

# Beam Based Impedance Measurements

CAS - Intensity Limitations in Particle Beams

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CERN

# Why do we need to measure impedance with beam?

Indeed beam coupling impedance of various machine elements can be estimated using

- advanced EM simulations (various codes)
- bench measurements – see previous talks!

→ **To verify** how good is the existing impedance model since

- there are elements - difficult for measurements and calculations,
- material properties are not always well known,
- nonconformities also exist...

→ **To identify** the offending impedance driving instability or posing some other intensity limitations

# Outline of the talk:

## impedance measurements with

- **Stable beam:**
    - synchrotron (and betatron) frequency shifts
    - change in debunching time
    - bunch lengthening
    - synchronous phase shift
  - **Unstable beam:** instability characteristics
    - spectra
      - single bunch (RF off)
      - multi-bunch (RF on)
    - growth rates
    - thresholds
- Practically **all intensity effects can be used** for impedance evaluation by comparison of measurements with simulations and/or analytical formulas!

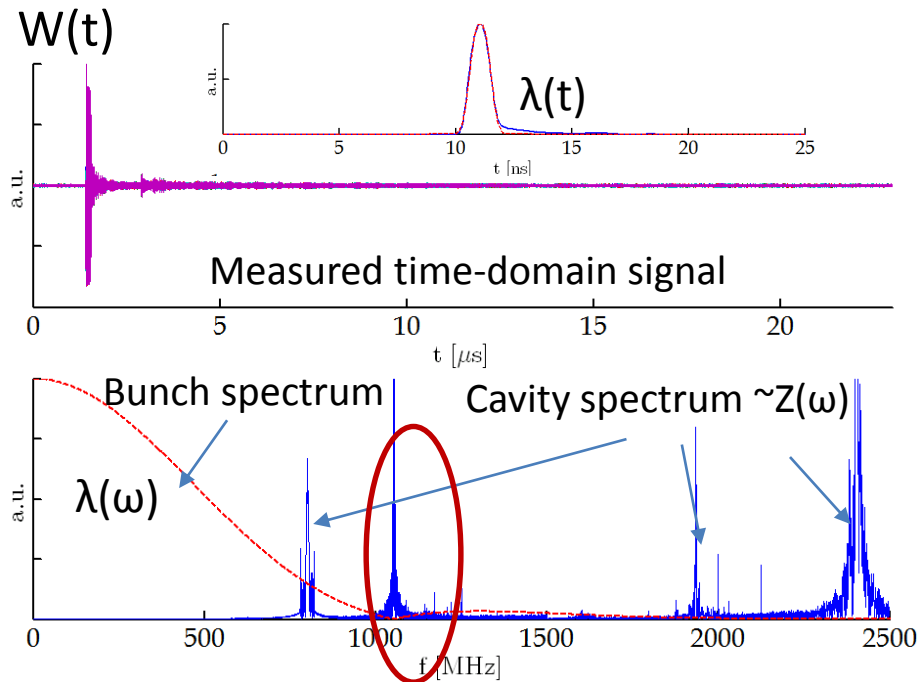
} → reactive impedance ( $\text{Im}Z$ )  
→ resistive impedance ( $\text{Re}Z$ )

**Below:** circular, proton, high energy ( $> \text{GeV}$ ) accelerators → relatively long bunches (ns) – very different from ps or even fs bunches for the impedance range of interest

# Measurements with stable beam

# Beam measurements: intermediate case

Impedance of particular element in the ring (cavity, ...) can be evaluated from the signal excited by a single bunch with known (measured) profile.



$$Z(\omega) \sim W(\omega)/\lambda(\omega)$$

Example [1]:  $\sim 2$  ns long bunch excites fundamental and HOM modes in the probe of the SPS 800 MHz TW cavity ( $f_0 = 43.4$  kHz,  $n_r = 4 \times 4620$ )

Absolute values depend on RF probe characteristics, but can be evaluated for the SW structures

[1] J. Varela et al., CERN ABT-Note-2015, to be published, 2015

[2] See also J. M. Byrd et al., NIM A 455, 2 (2000) and article in Handbook of Accelerator Physics and Engineering, 2<sup>nd</sup> edition, edited by A. Chao et al.

# Potential well distortion

In **equilibrium** the particle distribution is a function of Hamiltonian  $H$ :  $F = F(H)$  with potential well defined by the total voltage seen by a particle:

$$V(\varphi) = V_{\text{rf}}(\varphi) + V_{\text{ind}}(\varphi),$$

where  $V_{\text{ind}} = -e \omega_0 \sum_n G_n Z_n e^{in\theta}$ ,  $\omega_0 = 2\pi f_0$  is revolution frequency,  $Z_n = Z(n\omega_0)$   
 $G_n$  is the  $n$ -th Fourier harmonic of the bunch line density  $\lambda$  in equilibrium

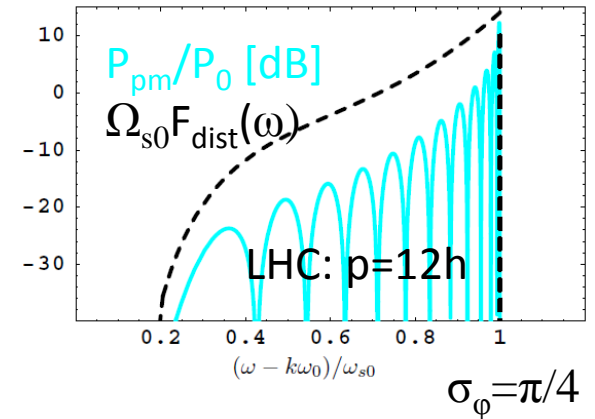
- Modified synchrotron frequency distribution (not only the shift)
- Bunch lengthening (or shortening – depends on sign of  $\eta \text{Im}Z$ )
  - Haissinski equation for **electron bunches** in equilibrium
  - Arbitrary distribution function for **proton bunches**
- Synchronous phase shift

# Measurements of incoherent synchrotron frequency

- **Longitudinal Schottky**: spectral density of current fluctuations [1]:

$$P(\omega) = \frac{e^2 N \omega_0^2}{2\pi} \sum_{k=-\infty}^{\infty} \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} \frac{1}{|m|} F\left(\frac{\omega - k\omega_0}{m}\right) |I_{mk}(J)|^2,$$

for short bunches  $I_{mk} \simeq i^m J_m(k\phi_a/h)$

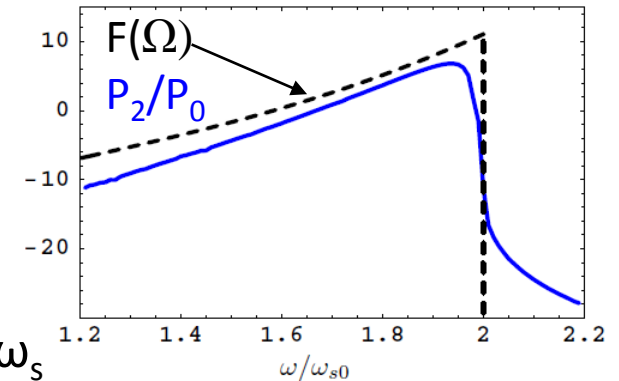


- **Peak detected Schottky**: power spectral density (quadrupole line) [2]

Frequency distribution

$$P_U(\omega) = \frac{P_0}{\omega_{s0}^2} \sum_{m=1}^{\infty} \int \Omega^2 F(\Omega) |A_m(\Omega)|^2 |S(\omega - m\Omega)|^2 d\Omega,$$

The PD Schottky spectrum deviates from distribution function  $F(\Omega)$  mainly due to form-factor  $A_m(\Omega)$



- **Bunch excitation** by phase modulation at  $\omega_{\text{mod}} \sim \omega_s$

[1] S. Chattopadhyay, CERN-84-11, 1984

[2] E. S., T. Bohl, T. Linnear, Proc. HB2010

# Incoherent synchrotron frequency shift and bunch lengthening

The total voltage seen by the bunch is

$$V(\varphi) = V_{\text{rf}}(\varphi) + V_{\text{ind}}(\varphi),$$

For inductive impedance  $V_{\text{ind}} = -L \, dI/dt = -e/\omega_0 \, \text{Im}Z/n \, d\lambda(t)/dt$

For small amplitude synchrotron motion

$$V \approx [V_0 \cos\varphi_s - e \, \text{Im}Z/n \, d^2\lambda/dt^2 / (h\omega_0)] \varphi$$

For a parabolic bunch  $\lambda = \lambda_0 (1 - 4t^2/\tau^2)$  with  $\lambda_0 = 3N/(2\tau)$ , N - number of particles

**Synchrotron frequency:**

$$\omega_s^2 = \omega_{s0}^2 \left[ 1 + \frac{3 I_b \text{Im}Z/n}{\pi^2 V_0 \cos\varphi_s (f_0 \tau)^3} \right], \text{ where bunch current } I_b = N e f_0$$

→ Strong dependence on bunch length

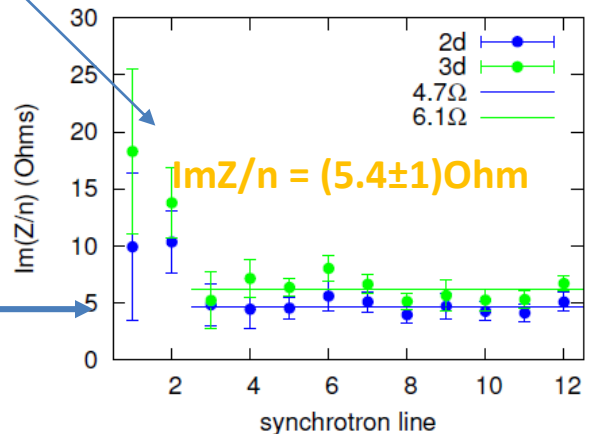
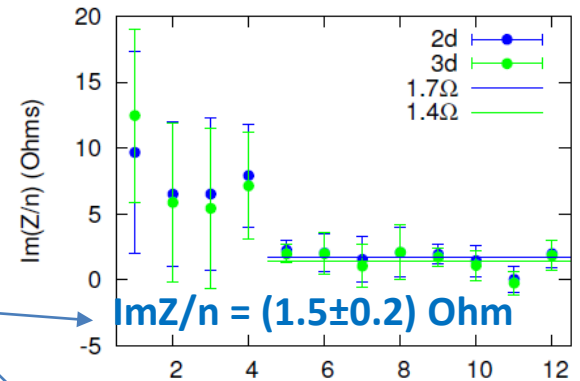
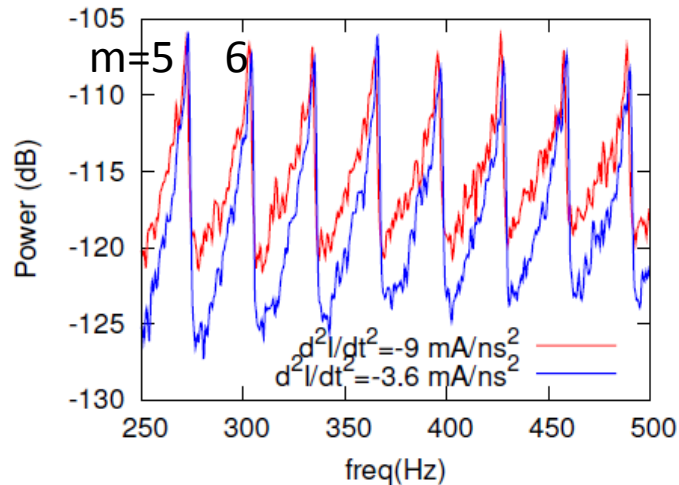
→ Defocusing effect above transition ( $\cos \varphi_s < 0$ ) for  $\text{Im}Z/n > 0$

**Bunch lengthening** is described by equation:  $1 = (\tau/\tau_0)^4 + (\tau/\tau_0) [\omega_s^2(\tau_0) - \omega_{s0}^2] / \omega_{s0}^2$



# Incoherent synchrotron frequency shift from longitudinal Schottky spectrum

- Measure the distance between positive and negative sidebands  $2m\Delta f_s$  for different  $m$  with time (intensity decay)
- Fit parabolas to top 30% of averaged bunch profiles to find  $\lambda''(t)$  and use it in the fit
- Blue and yellow RHIC rings are very similar, the source of the difference is not known yet



# Incoherent synchrotron frequency shift: LHC at 450 GeV

LHC Design Report:  $\text{Im}Z/n=0.1 \text{ Ohm}$

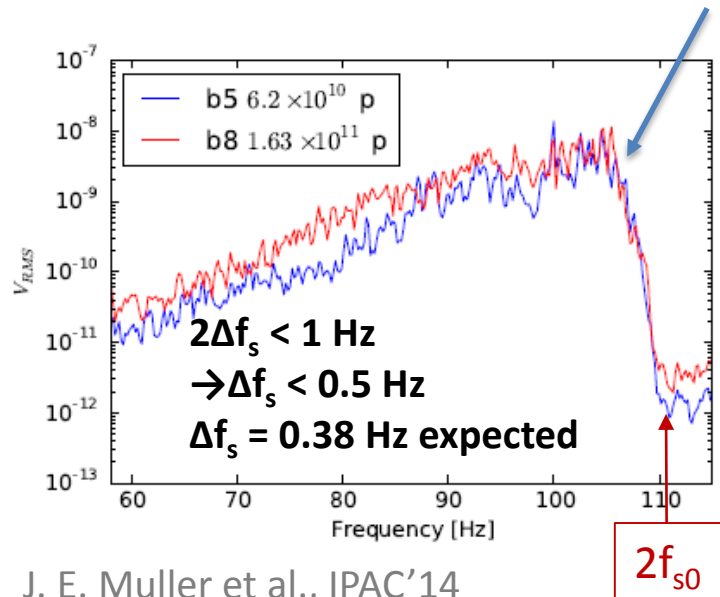
Measurements of the LHC impedance:  $\rightarrow$

(1) Phase modulation ( via cavity set point):

$\varphi = \varphi_0 \sin(\omega_{\text{mod}} t) \rightarrow$  scan  $\omega_{\text{mod}}$  from above to see bunch excitation as a function of intensity

(2) Peak detected Schottky spectrum

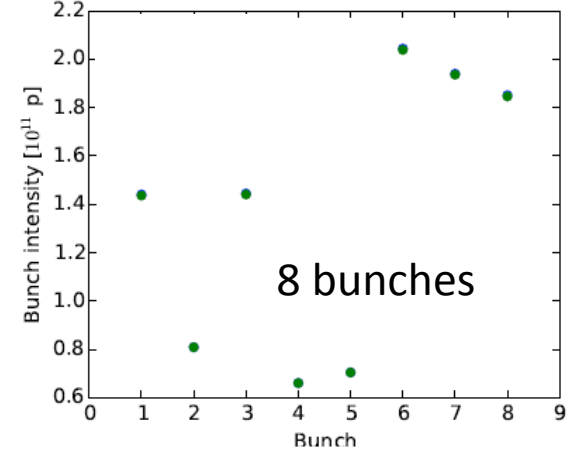
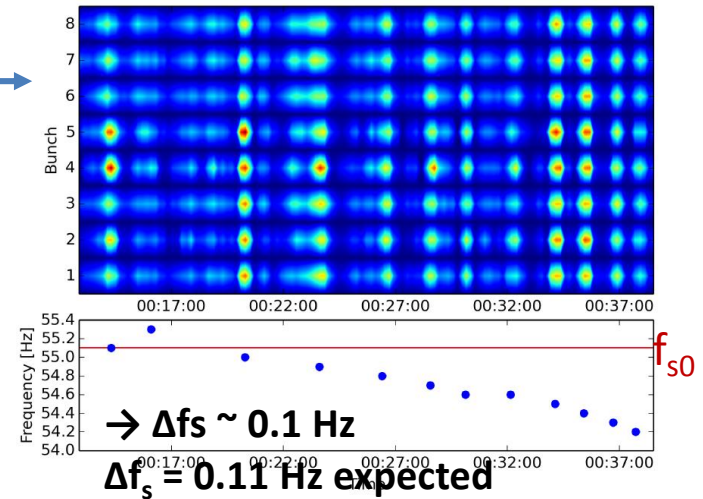
$m=2$  line  $\sim$  synchrotron frequency distribution



J. E. Muller et al., IPAC'14

(3) Loss of Landau damping is the most accurate estimation so far (see below)

