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Synchrotron Light Machines

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In this lecture, we shall:

- look briefly at the main parameters used to characterise the performance of a synchrotron light source;

- discuss how the synchrotron radiation properties are related to the machine parameters of a synchrotron storage ring;

- develop an outline design for a synchrotron storage ring in a third generation light source, to meet certain specifications for the synchrotron light output.

Free electron lasers (used in fourth-generation light sources) will be discussed in a further lecture.

In this lecture, we shall focus on aspects of the storage ring design closely associated with the beam dynamics.

We shall not consider in detail technical aspects of the various subsystems, which include:

- source and booster linac or synchrotron,
- injection system,
- vacuum system,
- personnel safety system,
- control system.

These are all important and interesting, but we have to draw the line somewhere!

Key Performance Parameters

Photon beam property	Related machine properties
spectral range	energy, magnetic field strength
photon flux	energy, current, fields
brightness	energy, current, fields, emittance
polarisation	magnetic field shape
time structure	bunch spacing
stability	correction/feedback, beam lifetime
beamline capacity	number of dipoles/insertion devices

Some of the key photon beam properties are specified by a range rather than a single value: the machine design must be flexible enough to cover the desired range.

The photon beam properties are related to the machine properties in a complicated way, and some compromises are required (e.g. higher current gives higher brightness, but can limit stability).

Compare the key parameters of some "typical" third generation light sources:

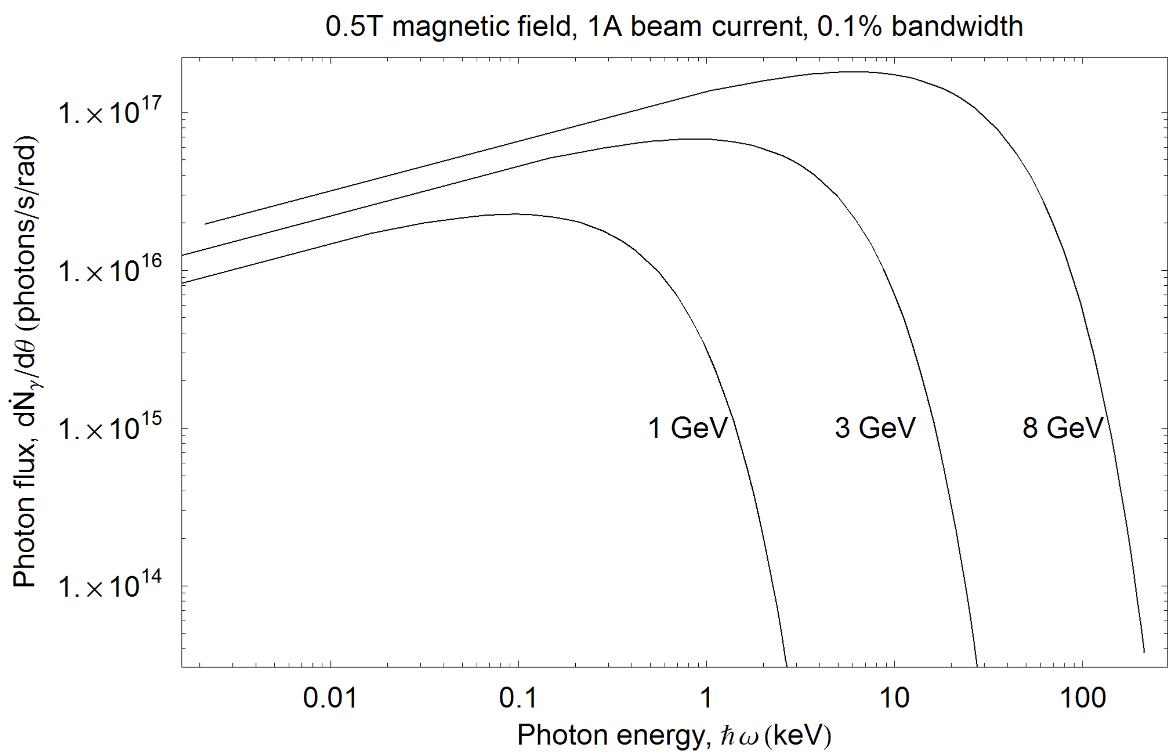
	Elettra	ALBA	DLS	ESRF	APS	SPring-8
Energy	2 GeV	3 GeV	3 GeV	6 GeV	7 GeV	8 GeV
Circumference	259 m	269 m	562 m	845 m	1104 m	1436 m
Lattice type	DBA	DBA	DBA	DBA	DBA	DBA
Current	300 mA	400 mA	300 mA	200 mA	100 mA	100 mA
Hor. emittance	7.4 nm	4.4 nm	2.7 nm	4 nm	3.1 nm	3.4 nm

Note that many machines have flexibility in their operating parameters, so the values in the table should be taken as being representative of the machine, rather than definitive.






There is a large user community requiring hard x-rays, with photon energy of 12 keV or greater.



The flux of hard x-ray photons from a magnet with field strength of order 0.5 T is substantially larger for beam energies above about 3 GeV, compared with lower beam energies.

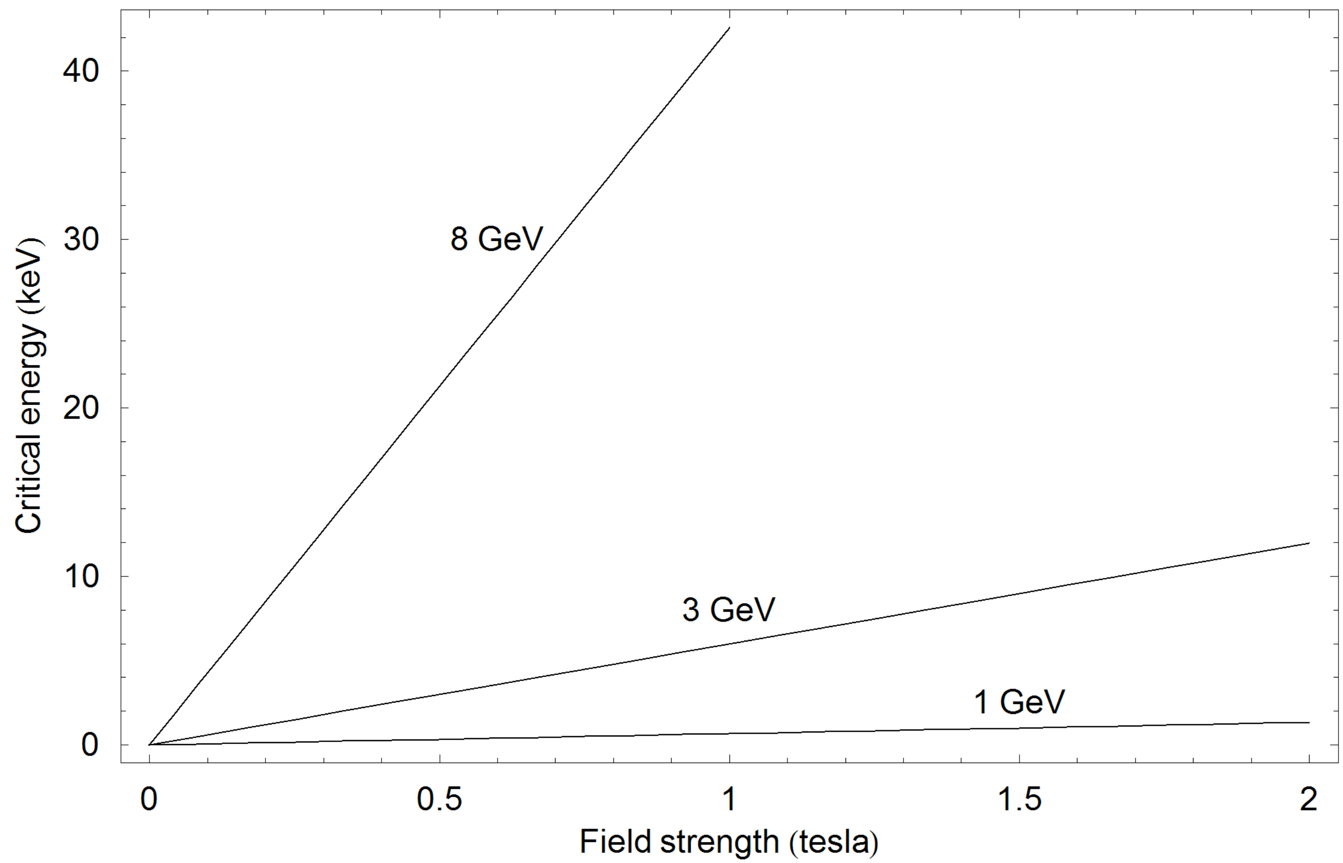
The formula for the photon flux is:

