,\ \tau\}$$



$$\left\{\frac{\alpha\varepsilon}{E_0},\ \Omega\tau\right\}$$





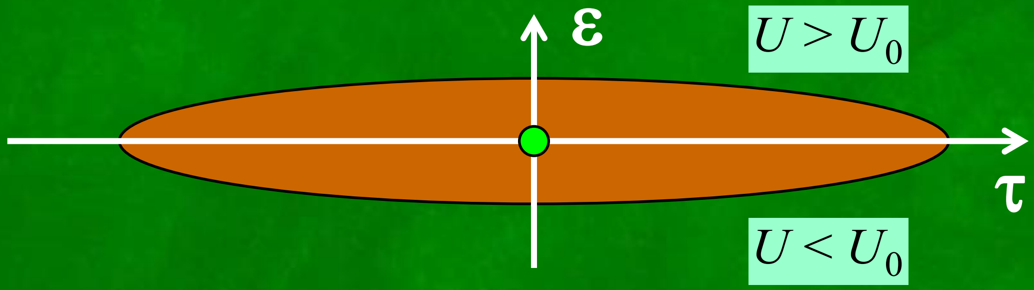
Longitudinal Phase Space

Longitudinal motion: damping of synchrotron oscillations

$P_\gamma \propto \boldsymbol{E}^2 B^2$

During one period of synchrotron oscillation:

- when the particle is in the upper half-plane, it loses more energy per turn, its energy gradually reduces



- when the particle is in the lower half-plane, it loses less energy per turn, but receives U_0 on the average, so its energy deviation gradually reduces

The synchrotron motion is damped

- the phase space trajectory is spiraling towards the origin

Robinson theorem: Damping partition numbers

- Transverse betatron oscillations are damped with

$$\tau_x = \tau_z = \frac{2ET_0}{U_0}$$

- Synchrotron oscillations are damped twice as fast

$$\tau_\varepsilon = \frac{ET_0}{U_0}$$

- The total amount of damping (Robinson theorem) depends only on energy and loss per turn

$$\boxed{\frac{1}{\tau_x} + \frac{1}{\tau_y} + \frac{1}{\tau_\varepsilon} = \frac{2U_0}{ET_0} = \frac{U_0}{2ET_0}\left(J_x + J_y + J_\varepsilon\right)}$$

the sum of the partition numbers $\boxed{J_x + J_z + J_\varepsilon = 4}$

Radiation loss

$$P_\gamma \propto E^2 B^2$$

Displaced off the design orbit particle sees fields that are different from design values

- **energy deviation ε**
  - different energy: $P_\gamma \propto E^2$
  - different magnetic field **B**
    particle moves on a different orbit, defined by the **off-energy** or **dispersion** function D_x

both contribute to linear term in $P_\gamma(\varepsilon)$

- **betatron oscillations**: zero on *average*

Radiation loss

$$\mathbf{P_\gamma \propto E^2 B^2}$$

To first order in $\boldsymbol{\varepsilon}$

$$\mathbf{U_{rad} = U_0 + U' \cdot \varepsilon}$$

electron energy changes slowly, at any instant it is moving on an orbit defined by $\mathbf{D_x}$

$$\mathbf{U' \equiv \left.\frac{dU_{rad}}{dE}\right|_{E_0}}$$

after some algebra one can write

$$U' = \frac{U_0}{E_0}\left(2 + \mathcal{D}\right)$$

$$\mathcal{D} \neq 0 \quad \textit{only when} \quad \frac{k}{\rho} \neq 0$$

Damping partition numbers

$$\boxed{J_x + J_z + J_\varepsilon = 4}$$

- Typically we build rings with no vertical dispersion

$$\boxed{J_z = 1} \qquad \boxed{J_x + J_\varepsilon = 3}$$

- Horizontal and energy partition numbers can be modified via $\mathcal{D}$:

$$\boxed{J_x = 1 - \mathcal{D}} \qquad \boxed{J_\varepsilon = 2 + \mathcal{D}}$$

  - Use of combined function magnets
  - Shift the equilibrium orbit in quads with RF frequency

EQUILIBRIUM BEAM SIZES

Radiation effects in electron storage rings

Average radiated power restored by RF

$$U_0 \cong 10^{-3} \text{ of } E_0$$

- Electron loses energy each turn
- RF cavities provide voltage to accelerate electrons back to the nominal energy

$$V_{RF} > U_0$$

Radiation damping

- Average rate of energy loss produces DAMPING of electron oscillations in all three degrees of freedom (if properly arranged!)

Quantum fluctuations

- **Statistical fluctuations in energy loss (from quantised emission of radiation) produce RANDOM EXCITATION of these oscillations**

Equilibrium distributions

- **The balance between the damping and the excitation of the electron oscillations determines the equilibrium distribution of particles in the beam**

Quantum nature of synchrotron radiation

Damping only

- If damping was the whole story, the beam emittance (size) would shrink to microscopic dimensions!*
- Lots of problems! (e.g. coherent radiation)

- How small? On the order of electron wavelength

$$E = \gamma mc^2 = h\nu = \frac{hc}{\lambda_e} \quad \Rightarrow \quad \lambda_e = \frac{1}{\gamma}\frac{h}{mc} = \frac{\lambda_C}{\gamma}$$

$\lambda_C = 2.4 \cdot 10^{-12} m$ – Compton wavelength

Diffraction limited electron emittance $\boxed{\varepsilon \geq \frac{\lambda_C}{4\pi\gamma}\left(\times N^{1/3} - \text{fermions}\right)}$

Quantum nature of synchrotron radiation

Quantum fluctuations

- Because the radiation is emitted in quanta, radiation itself takes care of the problem!
- It is sufficient to use quasi-classical picture:
  - » *Emission time is very short*
  - » *Emission times are statistically independent (each emission - only a small change in electron energy)*

Purely stochastic (Poisson) process

Visible quantum effects

I have always been somewhat amazed that a purely quantum effect can have gross macroscopic effects in large machines;

and, even more,

that Planck's constant has just the right magnitude needed to <u>make practical</u> the construction of large electron storage rings.

A significantly larger or smaller value of $\hbar$

would have posed serious -- perhaps insurmountable -- problems for the realization of large rings.

Mathew Sands

Quantum excitation of energy oscillations

Photons are emitted with typical energy $u_{ph} \approx \hbar\omega_{typ} = \hbar c \frac{\gamma^3}{\rho}$

at the rate (photons/second) $\mathcal{N} = \frac{P_\gamma}{u_{ph}}$

Fluctuations in this rate excite oscillations

During a small interval Δt electron emits photons $N = \mathcal{N} \cdot \Delta t$

losing energy of $N \cdot u_{ph}$

Actually, because of fluctuations, the number is $N \pm \sqrt{N}$

resulting in **spread in energy loss** $\pm\sqrt{N} \cdot u_{ph}$

For large time intervals RF compensates the energy loss, providing damping towards the design energy E_0

Steady state: typical deviations from E_0
≈ typical fluctuations in energy during a damping time τ_ε

Equilibrium energy spread: rough estimate

We then expect the rms energy spread to be $\sigma_\varepsilon \approx \sqrt{N \cdot \tau_\varepsilon \cdot u_{ph}}$

and since $\tau_\varepsilon \approx \dfrac{E_0}{P_\gamma}$ and $P_\gamma = N \cdot u_{ph}$

$\sigma_\varepsilon \approx \sqrt{E_0 \cdot u_{ph}}$ geometric mean of the electron and photon energies!

