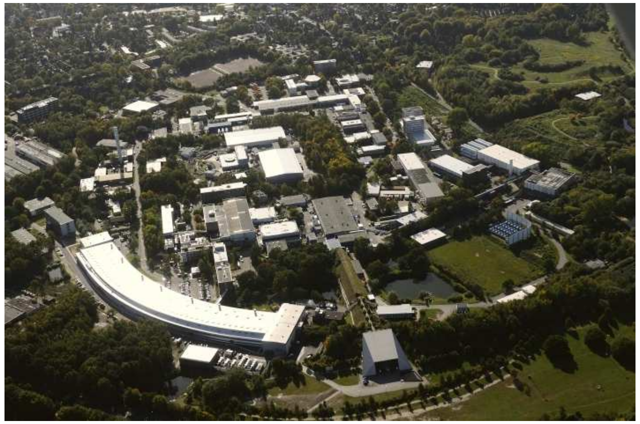
Impedance & Instabilities

- The concept of wakefields and impedance
- Wakefield effects and their relation to important beam parameters
- Beam-pipe geometry and materials and their impact on impedance
- An introduction to beam instabilities (including ion effects)



Rainer Wanzenberg

CAS Vacuum for Particle Accelerator
June 7 , 2017
Hotel Orenas Slott, Glumslov, Sweden

HELMHOLTZ ASSOCIATION

The CERN Accelerator School

DESY

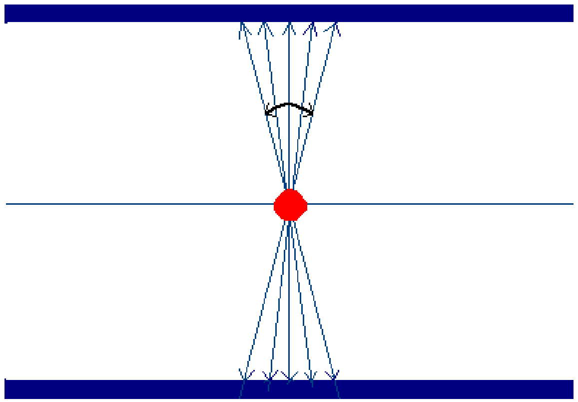
The concept of Wakefields and Impedance

Wake Field = the track left by a moving body (as a ship) in a fluid (as water); broadly : a track or path left

Impedance = Fourier Transform (Wake Field)

Electric Field of a Bunch

Point charge in a beam pipe



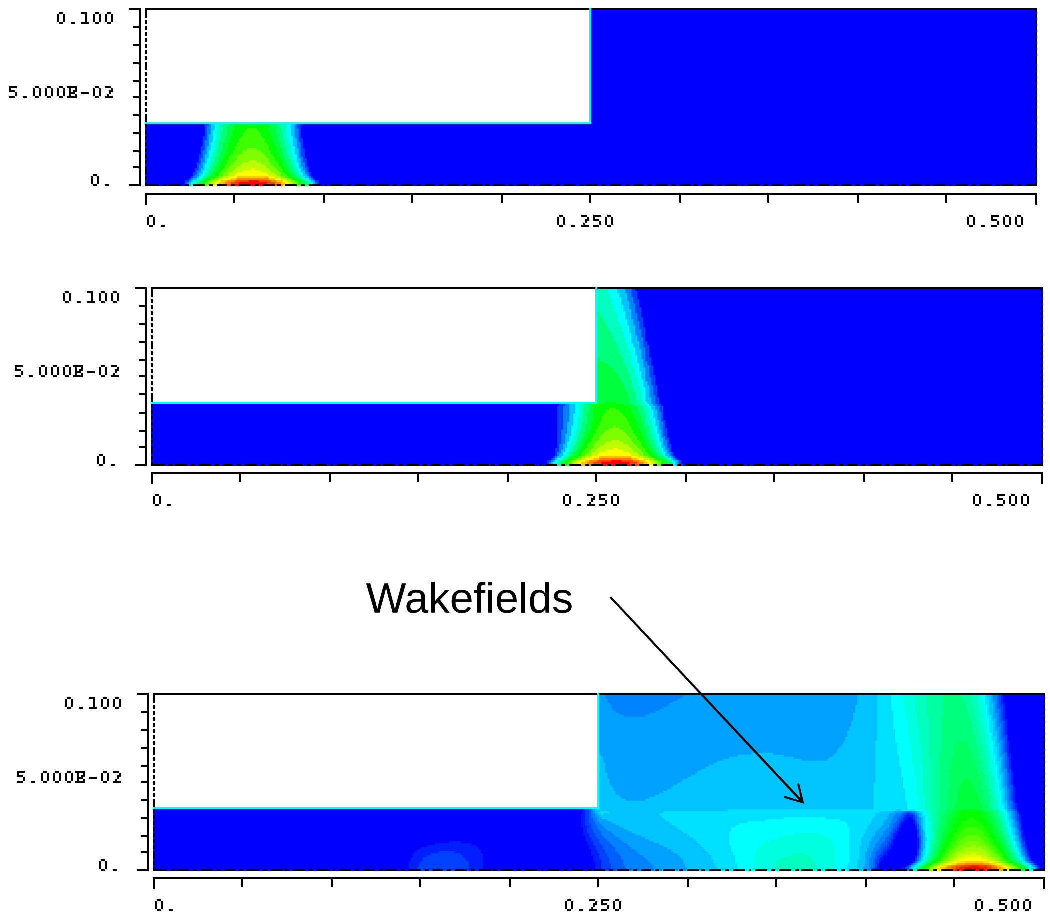
Opening angle

$$\phi = \frac{0.511\,\mathrm{MeV}}{E}$$

$$E = 10\,\mathrm{MeV}, =>$$

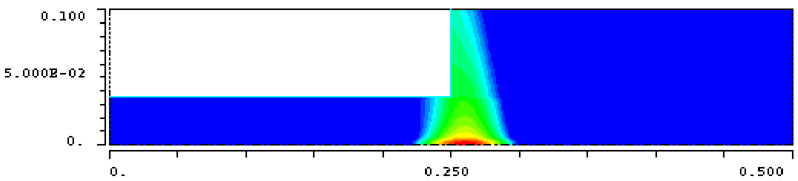
$$\phi = 50\,\mathrm{mrad} = 2.89^\circ$$

Gaussian Bunch, Step out transition

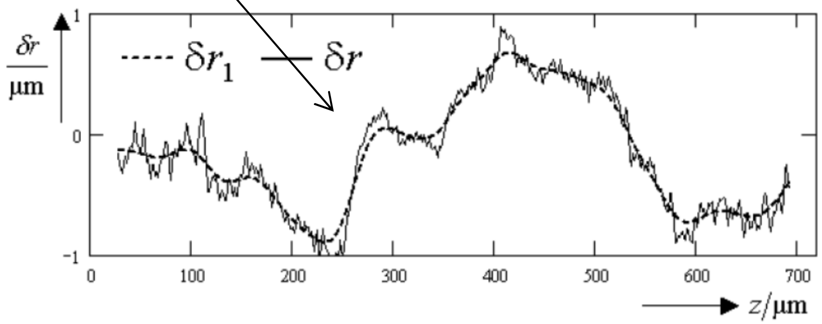


Geometric wakefields and resistive wall wakes

Geometric Wake

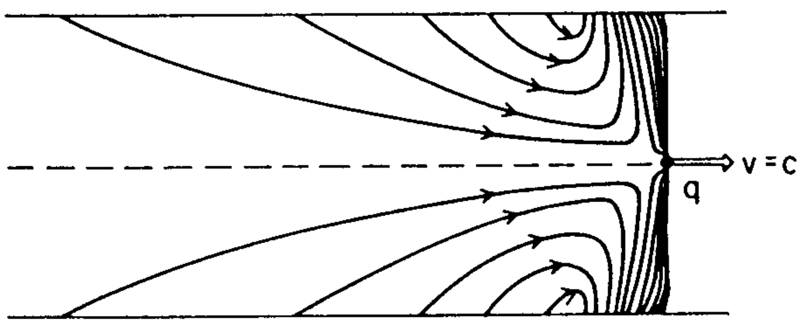


Material roughness, Oxide layers



M. Dohlus, M.I. Ivanyan, V.M. Tsakanov
Surface Roughness Study for the TESLA-FEL,
DESY-TESLA-FEL-2000-26

Resistive Wall Wake



$$\sigma = 57 \cdot 10^6 (\Omega \mathrm{m})^{-1} \ \ \mathrm{Cu}$$

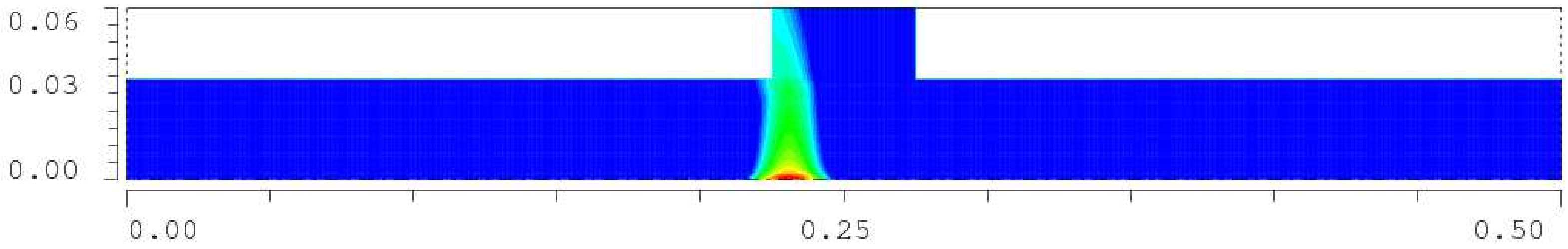
$$\sigma = 37 \cdot 10^6 (\Omega \mathrm{m})^{-1} \ \ \mathrm{Al}$$

$$\sigma = 1.5 \cdot 10^6 (\Omega \mathrm{m})^{-1} \ \ \mathrm{Steel}$$

$$(1 \ \ \Omega \, \mathrm{m} = 10^6 \ \Omega \frac{\mathrm{mm}^2}{\mathrm{m}})$$

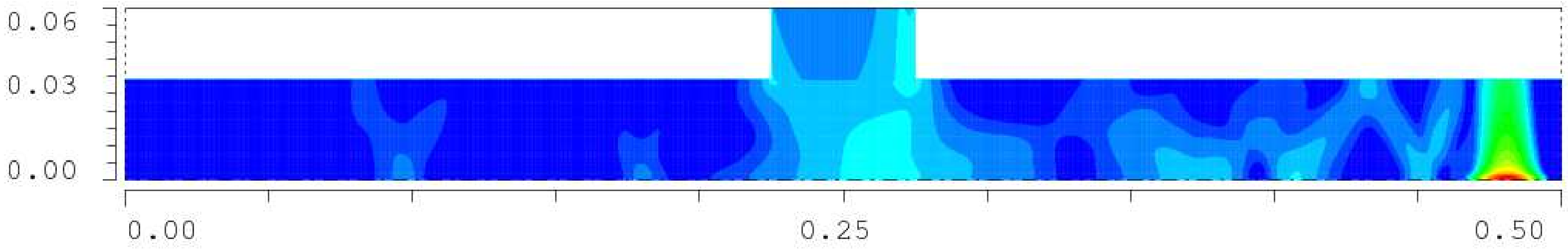
Short and Long Range Wakefields

Example: small cavity



Transient effect: within one bunch, head interacts with the tail

Long range wakefields:



One or many modes are excited in the cavity
bunch interacts with other bunches

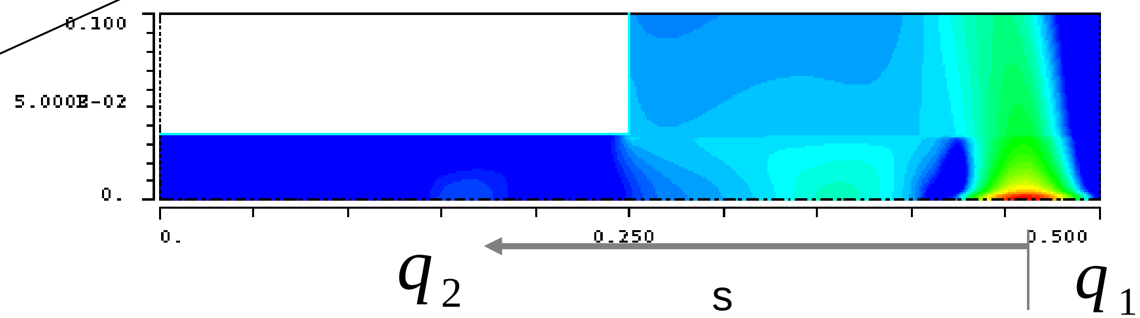
Effects on a test charge

Change of energy

$$\Delta\mathcal{E}(s) = q_2 \int dz\, E_z(r, z, t = (s+z)/c)$$

Bunch charge q_1
Test charge q_2

$E_z(r, z, t)$ depends on
the total bunch charge q_1



Equation of motion

q_1: $z = c\,t$

q_2: $z = c\,t - s$

Effects on a test charge (cont.)

