Lifetime, cross sections and activation

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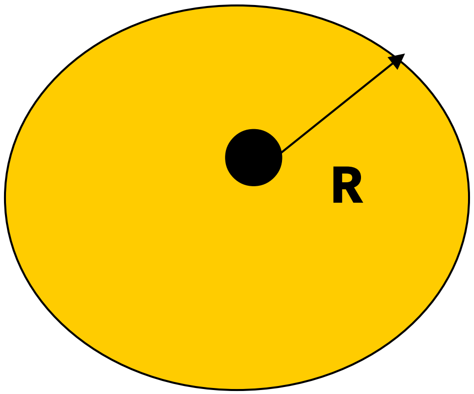
Introduce the concept of cross section

- Given a particle A approaching a particle B

The probability that A interacts with B is called the Cross Section for that specific process

This is the most general definition of cross section i.e. the cross section corresponds to the probability that particle A interacts with particle B

Simple geometrical interpretation of the concept of cross section



$\sigma = \pi \times R^2$

The cross section (σ) is the area within which a reaction will take place

⇒ The units are those of an area

In the early days of nuclear physics the following definition was introduced

1 barn = 10^{-24} cm^2

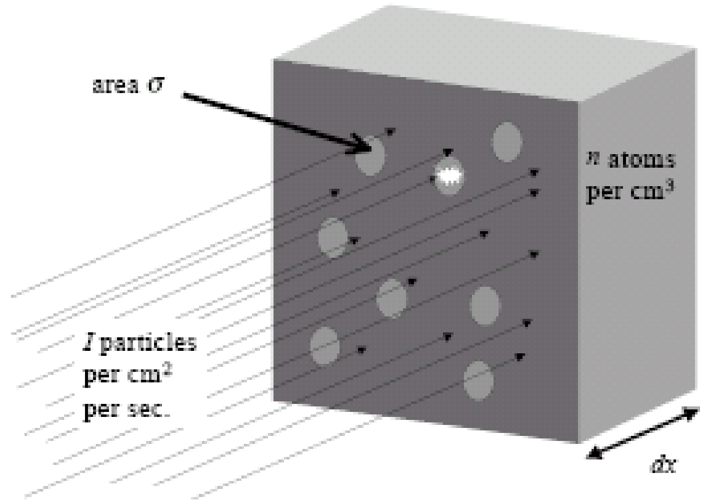
Why "barn"

During early experiments the physicists discovered that interactions were far more probable than expected;

The nucleus were "as big as a barn".

Relation between the cross section and the life time of a beam

- Look at a beam of particles hitting a target



I particle /sec /cm^2

n target atoms/cm^3

dx cm long target

The number of beam particle interacting and disappearing from the beam is then

$dI = I \cdot n \cdot dx \cdot \sigma$

The target thickness traversed

$dx = v \cdot dt$ where v is the velocity of the particle

Combine with the $dI = I \cdot n \cdot dx \cdot \sigma$

$dI/dt = I \cdot n \cdot \sigma \cdot v$ — $1/\tau$

$I = I_0 \exp(-t/\tau)$

where $\tau = 1/(n \cdot \sigma \cdot v)$

Normally we have a gas mixture e.g. H_2, CH_2, CO, CO_2...

Then $n \cdot \sigma \Rightarrow \sum n_i \cdot \sigma_i$

$1/\tau_{total} = 1/\tau_{H2} + 1/\tau_{CH2} + 1/\tau_{CO} + 1/\tau_{CO2}$

What are the typical values of cross sections ?

- **There are NO typical values**

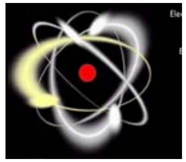
- Depends on many factors
  - Target particle i.e. rest gas
  - Incident particle
  - Energy of incident particle
  - Type of interaction

- **Unfortunately NO simple rule**
  - Look at each dependence separately

Dependence on target particle

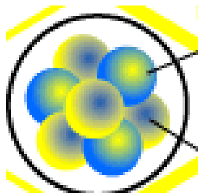
- *Again simple geometrical consideration*

Atom



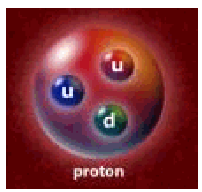
R~ 1 *Angstrom* = 10^{-8} cm ⇒ $\sigma = \pi R^2 \sim$ *Mega* barns

Nucleus



R~ 10 fermi = 10^{-12} cm ⇒ $\sigma = \pi R^2 \sim$ barns

Proton



R~ 1 fermi = 10^{-13} cm ⇒ $\sigma = \pi R^2 \sim$ mbarns

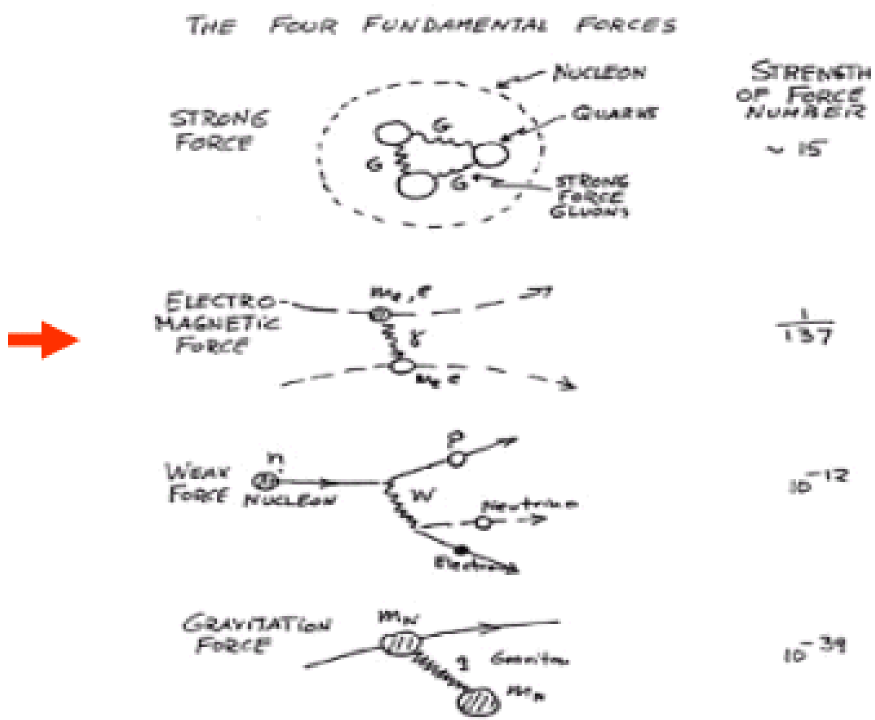
Dependence on incident particle

- In accelerators we mainly have to deal with protons, electrons or ions.