Derivative of the 400 MHz component



# Coherent synchrotron frequency shift

- Dipole oscillations → negligible effect
- Quadrupole oscillations → mismatched bunches

The shift of quadrupole oscillation frequency:

$$\omega_{2s}(N) = \Delta\omega_{inc}(N) + \Delta\omega_{coh}(N) + 2\omega_{s0},$$

where  $\Delta\omega_{inc} \sim \text{Im}Z_1$ ,  $\Delta\omega_{coh} \sim \text{Im}(Z/n)^{m=2}_{eff}$  and

$$\text{Im}(Z/\omega)^m_{eff} = \frac{\sum_{p=-\infty}^{\infty} h_m(\omega_p) Z(\omega_p) / \omega_p}{\sum_{p=-\infty}^{\infty} h_m(\omega_p)}, \quad \omega_p = p\omega_0 + m\omega_s$$

For a Gaussian bunch  $h_m(\omega) = (\omega\sigma)^{2m} e^{-\omega^2\sigma^2}$ ;  $Z_1 \simeq \sum_{p=-\infty}^{\infty} p \text{Im}Z(\omega_p) e^{-\omega_p^2\sigma^2/2}$

- **Loss of Landau damping:**  $\Delta\omega_{coh}^m > \Delta\omega_s \rightarrow$

$\Delta\omega_s$  - synchrotron frequency spread inside the bunch

$\eta$  - slip factor

$$|\text{Im}Z|/n < \frac{|\eta|E}{eI_b\beta^2} \left(\frac{\Delta E}{E}\right)^2 \frac{\Delta\omega_s}{\omega_s} f_0\tau$$

F. Sacherer, IEEE Trans. Nucl. Sci.  
NS-20, p.825, 1973

# Quadrupole synchrotron frequency shift

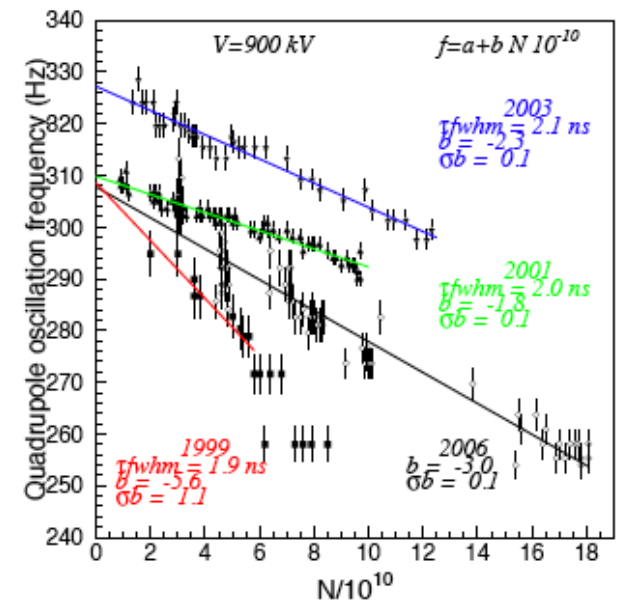
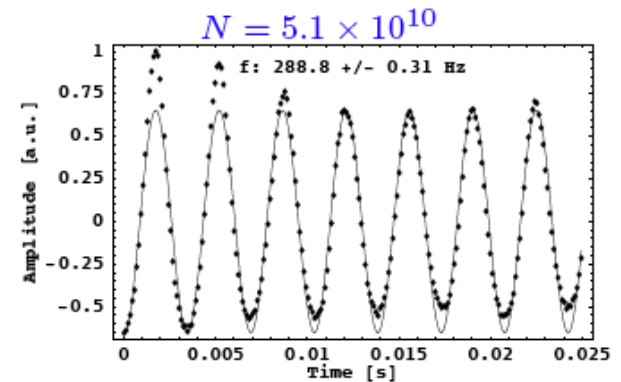
Measurements of **quadrupole oscillation frequency** of bunches injected with variable intensity and constant length (SPS, 26 GeV/c) from bunch length, peak amplitude and Schottky signals:

$$f_{2s}(N) = a + b \times N/10^{10}$$

- 1999: before impedance reduction: **b = -5.6**
  - 2001: SPS impedance reduction: **b = -1.8**
  - 2003: installation of 4 extraction kickers: **b = -2.3**
  - 2006: 5 more kickers installed: **b = -3.0**
- Successful reference measurements

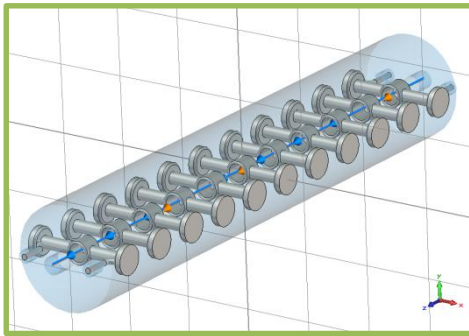
2007: a few kickers serigrafed (shielded) – but effect was not measurable anymore (increase of b)!

Why?

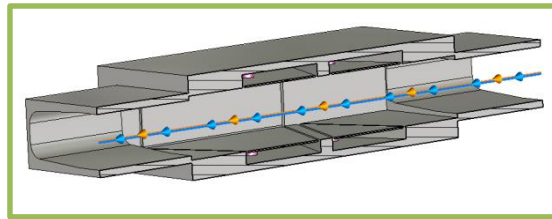


# Realistic impedance model (CERN SPS)

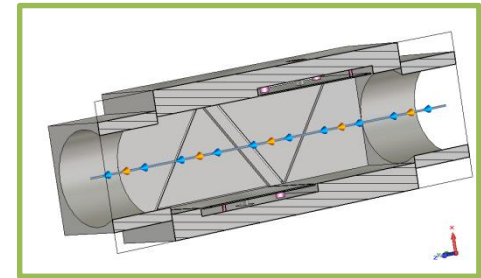
TW RF cavities:  
200 MHz and 800 MHz



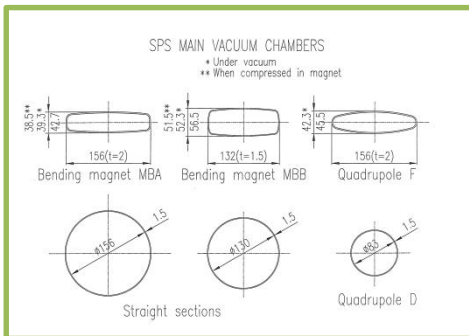
Beam position  
monitor H



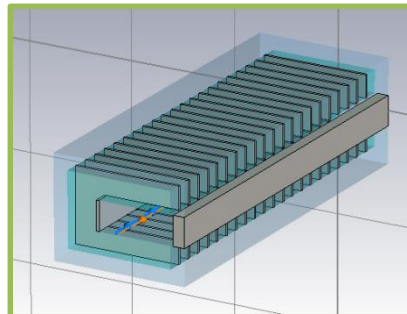
Beam position  
monitor V



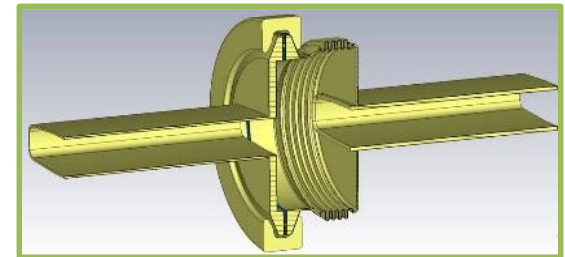
Vacuum chambers



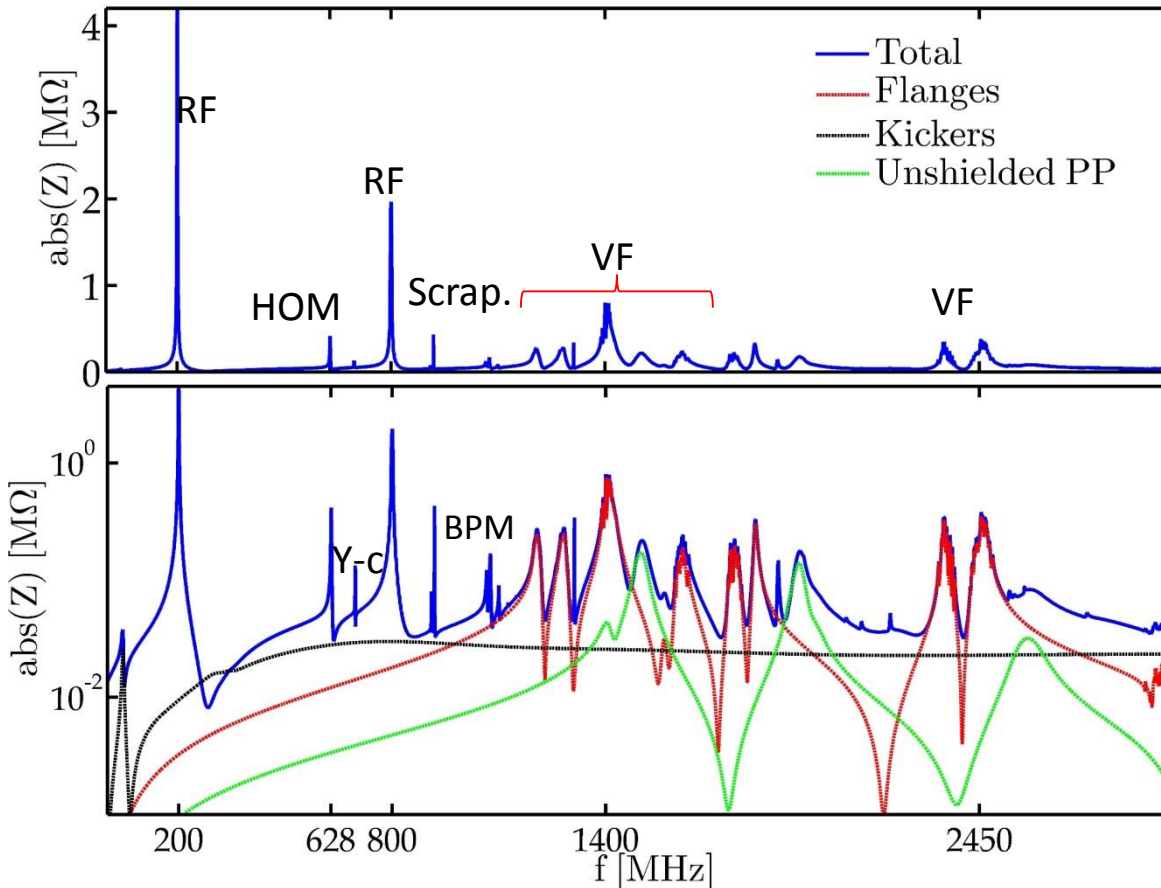
Kickers



Vacuum flanges



# Present SPS impedance model



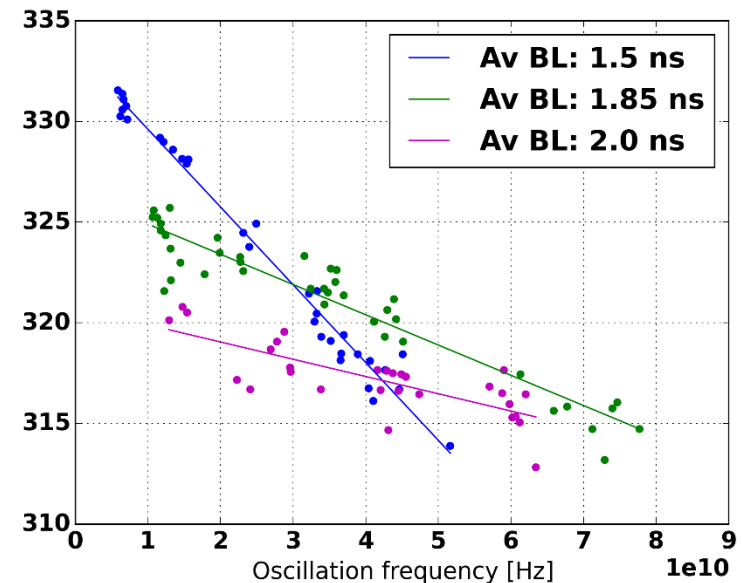
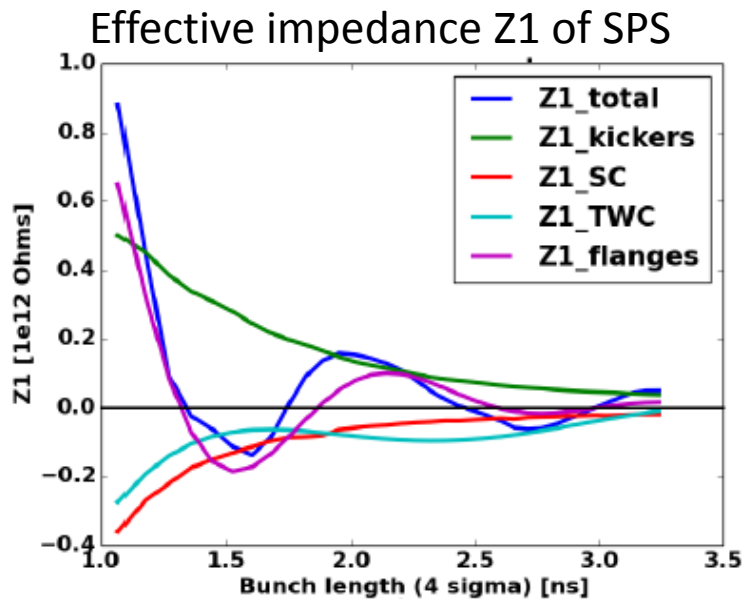
J. Varela, C. Zannini et al., 2015

## Model includes:

- 200 MHz cavities (2+2) + HOMs
- 800 MHz cavities (2)
- Kicker magnets (8 MKEs, 4 MKPs, 5 MKDs, 2 MKQs)
- Vacuum flanges (~500) + DR
- BPMs: BPH&BPV (~200)
- Unshielded pumping ports (~ 16 similar + 24 various)
  - non-conformal assumed 0
- Y-chambers (2 COLDEX + 1)
- Beam scrappers (3 S + 4 UA9)
- Resistive wall
- AEPs (RF phase PUs, 2) ~ 0
- 6 ZSs + PMs
- 25 MSE/MST + PMs

# Synchrotron frequency shift: effective impedance

**Realistic** ring impedance usually cannot be approximated by constant  $\text{Im}Z/n$  since its frequency dependence has a complicated structure



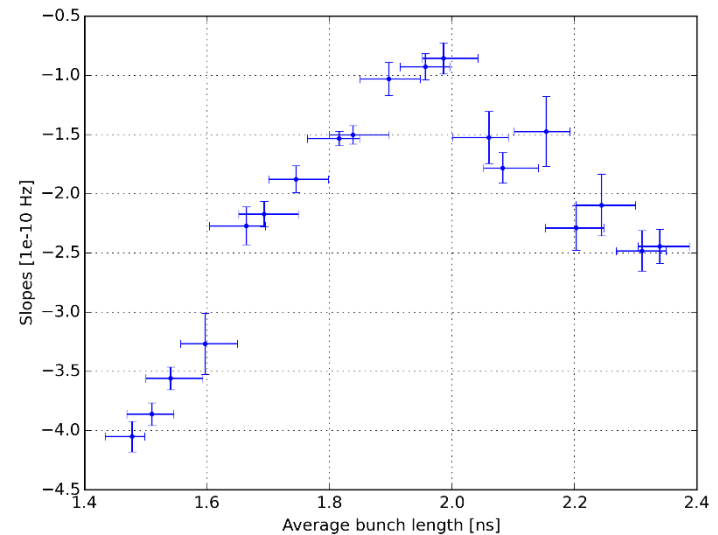
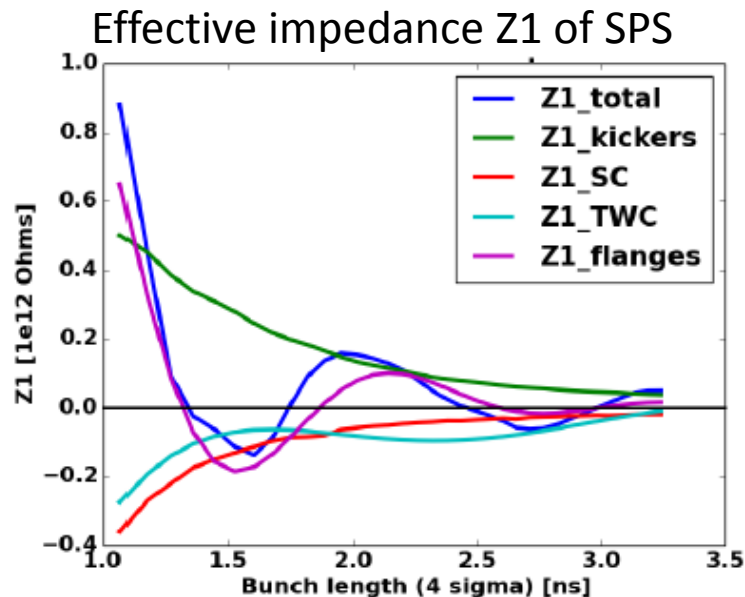
A. Lasheen et al., 2015

