Coherent beam-beam effects

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The CERN Accelerator School

Content

- Coherent vs. incoherent
  - Self-consistent solutions
- Coherent modes of oscillation
  - Decoherence
  - Impedance driven instabilities
- Summary

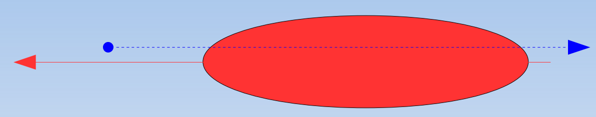
Weak-strong treatment

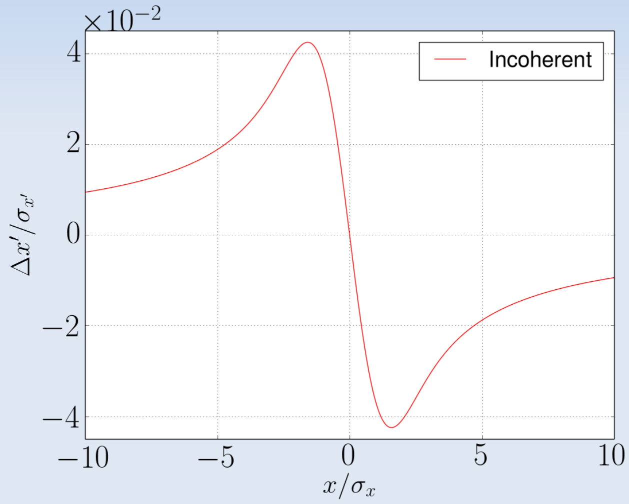
- The electromagnetic interaction felt by a particle traveling through a counter rotating beam is very non-linear

  → resonances, losses, emittance growth

- The other beam is not perturbed by the passage of the particle

  → weak-strong approximation





$$\Delta x'(x) = \frac{-2 r_0 N}{\gamma_r} \frac{1}{x} \left(1 - e^{\frac{-x^2}{2\sigma^2}}\right) \approx 4\pi \xi x$$

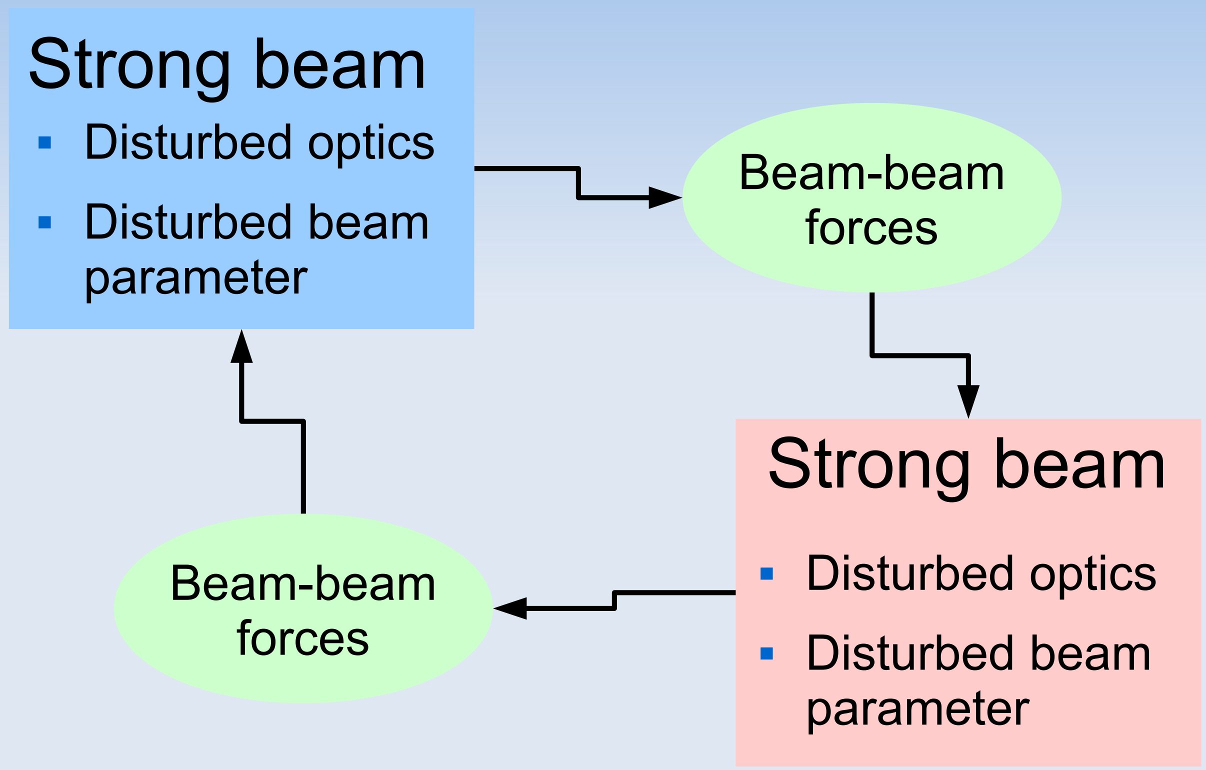
Self-consistent solutions

Strong beam
- Optics
- Beam parameters

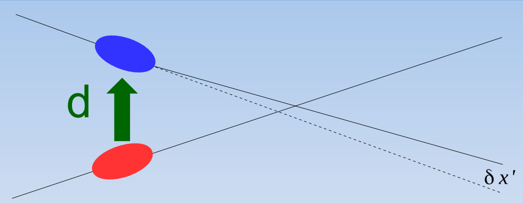
→ **Beam-beam forces** →

Weak beam
- Disturbed optics
- Disturbed beam parameter

Self-consistent solutions



Self-consistent solutions



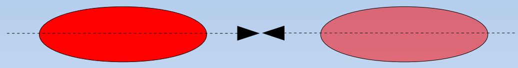
$$\delta x = \delta x' \beta \cot(\pi Q)$$

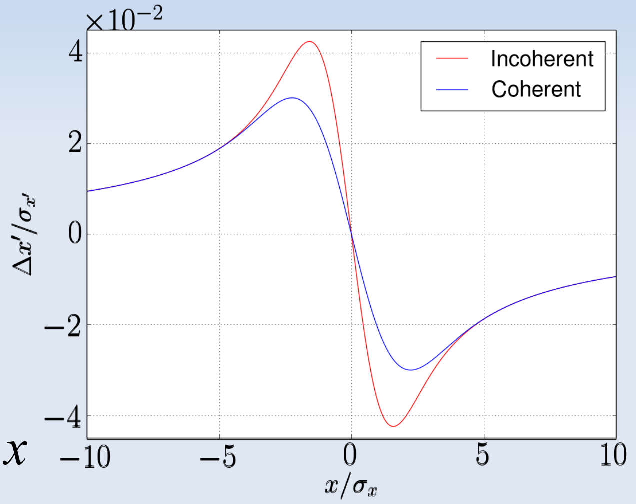
- Weak-strong :

$$\delta x = \Delta x_{coh}'(d) \beta \cot(\pi Q)$$

Coherent beam-beam force

- The average force felt by the particles in the beam is called the coherent force (1)



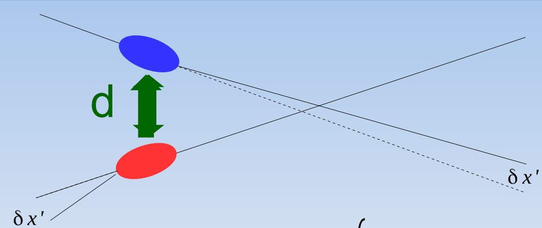


$$\Delta x'_{coh}(\Delta x)=\int_{-\infty}^{\infty}\Delta x'(\Delta x-X)\rho(X)\,dX$$

$$=\frac{-2r_0N}{\gamma_r}\frac{1}{\Delta x}\left(1-e^{\frac{-\Delta x^2}{4\sigma^2}}\right)\approx\frac{4\pi\xi}{2}\Delta x$$

$$\Delta x'(x)=\frac{-2r_0N}{\gamma_r}\frac{1}{x}\left(1-e^{\frac{-x^2}{2\sigma^2}}\right)\approx 4\pi\xi x$$

Self-consistent solutions



$$\delta x=\delta x'\beta\cot(\pi Q)$$

- Weak-strong :

$$\delta x=\Delta x_{coh}'(d)\beta\cot(\pi Q)$$

- Strong-strong :

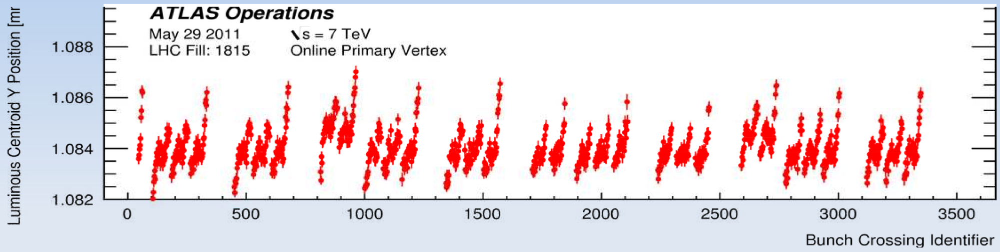
$$\begin{cases}\delta x_{B1}=\Delta x_{coh}'(d+\delta x_{B1}+\delta x_{B2})\beta_{B1}\cot(\pi Q_{B1})\\ \delta x_{B2}=\Delta x_{coh}'(d+\delta x_{B1}+\delta x_{B2})\beta_{B2}\cot(\pi Q_{B2})\end{cases}$$

- Similar treatment applies to the optical functions (e.g. dynamic β effect (2))
- These effects were already covered in T. Pieloni's lectures, but :

  → Simple formulas become non-linear system of equations

  - Iterative methods are used to evaluate these effects (3)
  - Prohibits several single beam measurement techniques
- The solution of the non-linear equations is not always unique

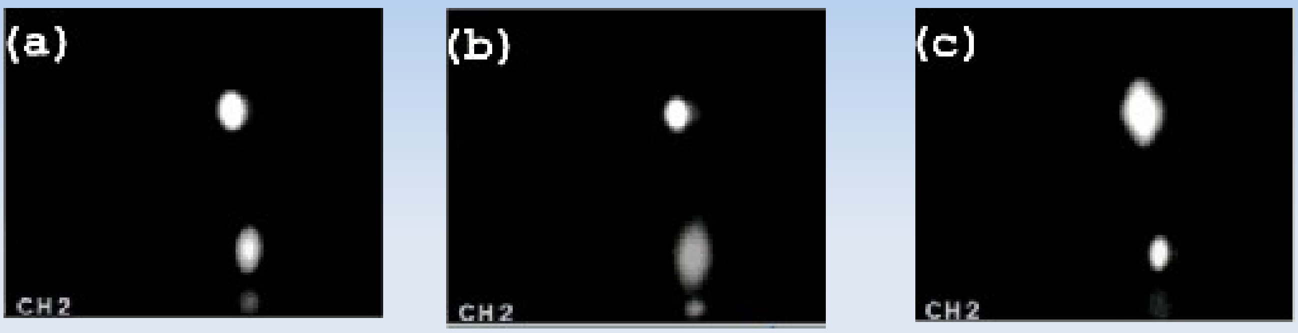
Observations
Orbit effect



- Displacement of the luminous region
  - Different bunches experience different beam-beam long-range interactions → they have different orbits
- Also observed in LEP with bunch trains

Observations
Dynamic β : Flip-flop

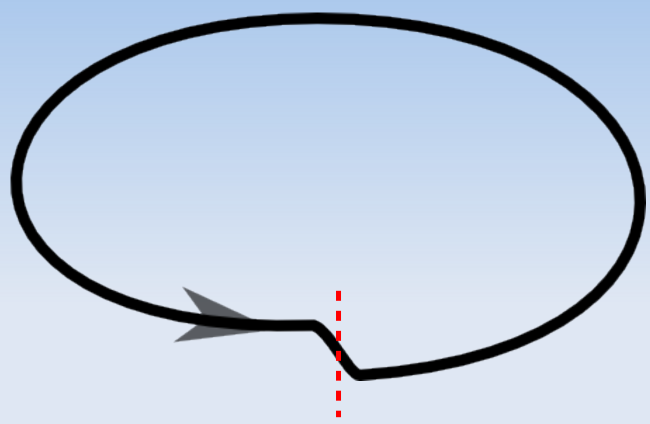
VEPP-2000 (4) :



- Low ξ : The two beams have identical transverse sizes
- High ξ : Two equivalent equilibrium configurations :
  - Electron beam is blown up
  - Positron beam is blown up

Coherent modes of oscillation

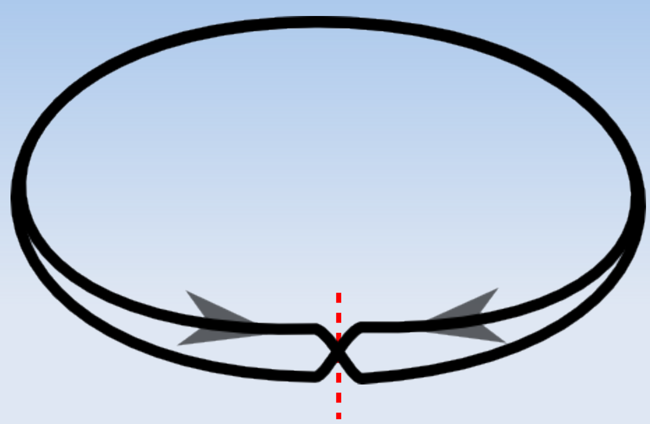
Rigid bunch model



$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix}_{t+1} = \begin{pmatrix} \cos(2\pi Q) & \sin(2\pi Q) \\ -\sin(2\pi Q) & \cos(2\pi Q) \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}_t$$

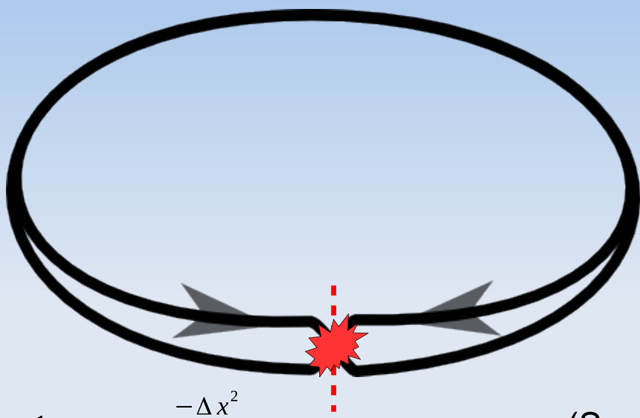
Coherent modes of oscillation

Rigid bunch model



$$\begin{pmatrix} x_{B1} \\ x_{B1}' \\ x_{B2} \\ x_{B2}' \end{pmatrix}_{t+1} = \begin{pmatrix} \cos(2\pi Q) & \sin(2\pi Q) & 0 & 0 \\ -\sin(2\pi Q) & \cos(2\pi Q) & 0 & 0 \\ 0 & 0 & \cos(2\pi Q) & \sin(2\pi Q) \\ 0 & 0 & -\sin(2\pi Q) & \cos(2\pi Q) \end{pmatrix} \begin{pmatrix} x_{B1} \\ x_{B1}' \\ x_{B2} \\ x_{B2}' \end{pmatrix}_t$$

Coherent modes of oscillation
Rigid bunch model

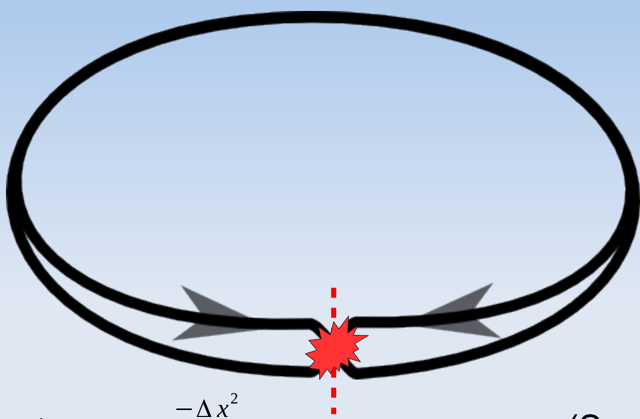


$$\Delta x'_{B1} = \frac{-2 r_0 N}{\gamma_r} \frac{1}{\Delta x}\left(1 - e^{\frac{-\Delta x^2}{4\sigma^2}}\right) \approx k\left(x_{B1} - x_{B2}\right)$$

(Small amplitude approximation)

$$\begin{pmatrix} x_{B1} \\ x_{B1}' \\ x_{B2} \\ x_{B2}' \end{pmatrix}_{t+1} = \begin{pmatrix} \cos(2\pi Q) & \sin(2\pi Q) & 0 & 0 \\ -\sin(2\pi Q) & \cos(2\pi Q) & 0 & 0 \\ 0 & 0 & \cos(2\pi Q) & \sin(2\pi Q) \\ 0 & 0 & -\sin(2\pi Q) & \cos(2\pi Q) \end{pmatrix} \begin{pmatrix} x_{B1} \\ x_{B1}' \\ x_{B2} \\ x_{B2}' \end{pmatrix}_{t}$$

Coherent modes of oscillation
Rigid bunch model



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(Small amplitude approximation)

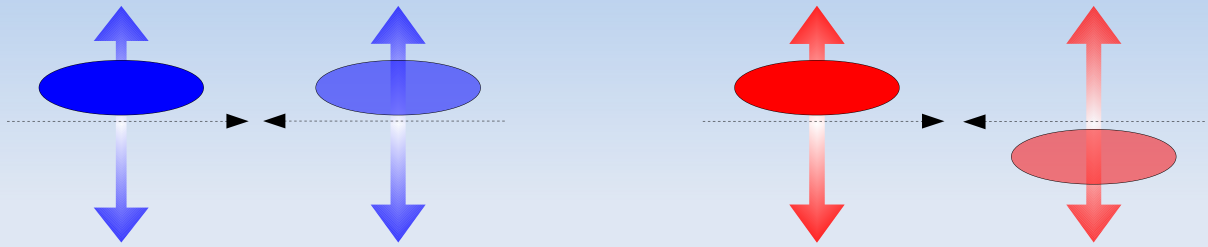
$$\begin{pmatrix} x_{B1} \\ x_{B1}' \\ x_{B2} \\ x_{B2}' \end{pmatrix}_{t+1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ +k & 1 & -k & 0 \\ 0 & 0 & 1 & 0 \\ -k & 0 & +k & 1 \end{pmatrix} \cdot M_{lattice} \begin{pmatrix} x_{B1} \\ x_{B1}' \\ x_{B2} \\ x_{B2}' \end{pmatrix}_{t}$$

Coherent modes of oscillation

Rigid bunch model

In-phase oscillations
→ σ mode

Out of phase oscillations
→ π mode



- $x_1 = x_2$ at every interaction

  $\rightarrow Q_\sigma = Q$

- $x_1 = -x_2$ at every interaction

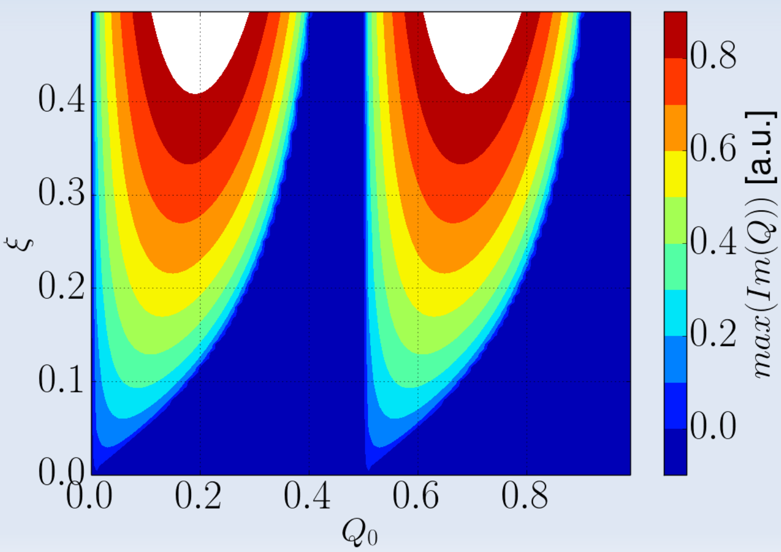
  $\rightarrow Q_\pi \sim Q - \xi$ (*)

(*) ξ << 1 and for tunes away from resonances

Collective resonance

$$\begin{pmatrix} x_i \\ x_i' \end{pmatrix}_{t+1} = M_{lattice} \cdot M_{BB} \begin{pmatrix} x_i \\ x_i' \end{pmatrix}_t$$

Resonance conditions : $Q_\sigma = n/2$, $Q_\pi = n/2$



- The rigid dipole mode can be unstable under resonant conditions
- Higher order resonances can also drive the beam-beam coherent modes unstable (2)

Coherent modes of oscillation

Vlasov perturbation theory (5)

Rigid bunch model :

Each beam centroid position and momentum x_1, x'_1 and x_2, x'_2

- Equation of motion :

$$\begin{pmatrix} x_i \\ x_i' \end{pmatrix}_{t+1} = M_{lattice} \cdot M_{BB} \begin{pmatrix} x_i \\ x_i' \end{pmatrix}_t$$

- Non Linear beam-beam map :

$$\Delta x'_{coh} = \frac{-2 r_0 N}{\gamma_r} \frac{1}{\Delta x} \left(1 - e^{\frac{-\Delta x^2}{4\sigma^2}}\right)$$

- Linearized kick :

$$\Delta x'_{coh} = \frac{4\pi\xi}{2} \Delta x$$

- Write one turn matrix and find eigenvalues / eigenvectors

Vlasov perturbation theory :

Each beam phase space distribution

$F^{(1)}, F^{(2)}$

- Liouville's thorem :

$$\begin{cases} \frac{\partial F^{(1)}}{\partial t} + [F^{(1)}, H(F^{(2)})] = 0 \\ \frac{\partial F^{(2)}}{\partial t} + [F^{(2)}, H(F^{(1)})] = 0 \end{cases}$$

- Hamiltonian (lattice + beam-beam)

$$H(J^{(1)}, \Psi^{(1)}, F^{(2)}, t)$$

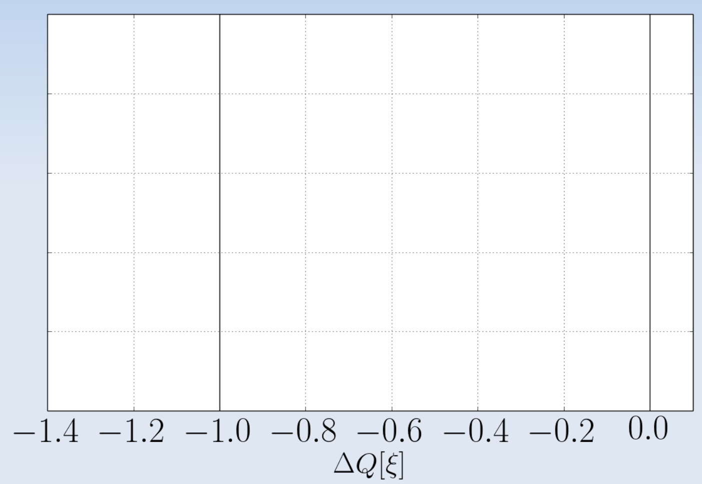
- First order perturbation

$$F^{(i)} = F_0 + F_1^{(i)}$$

- Formulate the linearized system as a linear operator → find eigenvalues / eigenfunctions

Coherent mode spectrum

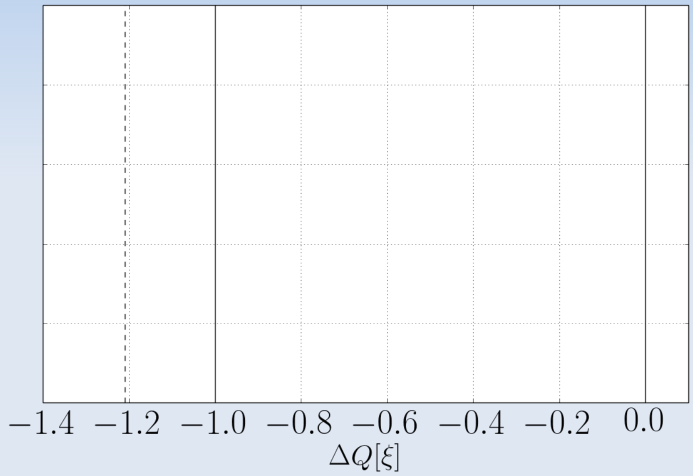
Rigid bunch : $Q_\pi = Q - \xi$ $Q_\sigma = Q$



- The Yokoya factor Y is usually between 1.0 and 1.3 depending on the type of interaction (Flat, round, asymmetric, long-range, …) (5)

Coherent mode spectrum

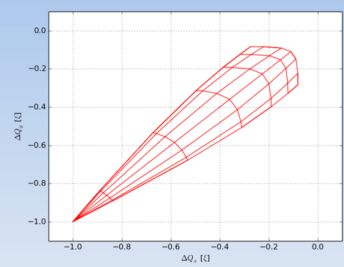
Self-consistent Model : $Q_\pi = Q - Y\xi$ $\quad$ $Q_\sigma = Q$



- The Yokoya factor Y is usually between 1.0 and 1.3 depending on the type of interaction (Flat, round, asymmetric, long-range, …) (5)

Incoherent spectrum

- The non-linearity of beam-beam interactions result in a strong amplitude detuning
- The single particles generate a continuum of modes, the *incoherent spectrum*



