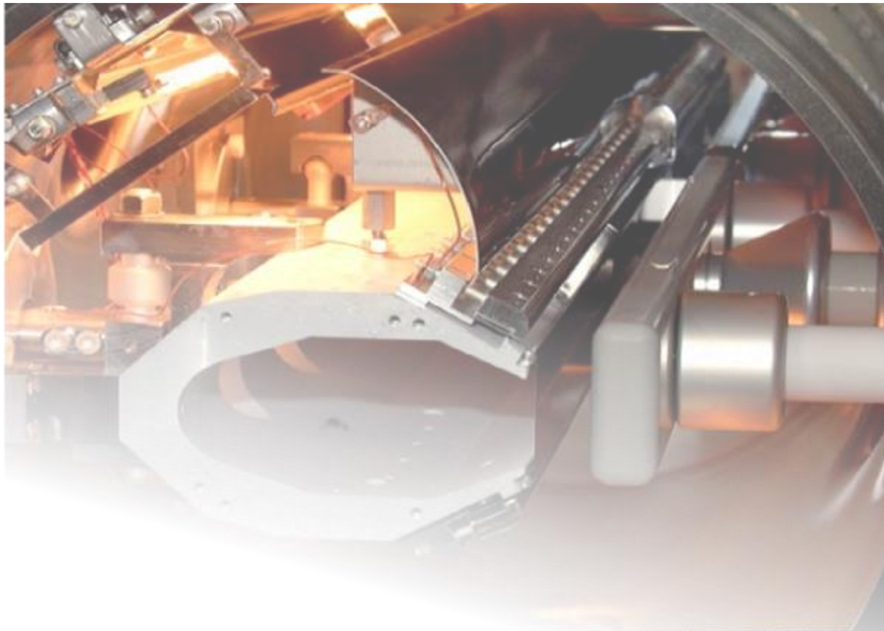
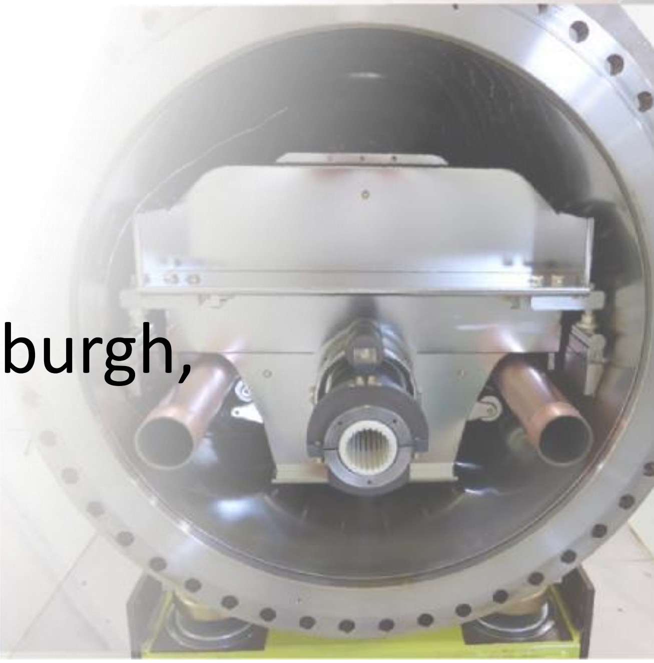
Septa,Kickers and Transfer Lines



Wolfgang Bartmann

CERN

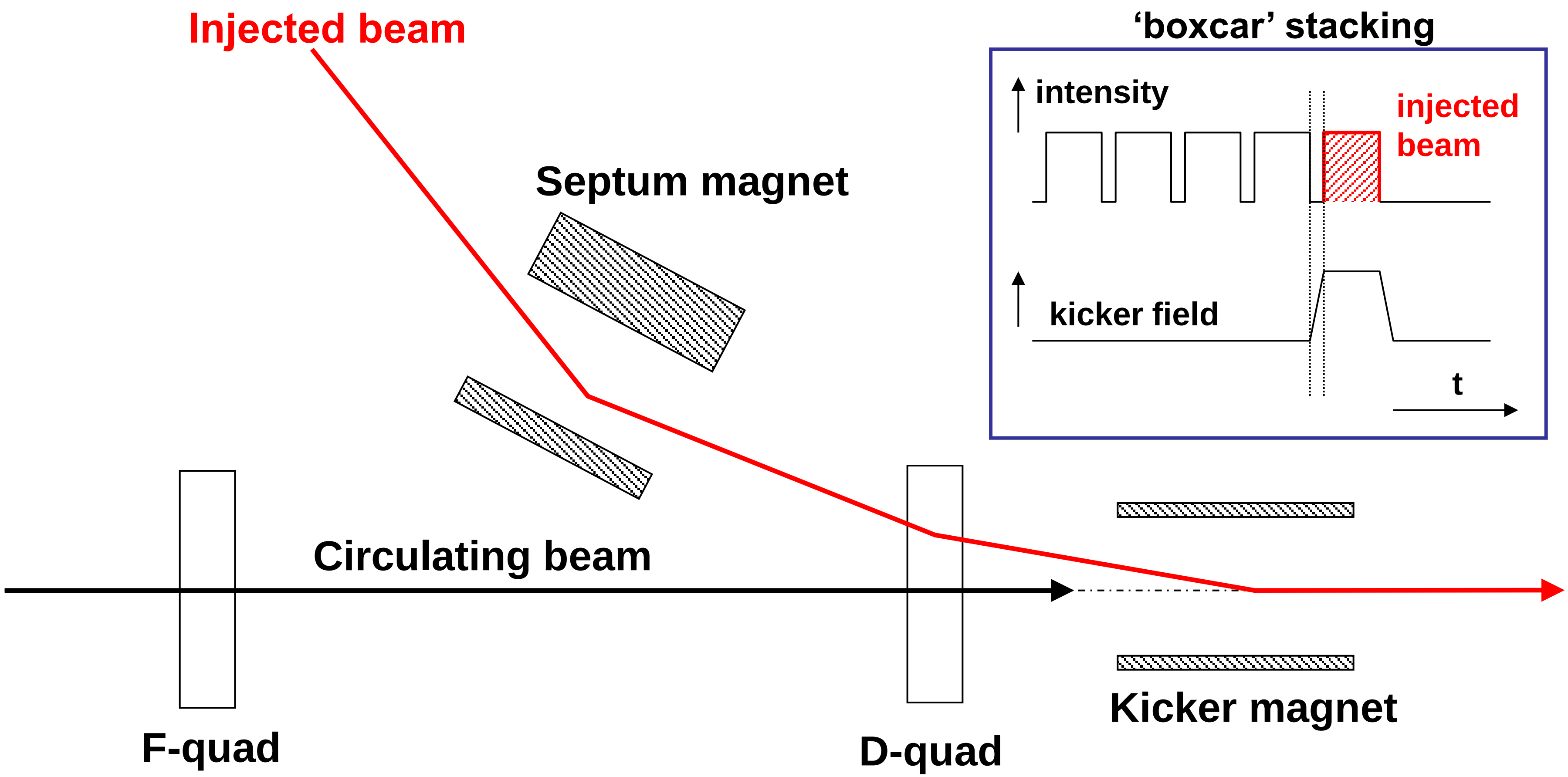
(based on lectures by M.J. Barnes, J. Borburgh,
B. Goddard, V. Kain and M. Meddahi)



Septa, Kickers and Transfer Lines

- Beam transfer devices
  - Septa
  - Kickers
- Transfer lines
  - Geometric link between machines/experiment
  - Match optics between machines/experiment
  - Preserve emittance
  - Change particles' charge state (stripping foils)
  - Measure beam parameters (measurement lines)
  - Protect downstream machine/experiment

Single-turn injection – septum and kicker



- Septum deflects the beam onto the closed orbit at the centre of the kicker
- Kicker compensates for the remaining angle
- Septum and kicker either side of D quad to minimise kicker strength

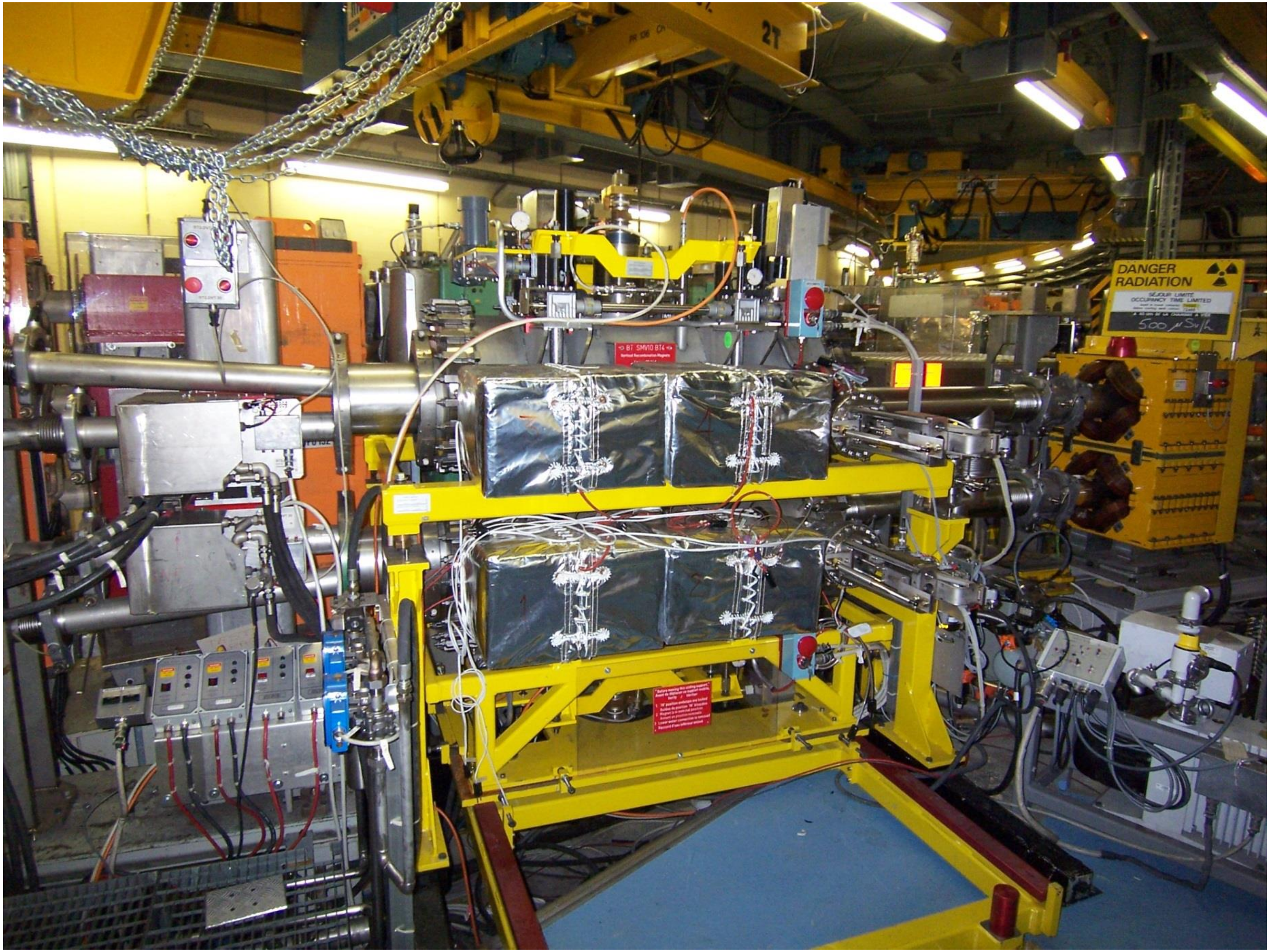
Example Parameters for Septa at CERN

Septum Location	Beam momentum (GeV/c)	Gap Height (mm)	Max. Current (kA)	B (T)	Deflection (mrad)	Septum thickness (mm)
LEIR/AD/CTF (13 systems)	Various	25 to 55	1 DC to 40 pulsed	0.5 to 1.6	up to 130	3 - 19.2
PS Booster (6 systems)	1.4	25 to 50	28 pulsed	0.1 to 0.6	up to 80	1 – 15
PS complex (8 systems)	26	20 to 40	2.5 DC to 33 pulsed	0.2 to 1.2	up to 55	3 - 11.2
SPS Ext.	450	20	24	1.5	2.25	4.2 - 17.2

Example Parameters for Kickers at CERN

Kicker Location	Beam momentum (GeV/c)	# Magnets	Gap Height [V_{ap}] (mm)	Current (kA)	Impedance (Ω)	Rise Time (ns)	Total Deflection (mrad)
CTF3	0.2	4	40	0.056	50	~4	1.2
PS Inj.	2.14	4	53	1.52	26.3	42	4.2
SPS Inj.	13/26	16	54 to 61	1.47/1.96	16.67/12.5	115/200	3.92
SPS Ext. (MKE4)	450	5	32 to 35	2.56	10	1100	0.48
LHC Inj.	450	4	54	5.12	5	900	0.82
LHC Abort	450 to 7000	15	73	1.3 to 18.5	1.5 (not T-line)	2700	0.275

Septa

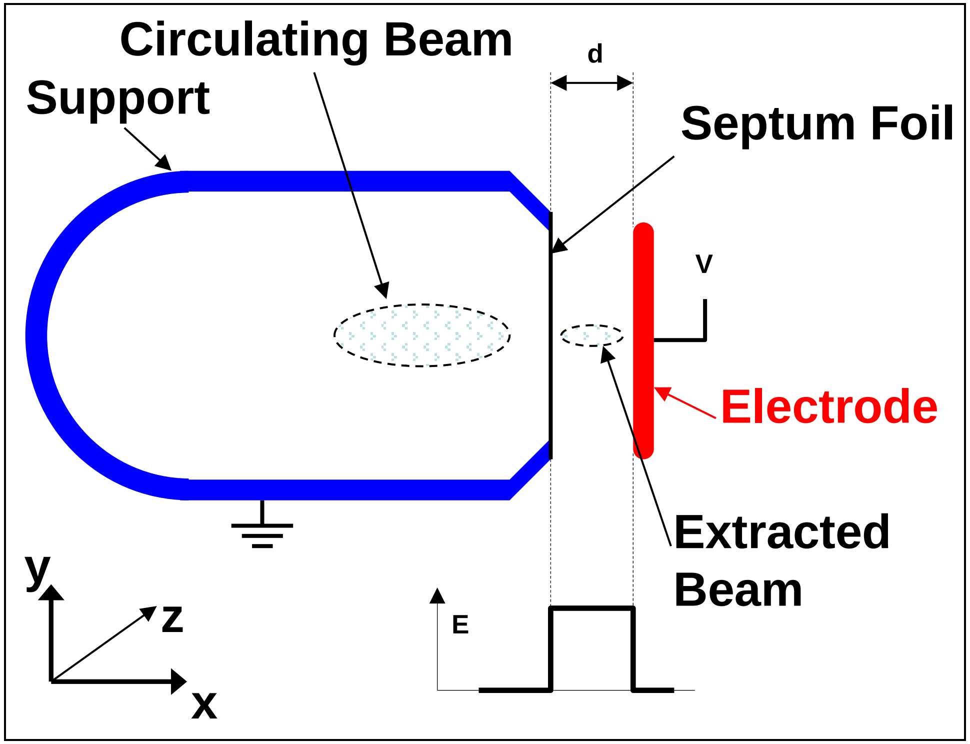


Septa

- Main Types:
  - Electrostatic Septum (DC)
  - DC Magnetic Septum
  - Direct Drive Pulsed Magnetic Septum
  - Eddy Current Septum
  - Lambertson Septum (deflection orthogonal to kicker deflection)

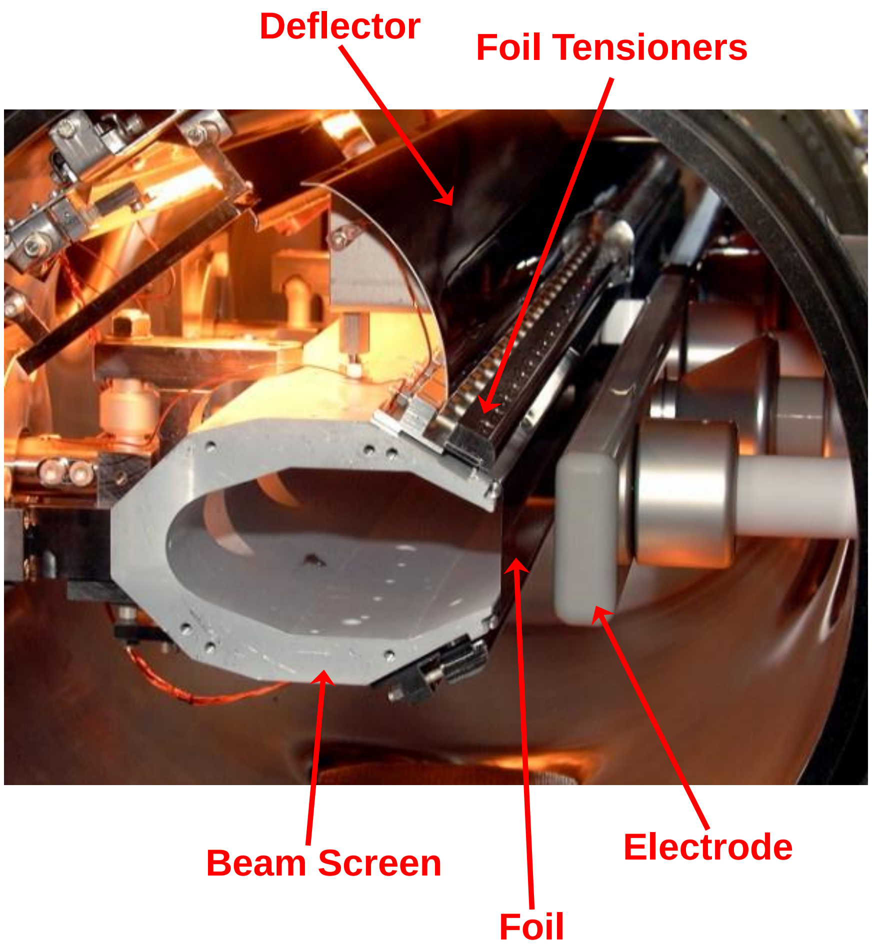
- Main Difficulties:
  - associated with Electrostatic septa is surface conditioning for High Voltage
  - associated with Magnetic septa are not electrical but rather mechanical (cooling, support of this septum blades, radiation resistance)