- Only **effective impedance** can be measured with the beam
- Strong dependence of the synchrotron frequency shift on **bunch length**



# Synchrotron frequency shift: effective impedance

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A. Lasheen et al., 2015

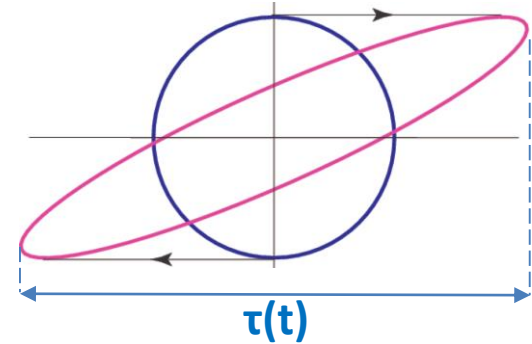
- Only **effective impedance** can be measured with the beam
- Strong dependence of the synchrotron frequency shift on **bunch length**



# Measurements with RF off: debunching time (1/3)

For a parabolic bunch injected into the ring with **RF off** variation of bunch length  $\tau(t)$  and peak line density  $\lambda_p(t)$  with time (debunching process)

$$\frac{\tau(t)}{\tau(0)} = r(t) \quad \text{and} \quad \frac{\lambda_p(t)}{\lambda_{p0}} = \frac{1}{r(t)}$$



can be described using an exact analytical solution of equation ( $\text{Im}Z/n = \text{const}$ ):

$$\frac{\dot{r}^2}{2} + U(r) = 0, \quad U(r) = -\frac{\Omega^2(r-1)(1+ar)}{2r^2} \quad \text{where}$$

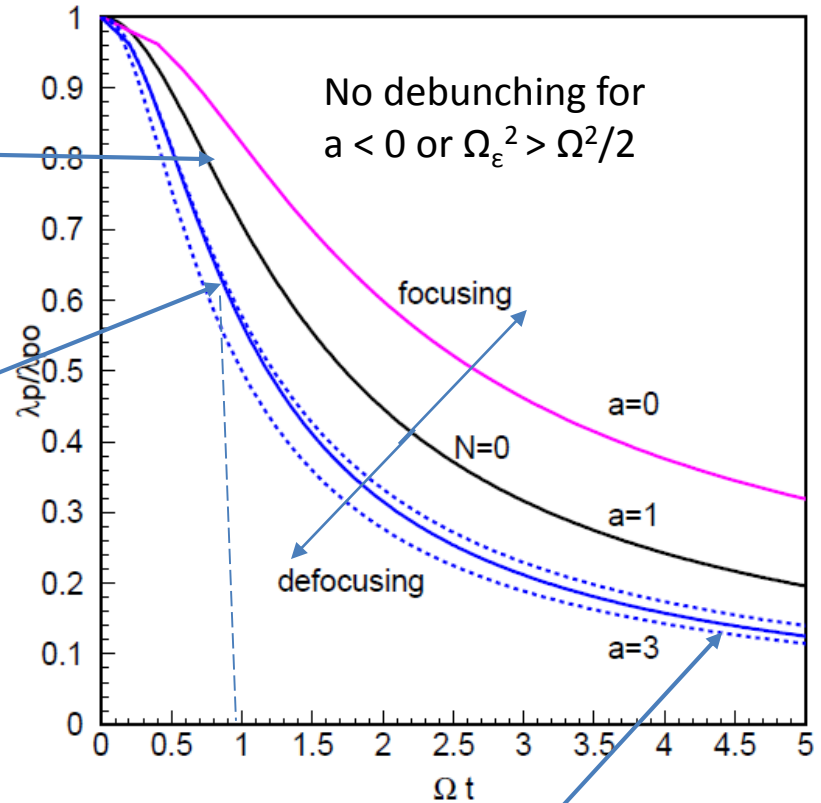
$$a = 1 + 2\text{sgn}(\eta\text{Im}Z)\frac{\Omega_\epsilon^2}{\Omega^2} \quad \Omega = \frac{2\eta}{\tau} \frac{\Delta p_{\max}}{p} \quad \text{and} \quad \Omega_\epsilon = \left( \frac{6Ne^2\eta}{\pi E_s \tau^3} \frac{\text{Im}Z}{n} \right)^{1/2}$$

# Debunching time (2/3)

For  $N=0$ :  $\tau(t) = \tau(0) [1 + \Omega^2 t^2]^{1/2}$   
 with debunching time  $t_d = 1/\Omega$  ( $a=1$ ).  
 For matched bunch  $\Omega = \omega_{s0}$   
 (otherwise defined by RF in injector)

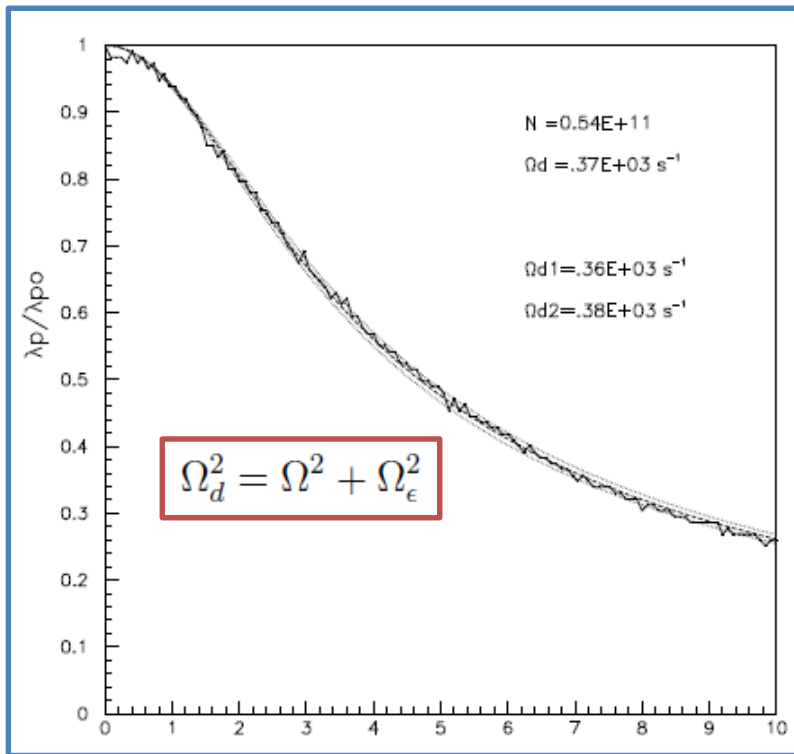
For  $t \leq 1/\Omega$   
 $\tau(t) = \tau(0) [1 + (\Omega^2 \pm \Omega_\epsilon^2) t^2]^{1/2}$

But... if RF is switched off for matched bunch,  
 $\Omega^2 = \omega_{s0}^2 - \Omega_\epsilon^2$  due to potential well distortion and  $t_d \approx 1/\omega_{s0}$   
 → no effect can be measured



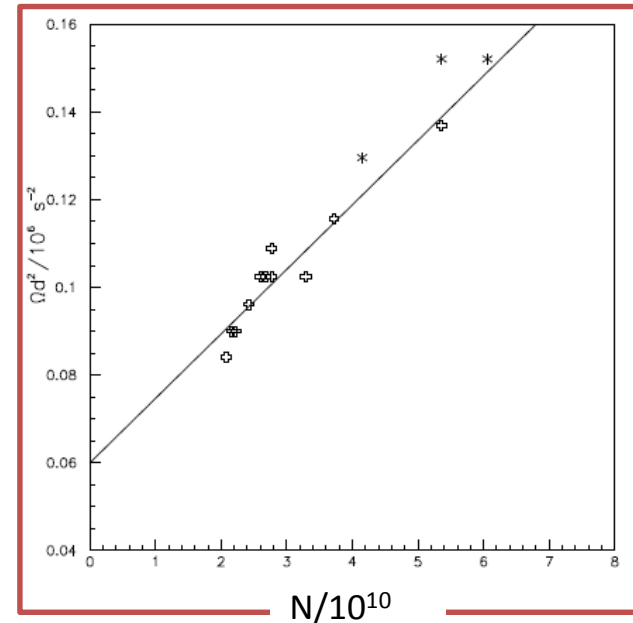
Asymptotic solution for  $t \gg 1/\Omega$ :  
 $\tau(t) \approx \tau(0) [1 + (\Omega^2 \pm 2\Omega_\epsilon^2) t^2]^{1/2}$

# Debunching time (3/3): measurements in the CERN SPS



Time [ms]

Bunch length after rotation can also be used for impedance estimation!



Measured  $\text{Im}Z/n = 18.7 \text{ Ohm}$  is slightly higher than values found by other methods (with RF on) at that time (before impedance reduction), most probably due to longer bunches during debunching

# Transverse (reactive) impedance: betatron tune shift measurements (1/2)

Measurements of coherent betatron tune shift due to effective impedance:

$$(Z_{\perp})_{\text{eff}}(\omega_{\xi}) = \int_{-\infty}^{\infty} Z_{\perp}^{\perp}(\omega) h_m(\omega - \omega_{\xi}) d\omega$$

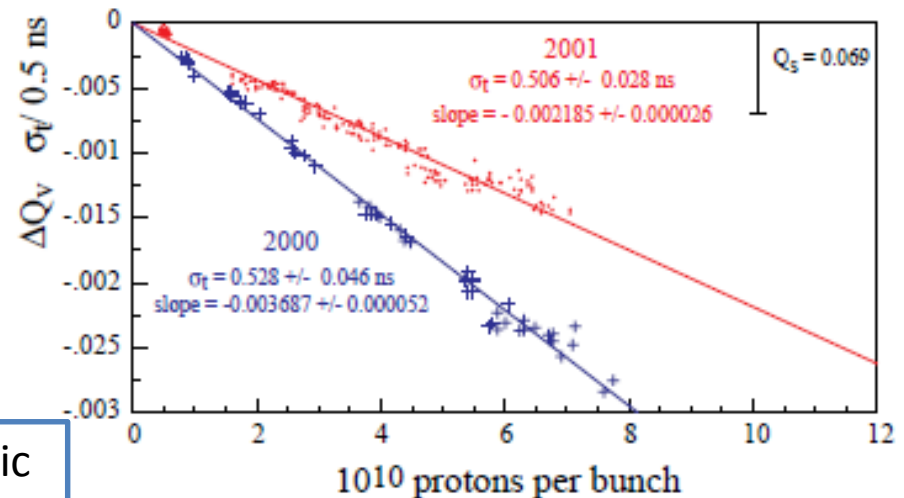
where  $\omega_{\xi} = \xi \frac{\omega_{\beta}}{\eta}$

For Gaussian bunch and  $m=0$

$$h_0(\omega) = \frac{\sigma_t}{\sqrt{\pi}} e^{-(\omega\sigma_t)^2}$$

→ Tune shift with intensity is one of the basic measurements of total transverse impedance  
Results of the SPS impedance reduction are visible in the [reference measurements \[2\]](#)

$$\Delta\omega_{\beta} = \frac{N e c}{4\sqrt{\pi} \omega_{\beta} (E/e) T_0 \sigma_t} i(Z_{\perp})_{\text{eff}}$$



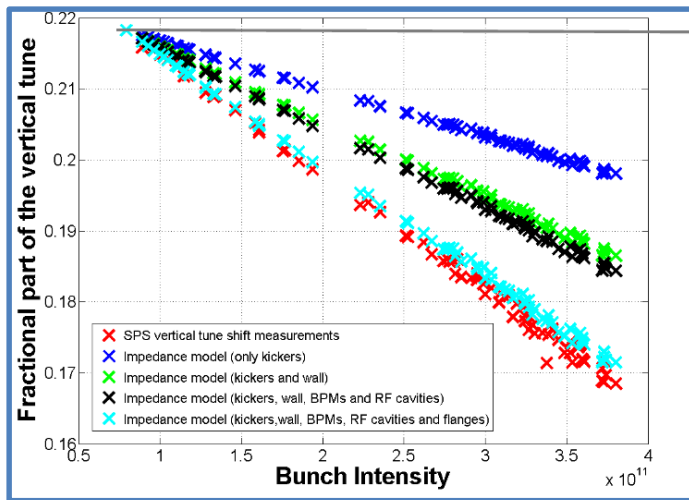
year	$\text{Im } Z_x$ MΩ/m	$\text{Im } Z_y$ MΩ/m
2000	$-0.9 \pm 1.8$	$26 \pm 3$
2001	$-0.35 \pm 0.53$	$18.4 \pm 0.5$

[1] F. Sacherer, Proc. 1976 Erice School, 1977

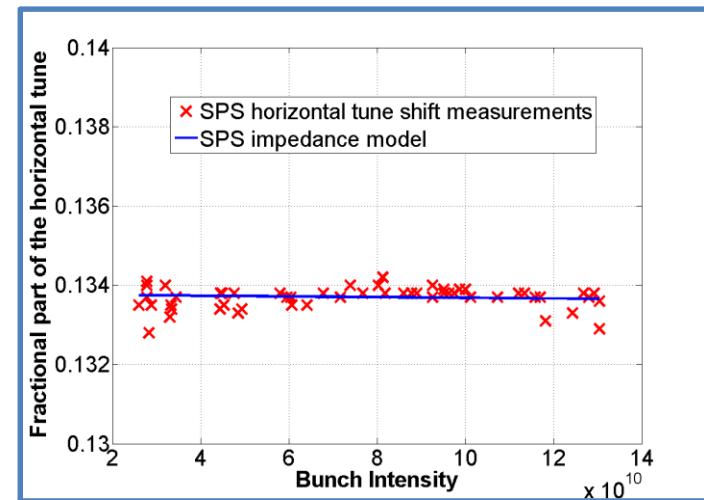
[2] H. Burkhardt, G. Rumolo, F. Zimmermann, PAC'01

# Transverse (reactive) impedance: betatron tune shift measurements (2/2)

SPS vertical tune shift @ 26 GeV/c



SPS horizontal tune shift



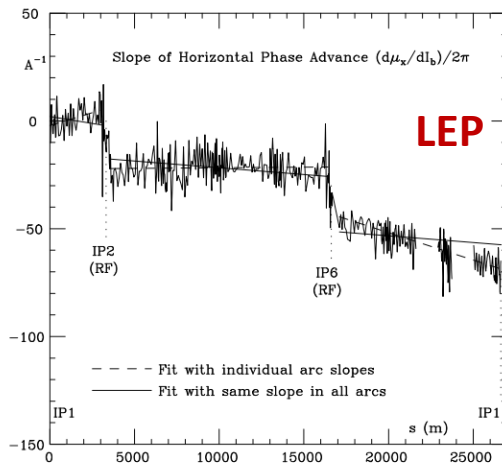
→ Present SPS impedance model reproduces about 90% of the vertical tune measured in the present Q20 optics

C. Zannini et al., PAC'15

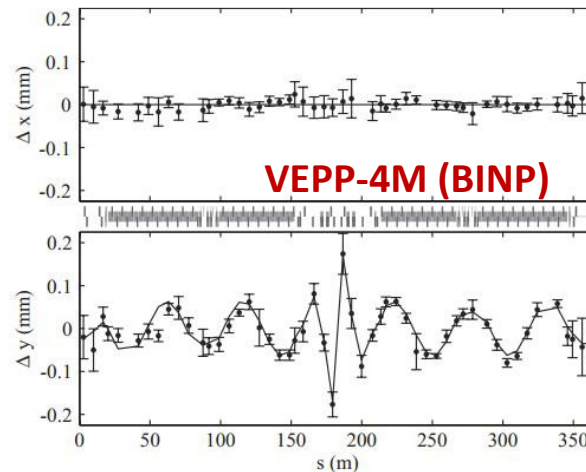
# Local transverse impedance measurements

- Betatron frequency depends on current so phase advance does → local phase advance can be measured by BPMs for excited betatron motion
- Orbit bump method (effect on the orbit)
- Orbit Response Matrix fit: small lattice changes due to defocusing effect of impedance can be found (current dependent focusing errors)

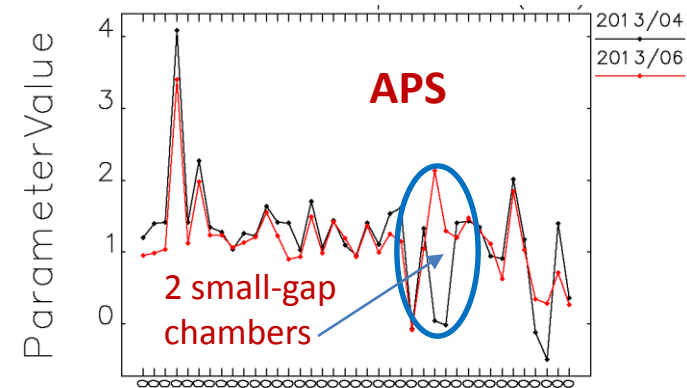
Issues: BPMs(N), orbit drifts...



