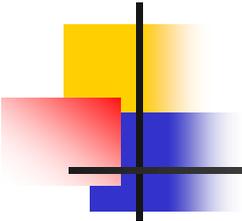


# Introduction to Multi-Particle Effects

D. Brandt, CERN

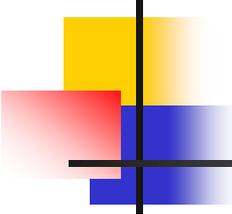


All the material presented in these lectures has been directly taken from the numerous lectures given at CAS by **Professor Albert Hofmann**.

I would like to sincerely thank Albert for all his efforts in producing such highly professional and pedagogical lectures for CAS.

Similarly, some slides have been borrowed from his successors at CAS, namely:

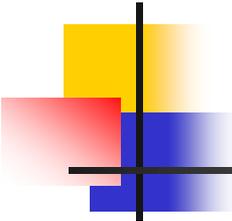
- **Giovanni Rumolo** and **Karl-Heinz Schindl** from CERN
- **Massimo Ferrario** INFN-Frascati
- **Marco Lonza** Elettra -Trieste
- Thank you to all of them for allowing me to use these slides.



# Aim of the lectures:

---

- Introduction to a few basic concepts
- Introduction to the “jargon”
- Allow you to follow basic discussions with colleagues
- Help you to distinguish between the different types of problems, their possible implications and cures
- Very intuitive approach

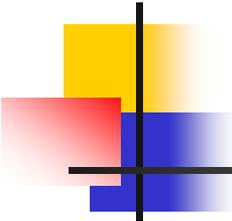


# A few general statements...

---

- So far, beam dynamics presented for a **single particle** case.
- Real beam composed of a lot of **charged particles moving** → self generated e.m. fields, e.m. forces, and interactions with the environment

**Multi-particle effects** are usually referred to as **Collective Effects**.  
These effects are very important because, usually, they are responsible for the limitation of the performance of the machine!



# A few general statements...

---

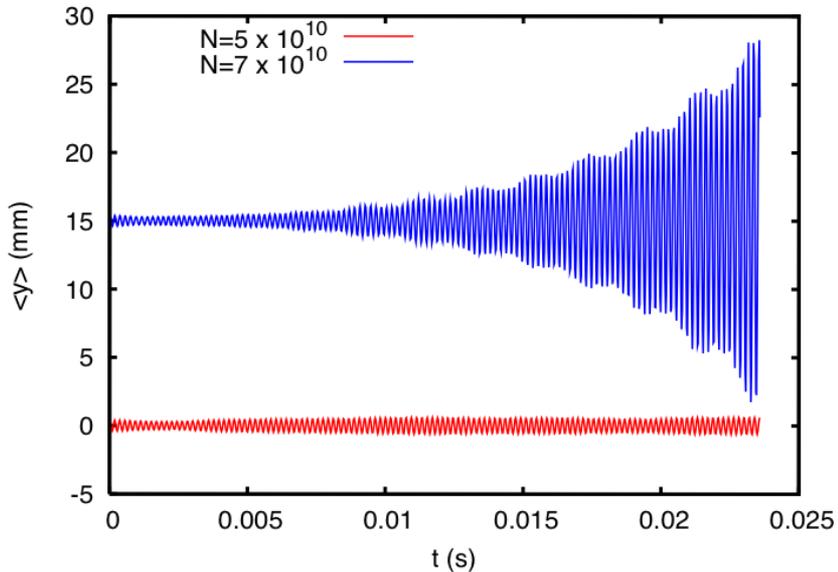
- These effects can be:

- **Incoherent**: affect the particles individually and cause an emittance growth, tune spread, bad lifetime. Usually slow processes.

- **Coherent**: affect the centre-of-mass motion, **visible on BPMs**, cause tune shifts, instabilities. Usually fast processes with rapid beam loss.

- **Require a second set of particles**: beam-beam, electron cloud, collisions with residual gases. Coherent or incoherent.

# Examples:

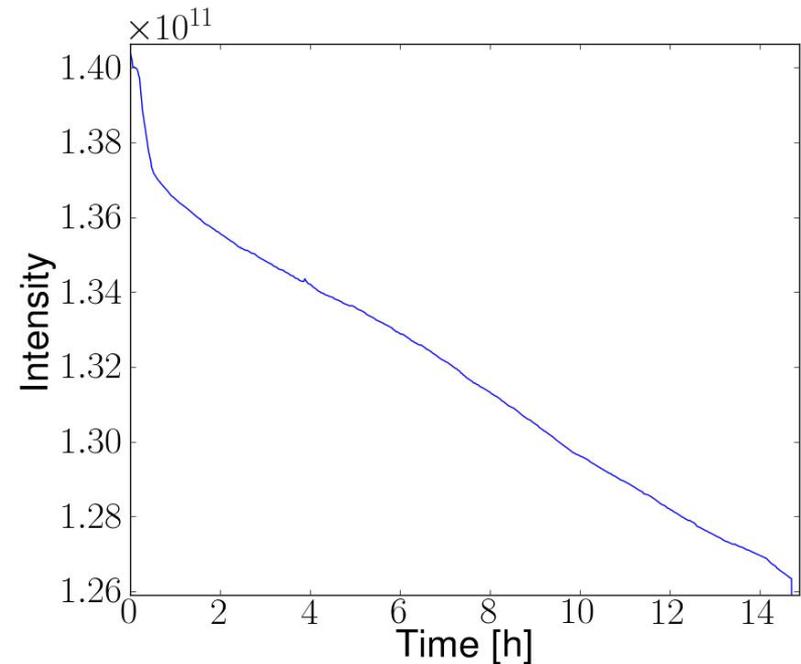


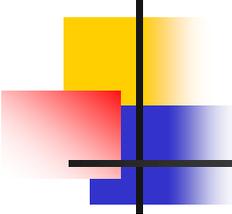
## Coherent effect:

When the bunch current exceeds a certain limit (current threshold), **the centroid of the beam**, e.g. as seen by a BPM, exhibits an exponential growth and the beam is lost within few milliseconds (simulation of an SPS bunch)

## Incoherent effect:

Slow beam loss. Blow-up due to nonlinearities (beam-beam) and subsequent loss at collimators.



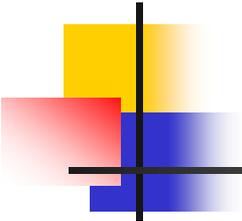


# Multi-particle effects:

---

In these two lectures:

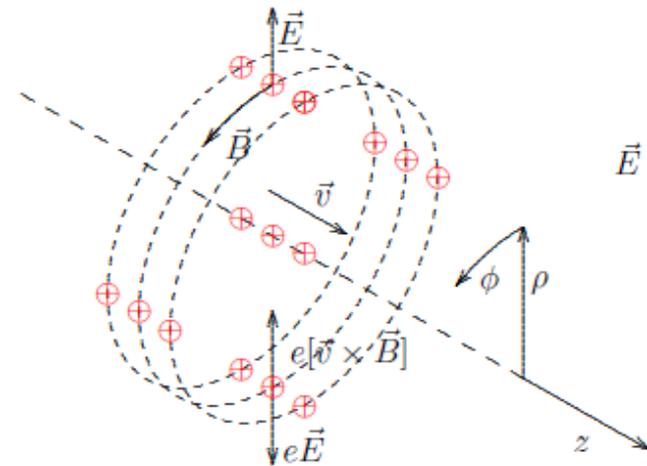
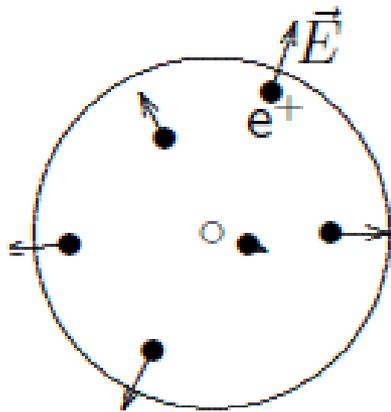
- Interaction between the charged particles within the bunch (**space charge effects**)
- Interaction between the bunch and the environment (**Impedance – wakes**)
- Interaction between the bunches via the impedance (**coupled-bunch effects**)



# Space charge effects

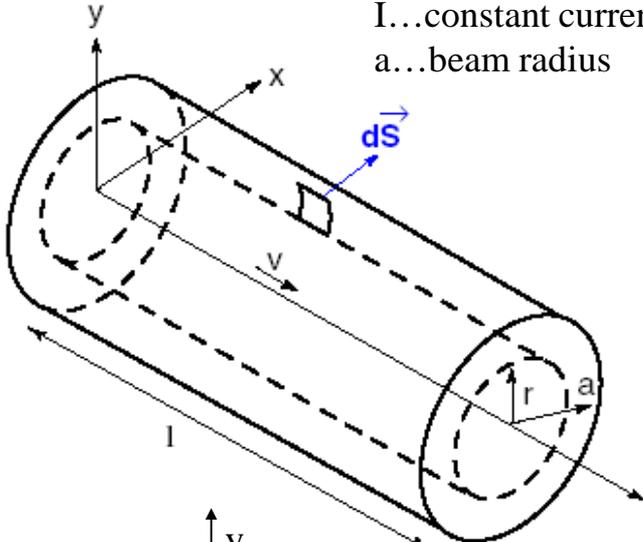
# Space charge effects:

- We deal with charged particles ( $\vec{E}$ -field)  $\rightarrow$  repelling effect  $\rightarrow$  reduces the focusing
- Charged particles are moving  $\rightarrow$   $\vec{B}$ -field  $\rightarrow$  Lorentz force



# Space charge: Fields

$\eta$ ...charge density in Cb/m<sup>3</sup>  
 $\lambda$ ... constant line charge  $\pi a^2\eta$   
 $I$ ...constant current  $\lambda\beta c = \pi a^2\eta\beta c$   
 $a$ ...beam radius



**Electric**

$$\vec{E} = E_r$$

$$\text{div } \vec{E} = \frac{\eta}{\epsilon_0}$$

**Magnetic**

$$\vec{B} = B_\phi$$

$$\text{curl } \vec{B} = \mu_0 \vec{J}$$

Current density ( $\beta c \eta$ )

$$\iiint \text{div } \vec{E} \, dV = \iint \vec{E} \, d\vec{S} \quad \oint \vec{B} \cdot r \, d\phi = \iint \text{curl } \vec{B} \, d\vec{A}$$

Apply these integrals over

cylinder radius  $r$   
length  $l$

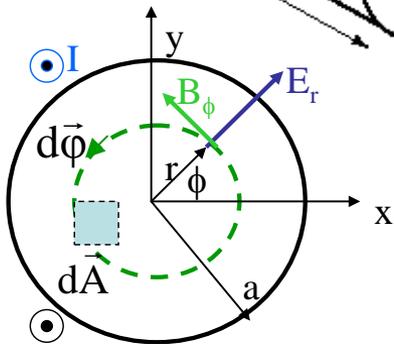
cross section  
radius  $r$

$$r^2 \pi l \frac{\eta}{\epsilon_0} = E_r 2r\pi l$$

$$B_\phi 2r\pi = \mu_0 r^2 \pi \beta c \eta$$

$$E_r = \frac{I}{2\pi\epsilon_0\beta c} \frac{r}{a^2}$$

$$B_\phi = \frac{I}{2\pi\epsilon_0 c^2} \frac{r}{a^2}$$



# Space charge: Forces

$$\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$$

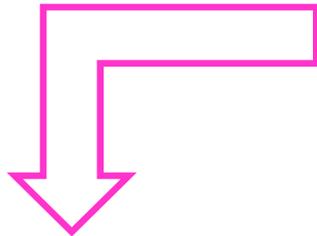
$$F_r = e(E_r - v_s B_\phi)$$

$$F_r = \frac{eI}{2\pi\epsilon_0\beta c} (1 - \beta^2) \frac{r}{a^2} = \frac{eI}{2\pi\epsilon_0\beta c^2} \frac{1}{\gamma^2} \frac{r}{a^2}$$

Electric

magnetic

$$\beta=1 \rightarrow F_r=0$$

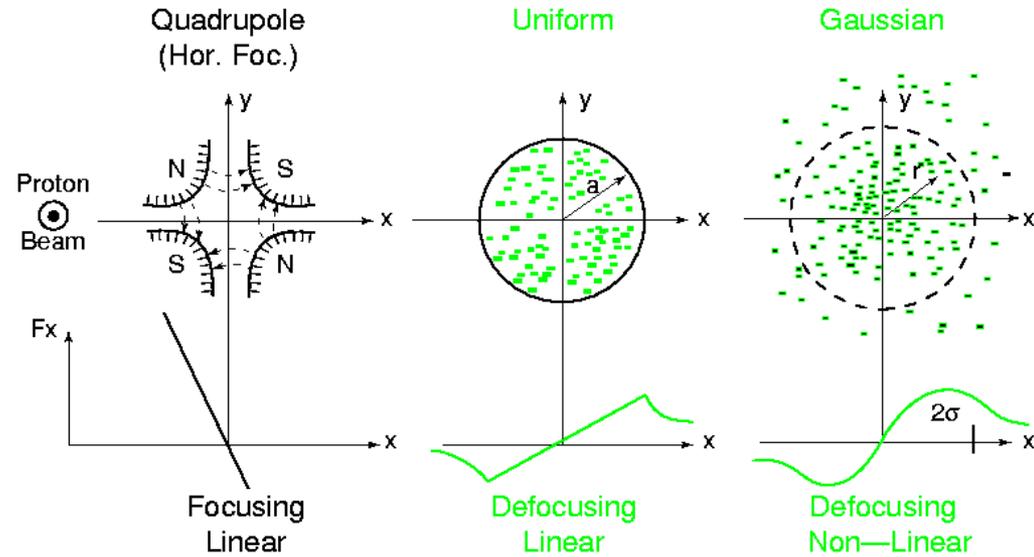


$$F_x = \frac{eI}{2\pi\epsilon_0\beta c\gamma^2 a^2} x$$

$$F_y = \frac{eI}{2\pi\epsilon_0\beta c\gamma^2 a^2} y$$

Space charge forces

- circular beam
- uniform charge density
- $F_x, F_y$  linear in  $x, y$
- defocusing lens** in either plane



# Space charge: Tune Shift

- ❑ **Beam not bunched** (so no acceleration)
- ❑ **Uniform density** in the circular x-y cross section (not very realistic)

$$x'' + (\mathbf{K}(s) + \mathbf{K}_{SC}(s))x = 0 \quad \rightarrow \quad Q_{x0} \text{ (external)} + \Delta Q_x \text{ (space charge)}$$

For small “gradient errors”  $k_x$

$$\Delta Q_x = \frac{1}{4\pi} \int_0^{2R\pi} k_x(s) \beta_x(s) ds = \frac{1}{4\pi} \int_0^{2R\pi} \mathbf{K}_{SC}(s) \beta_x(s) ds$$

$$\Delta Q_x = -\frac{1}{4\pi} \int_0^{2R\pi} \frac{2r_0 I}{e\beta^3 \gamma^3 c} \frac{\beta_x(s)}{a^2} ds = -\frac{r_0 R I}{e\beta^3 \gamma^3 c} \left\langle \frac{\beta_x(s)}{a^2(s)} \right\rangle = -\frac{r_0 R I}{e\beta^2 \gamma^2 c \epsilon_{xn}}$$

$$\Delta Q_{x,y} = -\frac{r_0 N}{2\pi \epsilon_{x,y,n} \beta \gamma^2}$$

using  $I = (Ne\beta c)/(2R\pi)$  with  
 $N$ ...number of particles in ring  $\epsilon_{x,y}$ ....normalised emittance

- ❑ “Direct” space charge, unbunched beam
- ❑ Important for **low-energy** machines
- ❑ **Independent of machine size**  $2\pi R$  for a given  $N$

# Direct space charge:

This force:

$$\vec{F} = e \left( \vec{E} + [\vec{v} \times \vec{B}] \right)$$

changes the slopes of the **individual particles** and produces a defocusing effect:

$$\Delta Q_{xy} = - \frac{r_0 R I}{e c \epsilon_{xyn} \beta^2 \gamma^2}$$

or:

$$\Delta Q_{xy} = - \frac{r_0 N_b}{2 \pi \epsilon_{xyn} \beta \gamma^2}$$

!!!

# Direct space charge:

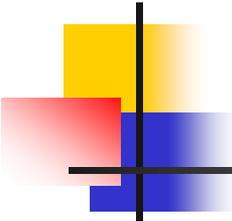
- Direct space charge is a purely **INCOHERENT** effect

- For **bunched** beams, the current depends on the **position** in the bunch ( $I(s)$ ). This leads to a **tune spread** and a **tune modulation** (synchrotron oscillations).

$$\Delta Q_{xy} = - \frac{r_0 R I(s)}{e c \epsilon_{xyn} \beta^2 \gamma^2}$$

!!!

- This tune spread is very important for stability with **Landau damping**.



# Important effect:

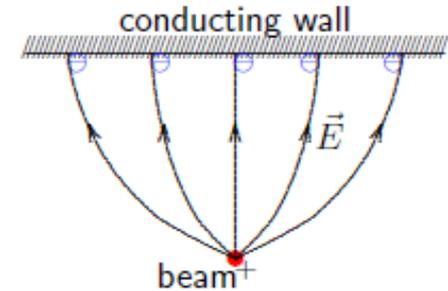
---

- This **tune spread** is very important for the stability of the beam, because it induces a fundamental effect called **Landau damping**.

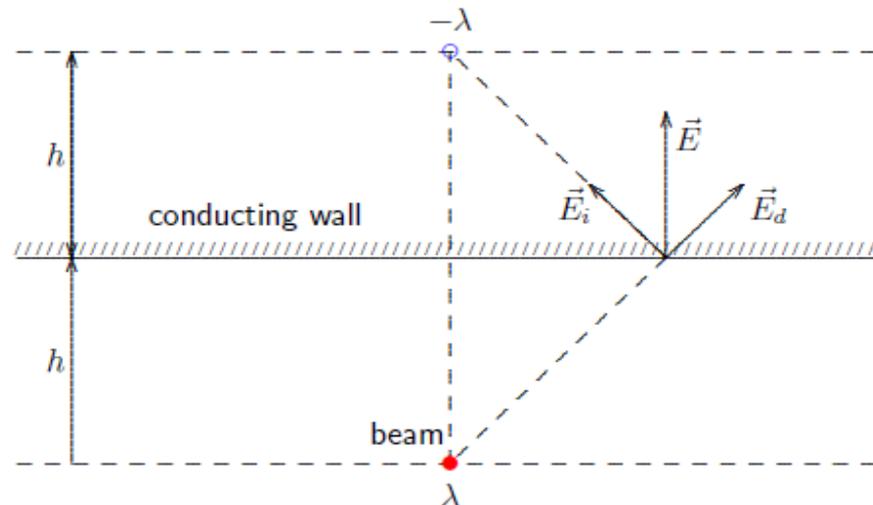
At the same time, **this effect is at the origin of a serious dilemma**, because a large spread is good for **Landau damping** but is very bad for **resonances** !

# Effect of the vacuum chamber:

- A perfectly conducting vacuum chamber imposes a **perpendicular electric field** as boundary condition on the surface ( $E_{\parallel} = 0$ ).



- To compute the field, one has to introduce an image charge ( $-\lambda$ ) at a distance  $h$  from the beam:

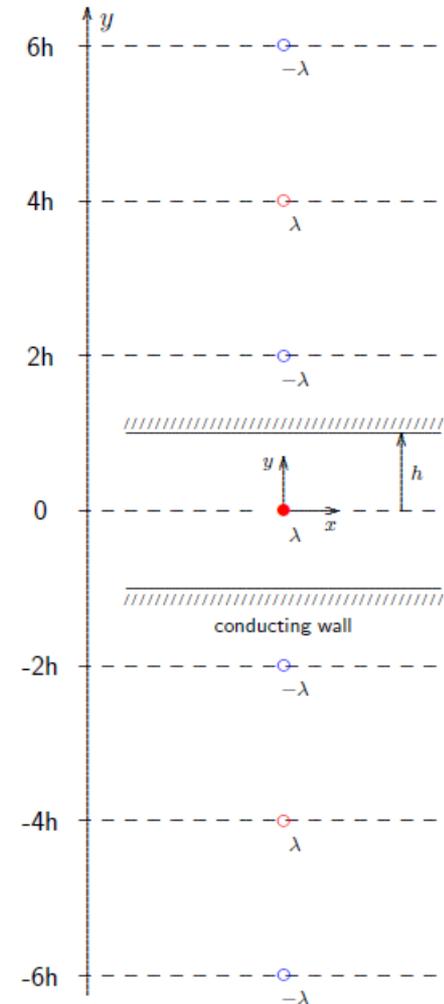


# Effect of vacuum chamber:

## Indirect space charge

- Actually, the vacuum chamber represents **2 conducting boundaries** at  $\pm h$ . To satisfy  $E_{//}=0$ , the procedure is a little bit more complicated:

- Compute the **fields**  $E_i$  due to each line charge, sum the fields, compute the **force**, new **focusing term** in equation of motion, compute  $\Delta Q$  !



# Total incoherent effect:

The **direct** and **indirect incoherent** space charge effects are given by:

$$\Delta Q_{xy} = - \frac{2r_0 I R \langle \beta \rangle_{xy}}{ec \beta^2 \gamma} \left\{ \frac{1}{2a^2 \gamma} \mp \frac{\pi^2}{48h^2} \right\}$$

direct

different signs!

indirect

- Indirect space charge still exists at high energy !
- $1/\gamma$  effect due to rigidity of the beam at higher energy

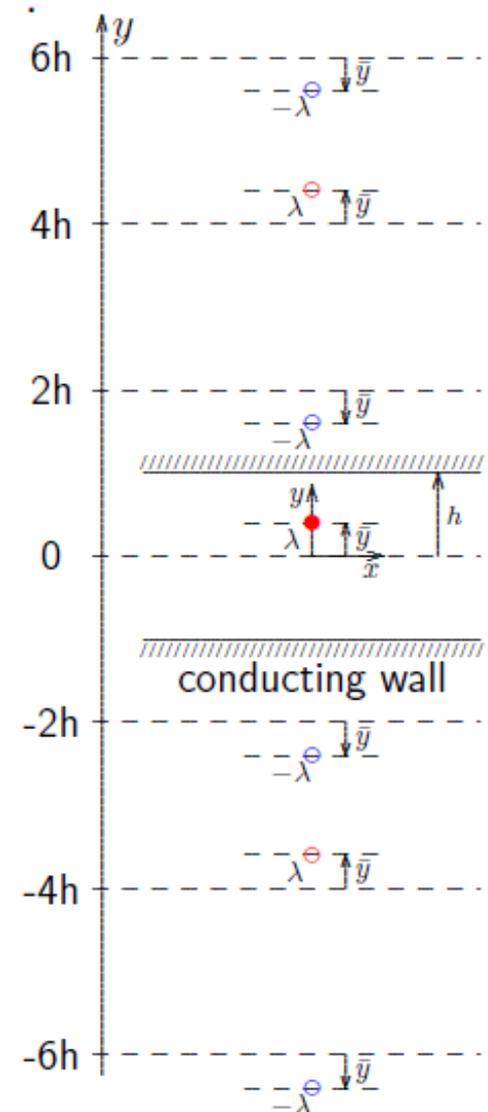
# Coherent space charge:

- If the beam has a **coherent** motion, then:
  - Direct space charge unaffected,  $\Delta Q_{coh} = 0$
  - Indirect space charge is modified:

$$(\Delta Q_{xy})_{coh} = -\frac{r_0 I R \langle \beta \rangle_{xy}}{ec \beta^3 \gamma h^2}$$

- **Always negative** (defocusing) !