Relative energy spread can be written then as:

$$\frac{\sigma_\varepsilon}{E_0} \approx \gamma \sqrt{\frac{\lambda\!\!\!^{-}_e}{\rho}} \qquad \lambda\!\!\!^{-}_e = \frac{\hbar}{m_e c} \simeq 4 \cdot 10^{-13} m$$

it is roughly constant for all rings

- typically $\rho \propto E^2$

$$\frac{\sigma_\varepsilon}{E_0} \sim const \sim 10^{-3}$$

Equilibrium energy spread

More detailed calculations give

- for the case of an 'isomagnetic' lattice

$$\rho(s) = \begin{matrix} \rho_0 & \text{in dipoles} \\ \infty & \text{elsewhere} \end{matrix}$$

$$\boxed{\left(\frac{\sigma_\varepsilon}{E}\right)^2 = \frac{C_q E^2}{J_\varepsilon \rho_0}}$$

with $$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar c}{\left(m_e c^2\right)^3} = 1.468 \cdot 10^{-6} \left[\frac{\text{m}}{\text{GeV}^2}\right]$$

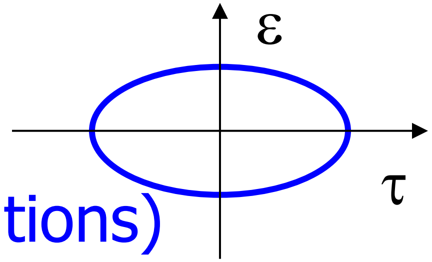
It is difficult to obtain energy spread < 0.1%

- limit on undulator brightness!

Equilibrium bunch length

Bunch length is related to the energy spread

- Energy deviation and time of arrival (or position along the bunch) are conjugate variables (synchrotron oscillations)



$$\hat{\tau} = \frac{\alpha}{\Omega_s}\left(\frac{\hat{\varepsilon}}{E}\right)$$

$$\boxed{\sigma_\tau = \frac{\alpha}{\Omega_s}\left(\frac{\sigma_\varepsilon}{E}\right)}$$

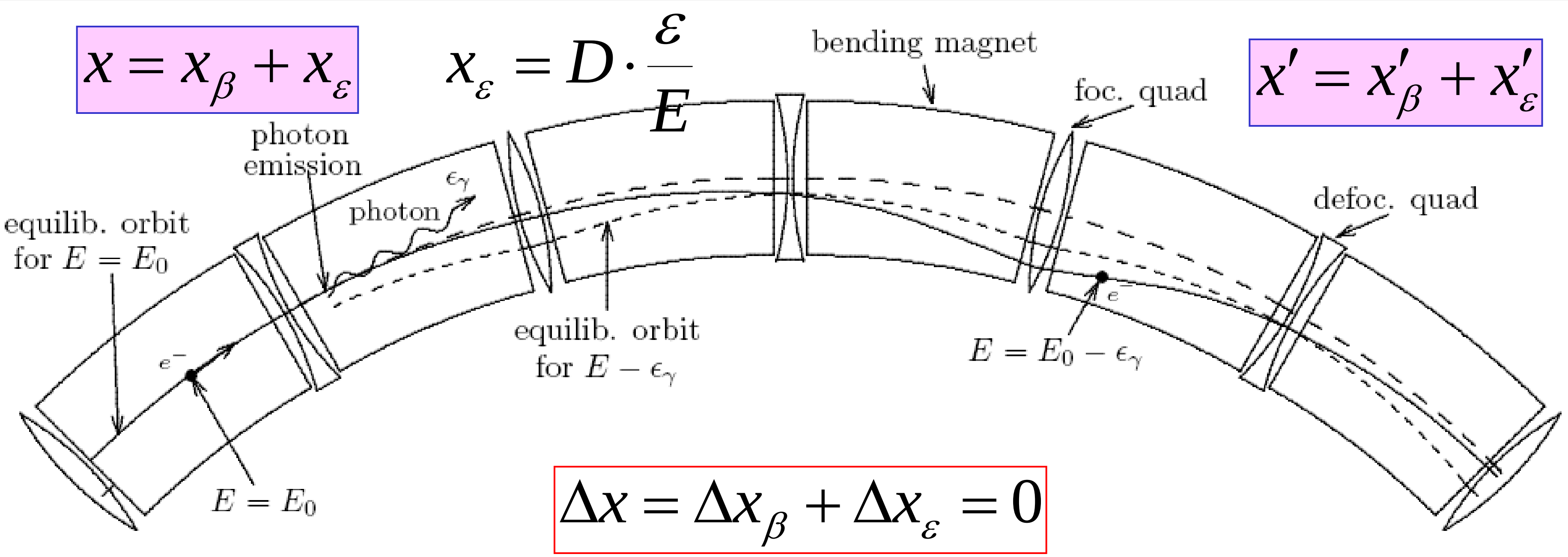
- recall that $\Omega_s \propto \sqrt{V_{RF}}$

Two ways to obtain short bunches:

- RF voltage (power!) $\sigma_\tau \propto 1/\sqrt{V_{RF}}$
- Momentum compaction factor in the limit of $\alpha = 0$
  isochronous ring: particle position along the bunch is frozen $\sigma_\tau \propto \alpha$

Excitation of betatron oscillations

$$x = x_\beta + x_\varepsilon \qquad x_\varepsilon = D \cdot \frac{\varepsilon}{E} \qquad x' = x'_\beta + x'_\varepsilon$$



$$\Delta x = \Delta x_\beta + \Delta x_\varepsilon = 0$$

$$\Delta x_\beta = -D \cdot \frac{\varepsilon_\gamma}{E}$$

Courant Snyder invariant

$$\Delta x'_\beta = -D' \cdot \frac{\varepsilon_\gamma}{E}$$

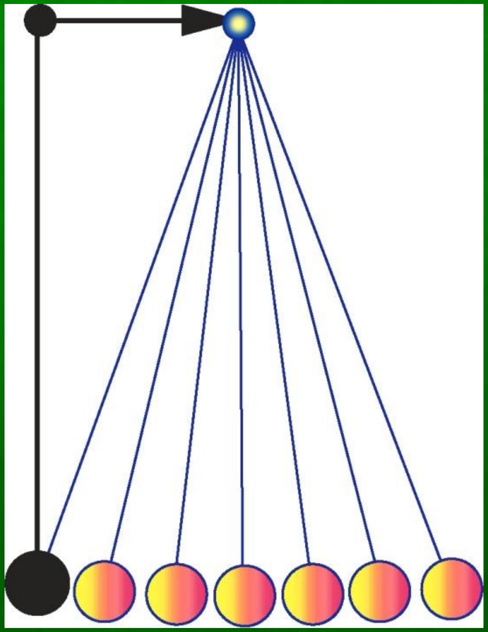
$$\Delta\varepsilon = \gamma \Delta x_\beta^2 + 2\alpha \Delta x_\beta \Delta x'_\beta + \beta \Delta x'^2_\beta = \left[\gamma D^2 + 2\alpha D D' + \beta D'^2\right] \cdot \left(\frac{\varepsilon_\gamma}{E}\right)^2$$

Excitation of betatron oscillations

Electron emitting a photon

- at a place with non-zero dispersion
- starts a betatron oscillation around a new reference orbit

$$\boxed{x_\beta \approx D \cdot \frac{\varepsilon_\gamma}{E}}$$



Horizontal oscillations: equilibrium

Emission of photons is a random process

- Again we have random walk, now in **x**. How far particle will wander away is limited by the radiation damping
- The balance is achieved on the time scale of the damping time $\tau_x = 2\ \tau_\varepsilon$

$$\sigma_{x\beta} \approx \sqrt{\mathcal{N} \cdot \tau_x} \cdot D \cdot \frac{\varepsilon_\gamma}{E} = \sqrt{2} \cdot D \cdot \frac{\sigma_\varepsilon}{E}$$

- Typical horizontal beam size ~ 1 mm

Quantum effect visible to the naked eye!

