

# Superconducting RF Cavities

Rama Calaga, CERN, 2016

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Superconductivity & SC-RF Basics

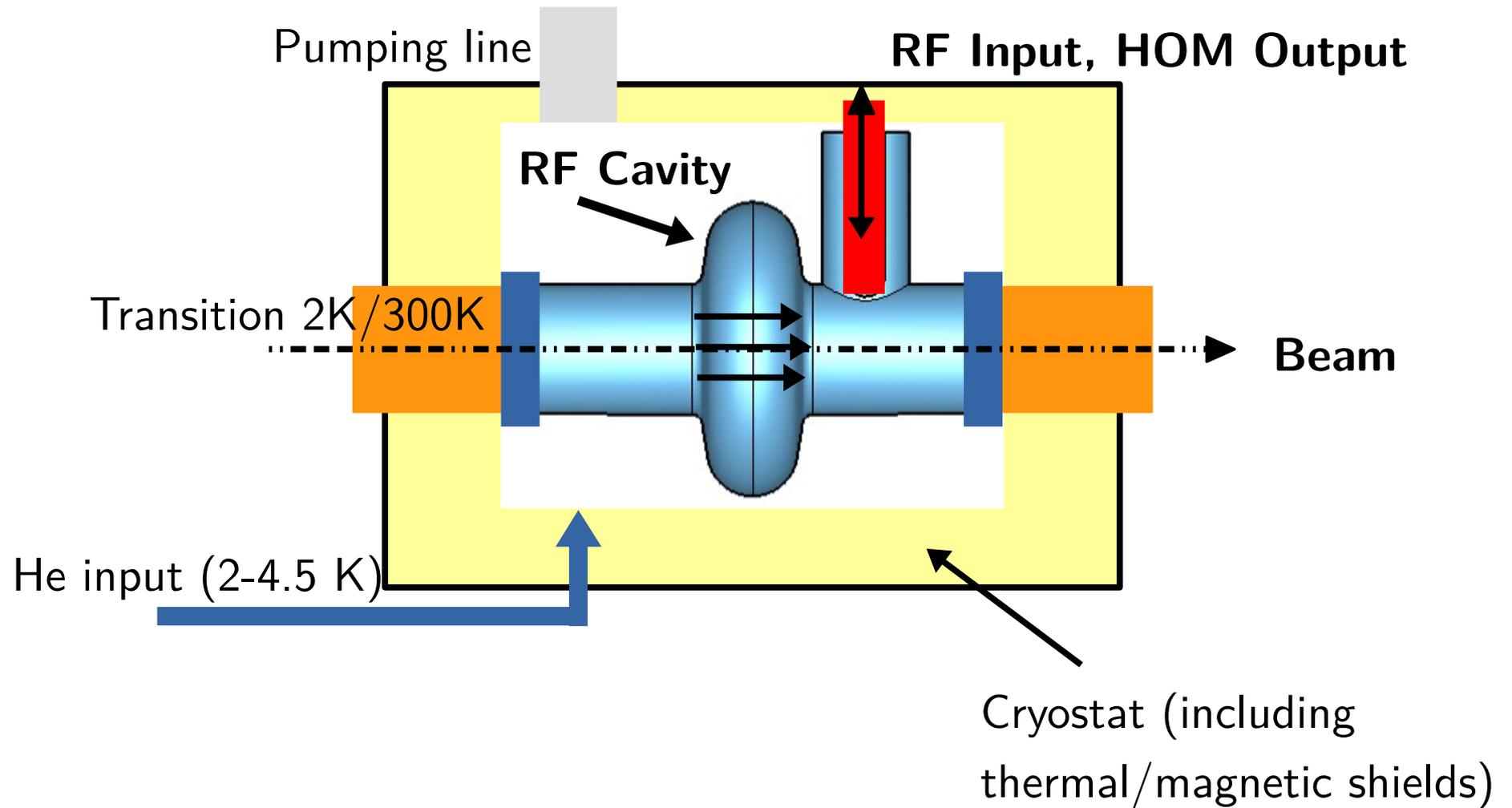
Practical Aspects I & II

<sup>†</sup>Note: For a detailed treatment, see references (slide 2)

# Some References

1. RF Superconductivity: Science, Technology, and Applications  
H. Padamsee et al., Wiley-VCH (2009).
2. SRF Conferences & Tutorials (link for [SRF2015](#))
3. CAS (1992), USPAS (2013, 2015), JUAS (2015) ...

# Outline



# SC-RF, European XFEL

2.5-20 GeV electron linac, 800 SC-RF Cavities (2.1 km)

Will be one of the largest SC-RF Linear Accelerator in the world

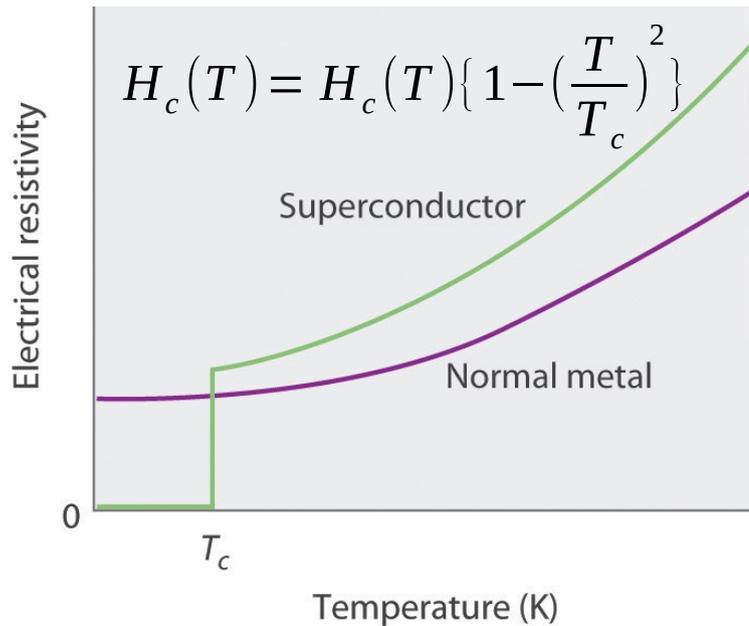


SC-RF is the basis of practically all high energy accelerators  
& w/o which ERLs probably cannot be realized

# Superconductivity & RF (Qualitative Look)

# Superconductivity

A thermodynamic phase transition below  $T=T_c$ , a macroscopic quantum phenomena

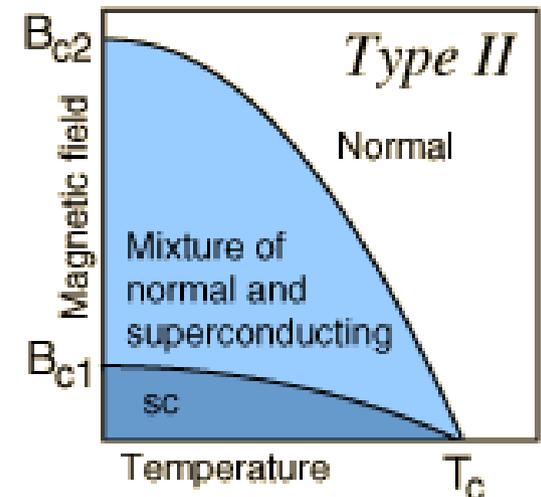
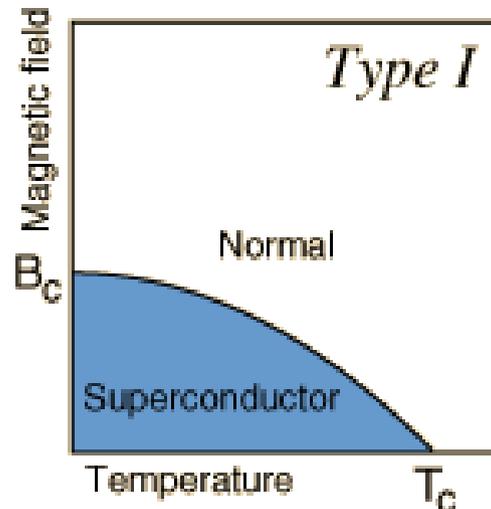


Can be qualitatively understood using two-fluid model ( $J_n, J_s$ ), where below  $T_c$  electrons pair-up into cooper pairs (condensation)

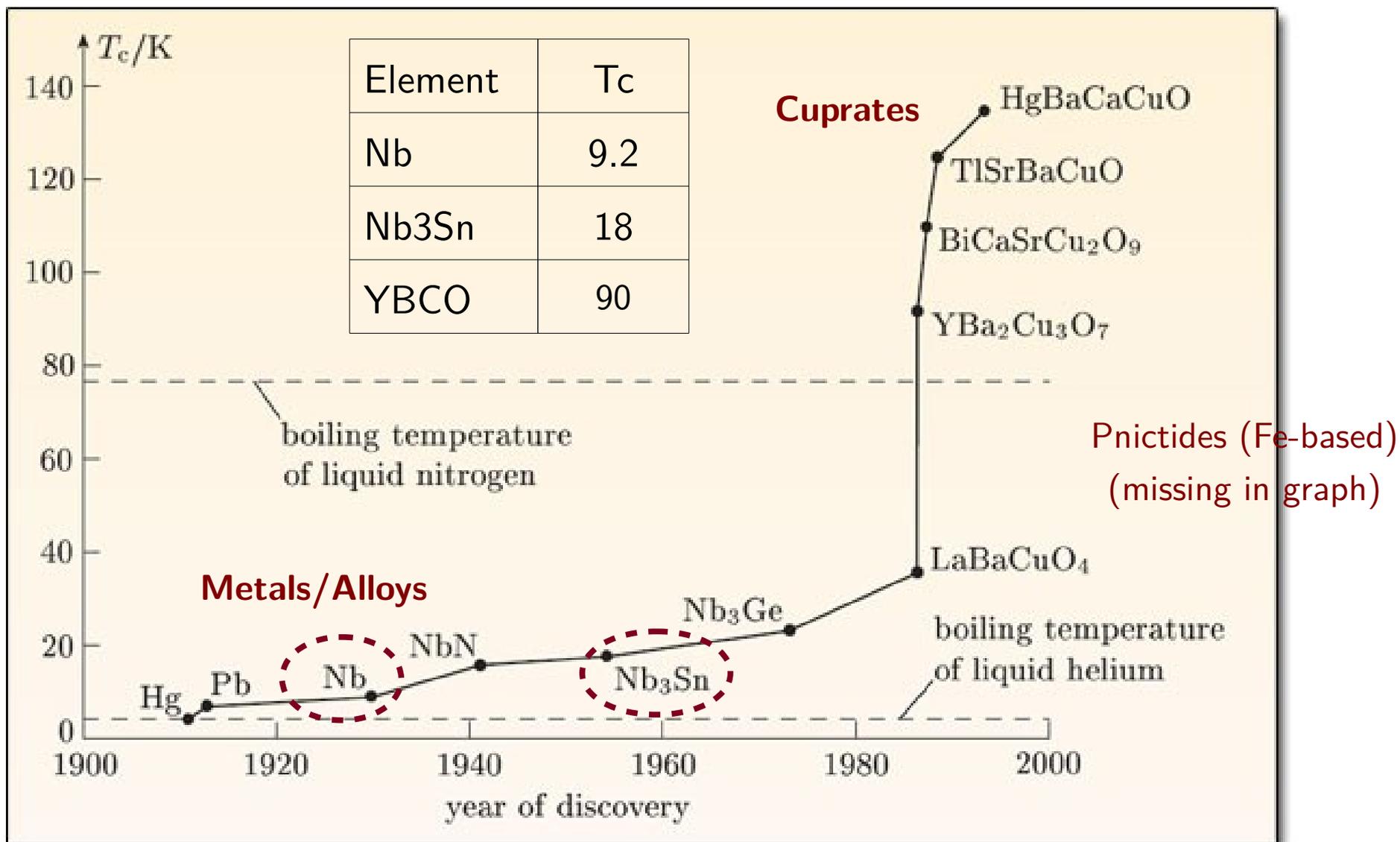
Type I – Complete flux expulsion

Type II – Mixed states with flux penetration in quantized vortices

$$\phi_0 = \frac{\hbar}{2e}$$



# SC-Elements, Evolution



For Nb (type II):  $B_{c1} \sim 180$  mT,  $B_{c2} \sim 400$  mT

# Characteristic Length Scales

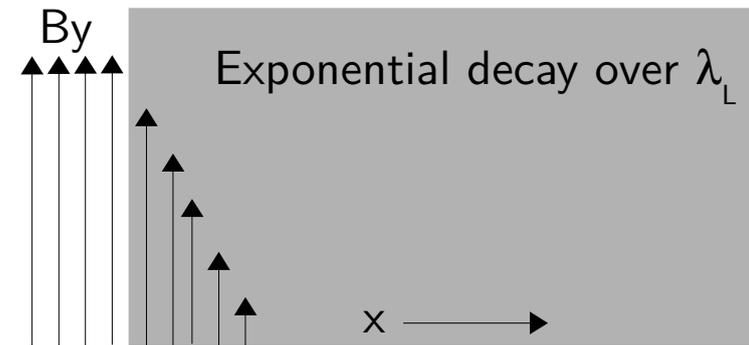
**London penetration depth** – length scale over which the B-field decays in SC

**Coherence length** – distance over the which cooper pairs are correlated

$$B_y(x) = B_0 e^{-\frac{x}{\lambda_L}}$$

$$\lambda_L = \sqrt{\frac{m_e}{\mu_0 n_s e^2}}$$

$$\xi_c = \frac{\hbar v_F}{\Delta}$$



Critical Field:

$$B_c = \frac{\phi_0}{2\sqrt{2}\pi\lambda_L\xi}$$

Element	$\lambda(0)$
Pb	390
Nb	470
Sn	510
YBCO	1700

# Surface Resistance

Av. power dissipated on the surface:  $P_d = \frac{1}{2} R_s H_0^2$

## Normal-Conductor

$$R_s = \frac{1}{\sigma \delta} = \sqrt{\frac{\mu_0 \omega}{2 \sigma}}$$

For Copper:

$$\sigma = 5.85 \times 10^7 \text{ S/m (300 K)}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$$

$$\omega = 2\pi \times 1 \text{ GHz}$$

$$\delta = \underline{2 \mu\text{m}}$$

$$R_s = \mathbf{8.2 \text{ m}\Omega}$$

(Don't forget anomalous skin effect)

## Super-Conductor

$$R_s = \frac{1}{(\sigma_n - i\sigma_s)\lambda_L} = \frac{1}{2} \sigma_n \omega^2 \mu_0^2 \lambda_L^3$$