$$\frac{d\dot{N}_\gamma}{d\theta} = \frac{\sqrt{3}}{2\pi}\frac{\alpha}{em_e c^2} E I \frac{\Delta\omega}{\omega}\frac{\omega}{\omega_c}\int_{\omega/\omega_c}^{\infty} K_{5/3}(x)\,dx, \qquad (1)$$

where:

- $d\dot{N}_\gamma/d\theta$ is the number of photons in frequency range $\Delta\omega$ produced per unit time per unit bend angle θ,
- E is the electron beam energy,
- I is the electron beam current,
- $\omega_c = \frac{3}{2}c\gamma^3/\rho$ is the photon critical frequency (where γ is the relativistic factor for the electrons, and ρ is the bending radius),
- $\alpha \approx 1/137$ is the fine structure constant.

With electron beam energies below about 3 GeV, very strong magnetic fields are needed to produce photons with energy of 12 keV and above.



Advantages of higher electron beam energy:

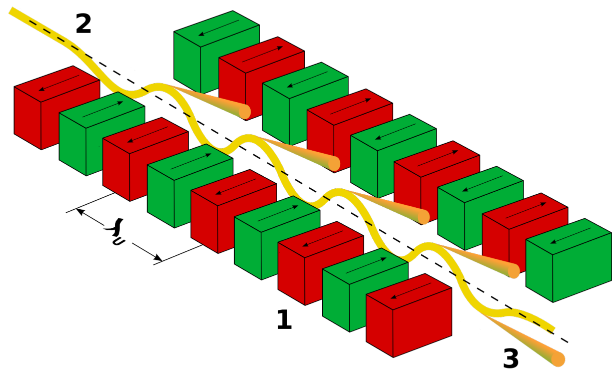
- Easier to produce high-energy photons (hard x-rays).
- Better beam lifetime.
- Easier to achieve higher current without encountering beam instabilities.

Disadvantages of higher electron beam energy:

- Higher energy beams have larger emittances (reduced brightness) for a given lattice.
- Stronger (more expensive) magnets are needed to steer and focus the beam.
- Larger rf system needed to replace synchrotron radiation energy losses.

A beam energy of 3 GeV seems a good compromise.

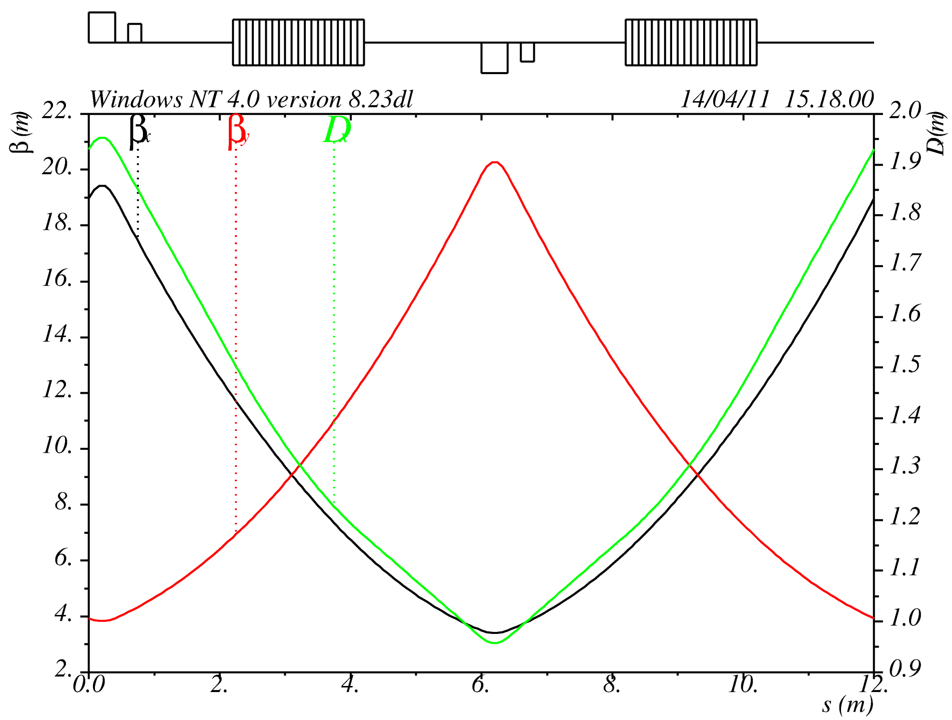
We can generate hard x-rays from appropriate insertion devices (undulators and wigglers).



The main dipoles will probably generate softer x-rays, but this radiation can still be useful.

Now let us consider the lattice design. The simplest option would be a straightforward FODO lattice...

$$\text{brightness} \propto \frac{\text{photon flux}}{\text{electron beam emittance}} \tag{2}$$



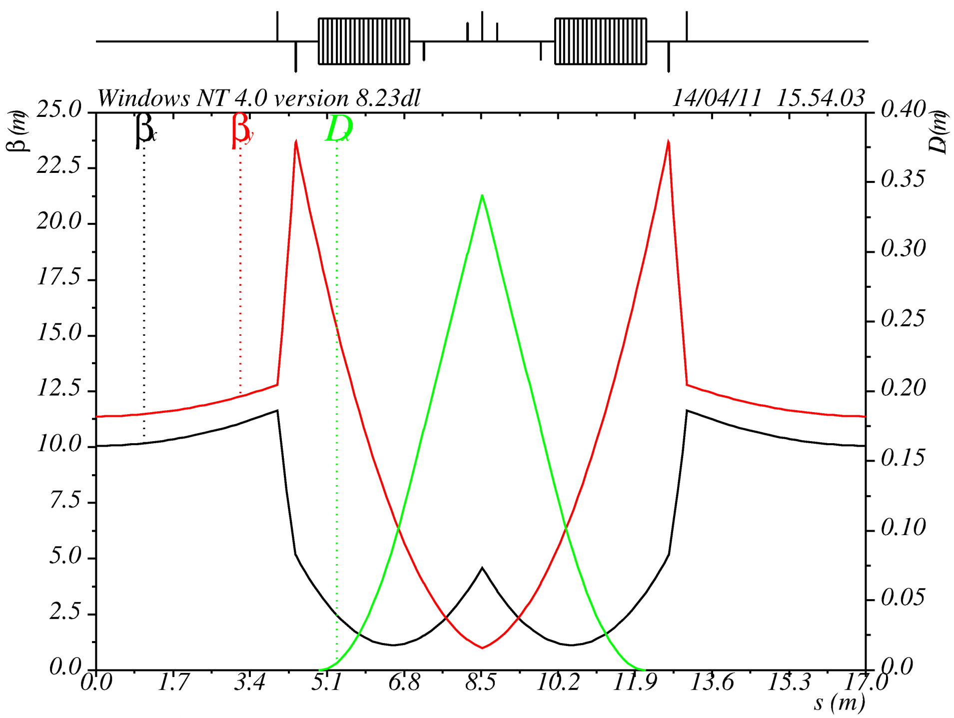
In a FODO lattice, the minimum emittance is given by:

$$\varepsilon_0 \approx 1.2 C_q \gamma^2 \theta^3, \tag{3}$$

where γ is the relativistic factor, θ the dipole bending angle, and $C_q \approx 3.832 \times 10^{-13}\,\text{m}$.