- Vertical size - determined by coupling

Beam emittance

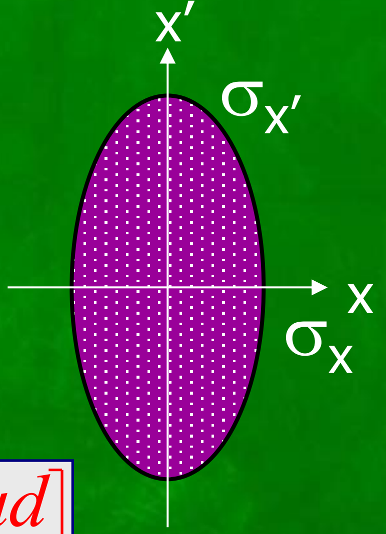
$$\text{Area} = \pi \cdot \varepsilon$$

Betatron oscillations

- Particles in the beam execute betatron oscillations with different amplitudes.

Transverse beam distribution

- Gaussian (electrons)
- "Typical" particle: 1 - σ ellipse
  (in a place where $\alpha = \beta' = 0$)



Units of ε $[m \cdot rad]$

$$\text{Emittance} \equiv \frac{\sigma_x^2}{\beta}$$

$$\varepsilon = \sigma_x \cdot \sigma_{x'}$$

$$\sigma_x = \sqrt{\varepsilon\,\beta}$$

$$\sigma_{x'} = \sqrt{\varepsilon / \beta}$$

$$\beta = \frac{\sigma_x}{\sigma_{x'}}$$

Equilibrium horizontal emittance

Detailed calculations for isomagnetic lattice

$$\varepsilon_{x0} \equiv \frac{\sigma_{x\beta}^2}{\beta} = \frac{C_q E^2}{J_x} \cdot \frac{\langle \mathcal{H} \rangle_{mag}}{\rho}$$

where

$$\begin{aligned} \mathcal{H} &= \gamma D^2 + 2\alpha D D' + \beta D'^2 \\ &= \frac{1}{\beta}\left[D^2 + (\beta D' + \alpha D)^2\right] \end{aligned}$$

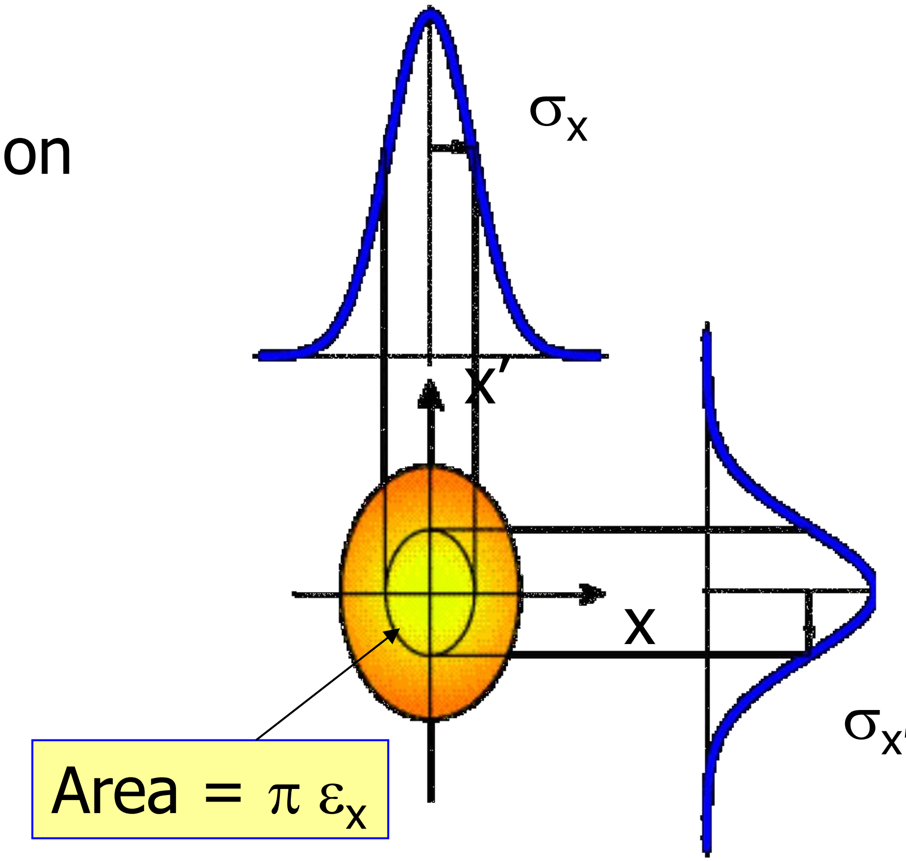
and $\langle \mathcal{H} \rangle_{mag}$ is average value in the bending magnets

2-D Gaussian distribution

Electron rings emittance definition

- 1 - σ ellipse

$$n(x)\,dx = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-x^2/2\sigma^2} dx$$