For Niobium:

$$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$$

$$\omega = 2\pi \times 1 \text{ GHz}$$

$$\xi = 39 \text{ nm}$$

$$\lambda_L (T=0\text{K}) = \underline{36 \text{ nm}}$$

$$R_s \sim \mathbf{n\Omega}$$

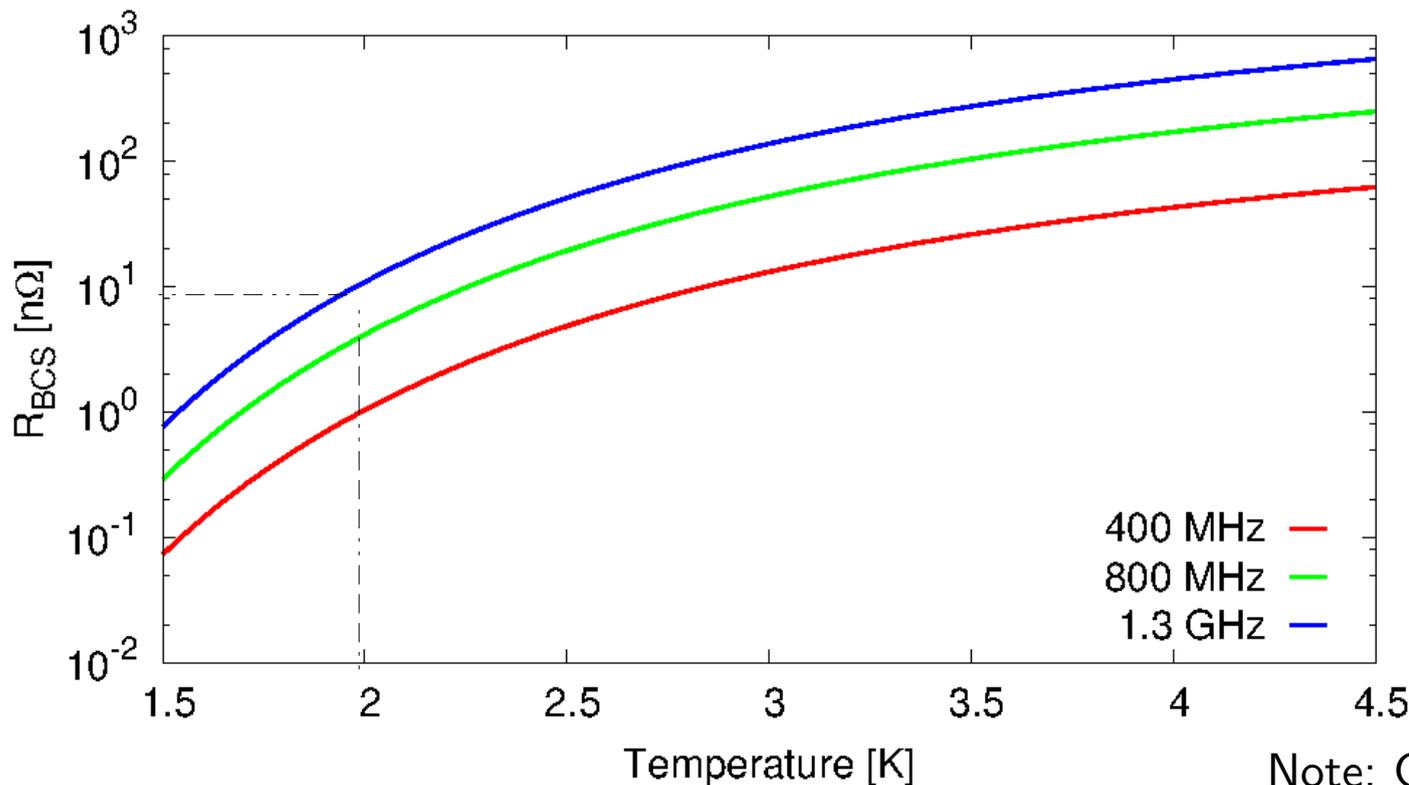
# Surface Resistance from BCS

Mattis-Bardeen approx (BCS theory):

$$R_{BCS} = A \frac{\omega^2}{T} e^{-\frac{\Delta}{k_B T}}$$

$(\lambda_L^4, \xi, l, \rho^{0.5})$   
 ← energy gap

For Niobium:  $R_{BCS} \approx 2.4 \times 10^{-4} \left( \frac{f [MHz]}{1500} \right)^2 \frac{1}{T} e^{-\frac{17.67}{T}}$



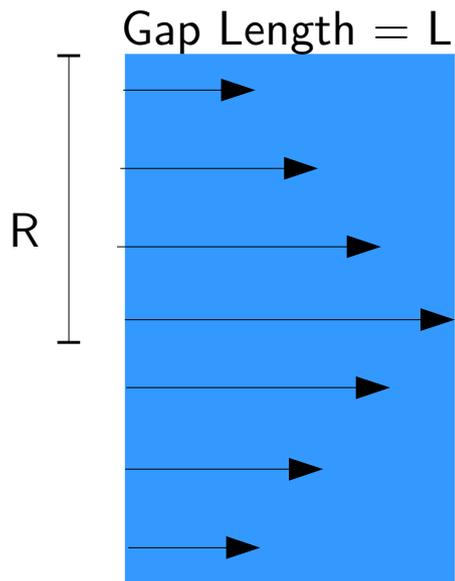
$$R_s = R_{BCS} + R_{res}$$

There is also temperature independent residual resistance which is lower limit

Note: Cryogenic to wall plug power

# Cylindrical Cavity

Standing waves of TM & TE



$$\omega_{mnp} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \sqrt{\left(\frac{p_{mn}}{r}\right)^2 + \left(\frac{p\pi}{l}\right)^2} \quad - \text{ (TM)}$$

$$\omega_{mnp} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \sqrt{\left(\frac{p'_{mn}}{r}\right)^2 + \left(\frac{p\pi}{l}\right)^2} \quad - \text{ (TE)}$$

$$E_z = E_0 J_0(\omega_0 r / c) \cos(\omega_0 t)$$

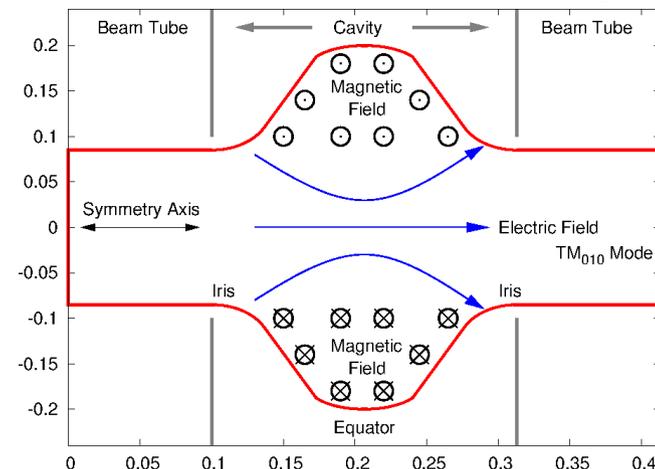
$$H_\phi = -\frac{1}{\mu_0 c} E_0 J_0(\omega_0 r / c) \sin(\omega_0 t)$$

$$\omega = \frac{2.405 c}{R}$$

1<sup>st</sup> mode (m=0, n=1, p=0) suited for acceleration with field lines uniform over z

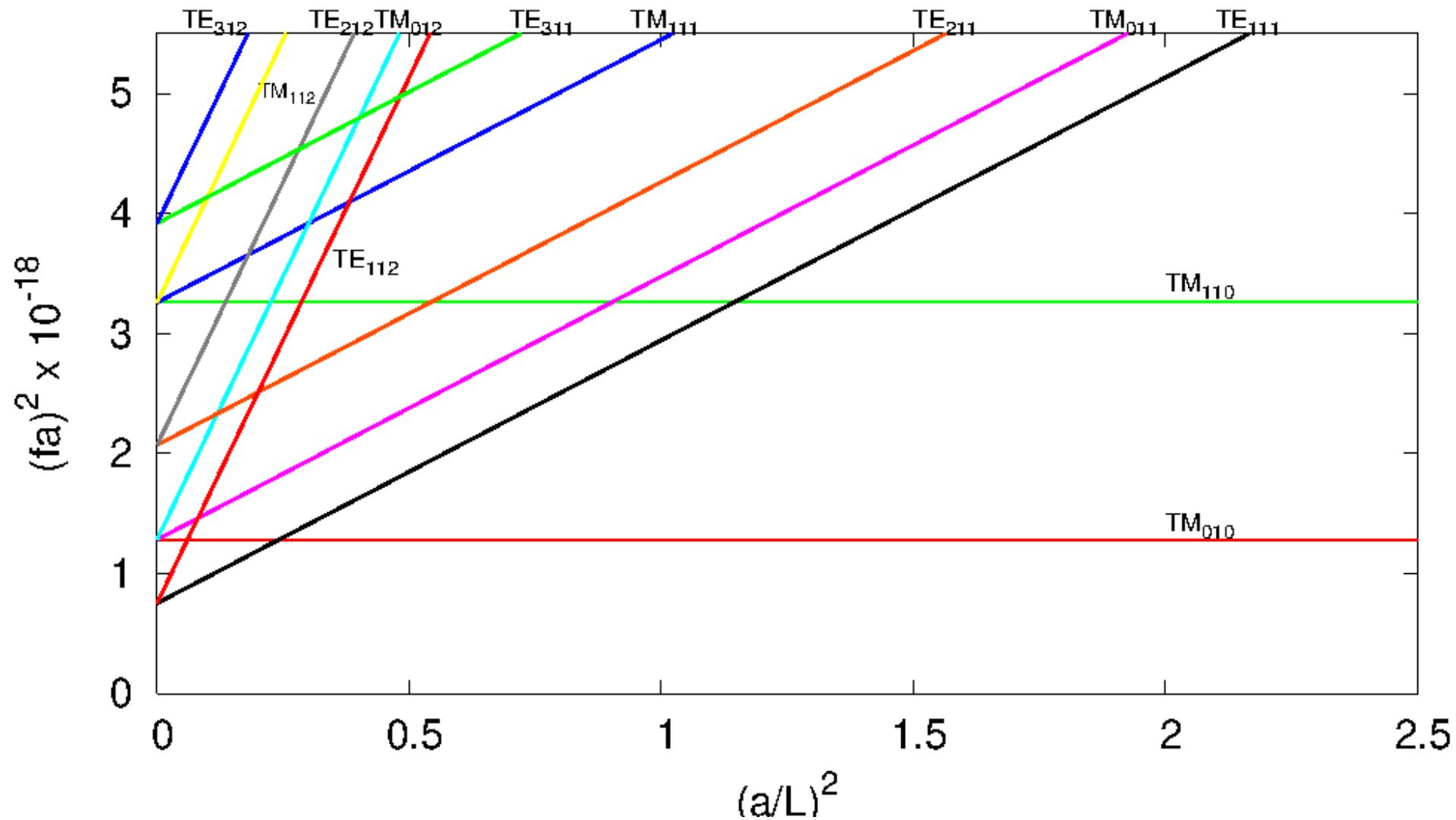
The frequency is only dependent on radius and

Standard SC-RF Cavity



# Mode Spectrum Vs Geometry

Cylindrical Cavity



# Figures of Merit I

Voltage:

$$V_{acc} = \left| \int_{z=0}^{z=l} E_z e^{i\omega_0 z/c} dz \right|$$

Transit Time:

( $\beta c T$  = distance covered in a RF period)

$$T = \frac{\int_0^l E_0 e^{i\omega z/c} dz}{\int_0^l E_0 dz}$$

Stored Energy:

$$U = \frac{1}{2} \epsilon_0 \int_V |\vec{E}|^2 dv = \frac{1}{2} \mu_0 \int_V |\vec{H}|^2 dv$$

# Figures of Merit II

Quality Factor:  $Q_0 = \frac{\omega_0 U(t)}{P_d(t)}$        $U(t) = U_0 e^{-t/\tau}$

Shunt Impedance:  $R_{shunt} = \frac{V^2}{P_d}$        $\frac{R_a}{Q_0} = \frac{V^2}{\omega U}$

Geometric Factor:  $G = R_s Q_0 = \frac{\omega_0 \mu_0 \int |\vec{H}|^2 dv}{\int_S |\vec{H}|^2 ds}$

Power dissipated:  $P_{walls} = \frac{V_c^2}{R/Q \cdot G} R_s$

Material 

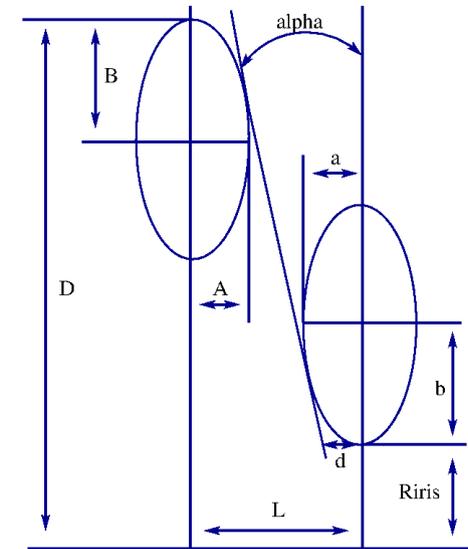
Geometry 

# Cavity Design, TM-Class

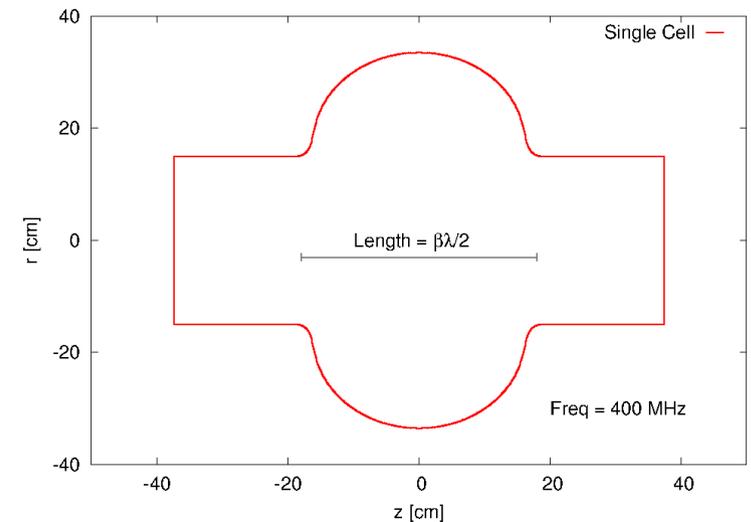
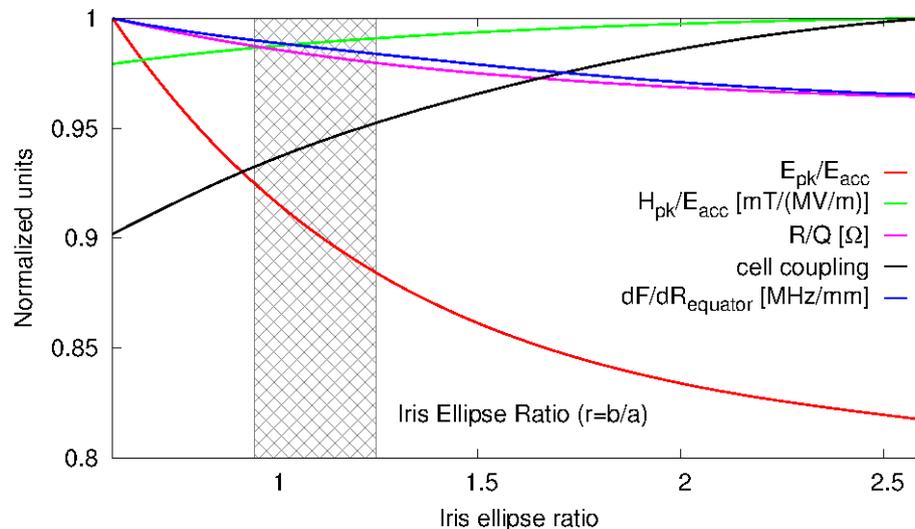
## Standard Criteria:

- Minimize peak surface fields (E, B)
- Optimum R/Q based upon application
- Optimum mechanical stiffness (tuning vs. de-tuning)
- Strong cell-to-cell coupling (multi-cell)

$\frac{1}{2}$ -cell parametrization  
(P. Pierini et al.)