- Fundamentally different particles:

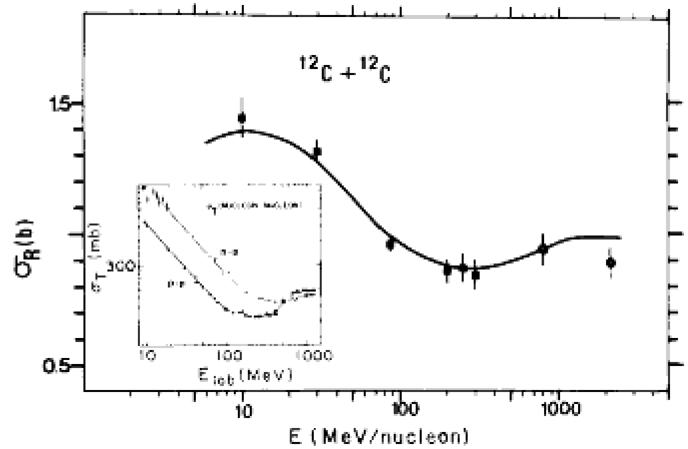
  - Size
  - Mass
  - Compositeness
  - interactions

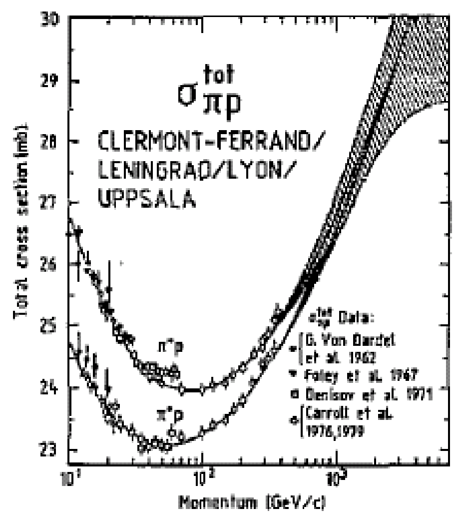
Short reminder of different type of interactions

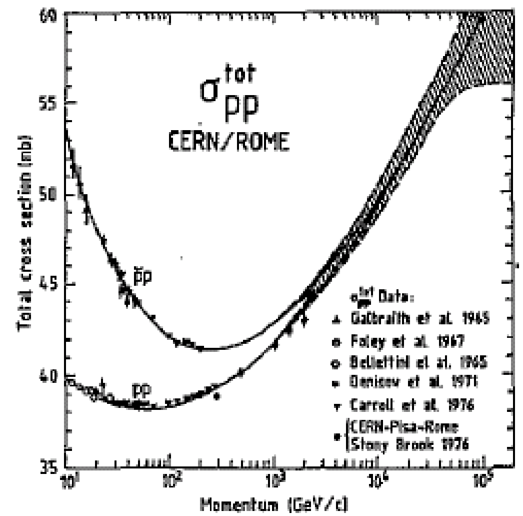


- Strong interaction for head on collisions
- Electromagnetic dominates for peripheral collisions

Example of energy dependence







Classify interaction between charged particles and rest gas

- Elastic

  A collision is called elastic if the particles do not change identity during the interaction-like collisions of billiard balls

  - **Electromagnetic-both particles charged**
    - Single
    - Multiple

  - **Strong nuclear force –basically independent of charge**

Classify interaction between charged particles and rest gas

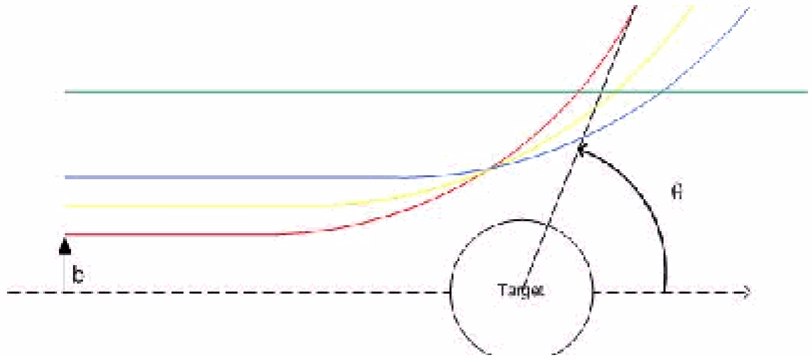
- Inelastic- everything that is not elastic

  Change of nature of the particles and also creation of new particles
  - **Electromagnetic**
    - Bremsstrahlung
    - Ionization
    - Electron capture
    - Electron loss
  - **Strong**
    - Nuclear reactions
    - Particle break up
    - Particle creation

Look at some of those-elastic and inelastic- more in detail

Elastic scattering-electromagnetic interactions (Coulomb)

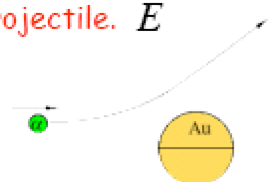
- Incoming particle interacts with rest gas nucleus. The scattering angle depends on the impact parameter.



Rutherford scattering

α scattering off Gold foil

Kinetic energy of projectile. E

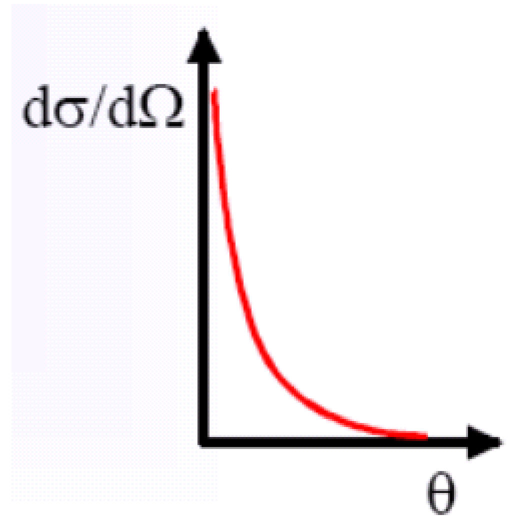


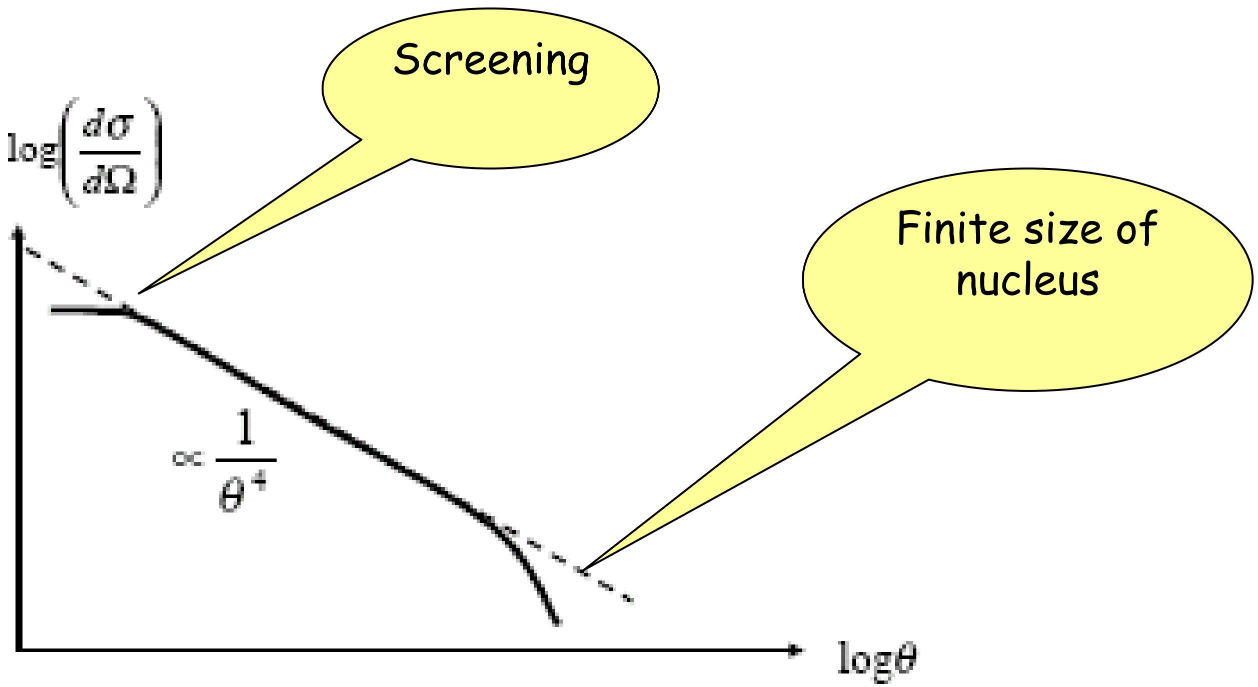
Charges of projectile and target. $Ze, Z'e$