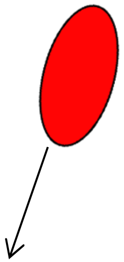
Lorentz force:

$$\vec{F} = q_2\left(\vec{E} + c\ \vec{u}_z \times \vec{B}\right)$$

The electric and magnetic fields
are generated by the bunch charge q_1

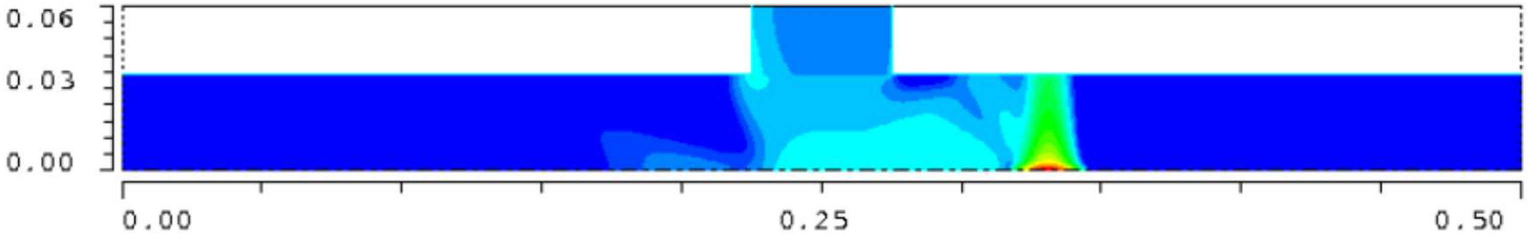
test charge q_2

- change of energy
- **transverse kick**

Bunch:  q_1

Approximations

Numerical calculation of a wakefield of a Gaussian bunch traversing a cavity



The concept of a Wakefield assumes the
Rigid Beam Approximation

- The wakefield does not affect the motion of the beam
- The wakefield does not affect the motion of the test charge
  (only the energy or momentum change is calculated)

The interaction of the beam with the environment is *not* self consistent.

Nevertheless, one can use results from wake field calculations for
a turn by turn tracking code - sometimes cutting a beam into slices.

Wakefield effects and their relation to important beam parameters

Beam Parameter:

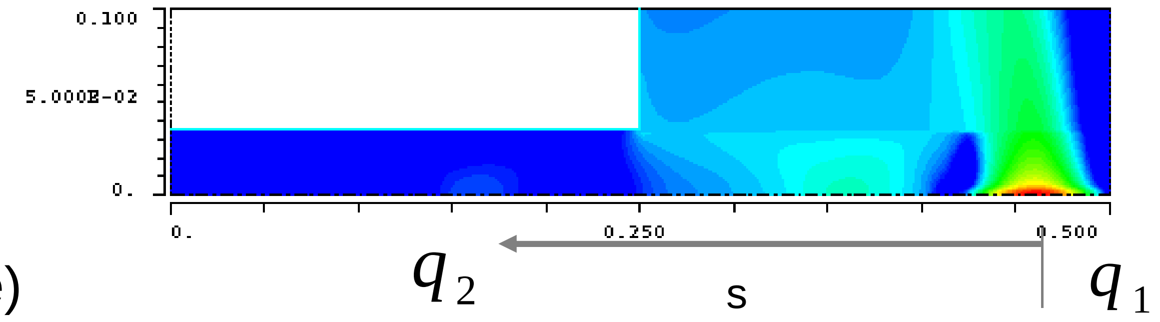
Total bunch charge q_1

Bunch length (and shape)

Transverse dimensions
(Emittance, Beta-function)

Number of bunches
Total beam current

Synchrotron tune
Betatron tune
Damping time (Synchrotron Rad.)
Chromaticity



$$\Delta\mathcal{E}(s) = q_2 \int dz\, E_z(r, z, t = (s+z)/c)$$

Transient effects:

Wakefield depends on

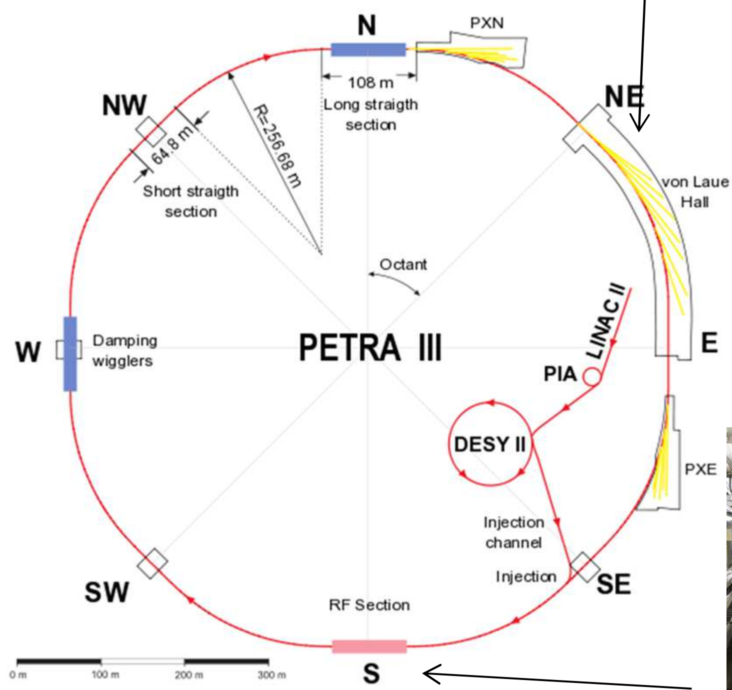
- **bunch charge and charge distribution**
- **geometry change or material properties**

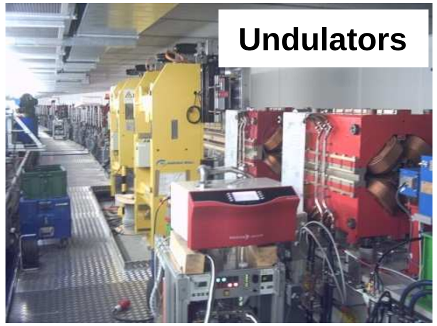
Example: Beam parameters of PETRA III

Design Parameter	PETRA III	
Energy / GeV	6	
Circumference /m	2304	
RF Frequency / MHz	500	
RF harmonic number	3840	
RF Voltage / MV	20	
Momentum compaction	$1.22\ 10^{-3}$	
Synchrotron tune	0.049	
Total current / mA	**100**	
Number of bunches	**960**	**40**
Bunch population / 10^{10}	**0.5**	**12**
Bunch separation / ns	8	192
Emittance (horz. / vert.) /nm	1.2 / 0.01	
Bunch length / mm	**12**	
Damping time H/V/L / ms	16 / 16 / 8	

Coupled bunch instabilities threshold current ~ 10 mA

→ powerful broadband feedback neccessary





Wakepotential

Longitudinal Wakepotential:

$$\mathcal{W}_{||}(s) = \frac{1}{q_1} \int dz\, E_z(r, z, t(s, z))$$

(Unit V/C)

Electric field of a Gaussian bunch

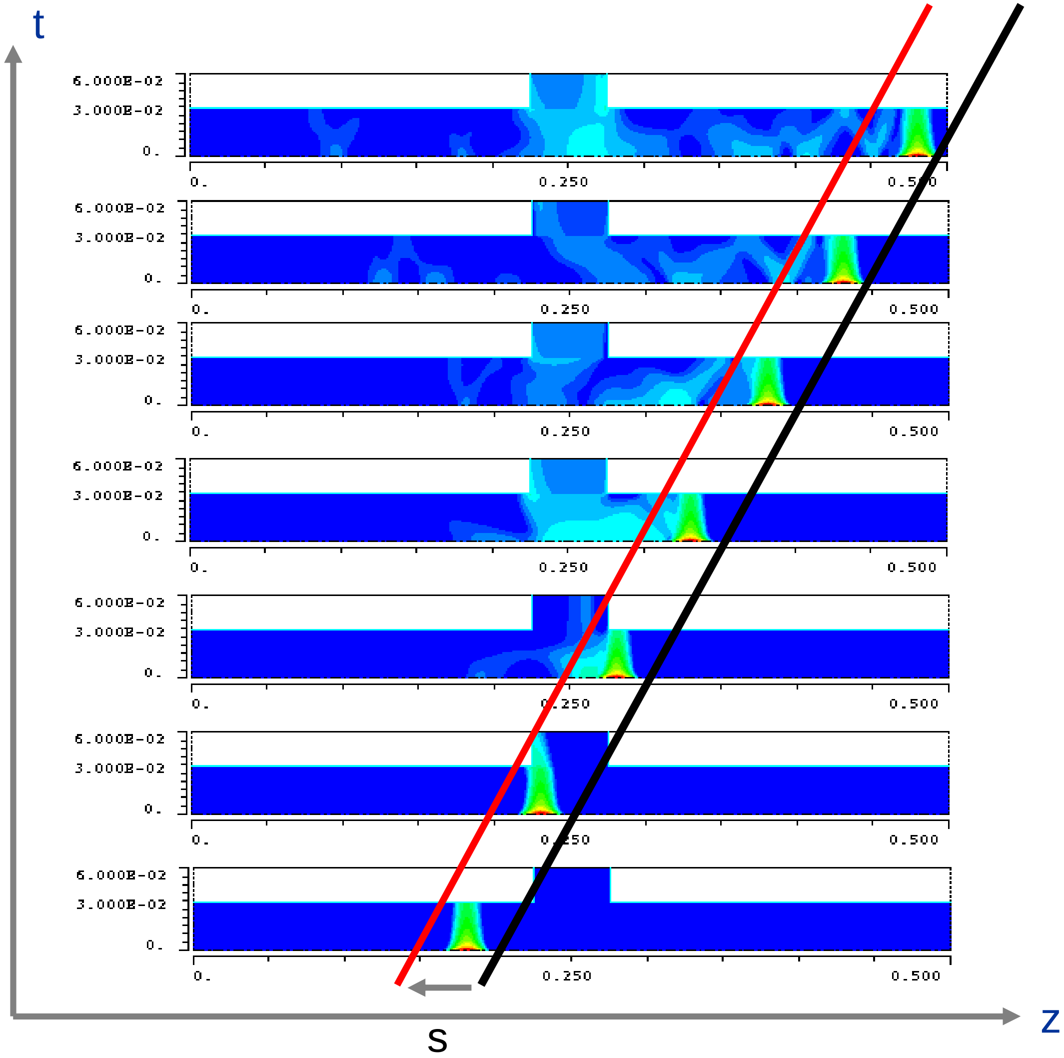
time of the test charge

head: t = z/c

tail: t = (s+z)/c
(or test charge)

energy loss/gain:

$$\Delta E = e\ q_1\ W_z(s)$$



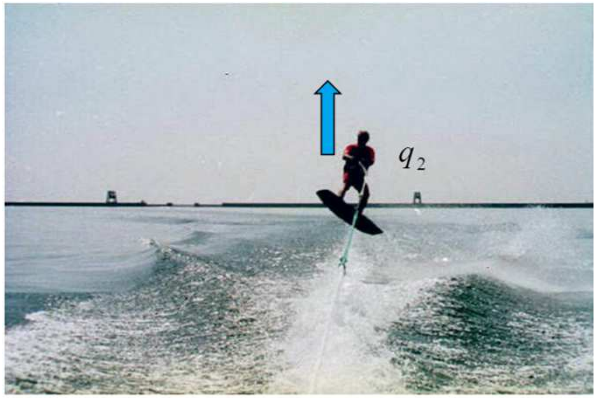
Transverse Wakepotential

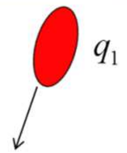
Lorentz Force

$$\boldsymbol{F}(\boldsymbol{r_2}, t) = q_2\,(\boldsymbol{E}(\boldsymbol{r_2}, t) + \boldsymbol{v} \times \boldsymbol{B}(\boldsymbol{r_2}, t))$$

Longitudinal Wakepotential

$$\mathcal{W}_{||}(s) = \frac{1}{q_1} \int dz\, E_z(r, z, t(s, z))$$





<u>Transverse Wakepotential</u>

$$\boldsymbol{\mathcal{W}}_{\perp}(\boldsymbol{r}_{2\perp}, s) = \frac{1}{q_1} \int_{-\infty}^{\infty} dz [\boldsymbol{E}_{\perp}(\boldsymbol{r}_{2\perp}, z, t) + c\,\boldsymbol{u}_z \times \boldsymbol{B}_{\perp}(\boldsymbol{r}_{2\perp}, z, t)]_{t=(s+z)/c}$$