Example scan vs iris ratio

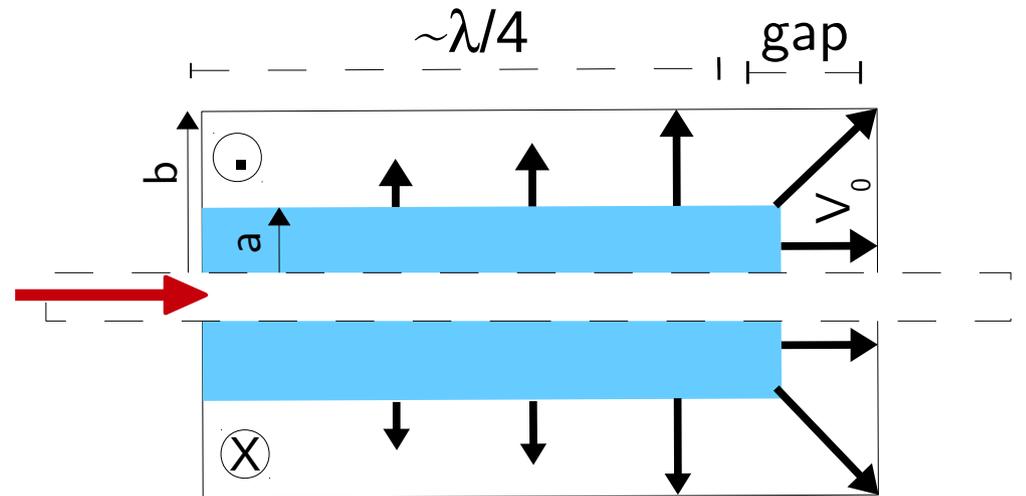


Note: No single optimum shape for everything

# TEM Class Resonators

Another important class are using uniform transmission lines

$\lambda/4$  being one of the simplest form



$$V_z = V_0 \sin(kz)$$

$$I_z = I_0 \cos(kz)$$

Line Impedance:  $Z_0 = \frac{\eta}{2\pi} \ln \frac{b}{a} \longrightarrow \frac{R}{Q} = 4 \frac{Z_0}{\pi}$

Geometric Factor:  $G = \frac{2\pi\eta}{\lambda} \frac{\ln b/a}{a^{-1} + b^{-1}}$

Widely used in low velocity (protons, ions) applications for compactness

# Cavity Design, Numerical

Almost all practical applications requires a deviation from idealized cavity. Therefore, spatial discretization of the structure and solve Maxwell's equation numerically.

Frequency Domain (eigenvalue problem)

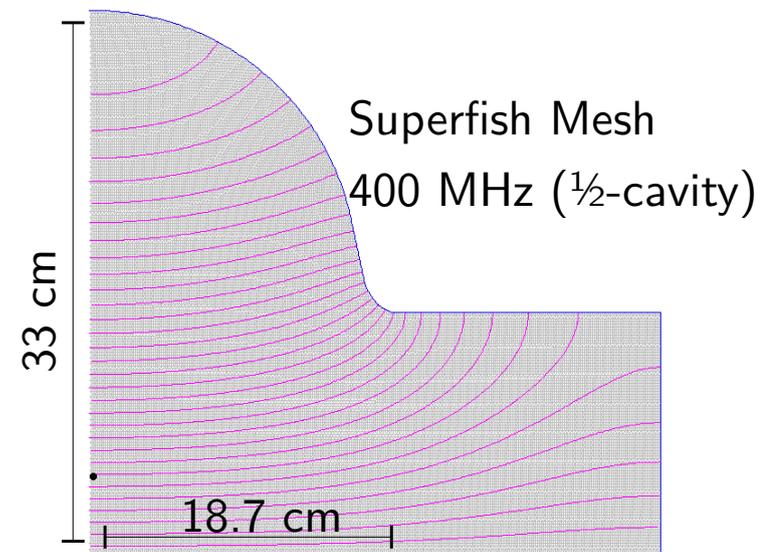
Time Domain (transient response)

Different methods: Finite (difference, integration, element)

Generally used (but not comprehensive):

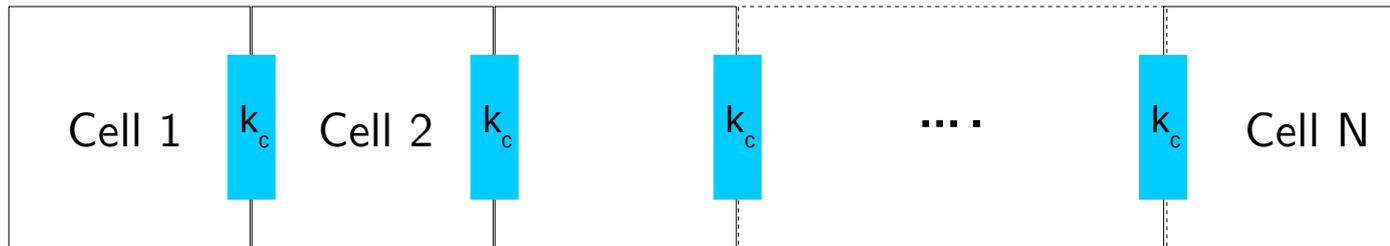
2D: **Superfish**, SLANS, **ABCI**

3D: **CST**, **HFSS**, **ACE3P**, **GdfidL**

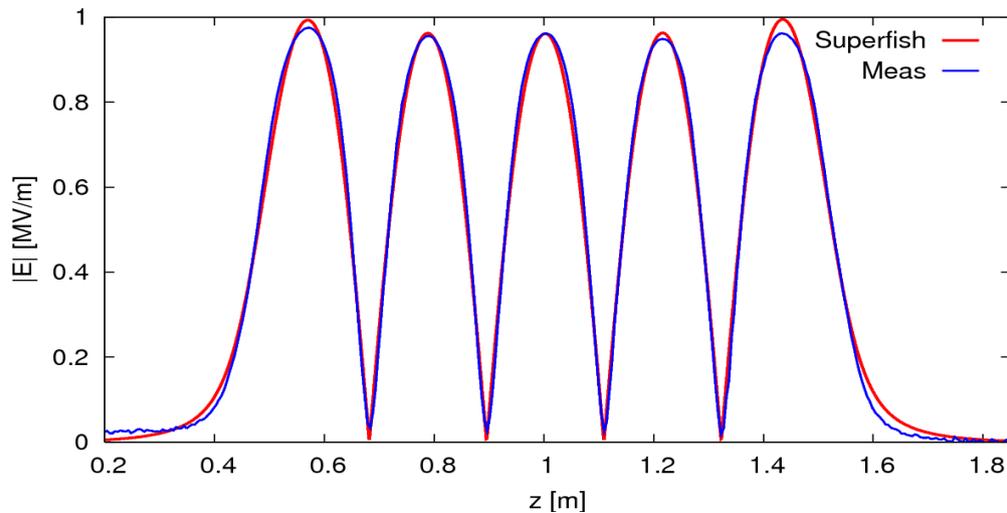
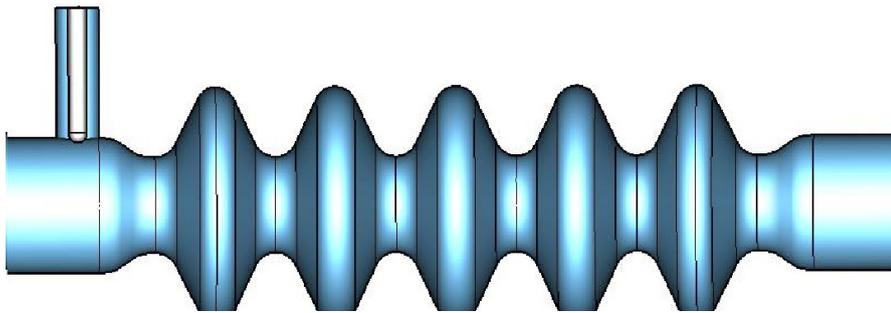


# Multi-Cells (mainly for Linacs)

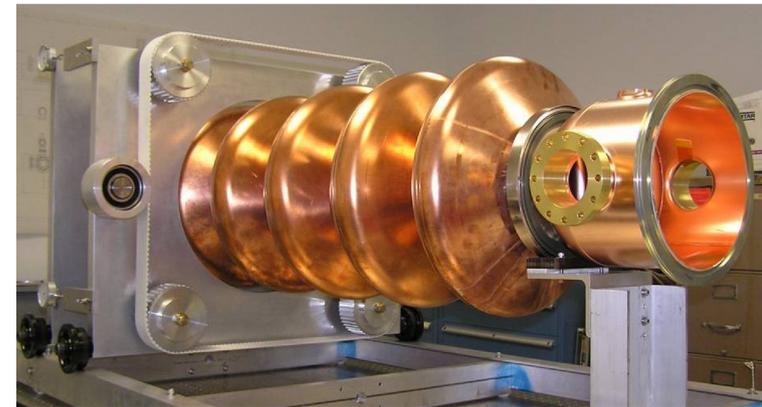
To improve the “real-estate” gradient & ancillary equipment (couplers, flanges, warm-cold transitions, etc...) it is often efficient to go to N-coupled cells



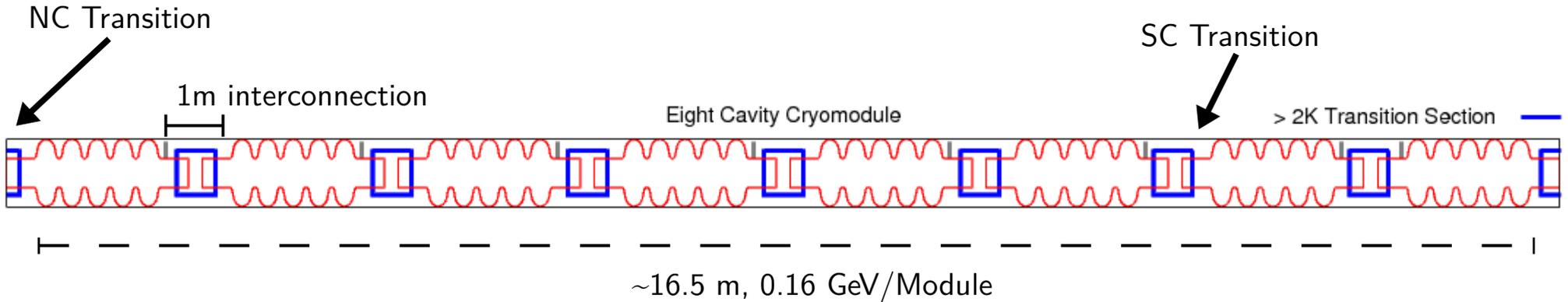
$$\left(\frac{\omega_n}{\omega_0}\right)^2 = 1 + 2k_c \left[1 - \cos\left(\frac{n\pi}{N}\right)\right]$$



$$a = \frac{N^2}{k_c}$$



# Towards a Compact Footprint



Example 5-cell cavity at 700 MHz:

In the above, 8-cavity cryomodule: 0.16 GeV, 16.5 m

For 20 GeV, LINAC-FEL  $\rightarrow$  2 km

Homework :

Calculate the real estate length assuming only single cells/cavity

# Practical Aspects I

(Measurements, Freq. Detuning, HOMs)

# Field Measurements

Standard practice to use Cu-models for fabrication trials & RF measurements

Slater's theorem:  $\Delta\omega/\omega \approx \mu \Delta U/U$  (for small perturbations)

Bead inside a cavity:

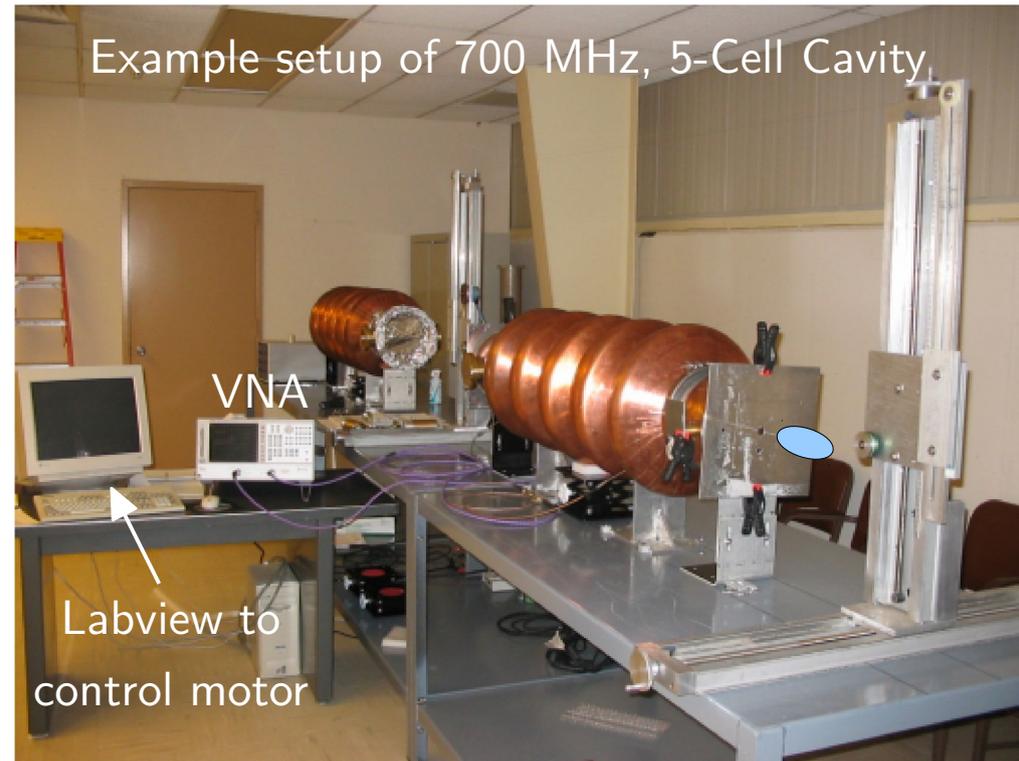
$$\frac{\delta\omega}{\omega_0} = \begin{cases} -\frac{\pi r^3}{U} (\epsilon_0 \frac{\epsilon_r + 2}{\epsilon_r - 1} E_0^2) & : \text{ dielectric} \\ -\frac{\pi r^3}{U} (\epsilon_0 E_0^2 - \frac{\mu_0}{2} H^2) & : \text{ metal} \end{cases}$$