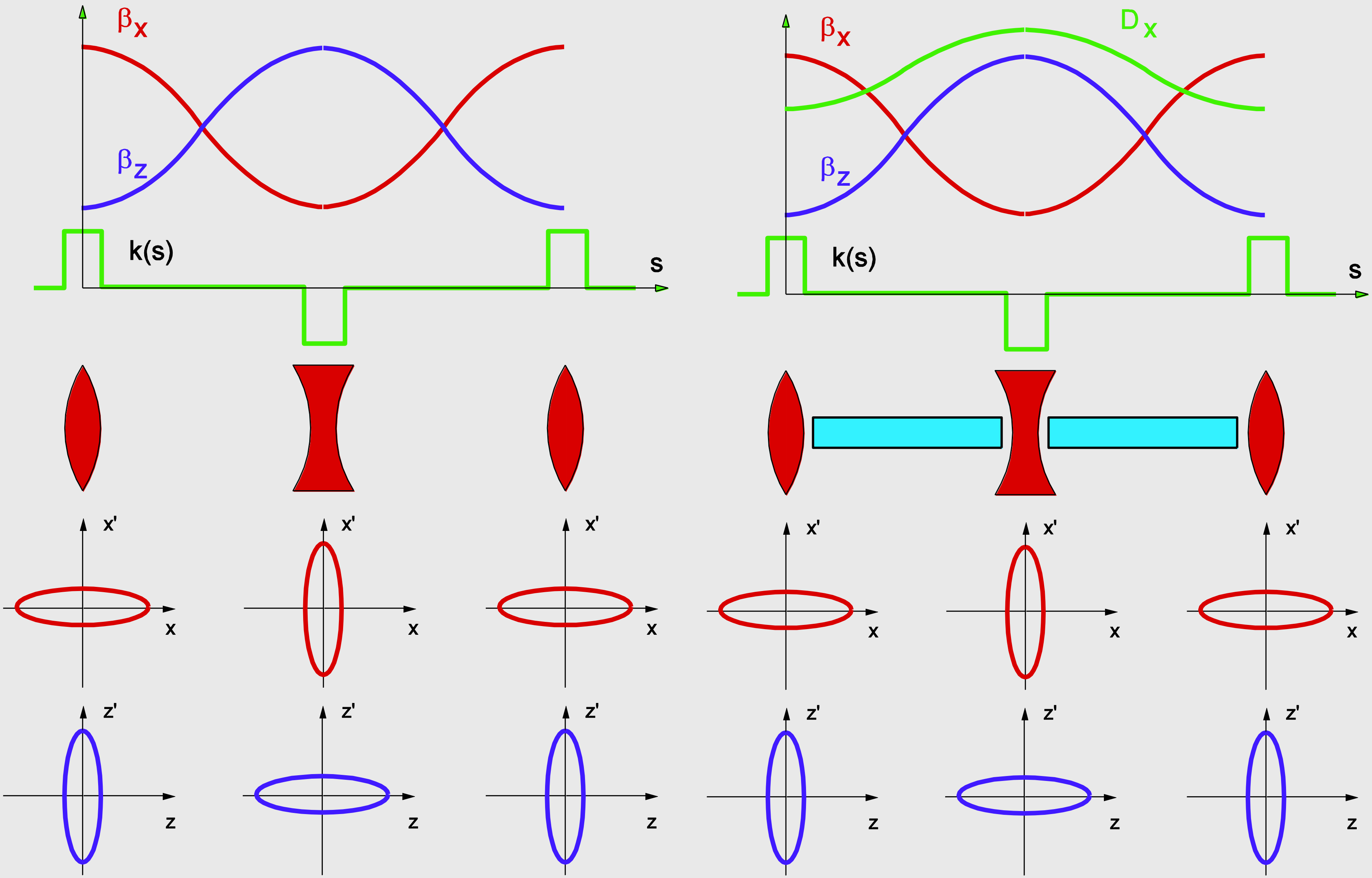
- Probability to be inside 1-σ ellipse

$$P_1 = 1 - e^{-1/2} = 0.39$$

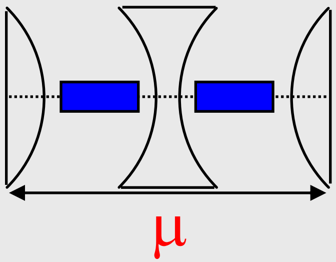
- Probability to be inside n-σ ellipse

$$P_n = 1 - e^{-n^2/2}$$

FODO cell lattice



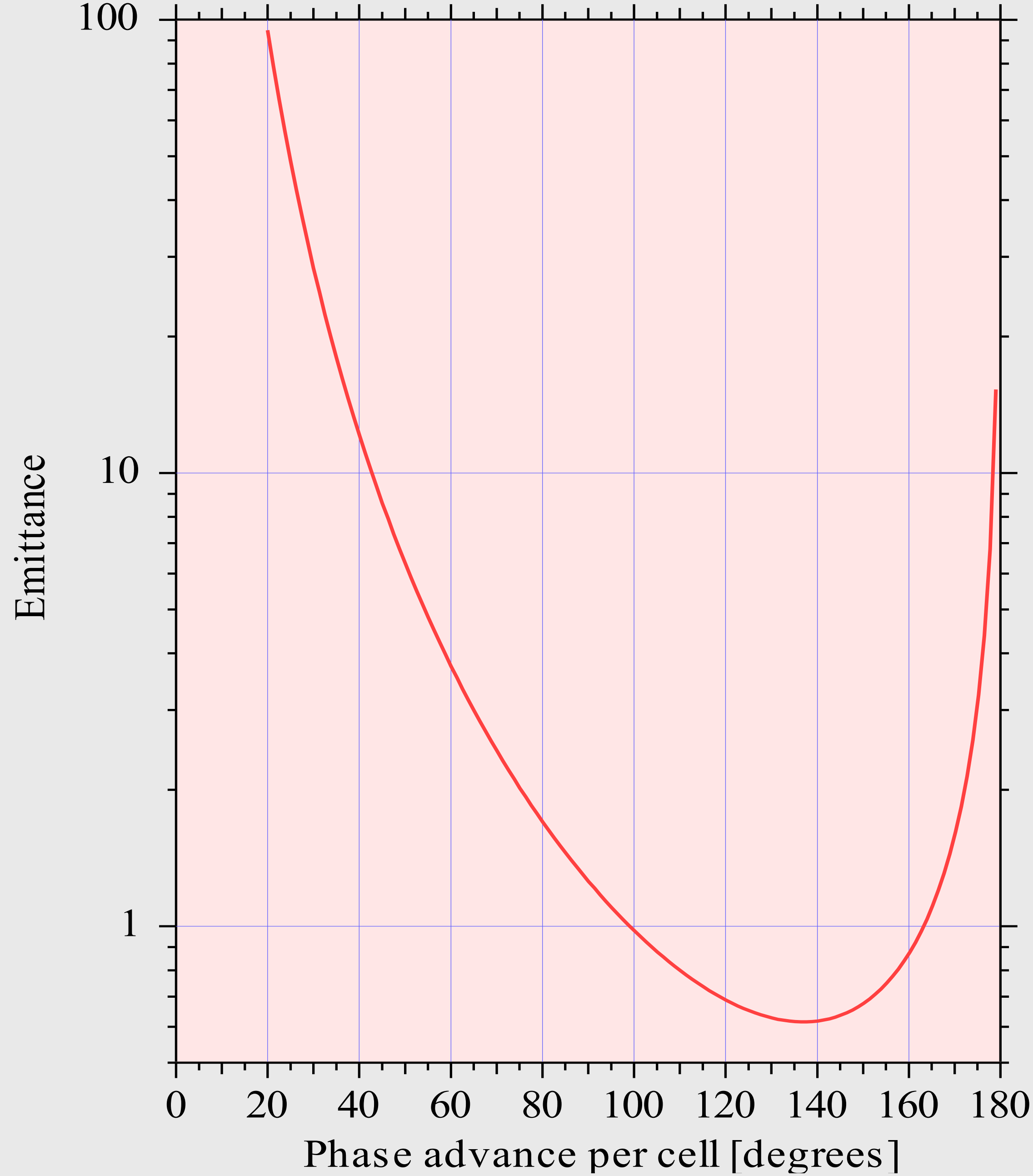
FODO lattice emittance



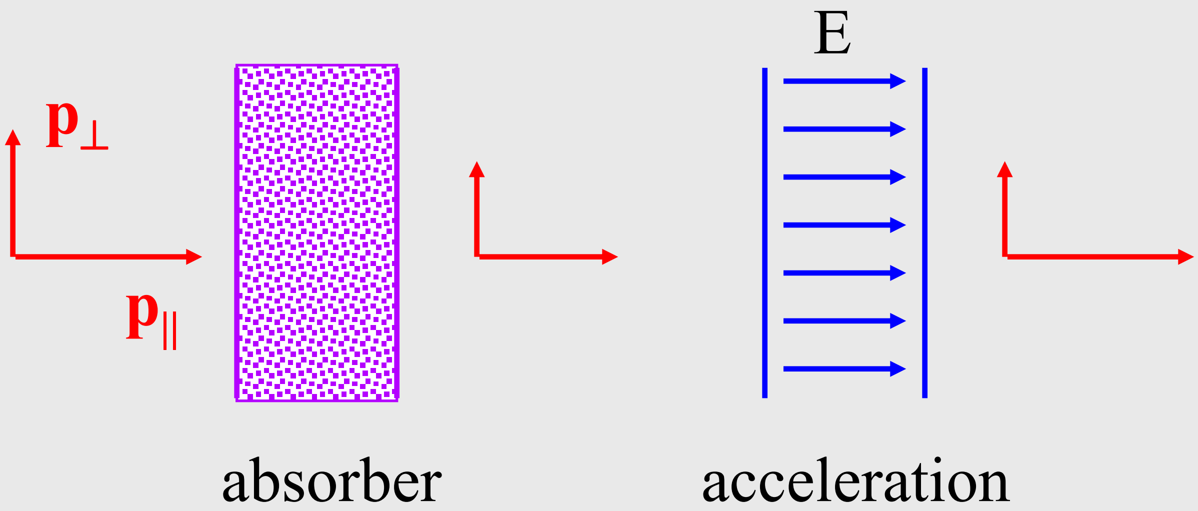
$$\boxed{\mathcal{H} \sim \frac{D^2}{\beta} \sim \frac{R}{Q^3}}$$

$$\boxed{\varepsilon_{x0} \approx \frac{C_q E^2}{J_x} \cdot \frac{R}{\rho} \cdot \frac{1}{Q^3}}$$