Cross section per unit solid angle as a function of the scattering angle, θ

$$\frac{d\sigma}{d\Omega}(\theta) = \left(\frac{ZZ'e^2}{4E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}$$

- Deviation from simple formulae





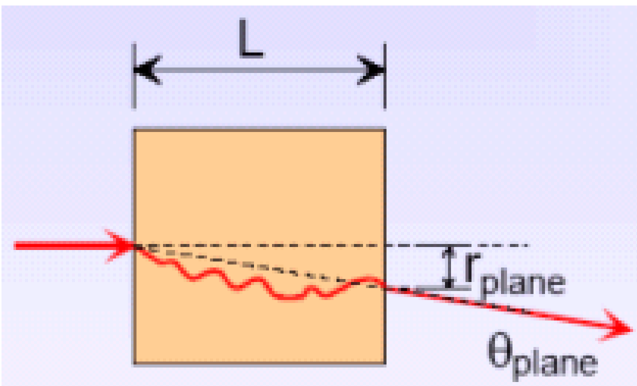
Elastic Scattering-Multiple Coulomb scattering

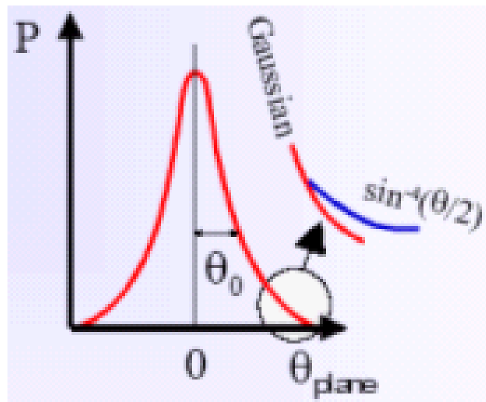
- Consider here cumulative effect of many small Coulomb deviations.

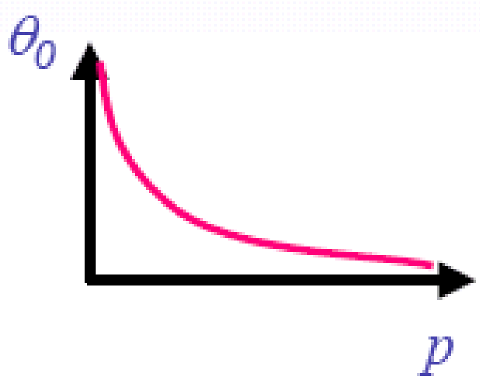
Approximation:

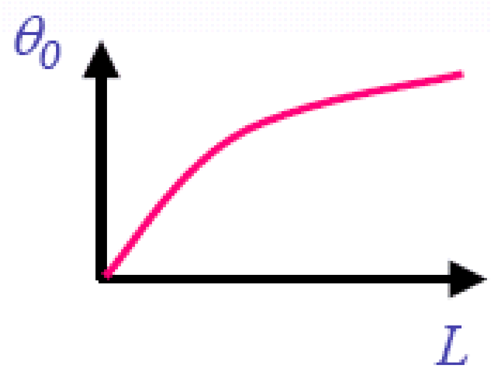
$$\theta_0 \propto \frac{1}{p}\sqrt{\frac{L}{X_0}}$$

X_0 = “radiation length”



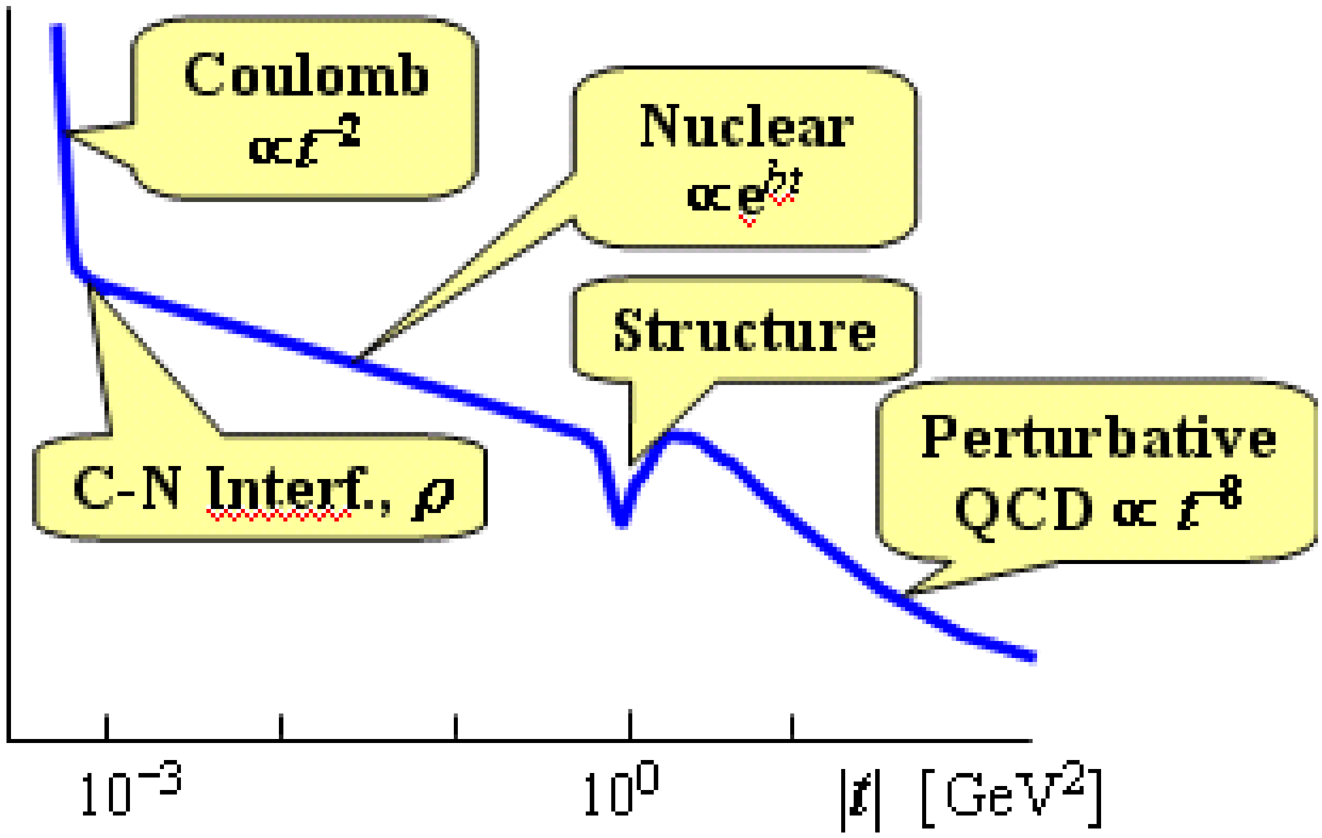






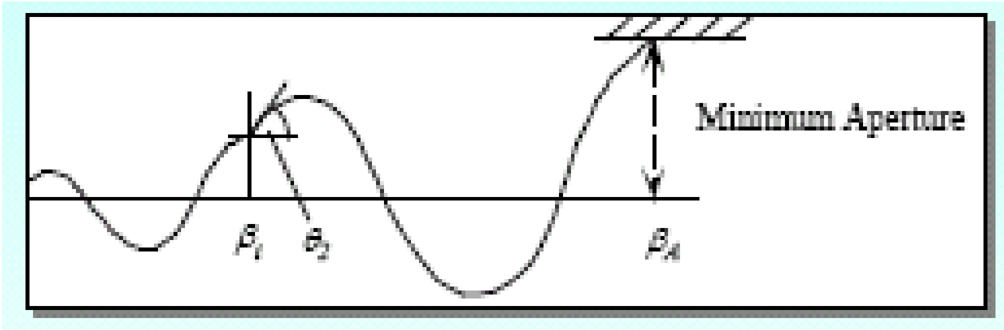
Elastic scattering – strong interactions

- Dominates at large angles



When does elastic scattering implies loss of beam particles ?

