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TRANSVERSE OPTICS IN THE ERL ARCS

31 May - 10 June 2016

Cern Accelerator School
Free Electron Lasers and Energy Recovery Linacs

from previous lectures…

WHAT HAVE YOU LEARNED SO FAR?

Concept of ENERGY RECOVERY

✧ The electron-beam energy, produced by accelerating electrons in electromagnetic fields of particle accelerators, can be recycled

✧ The RF fields, by proper choice of the time of arrival of the electron bunches in the linac beam, are used to both accelerate and decelerate the same beam

Potential of ENERGY RECOVERY LINAC

✧ Combines the two worlds of storage rings and linacs and it features some advantages of both arrangements

✧ Flexible modes of operation adaptable to user requirements

Applications of ENERGY RECOVERY LINAC

✧ Many projects and proposals worldwide
Light sources, colliders, fixed target and gas target experiment, electron cooling, compact sources…

WHAT HAVE YOU LEARNED SO FAR?

Main features to be addressed when designing an ERL accelerator

✧ Choice of injection energy

✧ Number of passes through the linac

✧ General features of the linac topology, such as the use of single or multiple linacs, the use of asymmetric gains in multiple linacs, and the connectivity of the recirculation path

✧ Details of phase-space management, such as the degree of functional modularity and specific schemes for longitudinal and transverse matching

✧ Phase-space preservation throughout the acceleration and energy-recovery cycle

✧ Control of beam halo

ERL Beam Dynamics: potentials and issues

✧ Goal Beam Parameter & Energy Recovery Issues
- efficient energy recovery
- beam quality
- Bunch manipulation

✧ Linear Beam Optics

✧ Nonlinear Beam Optics

✧ Collective Effects
- Space charge effects
- Coherent synchrotron radiation
- Geometric waves

✧ BBU –beam break up

✧ Unwanted Beam: dark current & halo

✧ Ion trapping

WHAT WILL YOU LEARN NOW?

Optics in the ERL design

- ✧ **Optics design and optimization**
  Linac
  Arcs
  Spreader and Re-combiner

- ✧ Per each component, constraints, issues and possible solutions will be showed along with examples from studies on existing/proposed machines

- ✧ The idea is to give you a method, a possible way to proceed

- ✧ **Simulation tools**

ERL-general features

EXAMPLE OF A MULTI-PASS ERL

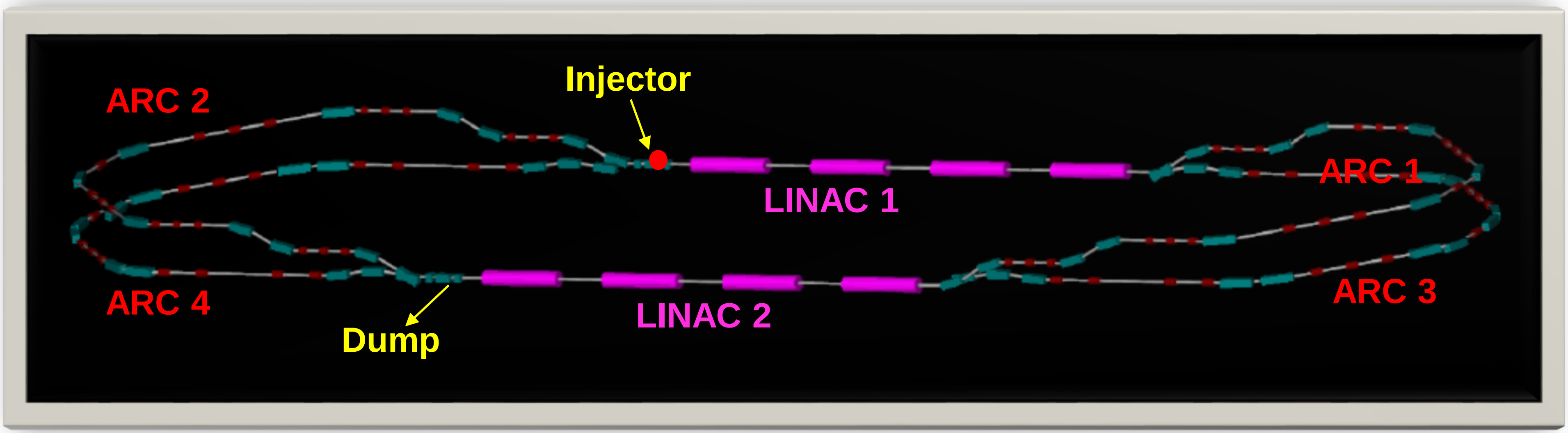
Main components

Injector

Linacs

Optics transport lines

Beam dump

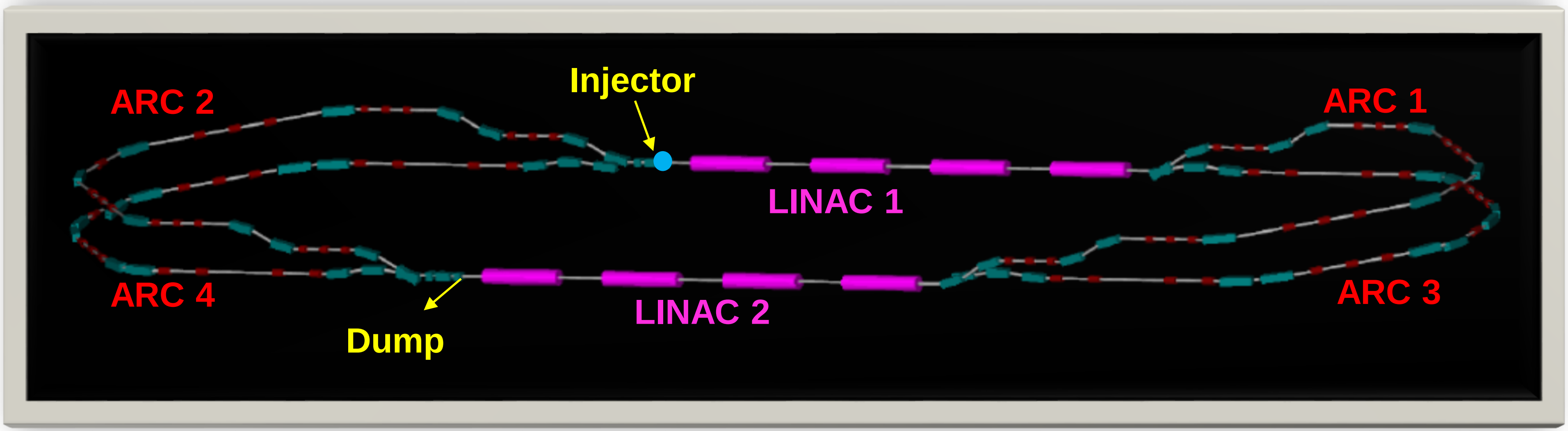


Two passes ‘up’ for acceleration

EXAMPLE OF A MULTI-PASS ERL

After acceleration the beam is phase shifted by 180°
and then sent back through the recirculating linac at a decelerating RF phase

Two passes 'down' for deceleration



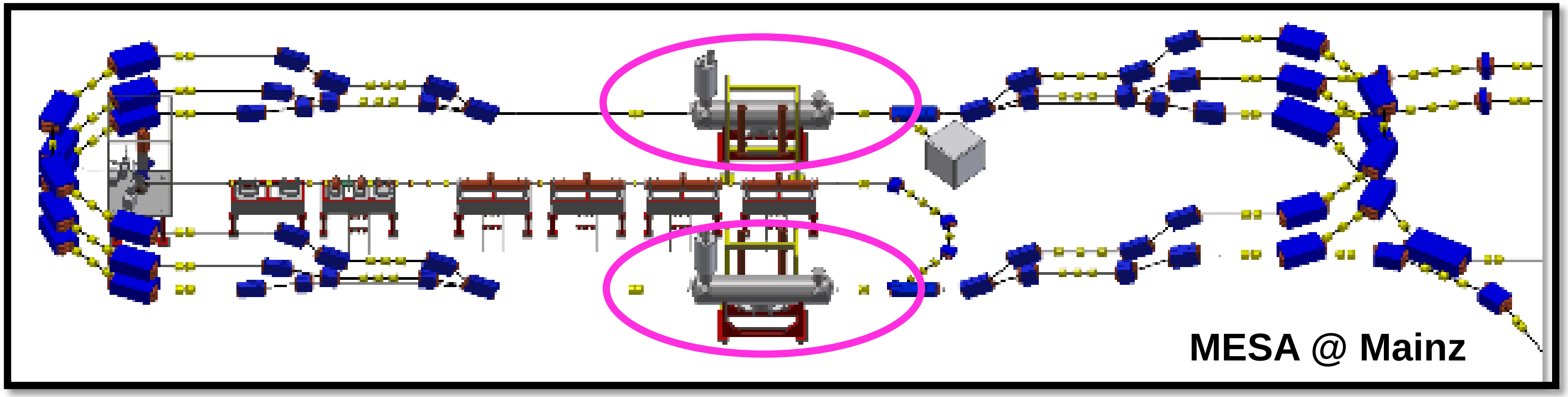
During deceleration the energy stored in the beam is reconverted to RF energy
and the final beam, at its original energy, is directed to a beam dump

Transport Optics

TRANSPORT OPTICS

Appropriate recirculation optics are of fundamental concern in a multi-pass machine to preserve beam quality

The design comprises different regions:
the linac optics



The focusing strength of the quadrupoles along the linac needs to be set to transport co-propagating beams of different energy and to (eventually) support a large number of passes

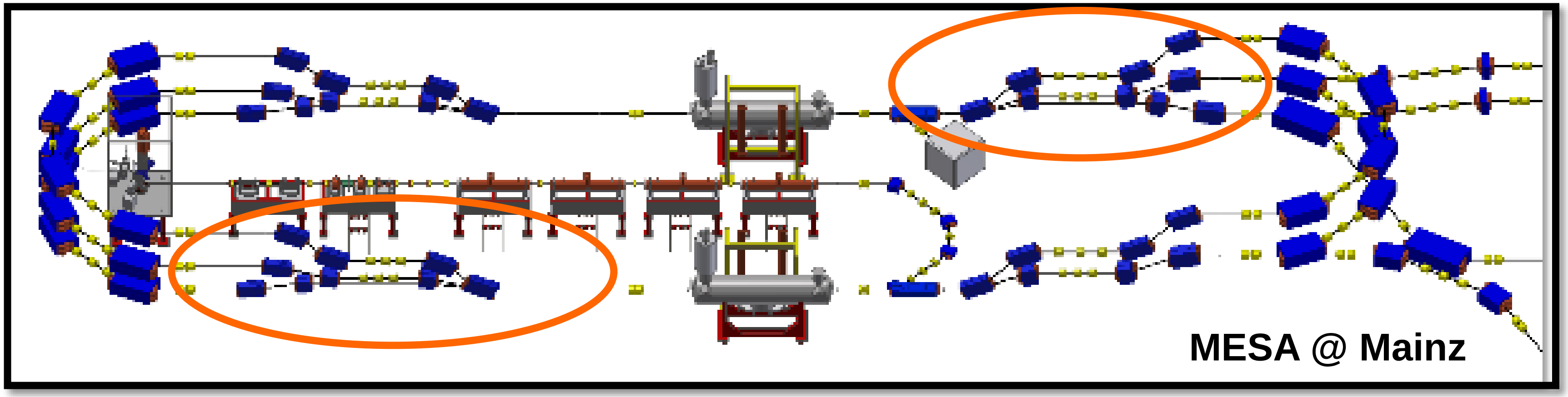
The focusing profile depends on many parameters such as number of cavity, BBU threshold current, beam properties…

TRANSPORT OPTICS

Appropriate recirculation optics are of fundamental concern in a multi-pass machine to preserve beam quality

The design comprises different regions:

The Spreader optics



At the end of the linac the beams need to be directed into the appropriate energy dependent arc

Spreaders separate horizontally or vertically beams and match optics functions to arcs

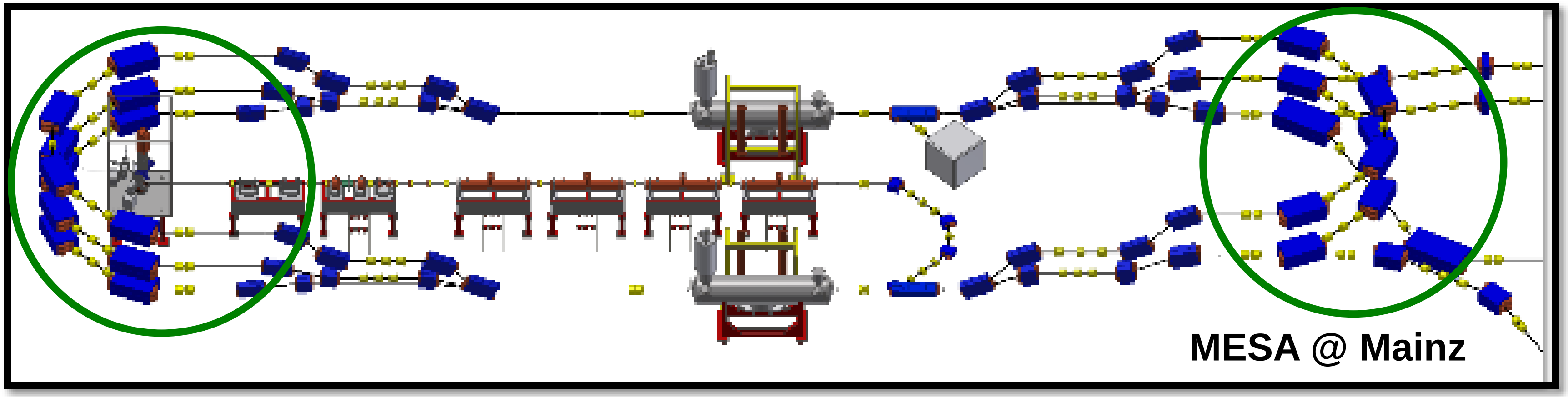
Important parameters: energy loss and β values

TRANSPORT OPTICS

Appropriate recirculation optics are of fundamental concern in a multi-pass machine to preserve beam quality

The design comprises different regions:

The Arc optics



Choice of the base cell depends on main parameters

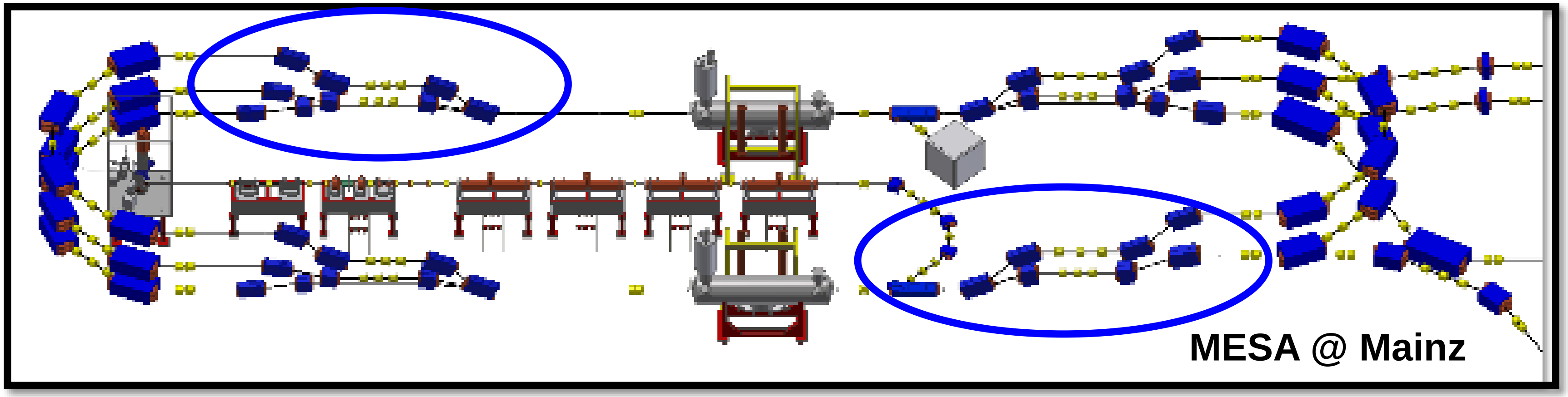
e.g. In high-energy machines disturbing effects on the beam phase-space such as cumulative emittance and momentum growth have to be counteracted through a pertinent choice of the basic optics cell

TRANSPORT OPTICS

Appropriate recirculation optics are of fundamental concern in a multi-pass machine to preserve beam quality

The design comprises different regions:

The Re-combiner optics



Spreader and combiners are mirror symmetric

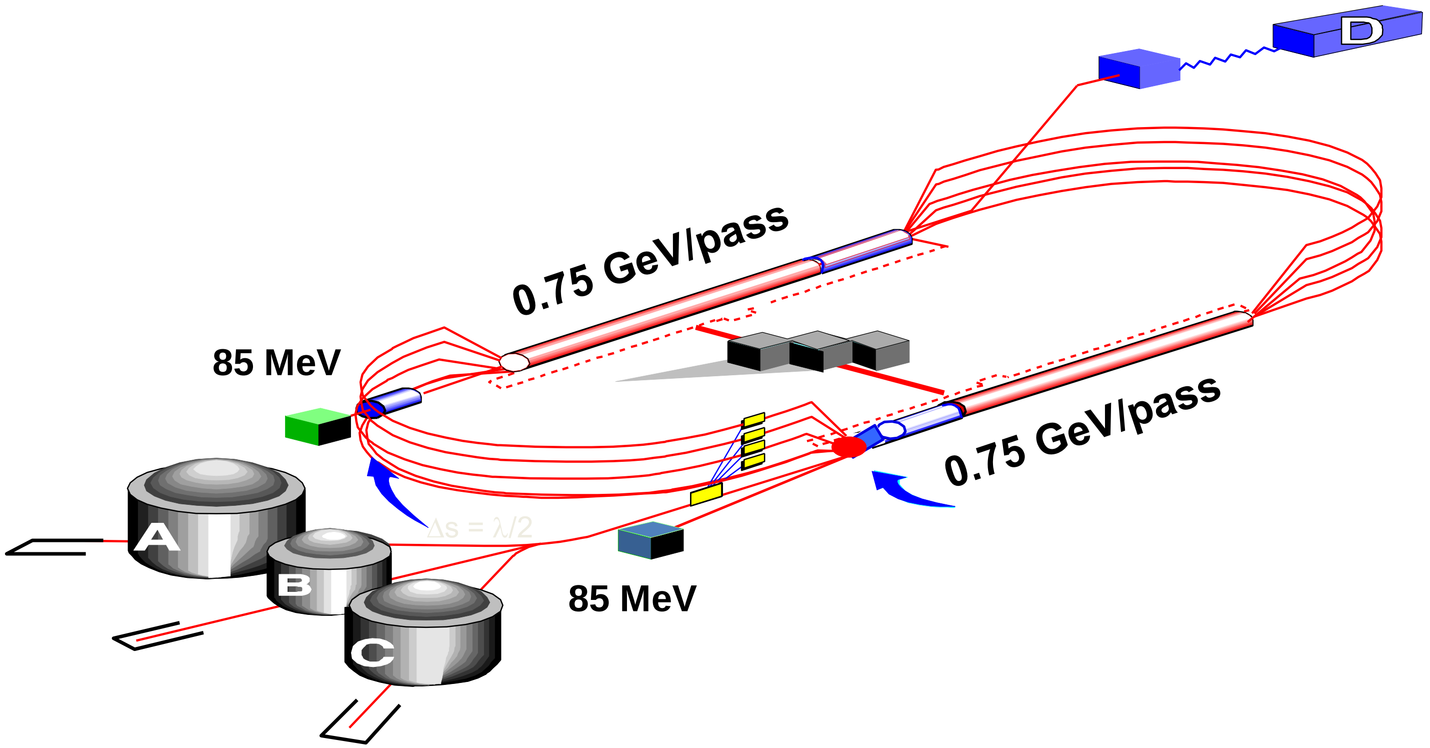
Combiners merge horizontally or vertically beams and match optics functions to linac

Linac Optics

OPTICS CONSTRAINTS

Define the main constraints in the optics design for the linac

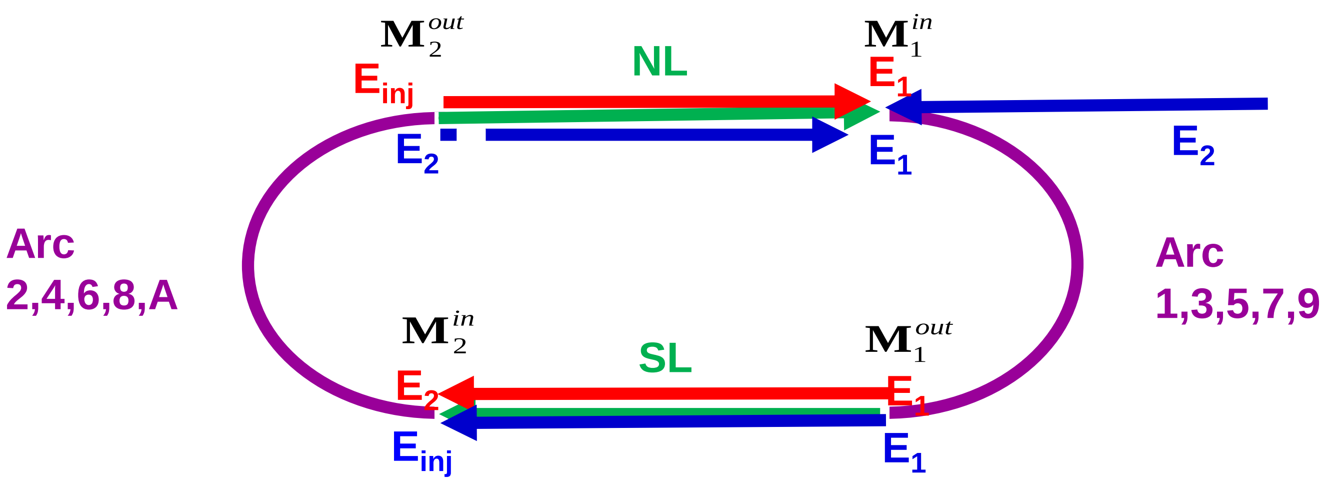
How linac optics has been designed?



CEBAF recirculating linear accelerator
5-pass ERL

OPTICS CONSTRAINTS

When decelerating, the beam keeps turning in the same direction, therefore any possible arc matching aiming at optimising the Twiss functions at each linac injection during the acceleration, would cause a mismatch during the deceleration

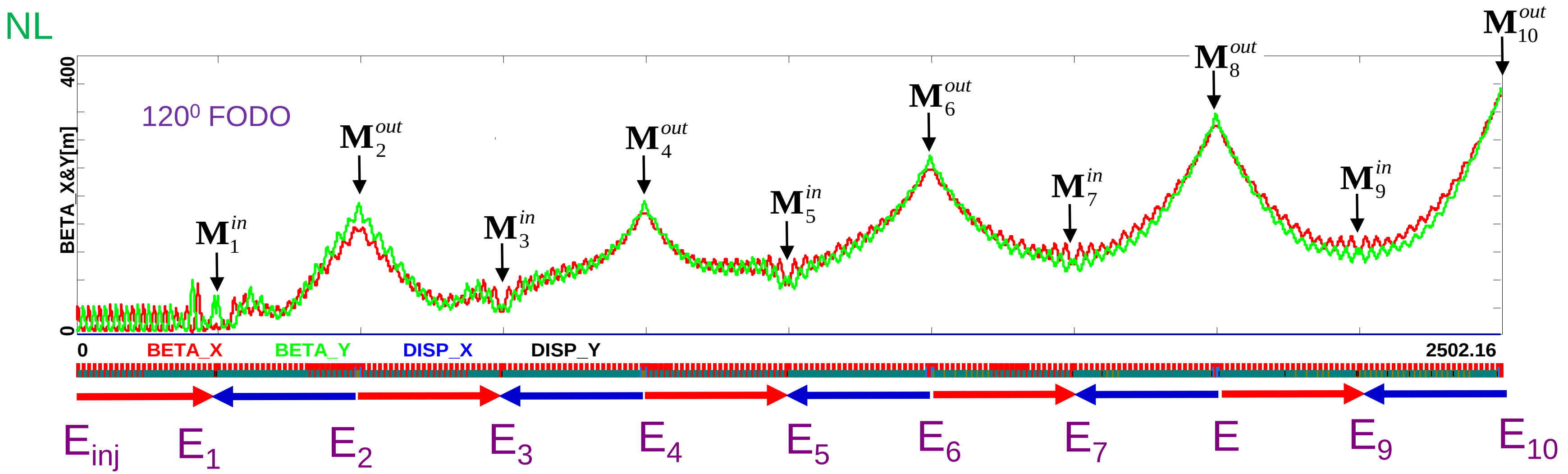


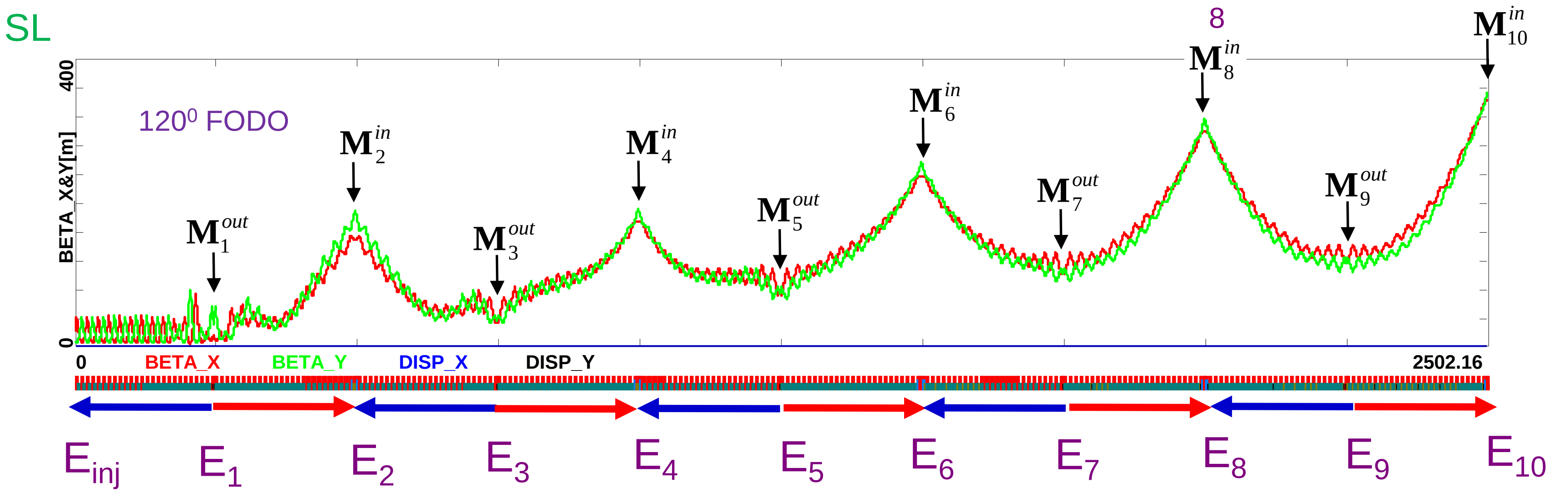
$$\mathbf{M} = \begin{bmatrix} \beta_x \\ -\alpha_x \\ \beta_y \\ -\alpha_y \end{bmatrix}$$

The Twiss functions must be preserved, with the only exception of the sign of $\beta' = -2\alpha$

MULTI-PASS LINAC OPTICS

The optics of the two linacs are symmetric, the first being matched to the first accelerating passage and the second to the last decelerating one





OPTIMIZATION CRITERIA

1. **The optimization of the linac optics aims at mitigating the impact of imperfections and collective effects such as wake-fields driven by**

$$\left\langle \frac{\beta}{E} \right\rangle = \int_{\text{Acceleration}} \frac{\beta}{E} ds,$$

MINIMUM →

Free parameters:
Input optics functions
(β function and its derivative)
Quads Strength profile

2. **One should also consider the interaction of bunches at different turns, resulting in the integrals**

$$I_{ij} = \int_{\text{Linac1,2}} \frac{\sqrt{\beta_i \beta_j}}{\sqrt{E_i E_j}} ds,$$