To achieve a natural emittance of 6 nm in a 3 GeV FODO lattice, we need a minimum of 86 dipoles (43 FODO cells).

A further drawback of the FODO lattice is that there are no good locations for insertion devices. If these are placed where there is large dispersion, they will blow up the emittance by quantum excitation.

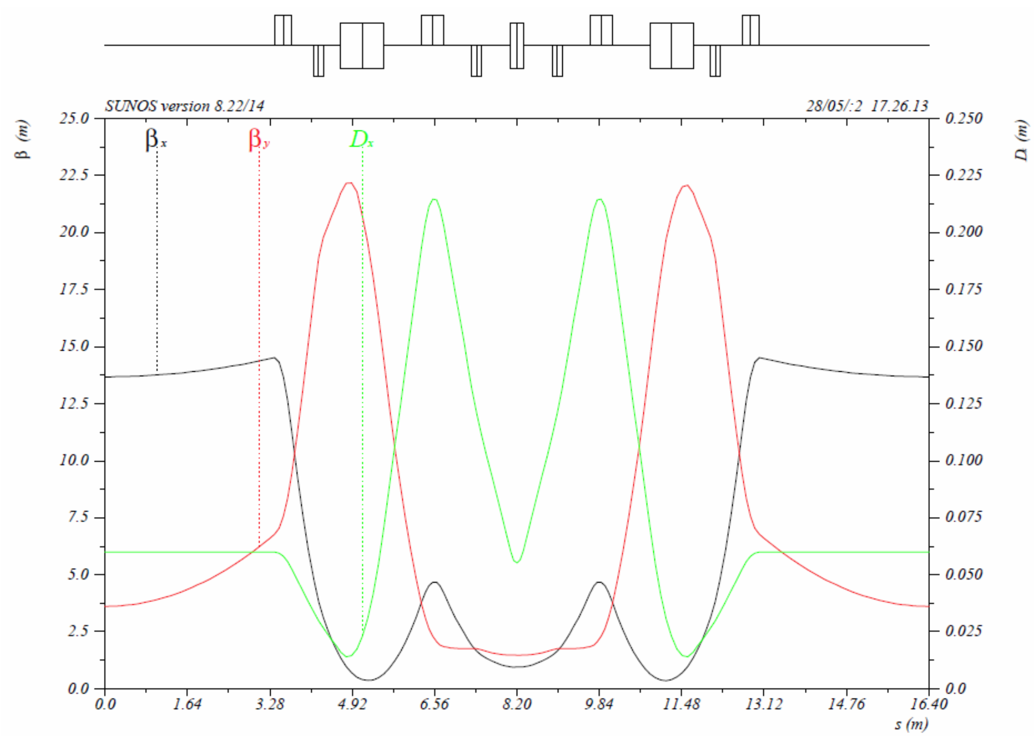


In a Double Bend Achromat (DBA) lattice, the minimum emittance is given by:

$$\varepsilon_0 \approx \frac{1}{4\sqrt{15}} C_q \gamma^2 \theta^3. \quad (4)$$

To achieve a natural emittance of 6 nm in a 3 GeV DBA lattice, we need a minimum of 32 dipoles (16 DBA cells).

By detuning to allow some dispersion to "leak" outside the achromat, the dispersion can be further reduced (despite enhanced quantum excitation from the insertion devices).



Superbend detuned Triple Bend Achromat (TBA) in the Advanced Light Source. *C. Steier et al, Proceedings of EPAC'02, Paris, France (2002)*.

Based on the desire for high brightness, hard x-ray output, we have decided on:

- 3 GeV beam energy,
  - 3 keV photon critical energy in a 0.5 T magnetic field,
  - 12 keV photon critical energy in a 2 T magnetic field;
- DBA lattice with (about) 16 cells,
  - approximately 270 m circumference,
  - around 6 nm natural emittance.

The next important step is to decide the dipole field.

The Dipole Field

The dipole field is important, because it is closely associated with:

- the properties of the synchrotron radiation from the dipole,
- the natural energy spread of the beam,
- the natural bunch length,
- the rf parameters,
- the beam lifetime.

Let us first make a rough estimate of a "reasonable" dipole field, and see what it means for the beam properties.

With 16 DBA cells, there are 32 dipoles, so the bending angle of each dipole is:

$$\theta = \frac{2\pi}{32}. \tag{5}$$

At 3 GeV beam energy, the beam rigidity is:

$$B\rho = \frac{p}{e} \approx \frac{E}{ec} \approx 10\,\mathrm{Tm}. \tag{6}$$

So the dipole field is related to the length L by:

$$B = \frac{B\rho}{\rho} = B\rho\frac{\theta}{L} \approx \frac{1.96\,\mathrm{Tm}}{L}. \tag{7}$$

Very strong fields require a lot of current (and iron) in the dipole. Very long dipoles are difficult to handle. A reasonable compromise is $B \approx 1.6\,\mathrm{T}$, and $L \approx 1.25\,\mathrm{m}$.

With 3 GeV beam energy, and 1.6 T field, the photon critical energy is:

$$\hbar\omega_c = \frac{3}{2}\hbar c\frac{\gamma^3}{\rho} \approx 9.6\,\text{keV}. \tag{8}$$

This is approaching the energy for hard x-rays.

With the dipole field, we can calculate the values of the first three synchrotron radiation integrals.

The first synchrotron radiation integral is related to the momentum compaction factor, α_p:

$$\alpha_p = \frac{\Delta C}{\Delta \delta} = \frac{1}{C_0} I_1, \qquad \text{where} \qquad I_1 = \oint \frac{\eta_x}{\rho}\, ds. \tag{9}$$

The second synchrotron radiation integral is related to the energy loss per turn, U_0:

$$U_0 = \frac{C_\gamma}{2\pi} E^4 I_2, \qquad \text{where} \qquad I_2 = \oint \frac{1}{\rho^2}\, ds. \tag{10}$$

The third synchrotron radiation integral is related to the natural energy spread, σ_δ:

$$\sigma_\delta^2 = C_q \gamma^2 \frac{I_3}{j_z I_2}, \qquad \text{where} \qquad I_3 = \oint \frac{1}{|\rho|^3}\, ds. \tag{11}$$

In the above, C is the circumference of the closed orbit, $\delta = \Delta E/E$ is the energy deviation, $C_\gamma \approx 9.846 \times 10^{-5}\,\mathrm{m/GeV^3}$.