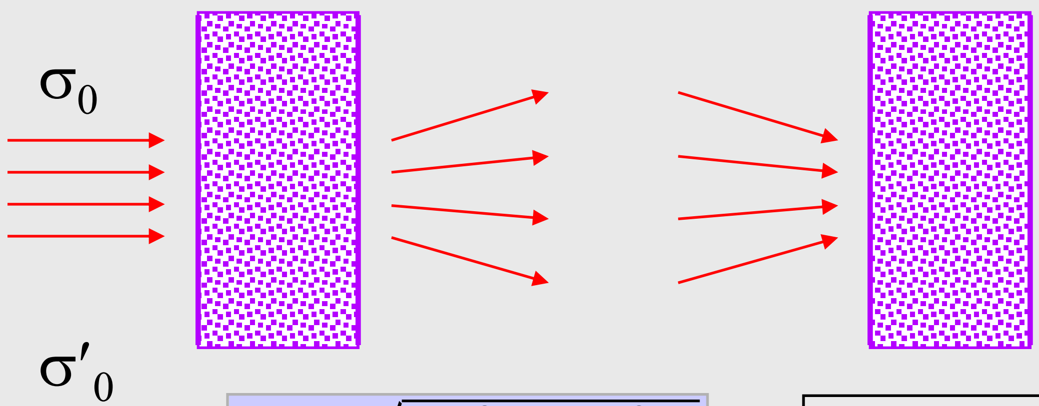
$$\boxed{\boldsymbol{\varepsilon} \propto \frac{\mathbf{E}^2}{\mathbf{J_x}} \theta^3 \mathrm{F}_{\mathrm{FODO}}(\mu)}$$



Ionization cooling



similar to radiation damping, but there is multiple scattering in the absorber that blows up the emittance

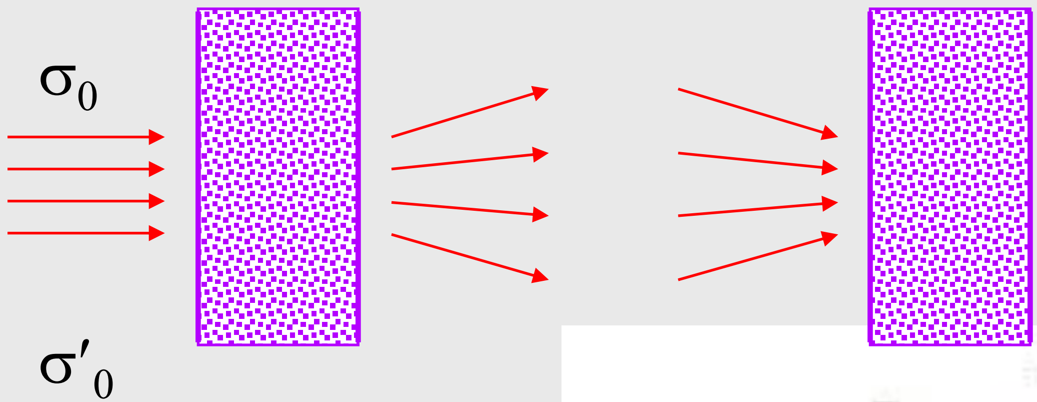


$$\sigma' = \sqrt{\sigma'^2_0 + \sigma'^2_{MS}}$$

$$\boxed{\sigma'_0 >> \sigma'_{MS}}$$

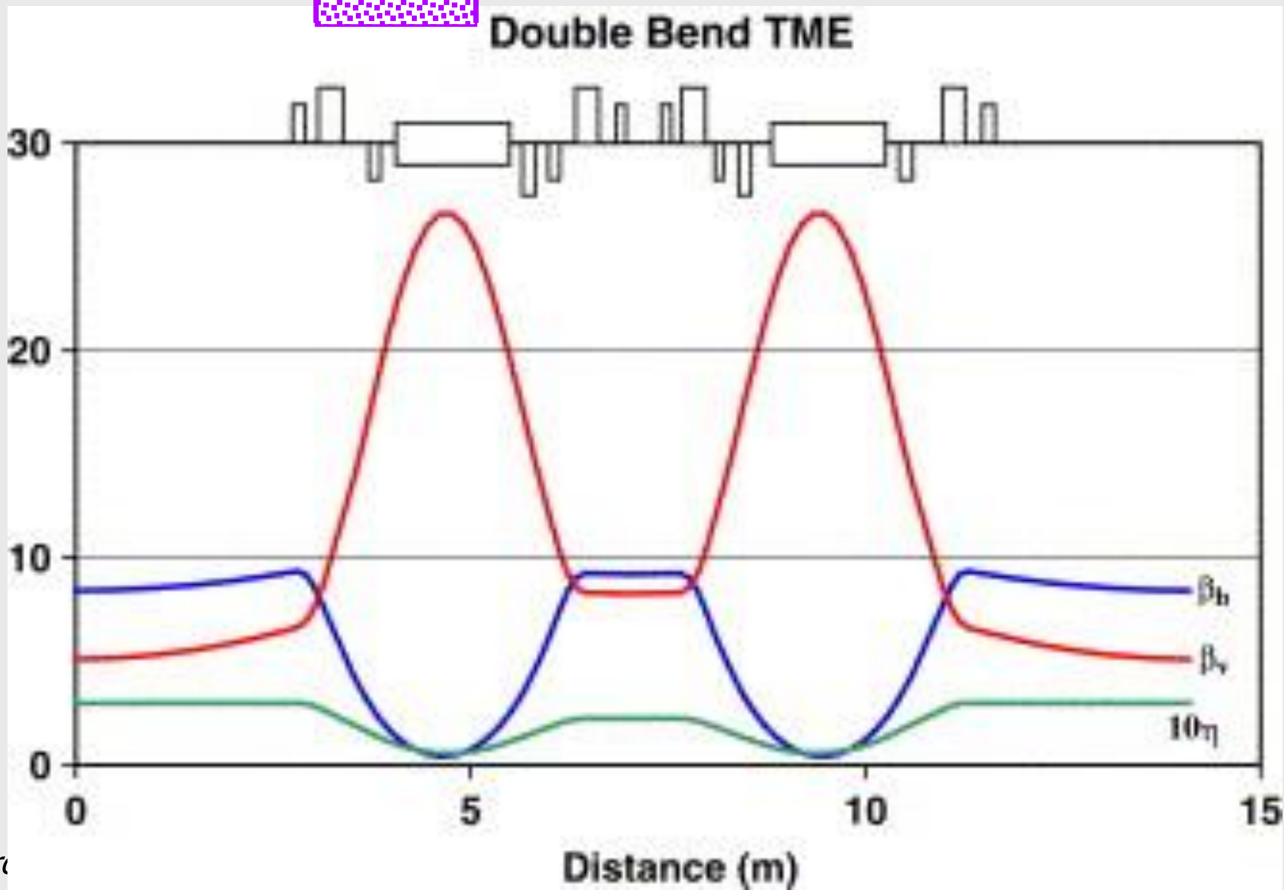
to minimize the blow up due to multiple scattering in the absorber we can **focus** the beam

Minimum emittance lattices



$$\varepsilon_{x0} = \frac{C_q E^2}{J_x} \cdot \theta^3 \cdot F_{latt}$$

$$F_{min} = \frac{1}{12\sqrt{15}}$$



Quantum limit on emittance

- Electron in a storage ring's dipole fields is accelerated, interacts with vacuum fluctuations: «accelerated thermometers show increased temperature»
- synchrotron radiation opening angle is ~ 1/γ-> a lower limit on equilibrium vertical emittance
- independent of energy

$$\epsilon_y = \frac{13}{55} C_q \frac{\oint \beta_y(s)|G^3(s)|ds}{\oint G^2(s)ds}$$