Electrostatic Septum



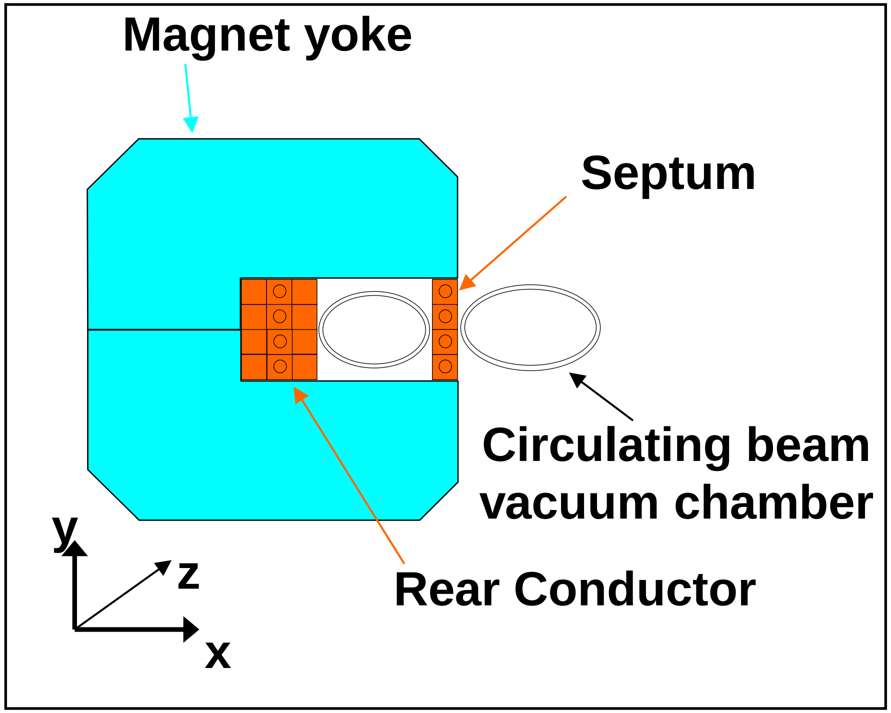
- Thin septum < 0.1 mm
- Vacuum as insulator between septum and electrode → vacuum tank
- Remote positioning system

Electrostatic Septum



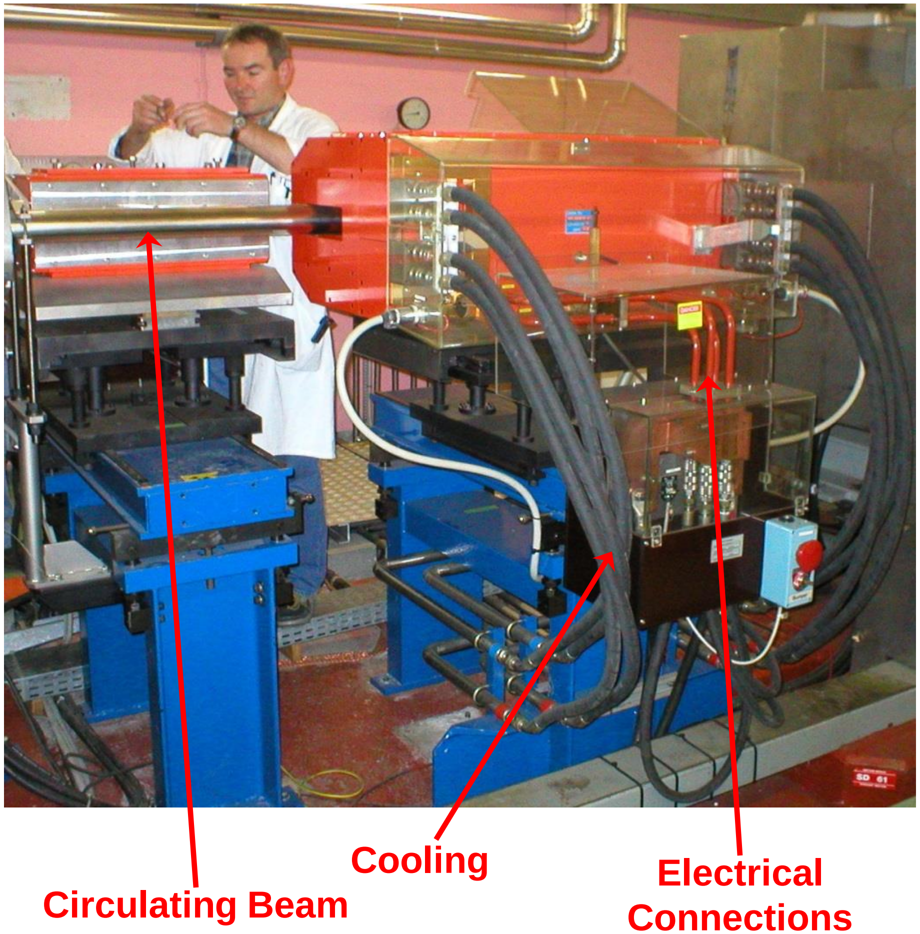
- Variable gap width: 10 - 35 mm
- Vacuum: 10^{-9} to 10^{-12} mbar range
- Voltage: up to 300 kV
- Electric field strength: up to 10 MV/m;
- Septum Molybdenum foil or Tungsten wires
- Electrodes made of anodised aluminium, stainless steel or Titanium

DC Magnetic Septum



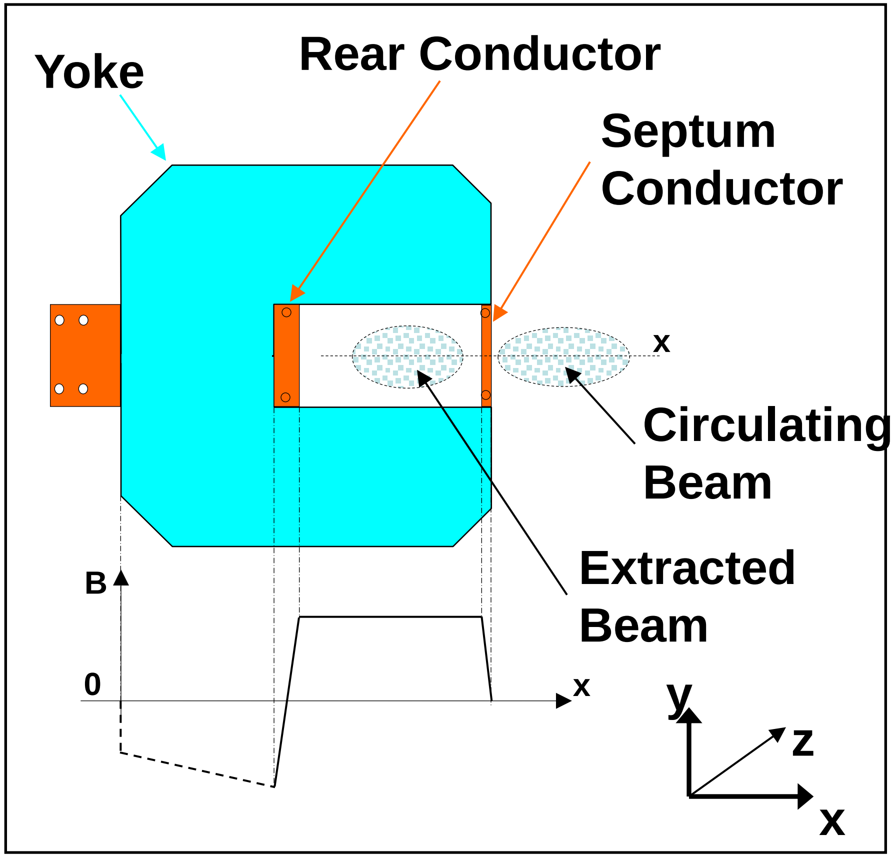
- Continuously powered
- Usually multi-turn coil to reduce the current needed
- Coil and the magnet yoke can be split for installation and maintenance
- Rarely under vacuum

DC Magnetic Septum



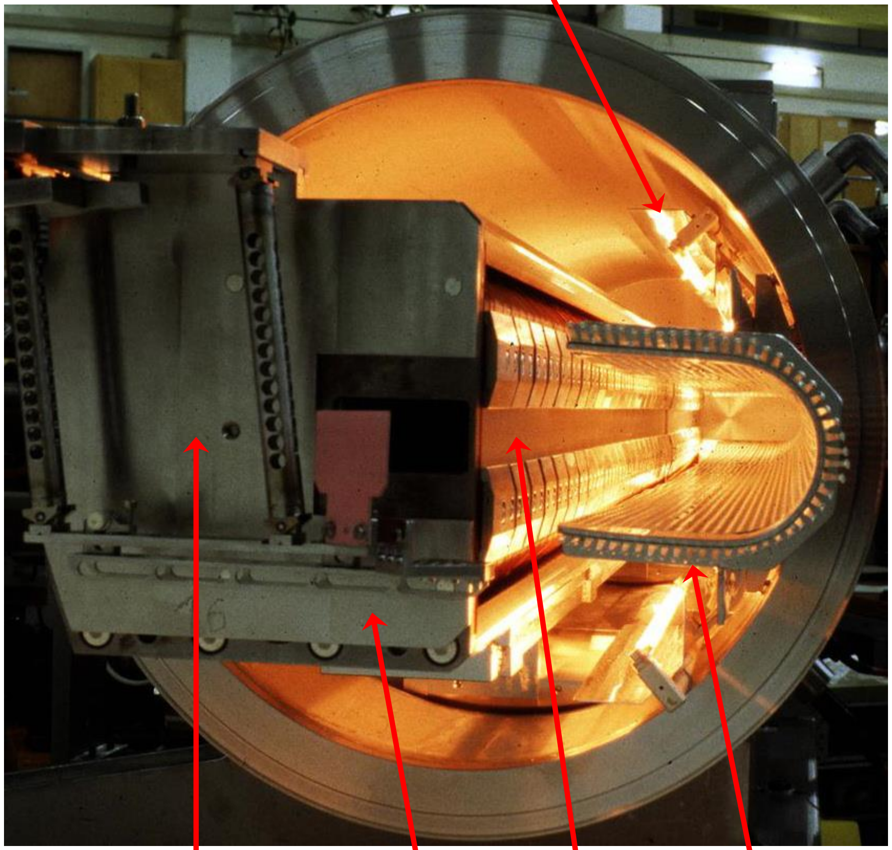
- Gap height: 25 - 60 mm
- Septum thickness: 6 - 20 mm
- Outside vacuum;
- Laminated steel yoke;
- Coil water cooling circuits (12 - 60 l/min.)
- Current range: 1 - 10 kA;
- Power consumption: 10 - 100 kW !

Direct Drive Pulsed Magnetic Septum



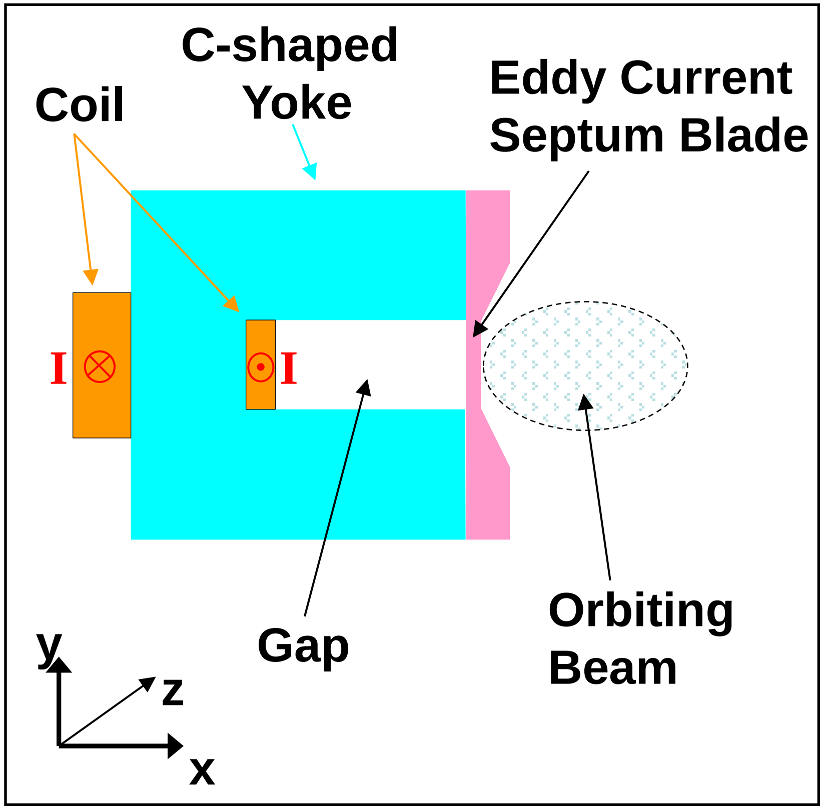
- Powered with a half sine wave current of a few ms
- Single turn coil to minimize magnet self-inductance
- Under vacuum

Direct Drive Pulsed Magnetic Septum



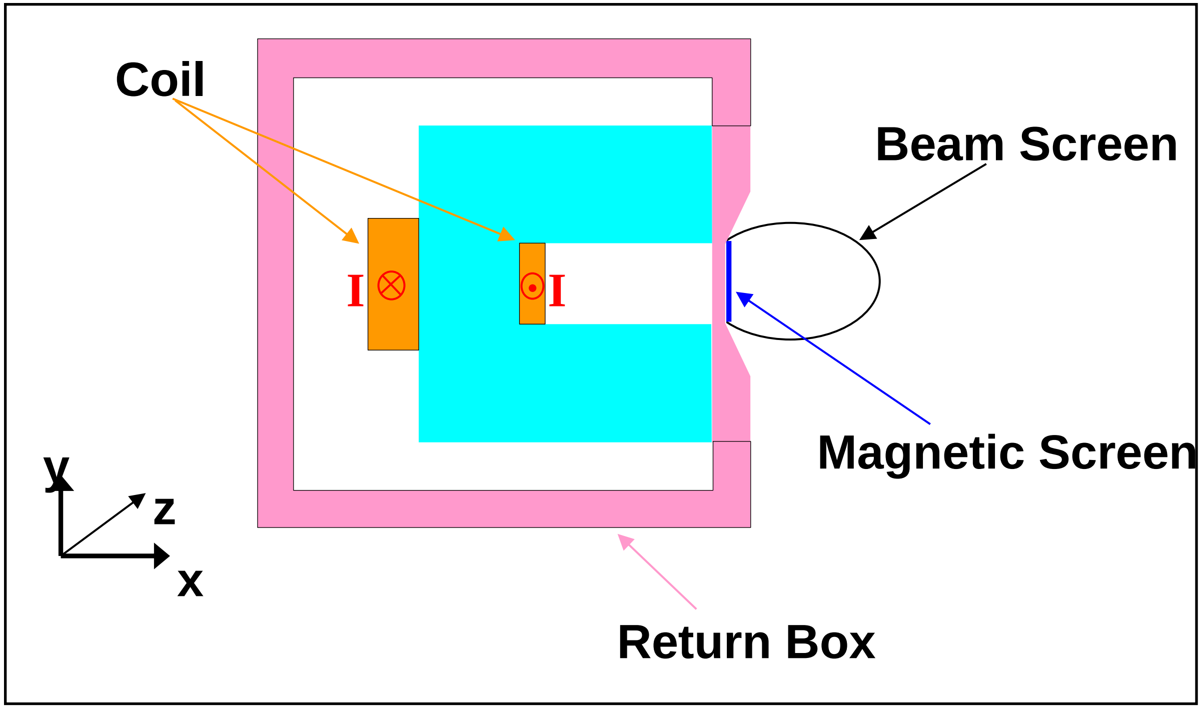
- Septum thickness: 3 - 20 mm
- Vacuum ~10^{-9} mbar
- Laminated steel yoke of 0.35 mm - 1.5 mm thick laminations
- Water cooling circuits 1 - 80 l/min
- Current: half-sine 7 - 40 kA, half-period ~3 ms;
- Power supplied by capacitor discharge
- Transformer between power supply and magnet

Eddy Current Septum



- Powered with a half or full sine wave current with a period of typically 50 μs.
- Single turn coil to minimize magnet self-inductance
- Coil dimensions not critical
- Magnetic field induces eddy currents in the septum blade → counteracting the fringe field
- Long decay time of eddy currents
- Thin septum