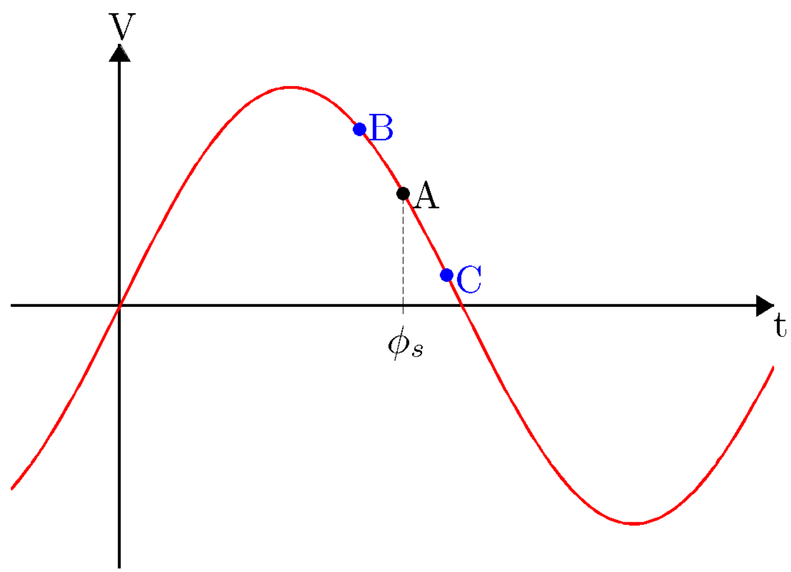
Together with the rf system (voltage and frequency), the dipole field determines the longitudinal dynamics of the beam, including critical properties such as bunch length, beam lifetime, and some instability thresholds (which can limit the total current).

Let us consider first the rf system. We will aim to work out appropriate values for the voltage and frequency.

The rf cavities must replace the energy lost by synchrotron radiation. Using equation (10), for a 3 GeV beam with 1.6 T dipole field, we find:

$$U_0 \approx 1.28\,\mathrm{MeV}. \tag{12}$$

The rf cavity voltage must be at least 1.28 MV. However, some "overvoltage factor" is needed to ensure stability of longitudinal oscillations...



ϕ_s is the synchronous phase: this is the phase at which the rf cavities exactly replace the energy lost by synchrotron radiation:

$$\phi_s = \pi - \sin^{-1}\left(\frac{U_0}{eV_{\mathrm{rf}}}\right). \tag{13}$$

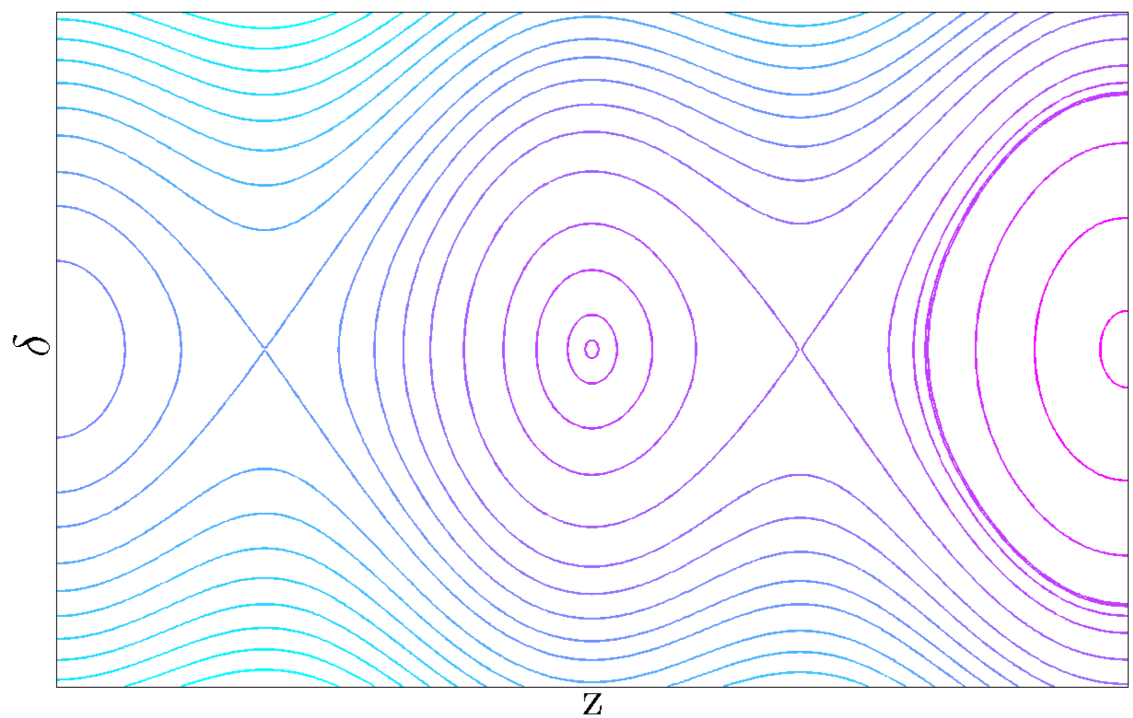
The *maximum* energy deviation for which the synchrotron oscillations remain stable is the *rf acceptance*:

$$\delta_{\mathrm{max,rf}} \approx \frac{2\nu_s}{h\alpha_p}\sqrt{1 + \left(\phi_s - \frac{\pi}{2}\right)\tan(\phi_s)}, \tag{14}$$

where ν_s is the synchrotron tune, h is the harmonic number, and α_p is the momentum compaction factor.

The longitudinal dynamics can be represented by contours in longitudinal phase space (a plot of energy deviation versus longitudinal position within a bunch).

The exact shape of the phase space is determined by the RF frequency and voltage, and the momentum compaction factor.



To achieve a good beam lifetime, we need a large rf acceptance. However, there is no point making it too large, because at some point the energy acceptance becomes limited by nonlinear effects in the lattice.

Typically, the rf acceptance should be at least 4%.

The synchrotron tune is given by:

$$\nu_s^2 = -\frac{eV_{\mathsf{rf}}}{E}\cos(\phi_s)\frac{\alpha_p h}{2\pi}. \tag{15}$$

The harmonic number h is the circumference of the ring divided by the rf wavelength:

$$h = \frac{C f_{\mathsf{rf}}}{c}, \tag{16}$$

where f_{rf} is the rf frequency.

If we can find a value for α_p, then from equations (13), (14), (15) and (16) we can work out values for the rf voltage and frequency to give a reasonable energy acceptance.

To find the momentum compaction factor, we need to integrate the dispersion through a dipole. This can be done as follows.