$G(s)$ =curvature, C_q = 0.384 pm

- in case of SLS: 0.2 pm

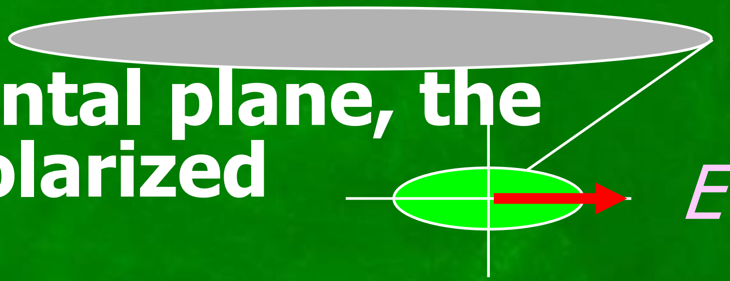
isomagnetic lattice

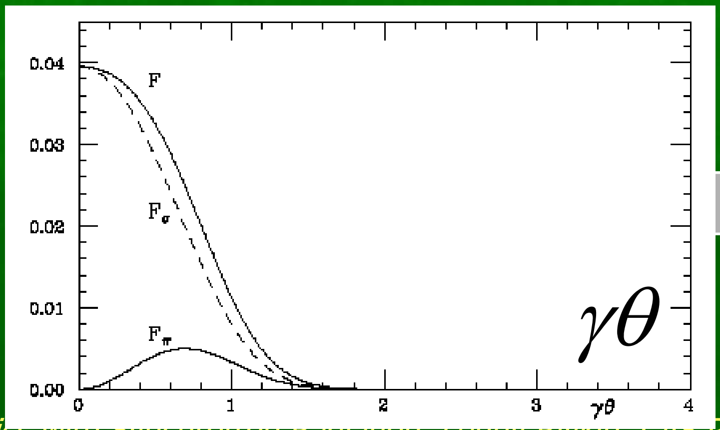
$$\varepsilon_y = 0.09\,\mathrm{pm} \cdot \frac{\langle \beta_y \rangle_{\mathrm{Mag}}}{\rho}$$

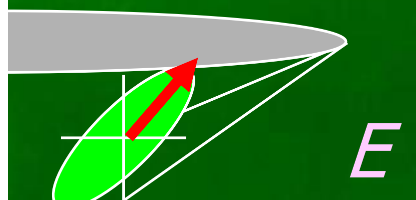
Polarisation

Synchrotron radiation observed in the plane of the particle orbit is horizontally polarized, i.e. the electric field vector is horizontal

Observed out of the horizontal plane, the radiation is elliptically polarized





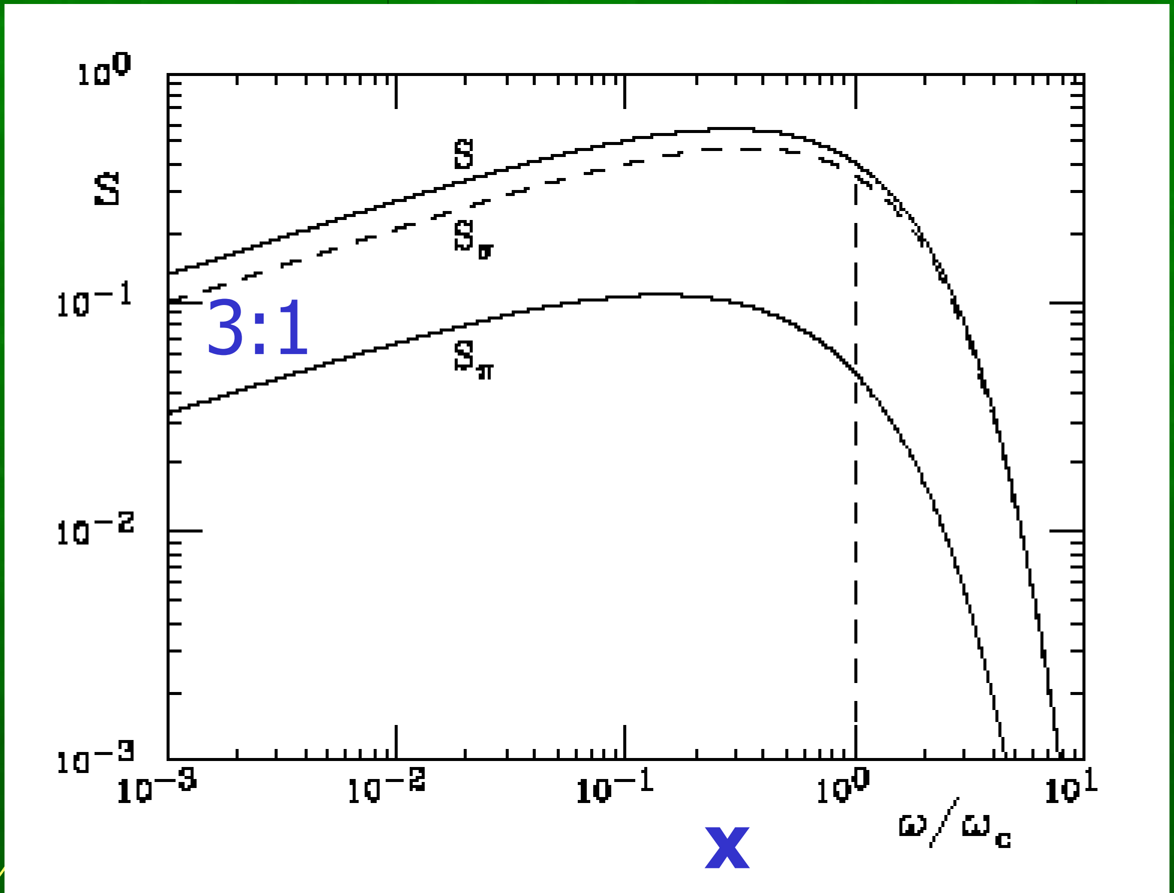


Polarisation: spectral distribution

$$\frac{dP}{d\omega} = \frac{P_{tot}}{\omega_c} S(x) = \frac{P_{tot}}{\omega_c}\left[S_\sigma(x) + S_\pi(x)\right]$$