D. Brandt et al., PAC'95



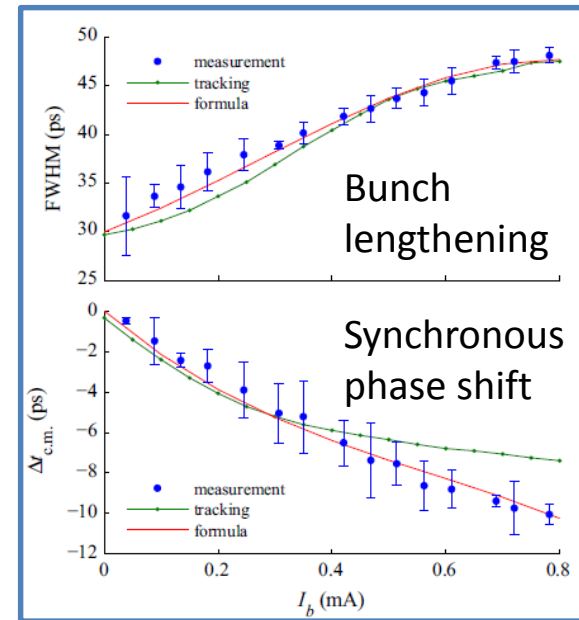
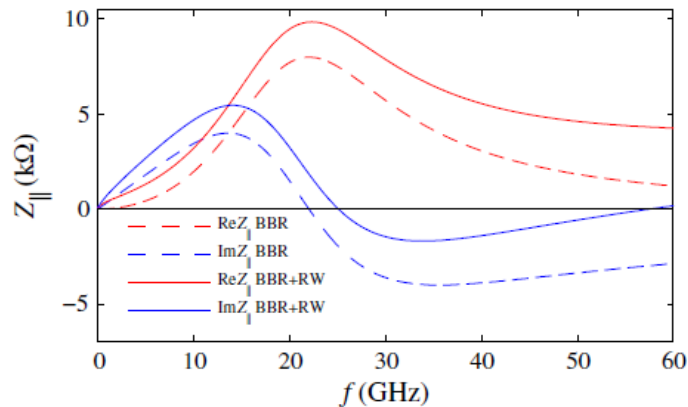
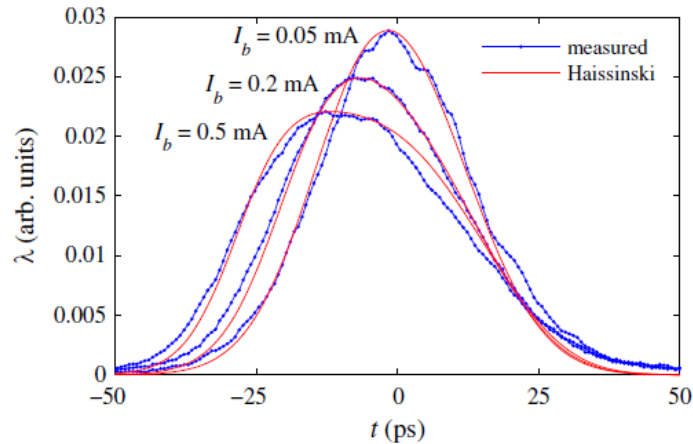
V. Kiselev, V. Smaluk, EPAC'98



V. Sajaev, PAC 2003,  
AOP-TN-2012-046

# Longitudinal impedance: bunch lengthening

The measured bunch profiles and Haissinski equation fit

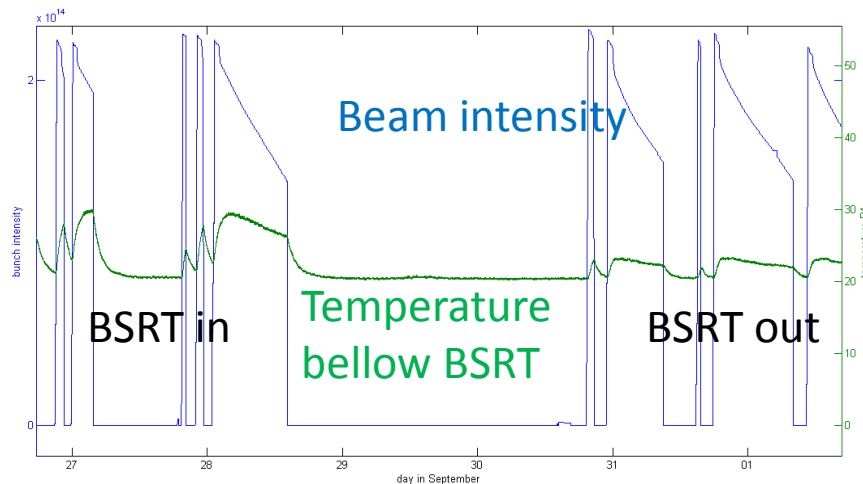
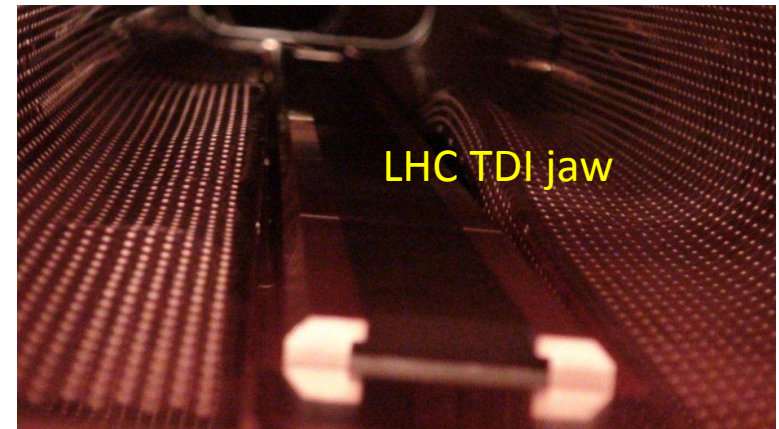
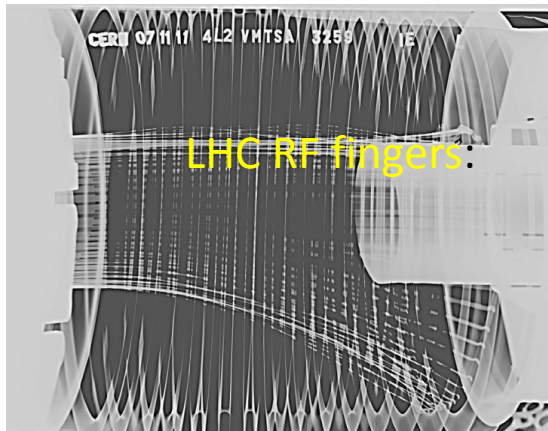


Diamond Light Source (3 GeV): beam-based (longitudinal and transverse) impedance models: **broad-band resonators with  $Q=1$**  ( $R_{sh}$  and  $\omega_r$  – are fitting parameters)

V. Smaluk et al., PRST AB 18, 2015

# Resistive impedance measurements: beam induced heating (damage?)

Comparison of expected and measured heating: not very accurate, but  
**very efficient in case of problems**





# Measurement of resistive impedance: synchronous phase shift

The energy loss of the bunch per turn and per particle is defined by **loss factor k**

$$U_b = -e^2 N \mathbf{k} = -e^2 N \sum_n k_n(\sigma),$$

For a Gaussian bunch the loss factor  $k_n$  due to the longitudinal impedance  $Z_n(\omega)$

$$k_n(\sigma) = \frac{\omega_0}{\pi} \sum_{p=0}^{\infty} \operatorname{Re} Z_n(p\omega_0) \exp[-(p\omega_0\sigma)^2]. \quad \text{For resonator: } k = \omega_r R_{sh} / (2Q) \text{ for } \omega_r \tau \ll 1$$

Any energy loss is compensated by the RF system. The shift of the synchronous phase  $\varphi_s$  due to energy loss  $U_b$ :  $\Delta\varphi_s = U_b / (eV_{rf} \cos\varphi_s) = -eNk / (V_{rf} \cos\varphi_s)$  **can be measured**

(1) as a distance between two bunches

or as a variation of phase between the beam signal and

(2) **reference RF signal** (sent from the power amplifiers to the cavity)

→ energy loss due to cavity fundamental impedance is **included**

(3) **probe in the cavity** which contains information from both applied RF voltage and the induced beam-loading voltage → beam loading is **excluded**

# Synchronous phase shift: distance between two bunches

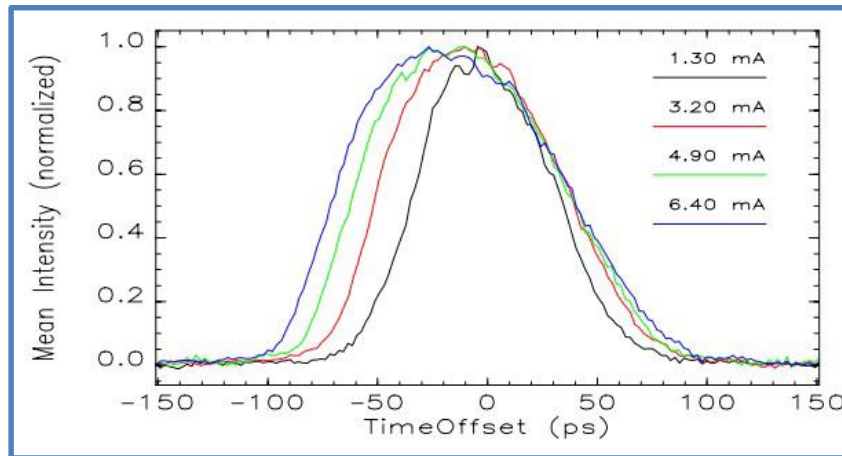
Measuring the distance between two bunches (separated by 1/2 ring) – similar to the reference RF signal (beam loading is included):

Bunch (1) - a time reference bunch (low intensity)

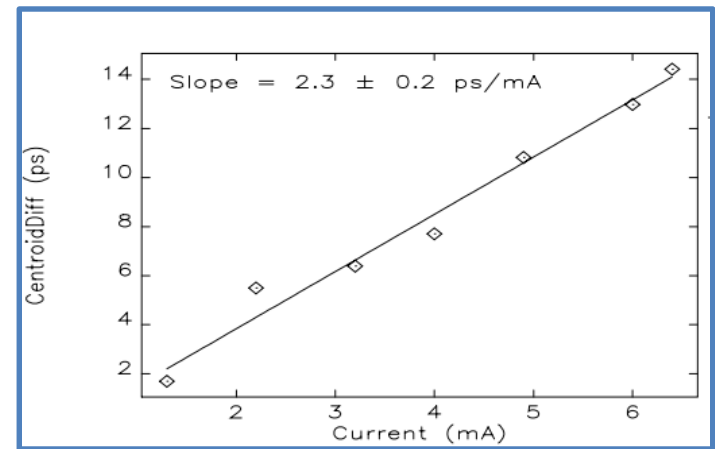
Bunch (2) with varied intensity

→ RF cavities are responsible for 70% of the total measured loss factor

Bunch profile



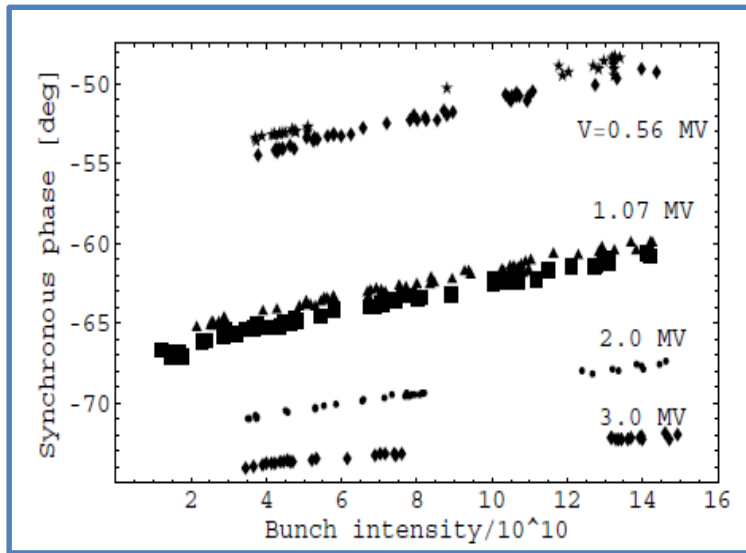
Bunch centroid shift



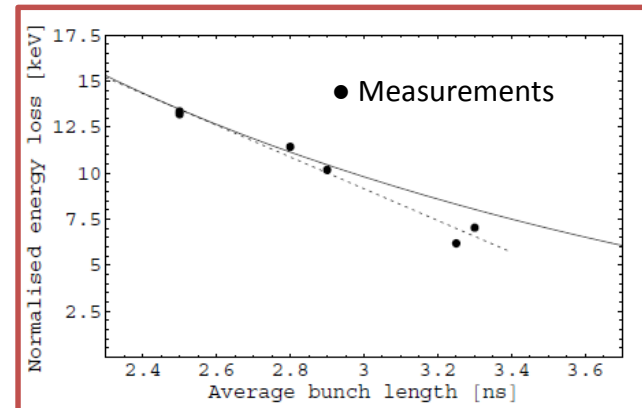
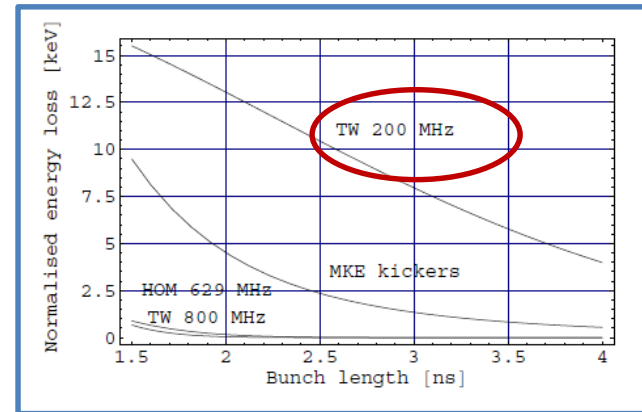
[1] N. Sereno et al., Proc. PAC'97

# Synchronous phase shift: beam phase relative to the RF reference

Measurements in the SPS @ 26 GeV/c  
Single bunches injected in 4 different RF voltages → dependence on bunch length

