CSR and Microbunching Instability

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Outline

- ❑ *Coherent Synchrotron Radiation*
  - Longitudinal electric field
  - Transverse emittance growth

- ❑ *Microbunching Instability*
  - Longitudinal Space Charge
  - Gain
  - Laser Heater

- ❑ *Challenges in ERL Arcs*

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Credits and References

- ***Main References:***

  Ya.S. Derbenev et al., *TESLA-FEL 95-05, DESY, Hamburg, Germany (1995).*

  E. L.Saldin, E. A.Schneidmiller, M. V. Yurkov, NIM A 490, 1 (2002).

- ***Lectures:***

  S. Di Mitri & M. Venturini, *USPAS Course (2013, 2015)*

- ***Technical Notes:***

  Beam Dynamics Newletter No. 38 (2005)

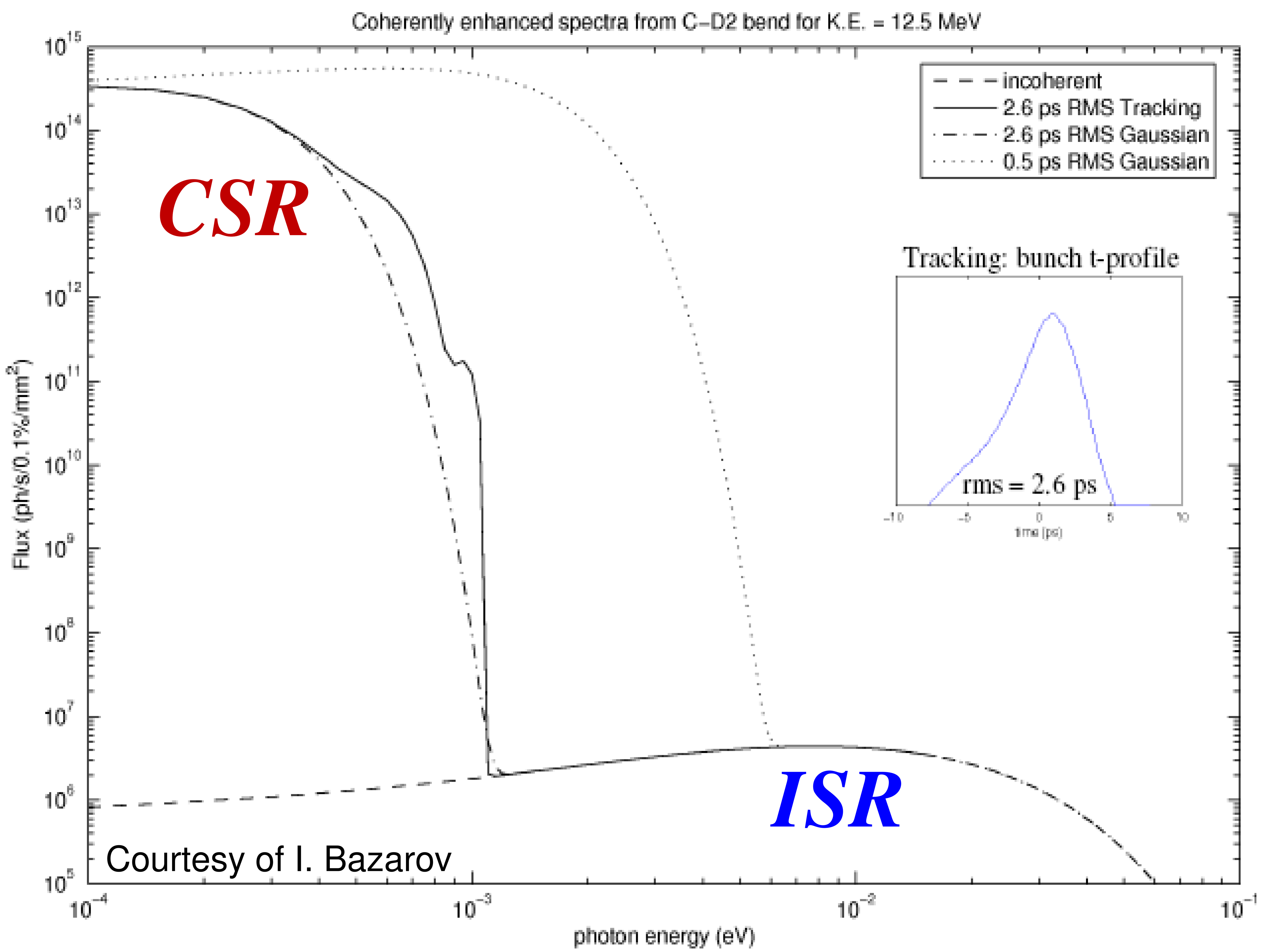
  S. Di Mitri & M. Cornacchia, *Physics Reports 539 (2014)*

- ***Acknowledgment:***

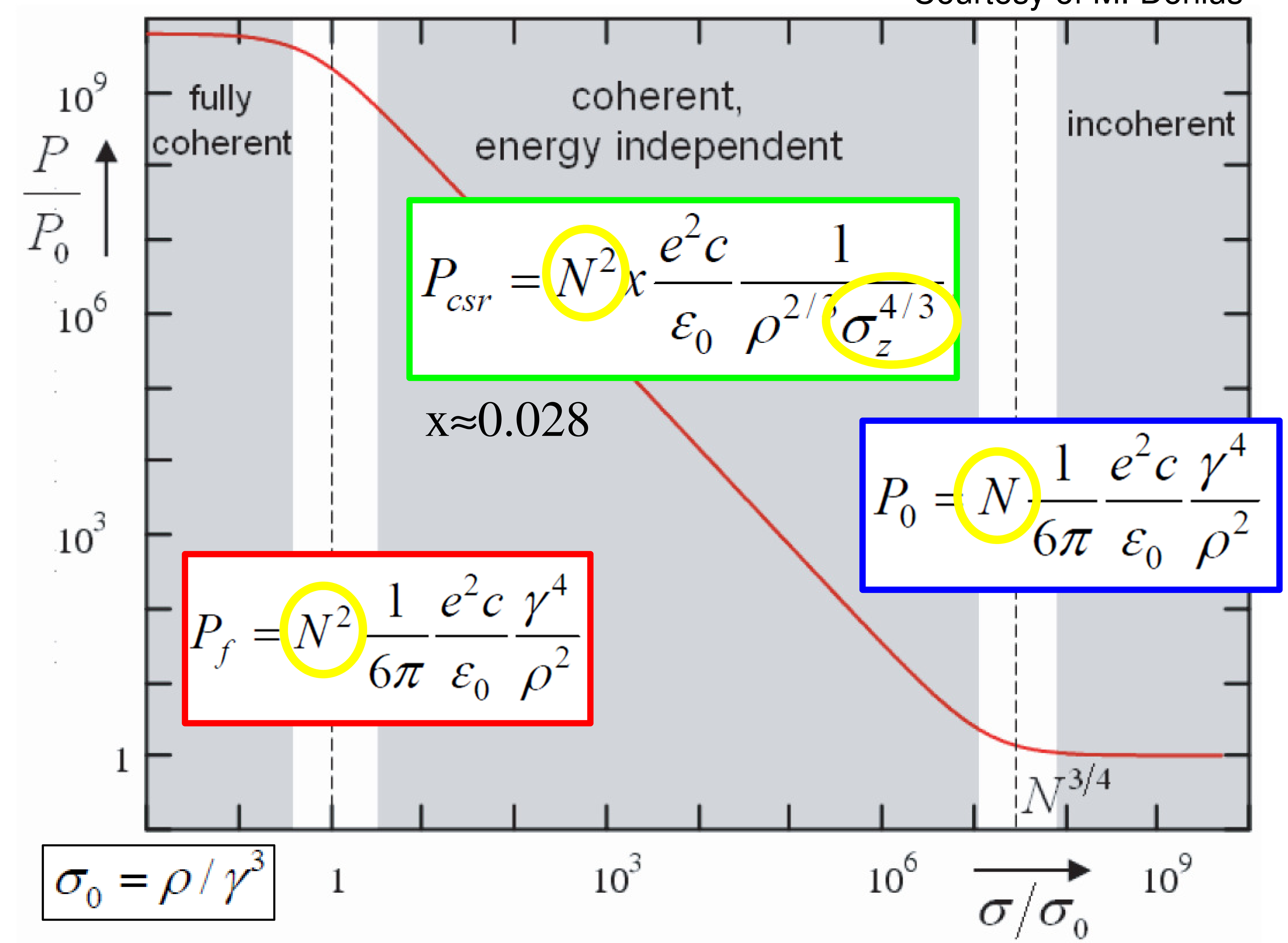
  M. Venturini, for valuable guidance and figures

Radiation Peak Power

For light wavelengths longer than the bunch length, the radiation intensity is coherently enhanced ($P \propto N^2$).



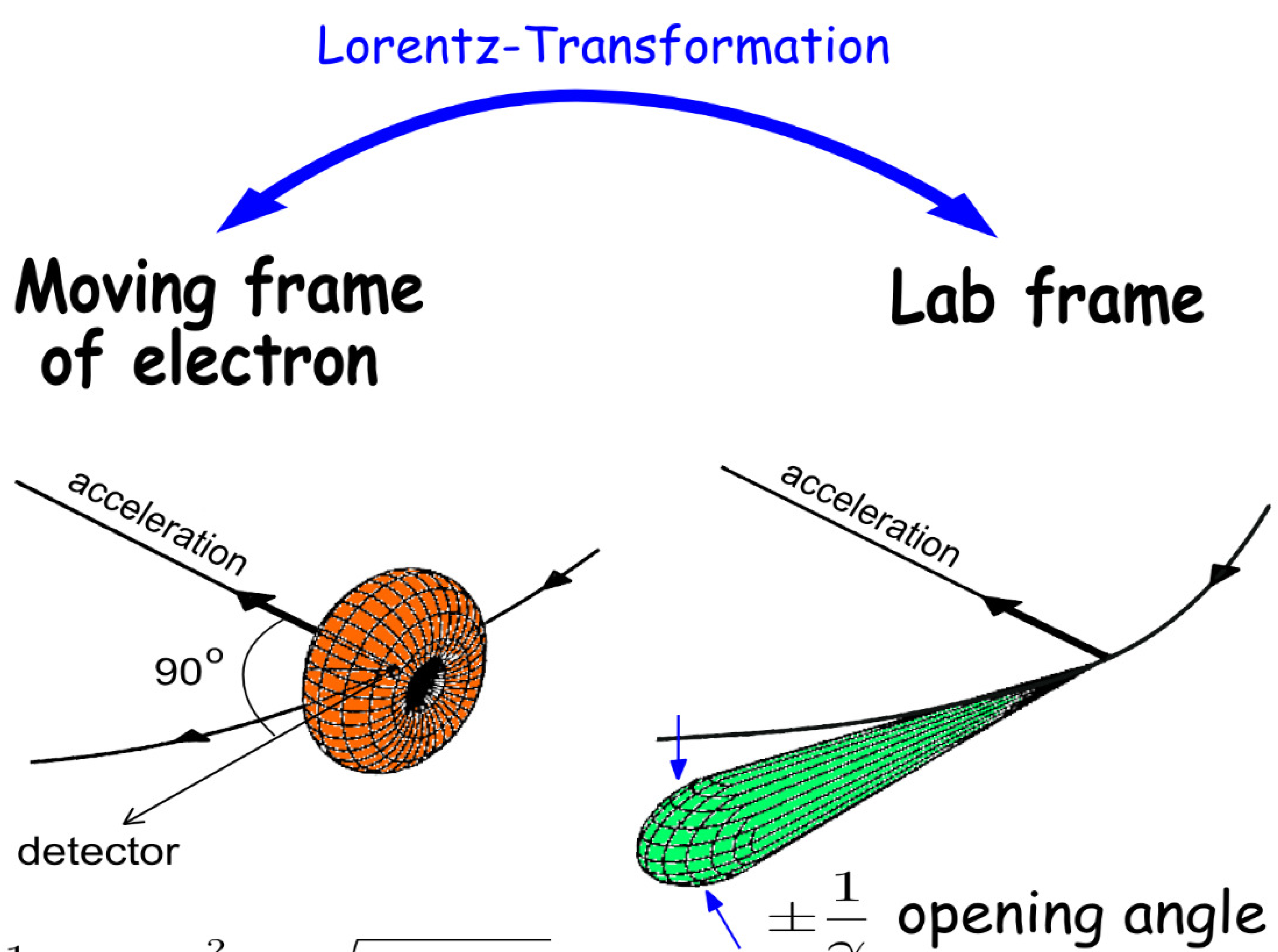
Courtesy of M. Dohlus



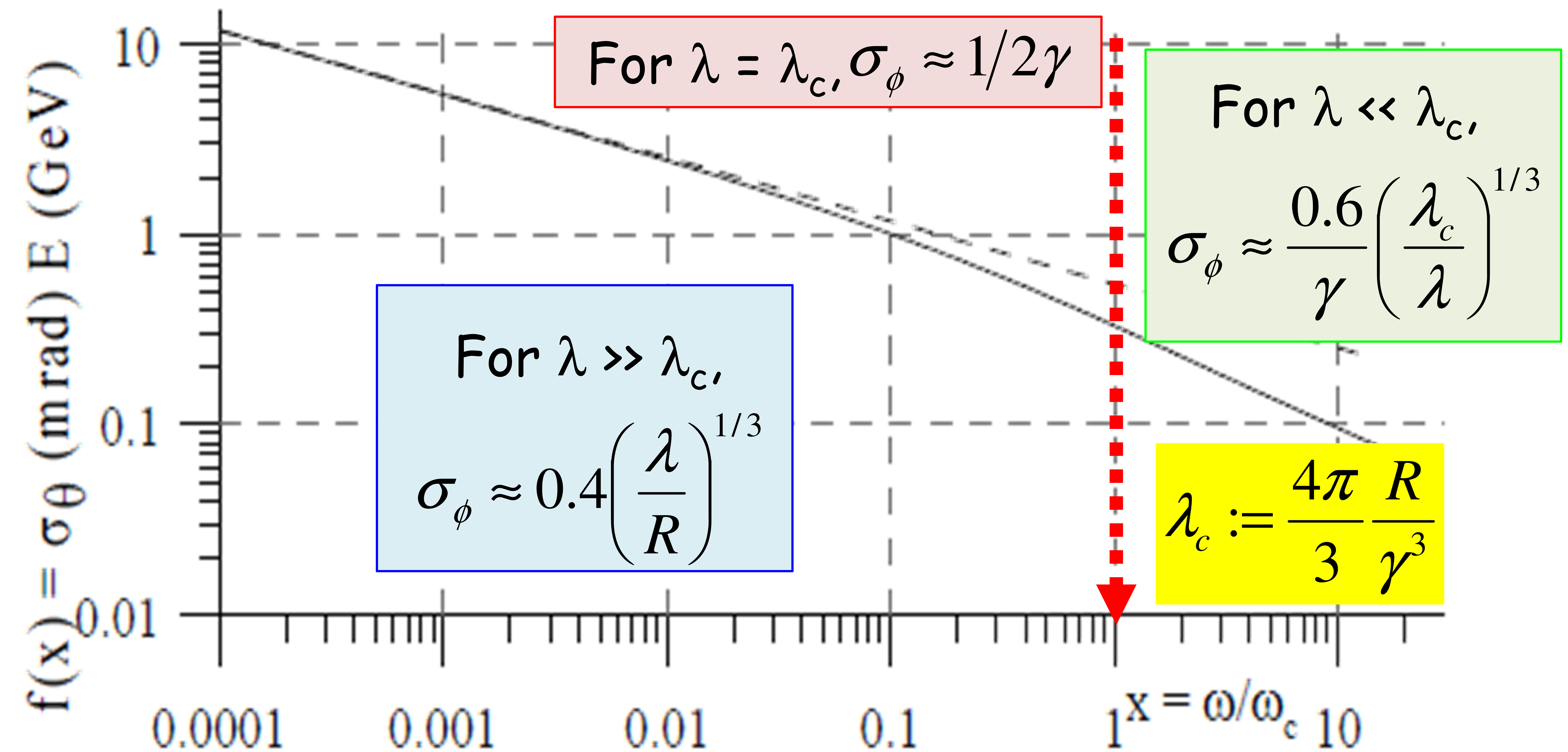
For typical bunch lengths, energies and compressors geometry in linacs for FELs, P_{CSR} is in the "energy-independent" regime (above, Gaussian beam).