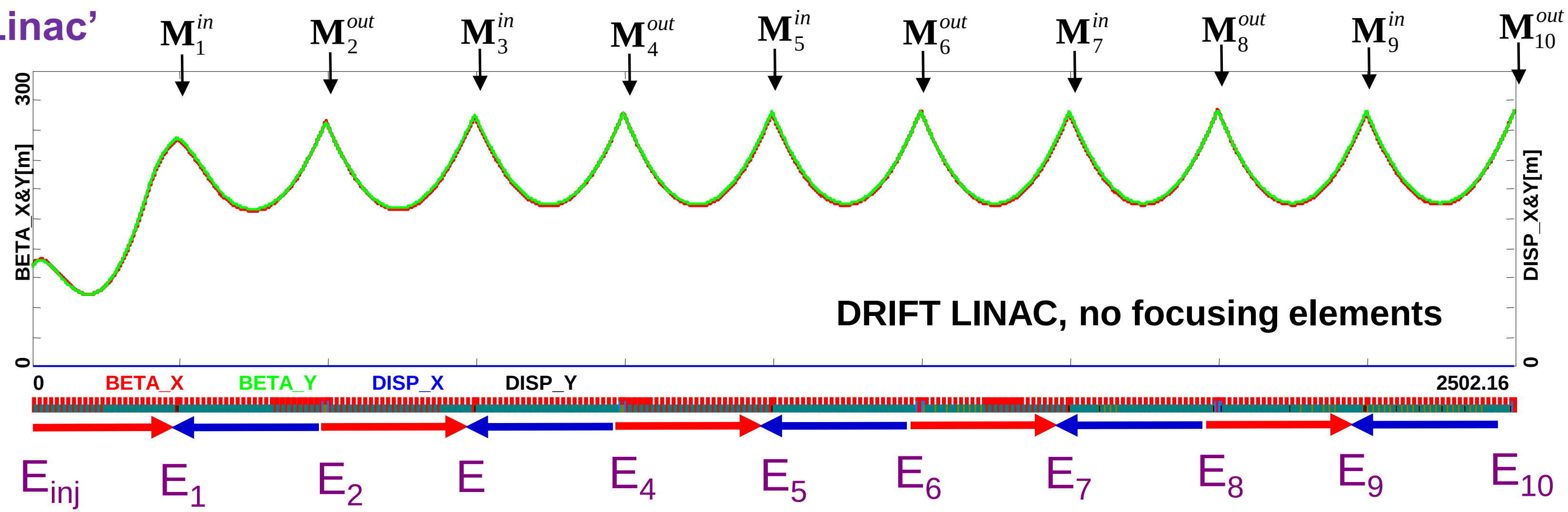
where the energy and the β functions need to be evaluated for the different turn numbers: i, j

$$F = \sqrt{(I_{11} + I_{22} + I_{33})^2 + 2(I_{12} + I_{23})^2 + 2(I_{13})^2}.$$

Merit function (for acceleration only) to be minimised

MULTI-PASS LINAC OPTICS: OPTIMIZATION

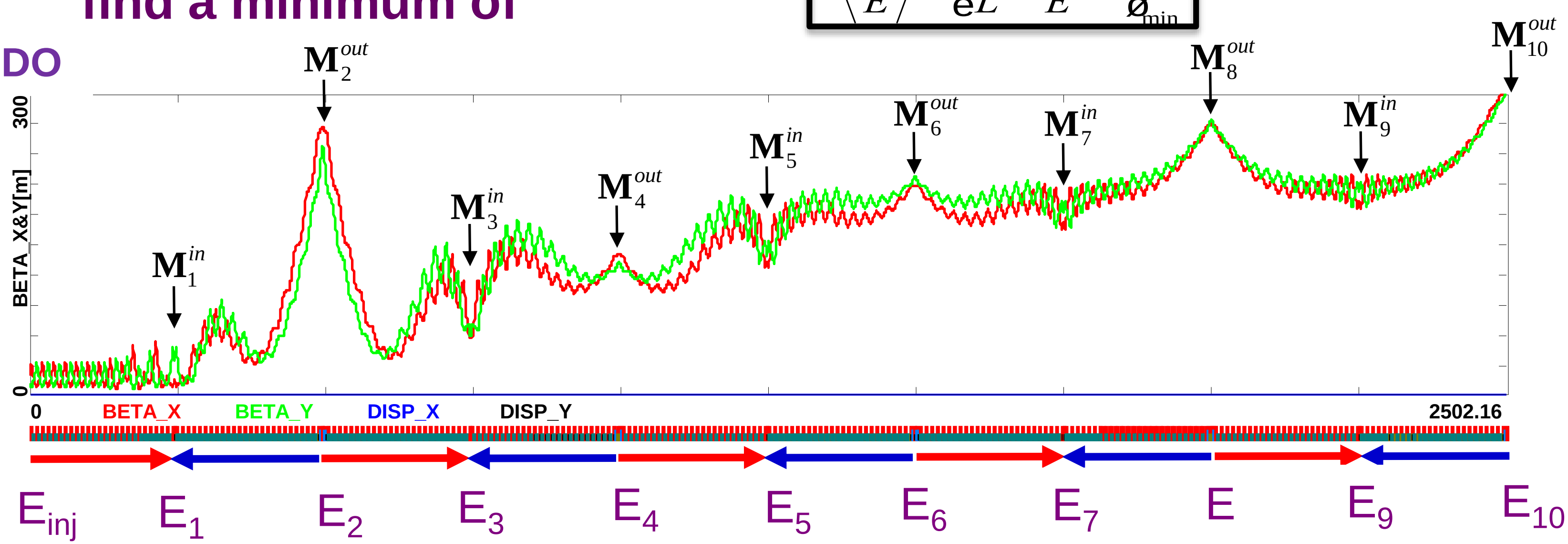
'Drift Linac'



Evaluate differents optics to find a minimum of

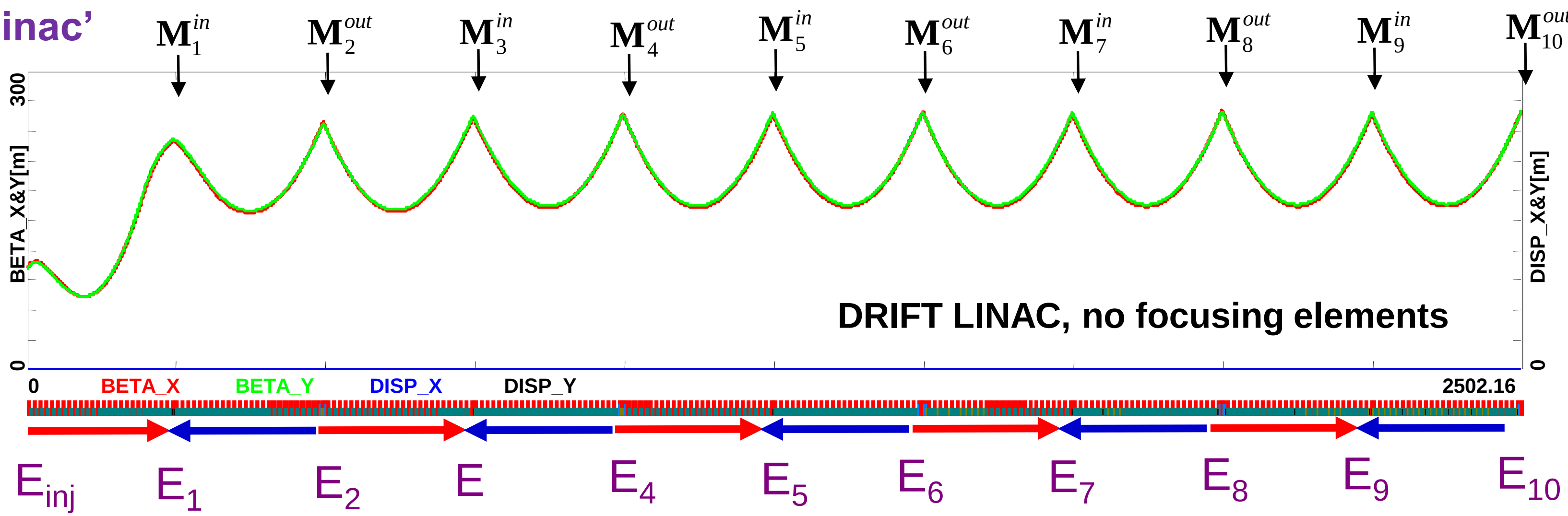
$$\left\langle \frac{\beta}{E} \right\rangle = \left(\frac{1}{L} \int \frac{\beta}{E}\, ds \right)_{min}$$

90⁰ FODO



MULTI-PASS LINAC OPTICS: OPTIMIZATION

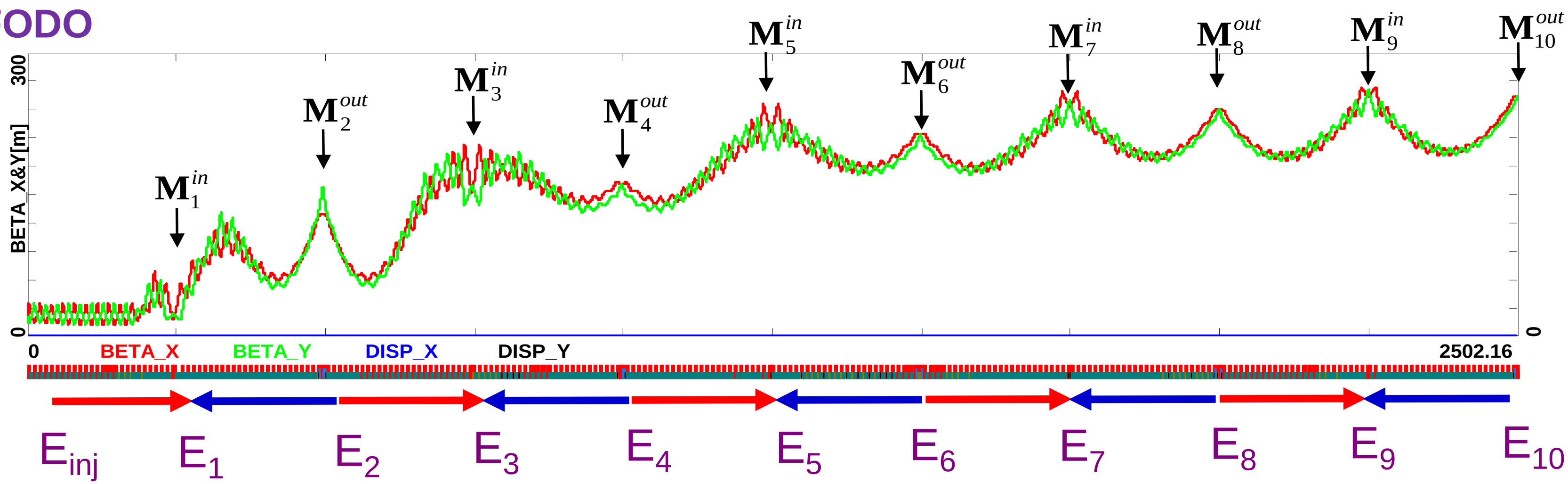
‘Drift Linac’



Evaluate differents optics to find a minimum of

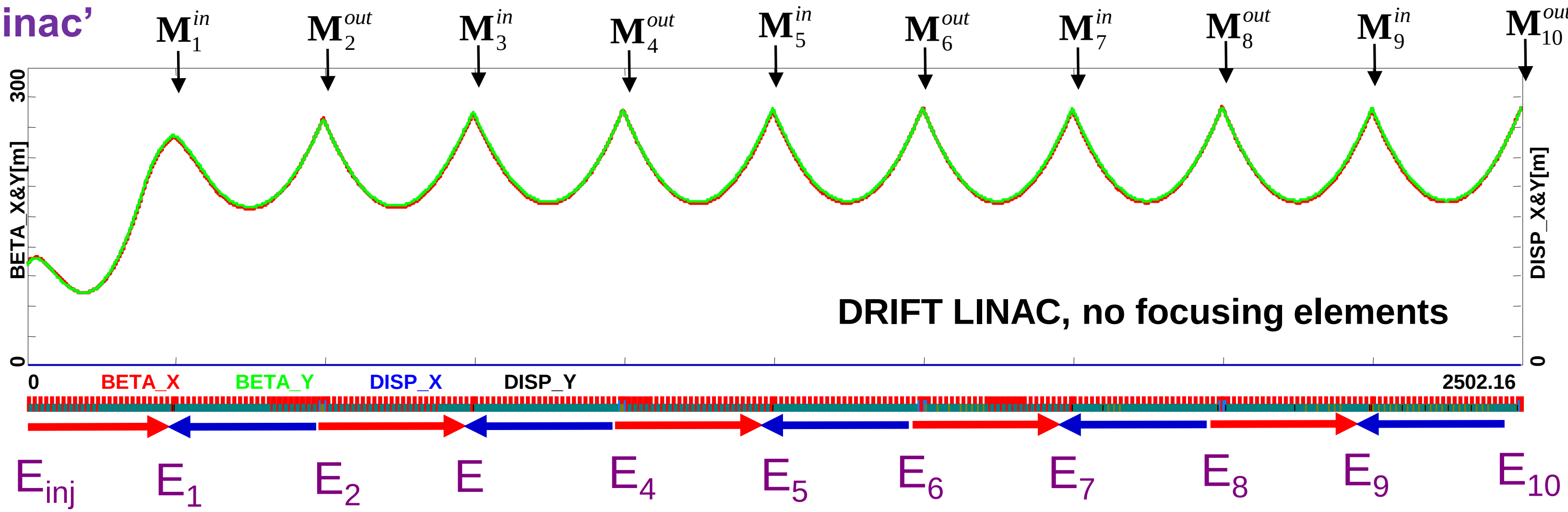
$$\left\langle \frac{\beta}{E} \right\rangle = \left(\frac{1}{L} \int \frac{\beta}{E}\, ds \right)_{\min}$$

60^0 FODO



MULTI-PASS LINAC OPTICS: OPTIMIZATION

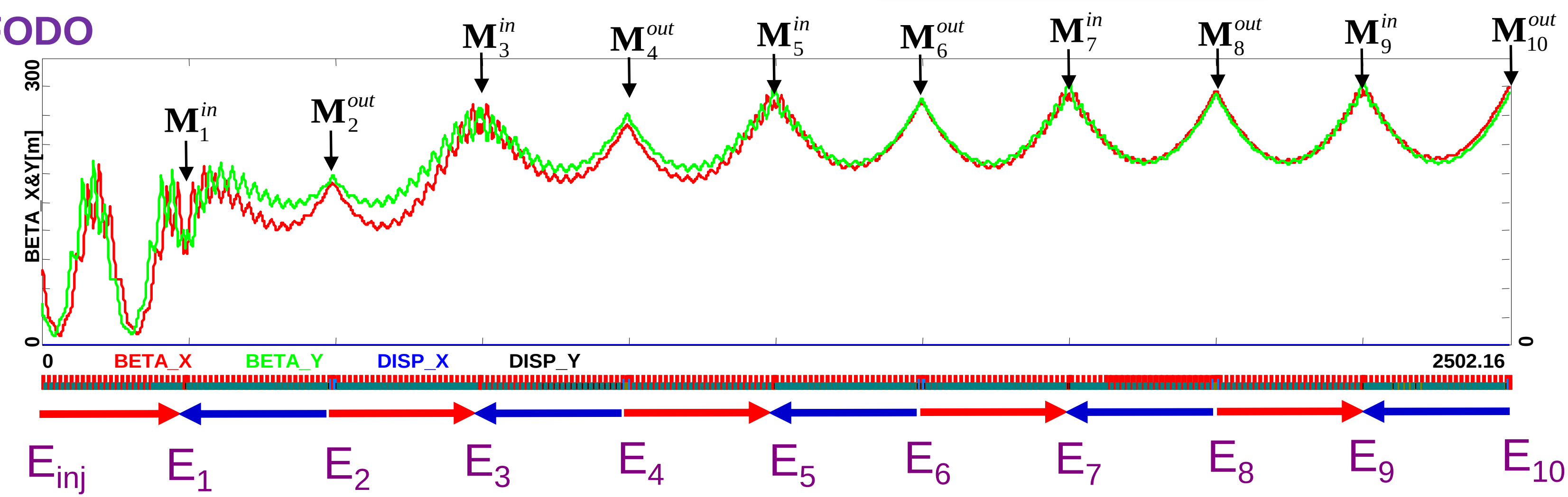
'Drift Linac'



Evaluate differents optics to find a minimum of

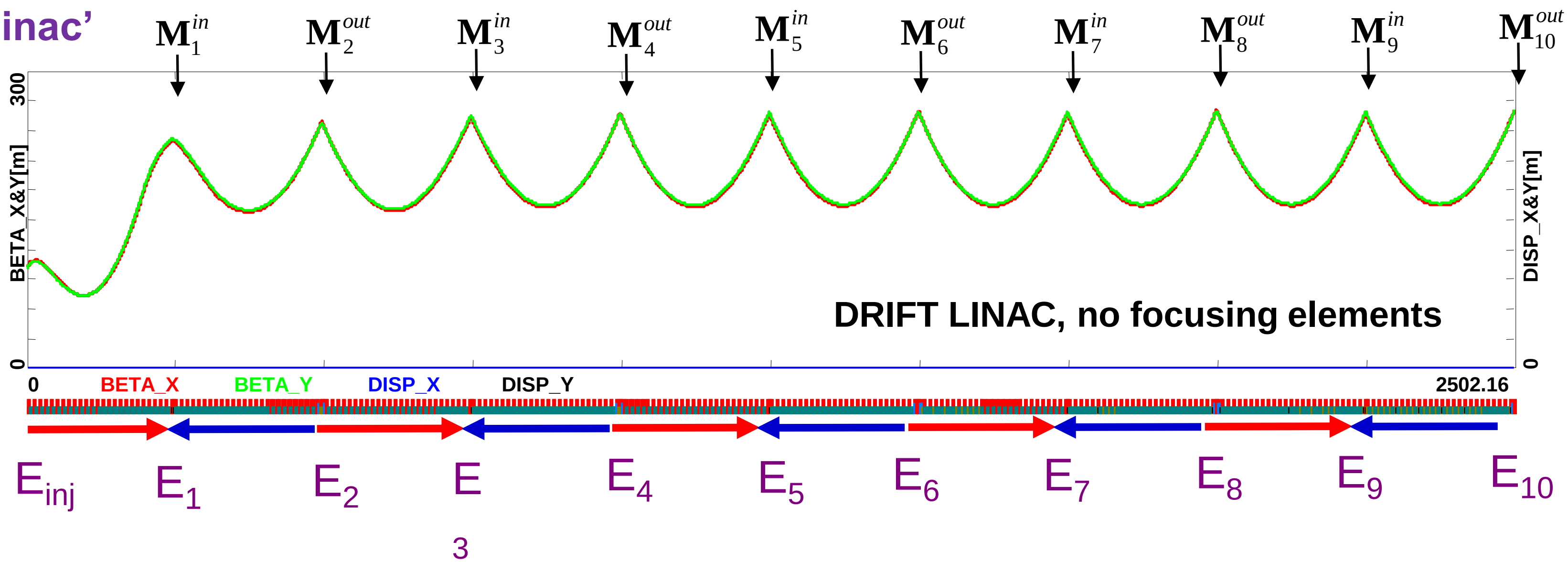
$$\left\langle \frac{\beta}{E} \right\rangle = \left(\frac{1}{L} \int \frac{\beta}{E}\, ds \right)_{\min}$$

30^0 FODO



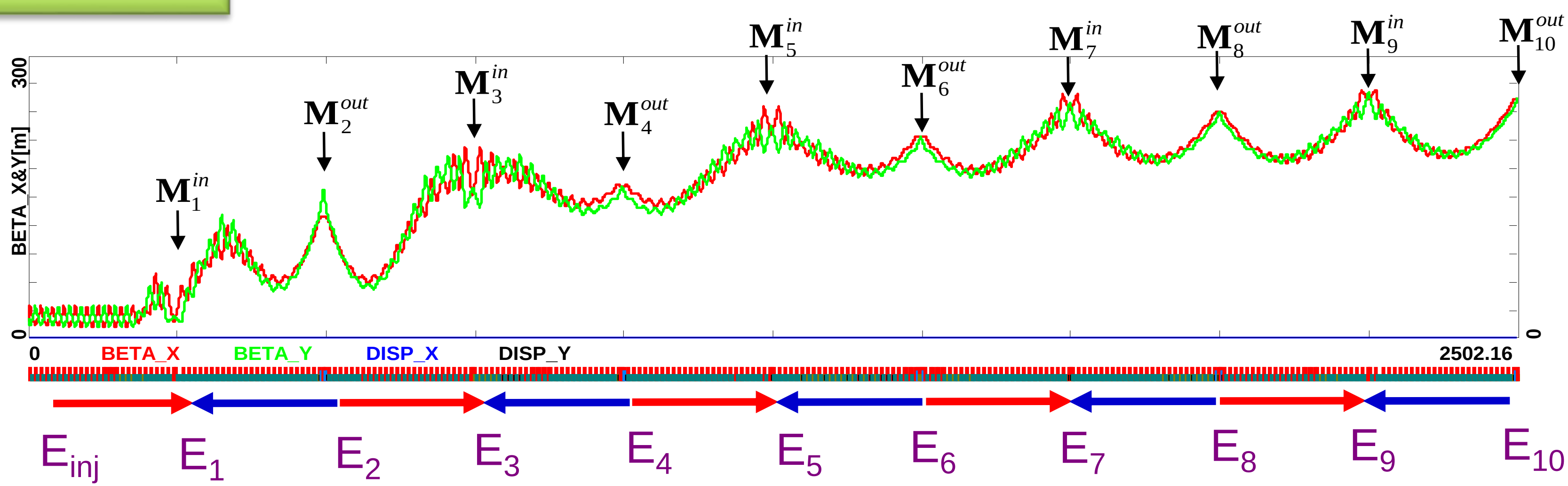
MULTI-PASS LINAC OPTICS: OPTIMIZATION

'Drift Linac'



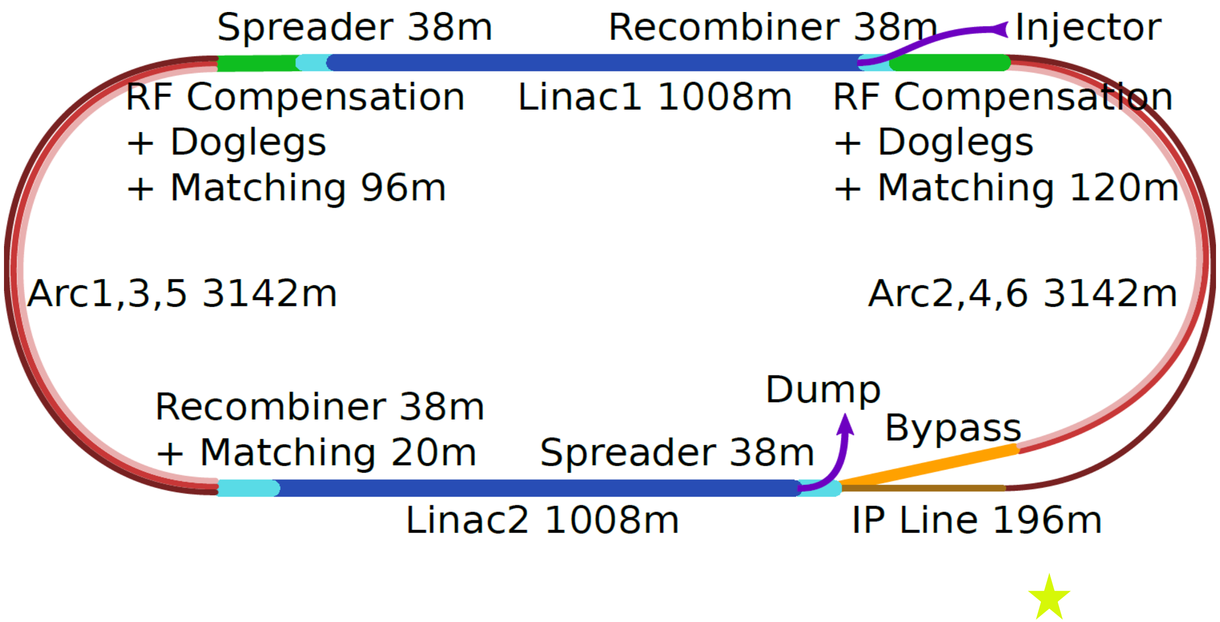
FIND A SOLUTION

60^0 FODO



LINAC OPTICS: EXAMPLE

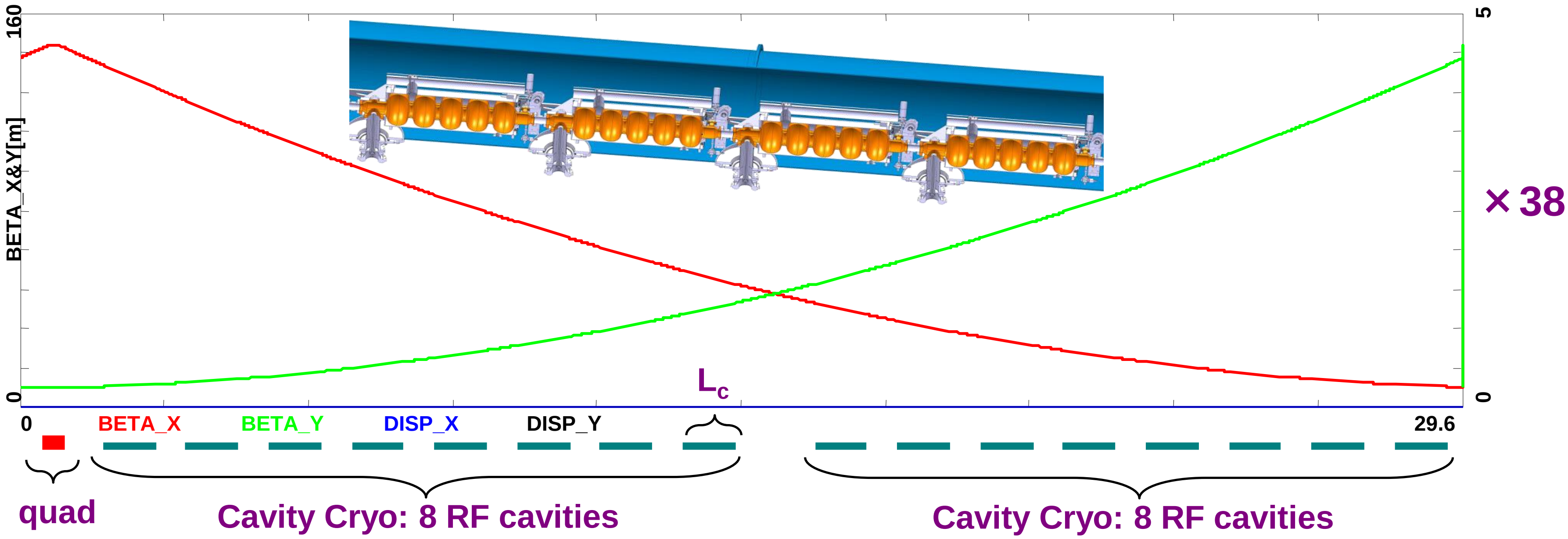
LHeC recirculating linear accelerator
3-pass ERL

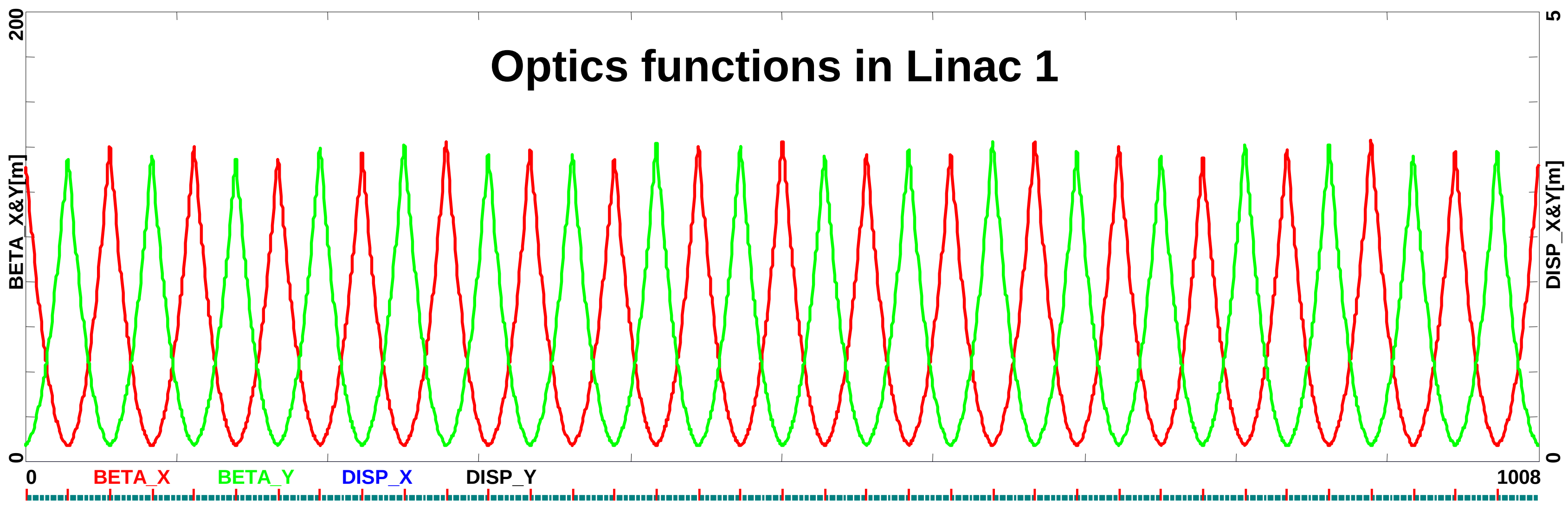


RECIRCULATOR COMPLEX

1. 0.5 Gev injector
2. Two 1km SCRF linacs (10 GeV per pass)
3. Six 180° arcs, each arc 1 km radius
4. Re-accelerating stations
5. Switching stations
6. Matching optics
7. Extraction dump at 0.5 GeV