Vector Network Analyzer  $\rightarrow$  S-parameters

$$S_{21} = \frac{2\sqrt{\beta_1\beta_2}}{(1 + \beta_1 + \beta_2) + iQ_0 \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)}$$

$\beta_1, \beta_2$  are coupling factors for antenna's, assuming they are small:

$$\frac{\delta\omega}{\omega_0} \approx -\frac{1}{2Q_L} \tan(\phi)$$



# Detuning from Lorentz Force

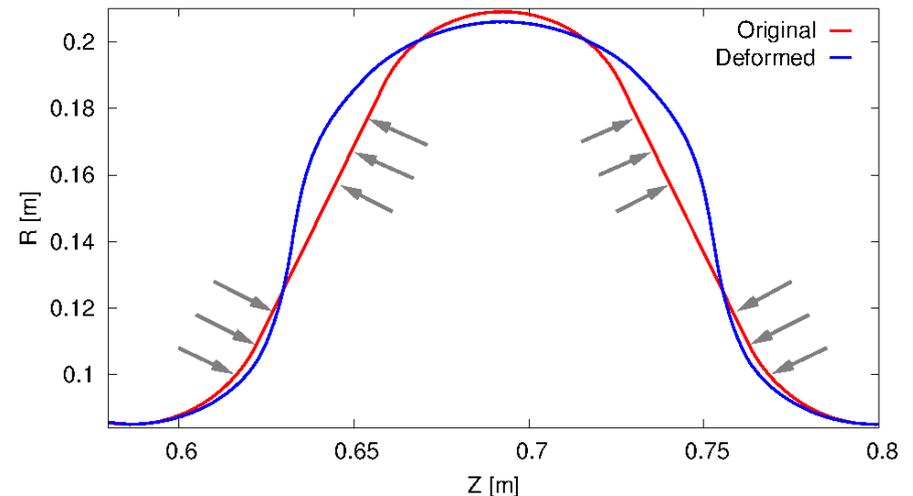
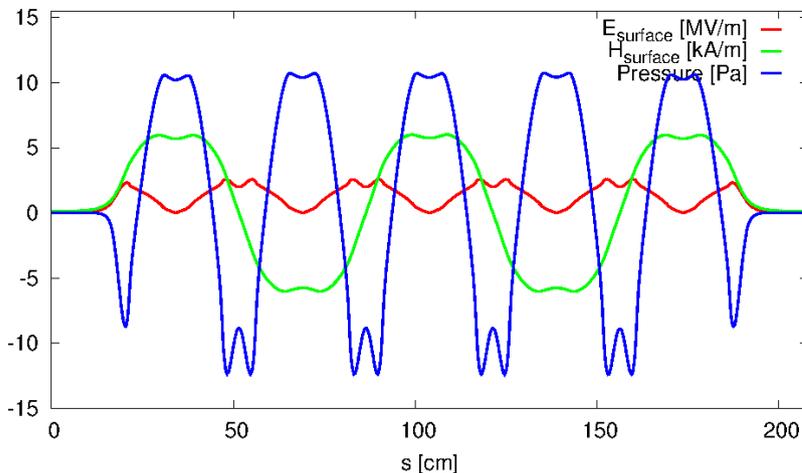
SC-Cavities are operated typically with narrow bandwidth  
It implies careful control/tracking of frequency

Radiation pressure from the very high electro-magnetic fields will distort the cavity shape and therefore the frequency

$$\delta f \propto \frac{f_0}{4U} \int_{\delta V} (\epsilon_0 \vec{E}^2 - \mu_0 \vec{H}^2) dV$$

inductive

capacitive

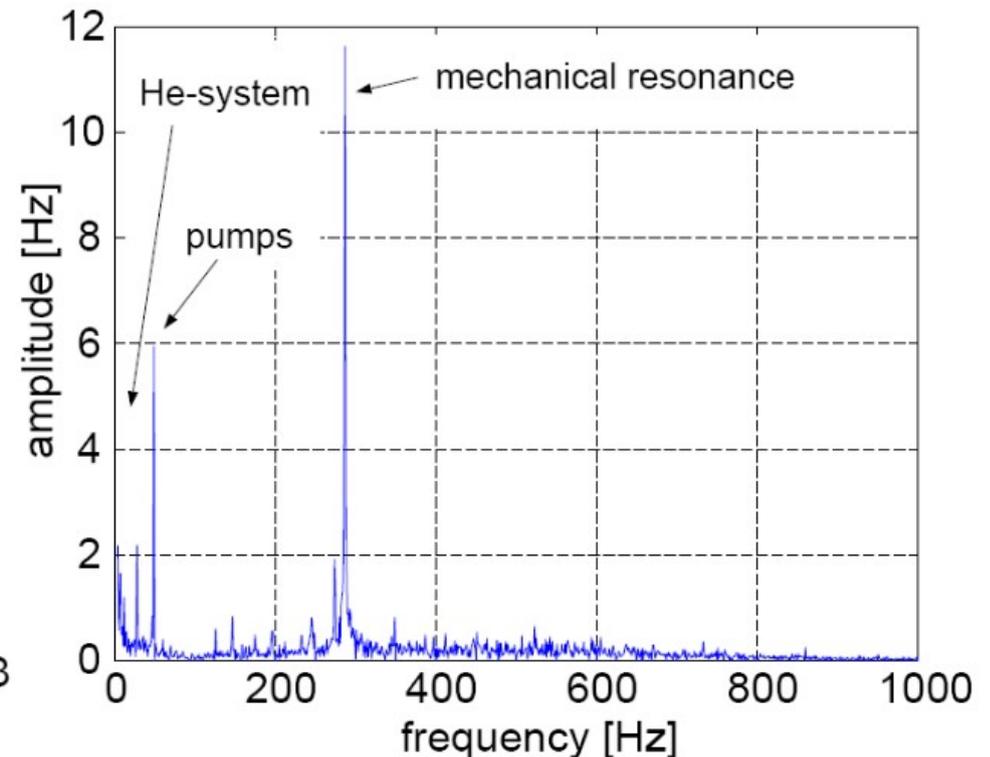
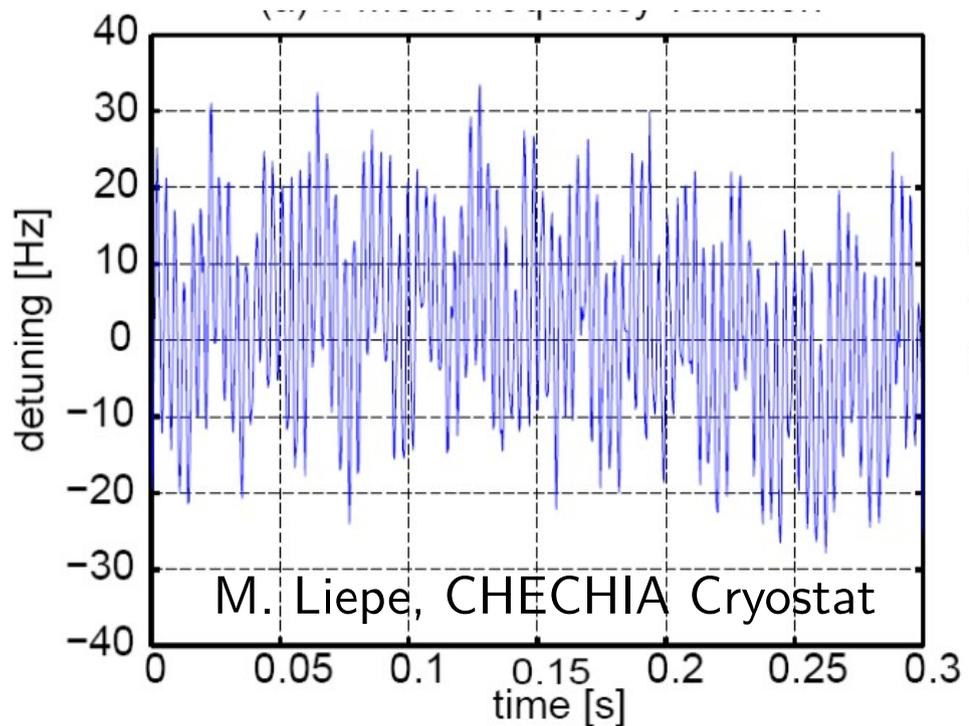


# Detuning From External Forces

External noise can be transferred to cavity via the cryostat (Microphonics)

## Mitigation

Tuning system: Mechanical - slow and/or electro-mechanical - moderate  
RF feedback - fast, BW limited ( $\Delta\omega$ )



# RF Power

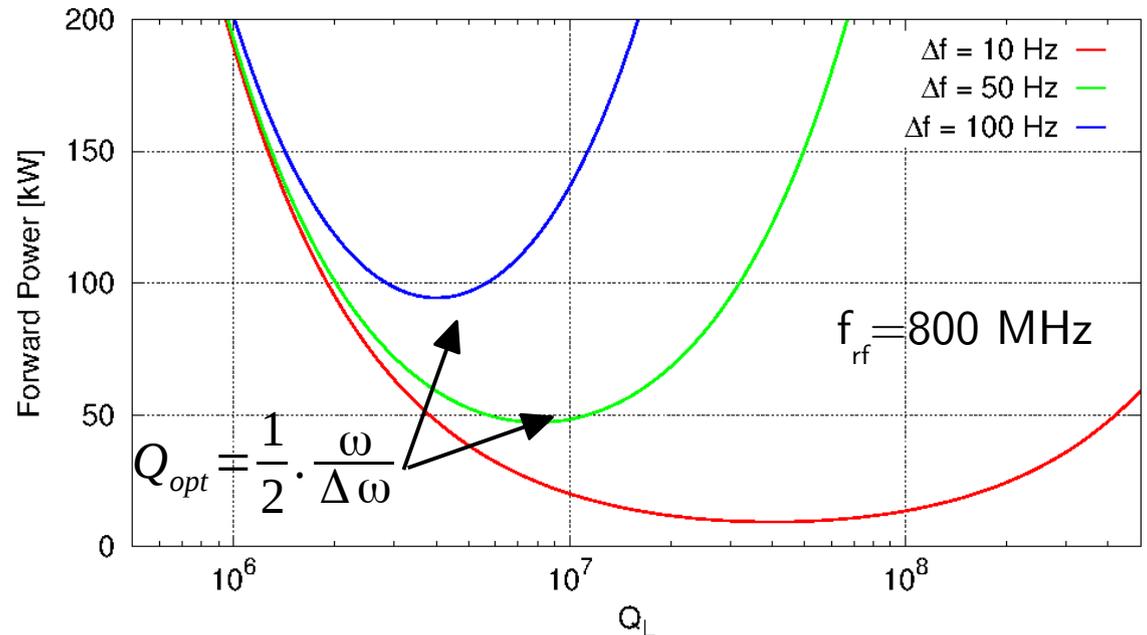
$$P_f = \frac{R_s}{4\beta} \left[ \frac{(1 + \beta)^2 V^2}{R_s^2} + \left( \frac{V}{X} - I_b \right)^2 \right]$$

$$Z(\omega) = \frac{R/Q \cdot Q_L}{1 + i \tan \psi} \quad Q_L = \frac{Q_0}{(1 + \beta)} \quad X = \frac{R}{Q} \frac{\omega}{\Delta \omega} \quad \tan(\psi) = 2 Q_L \frac{\Delta \omega}{\omega}$$

Assuming no beam-loading (ERL), one can show

$$P_f = \frac{V^2}{R/Q} \cdot \frac{\Delta \omega}{\omega}$$

To maintain a constant gap voltage, the input power scales linearly with detuning



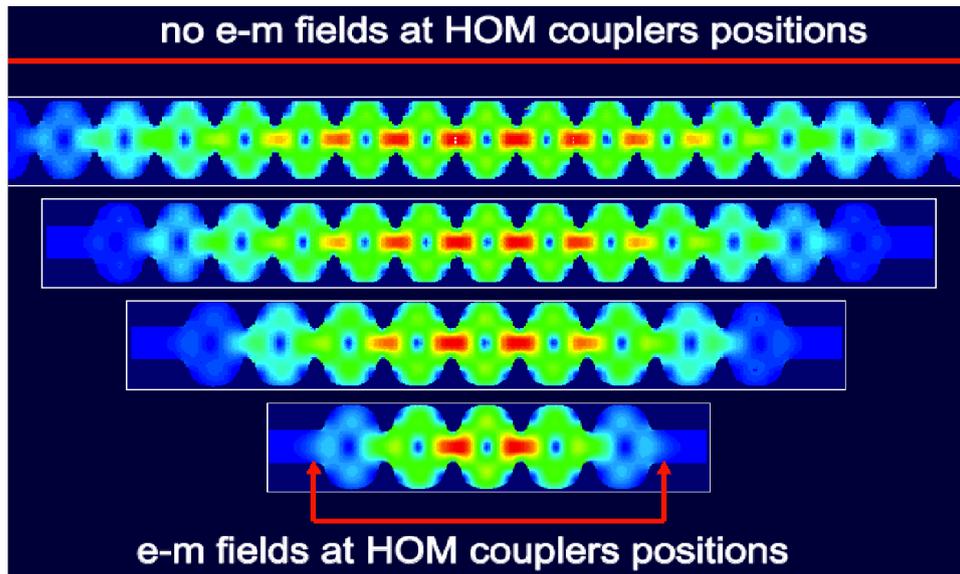
# Higher Order Modes

Beyond the fundamental (accelerating) mode, there exists infinite eigenmodes

$$\omega_{mnp} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \sqrt{\left(\frac{p_{mn}}{r}\right)^2 + \left(\frac{p\pi}{l}\right)^2} - \text{(TM)}$$

$$\omega_{mnp} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \sqrt{\left(\frac{p'_{mn}}{r}\right)^2 + \left(\frac{p\pi}{l}\right)^2} - \text{(TE)}$$