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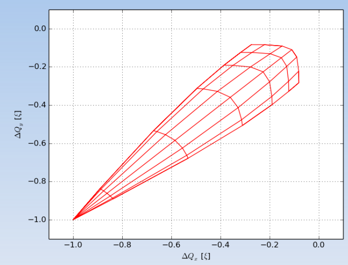
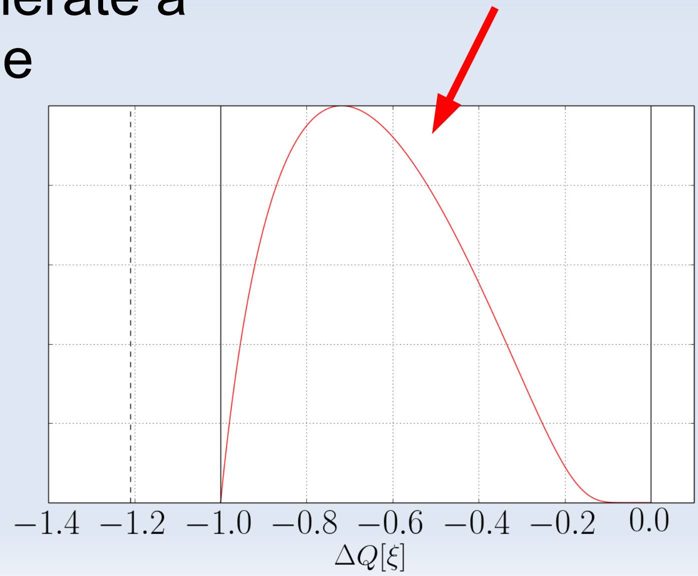
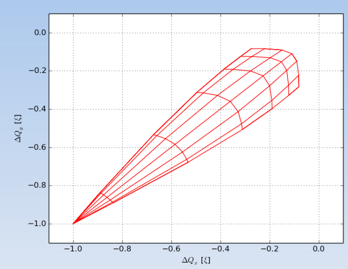
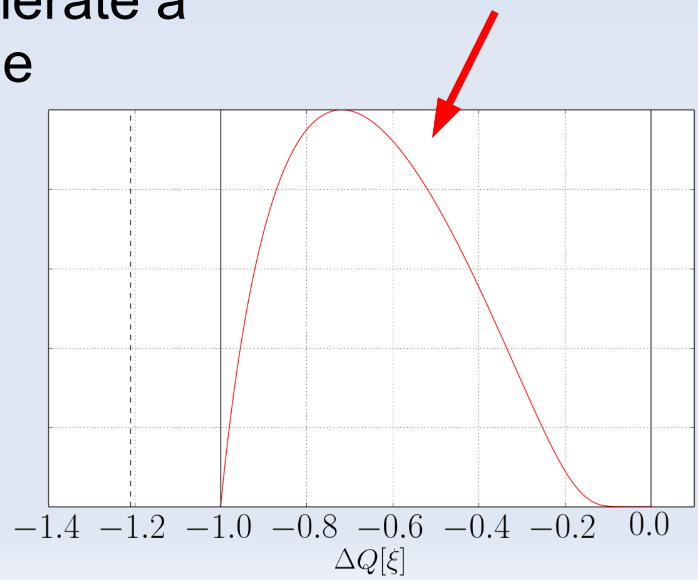




Incoherent spectrum

- The non-linearity of beam-beam interactions result in a strong amplitude detuning
- The single particles generate a continuum of modes, the *incoherent spectrum*
- Both the σ and π mode are outside the incoherent spectrum

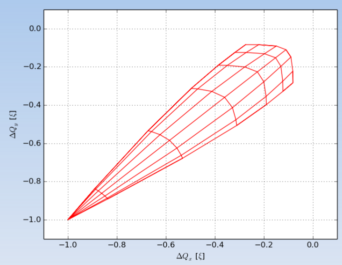


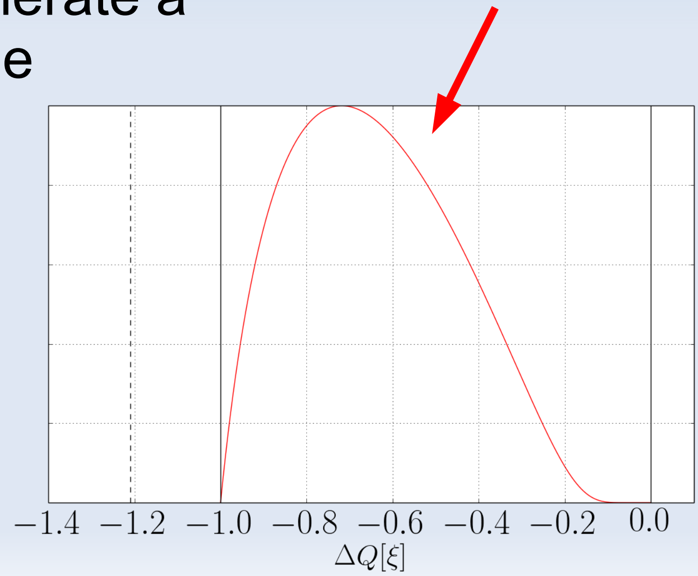


Incoherent spectrum

- The non-linearity of beam-beam interactions result in a strong amplitude detuning
- The single particles generate a continuum of modes, the *incoherent spectrum*
- Both the σ and π mode are outside the incoherent spectrum

→ Absence of Landau damping !

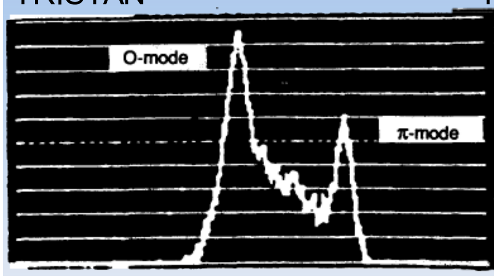




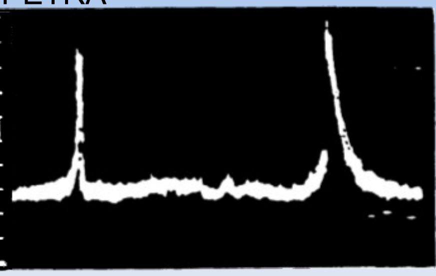
Observations

(6)

TRISTAN

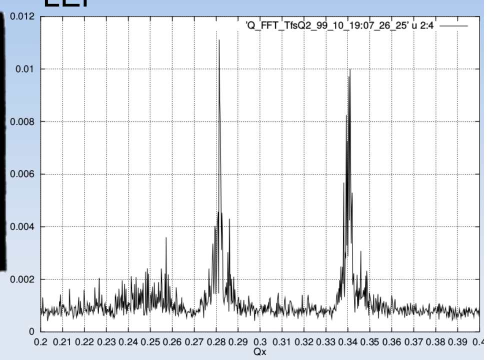


PETRA

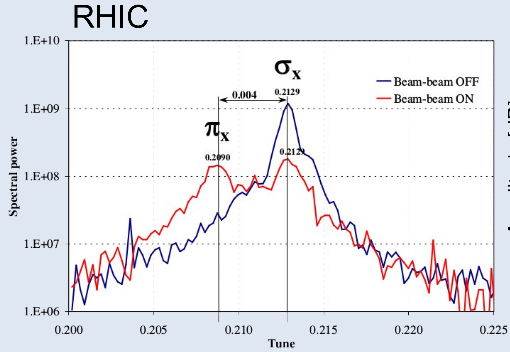


Vertical eigenfrequencies of two colliding bunches.

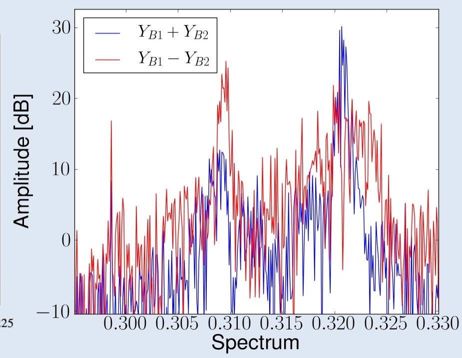
LEP



RHIC



LHC



- Perfect agreement with fully self-consistent models
- $\mathrm{Sp\bar{p}S}$?
- Tevatron ?

Multiparticle tracking

(see K. Li's lectures)

- Model the beam distribution with a discrete set of macro-particles
- Track the particles, solving for each beam's fields at each interaction

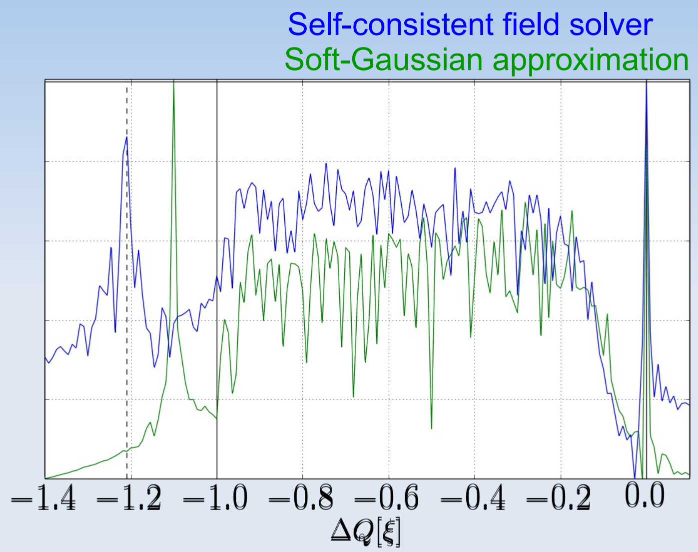
$$\begin{pmatrix} x_i \\ x_i' \end{pmatrix}_{t+1} = M_{lattice} \cdot M_{BB} \begin{pmatrix} x_i \\ x_i' \end{pmatrix}_t$$

- Non-linear beam-beam map
  - Gaussian fit : soft-Gaussian approximation

$$\Delta x'_i = \frac{-2 r_0 N}{\gamma_r} \frac{1}{x_i} \left(1 - e^{\frac{-x_i^2}{2\sigma^2}}\right)$$

- Numerical Poisson solver

Beam-beam coherent mode spectrum



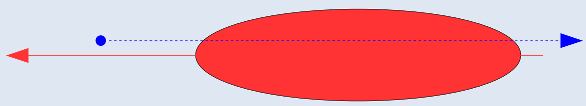
- The soft-Gaussian approximation underestimate the Yokoya factor

  → Need to fully resolve the particles distribution

Decoherence : weak-strong

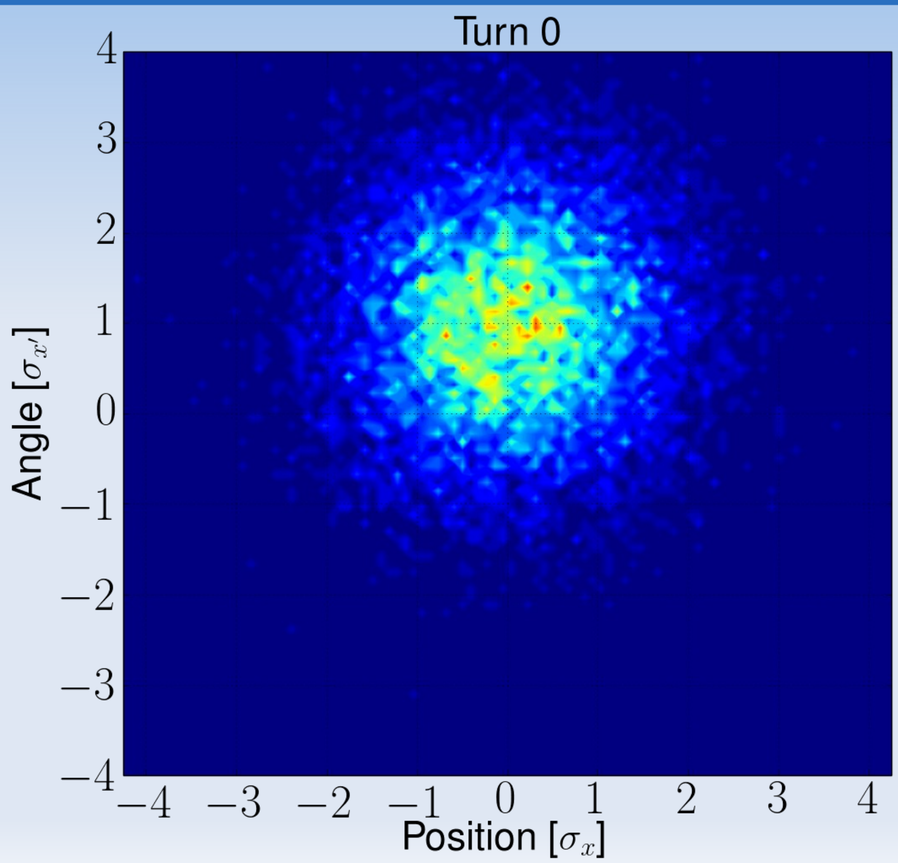
- Multiparticle tracking simulation, with a single beam and a fixed beam-beam interaction

  → weak-strong regime :

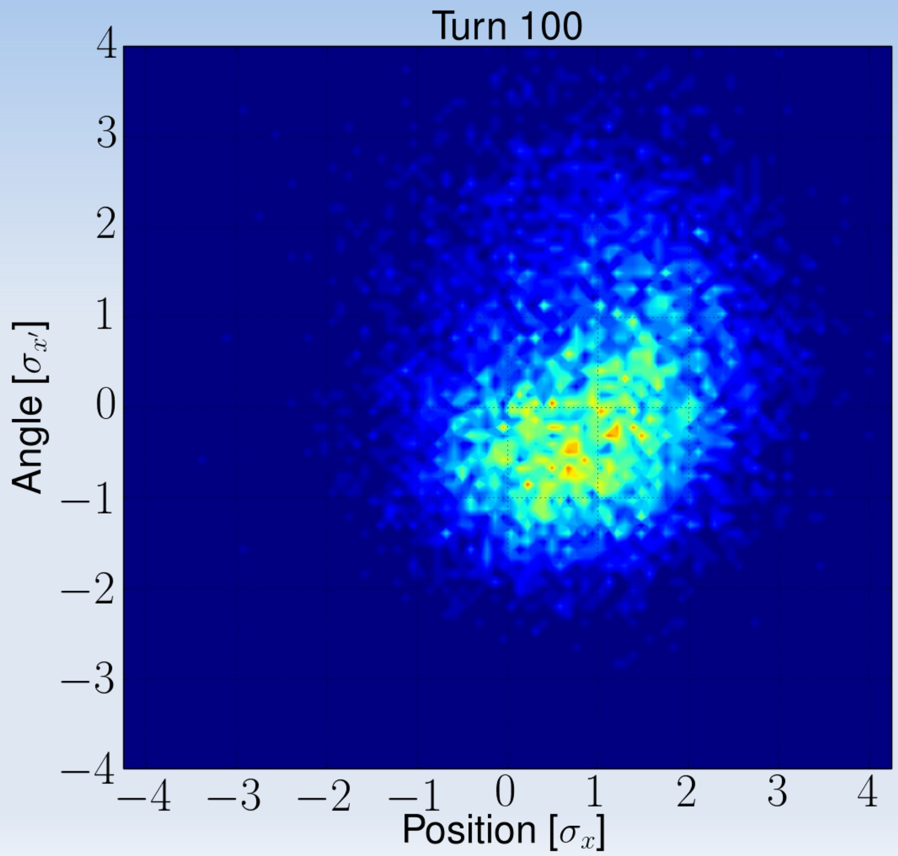


- Start the simulation with a beam offset with respect to the closed orbit and let it decohere

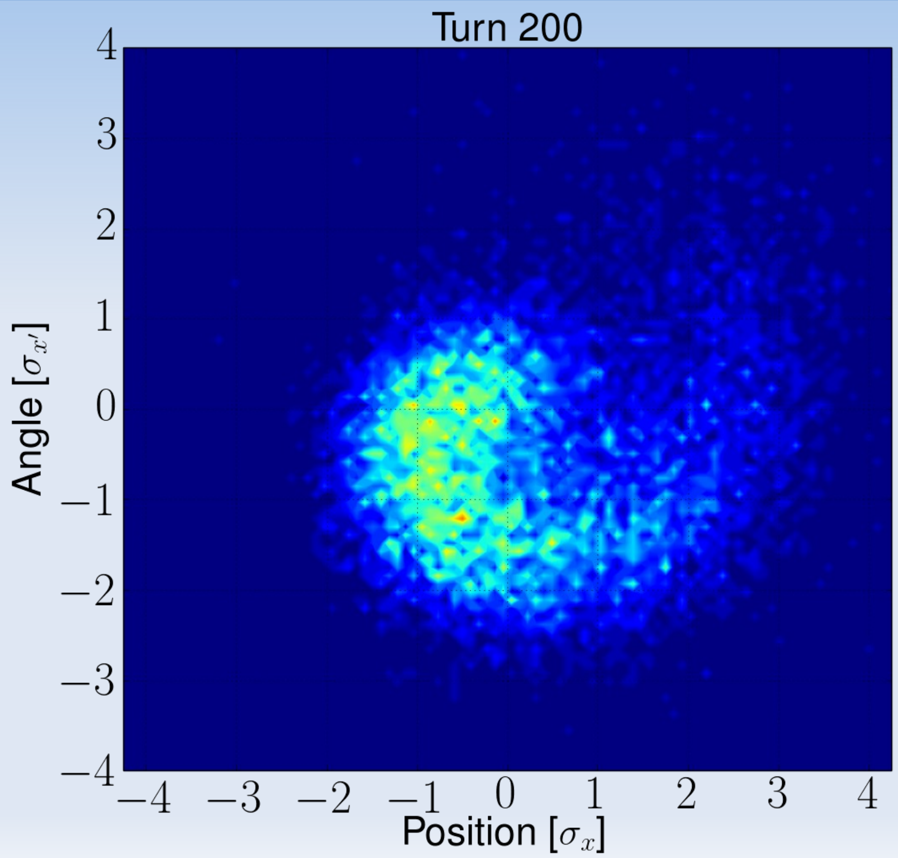
Decoherence : weak-strong



Decoherence : weak-strong



Decoherence : weak-strong



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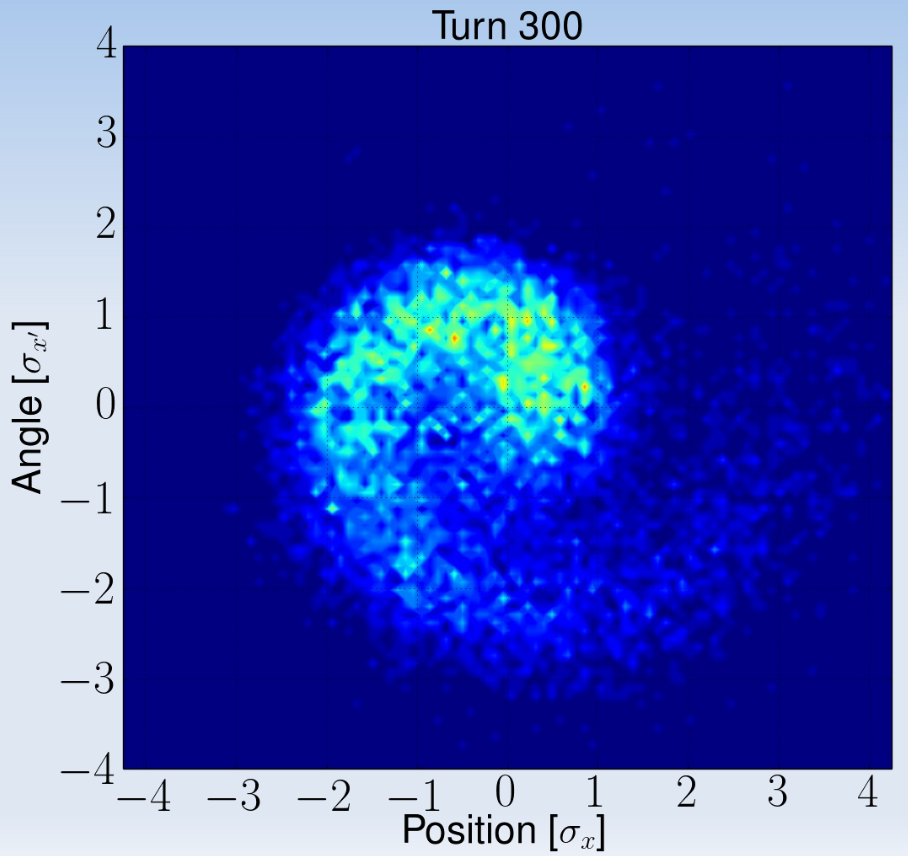
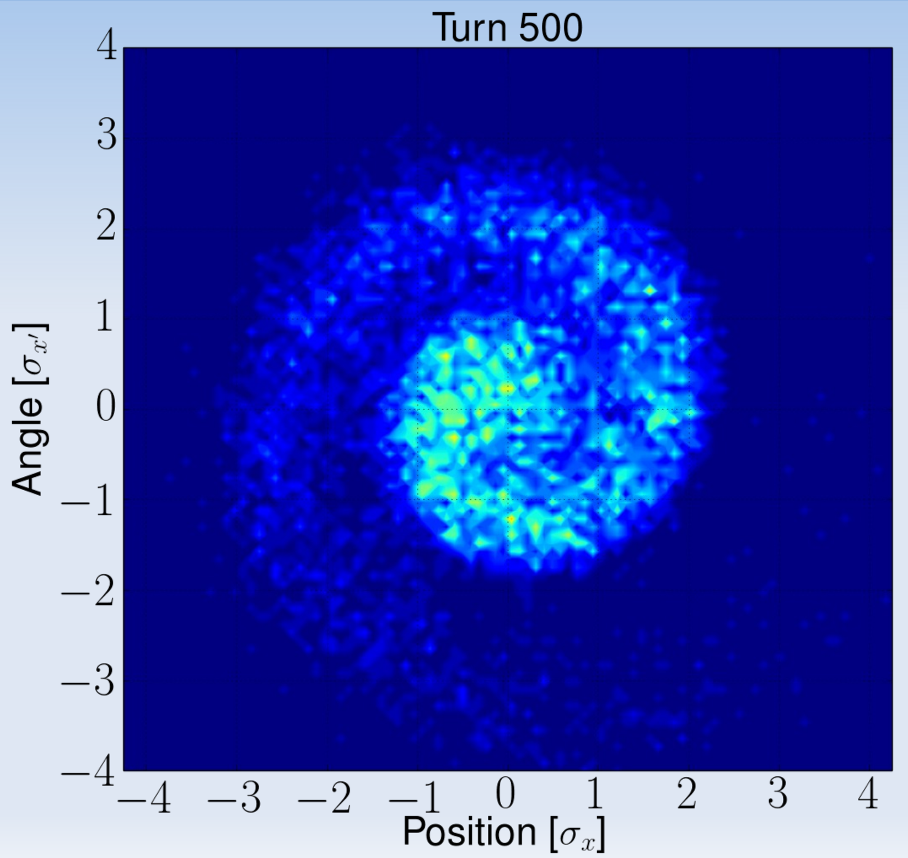


Decoherence : weak-strong



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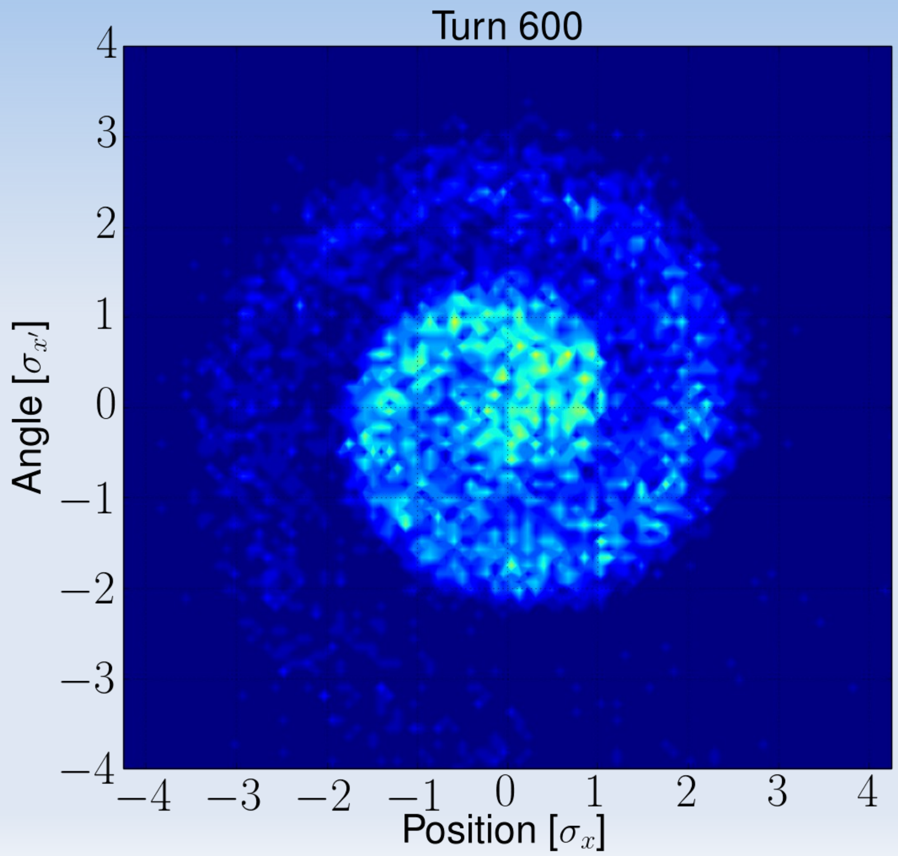