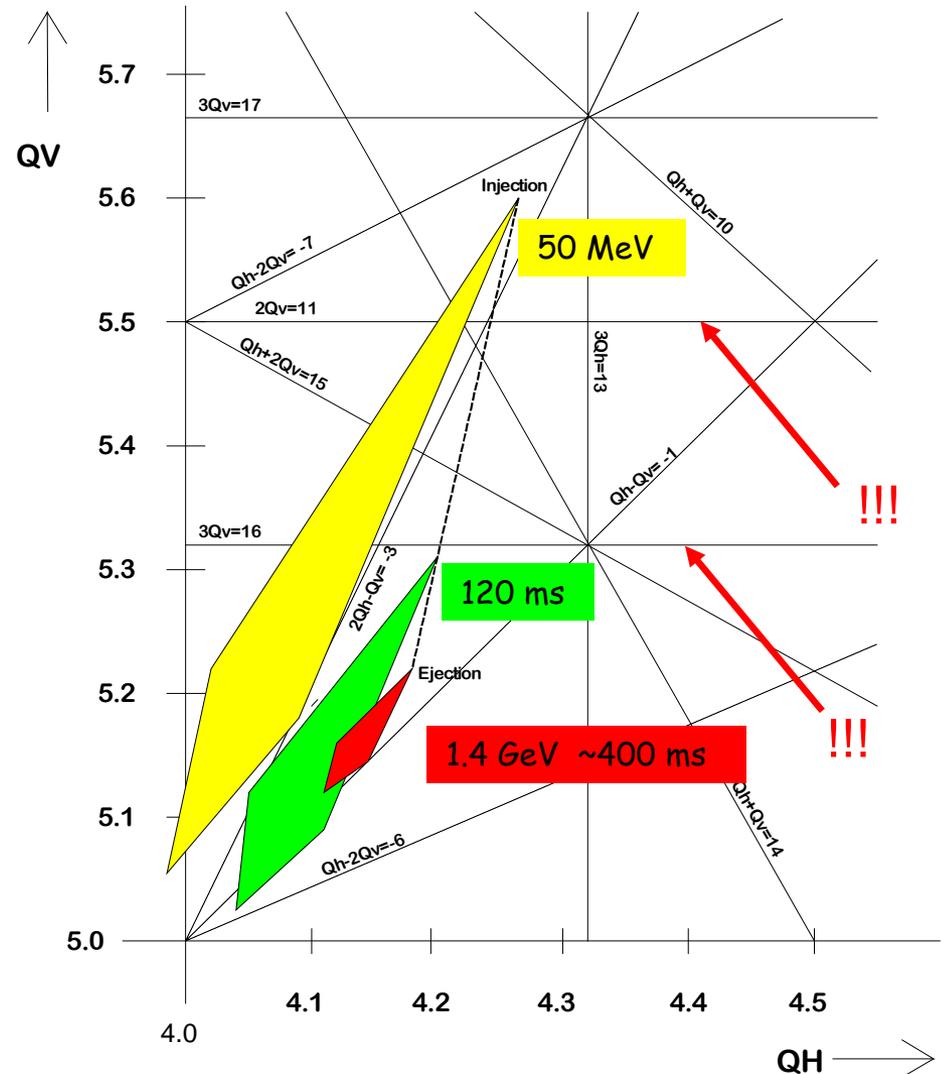
- Since  $\Delta Q_{coh} \neq \Delta Q_{incoh}$ , space charge makes it very difficult to avoid resonances !

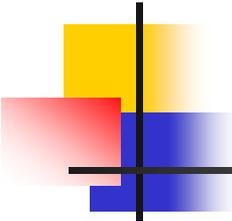


# A space charge limited accelerator:

- Since  $\Delta Q_{\text{coh}} \neq \Delta Q_{\text{incoh}}$ , space charge makes it very difficult to know position of tune spread in order to avoid resonances !

- Direct space charge tune spread  **$\sim 0.55$  at injection**, covering 2<sup>nd</sup> and 3<sup>rd</sup> order stop-bands
- "necktie"-shaped tune spread **shrinks rapidly** due to the  $1/\beta\gamma^2$  dependence
- Enables the working point to be moved **rapidly** to an area clear of strong stop-bands



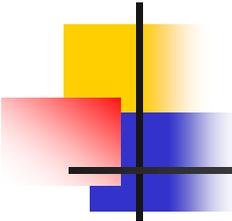


# Space charge summary:

---

Direct space charge (self-field of the beam):

- Affects the **incoherent** motion (but not the coherent one)
- Is proportional to the **total number of particles in the beam  $N_b$  and beam emittance.**
- Is **defocusing** in both transverse planes
- Scales with  **$1/\gamma^2$**  (negligible for high energy or leptons)

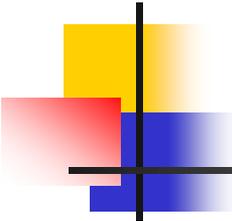


# Space charge summary:

---

## Indirect space charge (vacuum chamber):

- Is proportional to the total **number of particles** in the beam  $N_b$  **but not to the emittance.**
- Does not disappear with increasing energy !
- Affects the **incoherent** motion, but with **different signs** for the two transverse planes !
- Also has an effect on the **coherent** motion !
- Since  $\Delta Q_{\text{coh}} \neq \Delta Q_{\text{incoh}}$ , space charge makes it very difficult to avoid resonances !



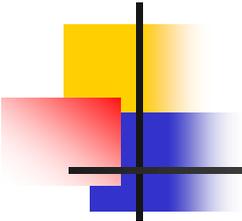
# Space charge summary:

## Bunched beams:

- Space charge depends on particle position in the bunch → **tune spread** and tune modulation.
- Space charge is a **hard limit** on intensity/emittance ratio.
- Very difficult to go above  $\Delta Q_y \sim 0.5$  → **limit for N**

## How to avoid it (for a given final energy)?

- Increase N by raising the injection energy in last machine
- Make a longer Linac (more tanks) or add a small booster
- Seems obvious but do not forget the **financial aspect !!!**

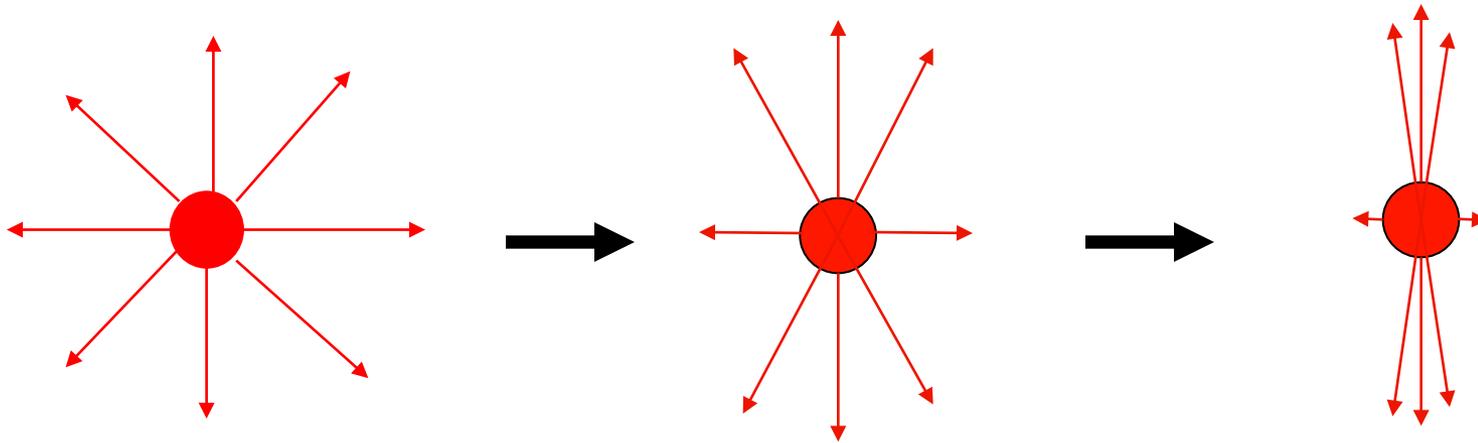


---

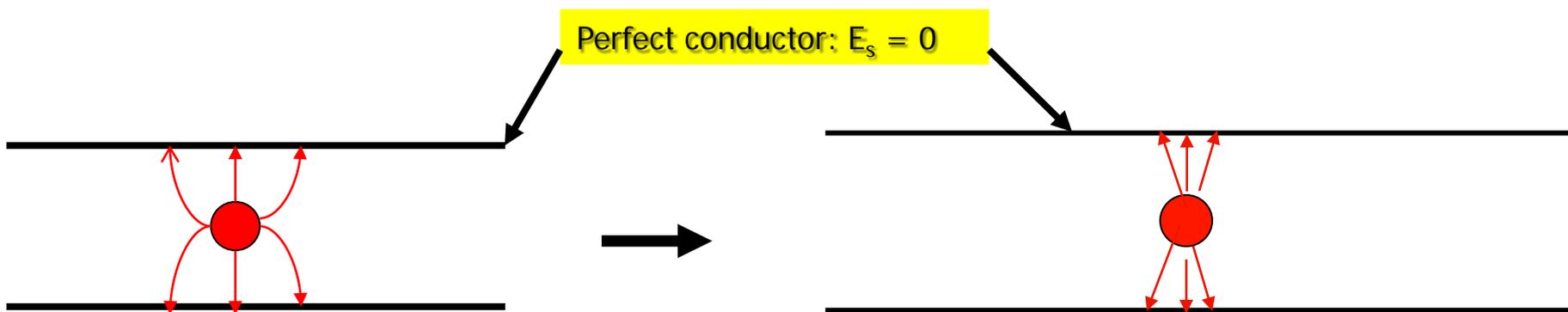
# Bunch-surroundings effects

## Impedance - Wakes

# Interaction beam – vacuum chamber:

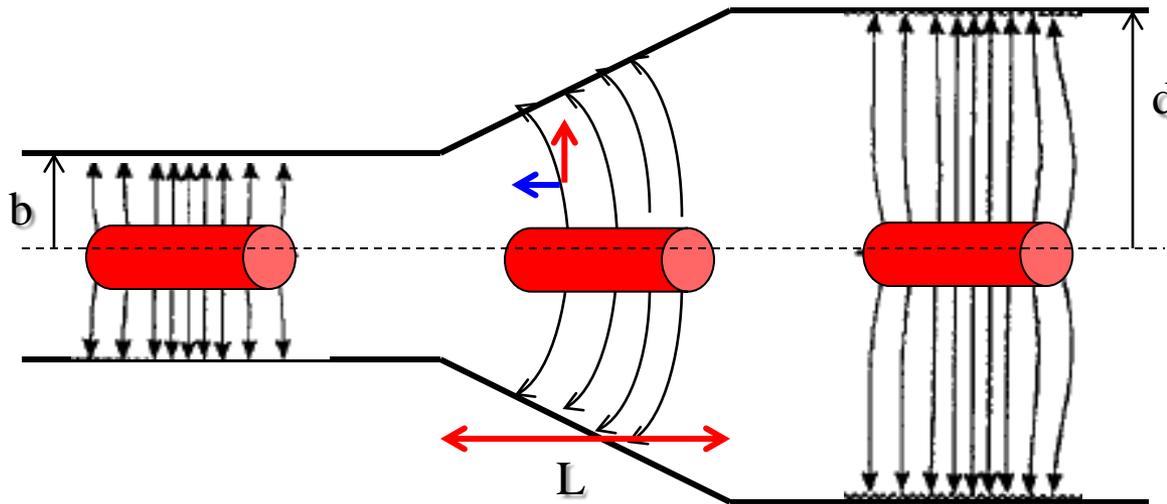


Perfect conductor:  $E_s = 0$



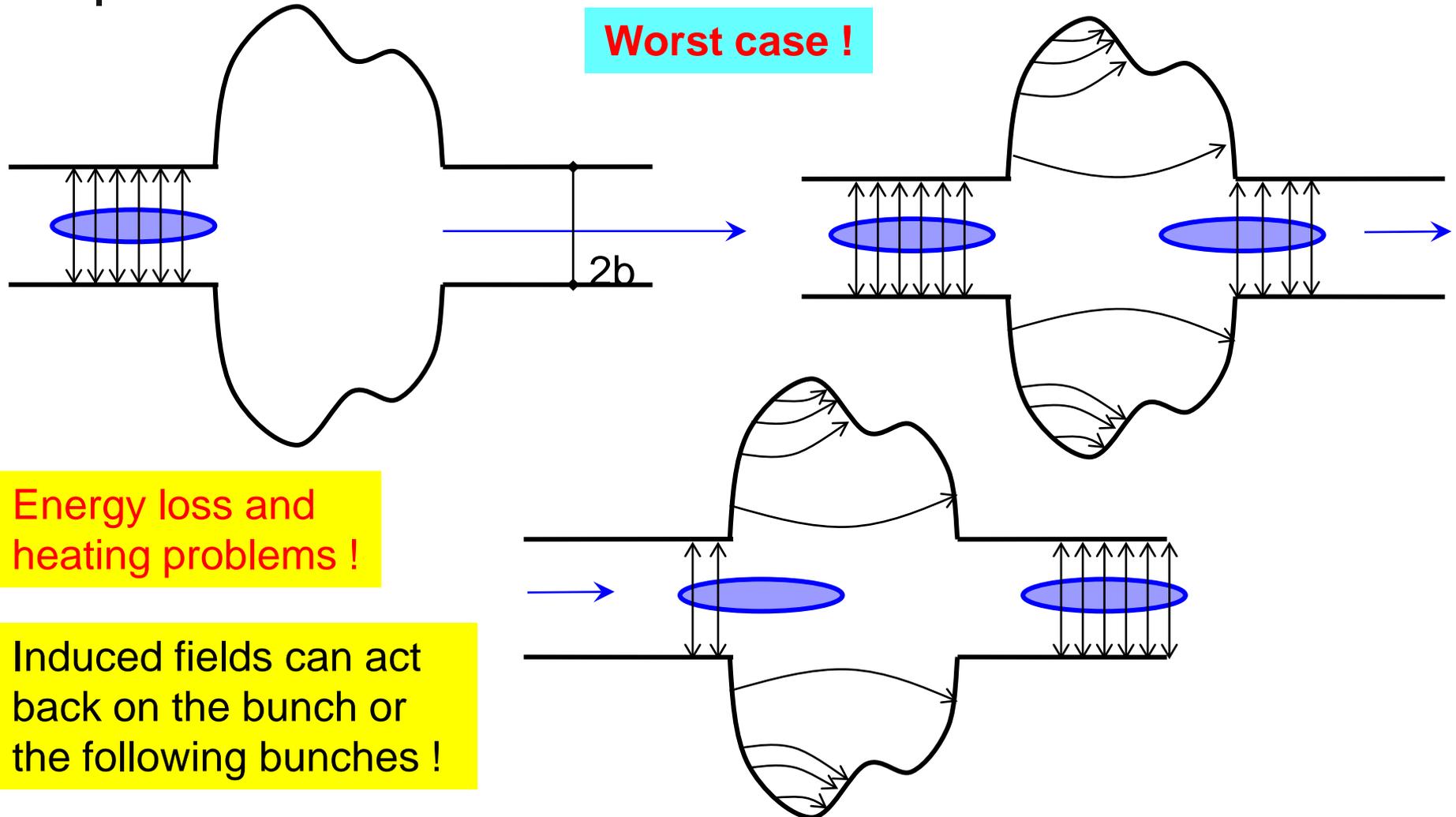
# Change in the cross-section

- If conductor is **not perfect**, or, even worse, if  **$b \neq \text{const.}$**



**$E_s \neq 0$**  => there is an interaction between the beam and the wall!

# Beam pipe discontinuity:



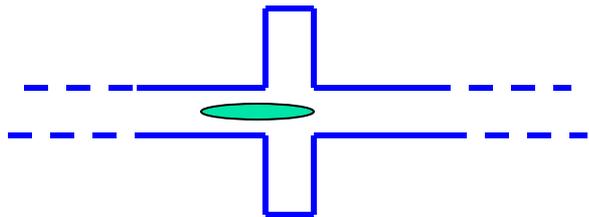
Energy loss and heating problems !

Induced fields can act back on the bunch or the following bunches !

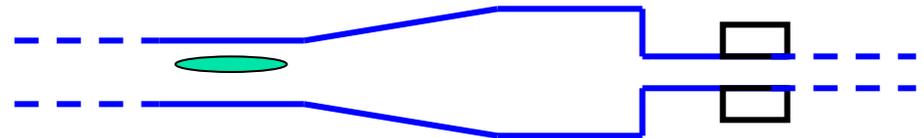
# Interaction beam – vacuum chamber

Different types of interactions:

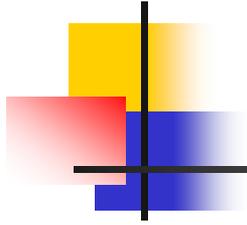
**Resistive wall effect:**  
Finite conductivity



**Narrow-band resonators:**  
Cavity-like objects

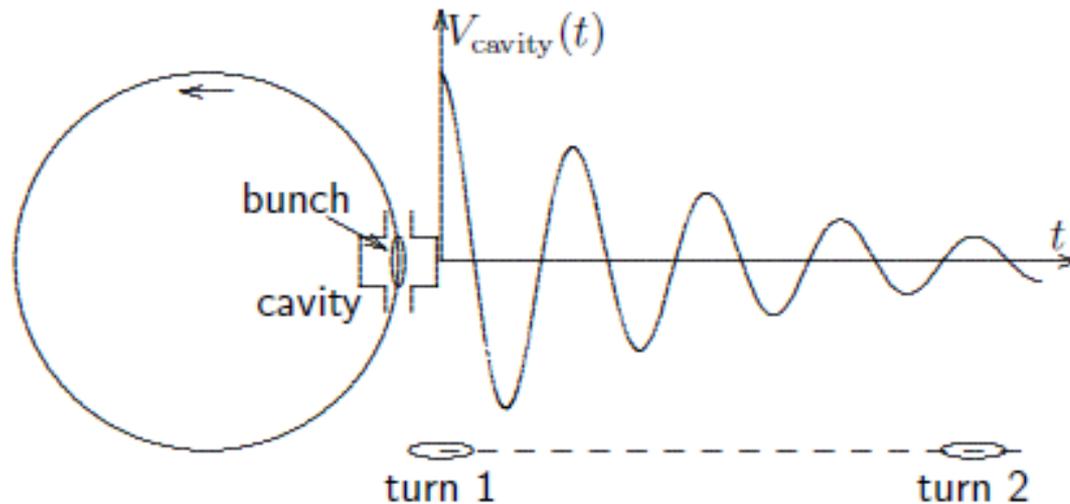


**Broad-band resonators:**  
Tapers, other non-resonant structures



# 1. Cavity-like objects

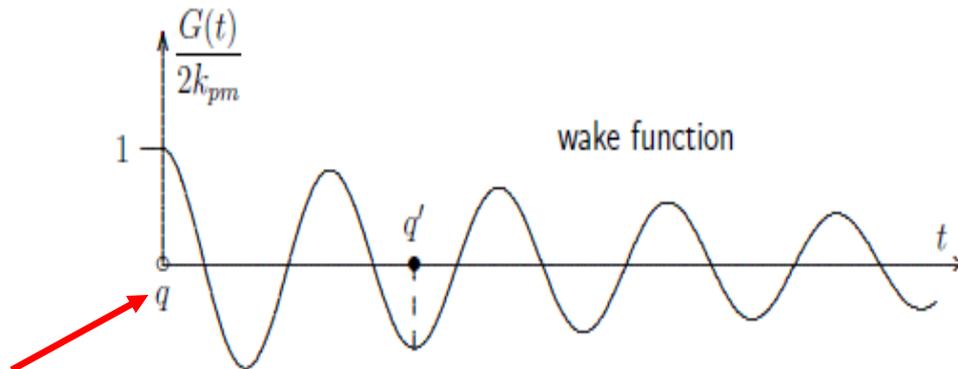
# Bunch traversal of a cavity-like object



- The bunch induces a voltage  $V(t)$  oscillating in the cavity

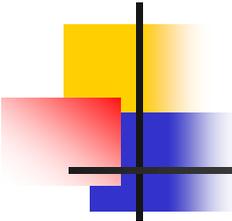
# 1<sup>st</sup> definition: the wake function $G(t)$

$G(t) = V(t)/q =$  Green's or *Wake* function



$$G(t < 0) = 0$$

A voltage induced by a charge  $q$  at  $t=0$  changes the energy of a second charge  $q'$  traversing the cavity at  $t$  by  $U = -q'V(t) = -qq'G(t)$



# The importance of the wake function:

---

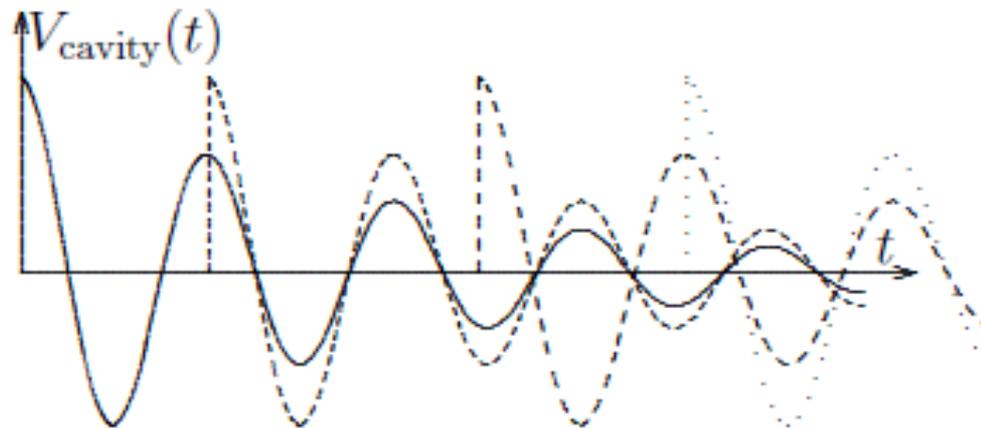
If we were able to compute the **wake function**  $G(t)$ , we would be able to solve all the problems related to the **interactions** between the **beam** and the **cavity-like** objects !

**Unfortunately ...** this is not possible and we have to use some approximations.

A few important properties related to the wake function...

# Multi traversals or multi bunches...

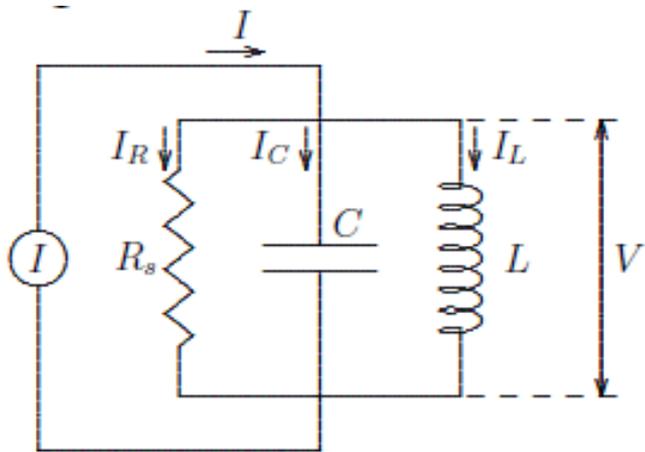
- Let us consider a single bunch coming back through the cavity or different bunches crossing the cavity:



- It is intuitively clear that the voltages induced during the **different passages** can **add** or **compensate** each other.
- This can lead to **growing oscillations** of the particle motion and result in an **instability** (or damping).

# The concept of wake-impedance

- Each cavity-like object has (many) **narrow-band oscillation modes** which are interpreted as **RLC-circuits**:



$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$Q = R_s C \omega_r$$

$$\alpha = \frac{\omega_r}{2Q}$$

$$L = \frac{R_s}{Q \omega_r}$$

$$C = \frac{Q}{R_s \omega_r}$$

- Driving this circuit with a current  $I$  gives the voltages and currents across the elements.