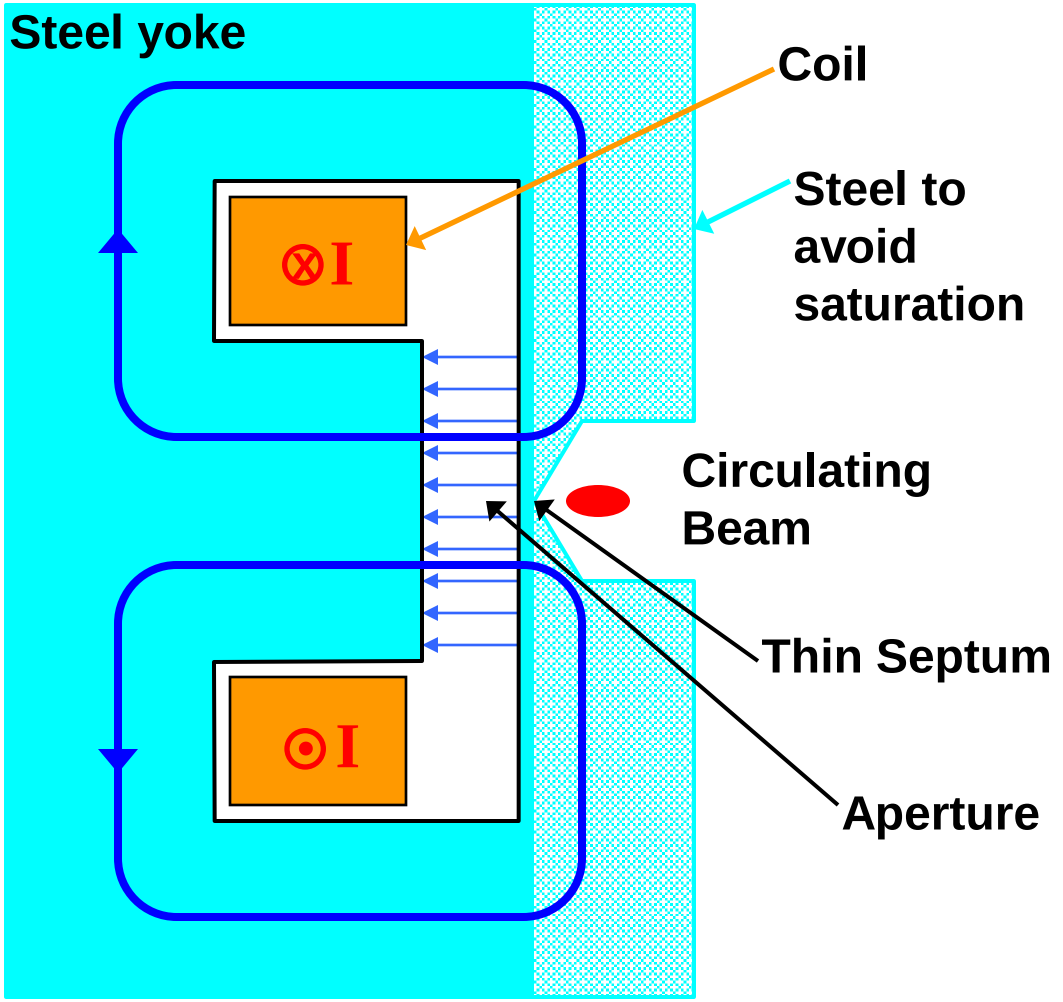
Eddy Current Septum



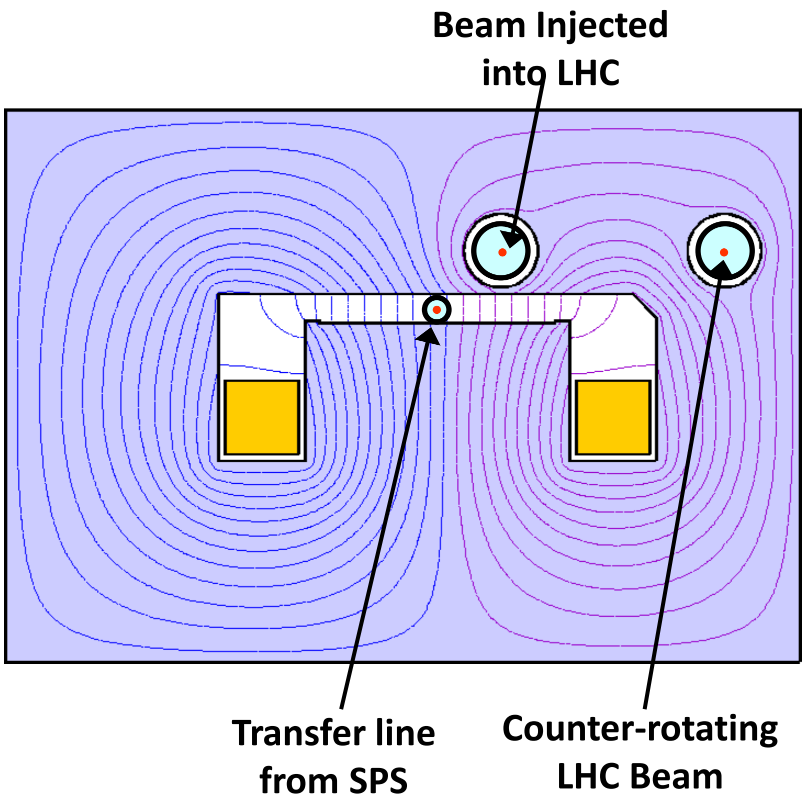
- Return box allows to reach better fringe field compensation (~10^{-3} of main field) and improves heat transfer
- Magnetic screen for circulating beam shielding

Lambertson Septum

- DC or pulsed
- Conductors are enclosed in steel yoke, “well away” from beam
- Thin steel yoke between aperture and circulating beam – however extra steel required to avoid saturation
- Lambertson deflects beam orthogonal to kicker deflection

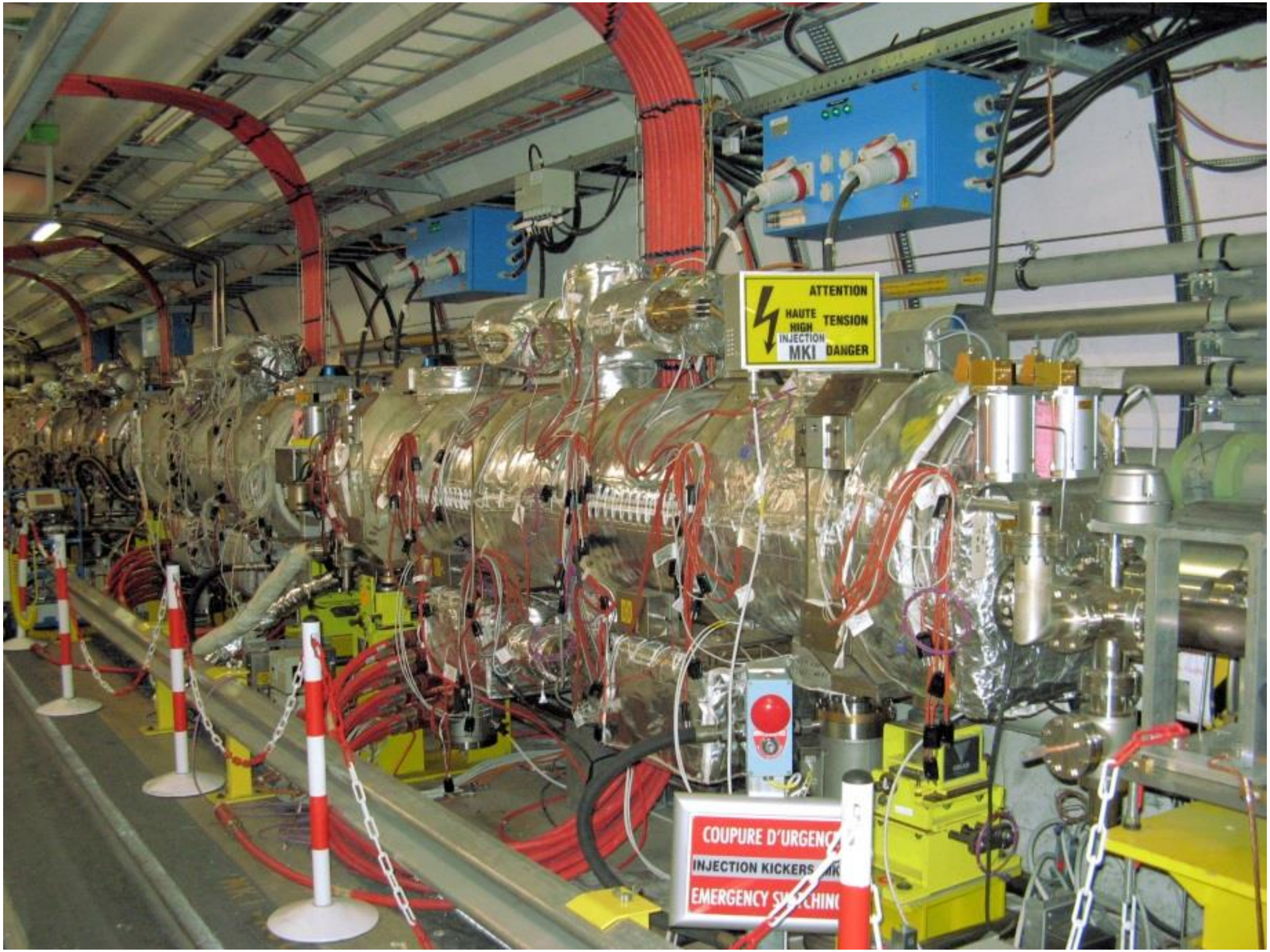


Lambertson Septum



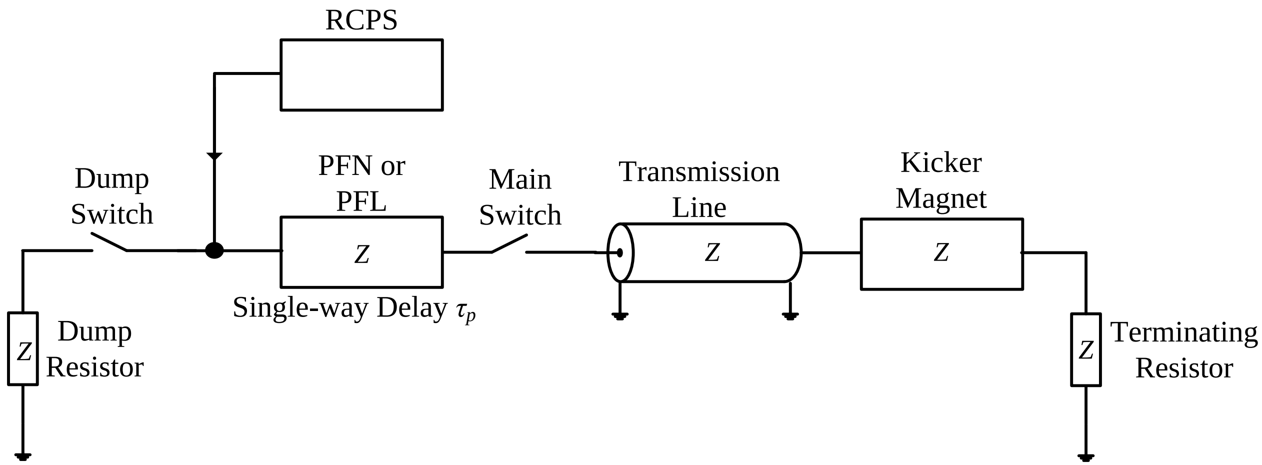
- Septum deflects beam horizontally to the right
- Kicker deflects beam vertically onto central orbit

Kickers



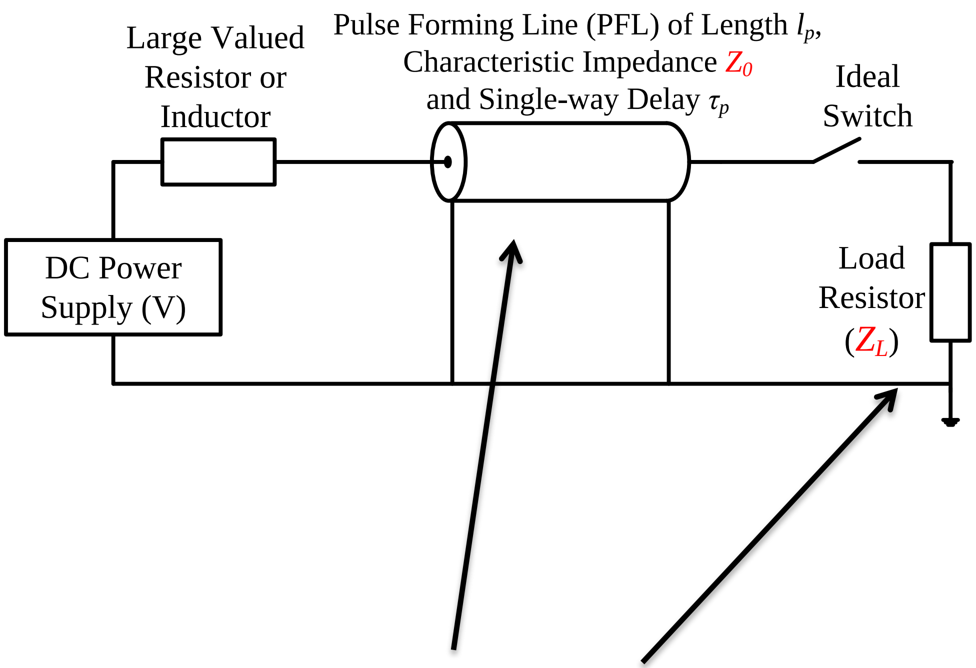
ATTENTION
HAUTE TENSION
HIGH TENSION
INJECTION
MKI
DANGER
COUPURE D'URGENC
INJECTION KICKERS
EMERGENCY S

Simplified kicker schematic

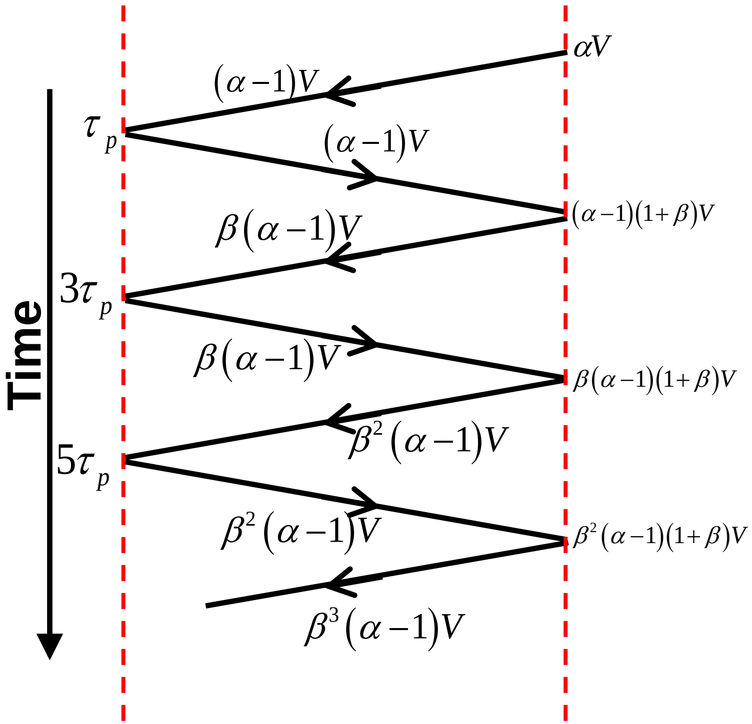


- Pulse forming network or line (PFL/PFN) charged to voltage Vp by the resonant charging power supply (RCPS)
- Close main switch → voltage pulse of Vp/2 through transmission line towards magnet
- Once the current pulse reaches the (matched) terminating resistor full-field has been established in the kicker magnet
- Pulse length control with dump switch

Reflections



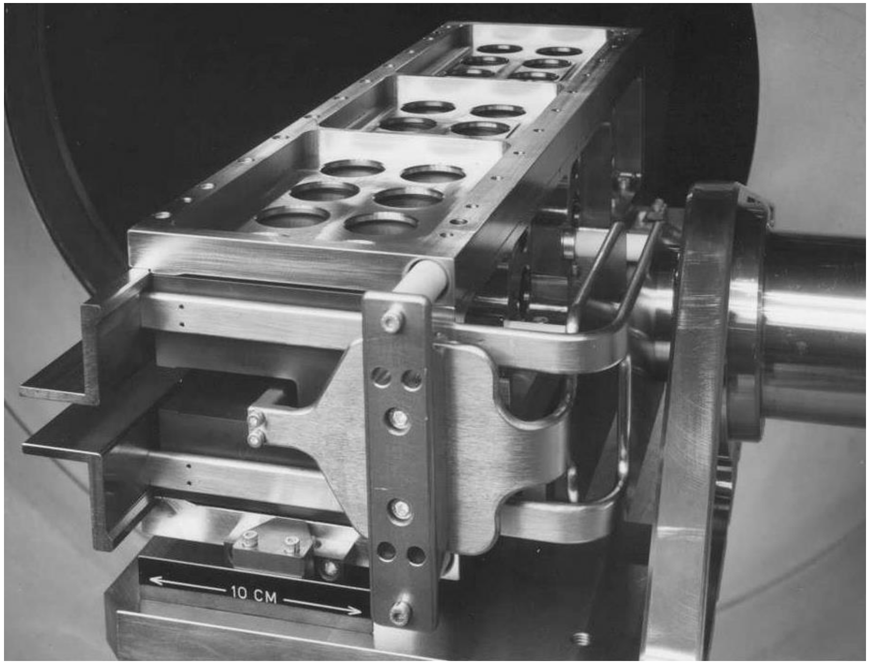
Match impedances to avoid reflections



$$V_L = V \cdot \left(\frac{Z_L}{Z_0 + Z_L} \right) = \alpha V$$

$$\Gamma = \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) = \beta$$

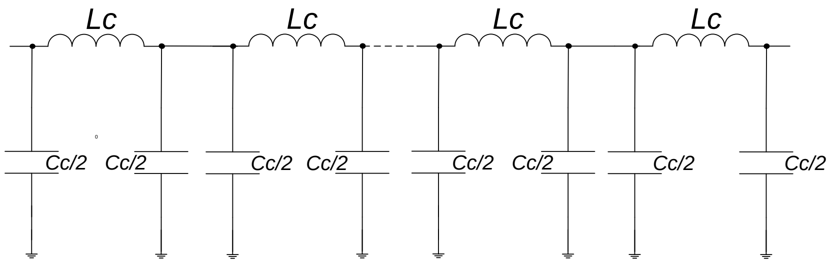
Magnets - historic

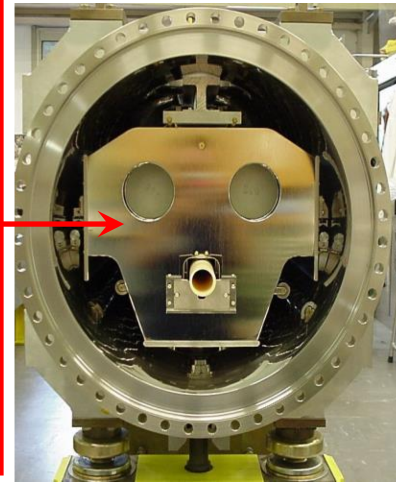


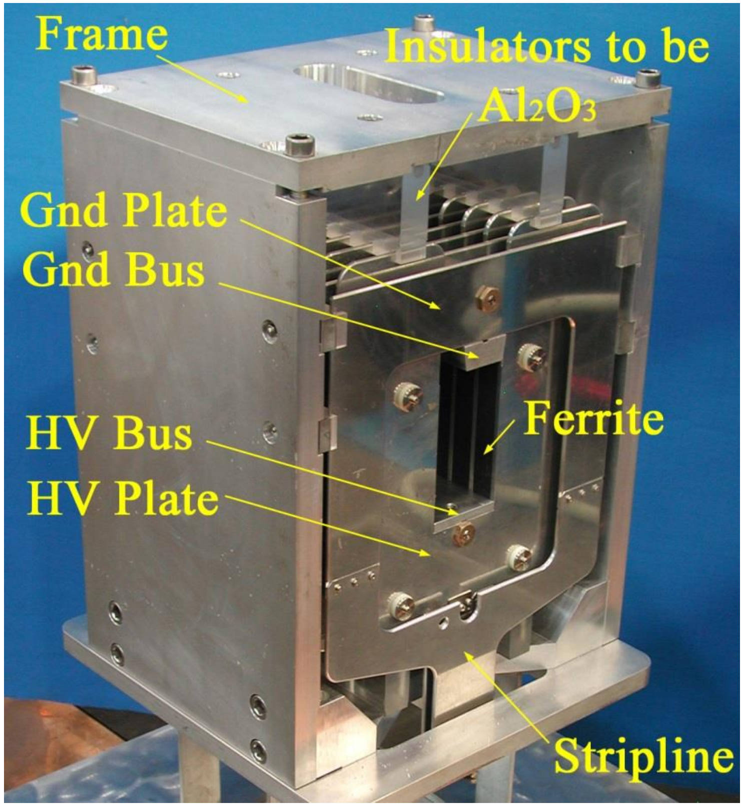
- Kicker magnets in the 1960’s (AA accumulator ejection)
- Current pulses were limited → small aperture to reach required field and kick angle
- Needed to be operated hydraulically to put the kicker around the beam when the beam size at extraction was small enough…