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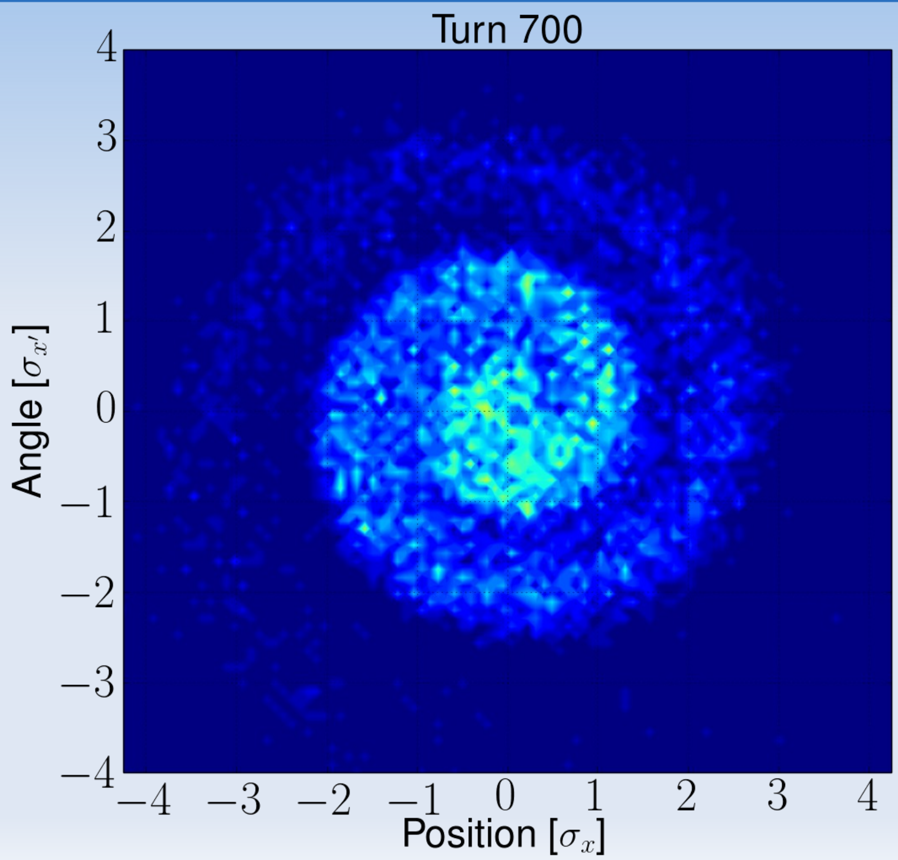
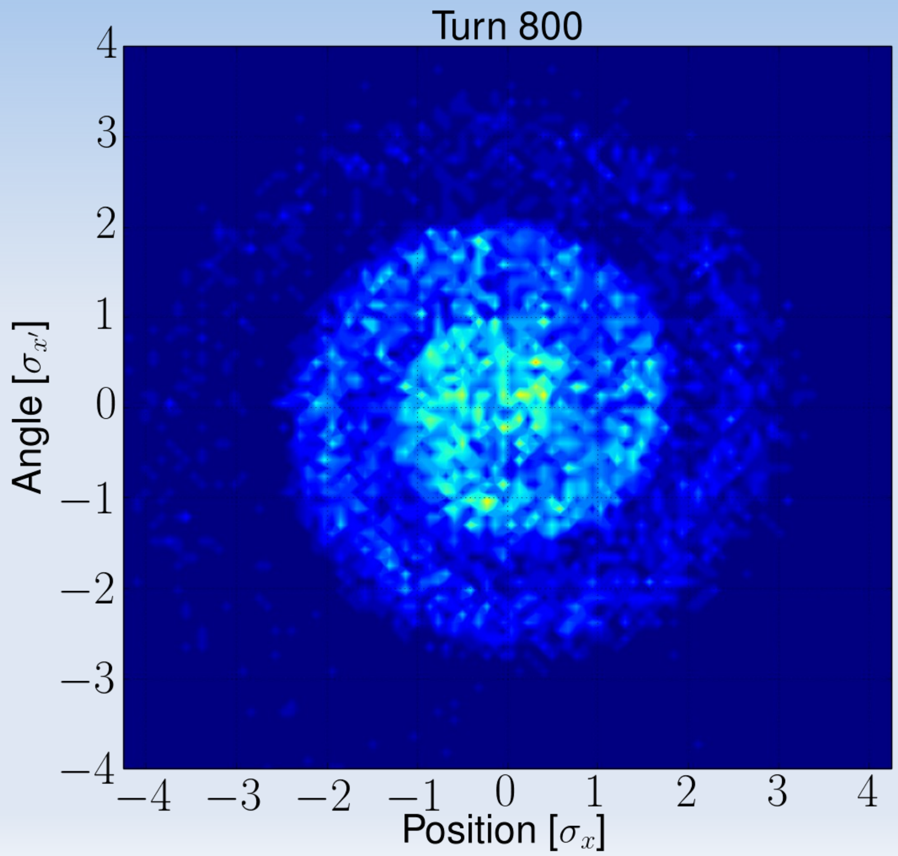


Decoherence : weak-strong



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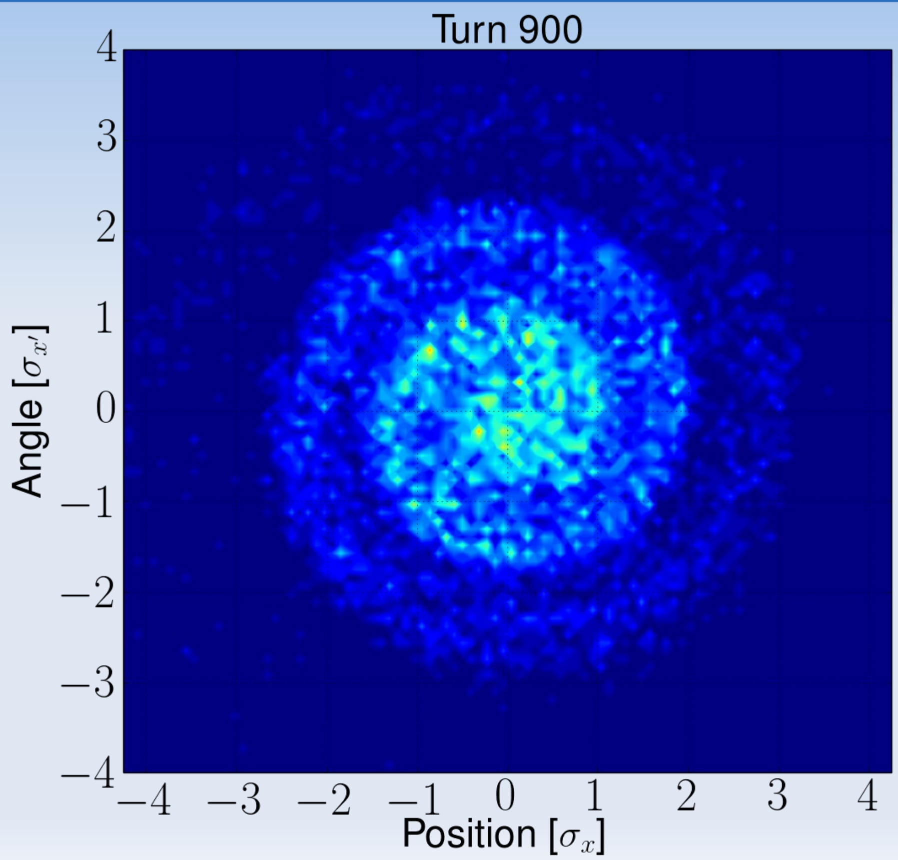

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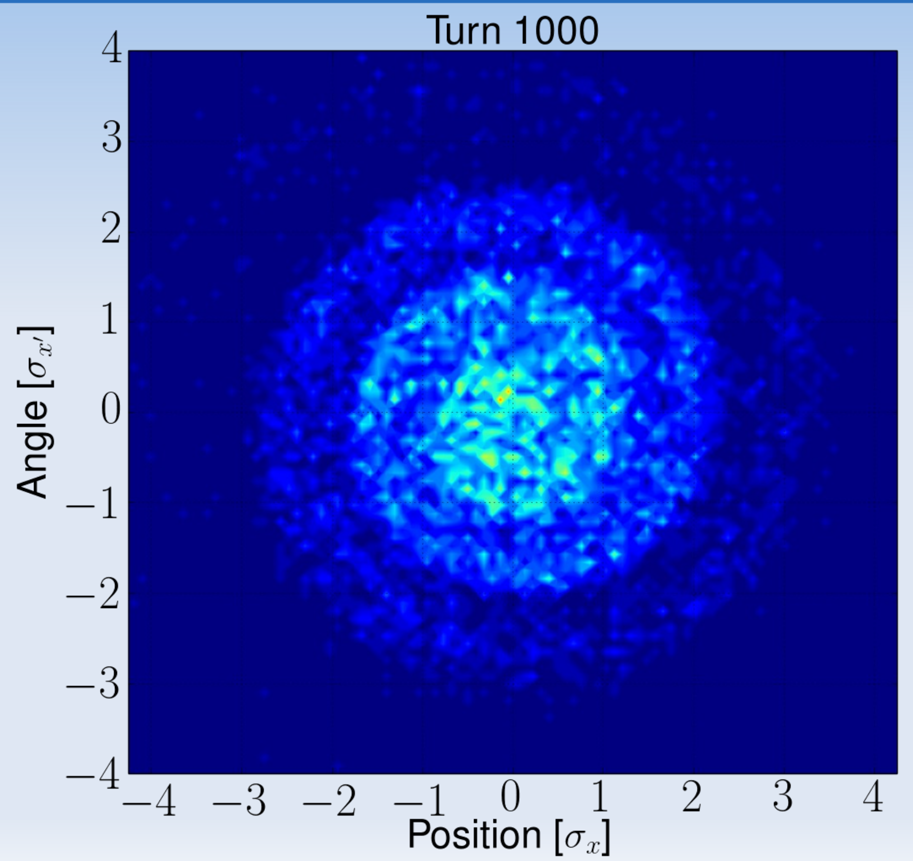

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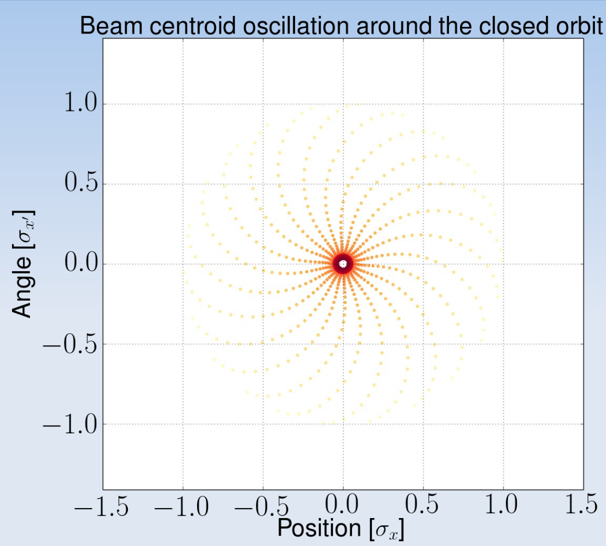
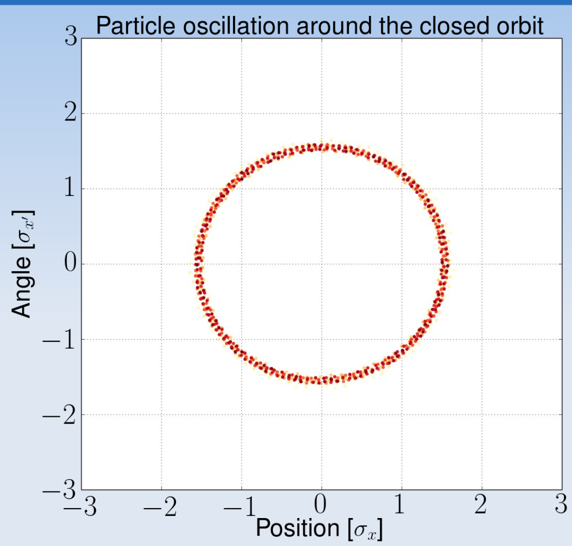


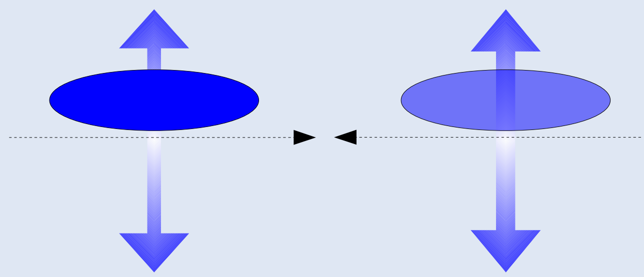


- The amplitude detuning due to beam-beam interaction leads to decoherence identically to other lattice non-linearities

  $\rightarrow \frac{1}{\epsilon_0}\frac{d\epsilon}{dt}=\frac{\Delta^2}{2}$
- Decoherence time ~1/ξ

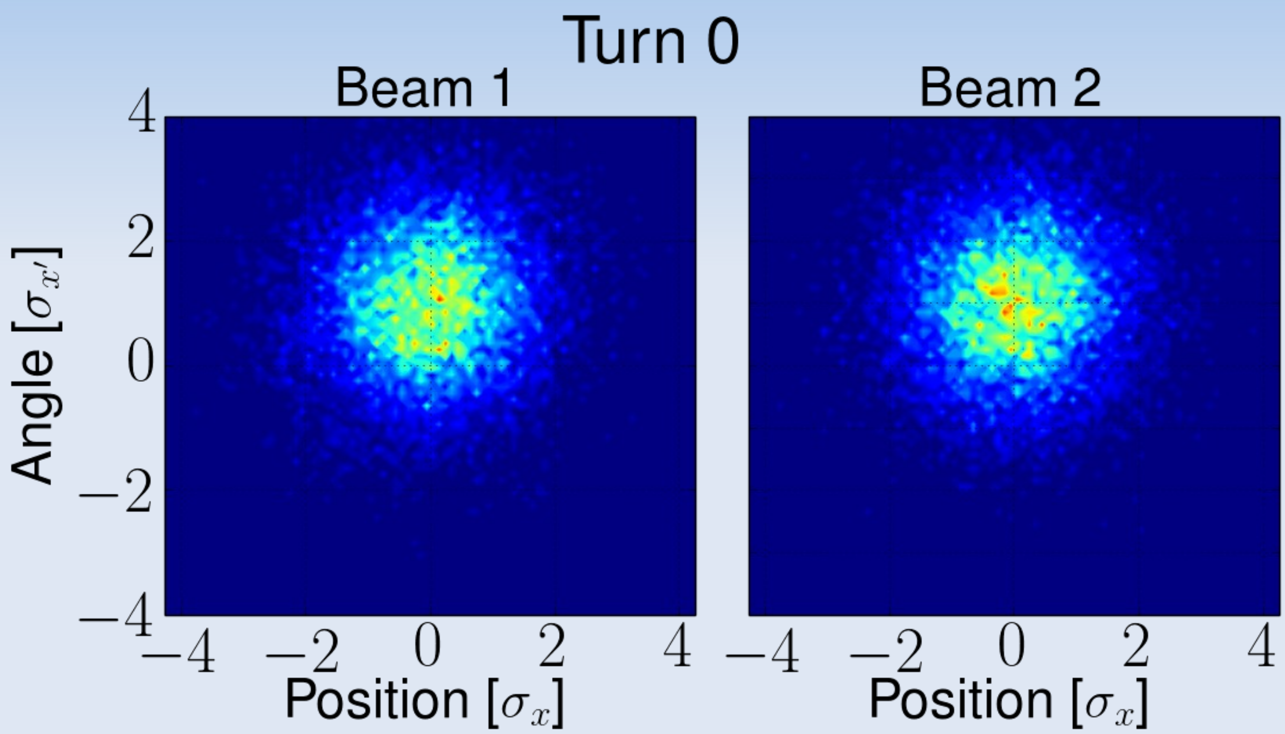
Decoherence : strong-strong

- Similar setup but :
  - Two independent beams
  - Non-linear beam-beam map based on the charge distribution

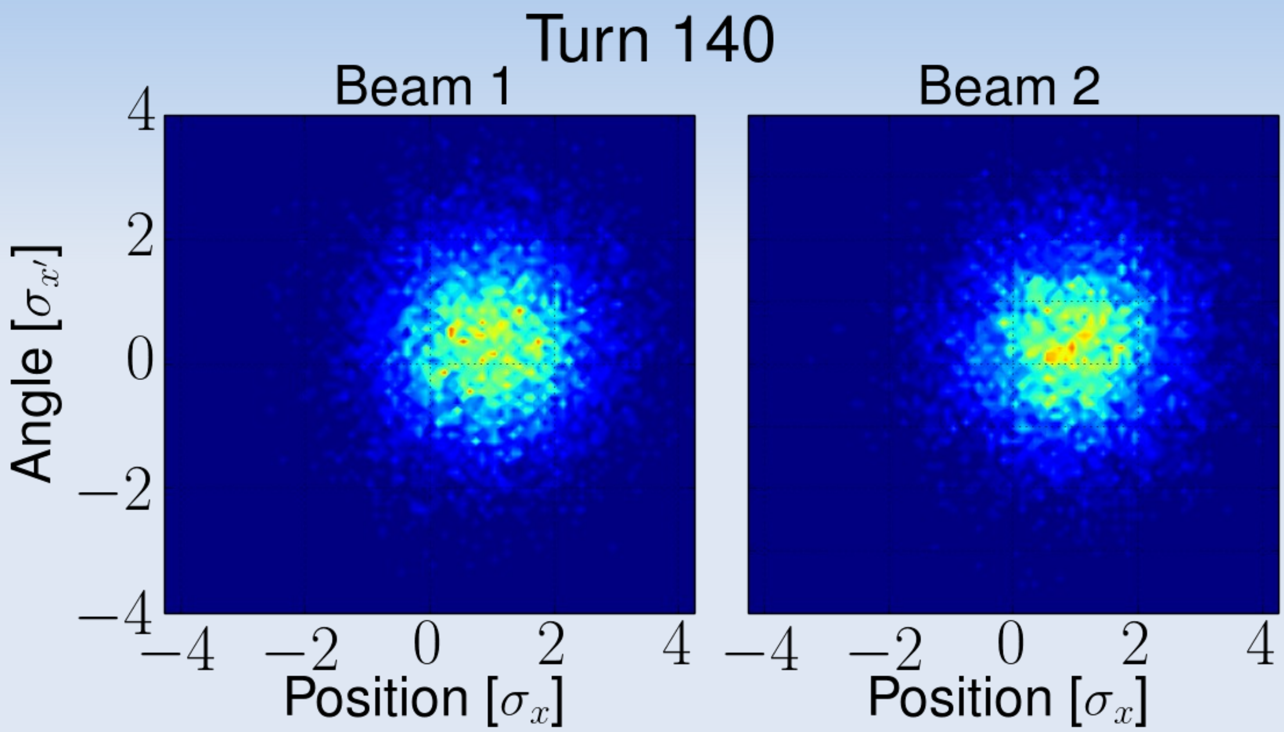


- Start the simulation with both beams offset in the same direction with respect to the closed orbit
- Let the mode decohere ?