They can be excited by the beam which typically has a wide frequency range ( $1/\sigma_z$ ) depending on the synchronism condition



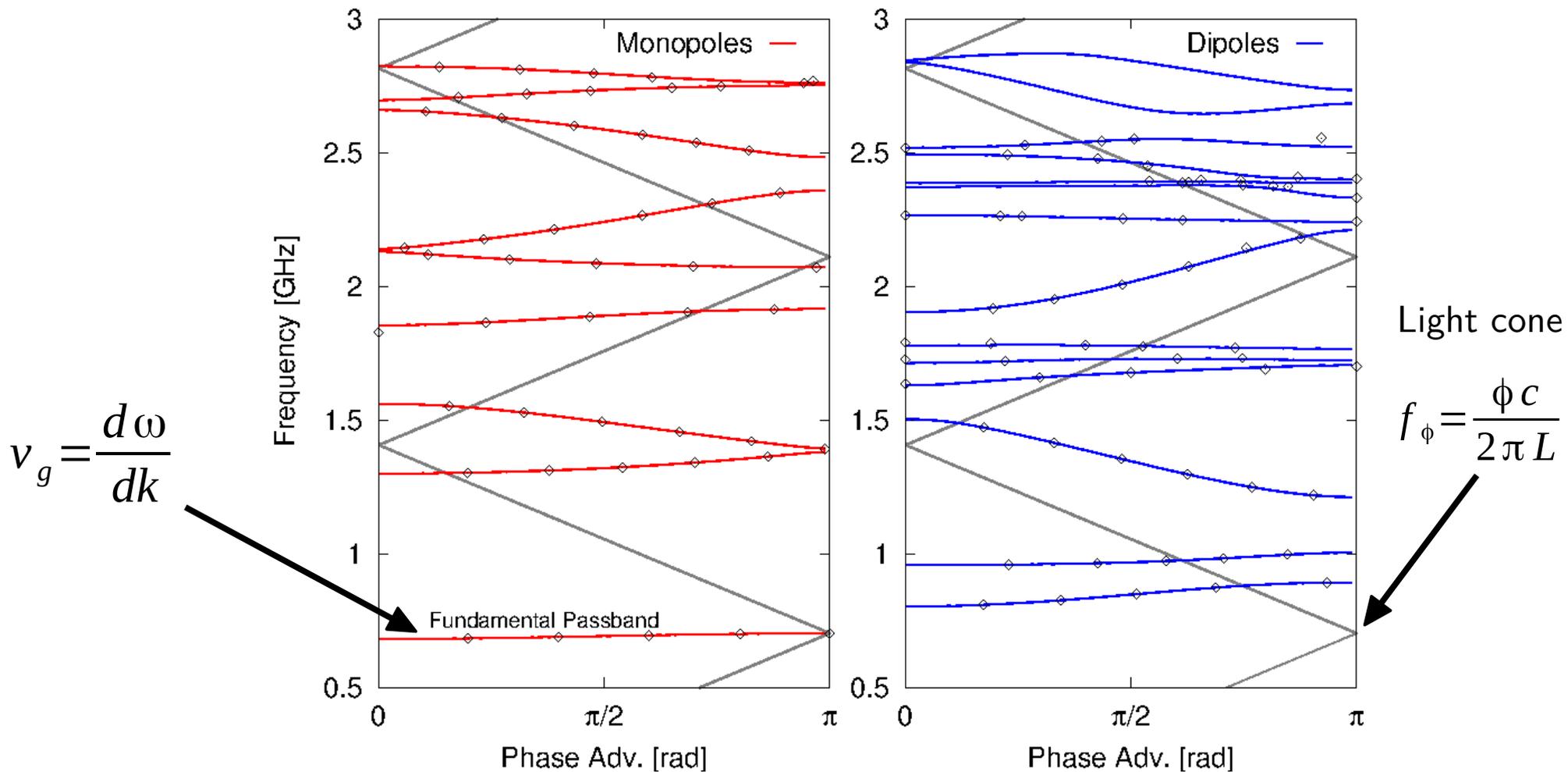
Multi-Cells have N-times the number of HOMs (passbands)

Strong damping of the HOMs is often key to aspect to reach high currents and beam quality

# Dispersion Curves

Approximation of an infinitely periodic structure

Modes with phase velocity =  $\beta c$  are strongly excited (also high R/Q)

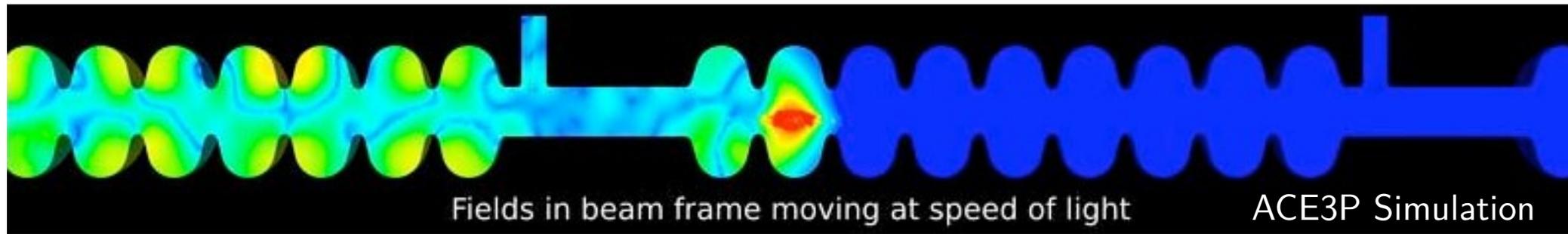


# Loss Factor

Bunch traversing the structure losses energy and leaves behind a wakefield into parasitic modes, which can be characterized by loss factor ( $k$ )

For pulsed linacs (FELs), resulting energy spread & emittance growth

High current CW (storage rings, ERLs) limited by power

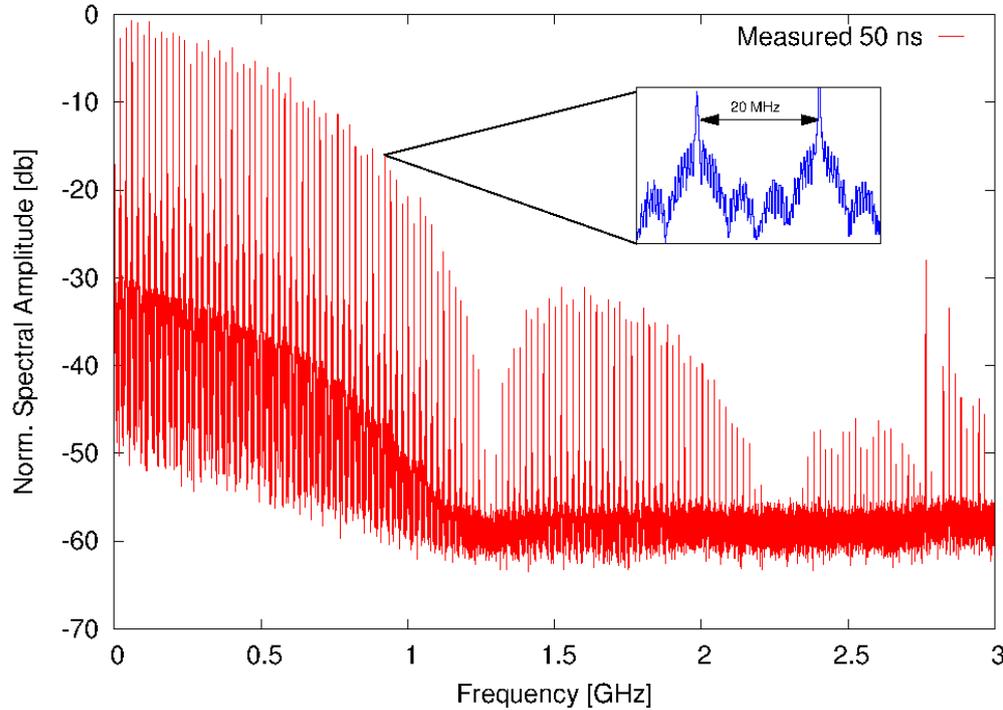
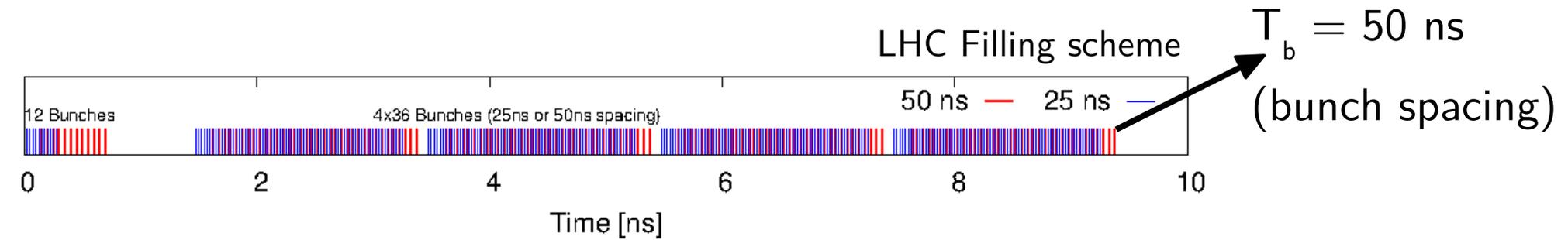


Energy loss:  $\Delta U = k_n q^2$        $k_n = \frac{\omega}{2} \cdot \frac{R}{Q}$  (loss factor/mode assuming TM-like)

$$k_{||}(n) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{Re} Z_{||}(\omega) d\omega.$$

Analogous transverse loss factor  $k_t \rightarrow$  emittance growth

# HOM Losses



Non-Resonant Case:

$$P_{HOM} = (\sum k_n - k_0) \cdot q \cdot I_b$$

Fundamental

Resonant Case:

$$P_{HOM} = I_b^2 \cdot \frac{R}{Q} \cdot Q_L \cdot F_n^2$$

Damping essential  
for SC-Cavities

## Homework:

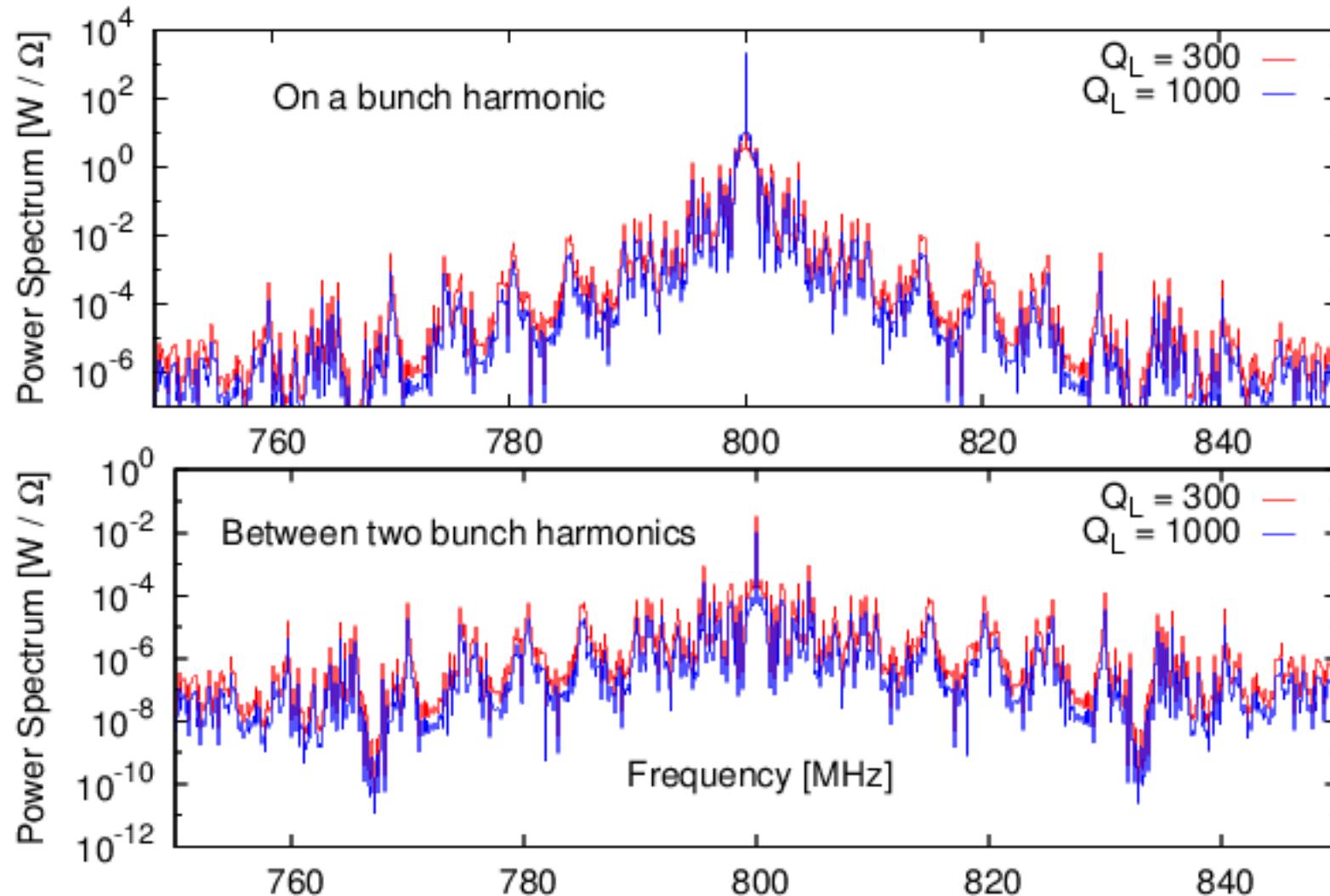
Calculate energy spread, HOM power/cavity for 6-pass ERL

$q=1$ nC,  $f_{rev} = 1$  MHz,  $k = 1$  V/pC

# HOM Power Contd.

In reality one integrates numerically the HOM impedance over the bunch spectrum/filling scheme