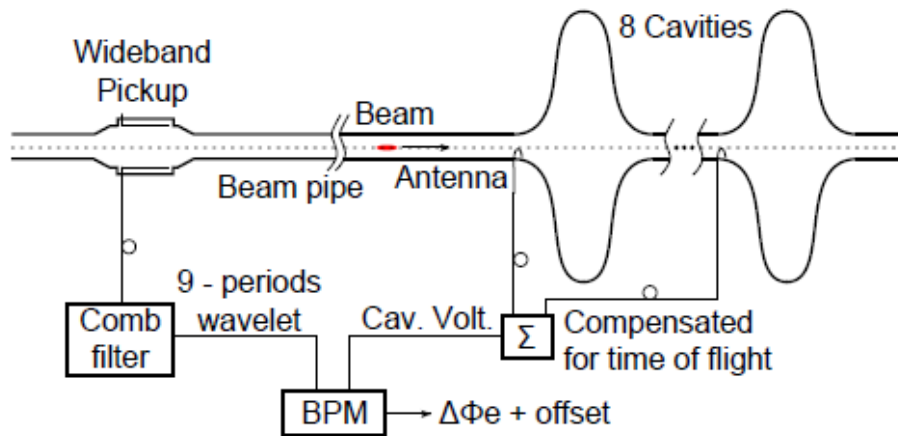


$$\phi_{si} = \phi_{0i} + b_i \times (N/10^{10}) \quad [\text{deg}]$$



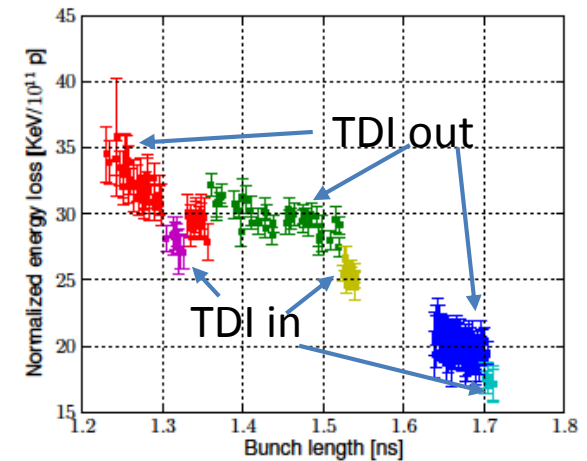
→ Losses are dominated by the main RF impedance of the 200 MHz TW system

# Synchronous phase shift relative to measured RF phase: CERN LHC (1/4)



A 400 MHz wavelet is generated from the wideband PU signal and compared in phase to the vector sum of the RF voltages in Beam Phase Module (BPM)

Measured **relative effect** of the TDI impedance (in - out)



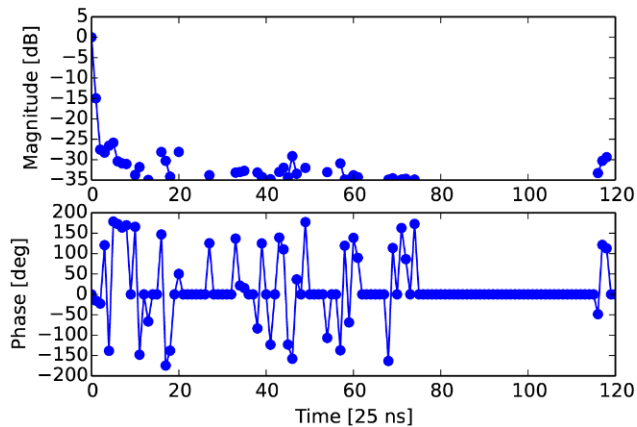
→ Effect of e-cloud on beam is similar to impedance: it causes instabilities, emittance blow-up, losses and **heat load!** The e-cloud density can be estimated using bunch-by-bunch synchronous phase shift (J. E. Muller et al., IPAC'14)

**Very high accuracy is required to measure small shifts < 1 deg!**

# Bunch-by-bunch synchronous phase shift: measurements in LHC (2/4)

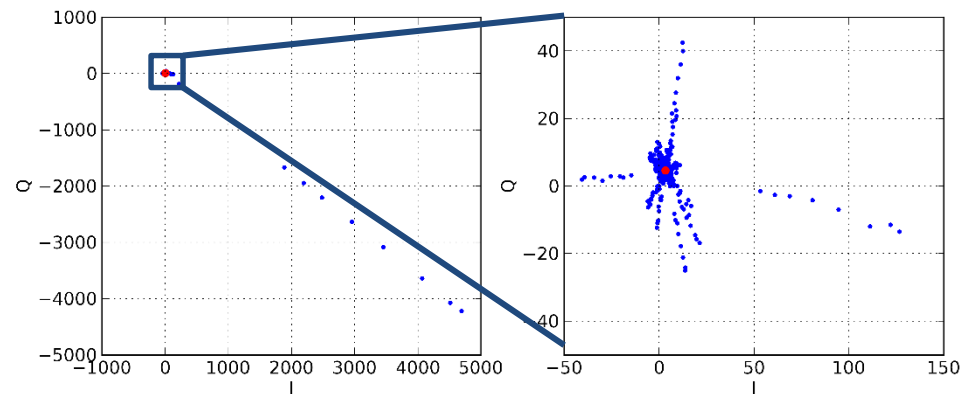
## Corrections for systematic errors

(1) Reflections in the cables:  
affect subsequent bunches



→ Transfer function measured with a  
single bunch and used for correction

(2) Offset in the IQ plane (vector  
representation): affects single bunch

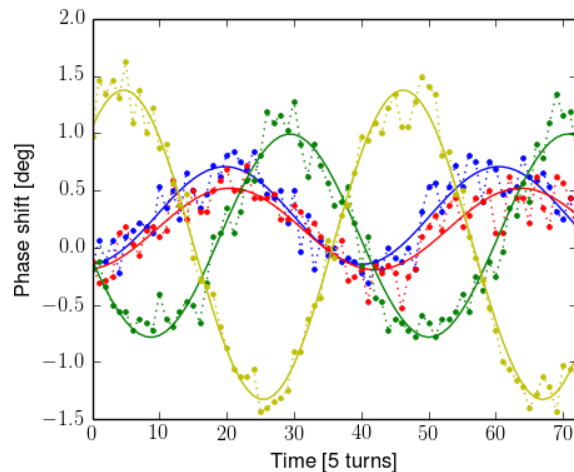


→ Measured from the noise in the empty  
buckets to correct the origin displacement

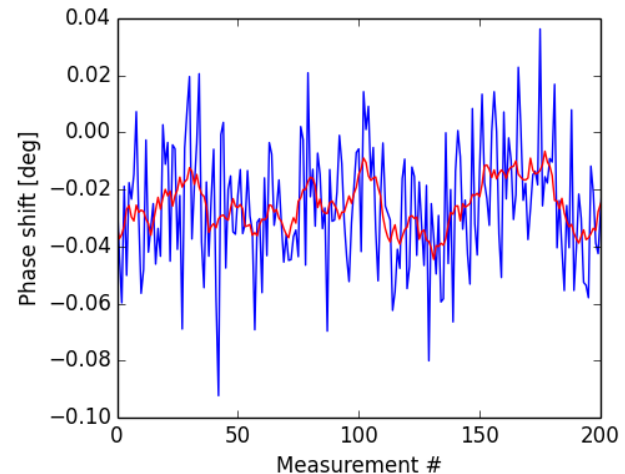
# Bunch-by-bunch synchronous phase shift: measurements in LHC (3/4)

## Data post-processing

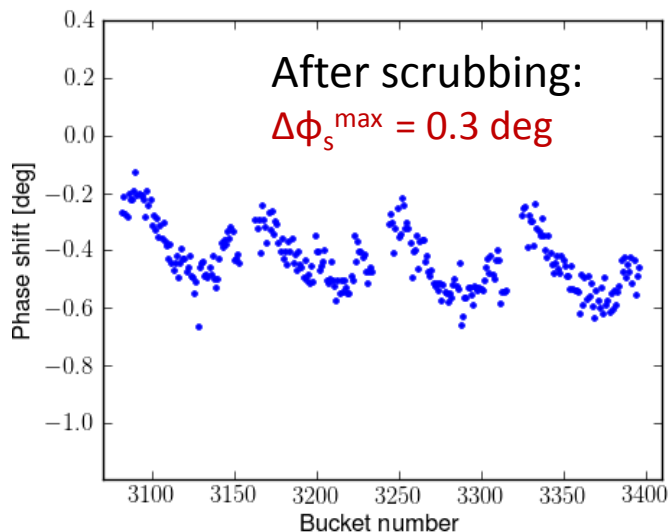
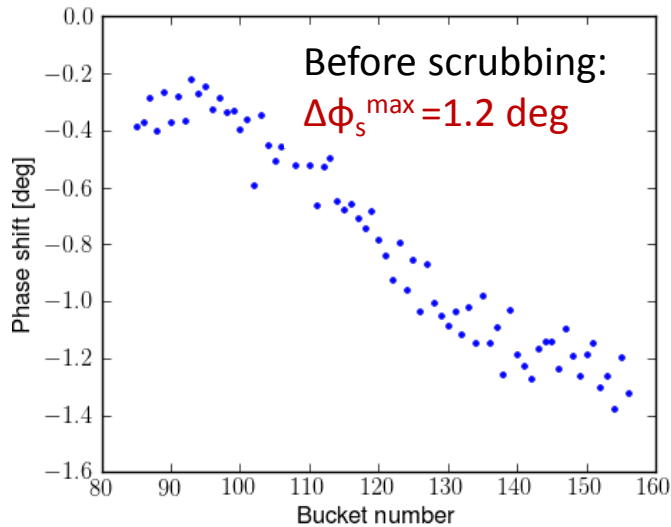
(1) Sine-wave fit of the  
synchrotron oscillations



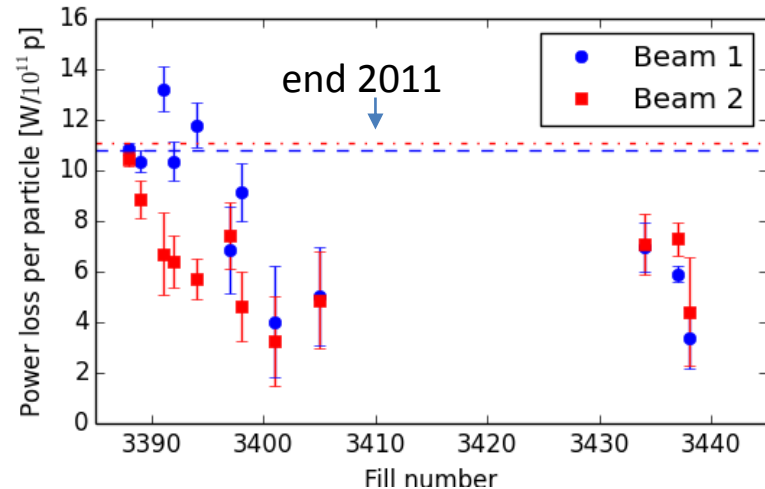
(2) Smoothing phase of each  
bunch over time



# Synchronous phase shift: measurements in LHC for e-cloud (4/4)



Scrubbing effect seen from the maximum power loss per particle (2012)

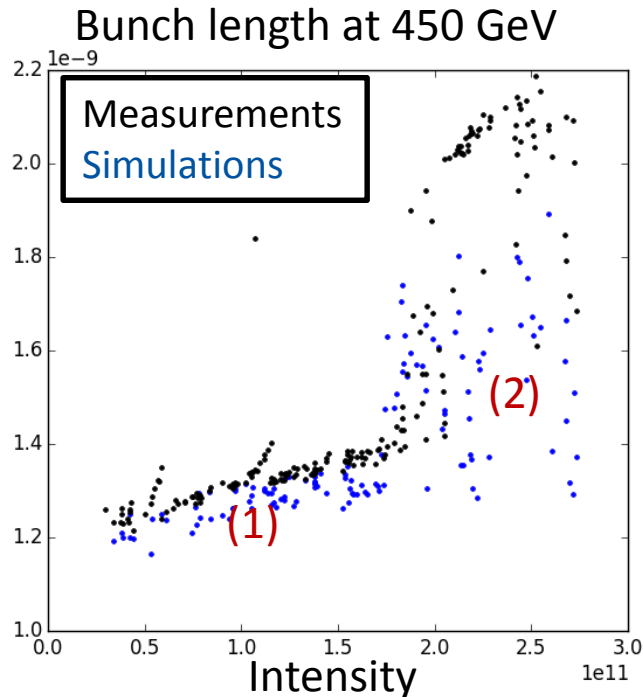


→ From 2015 this is an operational tool available in the CERN Control Center  
Comparison with simulations gives good estimate of e-cloud density (see talk of G. Rumolo)

# Measurements with unstable beam



# Bunch lengthening: single bunch instability



SPS at 450 GeV (single 200 MHz RF system)

→ Two very different slopes in dependence of bunch length on intensity:

(1) Potential well distortion

(2) Emittance blow-up due to instability

What could be a source of this instability?

Simulations of the whole acceleration cycle using full SPS impedance model

A. Lasheen et al., 2015

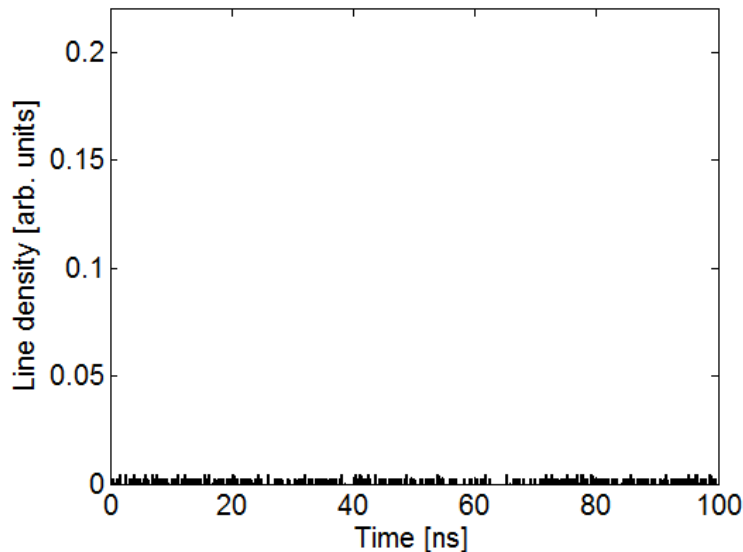
# Spectrum of unstable single bunches (1/4)

## Method of measurement:

- Inject **long** single bunches into ring with **RF off**
- Bunches with low momentum spread: slow debunching and fast instability
- Measure bunch profiles or spectrum amplitude at given frequency
- Use projection of spectra to see **longitudinal impedances with high R/Q**

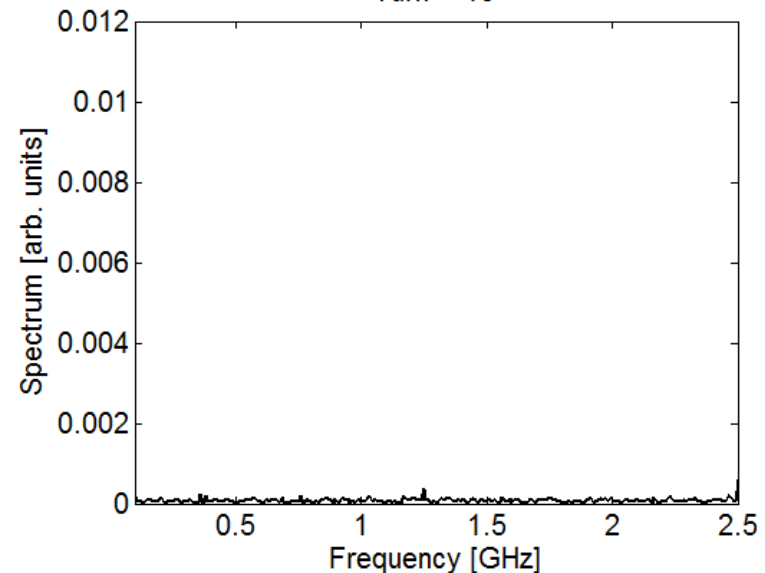
Bunch profile (SPS 26 GeV/c)

Turn = 10



Spectrum of unstable bunch

Turn = 10



# Single bunch instability with RF off

The linearised Vlasov equation for the line density perturbation  $\rho(\theta, t) = e^{-i\Omega t} \sum_n \rho_n e^{in\theta}$

$$\rho_n = -i \frac{\eta n \omega_0}{2\pi E_0} \left( \frac{e\omega_0}{\Omega} \right)^2 \sum_{n'} G_{n-n'} Z_{n'} \rho_{n'}$$

describes a fast microwave instability, assuming particle distribution  $F(\theta, \dot{\theta}) = G(\theta) \delta(\dot{\theta})$

For a coasting beam  $G_{n-n'} = N \delta_{n,n'} / (2\pi)$  and  $\Omega_n^2 = -i \frac{(en\omega_0)^2 N \omega_0}{2\pi E_0} \eta \frac{Z_n}{n}$

→ the negative mass instability.