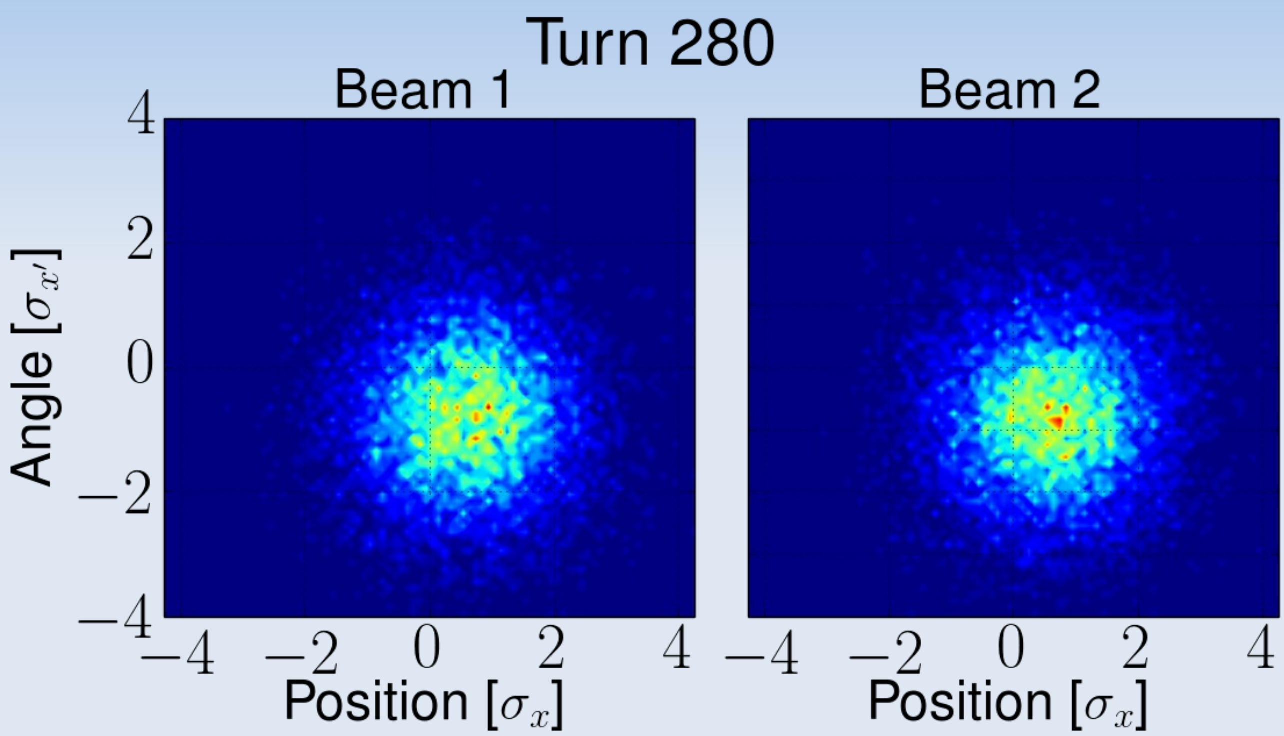
Decoherence : σ-mode



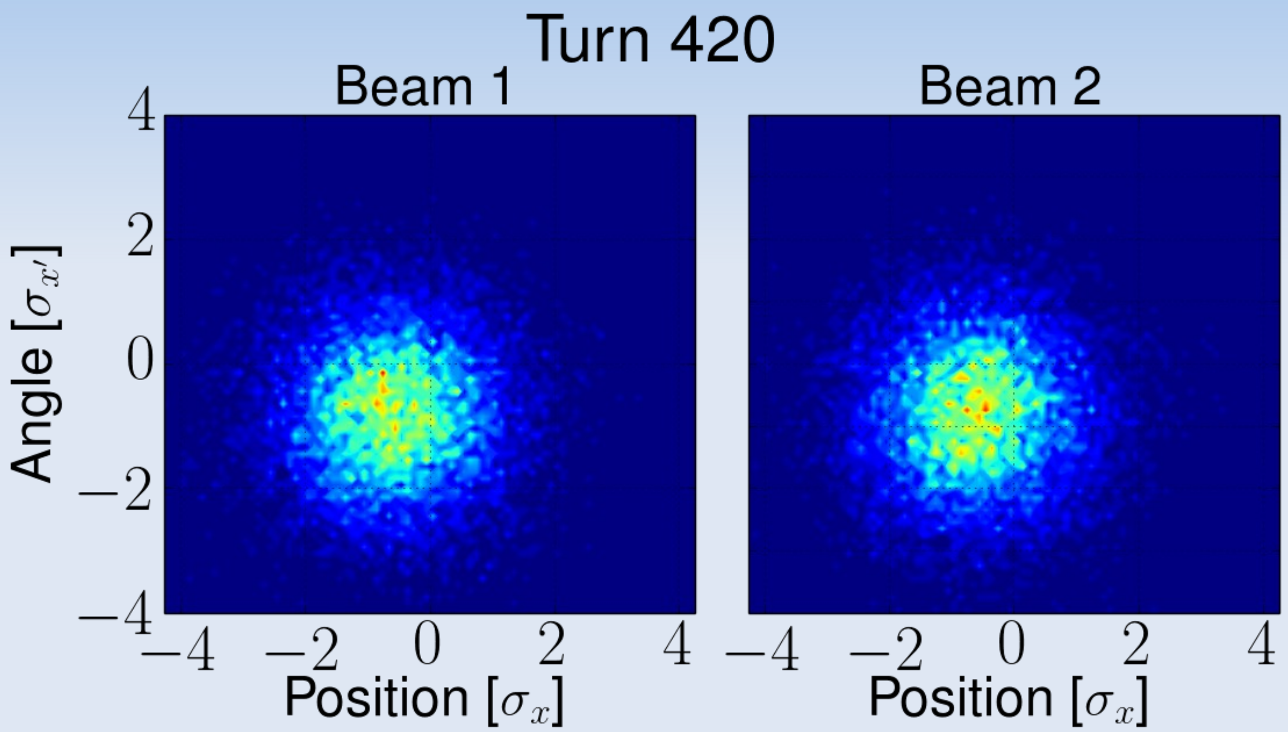
Decoherence : σ-mode



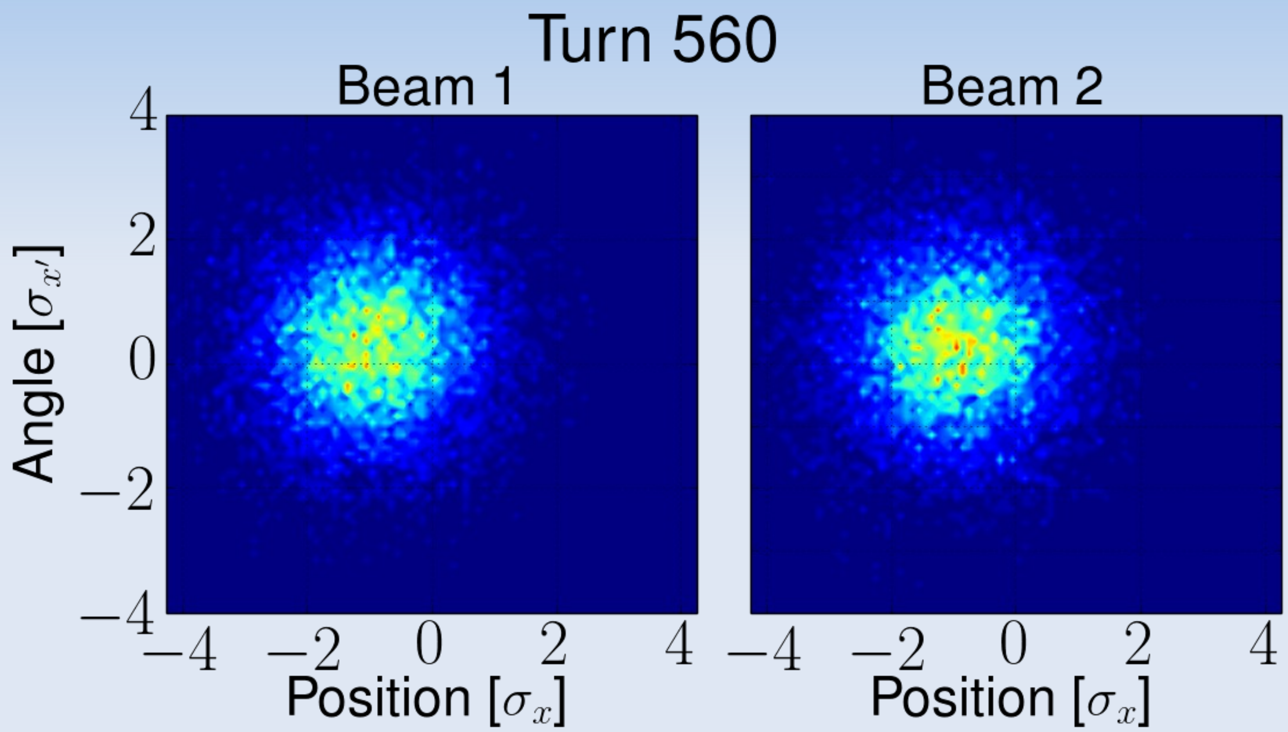
Decoherence : σ-mode



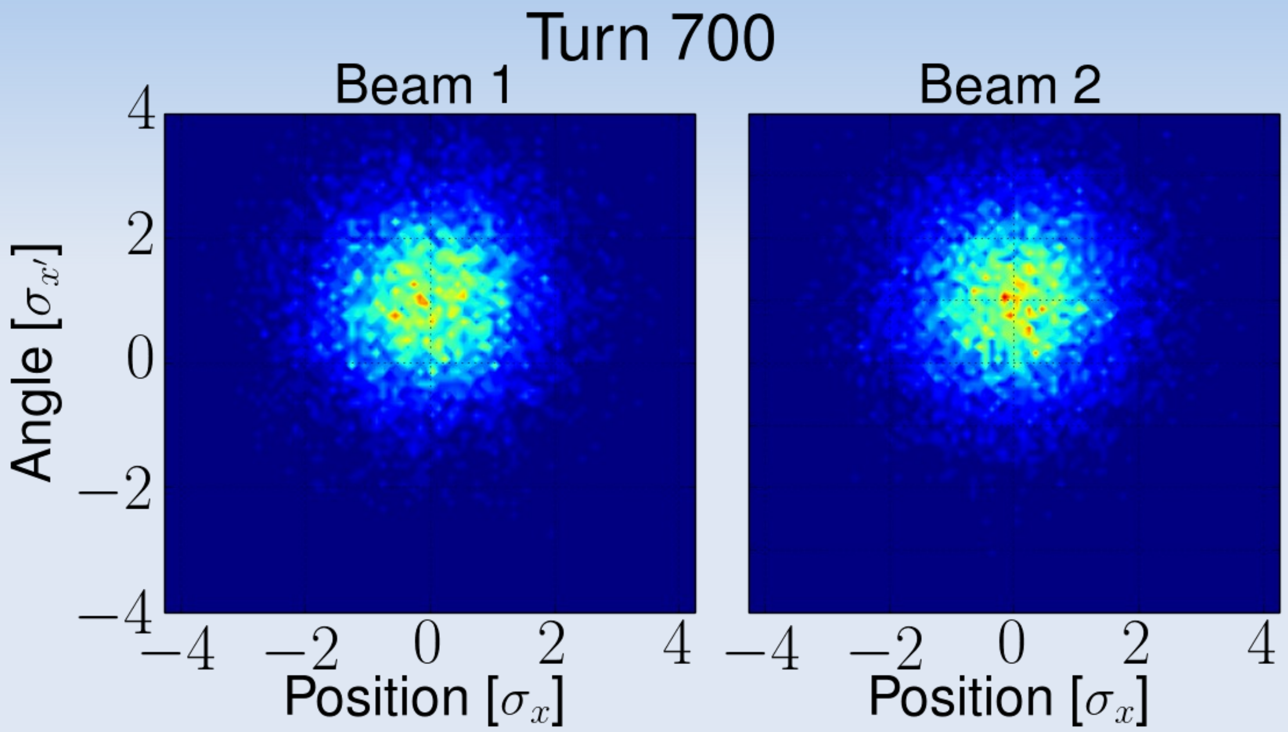
Decoherence : σ-mode



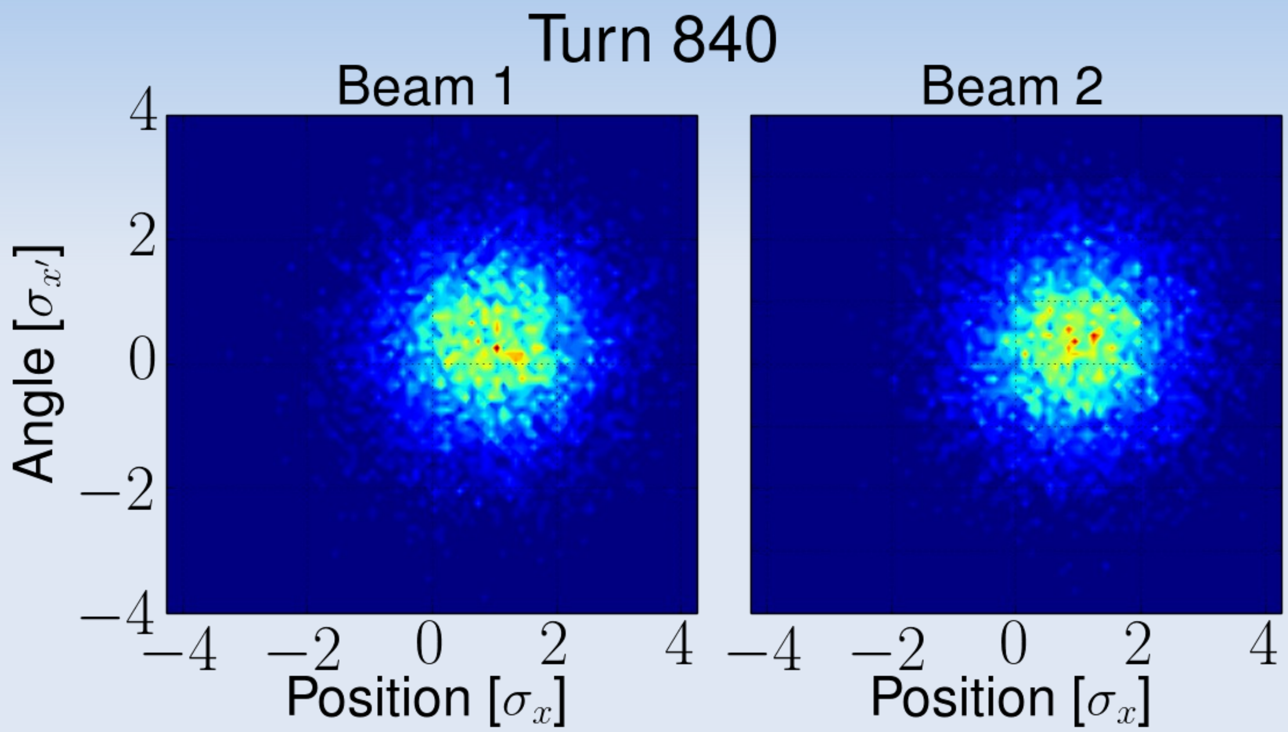
Decoherence : σ-mode



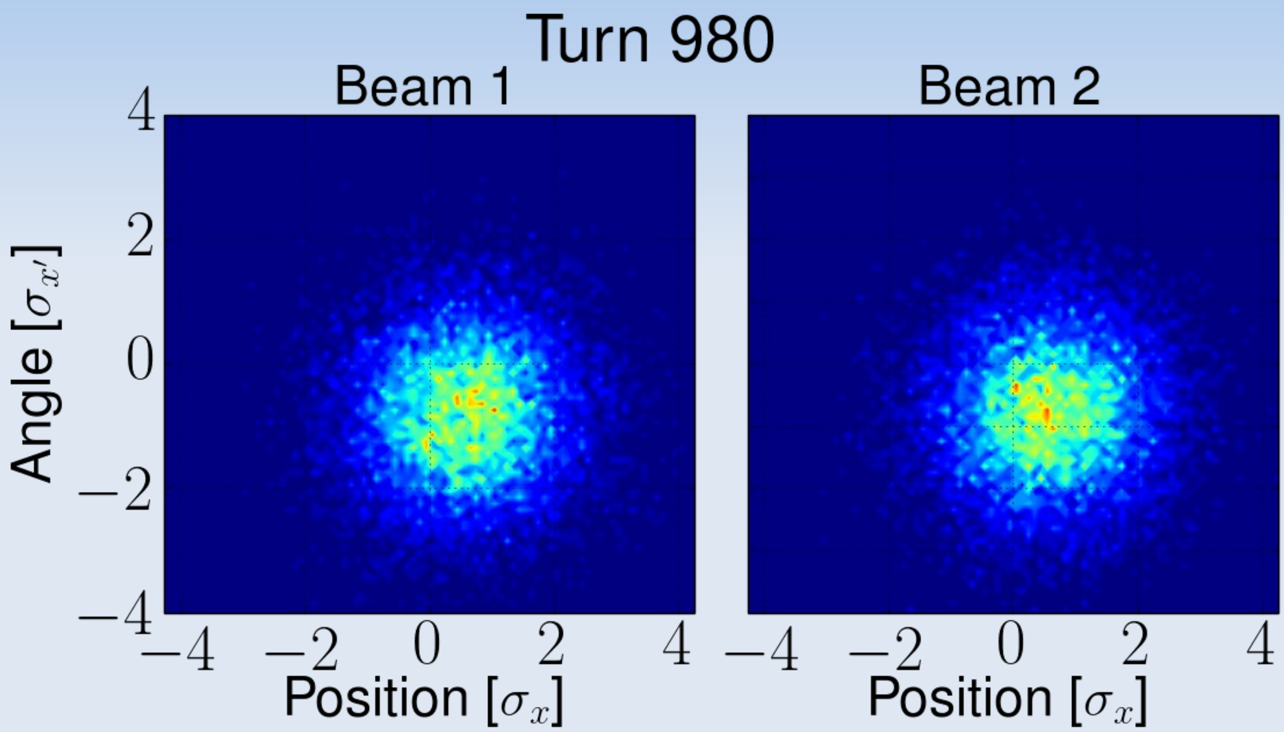
Decoherence : σ-mode



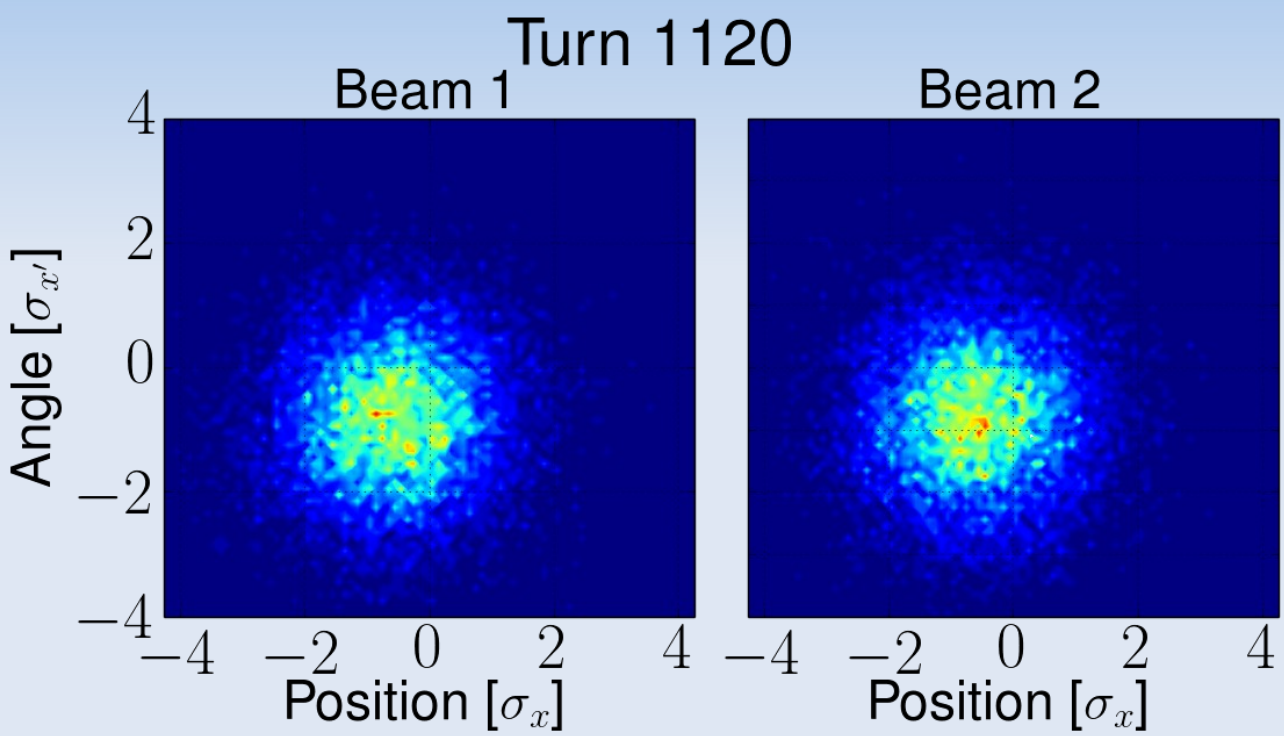
Decoherence : σ-mode



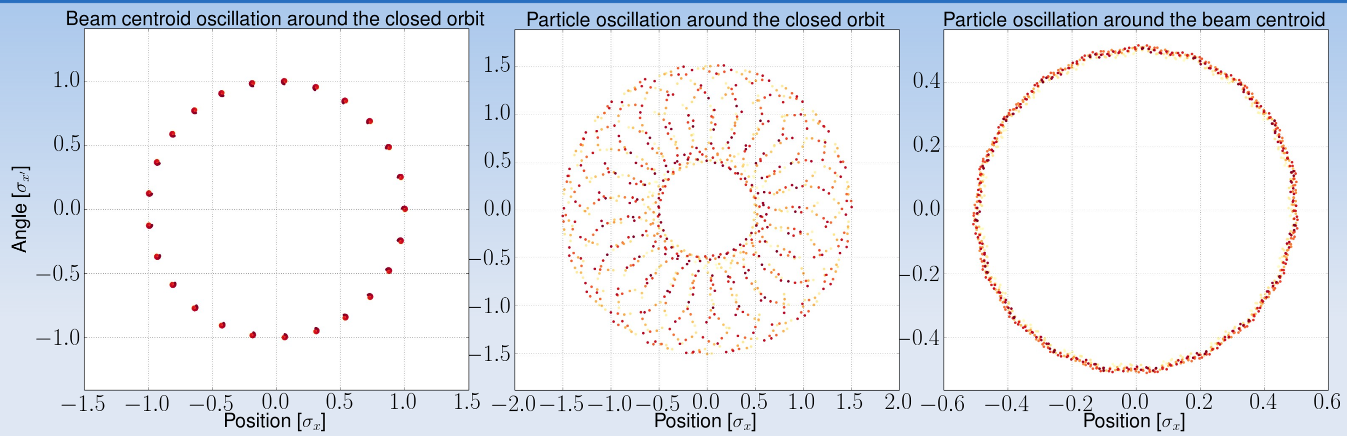
Decoherence : σ-mode



Decoherence : σ-mode



Decoherence of the σ mode



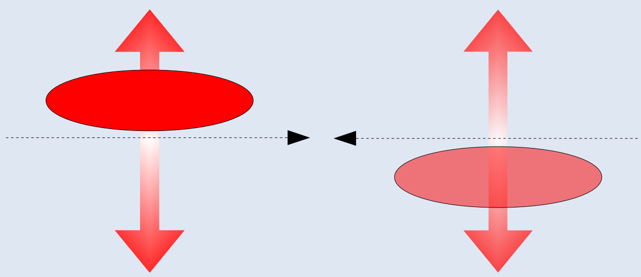
- The single particle motion is the linear composition of the centroid position and the position with respect to the centroid position

  → The single particle motion does not change the coherent force

- The incoherent and coherent motion are decoupled → Absence of decoherence

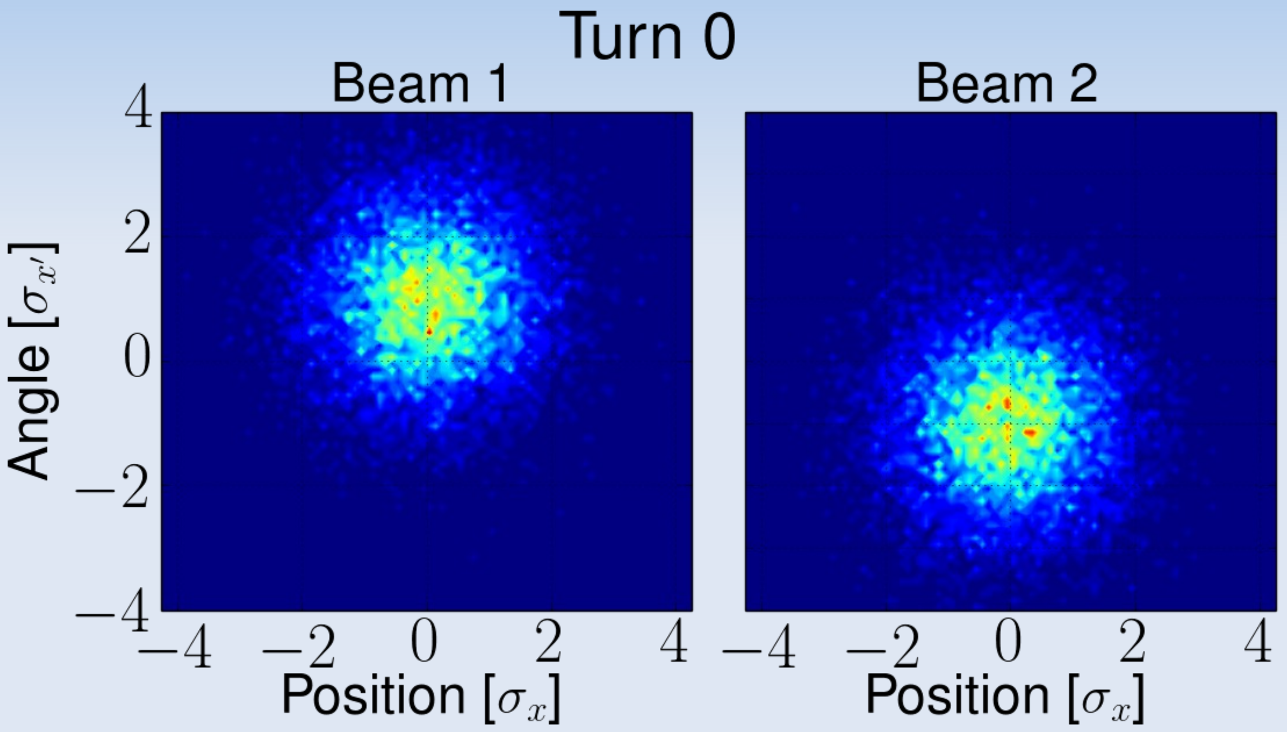
Decoherence : strong-strong

- Identical setup :
  - Two independent beams
  - Non-linear beam-beam map based on the charge distribution

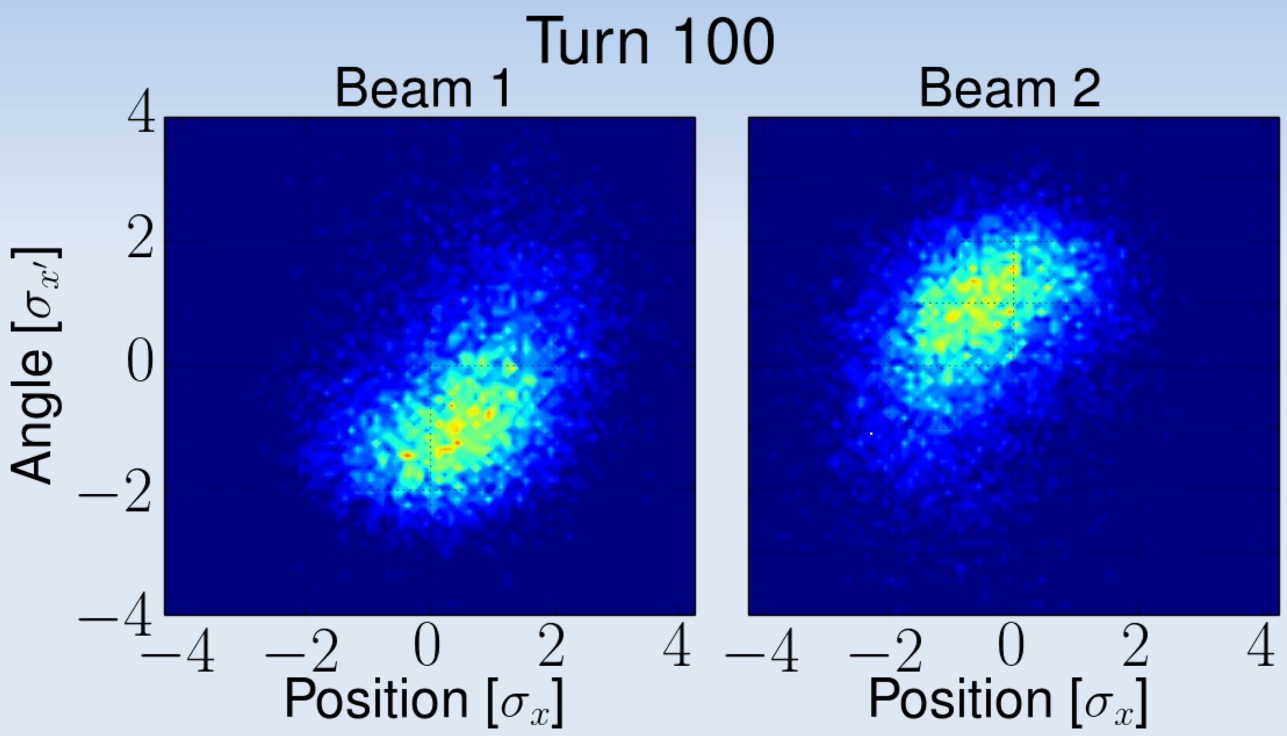


- Start the simulation with both beams offset in opposite directions with respect to the closed orbit
- Let the mode decohere ?