Magnets – transmission line

- Todays fast kickers are generally **ferrite loaded transmission line magnets**
- Consists of many cells to approximate a broadband coaxial cable

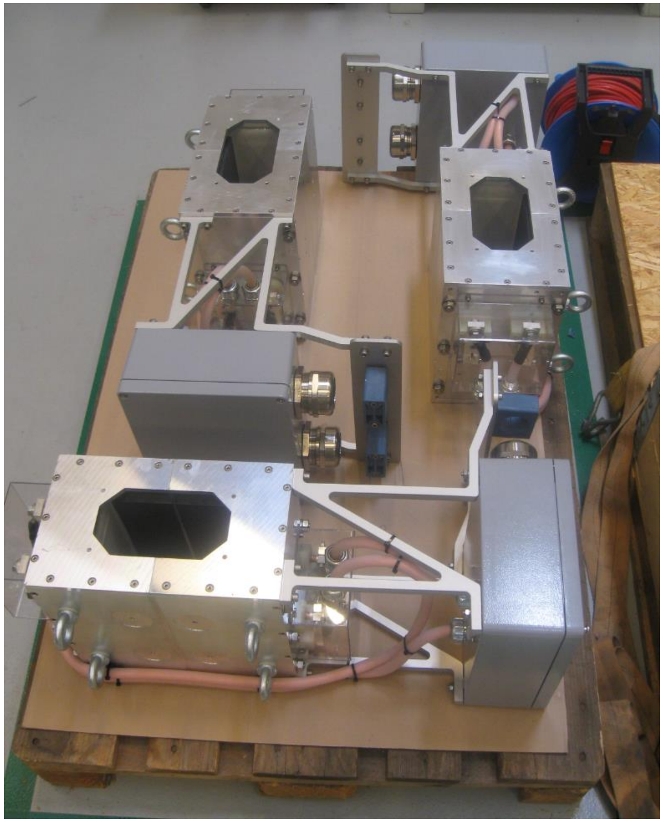






Magnets – lumped inductance

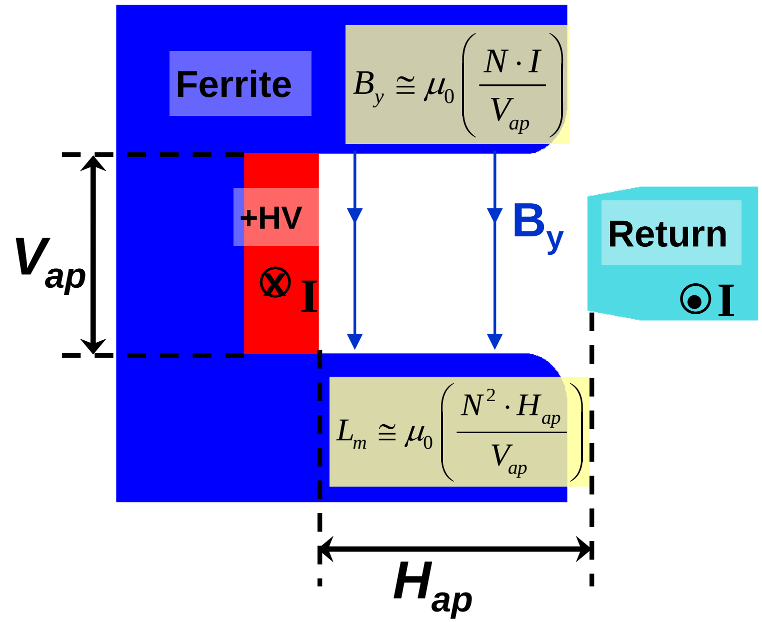
Robust and cheap construction
BUT impedance mismatch and slow response



Magnets – in/outside vacuum

Why put the magnet under vacuum:

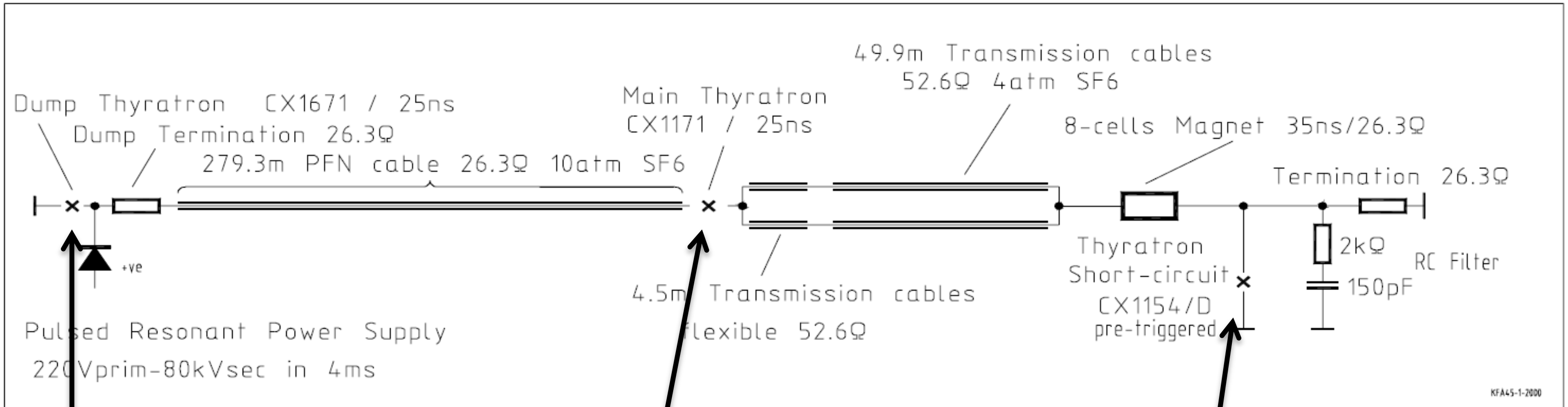
- Reduce aperture and therefore voltage and current
- Machine vacuum is a reliable dielectric (70 kV/cm OK)
- Recovers after a flashover



Drawbacks:

- Costly to construct: bake-out, vacuum tank, pumping, cooling
- A suitably treated chamber (ceramics) anyway needed for coupling impedance to beam

Terminated vs. Short circuit



Dump switch: Control pulse length

Main switch: trigger voltage pulse

Short circuit switch: when fired magnet current is doubled

Short-circuit mode allows to reach almost double the deflection angle at the expense of also a factor two longer rise/fall time

Switches

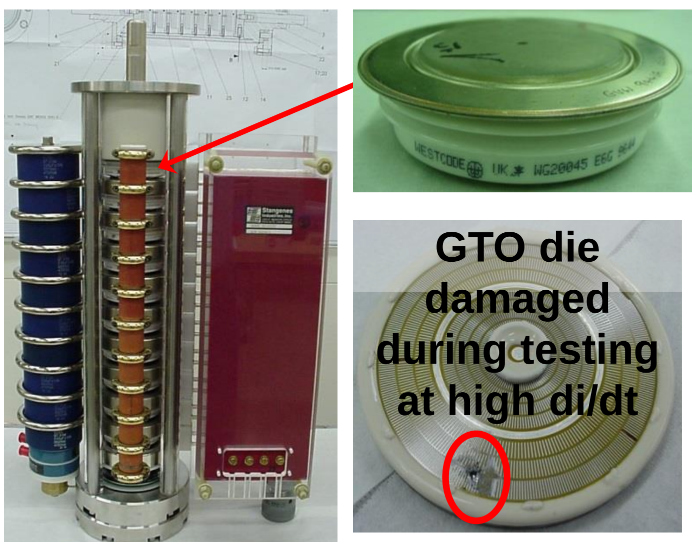
Thyratrons:

- can hold off 80 kV and switch 6 kA within 30 ns
- BUT: housing, insulation, erratics

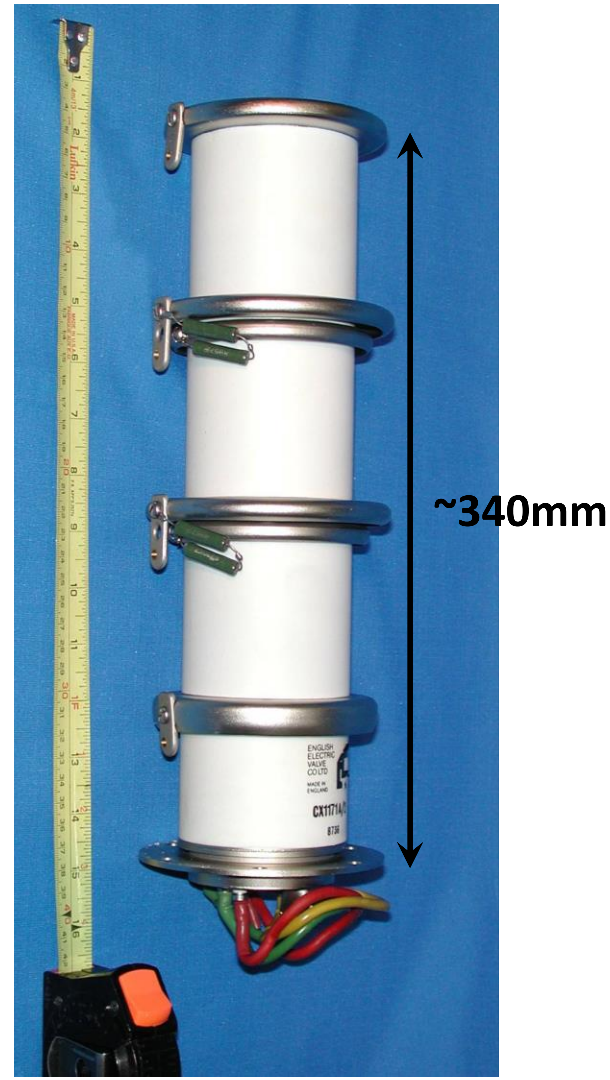
Semiconductors:

- Allows beam energy tracking, eg. LHC dump kickers
- Rise time > 1μs
- Low maintenance

Semiconductor



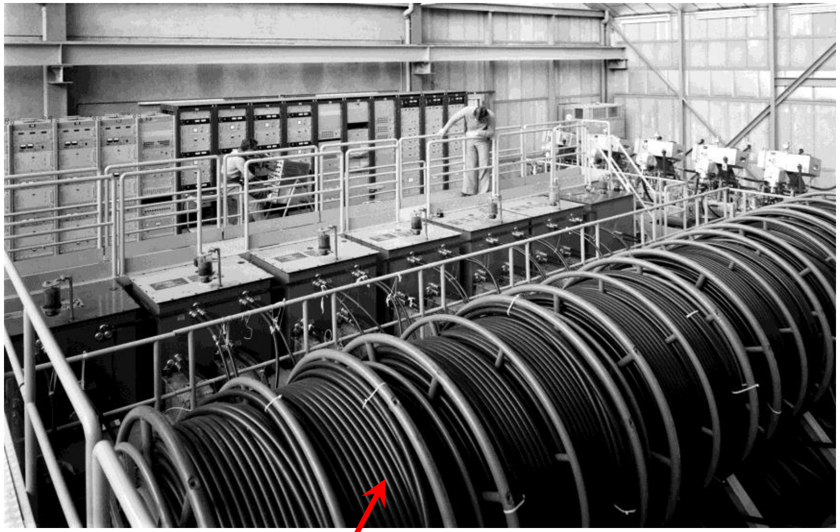
Thyratron



PFN/PFL

Pulse forming line

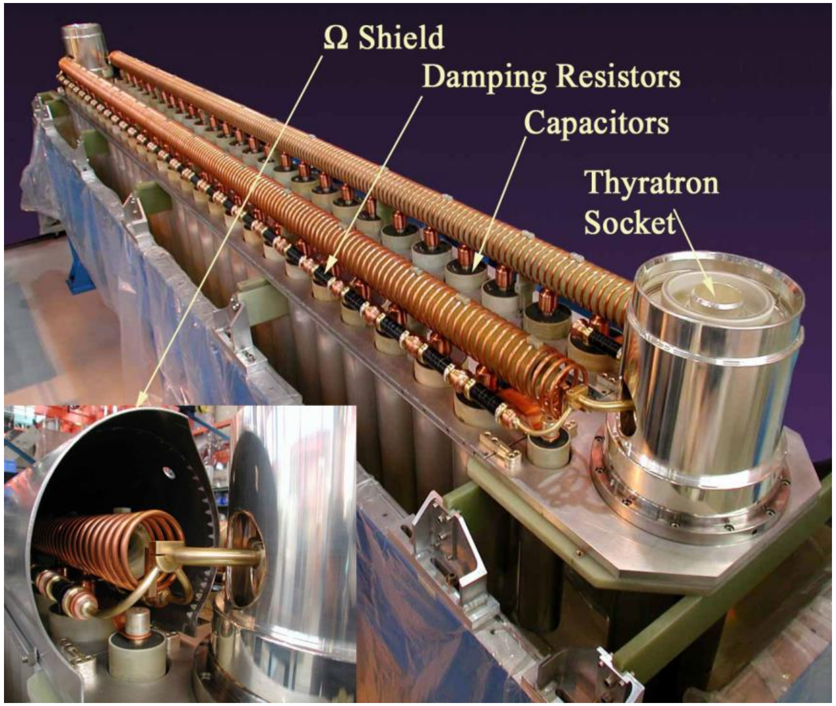
- Coaxial cable charged to double the required pulse voltage
- Short pulses (< 3 μs)
- Low attenuation required to minimize droop → above 50 kV SF6 pressurized cables
- Bulky!