- Look at single Coulomb scattering as an example



- Kick θ_i results in an oscillation

$$u(s) = \theta_i \sqrt{\beta(s)\,\beta_i}\ \sin(\varphi(s) - \varphi_i)$$

- At the position of minimum aperture the maximum allowed amplitude is:

$$\Theta_i \sqrt{\beta_A \beta_i}$$

- Particle lost if

$$\Theta_i \sqrt{\beta_A \beta_i} > A$$

where A = half aperture of dynamic aperture

- Average over the circumference of the machine

$$\Theta_{max}=A/\sqrt{\beta_A\beta_{average}}$$

- To get the loss cross section σ_{loss} we just need to integrate

$$\frac{d\sigma}{d\Omega}(\theta)=\left(\frac{ZZ'e^2}{4E}\right)^2\frac{1}{\sin^4\frac{\theta}{2}}$$

from θ_{max} to π and we get

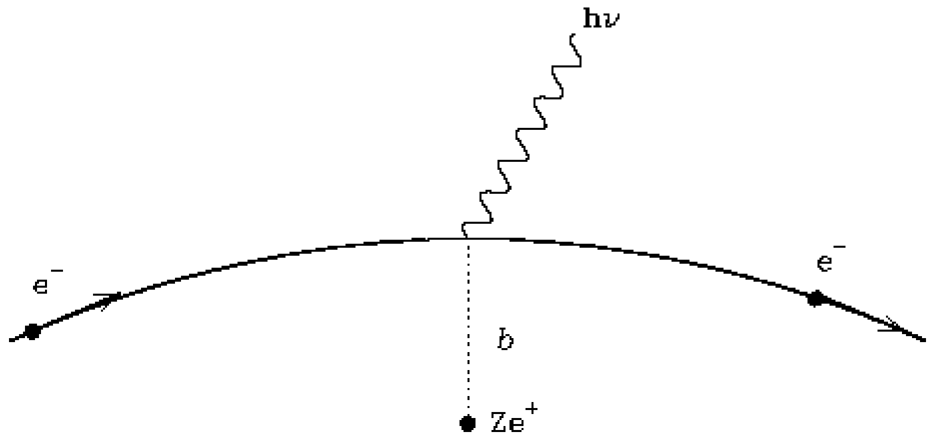
$$\sigma_{loss}\propto Z'^2/E^2\ 1/\Theta^2_{max}=Z'^2/E^2\ \beta_A\beta_{average}/A^2$$

This is the cross section to be used for life time calculation

Inelastic cross sections - electron beam
Bremsstrahlung

Charge particle accelerated ⇒ Electromagnetic radiation emitted

Charge particle accelerated by the field of an atomic nuclei
⇒ photons are emitted



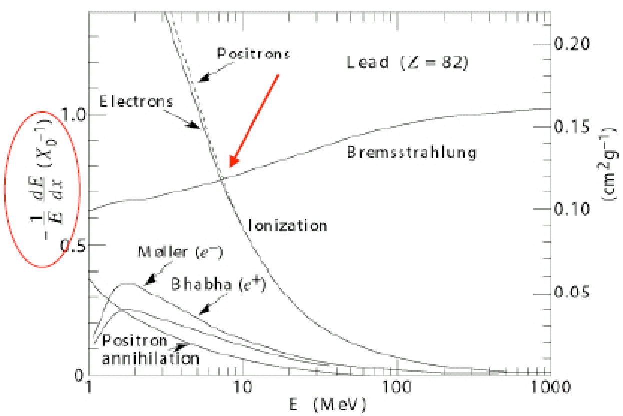
Called Bremsstrahlung –German for "braking" radiation

The energy emitted by an accelerated particle $\propto 1/m^2$

$\Rightarrow$ Bremsstrahlung important for electrons and positrons

The energy loss is roughly proportional to the energy of the particle $\Rightarrow$ Bremsstrahlung important at high energies

Energy loss of electrons in matter



Energy loss can be described approximately as:

$$\left(\frac{dE}{dx}\right) = \frac{E}{X_0}$$

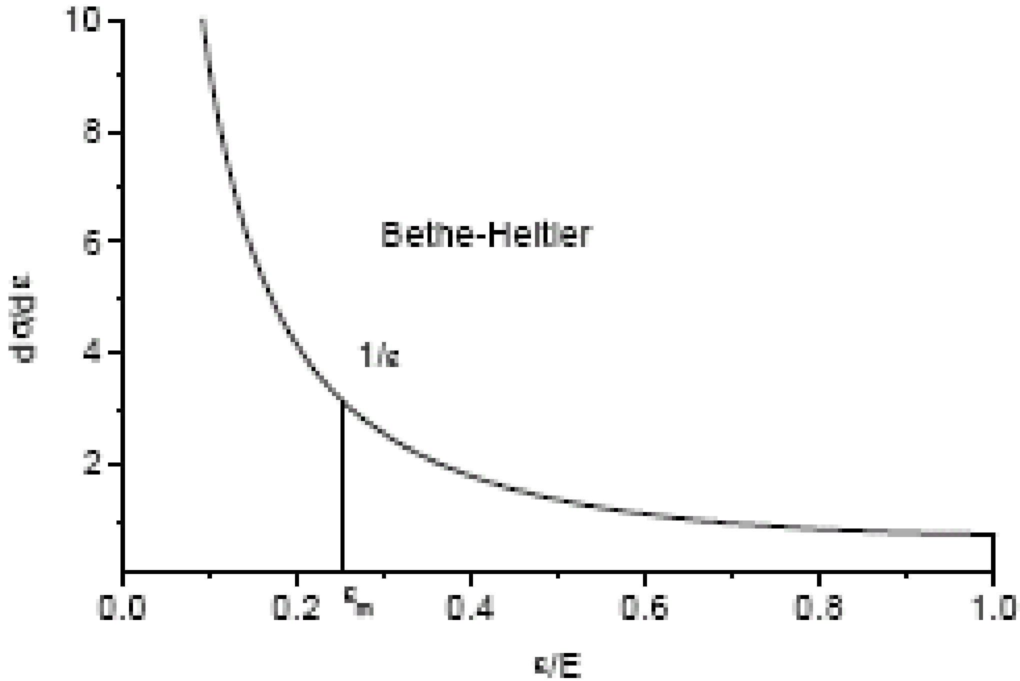
X_0 = 'radiation length'; Unit: mass/unit area (g/cm^{-2})

The average energy loss is

$$\langle E \rangle = E_0 \cdot \exp\left(-\frac{x}{X_0}\right)$$

$$1/x_0 \sim Z_t^2$$

How does the photon spectrum look ?