Change of momentum of a test charge q_2

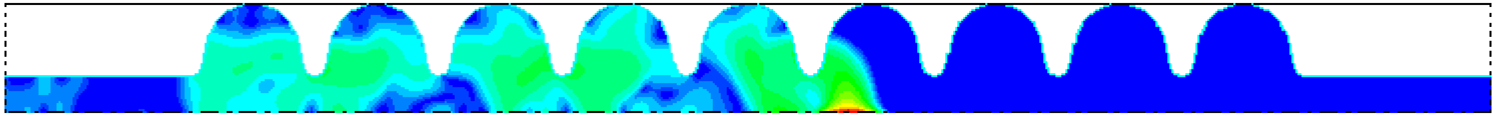
$$\Delta\boldsymbol{p}(\boldsymbol{r_2}_{\perp}, s) = \frac{1}{c}\, q_2\, q_1\, \boldsymbol{\mathcal{W}}(\boldsymbol{r_2}_{\perp}, s)$$

Kick on an electron (q_2) due to the bunch charge q_1

$$\boxed{\boldsymbol{\theta}(\boldsymbol{r_2}_{\perp}, s) = \frac{e}{E}\, q_1\, \boldsymbol{\mathcal{W}}_{\perp}(\boldsymbol{r_2}_{\perp}, s)}$$

beam energy

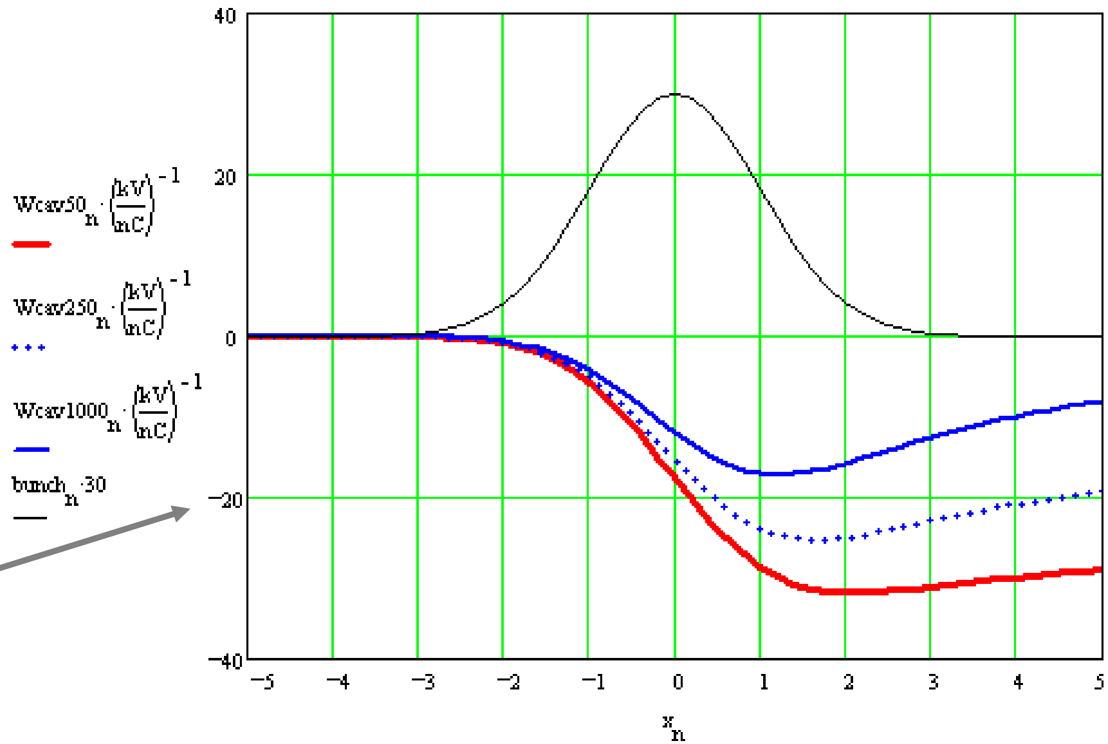
Example: TESLA cavity



cavity length ~ 1 m

rms bunch length

σ = 50 μm
σ = 250 μm
σ = 1 mm



ΔE= e 1 nC (-20 kV/nC)
= 20 keV

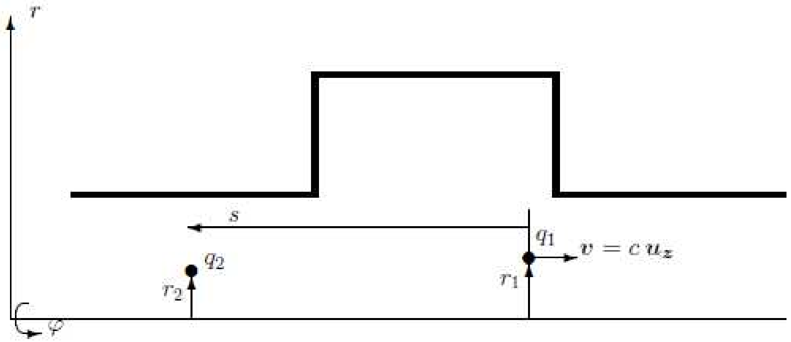
s/σ

Energy spread: 20 keV / 20 MeV = 0.1 %

assuming an accelerating gradient of 20 MV/m

Wake: point charge versus bunch

Longitudinal Wakepotential



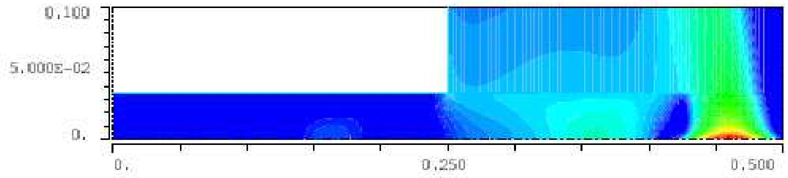
Point charge

$$W_{||}(s) = \frac{1}{q_1} \int dz\, E_{z,p.c.}(r, z, t(s,z))$$

Bunch

$$\mathcal{W}_{||}(s) = \frac{1}{q_1} \int dz\, E_z(r, z, t(s,z))$$

$$\mathcal{W}_{||}(r_{2\perp}, s) = \int_0^\infty ds'\, \lambda(s-s')\, W_{||}(r_{2\perp}, s')$$



The fields E and B are generated by the charge distribution ρ and the current density j. They are solutions of the Maxwell equations and have to obey several boundary conditions.

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j} + \frac{1}{c^2}\frac{\partial}{\partial t}\boldsymbol{E} \qquad \nabla \cdot \boldsymbol{B} = 0 \qquad \boldsymbol{j} = c\, \boldsymbol{u}_z\, \rho$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \qquad \nabla \cdot \boldsymbol{E} = \frac{1}{\epsilon_0}\rho \qquad \rho = q_1\, \lambda_\perp\, \lambda$$

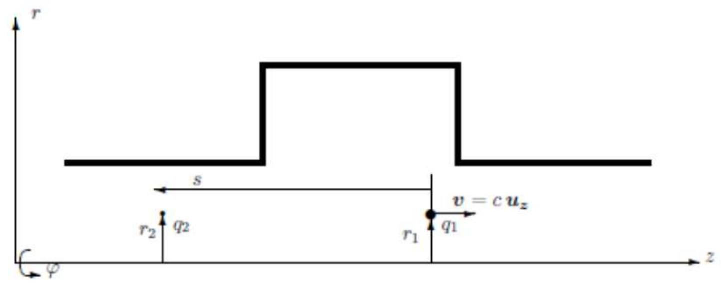
Line charge density

$$\lambda(s) = \frac{1}{\sigma_z \sqrt{2\,\pi}} \exp\left(-\frac{1}{2}\frac{s^2}{{\sigma_z}^2}\right)$$

Longitudinal Impedance

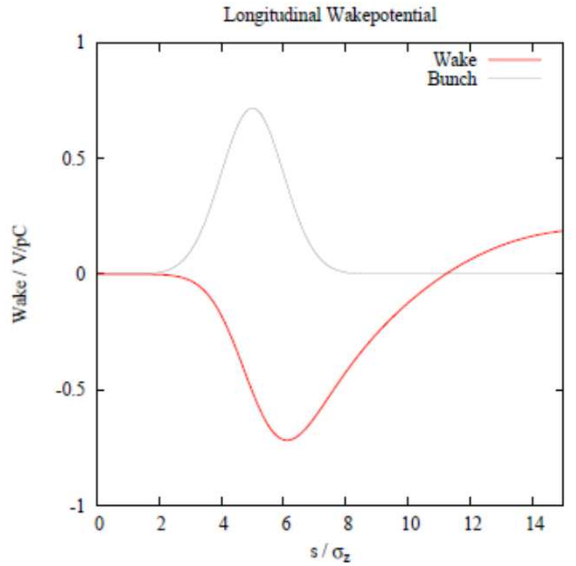
Longitudinal Impedance
= Fourier transform of the point charge wake

$$\boxed{Z_{\|}(r_{2\perp},\omega) = \frac{1}{c}\int_{-\infty}^{\infty} ds\, W_{\|}(r_{2\perp},s)\, \exp(-i\frac{\omega}{c}\, s)}$$



Wakepotential of a Gaussian bunch
= convolution of point charge wake and charge distribution

$$\mathcal{W}_{\|}(r_{2\perp},s) = \int_0^{\infty} ds'\, \lambda(s-s')\, W_{\|}(r_{2\perp},s')$$



Fourier transform of the wakepotential of a Gaussian bunch

$$\widetilde{\lambda}(\omega)\; Z_{\|}(r_{2\perp},\omega) = \frac{1}{c}\int_{-\infty}^{\infty} ds\, \mathcal{W}_{\|}(r_{2\perp},s)\, \exp(-i\frac{\omega}{c}\, s)$$

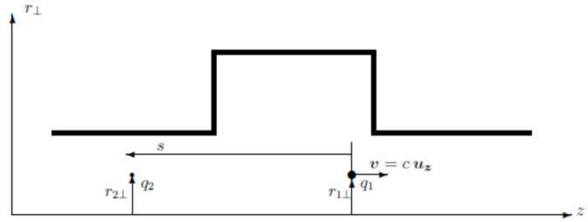
Impedance

Fourier transform of the charge distribution

Transverse Impedance

Transverse Impedance
= Fourier transform of the point charge transverse wake

$$\boldsymbol{Z}_{\perp}(r_{2\perp},\omega) = \frac{-i}{c}\int_{-\infty}^{\infty} ds\, \boldsymbol{W}_{\perp}(r_{2\perp},s)\,\exp(-i\frac{\omega}{c}\,s)$$



Panofsky-Wenzel-Theorem

$$\frac{\partial}{\partial s}\boldsymbol{W}_{\perp}(\boldsymbol{r}_{2\perp},s) = -\boldsymbol{\nabla}_{2\perp}W_{||}(\boldsymbol{r}_{2\perp},s) \qquad \frac{\omega}{c}\,\boldsymbol{Z}_{\perp}(\boldsymbol{r}_{2\perp},\omega) = \boldsymbol{\nabla}_{\perp}\,Z_{||}(\boldsymbol{r}_{2\perp},\omega)$$

Relation between the transverse and longitudinal Wakepotential

$$\frac{\partial}{\partial s}\boldsymbol{W}_{\perp}(\boldsymbol{r}_{2\perp},s) = \frac{1}{q_1}\int_{-\infty}^{\infty} dz \left[\frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{E}_{\perp}(\boldsymbol{r}_{2\perp},z,t) + c\,\boldsymbol{e}_z \times \frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{B}(\boldsymbol{r}_{2\perp},z,t)\right]_{t=(s+z)/c}$$

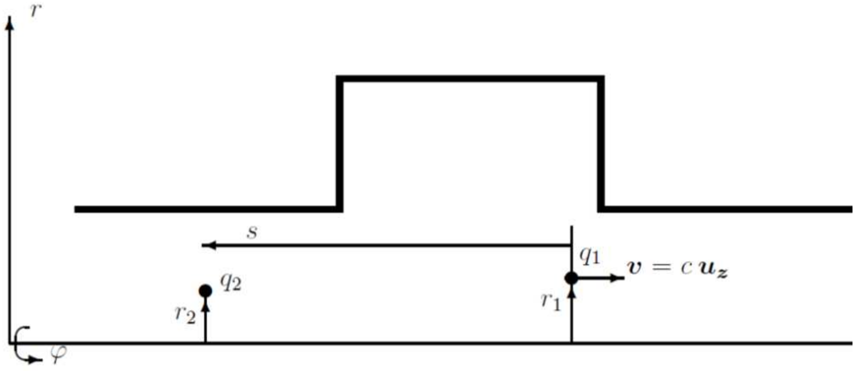
Maxwell equation

$$\boldsymbol{\nabla}\times\boldsymbol{E} = -\tfrac{\partial}{\partial t}\boldsymbol{B} \quad\Rightarrow\quad \boldsymbol{e}_z \times \frac{\partial}{\partial t}\boldsymbol{B} = \frac{\partial}{\partial z}\boldsymbol{E}_{\perp} - \boldsymbol{\nabla}_{\perp}E_z$$