The dispersion satisfies the equation:

$$\eta_x'' + \left(\frac{1}{\rho_2} + k_1\right)\eta_x = \frac{1}{\rho}. \tag{17}$$

In a DBA lattice, the dispersion η_x and its gradient η_x' are zero at one end of the dipole. Assuming that there is no quadrupole gradient (i.e. $k_1 = 0$), integrating the equation of motion then gives:

$$\eta_x = \rho\left(1 - \cos\left(\frac{s}{\rho}\right)\right). \tag{18}$$

We then find for the integral in one dipole:

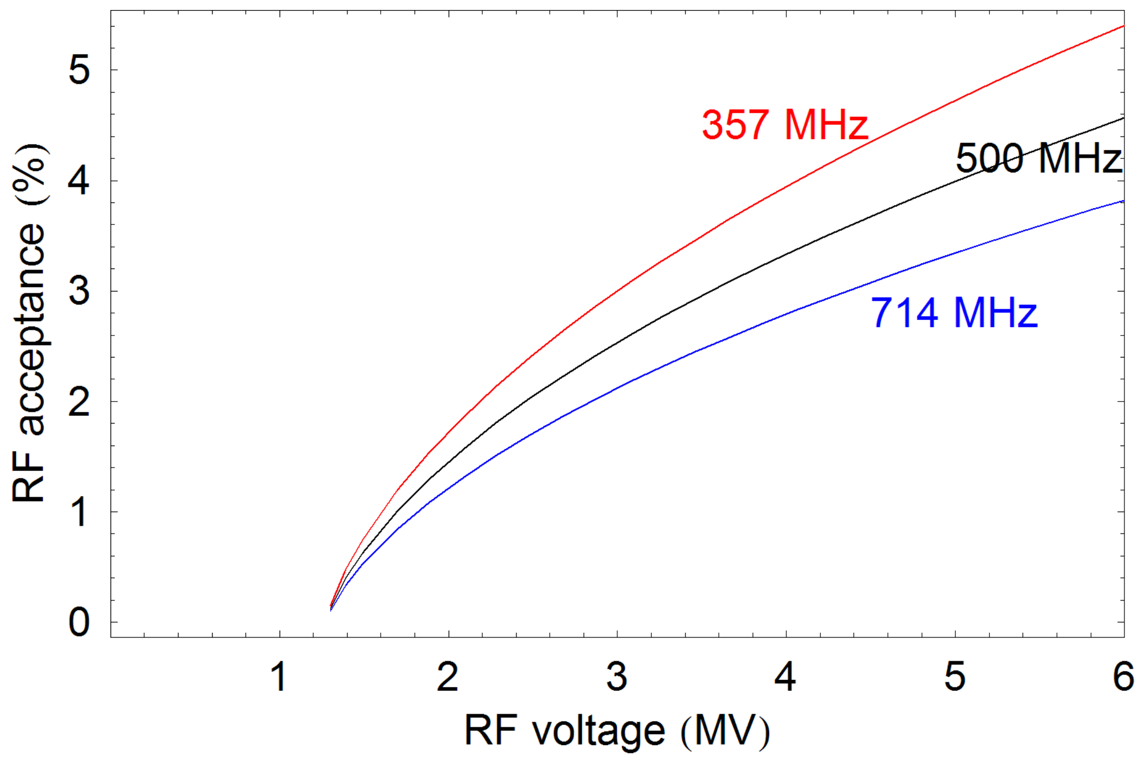
$$\int_0^L \frac{\eta_x}{\rho} ds = L - \rho \sin(\theta). \tag{19}$$

Using the parameters for our dipoles, and assuming a circumference $C = 270\,\mathrm{m}$, we then find:

$$\alpha_p = \frac{N}{C} \left(L - \rho \sin(\theta) \right) \approx 9.3 \times 10^{-4}, \tag{20}$$

where $N = 32$ is the number of dipoles in the lattice.

Now that we have the momentum compaction factor for our lattice, we can plot the energy acceptance as a function of rf voltage, for different rf frequencies.



For a given rf voltage, reducing the rf frequency increases the rf acceptance.

The rf voltage and frequency also affect the equilibrium bunch length. A long bunch is desirable, since longer bunches have lower particle density for a given bunch charge, and lower peak current. This improves lifetime and stability.

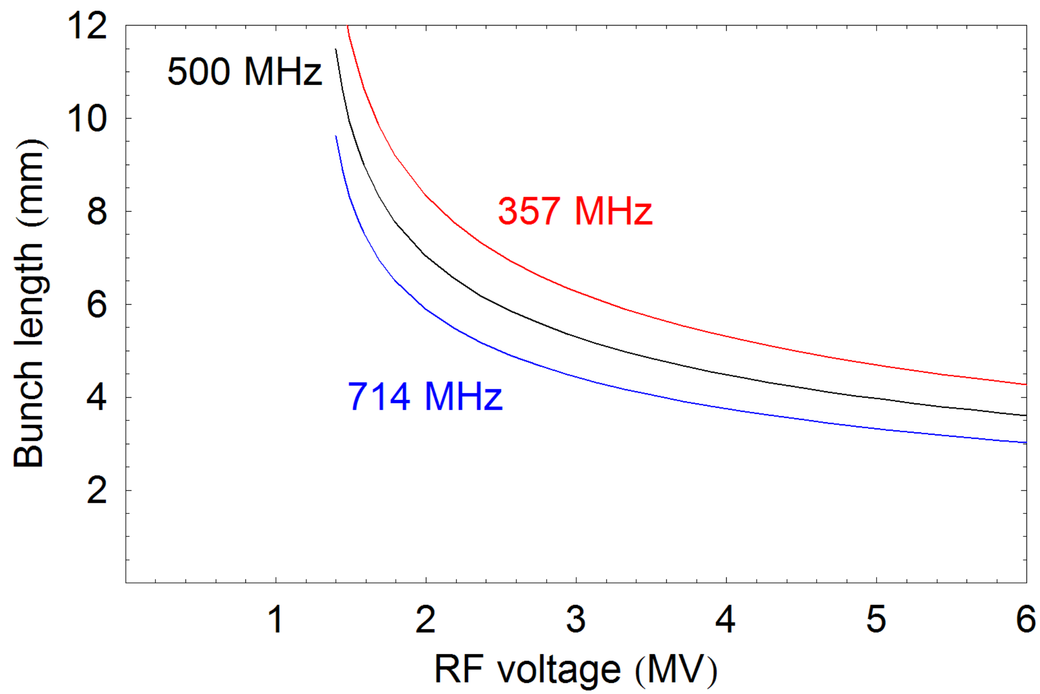
The equilibrium bunch length is given by:

$$\sigma_z = \frac{\alpha_p \sigma_\delta C}{2\pi\nu_s}, \tag{21}$$

where the equilibrium energy spread is given by equation (11).

Again, we can plot the equilibrium bunch length as a function of rf voltage, for different rf frequencies...

Equilibrium Bunch length



If the frequency is too low, the waveguide and cavities take up a lot of space. Also, it is convenient to use industry standard frequencies.

500 MHz is a common choice for the rf frequency in light sources. If we choose this frequency, then a voltage of 5 MV will give a bunch length of 4 mm and an rf acceptance of 4%.

The final parameter we need to complete our "design" is the beam current. The higher the beam current, the higher the photon flux and brightness of the synchrotron radiation.

To estimate the maximum current that we can store in our light source, there are two effects we need to consider:

- Touschek scattering: particles within a bunch collide as they make betatron and synchrotron oscillations. If the change in momentum of a particle resulting from a collision is outside the energy acceptance, the particle will be lost from the beam.

- Instabilities: particles generate electromagnetic fields that can act back on the beam. If the current is very high, the fields can become strong enough to destabilise the beam.

Touschek scattering leads to a decay of the charge of a bunch. The decay rate depends on the number of particles in the bunch, the bunch dimensions (transverse and longitudinal), and the energy acceptance.

We have already determined the bunch length and energy acceptance. We also know the horizontal emittance. If we make some assumptions for the beta functions, then we can estimate the Touschek lifetime as a function of bunch current and vertical emittance.