Let's consider a resonant impedance with bandwidth  $\Delta\omega_r = \omega_r / 2Q$ . **Two regimes exist:**

**(1) Narrow-band impedance:**  $\Delta\omega_r \ll 1/\tau$

→ We can assume for  $n' > 0$ :  $G_{n-n'} \simeq G_{n-n_r}$

Then growth rate  $\frac{\text{Im}\Omega}{\omega_r} \simeq \left( \frac{N e^2 \omega_0 |\eta| R_{sh}}{16\pi E_0 Q} \right)^{\frac{1}{2}}$

Instability spectrum:  $\rho_n \sim n G_{n-n_r}$

→ centered at  $n \sim n_r$  with width  $\sim 1/\tau$

**(2) Broad-band impedance:**  $\Delta\omega_r > 1/\tau$

→ For a long Gaussian bunch assume

$$G_{n-n'} = \frac{N}{2\pi} \exp\left(-\frac{(n-n')^2 \sigma^2}{2}\right) \approx \frac{N}{\sqrt{2\pi}\sigma} \delta_{n,n'}$$

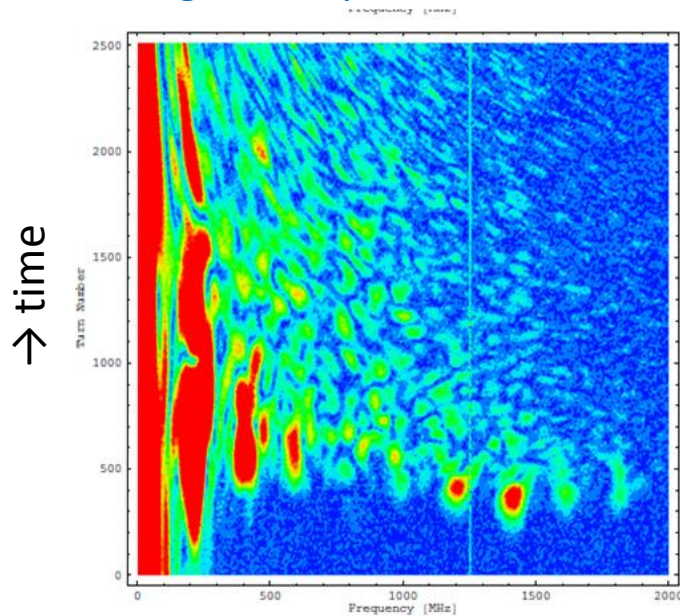
Growth rate  $\Omega_n^2 \simeq -i \frac{(en\omega_0)^2 N}{(2\pi)^{3/2} E_0 \sigma_t} \eta \frac{Z_n}{n}$

similar to a coasting beam where average current is replaced by peak.

→ Spectrum width  $\sim$  impedance  $\Delta\omega_r$

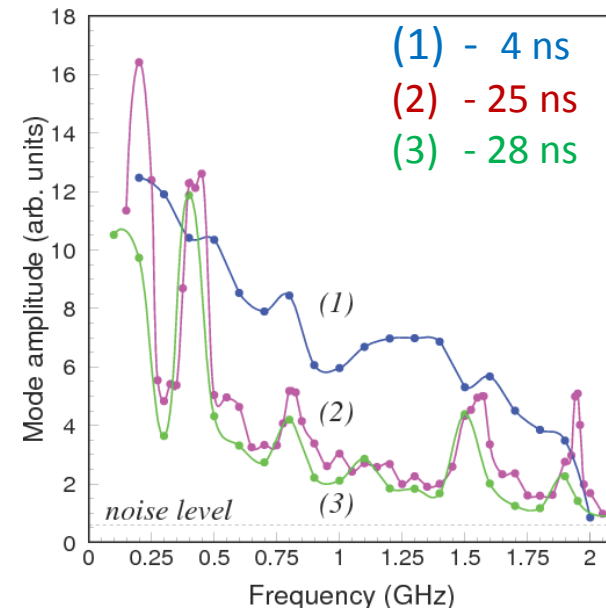
# Spectrum of unstable single bunches (2/4)

Spectrum of unstable bunch during development of instability



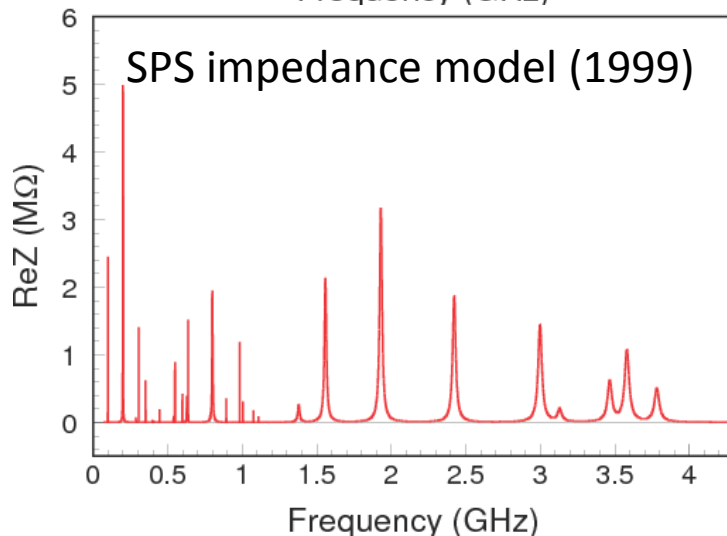
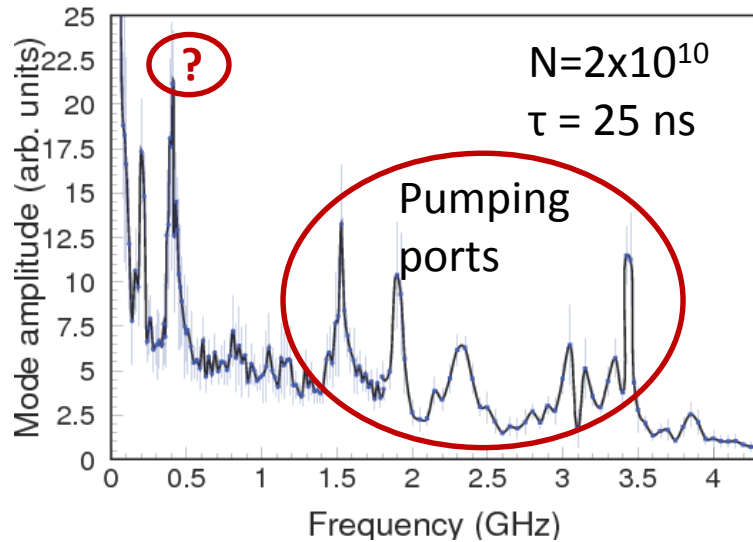
→ To have a good frequency resolution bunches should not debunch too fast: instability growth time  $\ll$  debunching time  $t_d$

Projection of unstable spectrum measured with short and long bunches



→ Frequency resolution is defined by bunch length (SPS, 1997)

# Unstable bunch spectrum: narrow-band impedances (3/4)



The  $TM_{mnl}$  modes of a cylindrical cavity with radius  $r_c$  and length  $z$ :

$$f_r = \frac{\omega_r}{2\pi} = \frac{c}{2} \sqrt{\left(\frac{l}{z}\right)^2 + \left(\frac{u_{m,n}}{\pi r_c}\right)^2}$$

$u_{m,n}$  is the  $n$ -th root of equation  $J_m(u) = 0$ .

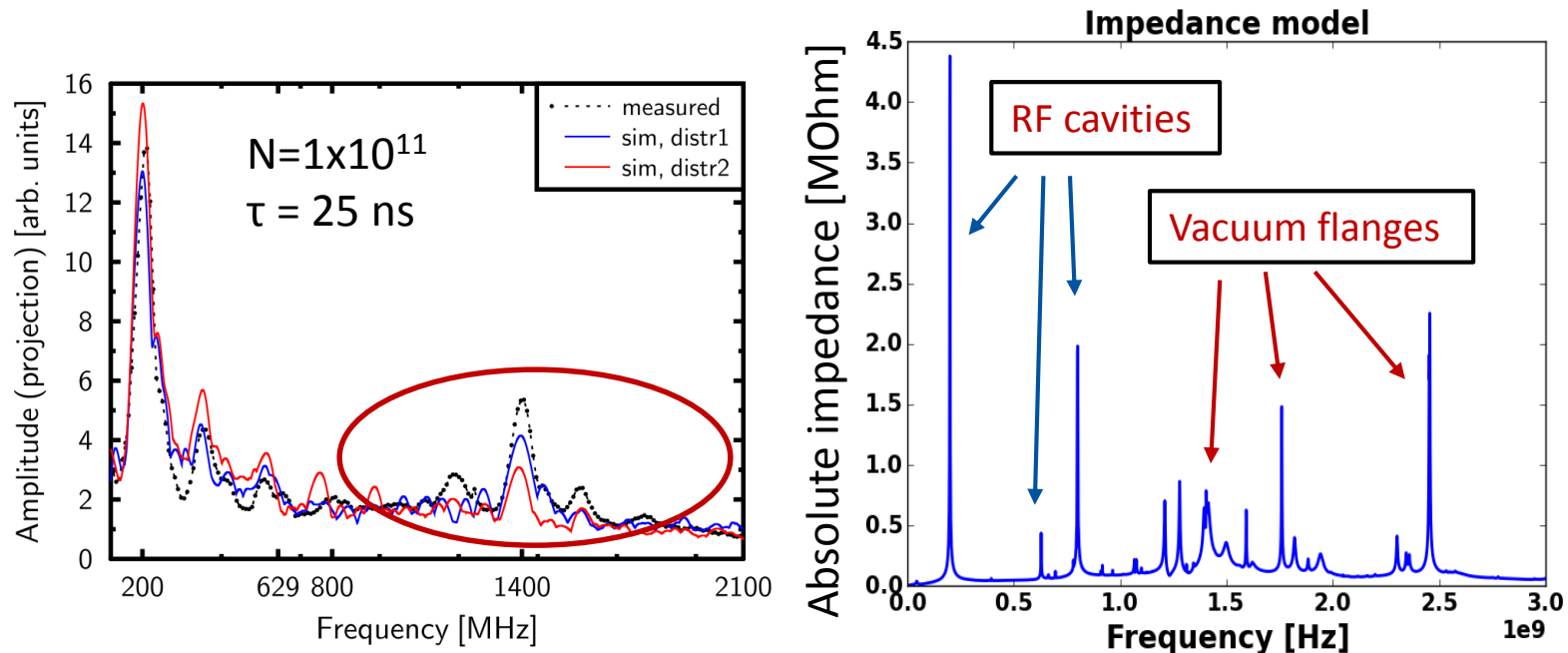
For  $r = 8.2 \text{ cm}$  and  $z = 20 \text{ cm}$  (typical SPS pumping port) the lowest frequencies of the  $TM_{0l}$  and  $TM_{2l}$  modes ( $l=0,1,2 \dots$ ):

1.4, 1.53, 1.88, 2.34, 2.87... GHz

→ all visible in unstable bunch spectrum

The peak amplitude depends on R/Q of the mode (SPS: for  $\sim 900$  elements with  $Q \sim 50$ , maximum R/Q  $\sim 40 \text{ k}\Omega$ )

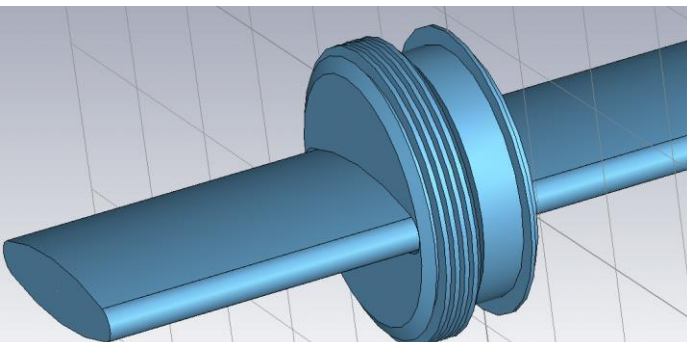
# Unstable bunch spectrum: narrow-band impedances (4/4)



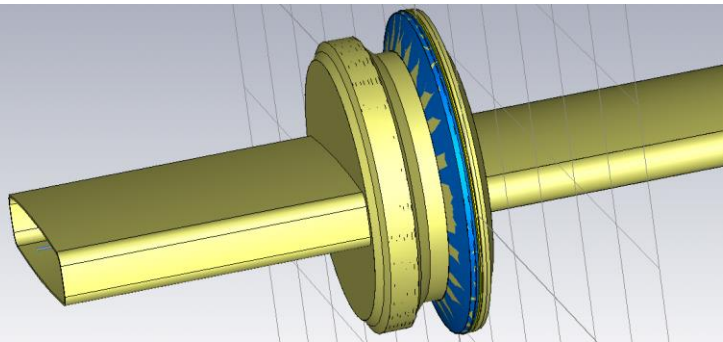
The main source of the **longitudinal instability** limiting the LHC beam intensity in the SPS has been identified → shielding of  $\sim 200$  vacuum flanges is planned during the next long shutdown (2019 - 2020) together with improved HOM damping

# The SPS vacuum flanges

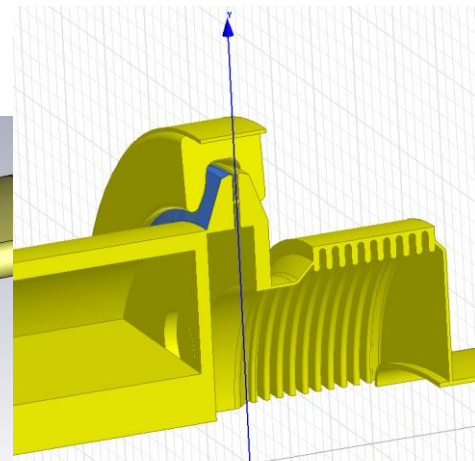
## Group I – 1.4 GHz



Non-enamelled QF - QF  $\approx 26$

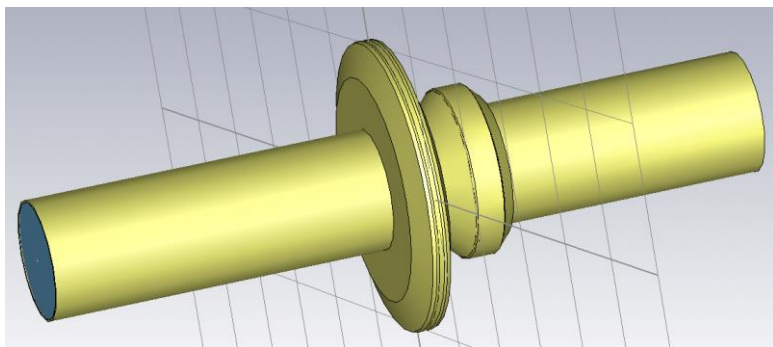


Enamelled QF - MBA  $\approx 97$

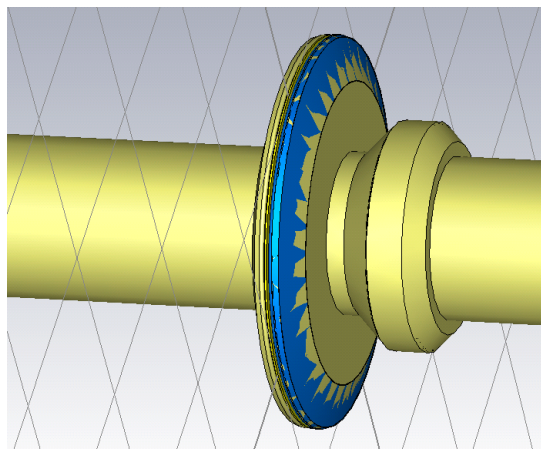


Non-shielded, enamelled  
BPH - QF  $\approx 39$

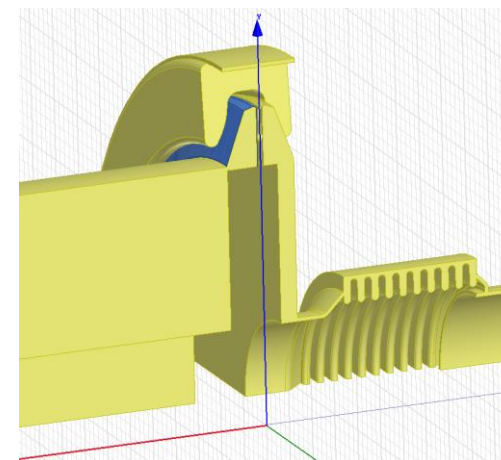
## Group II



Non-enamelled QD - QD  $\approx 75$



Enamelled QD - QD  $\approx 99$



Enamelled BPV - Q  $\approx 90$



# Spectrum of multi-bunch beam (1/2)

- Let's consider a narrowband resonant impedance at unknown  $\omega_r = \omega_0 p_r$ .
  - The unstable spectrum of multi-bunch beam has components at
$$\omega = (n + l M)\omega_0 + m\omega_s, \quad -\infty < l < \infty,$$