# Wake function with RLC circuit

$$G(t) = \frac{\omega_r R_s}{Q} e^{-\alpha t} \left( \cos \left( \omega_r \sqrt{1 - \frac{1}{4Q^2}} t \right) - \frac{\sin \left( \omega_r \sqrt{1 - \frac{1}{4Q^2}} t \right)}{2Q \sqrt{1 - \frac{1}{4Q^2}}} \right)$$

with

Loss factor  $k$

$$k = \frac{\omega_r R_s}{2Q}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

Compute resonant modes up to High frequencies !!!  
→ sum

# Fundamental relation:

$$Z(\omega) = \int_{-\infty}^{\infty} G(t) e^{-j\omega t} dt$$

$$G(t < 0) = 0$$

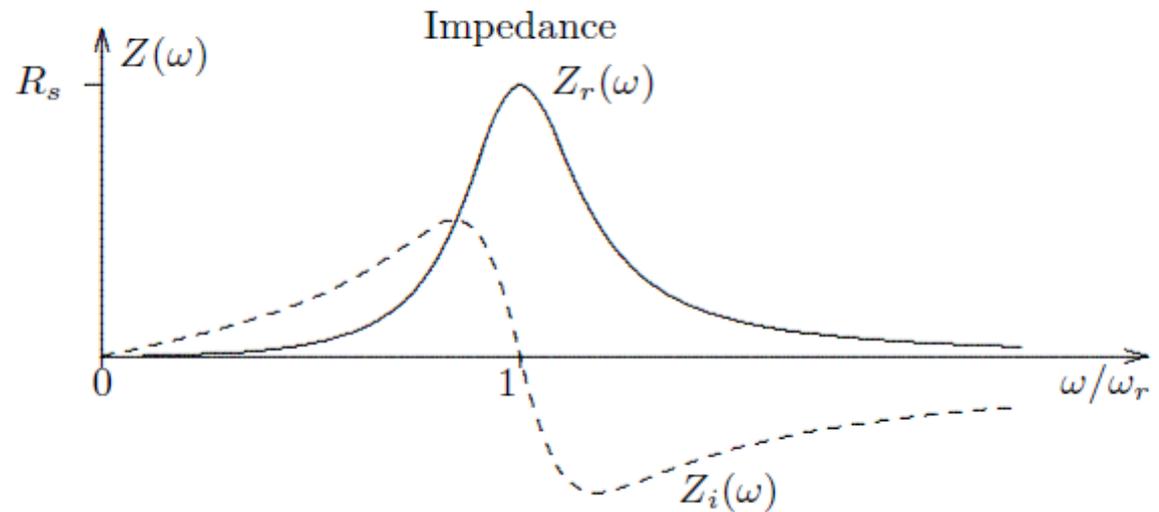
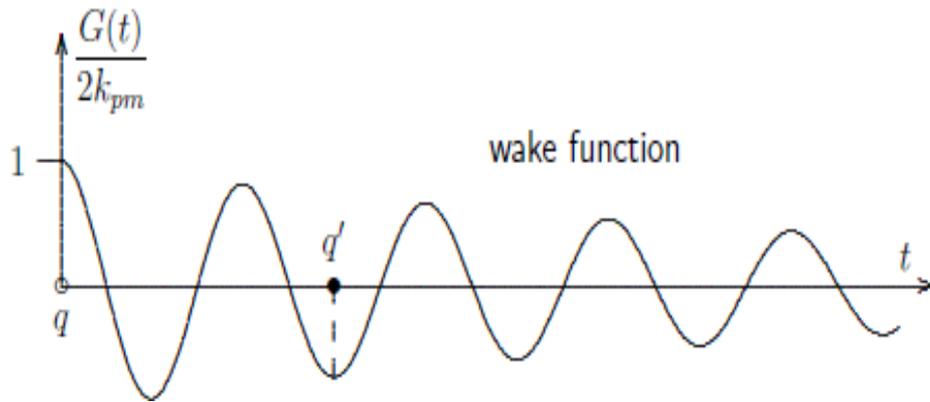
The **impedance**  $Z(\omega)$  is the Fourier transform of the **Green's Function**  $G(t)$  !

As a consequence, for collective effects, it is completely **equivalent** to work in the **time domain** (wake) or in the **frequency domain** (impedance) !

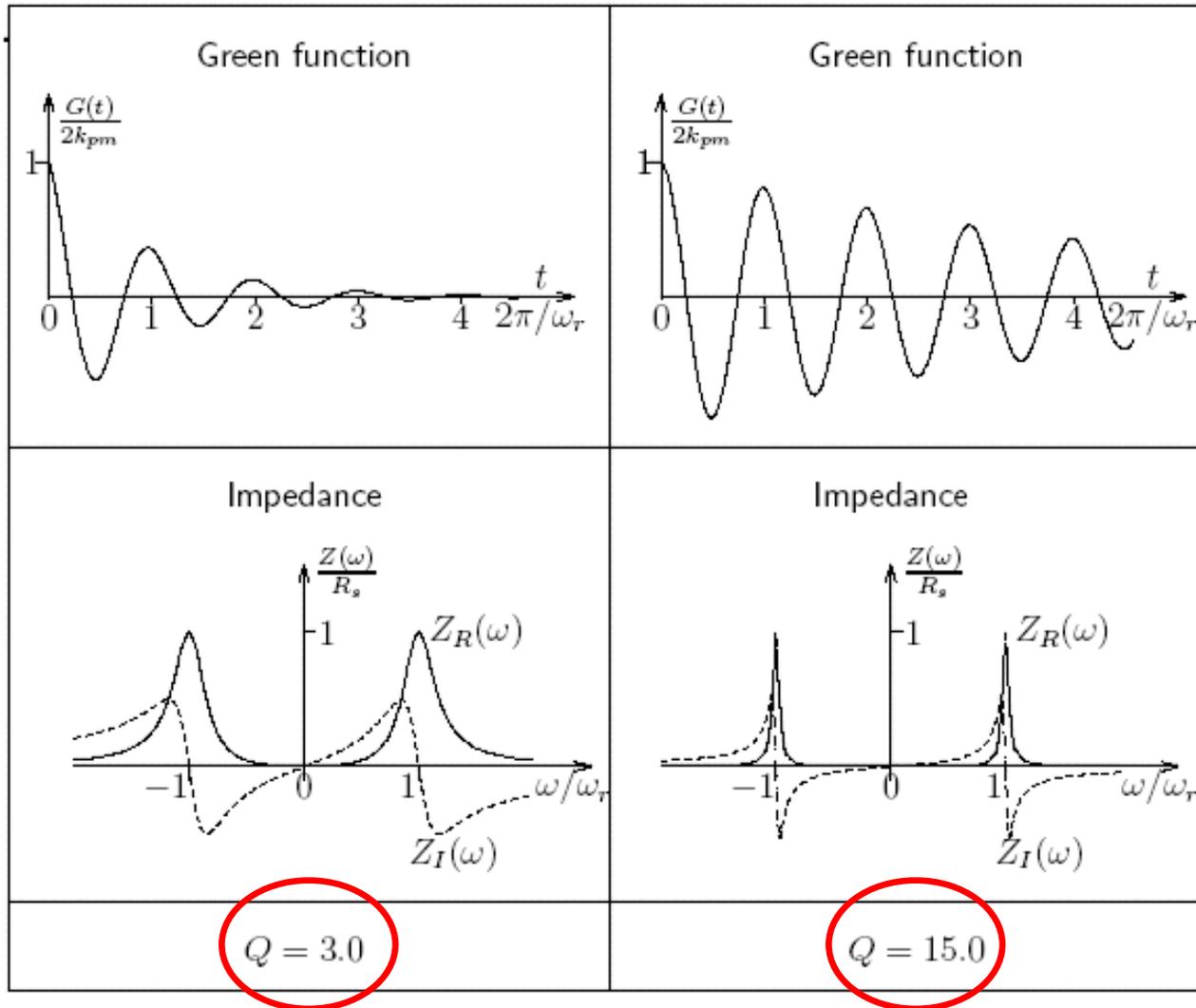
$$Z_R(\omega) = R_s \frac{1}{1 + Q^2 \left( \frac{\omega_r^2 - \omega^2}{\omega_r \omega} \right)^2}$$

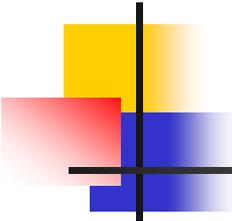
$$Z_I(\omega) = -R_s \frac{Q \frac{\omega^2 - \omega_r^2}{\omega_r \omega}}{1 + Q^2 \left( \frac{\omega^2 - \omega_r^2}{\omega_r \omega} \right)^2}$$

# Equivalent representations:



# Equivalent representations:





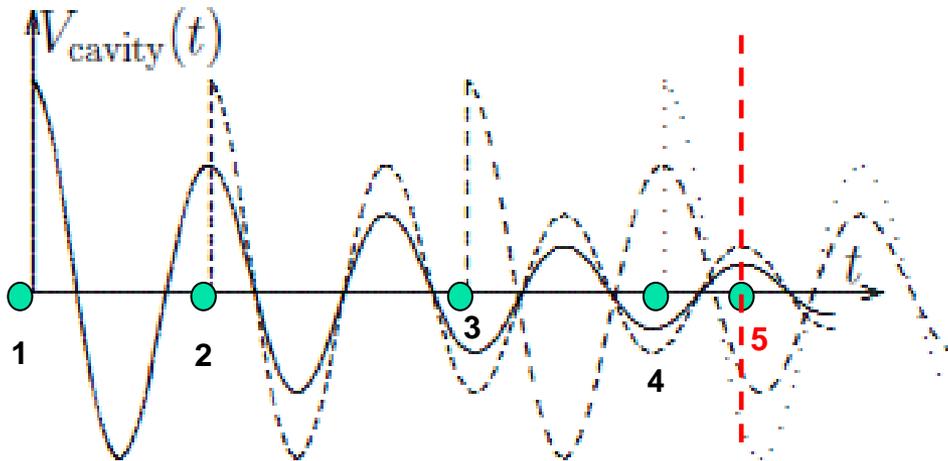
# How to approximate $G(t)$ ?

---

Since we cannot obtain the wake function  $G(t)$ , one has to think about appropriate ways to approximate it...

→ The concept of **Wake Potential** !

# Bunch of 5 macro-particles



$$W_1 = V(0)/2$$

$$W_2 = V(0)/2 + V(\Delta_{12})$$

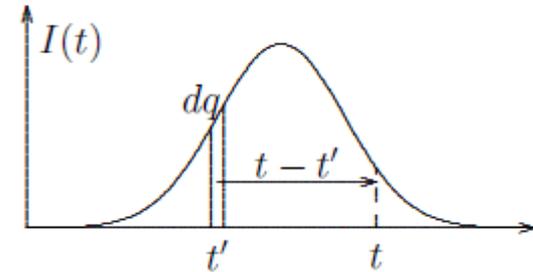
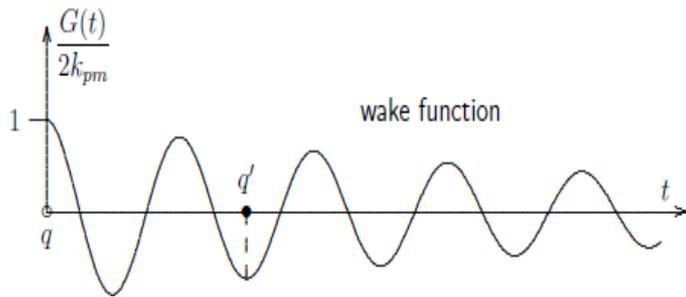
$$W_3 = V(0)/2 + V(\Delta_{13}) + V(\Delta_{23})$$

$$W_4 = V(0)/2 + V(\Delta_{14}) + V(\Delta_{24}) + V(\Delta_{34})$$

$$W_5 = V(0)/2 + V(\Delta_{15}) + V(\Delta_{25}) + V(\Delta_{35}) + V(\Delta_{45})$$

In practice, sum becomes an integral over the bunch distribution !

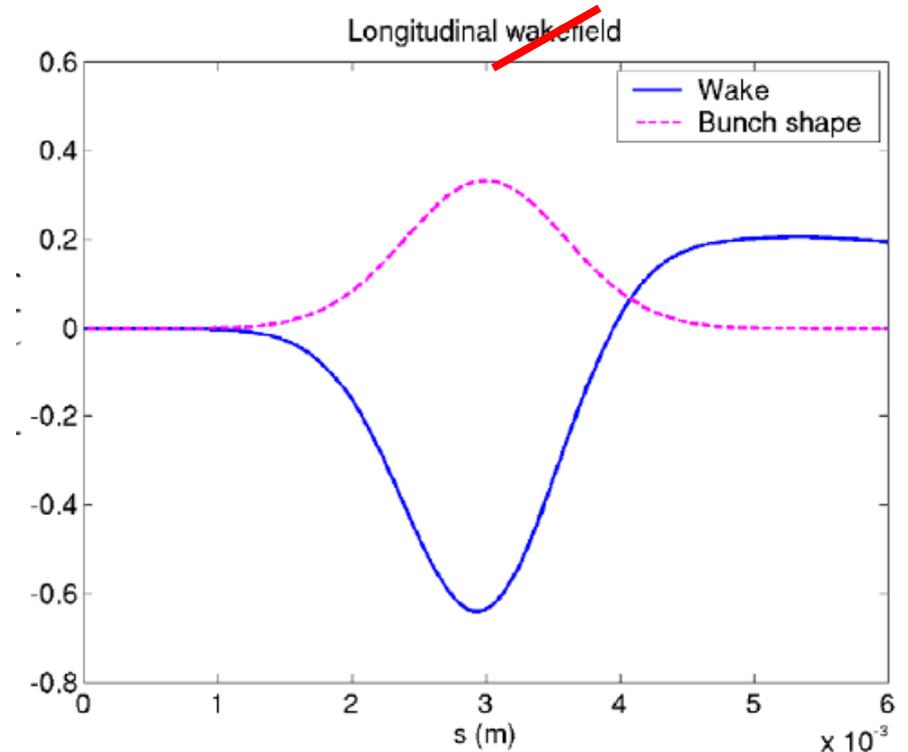
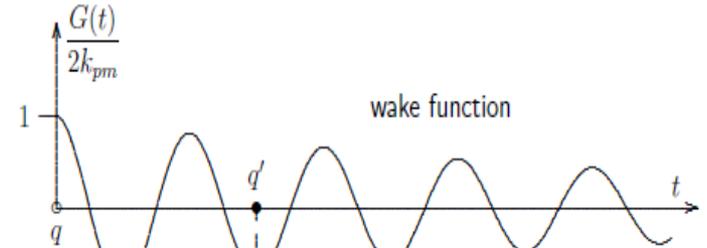
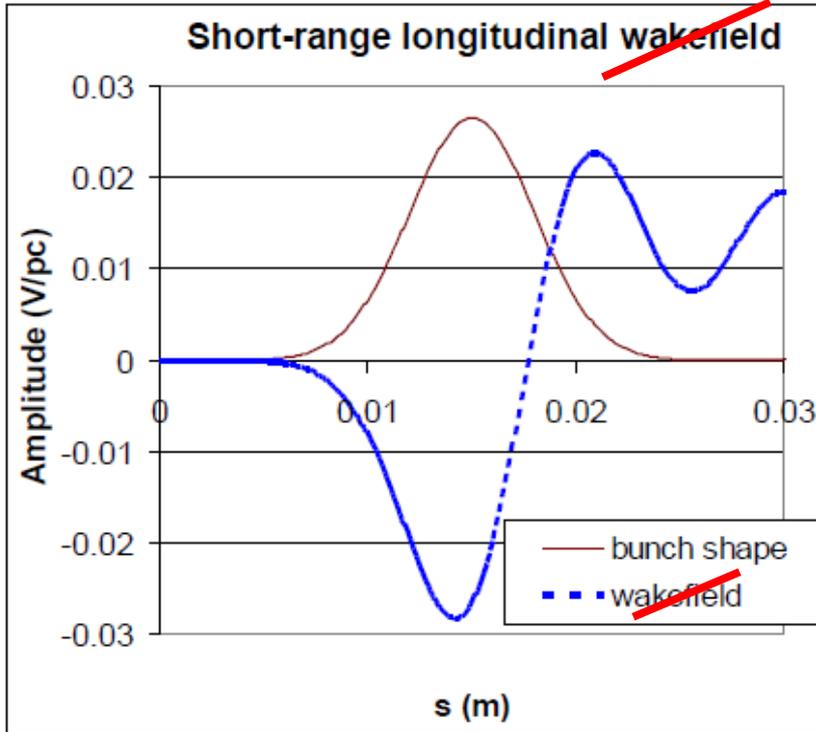
# The wake potential

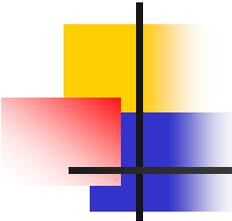


The wake potential (what we obtain from time-domain codes - V/pC):

$$W(t) = \int_{-\infty}^t I(t')G(t')dt'$$

# Examples of wake potentials:





# In practice...

---

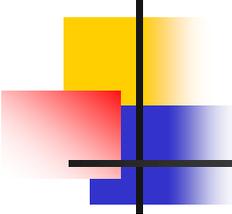
In practice, for a given object, it is **not possible to obtain the Green's function:**

**Frequency domain:**  
(HFSS, URMEL,...)

- Compute all the resonances up to the **highest possible frequency**
- Build the appropriate sum and use this as a **pseudo Green's function**

**Time domain:**  
(Particle Studio, MAFIA,...)

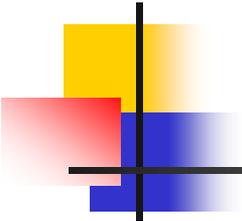
- Evaluate the wake potential for the **smallest possible bunch length**
- **Assume** this wake potential is the **Green's function** (wake field !)
- Perform a Fourier transform to get the impedance  **$Z(\omega)$**



# Brief – side remark (1):

---

- **Wake potentials** in a general structure may be most accurately obtained via **numerical solution of Maxwell's equations**.
- in the '80s the first **2D and 3D codes** were developed to solve numerically the Maxwell equations in given (simplified) geometries (time or frequency domain)
  - TBCI, **MAFIA**, ABCI, NOVO, XWAKE, ....
  - More recently: **GdfidL**, **HFSS**, Microwave Studio, **Particle Studio**



---

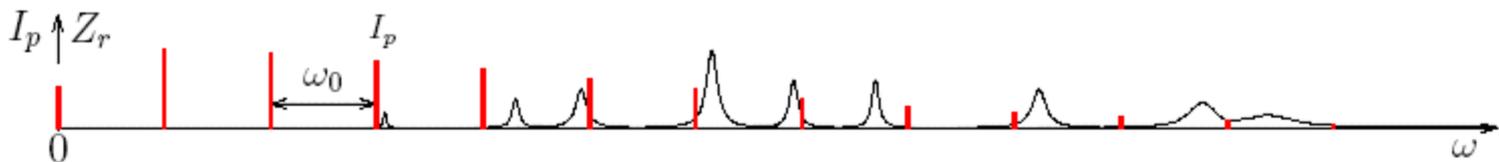
## Collective effects from the control room...

# General rules (1):

- If you deal with high-Q resonators (long memory), then you have to deal with **each resonator separately**.

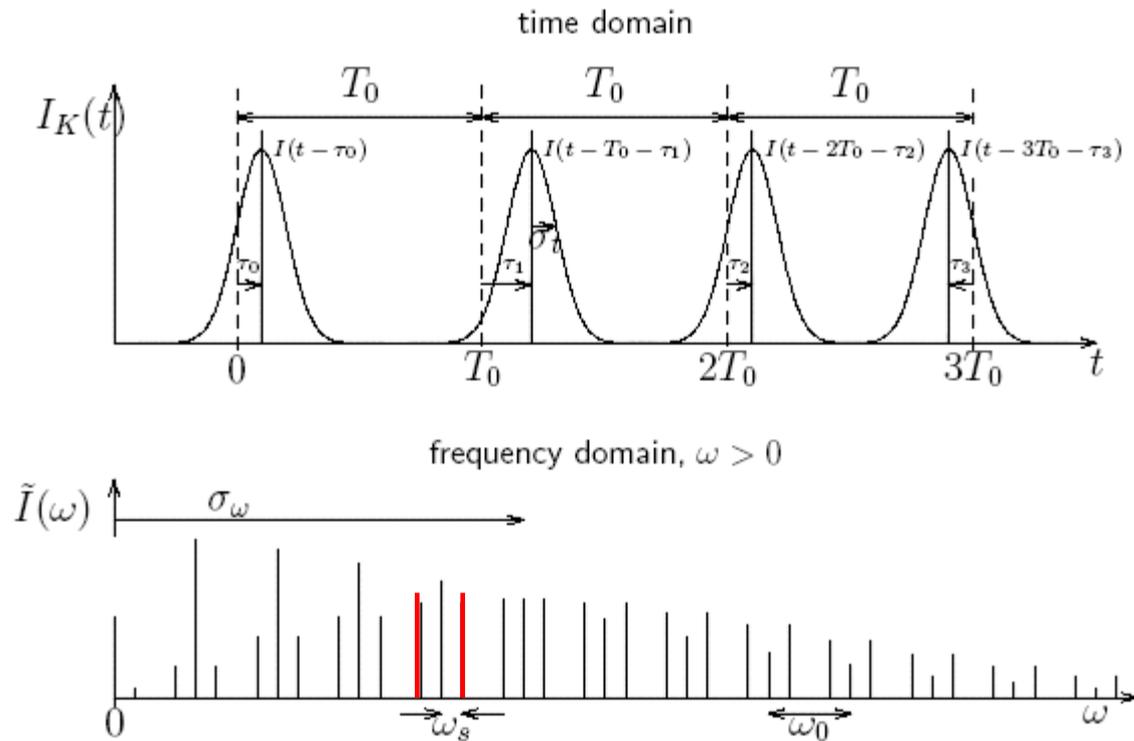
→ **For each resonance**, study the interaction with the beam

→ Study the interactions where the frequencies seen by the bunch and those of the impedance do overlap !