The formula for the Touschek lifetime is:

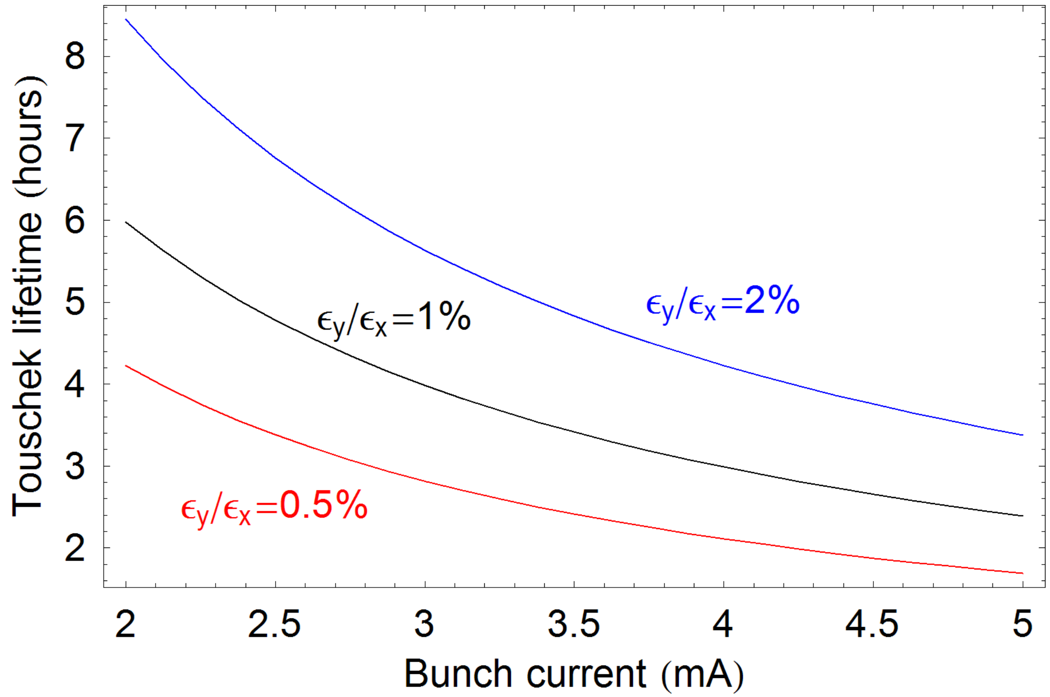
$$\frac{1}{\tau} = \frac{r_e^2 c N}{8\pi \sigma_x \sigma_y \sigma_z \gamma^2 \delta_{\mathrm{max}}^3} D(\zeta), \tag{22}$$

where:

$$\zeta = \left(\frac{\delta_{\mathrm{max}} \beta_x}{\gamma \sigma_x} \right)^2, \tag{23}$$

and:

$$\begin{aligned} D(\zeta) = \quad & \sqrt{\zeta} \left(-\frac{3}{2} e^{-\zeta} + \frac{\zeta}{2} \int_\zeta^\infty \frac{\ln(u)}{u} e^{-u} du \right. \\ & \left. + \frac{1}{2} (3\zeta - \zeta \ln(\zeta) + 2) \int_\zeta^\infty \frac{e^{-u}}{u} du \right). \end{aligned} \tag{24}$$

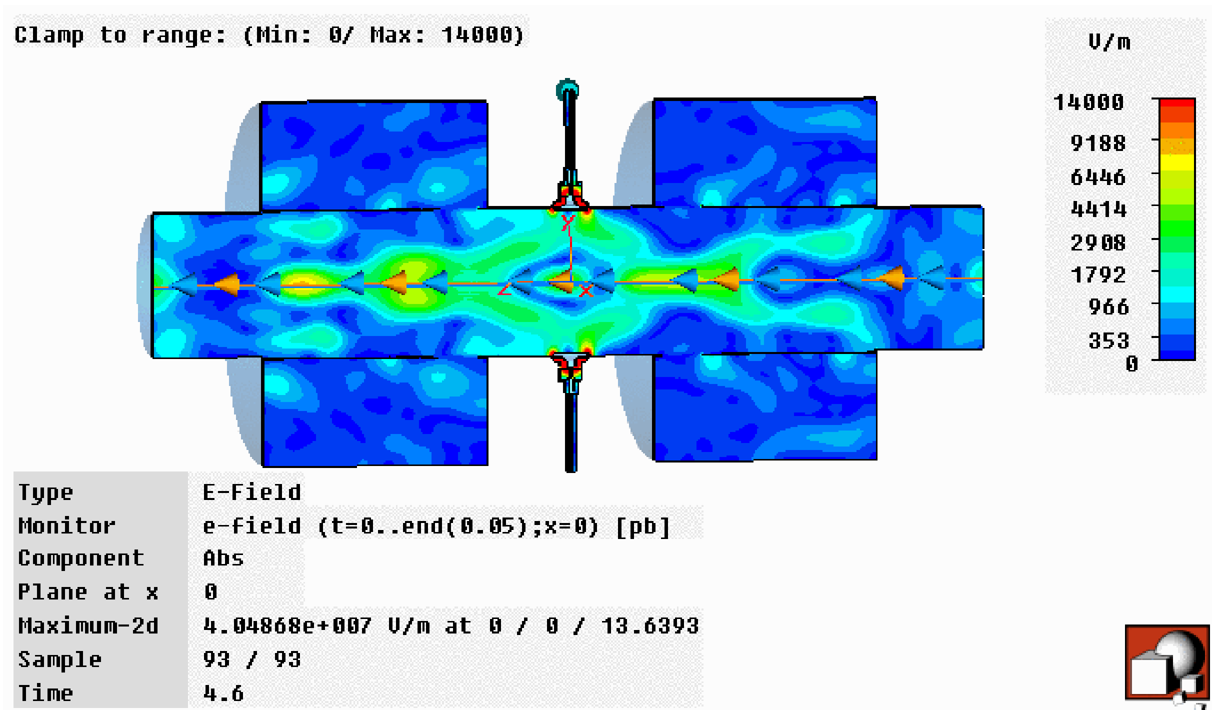


To keep a lifetime of more than 20 hours with 1% emittance ratio, the bunch current must be below 5 mA.

The harmonic number of our storage ring is about 450. This means we can store a maximum of 450 bunches.

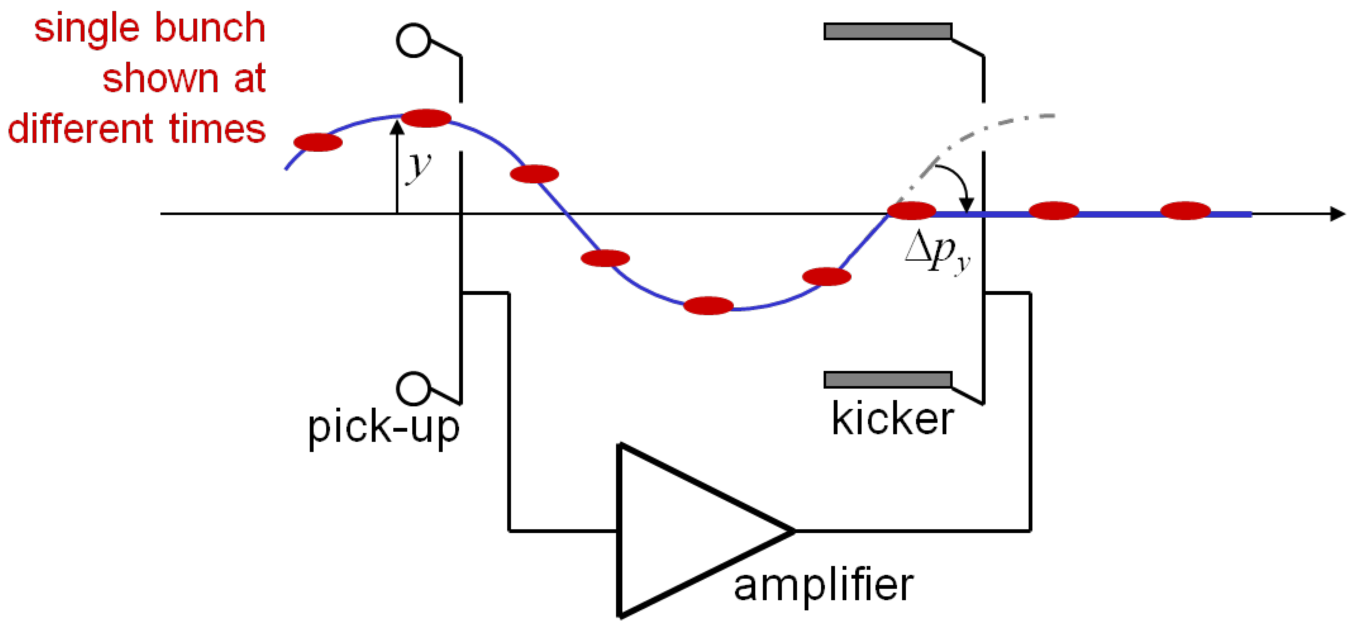
If every bunch contributed 5 mA of current, the total current would be more than 2 A: this seems rather large!