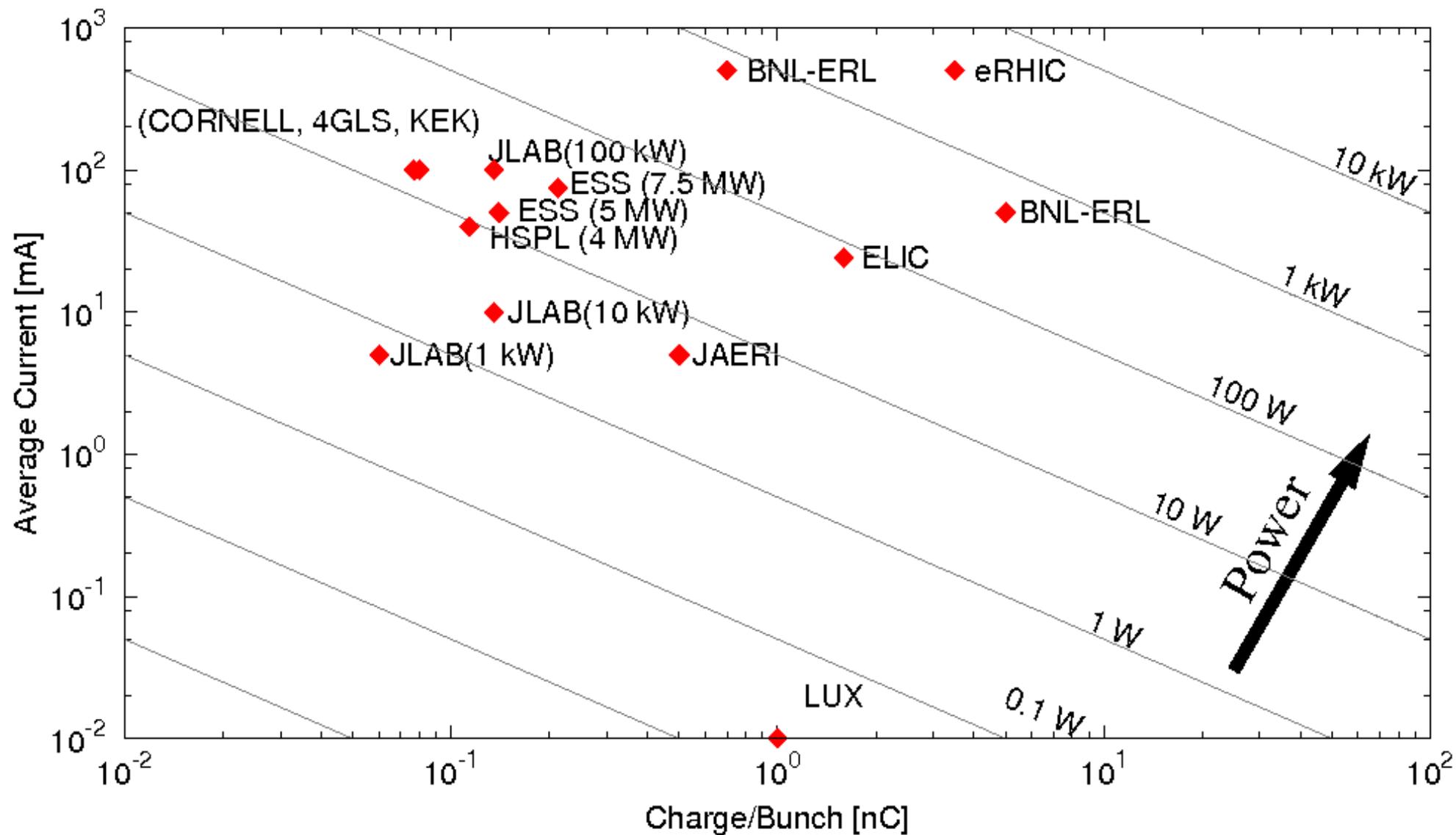
LHC example with an 800 MHz HOM ( $1 \Omega$ )



# HOM Power, ERLs

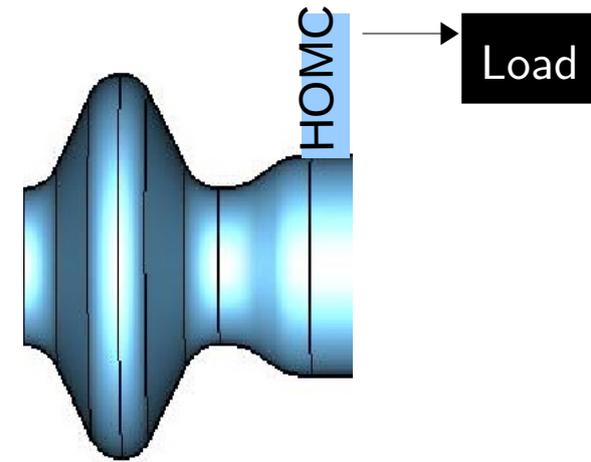
$$P_{avg} = k_L Q_b I_a$$

Power to be extracted but NOT into 2K

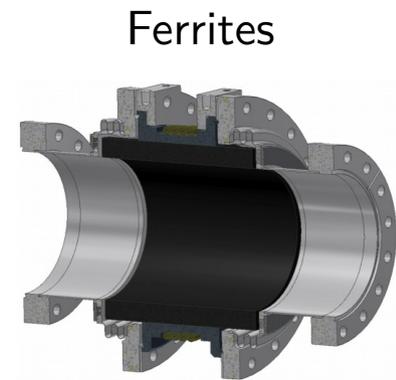
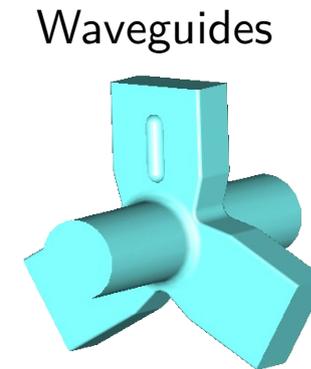
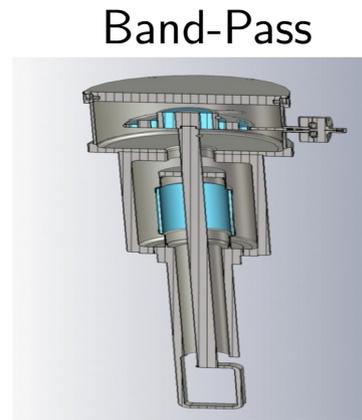
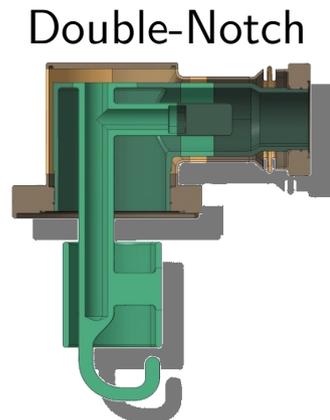
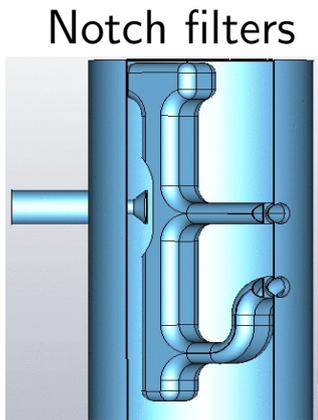


# HOM Damping & Extraction

- Notch Filters → Narrow-band & targeted damping
- Waveguides → Higher frequencies more suitable
- Ferrites → Broadband room temp



SC → NC

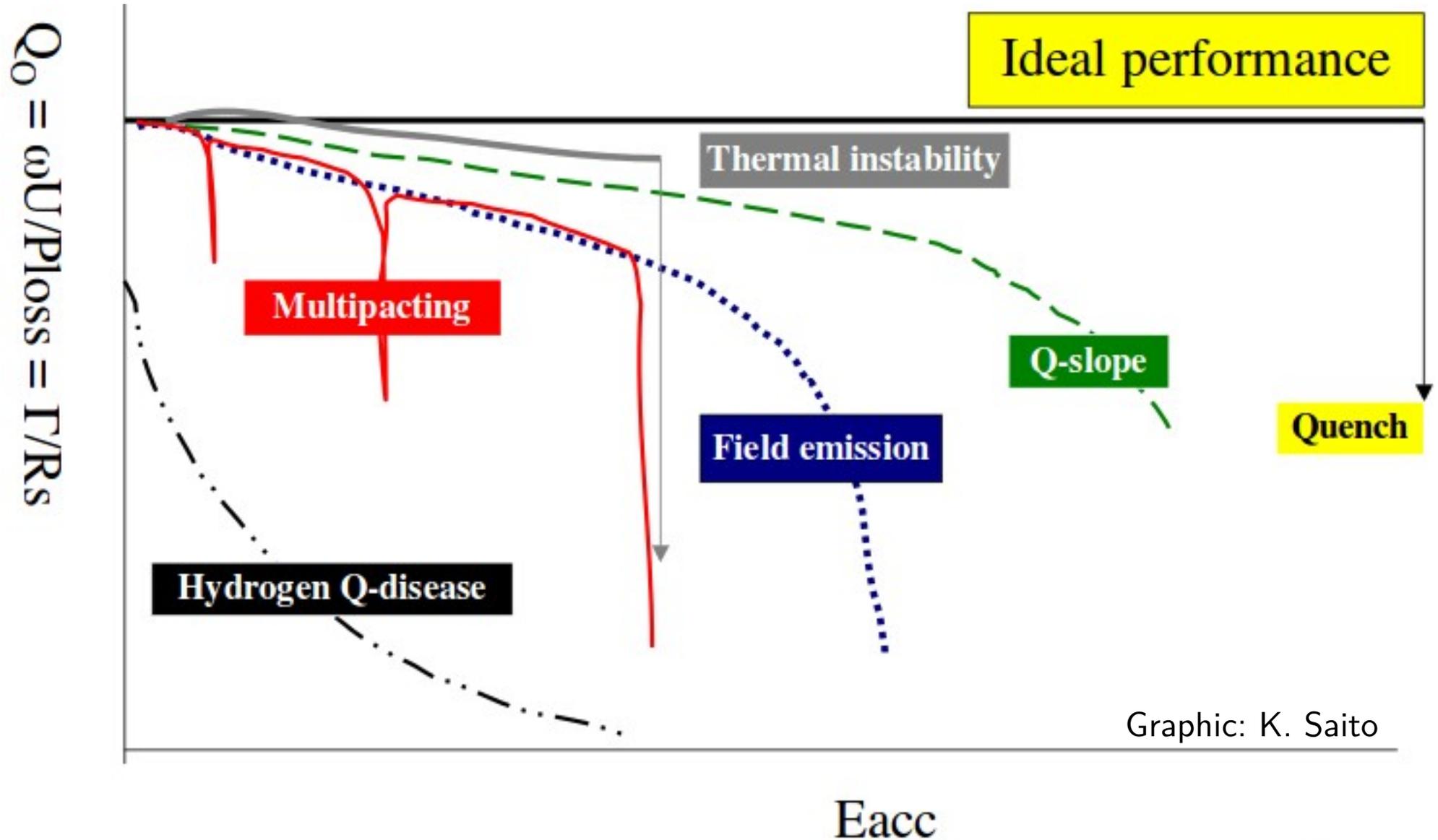


Main objective is have high impedence for the fundamental mode while high transmission for HOMs

# Practical Aspects II

(Surface Treatment & Cold Measurements)

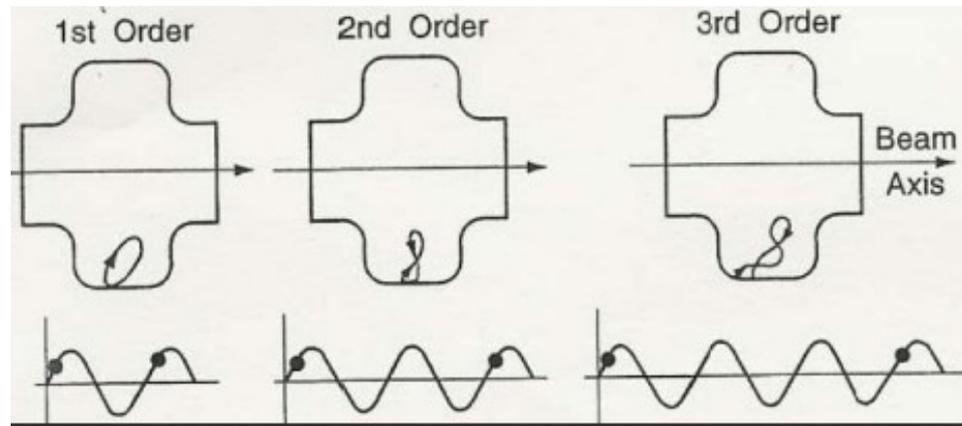
# SC Cavity Performance Limitations



Graphic: K. Saito

# Multipacting

Resonant electron multiplication of electrons from the cavity surface impacting back in integer RF cycles with a surface emission coefficient (SEY)  $> 1$



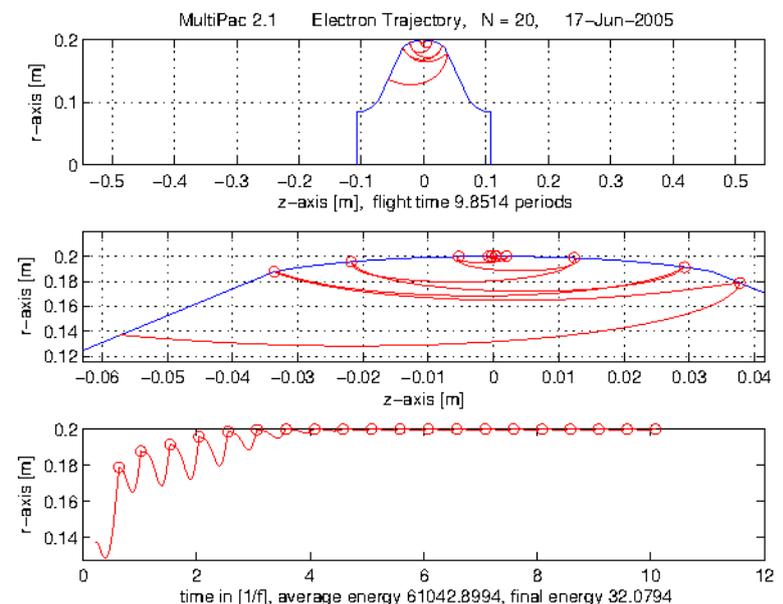
## Consequence

An electron avalanche of electrons absorbing all RF power, leading a thermal breakdown

## Mitigation

It is field, phase and SEY dependent.