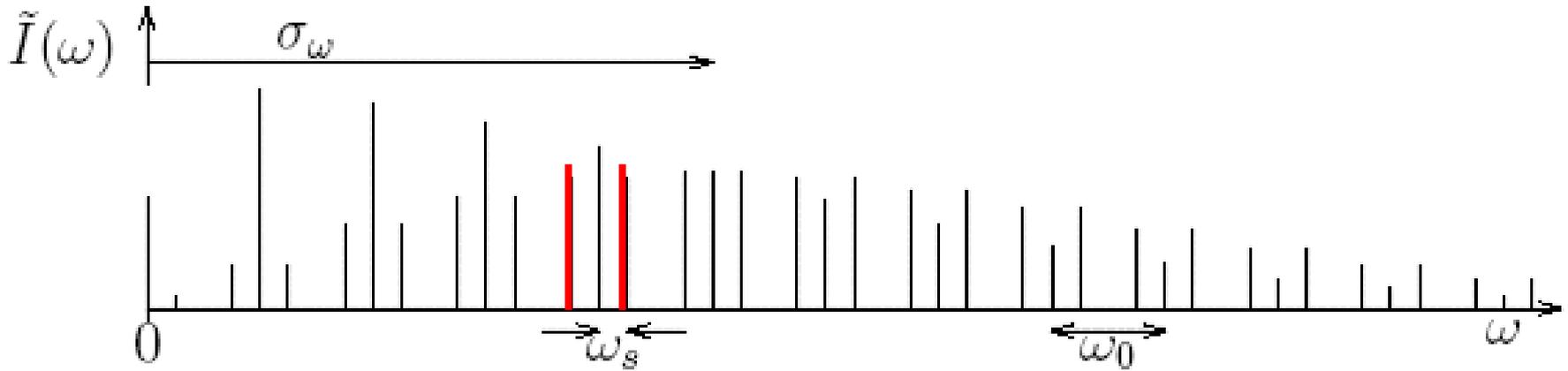
- Bunch executes synchrotron oscillations → modulation of the signals → sidebands in the spectrum,  $Q_s$  apart from the carriers

# Oscillating bunch (longitudinal):



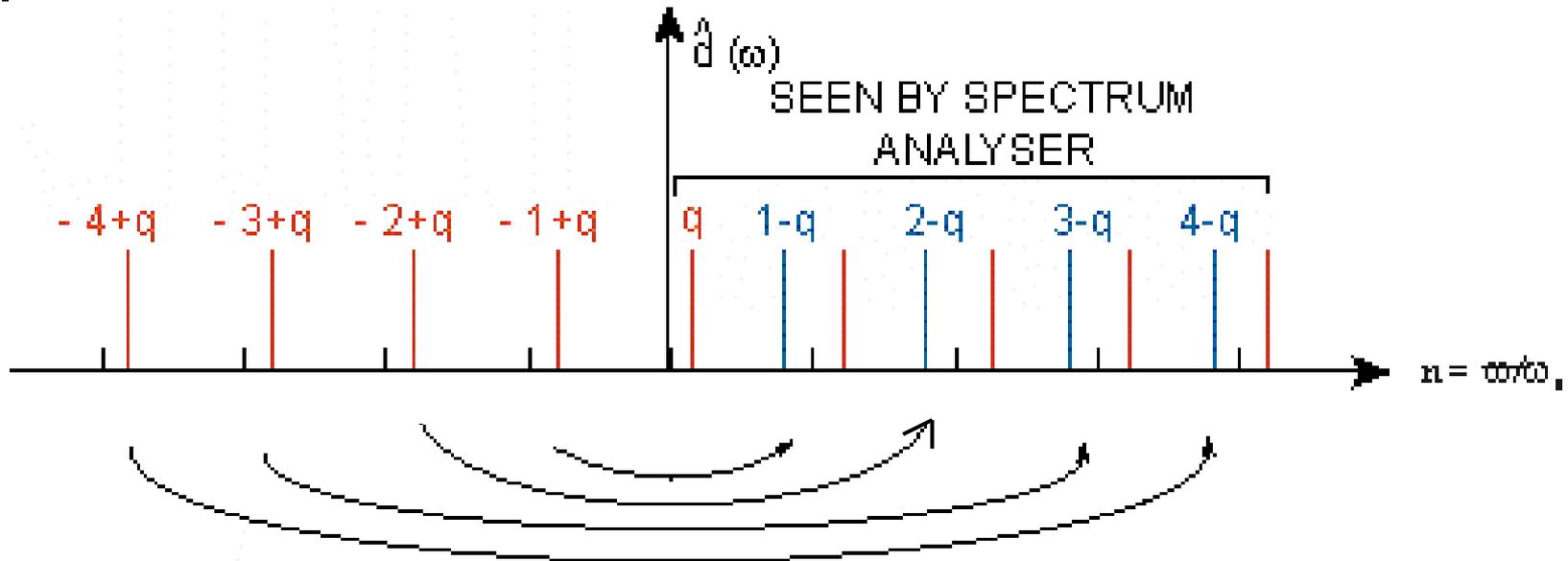
- The two (red) lines around  $\omega_0$  and separated by  $\pm \omega_s$  are called sidebands.

# Oscillating bunch (longitudinal):



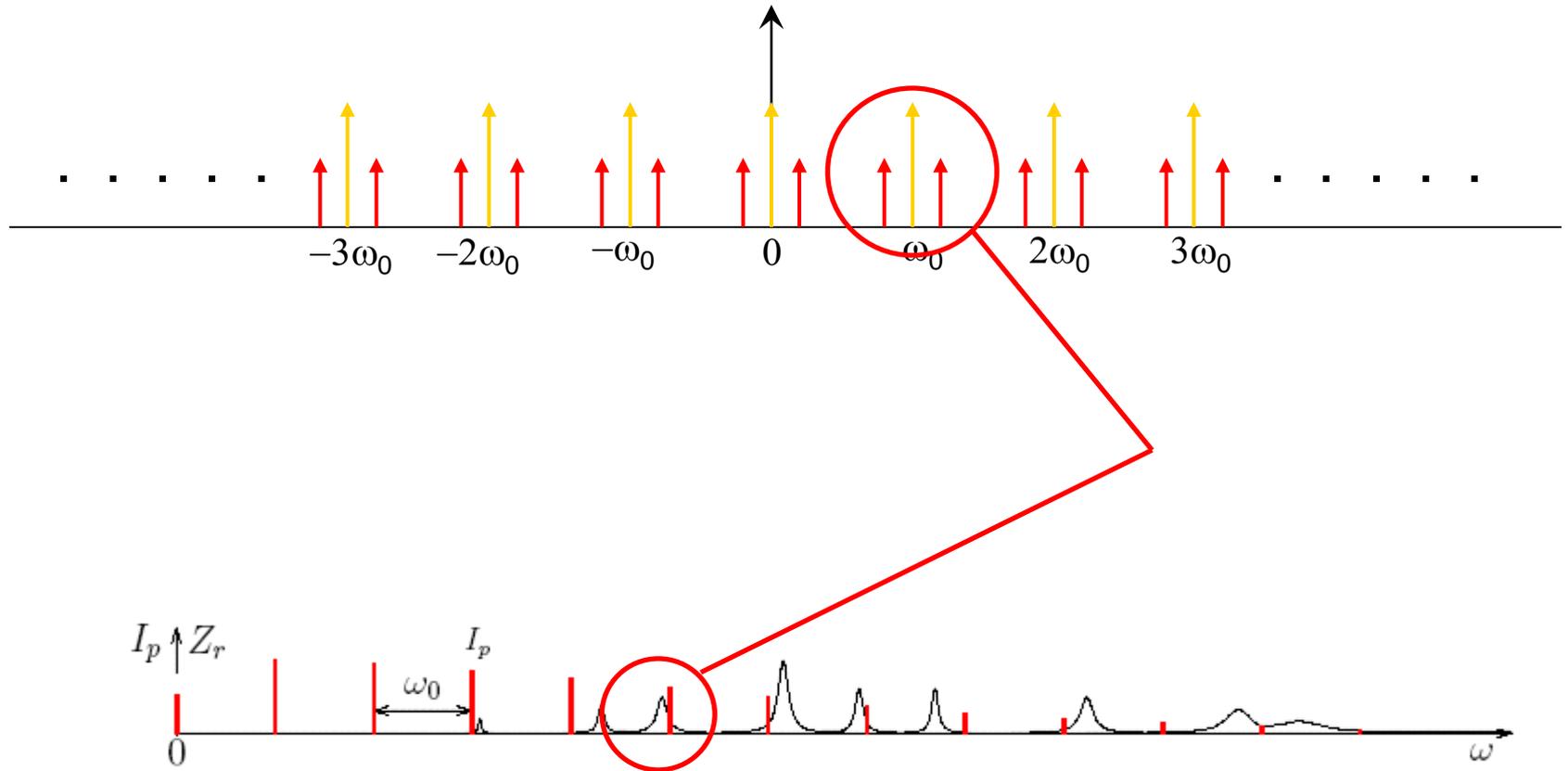
- Picture is similar in **the transverse planes**, but sidebands separated by the **non-integer part** of the betatron tune (**q**)

# Transverse plane ( $q=0.2$ )



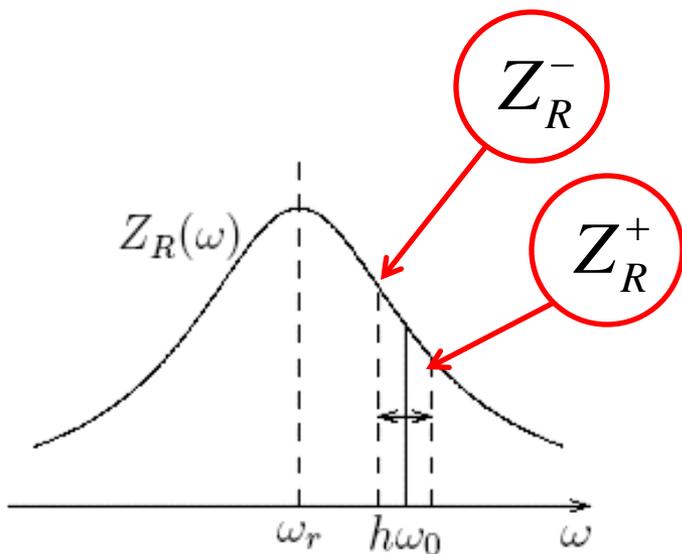
- On the spectrum analyser, we observe the « upper side bands » (USB) and the « lower side bands » (LSB (blue)).

# Stability with Side Bands:



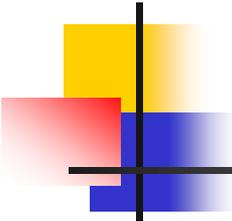
# Intuitive picture:

## Example: Longitudinal – Robinson instability



### Above transition:

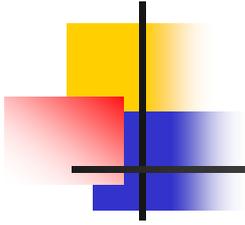
- $\omega_0$  smaller when energy is high
  - if  $\omega_r < h\omega_0$ , bunch sees more impedance if it has an energy **excess** (more losses).
  - **Less losses** if it has a **lack** of energy
- **damping !**
- Situation **reversed below transition !**



# First recipe (Golden Rule 1):

Conditions for stability (damping) SB:

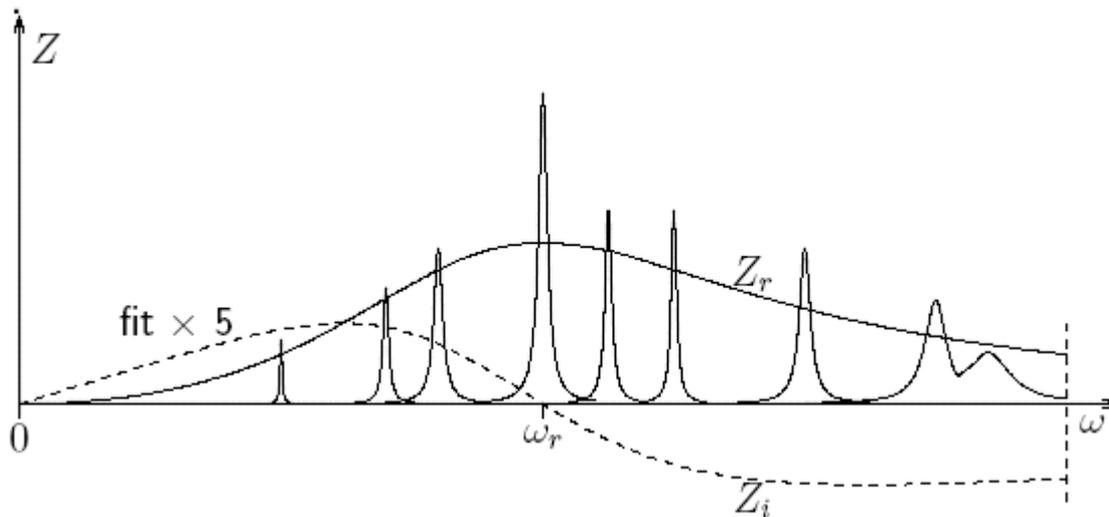
	Below transition	Above transition
Longitudinal	$Z_R^+ > Z_R^-$	$Z_R^+ < Z_R^-$
Transverse	$Z_{RT}^+ > Z_{RT}^-$	$Z_{RT}^+ > Z_{RT}^-$
Chromaticity	$Q' < 0$	$Q' > 0$



## 2. Broad-band model

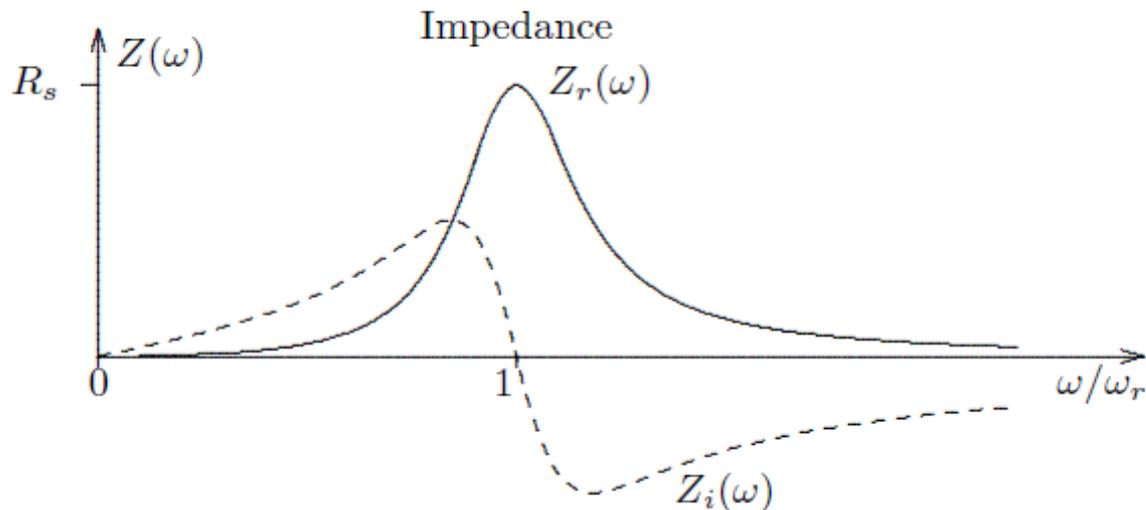
## General rules (2):

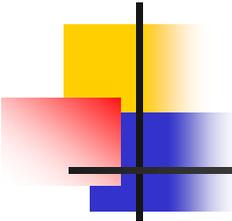
- If you are interested in **single traversal stability**, it is very useful and practical to move to the Broad Band Resonator Model (BBR). Only 3 parameters to be defined to obtain  $Z_R(\omega)$ ,  $Z_I(\omega)$  and  $G(t)$ . Usually, one takes a value of  $Q = 1$ .



# BBR model:

- With the BBR model, you treat the whole machine as a single resonator, for which the expressions for the impedance (wake field) are known.
- Same wake function as for the RLC circuit but only one set of parameters.

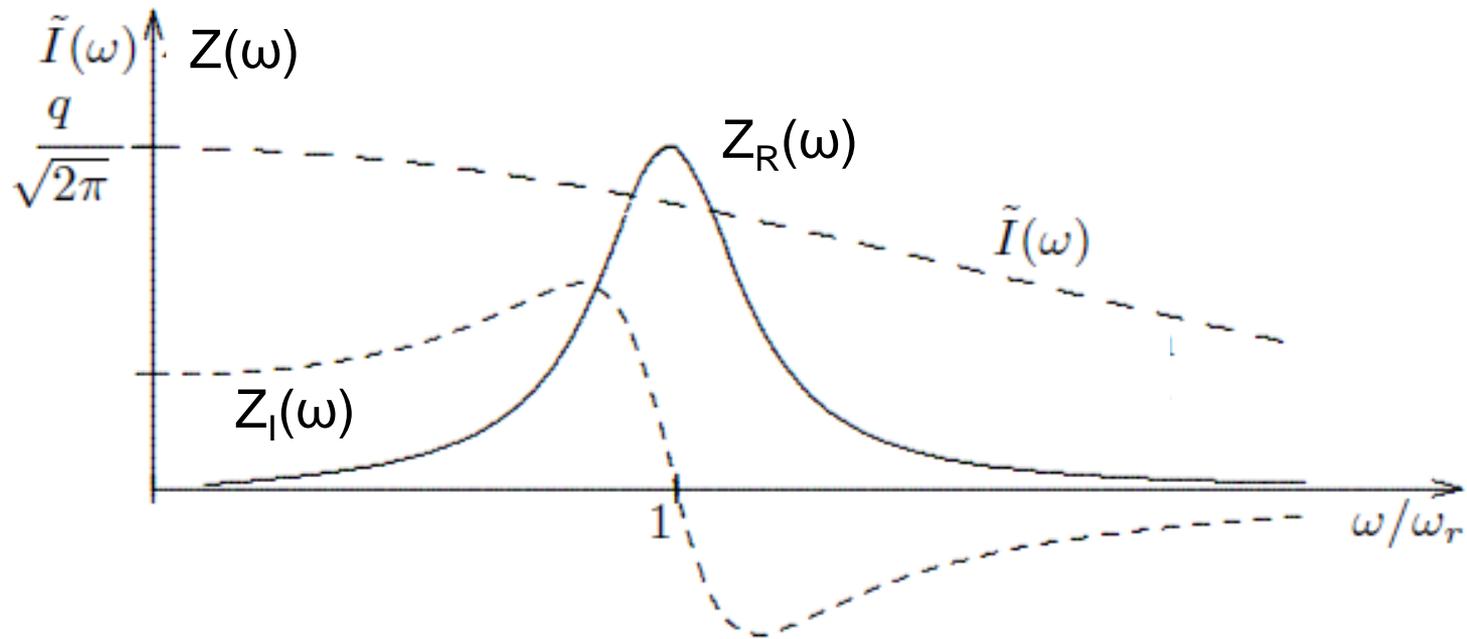


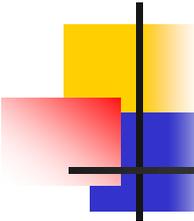


# Longitudinal impedance: $|Z/n|$

- Usually, the longitudinal impedance of a machine is rather characterised by the value  $|Z/n|$  (sum of the inductive parts at low frequencies divided by  $n=\omega/\omega_0$ ).  $|Z/n| = L\omega_0$
- This has the advantage that the resulting value becomes **independent of the size of the machine** and allows therefore for an easy **comparison between different machines**.
- In addition, by plotting  $(Z/n)$  rather than  $Z(\omega)$ , it has the advantage that the plots for longitudinal and transverse impedances exhibit a very similar behaviour.

# $Z/n$ or $Z_T$

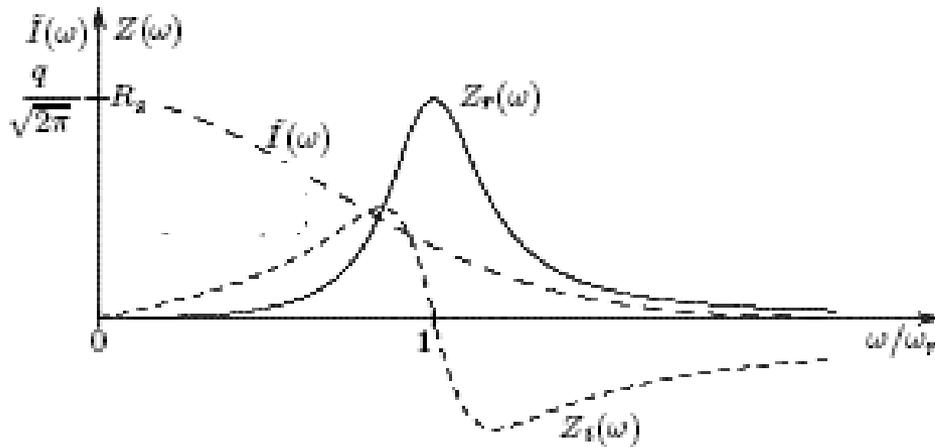




# $|Z/n|$ as a function of time:

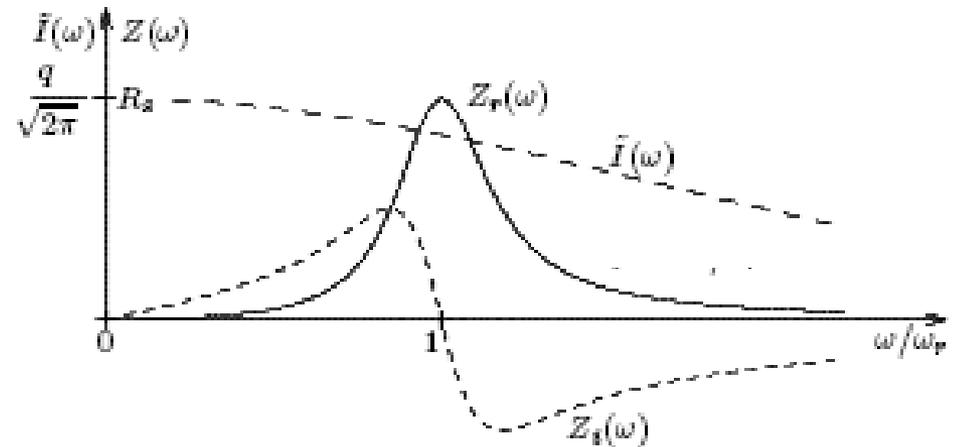
Machine	$ Z/n $ [ $\Omega$ ]
PS (~ 1960)	> 50
SPS (~ 1970)	~ 20
LEP (~ 1990)	~ 0.25 (1.0)
LHC (~ 2010)	~ 0.10 (0.25)

# Effect of the bunch length:

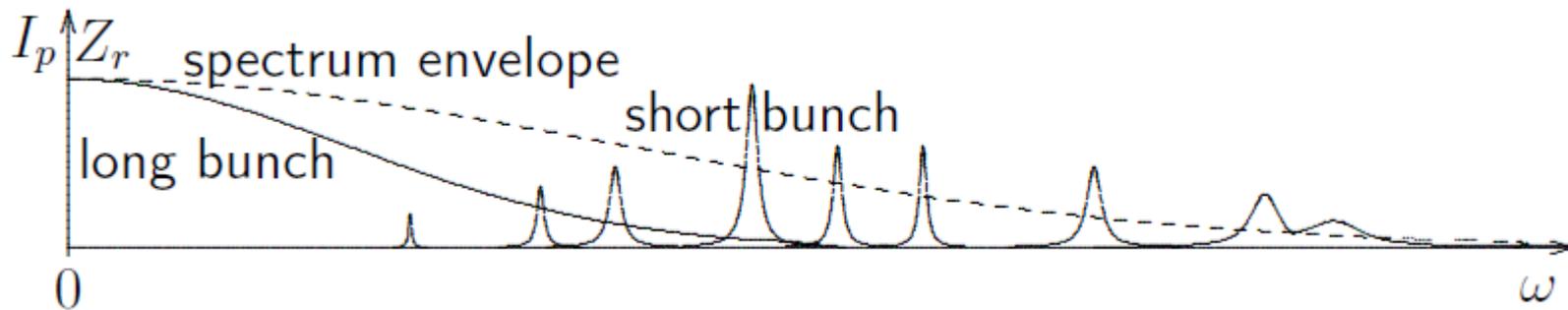


- Long bunch

- Short bunch

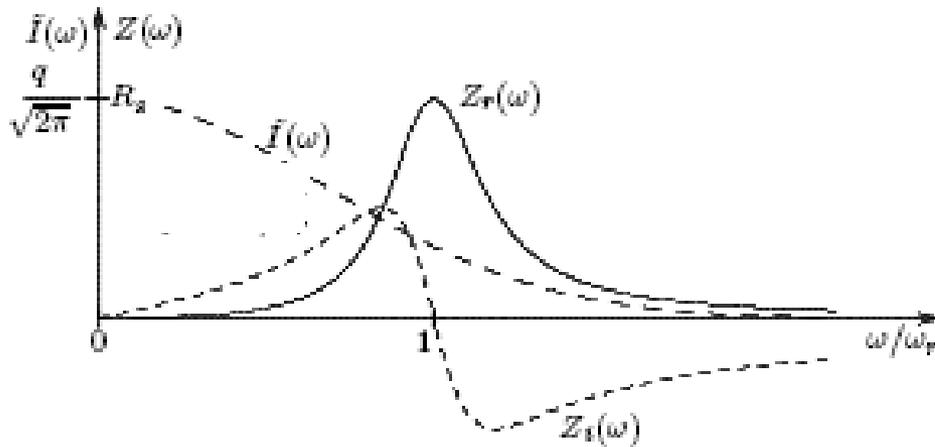


# Shortening the bunch...



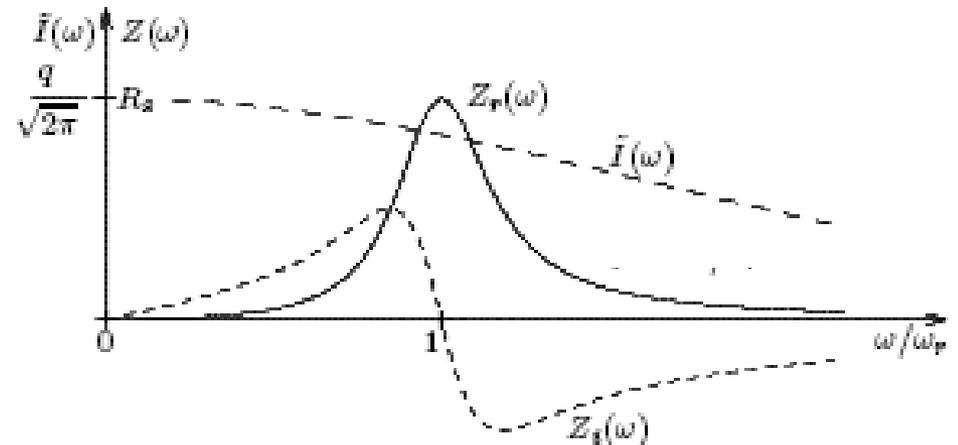
Making the bunch shorter does not change its spectrum at low frequencies, but extends it to higher frequencies than before. The bunch interact therefore with an extended impedance, leading to a larger energy loss!