Radiation Opening Angle

ISR at the <u>critical frequency</u> is emitted in dipoles within an angular cone ~ $1/\gamma$ (in the lab frame). *What about CSR ?*



$$\frac{1}{\gamma} = \frac{m_o c^2}{E} = \sqrt{1 - (\frac{v}{c})^2}$$



Example (FERMI BC1):

E = 300 MeV ($1/\gamma \approx 1.7$ mrad)

L = 0.3 m, θ = 100 mrad

R = 3.0 m (B = 0.3 T) $\rightarrow \lambda_c \cong 60\ nm$

I_b = 3 mm -> 0.1 mm $\rightarrow \lambda_{CSR} \geq 10\ \mu m$

$\Rightarrow \sigma_{\phi,CSR} \geq 6\ mrad$

CSR Effect on e-Beam

❑ Lienard-Wiechert retarded fields:

"velocity-" or "near-" field "acceleration-" or "far-" field

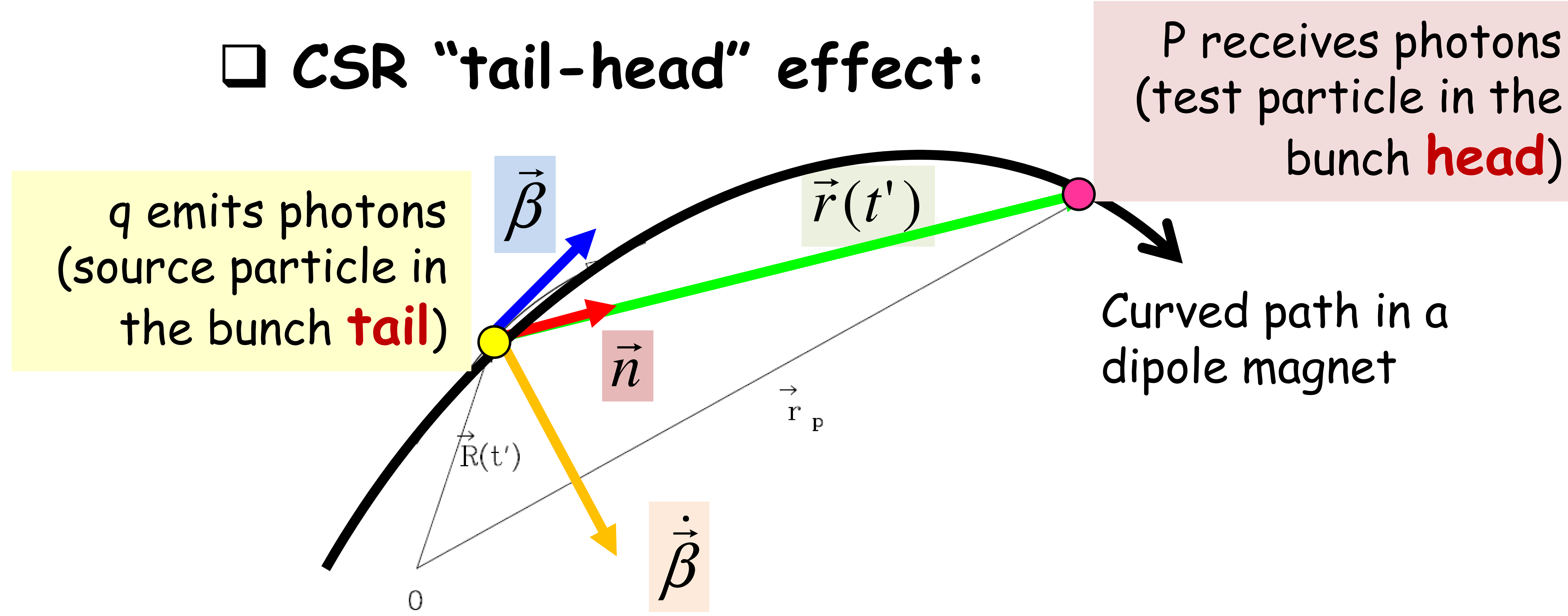
$$\vec{E}(t) = \cancel{\frac{q}{4\pi\varepsilon_0\gamma^2}\left[\frac{\vec{n}-\vec{\beta}}{r^2\left(1-\vec{n}\vec{\beta}\right)^3}\right]_{ret}} + \frac{q}{4\pi\varepsilon_0}\left\{\frac{\vec{n}\times\left[\left(\vec{n}-\vec{\beta}\right)\times\dot{\vec{\beta}}\right]}{cr\left(1-\vec{n}\vec{\beta}\right)^3}\right\}_{ret}$$

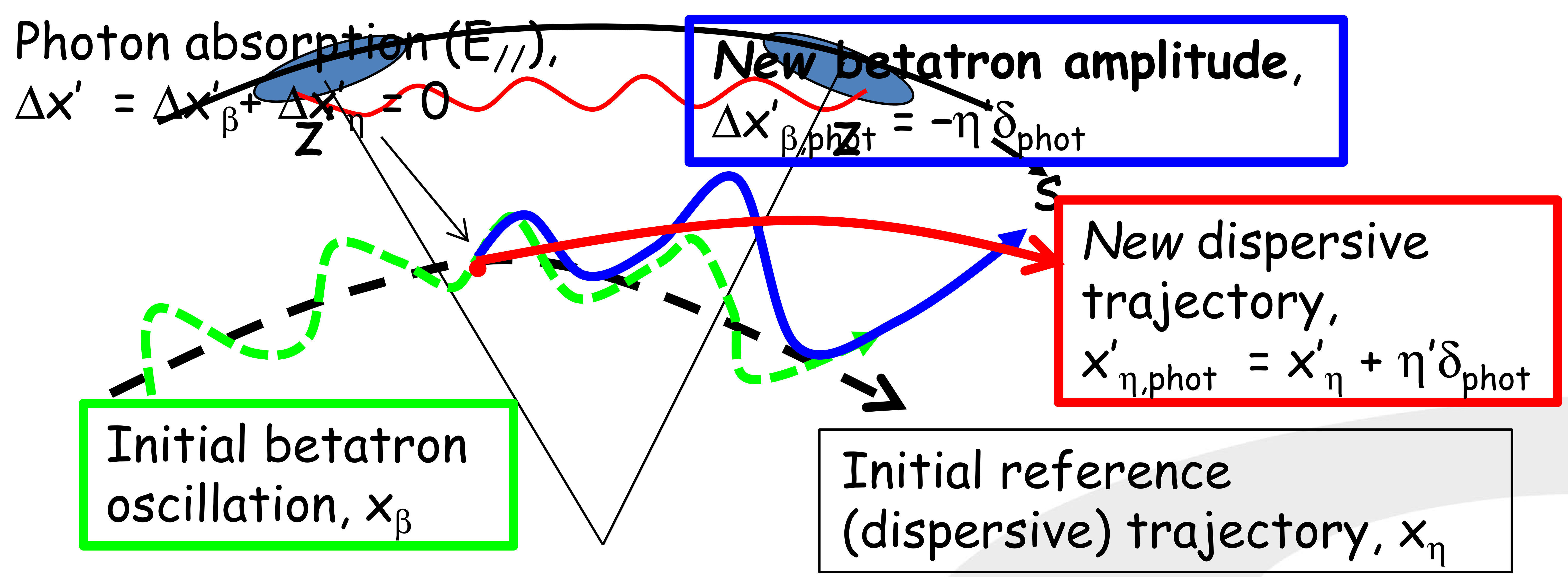
$$\vec{B}(t) = \frac{\vec{n}\times\vec{E}(t)}{c}$$

We consider range of parameters in which the near-field is suppressed by the $1/(r\gamma)^2$ dependence.

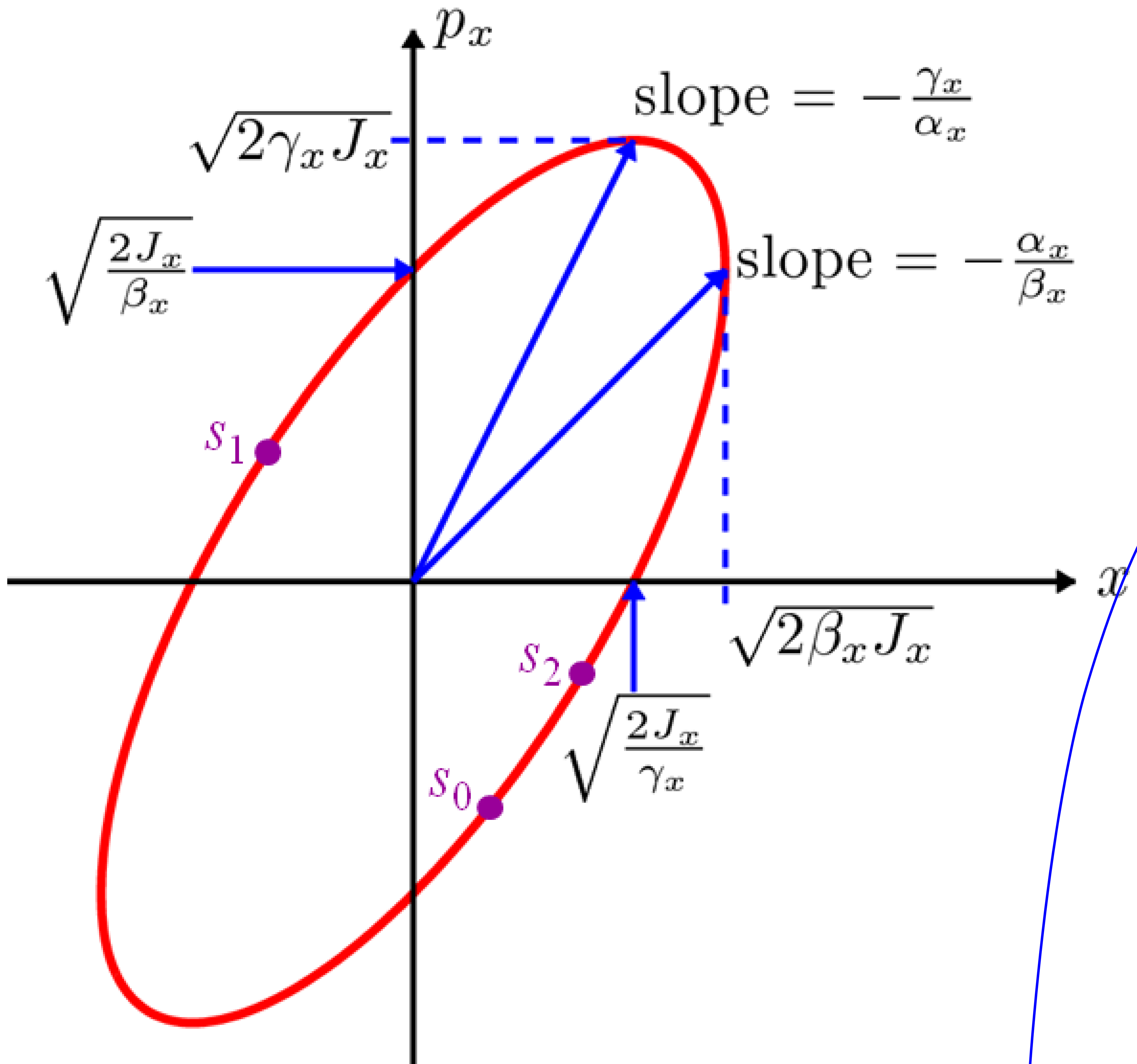
1. B and E are orthogonal.
2. We are interested to the E(t)-field component $E_{//}$ along **the *longitudinal velocity* of a test particle** ("observer").
3. Since **$E_{//}$ changes p_z** of a test particle ***in a dispersive* region** (e.g., in a dipole magnet), the particle transverse motion is also perturbed.

❑ CSR "tail-head" effect:





Phase Space Ellipse



Area of ellipse = $2\pi J_x$ is **invariant** under **linear motion** and **absence of frictional forces** (e.g., radiation emission).

For a particle beam,
$2J_x$ -> ε_x, EMITTANCE

- Twiss parameters and 2nd order beam momenta are connected:

$$\langle x_\beta^2\rangle = \beta_x \varepsilon_x \qquad \langle x'^2_\beta\rangle = \gamma_x \varepsilon_x \qquad \langle x_\beta x'_\beta\rangle = -\alpha_x \varepsilon_x$$

$$\varepsilon_x = \sqrt{\langle x_\beta^2\rangle\langle x'^2_\beta\rangle - \langle x_\beta x'_\beta\rangle^2}$$

RMS EMITTANCE

- Particle coordinates transform according to:

$$x(s) = x_\beta(s) + \boxed{R_{16}(s_0 \to s)}\boxed{\delta(s)} \equiv x_\beta + \Delta x$$
$$x'(s) = x'_\beta(s) + \boxed{R_{26}(s_0 \to s)}\boxed{\delta(s)} \equiv x'_\beta + \Delta x'$$

Superposition of Betatron motion + Dispersive motion

~ Energy dispersion functions

Change of longitudinal momentum by absorption of radiation

- For a single "energy-kick" at s, the beam horizontal emittance becomes:

$$\varepsilon_x^2 = \varepsilon_{x,0}^2 + \varepsilon_{x,0}\left(\beta_x\langle\Delta x'^2\rangle + 2\alpha_x\langle\Delta x\Delta x'\rangle + \gamma_x\langle\Delta x^2\rangle\right) + \left(\langle\Delta x^2\rangle\langle\Delta x'^2\rangle - \langle\Delta x\Delta x'\rangle^2\right)$$

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Projected Emittance Growth

- If δ is due to independent (i.e., z-uncorrelated or "<u>incoherent</u>") photon emissions, the emittance growth is a non-reversible process:

$$\varepsilon_x^2 \propto \left\langle \Delta x^2 \right\rangle_{inc} = \int_{s_0}^{s} ds' R_{16}^2(s') \frac{d\sigma_\delta^2(s')}{ds'} ds'$$

- *σ_δ is the sum of uncorrelated energy-kicks along the bunch.*
- *Integral is always positive (similar for $\Delta x'$).*

- If δ is <u>correlated</u> with z along the bunch – that applies to CSR - the emittance growth can be made *small or null*:

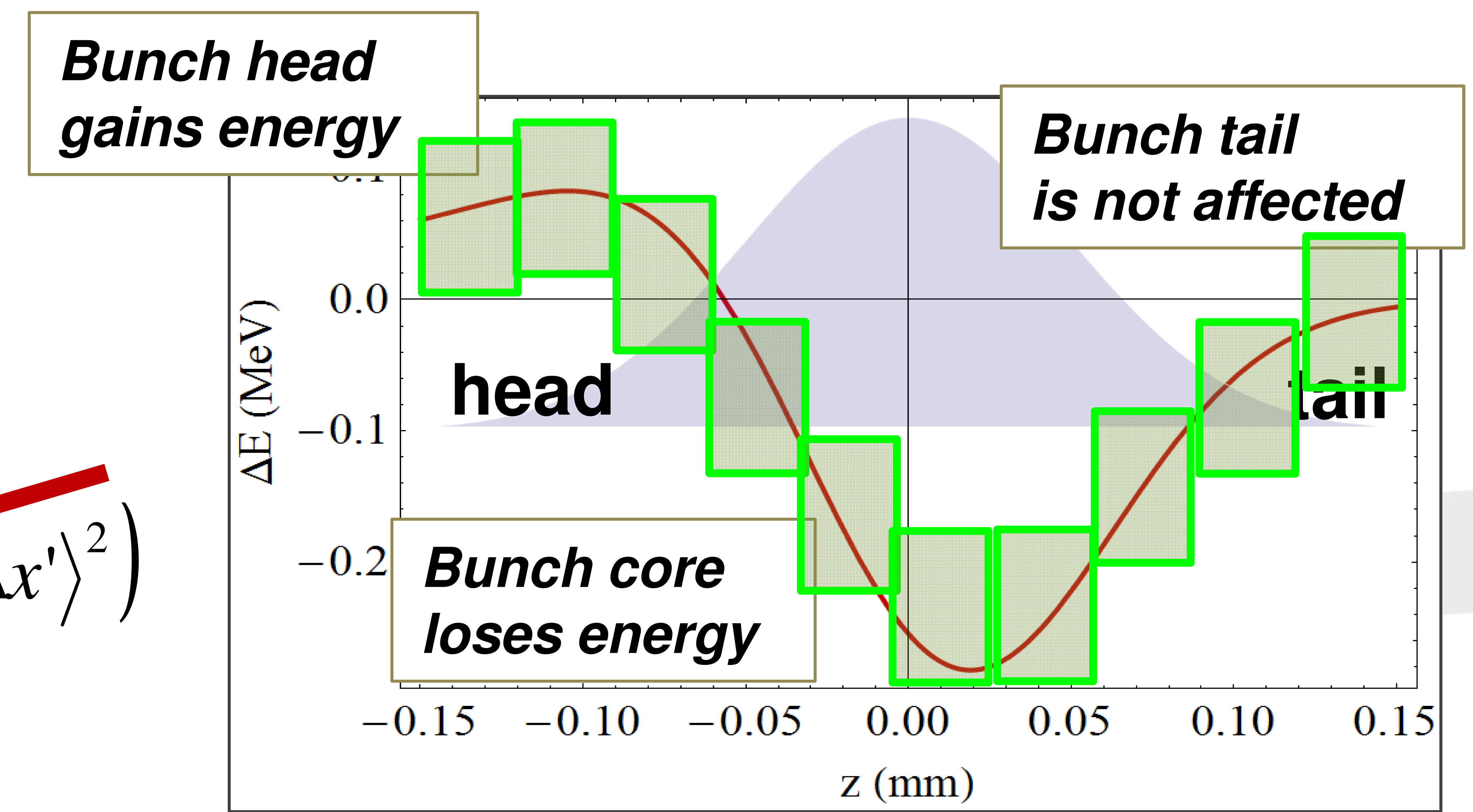
$$\varepsilon_x^2 \propto \left\langle \Delta x^2 \right\rangle_{csr} = \left[\int_{s_0}^{s} ds' R_{16}(s') \frac{d\sigma_\delta(z,s')}{ds'} ds' \right]^2$$

- *σ_δ is correlated with z along the bunch.*
- *We may design an optics that makes the integral small or zero (similar for $\Delta x'$).*

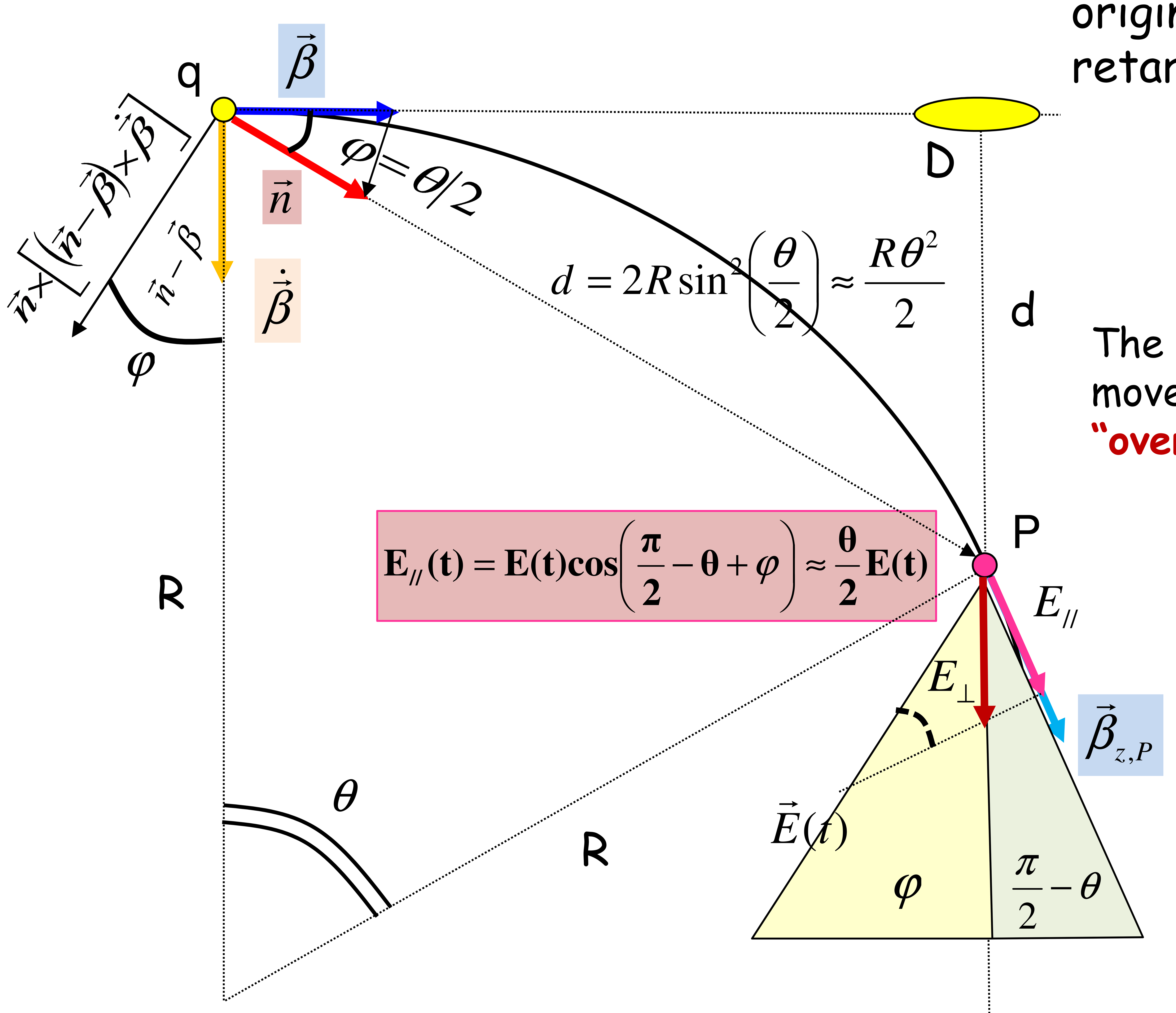
- Longitudinal bunch slices follow different trajectories:
  - each slice acquires a new betatron invariant,
  - the beam **projected emittance** (integrated over z) **grows…**,
  - …although the *slice* emittance might still be preserved.

$$\varepsilon_x^2 = \varepsilon_{x,0}^2 + \varepsilon_{x,0}\left(\beta_x\left\langle \Delta x'^2 \right\rangle + 2\alpha_x \left\langle \Delta x \Delta x' \right\rangle + \gamma_x \left\langle \Delta x^2 \right\rangle\right) + \left(\left\langle \Delta x^2 \right\rangle\left\langle \Delta x'^2 \right\rangle - \left\langle \Delta x \Delta x' \right\rangle^2\right)$$

Since Δx and $\Delta x'$ from CSR field are correlated, this goes to 0.