RF conditioning and/or geometrical shaping to suppress the resonant behavior (ex: elliptical shape)



# Field Emission

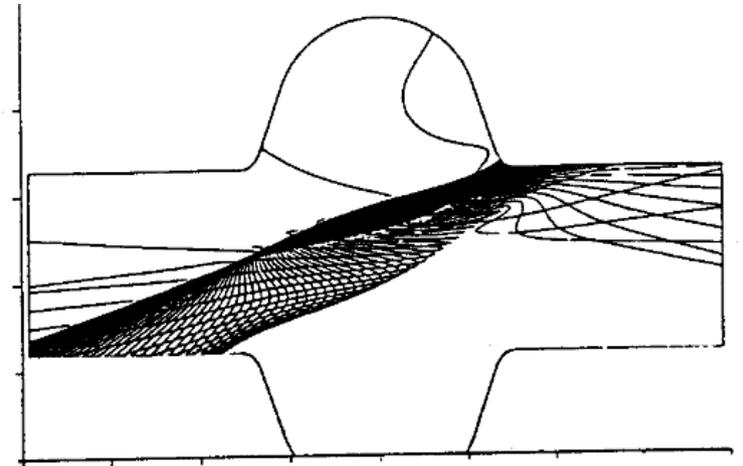
An electron emitting site on the cavity surface (due to impurities, surface defects etc..) with a sufficient local field enhancement ( $10^2 - 10^3$ ).

They get accelerated/bent by the strong RF field and impact elsewhere on the surface with the typical signature of strong x-rays leading to vacuum and/or thermal breakdown (“hot zones”)

Explained by modified F-N theory  
( $\beta$ -enhancement factor)

$$j = \frac{A \cdot \beta^2 \cdot E^2}{\Phi} e^{-B \frac{\Phi^{3/2}}{\beta E}}$$

Mitigation by surface smoothness, cleanliness  
(HPR + Cleanroom) and RF conditioning



# Surface Treatment(s)

Cavity surfaces are typically formed by mechanical means which leave a damaged cortical layer (impurities, inclusions, hydroxides, oxides...)

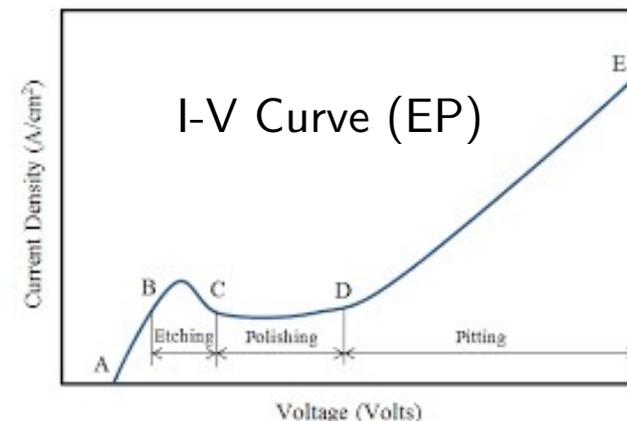
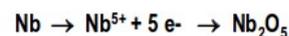
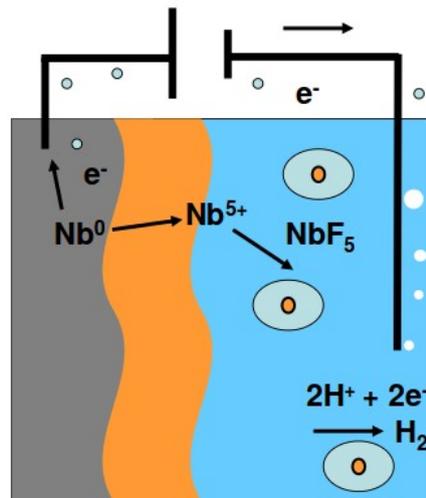
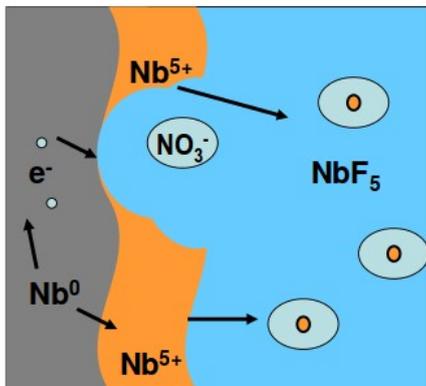
Standard practice (after degreasing) is to remove 100-200  $\mu\text{m}$  by oxidation & reduction:

- Buffer Chemical Polishing ( $\text{HNO}_3$ ,  $\text{HF}$ ,  $\text{H}_3\text{PO}_4$ )
- Electro-Polishing ( $\text{HF}$ ,  $\text{H}_2\text{SO}_4$ ) – roughness  $\sim$ micron level
- Mechanical Barrel Polishing



G. Ciovati, USPAS Lecture, 2015

C. Antoine Arxiv:1501.03343



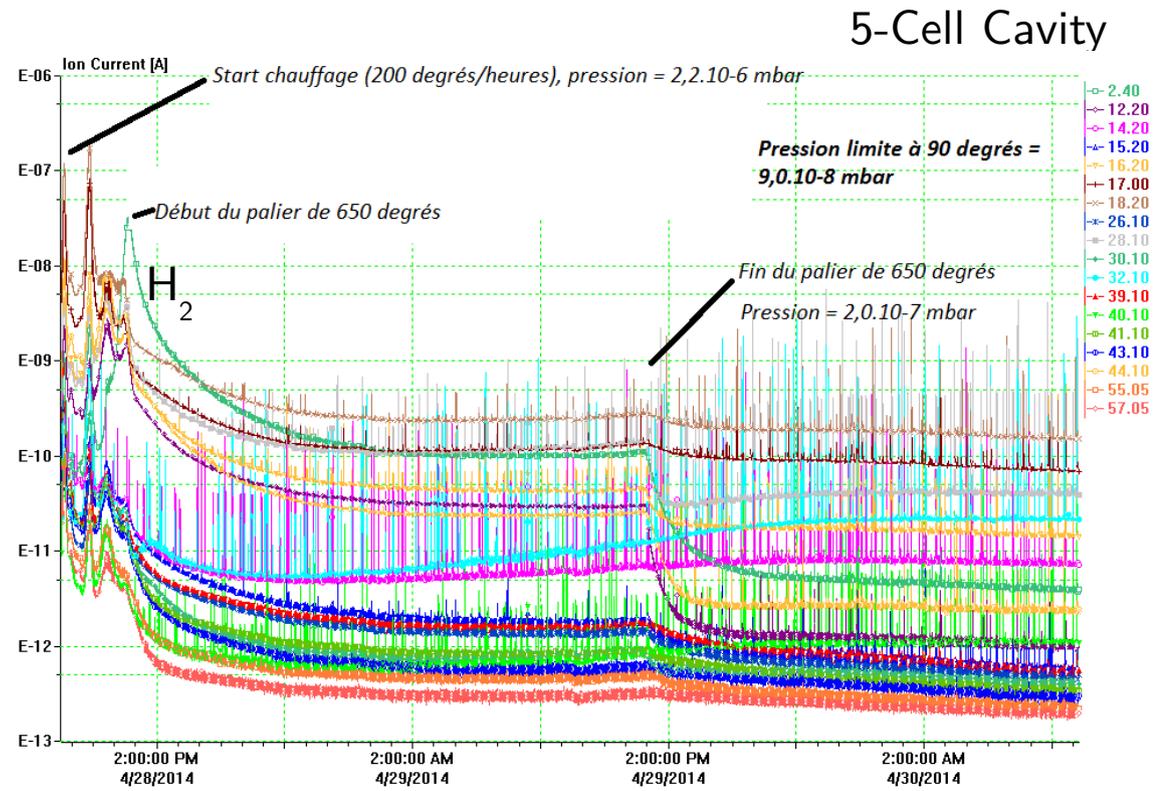
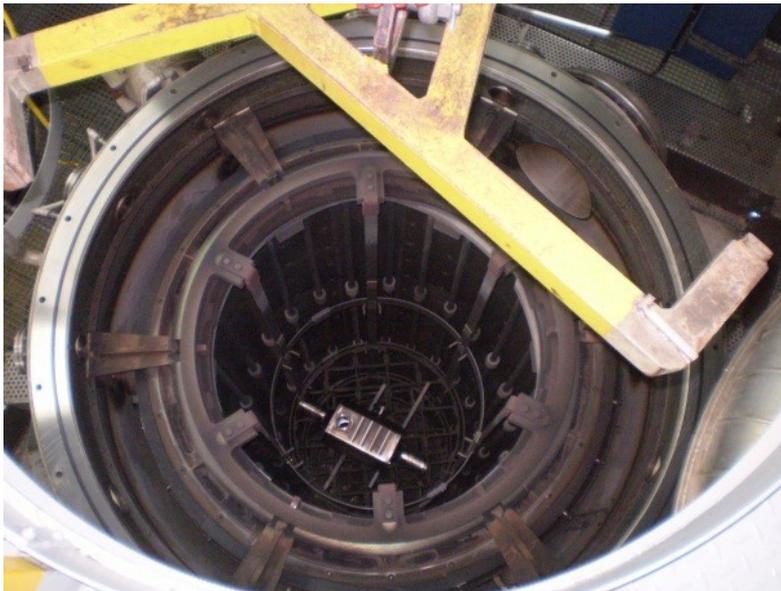
Int. J. Electrochem. Sci.,7 (2012)

# Heat Treatment

Substantial Hydrogen concentration is shown to yield Q-disease due to Hydrogen dissolution into the Niobium bulk during chemistry

Heat treatment (UHV) at 600-800 °C (10-24 hrs) – Removal of the H<sub>2</sub>  
Niobium is a strong getter above 250 °C → Requires a light chemistry after +  
(high pressure) water rinsing

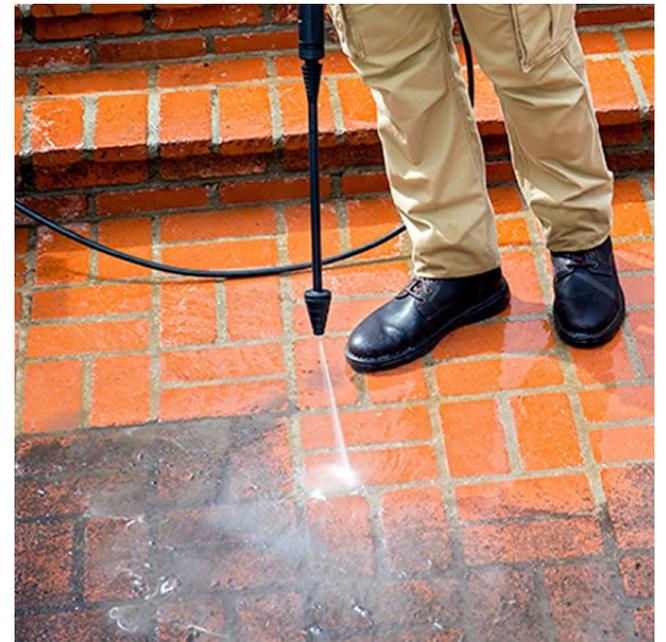
CERN UHV Furnace



# HPR & Clean Room

High Pressure Rinsing ( $\rho=18\text{ M}\Omega\text{cm}$ ) and clean room assembly (ISO4) have shown great success in suppressing field emission & improve cavity performance.

Additional low temp baking ( $120\text{ }^{\circ}\text{C}$ ) shown to improve high field Q-drop



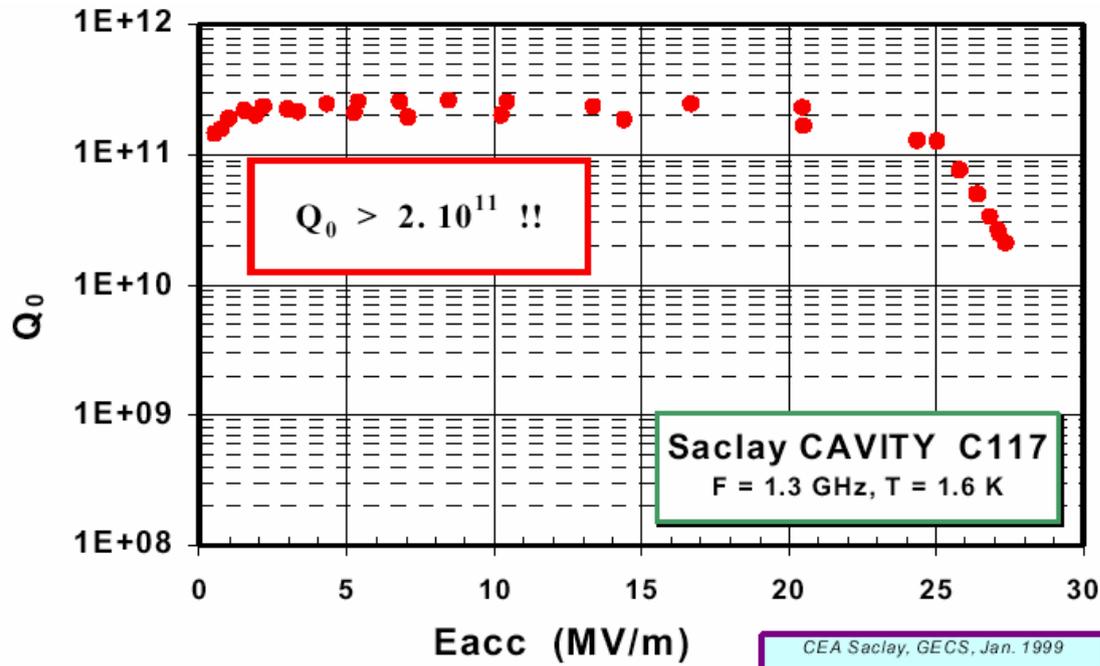
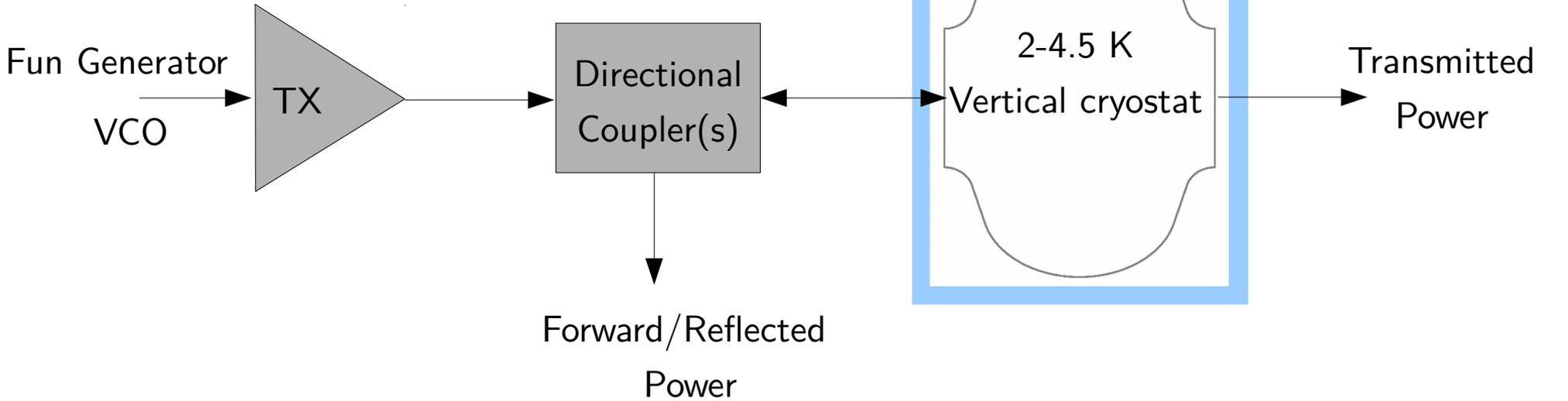
Clean Room Assembly (ISO4)