# Convolution bunch with impedance:



- Long bunch

- Short bunch



## Second recipe (longitudinal):

### Golden Rule 2

The convolution between the bunch spectrum and the **real part** of the longitudinal impedance yields the **energy loss** of the bunch interacting with the impedance

The convolution between the bunch spectrum and the **imaginary part** of the longitudinal impedance yields the **tune shift  $\Delta Q$**  resulting from the interaction

$$k_{pm} \approx \frac{1}{q^2} \int Z_R(\omega) |\tilde{I}(\omega)|^2 d\omega$$

$$\Delta Q \propto \int Z_I(\omega) |\tilde{I}(\omega)|^2 d\omega$$

# Transverse impedance:

For the BBR-model, there exists a very handy relation for the transverse impedance  $Z_T(\omega)$ , namely:

$$Z_T(\omega) = \frac{2R}{b^2} \left| \frac{Z_L(\omega)}{n} \right|$$

$$n = \frac{\omega}{\omega_0}$$

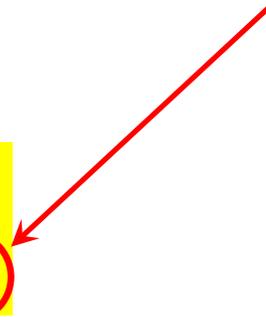
$$Z_T = \frac{\Omega}{m}$$

Watch out !!!

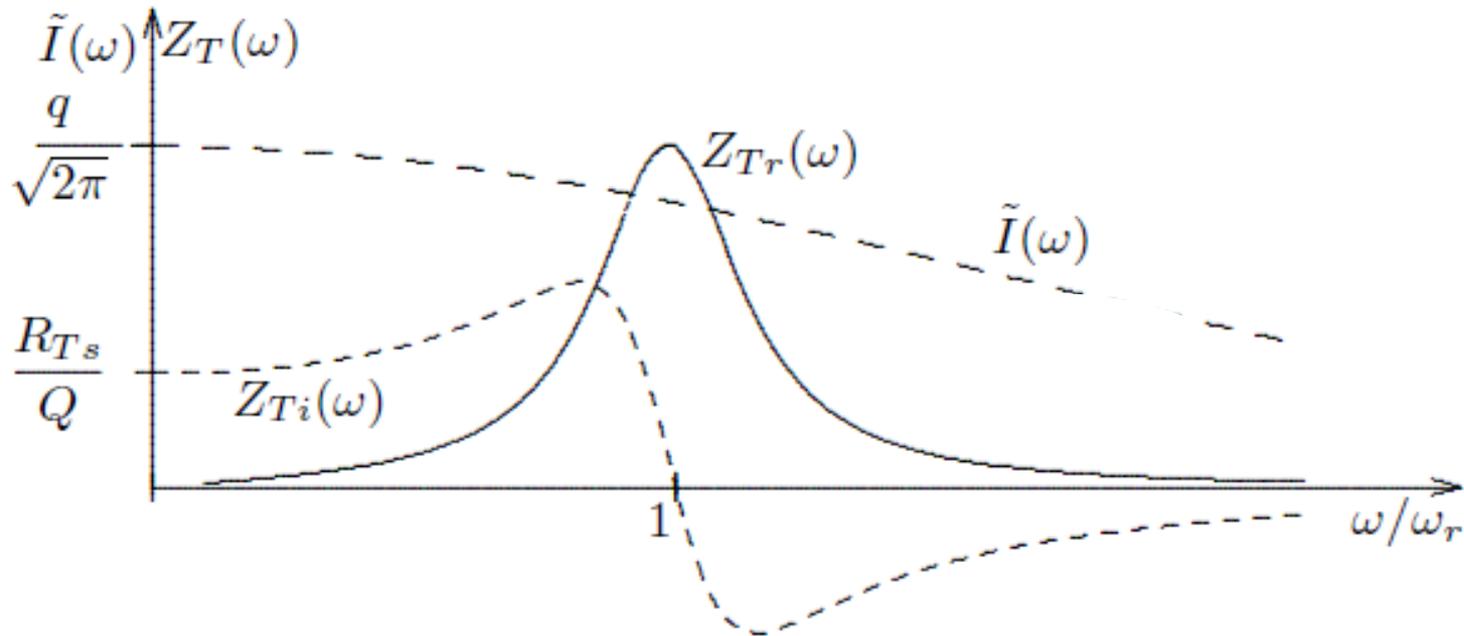
$$Z_L(\omega) \propto \frac{1}{b}$$



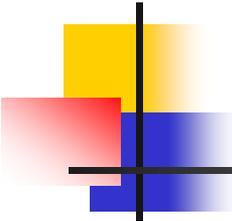
$$Z_T(\omega) \propto \frac{1}{b^3}$$



# Transverse impedance ( $\Omega/m$ )



Here again, perform convolution between bunch spectrum and impedance



# Third recipe (transverse):

## Golden Rule 3

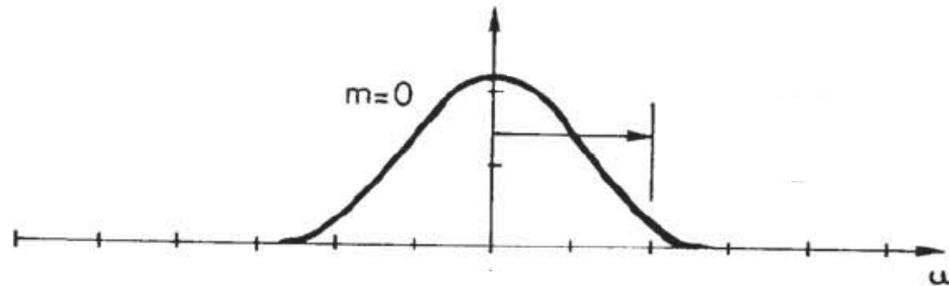
The convolution between the bunch spectrum and the real part of the transverse impedance yields the growth time of the instability

The convolution between the bunch spectrum and the imaginary part of the transverse impedance yields the tune shift  $\Delta Q$  resulting from the interaction

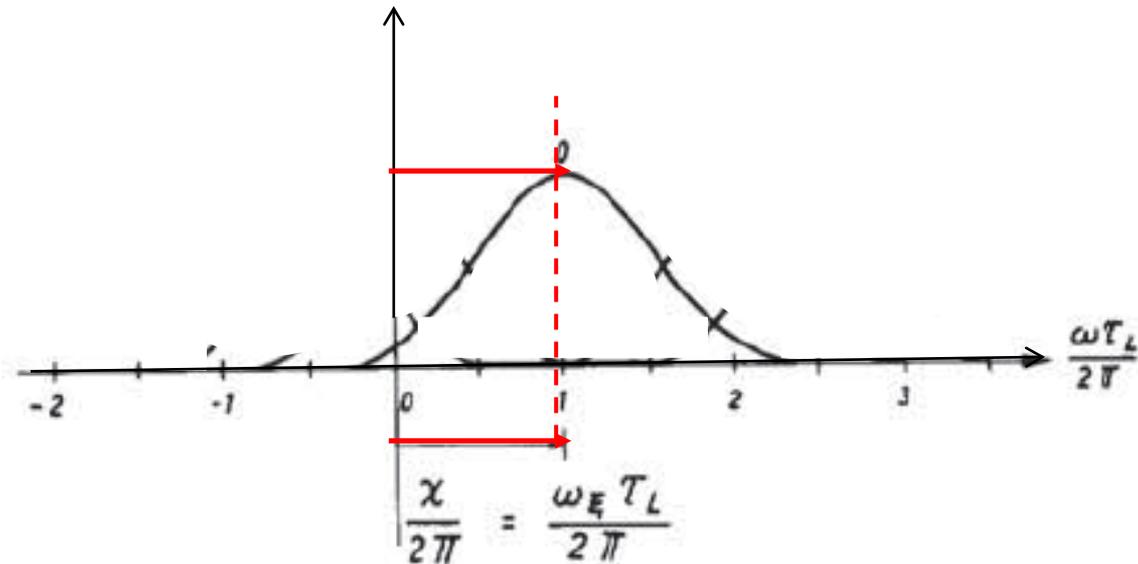
- **Watch out:** a **change** in chromaticity  $Q'$  can have **beneficial** or **dramatic** effects on the situation. **Why this ?**

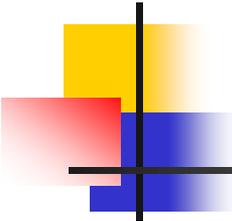
# Spectrum of the modes:

Normal:  $Q' = 0$



New:  $Q' > 0$





# With shifted spectrum:

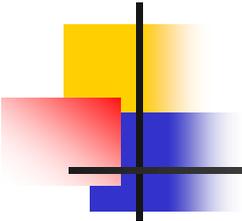
---

- Since bunch spectrum is shifted → convolution integrals are modified. The main usual consequences are:

- **Convolutions with** the longitudinal and transverse real parts of the impedances increase → **energy loss** ↗ and **instability growth time** ↘

- **Convolutions with** the longitudinal and transverse imaginary parts of the impedances decrease → **measured tune shifts smaller** → could yield **misleading conclusions** on the effective impedance !

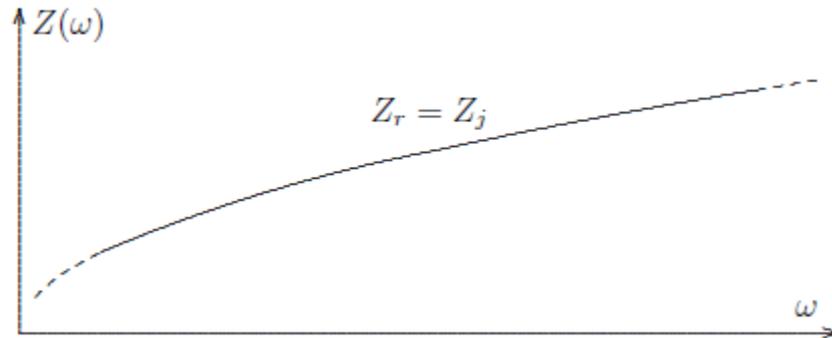
- A too large positive increase of  $Q'$  in order to stabilize the  $m=0$  head-tail mode (above transition) might lead the  $m=1$  mode becoming unstable (LEP, LHC ?).



## 3. Resistive wall

# Resistive wall impedance:

Longitudinal



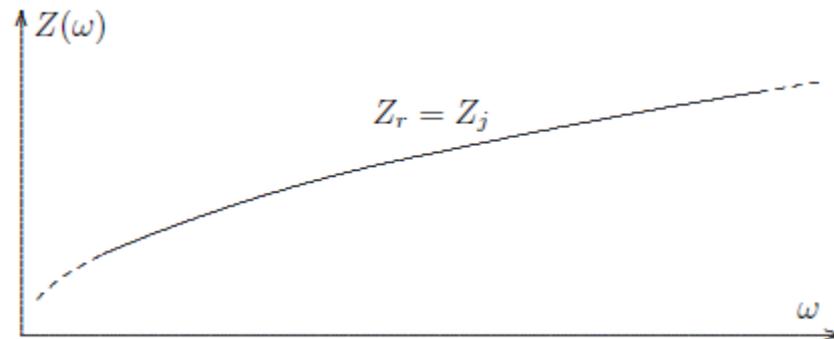
$$Z(\omega) = \frac{(1+j)R}{b} \sqrt{\frac{\mu_0 \omega}{2\sigma_c}}$$

$\mu$  = permeability  
 $\sigma_c$  = conductivity

- Resistive and Inductive parts have equal magnitude
- $Z(\omega)$  is a steadily growing function ( $\omega^{1/2}$ )

# Resistive wall impedance:

Longitudinal



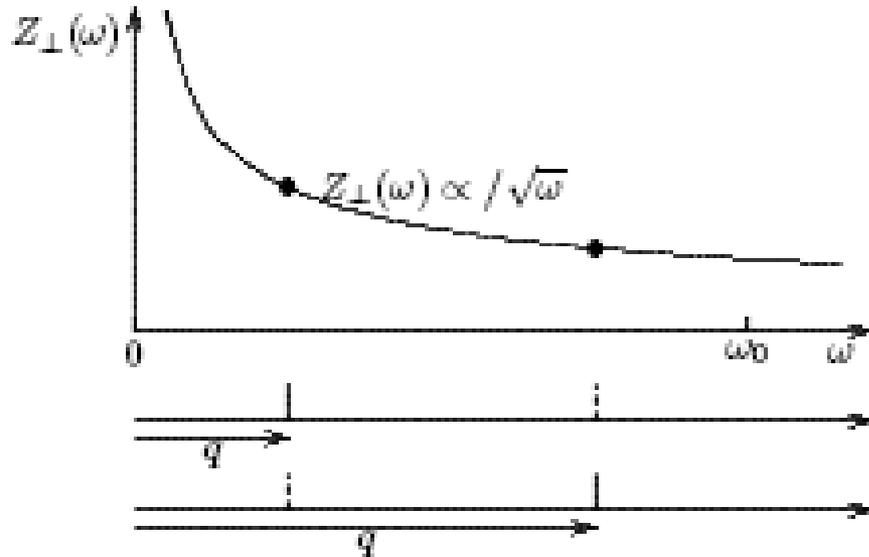
•  $Z_R^+(\omega) > Z_R^-(\omega) \rightarrow$  no damping above transition (Rule 1) !

$\rightarrow$  Potential source of instability ! Select good material !

Need Landau damping or feedback system !

... and what about the transverse plane ?  $Z_T \propto Z_L/\omega$  !!!

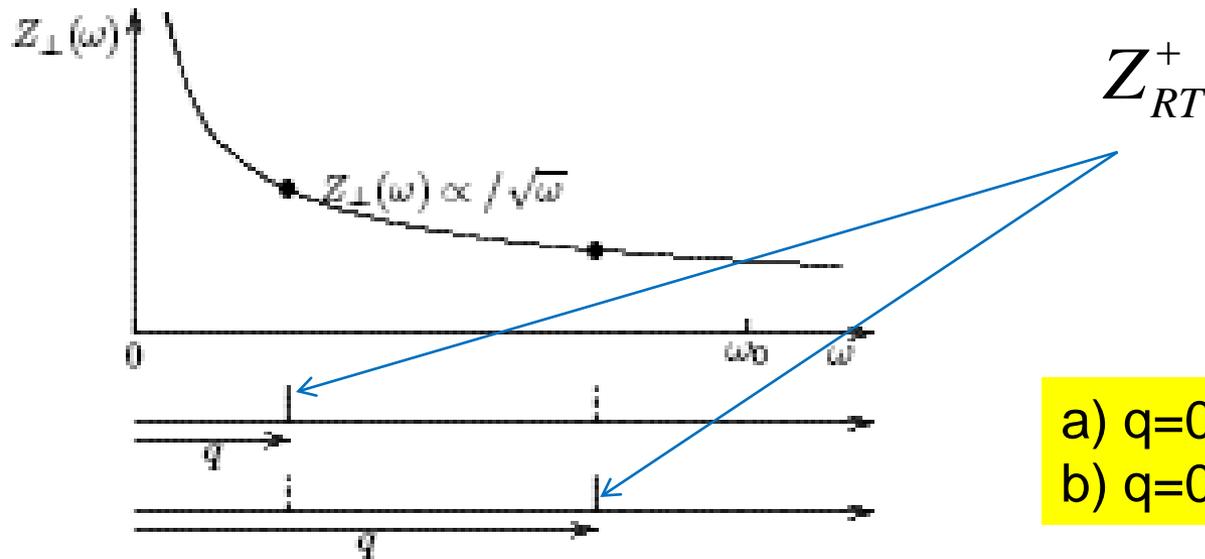
# Resistive wall (transverse)



a)  $q=0.2$   
b)  $q=0.8$

- Transverse stability:  $Z_{RT}^+ > Z_{RT}^-$  !

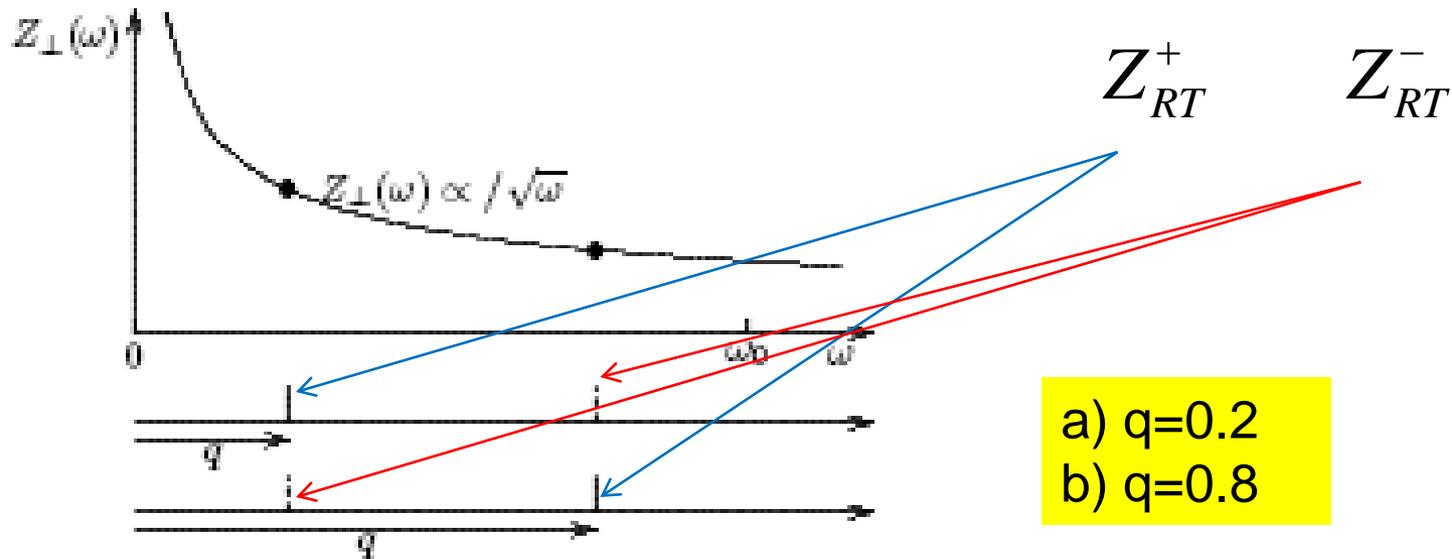
# Resistive wall (transverse)



a)  $q=0.2$   
b)  $q=0.8$

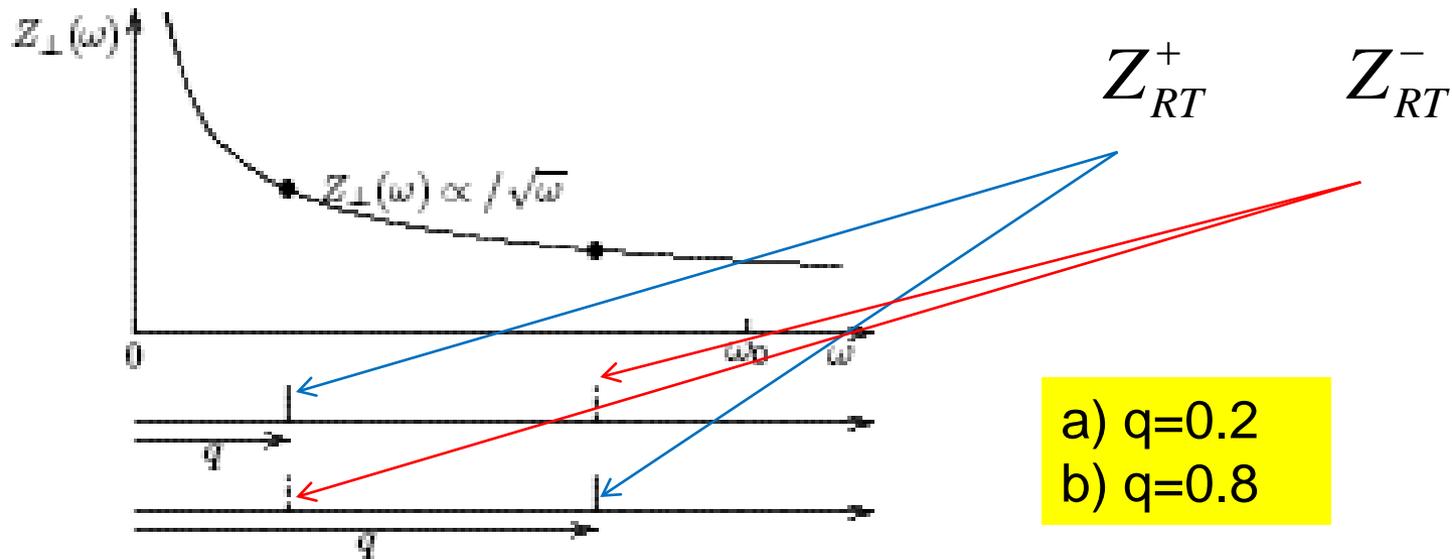
- Transverse stability:  $Z_{RT}^+ > Z_{RT}^-$  !

# Resistive wall (transverse)



- Transverse stability:  $Z_{RT}^{+} > Z_{RT}^{-}$  !

# Resistive wall (transverse)



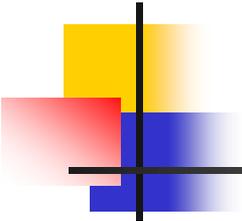
- Transverse stability:  $Z_{RT}^+ > Z_{RT}^-$  !

→ Select a tune  $q$  below the half integer ( $q < 0.5$ ) for stability !

# LHC Beam-Screen (material)

- Without proper Cu-coating of the beam-screen, nominal intensity foreseen for the LHC could not circulate in the machine!