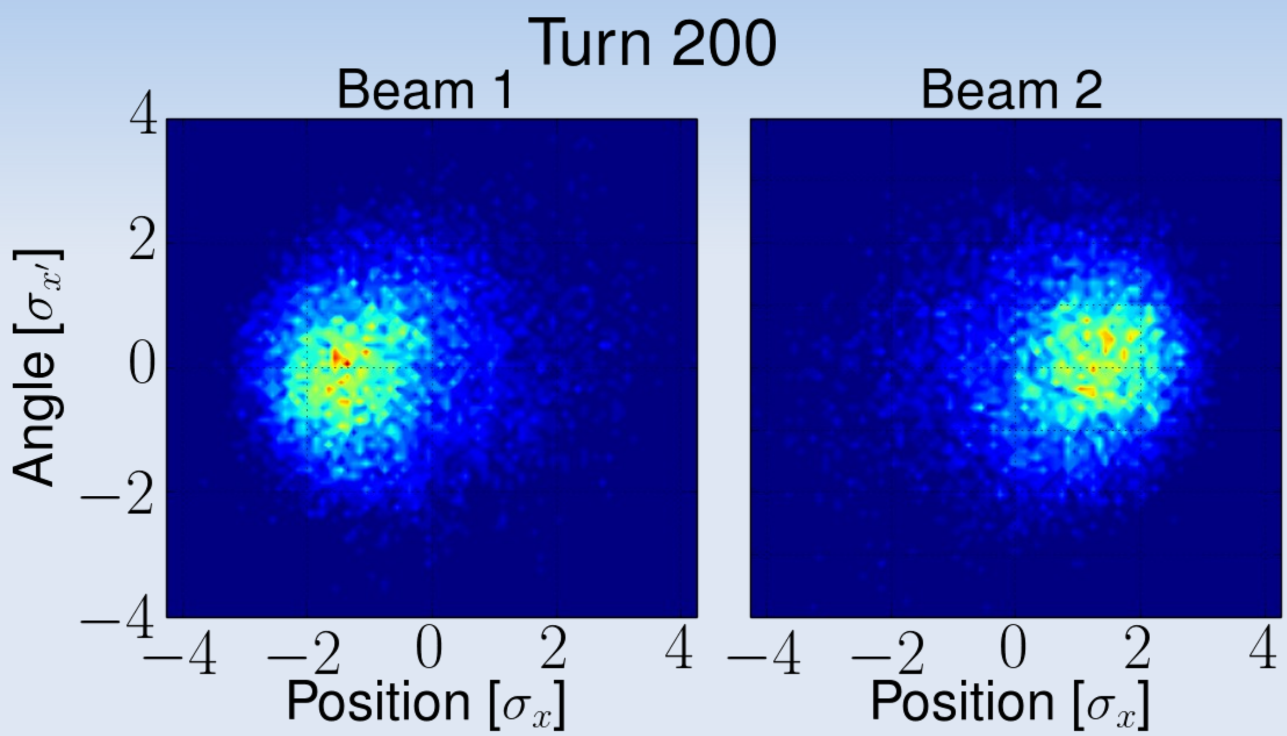
Decoherence of the π mode



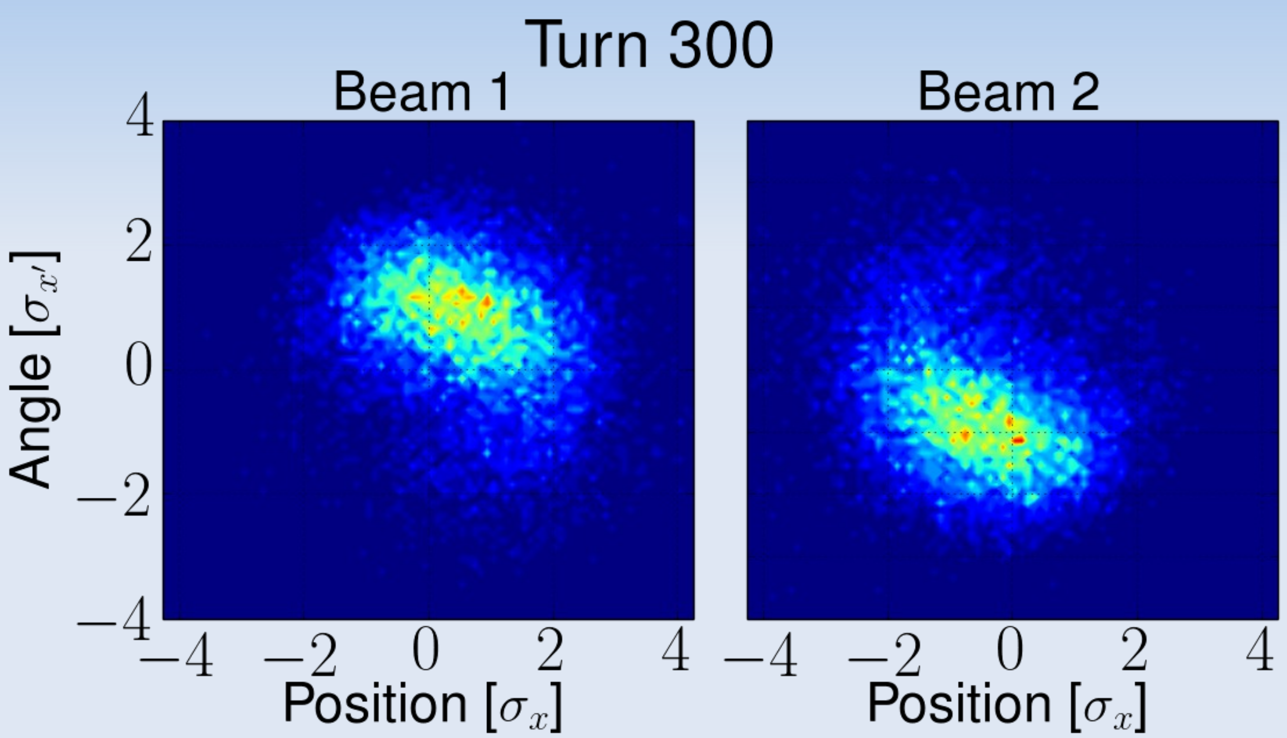
Decoherence of the π mode



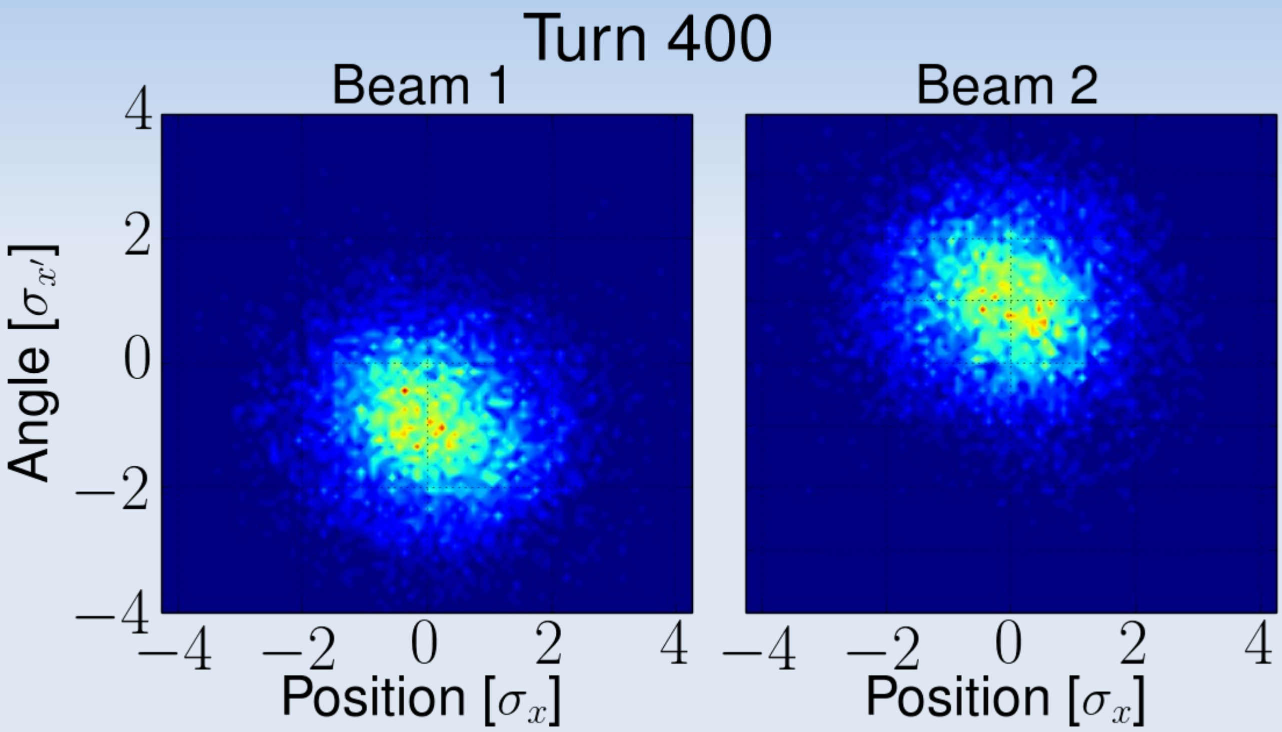
Decoherence of the π mode



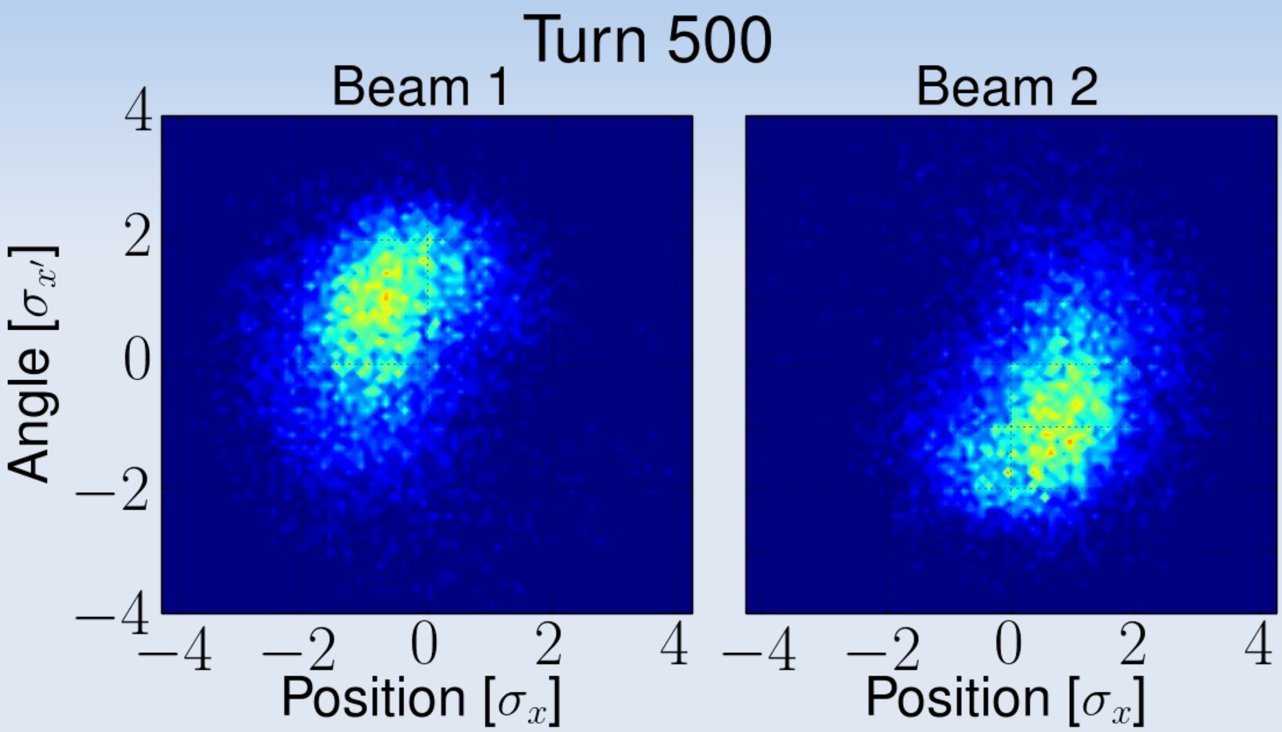
Decoherence of the π mode



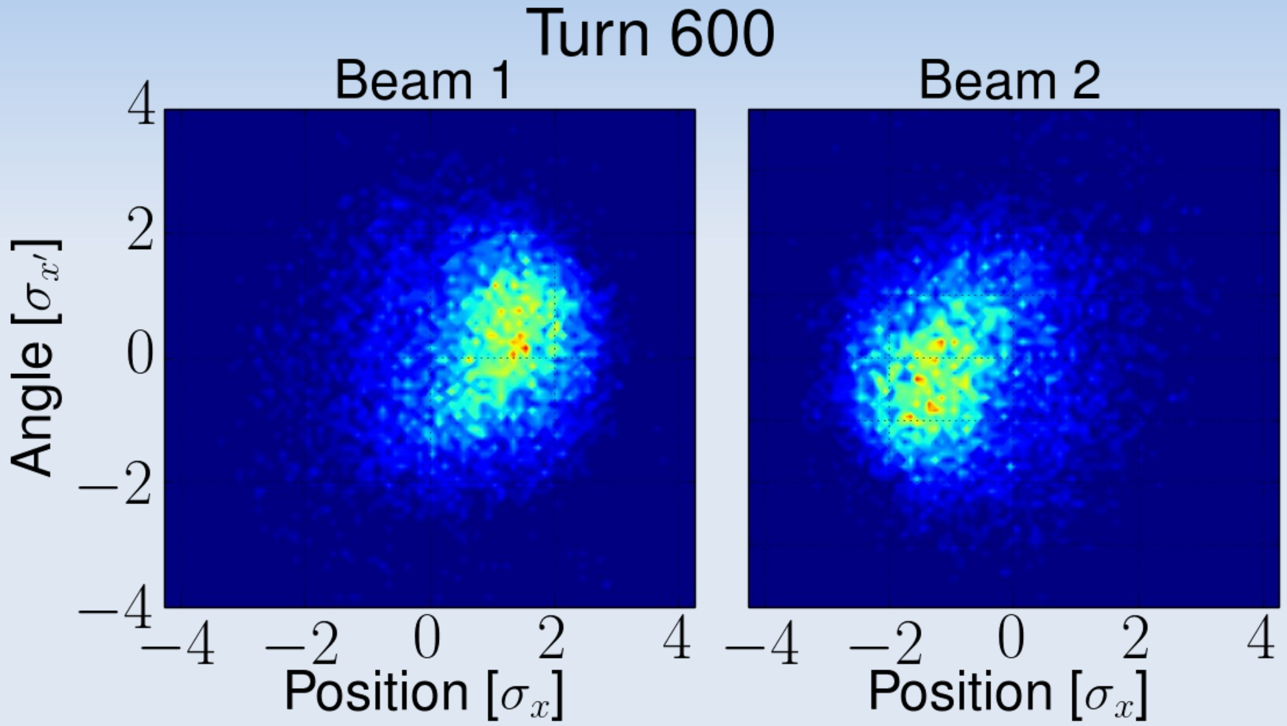
Decoherence of the π mode



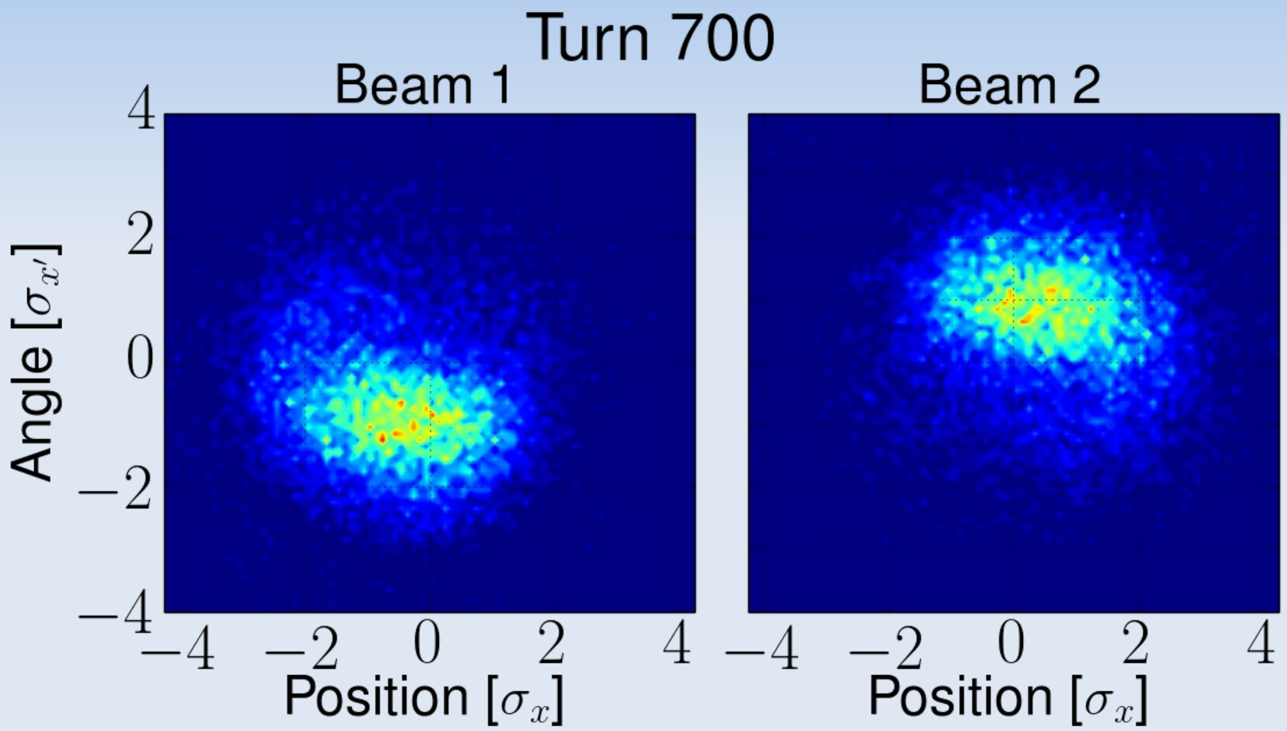
Decoherence of the π mode



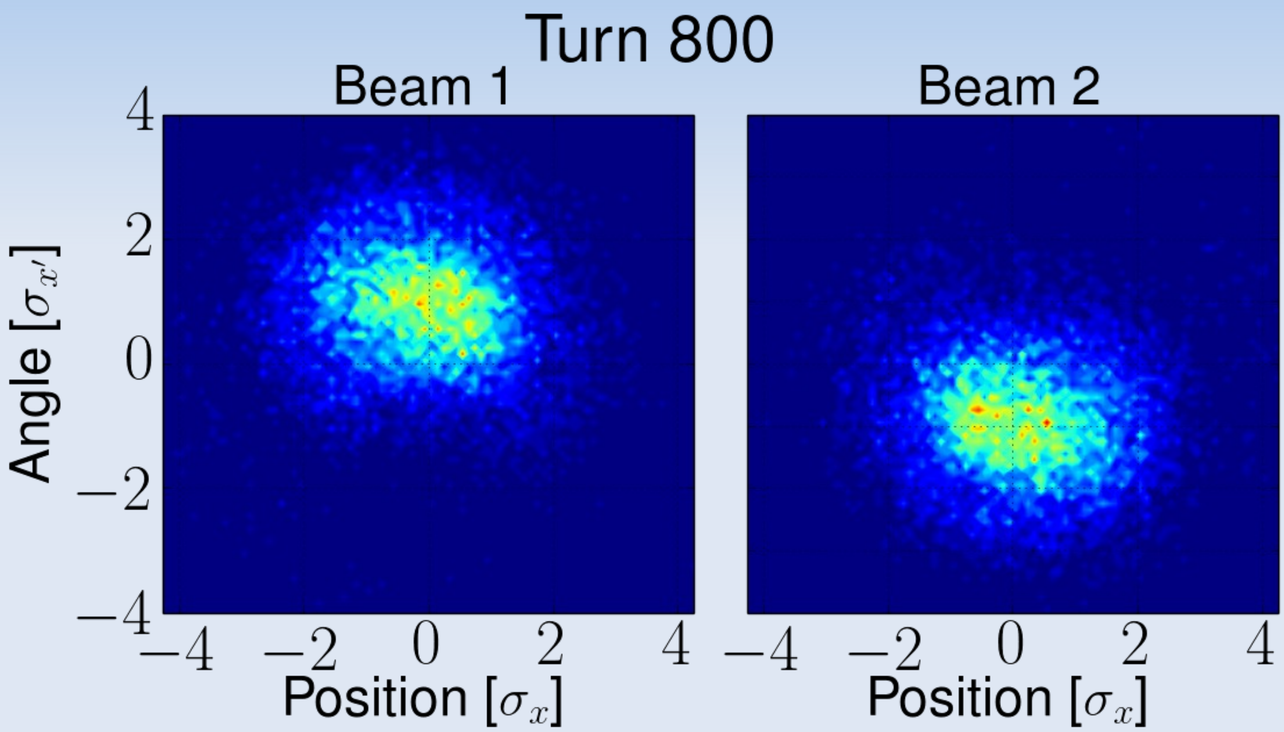
Decoherence of the π mode



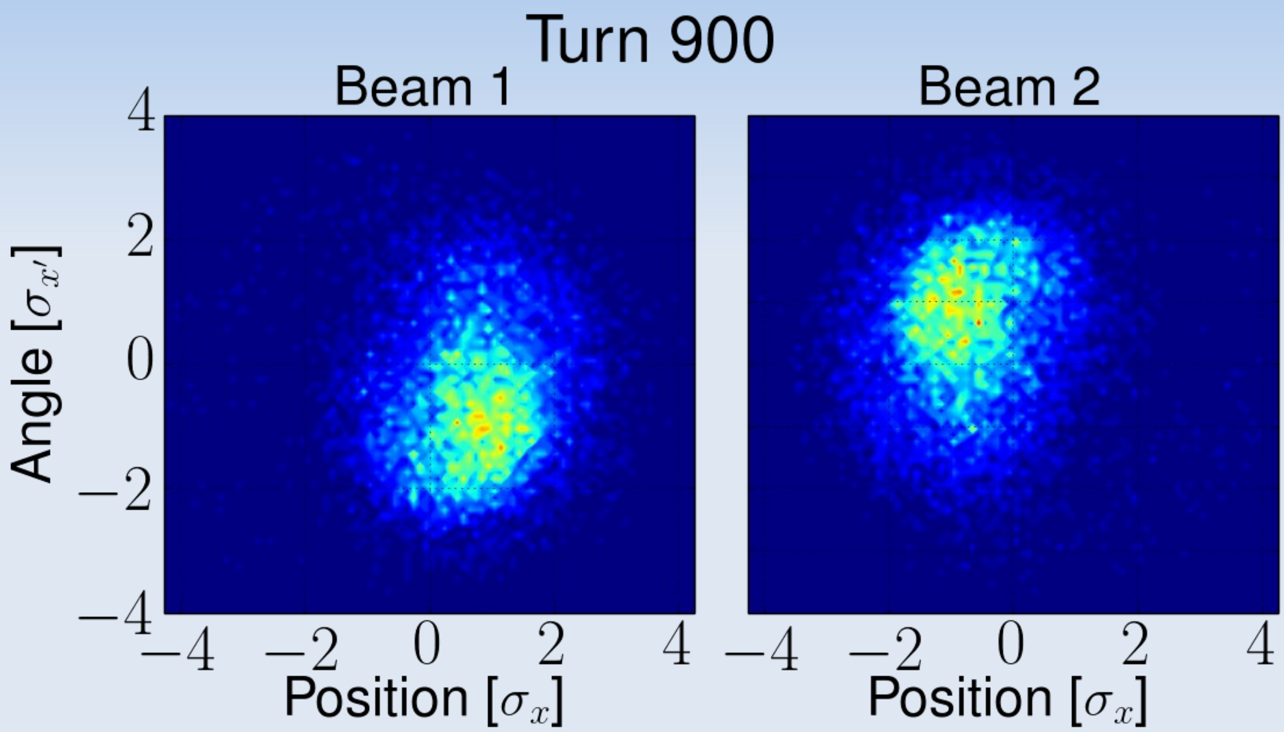
Decoherence of the π mode



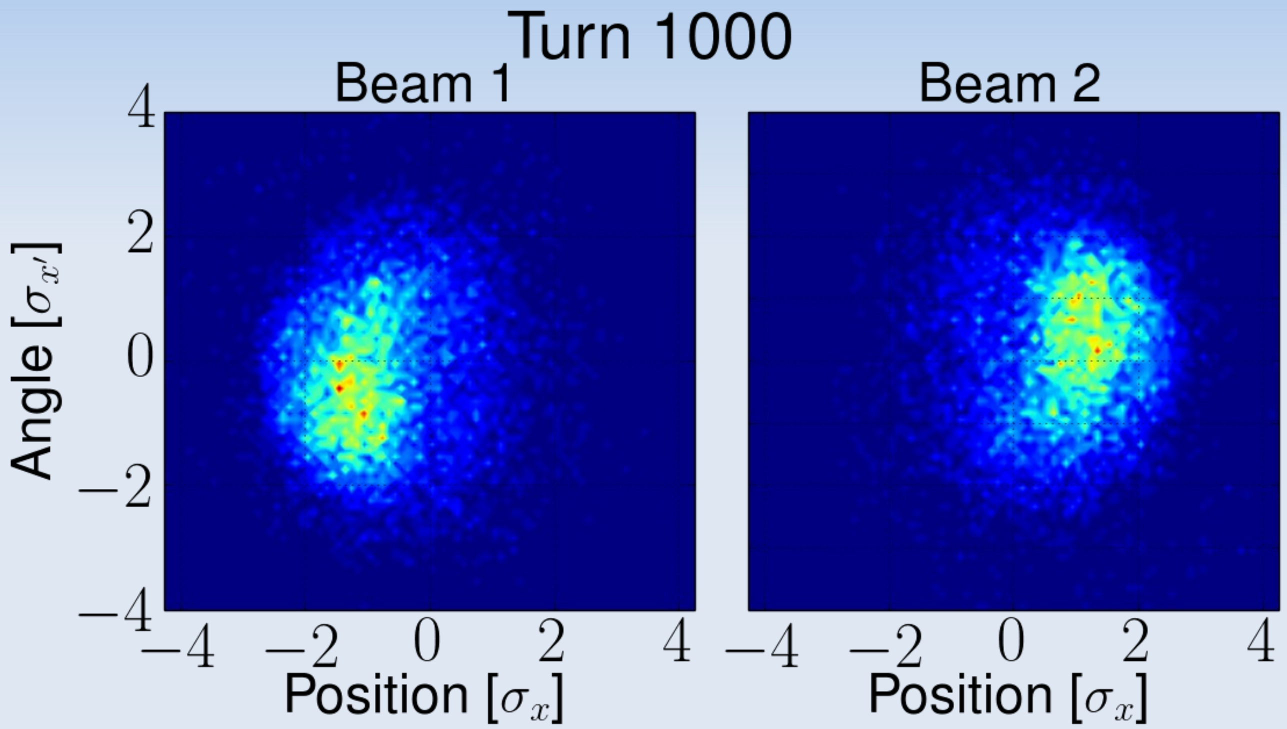
Decoherence of the π mode



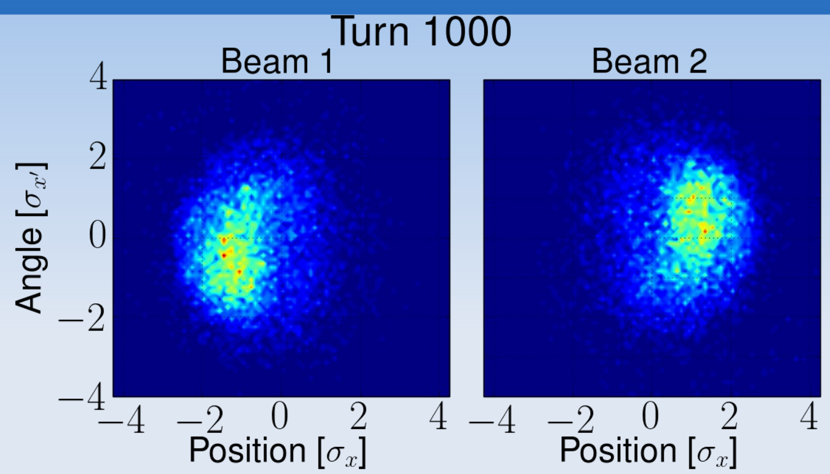
Decoherence of the π mode



Decoherence of the π mode



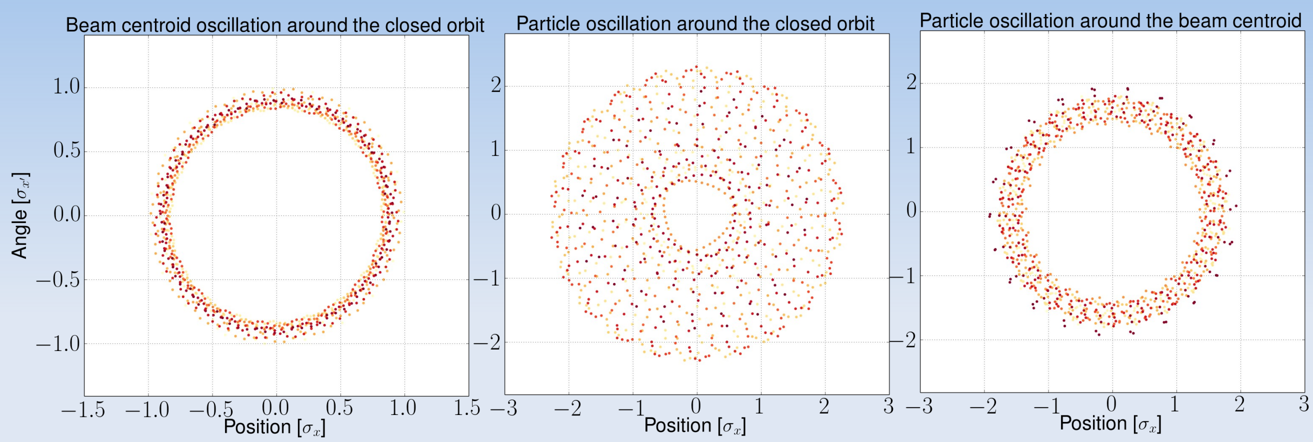
Decoherence of the π mode



- Due to the particles frequency spread, the beam distribution is distorted (i.e. non-Gaussian)
- The bunch centroids remain out of phase