Retarded Electric Field (Longitudinal)



For $\theta<<1$, $E(t) \approx E_{\perp}$, i.e., the field direction is as if it originated from position D of the bunch line charge, at the retarded time t:

$$\vec{E}(t) \approx \frac{q}{4\pi\varepsilon_0}\left\{\frac{\vec{n}\times\left[\left(\vec{n}-\vec{\beta}\right)\times\dot{\vec{\beta}}\right]}{cr\left(1-\vec{n}\vec{\beta}\right)^3}\right\}_{ret} \rightarrow (\theta<<1)\; E_{\perp} = \frac{\lambda_z}{2\pi\varepsilon_0 d},$$

The maximum length of electron-field ***interaction*** as the bunch moves along the dipole, is the electron-photon **slippage** (also, **"overtaking length"**):

$$s = R\theta - 2R\sin(\theta/2) \approx \frac{R\theta^3}{24}$$

The **longitudinal electric field at P** becomes (use defs. for "s(θ)" and "d(θ)"):

$$E_{//} \approx E_{\perp}\theta = \frac{\lambda_z\theta}{2\pi\varepsilon_0 d} \approx \frac{1}{3^{1/3}2\pi\varepsilon_0}\frac{\lambda_z}{R^{2/3}s^{1/3}}$$

The **single particle energy loss per unit length** is (<u>uniform</u> charge distribution, $\lambda_z=Q/l_b$):

$$\frac{dE_e}{dz} \approx -\frac{1}{3^{1/3}2\pi\varepsilon_0}\frac{eQ}{R^{2/3}l_b^{4/3}}$$

CSR-Induced Energy Loss

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- Two particles on same circular path in a dipole magnet (**1-D model**),
- $\gamma^3 s/R \gg 1$ (**steady-state approximation**),

ENERGY CHANGE ALONG BUNCH, per METER:

$$\frac{dU_e(z)}{dz} \cong -\frac{Ne^2}{3^{1/3}2\pi\varepsilon_0}\frac{1}{R^{2/3}}\int_z^{\infty}\frac{dz'}{(z'-z)^{1/3}}\frac{d\lambda(z')}{dz'}$$

Current spikes or fast rises enhance the z-CSR field.

- Gaussian bunch:

$$U_{tot} = -0.028 \times e^2 Z_0 c N \frac{\theta R^{1/3}}{\sigma_z^{4/3}} = -0.16\ \mathrm{MeV}$$ **Total energy loss**

$$\langle\delta\rangle_{csr} \simeq -0.35 \times r_e \frac{N}{\gamma}\frac{\theta R^{1/3}}{\sigma_z^{4/3}} = 0.20\%$$ **Relative mean energy loss**

$$\sigma_{\delta,csr} \simeq 0.7 \times |\langle\delta\rangle_{csr}| = 0.14\%$$ **Relative rms energy spread**

$\sigma_z = 50\mu m$
$Q = 300pC$
$L_B = 1m$
$\theta_B = 10^o$
$R = 5.7m$
$I_{pk} = 715\ A$
$E = 700\ MeV$

CSR effect is larger for:
- **high charge**
- **low beam energy**
- **short bunch length**
- **large bending angle**

Single Kick in a 4-Dipoles Chicane

- In a 4-dipoles chicane, the CSR effect is ***stronger*** in the last dipole, where the bunch is ***shorter:***

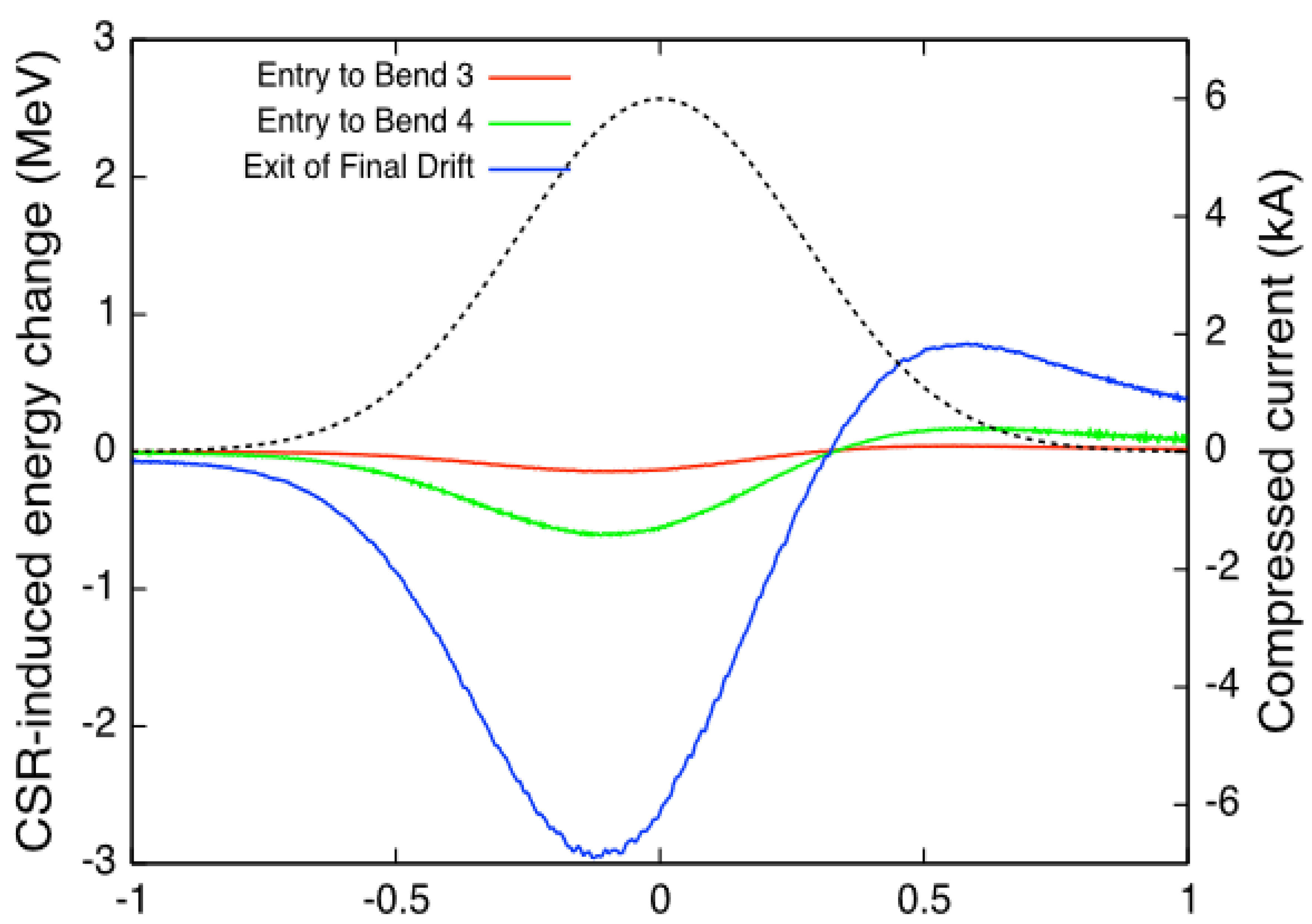
$$\boxed{\varepsilon_x^2 = \varepsilon_{x,0}^2 + \varepsilon_{x,0}\left(\beta_x\left\langle \Delta x'^2\right\rangle + 2\alpha_x\left\langle \Delta x \Delta x'\right\rangle + \gamma_x\left\langle \Delta x^2\right\rangle\right)} \Rightarrow \boxed{\varepsilon_x^2 = \varepsilon_{x,0}^2 + \varepsilon_{x,0} H_x(s)\sigma_{\delta,CSR}^2(s)} \Rightarrow \boxed{\approx \varepsilon_{x,0}^2 + \varepsilon_{x,0}\left(\breve{\beta}_x \theta^2 \sigma_{\delta,CSR}^2\right)_{4th\ dipole}}$$

$$\left\langle \Delta x^2\right\rangle = \eta_x^2 \sigma_{\delta,CSR}^2, \left\langle \Delta x'^2\right\rangle = \eta'^2_x \sigma_{\delta,CSR}^2$$

$$H_x \equiv \left[\eta_x^{\ 2} + (\beta_x \eta_x' + \alpha_x \eta_x)^2\right] / \beta_x$$