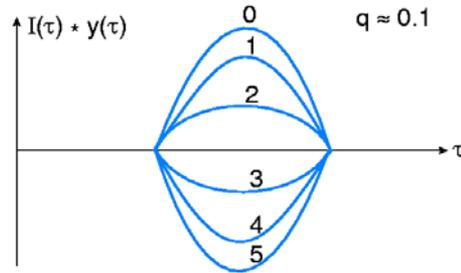




---

## 4. The Head – Tail Instability

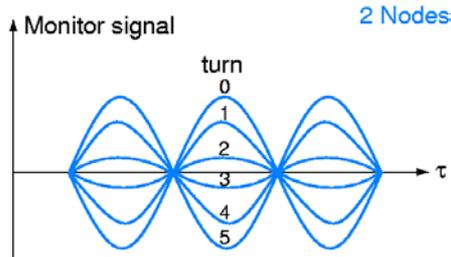
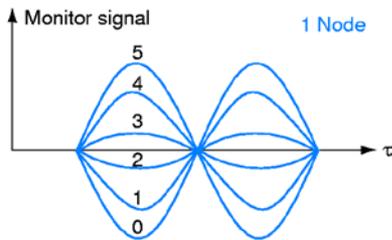
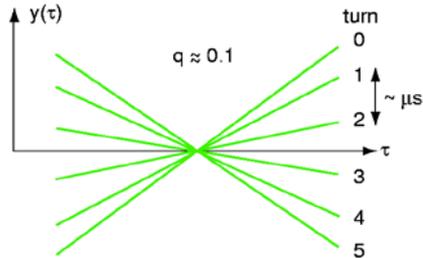
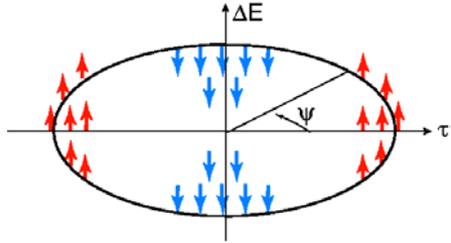
# Head-tail modes $m$ (single bunches)



**m=0**

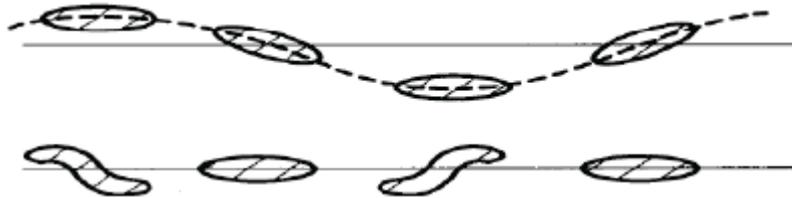
**m=2**

**m=1**

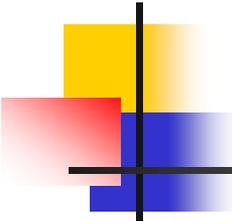


# m=0 and m=1

- What is the mode m=0 ? Nothing but the **tune** of the machine !



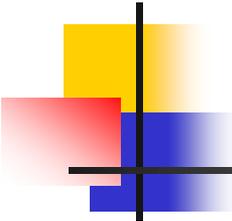
- What is the mode m=1 ? Dipole oscillation with head and tail of the bunch out of phase by  $\pi$ . On top of this, the particles perform a synchrotron oscillation.



# General principle:

---

- The **Head – Tail instability** is directly related to the **chromaticity ( $Q'$ )** of the machine.
- As already mentioned , there could be some combinations of beam parameters and impedances such that the tail of the bunch is influenced by the wakes created by the head of the same bunch.
- Since the particles in the bunch perform synchrotron oscillations, there could be a situation where the synchrotron motion yields an accumulation of the effect and result in an instability of the bunch (usually very fast).



# Reminder: chromaticity

---

$$Q' = \Delta Q / (\Delta p/p)$$

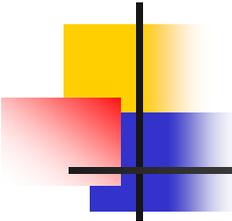
- Take a particle and slightly **increase** its momentum:

$$\rightarrow \Delta p/p > 0 \rightarrow \Delta Q < 0 \rightarrow Q' < 0$$

- Take a particle and slightly **decrease** its momentum:

$$\rightarrow \Delta p/p < 0 \rightarrow \Delta Q > 0 \rightarrow Q' < 0$$

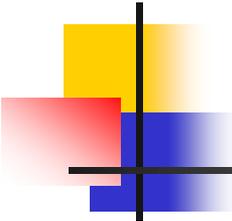
The natural chromaticity  
of the machine  $Q'$  is  
**always negative !**



# Head-Tail instability 1 (and GR 1):

---

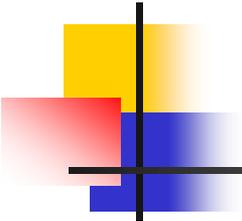
- The most dangerous mode is the mode  $m=0$ :
  - It is unstable below transition ( $\gamma < \gamma_t$ ), if the chromaticity is positive ( $Q' > 0$ )
  - It is unstable above transition ( $\gamma > \gamma_t$ ), if the chromaticity is negative ( $Q' < 0$ )
- Higher order modes ( $m \geq 1$ ) are unstable for **negative chromaticities below transition** and for **positive chromaticities above transition**. However, they are much slower and they can be naturally damped by other sources of tune spread, or can be suppressed with a damper.
- As a consequence, it is critical to control the mode  $m=0$  by operating the machine with the correct sign of chromaticity (and keep  $Q'$  low to avoid  $m=1$  instability).



# Head-Tail instability 2

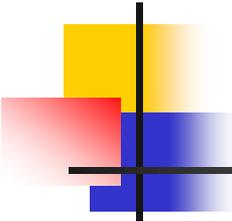
---

- Machines that **run always below their transition energy** (usually hadron machines) must have **negative chromaticity** (e.g., the CERN-PSB, GSI-SIS) and they can live with their **natural chromaticity**. These machines can also avoid to use sextupoles for chromaticity correction.
- Machines that run always **above transition energy** (lepton machines, CERN-LHC, BNL-RHIC with protons) need **chromaticity correction** (and therefore many sextupoles families) in order to make their chromaticity slightly positive.
- Machines that **cross transition** (CERN-PS, CERN-SPS, BNL-RHIC with ions) need a scheme of **synchronized swap of the sign of chromaticity** at transition crossing (in addition to the required RF swap).



---

## 5. A few general remarks...



# More generally:

---

Very important application:

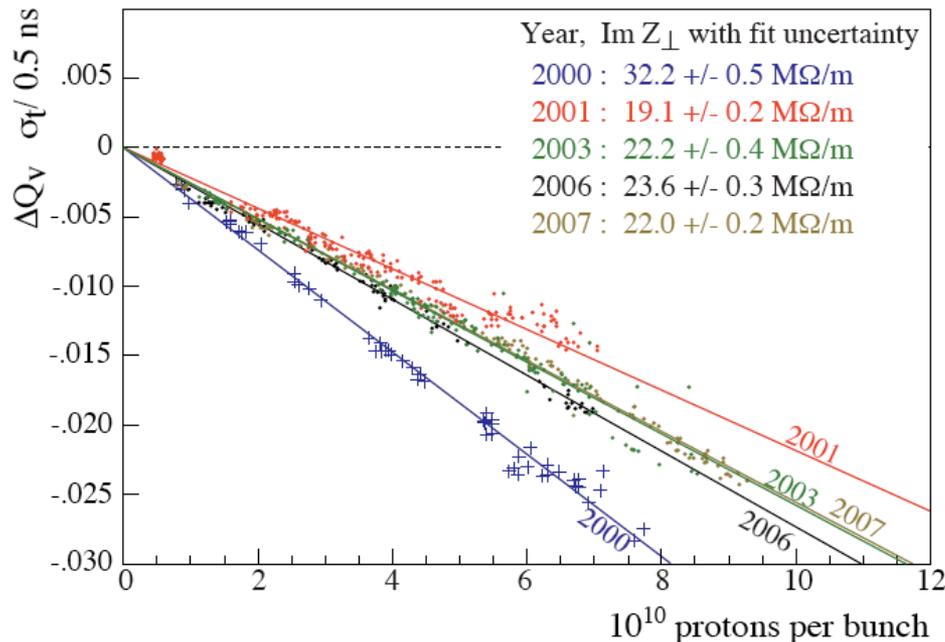
- According to **Golden Rules 2 and 3**, the interaction between the bunch spectrum and the imaginary part of the impedance is responsible for a tune shift  $\Delta Q$  (measurable).
- It is equivalent to say that the observed tune shift contains very important information on the actual impedance!

Essential for:

- Does the impedance correspond to the design (**commissioning**) ?
- Was the **scheduled improvement** campaign successful ?
- **Measurements of tunes vs. intensity done as soon as possible !**

# Examples of tune shift measurements

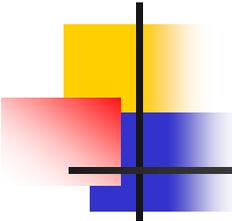
Vertical coherent tune shift with intensity at 26 GeV, scaled to 0.5 ns



- Measurements of coherent tune shift as function of intensity in the CERN-SPS (H. Burkhardt, G. Rumolo, F. Zimmermann)

⇒ From the slope of the tune shift one can infer the low frequency imaginary part of the machine impedance ( $iZ_{\text{eff}}$ ). Machines with flat beam pipes show usually no tune shift in the horizontal plane and significant tune shift in the vertical plane

⇒ Tune shift measurements done with high longitudinal emittance bunches can extend to high intensities because the TMCI threshold is higher

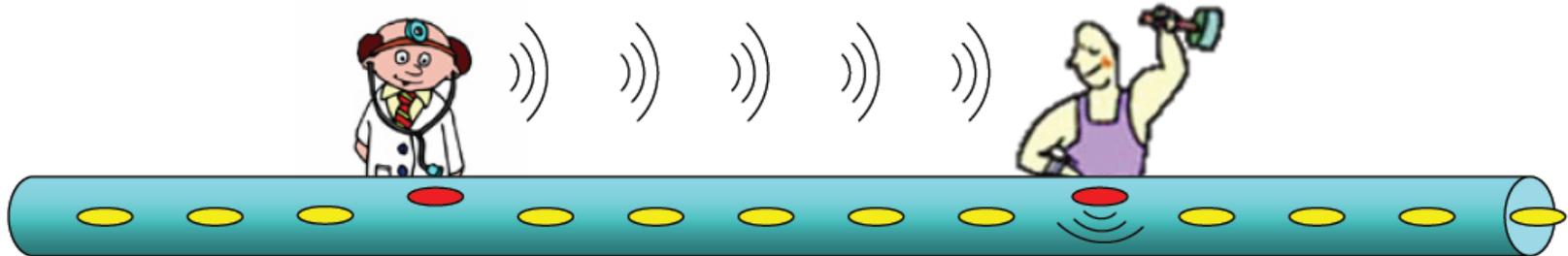


# Cures from a general point of view:

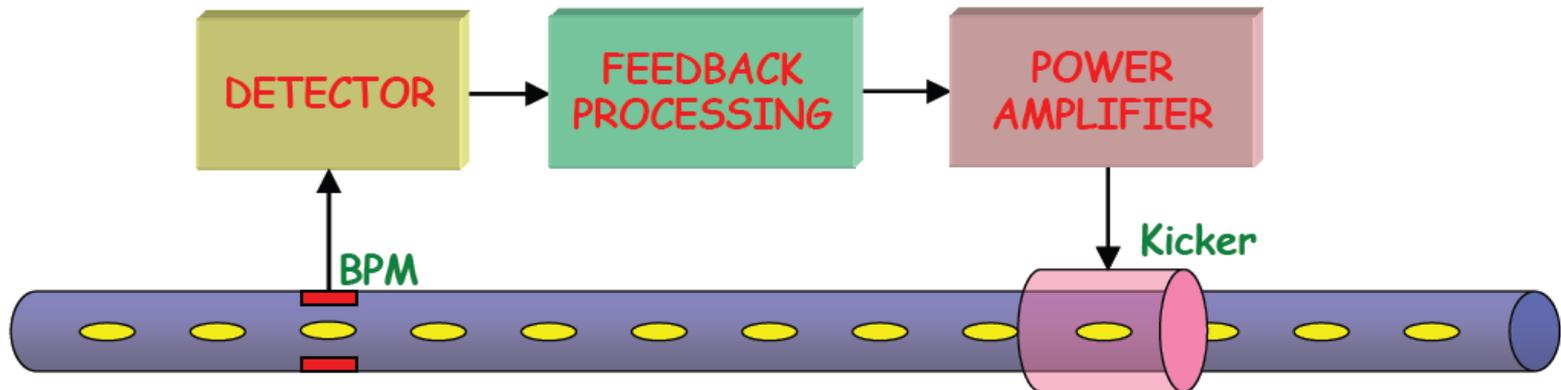
---

- Damp Higher Order modes (**HOM** = resonances) in the cavities by **HOM Dampers** (unwanted mode picked up by an antenna and sent to a damping resistor).
- Protect from unwanted « **cavities** » in the beam pipe with adequate **RF-shielding** (mimick a smooth beam pipe).
- Avoid any **abrupt changes** in the beam pipe cross section.
- Use **highly conductive materials** whenever possible.
- Use **feedback systems** (both for longitudinal and transverse planes).
- Study carefully « **Landau Damping** » effects (Intermediate CAS).

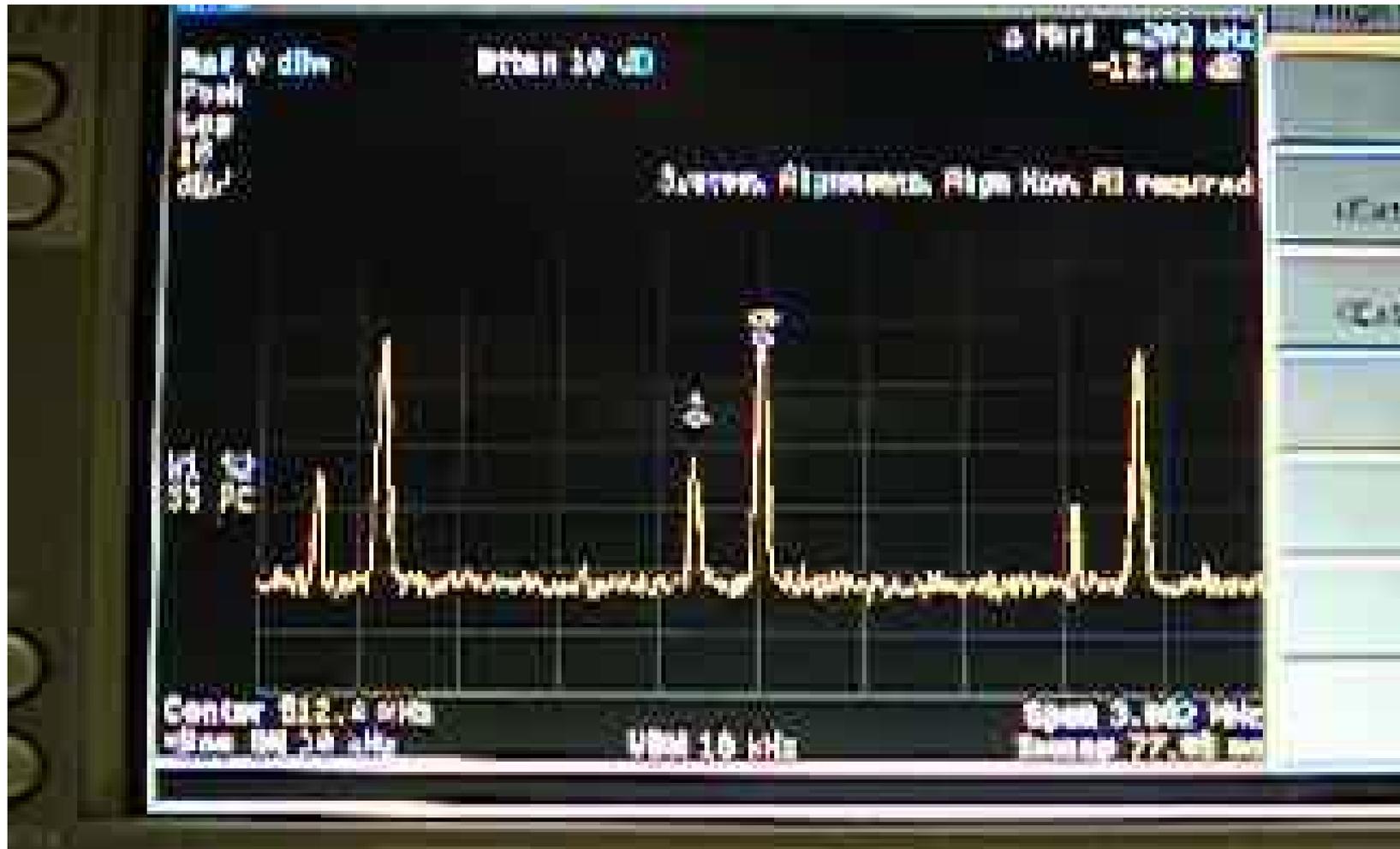
# Feedback systems:



Since the oscillation is a sinusoidal, one measures the **position** with a BPM and correct the **slope** with a kicker located at  $k\pi/2$  ( $k$  odd) in phase advance.

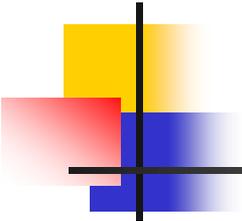


# Feedback at Elettra (Trieste)



# Feedback effect from SR

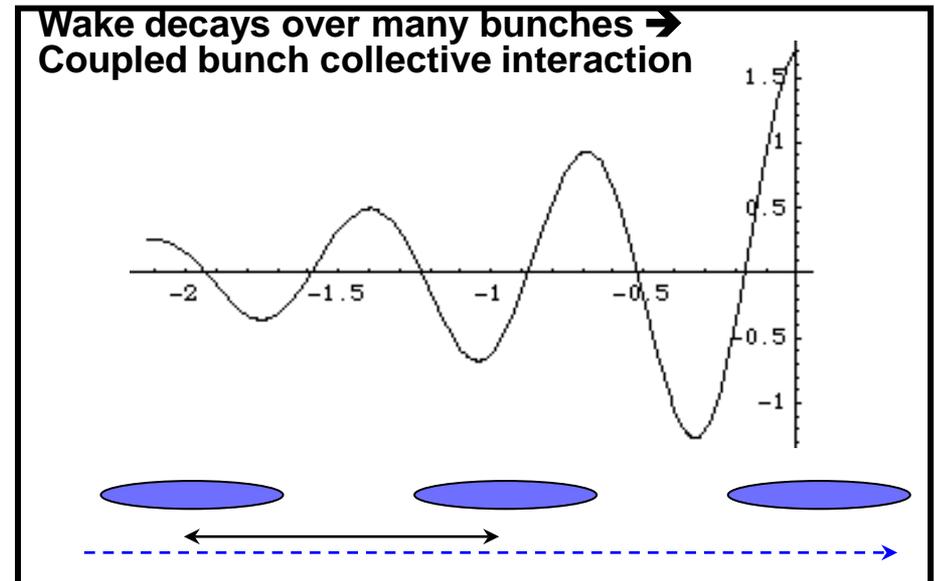
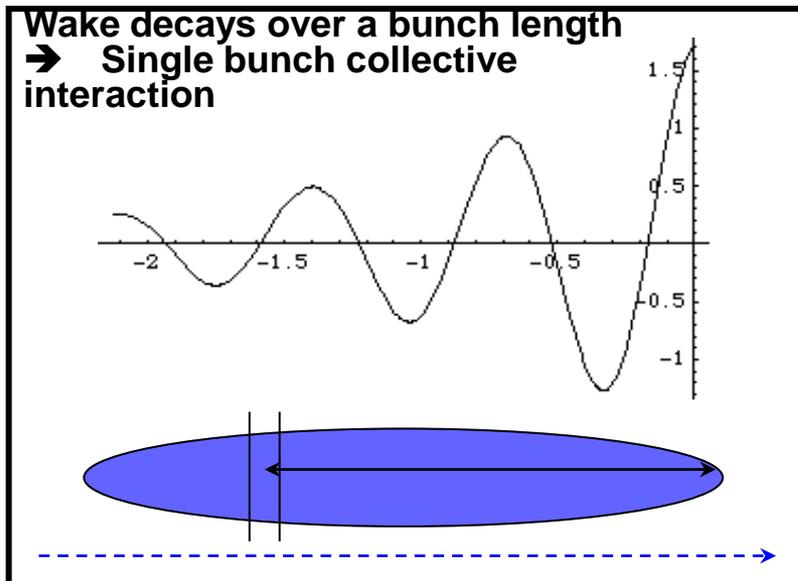


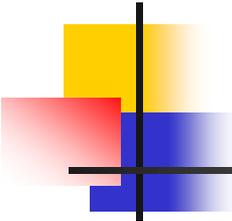


# Multi-bunch effects

# Multi-bunch effects

- Multi-bunch effects require long lived excitations.
- The e.m. effects created by one bunch are transferred to the other bunches via the impedance.





# Coupled-bunch effects

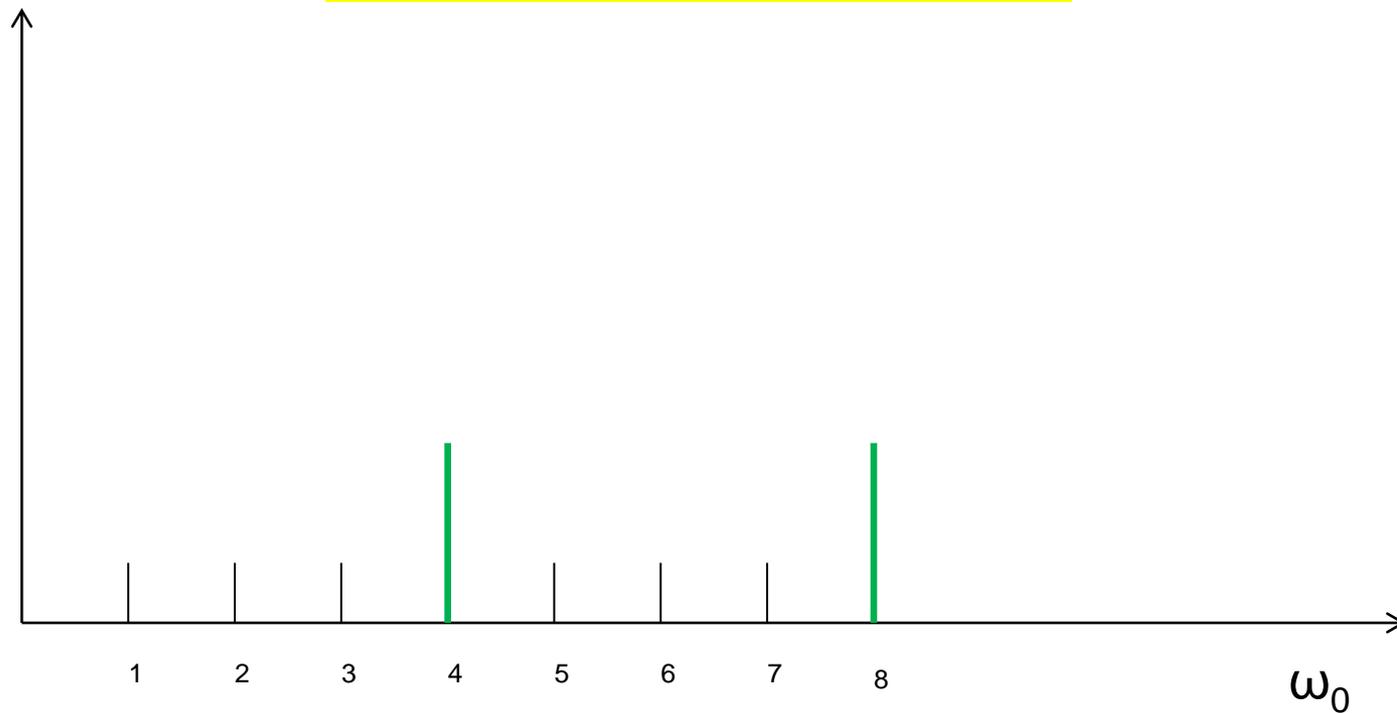
---

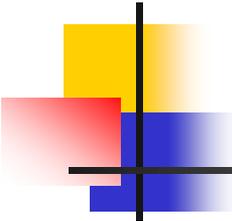
- With **N** bunches in the machine, there exist **N** different possible modes of oscillations ( $n=0, 1, 2 \dots N-1$ ).