$n=0, 1 \dots M-1$  is the coupled-bunch mode number,  $M$  is number of equidistant bunches in the ring and  $m=1, 2, \dots$  is the multipole number
  - On the spectrum analyzer negative  $\omega$  appear at  $[(l+1) M - n]\omega_0 - m \omega_s$
  - Measured mode  $n$  is not sufficient to determine  $\omega_r$  since  $n + l M \approx \pm p_r$  and  $l$  is not known. Smaller is  $M$ , more possibilities exist
  - Similar spectrum at  $n + l M$  and  $(l+1) M - n$ , but above transition internal synchrotron sidebands correspond to impedance at higher frequency and external – at lower (the opposite for  $\gamma < \gamma_t$ )  $\rightarrow$  high frequency resolution
- $\rightarrow$  Measuring  $n$  for different  $M$  (with  $M_1 \neq kM_2$ ) can help to determine  $\omega_r$

[1] F. Sacherer, F. Pedersen, Theory and performance of the longitudinal active damping system for the CERN PS Booster, NS-24, N3, p.1396, 1977



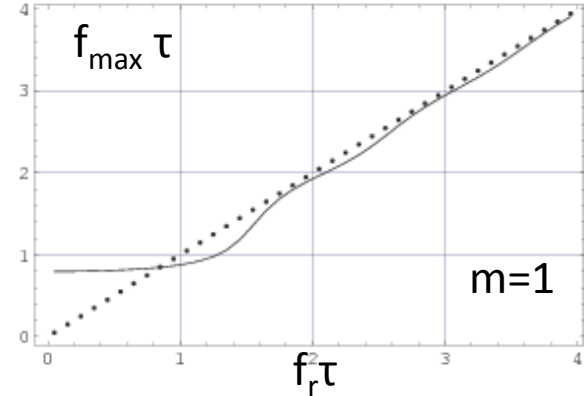
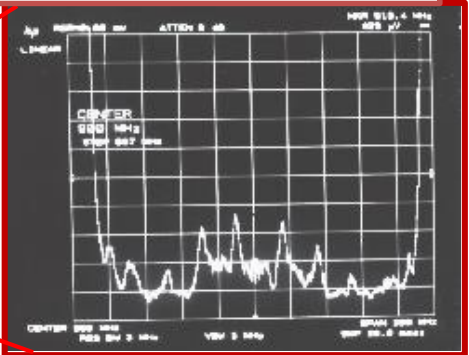
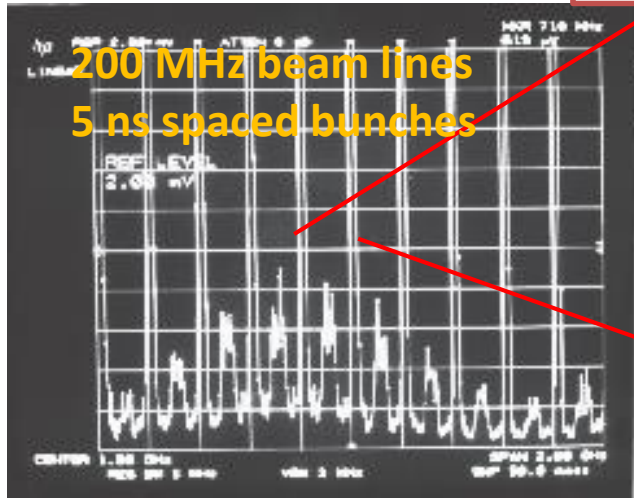
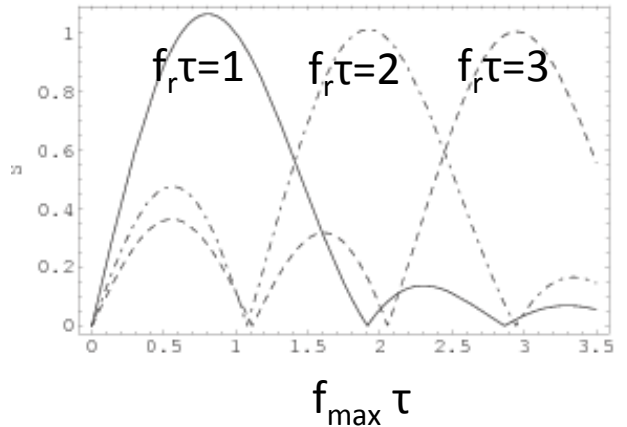
# Spectrum of multi-bunch beam (1/2)

The amplitude of spectral lines:  $j_k = g_{kp} / g_{pp}$   
 in single RF:  $g_{kp}^m \sim \int F' J_m(kr) J_m(pr) dr$  ( $r_{max} \approx \omega_0 \tau / 2$ )  
 Spectrum envelope with maximum at  $f_{max}$ :

if  $f_{max} \tau < 1 \rightarrow f_r < 1/\tau$   
 if  $f_{max} \tau > 1 \rightarrow f_r \sim f_{max}$

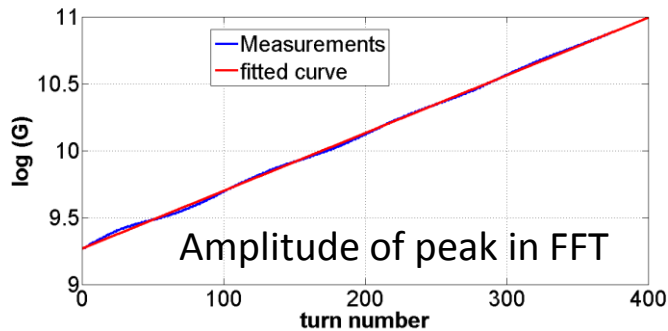
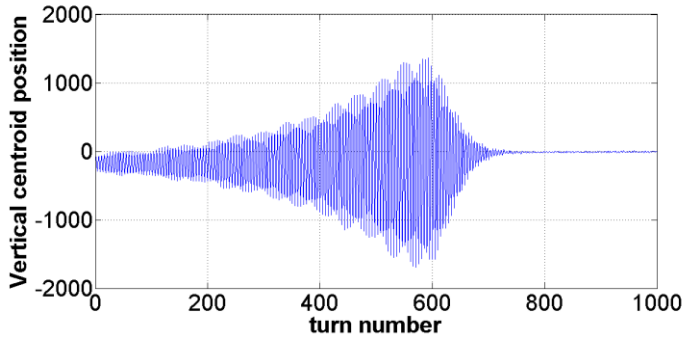
$\rightarrow f_r$  can be identified  
 $\rightarrow R_{sh}$  from growth rate measurements

Spectrum envelope ( $m=1$ )



SPS: known HOMs at 629, 912 MHz...

# Head-tail growth rate as a function of chromaticity



Effective impedance is a convolution of  $Z$  with longitudinal spectral density

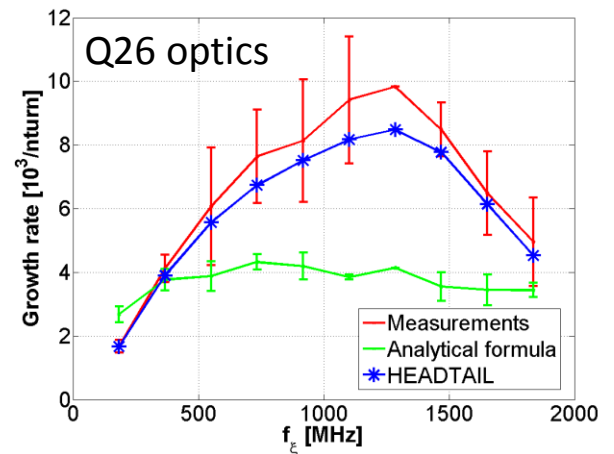
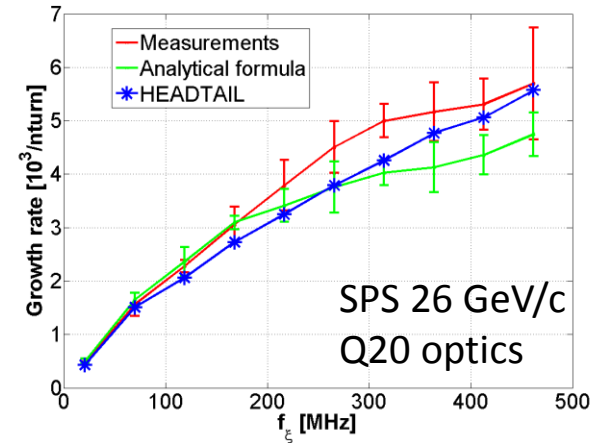
$$h_0(\omega) = \frac{\sigma_t}{\sqrt{\pi}} e^{-(\omega\sigma_t)^2}$$

where  $\omega \rightarrow (\omega - \omega_\xi)$  and chromatic frequency is

$$\omega_\xi = 2\pi f_\xi = \omega_\beta \xi / \eta,$$

$\xi$  is chromaticity,  $\eta$  - slip factor

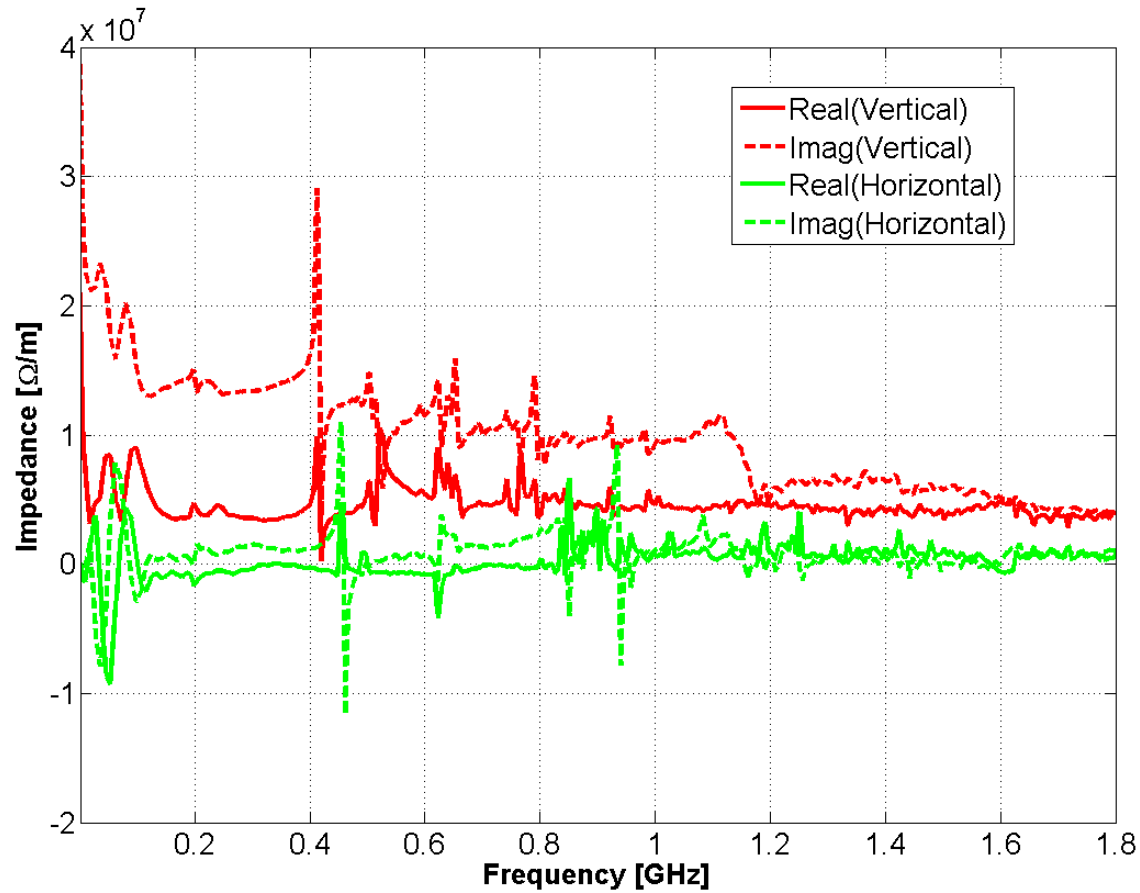
→ By varying  $\xi$  one can sample **frequency dependence of the transverse impedance**  
 → Good agreement with **simulations**



$$\tau^{-1} = \Gamma \left( \frac{1}{2} \right) \frac{\text{Re} [Z_{\perp, dip}^{eff}(\xi)] N r_0 c^2}{8\pi^2 \gamma Q_{\perp} \sigma_z}$$

- [1] F. Sacherer, Erice School, 1976
- [2] A. Chao, Physics of collective beam instabilities
- [3] C. Zannini et al., IPAC'15, talk at CERN

# SPS transverse impedance model



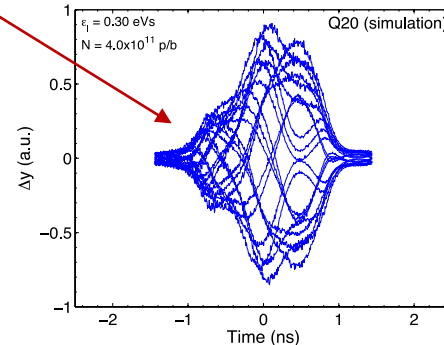
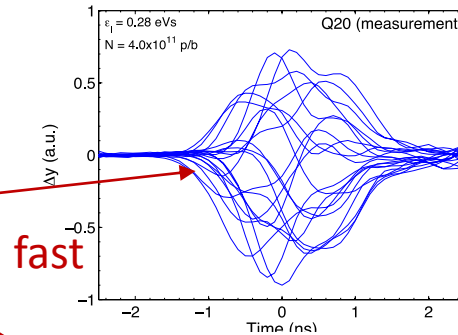
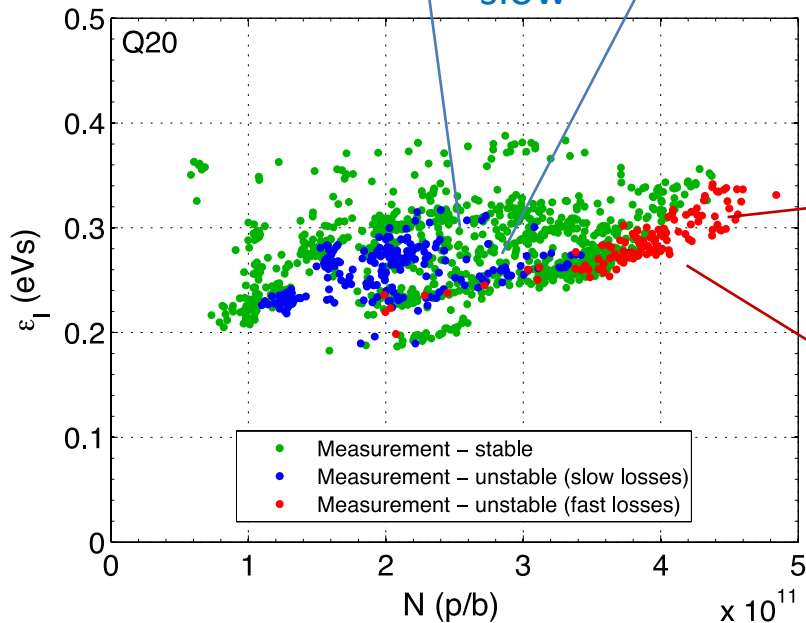
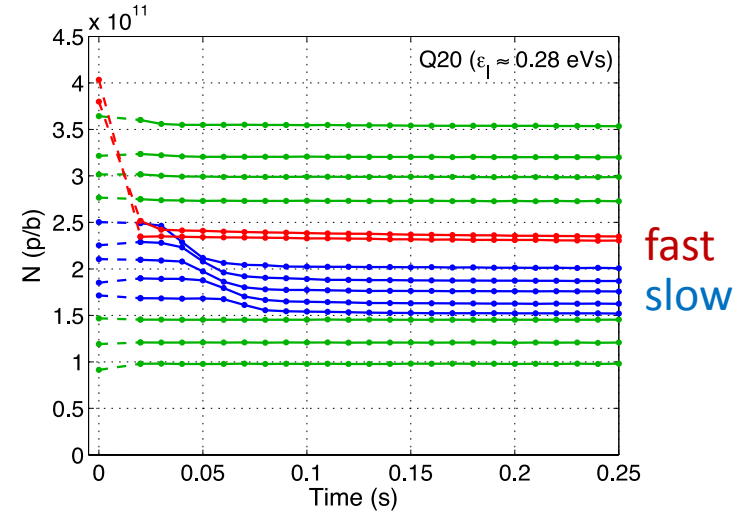
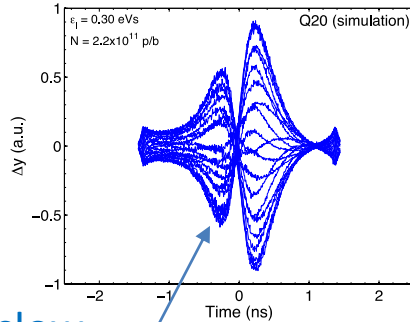
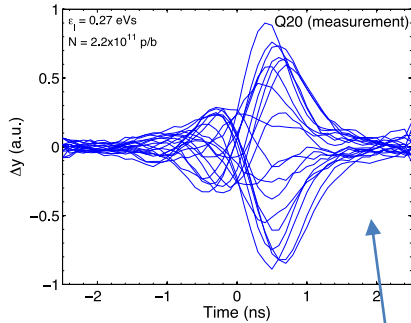
C. Zannini et al.