10 GEV LINAC OPTICS: FOCUSING PROFILE

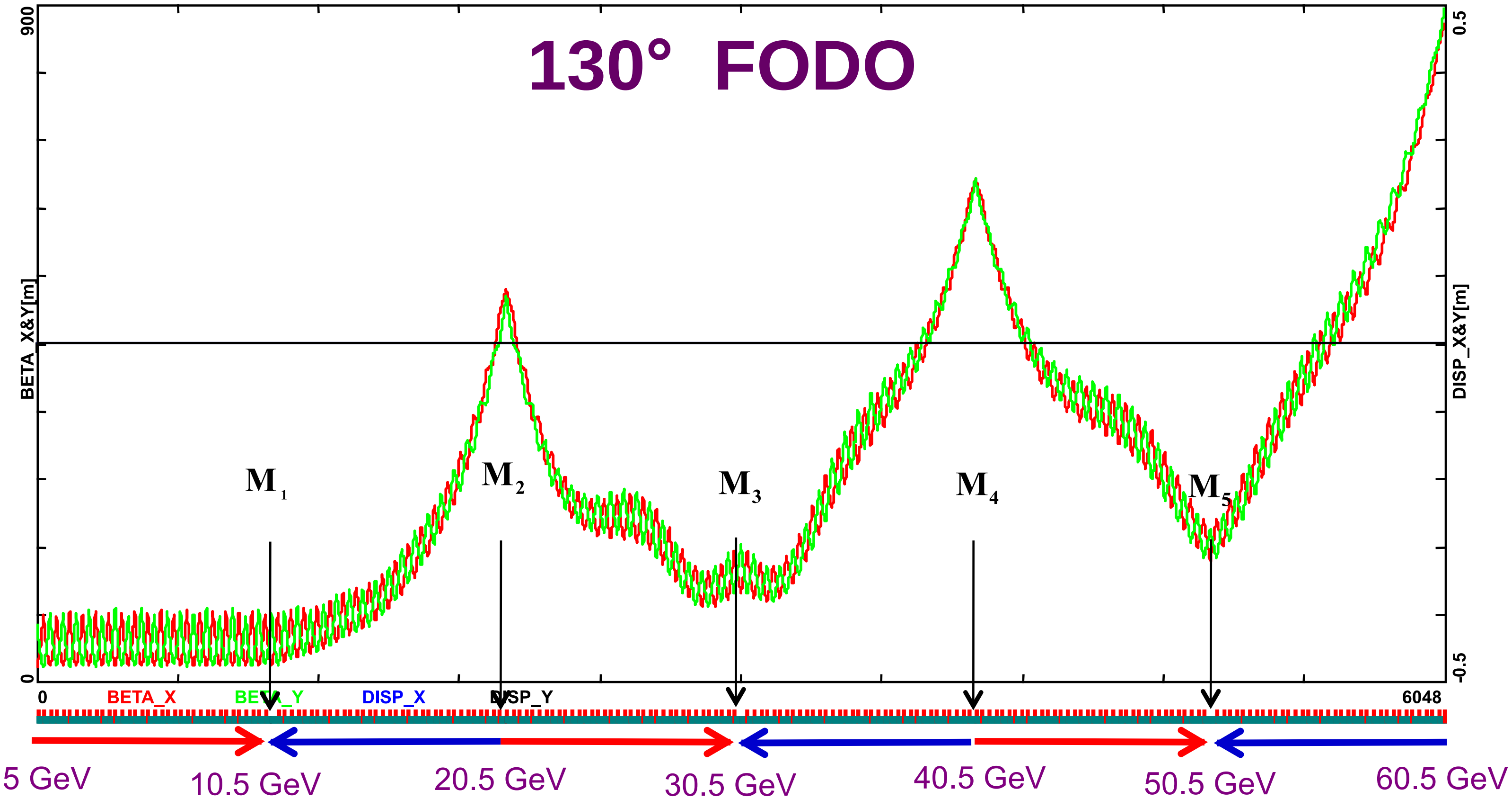




10 GeV LINAC OPTICS: FOCUSING PROFILE

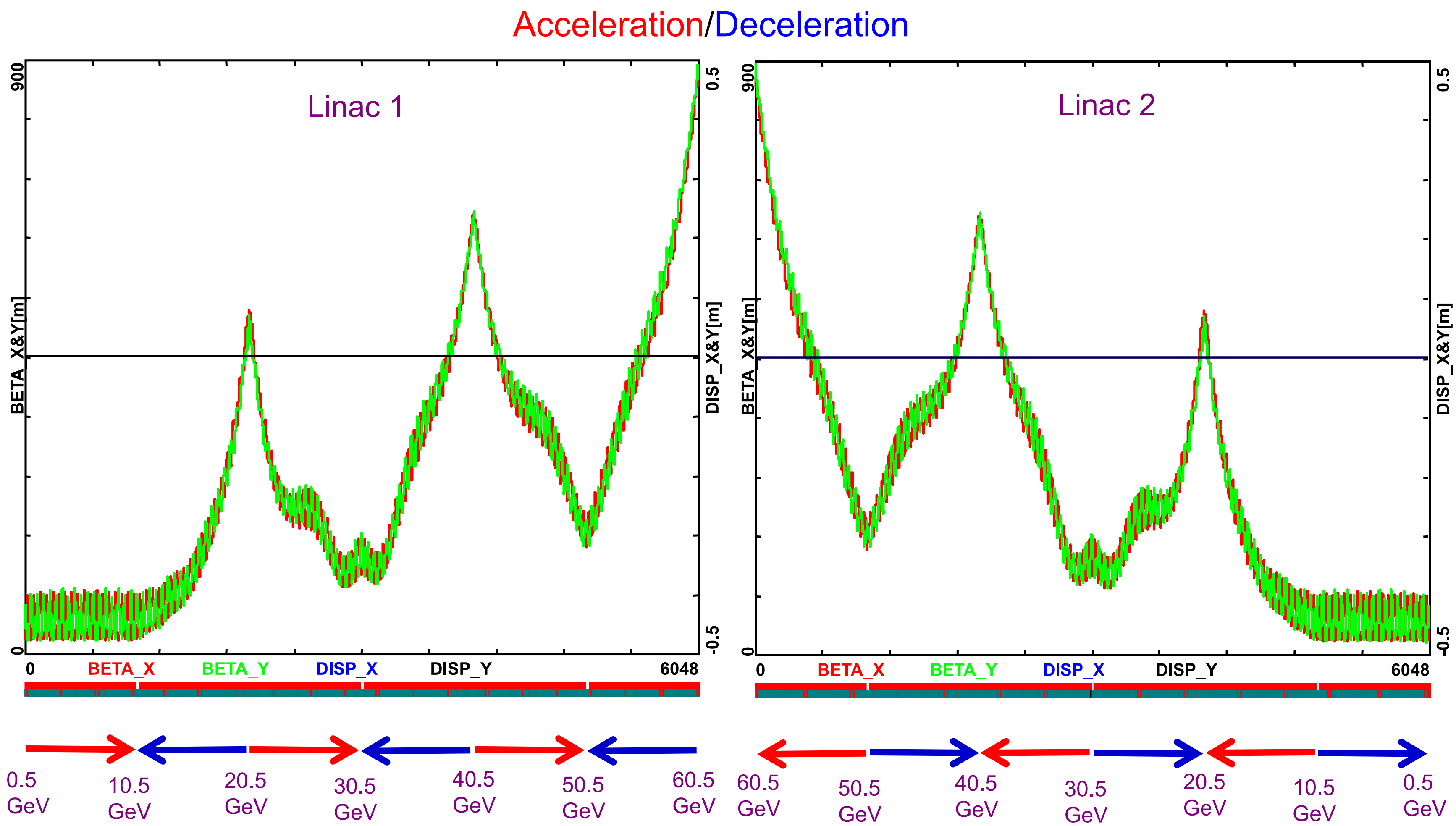
MINIMUM →

$$\left\langle \frac{\beta}{E} \right\rangle = \int_{\text{Acceleration}} \frac{\beta}{E}\, ds,$$



Optics function for 3 passes up for acceleration
and 3 passes down for deceleration

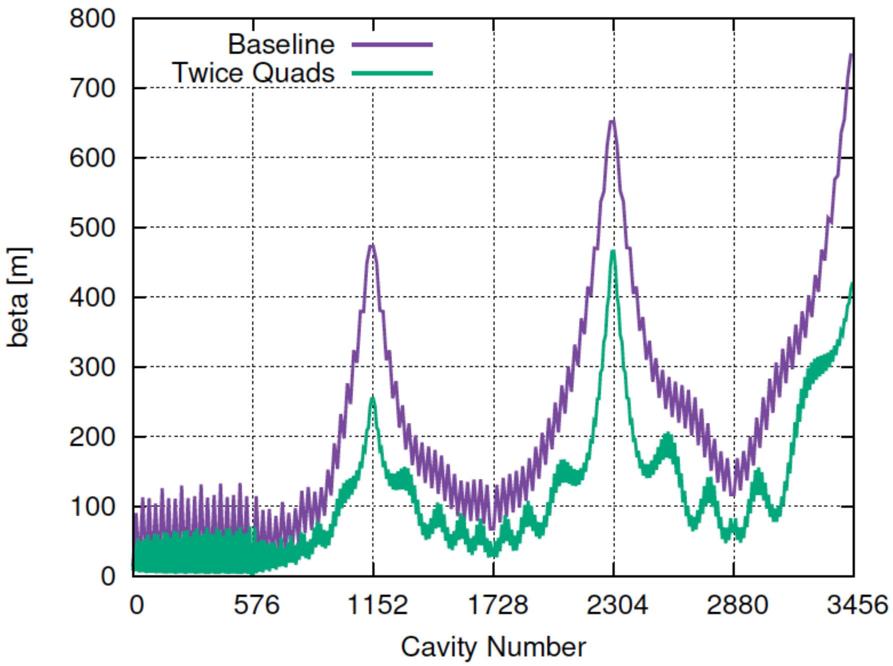
LINAC 1 AND 2: MULTI-PASS OPTICS



The optics of the two linacs are symmetric, the first being matched to the first accelerating passage and the second to the last decelerating one

LINAC OPTICS: FOCUSING PROFILE

Solution is never unique!



Substantial improvements have been obtained doubling the number of quadrupoles (placing a quadrupole after every cryomodule instead of every two)

In this case the merit function is almost halved but the number of quads is doubled

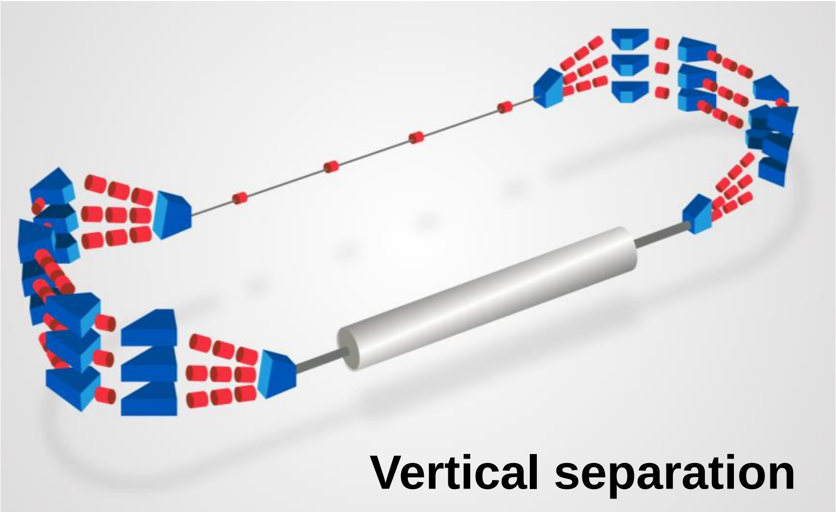
Possible compromise?

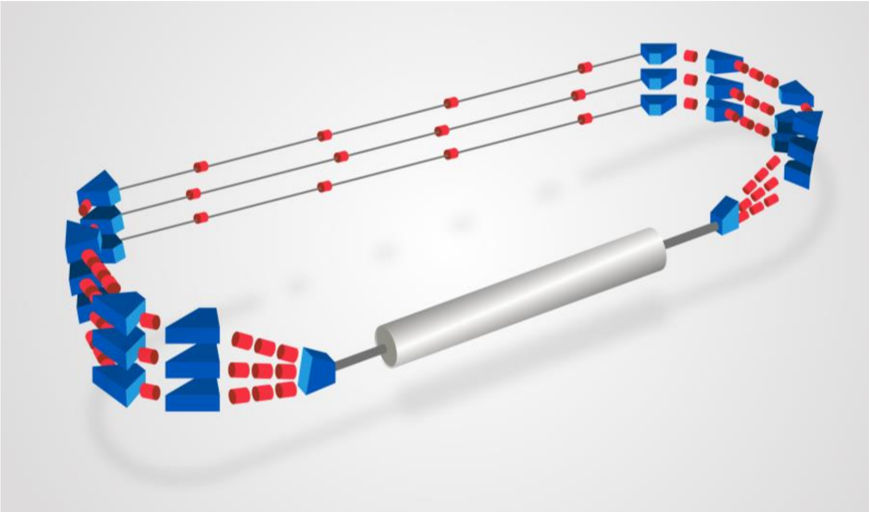
As most of the contribution to the merit function comes from the very low energies, the additional quads could be inserted only in the initial/final part of Linac1/Linac2

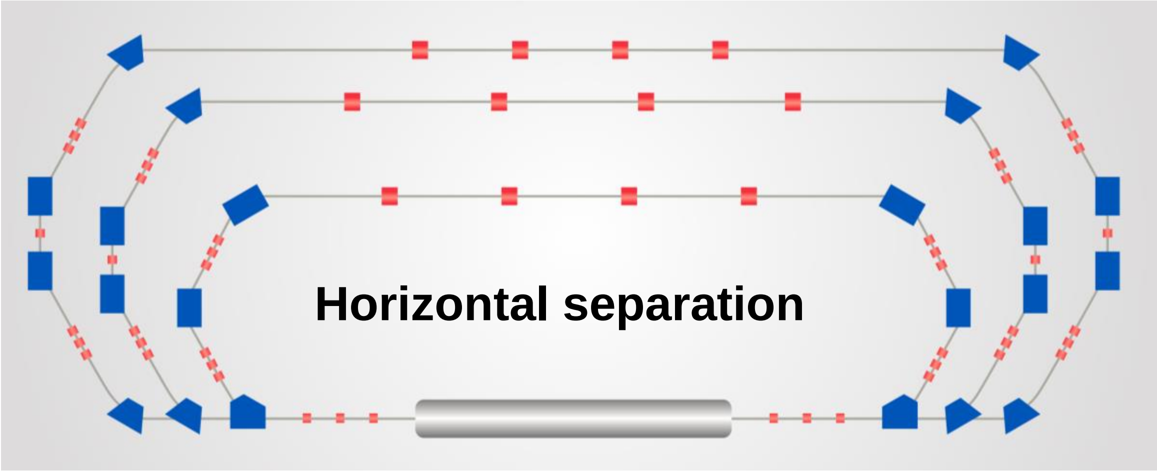
Optics in spreader and combiner

OPTICS IN SEPARATION REGIONS

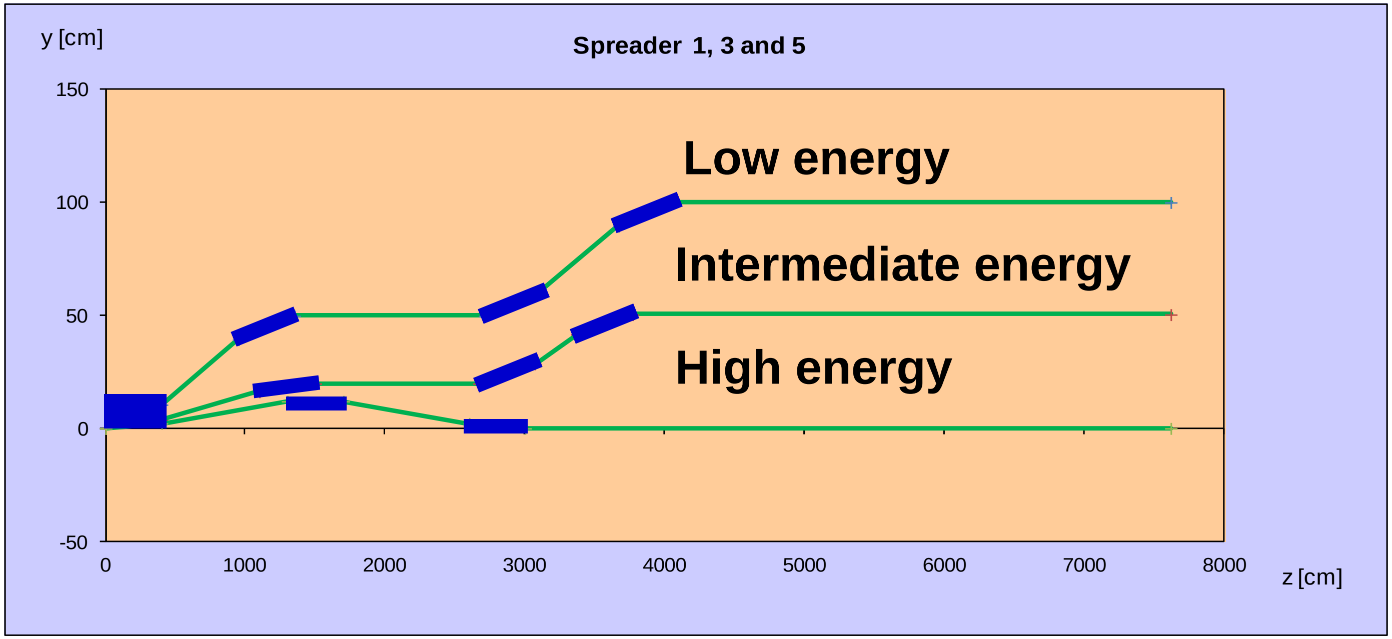
Many possible solutions for different schematics







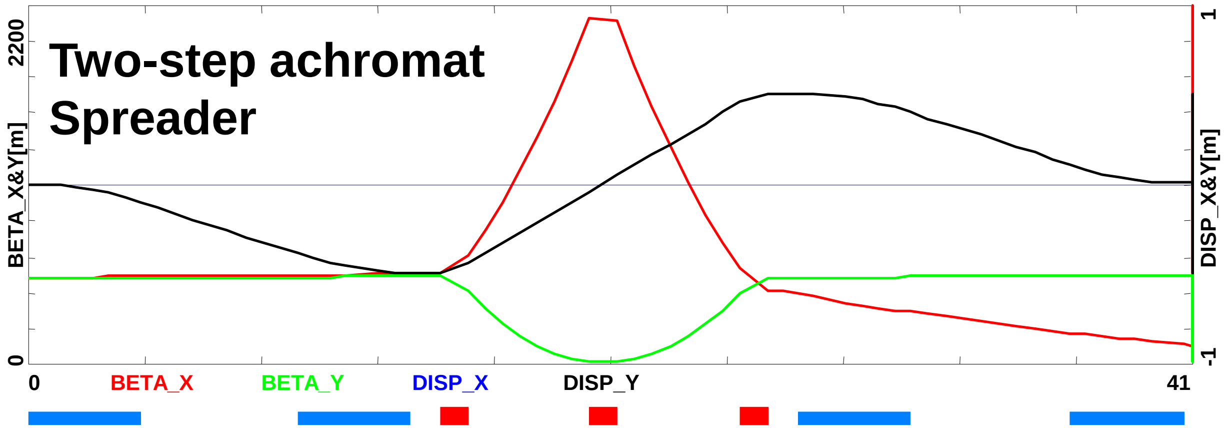
SEPARATION REGIONS: POSSIBLE SOLUTIONS



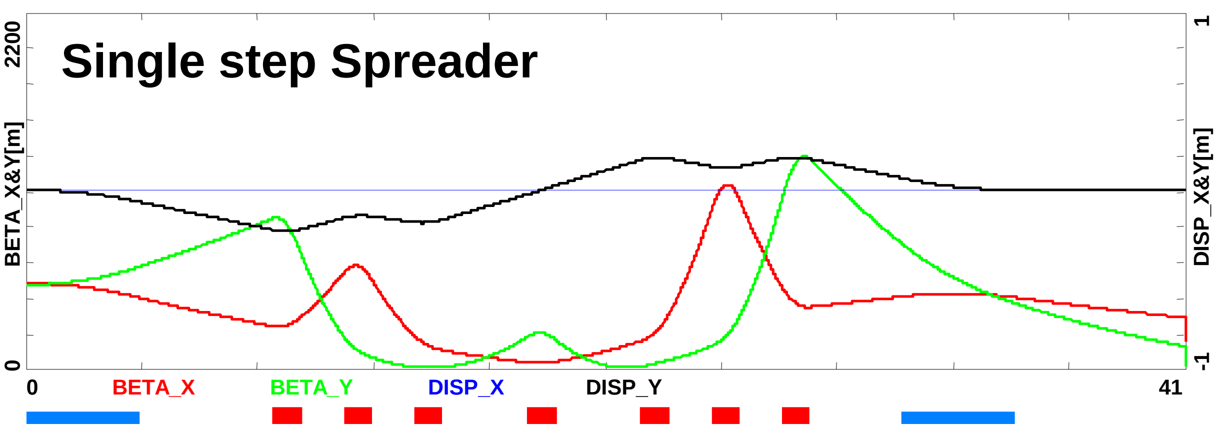
Solution adopted at LHeC, CEBAF, MESA

- ✧ The spreader consists of a vertical bending magnet that initiates the separation
- ✧ The highest energy, at the bottom, is brought back to the horizontal plane with a chicane
- ✧ The lower energies are captured with two-steps vertical bendings

SEPARATION REGIONS: POSSIBLE SOLUTIONS

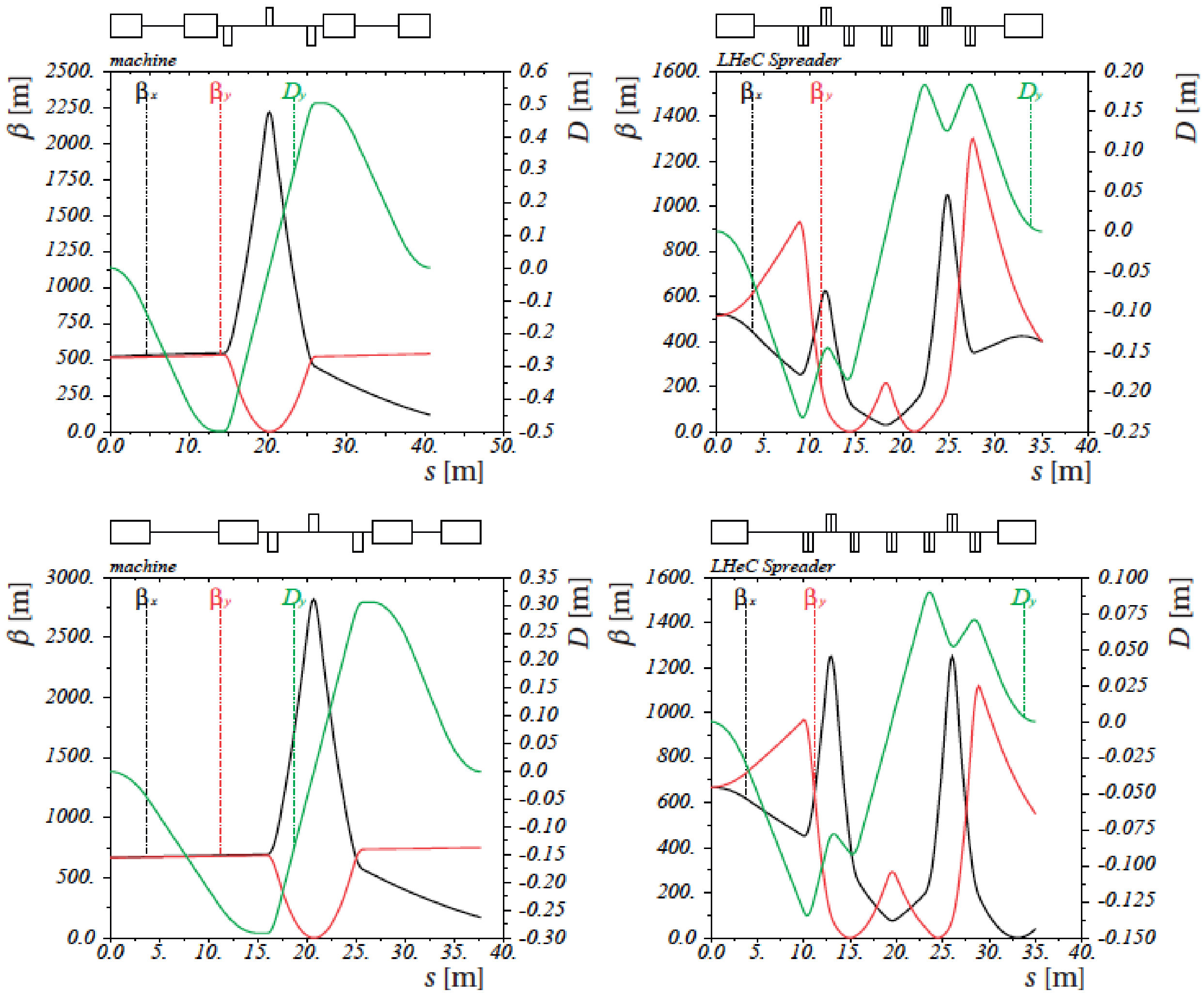


The vertical dispersion introduced by the first step bend is suppressed by two quadrupoles located appropriately between the two stages



The single step spreader starts with a dipole and then a defocusing quad to bring back the vertical dispersion. The quadrupole triplet focuses the beam.
The next quadrupole does not affect the dispersion as it is placed where it crosses the zero, it offers an extra degree of freedom to control the beta functions.