- If the **bunches** are **stable**, then there exist only **lines** at the multiple of  $N\omega_0$ .

# On the screen:

4 bunches – all modes stable

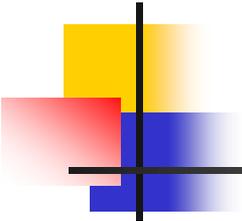




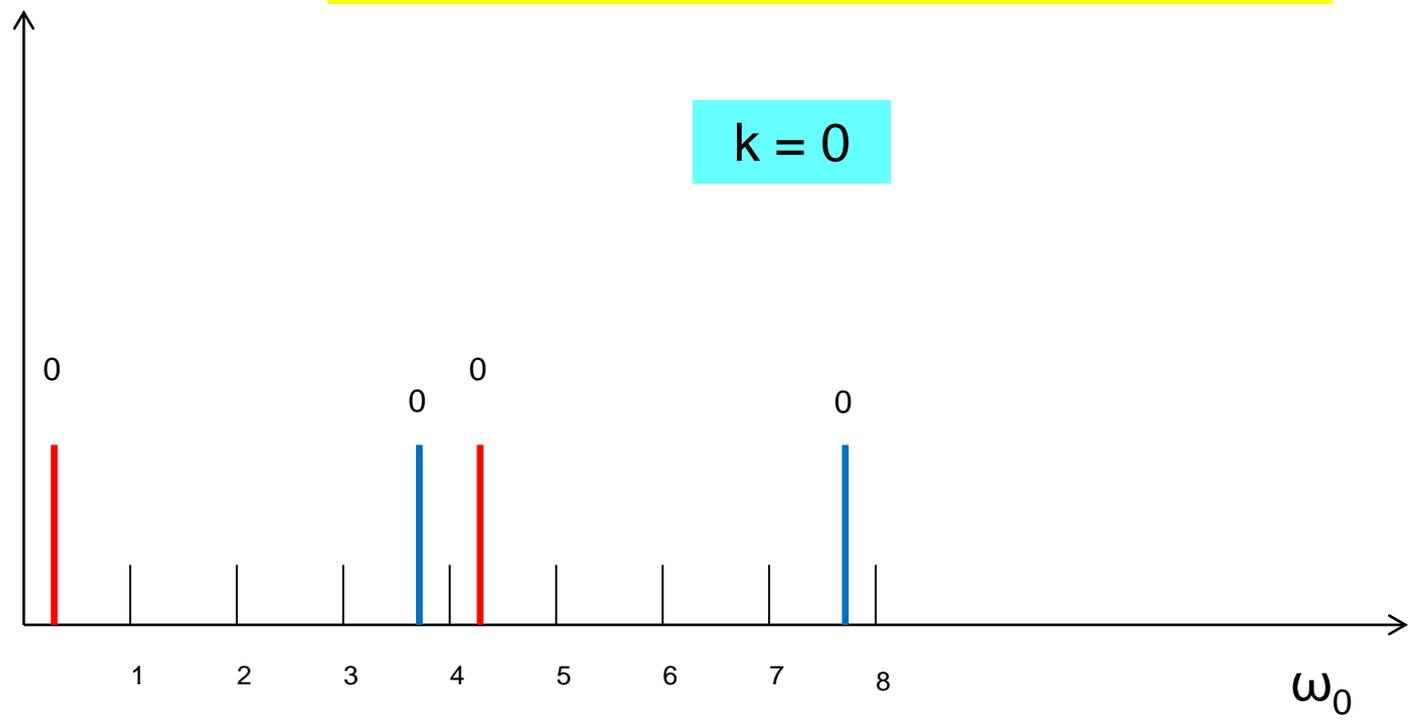
# Coupled-bunch effects

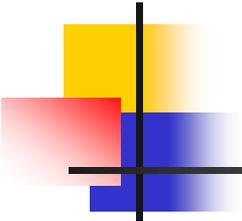
---

- With  $N$  bunches in the machine, there exist  $N$  different possible modes of oscillations ( $n=0, 1, 2 \dots N-1$ ).
- If the **bunches** are **stable**, then there exist only **lines** at the multiple of  $N\omega_0$ .
- If one mode (e.g.  $k$ ) is unstable, then lines at  $k\omega_0$  will appear with both **USB** and **LSB**.

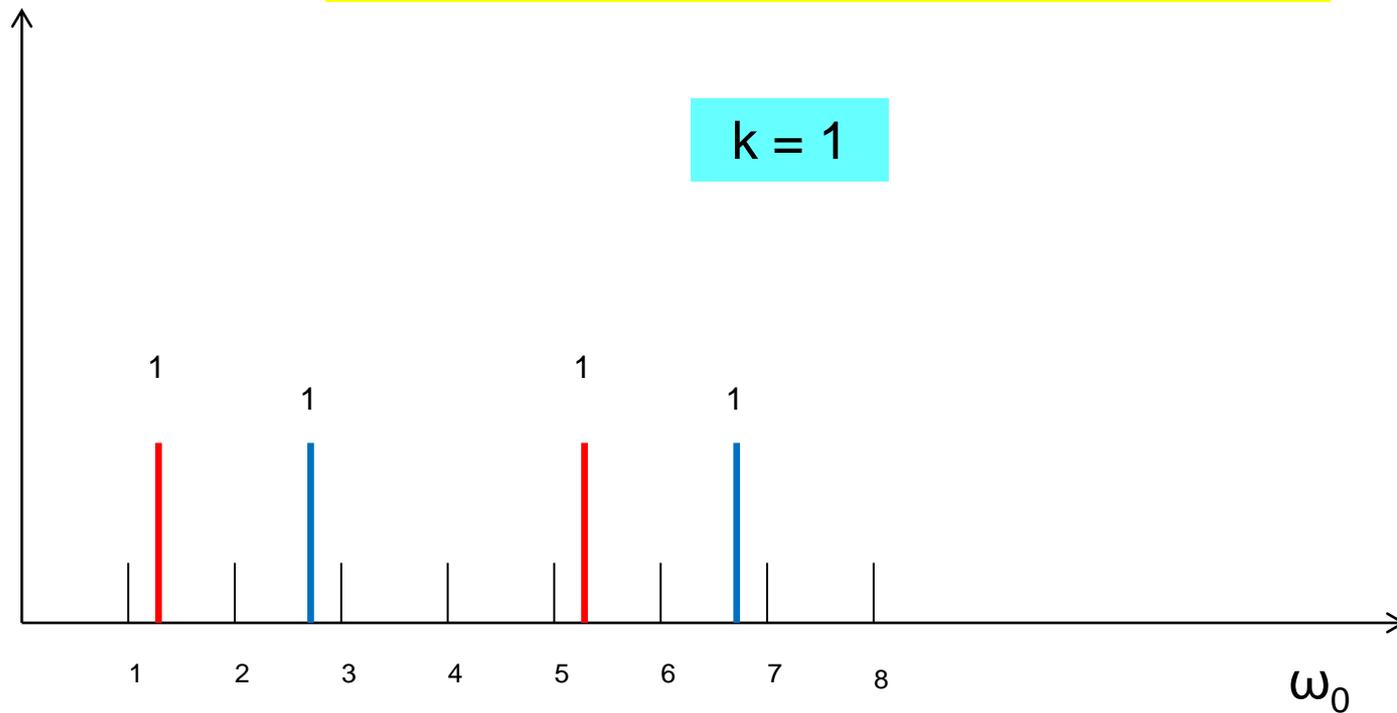


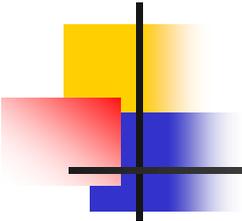
4 bunches – individual modes unstable





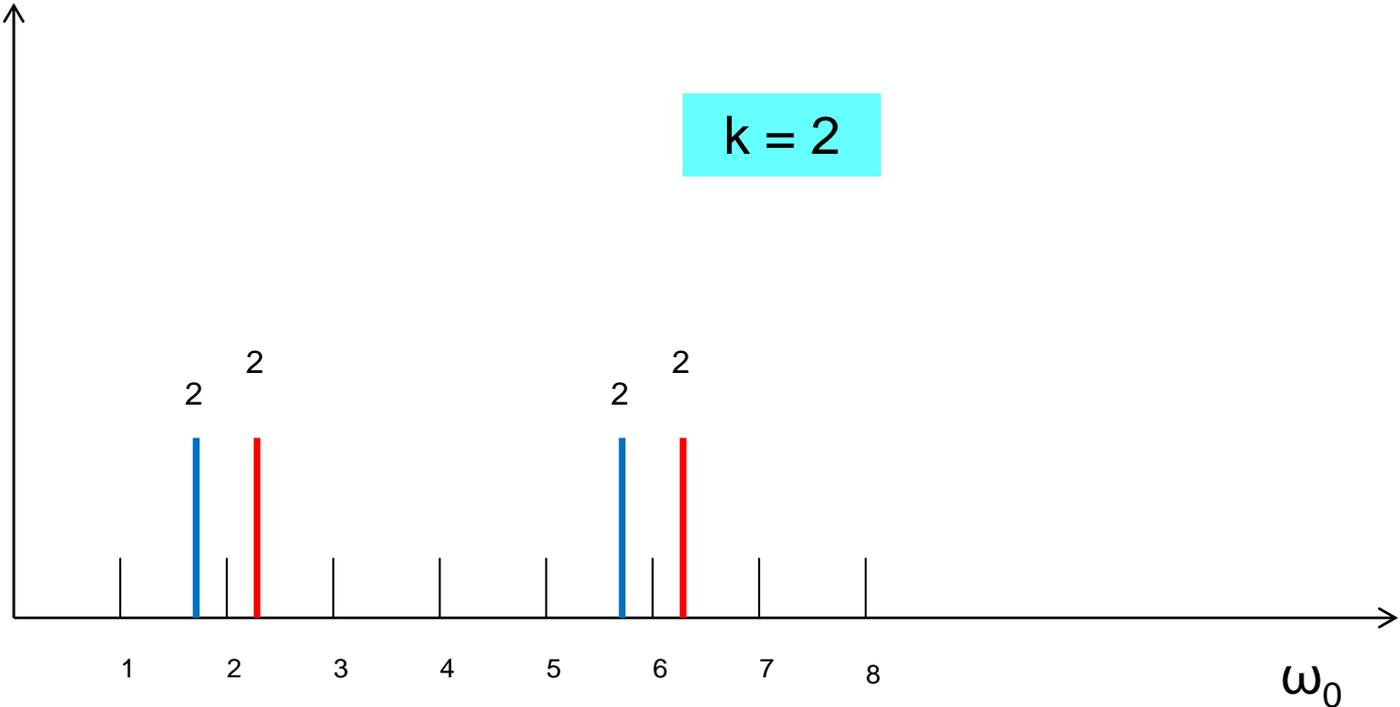
4 bunches – individual modes unstable



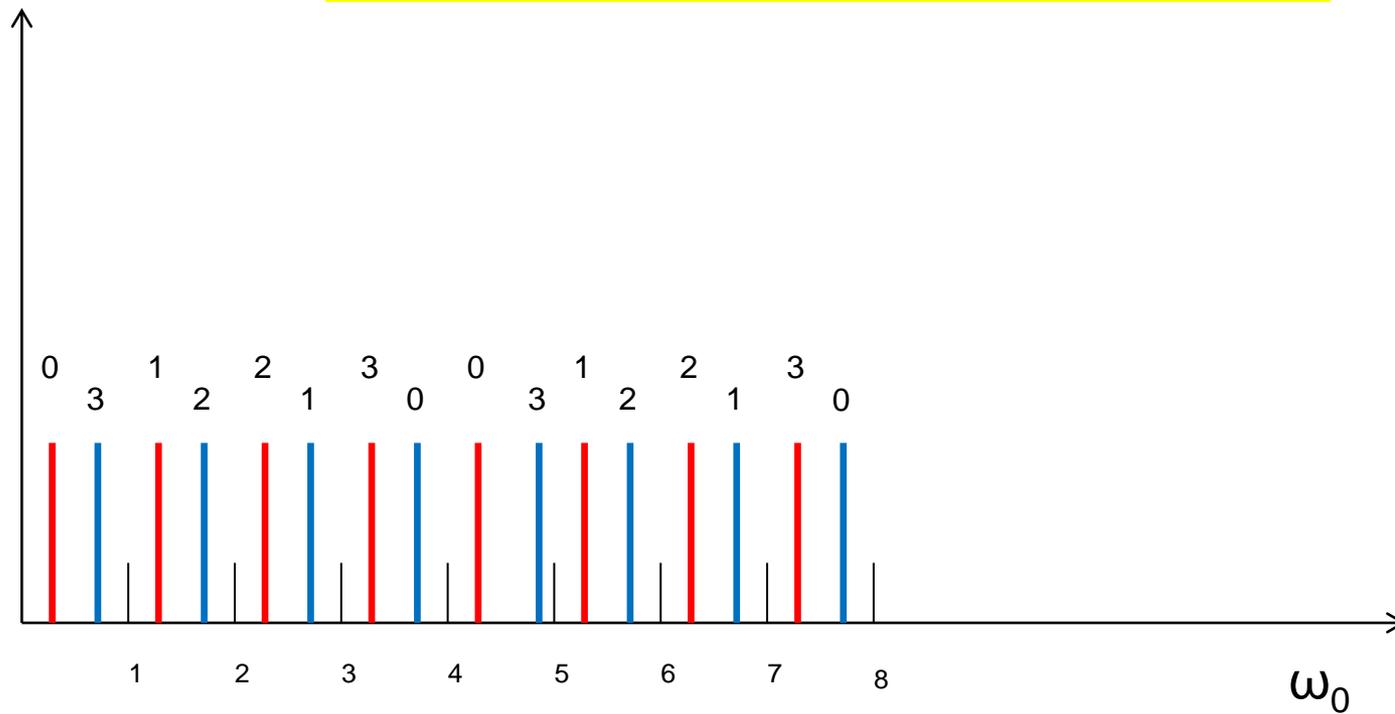


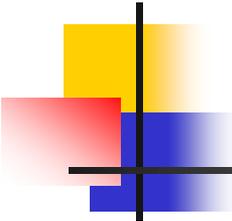
4 bunches – individual modes unstable

$k = 2$



4 bunches – individual modes unstable





# Coupled-bunch effects

---

- With  $N$  bunches in the machine, there exist  $N$  different possible modes of oscillations ( $n=0, 1, 2 \dots N-1$ ).

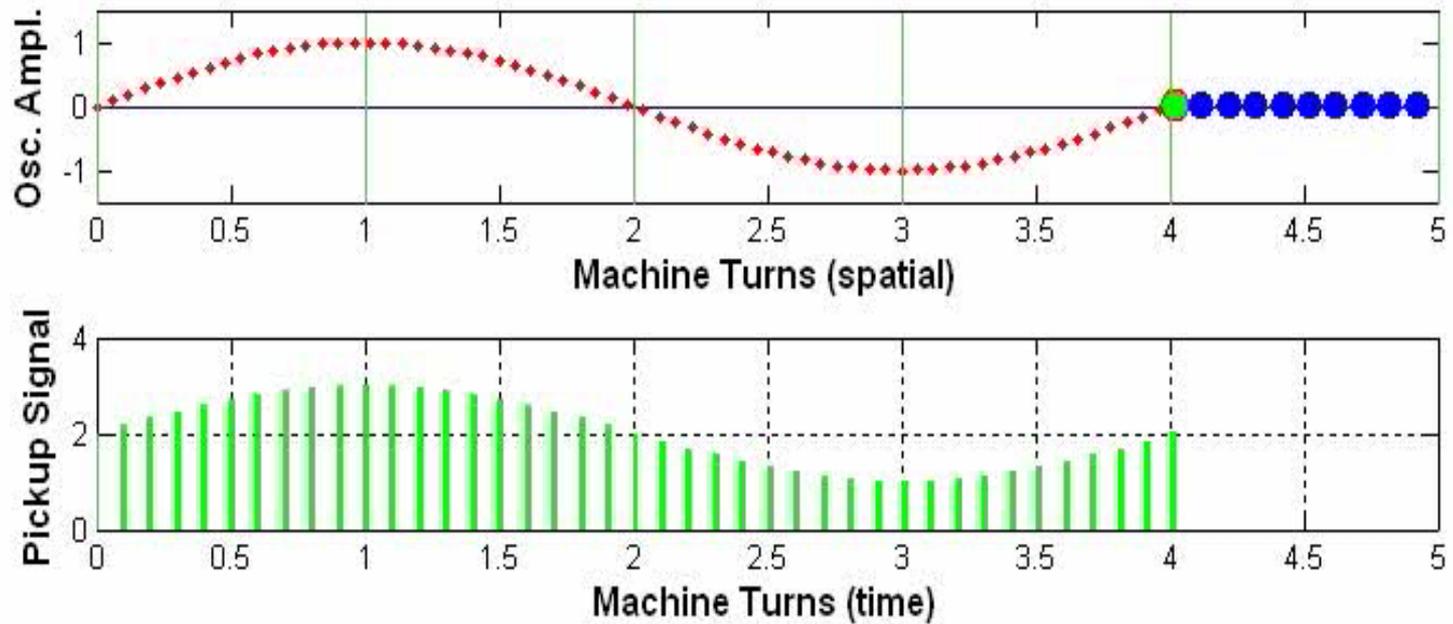
- If the **bunches** are **stable**, then there exist only **lines** at the multiple of  $N\omega_0$ .

- If one mode (e.g.  $k$ ) is unstable, then lines at  $k\omega_0$  will appear with both **USB** and **LSB**.

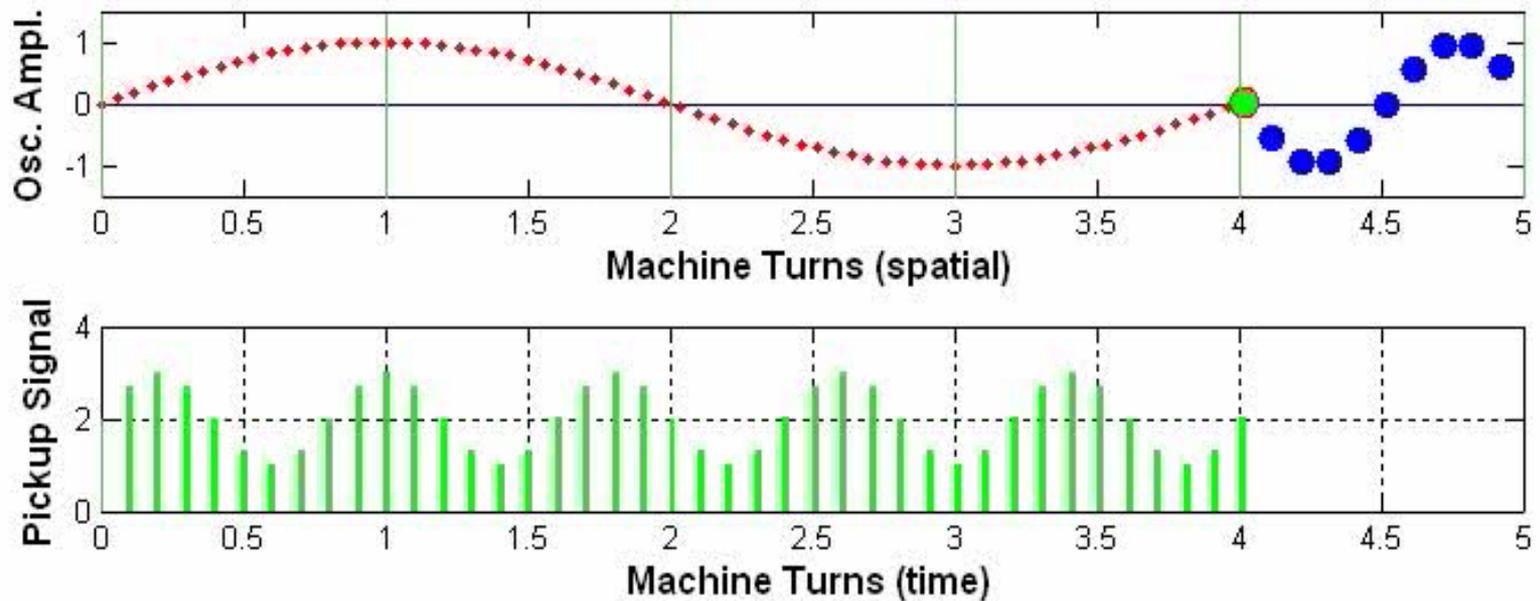
- **Usually** there are lines (**USB and LSB**) at **most of the multiple of the revolution frequency**.