Multipole expansion of the wake

Longitudinal Wakepotential

$$W_{||}(r_1, r_2, \varphi_1, \varphi_2, s) = \sum_{m=0}^{\infty} {r_1}^m \, {r_2}^m \, W_{||}^{(m)}(s) \; \cos\left(m\left(\varphi_2 - \varphi_1\right)\right)$$



Multipole expansion in Cartesian coordinates:

$$\begin{aligned} W_{||}(x_1, y_1, x_2, y_2, s) \quad &\approx \quad W_{||}^{(0)}(s) \\ &+ \; (x_2\; x_1 + y_2\; y_1)\; W_{||}^{(1)}(s) \\ &+ \; \left((x_2{}^2 - y_2{}^2)\,(x_1{}^2 - y_1{}^2) + 2\,x_2\,y_2\; 2\,x_1\,y_1\right)\; W_{||}^{(2)}(s) \end{aligned}$$

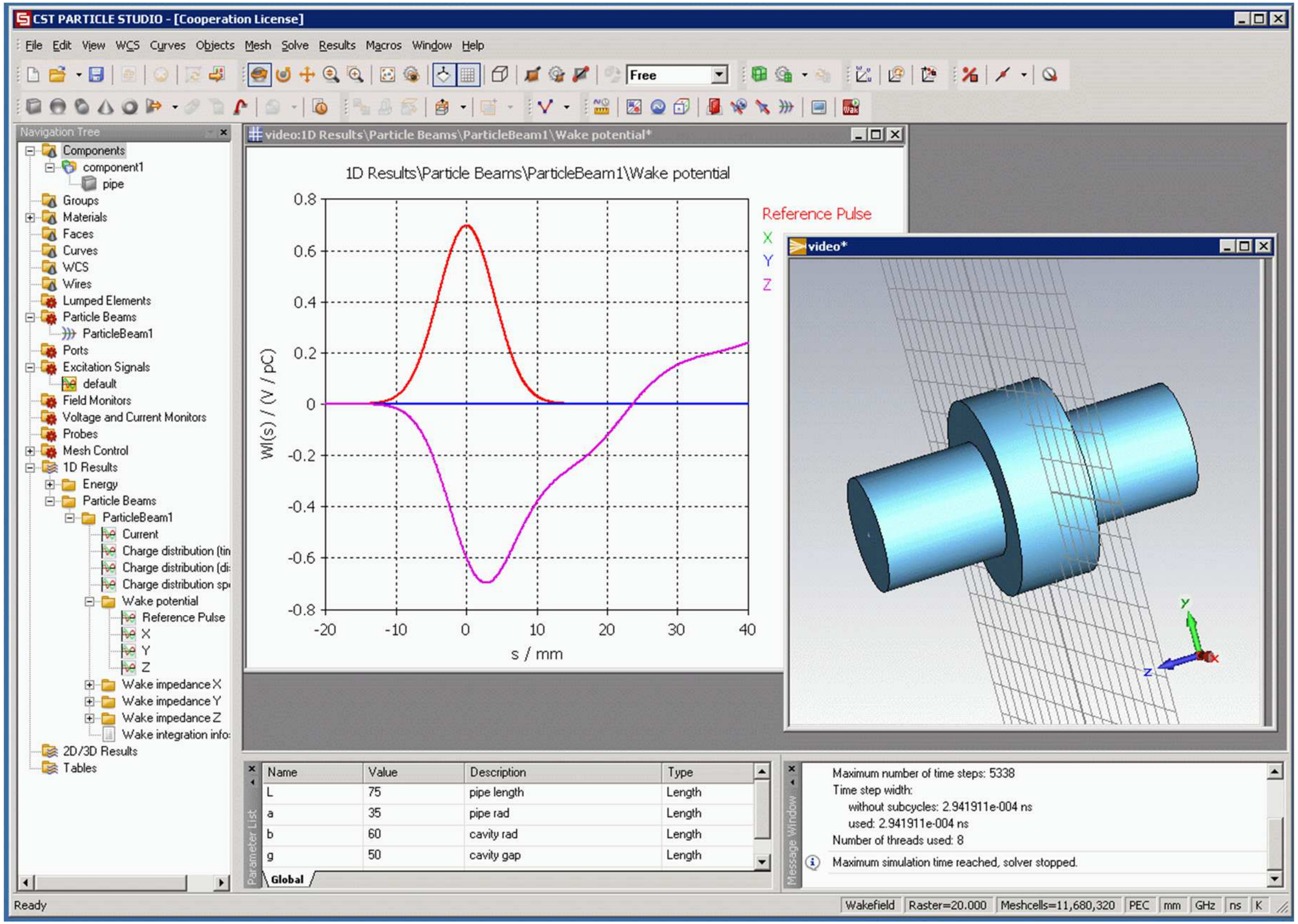
Using the Panofsky-Wenzel-Theorem: $W_{\perp}^{(m)}(s) = -\int_{-\infty}^{s} ds'\, W_{||}^{(m)}(s')$

$$\begin{aligned} \boldsymbol{W}_{\perp}(x_1, y_1, x_2, y_2, s) \quad &\approx \quad (x_1\;\boldsymbol{u_x} + y_1\;\boldsymbol{u_y})\; W_{\perp}^{(1)}(s) \\ &+ (x_2\;\boldsymbol{u_x} - y_2\;\boldsymbol{u_y})\; 2\; (x_1{}^2 - y_1{}^2)\; W_{\perp}^{(2)}(s) \\ &+ (y_2\;\boldsymbol{u_x} + x_2\;\boldsymbol{u_y}) 2\; (2\,x_1\,y_1)\; W_{\perp}^{(2)}(s). \end{aligned}$$

Transverse dipole wake

Wakepotential of a cavity

Numerical calculation with the CST Studio suite (commercial 3D code)



Numerical calculations

There exist several numerical codes to calculate wakefields.
Examples are:

Non commercial codes (2D, r-z geometry)

- ABCI, Yong Ho Chin, KEK
  http://abci.kek.jp/abci.htm

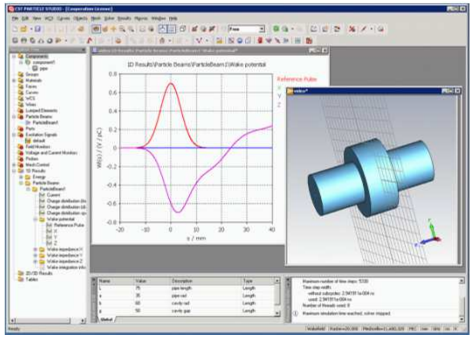
- Echo 2D, Igor Zagorodnov, DESY
  http://www.desy.de/~zagor/WakefieldCode_ECHOz/

  Recently development: Echo3D (version 1.0)

Commercial codes (3D)

- GdfidL

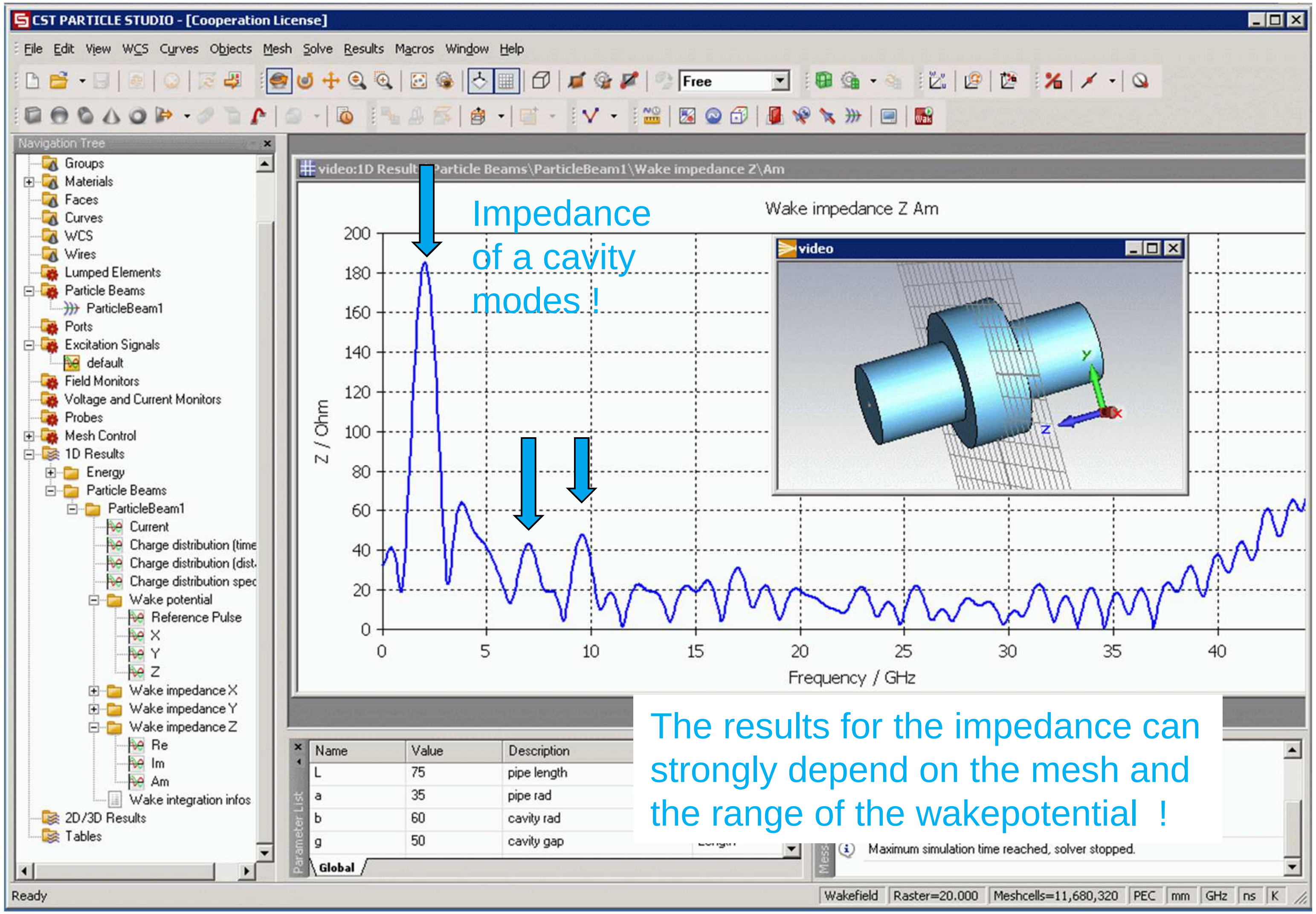
- *CST (Particle Studio, Microwave Studio)* →



The Maxwell equations are solved on a grid

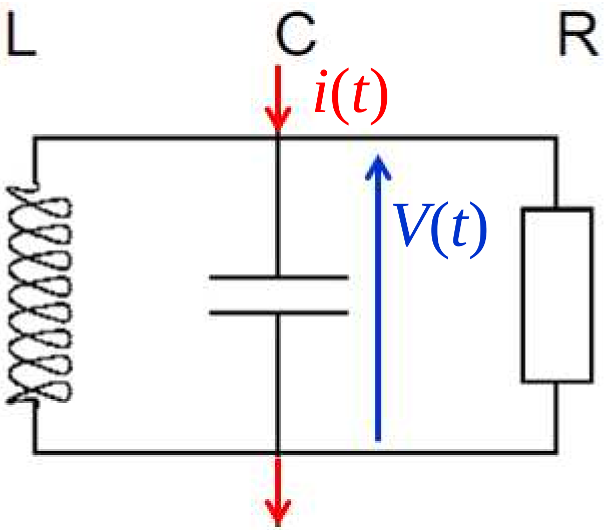
Impedance of a cavity

Fourier transform of the time domain calculation



The results for the impedance can strongly depend on the mesh and the range of the wakepotential !