SEPARATION REGIONS: POSSIBLE SOLUTIONS



The two-steps design simplifies the suppression of the vertical dispersion, but could also induces a non negligible energy loss, moreover it raises the horizontal β function to very high values

In the single step spreader the energy loss is reduced by a factor 5 and at the same time both the dispersion and the β functions are mitigated

Arc Optics

ARC OPTICS

From the previous lecture

Main design Considerations

✧ Length and Hardware
✧ Flexibility* and tunability
✧ Chromatic properties
✧ Coherent and Incoherent synchrotron radiation

High Transmission

Variable Momentum compaction

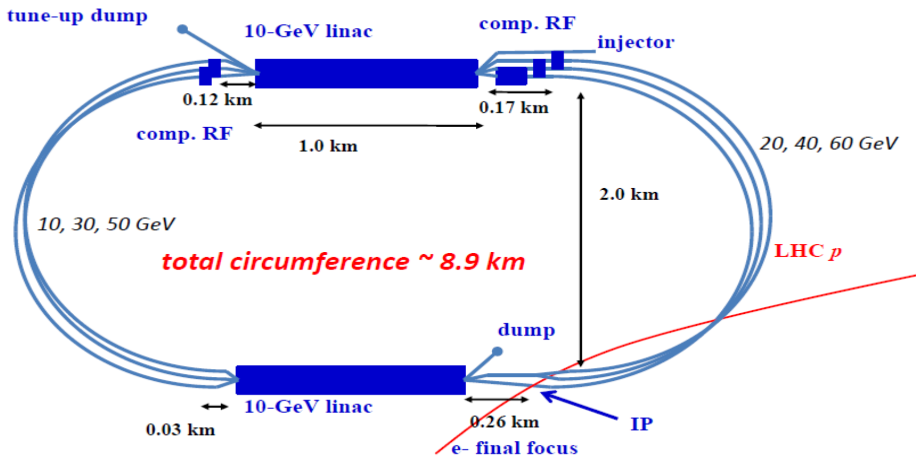
Betatron phase advance

Various options, DBA, TBA, Bates Arc …

*Operational flexibility is a fundamental aspect. The intention is to come up with a system design that gives an independent handle on as many different parameters as possible, without adversely influencing others

EXAMPLE OF A HE MACHINE

In the LHeC there are 6 arcs from 10 to 60 GeV



Three different arcs design have been developed
due to the need of controlling
emittance increase, momentum spread growth and isochronicity

In the design for the lowest energy turns, beta-functions are kept small in order to limit the required vacuum chamber size and consequently the magnet aperture

At the highest energy, the lattice is optimized to keep the emittance growth from synchrotron radiation limited

EMITTANCE INCREASE AND ISOCHRONICITY

Growth of normalized emittance

$$\Delta\varepsilon^N = \frac{2}{3} C_q \, r_0 \, \gamma^6 \, I_5$$

$$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar c}{mc^2} = 3.8319 \times 10^{-13} \text{ m},$$

$$r_0 = 2.818 \times 10^{-15} \text{ m},$$

$$I_5 = \int_0^L \frac{H}{|\rho|^3} ds = \frac{\theta \langle H \rangle}{\rho^2},$$

$$H = \gamma D^2 + 2\alpha D D' + \beta D'^2$$

$$\Delta\varepsilon^N = \frac{2}{3} C_q \, r_0 \, \gamma^6 \langle H \rangle \frac{\theta}{\rho^2}$$

Momentum compaction

$$M_{56} = -\int \frac{D}{\rho} ds = -\theta_{bend} \langle D \rangle$$

$$\Delta C = -M_{56} \frac{\Delta p}{p}$$

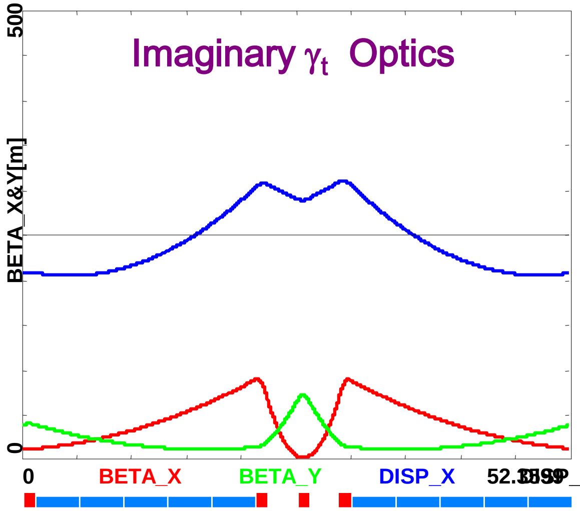
$$\Delta\phi_{RF} = \frac{360 \times \Delta C}{\lambda_{RF}} = -\frac{360}{\lambda_{RF}} N_{cell} M_{56}^{cell} \frac{\Delta p}{p}$$

Momentum spread growth

$$\frac{\Delta\sigma_E^2}{E^2} = \frac{55\alpha}{48\sqrt{3}} \left(\frac{\hbar c}{mc^2}\right)^2 \gamma^5 \int_0^L \frac{1}{\rho^3} ds \qquad \int_0^L \frac{1}{|\rho|^3} ds = \frac{\theta}{\rho^2}, \qquad \frac{\Delta\sigma_E^2}{E^2} = \frac{55\alpha}{48\sqrt{3}} \left(\frac{\hbar c}{mc^2}\right)^2 \gamma^5 \frac{\theta}{\rho^2}$$

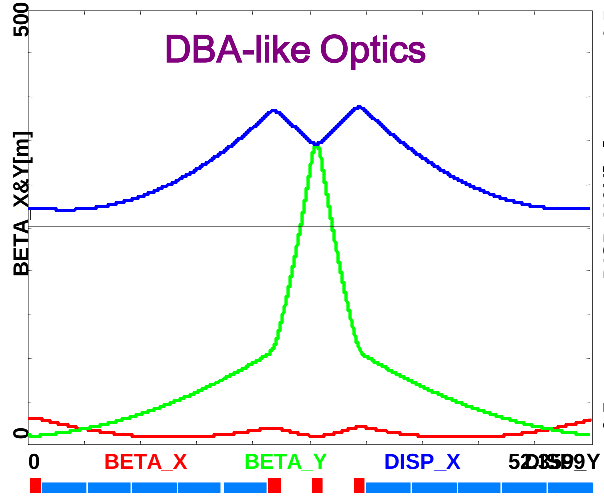
ARC OPTICS: FMC CELL

Arc 1 , Arc2



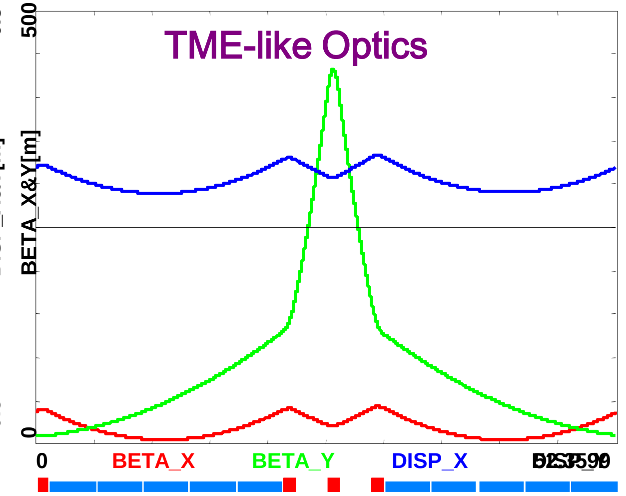
$$\langle H \rangle = 8.8 \times 10^{-3}\ m$$

Arc 3, Arc 4



$$\langle H \rangle = 2.2 \times 10^{-3}\ m$$

Arc5, Arc 6



$$\langle H \rangle = 1.2 \times 10^{-3}\ m$$

factor of 20 smaller than FODO

ARC 1, ARC 2

At the lowest energy it is possible to compensate for the bunch elongation with a negative momentum compaction setup which, additionally, reduces the beam size

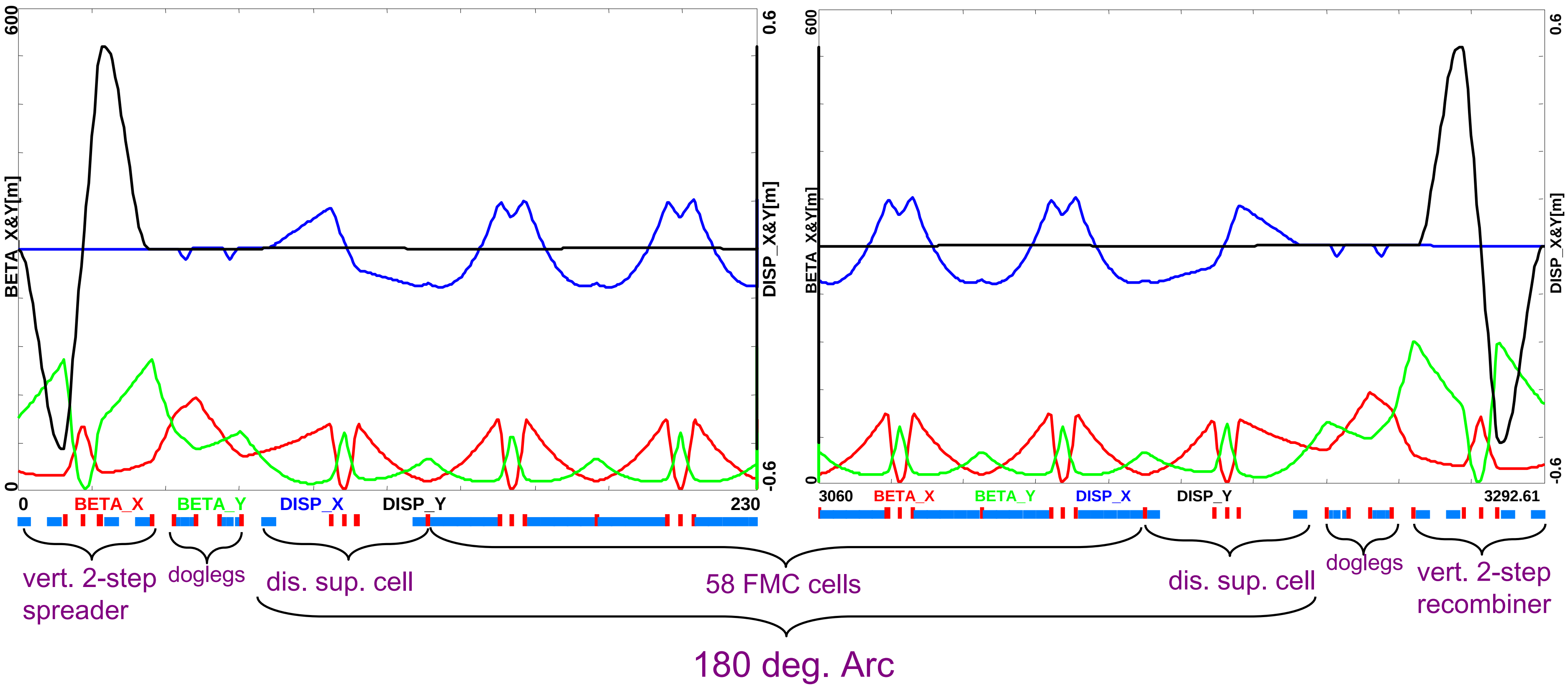
ARC 5, ARC 6

The cells are tuned to contain the dispersion in the bending sections, as in a theoretical minimum emittance lattice

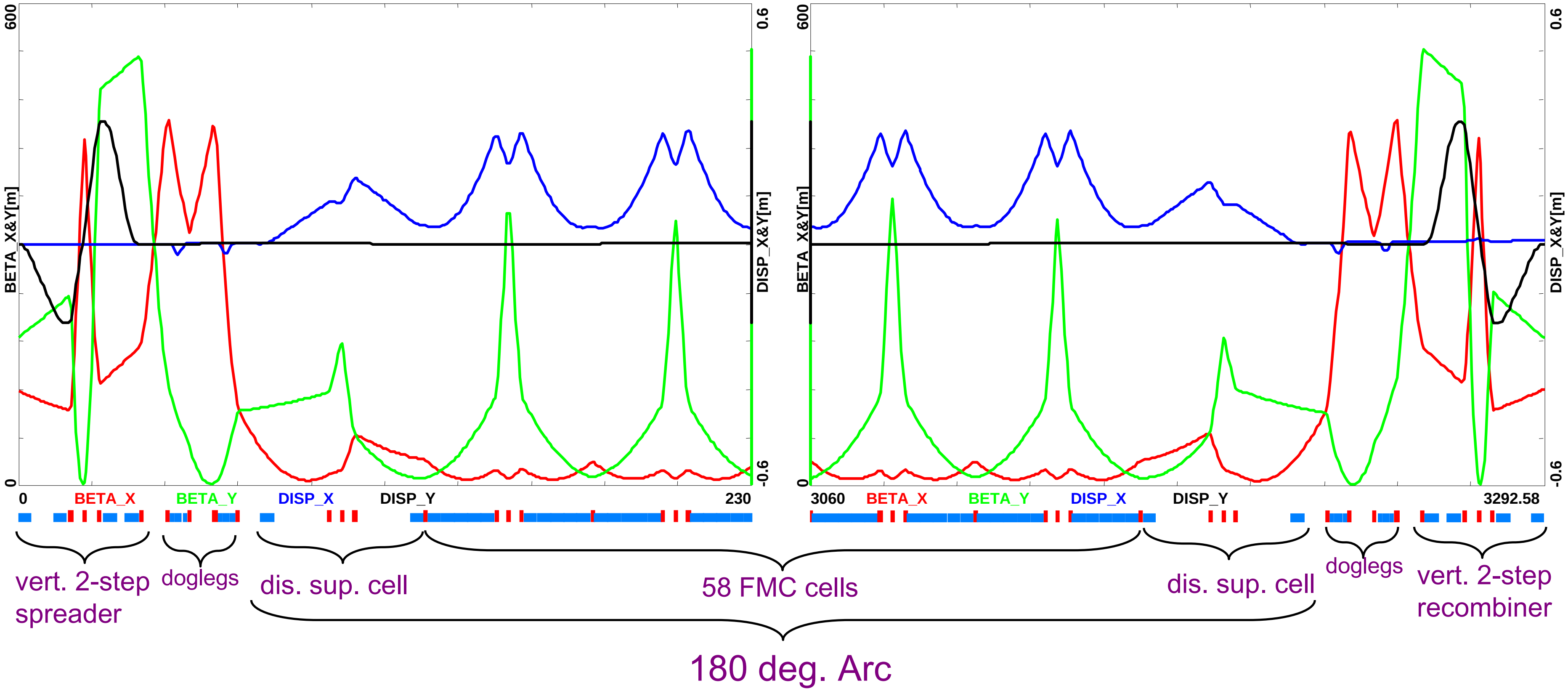
ARC 3, ARC 4

The intermediate energy arcs are tuned to a DBA-like lattice, offering a compromise between bunch lengthening and emittance dilution

ARC 1 OPTICS (10 GeV)



ARC 3 OPTICS (30 GeV)



Arc dipoles:

$Lb=400 cm

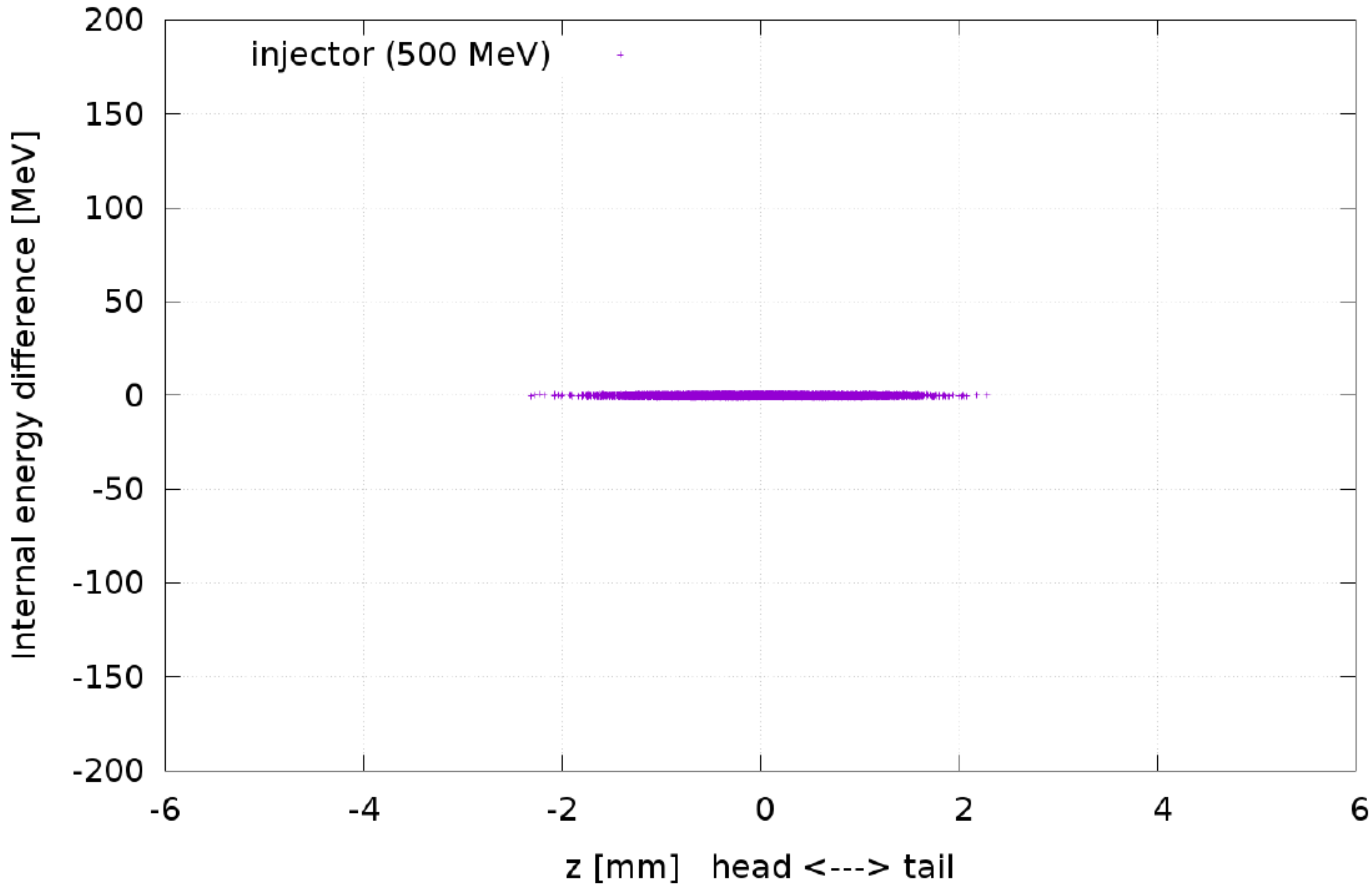
$B=1.37 kGauss

SINGLE BUNCH TRACKING

The transport of a single bunch from the injector to the dump is the first step to validate the machine design

Effect of synchrotron radiation on the **emittance** and on the **induced energy spread**

EVOLUTION OF LONGITUDINAL PHASE SPACE



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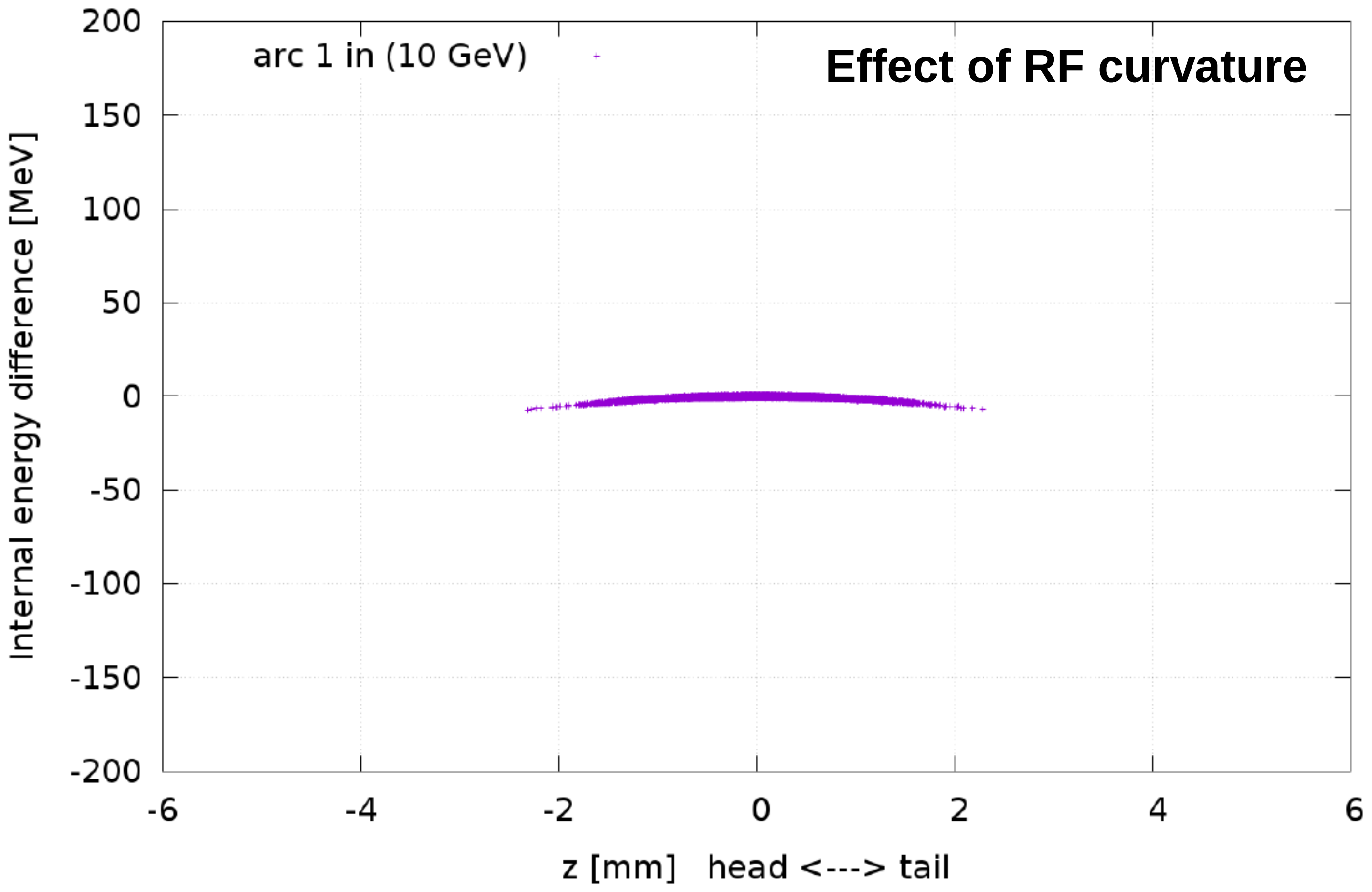

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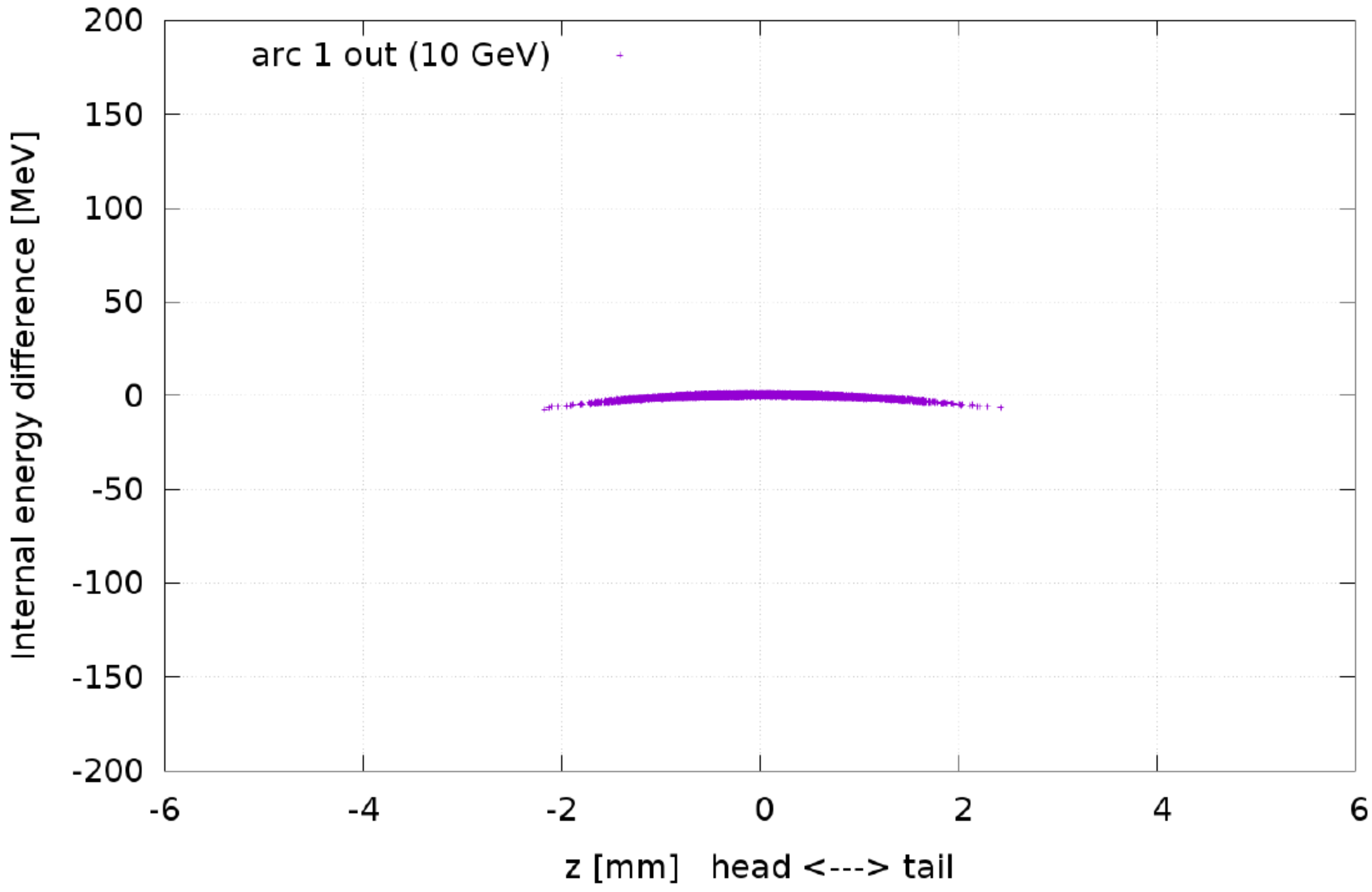

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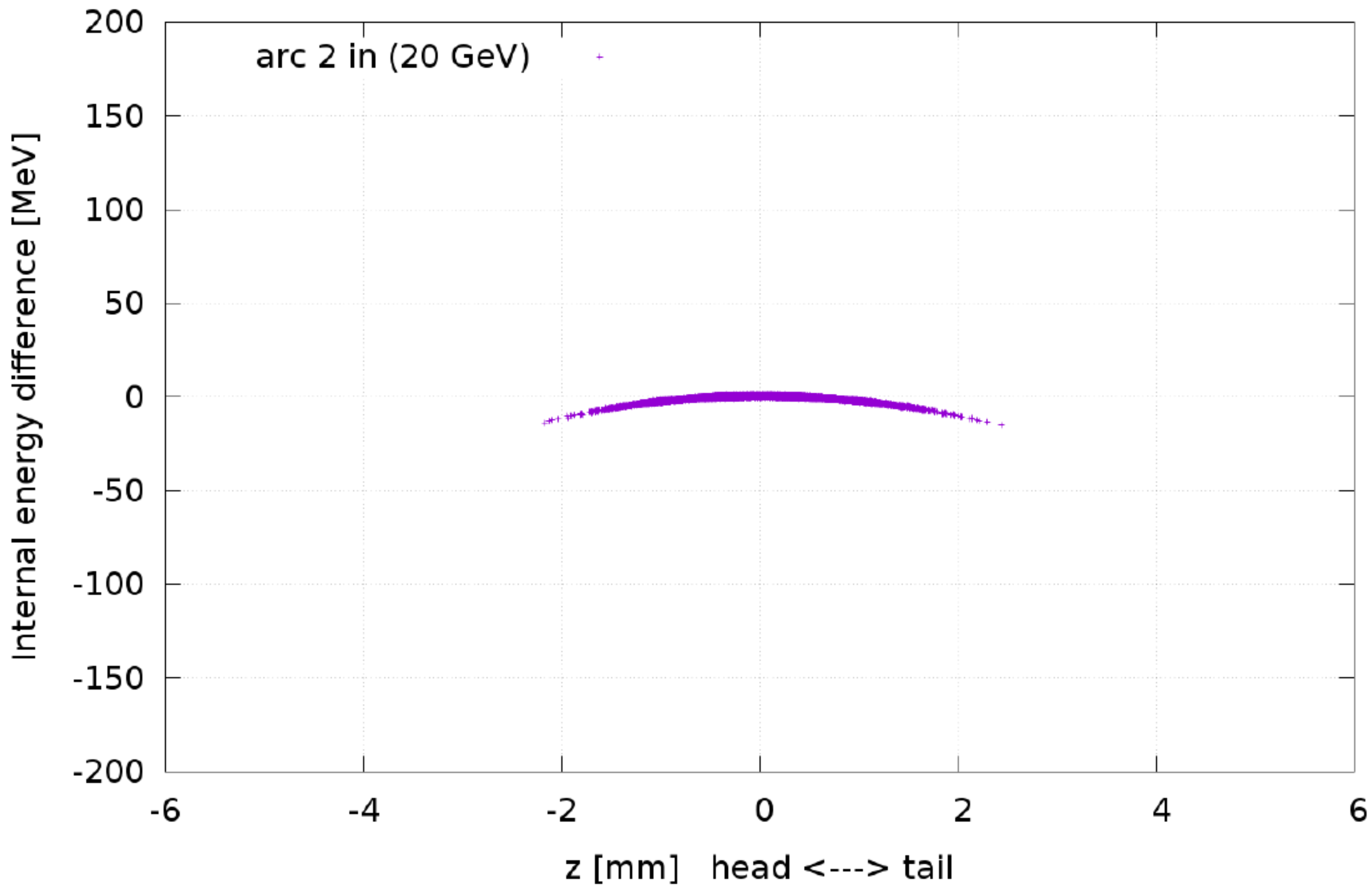