Reels of PFL

Pulse forming network

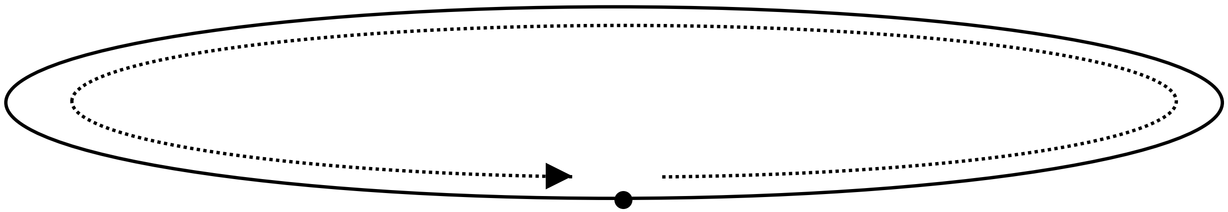
- For low droop and long pulses (> 3 μs)
- Artificial coaxial cable made of lumped elements



LHC Injection PFN

Transfer lines

Circular Machine



One turn

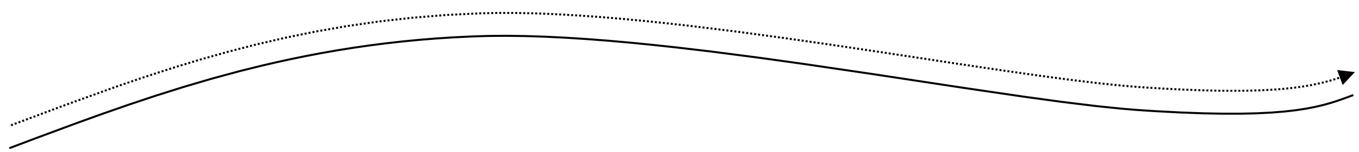
$$\mathbf{M}_{1\rightarrow 2} = \mathbf{M}_{0\rightarrow L} = \begin{bmatrix} \cos 2\pi Q + \alpha \sin 2\pi Q & \beta \sin 2\pi Q \\ -1/\beta \left(1+\alpha^2\right) \sin 2\pi Q & \cos 2\pi Q - \alpha \sin 2\pi Q \end{bmatrix}$$

- The solution is *periodic*
- Periodicity condition for one turn (closed ring) imposes $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $D_1 = D_2$
- This condition *uniquely* determines $\alpha(s)$, $\beta(s)$, $\mu(s)$, $D(s)$ around the whole ring

Transfer line

One pass: $$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$

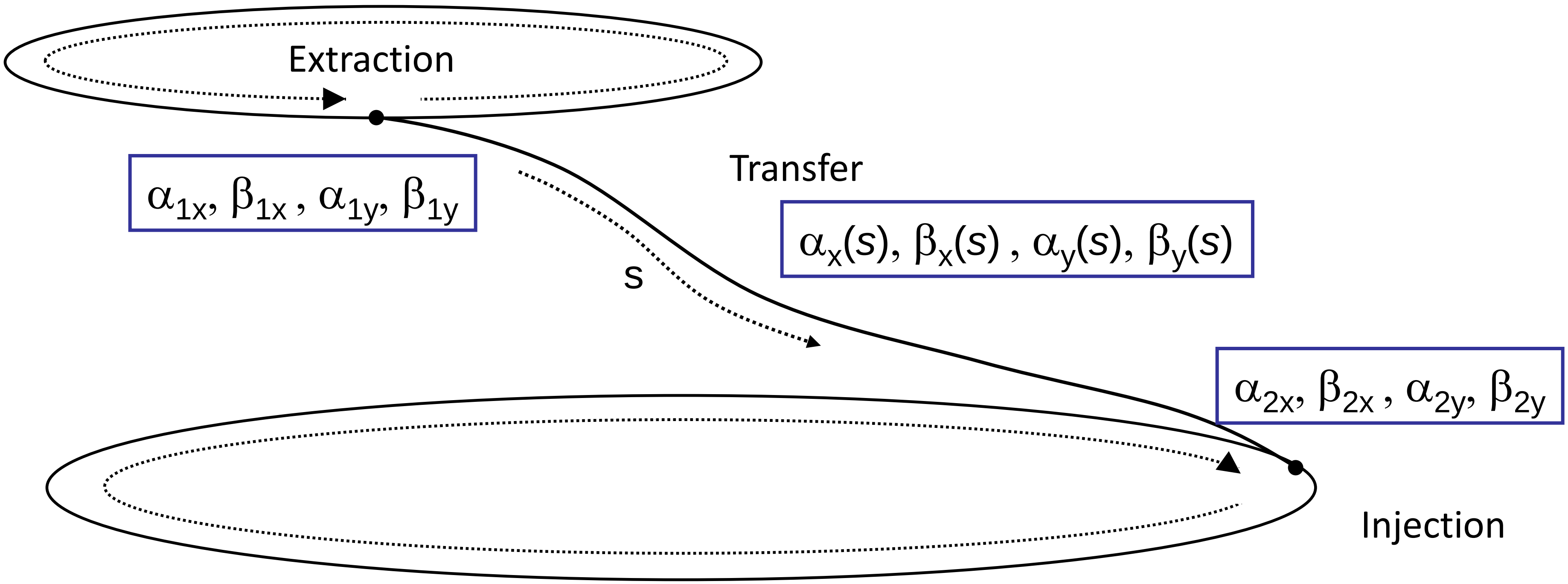


$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix}$$

$$\mathbf{M}_{1\to 2} = \begin{bmatrix} \sqrt{\beta_2/\beta_1}\left(\cos\Delta\mu + \alpha_1 \sin\Delta\mu\right) & \sqrt{\beta_1\beta_2}\sin\Delta\mu \\ \sqrt{1/\beta_1\beta_2}\left[(\alpha_1 - \alpha_2)\cos\Delta\mu - (1+\alpha_1\alpha_2)\sin\Delta\mu\right] & \sqrt{\beta_1/\beta_2}\left(\cos\Delta\mu - \alpha_2 \sin\Delta\mu\right) \end{bmatrix}$$

- No periodic condition exists
- The Twiss parameters are simply propagated from beginning to end of line
- At any point in line, $\alpha(s)$ $\beta(s)$ are functions of α_1 β_1

Linking Machines



The Twiss parameters can be propagated when the transfer matrix **M** is known

$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$

$$\begin{bmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} C^2 & -2CS & S^2 \\ -CC' & CS'+SC' & -SS' \\ C'^2 & -2C'S' & S'^2 \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{bmatrix}$$

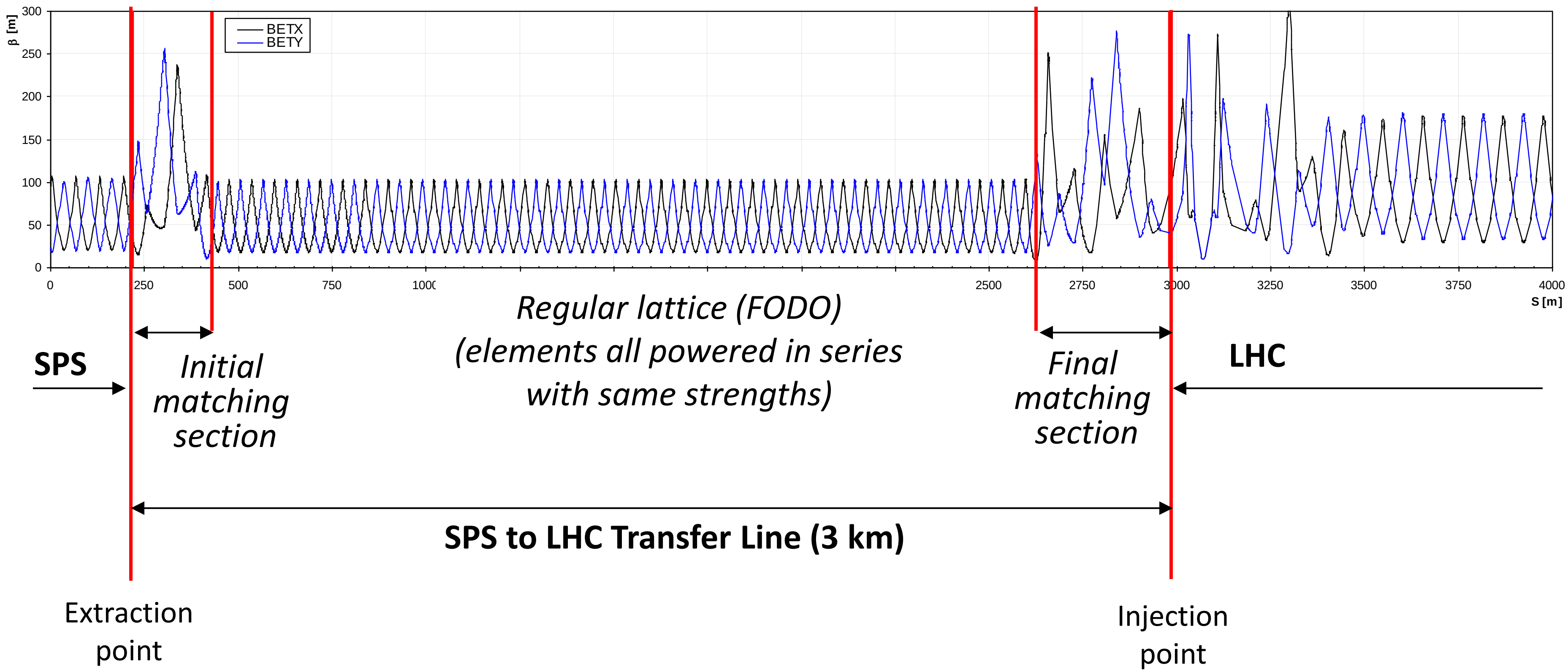
Optics Matching

- Need to “match” 8 variables (α_x β_x D_x D'_x and α_y β_y D_y D'_y)
  - Independently powered quadrupoles

- Maximum β and D values are imposed by magnet apertures

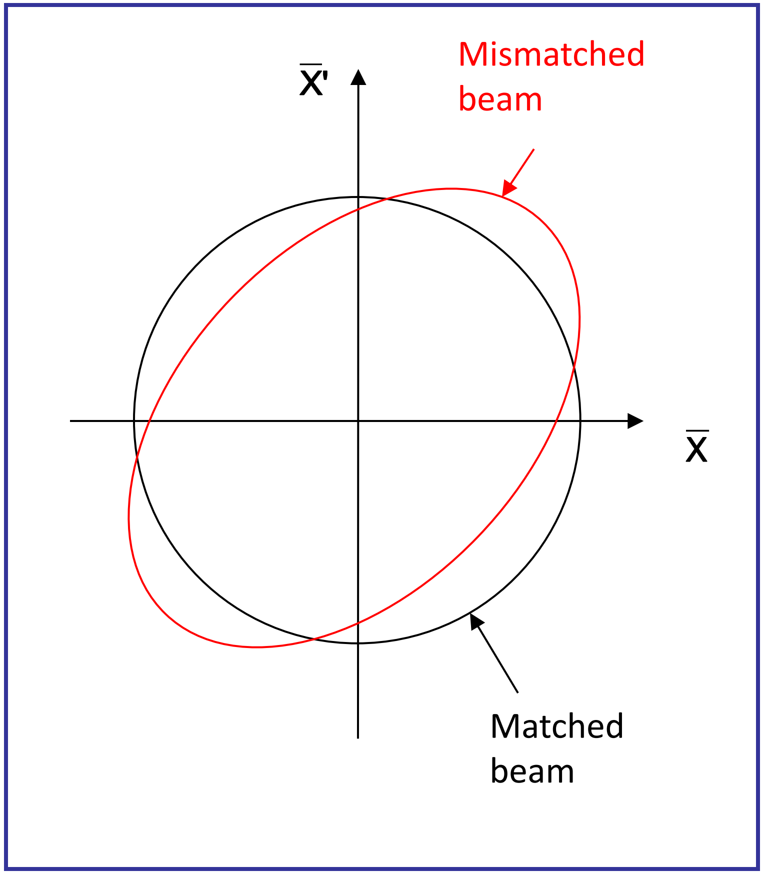
- Other constraints can exist
  - phase conditions for collimators,
  - insertions for special equipment like stripping foils

Optics Matching



Blow-up from betatron mismatch

- Optical errors occur in transfer line and ring, such that the beam can be injected with a mismatch.
- Filamentation will produce an emittance increase.
- In normalised phase space, consider the matched beam as a circle, and the mismatched beam as an ellipse.



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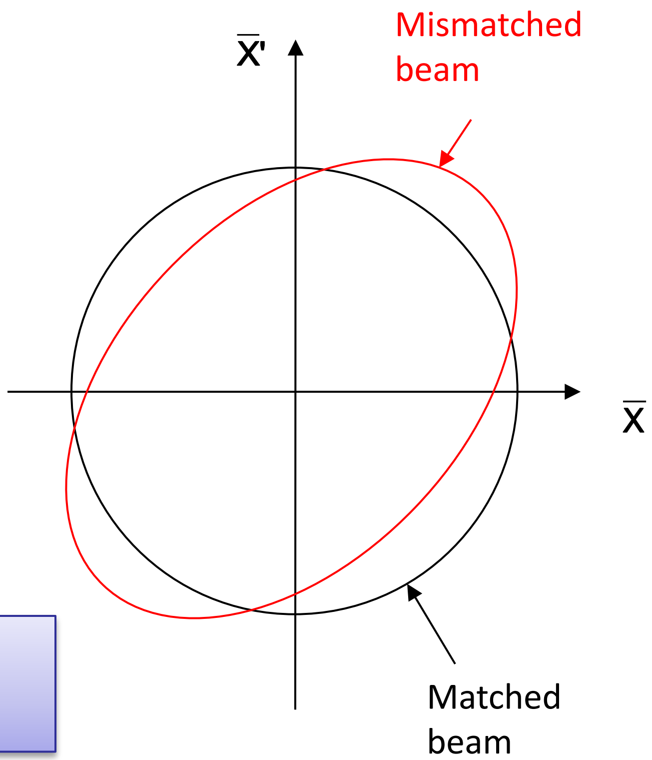


Resulting emittance after filamentation:
(see Appendix for derivation)

$$\varepsilon_{new} = \frac{1}{2}\varepsilon_0\left(\lambda^2 + \frac{1}{\lambda^2}\right) = H\varepsilon_0 = \frac{1}{2}\varepsilon_0\left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2 + \frac{\beta_2}{\beta_1}\right)$$

Blow-up from betatron mismatch

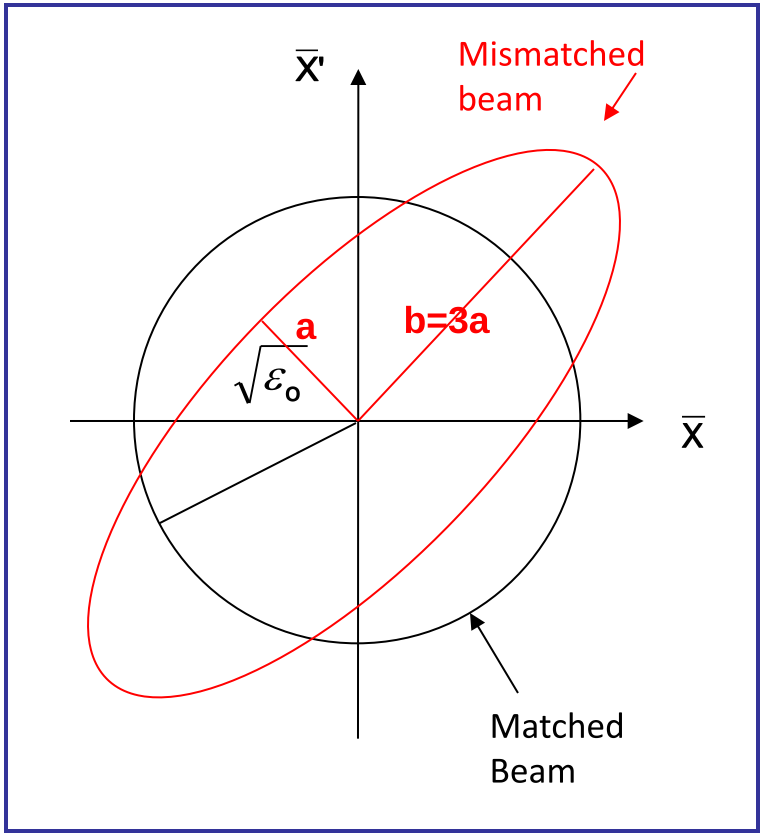
A numerical example….consider *b = 3a* for the mismatched ellipse

$$\lambda = \sqrt{b/a} = \sqrt{3}$$

Then

$$\varepsilon_{new} = \frac{1}{2}\varepsilon_0\left(\lambda^2 + 1/\lambda^2\right)$$

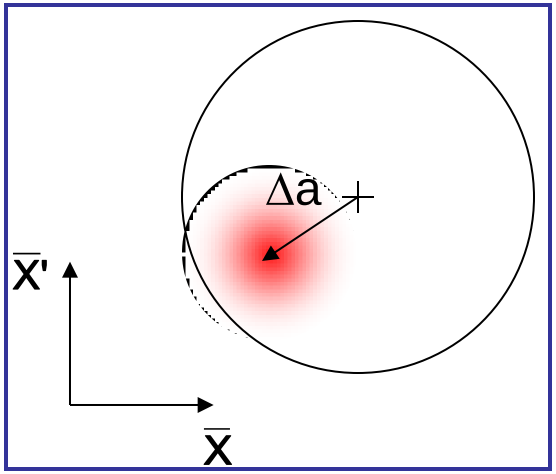
$$= 1.67\,\varepsilon_0$$

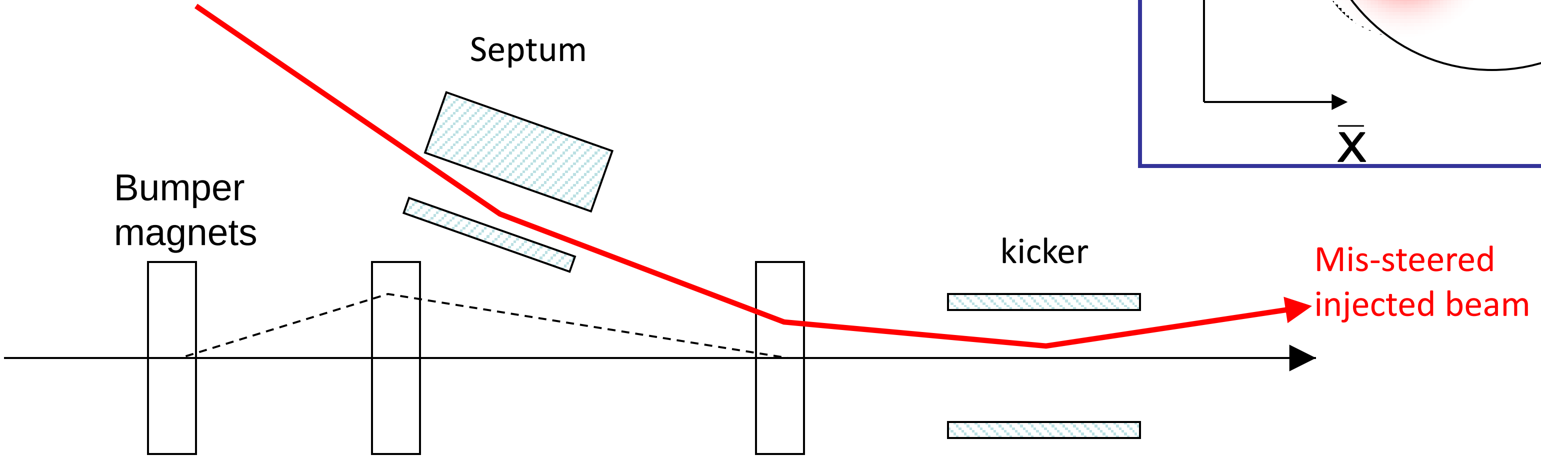


Steering (dipole) errors

- Precise delivery of the beam is important.
  - To avoid **injection oscillations** and emittance growth in rings
  - For stability on secondary particle production targets
- Convenient to express injection error in s (includes x and x' errors)

$$\Delta a\ [\sigma] = \sqrt{(X^2+X'^2)/\varepsilon} = \sqrt{(\gamma x^2 + 2\alpha x x' + \beta x'^2)/\varepsilon}$$