Equivalent circuit model for the longitudinal wake



with $C = (2k)^{-1}$

$LC = \omega_m^{-2}$

loss parameter k

$\Delta E = q^2\ k$

no losses: $R \to \infty$

bunch current: $i(t) = q\delta(t)$

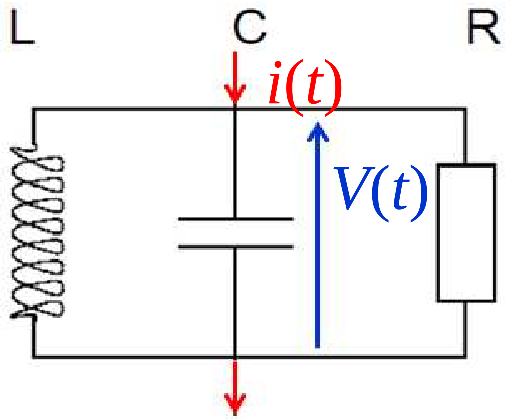
$\to$ voltage: $V(t>0) = -2\ k\cos(\omega_m\ t)$

with losses: $R = \dfrac{Q}{\omega_m\ C} = Q\,\omega_m\ L = \dfrac{2k\ Q}{\omega_m}$ shunt impedance

longitudinal impedance (one mode):

$$Z(\omega) = \frac{V(\omega)}{I(\omega)} = -\left(\frac{1}{i\omega L} + i\omega C + \frac{1}{R}\right)^{-1} = -\frac{2kQ}{\omega_m}\left(1 + i\ Q\left(\frac{\omega}{\omega_m} - \frac{\omega_m}{\omega}\right)\right)^{-1}$$

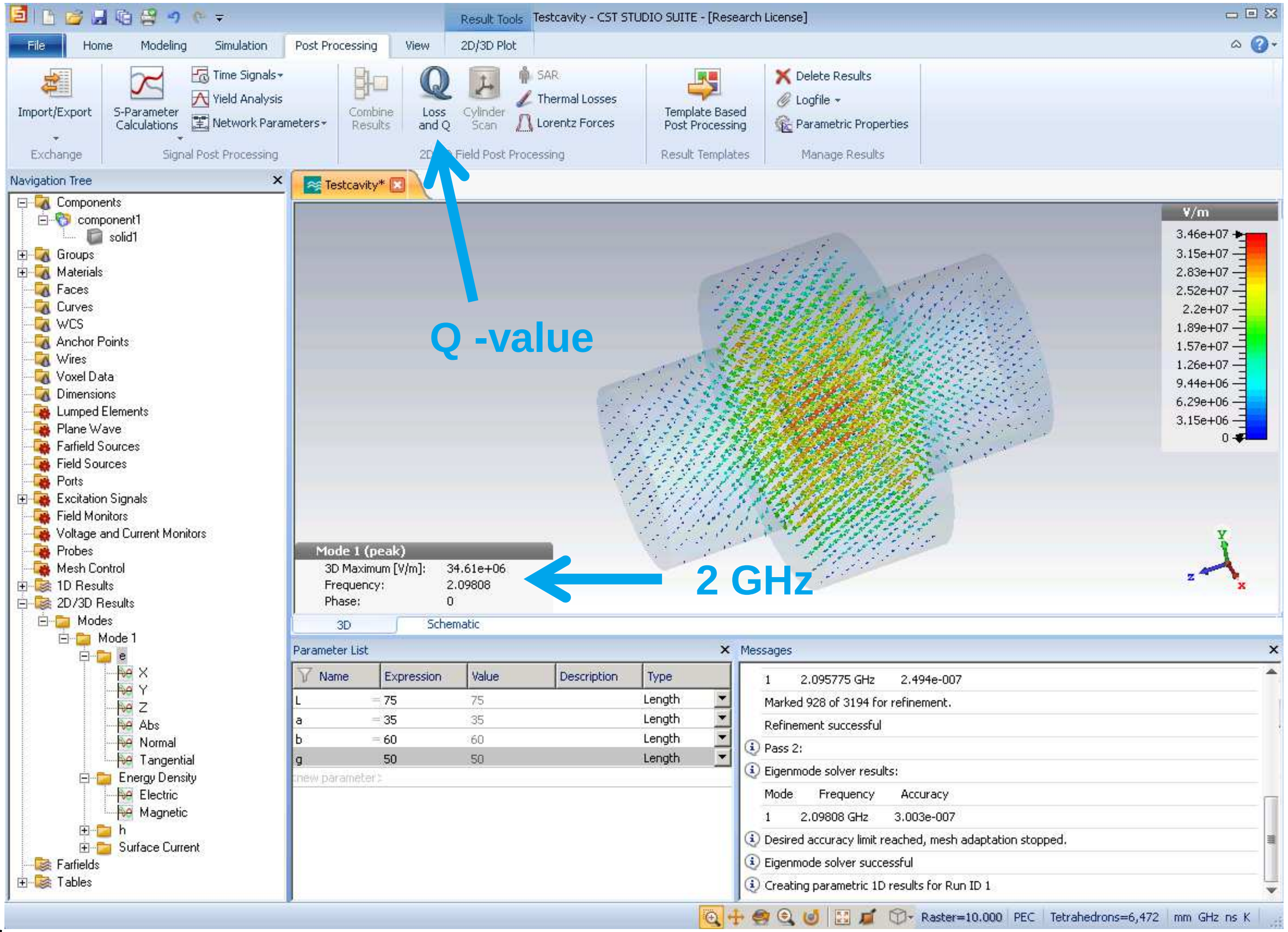
Frequency domain analysis



$$C = (2\,k)^{-1}$$

$$LC = \omega_m^{-2}$$

$$R = \frac{Q}{\omega_m\, C}$$



frequency $\omega_m = 2\pi\ f_m$

loss parameter $k = V^2 / (4\,U)$

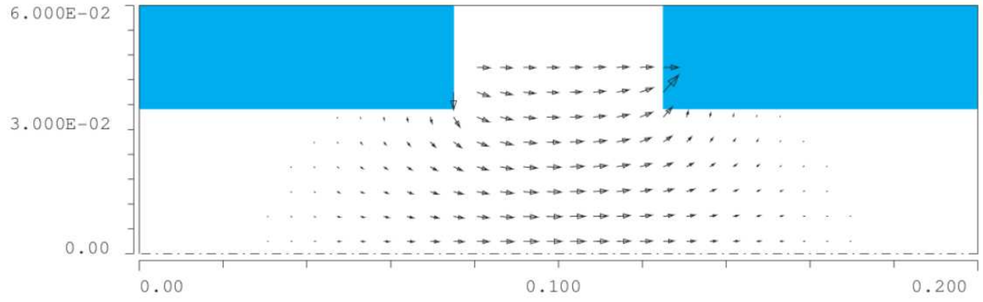
quality Q

„R/Q“ $R/Q = 2k/\omega_m$

Frequency domain post processing

How to obtain the $k = V^2/(4\,U)$ loss parameter of a mode

$$E_z(r,z,t) = E_z(r,z)\,\exp(i\,\omega\,t)$$



$$k_{\|} = \frac{|V|^2}{4\,U}$$

Voltage = integrated electric field with phase factor

$$\Delta E = q^2\;k_{\|}$$

$$V = \int_{-\infty}^{\infty} dz\,E_z(r,z)\,\exp(i\,\omega\,z/c)$$

Stored energy

Point cahrge wake potential
of one mode (s > 0)

$$U = \frac{\epsilon_0}{2}\int d^3r\,|\boldsymbol{E}(\boldsymbol{r})|^2$$

$$W_{\|}(s) \approx -2\,k_{\|}\,\cos(\omega_r\,\frac{s}{c})\,\exp(-\frac{\omega_r}{2\,Q}\,\frac{s}{c})$$

Frequency domain post processing: Dipole modes

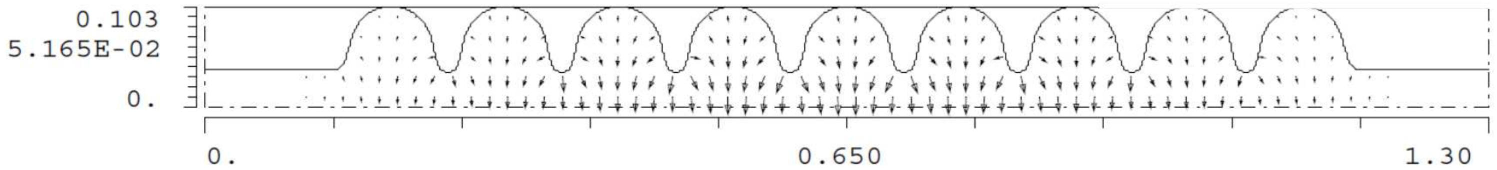
- Numerical Solution of the Maxwell Equations
  - Mode frequency
  - Electric and magnetic field

- Basic post processing
  - Q-value
  - stored energy in the E, B field of the mode
  - voltage with phase factor

- Loss parameter **at beam offset r**, R/Q. R
  - $\boxed{k_{\|}(r) = \frac{|V(r)|^2}{4\,U}, \text{ unit V/C}}$
  - $\frac{R}{Q} = \frac{2\,k_{\|}(r)}{\omega}$, unit Ohm
  - $R = \frac{R}{Q}\,Q = \frac{2\,k_{\|}(r)\,Q}{\omega}$, unit Ohm

- **For Dipole modes**
  - $\frac{R^{(1)}}{Q} = \frac{1}{r^2}\frac{2\,k_{\|}(r)}{\omega}$, unit $\mathrm{Ohm/m^2}$
  - $\left(\frac{R}{Q}\right)' = \frac{R}{Q}\frac{1}{(r\,\omega/c)^2}$, unit Ohm
  - $Z_\perp = \frac{R^{(1)}}{Q}\frac{1}{\omega/c}\,Q$, unit Ohm
  - $R_\perp = \left(\frac{R}{Q}\right)'\,Q$

    $Z_\perp = \frac{\omega}{c}\,R_\perp$



Tesla cavity, 1.62 GHz Dipole mode

Wakefield effects and their relation to important beam parameters

Beam Parameter:

Total bunch charge q_1
Bunch length (and shape)

Transverse dimensions
(Emittance, Beta-function)

Number of bunches
Total beam current

Synchrotron tune
Betatron tune
Damping time (Synchrotron Rad.)
Chromaticity

$$\mathcal{W}_\perp(r_{2\perp}, s) = \frac{1}{q_1}\int_{-\infty}^{\infty} dz [E_\perp(r_{2\perp}, z, t) + c\, u_z \times B_\perp(r_{2\perp}, z, t)]_{t=(s+z)/c}$$

$$Z_\perp(r_{2\perp}, \omega) = \frac{-i}{c}\int_{-\infty}^{\infty} ds\, W_\perp(r_{2\perp}, s)\, \exp(-i\frac{\omega}{c}\, s)$$

Kick on the test charge:

$$\boxed{\theta(r_{2\perp}, s) = \frac{e}{E}\, q_1\, \mathcal{W}_\perp(r_{2\perp}, s)}$$

$$y(s) \sim \exp(-i\,\Omega\, s/c)$$

$$\Omega = \omega_\beta + i\,\tau^{-1}$$

Betatron frequency

growth / damping rate

Equation of motion (rigid beam approximation)

$$\frac{d^2}{ds^2} y(s) + \left(\frac{\omega_\beta}{c}\right)^2 y(s) = \frac{e\ q}{mc^2\,\gamma} \sum_{n=0}^{\infty} \frac{1}{C}\, y(s - n\,C) \quad W_\perp^{(1)}(n\,C)$$

sum: n turns

C = Circumference of the storage ring

dipole wake

ansatz for the solution: $y(s) \sim \exp(-i\,\Omega\,s/c)$

Equation of motion is transformed: $\sum_{n=0}^{\infty} \exp(i\ n\ \Omega\,T_0)\ W_\perp^{(1)}(n\,C) = \frac{i}{T_0} \sum_{p=-\infty}^{\infty} Z_\perp^{(1)}(\Omega + p\omega_0)$

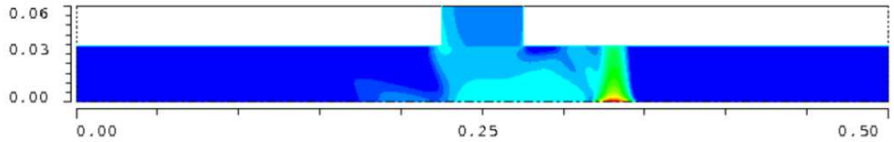
$$\Omega^2 - \omega_\beta^2 = -i\,\frac{e\ q}{mc^2\,\gamma}\,\frac{c}{T_0^2} \sum_{p=-\infty}^{\infty} Z_\perp^{(1)}(\Omega + p\omega_0) \qquad \Omega + \omega_\beta \approx 2\,\omega_\beta$$

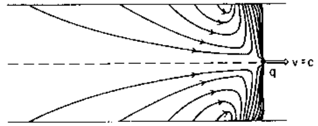
Impedance is related to betatron frequency shift and instability growth rate

$$\Omega - \omega_\beta = -i\,\frac{1}{2\omega_\beta}\,\frac{e\ q}{mc^2\,\gamma}\,\frac{c}{T_0^2} \sum_{p=-\infty}^{\infty} Z_\perp^{(1)}(\omega_\beta + p\omega_0)$$