More stringent limits on the beam current come from the wake fields generated by particles in the beam in different parts of the accelerator.



Wake fields can be short-range (acting over the length of a single bunch) or long-range (acting from one bunch to the following bunches), and can "kick" particles longitudinally or transversely.

The effects of long-range wake fields can be suppressed using bunch-by-bunch feedback systems.



Calculating the effects of the wake fields is a complex task. However, the currents induced in the vacuum chamber often make an important contribution to the wake fields, and the effects of these currents can be estimated using a relatively simple formula.

The (transverse) resistive-wall growth rate (in turns^{-1}) is given by:

$$\Gamma \approx \frac{c}{\gamma\sqrt{2\pi(1-\Delta)}}\frac{I}{I_A}\oint \frac{\beta}{b^3}\sqrt{\frac{4\pi\varepsilon_0}{\omega_0\sigma}}\,ds, \tag{25}$$

where:

- Δ is the fractional part of the betatron tune,
- $\beta \approx 10\,\mathrm{m}$ is the beta function,
- the vacuum chamber has conductivity σ and circular cross-section with radius b,
- ω_0 is 2π times the revolution frequency,
- $I_A \approx 17\,\mathrm{kA}$ is the Alfven current.

To estimate the resistive-wall growth rate, we need to know the conductivity and aperture of the vacuum chamber.

Using a material with high conductivity helps to reduce the growth rates. Gold or silver would be nice: aluminium is more practical, and also has suitable mechanical properties. Vacuum chambers are also sometimes made of copper, or (less ideal for resistive-wall) stainless steel.

The conductivity of aluminium is $3.55 \times 10^7\,\Omega^{-1}\mathrm{m}^{-1}$.

The chamber aperture can be estimated from the dipole field...

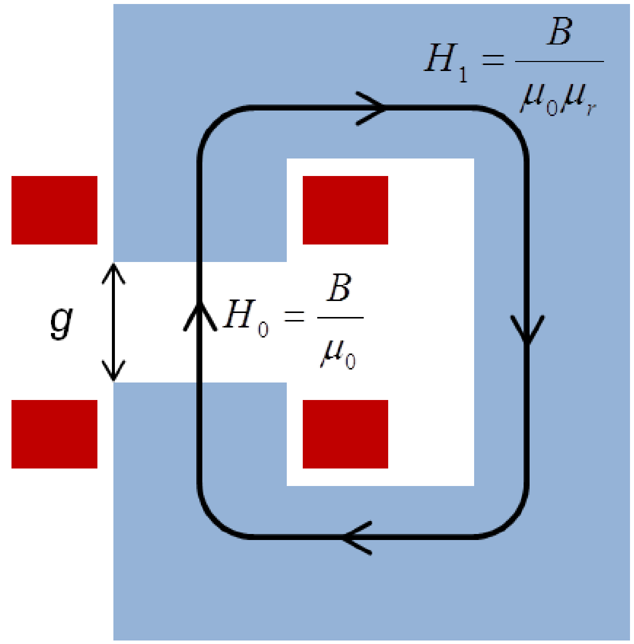
From Maxwell's equation:

$$\nabla \times \vec{H} = \vec{J}, \qquad \oint \vec{H} \cdot d\vec{\ell} = NI, \quad (26)$$

the number of ampere-turns NI on a dipole with pole gap g and field B is:

$$NI = g\frac{B}{\mu_0}. \quad (27)$$

We assume infinite permeability in the iron core.



Let us assume a limit $NI \approx 3 \times 10^4$ A.

Then, for a field of 1.6 T, the gap is about 25 mm.

Allowing some margin for the chamber wall, a reasonable estimate of the inner radius of the vacuum chamber is $b \approx 10$ mm.

For a given beam current, we can now use equation (25) to estimate the resistive wall growth rate.

If we do not use a feedback system, we will need to rely on the natural damping from synchrotron radiation to keep the beam stable. The current will be limited by the condition that the resistive-wall growth time must be longer than than the radiation damping time.

In the transverse planes, the radiation damping time is given by:

$$\frac{\tau}{T_0} = 2\frac{E}{U_0}. \tag{28}$$

For our storage ring, with 3 GeV beam energy and 1.28 MeV loss per turn, the damping time is about 4700 turns.

Using equation (25), and assuming $\Delta \approx 0.5$, the maximum beam current we can store with resistive-wall growth time longer than 4700 turns, is approximately 70 mA.

This is not very much current!

If we want to achieve the currents (and hence the brightness) of many third-generation synchrotron light sources, we will need a bunch-by-bunch feedback system to suppress coupled-bunch instabilities.

It is reasonable to assume that the feedback system will allow the machine to operate with currents three or four times above the resistive-wall threshold.

This would mean a current of between 200 and 300 mA.

A current of 300 mA would require the feedback system to provide a damping time of 1000 turns.

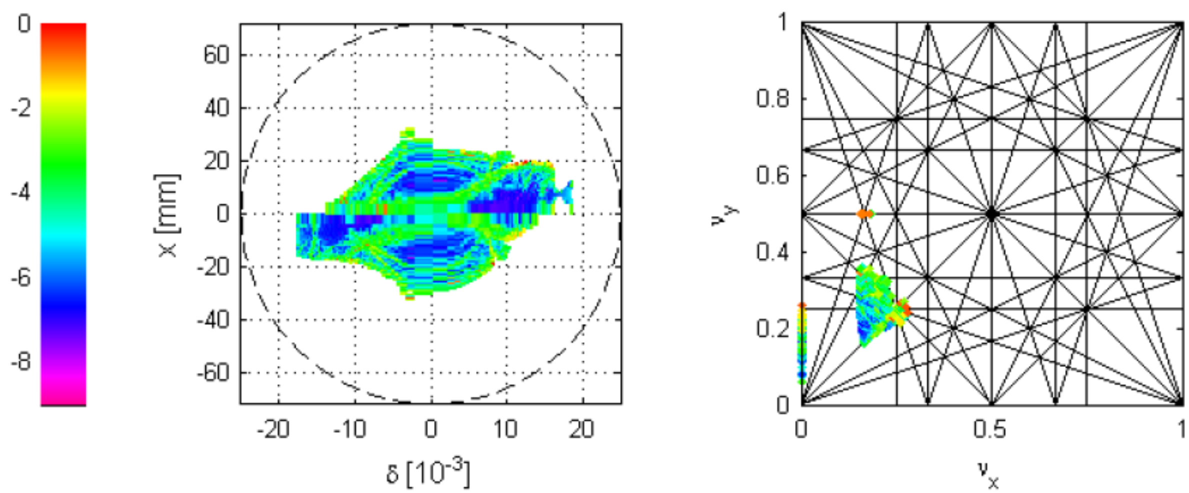
This is comfortably within the capability of modern fast feedback systems; but it should be remembered that there will be many other contributions to the long-range wake fields (e.g. higher-order modes in the rf cavities).