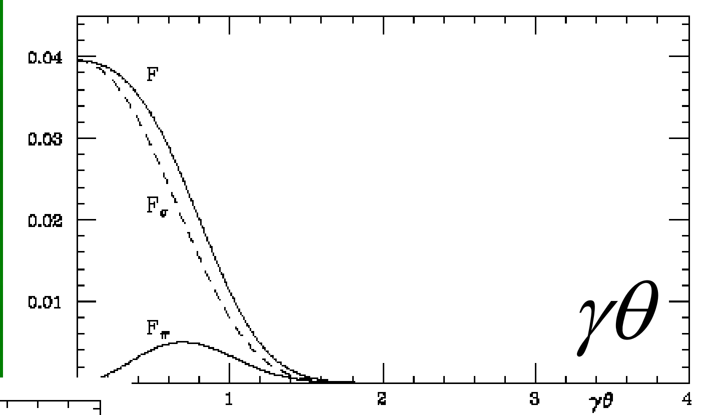
$$S_\sigma = \frac{7}{8} S$$

$$S_\pi = \frac{1}{8} S$$

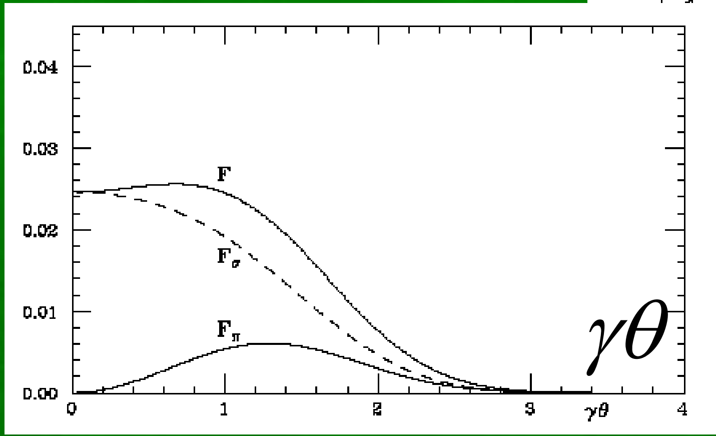


Angular divergence of radiation

- at the critical frequency



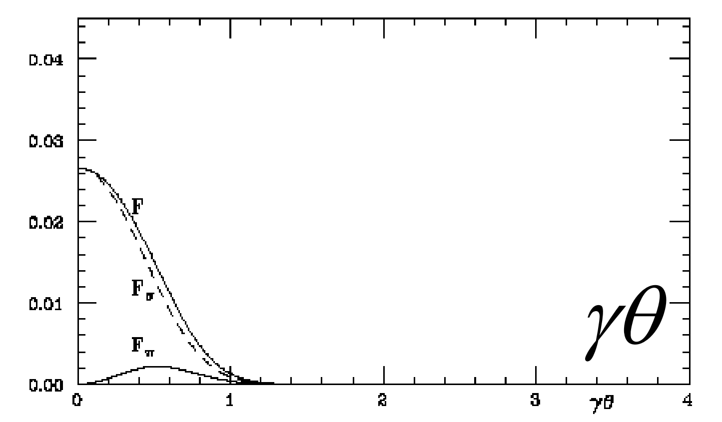
- well below



$$\omega = 0.2\,\omega_c$$

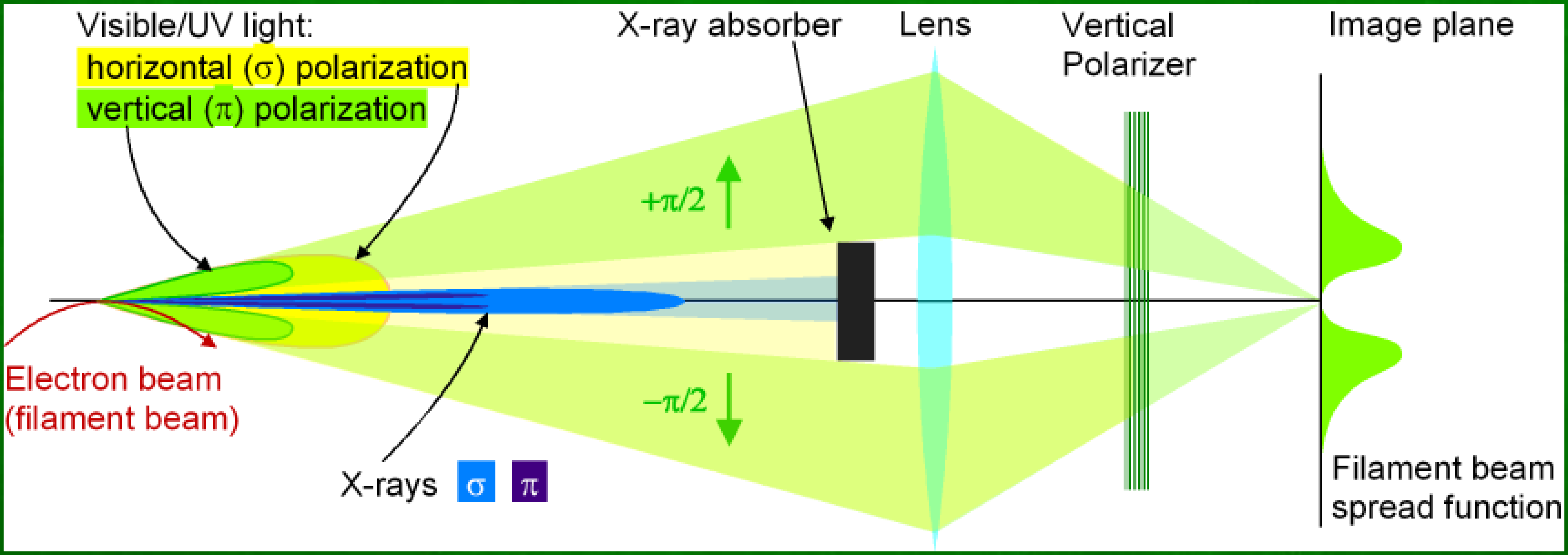
- well above

$$\omega = 2\,\omega_c$$



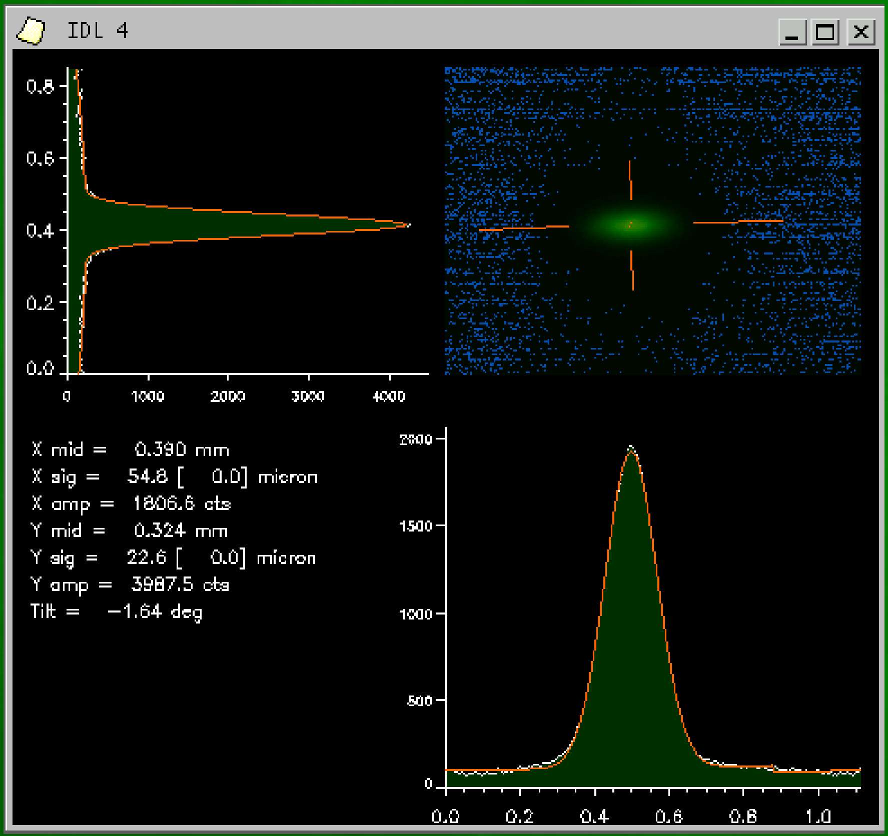
Seeing the electron beam (SLS)

Making an image of the electron beam using the vertically polarised synchrotron light



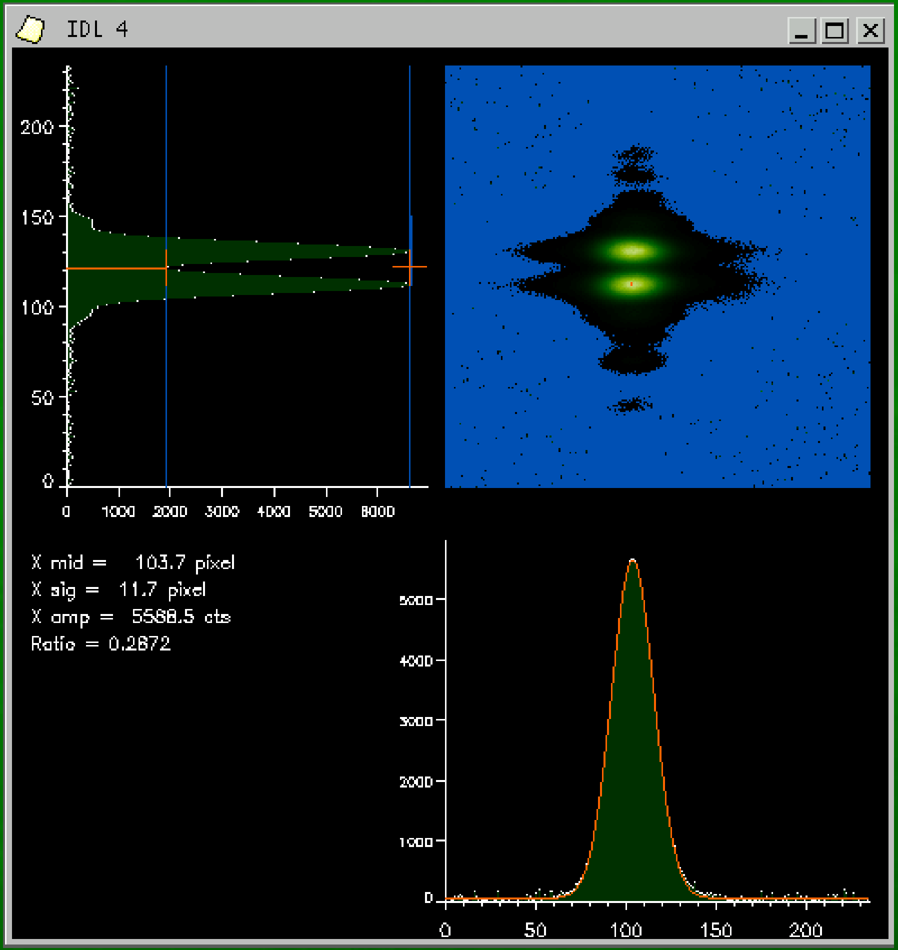
Seeing the electron beam (SLS)