Beam-pipe geometry and materials and their impact on impedance

Geometric Wake
Resistive Wall Wake

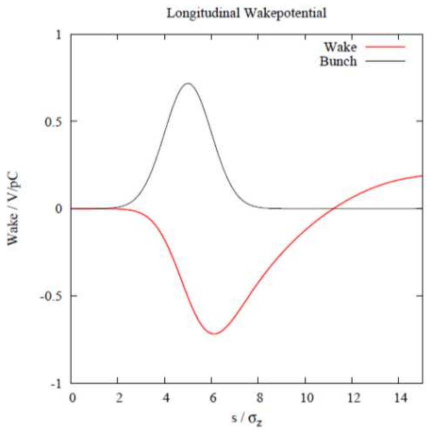




Loss parameter **and** **Kick parameter**

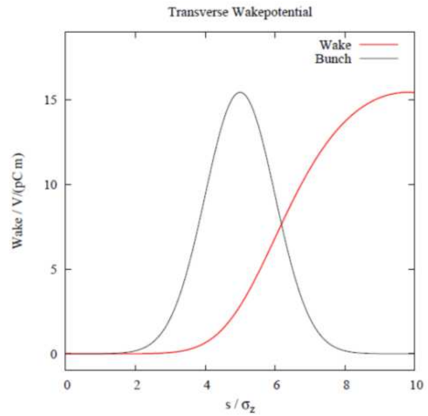
$$k_{tot} = \int ds\, \lambda(s)\, \mathcal{W}(s)$$

$$k_\perp = \int ds\, \lambda(s)\, \frac{1}{r_2} \mathcal{W}_\perp(r_2, s)$$



Line charge density:

$$\lambda(s) = \frac{1}{\sigma_z \sqrt{2\,\pi}} \; \exp\left(-\frac{1}{2}\, \frac{s^2}{{\sigma_z}^2}\right)$$



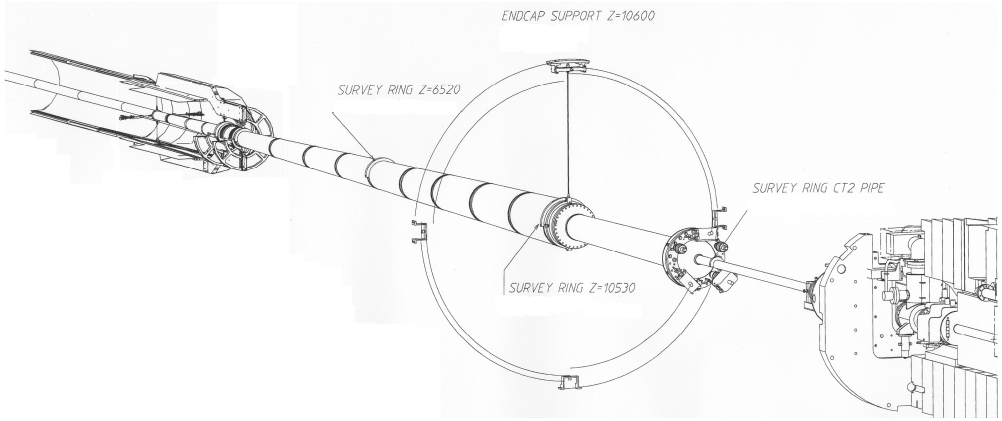
Loss parameter of a mode: $2\, k_{||} = \omega_r \, \frac{R}{Q}$

$$k_{tot} \approx k_{||} \exp\left(-{\omega_r}^2 (\frac{\sigma_z}{c})^2\right)$$

Kick parameter and dipole wake

$$\boldsymbol{k_\perp = \int ds\, \lambda(s)\, \mathcal{W}_\perp^{(1)}(s)}$$

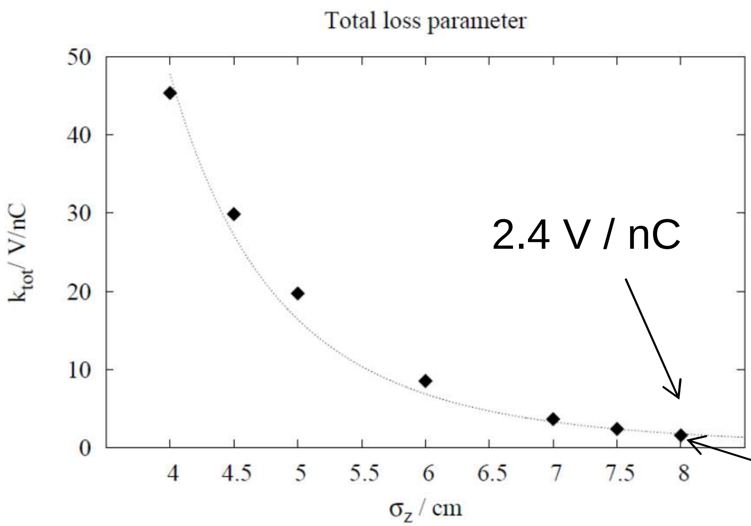
Example: CMS experimental chamber



$$k_{tot} = \int ds\, \lambda(s)\, \mathcal{W}(s)$$

$$k^{(0)}_{||\mathrm{tot}} = \frac{1}{\pi}\int_0^\infty d\omega\ h(\omega)\, \Re(Z_{||})(\omega)$$

$$h(\omega) = \exp\left(-\left(\frac{\sigma_z}{c}\omega\right)^2\right)$$



2.4 V / nC

rms bunch length →

Transient heating

revolution frequency

$$P = N_b\, f_R\, {q_b}^2\, k^{(0)}_{||\mathrm{tot}}$$

Number of bunches

single bunch charge

HL-LHC parameters: P ~ 100 W
(2800 bunches, 35 nC)

CERN-ATS-Note-2013-018 TECH

Example: PETRA III Vacuum chamber

Resistive wall impedance

analytic formula (valid for long bunches)

$$Z_{\parallel}^{(0)}(\omega) = \left(1 - \frac{\omega}{|\omega|}\, i\right)\, L\, \frac{1}{2\pi b} \sqrt{\frac{|\omega|\, \mu}{2\,\sigma}}$$

$$Z_{\perp}^{(1)}(\omega) = \frac{c}{\omega}\, \frac{2}{b^2}\, Z_{\parallel}^{(0)}(\omega)$$

$\sigma = 57 \cdot 10^{6} (\Omega\mathrm{m})^{-1}$ Cu
$\sigma = 37 \cdot 10^{6} (\Omega\mathrm{m})^{-1}$ Al
$\sigma = 1.5 \cdot 10^{6} (\Omega\mathrm{m})^{-1}$ Steel
$(1\ \ \Omega\,\mathrm{m} = 10^{6}\ \Omega \frac{\mathrm{mm}^2}{\mathrm{m}})$

b = pipe radius
L = pipe length

$$\delta_s = \sqrt{\frac{2}{|\omega|\, \mu\, \sigma}}$$

Arc: **Al**, 80 mm x 40 mm

Loss and Kick parameter/length:

$$k'_{\parallel} = \frac{c}{4\,\pi^2\, b\, {\sigma_z}^{3/2}} \sqrt{\frac{Z_0}{2\,\sigma}}\, \Gamma(\frac{3}{4})$$

$$k'_{\perp} = \frac{c}{2\,\pi^2\, b^3\, \sqrt{\sigma_z}} \sqrt{\frac{Z_0}{2\,\sigma}}\, \Gamma(\frac{1}{4})$$

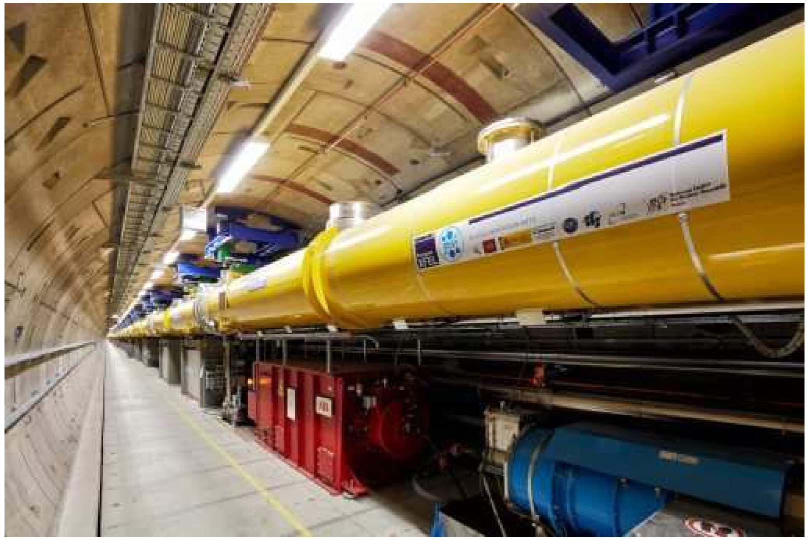
$$\Gamma(\frac{3}{4}) = 1.225 \qquad \Gamma(\frac{1}{4}) = 3.626$$

b = 2 cm
Al chamber
σ_z = 10 mm

Loss parameter

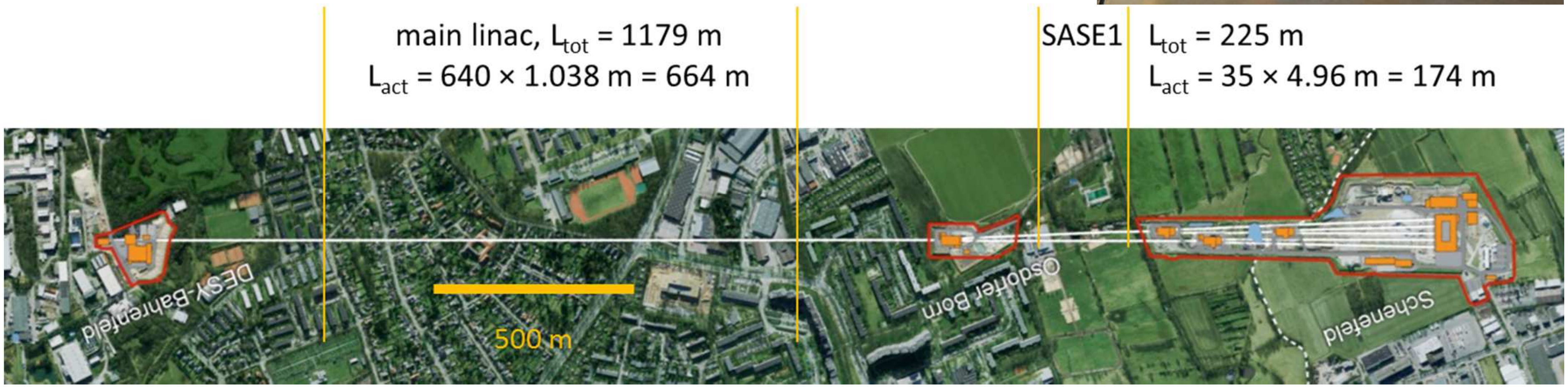
1.1 V / (nC m)

Example: European XFEL

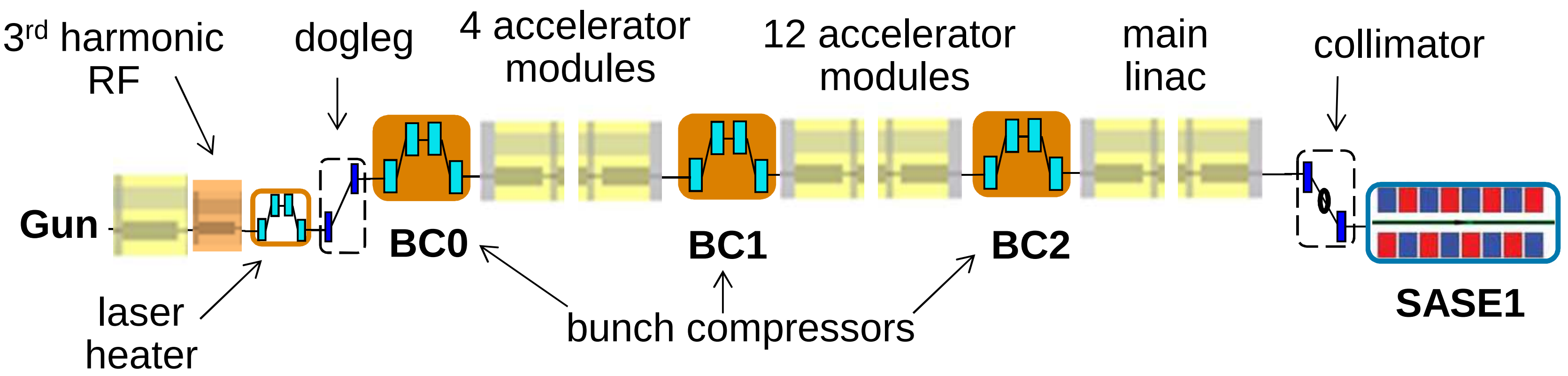


http://www.xfel.eu/

First laser light May 4, 2017

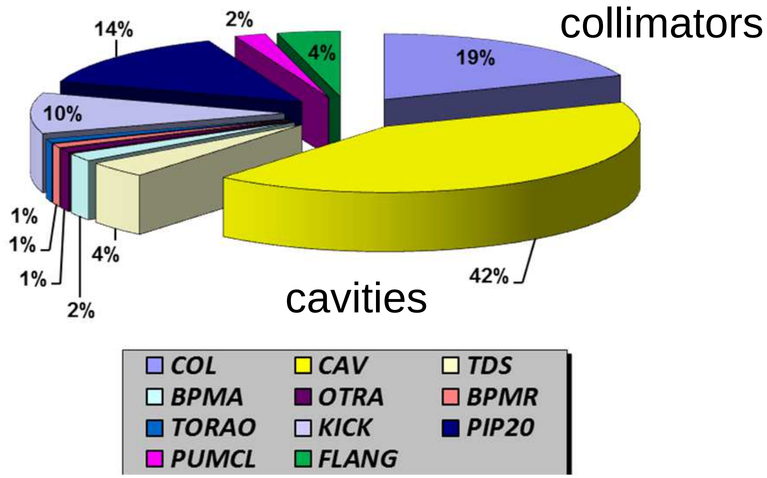


Example: European XFEL



Before the Undulator: Impedance

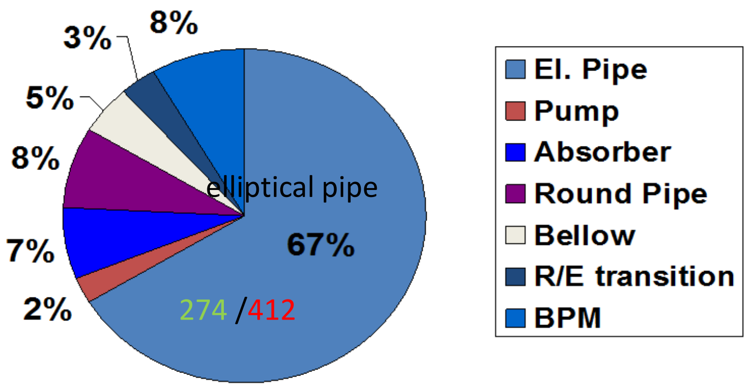
total energy **loss** ≈ 35.3 MeV
total energy spread ≈ 15.4 MeV