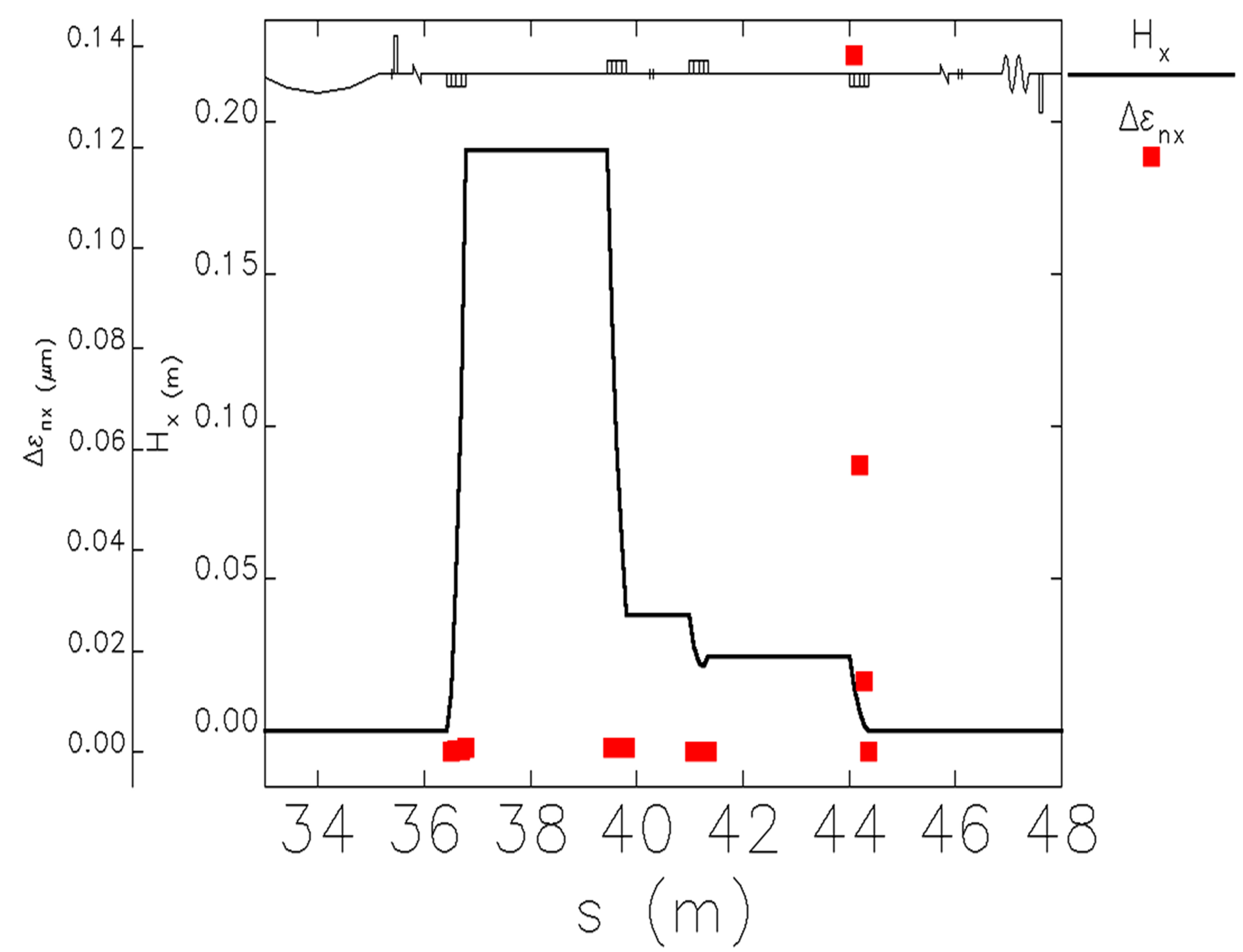
β_x minimum at 4th dipole, $\theta <<1$

Energy loss at chicane's dipoles

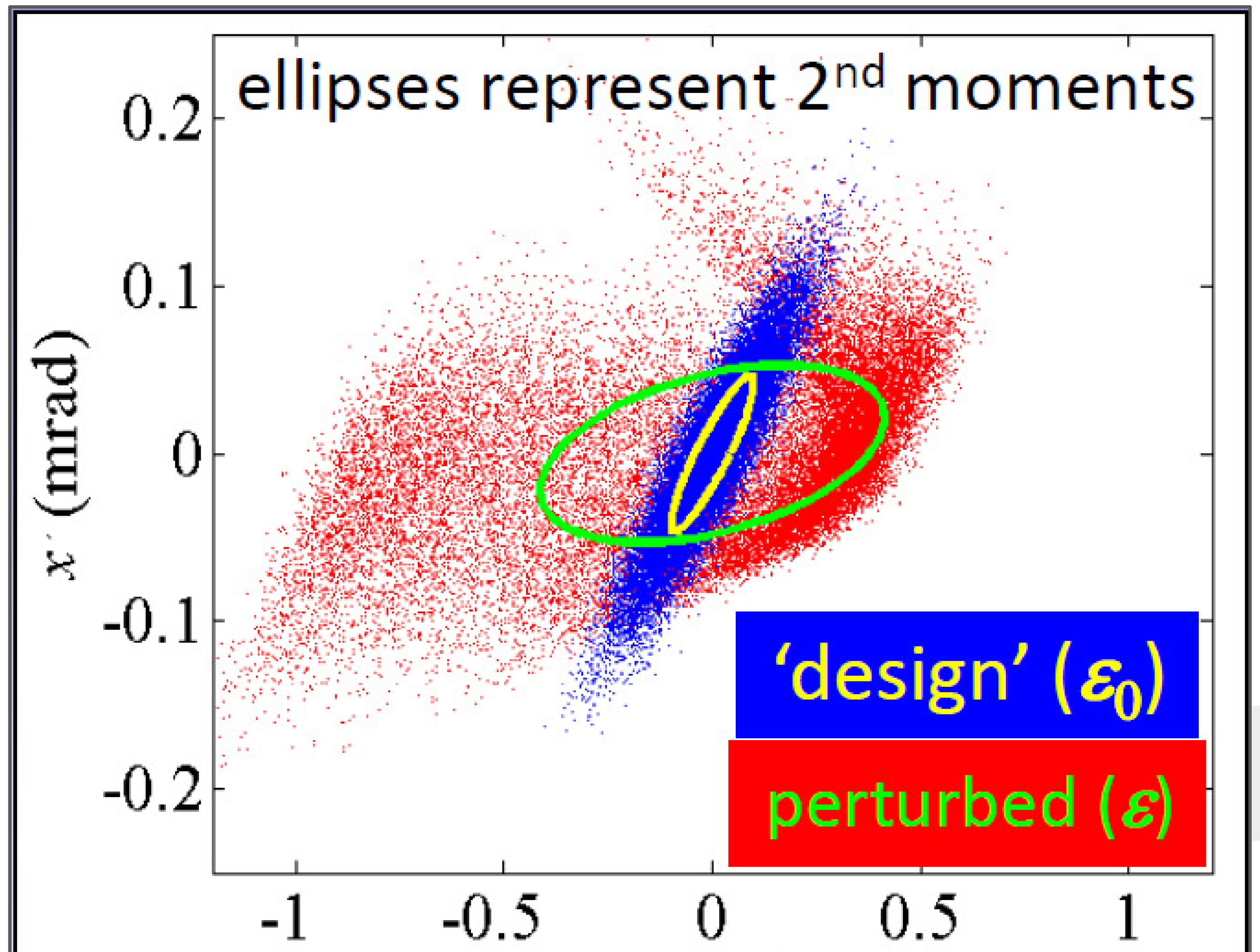


Courtesy of J. MItchell

Projected emittance along chicane



Phase space at chicane's exit



Courtesy of P. Emma

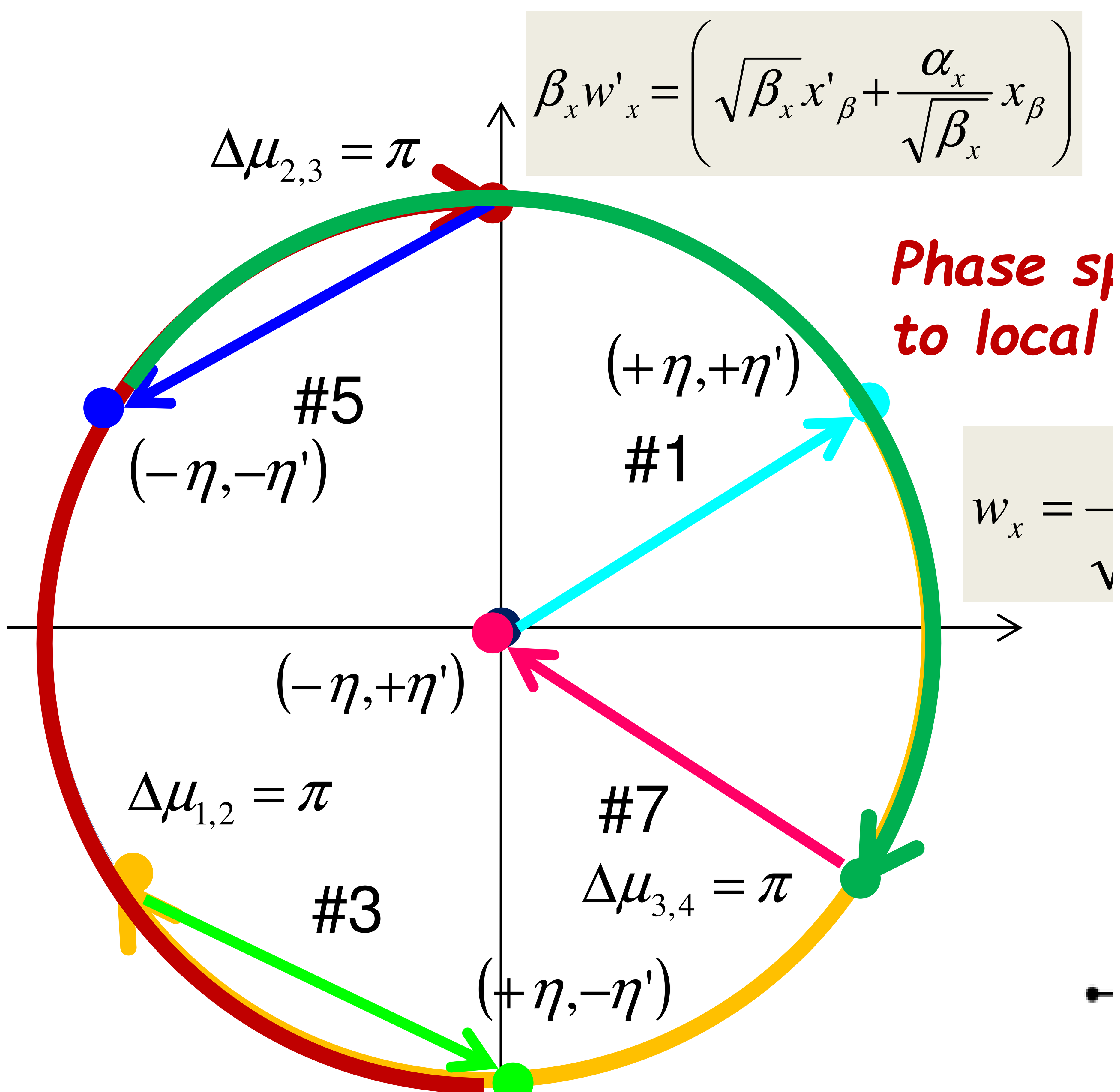
Multiple Kicks in an Isochronous Line

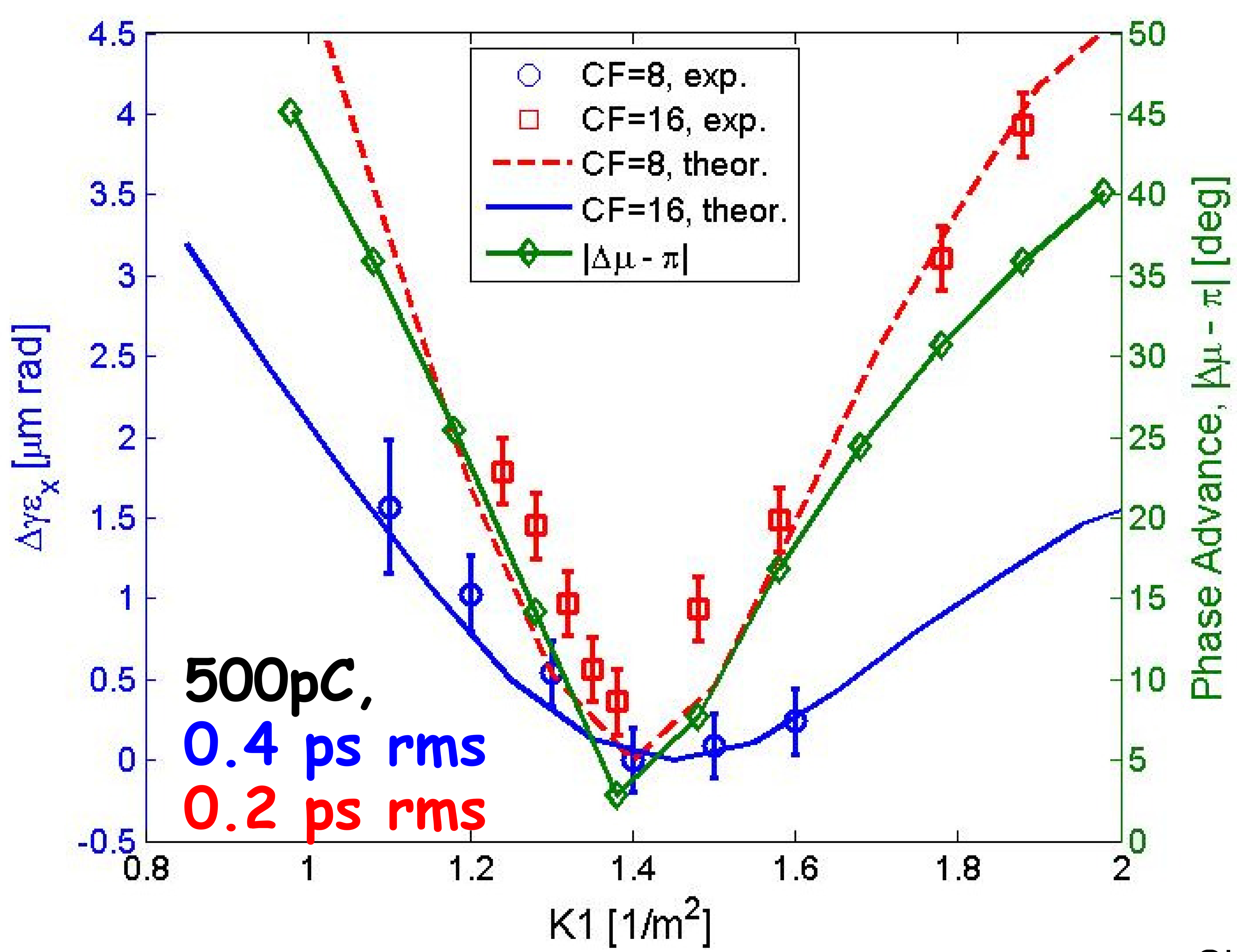
- If the bunch has ***same shape and length*** at identical dipoles, the CSR energy kick is the same at all points of emission.

$$\varepsilon_x^2 = \varepsilon_{x,0}^2 + \varepsilon_{x,0}\left(\beta_x\langle\Delta x'^2\rangle + 2\alpha_x\langle\Delta x\Delta x'\rangle + \gamma_x\langle\Delta x^2\rangle\right) \quad \Rightarrow \quad \varepsilon_x^2 = \varepsilon_{x,0}^2 + \varepsilon_{x,0} H_x(s_1)\sigma_{\delta,CSR}^2 X\left(\alpha_x, \beta_x, \Delta\mu_x\right)_{s_f}$$

This is at the first location of CSR emission (e.g., **first dipole**).

This is function of **Twiss parameters** along the line.





THEOREM:

- If optics (Twiss *and* dispersion) is (anti-)symmetric at dipoles' location, X_{if} = 0 for $\Delta\mu_x$ = (2)π between dipoles.

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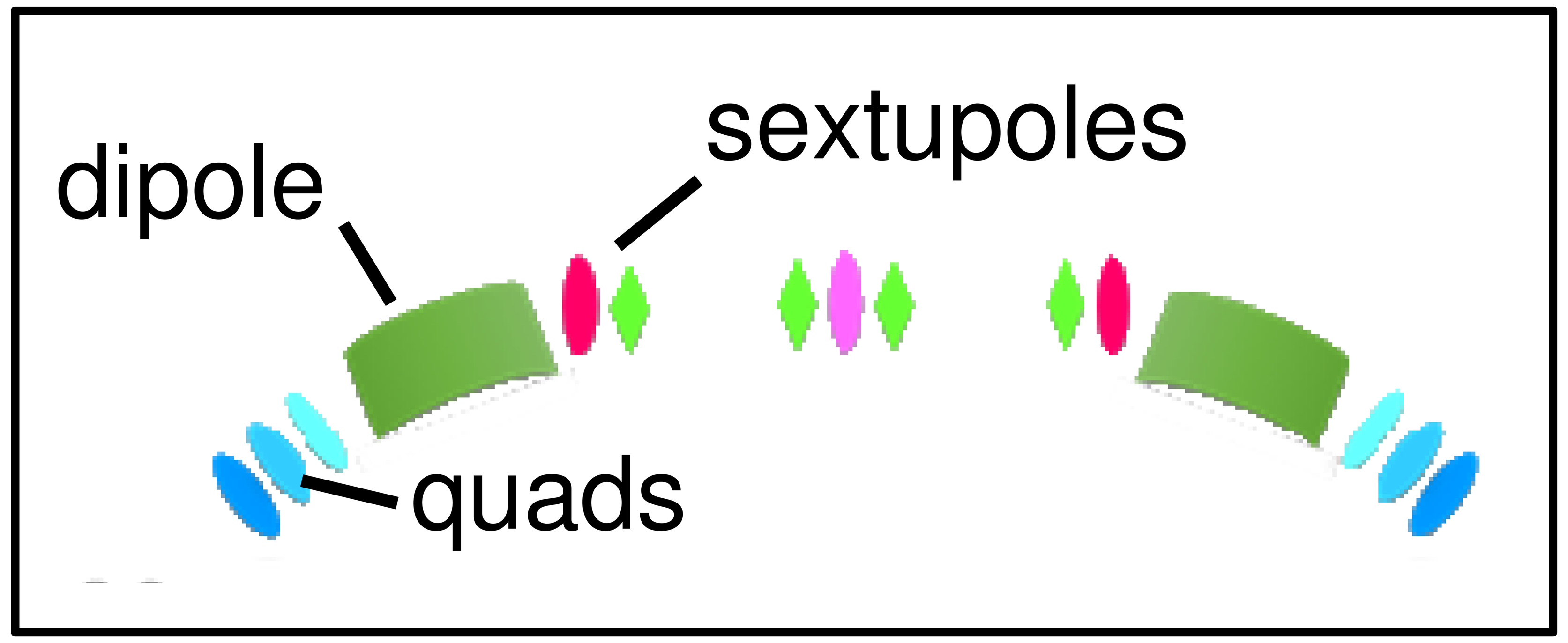
Arc Compressor

❑ Extend the multiple-kick scenario to the case of varying bunch length, e.g., the beam has an initial energy chirp and is **compressed** in a **180 deg arc (= 6 DBA cells) with nonzero R_{56}.**

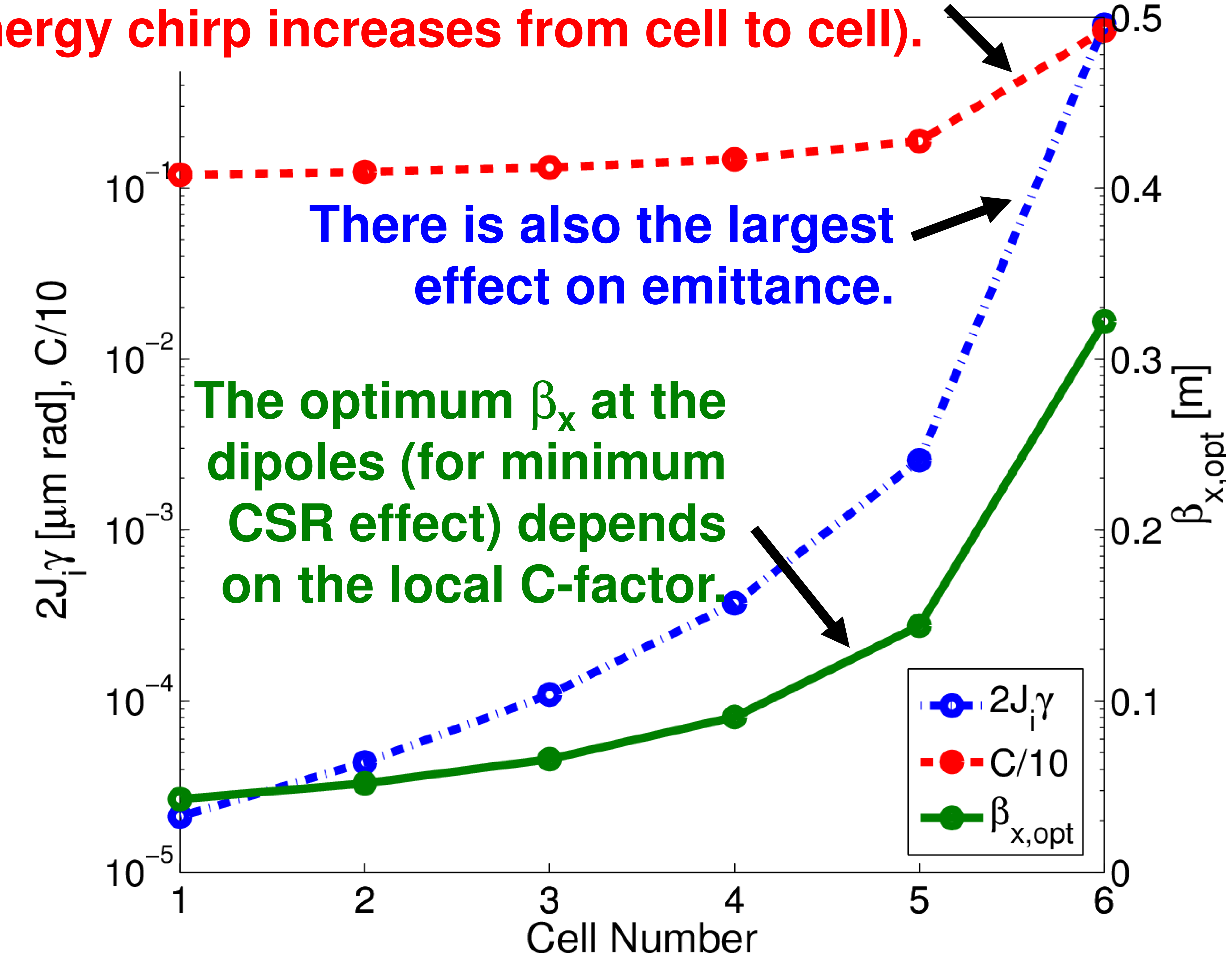
DBA cell



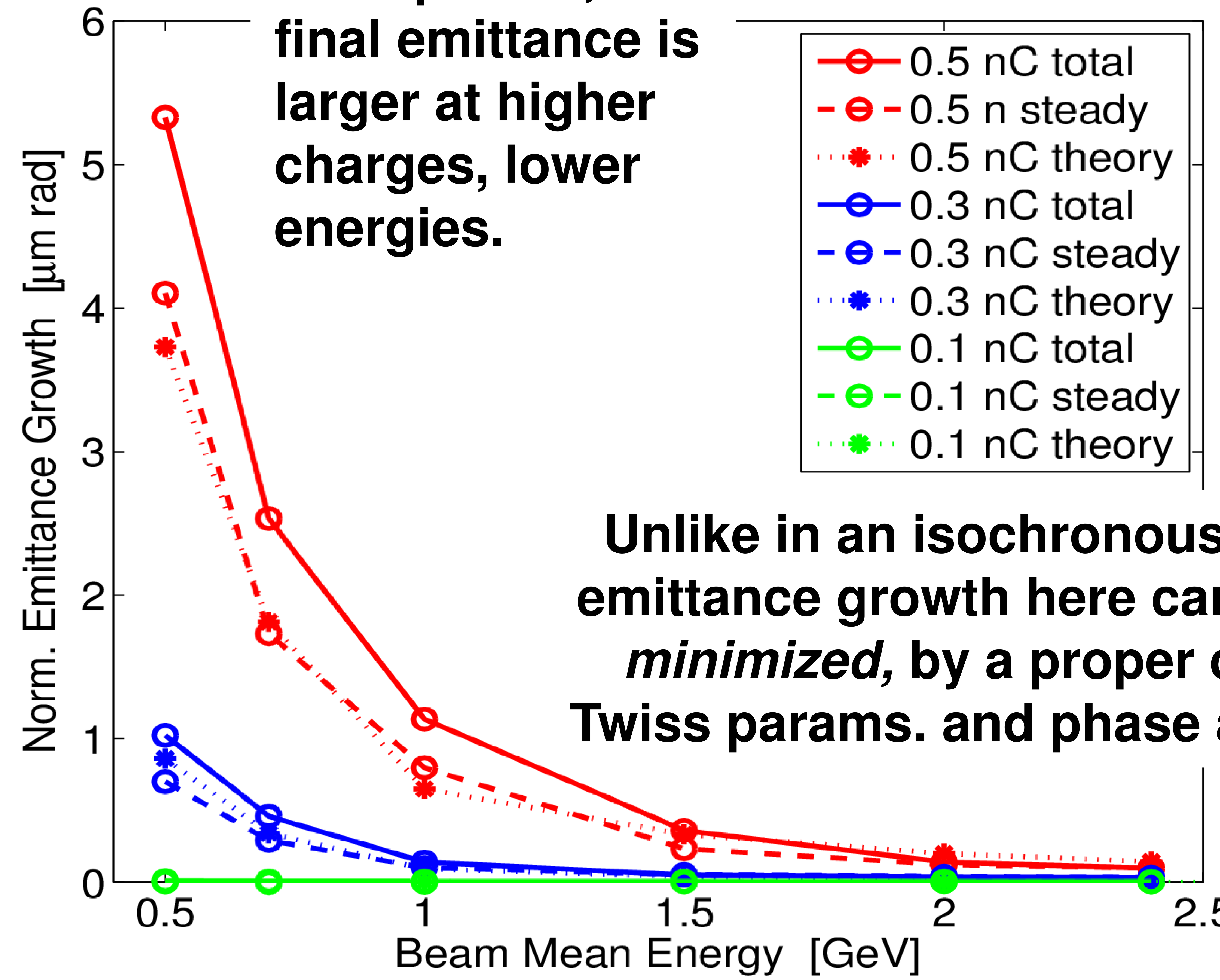
Most of compression work happens in the very last cell (R_{56} is same for all cells, while the energy chirp increases from cell to cell).

There is also the largest effect on emittance.

The optimum β_x at the dipoles (for minimum CSR effect) depends on the local C-factor.



As expected, the final emittance is larger at higher charges, lower energies.