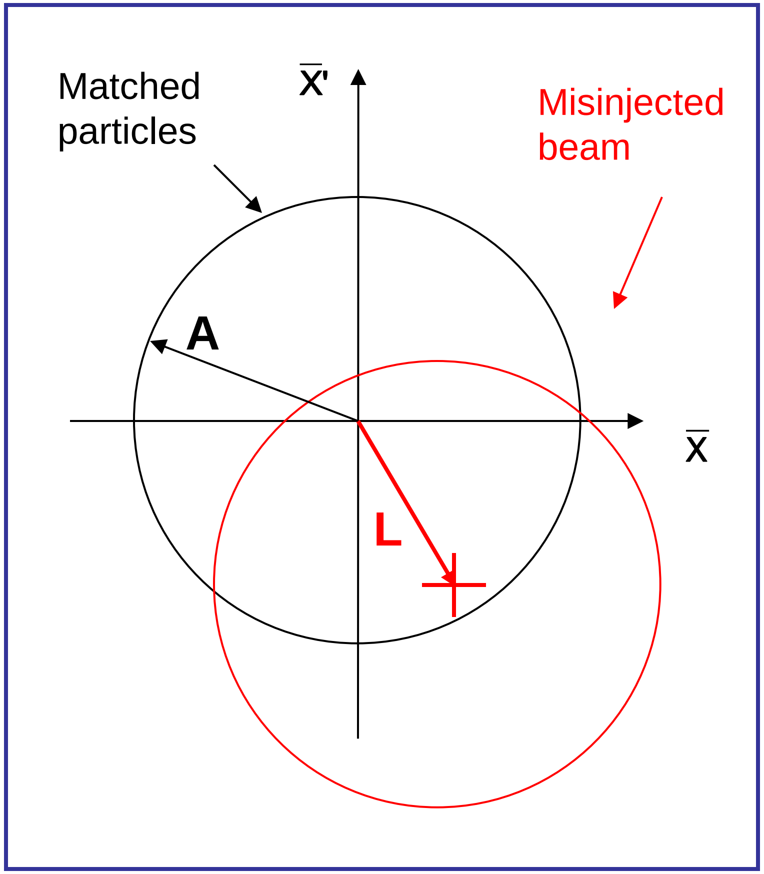
Blow-up from steering error

- Consider a collection of particles with max. amplitudes A
- The beam can be injected with a error in angle and position.
- For an injection error Δa_y (in units of sigma = $\sqrt{\beta}$) the mis-injected beam is offset in normalised phase space by $L = \Delta a_y \sqrt{\varepsilon}$

Resulting emittance after filamentation:
(see Appendix for derivation)

$$\varepsilon_{new} = \left\langle A_{new}^{2} \right\rangle / 2 = \varepsilon_0 + L^2 / 2$$

$$= \varepsilon_0 \left(1 + \Delta a^2 / 2\right)$$

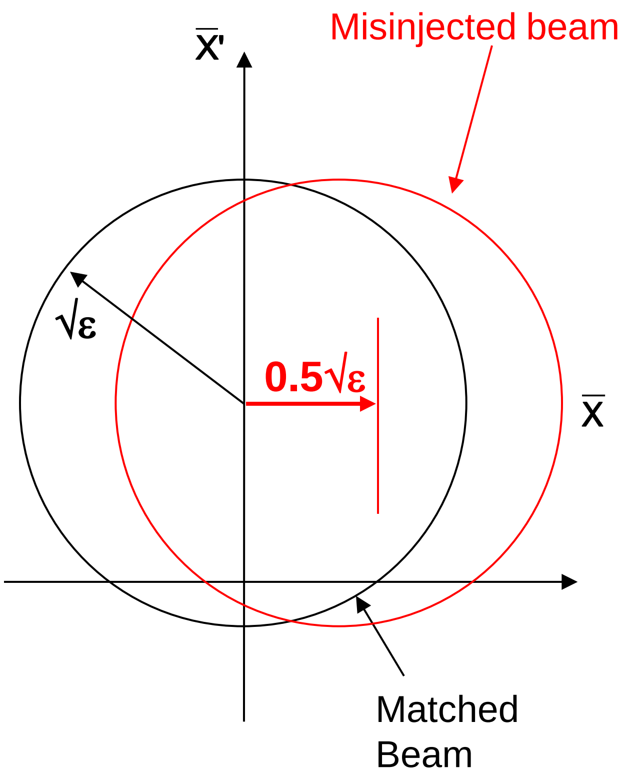


Blow-up from steering error

A numerical example....

Consider an offset Δa of 0.5 sigma for injected beam

$$\varepsilon_{new} = \varepsilon_0 \left(1 + \Delta a^2 / 2\right)$$
$$= 1.125\varepsilon_0$$

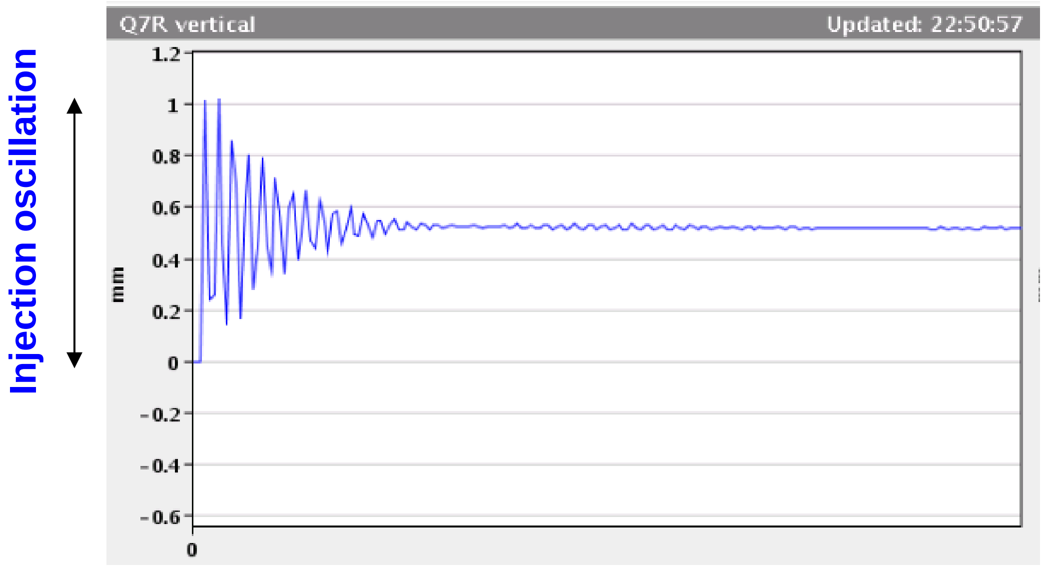


For nominal LHC beam:

allowed growth through LHC cycle ~ 10 %

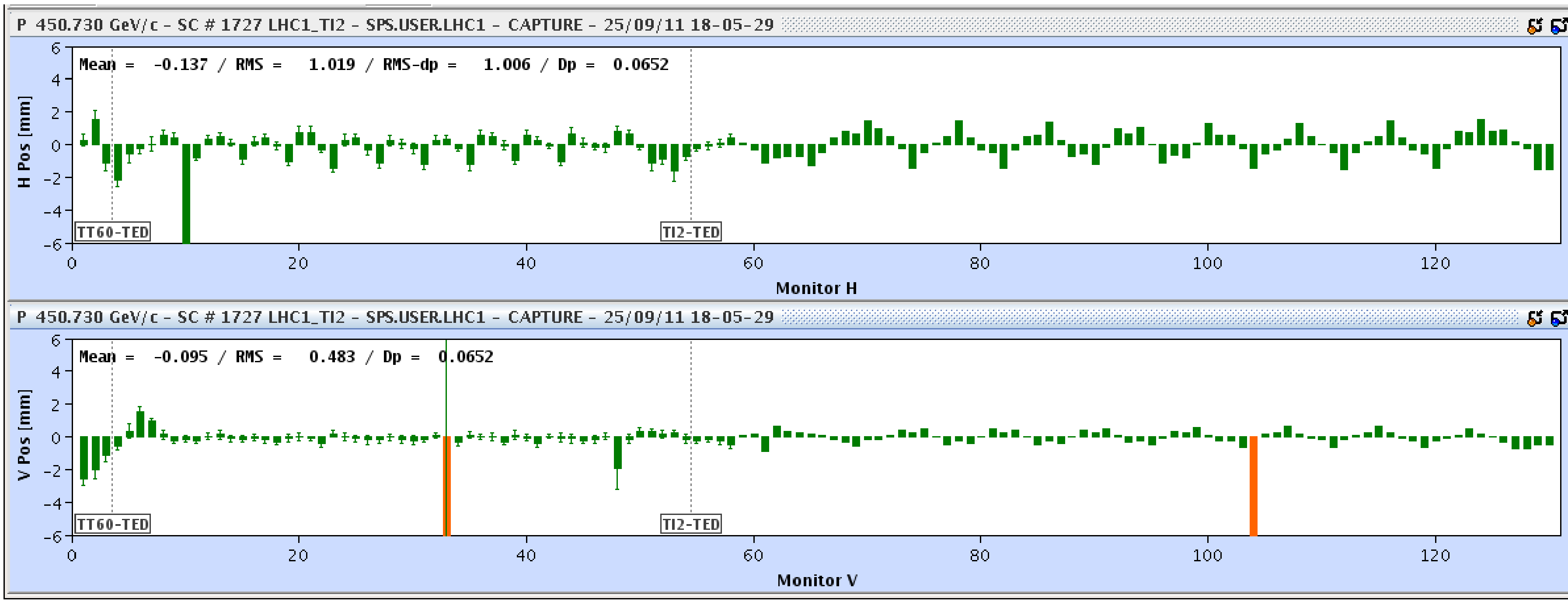
Damping of injection oscillations

- **Residual transverse oscillations lead to an emittance blow-up through filamentation**
- **"Transverse damper" systems used to damp injection oscillations - bunch position measured by a pick-up, which is linked to a kicker**
- **Damper measures offset of bunch on one turn, then kicks the bunch on a subsequent turn to reduce the oscillation amplitude**



Example: LHC injection of beam 1

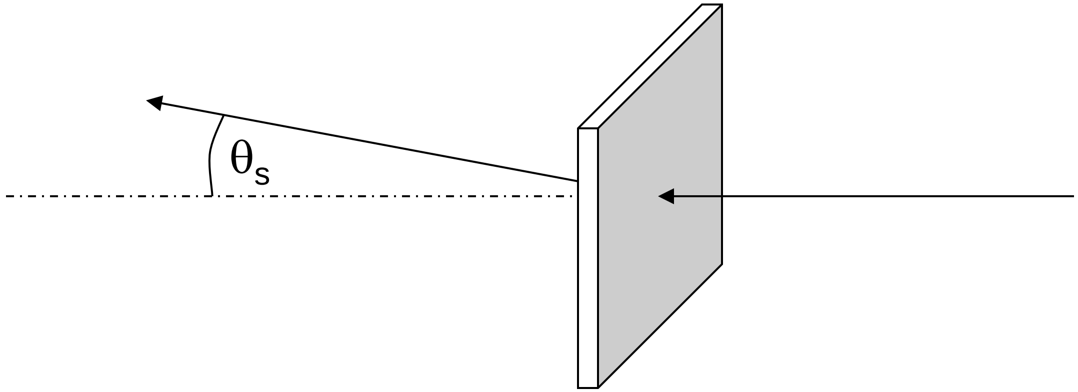
- Oscillation down the line has developed in horizontal plane
- Injection oscillation amplitude > 1.5 mm
- Good working range of LHC transverse damper +/- 2 mm



- Aperture margin for injection oscillation is 2 mm

Blow-up from thin scatterer

- Scattering elements are sometimes required in the beam
  - Thin beam screens (Al_2O_3,Ti) used to generate profiles.
  - Metal windows also used to separate vacuum of transfer lines from vacuum in circular machines.
  - Foils are used to strip electrons to change charge state
- The emittance of the beam increases when it passes through, due to multiple Coulomb scattering.



rms angle increase:

$$\sqrt{\langle \theta_s^2 \rangle}[mrad] = \frac{14.1}{\beta_c p[MeV/c]} Z_{inc} \sqrt{\frac{L}{L_{rad}}} \left(1 + 0.11 \cdot \log_{10} \frac{L}{L_{rad}}\right)$$

β_c = v/c, p = momentum, Z_{inc} = particle charge /e, L = target length, L_{rad} = radiation length

Blow-up from thin scatterer

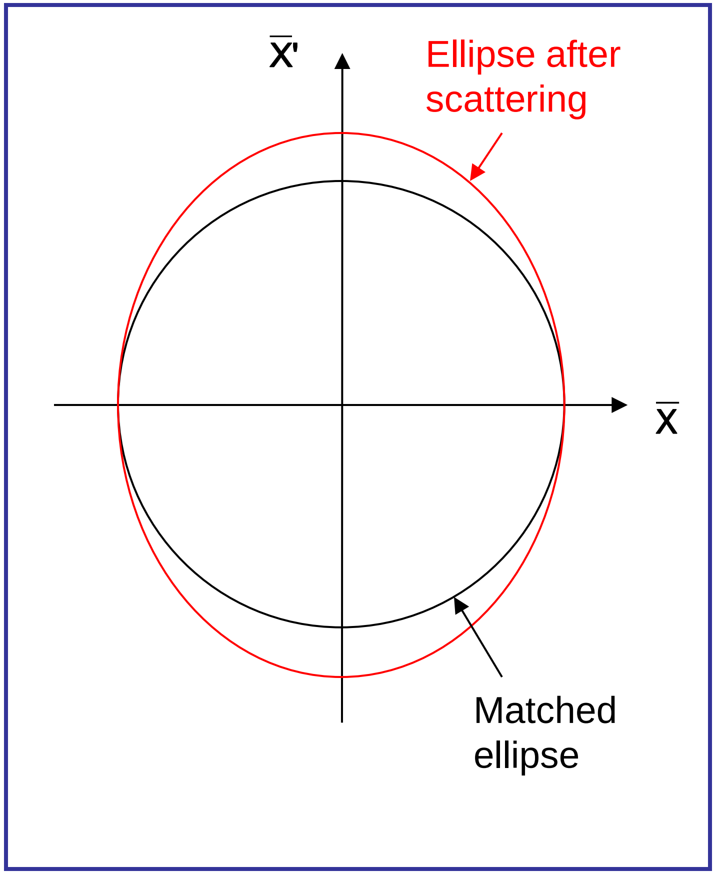
Each particles gets a random angle change q$_s$ but there is no effect on the positions at the scatterer

$$\overline{X}_{new} = \overline{X}_0$$

$$\overline{X}'_{new} = \overline{X}'_0 + \sqrt{\beta}\theta_s$$

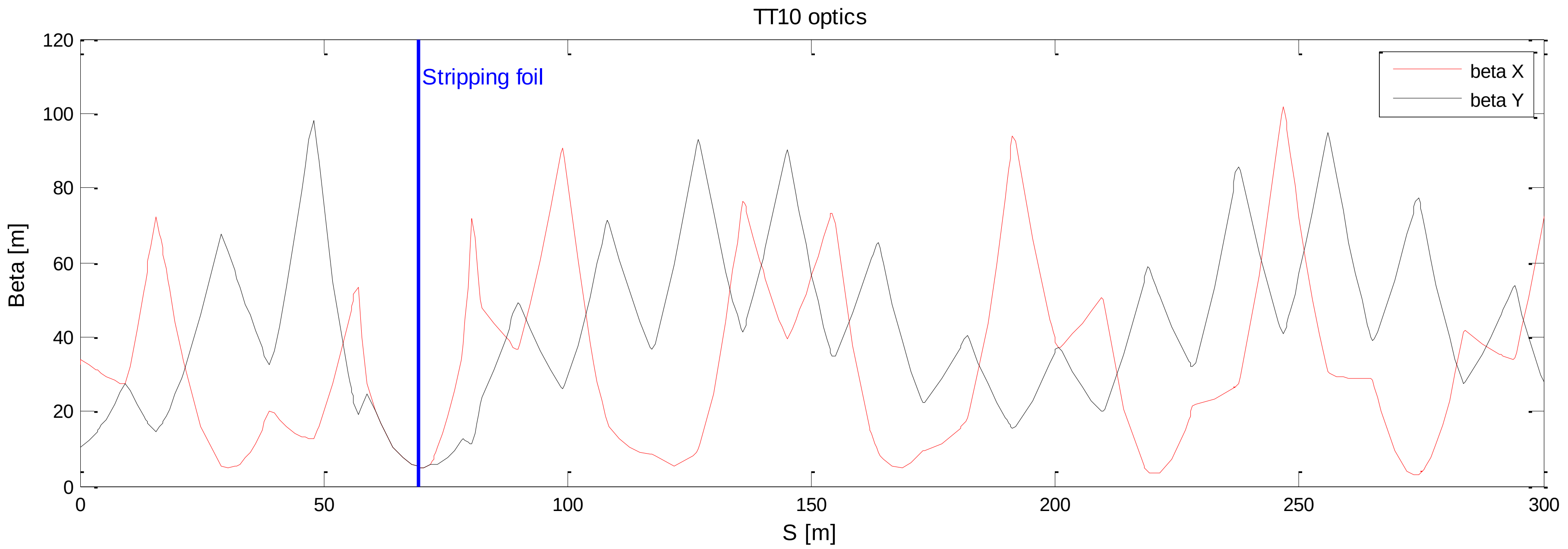
After filamentation the particles have different amplitudes and the beam has a larger emittance

$$\varepsilon_{new} = \varepsilon_0 + \frac{\beta}{2}\langle \theta_s^2 \rangle$$