In the Undulator: Impedance

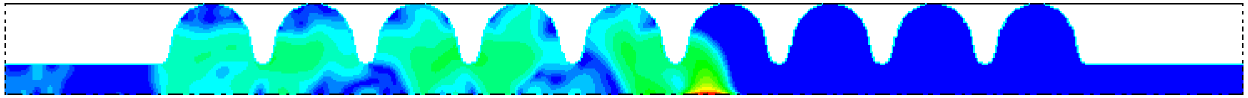
80% is related to material properties

energy spread 14 MeV (35 sections)



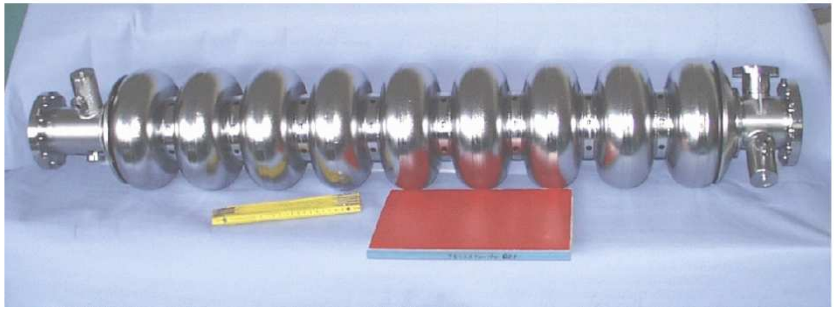
An Introduction to beam instabilities

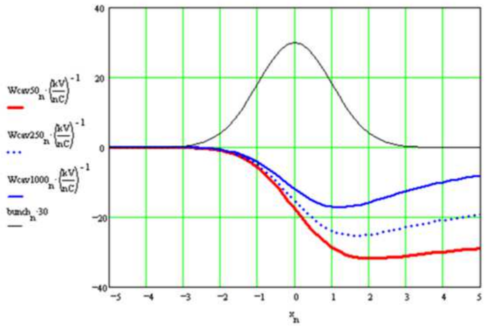
Longitudinal



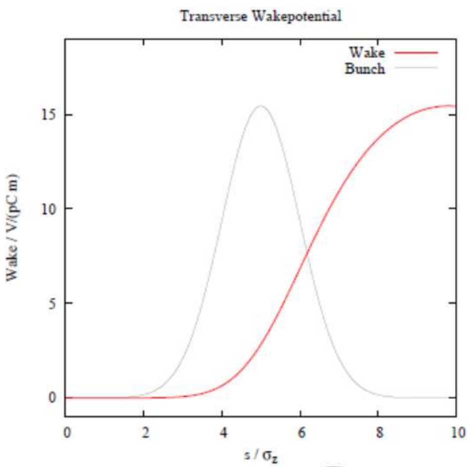
Energy spread

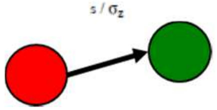
TESLA cavity



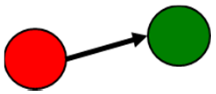


Transverse





Head → Tail

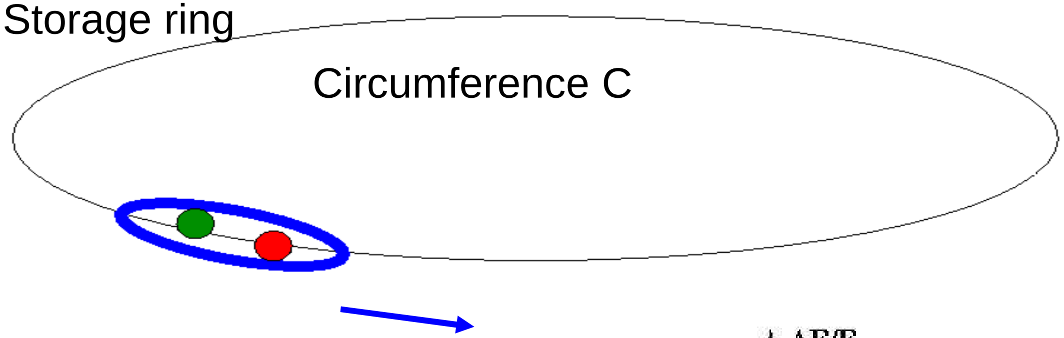


Kick on tail due to the wake of the head

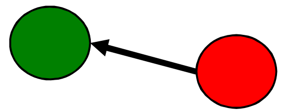
$$\boxed{\boldsymbol{\theta}(\boldsymbol{r_2}_\perp, s) = \frac{e}{E}\, q_1\, \boldsymbol{\mathcal{W}}_\perp(\boldsymbol{r_2}_\perp, s)}$$

Head tail instability

Storage ring

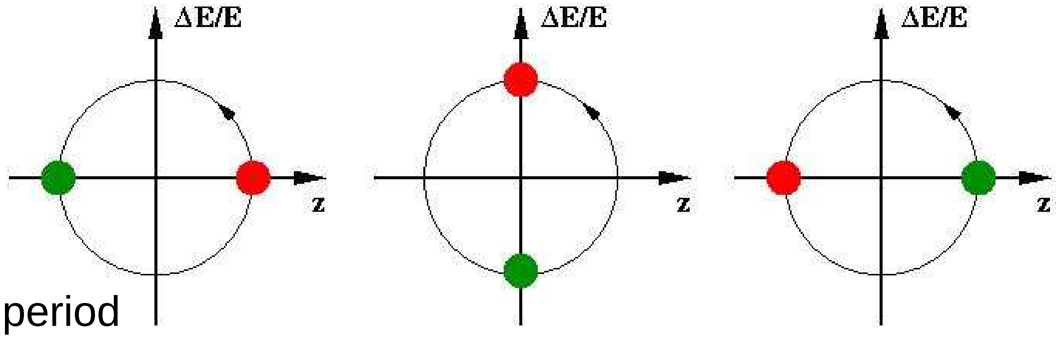


Wakefield: Tail ← Head



Synchrotron oscillations:

Positions of
Head and Tail are exchanged
after one synchrotron oscillation period



<u>Equation of motion 0 … $T_s/2$</u> (time for a synchrotron period)

$$y''_1 + \left(\frac{\omega_\beta}{c}\right)^2 y_1 = 0$$

N/2 = bunch population of the head

Wake potential of the storage ring
(also the total kick parameter could be used here)

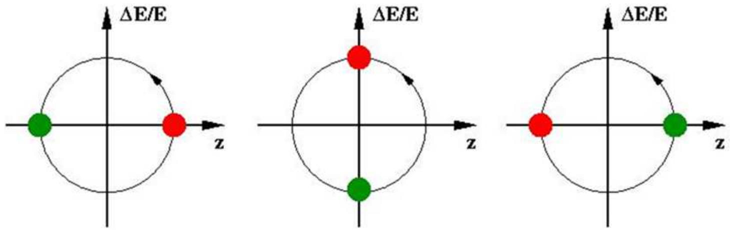
$$y''_2 + \left(\frac{\omega_\beta}{c}\right)^2 y_2 = \frac{N\, r_0}{2\gamma\, C} \mathcal{W}_\perp\, y_1$$

$$r_0 = \frac{1}{4\pi\,\epsilon_0} \frac{e^2}{m_0\, c^2} = 2.818 \cdot 10^{-15}\,\mathrm{m}$$

Head Tail Instability (cont.)

phasor notation:

$$\tilde{y}_{1,2} = y_{1,2} + i\,\frac{c}{\omega_\beta}{y'}_{1,2}$$



0 … $T_s/2$

$$\begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix}_{s=c\,T_s/2} = \exp(-i\;\omega_\beta\,T_s/2) \begin{pmatrix} 1 & 0 \\ i\,\Upsilon & 1 \end{pmatrix} \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix}_{s=0} \qquad \Upsilon = \frac{\pi\,N\,r_0\,c^2}{4\gamma\,C\omega_\beta\omega_s}\mathcal{W}_\perp$$

0 … T_s

$$\begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix}_{s=c\,T_s} = \exp(-i\;\omega_\beta\,T_s) \underbrace{\begin{pmatrix} 1 & i\,\Upsilon \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ i\,\Upsilon & 1 \end{pmatrix}} \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix}_{s=0}$$

Stability requires pure imaginary eigen values of the matrix which can be translated into a criteria for:

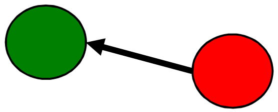
$$\Upsilon < 2$$

Single bunch instability: TMCI

TMCI = Transverse Mode Coupling Instability

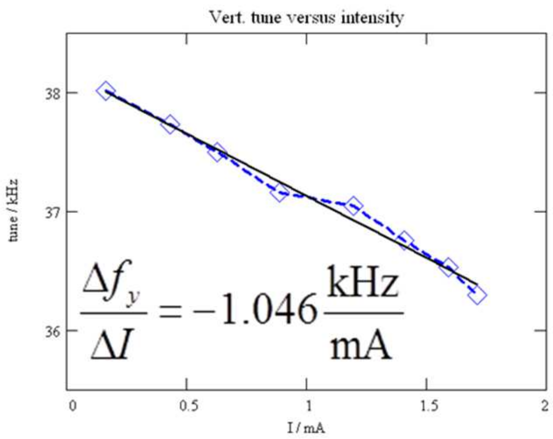
Macro particle model

Tail ← Head



Modes of the charge distribution
(Vlasov equation with wakefields)