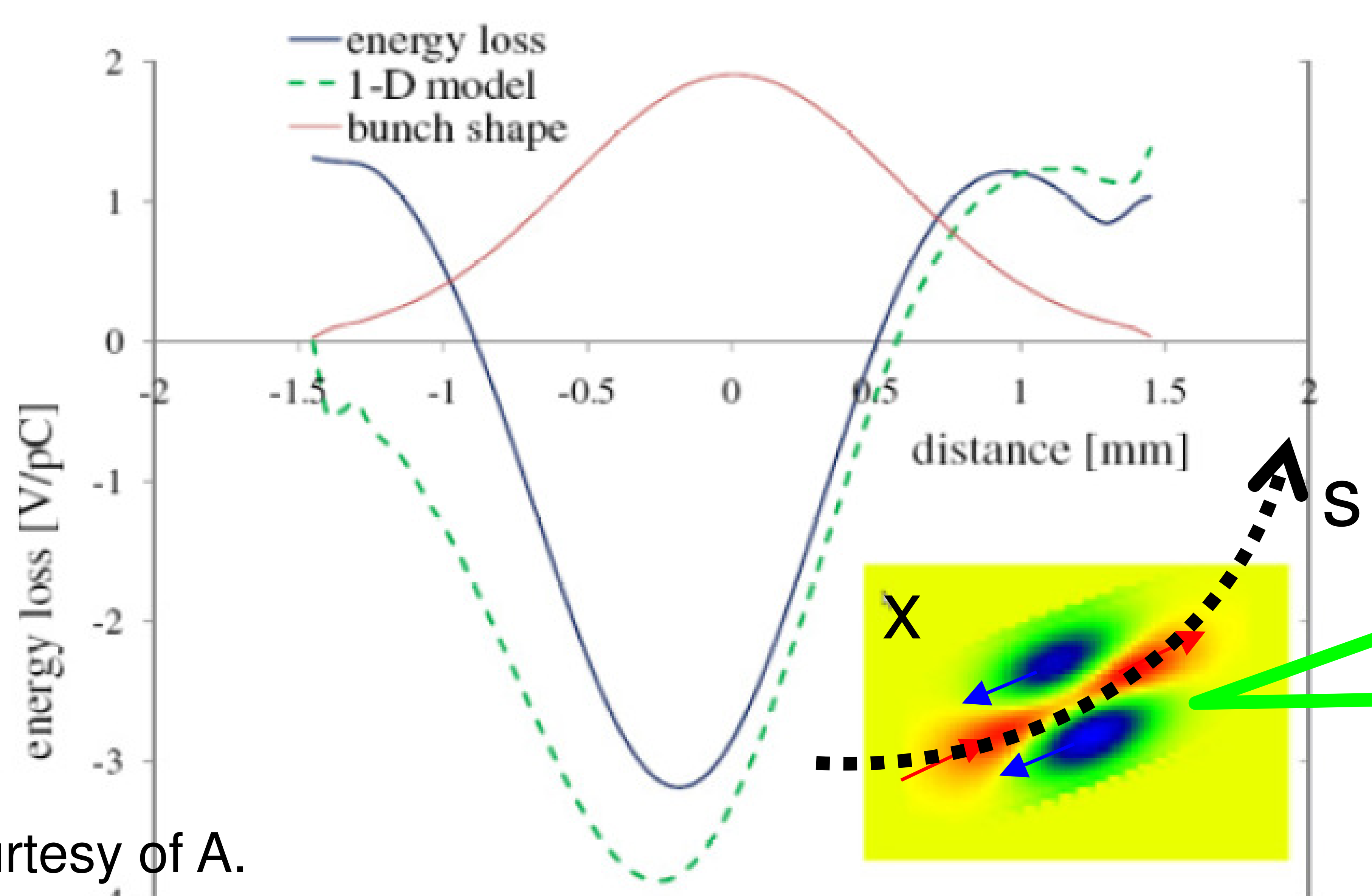
Unlike in an isochronous line, the emittance growth here can only be *minimized*, by a proper choice of Twiss params. and phase advance.

Limitations to the 1-D, Steady-State Model

❑ Steady-state model doesn't account for **transient effects** (in and out of dipoles):

Length scale for transient effects is $L_t \simeq (24R^2\sigma_z)^{1/3}$

❑ 1-D model does not account for CSR field **radial dependence**:



Courtesy of A. Novokatski

Transverse effects become important when $\sigma_\perp \gg (R\sigma_z{}^2)^{1/3}$

2-D CSR field modifies the beam energy distribution. Energy spread is correlated both along z and x.

- CSR longitudinal force is usually far more important than the transverse one.
- 1-D model including transient effects allows relatively fast particle tracking.
- 2-D or 3-D effects are presently treated in few codes devoted to high accuracy simulations.

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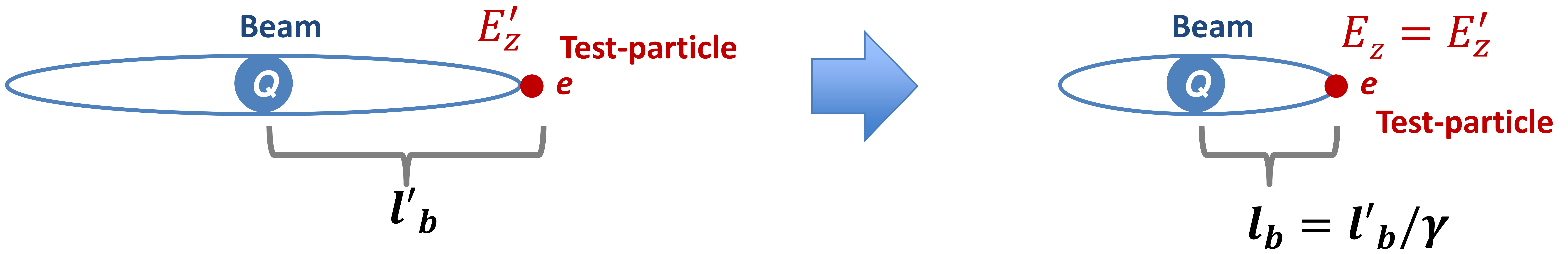
Longitudinal Space Charge

Consider a beam of length $2l_b$, with charge $Q = -eN$ and a test electron $q = -e$ close to the beam head. The beam is in relativistic motion with respect to the lab.

Beam co-moving frame **Lab frame**



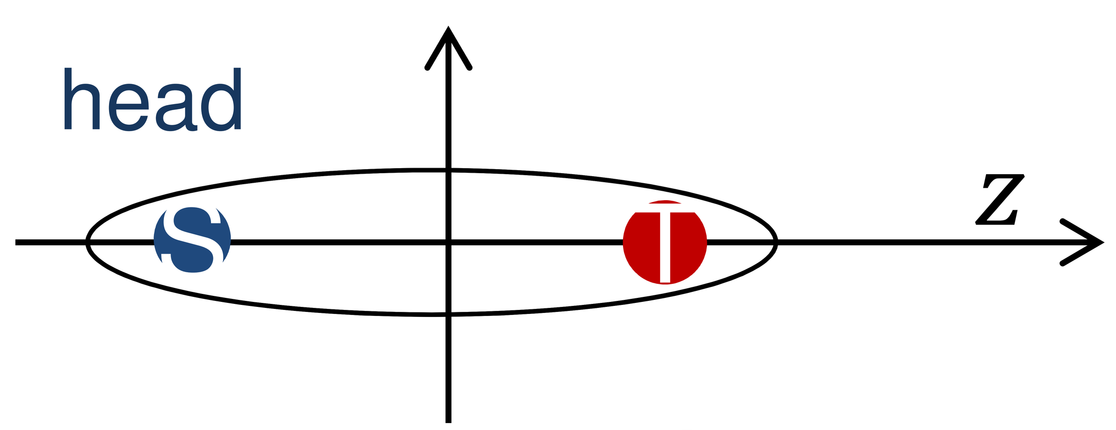
E-field experienced by test electron: $\boldsymbol{E_z = E'_z = \frac{1}{4\pi\varepsilon_0}\frac{Q}{l'^2_b} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{l_b^2\gamma^2}}$ Longitudinal space charge field

Work done by space charge over distance L: $\Delta U = qE_zL = \frac{1}{4\pi\varepsilon_0}\frac{e|Q|}{l_b^2\,\gamma^2}L = \frac{Z_0c}{4\pi l_b^2}\frac{e|Q|}{\gamma^2}L > 0$ *i.e.* test-electron gains energy

- Only at 10s of MeV energy or lower (i.e. in the injector) space charge effects over **bunch-length scale** are significant.

- At energies > 10s MeV (even GeV), space charge can become relatively large (and dominant) either for very short bunches or on **shorter length scales**.

Wakefield and Impedance



$\Delta z \equiv z - z'$

Coordinate of test particle

Coordinate of source particle

"-" sign: a matter of convention

E-field experienced by test particle (along direction of motion)

$$w_z(|\Delta z|) = -\frac{1}{q_s}\int_0^L ds\, E_z\left(s, t = \frac{s+|\Delta z|}{c}\right)$$

Long. separation between source and test particles

Source-particle charge

Longitudinal wake field (wake function)

(space-integrated e-field over source charge)

MKS units: $\frac{1}{C} \times m \times \frac{V}{m} = \frac{V}{C}$

WARNING! In the following, w_z is wakefield *per unit length* [V/C/m].
⇒ Z(k) becomes LSC impedance *per unit length* [Ω/m].

$$V(z) = Q\int_{-\infty}^{z} dz' w_z(z-z')\lambda(z')$$

bunch charge $= Nq$

Longitudinal bunch density (no. part/m) normalized to unity $\int dz'\,\lambda(z') = 1$

Longitudinal wake potential

(voltage drop experienced by test charge, generated from whole bunch)

MKS units: of $C \times m \times \frac{V}{C} \times \frac{1}{m} = V$

- The wakefield **impedance** is the Fourier transform of the wake function (k is wave vector):

$$Z(k) = \frac{1}{c}\int_{-\infty}^{\infty} dz\, w_z(z) e^{-ikz}$$

MKS: units: $\frac{1}{m/s} \times m \times \frac{V}{C} = \frac{V}{A} = \Omega$

In the Fourier space we can write:

$$Z(k) = -\frac{V(k)}{I(k)}$$

LSC Instability

Microbunching Gain (one compression stage)

$I(z) = I_0[1 + A\cos(kz)]$
$\Delta I_0 \sim I_0 A$

LSC

$\Delta\gamma(z) = 4\pi A \frac{I_0}{I_A}\frac{|Z(k)|}{Z_0} L_s \sin(kz)$

R_{56}

$\Delta I_f \sim R_{56}\Delta\gamma$

Gain

$|\frac{\Delta\hat{I}_f}{\Delta\hat{I}_i}| \sim \frac{k|R_{56}|\Delta\gamma}{A} = 4\pi\frac{I_0}{I_A}\frac{|Z(k)|}{Z_0} L_s k\,|R_{56}|$

$$G(k_0) = \frac{relat.\,ampl.\,of\ final\ density\ perturbation}{relat.\,ampl.\,of\ initial\ density\ perturbation} = \frac{\Delta\hat{\rho}_f/\rho_f}{\Delta\hat{\rho}_i/\rho_i} \simeq 4\pi \frac{I}{I_A}\frac{L_s}{\gamma_{BC}}\frac{|Z(k_0)|}{Z_0}|R_{56}|Ck_0\, e^{-(Ck_0R_{56}\sigma_{\delta,0})^2/2}$$

G is larger for higher peak current, longer path, lower beam energy

Z generates energy modulation at initial wave number k_0

R_{56} transforms energy- to density modulation (enhances gain)

What is Z(k) for the LSC field?

Bessel function

$$Z(k) = \frac{iZ_0}{\pi\gamma r_b}\frac{1 - \xi_b K_1(\xi_b)}{\xi_b}$$

Peak is at $\frac{r_b k}{\gamma} \simeq 1$

Effective radius for Gaussian bunches:
$r_b \simeq 1.7(\sigma_x + \sigma_y)/2$

$\xi_b = kr_b/\gamma$

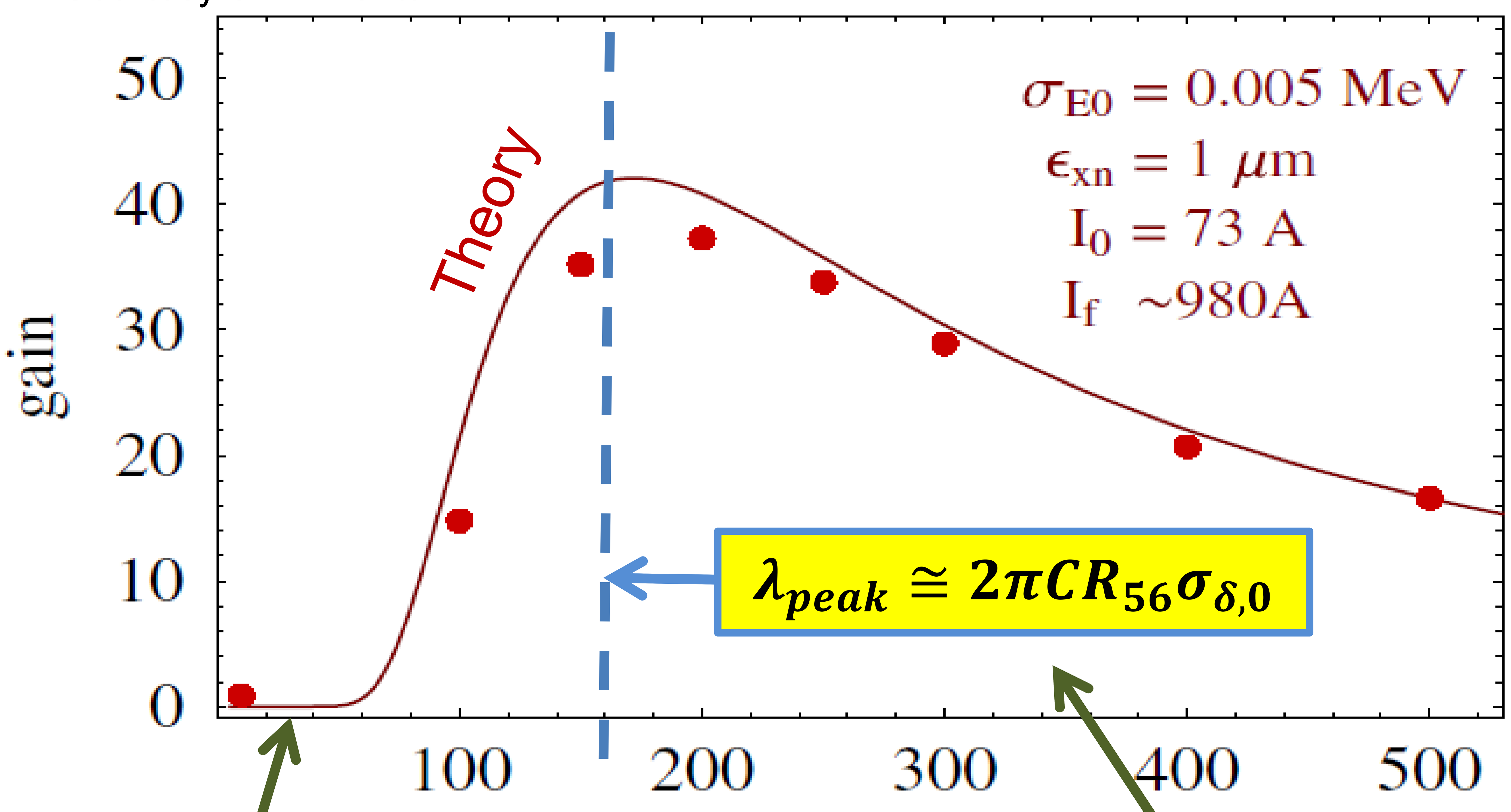
$\sigma_{\delta,0}$ is initial **uncorrelated energy spread**. The slippage associated to $R_{56}\sigma_{\delta,0}$ smears the microbunching (energy-Landau damping).

Wavelengths of Interest

Theory vs. PIC-Simulation

(LBNL FEL proj., end of BC)

Courtesy of M. Venturini

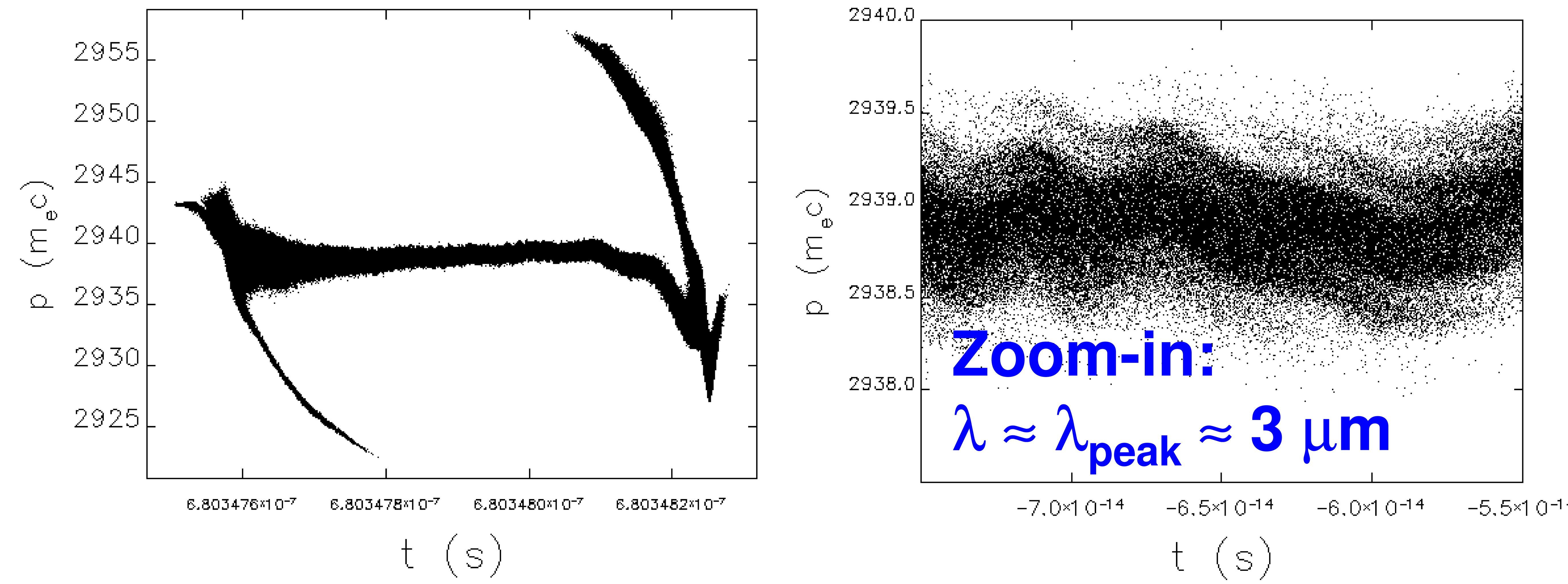


Gain is exponentially suppressed at short wavelengths

Wavelength of modulation Before compression

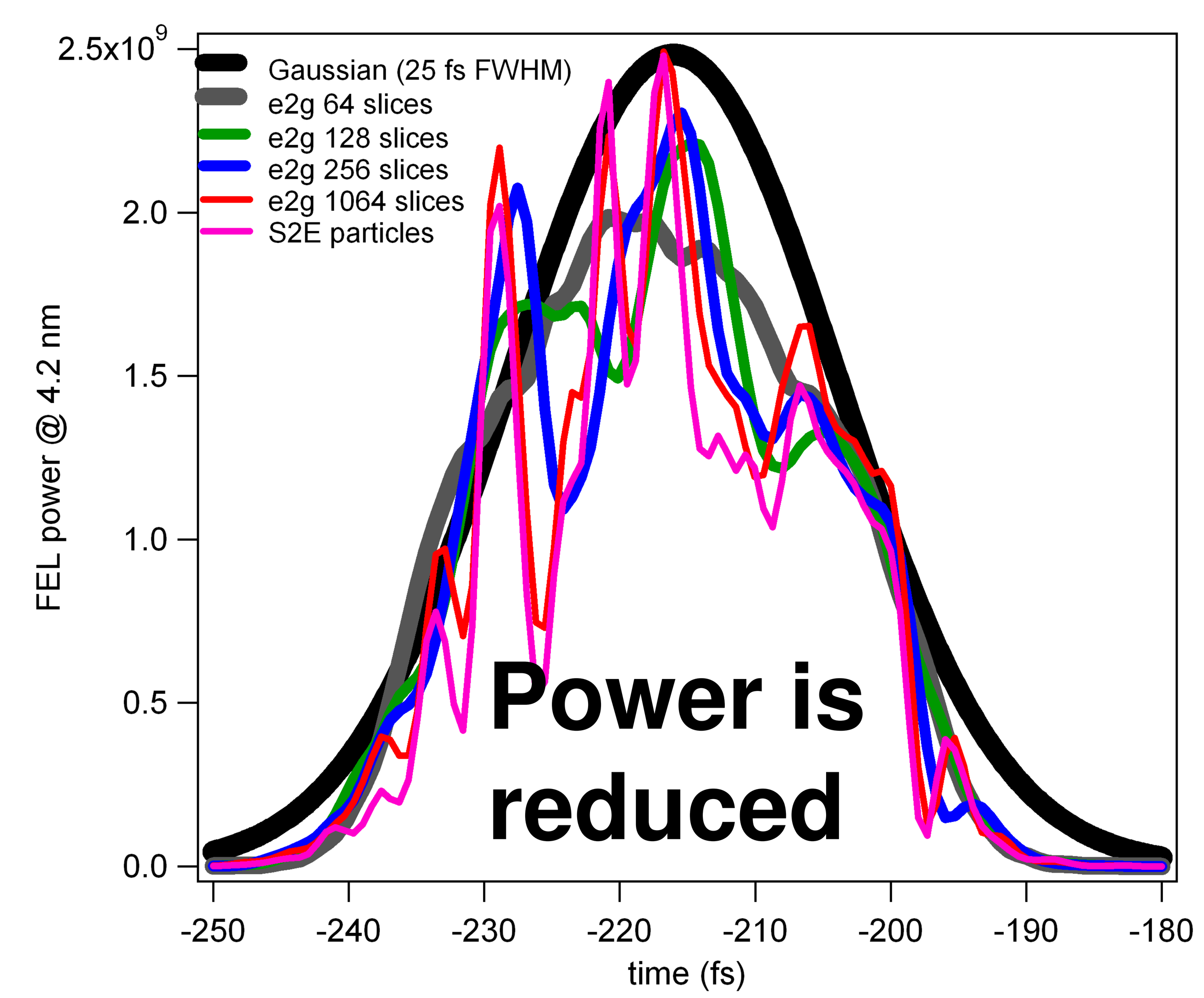
Gain is typically broadband

Longitudinal phase-space (exit of FERMI Linac)