  → The coherent force is *(almost)* unaffected
- This is a consequence of the decoupling of the incoherent and coherent motion, as they have different frequencies

Decoherence of the π mode

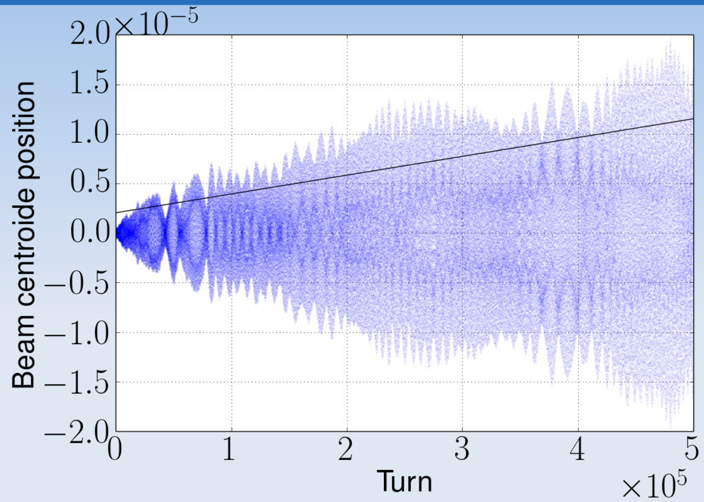


- Again, the single particle motion is 'regular' with respect to the bunch centroid

  → Absence of decoherence

  - A slight emittance growth still exists due to the mismatch of the distribution

External excitation



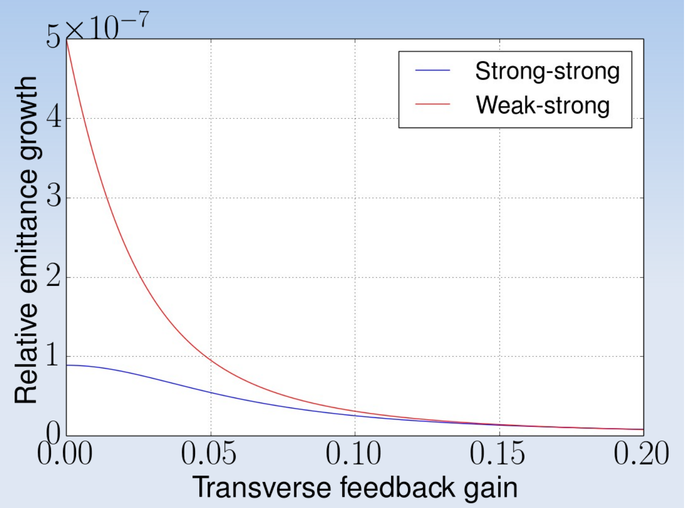
- Since there is no decoherence, any external source of excitation around the mode frequency (Field ripple, ground motion, RF noise, transverse feedback, ...) leads to an unbound growth of the oscillation amplitude

  → Need stabilisation mechanisms (Passive mitigation, other sources of detuning, synchrotron radiations, feedback systems)

Emittance growth due to decoherence
Weak-strong vs strong-strong (7)

$$\frac{1}{\epsilon_0}\frac{d\epsilon}{dt}=\left\langle\frac{\Delta^2}{2}\frac{4\pi^2\left(1-\frac{g}{2}\right)^2\Delta Q^2}{4\pi^2\left(1-\frac{g}{2}\right)^2\Delta Q^2+\left(\frac{g}{2}\right)^2}\right\rangle$$

$$\frac{1}{\epsilon_0}\frac{d\epsilon}{dt}=\frac{\Delta^2(1-s_0)}{4\left(1+\frac{g}{2\pi\xi}\right)^2}$$

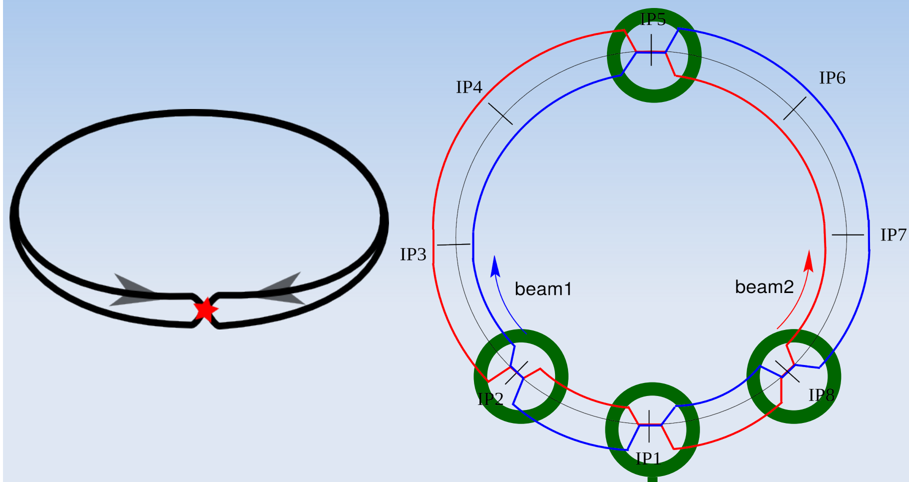


- When ξ << g : The strong-strong mechanism slowed down the decoherence time, which result in a mitigation of the growth
- When g >> ξ: Both formalisms lead to similar results

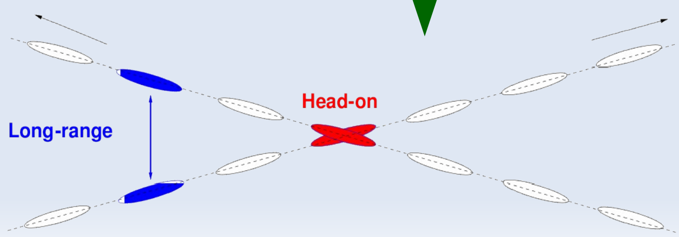
From theory to application

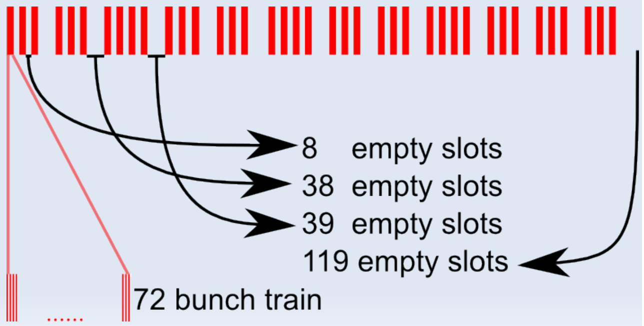
- Assumptions :
  - First order perturbation in ξ
    → Non-linear coupling terms due to beam-beam are neglected
  - Absence of other sources of amplitude detuning (Chromaticity, lattice, space charge, ...)
  - Symmetric optics and beam parameters
- Real configurations are not that simple !

Example : The Large Hadron Collider



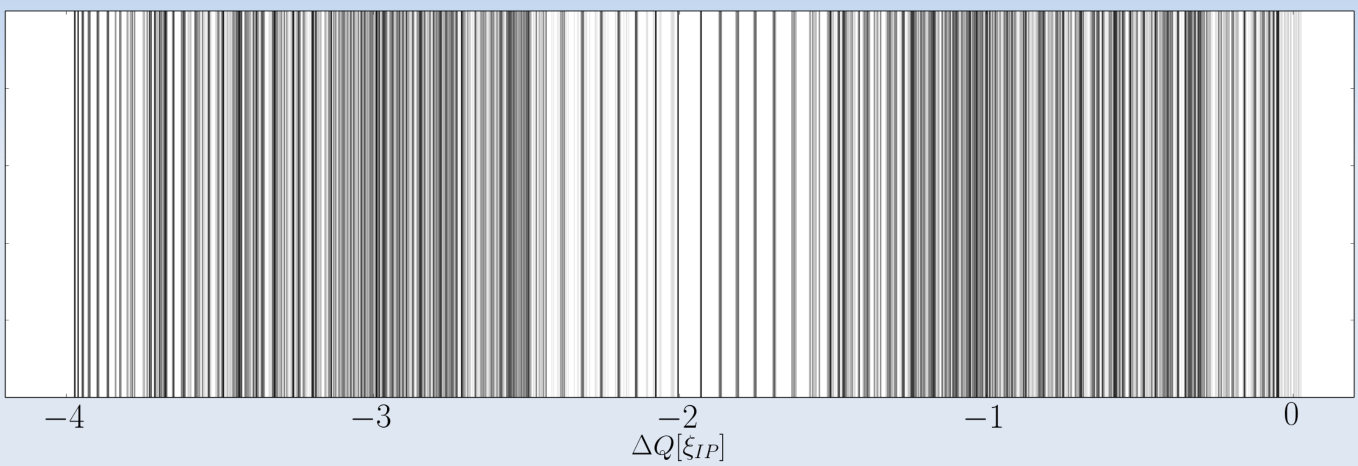
- 2808 bunches
- 4 head-on collisions
- ~120 long-range collisions





Multiple interactions points / many bunches

- Rigid bunch model : Find the eigenvalues of the ~10^4X10^4 matrix (one plane only !)



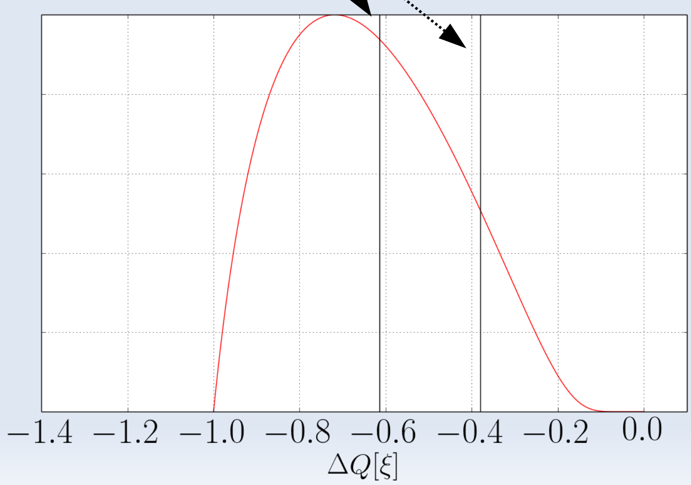
- The σ/π modes become a forest of modes with intermediate frequencies

  → No coherent modes observed in such conditions

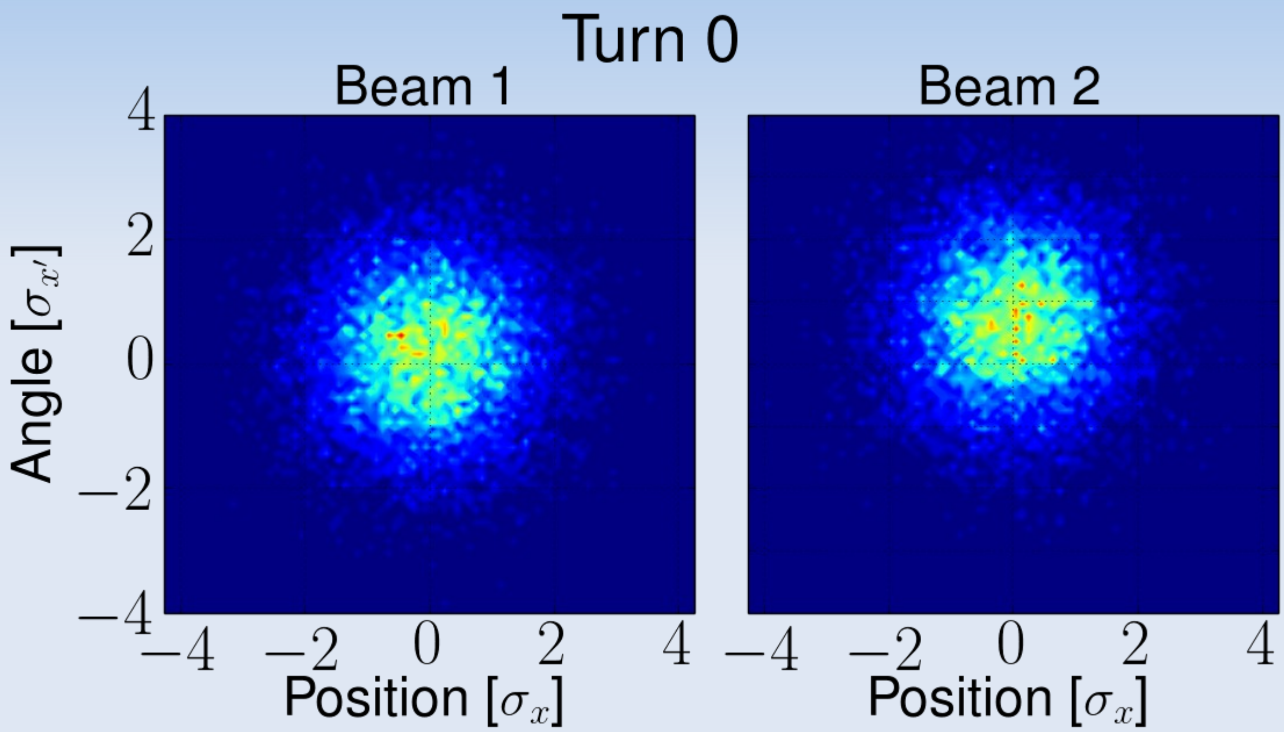
Beams with different tunes

$$\begin{pmatrix} x_{B1} \\ x_{B1}' \\ x_{B2} \\ x_{B2}' \end{pmatrix}_{t+1} = \begin{pmatrix} \cos(2\pi Q_1) & \sin(2\pi Q_1) & 0 & 0 \\ -\sin(2\pi Q_1) & \cos(2\pi Q_1) & 0 & 0 \\ 0 & 0 & \cos(2\pi Q_2) & \sin(2\pi Q_2) \\ 0 & 0 & -\sin(2\pi Q_2) & \cos(2\pi Q_2) \end{pmatrix} \cdot M_{BB} \begin{pmatrix} x_{B1} \\ x_{B1}' \\ x_{B2} \\ x_{B2}' \end{pmatrix}_{t}$$

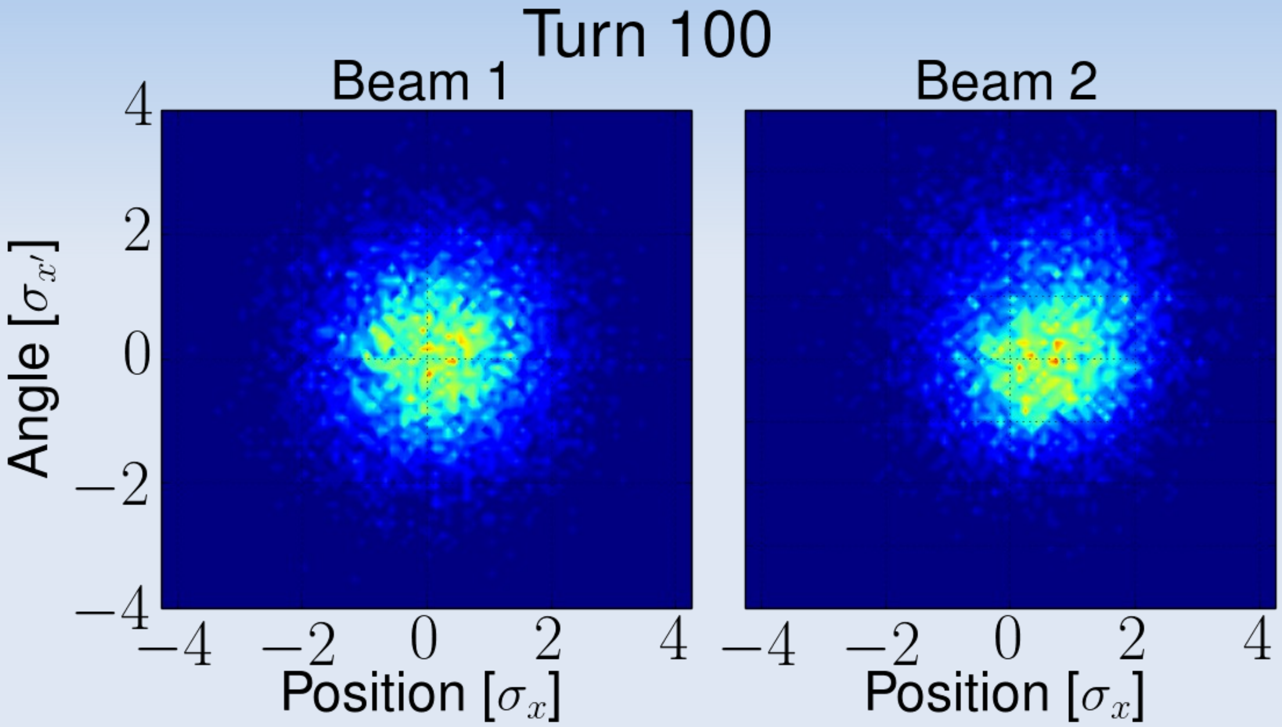
- The coherent modes are inside the incoherent spectrum