If we store a beam current of 300 mA in 400 bunches, the current from each bunch will be only 0.75 mA.

Using the formulae in Appendix A, the Touschek lifetime (assuming an energy acceptance of 4% and an emittance ratio of 1%) with a bunch current of 0.75 mA would be 130 hours.

However, the energy acceptance could be considerably less than the 4% limit from the rf system.

A major limitation on the energy acceptance comes from the sextupoles that will be needed to correct the chromaticity of the lattice. These introduce nonlinear effects, in particular resonances, that lead to betatron oscillations becoming unstable.



Depending on the details of the lattice design, the dynamic energy acceptance might be as small as 2%. Then, the Touschek lifetime with 0.75 mA bunch current would be only 16 hours.

Based on the specification for a high-brightness x-ray synchrotron radiation source, we have arrived at the following outline "design":

Beam energy	3 GeV
Circumference	270 m
Lattice style	16-cell DBA
Natural emittance	6 nm
Vertical/horizontal emittance ratio	1%
Dipole field	1.6 T
Dipole length	1.25 m
Dipole critical photon energy	9.6 keV
Energy loss per turn	1.28 MeV
RF frequency	500 MHz
RF voltage	5 MV
Momentum compaction factor	9.3×10^{-4}
RF acceptance	4%
Equilibrium bunch length	4 mm
Equilibrium energy spread	10^{-3}
Touschek lifetime	20 hours at 5 mA bunch current
Vacuum chamber material	aluminium
Vacuum chamber radius	10 mm
Beam current	300 mA

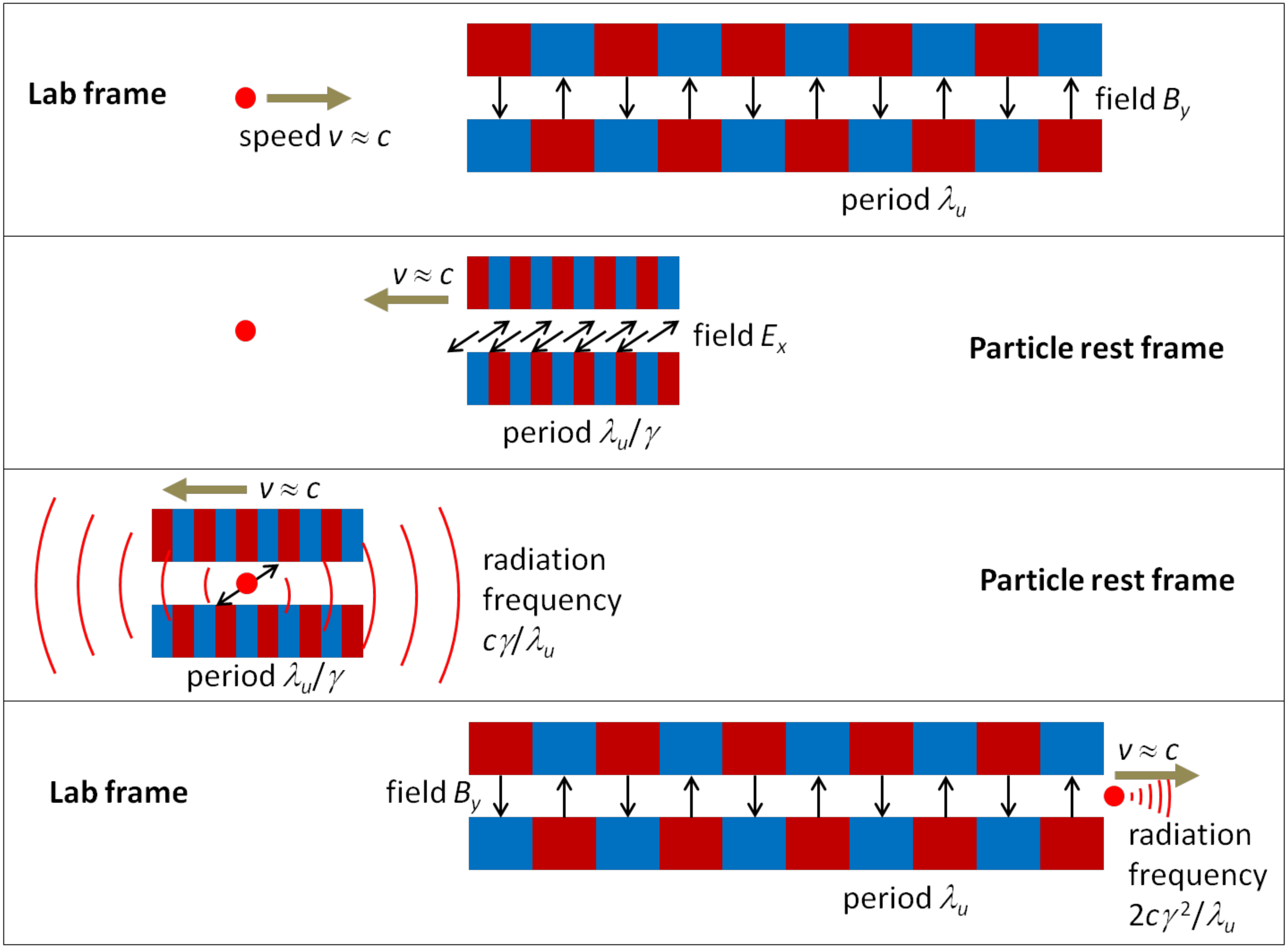
A benefit of a DBA lattice is the availability of long, low-dispersion straight sections that are ideal locations for insertion devices: undulators and wigglers.

Undulators and wigglers consist of periodic arrays of magnets designed to produce intense beams of synchrotron radiation with particular properties.

In a wiggler, the period is relatively long: the synchrotron radiation produced from a wiggler is similar in many respects to that produced by a dipole.

Undulators have short periods, which leads to the synchrotron radiation spectrum having sharp peaks at well-defined wavelengths.

Final Remarks: Insertion Devices



A detailed analysis of the undulator radiation gives the wavelength of the undulator radiation (observed along the axis of the undulator):

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right), \quad \text{where:} \quad K = \frac{eB\lambda_u}{2\pi m_e c}. \tag{29}$$

The parameter K is the *undulator parameter*. K/γ is the maximum angle of the particle trajectory with respect to the undulator axis.

An insertion device with $K \leq 1$ is called an undulator.

The radiation from an undulator has bandwidth $\Delta\omega/\omega = 1/2N_u$ (where N_u is the number of periods in the undulator), and is emitted in a cone with opening angle $1/\gamma$.

An insertion device with $K > 1$ is called a wiggler.

Synchrotron radiation from wigglers is similar to synchrotron radiation from dipoles: the spectrum is broad compared to an undulator, and the radiation is emitted in a wider fan than in an undulator (with opening angle K/γ).

The specification of the insertion devices (and associated synchrotron light beam lines) depends on the user community that the light source is intended to serve.

Usually, there will be quite a wide range of insertion devices, providing light with different properties for different applications.

A "Typical" Application: Protein Crystallography