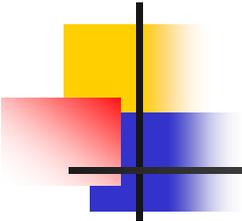
# Coupled-bunch oscillations modes $n$

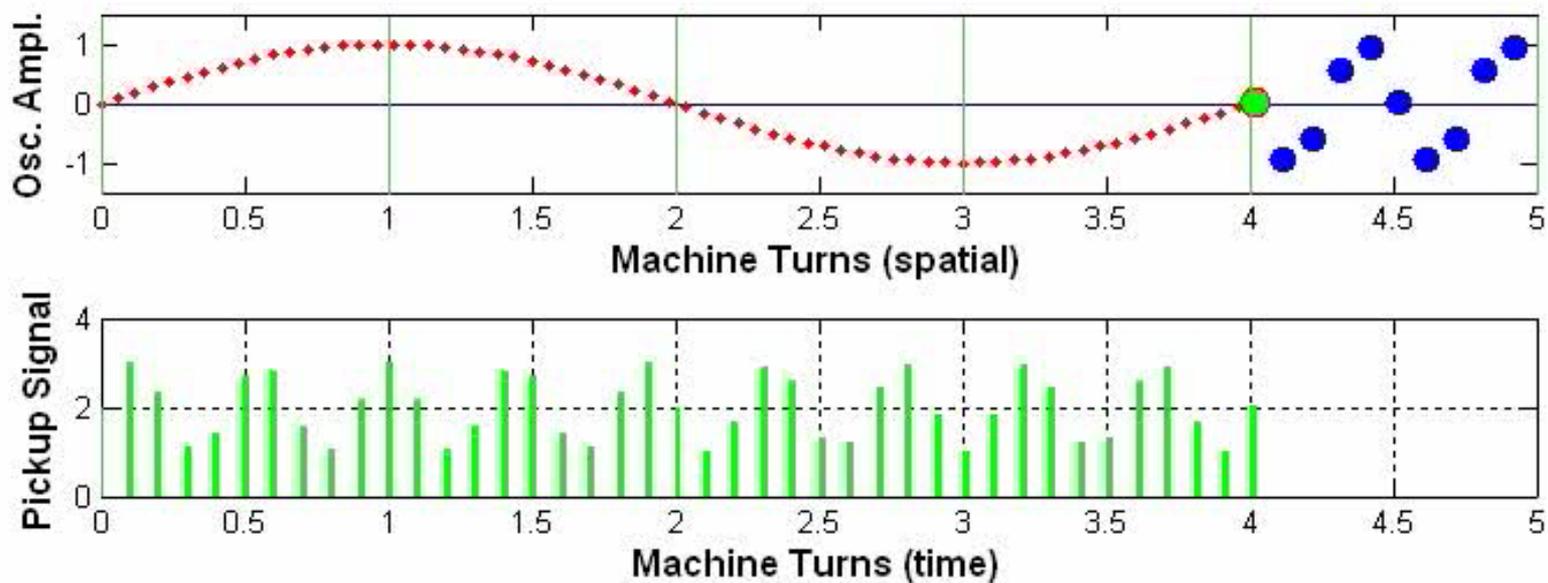
$n=0$



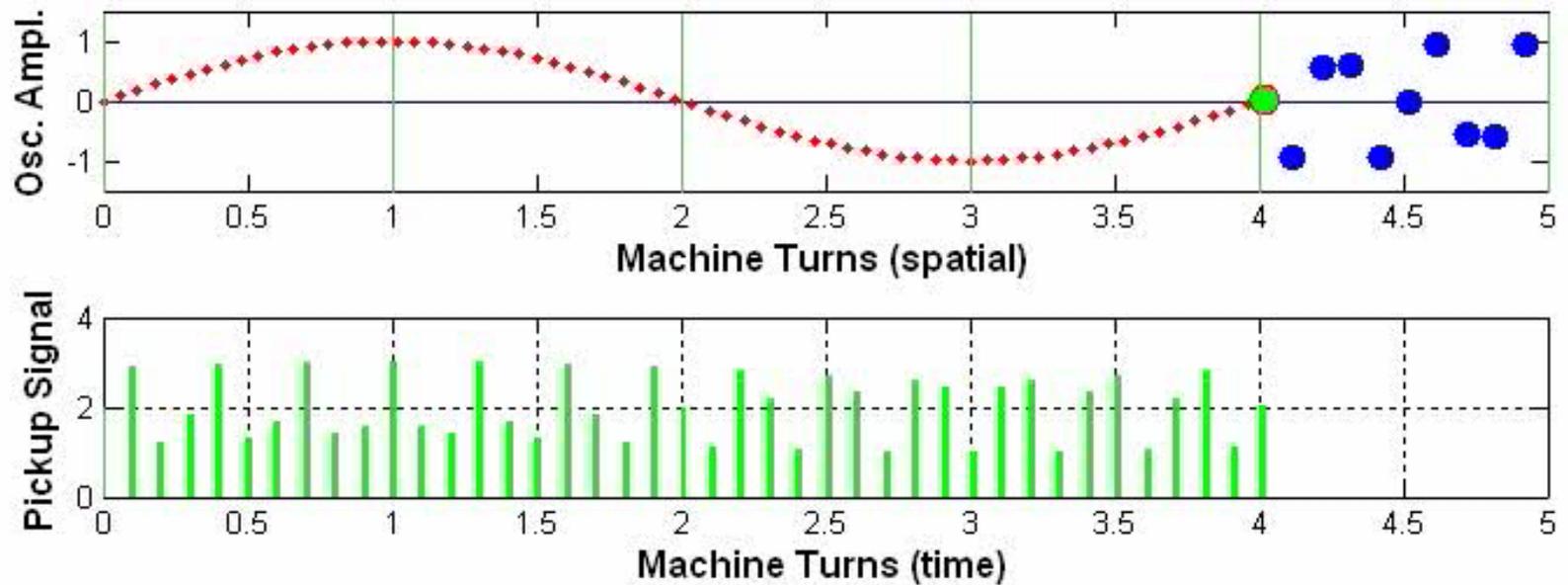
$n = 1$



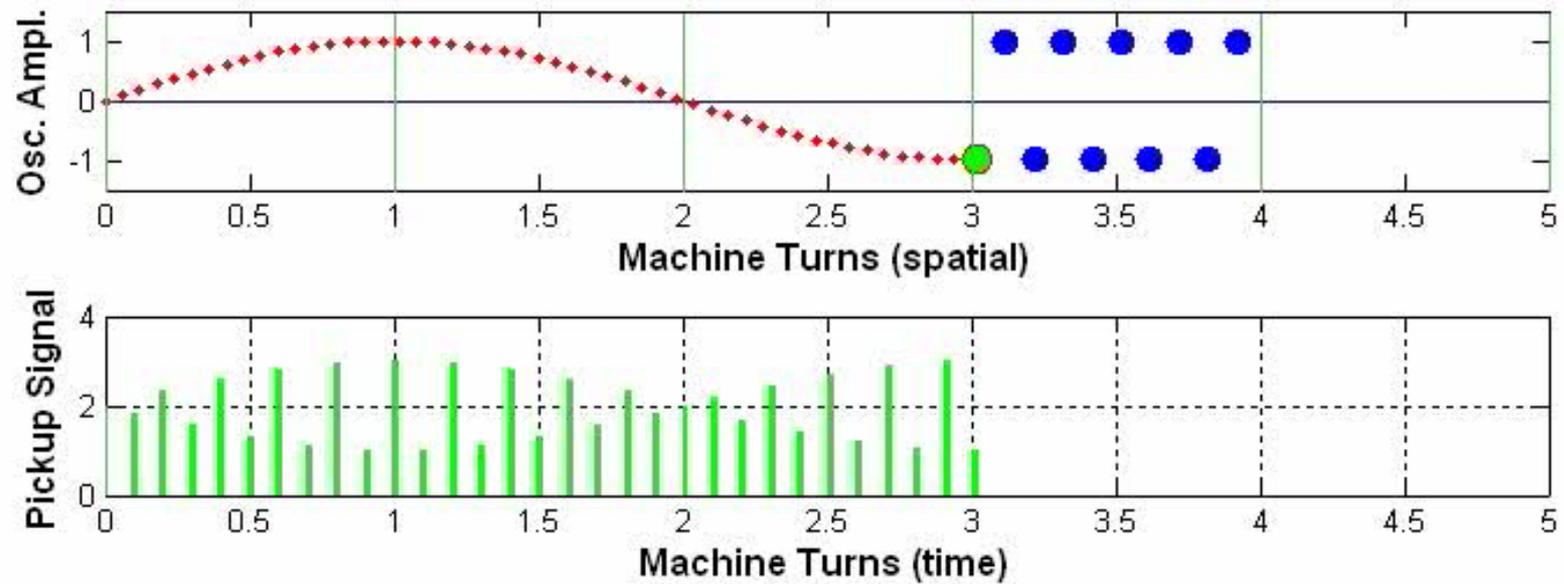

$$n = 2$$



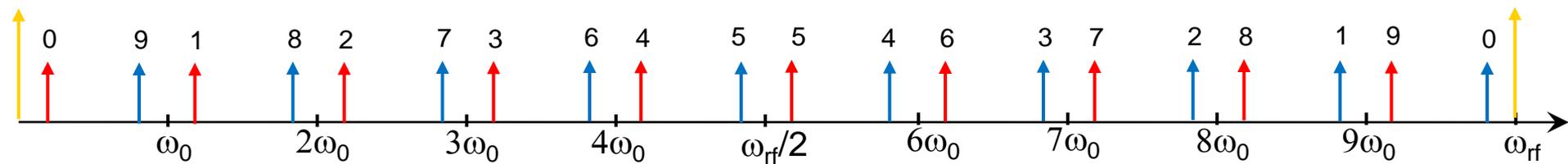
$n = 3$



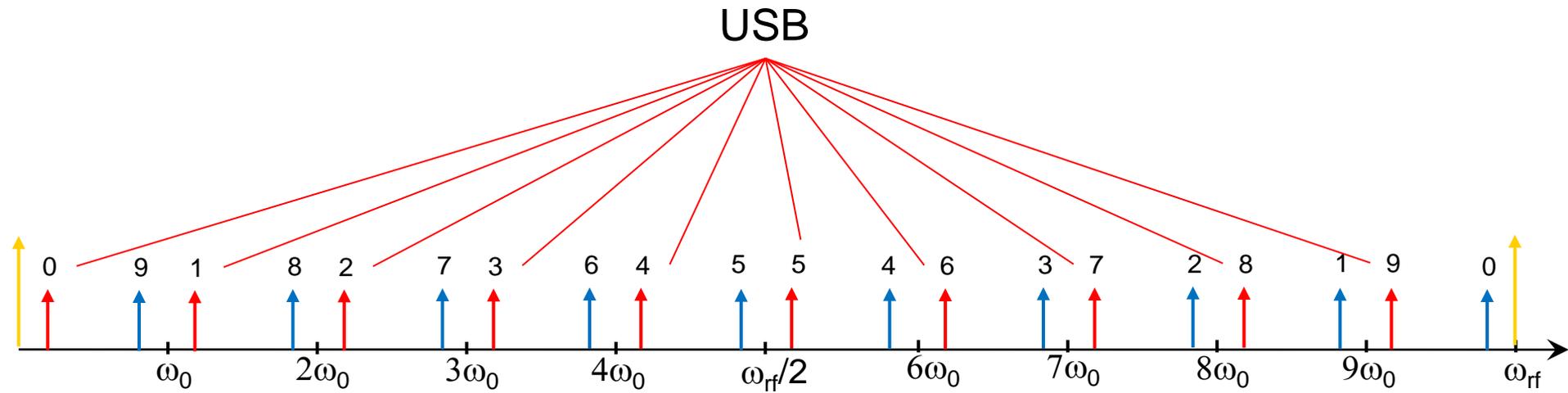
...  $n = 5$



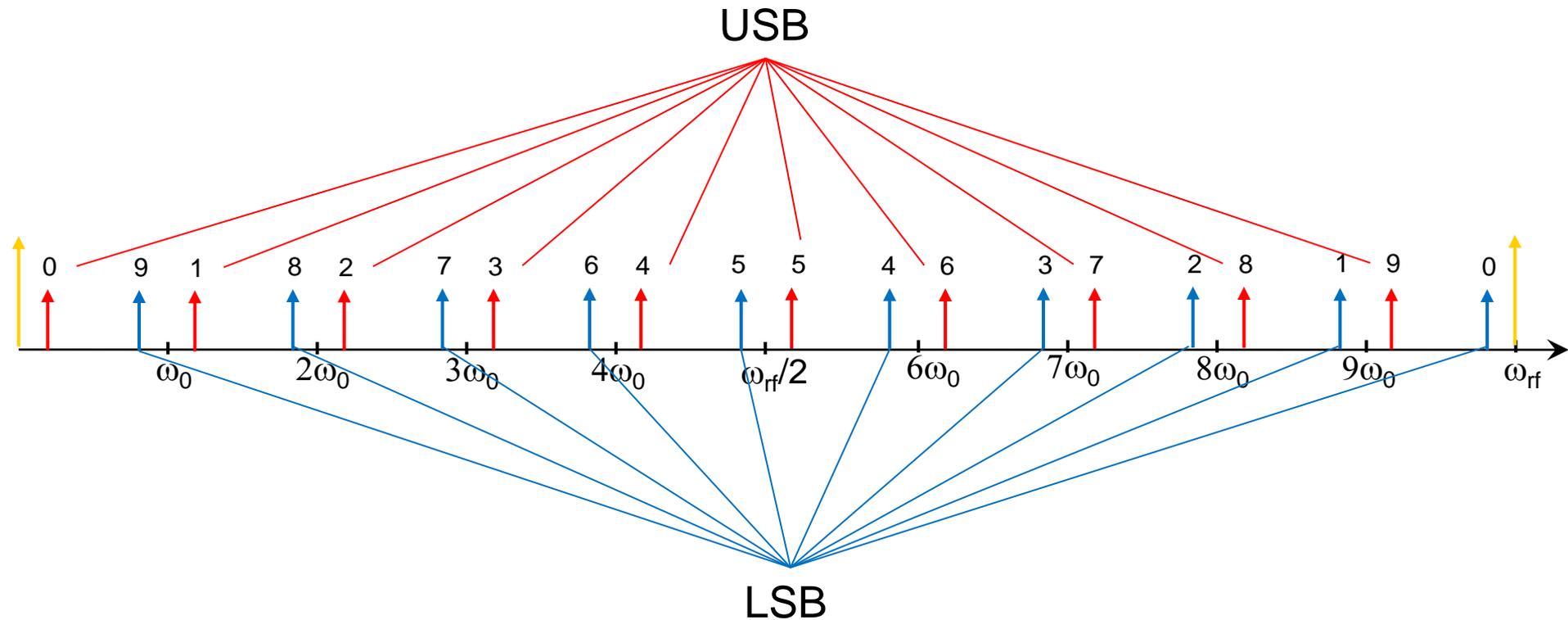
# Identification of modes $n$ ( $N=10$ ):

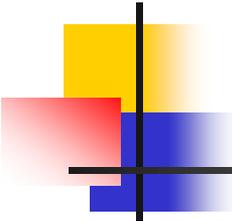


# Identification of modes n (N=10):



# Identification of modes $n$ ( $N=10$ ):

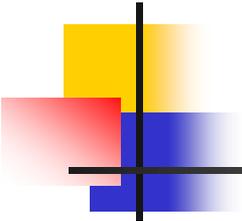




# Fourth recipe (Golden Rule 4):

Stability with side bands MB:

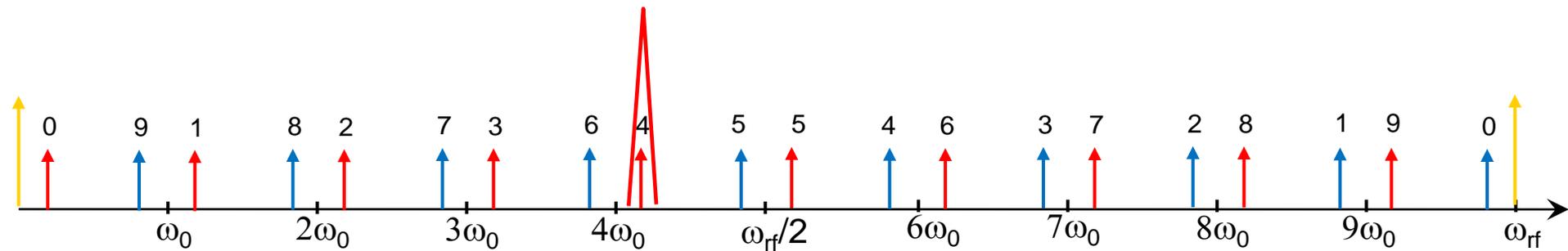
	<b>Below transition</b>	<b>Above transition</b>
Longitudinal	Upper SB	Lower SB
Transverse	Upper SB	Upper SB



## “Micro – Tutorial” ...

# What about...

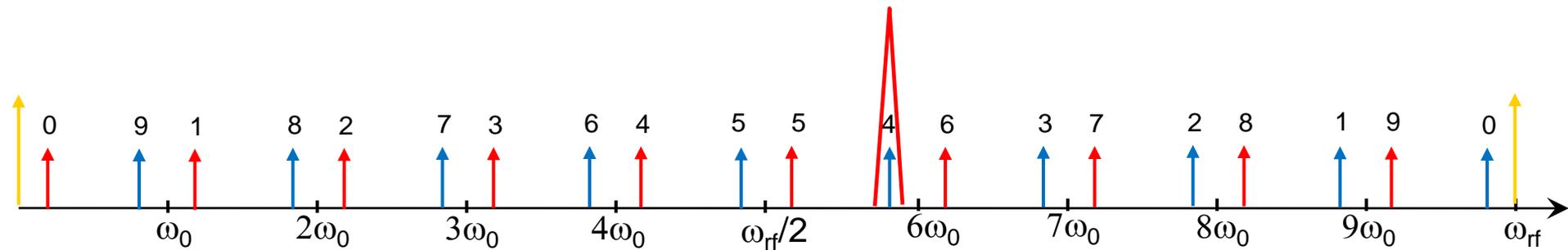
Longitudinal – below transition !



- No problem, on the Upper Side Band → stabilizing !

# What about...

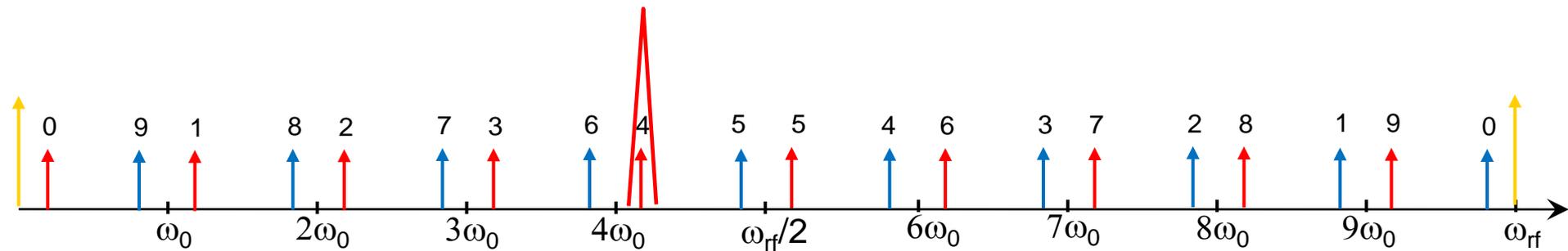
Longitudinal – below transition !



- Bad, on the Lower Side Band → unstable !

# What about...

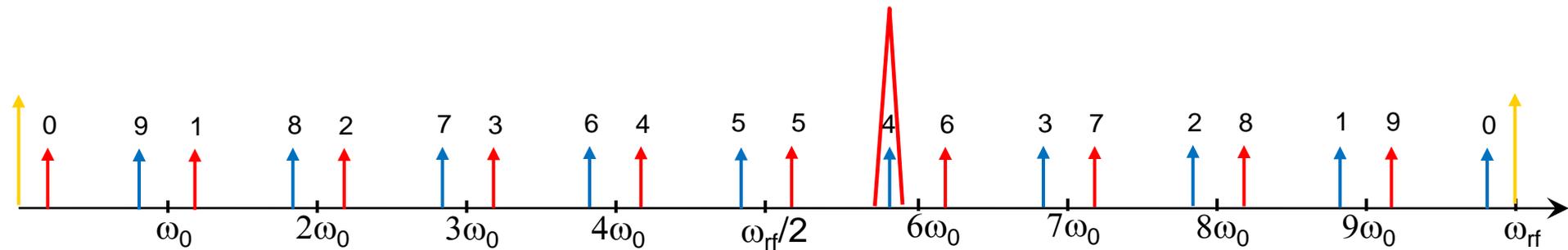
Longitudinal – above transition !



- Bad, on the Upper Side Band → unstable !

# What about...

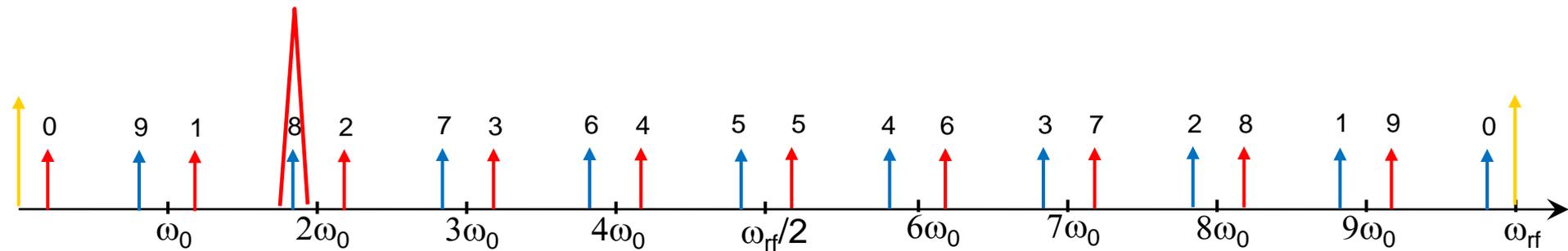
Longitudinal – above transition !



- No problem, on the Lower Side Band → stabilizing !

# What could we do...?

Longitudinal – below transition !

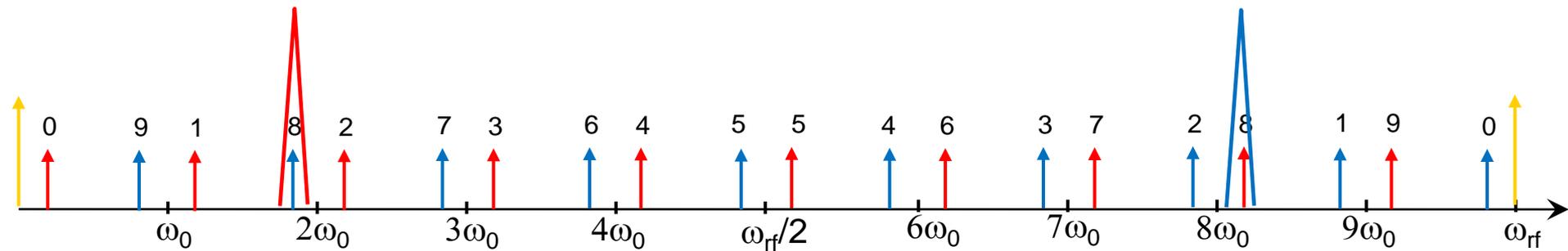


- Bad, on the LSB of the mode  $n=8$  !

- If problem is serious and cannot be solved....

# What could we do...?

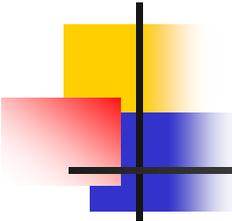
Longitudinal – below transition !



- Bad, on the LSB of the mode  $n=8$  !

- If problem is serious and cannot be solved....

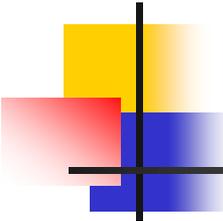
- Add an impedance (e.g. a cavity) tuned on the USB of  $n=8$  which will damp the instability !



# Collective effects: Summary

---

- Interaction between the particles **within a bunch** (space charge, watch out at injection energy!).
- Interaction between the **bunch and the environment** (impedance).
- Interaction between the **different bunches via the environment** (multi-bunch instabilities)
- There are other collective effects to be considered when the beams are colliding! (e.g. **Beam-Beam** , **Landau damping** - **CAS Intermediate course**)
- Taking into account the collective effects at the **design phase** is a relatively new procedure (~ LEP). The creation of an “**Impedance Police Team**” proved to be very useful for LEP and **vital for LHC!**



# Procedure:

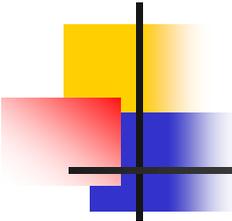
---

- Expected performance of the machine defined → **required intensities known.**
- Compute maximum longitudinal ( $Z/n$ ) and transverse ( $Z_T$ ) impedances which **allow for these intensities.**
- Make sure your Impedance Police Team has sufficient scientific credit to manage (unavoidable) conflicts with component designers and Finance Committee:

Remember:  $Z_T = (2R/b^2) \cdot (Z/n)$  (Broad-band Impedance)

Magnets + Finance want  $b \downarrow$  and Collective Effects want  $b \uparrow$

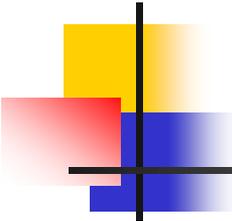
Remember about the vacuum chamber of insertion devices !!!



# The Impedance Police Team

---

- Every single object **visible by the beam** should be submitted for approval to the Impedance Police Team.
- The team evaluates by means of **dedicated programs** the **longitudinal and transverse impedances** of the object.
- The team **approves** or **proposes modifications** for the object.
- Once approved, the object is included in the **Impedance budget** of the machine, which is regularly **updated**.
- For each update, **ALL** the instability thresholds are **re-evaluated**.
- The **time domain** codes yield the corresponding **wakefields** to be used for further multi-particle simulations.
- The **frequency domain** codes yield **BB-impedances** or single **resonant modes** (narrow-band impedances) which will be used to compute **instabilities**, but also **power deposition** in the different elements of the machine (essential for SC machines).



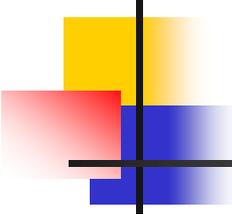
# C.E. and the real world...

---

- To operate safely a machine like the LHC, we need to control the Power Converters at the level of  $10^{-5}$  !

- To operate safely a machine like the LHC, we need to control the “q” at least at the level of the second digit (e.g. 59.32) !

- For **collective effects**, if your estimations are between **20-50%** of the measured effect, then this is more than **excellent** ! Never try to predict something at the per mil level with collective effects, this would simply be foolish...differences within a factor of two are not rare at all !



# Final Words...

---

My sincere thanks to **Werner Herr** for having discussed in great details with me about the optimal content of these lectures !!!

**Thank you for your attention !**