$d\sigma/d\varepsilon = 4/3 \cdot 1/X_0 \cdot 1/\varepsilon \cdot F(\varepsilon,E)$

$F(\varepsilon,E)$ is a slow varying function

$F(\varepsilon,E) \cong 1 - \varepsilon/E + \frac{3}{4} (\varepsilon/E)^2$

Energy acceptance of the accelerator determine the losses due to bremsstrahlung

Assume that all energies from the nominal E down to $E-\varepsilon_m$ is accepted $\Rightarrow$

energy losses > $\varepsilon_m \Longrightarrow$ the particle is rejected

To get σ_{loss} integrate from ε_m to E

$$\mathbf{d\sigma / d\varepsilon = 4/3 \cdot 1/X_0 \cdot 1/\varepsilon \cdot F(\varepsilon,E)}$$

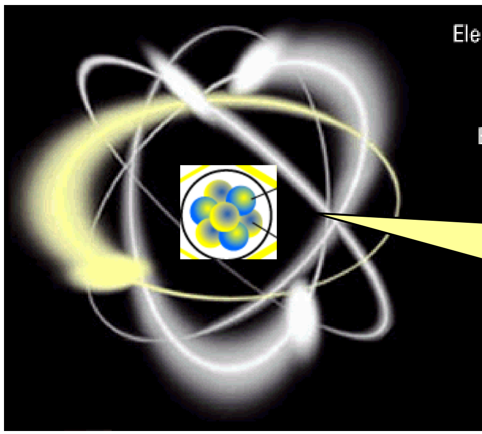
$$\sigma_{loss\ brems} = \mathbf{4/3 \cdot 1/X_0 \cdot (\ln E/\varepsilon_m - 5/8) \quad (\varepsilon_m << E)}$$

σ_{loss} strong dependence on atomic number of residual gas
weak dependence on the maximum energy acceptance

Simplified treatment excluding effect from electrons of the atom and screening effects pg1

Inelastic cross sections - Ion beams

- More complicated than electron and proton beams
- Two more degree of freedom



Z_i charge of nucleus

q total charge state

- Interaction with the rest gas atoms might either lead to
  - Capture of additional electrons
  - Loss or stripping of existing electrons
- Both cross sections are a function of q , Z_i, Z_T and β_i .

- In a simple and intuitive picture we compare β_i with the velocity of the outer most electron β_e

$$\beta_i \sim \beta_e \quad \Rightarrow \quad \sigma_C \sim \sigma_L$$

where σ_C =the cross section for electron capture

σ_L =the cross section for electron loss

$\beta_i > \beta_e \quad \Rightarrow \quad \sigma_L$ dominates

$\beta_i < \beta_e \quad \Rightarrow \quad \sigma_C$ dominates

- Notion of equilibrium charge important

$\langle q \rangle$= equilibrium charge state = state reached after many collisions with a given gas

$\langle q \rangle$ is the state for which $\beta_i \sim \beta_e$

if $q = \langle q \rangle$ then $\sigma_C \sim \sigma_L$

- Around q = ‹ q › there are simplified scaling rules i.e

$$\sigma_c(q) = \sigma_c(\overline{q})(\tfrac{q}{\overline{q}})^a$$

$$\sigma_l(q) = \sigma_c(\overline{q})(\tfrac{q}{\overline{q}})^b$$

where $a \simeq 4$ and $b \simeq -2.3$ for charges lower than the equilibrium charge state and $a \simeq 2$ and $b \simeq -4$ for higher charge states. Many other scaling relations exist.

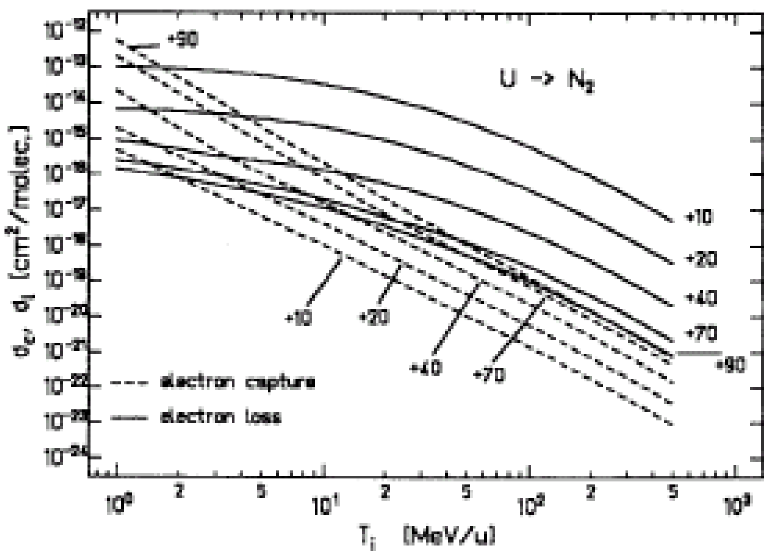


Figure 1: Capture and loss cross sections for uranium ions in N_2

Capture cross sections are according to (5) and loss cross sections are extrapolated by means of (1).

Inelastic cross sections - Ion beams – high energies

- RHIC and LHC

  High energies (100 GeV/A and 2.76 TeV/A) and bare ions

  $\Rightarrow$

  different mechanisms

- For peripheral collisions we have large electromagnetic cross sections

  high z $\Rightarrow$ strong field $\Rightarrow$ e+ e- pair production with

  $\sigma \sim$ 100 kbarn

However rather harmless inelastic reaction

no significant change of momentum of the ion

- Two other inelastic mechanisms dominates the losses
    - e+ e- pair production followed by electron capture
    - Electromagnetic dissociation of the nucleus
- Example: Lead on Lead at the LHC

Cross-sections	Pb ions	[barn]
Hadronic	σ_H	8
E.m. Diss.	σ_{emd}	225
e^-- capture	σ_{ec}	204

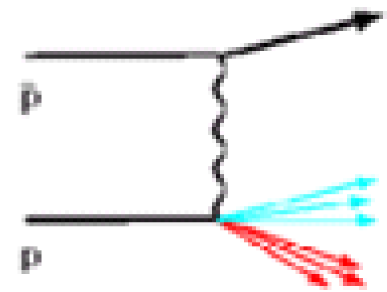
- Such big cross sections $\Rightarrow$ short life times from beam-beam interaction and not from beam-gas interactions

- Beam life times at LHC and RHIC from beam-beam are in the range of hours at high z - varies very fast with z

Ion	σ_H [b]	σ_{emd} [b]	σ_{ec} [b]	σ_{tot} [b]
Pb^{82}_{208}	8	225	204	437
Sn^{50}_{120}	5.5	44.5	18.5	68.5
Kr^{36}_{84}	4.5	15.5	3.0	23.0
Ar^{18}_{40}	3.1	1.7	0.04	4.84
O^{8}_{16}	1.5	0.13	$1.6\ 10^{-4}$	1.63

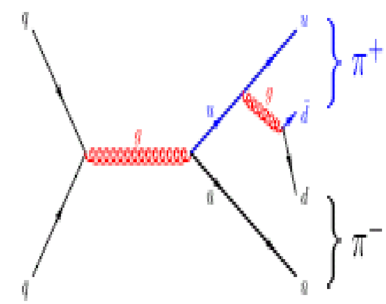
Inelastic cross sections – proton beams

- mp >>me $\Rightarrow$ Bremsstrahlung reduced
- Z =1 $\Rightarrow$ electron capture reduced
- Strong interaction of importance



Diffraction interactions: $p + A \rightarrow p\,\pi^{+}\pi^{-}K^{0}\ldots + A$

Non diffractive interactions

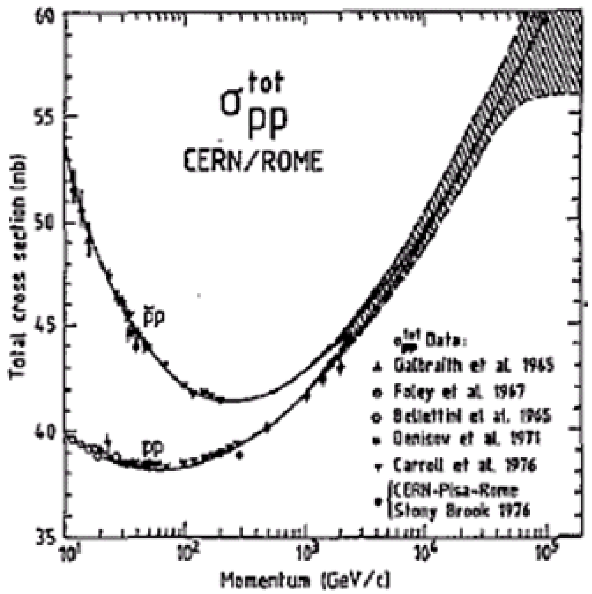


- In both cases the beam proton is completely lost
- The cross section depends strongly on energy