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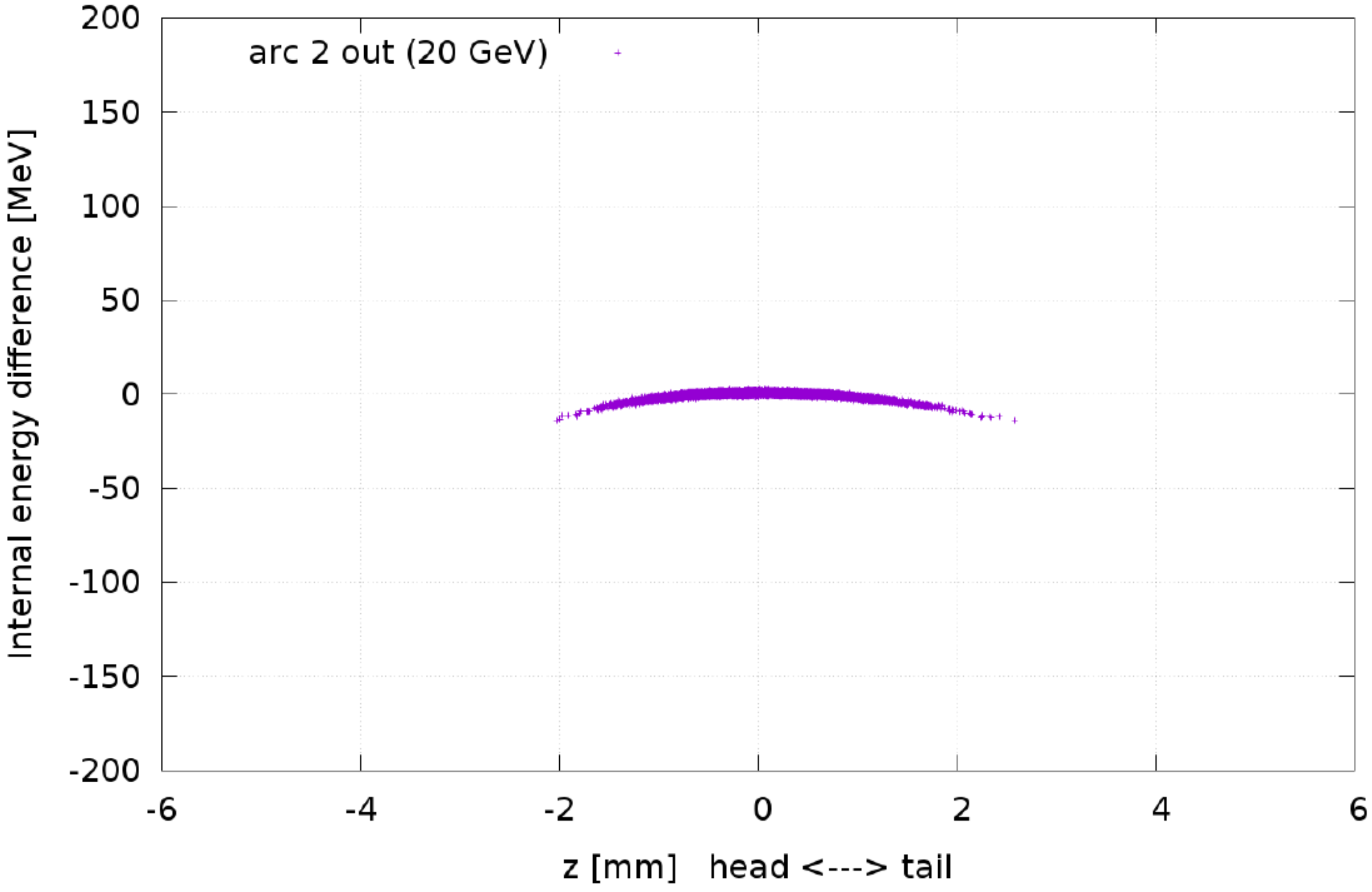

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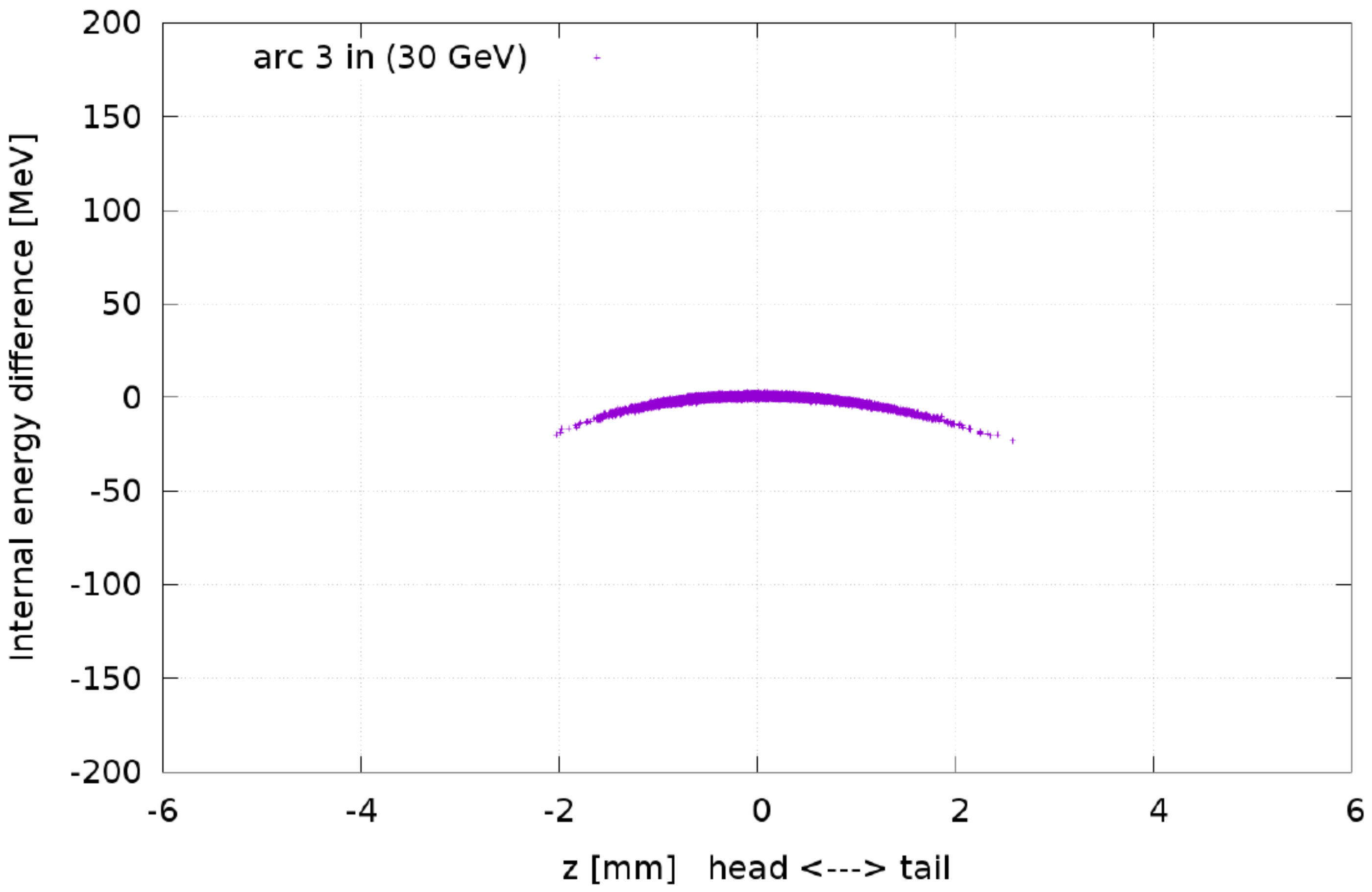
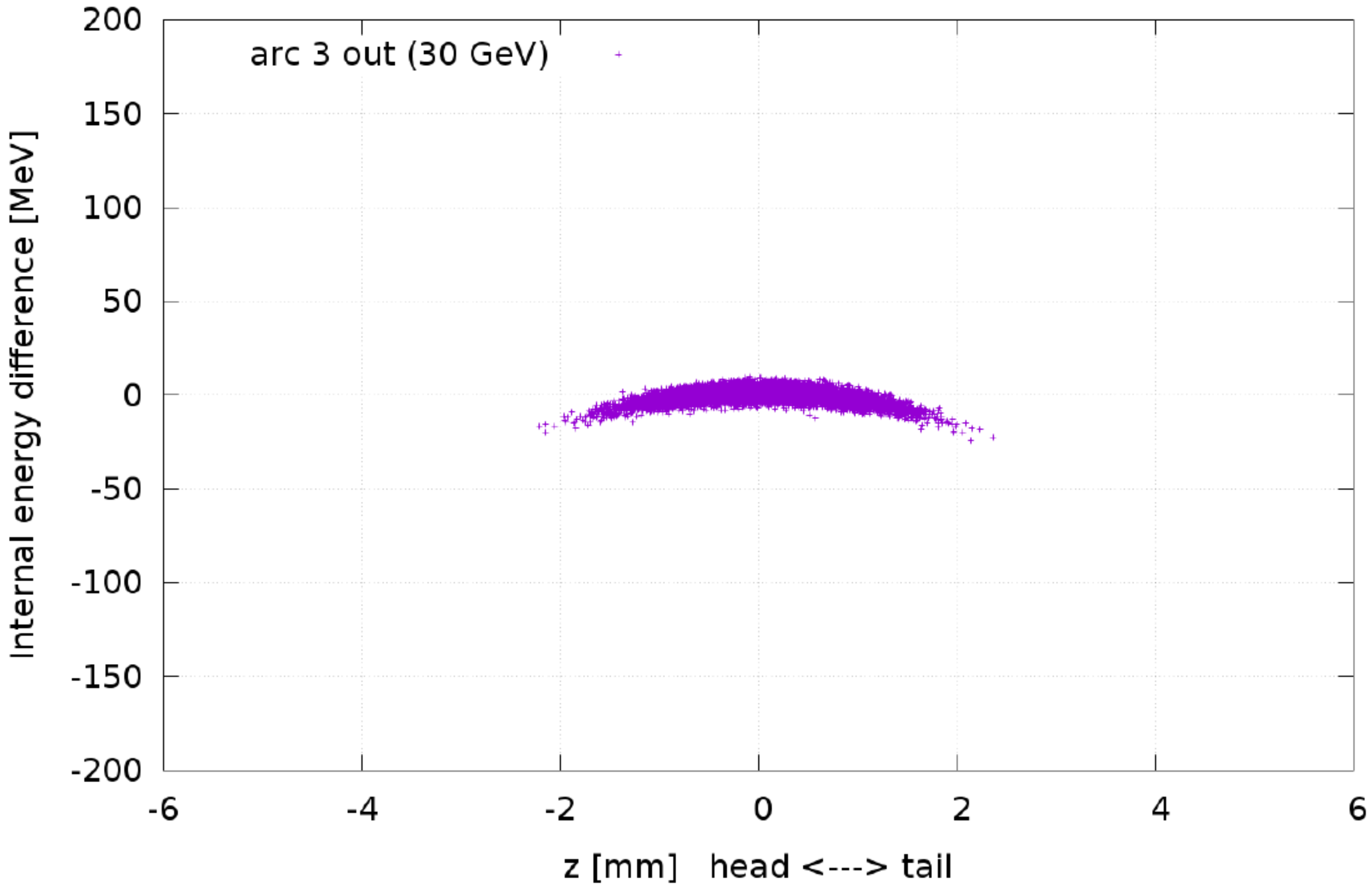
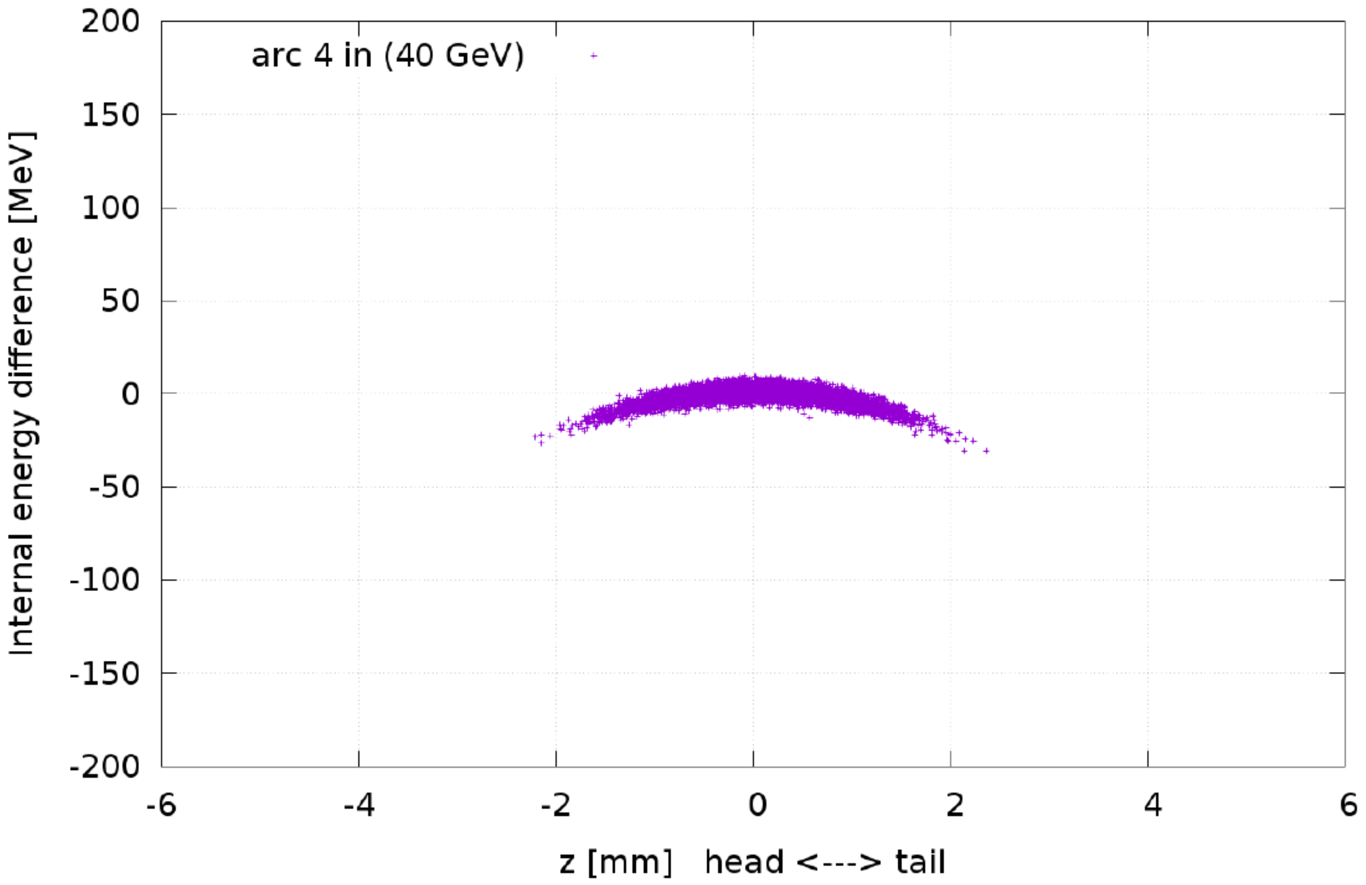