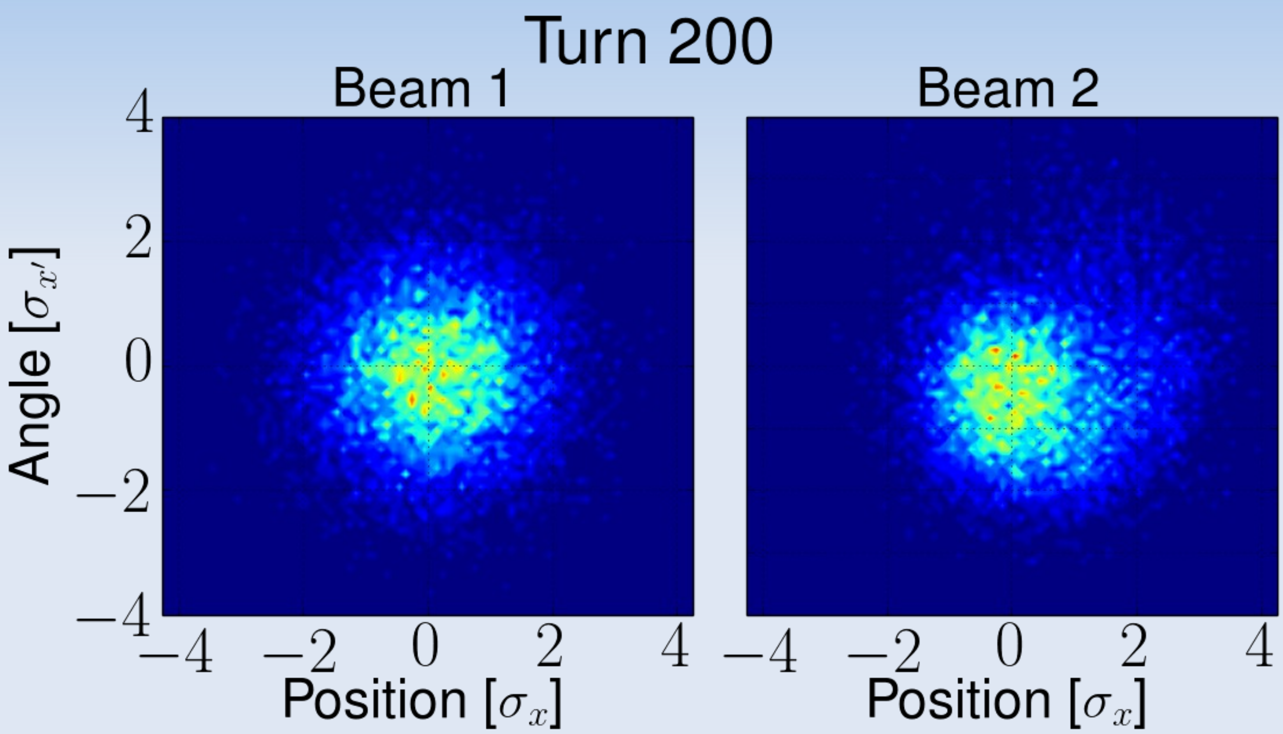
Decoherence



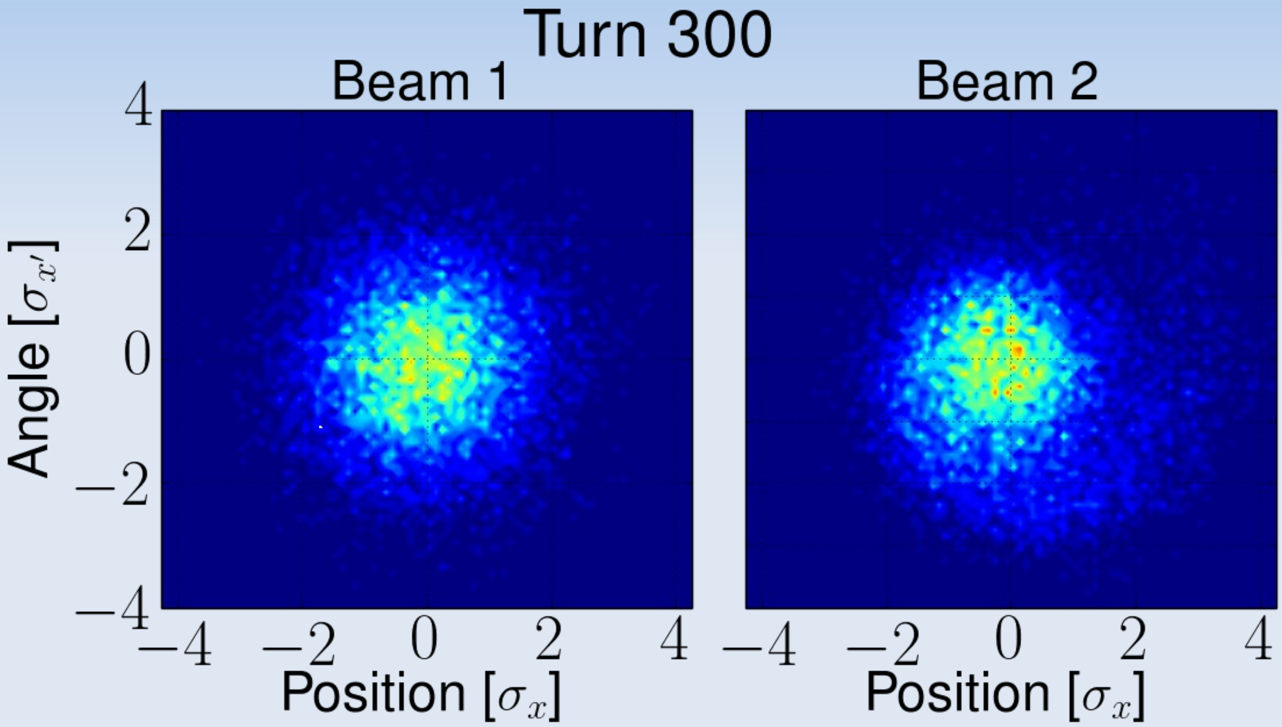
Decoherence



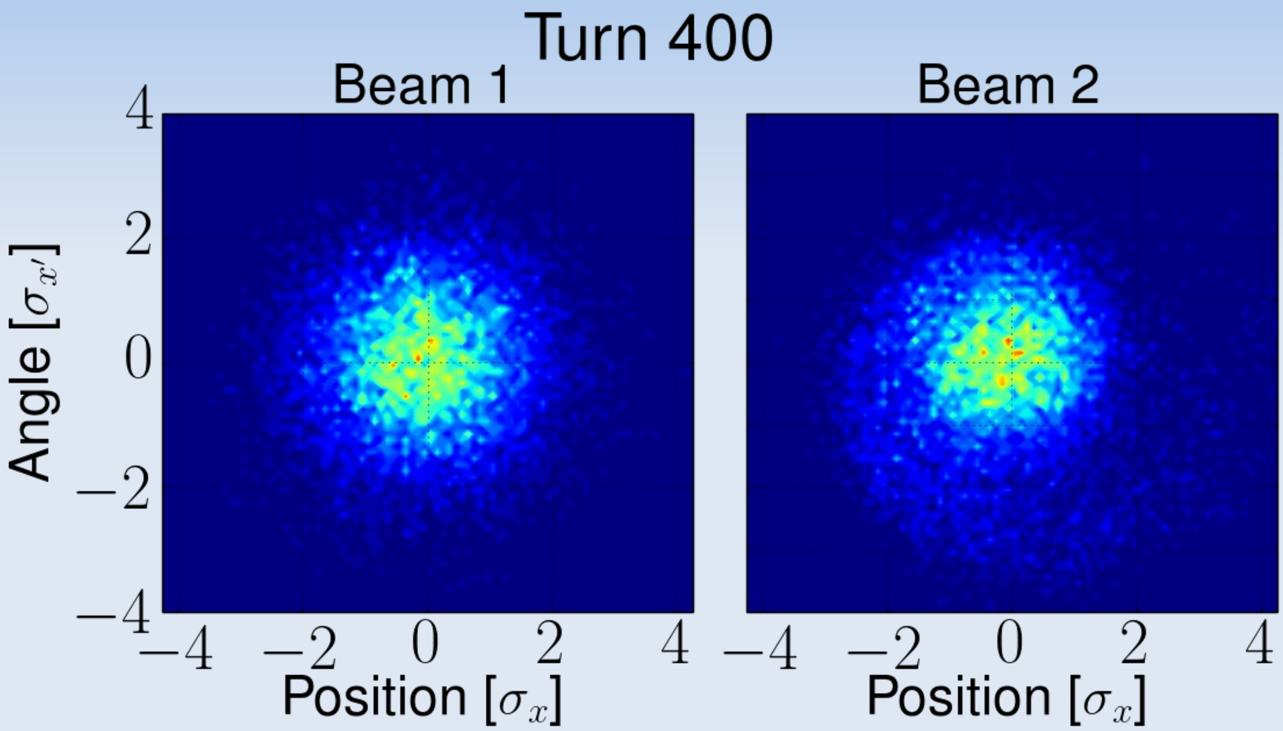
Decoherence



Decoherence



Decoherence



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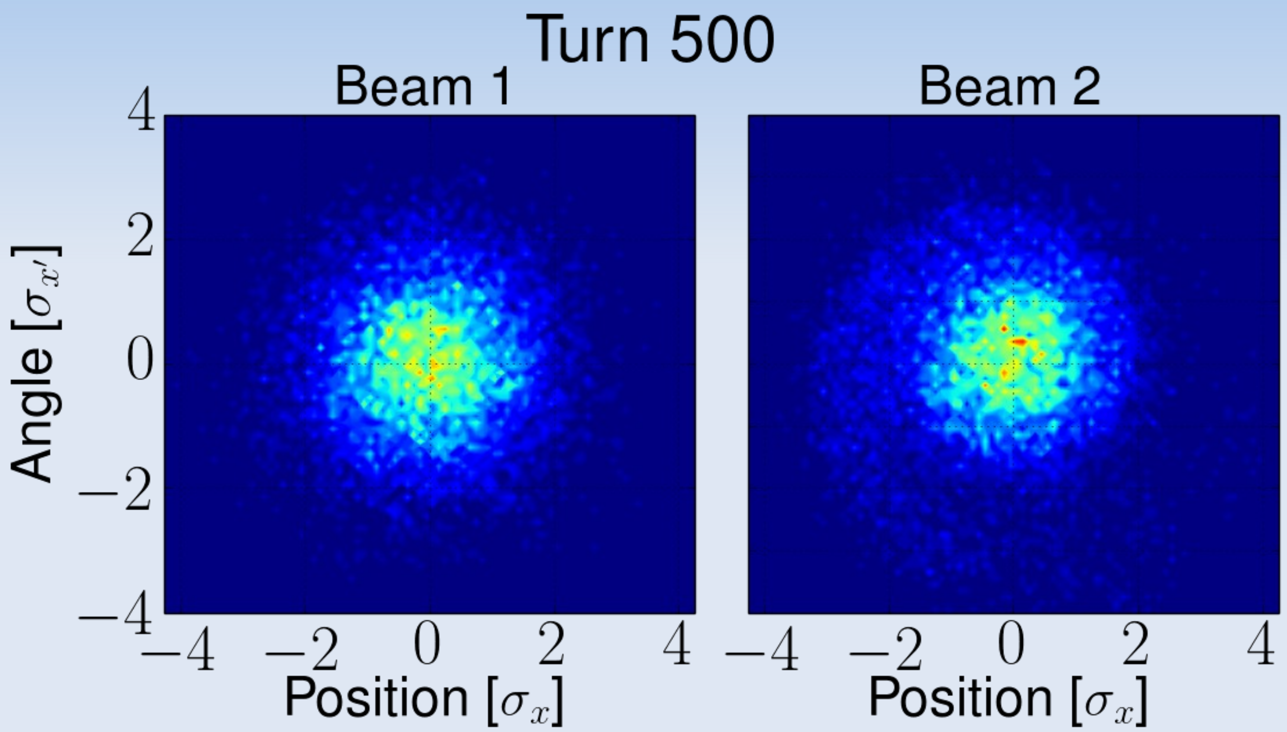
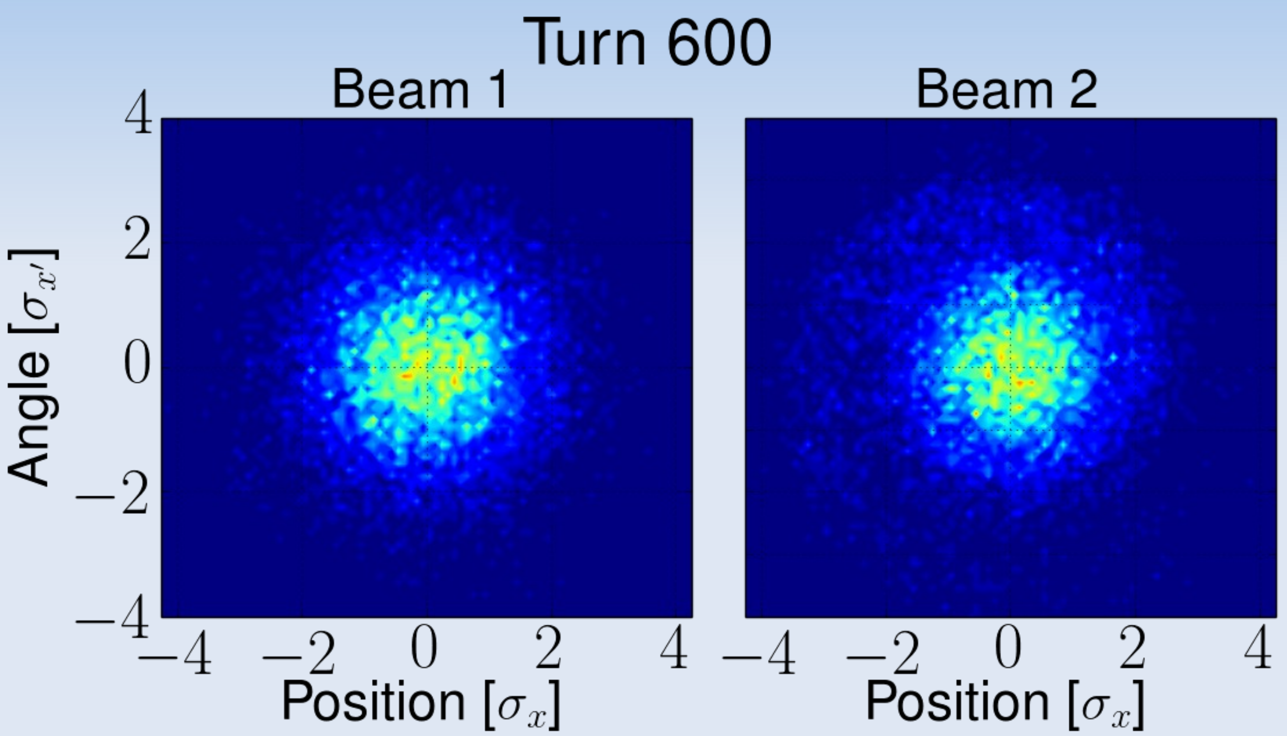
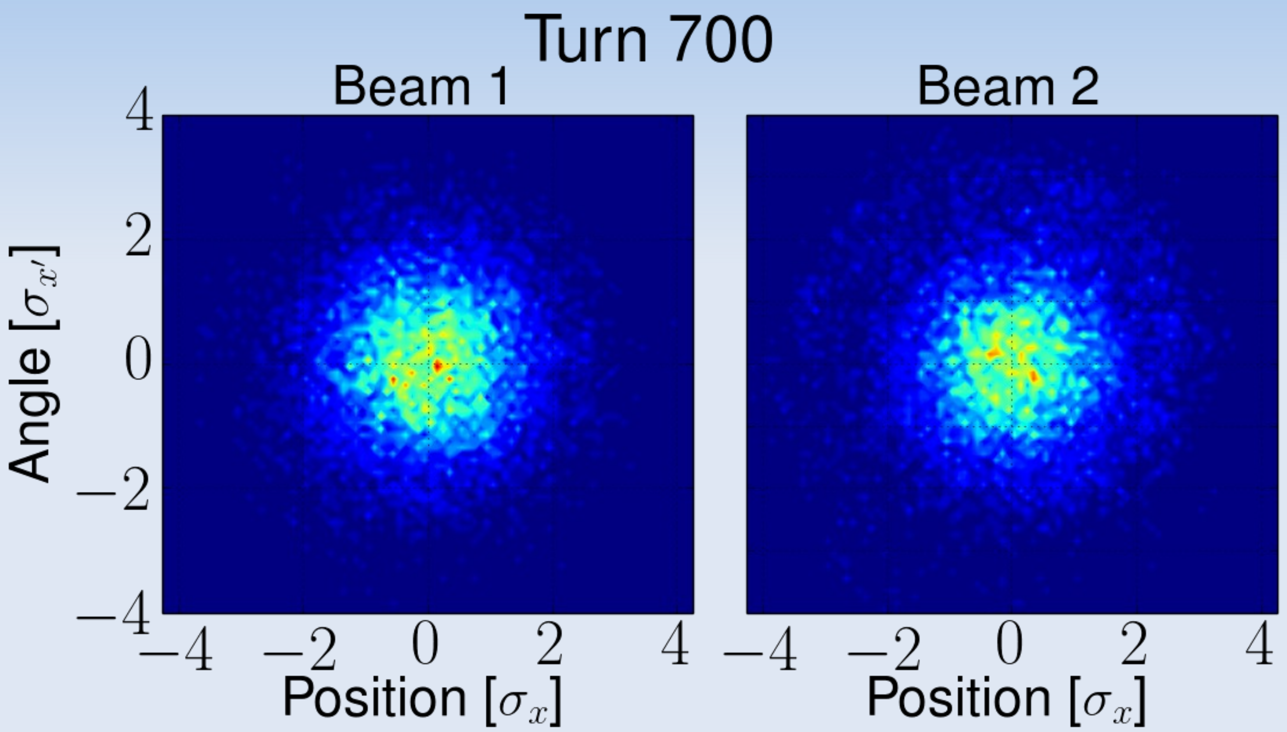


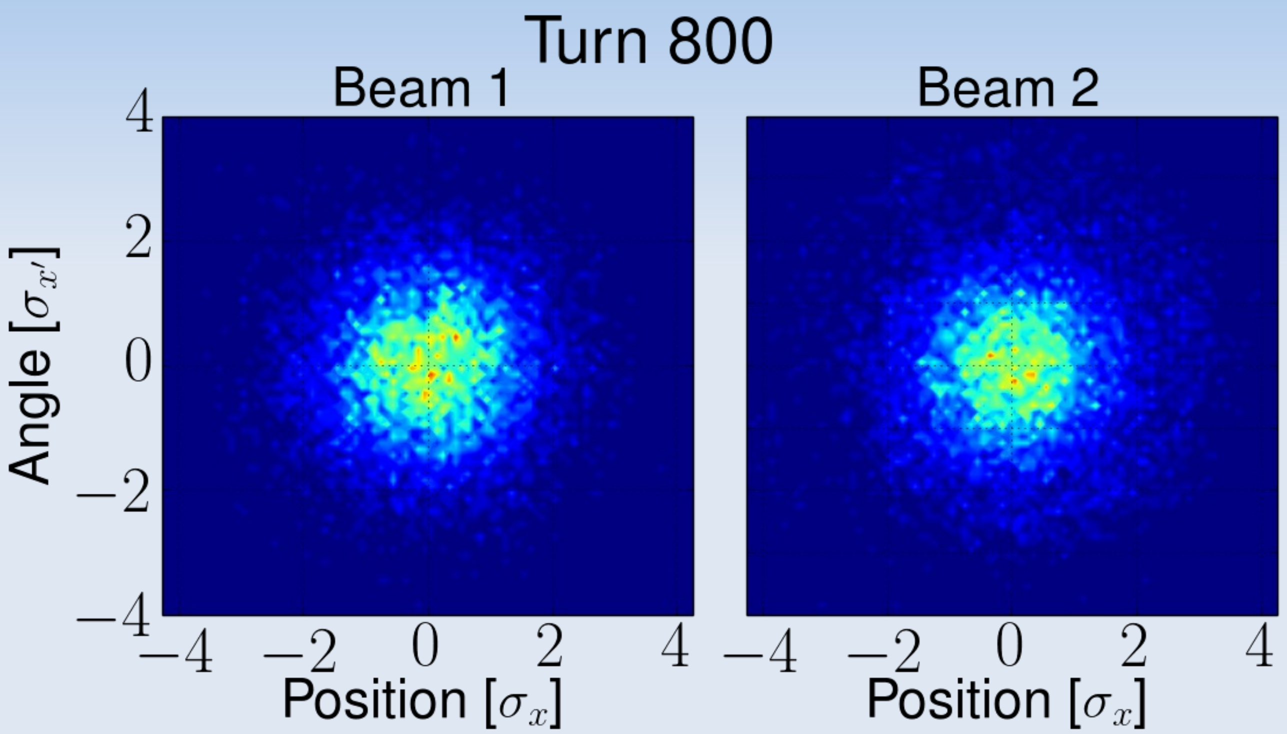
Decoherence



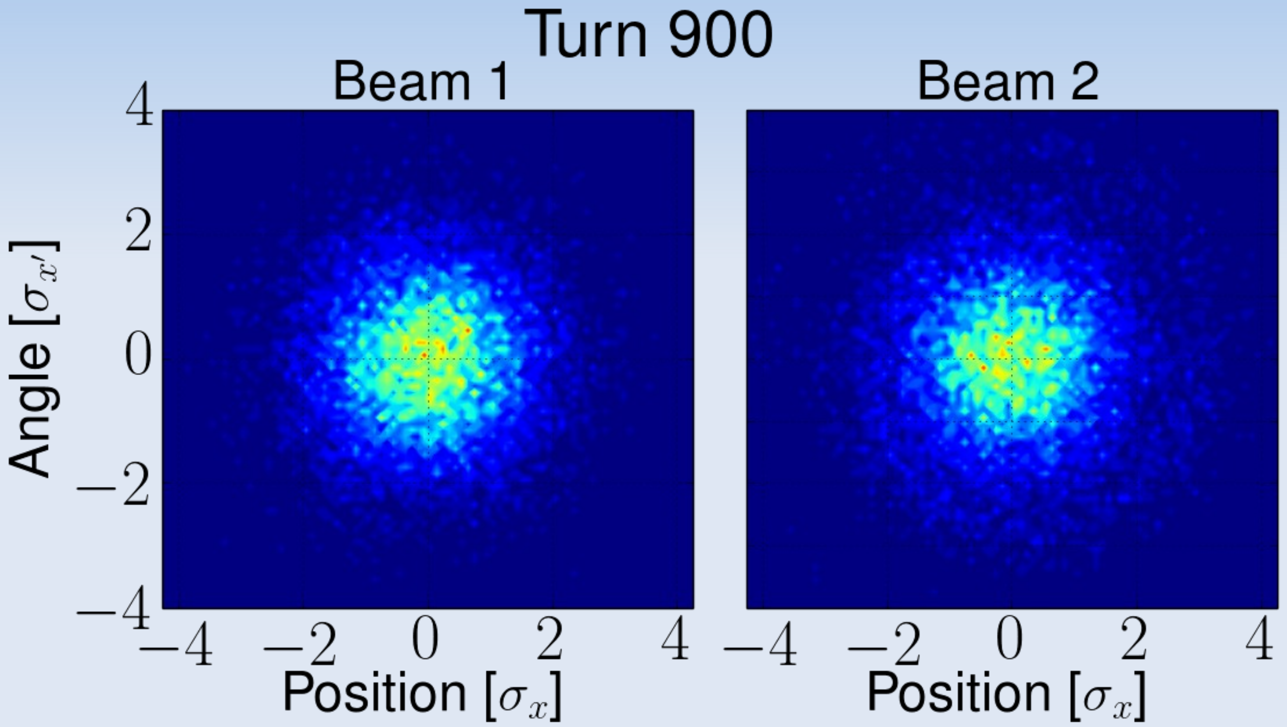
Decoherence



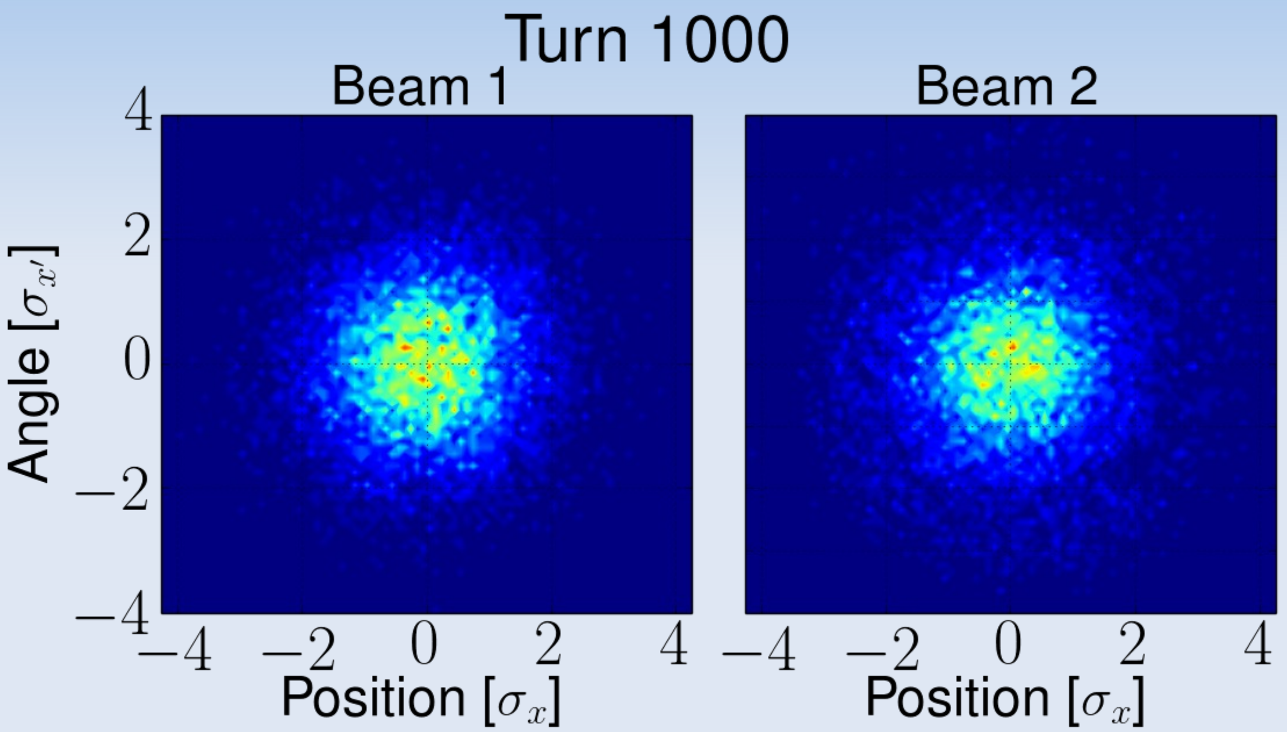
Decoherence



Decoherence



Decoherence



Decoherence

- Most symmetry breaking between the beams bring the beam-beam coherent modes towards the incoherent spectrum

  → break the coherence between the beams

- In realistic configurations, several parameters are not perfectly symmetric :
  - Intensities, emittances, β^*, tunes (phase advances between IPs), chromaticities

  → passive mitigation

Summary on coherent beam-beam modes

- Treating consistently the motion of the two beams (strong-strong) leads to a dynamic very different with respect to the single beam treatment (weak-strong)
  - Simple configuration : Two discrete coherent modes of oscillation outside of the incoherent spectrum

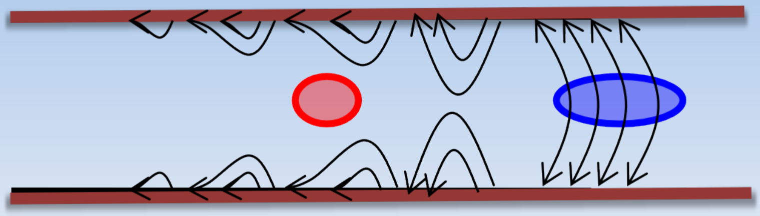
    → Absence of Landau damping and reduced decoherence

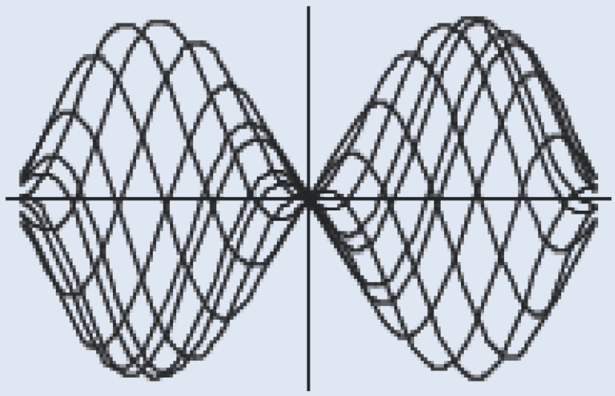
  - Complex configurations : Multiple coherent modes inside and outside of the incoherent spectrum

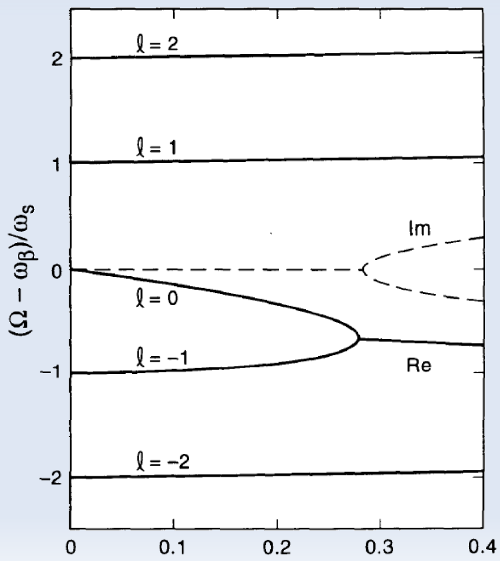
    → Landau damping and decoherence can be restored for most (all) of the modes

- What happens in the presence of beam coupling impedance ?