- How to go from p p cross section p+C or p+ N or p+O…

  Rule of thumb

  $\sigma_{pA}=\sigma_{pp} \cdot A^{0.7}$



- Example from early LHC studies

Gas	Cross Section mb
H_2	94
He	130
CH_4	568
H_2O	554
CO	840
CO_2	1300

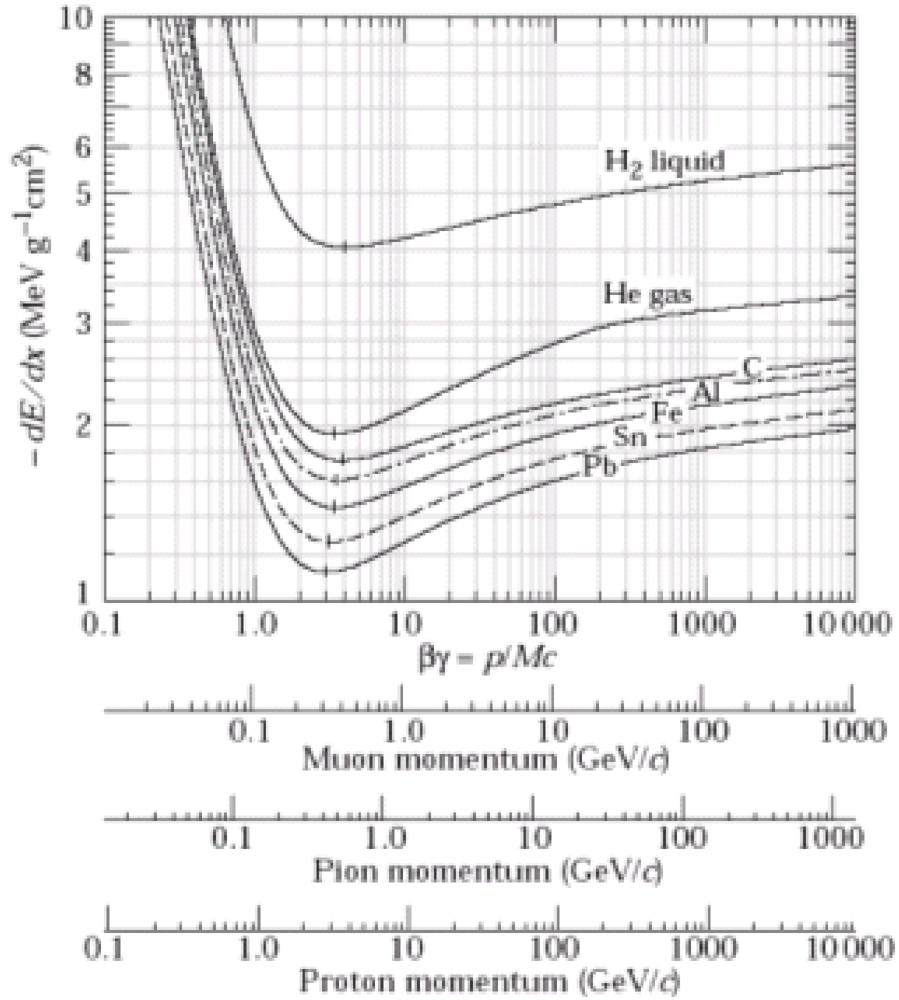
Inelastic cross sections – ionization of the rest gas

- Beam particle kick off electrons when passing through
  $\Rightarrow$ ionization energy loss
- Cross section big BUT very small energy loss
  $\Rightarrow$ beam particle hardly affected
- The energy loss is given by the well known Bethe-Block formula

$$\left\langle \frac{dE}{dx} \right\rangle = -4\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \gamma^2 \beta^2}{I^2} T^{\max} - \beta^2 - \frac{\delta}{2} \right]$$

- Note Z_i^2
  - energy loss can be important for bare ions with high Z

- Note $1/\beta^2$



$-dE/dx$ (MeV $g^{-1}cm^2$)
10
8
6
5
4
3
2
1
H_2 liquid
He gas
C
Al
Fe
Sn
Pb
0.1
1.0
10
100
1000
10 000
$\beta\gamma = p/Mc$
0.1
1.0
10
100
1000
Muon momentum (GeV/c)
0.1
1.0
10
100
1000
Pion momentum (GeV/c)
0.1
1.0
10
100
1000
10 000
Proton momentum (GeV/c)

- Cross section big

$Ne^{+10} \rightarrow H_2$			
Ion energy in MeV/u	Ioniz. cross sections in 10^{-18} cm^2/molec.		
	H^+	$2H^+$	free e^-
1	11.2	5.4	22.0
10	2.09	0.41	2.91
100	0.263	0.007	0.30
1000	–	–	0.072

$U^{+92} \rightarrow H_2$			
Ion energy in MeV/u	Ioniz. cross sections in 10^{-18} cm^2/molec.		
	H^+	$2H^+$	free e^-
1	155	211	583
10	78.2	42.3	163
100	14.4	4.56	23.5
1000	–	–	11.5

In general does NOT contribute to important losses.

Can be of importance at high Z_i

Activation

Activation = induced radioactivity

Making a material radioactive by bombardment of particles or radiation

OR

Transformation of a stable nucleus of an atom to one or several unstable nuclei

What is the origin of activation in accelerators ?

- Objects that are directly hit by the primary beam like dumps, targets, septa, collimators and so on. Dominating source

- Localized accidental beam losses

- Bad vacuum. In general not the main source

Observe:

Problem of activation significantly less severe in electron machines relative hadron machines.

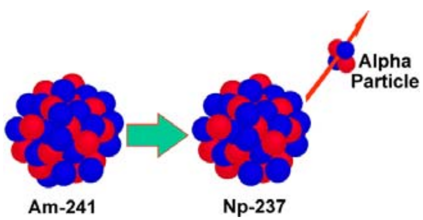
Different interactions $\Rightarrow$ Different process (photons $\Rightarrow$ photo nuclear reactions)

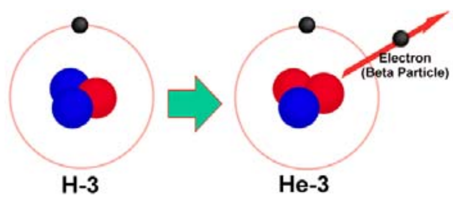
Two phenomena involved in activation

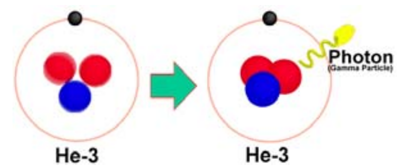
- The reaction that create the unstable nucleus

- The radioactive decay of the unstable nucleus

α β γ (fission)