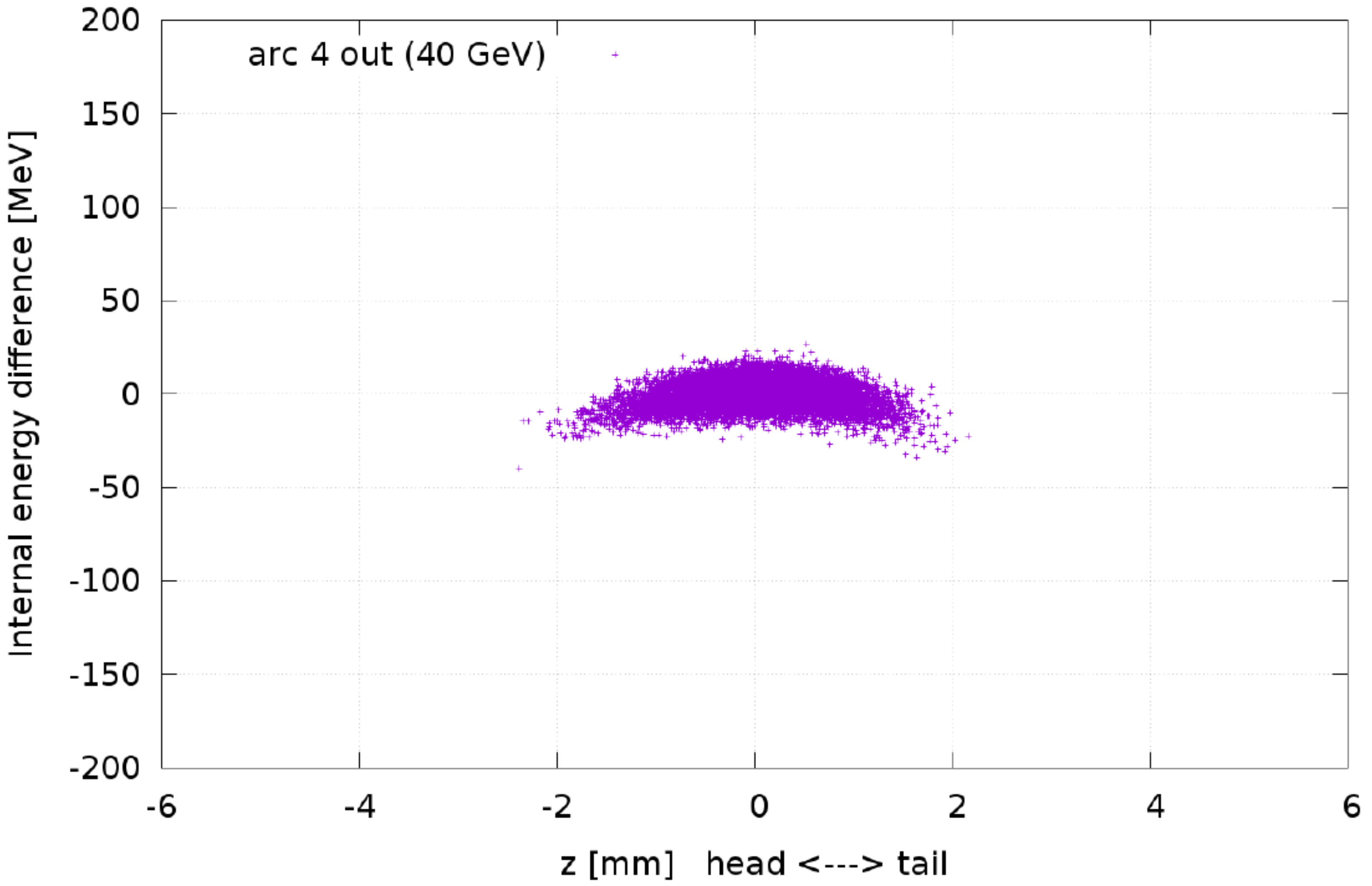
EVOLUTION OF LONGITUDINAL PHASE SPACE



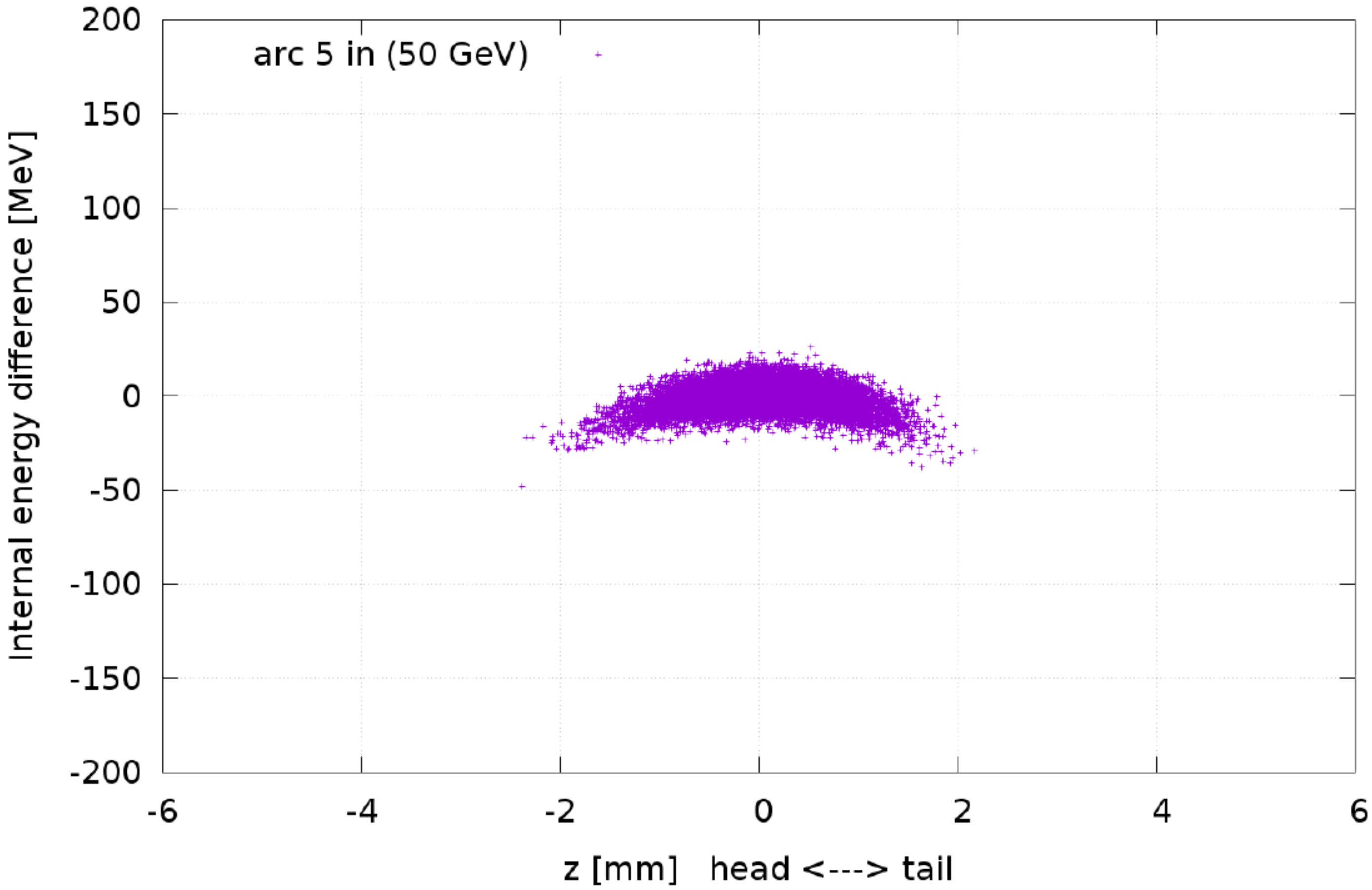
EVOLUTION OF LONGITUDINAL PHASE SPACE



EVOLUTION OF LONGITUDINAL PHASE SPACE

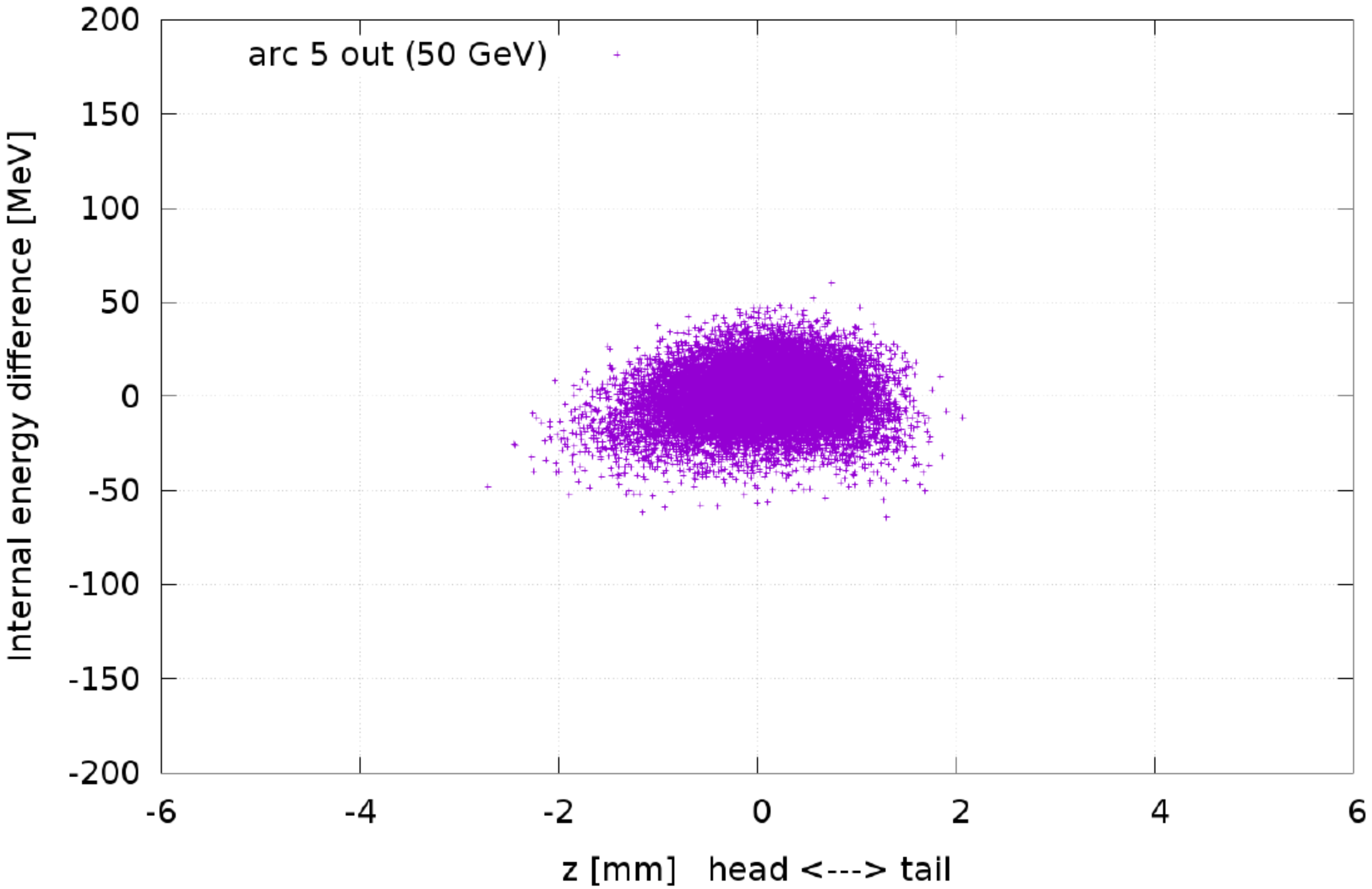


EVOLUTION OF LONGITUDINAL PHASE SPACE

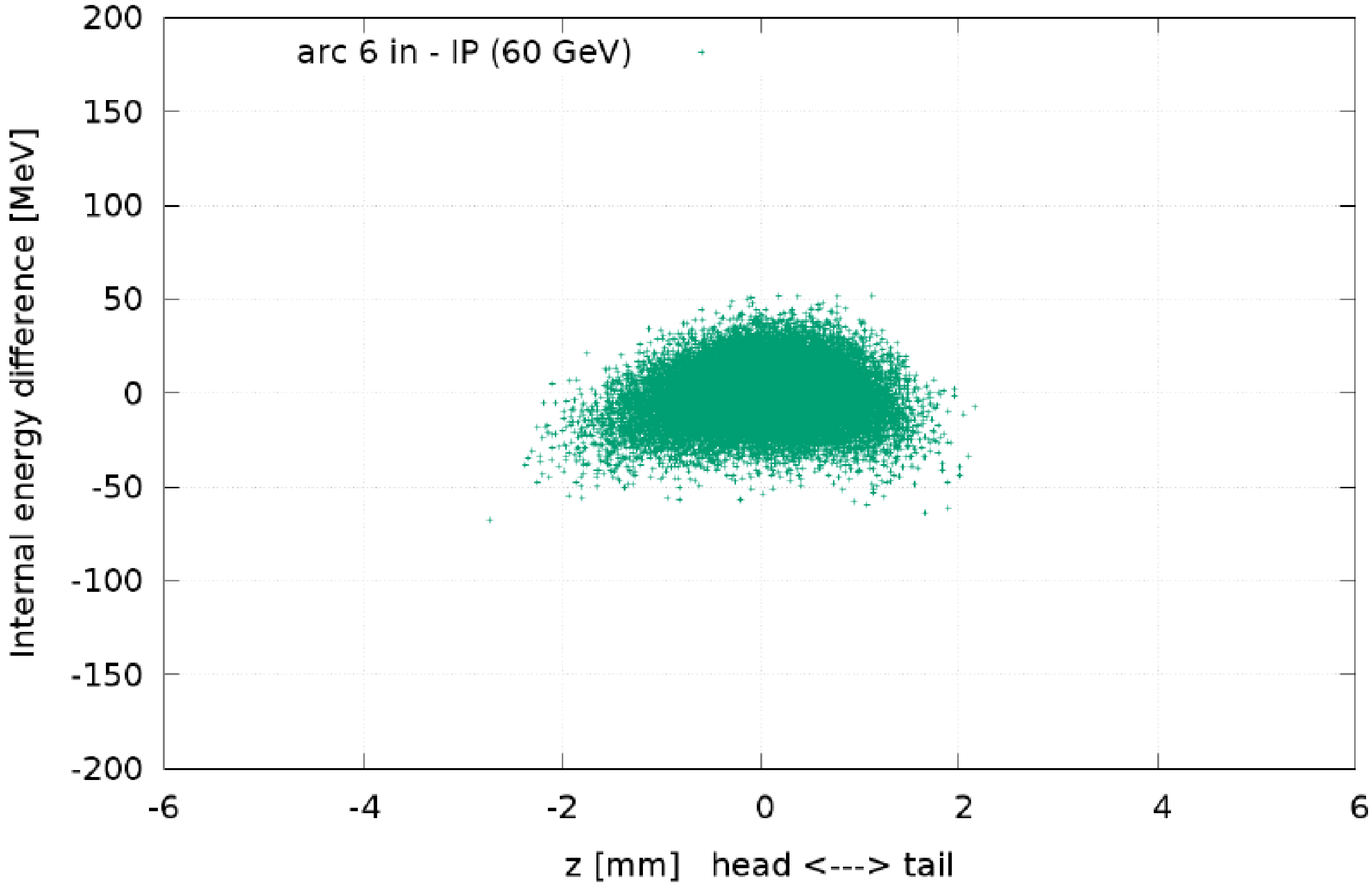


SYNCHROTRON RADIATION

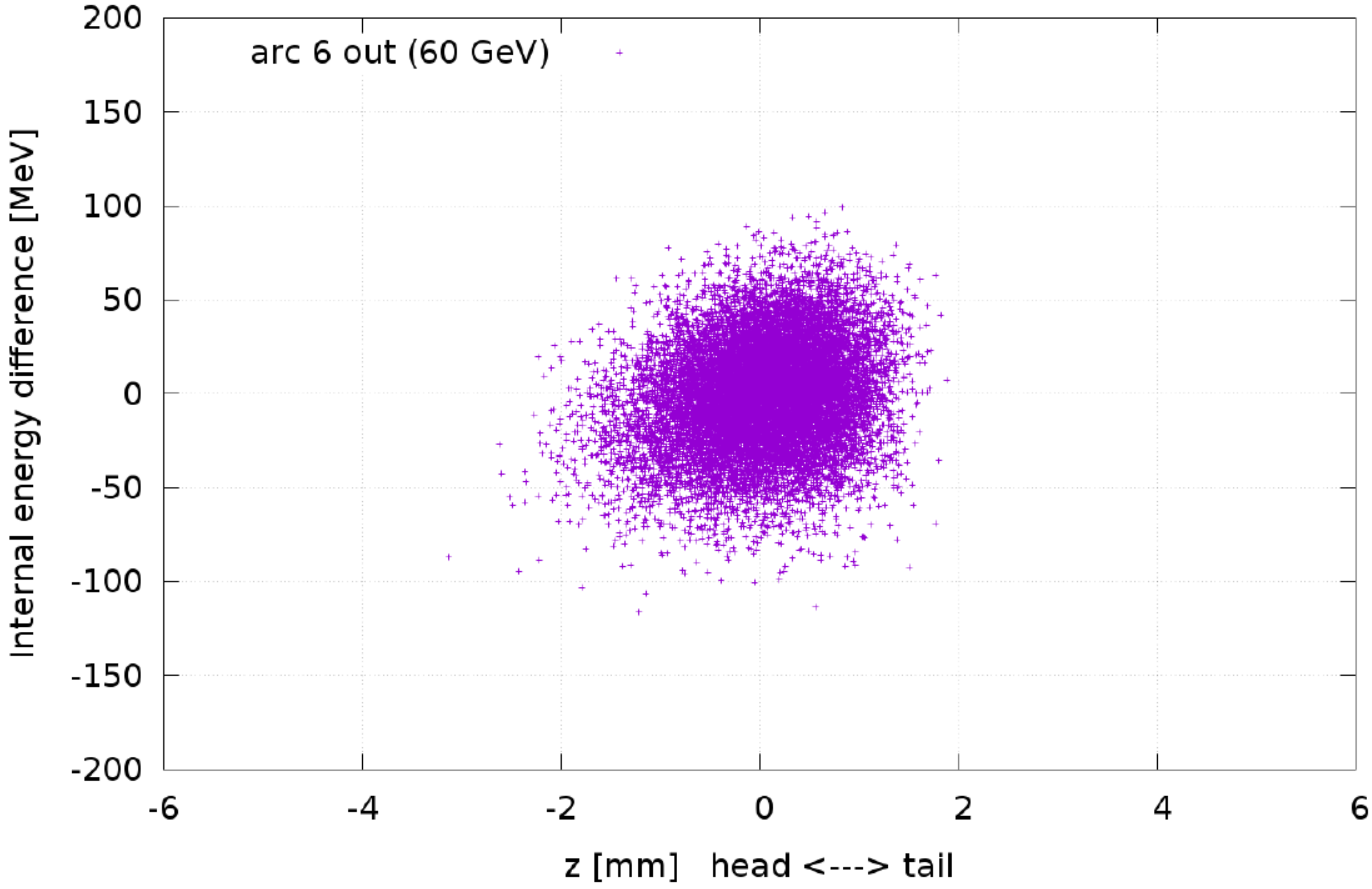
EVOLUTION OF LONGITUDINAL PHASE SPACE



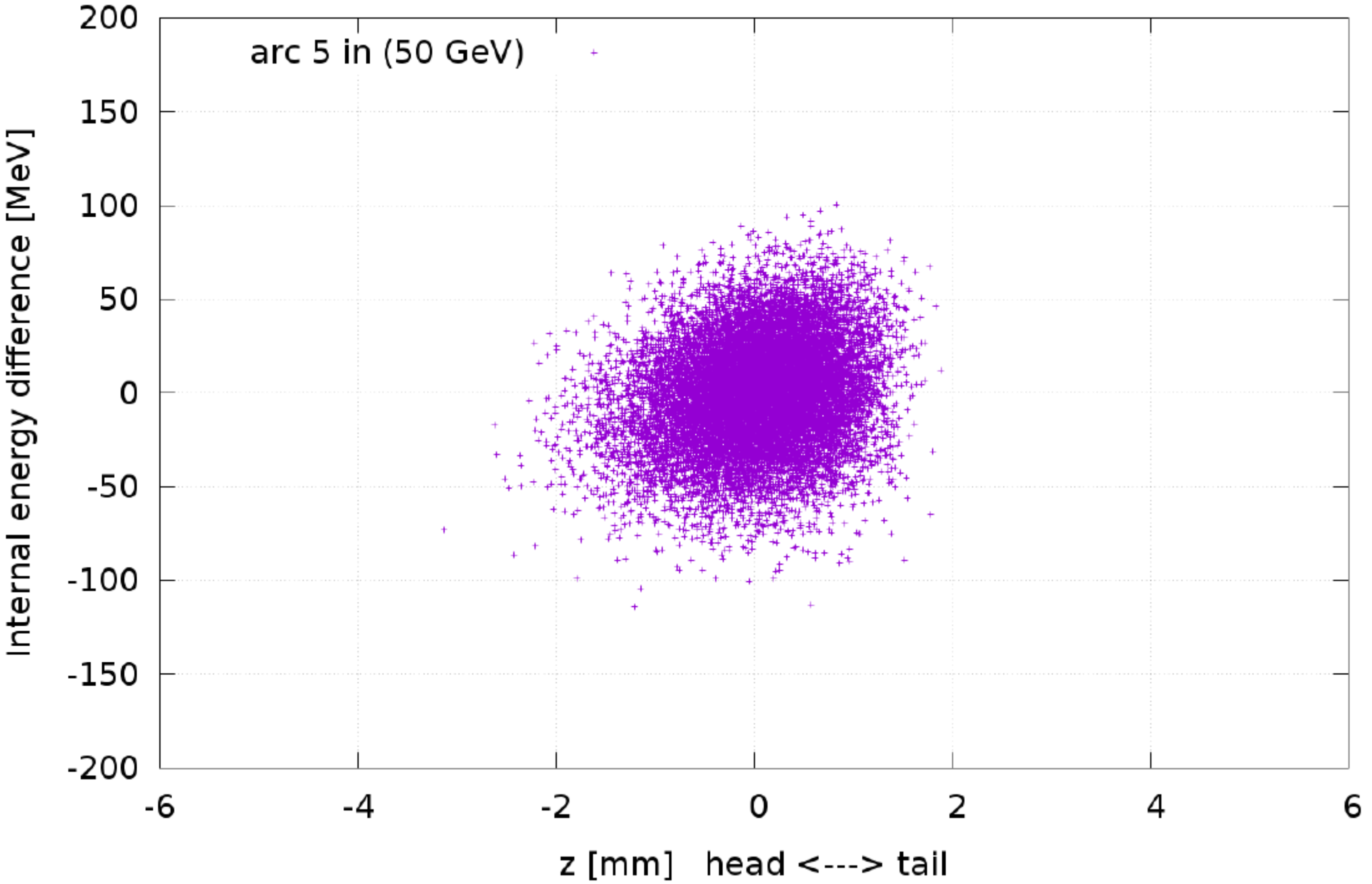
EVOLUTION OF LONGITUDINAL PHASE SPACE



EVOLUTION OF LONGITUDINAL PHASE SPACE

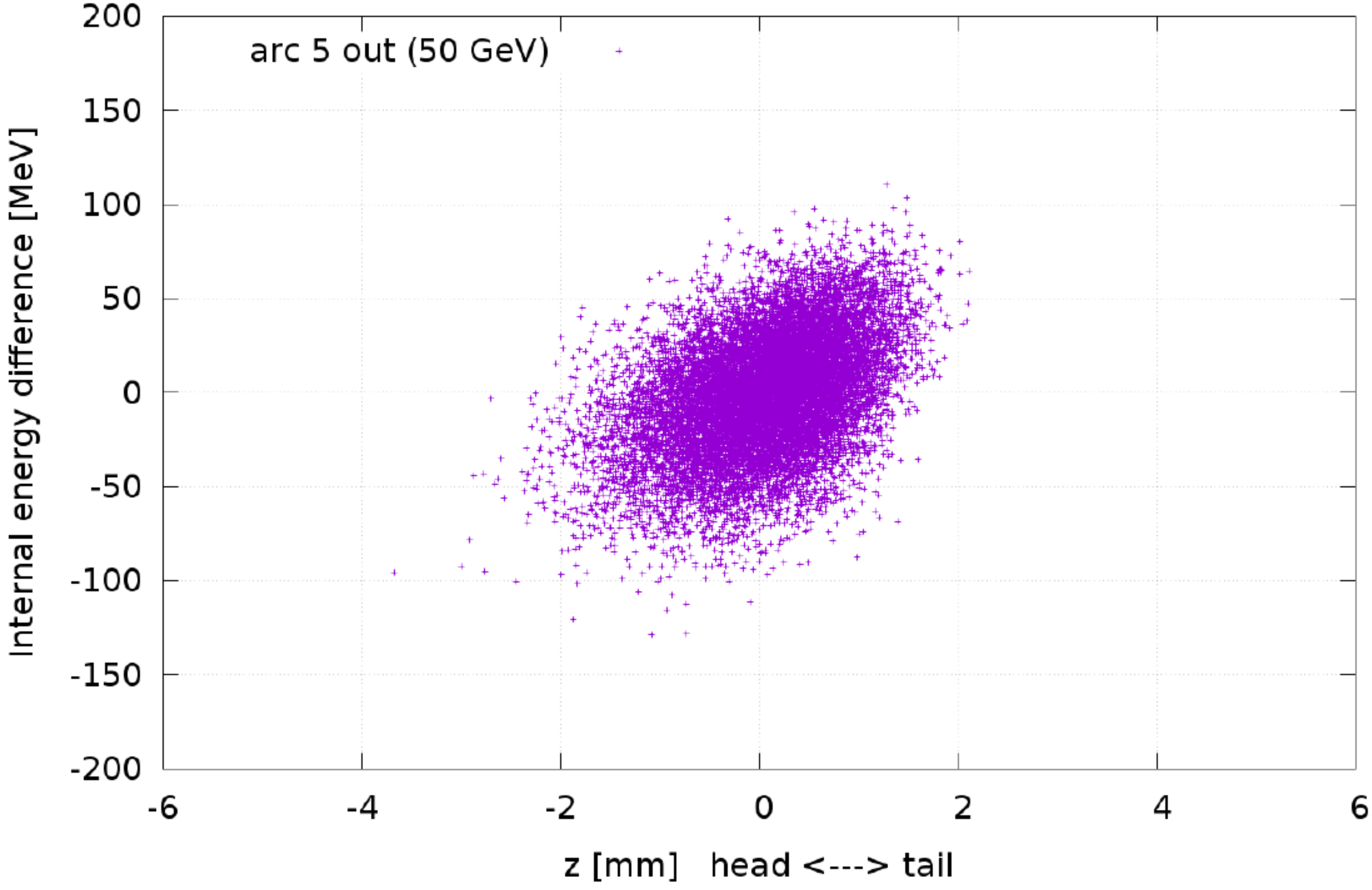


EVOLUTION OF LONGITUDINAL PHASE SPACE



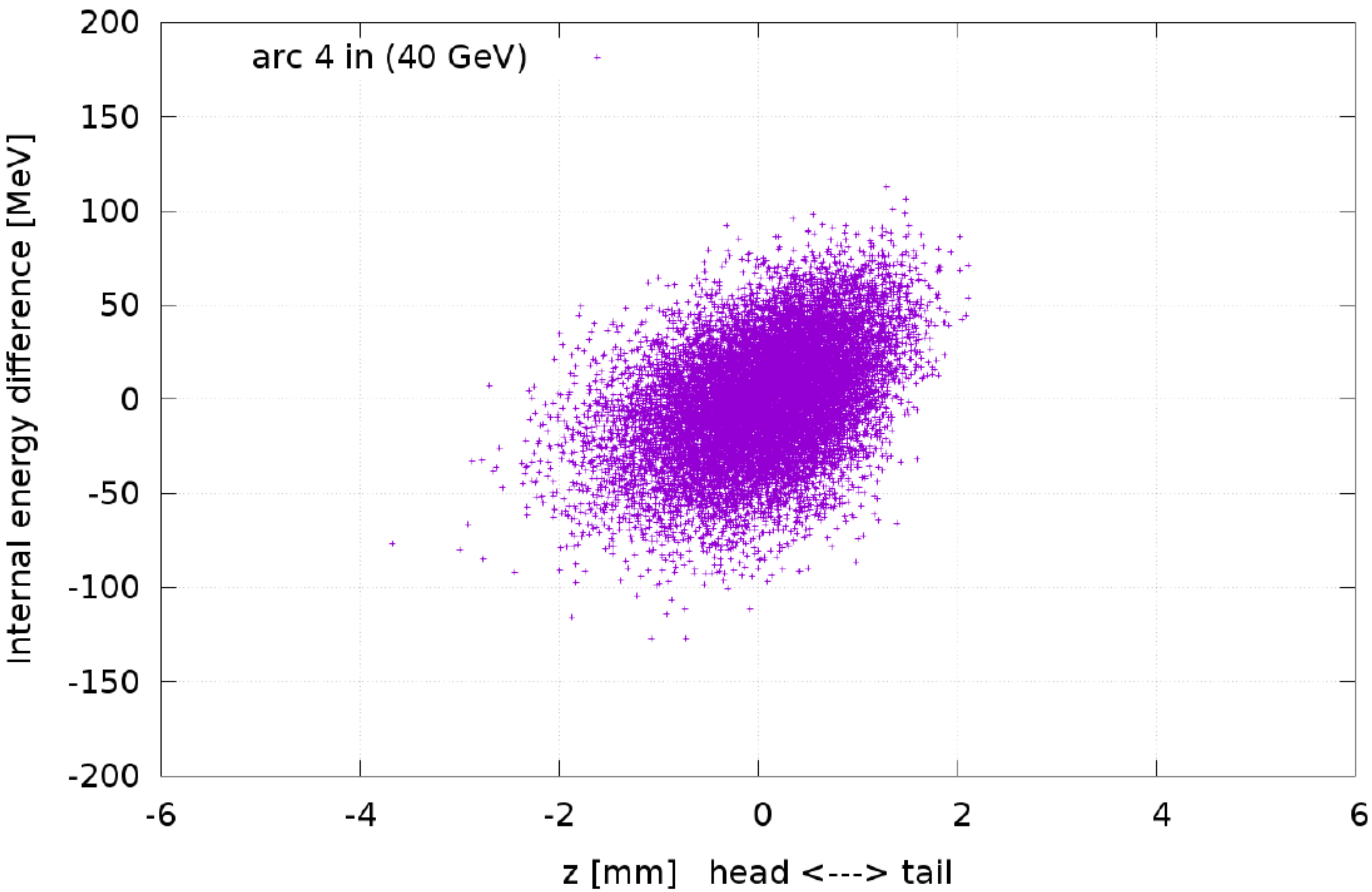
SYNCHROTRON RADIATION

EVOLUTION OF LONGITUDINAL PHASE SPACE

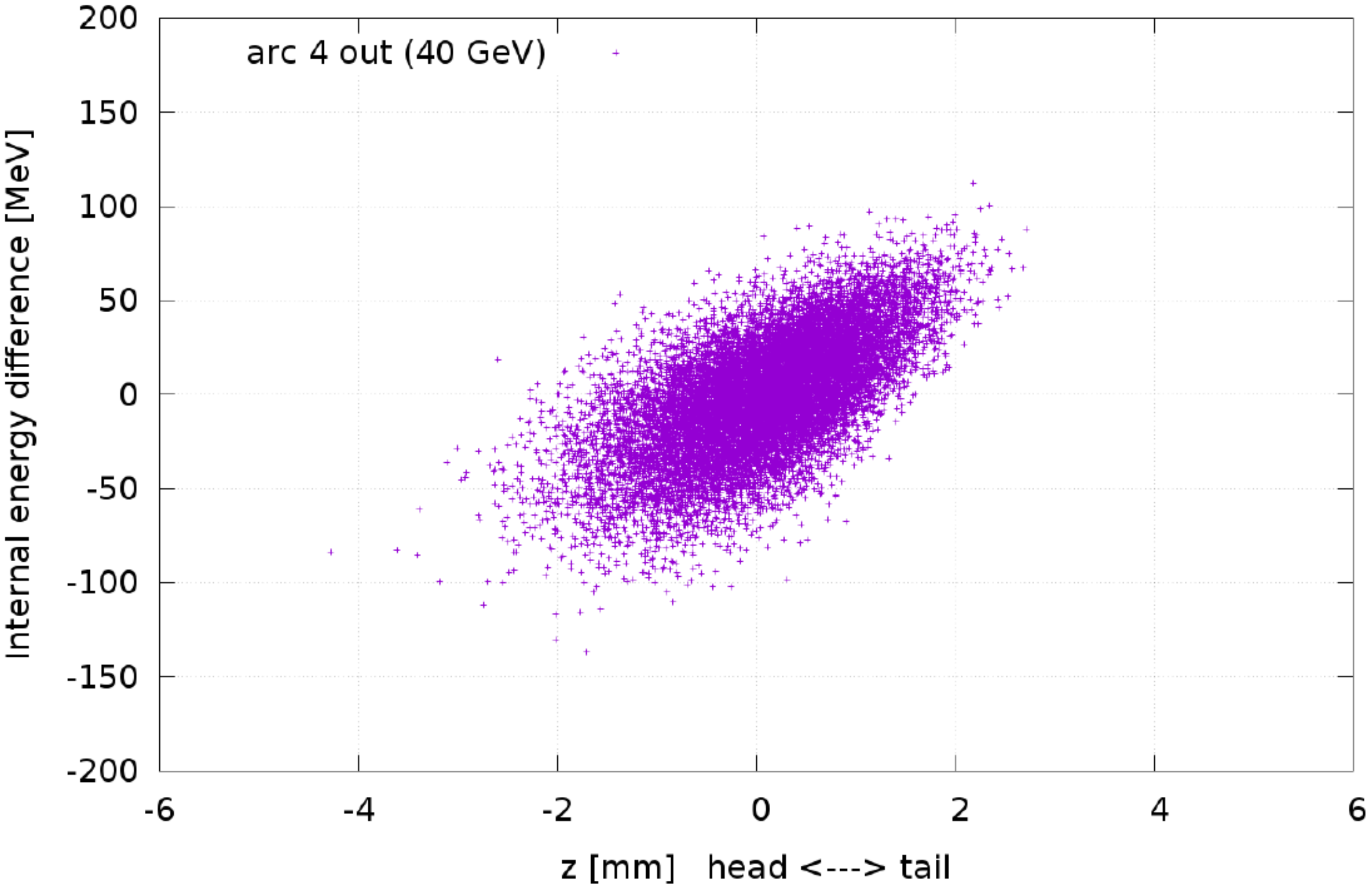


SYNCHROTRON RADIATION

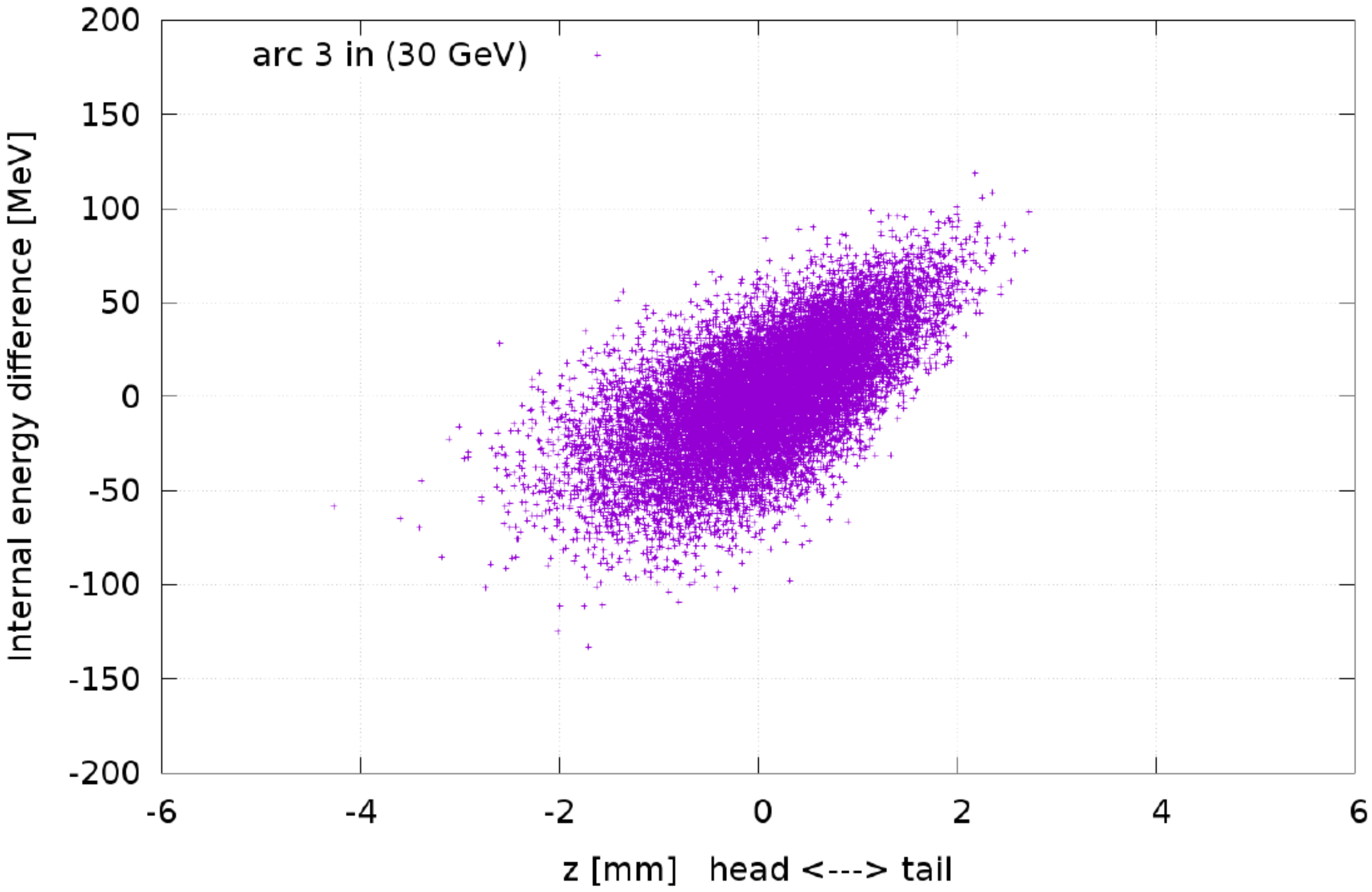
EVOLUTION OF LONGITUDINAL PHASE SPACE



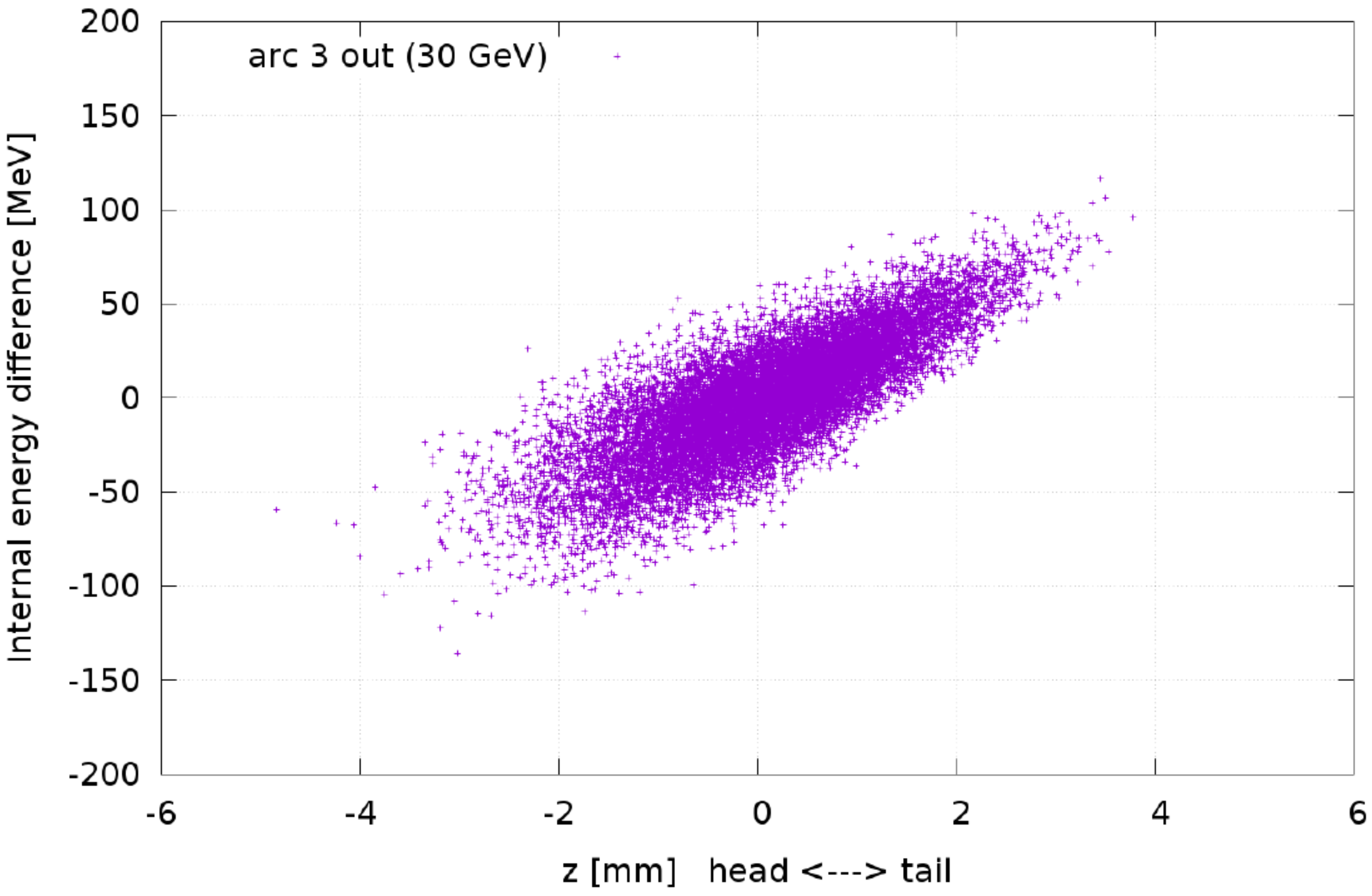
EVOLUTION OF LONGITUDINAL PHASE SPACE



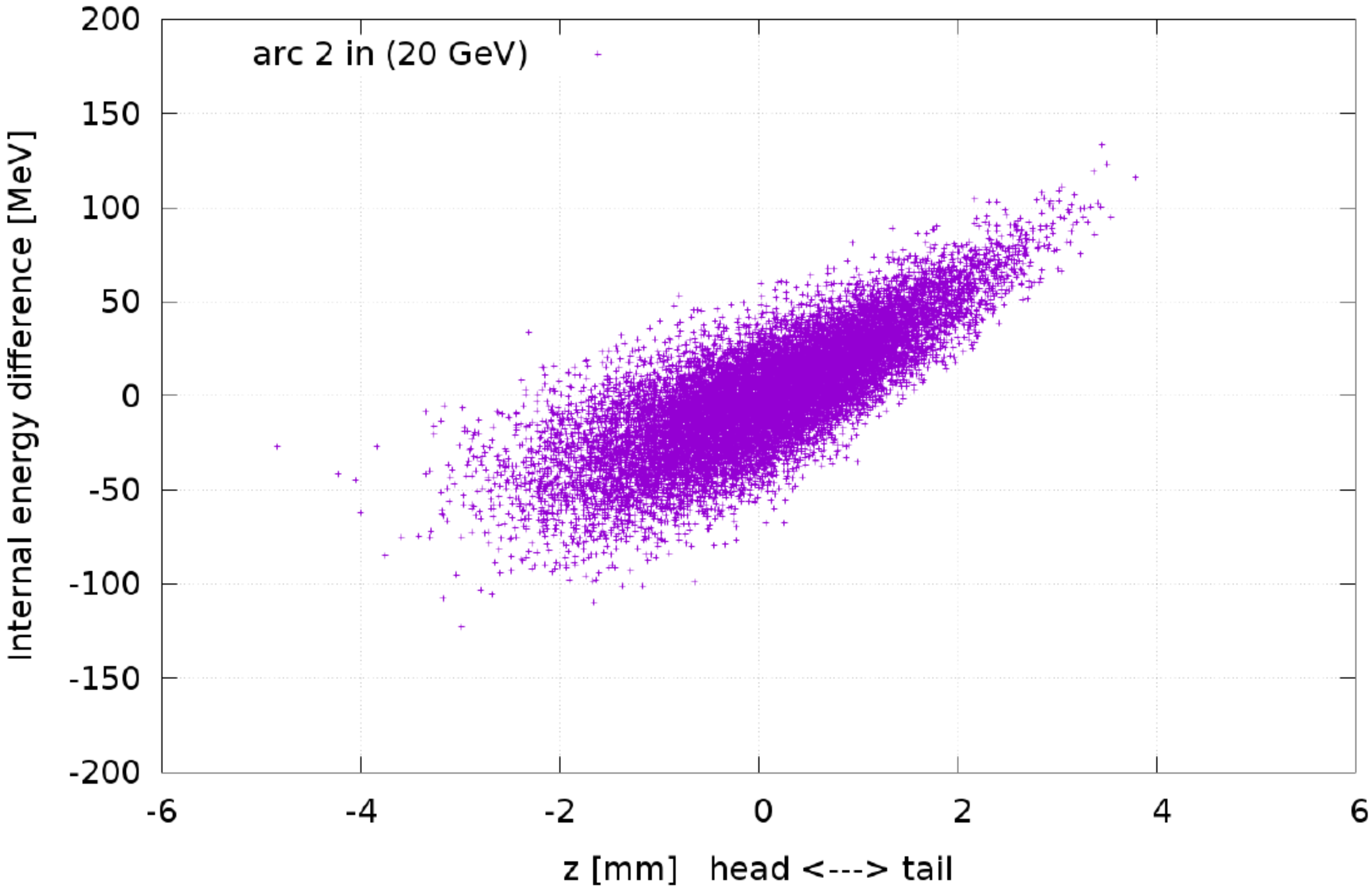
EVOLUTION OF LONGITUDINAL PHASE SPACE



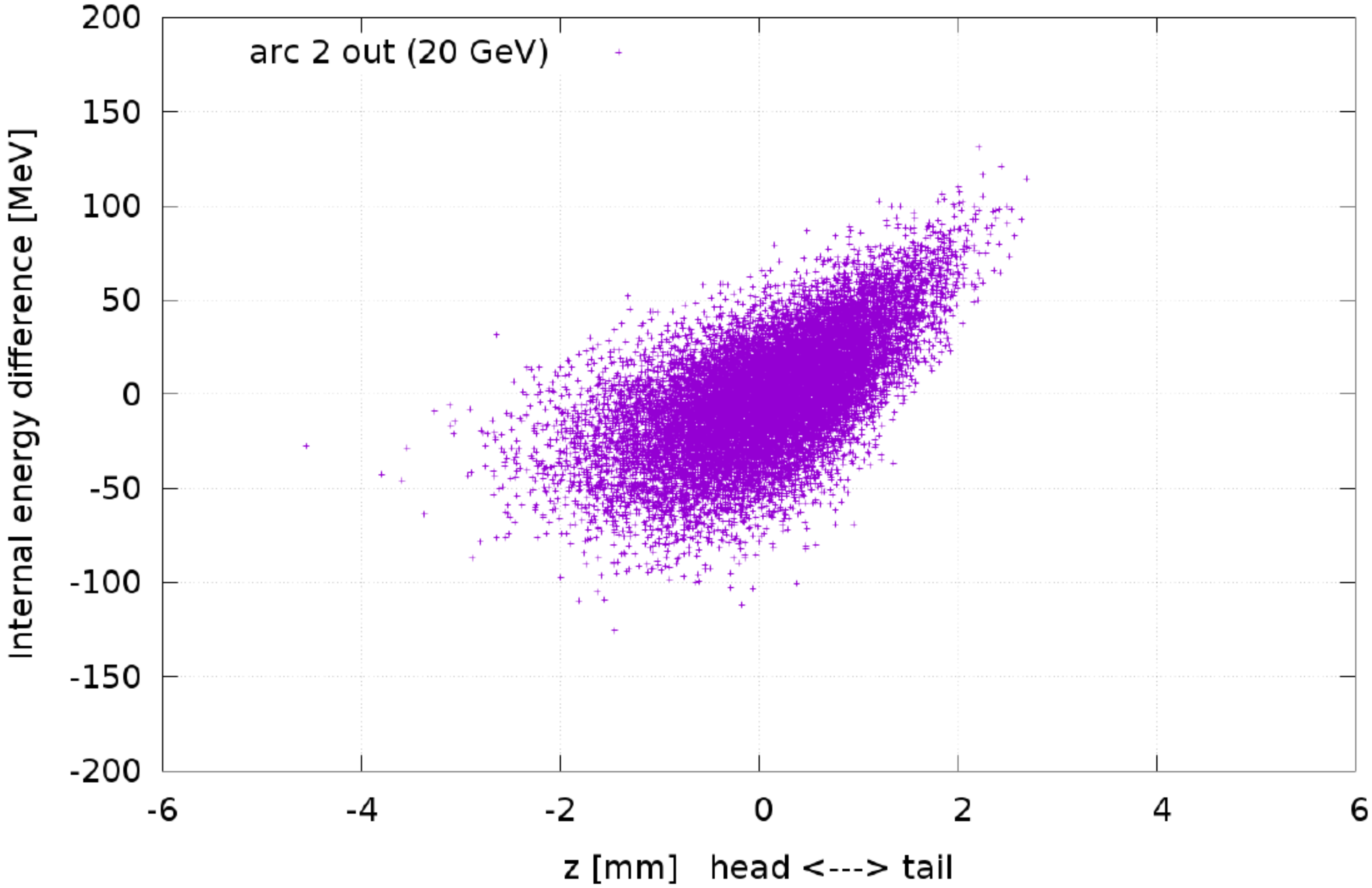
EVOLUTION OF LONGITUDINAL PHASE SPACE



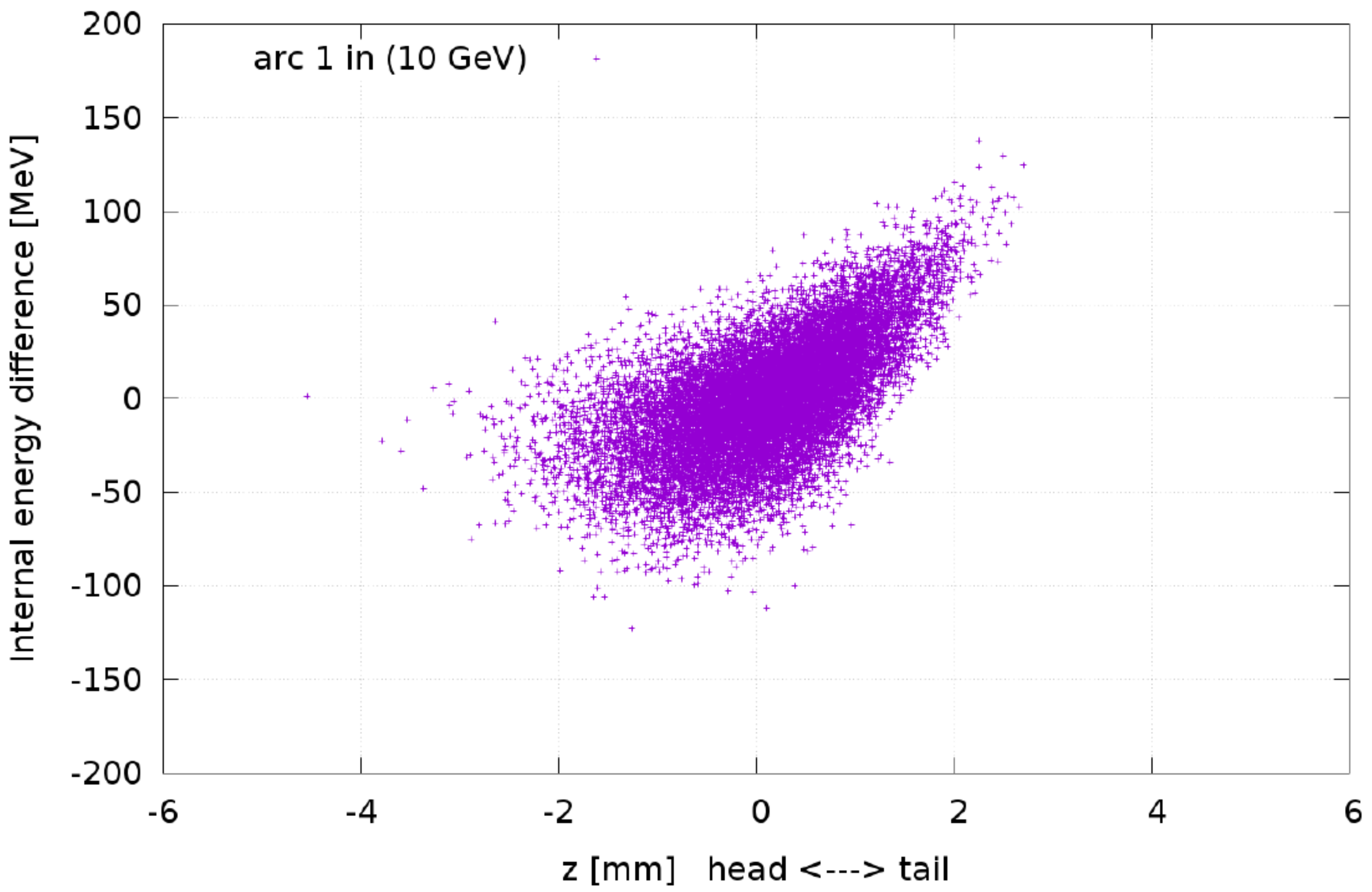
EVOLUTION OF LONGITUDINAL PHASE SPACE



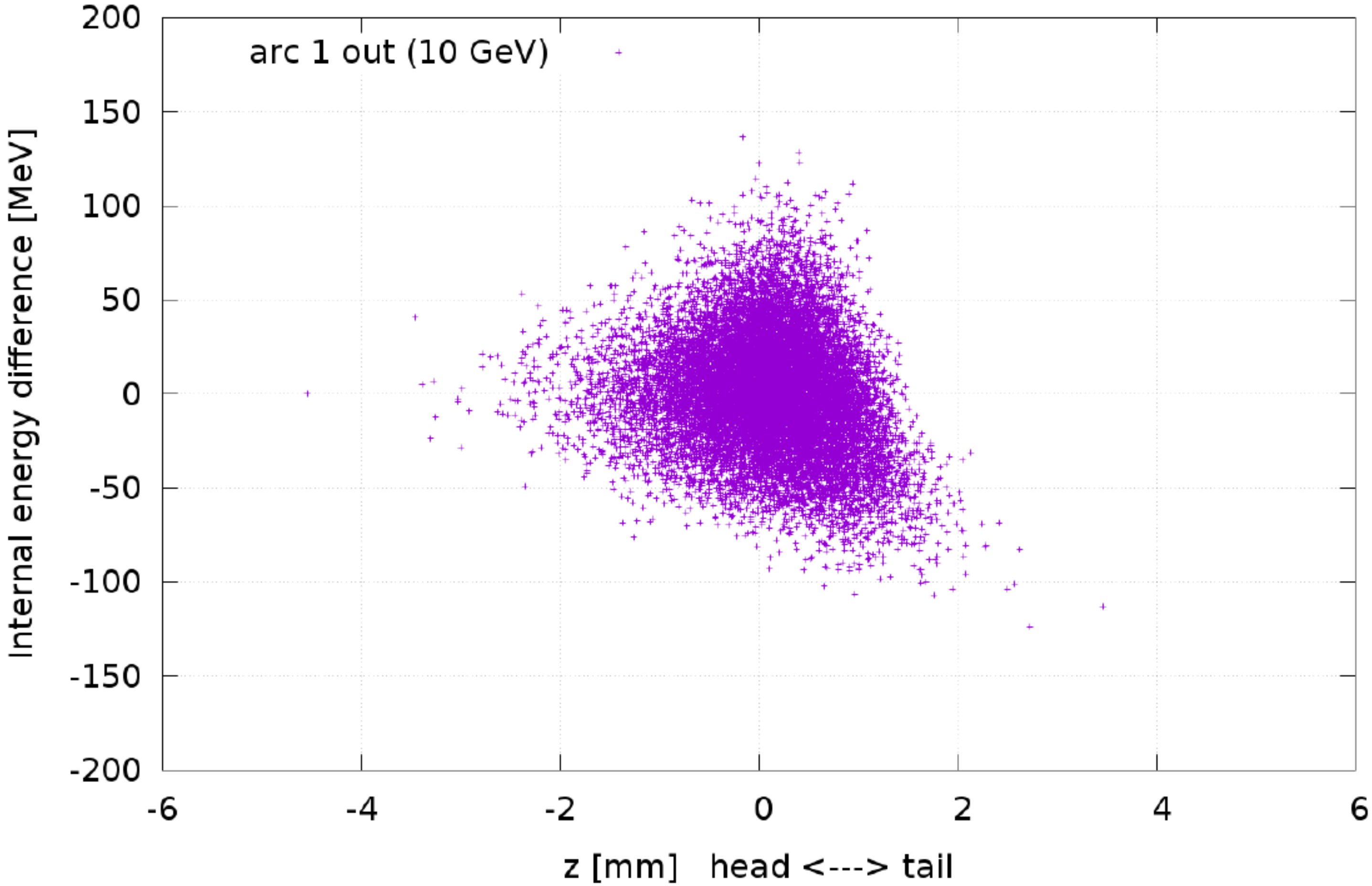
EVOLUTION OF LONGITUDINAL PHASE SPACE



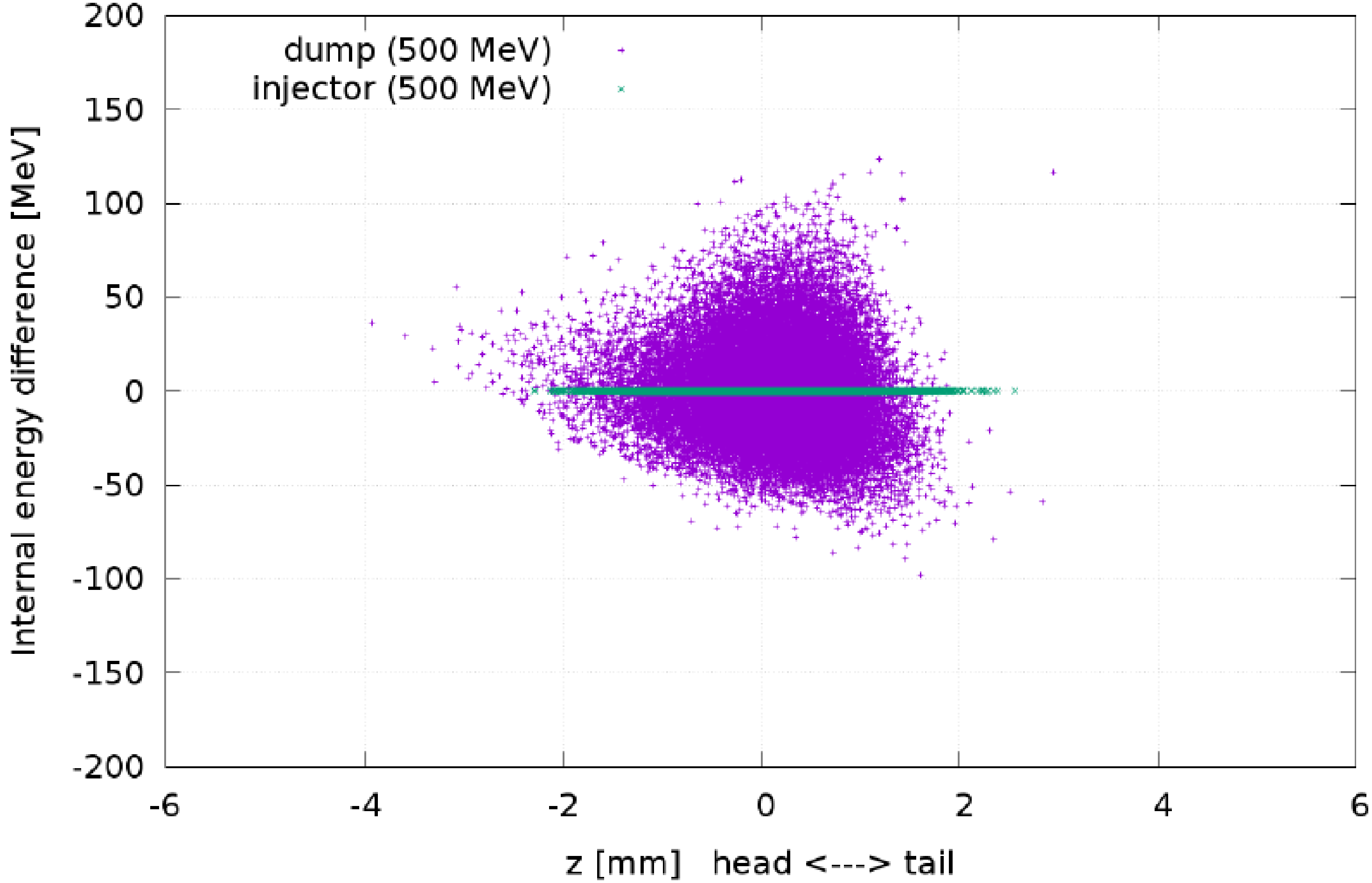
EVOLUTION OF LONGITUDINAL PHASE SPACE