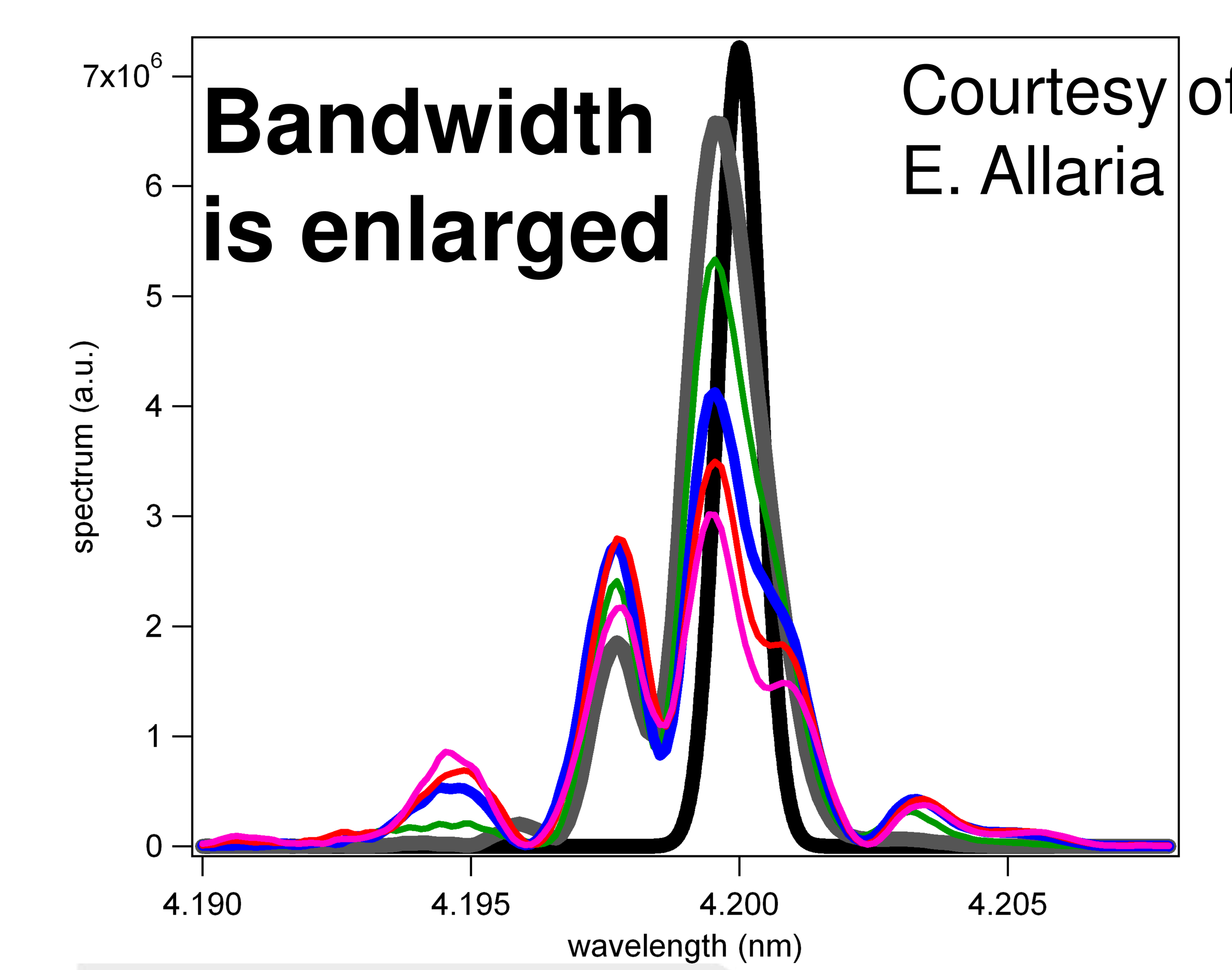


MBI is critical for FEL typically at $\lambda \approx$ cooperation length, (and even longer, i.e., $\approx$ undulator slippage length).

FEL POWER

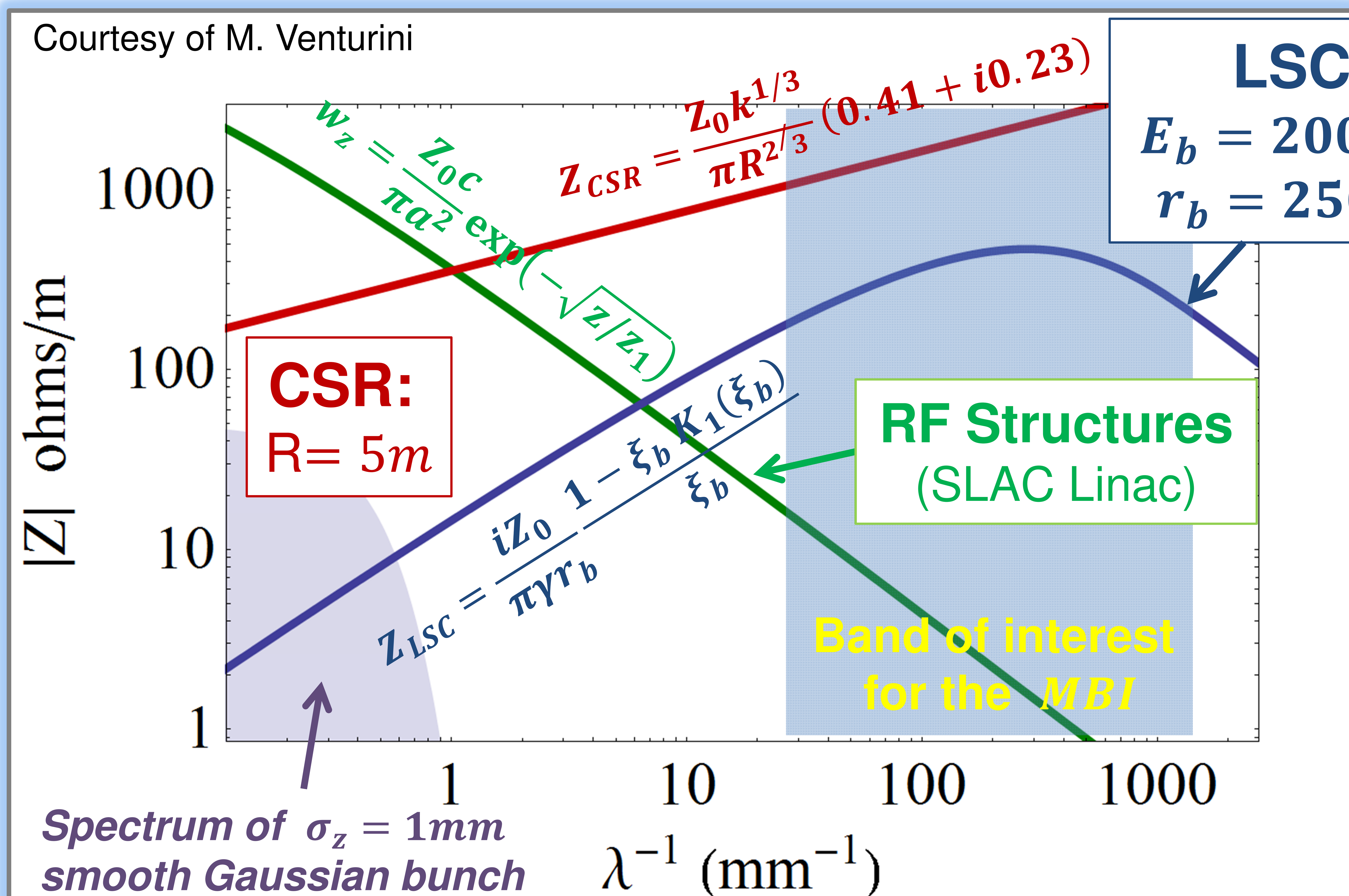


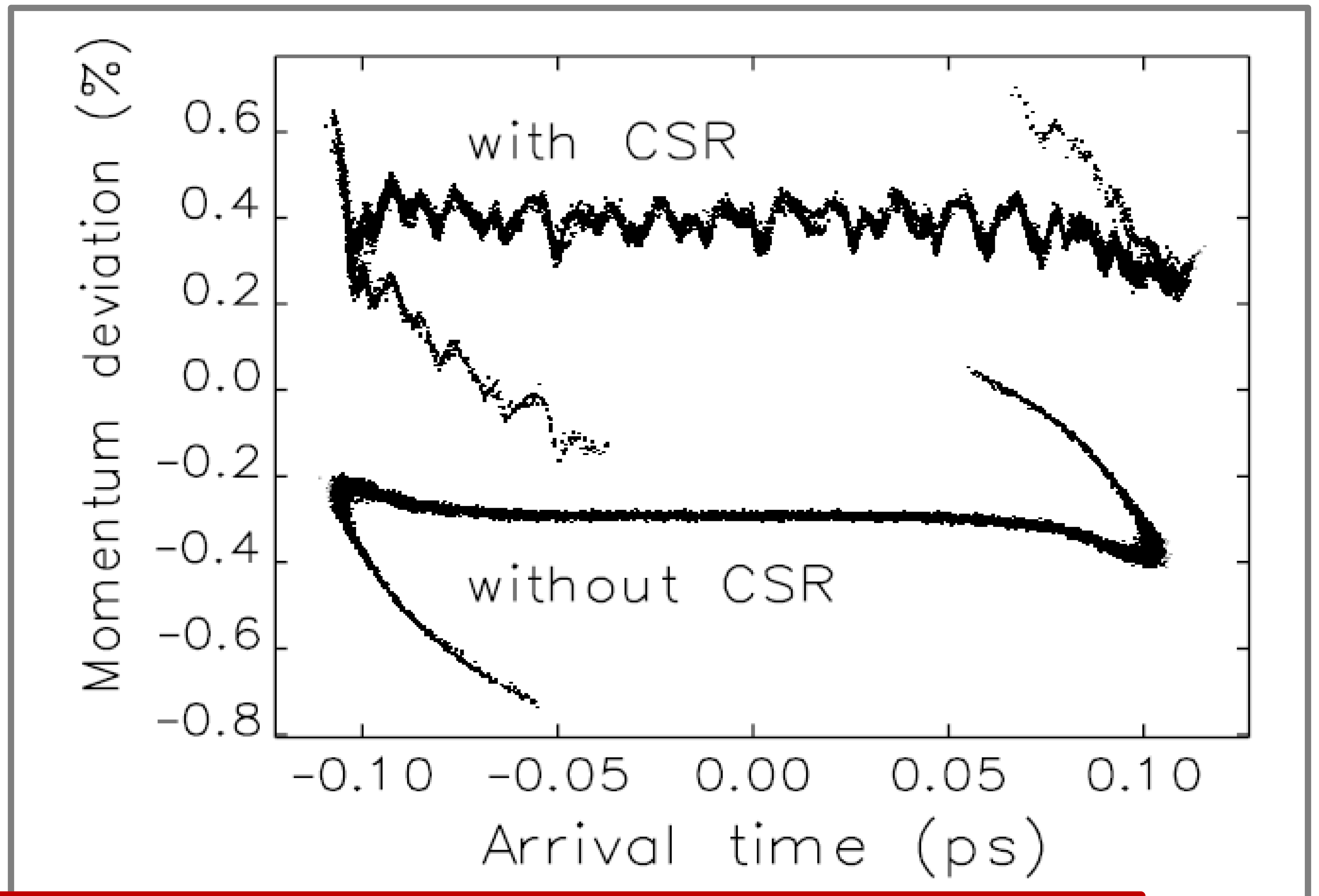
FEL SPECTRUM



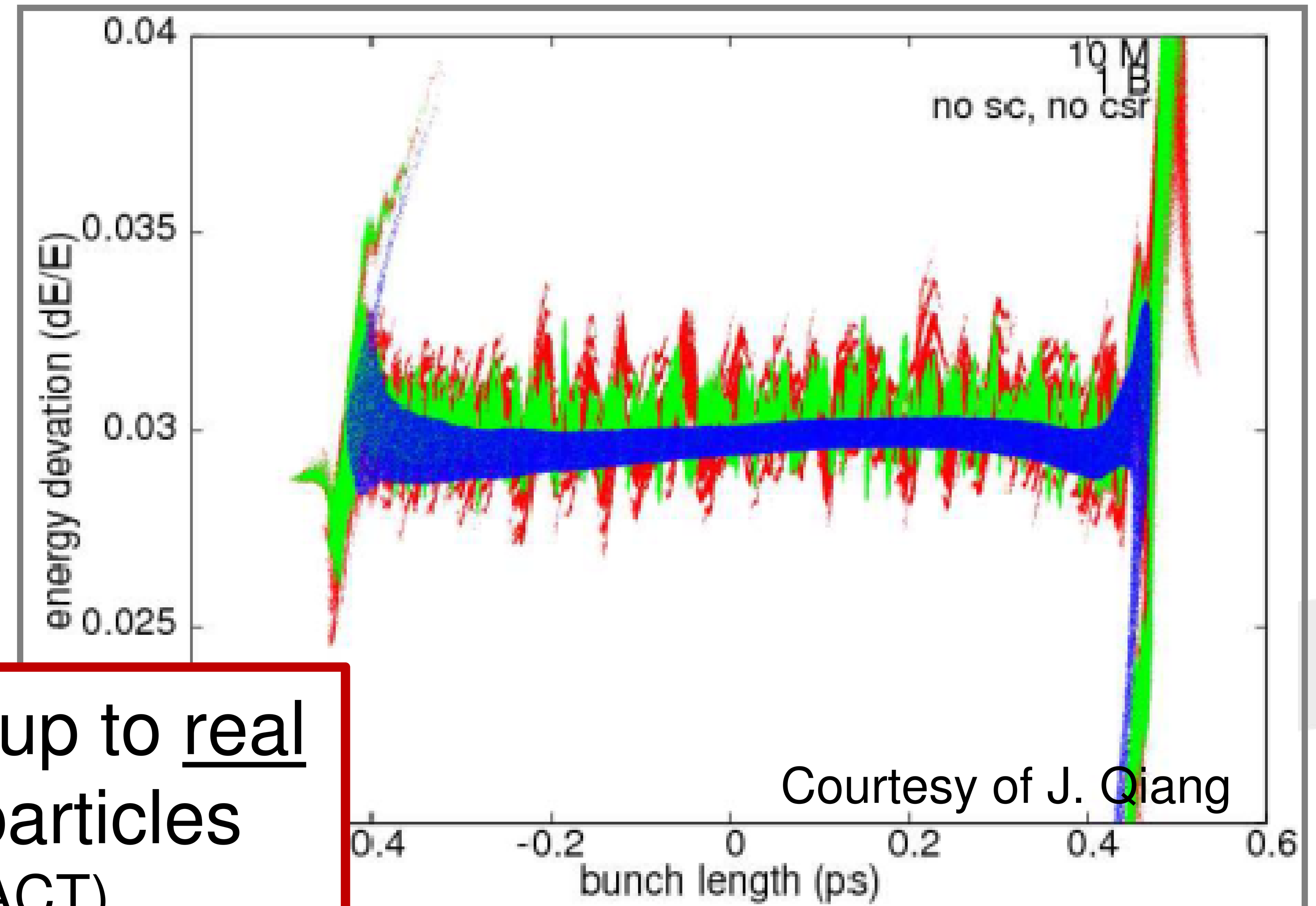
CSR + LSC

- **CSR enhances the MBI-gain** as it additionally contributes to the "energy-to-density" loop throughout the chicane.
- **CSR impedance** is the largest at high frequencies, but **overall its effect is smaller** than LSC (dipoles are short compared to rest of machine).





LCLS first S2E simulation *(M. Borland, 2001)*. Early physics model included CSR, not LSC.