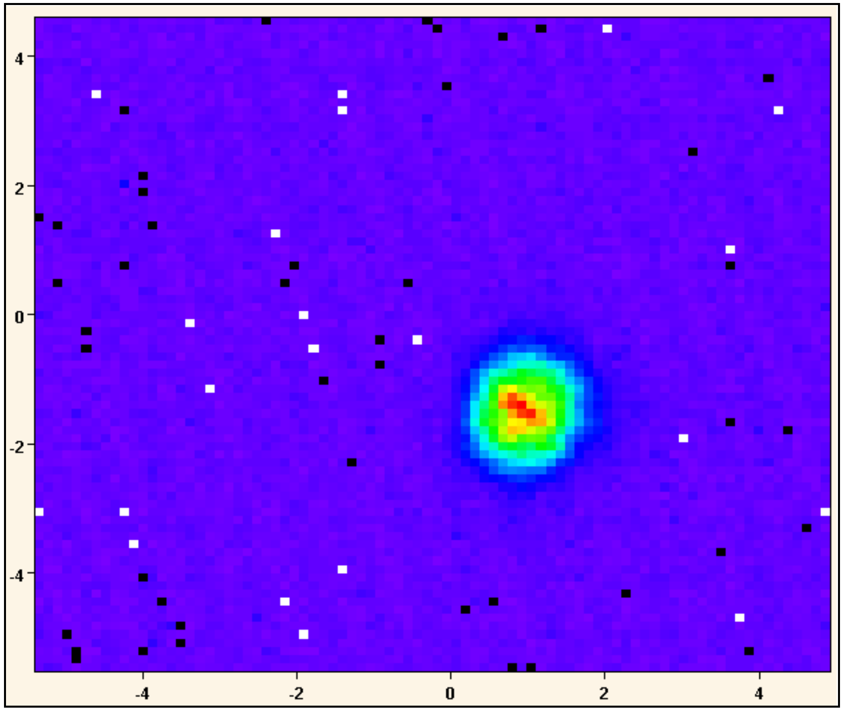
Blow-up from charge stripping foil

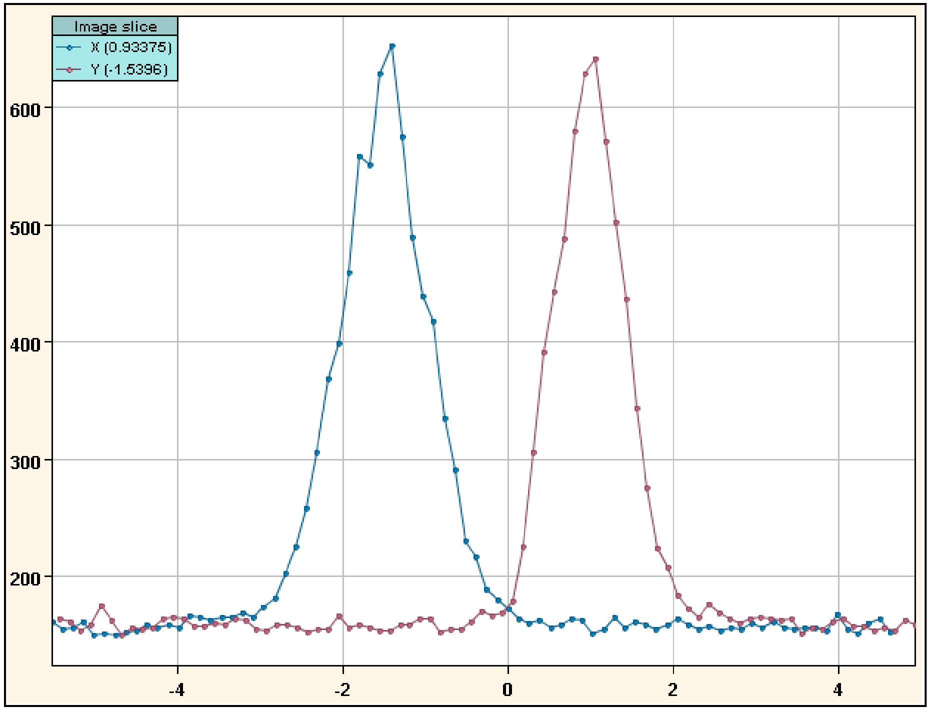
- For LHC heavy ions, Pb^{53+} is stripped to Pb^{82+} at 4.25GeV/u using a 0.8mm thick Al foil, in the PS to SPS line
- De is minimised with low-b insertion (b_{xy} ~5 m) in the transfer line
- Emittance increase expected is about 8%



Optics measurement with screens

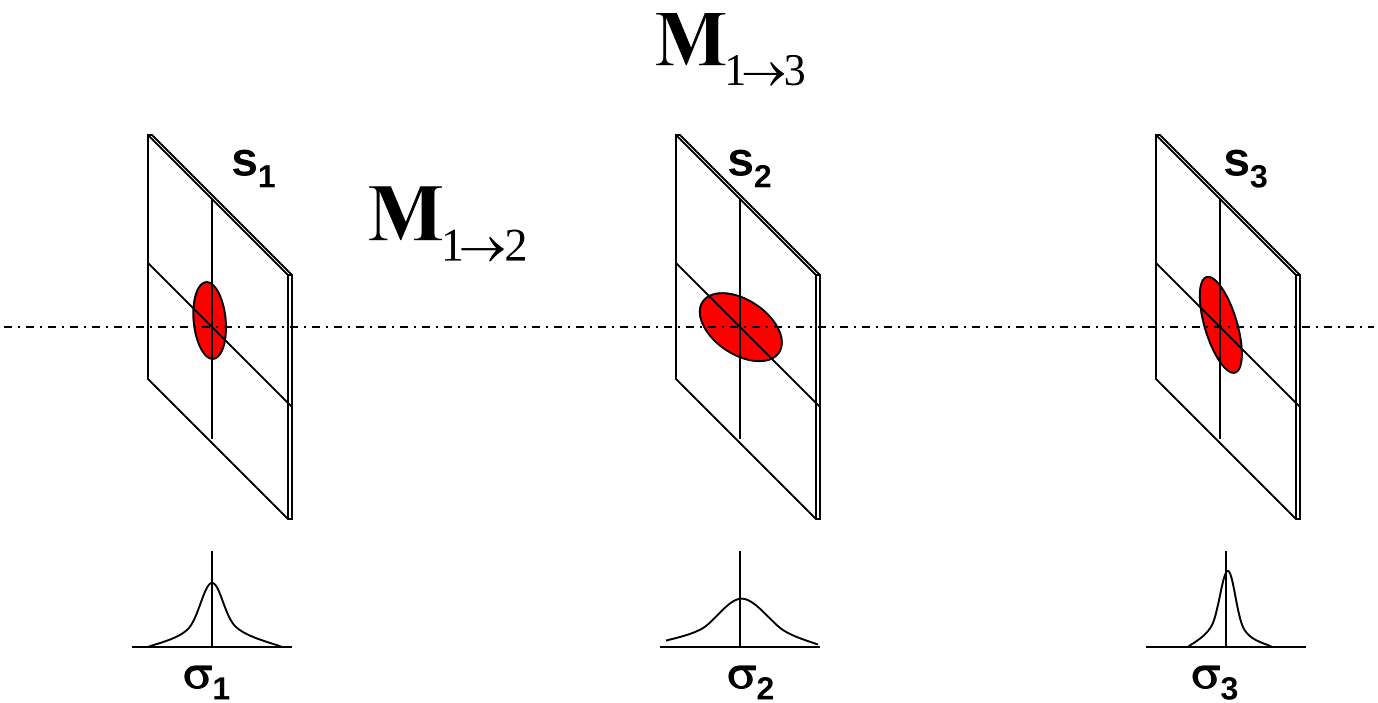
- A profile monitor is needed to measure the beam size
  - e.g. beam screen (luminescent) provides 2D density profile of the beam
- Profile fit gives transverse beam sizes σ.
- If optics is known, ε can be calculated from a single screen





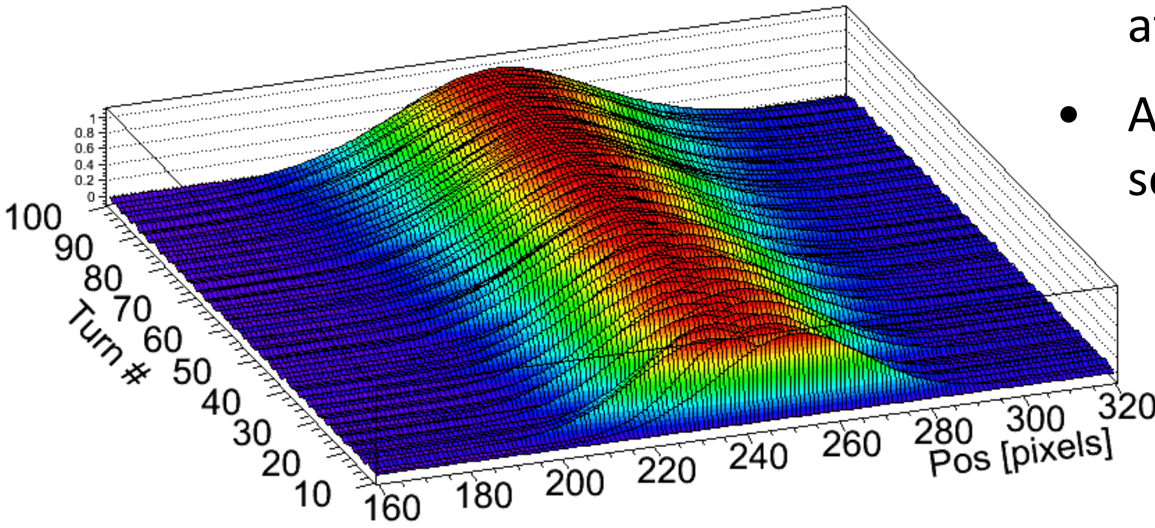
Optics Measurement with 3 Screens

- Assume 3 screens in a dispersion free region
- Measurements of s_1,s_2,s_3, plus the two transfer matrices M_{12} and M_{13} allows determination of ε, α and β



$$e=\frac{s_1^2}{b_1}=\frac{s_2^2}{b_2}=\frac{s_3^2}{b_3}$$

Matching screen



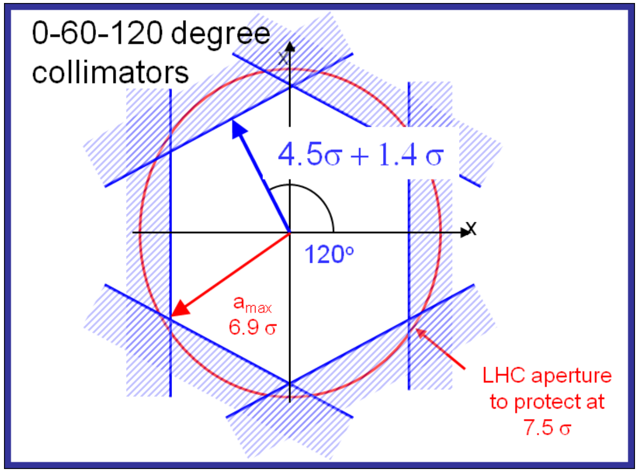
- 1 screen in the circular machine
- Measure turn-by-turn profile after injection
- Algorithm same as for several screens in transfer line

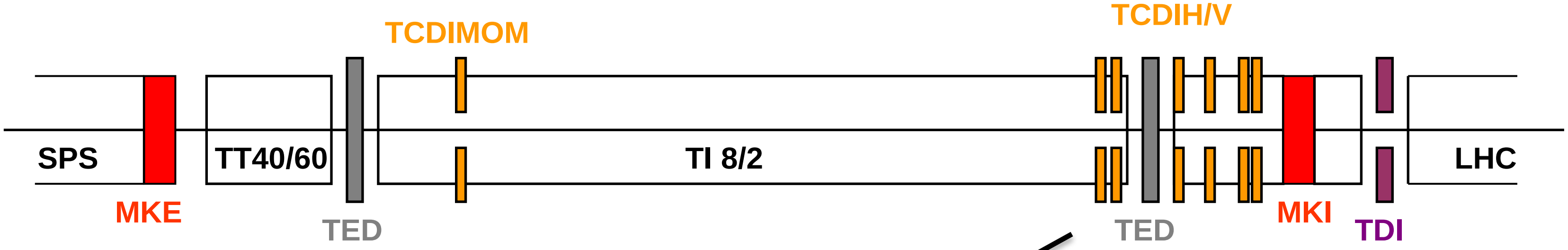
Profiles at matching monitor after injection with steering error

- Only allowed with low intensity beam
- Issue: radiation hard fast cameras

Injection protection

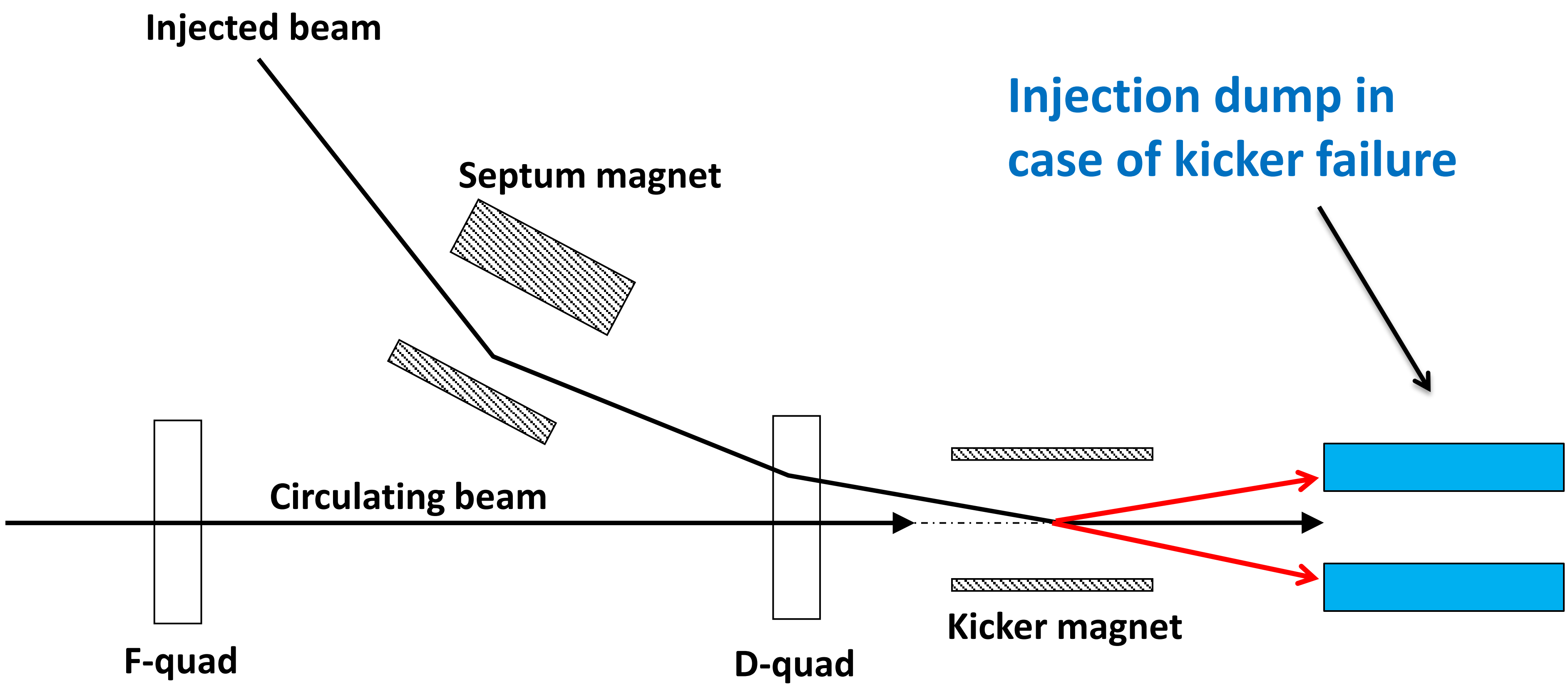
- If beam is powerful enough to destroy downstream machine elements
- Intercept large amplitude particles with collimators



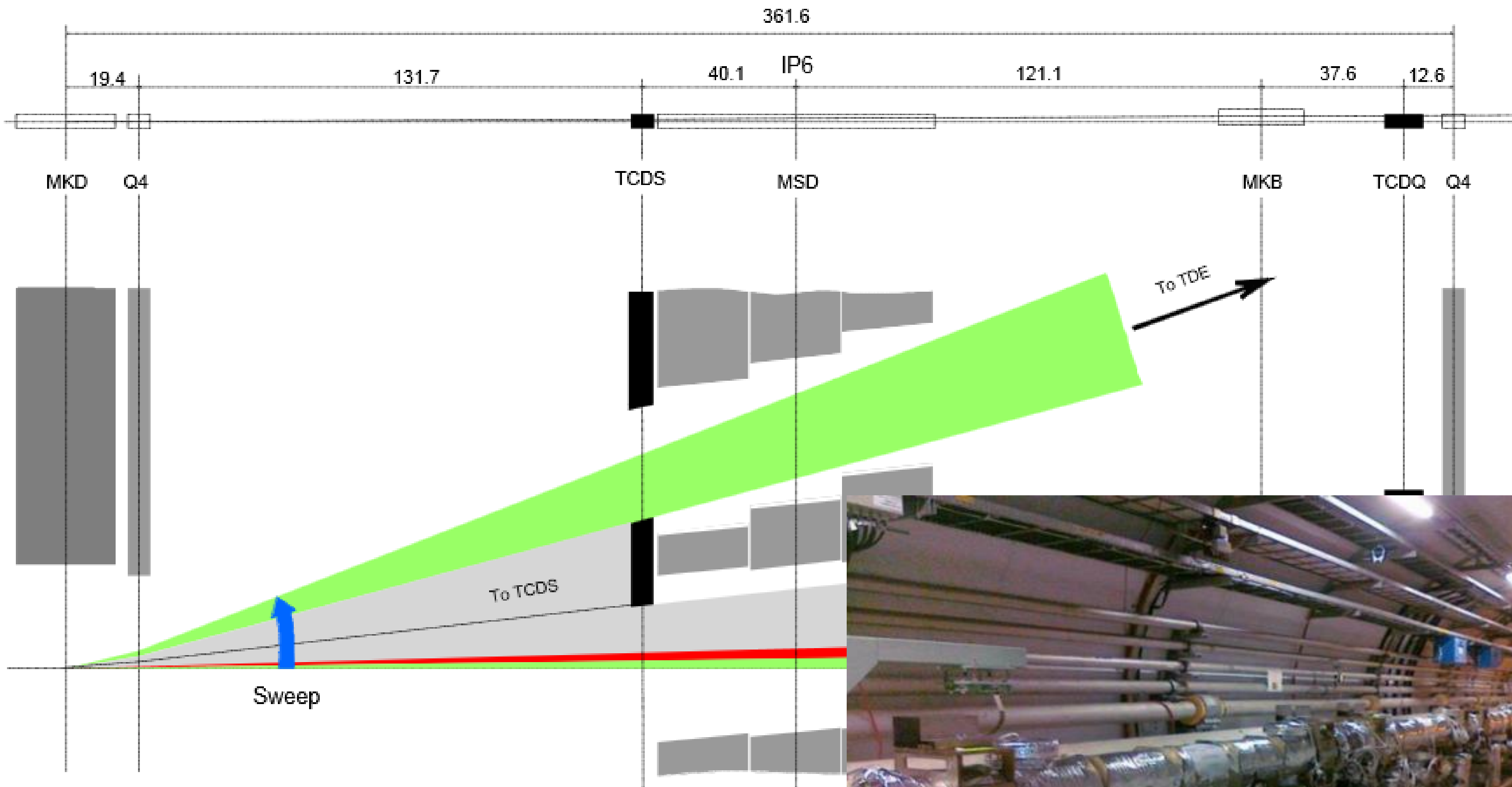


Gives additional constraints on optics and trajectory

Injection protection



Dump protection elements



Summary

- Depending on the injection/extraction concept chose dedicated septa and kickers
- Transfer lines present interesting challenges and differences from circular machines
  - No periodic condition mean optics is defined by transfer line element strengths and by initial beam ellipse
  - Matching at the extremes is subject to many constraints
  - Emittance blow-up is an important consideration, and arises from several sources
  - Measurement beam parameters important for understanding of optics and beam transfer process

Thank you for your attention!