Movable nozzle

Industrial cabinet for HPR

# SC-Cavity Measurements

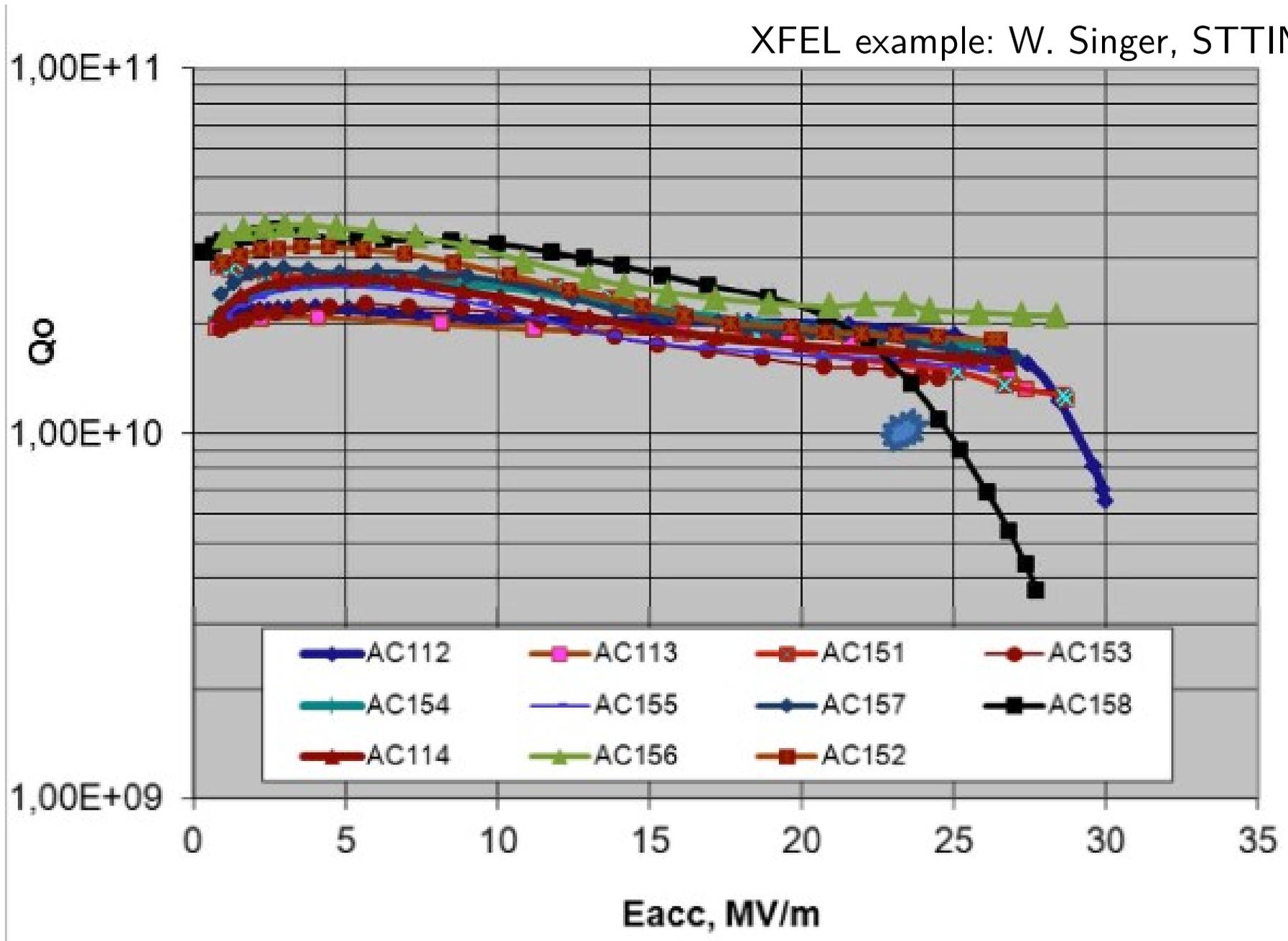


Estimate  $Q_0$  vs field from power & field decay measurements

Example single cell ( $R_s \sim 0.5$  n $\Omega$ )  
Requires very precise setup/calibration

# High Q, High Gradient

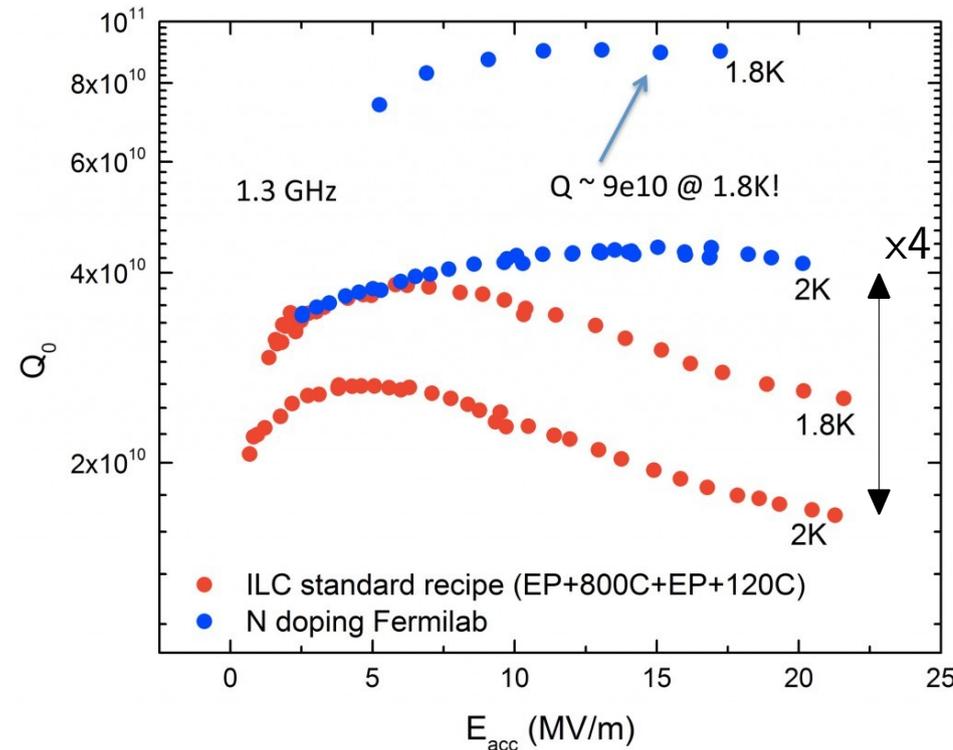
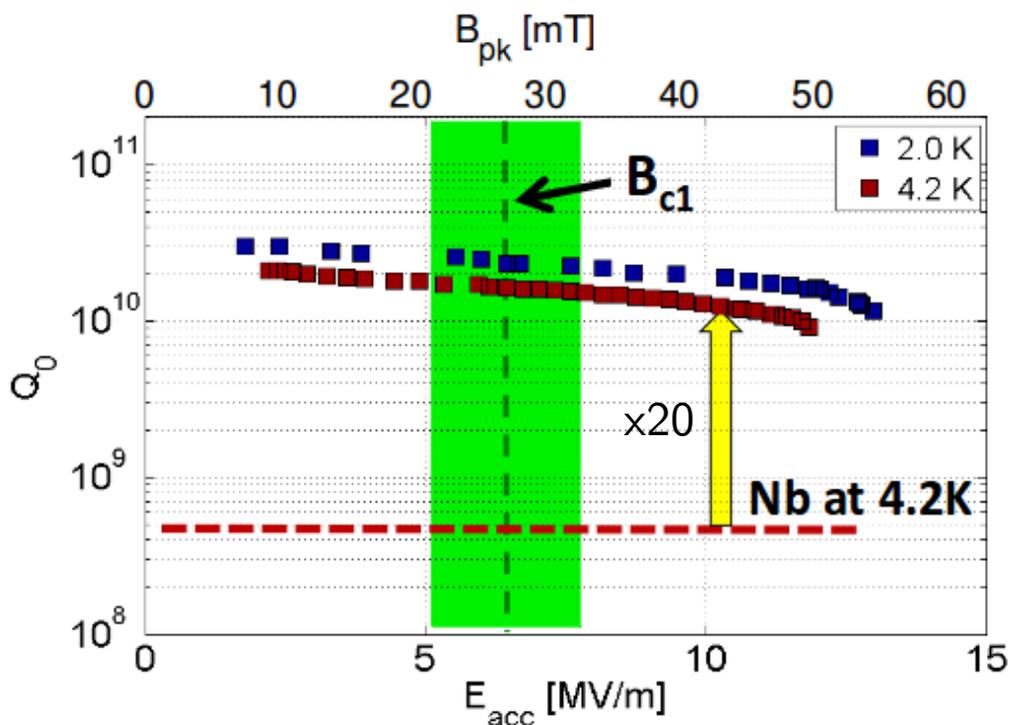
XFEL example: W. Singer, STTIN2010



# New Paths

Looking beyond state-of-the-art Niobium  
Nitrogen doping, Nb<sub>3</sub>Sn, Multi-layer..

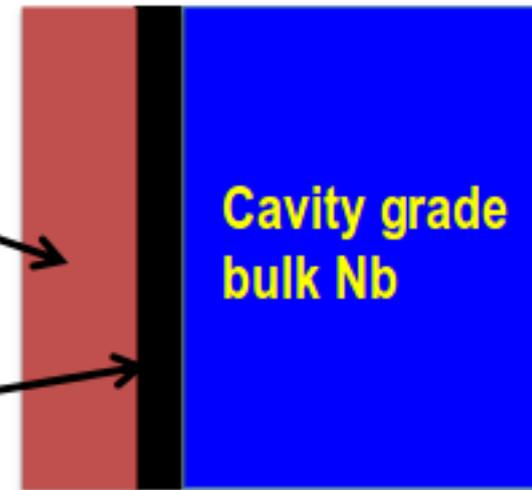
Controlled cool-down for better flux  
expulsion and good magnetic screening



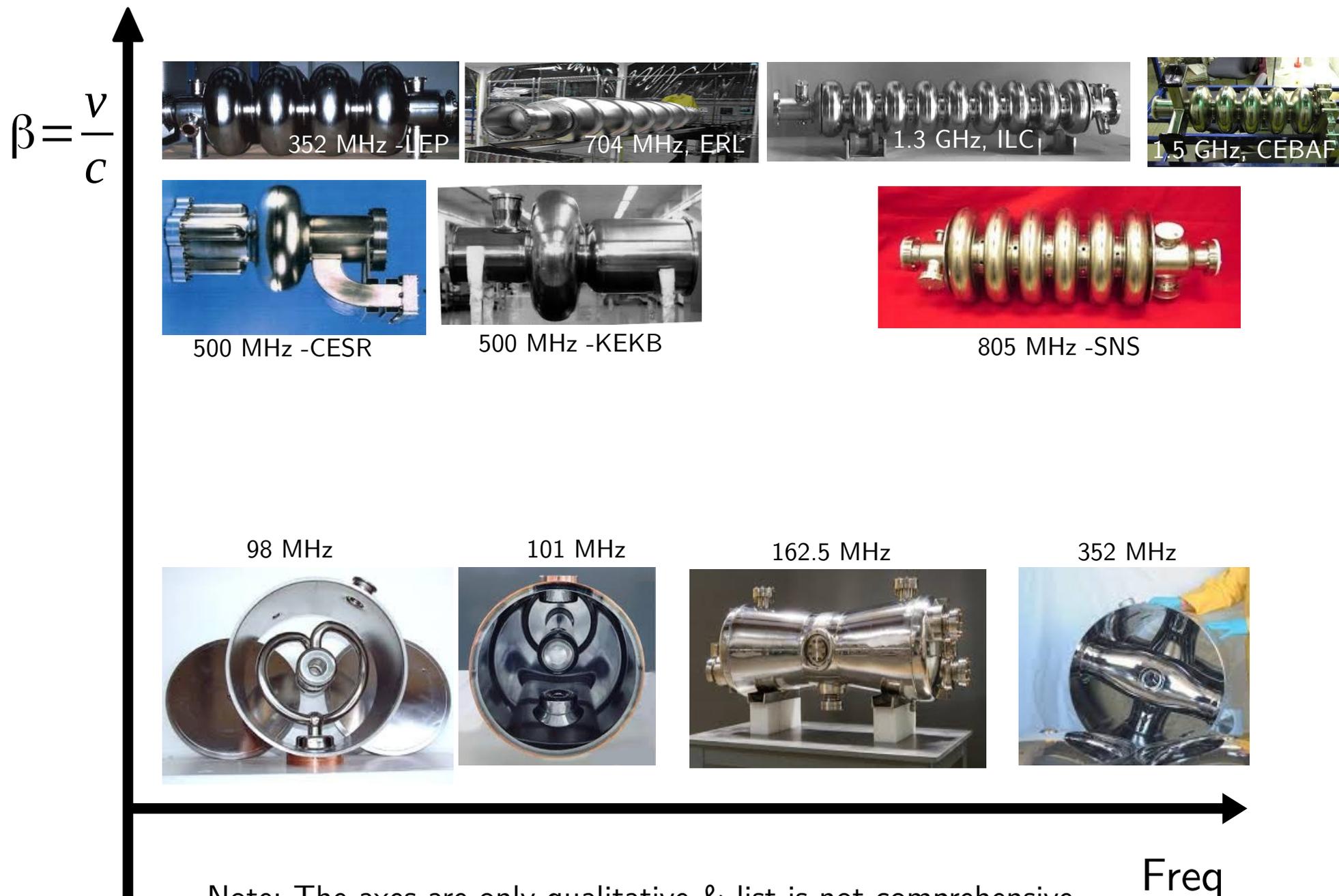
Alloyed Nb  
film of optimum  
thickness

A few nm thick  
 $Al_2O_3$  spacer

Cavity grade  
bulk Nb



# Finally, Some SC-Cavities in Real Life



Note: The axes are only qualitative & list is not comprehensive