# Transverse Mode Coupling Instability threshold

CERN SPS, 26 GeV/c

measurements:  $m=1$

HEADTAIL simulations



For long bunches  
TMCI threshold:

$$N_{th} \sim \epsilon_L |\eta| Z_{eff} / \beta_y$$

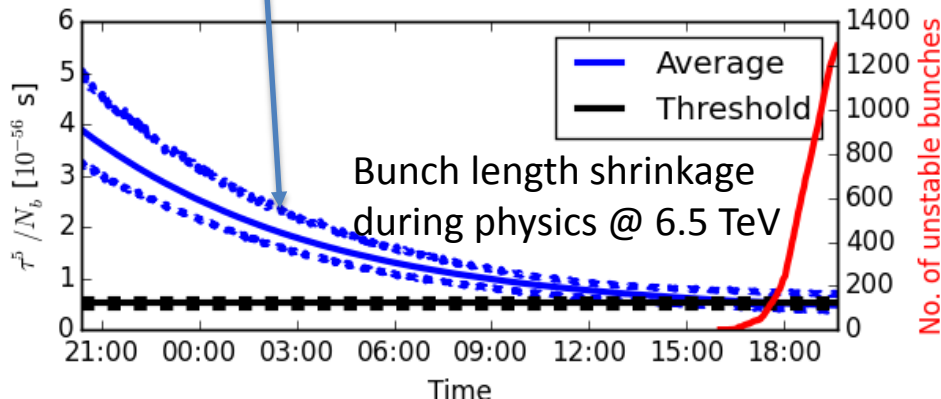
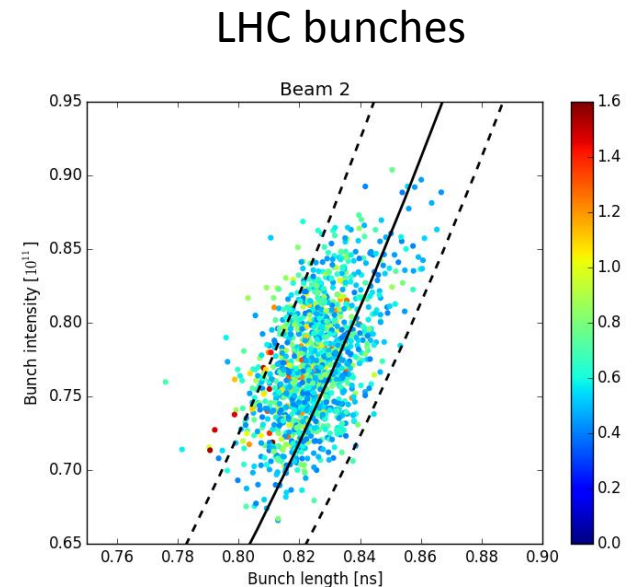
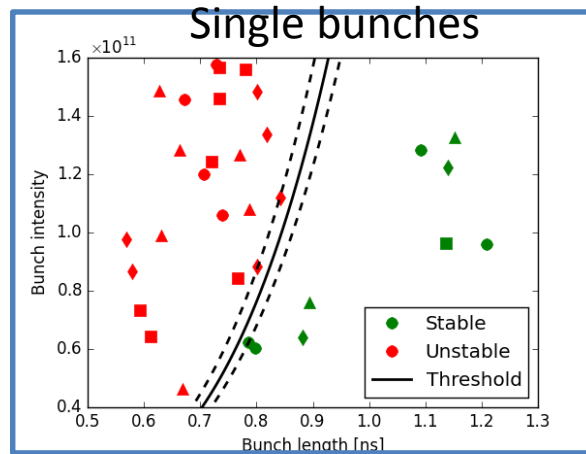
→ Instability island  
is reproduced in  
simulations

# Instability thresholds: loss of Landau damping in LHC

- Most accurate method to estimate longitudinal LHC impedance so far!
- Good agreement of measurements and simulations (full LHC model or  $\text{Im}Z/n = 0.08 \text{ Ohm}$ )

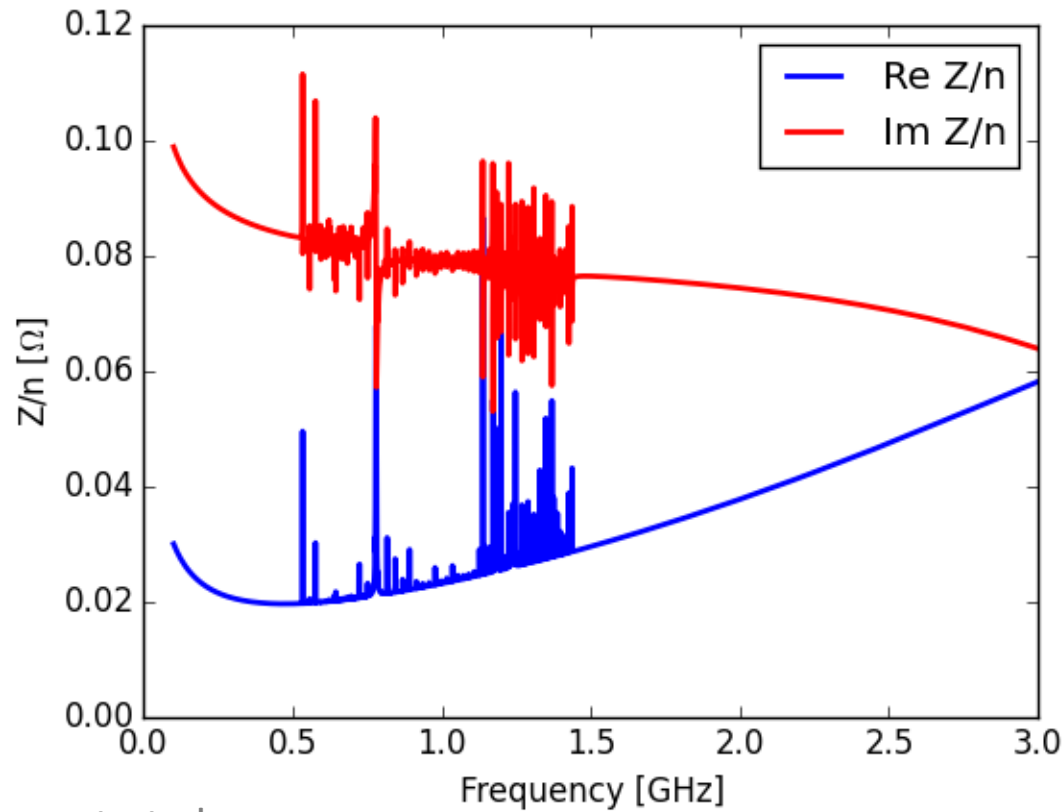
$$Z_{th} \propto \frac{\varepsilon^{\frac{5}{2}}}{N_b V^{\frac{1}{4}} E^{\frac{5}{4}}}$$

$$\rightarrow Z_{th} \propto \frac{\tau^5}{N_b}$$



J. E. Muller et al., 2015

# LHC longitudinal impedance model: $Z/n$



N. Mounet et al.

# Summary

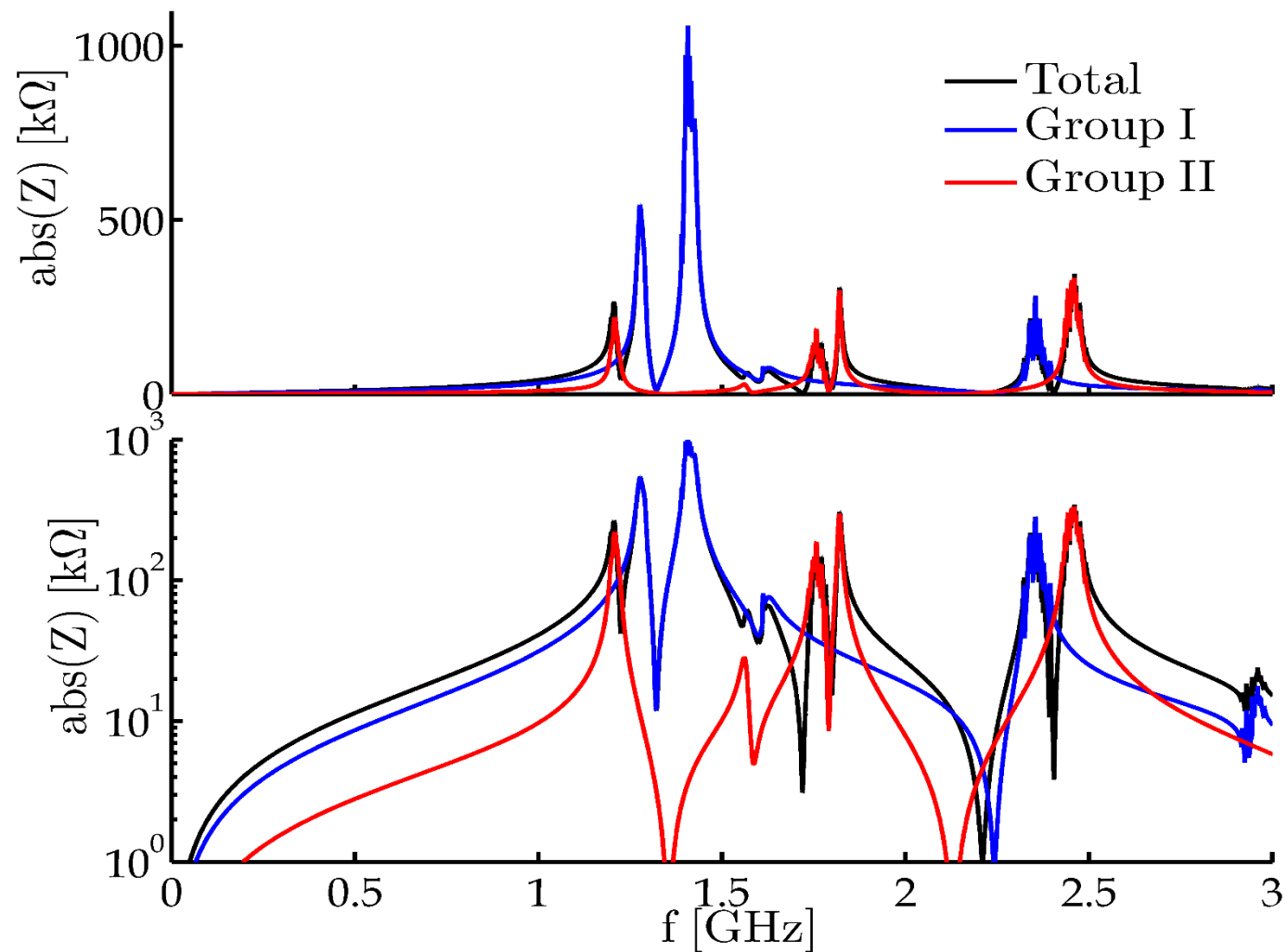
- Due to careful initial design ring impedances become smaller  
→ more elaborated methods are required to measure them with beam even in proton machines.
- Numerical simulations of various collective effects become more advanced and can be used for beam tests of impedance.
- Measurements with stable beam are mainly used for testing existing impedance models.
- Measurements with unstable beam may contain important information about parameters of the dominant impedances.

# Other methods (not discussed here)

- Transfer functions– D. Moehl and A. Sessler, 1971 (continuous beam)
- Longitudinal impedance variation with transverse displacement – G. Nassibian and F. Sacherer, NIM 159, 1979
- Direct wake-field measurement with 2 bunches and spectrometer (W. Cai, C. Jing, article in Handbook of Accelerator Physics and Engineering, 2<sup>nd</sup> edition, edited by A. Chao et al.)
- Localised loss factor or orbit distortion due to parasitic energy loss (TRISTAN, LEP)
- ...

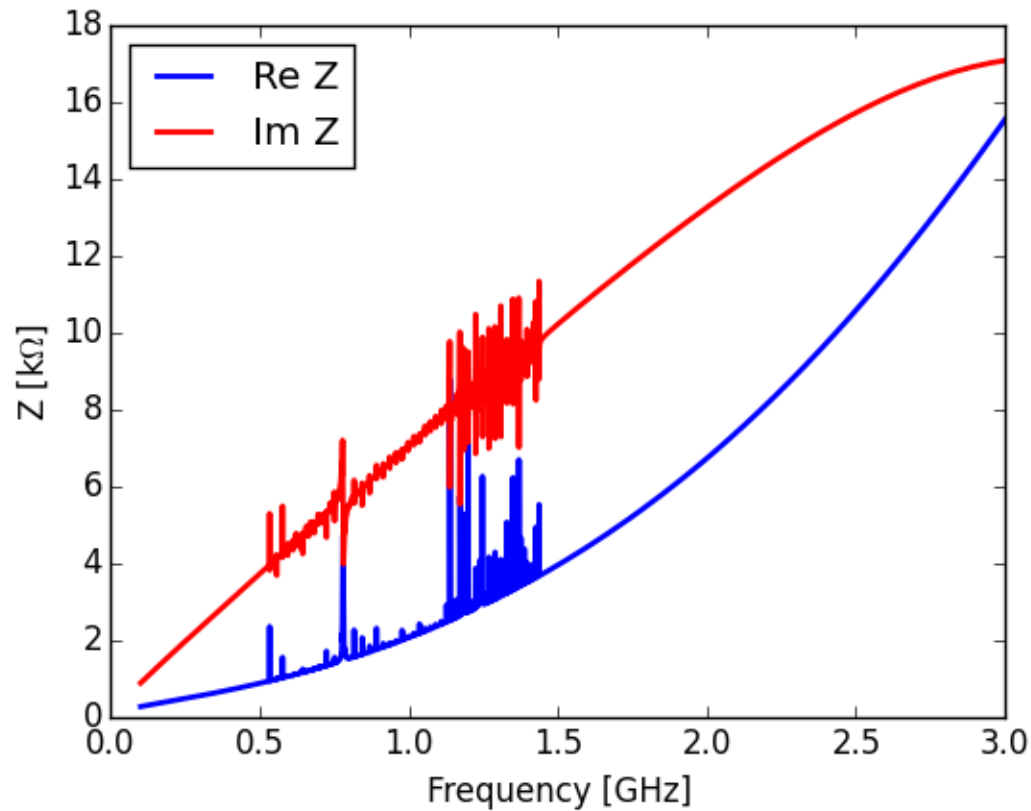


# The SPS vacuum flanges



Scattered resonances, J. Varela

# LHC longitudinal impedance model



N. Mounet et al.