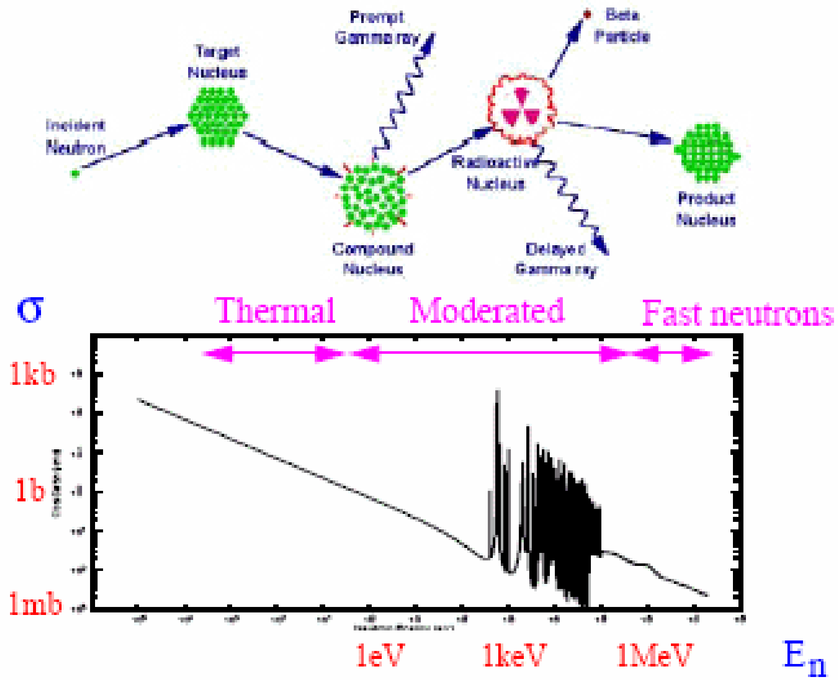






Neutron capture

Neutron capture (n, γ)



- Limited number of final states
- All reaction cross sections are well known but can vary from kbarns to mbarns !
- The activity depends very strongly on the material
- The activity can be calculated directly from the neutron flux and the cross sections.

- The activity depends very strongly on the material

Example:

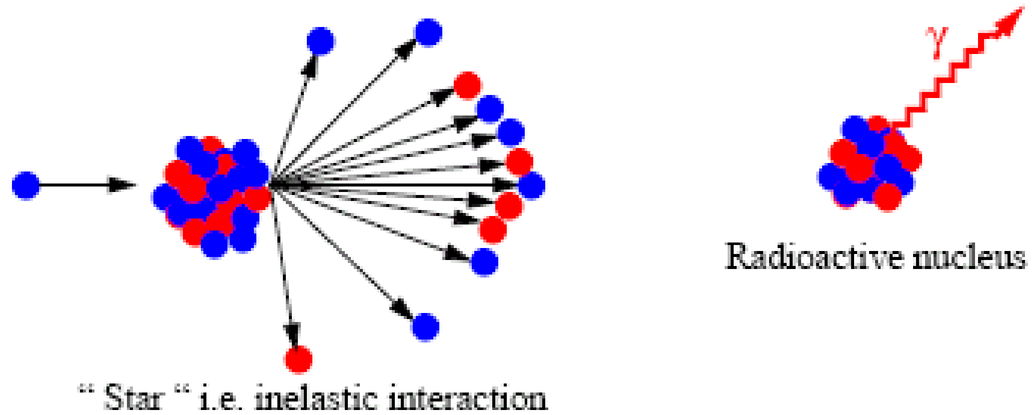
Ag-109 (48% of Ag) → Ag-110m
σ_{max} = 20000 barns
τ = 250 days

Pb-208 (52% of Pb) → Pb-209
σ_{max} = 3 barns
τ = 3.3 hours

The danger from a specific material depends on isotop composition, cross section and half life.