EVOLUTION OF LONGITUDINAL PHASE SPACE



EVOLUTION OF LONGITUDINAL PHASE SPACE



Arc length

RECOMBINATION PATTERN

In order to avoid boosting short-range wakefields, the lengths of the arcs should be tuned preventing the recombination of different bunches in the same bucket

The filling of the RF buckets can be controlled tuning the length of the arcs

Maximise the separation between the bunches at first and last turn

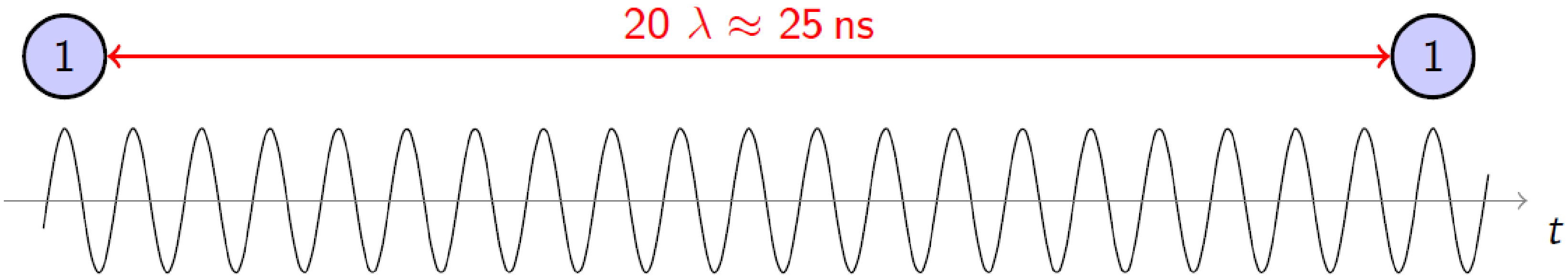
Multi – bunch effects are enhanced by the value of β/E

Low energy particles are more susceptible

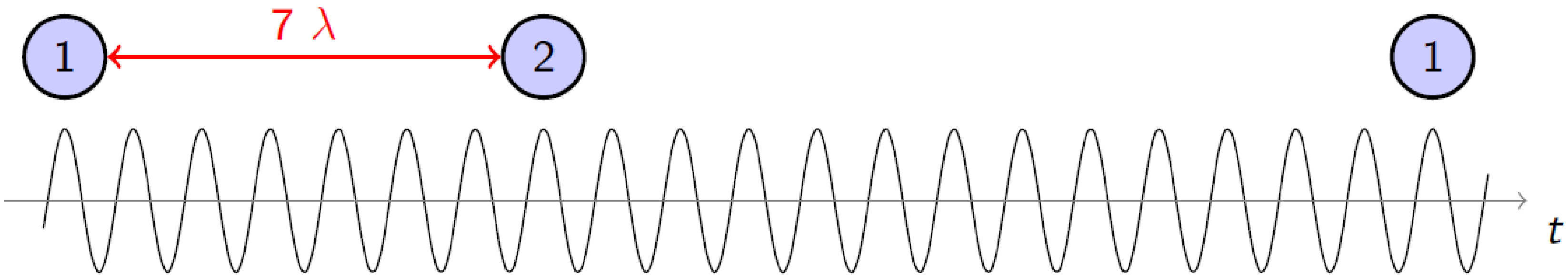
EXAMPLE CASE

The choice of 802 MHz RF frequency leads to 19 empty buckets between two injections at 25 ns, that can host the bunches at higher turn numbers

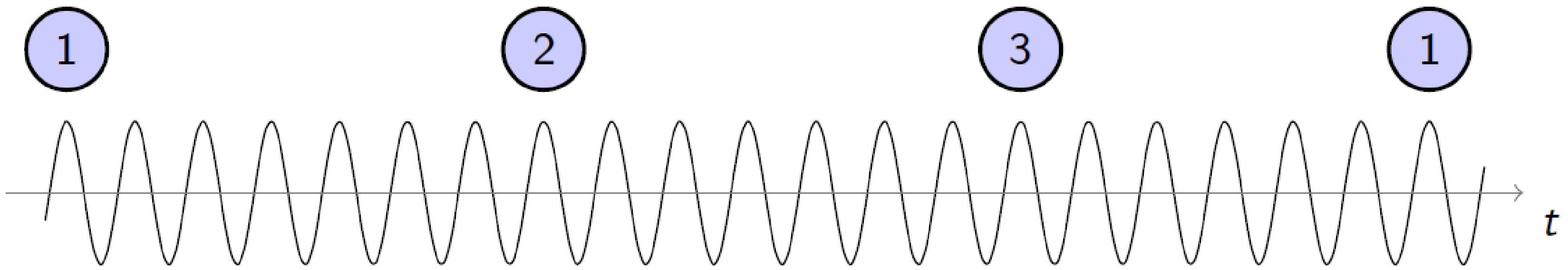
RECOMBINATION PATTERN



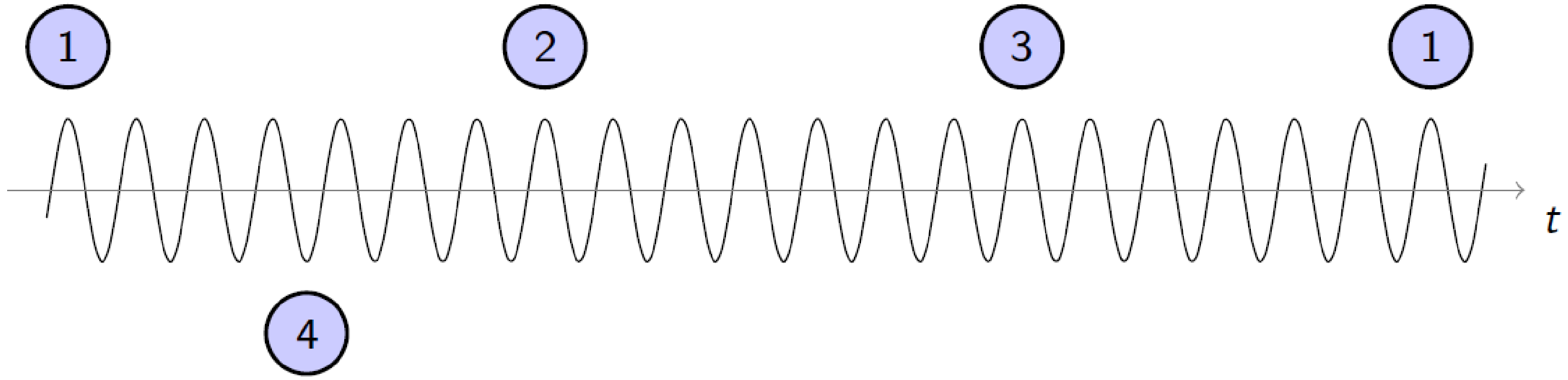
RECOMBINATION PATTERN



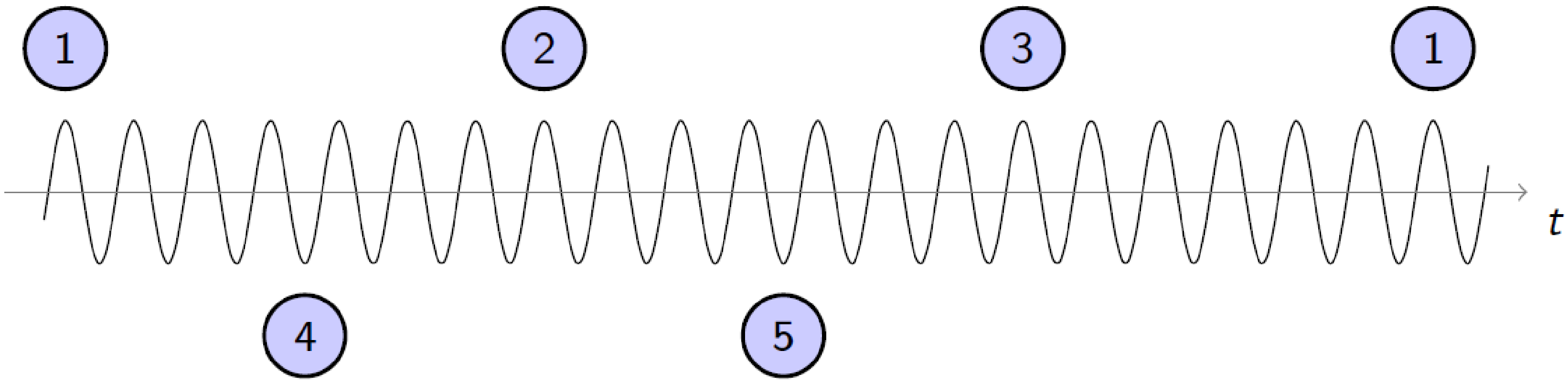
RECOMBINATION PATTERN



RECOMBINATION PATTERN

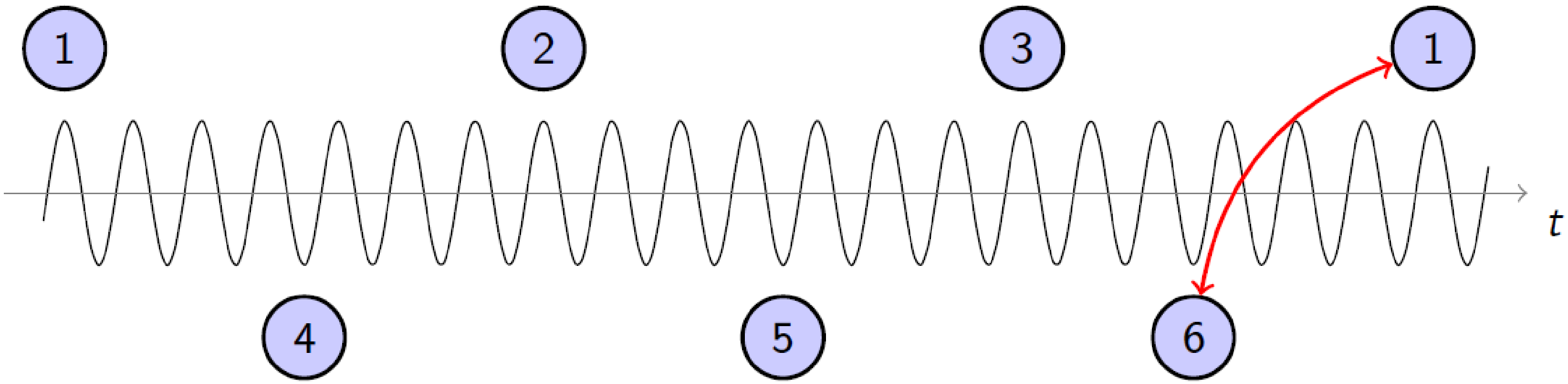


RECOMBINATION PATTERN



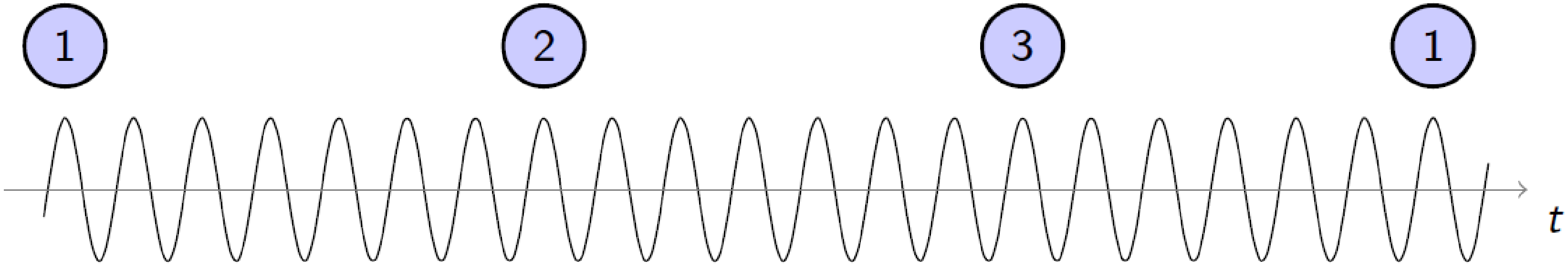
RECOMBINATION PATTERN

A good choice for the recombination pattern consists of almost equal spacing (compatibly with the RF) of the bunches in the RF buckets and a maximal separation between the bunches at the lowest energy that are more subjected to the kicks from the HOMs due to their lower rigidity