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Blow-up from betatron mismatch

General betatron motion

$$x_2 = \sqrt{a_2 b_2}\,\sin(j + j_o), \qquad x'_2 = \sqrt{a_2 / b_2}\left[\cos(j + j_o) - a_2 \sin(j + j_o)\right]$$

applying the normalising transformation for the matched beam

$$\begin{bmatrix} \overline{X}_2 \\ \overline{X}'_2 \end{bmatrix} = \sqrt{\frac{1}{\beta_1}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_1 & \beta_1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x'_2 \end{bmatrix}$$

an ellipse is obtained in normalised phase space

$$A^2 = \overline{X}_2^2 \left[\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2}\right)^2 \right] + \overline{X}'^2_2 \frac{\beta_2}{\beta_1} - 2\overline{X}_2 \overline{X}'_2 \left[\frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2}\right)\right]$$

characterised by g_{new}, b_{new} *and* a_{new}, where

$$\alpha_{new} = \frac{-\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2}\right), \qquad \beta_{new} = \frac{\beta_2}{\beta_1}, \quad \gamma_{new} = \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2}\right)^2$$

Blow-up from betatron mismatch

From the general ellipse properties

$$a = \frac{A}{\sqrt{2}}\left(\sqrt{H+1}+\sqrt{H-1}\right), \qquad b = \frac{A}{\sqrt{2}}\left(\sqrt{H+1}-\sqrt{H-1}\right)$$

where

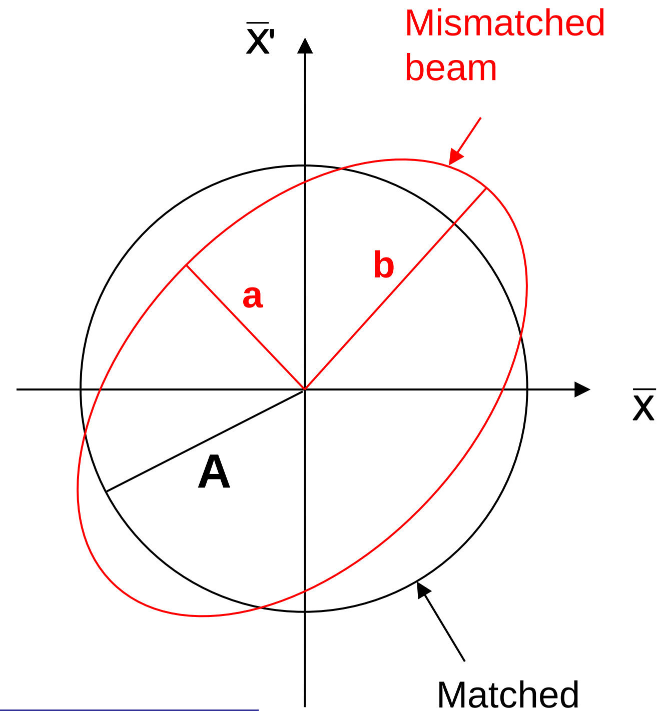
$$H = \frac{1}{2}\left(\gamma_{new}+\beta_{new}\right) \qquad \begin{matrix} a = \mathrm{A}/\lambda \\ b = \mathrm{A}\cdot\lambda \end{matrix}$$

$$= \frac{1}{2}\left(\frac{\beta_1}{\beta_2}+\frac{\beta_2}{\beta_1}\left(\alpha_1-\alpha_2\frac{\beta_1}{\beta_2}\right)^2+\frac{\beta_2}{\beta_1}\right)$$

giving

$$\lambda = \frac{1}{\sqrt{2}}\left(\sqrt{H+1}+\sqrt{H-1}\right), \qquad \frac{1}{\lambda} = \frac{1}{\sqrt{2}}\left(\sqrt{H+1}-\sqrt{H-1}\right)$$

$$\overline{\mathbf{X}}_{new} = \lambda\cdot\mathbf{A}\sin(\phi+\phi_1), \qquad \overline{\mathbf{X}}'_{new} = \frac{1}{\lambda}\mathbf{A}\cos(\phi+\phi_1)$$



generally

Blow-up from betatron mismatch

We can evaluate the square of the distance of a particle from the origin as

$$A_{new}^2 = \overline{X}_{new}^2 + \overline{X}'^2_{new} = \lambda^2 \cdot A_0^2 \sin^2(\phi+\phi_1) + \frac{1}{\lambda^2} A_0^2 \cos^2(\phi+\phi_1)$$

The new emittance is the average over all phases

$$\varepsilon_{new} = \frac{1}{2}\left\langle A_{new}^2 \right\rangle = \frac{1}{2}\left(\lambda^2 \left\langle A_0^2 \sin^2(\phi+\phi_1)\right\rangle + \frac{1}{\lambda^2}\left\langle A_0^2 \cos^2(\phi+\phi_1)\right\rangle \right)$$

$$= \frac{1}{2}\left\langle A_0^2 \right\rangle \left(\lambda^2 \left\langle \sin^2(\phi+\phi_1)\right\rangle + \frac{1}{\lambda^2}\left\langle \cos^2(\phi+\phi_1)\right\rangle \right)$$

0.5 0.5

$$= \frac{1}{2}\varepsilon_0 \left(\lambda^2 + \frac{1}{\lambda^2} \right)$$

If we’re feeling diligent, we can substitute back for l to give

$$\varepsilon_{new} = \frac{1}{2}\varepsilon_0 \left(\lambda^2 + \frac{1}{\lambda^2} \right) = H\varepsilon_0 = \frac{1}{2}\varepsilon_0 \left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right)^2 + \frac{\beta_2}{\beta_1} \right)$$

where subscript 1 refers to matched ellipse, 2 to mismatched ellipse.

Blow-up from steering error

- The new particle coordinates in normalised phase space are

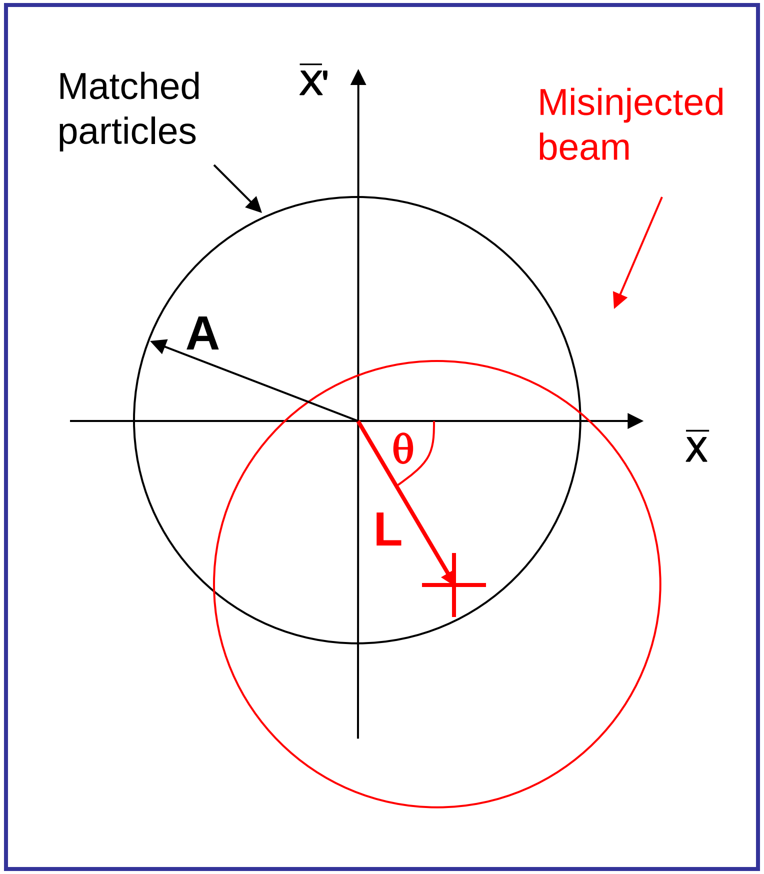
$$\overline{X}_{new} = \overline{X}_0 + L\cos\theta$$

$$\overline{X}'_{new} = \overline{X}'_0 + L\sin\theta$$

- For a general particle distribution, where A denotes amplitude in normalised phase space

$$A^2 = \overline{X}^2 + \overline{X}'^2$$

$$\varepsilon = \langle A^2 \rangle / 2$$



Blow-up from steering error

- So if we plug in the new coordinates….

$$A^2_{new} = \bar{X}^2_{new} + \bar{X}'^2_{new} = \left(\bar{X}_0 + L\cos\theta\right)^2 + \left(\bar{X}'_0 + L\sin\theta\right)^2$$

$$= \bar{X}_0^2 + \bar{X}_0'^2 + 2L\left(\bar{X}_0\cos\theta + \bar{X}'_0\sin\theta\right) + L^2$$

$$\left\langle A^2_{new}\right\rangle = \left\langle \bar{X}_0^2\right\rangle + \left\langle \bar{X}_0'^2\right\rangle + \left\langle 2L\left(\bar{X}_0\cos\theta + \bar{X}'_0\sin\theta\right)\right\rangle + \left\langle L^2\right\rangle$$

$$= 2\varepsilon_0 + 2L\left(\left\langle \cos\theta\bar{X}_0\right\rangle + \left\langle \sin\theta\bar{X}'_0\right\rangle\right) + L^2$$

(both averaged terms struck through and marked 0)

$$= 2\varepsilon_0 + L^2$$

- Giving for the emittance increase

$$\varepsilon_{new} = \left\langle \mathrm{A}_{new}{}^2\right\rangle / 2 = \varepsilon_0 + \mathrm{L}^2/2$$

$$= \varepsilon_0\left(1 + \Delta\mathrm{a}^2/2\right)$$

Blow-up from thin scatterer

$$A_{new}^2 = \overline{X}_{new}^2 + \overline{X}'^2_{new}$$

$$= \overline{X}_0^2 + \left(\overline{X}'_0 + \sqrt{\beta}\theta_s\right)^2$$

$$= \overline{X}_0^2 + \overline{X}'^2_0 + 2\sqrt{\beta}\left(\overline{X}'_0\theta_s\right) + \beta\theta_s^2$$

uncorrelated

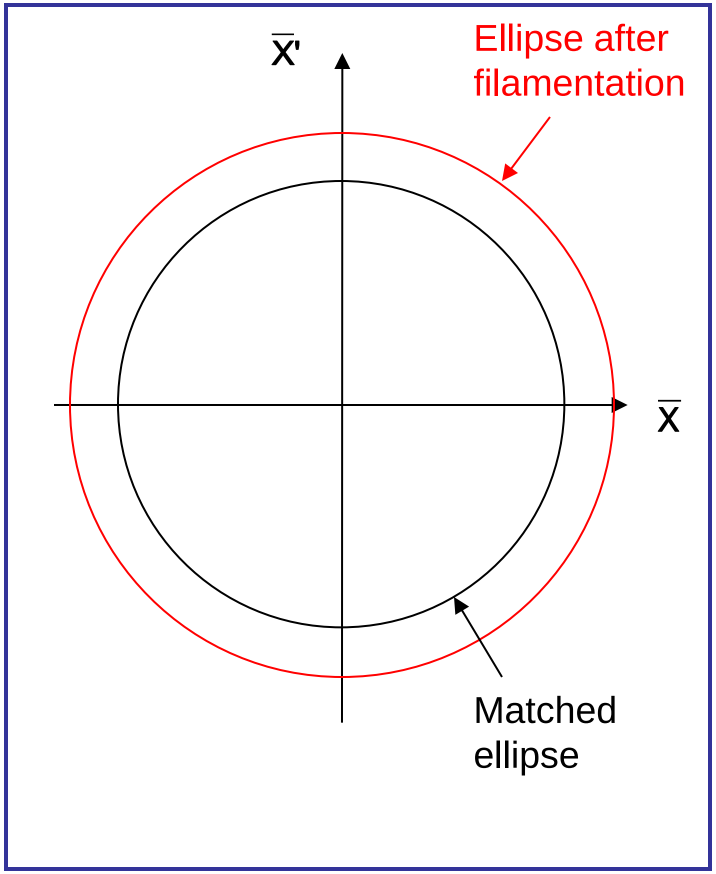
$$\left\langle A_{new}^2 \right\rangle = \left\langle \overline{X}_0^2 \right\rangle + \left\langle \overline{X}'^2_0 \right\rangle + 2\sqrt{\beta}\left\langle \overline{X}'_0\theta_s \right\rangle + \beta\left\langle \theta_s^2 \right\rangle$$

$$= 2\varepsilon_0 + 2\sqrt{\beta}\left\langle \overline{X}'_0 \right\rangle \left\langle \theta_s \right\rangle + \beta\left\langle \theta_s^2 \right\rangle$$

(the term $\left\langle \overline{X}'_0 \right\rangle$ is struck through and marked 0)

$$= 2\varepsilon_0 + \beta\left\langle \theta_s^2 \right\rangle$$

$$\varepsilon_{new} = \varepsilon_0 + \frac{\beta}{2}\left\langle \theta_s^2 \right\rangle$$



Need to keep b small to minimise blow-up (small b means large spread in angles in beam distribution, so additional angle has small effect on distn.)

Optics Measurement with 3 Screens

- Remember:

$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix}$$

$$\begin{bmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} C_2^2 & -2C_2S_2 & S_2^2 \\ -C_2C_2' & C_1S_1'+S_1C_1' & -S_2S_2' \\ C_2'^2 & -2C_2'S_2' & S_2'^2 \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{bmatrix}$$

$$\Rightarrow \left. \begin{aligned} \beta_2 &= C_2^2 \cdot \beta_1 - 2C_2S_2 \cdot \alpha_1 + S_2^2 \cdot \gamma_1 \\ \beta_3 &= C_3^2 \cdot \beta_1 - 2C_3S_3 \cdot \alpha_1 + S_3^2 \cdot \gamma_1 \end{aligned} \right| \times \varepsilon$$

$$\begin{aligned} \sigma_2^2 &= C_2^2 \cdot \beta_1\varepsilon - 2C_2S_2 \cdot \alpha_1\varepsilon + S_2^2 \cdot \gamma_1\varepsilon \\ \sigma_3^2 &= C_3^2 \cdot \beta_1\varepsilon - 2C_3S_3 \cdot \alpha_1\varepsilon + S_3^2 \cdot \gamma_1\varepsilon \end{aligned}$$

Square of beam sizes as function of optical functions at first screen

Optics Measurement with 3 Screens

$$s_1^2 = 1 \times b_1 e - 0 \times a_1 e + 0 \times g_1 e$$

$$s_2^2 = C_2^2 \times b_1 e - 2C_2 S_2 \times a_1 e + S_2^2 \times g_1 e$$

$$s_3^2 = C_3^2 \times b_1 e - 2C_3 S_3 \times a_1 e + S_3^2 \times g_1 e$$

- Build matrix

$$\mathrm{S} = \mathbf{N} \times \mathrm{P}$$

$$\mathrm{S} = \begin{pmatrix} s_1^2 \\ s_2^2 \\ s_3^2 \end{pmatrix}_{meas} \qquad \mathbf{N} = \begin{pmatrix} 1 & 0 & 0 \\ C_2^2 & -2C_2 S_2 & S_2^2 \\ C_3^2 & -2C_3 S_3 & S_3^2 \end{pmatrix} \qquad \mathrm{P} = \begin{pmatrix} b_1 e \\ a_1 e \\ g_1 e \end{pmatrix}$$

- We want to know Π

$$\mathrm{P} = \mathbf{N}^{-1} \times \mathrm{S}$$