X rays



$$\sigma_x \sim 55 \mu m$$

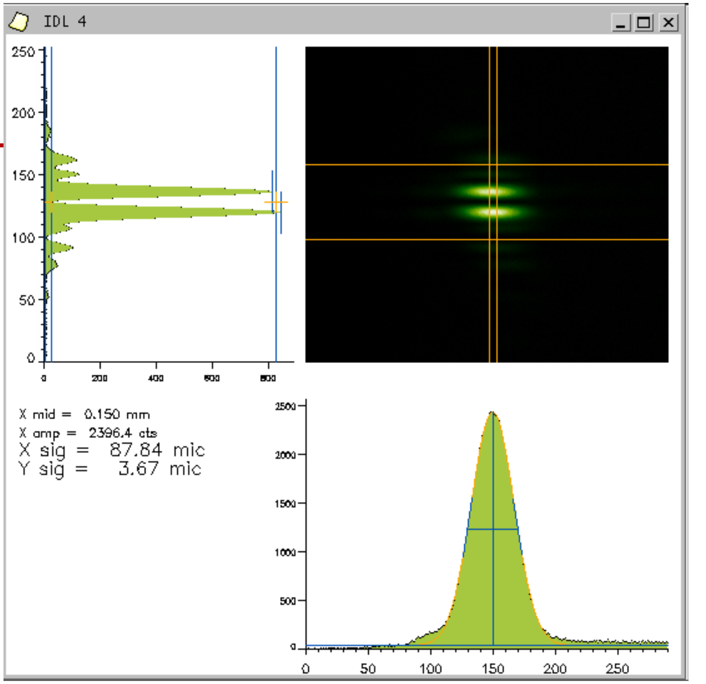
visible light, vertically polarised



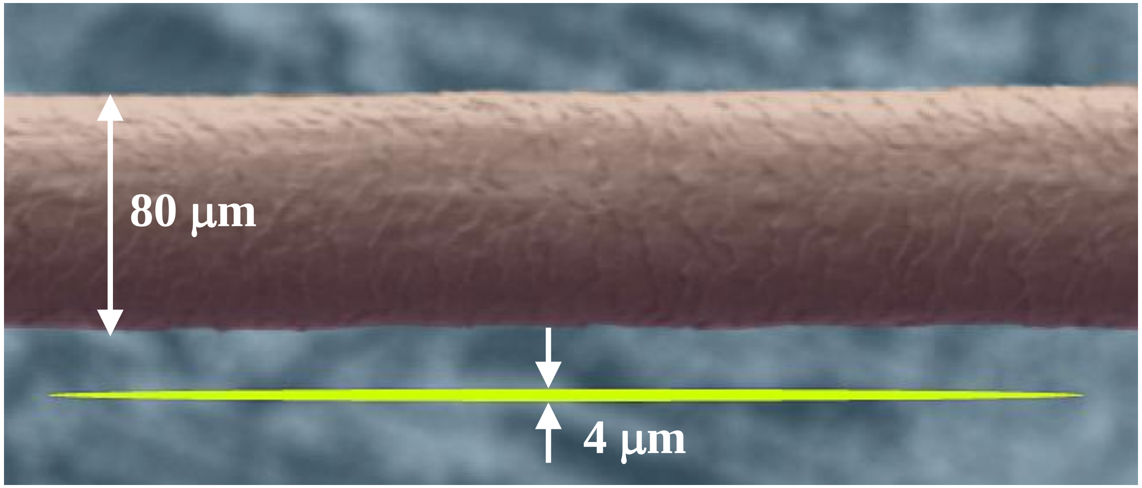
Vertical emittance record

Beam size **3.6 ± 0.6 μm**

Emittance **0.9 ± 0.4 pm**



SLS beam cross section compared to a human hair:



Summary of radiation integrals

Momentum compaction factor

$$\boxed{\alpha = \frac{I_1}{2\pi R}}$$

Energy loss per turn

$$\boxed{U_0 = \frac{1}{2\pi} C_\gamma E^4 \cdot I_2}$$

$$I_1 = \oint \frac{D}{\rho}\, ds$$

$$I_2 = \oint \frac{ds}{\rho^2}$$

$$I_3 = \oint \frac{ds}{|\rho^3|}$$

$$I_4 = \oint \frac{D}{\rho}\left(2k + \frac{1}{\rho^2}\right) ds$$

$$I_5 = \oint \frac{\mathcal{H}}{|\rho^3|}\, ds$$

$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5} \left[\frac{\text{m}}{\text{GeV}^3}\right]$$

Summary of radiation integrals (2)

Damping parameter

$$\boxed{\mathcal{D} = \frac{I_4}{I_2}}$$

Damping times, partition numbers

$$\boxed{J_\varepsilon = 2 + \mathcal{D}, \quad J_x = 1 - \mathcal{D}, \quad J_y = 1}$$

$$\boxed{\tau_i = \frac{\tau_0}{J_i}} \qquad \boxed{\tau_0 = \frac{2ET_0}{U_0}}$$

Equilibrium energy spread

$$\boxed{\left(\frac{\sigma_\varepsilon}{E}\right)^2 = \frac{C_q E^2}{J_\varepsilon} \cdot \frac{I_3}{I_2}}$$

Equilibrium emittance

$$\boxed{\varepsilon_{x0} = \frac{\sigma_{x\beta}^2}{\beta} = \frac{C_q E^2}{J_x} \cdot \frac{I_5}{I_2}}$$

$$I_1 = \oint \frac{D}{\rho}\, ds$$

$$I_2 = \oint \frac{ds}{\rho^2}$$

$$I_3 = \oint \frac{ds}{|\rho^3|}$$

$$I_4 = \oint \frac{D}{\rho}\left(2k + \frac{1}{\rho^2}\right) ds$$

$$I_5 = \oint \frac{\mathcal{H}}{|\rho^3|}\, ds$$

$$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar c}{(m_e c^2)^3} = 1.468 \cdot 10^{-6} \left[\frac{\mathrm{m}}{\mathrm{GeV}^2}\right]$$

$$\mathcal{H} = \gamma D^2 + 2\alpha D D' + \beta D'^2$$

Damping wigglers

Increase the radiation loss per turn U_0 with WIGGLERS

- reduce damping time

$$\boxed{\tau = \frac{E}{P_\gamma + P_{wig}}}$$

- emittance control

wigglers at high dispersion: blow-up emittance

e.g. storage ring colliders for high energy physics

wigglers at zero dispersion: decrease emittance

e.g. damping rings for linear colliders

e.g. synchrotron light sources (PETRAIII, 1 nm.rad)

END