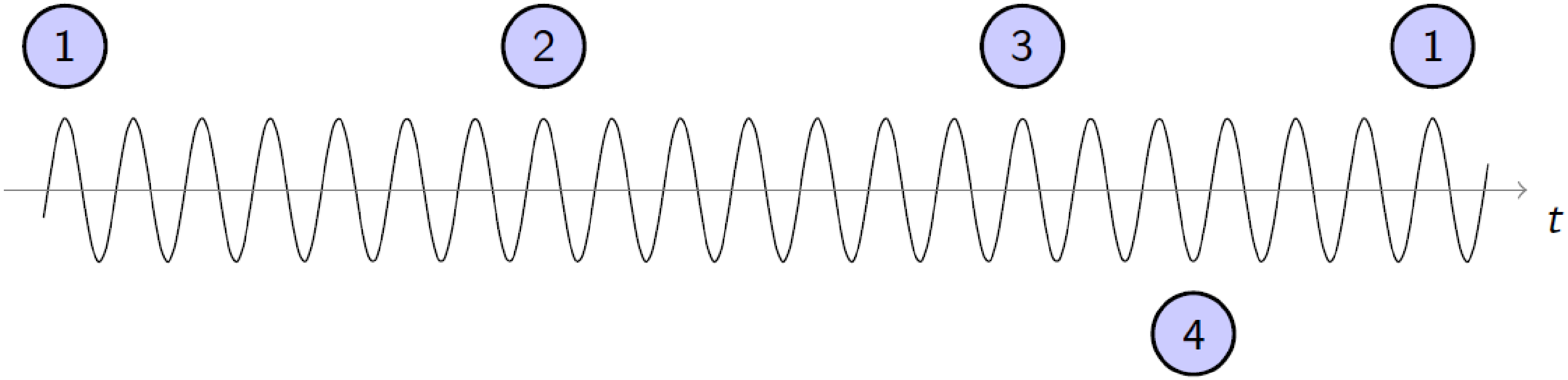
PATTERN 162435 is bad

RECOMBINATION PATTERN



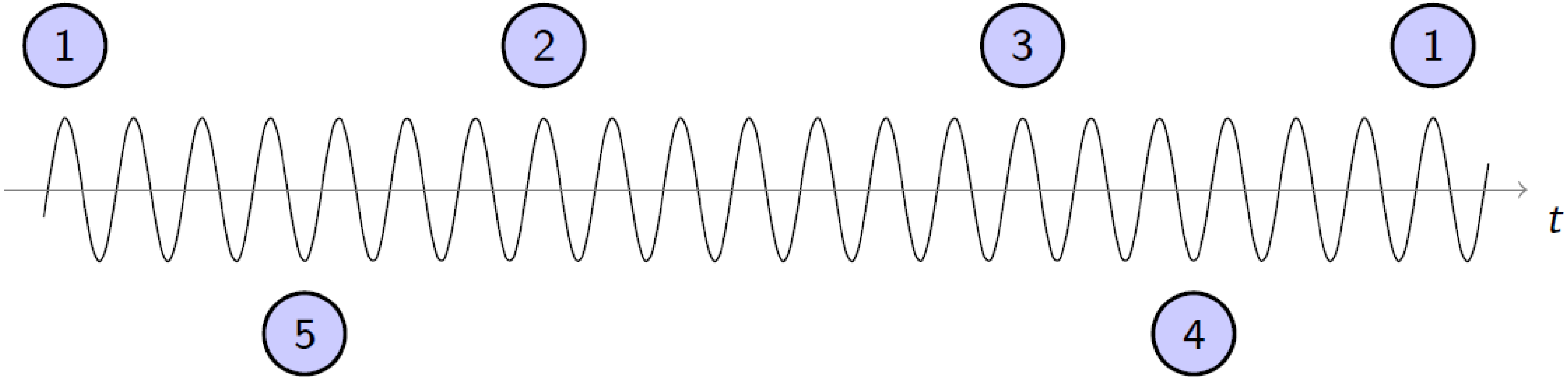
PATTERN 162435 is bad

RECOMBINATION PATTERN



PATTERN 162435 is bad

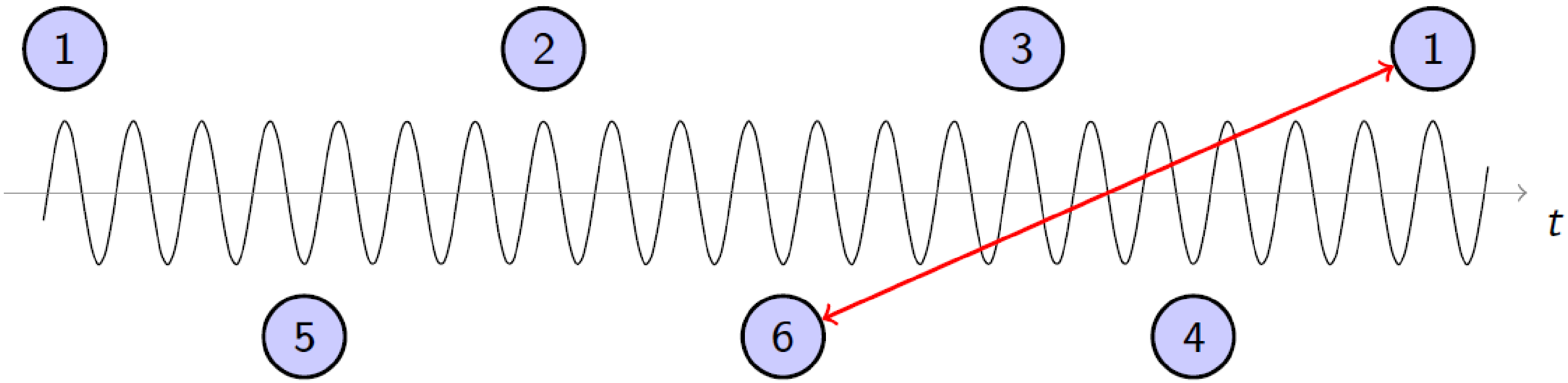
RECOMBINATION PATTERN



PATTERN 162435 is bad

RECOMBINATION PATTERN

Possible recombination pattern that maximises the separation between the bunches at first and last turn



PATTERN 162435 is bad
PATTERN 152634 is better

Simulation tools

SIMULATION TOOLS FOR ERLs

Several codes available for optics and beam dynamics simulations
MADX, OPTIM, ASTRA, ELEGANT, OPAL, PLACET2 …

As example, I will show a possible use of **OPTIM***, **Computer code for linear and non-linear optics calculations**

✧ **6D computations**
- large set of optics elements
- *x*-*y* coupling, acceleration (focusing in cavities is taken into account)..

✧ Similar to MADX but has **integrated GUI**

✧ Can **generate MAD and MADX** files from OptiM files

✧ It has been used for optics support of the following machines
- Jefferson lab (CEBAF – optics redesign, analysis of optics measurements…)
- Fermilab (Tevatron, Debuncher, Transfer lines, Electron cooler)
- LHeC …

* http://pbar.fnal.gov/organizationalchart/lebedev/OptiM/optim.htm

LHeC LINAC LAYOUT IN OPTIM

8 cavities

1 Cryomodule + 1 Cryomodule +1 Cryomodule + 1 Cryomodule + Optics 130° FODO

1 UNIT + 17 UNITs

18 UNITS = LINAC

oD00	qLF01	oD00	$Cryo1	$Cryo1	oD00	qLD01	oD00	$Cryo1	$Cryo1
oD00	qLF02	oD00	$Cryo1	$Cryo1	oD00	qLD02	oD00	$Cryo1	$Cryo1
oD00	qLF03	oD00	$Cryo1	$Cryo1	oD00	qLD03	oD00	$Cryo1	$Cryo1
oD00	qLF04	oD00	$Cryo1	$Cryo1	oD00	qLD04	oD00	$Cryo1	$Cryo1
oD00	qLF05	oD00	$Cryo1	$Cryo1	oD00	qLD05	oD00	$Cryo1	$Cryo1
oD00	qLF06	oD00	$Cryo1	$Cryo1	oD00	qLD06	oD00	$Cryo1	$Cryo1
oD00	qLF07	oD00	$Cryo1	$Cryo1	oD00	qLD07	oD00	$Cryo1	$Cryo1
oD00	qLF08	oD00	$Cryo1	$Cryo1	oD00	qLD08	oD00	$Cryo1	$Cryo1
oD00	qLF09	oD00	$Cryo1	$Cryo1	oD00	qLD09	oD00	$Cryo1	$Cryo1
oD00	qLF10	oD00	$Cryo1	$Cryo1	oD00	qLD10	oD00	$Cryo1	$Cryo1
oD00	qLF11	oD00	$Cryo1	$Cryo1	oD00	qLD11	oD00	$Cryo1	$Cryo1
oD00	qLF12	oD00	$Cryo1	$Cryo1	oD00	qLD12	oD00	$Cryo1	$Cryo1
oD00	qLF13	oD00	$Cryo1	$Cryo1	oD00	qLD13	oD00	$Cryo1	$Cryo1
oD00	qLF14	oD00	$Cryo1	$Cryo1	oD00	qLD14	oD00	$Cryo1	$Cryo1
oD00	qLF15	oD00	$Cryo1	$Cryo1	oD00	qLD15	oD00	$Cryo1	$Cryo1
oD00	qLF16	oD00	$Cryo1	$Cryo1	oD00	qLD16	oD00	$Cryo1	$Cryo1
oD00	qLF17	oD00	$Cryo1	$Cryo1	oD00	qLD17	oD00	$Cryo1	$Cryo1
oD00	qLF18	oD00	$Cryo1	$Cryo1	oD00	qLD18	oD00	$Cryo1	$Cryo1

LATTICE DESCRIPTION :
order of the elements in the lattice

LHeC LINAC LAYOUT IN OPTIM

```
# Main cavity parameters
#
$F=700e6; => 700000000
 $Lambda=$c/$F; => 42.827494
 $LCav=100; => 100
 $DE=$Grad*$LCav/100; => 17.3611111
#
$Cryo1="oCry A1 oCry A1 oCry A1 oCry A1 oCry A1 oCry A1 oCry A1 oCry A1 oCry ";
$LCry=60; => 60
$Lq=100; => 100
9*$LCry+8*$LCav); => 1340
$L00=(2800- $Lq -2*(9*$LCry+8*$LCav))/2; => 10
$FIdeg=0; => 0
$FI=$FIdeg/180*$PI; => 0
```

1 Cryomodule → 8 cavities

In 1 UNIT→ 4 Cryos → 32 cavities

$ΔE = energy gain per cavity = 17.36 MeV

$E00 = 500MeV (Injection Energy)

Energy gain/half unit :

$E01 = $E00 +16 *$DE*cos($Fi)

10 GeV Linac 1 :

500 MeV → 10500 MeV for the first pass

```
# Energy along machine
#
$E01 =$E00+16*$DE*cos($FI); => 777.777778
$E02 =$E01 +16*$DE*cos($FI); => 1055.55556
$E03 =$E02 +16*$DE*cos($FI); => 1333.33333
$E04 =$E03 +16*$DE*cos($FI); => 1611.11111
$E05 =$E04 +16*$DE*cos($FI); => 1888.88889
$E06 =$E05 +16*$DE*cos($FI); => 2166.66667
$E07 =$E06 +16*$DE*cos($FI); => 2444.44444
$E08 =$E07 +16*$DE*cos($FI); => 2722.22222
$E09 =$E08 +16*$DE*cos($FI); => 3000
$E10 =$E09 +16*$DE*cos($FI); => 3277.77778
#
$E11 =$E10+16*$DE*cos($FI); => 3555.55556
$E12 =$E11 +16*$DE*cos($FI); => 3833.33333
$E13 =$E12 +16*$DE*cos($FI); => 4111.11111
$E14 =$E13 +16*$DE*cos($FI); => 4388.88889
$E15 =$E14 +16*$DE*cos($FI); => 4666.66667
$E16 =$E15 +16*$DE*cos($FI); => 4944.44444
$E17 =$E16 +16*$DE*cos($FI); => 5222.22222
$E18 =$E17 +16*$DE*cos($FI); => 5500
$E19 =$E18 +16*$DE*cos($FI); => 5777.77778
$E20 =$E19 +16*$DE*cos($FI); => 6055.55556
#
$E21 =$E20+16*$DE*cos($FI); => 6333.33333
$E22 =$E21 +16*$DE*cos($FI); => 6611.11111
$E23 =$E22 +16*$DE*cos($FI); => 6888.88889
$E24 =$E23 +16*$DE*cos($FI); => 7166.66667
$E25 =$E24 +16*$DE*cos($FI); => 7444.44444
$E26 =$E25 +16*$DE*cos($FI); => 7722.22222
$E27 =$E26 +16*$DE*cos($FI); => 8000
$E28 =$E27 +16*$DE*cos($FI); => 8277.77778
$E29 =$E28 +16*$DE*cos($FI); => 8555.55556
$E30 =$E29 +16*$DE*cos($FI); => 8833.33333
#
$E31 =$E30+16*$DE*cos($FI); => 9111.11111
$E32 =$E31 +16*$DE*cos($FI); => 9388.88889
$E33 =$E32 +16*$DE*cos($FI); => 9666.66667
$E34 =$E33 +16*$DE*cos($FI); => 9944.44444
$E35 =$E34 +16*$DE*cos($FI); => 10222.2222
#
```

LHeC LINAC LAYOUT IN OPTIM

Quads strength profile

```
#
A1          L[cm]=$LCav  Ncell=5  Eff_L[cm]=10  A[MeV]=$DE  Phase[deg]=$FIdeg  WaveL[cm]=$Lambda
#
qLF01       L[cm]=$Lq       G[kG/cm]=$GF01          Tilt[deg]=0
qLF02       L[cm]=$Lq       G[kG/cm]=$GF02          Tilt[deg]=0
qLF03       L[cm]=$Lq       G[kG/cm]=$GF03          Tilt[deg]=0
qLF04       L[cm]=$Lq       G[kG/cm]=$GF04 Tilt[deg]=0
qLF05       L[cm]=$Lq       G[kG/cm]=$GF05          Tilt[deg]=0
qLF06       L[cm]=$Lq       G[kG/cm]=$GF06          Tilt[deg]=0
qLF07       L[cm]=$Lq       G[kG/cm]=$GF07          Tilt[deg]=0
qLF08       L[cm]=$Lq       G[kG/cm]=$GF08 Tilt[deg]=0
qLF09       L[cm]=$Lq       G[kG/cm]=$GF09          Tilt[deg]=0
qLF10       L[cm]=$Lq       G[kG/cm]=$GF10          Tilt[deg]=0
qLF11       L[cm]=$Lq       G[kG/cm]=$GF11          Tilt[deg]=0
qLF12       L[cm]=$Lq       G[kG/cm]=$GF12          Tilt[deg]=0
qLF13       L[cm]=$Lq       G[kG/cm]=$GF13          Tilt[deg]=0
qLF14       L[cm]=$Lq       G[kG/cm]=$GF14 Tilt[deg]=0
qLF15       L[cm]=$Lq       G[kG/cm]=$GF15          Tilt[deg]=0
qLF16       L[cm]=$Lq       G[kG/cm]=$GF16          Tilt[deg]=0
qLF17       L[cm]=$Lq       G[kG/cm]=$GF17          Tilt[deg]=0
qLF18       L[cm]=$Lq       G[kG/cm]=$GF18 Tilt[deg]=0
#
qLD01       L[cm]=$Lq       G[kG/cm]=$GD01          Tilt[deg]=0
qLD02       L[cm]=$Lq       G[kG/cm]=$GD02          Tilt[deg]=0
qLD03       L[cm]=$Lq       G[kG/cm]=$GD03          Tilt[deg]=0
qLD04       L[cm]=$Lq       G[kG/cm]=$GD04          Tilt[deg]=0
qLD05       L[cm]=$Lq       G[kG/cm]=$GD05          Tilt[deg]=0
qLD06       L[cm]=$Lq       G[kG/cm]=$GD06          Tilt[deg]=0
qLD07       L[cm]=$Lq       G[kG/cm]=$GD07          Tilt[deg]=0
qLD08       L[cm]=$Lq       G[kG/cm]=$GD08          Tilt[deg]=0
qLD09       L[cm]=$Lq       G[kG/cm]=$GD09          Tilt[deg]=0
qLD10       L[cm]=$Lq       G[kG/cm]=$GD10          Tilt[deg]=0
qLD11       L[cm]=$Lq       G[kG/cm]=$GD11          Tilt[deg]=0
qLD12       L[cm]=$Lq       G[kG/cm]=$GD12          Tilt[deg]=0
qLD13       L[cm]=$Lq       G[kG/cm]=$GD13          Tilt[deg]=0
qLD14       L[cm]=$Lq       G[kG/cm]=$GD14          Tilt[deg]=0
qLD15       L[cm]=$Lq       G[kG/cm]=$GD15          Tilt[deg]=0
qLD16       L[cm]=$Lq       G[kG/cm]=$GD16          Tilt[deg]=0
qLD17       L[cm]=$Lq       G[kG/cm]=$GD17          Tilt[deg]=0
qLD18       L[cm]=$Lq       G[kG/cm]=$GD18          Tilt[deg]=0
#
end list of elements
```

Gradient scaling

```
$GF01  = $GF00*$P00/$P00;  =>  0.01026216
$GD01  = $GD00*$P01/$P01;  =>  -0.0160967
$GF02  = $GF00*$P02/$P00;  =>0.0216529273
$GD02  = $GD00*$P03/$P01;  =>-0.0275867977
$GF03  = $GF00*$P04/$P00;  =>0.0330436927
$GD03  = $GD00*$P05/$P01;  =>-0.0390768946
$GF04  = $GF00*$P06/$P00;  =>0.0444344576
$GD04  = $GD00*$P07/$P01;  =>-0.0505669912
$GF05  = $GF00*$P08/$P00;  =>0.0558252224
$GD05  = $GD00*$P09/$P01;  =>-0.0620570877
$GF06  = $GF00*$P10/$P00;  =>0.0672159871
$GD06 = $GD00*$P11/$P01;  =>-0.0735471842
$GF07  = $GF00*$P12/$P00;  =>0.0786067517
$GD07 = $GD00*$P13/$P01;  =>-0.0850372806
$GF08  = $GF00*$P14/$P00;  =>0.0899975163
$GD08 = $GD00*$P15/$P01;  =>-0.0965273769
$GF09  = $GF00*$P16/$P00;  => 0.101388281
$GD09 = $GD00*$P17/$P01;  =>-0.108017473
$GF10  = $GF00*$P18/$P00;  => 0.112779045
$GD10 = $GD00*$P19/$P01;  => -0.11950757
$GF11  = $GF00*$P20/$P00;  =>  0.12416981
$GD11  = $GD00*$P21/$P01;  =>-0.130997666
$GF12  = $GF00*$P22/$P00;  => 0.135560574
$GD12  = $GD00*$P23/$P01;  =>-0.142487762
$GF13  = $GF00*$P24/$P00;  => 0.146951339
$GD13  = $GD00*$P25/$P01;  =>-0.153977859
$GF14  = $GF00*$P26/$P00;  => 0.158342104
$GD14  = $GD00*$P27/$P01;  =>-0.165467955
$GF15  = $GF00*$P28/$P00;  => 0.169732868
$GD15  = $GD00*$P29/$P01;  =>-0.176958051
$GF16  = $GF00*$P30/$P00;  => 0.181123633
$GD16 = $GD00*$P31/$P01;  =>-0.188448148
$GF17  = $GF00*$P32/$P00;  => 0.192514397
$GD17 = $GD00*$P33/$P01;  =>-0.199938244
$GF18  = $GF00*$P34/$P00;  => 0.203905162
$GD18 = $GD00*$P35/$P01;  => -0.21142834
```

MATCHING OF OPTICS FUNCTIONS

Fitting of beta-functions, dispersion and momentum compaction

```
BetaFitBlock dL[cm]=0.01 dB[kGs]=0.01 dG[kGs/cm]=1e-07
#Required parameters and their accuracy listed below(dPARM<=0. - no fitting)
#Maximum Betas[cm] and MomentumCompaction are on the next line
BtXmax=5000 dBtXmax=0 BtYmax=5000 dBtYmax=0 Alfa=0 dAlfa=0
#Fitting parameters at the end of the lattice
Beta_X[cm]=100  dBeta_X[cm]=0   Alfa_X=0      dAlfa_X=0
Beta_Y[cm]=100  dBeta_Y[cm]=0   Alfa_Y=0      dAlfa_Y=0
Disp_X[cm]=0    dDisp_X[cm]=0   D_prime_X=0   dD_prime_X=0
Disp_Y[cm]=0    dDisp_Y[cm]=0   D_prime_Y=0   dD_prime_Y=0
Qx=0.361111     dQx=1e-06
Qy=0.361111     dQy=1e-06
#Fit at element with number =2
#To create a Fitting at intermidiate element: uncomment the line above,
# write the correct element number and insert six lines describing the
# fit parameters. You can use up to 4 intermidiate points
#Each point has to be determined as described above
#
#Insert groups of elements below. Each group has to be located on one line.
#Start from the letter describing the type of changable parameter such as: L:, B:, G:
G: qLF01
G: qLD01
EndBetaFitBlock
```

The program uses the steepest descend method with automatically chosen step. The initial values of steps for length, magnetic field and its gradient are determined here

Required parameters and their accuracy. To calculate the fitting error (which is minimized in the course of the fitting) the program uses the accuracy parameters for each of fitting parameters

Elements can be organized in groups so that the elements in each group are changed proportionally during fitting

ARC OPTICS LAYOUT IN OPTIM

MATH HEADER : numeric variables and calculation

```
$R=100000; =>     100000
$N=60; =>       60
#
$Larc=$PI*$R; => 314159.265
$Lcell=$Larc/$N; => 5235.98776
$dp_p=3e-4; =>     0.0003
#
#$rho/$R; => 0.763943727
#
```

ARC RADIUS

CELLS NUMBER

ARC LENGTH

CELL LENGTH

```
#Arc dipole
#number of dipoles
$Ndip=$N*2*5; =>       600
#
 $Lb=400; =>       400
 $Ang_d= 180/$Ndip; =>       0.3
 $Ang_r=$Ang_d*$PI/180; =>0.00523598776
 $B=$Ang_r*$Hr/$Lb; => 2.20502593
 $rho=$Hr/$B; => 76394.3727
$rho/$R; => 0.763943727
#
```

TOTAL NUMBER OF DIPOLES

DIPOLE LENGTH

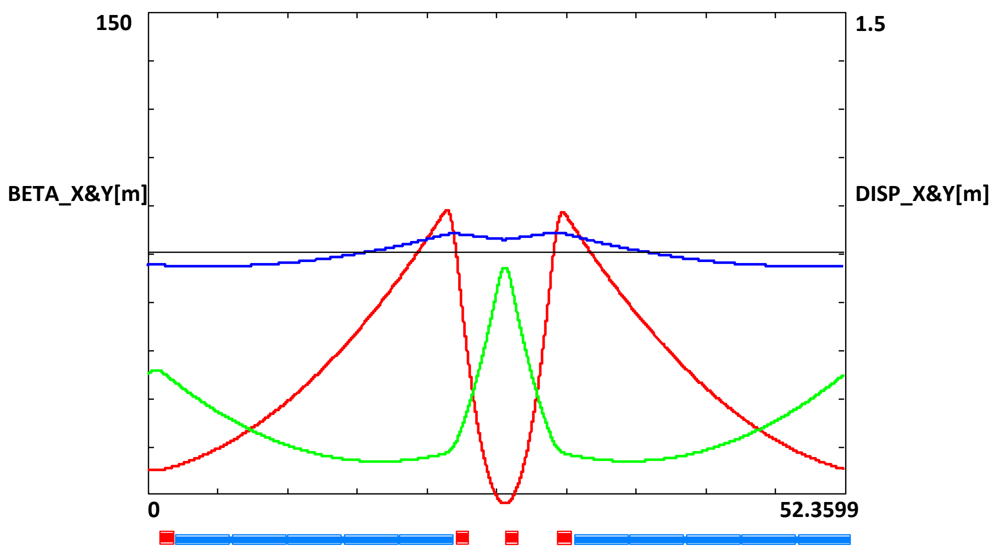
B=p/(ρc)

ARC OPTICS LAYOUT IN OPTIM

Quad singlet + 5 Dipoles + Quads triplet + 5 Dipoles

1 CELL

```
begin lattice. Number of periods=1
#
#
oD00 qQ0 oD00 $Dipole oD00 $Dipole oD00 $Dipole oD00 $Dipole oD00 $Dipole
oD00 qQ1 $D01 qQ2 $D00 qQ3 oD00 $Dipole oD00 $Dipole oD00 $Dipole oD00 $Dipole oD00 $Dipole
#
end lattice
```



Further readings and References

FOR FURTHER INFORMATION

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Many thanks for your attention

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