Impedance driven instabilities

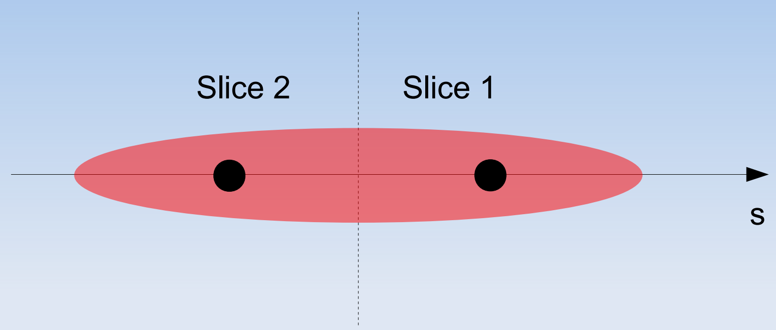






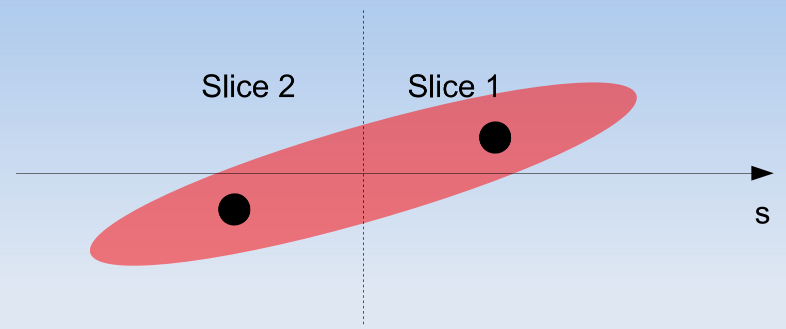
2 slices model
Linear transfer - Transverse

(9)



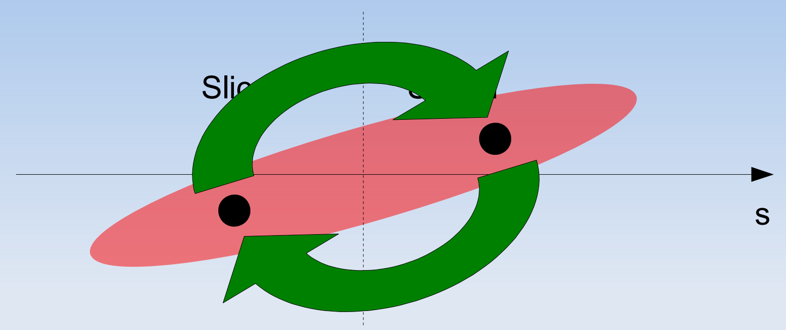
$$\begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \end{pmatrix}_{t+1} = \begin{pmatrix} \cos(2\pi Q) & \sin(2\pi Q) & 0 & 0 \\ -\sin(2\pi Q) & \cos(2\pi Q) & 0 & 0 \\ 0 & 0 & \cos(2\pi Q) & \sin(2\pi Q) \\ 0 & 0 & -\sin(2\pi Q) & \cos(2\pi Q) \end{pmatrix} \begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \end{pmatrix}_{t}$$

2 slices model
Linear transfer - Transverse (9)



$$\begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \end{pmatrix}_{t+1} = \begin{pmatrix} \cos(2\pi Q) & \sin(2\pi Q) & 0 & 0 \\ -\sin(2\pi Q) & \cos(2\pi Q) & 0 & 0 \\ 0 & 0 & \cos(2\pi Q) & \sin(2\pi Q) \\ 0 & 0 & -\sin(2\pi Q) & \cos(2\pi Q) \end{pmatrix} \begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \end{pmatrix}_{t}$$

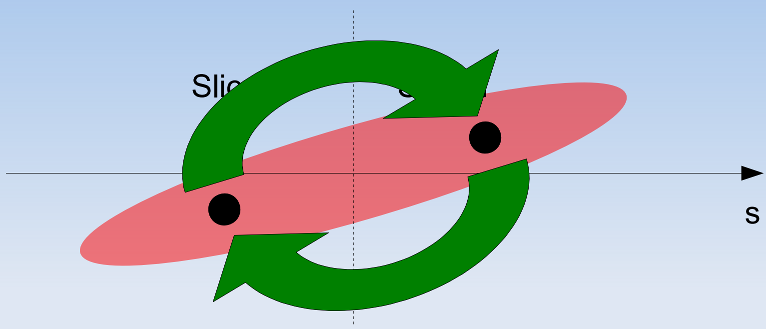
2 slices model
Linear transfer - Longitudinal (9)



$$\begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \end{pmatrix}_{t+1} = \begin{pmatrix} 0 & 0 & \cos(2\pi Q) & \sin(2\pi Q) \\ 0 & 0 & -\sin(2\pi Q) & \cos(2\pi Q) \\ \cos(2\pi Q) & \sin(2\pi Q) & 0 & 0 \\ -\sin(2\pi Q) & \cos(2\pi Q) & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \end{pmatrix}_{t}$$

- After half a synchrotron period, particles one and particles two have flipped positions

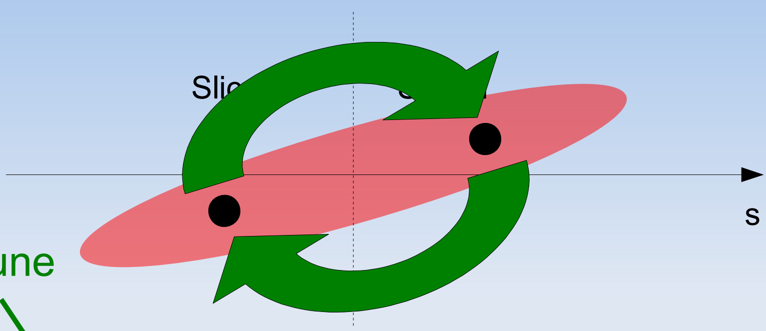
2 slices model
Linear transfer - Longitudinal (9)



$$\begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \end{pmatrix}_{t+1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes M_{lattice} \begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \end{pmatrix}_{t}$$

- After half a synchrotron period, particles one and particles two have flipped positions

2 slices model
Linear transfer - Longitudinal (9)



Synchrotron tune

$$\begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \end{pmatrix}_{t+1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^{2Q_s} \otimes M_{lattice} \begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \end{pmatrix}_{t}$$

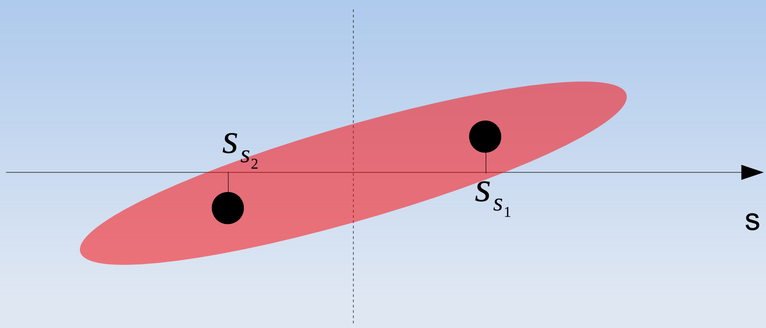
- After half a synchrotron period, particles one and particles two have flipped positions
- The effect over one turn is described with a fraction of a flip (2Qs)

2 slice model
Beam-beam kick

$$\Delta x'_{B1} = k(x_{B1} - x_{B2}) \longrightarrow \Delta x'_{B1s1} = k\left(x_{B1s1} - \frac{(x_{B2s1} + x_{B2s1})}{2}\right)$$

$$\begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \\ x_{B2s1} \\ x_{B2s1}' \\ x_{B2s2} \\ x_{B2s2}' \end{pmatrix}_{t+1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ k & 1 & 0 & 0 & -\frac{k}{2} & 0 & -\frac{k}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k & 1 & -\frac{k}{2} & 0 & -\frac{k}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -\frac{k}{2} & 0 & -\frac{k}{2} & 0 & k & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -\frac{k}{2} & 0 & -\frac{k}{2} & 0 & 0 & 0 & k & 1 \end{pmatrix} \cdot M_{lattice,SB} \begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \\ x_{B2s1} \\ x_{B2s1}' \\ x_{B2s2} \\ x_{B2s2}' \end{pmatrix}_{t}$$

2 slice model
Wake

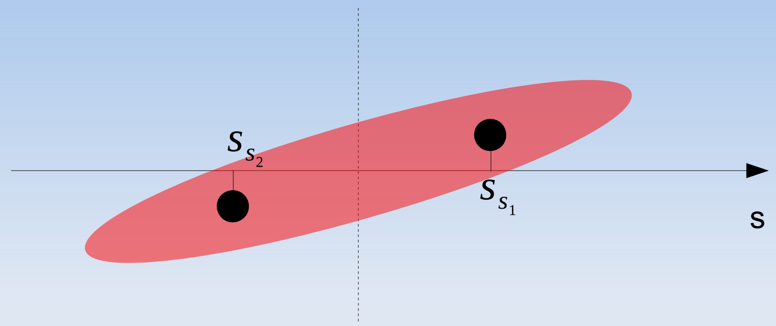


$$\Delta x_{B1s2}' = W_{dip}(s_{s2} - s_{s1})\, x_{B1s1}$$

$$\begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \end{pmatrix}_{t+1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ W_{dip} & 0 & 0 & 1 \end{pmatrix} \cdot M_{BB} \cdot M_{lattice,SB} \begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \end{pmatrix}_{t}$$

- Synchrotron motion is slow with respect to betatron motion → assume the longitudinal distribution is fixed over one turn and integrate the effect of the wake fields

2 slice model
Wake



$$\Delta x_{B1s2}' = W_{dip}(s_{s2}-s_{s1})\, x_{B1s1} + W_{quad}(s_{s2}-s_{s1})\, x_{B1s1}$$

$$\begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \end{pmatrix}_{t+1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ W_{dip} & 0 & W_{quad} & 1 \end{pmatrix} \cdot M_{BB} \cdot M_{lattice,SB} \begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \end{pmatrix}_{t}$$

- Synchrotron motion is slow with respect to betatron motion → assume the longitudinal distribution is fixed over one turn and integrate the effect of the wake fields

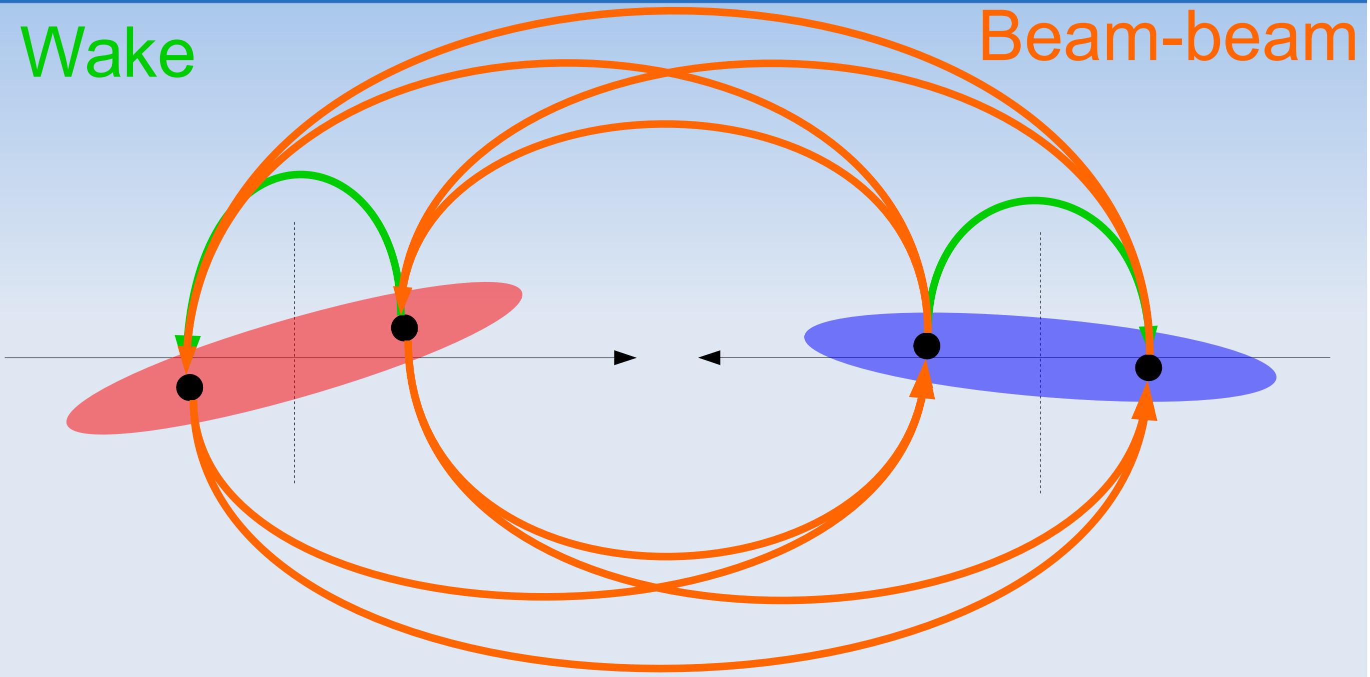
2 slice model

$$\begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \\ x_{B2s1} \\ x_{B2s1}' \\ x_{B2s2} \\ x_{B2s2}' \end{pmatrix}_{t+1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ W_{dip} & 0 & W_{quad} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & W_{dip} & 0 & W_{quad} & 1 \end{pmatrix} \cdot M_{BB} \cdot M_{lattice,SB} \begin{pmatrix} x_{B1s1} \\ x_{B1s1}' \\ x_{B1s2} \\ x_{B1s2}' \\ x_{B2s1} \\ x_{B2s1}' \\ x_{B2s2} \\ x_{B2s2}' \end{pmatrix}_{t}$$

$$\Rightarrow \vec{x}_{t+1} = M_{wake} \cdot M_{BB} \cdot M_{lattice,SB}\, \vec{x}_t \overset{\text{def}}{=} M_{one\,turn}\, \vec{x}_t$$

- The stability of the system is given by the normal mode analysis of $M_{\text{one turn}}$

2 slices / 2 beams model



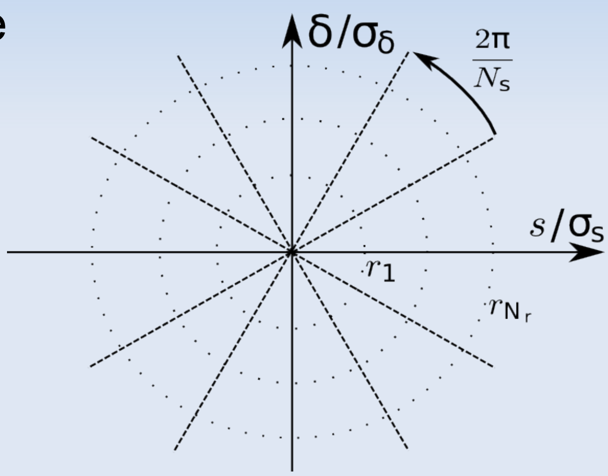
- The wake and the beam-beam force feeds back a perturbation of the bunch head to itself

→ Instability mechanism

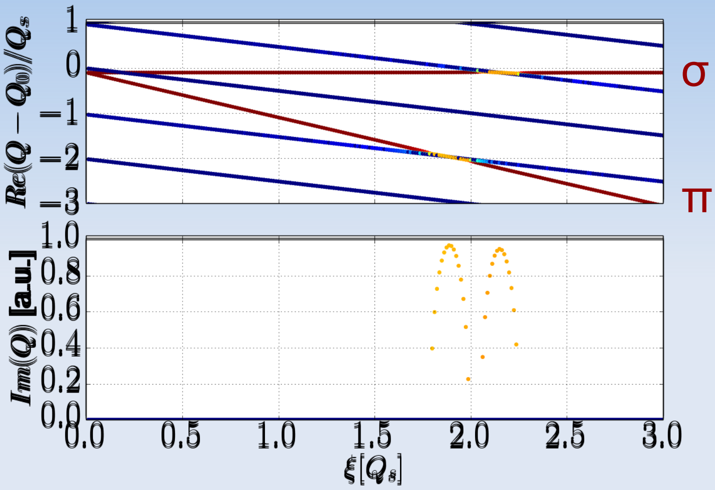
The circulant matrix model

(10)

- Identical formalism, with N slices and M rings, representing a more realistic discretisation of the longitudinal distribution
- Derive matrices for different elements (transfer maps, multiple beam-beam interactions, transverse feedback, …)
  - The flip matrix becomes a *circulant matrix*
- Analyse the stability of the one turn matrix with normal mode analysis



Mode coupling instability



- Rigid bunch model :

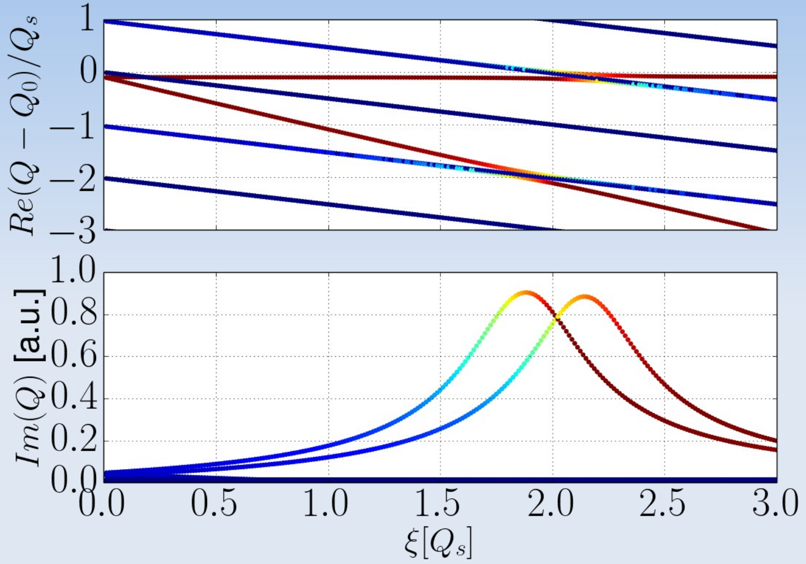
$$Q_\pi \approx Q_0 - \xi$$
$$Q_n = Q_0 \pm nQ_s$$
$$\rightarrow \xi_n = 2nQ_s$$

- Fully self-consistent model :

$$Q_\pi \approx Q_0 - Y\xi$$
$$\rightarrow \xi_n = \frac{nQ_s}{(Y - \frac{1}{2})}$$

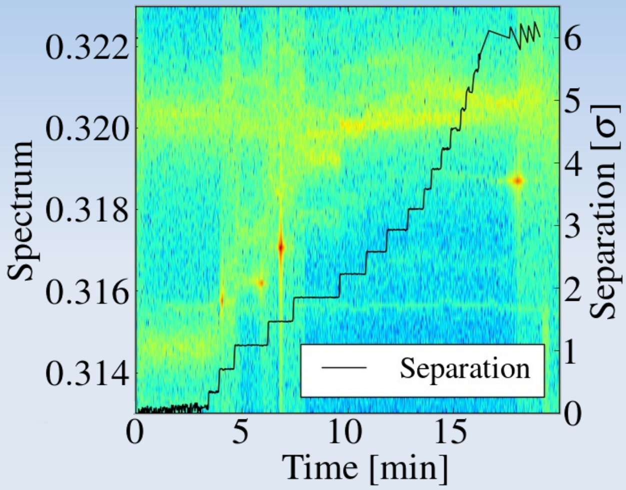
- The coupling of coherent beam-beam modes and head-tail mode leads to strong instabilities

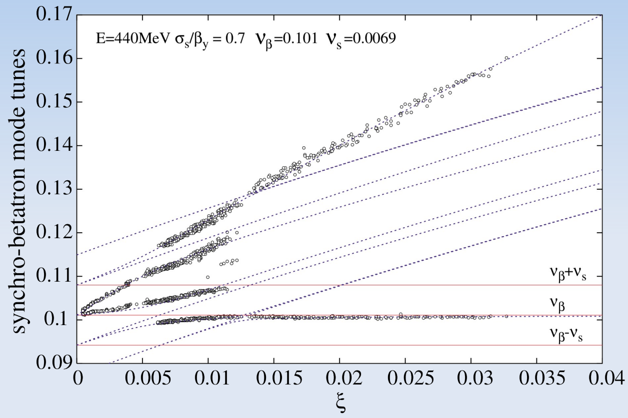
Beam-beam head-tail modes
(synchro-betatron beam-beam modes)



- At non-zero chromaticity, each beam is individually unstable (example : 2 units, above transition)
- The coherent beam-beam forces changes the nature of the modes

Observations



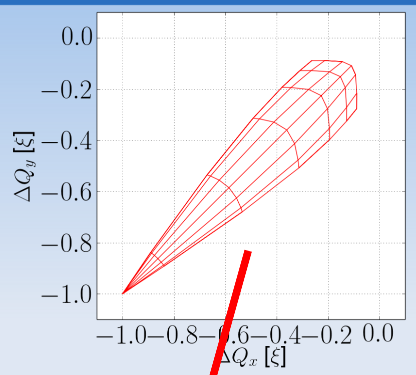


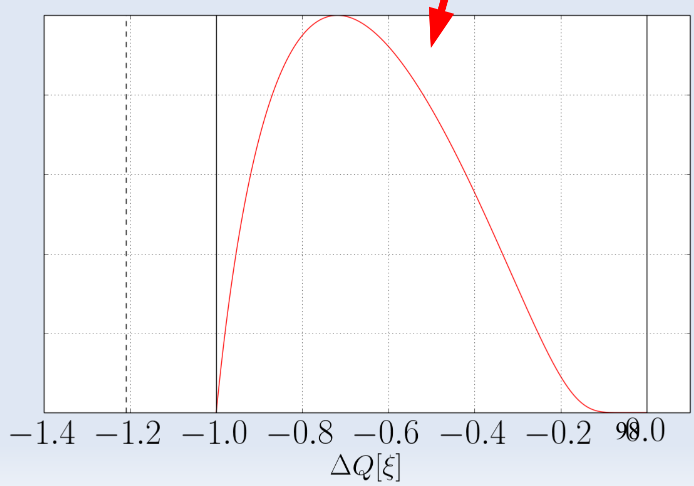
- Mode coupling instabilities were observed in dedicated experiments in the LHC
- Syncro-betatron beam-beam modes were observed at VEPP-2M in agreement with the models

Landau damping

- Single beam stability requires Landau damping
  - Usually through amplitude detuning arising from lattice non-linearities
- Lattice non-linearities are less effective against beam-beam head-tail modes

  → Passive mitigation may be very effective
- In specific cases, other synchrotron side bands can provide Landau damping (11)





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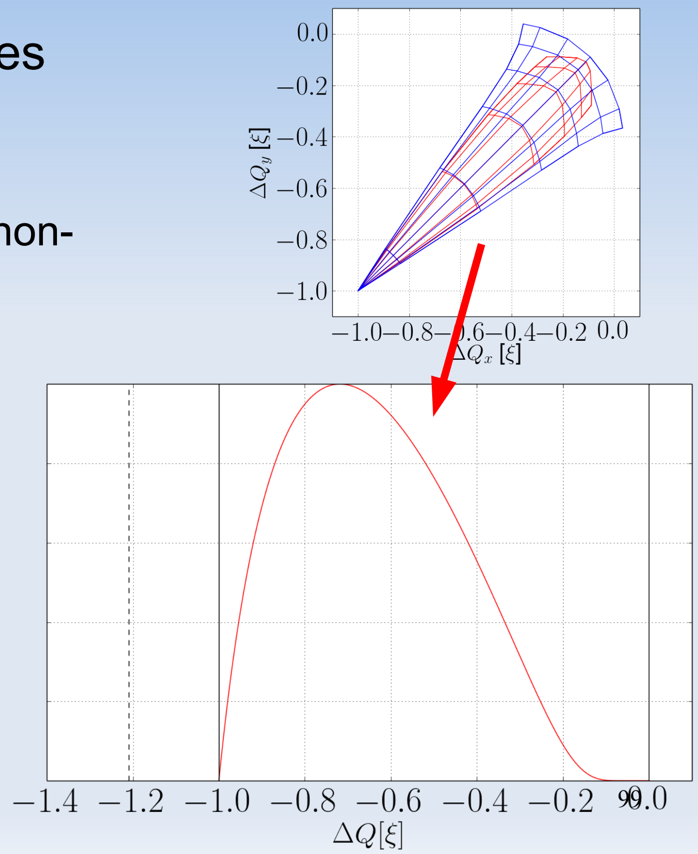

Summary
Coherent effects

- When colliding beams of similar strength (strong-strong) the effect of the two beams on each other needs to be considered in a self-consistent manner
  - Orbit effect
  - Dynamic β effect
  - Coherent beam-beam modes
- Several model exists to describe coherent beam-beam modes (Rigid bunch model, Vlasov perturbation theory, macro-particle tracking simulations)
  - Fully self-consistent treatment, allowing for non-Gaussian distributions, is needed to obtain an accurate description
  - The decoherence mechanism is very different in the strong-strong and weak-strong regime → different emittance growth

Summary
Intensity limitations

- Complicate the estimation (on paper and experimentally) of the optics disturbance caused by beam-beam interactions
- Coherent beam-beam modes may be driven unstable by :
  - Resonances
  - The beam coupling impedance
  - External excitations / noise
- Coherent beam-beam modes may break stabilisation mechanisms established for single beam stability (loss of landau damping)
- They were observed in several colliders, stabilised through :
  - Landau damping (asymmetric configurations, lattice non-linearities, chromaticity, ...)
  - Transverse feedback

References

(1) *E. Keil, Beam-beam dynamics, CERN Accelerator School, Rhodes, Greece, 1993*

(2) *Dynamic β effect*

A. Chao, Coherent Beam-Beam effects, SSCL-346, 1991

W. Herr, et al, Is LEP beam-beam limited at its highest energy, Proceedings of PAC99, New York, USA

(3) *Self-consistent methods*

E. Keil, Truly self-consistent treatment of the side effects with bunch trains, CERN SL/95-75 (1995)

H. Grote, et al, Self-consistent orbits with beam-beam in the LHC, Beam-beam workshop 2001, Fermilab

(4) *Flip-Flop effect*

M.H.R. Donald, et al, An Investigation of Flip-Flop beam-beam effect in SPEAR, IEEE Trans. Nuc. Sci. NS-26, 3580 (1979)

J.F Tennyson, Flip-flop modes in Symmetric and Asymmetric colliding beam storage rings, LBL-28013 (1989)

D.B. Shwartz, Recent beam-beam effect at VEPP-2000 and VEPP-4M, Workshop on beam-beam effects in hadron colliders, Geneva, Switzerland, 2013

References

(5) *Vlasov perturbation theory*

K. Yokoya, et al, Tune shift of coherent beam-beam oscillations, Part. Acc., ***27****, 181 (1990)*

Y. Alexahin, A Study of the coherent Beam-Beam effect in the framework of Vlasov perturbation theory, Nucl. Instrum. Methods Phys. Res. A ***480****, 253 (2002)*

(6) *Observations of beam-beam modes*

A. Piwinski, Observation of Beam-Beam effects in PETRA, IEEE Trans. Nuc. Sci. ***NS-26****, 3 (1979)*

H. Koiso, et al, Measurement of the coherent beam-beam tune shift in the TRISTAN accumulator ring, Part. Acc. ***27****, 83 (1990)*

W. Fisher, et al, Observation of coherent beam-beam modes in RHIC, Proceedings of the Particle Accelerator Conference 2003, Portland, USA

X. Buffat, et al, Coherent beam-beam mode in the LHC, Workshop on beam-beam effects in hadron colliders, Geneva, Switzerland, 2013

(7) *Emittance growth*

V.A. Lebedev, Emittance growth due to noise and methods for its suppression with the feedback system in large hadron colliders, AIP Conf. Proc. ***326****, 396 (1995)*

Y. Alexahin, On the Landau damping and decoherence of transverse dipole oscillations in colliding beams, Part. Acc. ***59****, 43 (1998)*

References

(8) *T. Pieloni, A study of beam-beam effects in hadron colliders with a large number of bunches, EPFL PhD thesis, 2008*

(9) *A.W, Chao, Physics of collective beam instabilities, John Wiley and Sons Inc, New York, 1993*

(10) *Circulant matrix model*

V.V. Danilov, et al, Feedback system for the elimination of transverse mode coupling instability, Nucl. Instum. Methods Phys. Res. ***A 391****, 77 (1997)*

E.A. Perevedentsev, et al, Simulation of the head-tail instability of colliding bunches, Phys. Rev. ST Accel. Beams ***4****, 024403 (2001)*

S. White, et al, Transverse mode coupling instability of colliding beams, Phys. Rev. ST Accel. Beams ***17****, 041002 (2014)*

(11) *W. Herr, et al, Landau damping of coherent beam-beam modes by overlap with synchrotron sidebands, LHC Project Note 304*