Measurement (2009) PETRA III

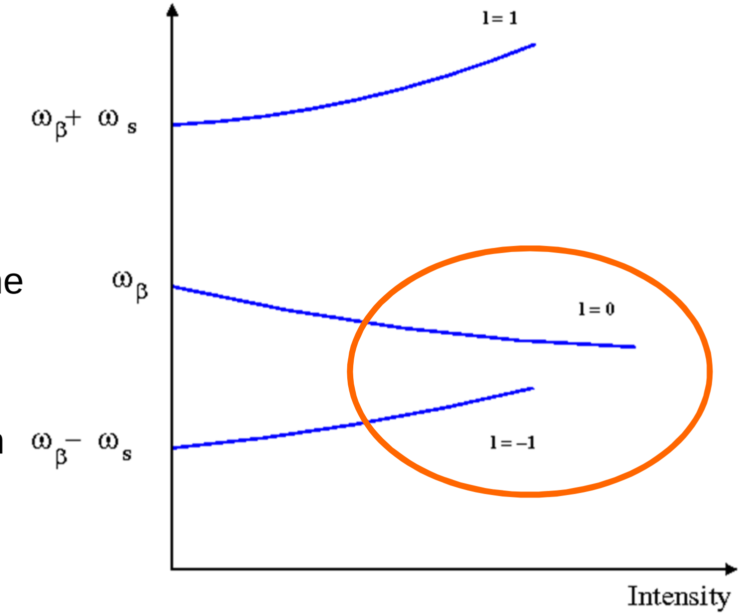


$Q_x = 37.13, \quad f_x = 16.9$ kHz
$Q_z = 30.31, \quad f_x = 39.5$ kHz
$Q_s = 0.049, \quad f_s = 6.1$ kHz

mode coupling

betatron tune

synchrotron side band



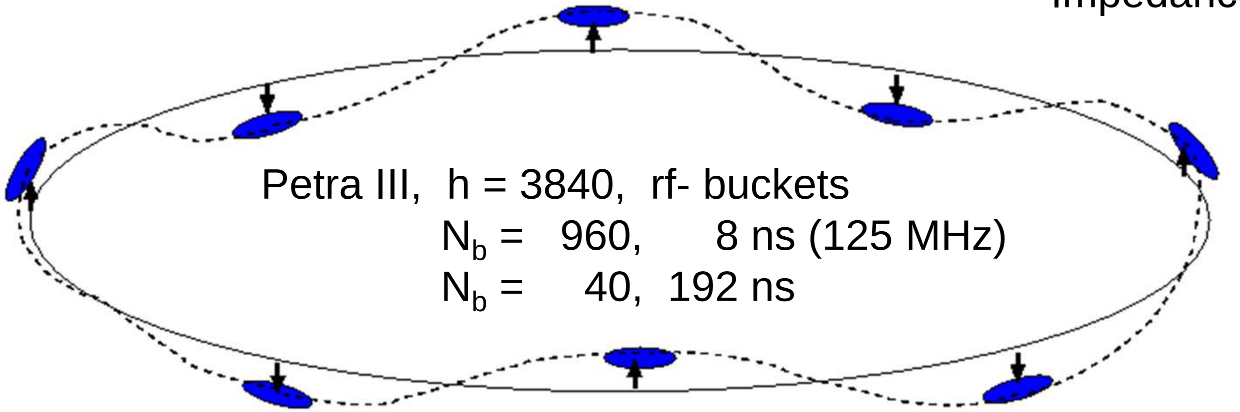
$$\Delta Q_\beta = \frac{I_B \; T_0}{4\pi E / e} \langle \beta \rangle k_\perp \qquad \Delta Q_\beta < Q_s \frac{1}{2}$$

Tune shift with intensity

Kick parameter

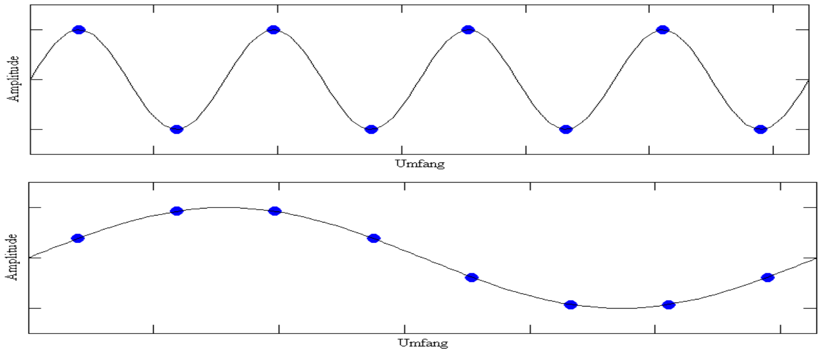
Multi bunch instabilities

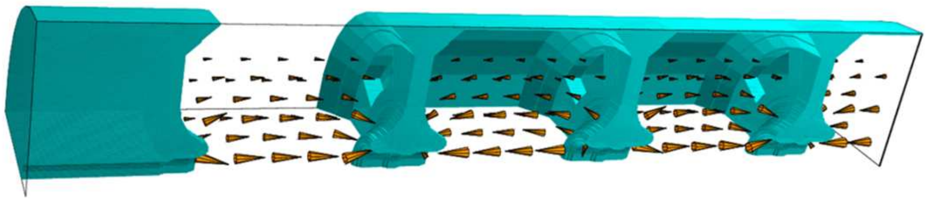
Impedance: HOMs in PETRA Cavities



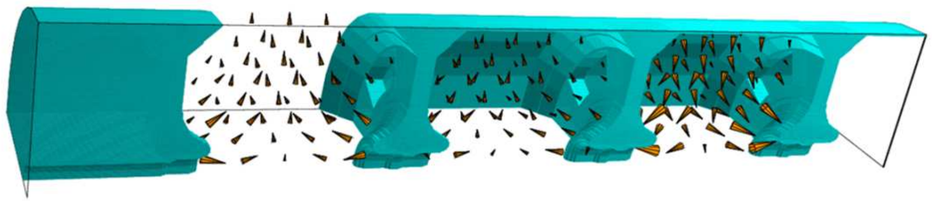
Petra III, h = 3840, rf- buckets
N_b = 960, 8 ns (125 MHz)
N_b = 40, 192 ns

Multi bunch modes





f = 500 MHz, R/Q = 830 Ohm



f = 728 MHz, R/Q = 89 Ohm

$\Rightarrow$ at PETRA III
feedback systems are needed

often it helps to detune HOMs

Ions

Equation for an ideal gas: $$p \cdot V = N \cdot k_B \cdot T$$

Boltzmann constant

$$R_{gas} = N_A \, k_B = 8.31447 \frac{\mathrm{J}}{\mathrm{K\ mol}}, \quad N_A = 6.0221367 \cdot 10^{23}$$

Residual gas density at room temperature (300 K)

$$d_{gas} = \frac{p_{gas} \; N_{Avo}}{R_{gas} \; 300\,\mathrm{K}} = 24.14 \cdot 10^6 \,\mathrm{cm}^{-3}, \quad p_{gas} = 1 \cdot 10^{-9} \;\mathrm{mbar}$$

Ion density

bunch population

$$\lambda_{ion} = d_{gas} \, \sigma_{ion} \, N_0 = 2 \,\mathrm{Mbarn} \, d_{gas} \, N_0$$

Typical cross section for ionization:
2 Mbarn = 2 x 10^{-18} cm^2

Example PETRA III: ion density

Residual gas density

$$d_{gas} = \frac{p_{gas}\, N_{Avo}}{R_{gas}\; 300\,\mathrm{K}} = 24.14 \cdot 10^6\,\mathrm{cm}^{-3}, \quad p_{gas} = 1 \cdot 10^{-9}\ \mathrm{mbar}$$

Ion density

$$\lambda_{ion} = d_{gas}\,\sigma_{ion}\,N_0 = 2\,\mathrm{Mbarn}\,d_{gas}\,N_0$$

= 0.24 ions/cm one bunch
with 5 x 10^9 electrons
(100 mA, 960 Bunche)

= 230 ions/cm 960 Bunchen
(after one turn)

compare with

20.8 x10^6 electrons /cm

average electron density

bunch to bunch distance 8 ns oder 2.4 m

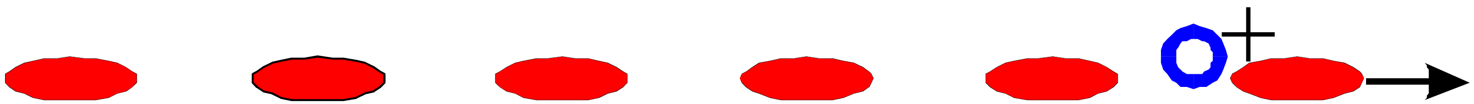
Ion Optics: linear model

Drift Quad (beam as a lense)

$$M = \begin{pmatrix} 1 & L_b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -a & 1 \end{pmatrix} \qquad \cos(\Phi) = \frac{1}{2} Tr(M) = 1 - \frac{a\, L_b}{2}$$

$$L_b = c\,\Delta t \qquad a = N_b \frac{2\, r_p}{\sigma_y\,(\sigma_x + \sigma_y)}\,\frac{1}{A}$$

Trapped ions in the beam:



critical ion mass number

$$\boxed{A > A_c = N_b\, L_b\, \frac{r_p}{2\,\sigma_y\,(\sigma_x + \sigma_y)}}$$

N_b = bunch population
σ_x, σ_y beam dimensions
r_p = 1.535 10^{-18} m

Ions:
A = 2 H_2
A = 16 CH_4
A = 18 H_2O
A = 28 CO, N_2
A = 32 O_2
A = 44 CO_2

Ion effects

Effects of trapped ions on the beam:
increased emittance, betatron tune shifts, reduced beam lifetime

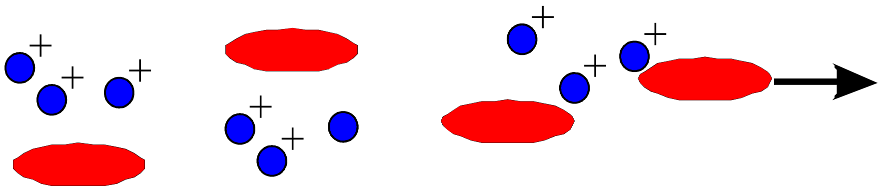
The effect of the ion cloud on the beam can be modelled as a broad band resonator wake field *) (linear approximation of the force)

$$W(z) = \hat{W} e^{-(\omega_i z/2Qc)} \sin\left(\frac{\omega_i z}{c}\right)$$

Non linear interaction between

beam ←→ ion cloud

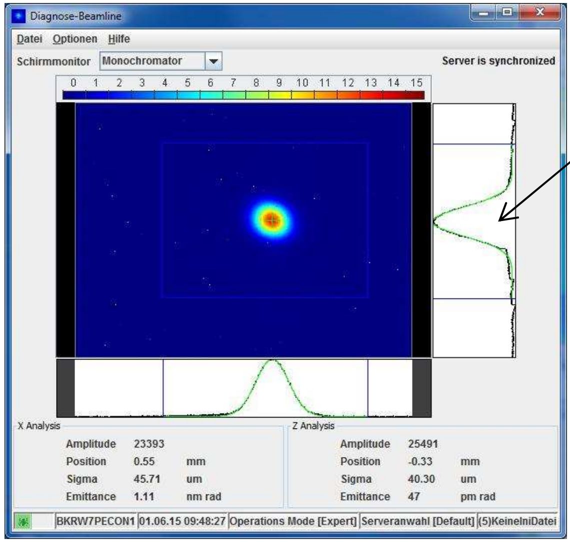
- Multi turn effects (trapped ions)
- Single pass effects
  (fast ion instability)



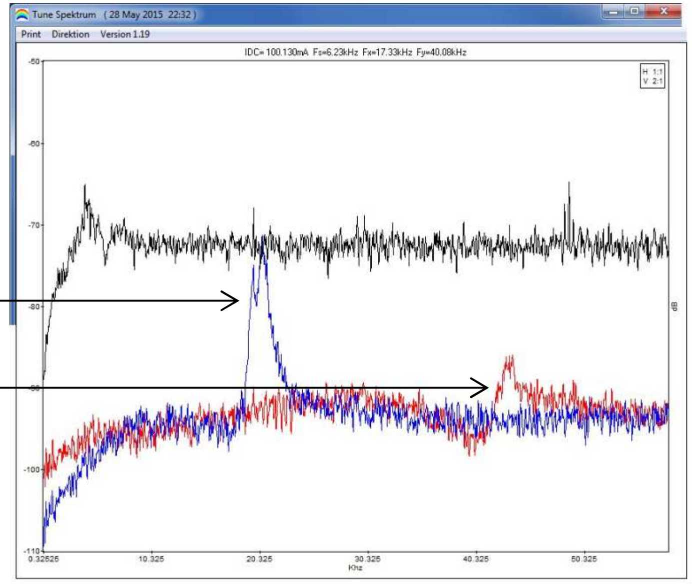
*) L. Wang et al. Phys. Rev. STAB 14, 084401 (2011)
Suppression of beam-ion instability in electron rings with multibunch train beam fillings

Example Ion effects at PETRA III

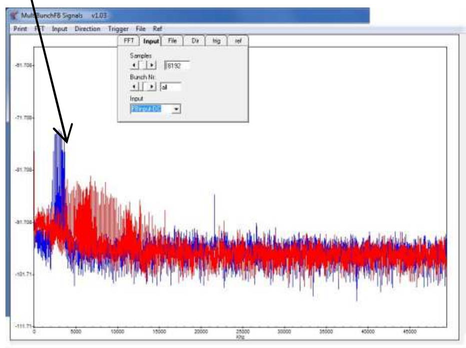
PETRA III vertical emittance increase (data from June 2015)



larger vert.
emittance
(~ factor 2)



“strange”
tune
spectrum
+
additional lines
in the multibunch
spectrum



2014: Installation of new vacuum chambers

The effects disappeared
with improved vacuum conditions
(conditioning)

Conclusion: Less is More

less ions
less impedance → more beam current
more luminosity
more brilliance

⇩

vacuum chamber design

small loss and kick parameters

$$k_{tot} = \int ds \, \lambda(s) \, \mathcal{W}(s)$$

$$\Delta Q_{\beta} = \frac{I_B \; T_0}{4\pi \, E/e} \; \langle \beta \rangle k_{\perp} < Q_s \frac{1}{2}$$

$$P = N_b \, f_R \, {q_b}^2 \, k^{(0)}_{||\mathrm{tot}}$$

Thank you for your attention !