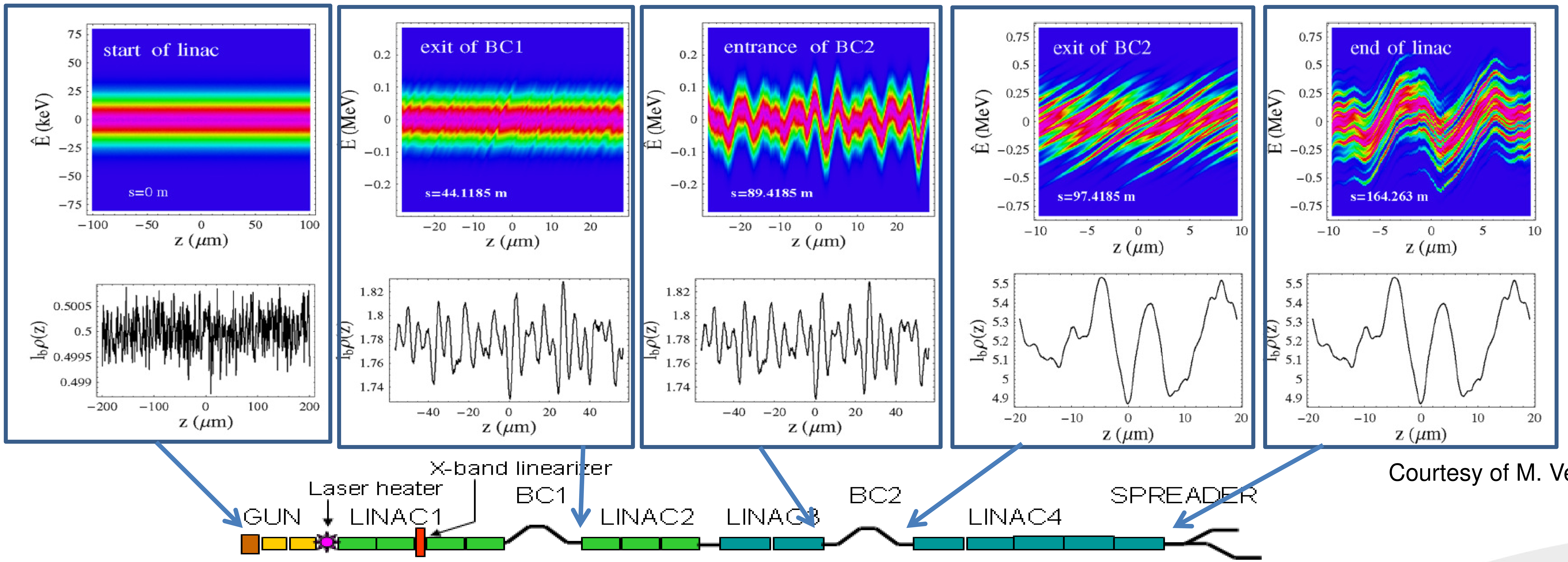


CSR+LSC, up to real number of particles (J. Qiang, IMPACT)

Evolution of Phase Space

- Effect compounded by repeated compression through BCs. In first approximation: $G_{tot} \simeq G_{BC1} \times G_{BC2} \times \cdots$
- If instability is large, effects beyond the linear approx. can become important: $G_{tot} > G_{BC1} \times G_{BC2} \times \cdots$

Study of μBI for FERMI: *Longitudinal phase space, Current profile at selected points*



Courtesy of M. Venturini

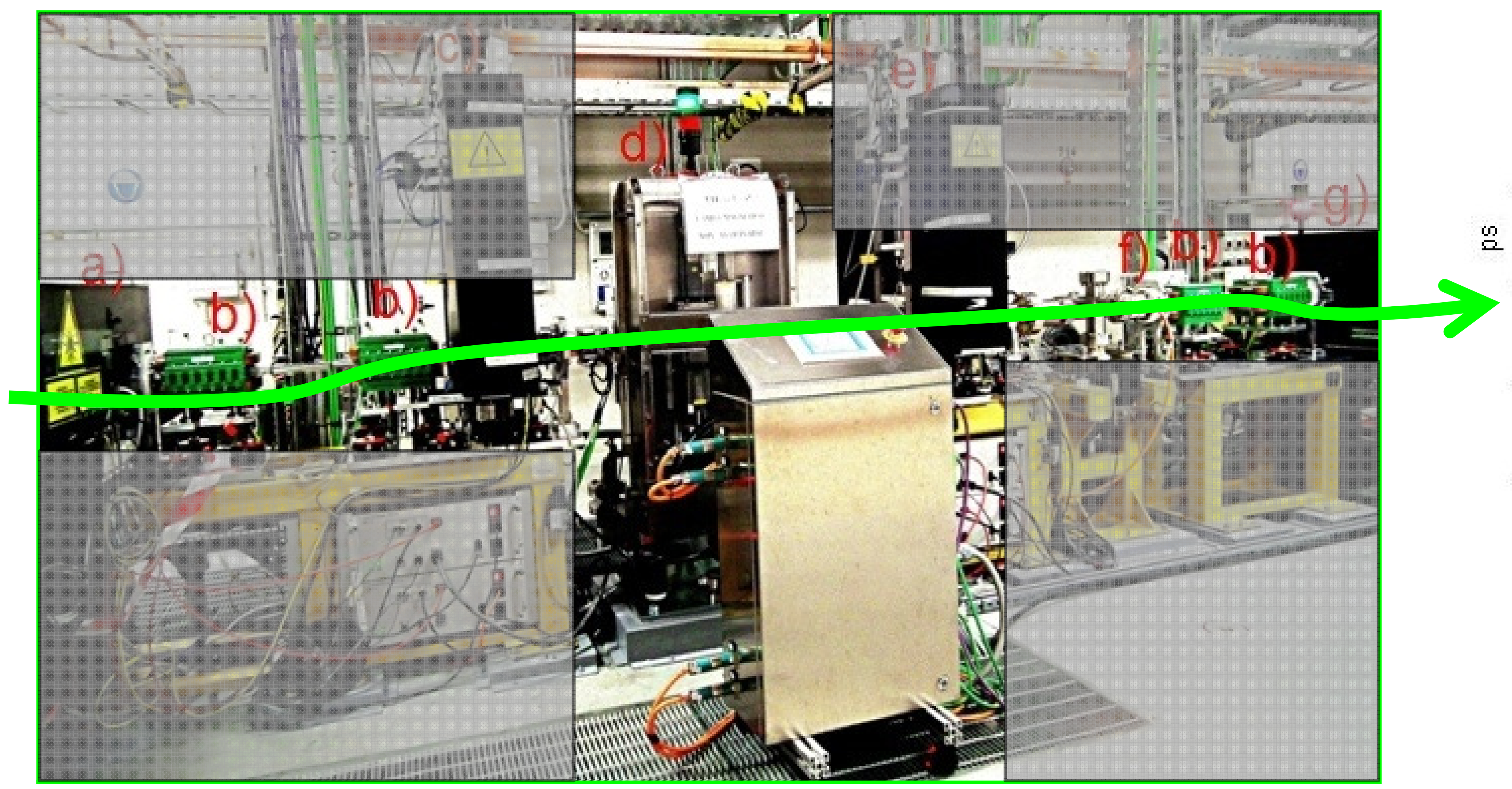
Main adverse effect of μBI instability is growth in energy spread: *it limits SASE performance, degrades harmonic generation in seeded FELs, and reduces longitudinal coherence of radiation*

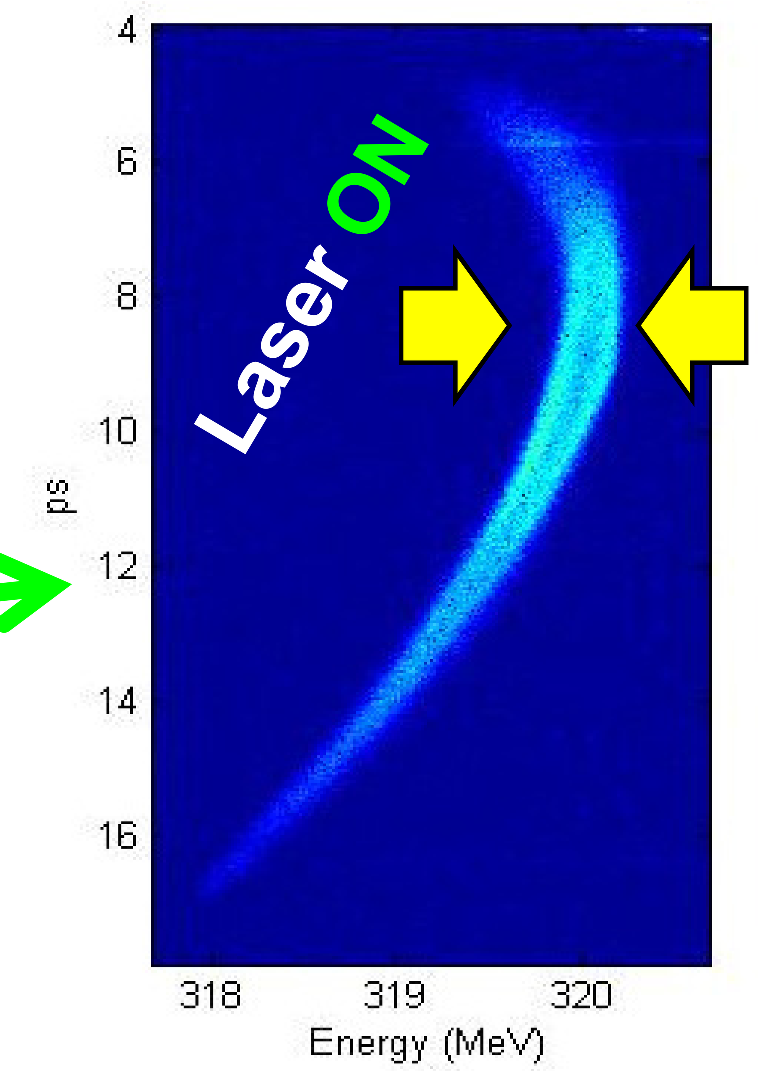
Laser Heater

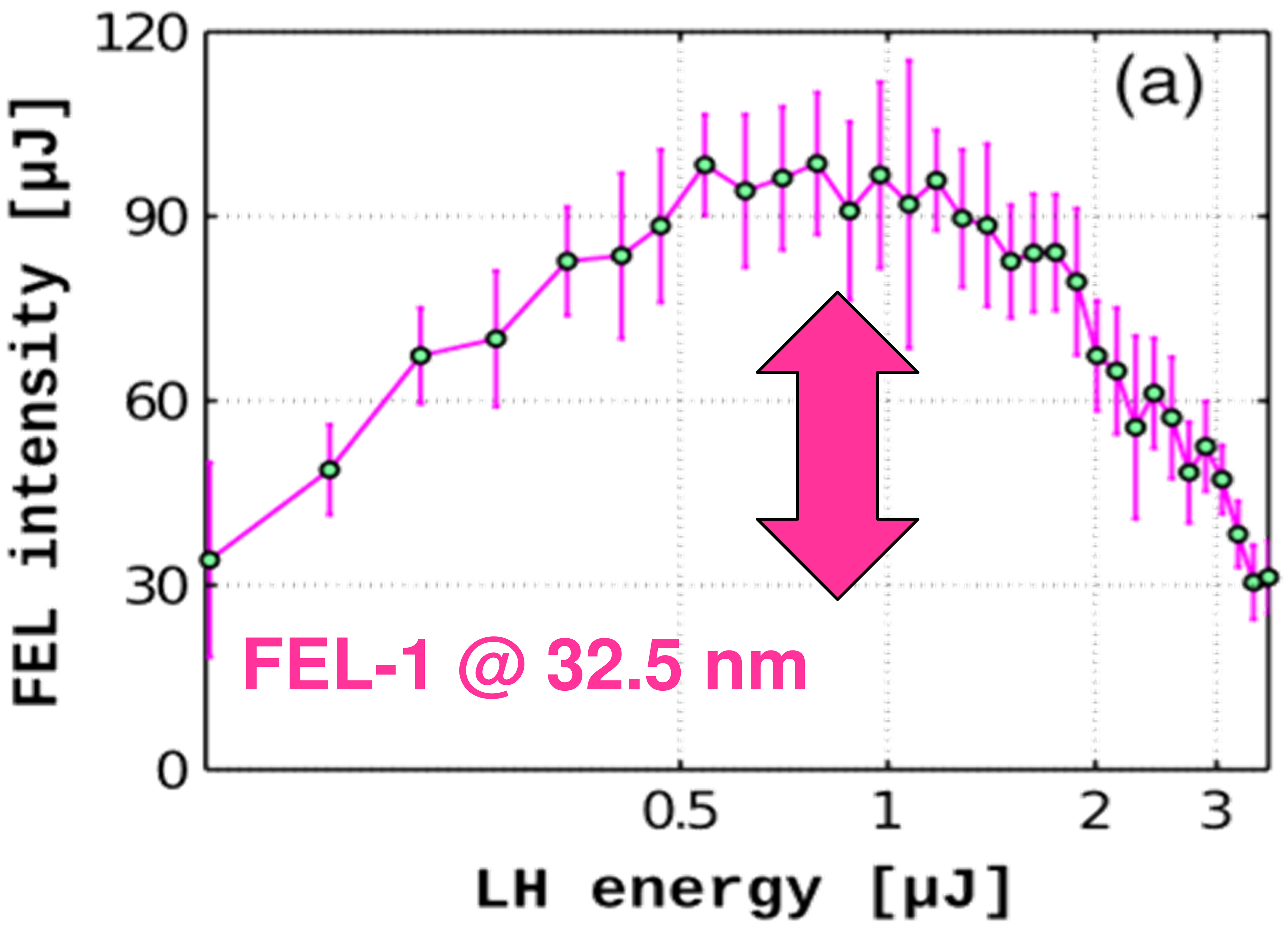
- The Laser Heater is a tool that enhances the energy-Landau damping of the μBI by increasing the beam uncorrelated energy spread before the μBI builds up (thereby it is typically installed before compression).

Impact on e-Beam and FEL

- **FERMI Laser Heater System**

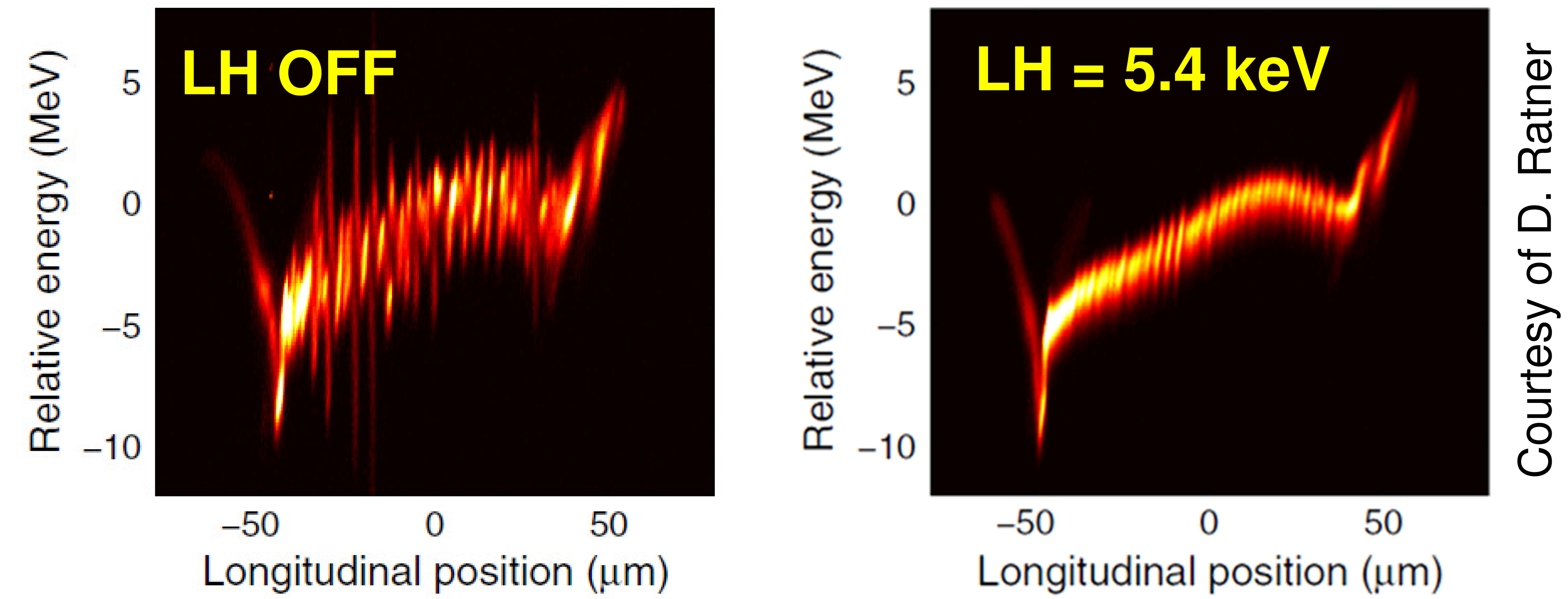




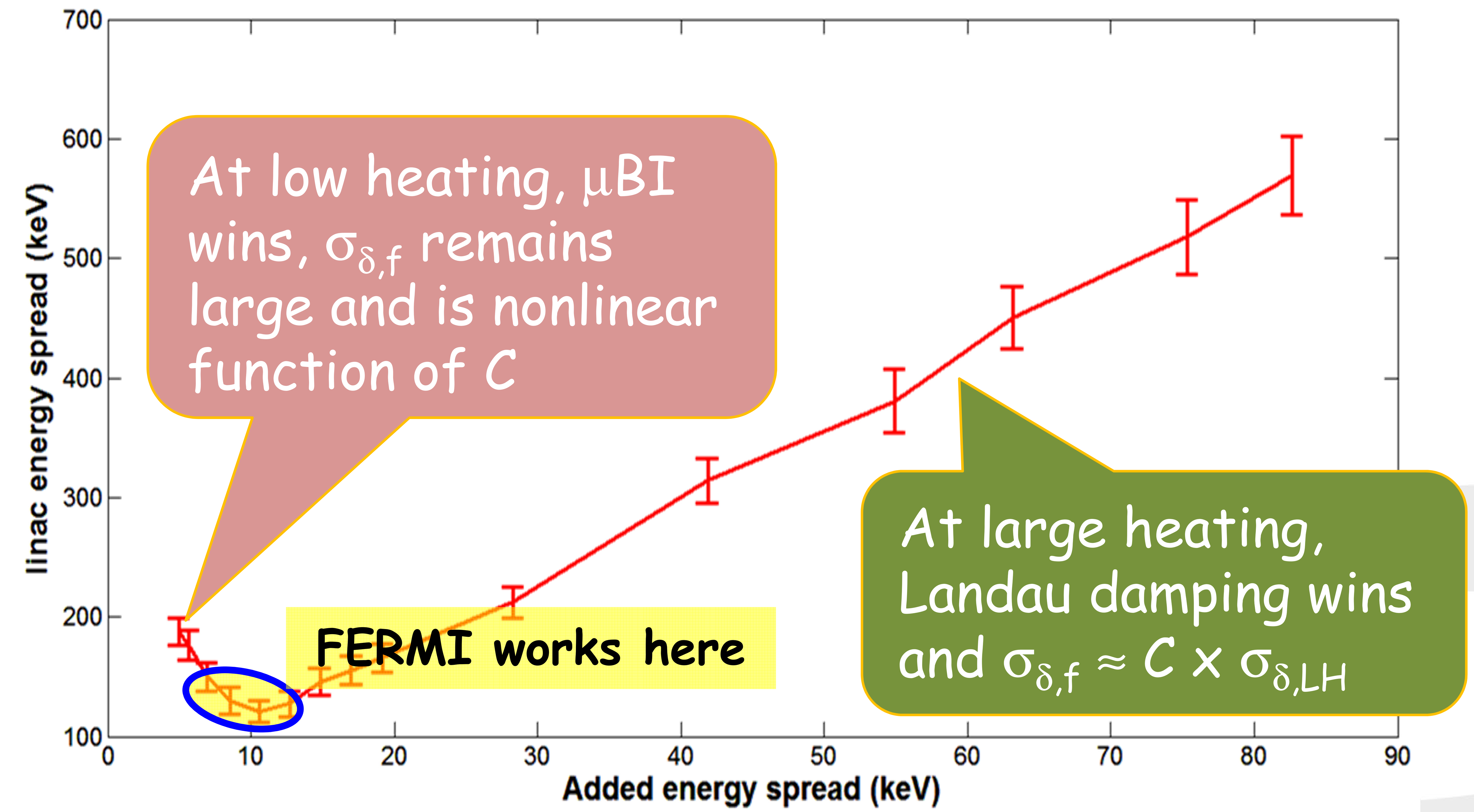


- *LH enhances FERMI FEL intensity by a factor ~3.*
- *LH narrows the FEL bandwidth.*