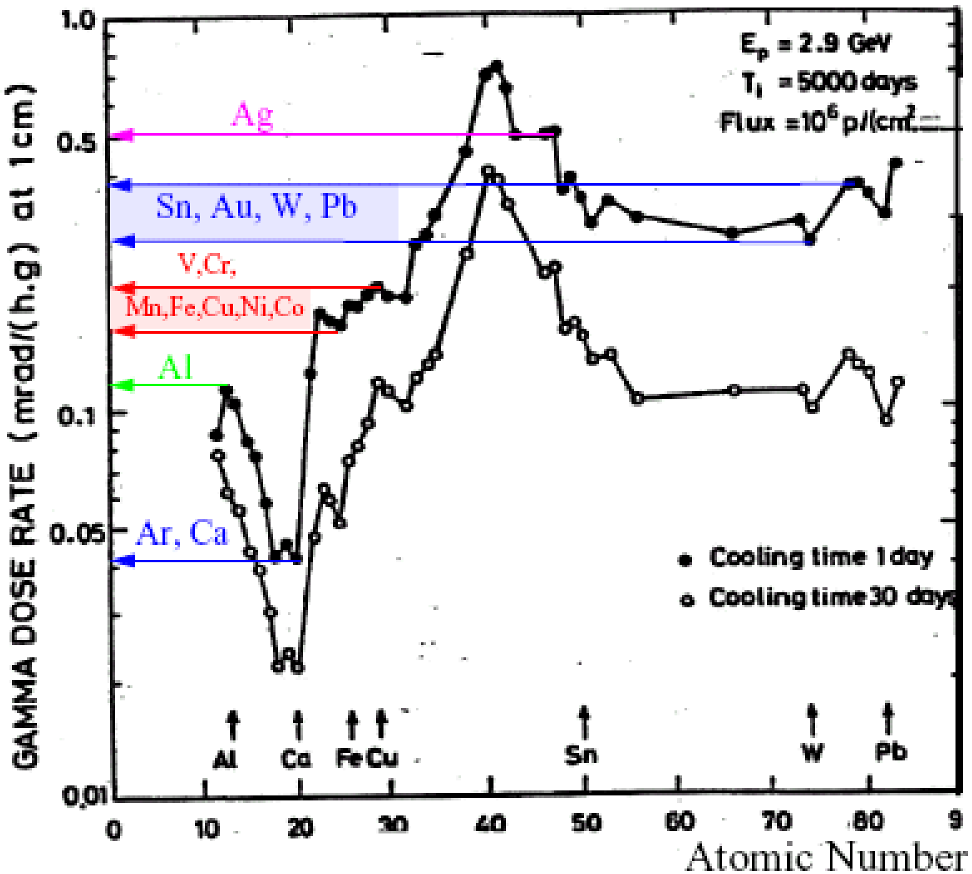
Activation by high energy hadrons

Activation by high energy hadrons

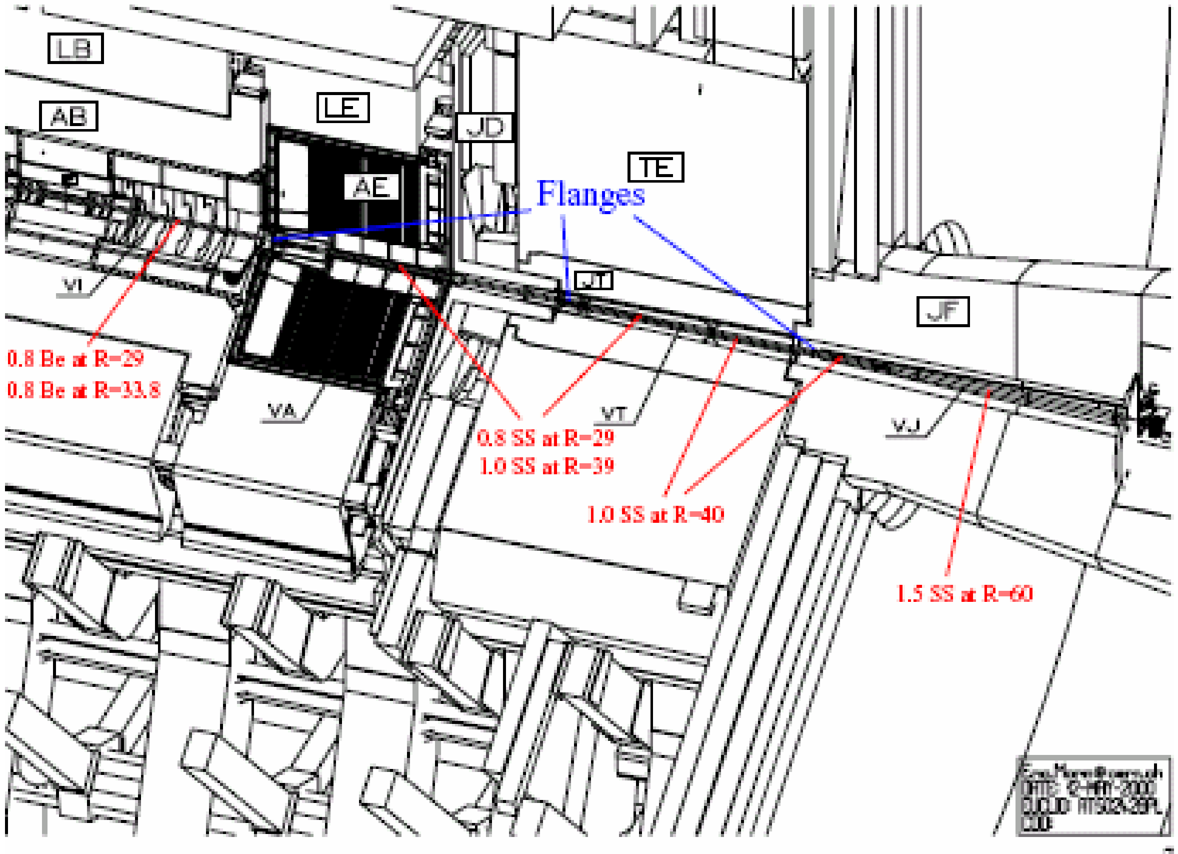


- Very large number of final states
- Not all reaction cross sections are well known
- The activity depends weakly on the material

Calculated activation by 2.9 GeV protons



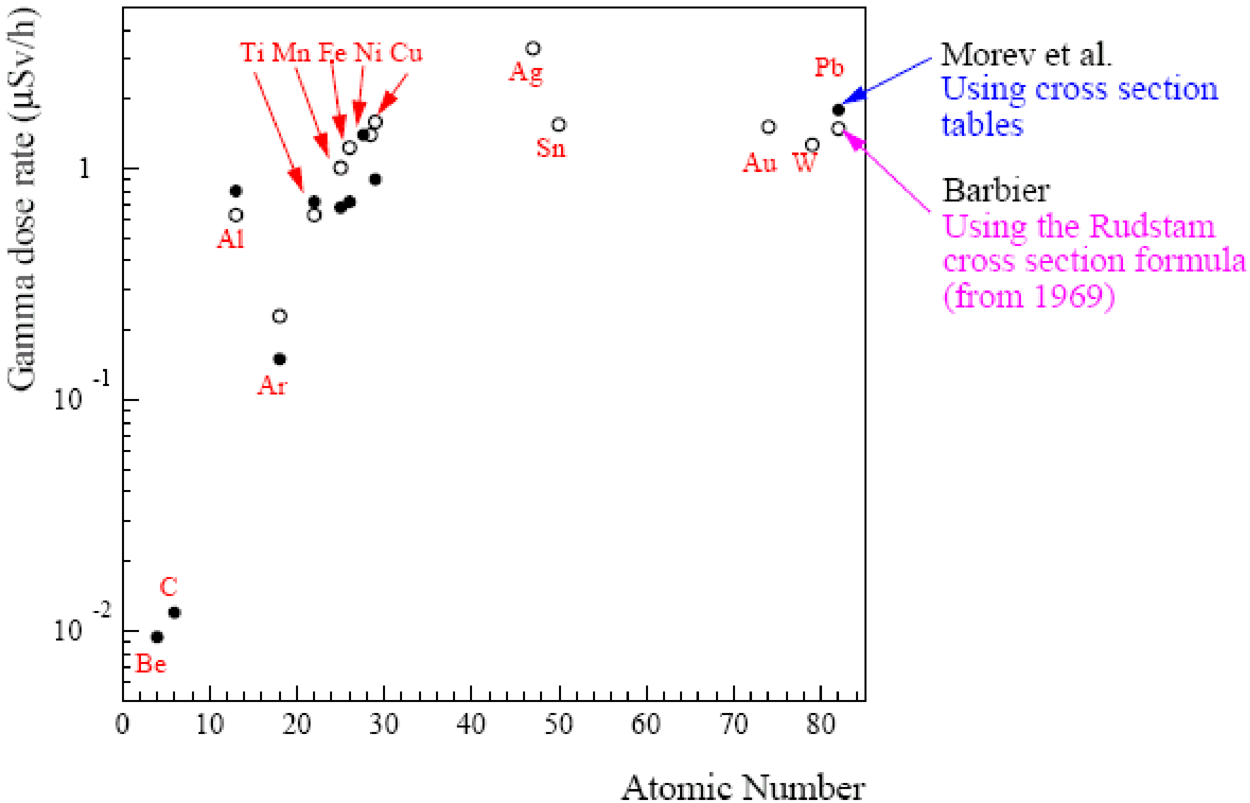
ATLAS example



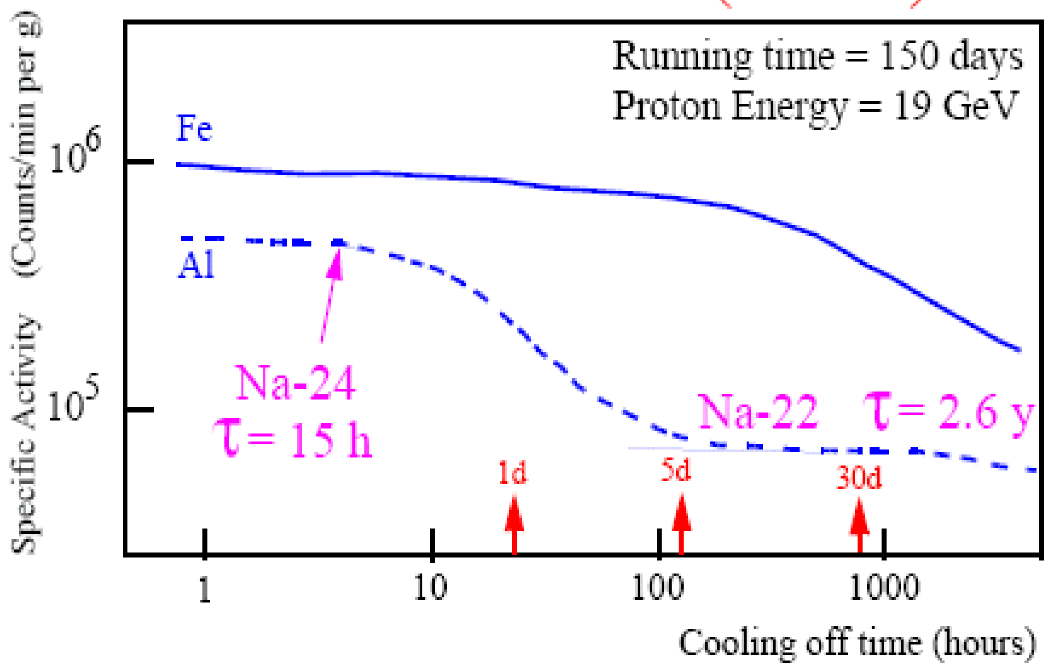
- How much is the activation reduced if beryllium, aluminum or carbon fiber is used instead of stainless steel?
  (beam-beam dominates)

Calculated dose rate from a point source of 1g at a distance of 1 cm.

The radioactivity was induced by 2.9 GeV protons (flux = 10^6 p/cm^2).
The doserate is after 5000 days of irradiation and 30 days of cooling off.



Measurement at the PS (Barbier)



Calculation (Morev)

The ratio of the doserate from a steel and an aluminium beampipe.

Cooling time	Running time 5000d	1000d	100d	30d
1 d:	4	7	12	12
5 d:	4	8	37	94
30 d:	2	3	12	21