- **Direct observation of μBI at LCLS (end of Linac)**



- **Strength of μBI depends on compression schemes. FERMI behaves better with BC1 only.**

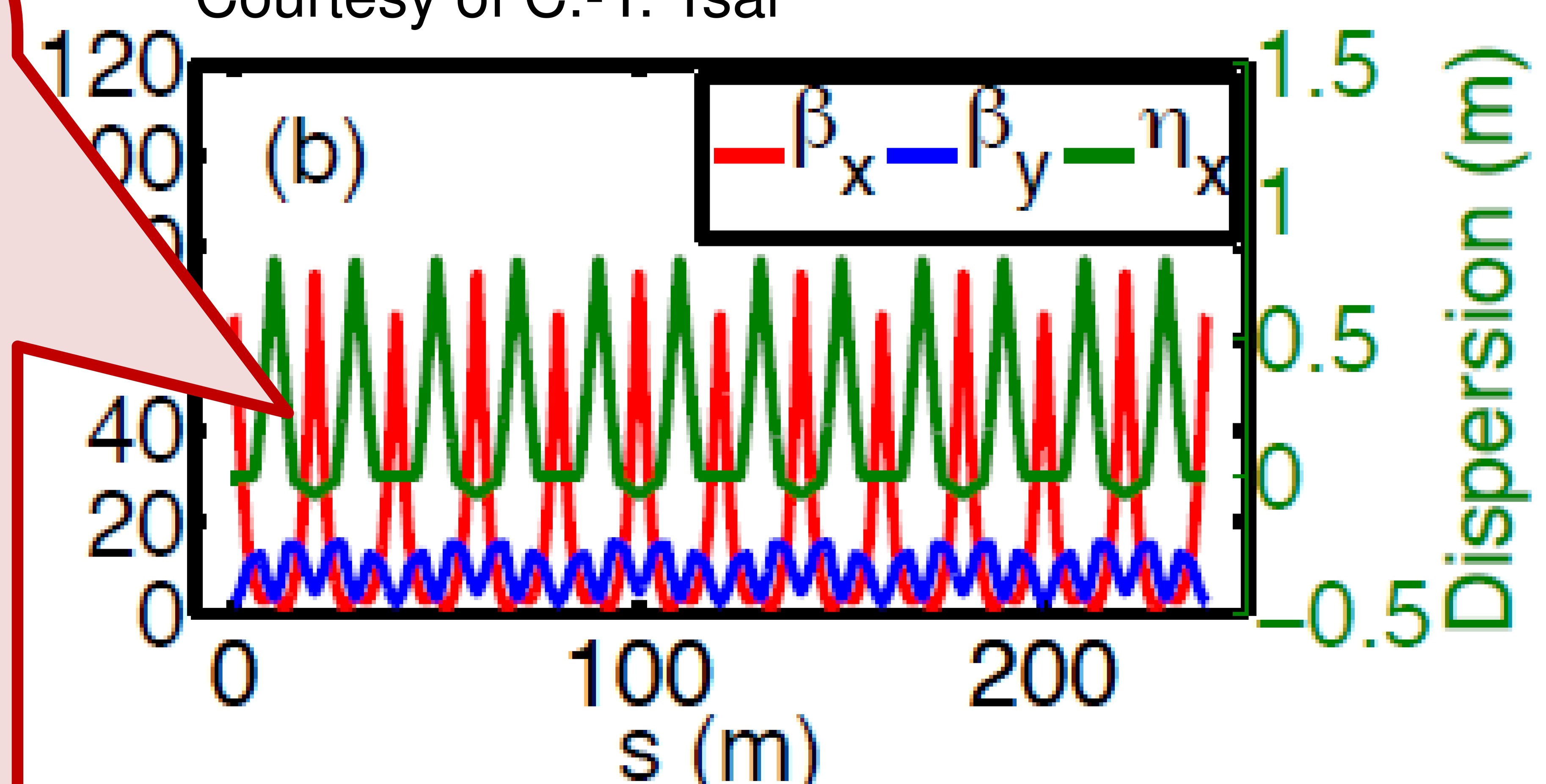


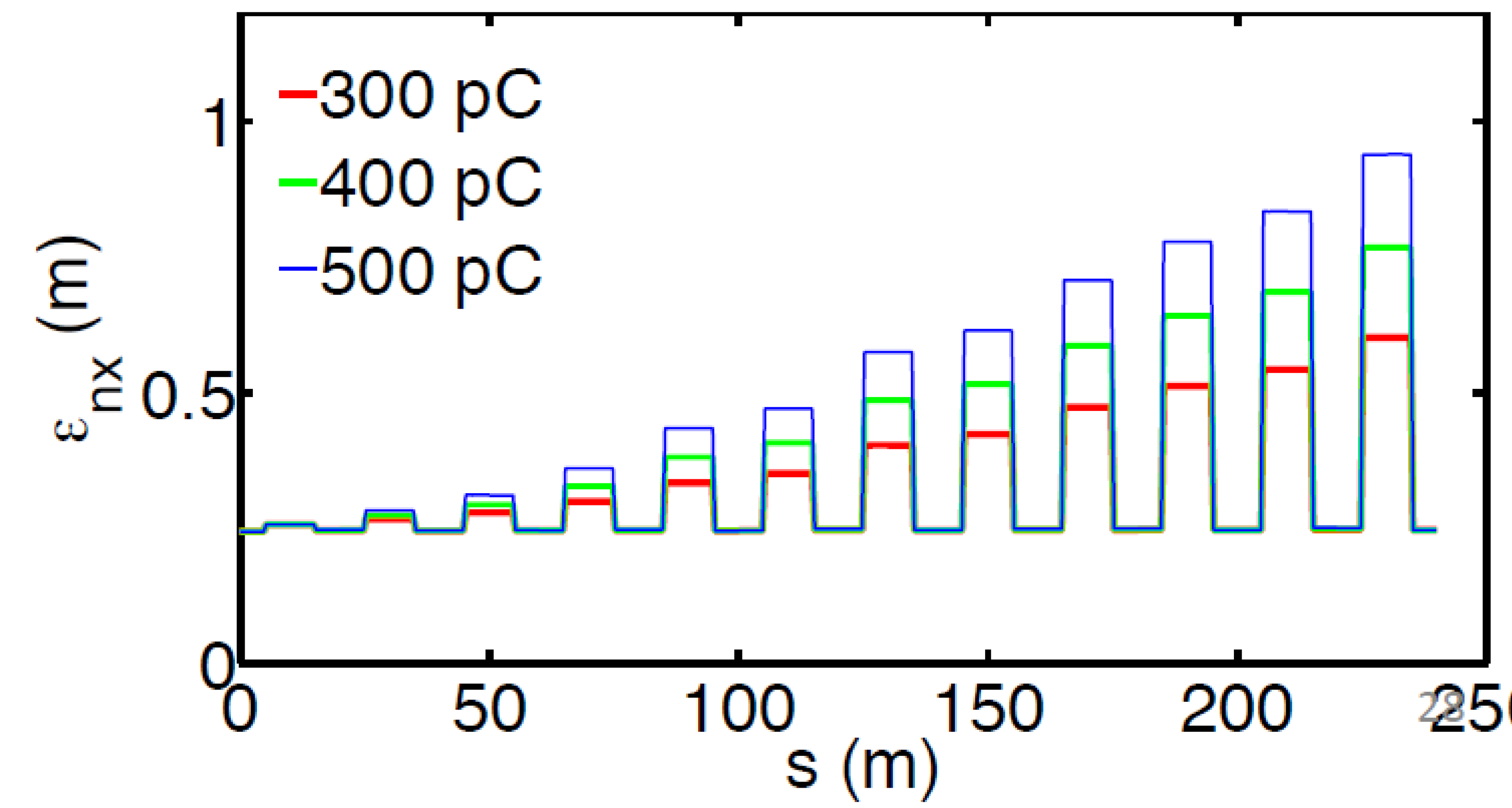
Fresh from Presses…

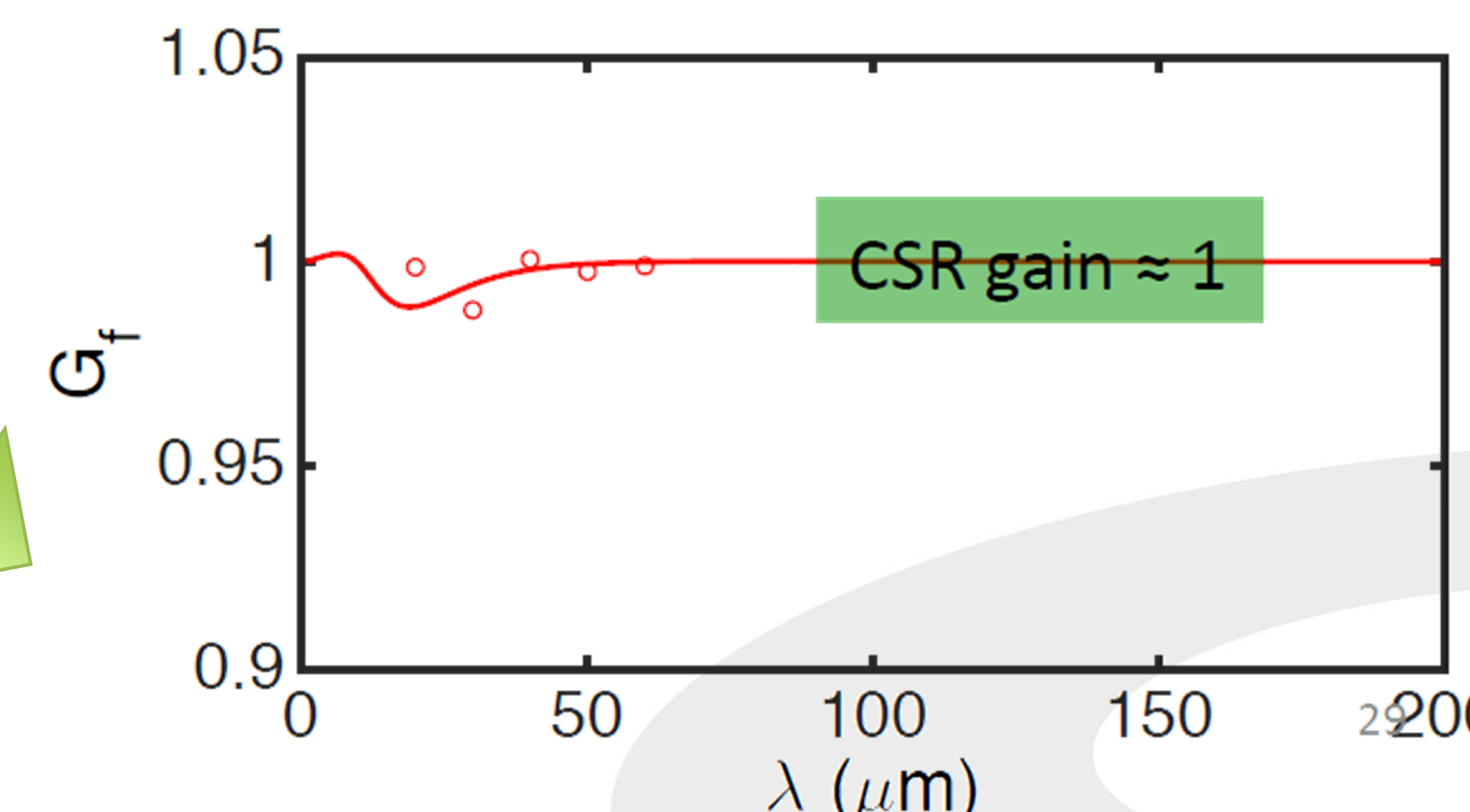
- Arcs might be a natural choice for bunch length compression in ERLs.
  - Because of many dipoles, CSR-induced emittance growth is a challenge.
  - Both dispersion and CSR contribute substantially to the μBI gain.
- Presently, optics conditions that simultaneously minimize emittance growth and μBI gain in **isochronous arcs** are under investigation……Possible extension to *arc compressors ?*

- **Locally isochronous**
- **Small β_x at the dipoles**
- **Large α_x at the dipoles**
- **2π phase advance between dipoles**

Courtesy of C.-Y. Tsai







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Summary

- **CSR** is the main source of **projected emittance growth** in beamlines traversed by high brightness electron beams. Emittance can be preserved through **optics design**.
  - CSR effect on emittance can be **cancelled** in **(locally) isochronous** transfer lines.
  - CSR effect on emittance can be **minimized** in **bunch length compressors**, e.g. chicanes and arcs.

- **LSC** turns out to dominate the dynamics of high brightness electron beams at spatial **scales much shorter** than the **bunch length**, in spite of GeV beam energies.
  - LSC is primarily responsible for **large slice energy spread** (energy modulation).
  - **CSR and LSC** play similar roles in setting up the μBI. Their contribution depends on the machine **layout** and **compression scheme**.

- **Laser Heater** is presently the favoured knob for keeping the μBI under control, and thereby optimizing the FEL performance.

Homework (1/3)

You should be able to work out all of the following ones, just looking to the presented slides. You are encouraged to work together, use books, and ask help if needed (I'll be around all night).

CAS "policy" adopted: homework are not mandatory, but your effort in doing them will be appreciated !

1. Calculate the total peak power emitted by a Gaussian electron bunch in the last dipole of a magnetic chicane, assuming the following parameters: Q=0.3 nC, σ_z=30 μm, E=1 GeV, B=0.3T. Similarly, calculate the total average power emitted by a train of 400 bunches at the repetition rate of 1 MHz. What is the dipole magnetic field that allows a reduction of the power by a factor of 2 ?
   - *Hint: slide 4*

2. Consider a 100 pC Gaussian bunch compressed by a factor of 30 in a 4-dipoles chicane; assume that the beam has a horizontal waist in the fourth dipole. The initial bunch duration is 2 ps rms, the initial normalized horizontal emittance (rms) is 0.3 mm mrad. The beam energy is 300 MeV, the dipole magnet is 0.3 m long, and has a filed of 0.3 T. Estimate the maximum horizontal betatron function in the fourth dipole of the chicane in order to limit the final normalized emittance to 0.4 mm mrad. What is the horizontal beam size at the entrance of that dipole? And at the exit? The chicane has an R_{56} = -41 mm.
   - *Hint: slide 10, 11 + Lesson on Bunch Compressors (Lorentz force, Energy Chirp)*

Homework (2/3)

3. Consider an arc compressor made of identical achromatic cells. Provide an expression for the compression factor along the arc, as a function of the cell number.
   - *Hint: slide 13 + Lesson on Bunch Compressors (Energy Chirp)*

4. CSR wakefield is shielded if the characteristic length of emission, e.g. the rms bunch length, is larger than $\frac{\Delta}{\pi}\sqrt{\frac{w}{R}}$, being Δ and w the vacuum chamber total height and width, respectively, and R the bending radius of the dipole magnet in which CSR is shielded. Estimate the suppression of energy loss due to shielding in a parallel plate model, for a beam with the following parameters: E = 60 MeV, σ_z = 170 μm, dipole length = 0.4m, bending angle = 0.35 rad, Δ = 5 mm. Use the following prescription:

$$\frac{\Delta E_{shield}}{\Delta E_{free}} \approx 4.2\left(\frac{n_{harm}}{n_c}\right)e^{-\frac{2n_{harm}}{n_c}}, \quad with \quad n_{harm} = \sqrt{\frac{2}{3}\left(\frac{\pi R}{\Delta}\right)^3}, \qquad n_c = \frac{R}{\sigma_z}$$

5. Consider a 300 pC charge bunch, 10 ps long (full width, flat-top current profile). Estimate the beam mean energy at which a single particle at the bunch edge will suffer of a 0.01% energy variation per meter, due to LSC force. What is the length scale at which a similar energy variation per meter may happen, if the beam mean energy is 100 MeV?
   - *Hint: slide 15*

Homework (3/3)

6. Consider a one-stage compression scheme (linac + chicane). Derive the expression for the modulation wavelength at which the gain is peaked (for simplicity, assume Z(k)=const.). Assume a microbunching peak gain of 1000 at the final (i.e., compressed) wavelength of 0.1 μm, at the beam energy of 1 GeV, for R_{56} = -41 mm in the chicane, and for a beam initial uncorrelated energy spread of 2 keV rms. What should the initial uncorrelated energy spread be in order to lower the gain to unity, assuming all the other beam, impedance and linac parameters unchanged ?
   - *Hint: slide 18.*

7. In Slide 23, the measurement of the beam longitudinal phase space at the end of the LCLS linac shows a larger slice energy spread for LH off w.r.t. LH on; why? The initial beam has an uncorrelated energy spread of approximately 2 keV rms. What should the final slice energy spread be with LH on (assume microbunching instability fully suppressed), if the total compression factor is 50 ?
   - *Hint: slide 23 + Lesson on Bunch Compressors (Uncorrelated vs. Slice Energy Spread)*

8. CSR-induced microbunching in a dipole magnet can be washed out by the particles transverse motion, if the beam emittance is non-zero (transverse Landau damping). Demonstrate that smearing is expected at wavelengths of the order of sqrt($\varepsilon_x H_x$), with H_x inside the dipole.
   - *Hint: consider the path length difference of a particle in a bend w.r.t. the straight path, as follows, and neglect terms at second order in the particle coordinates:*

$$\Delta l(s) = \int_0^s \frac{u(s')}{R(s')} ds' + \frac{1}{2}\int_0^s u'^2(s')ds', \quad with \;\; u(\theta) \cong u_0 \cos\theta + u'_0 R \sin\theta$$